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O`TAGANOVA UMIDA EGAMBERDI QIZI

”Ikkita qo`zg`almas tortishish markazi ta`siridagi moddiy nuqtaning harakati” mavzusidagi

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Bugun tarixiy bir davrda xalqimiz o`z oldiga ezgu va ulug` maqsadlar qo`yib, tinch osoyishta hayot kechirayotgan, avvalam bor o`z kuchi va imkoniyatlariga tayanib demokratik davlat va fuqarolik jamiyati qurish yo`lida ulkan natijalarni qo`lga kiritayotgan bir zamonda yashayapmiz.

"Kadrlar tayyorlash milliy dasturi" da shaxs milliy modelimizning asosiy tashkil etuvchisi sifatida belgilangan bo`lib, unga nisbatan qo`yiladigan talablarning amalga oshirilishi ta`lim islohatlarning pirovard natijasini belgilab beradi. O`z navbatida mazkur hujjatda shaxsning ijodkorligi va intellektual qobilyatlari barkamollikning asosiy belgilari sifatida qaraladi. Bundan kelib chiqadigan asosiy vazifa uzluksiz ta`lim tizimiga shaxsga ijodkorlik va intellektual qobilyatlarni rivojlantirish orqali uning barkamolligiga erishishdan iborat.

Kadrlar tayyorlash milliy modelining asosiy tarkibiy qismlari quydagilardan iboratdir: shaxs – kadrlar tayyorlash tizimining bosh subekti va obekti, ta`lim sohasidagi xizmatlarning istemolchisi va ularni amalga oshiruvchisi: davlat va jamiyat – talim va kadrlar tayyorlash tizimining faoliyatini tartibga solish va nazorat qilishni amalga oshiruvchi kadrlar tayyorlash va ularni qabul qilib olishning kadrlari; uzluksiz ta`lim malakali raqobatbardosh kadrlar tayyorlashning asosi bo`lib, ta`limning barcha turlarini, davlat ta`lim standartlarini, kadrlar tayyorlash tizimi tuzilmasi va uning faoliyat ko`rsatish muhitini o`z ichiga oladi; fan yuqori malakali mutaxassislar tayyorlovchi va ulardan foydalanuvchi, ilg`or pedagogik va axborot texnologiyalarini ishlab chiquvchi; ishlab chiqarish kadrlarga bo`lgan ehtiyojni, shuningdek, ularning tayyorgarlik sifati va saviyasiga nisbatan qo`yiladigan talablarni belgilovchi asosiy buyurtmachi, kadrlar tayyorlash tizimini moliya va moddiy texnika jihatidan ta`minlash jarayonining qatnashchisi.

Gravitatsion maydonda nuqta (kosmik kema KA massalar markazi) harakatini optimallashtirish masalasi mavjud. Variatsion masala ixtiyoriy gravitatsion maydonda harakatlanayotgan nuqta optimal trayektoriyasini va boshqaruvini (reaktiv kuchning qiymatini va yo`nalishini) aniqlashni o`z ichiga oladi.

Mavzuning dolzabligi: Sonli integrallashsiz, katta vaqt oralig`ida harakatni bashorat qilish mumkin bo`lgan o`ta murakkab hisoblarsiz, oldindan berilgan xossalar bilan KA orbitasi uchun boshlang`ich shartlarni tanlashga imkon beruvchi, qulay va iqtisodiy bo`lgan, yetarli daraja oddiy analitik nazariyani yaratish hisoblanadi.

**Tadqiqotning ob`yekti va predmeti**: Massasi o`zgaruvchan moddiy nuqta, analitik mexanika usullari, optimal trayektoriyalar aniqlash hisoblanadi.

Mavzuni maqsadi va vazifalari: Ikkita qo`zg`almas tortishish markazi ta`siridagi moddiy nuqtaning konus sirtida aylanadan farqli trayektoriyasini topish.

Ilmiy yangiligi: Moddiy nuqtaning koordinatalari, tezlik vektori, bazis vektori tashkil etuvchilarini maple 12 pragramma dasturi yordamida oldim.

Gravitasion maydonlarda moddiy nuqtaning harakatini optimallash masalasiga ko'plab ilmiy izlanishlar bag'ishlangan. Bazis-vektor va o'tish funksiyasini kiritishga asoslangan Louden usuli yordamida nol, oraliq va maksimal tortish qismlarida analitik yechimlarni aniqlash yopiq gamilton sistemasini integrallash masalasiga keltirilgan [1,2]. Shunday qvariatsion masalani yechishda gamilton sistemalari uchun o'rinli bo'lgan analitik mexanika usullaridan foydalanish imkonini berdi [2]. Ikkita qo`zg`almas tortishish markazidan biri cheksiz uzoqda joylashgan holda oraliq tortish qismlari uchun moddiy nuqtaning aylana bo`ylab harakati [4] ishda tahlil qilingan.

Tadqiqot ishida qo'llanilgan usullar: differensial tenglamalar sistemasi xususiy yechimlarini Dokshevich usulidan va ko'rilayotgan masala uchun mavjud integrallardan foydalaniladi.

Nazariy va amaliy ahamiyati: oraliq tortish qismlari uchun analitik yechimlarni topish va bu yechimlarni aniq masalalarga qo’llash muhim hisoblanadi.

Ushbu bitiruv malakaviy ish tuzulishining qisqacha tavsifi: 2 ta bob 6 ta paragrafdan iborat. I bobda massasi o`zgaruvchi moddiy nuqta harakat differensial tenglamasi keltirib chiqarilgan bo`lib, massani ajralishi va qo`shilishi hollari alohida keltiriladi. Ikkinchi paragrafda moddiy nuqta uchun variatsion masalaning qo’yilishi keltiriladi. Uchinchi paragrafda differensial tenglamalar sistemasining xususiy yechimlarini topishning Dokshevich usuli beriladi. II bobda ikkita qo’zg’almas tortishish markazining biri cheksiz uzoqlikda joylashgan holda moddiy nuqtaning harakati masalasi ko’rib chiqiladi. Birinchi paragrafda moddiy nuqtaning harakat differensial tenglamalari olinadi. Ikkinchi paragrafda Dokshevich usuli yordamida xususiy yechimlarni topish keltiriladi. Uchinchi paragrafda olingan natijalarni sonli hisobi keltiriladi.

I BOB. Markaziy tortishish maydonida moddiy nuqtaning harakat tenglamalari

1.1 O`zgaruvchan massali nuqtaning harakat differensial tenglamasi

Bir vaqtning o`zida zarrachalarning ajralishi va qo`shilishi natijasi o`zgaruvchan massali nuqtani harakat tenglamasini tuzamiz $t$ vaqt momentidagi nuqtaning barcha massasini $M(t)$ ko`rinishida belgilaymiz [5].

$M_1(t)$ orqali unga qo`shilgan zarrachalarining massasini va $M_2(t)$ orqali uning tarkibiga kiruvchi ajralgan zarrachalar massasini belgilaymiz bu yerda $M_1(t)$-o`suvchi funksiya $(dM_1 > 0)$ $M_2(t)$-kamayuvchi funksiya $(dM_2 < 0)$.

$M(t)$ funksiya esa o`suvchi ham, kamayuvchi ham, o`zgarmas ham bo`lishi mumkin.

Harakat miqdori o`zgarishi haqidagi teoremani $M(t)$ massali nuqta $\Delta t$ kichik vaqt oralig`ida unga qo`shilgan $\Delta M_1$ va bu vaqtda undan ajralgan $|\Delta M_2|$ massali zarrachalardan tashkil topgan sistemaga qo`llaymiz.

Yuqoridagi farazlarga qarab ko`rsatilgan zarrachalar va nuqta ta’sirlashadi, ya’ni mexanik sistemani faqat $t$ vaqt momentida tashkil qiladi (qo`shilguncha yoki ajralgandan keyin emas) shuning uchun bizni mulohazalarimiz natijasi $\Delta t \to 0$ deb qaralganda chegaraviy o`tishda o`rinli.

Ta’kidlangan teoremani beradi.

$$\overrightarrow{Q_1} - \overrightarrow{Q_0} = \vec{F}\Delta t \tag{1.1}$$

Bu yerda $\overrightarrow{Q_0}$ - t vaqt momentidagi sistemaning harakat miqdori , $\overrightarrow{Q_1}$ esa $t + \Delta t$ momentdagi $\vec{F}$ nuqtaga qo`yilgan kuchlarning teng tasir etuvchisi.

$M(t)$ massali nuqtaning absolyut tezligini $\vec{v}$ orqali ΔM_1 massali qo`shiluvchi va $|\Delta M_2|$ massali ajraluvchi zarralarning absolyut qiymatlarini mos ravishda $\overrightarrow{u_1}$ va $\overrightarrow{u_2}$ orqali belgilaymiz, shunda:

$$\overrightarrow{Q_0} = M\vec{v} + \Delta M_1 \cdot \overrightarrow{u_1}$$
$$Q_1 = (M + \Delta M_1 - |\Delta M_2|)(\vec{v} + \Delta\vec{v}) + |\Delta M_2|\overrightarrow{u_2}$$

bu yerda $\Delta\vec{v}$ - Δt vaqt oralig`idagi nuqta tezligining orttirmasi. Bu kattaliklarni (1.1) tenglamaga qo`yib va $|\Delta M_2| = -\Delta M_2$ (zarracha massasi – musbat qiymat, $M_2(t)$ esa kamayuvchi funksiya) ligini hisobga olib quyidagini olamiz

$$M\Delta\vec{v} + \Delta M_1(\vec{v} - \overrightarrow{u_1}) + \Delta M_2(\vec{v} - \overrightarrow{u_2}) + (\Delta M_1 + \Delta M_2)\Delta\vec{v} = \vec{F}\Delta t$$

bu yerdan tenglamaning ikki tomonini ham Δt ga bo`lib va $\Delta t \to 0$ da limitga o`tib ushbu tenglamaga ega bo'lamiz:

$$M\frac{d\vec{v}}{dt} = \vec{F} + (\overrightarrow{u_1} - \vec{v})\frac{dM_1}{dt} + (\overrightarrow{u_2} - \vec{v})\frac{dM_2}{dt} \qquad (1.2)$$

Bu o`zgaruvchan massali nuqta differensial harakat tenglamasi yoki bir vaqtning o`zida qo`shiluvchi va ajraluvchi zarrachali holat uchun tuzilgan I. V.Meshcherskiyni umumlashgan tenglamasi.

Agar faqat zarrachalarning qo`shilish (yoki faqat ajralish) sodir bo`lsa bu holda

$$\frac{dM_1}{dt} = \frac{dM}{dt} \text{ (yoki } \frac{dM_2}{dt} = \frac{dM}{dt}\text{)}$$

va (1.2) tenglama quyidagi ko`rinishga ega bo`ladi :

$$M\frac{dv}{dt} = \vec{F} + (\vec{u} - \vec{v})\frac{dM}{dt} \qquad (1.3)$$

bu yerda $\vec{u}$ qo`shiluvchi (ajraluvchi) zarraning absolyut tezligi. (1.3) ko`rinishdagi tenglama Meshcherskiy tomonida 1897-yili olingan, umumlashgan (1.2) ko`rinishda esa 1904-yildagi ishida e`lon qilingan.

$\overrightarrow{u_r} = (\vec{u} - \vec{v})$ kattalik zarrachalarning qo`shilish (ajralish) nisbiy tezligidir.

Agar bu kattalikni (1.3) tenglamaga kiritsak u quyidagi ko`rinishga keladi

$$M\frac{dv}{dt} = \vec{F} + \overrightarrow{u_r}\frac{dM}{dt} \qquad (1.4)$$

1.2 Variatson masalaning qo`yilishi

Massasi o`zgaruvchan nuqtani qaraymiz –bu kosmik apparatning massa markazidir. Sistema massa markazi harakati haqidagi teorema asosida va Meshcherskiy tenglamalaridan foydalanib bu harakatning differensial tenglamasini quyidagi ko`rinishda yozish mumkin:

$$M\frac{d\vec{v}}{dt} = \overrightarrow{\Phi} + M\vec{g} \qquad (1.5)$$

bu yerda M-massa, $\vec{g}$ –gravitatsion tezlanish, $\overrightarrow{\Phi}$–reaktiv kuch, bundan keyin uni tortishish kuchi deb ataymiz.

$$\overrightarrow{\Phi} = v_r\frac{dM}{dt}$$

bu yerda $\vec{v}_r$ –yonish mahsulotlarining tezligi, $\vec{v}_r = c\vec{e}$, $\vec{e}$–tortishish kuchi yo`nalishidagi birlik vector, $c$ ni esa doimiy kattalik deb hisoblaymiz (kimyoviy yonilg`ida ishlovchi raketalar uchun c nisbiy tezlik miqdori o’zgarmas).

$$\frac{dM}{dt} = -m < 0$$

(nuqtaning massasi kamayadi), m massani sekunddagi sarfi $0 \leq m \leq \bar{m}$. Shunday qilib, massasi o`zgaruvchan moddiy nuqtaning harakatning differensial tenglamasini (1.5) ni quyidagi kabi yozish mumkin.

$$\begin{cases} \dot{\vec{v}} = \dfrac{cm}{M}\vec{e} + \vec{g} \\ \dot{\vec{r}} = \vec{v} \\ \dot{M} = -m \end{cases}$$

bu yerda $\vec{r}$ nuqtaning radius vektori, $\vec{v}$ - nuqtaning tezligi.

Quyidagi variatsion masala qaraymiz: berilgan dastlabki holatdan yakuniy holatga o`tganda oldindan berilgan ma`lum funksionalni minimallashtiruvchi tortuvchi kuchning yo`nalishi va qiymatini topish ($m$ va $\vec{r}$) variatsion masalasini ko`rib chiqamiz.

Bizning holda minimallashtiruvchi funksional sifatida xarakteristik tezlikni olamiz

$$J = c\ln\frac{M_0}{M}.$$

Bu masala mexanik ma’nosiga ko`ra, massa sarfini minimallashtirish masalasi bilan bir xil kuchga ega.

$$J = \int_0^l \frac{cm}{M}dt = -\int_0^l \frac{c}{M}\frac{dM}{dt}dt = -c\int_0^l \frac{dM}{M}dt = c\ln\frac{M_0}{M}.$$

Masalaning qo`yilishiga ko`ra, tortish kuchining yo`nalishi va massa sarfi boshqaruvchi parametrlar rolini bajaradi. Bu holda tortish kuchini yo`nalishini belgilovchi bazis vektor $|\vec{\lambda}| = 1$, $\vec{e} = \vec{\lambda}$ kiritiladi [1]. Ma`lumki, optimal trayektoriyalar 3 ta qismdan iborat bo`ladi.

Nol tortishish qismi	(NT)	$m = 0$
Oraliq tortishish qismi	(OT)	$0 < m < \widetilde{m}$
Maksimal tortishish qismi	(MT)	$m = \widetilde{m}$

Statsionarlik va Veyershtrass shartlarini qo'llagan holda, bizning optimal boshqaruv masalasi o'n to'rtinchi tartibli yopiq Gamilton sistemasini NT, OT va MT oraliqlarda integrallash masalasiga keladi [1]. Bunda Gamilton funksiyasi quyidagi ko'rinishda bo'ladi:

$$H = \vec{\lambda}\left(\frac{cm}{M}\frac{\vec{\lambda}}{\lambda} + \bar{g}\right) + \overrightarrow{\lambda_r}\vec{v} - \lambda_7 m \qquad (1.6)$$

bu yerda λ_i-Lagranj ko'paytuvchilari.

Qo'shimcha funksiya – m boshqaruv parametrini fazoviy o'zgaruvchilar orqali kiritishimiz mumkin [1]. Bunda $\lambda, \lambda_r, \lambda_7$ – lar $\vec{v}, \vec{r}$ va M o'zgaruvchilarga mos ko'paytuvchilar.

1.3 Xususiy yechimlarni topishning Dokshevich usuli

Ko`rib o`tilgan masalani yechish uchun biz Dokshevich usulini qo`llaymiz, shuning uchun Dokshevich usuliga to`xtalib o`tsak. Dokshevich usuli oddiy differensial tenglamalar sistemasi uchun hali aniqlanmagan umumiy integralning tuzilishini oydinlashtirishga yordam beradi va xususiy integralni topishga xizmat qiladi.

Differensial tenglama sistemasi berilgan bo`lsin.

$$\begin{cases} \dot{x}_i = f(x, \lambda, t) \\ \dot{\lambda}_i = \varphi_i(x, \lambda, t) \end{cases} \quad (i = 1,2, \dots . n)$$

Aniqlash uchun $n = 5$ bo`lsin. Umumiy integrallar 6 o`zgaruvchiga bog`liq bo`lsin.

$$F(x_5, x_2, x_3, \lambda_2, \lambda_3, \lambda_6) = const \tag{1.8}$$

funksiya

$$(\frac{\partial F}{\partial x_1}) f_1 + \left(\frac{\partial F}{\partial x_2}\right) f_2 + \cdots + \left(\frac{\partial F}{\partial x_6}\right) \lambda_6 = 0 \tag{1.9}$$

tenglamani qanoatlantirishi kerak. Bu tenglama (1.8) ga kirmaydigan o`zgaruvchilar uchun ayniyat hisoblanadi. (1.9) tenglikdan x_1, x_4, λ_1, λ_4 ga mos koeffitsientlarni 0 ga tenglaymiz.

$$X_k(F) = 0 \quad (k = 1{,}2{,}3{,}4) \tag{1.10}$$

(1.10) tenglamalar soni (1.8) integraldagi o`zgaruvchilar sonidan kam $(5 > 4)$, yana ikkita tenglama kiritamiz

$$X_5 = (X_1, X_2) = 0$$
$$X_6 = (X_2, X_3) = 0$$

(boshqa (X_1, X_2), (X_1, X_4), (X_3, X_4) hollarni ham ko`rishimiz mumkin). Bu yerda

$$(X_k, X_i) = X_k\left(X_i(F)\right) - X_i(X_k(F)).$$

$X_k(F) = 0$ sistemaning determinanti 0 ga teng bo`lishi kerak. Bu shartdan foydalanib xususiy integralni (munosabatni) olishimiz mumkin. Agar bu integral ma`lum integral bilan invalyutsiyaga joylashsa xususiy yechimni topishda Levi-Chivita usulini qo`llashimiz mumkin.

II BOB. Ikkita qo'zg'almas tortishish markazining biri cheksiz uzoqlikda joylashgan holda moddiy nuqtaning harakati

2.1. Harakat differensial tenglamalari

Masalaning differensial tenglamalar sistemasi Gamilton sistemasi ko`rinishda yoziladi va ushbu ko`rinishga ega bo`ladi.

$$\dot{x}_i = \frac{\partial H}{\partial \lambda_i}, \dot{\lambda}_i = -\frac{\partial H}{\partial \dot{x}_i} (i = \overline{1,7}) \qquad (1.7)$$

bu yerda $\vec{v}(x_1 x_2 x_3)$, $\vec{r}(x_4 x_5 x_6)$, $M = x_7$ (3) sistema m boshqaruvchi funksiyaning ma`lum qiymatlarida yopiq sistemaga aylanadi. Biz OT qismlarini ko`rib chiqamiz, bu holda $0 < m(t) < \tilde{m}$.

Ikki qo`zg`almas grativatsiya markazi maydonida chegaralangan massa sarfi bilan harakatlanuvchi nuqta trayektoriyasini optimallash masalasini ko`rib chiqamiz. Agar moddiy nuqtaning harakati birinchi markaz atrofida yuz bersa, u holda kuch maydonini markaziy Nyuton maydoni va bir jinsli kuch maydonining birgalikdagi ta`sirini ifodalovchi maydon bilan almashtirish mumkin. Bu holda ikkinchi markazni cheksiz uzoqlikda joylashgan deb hisoblash imkonini beradi.

Nuqta harakatini tahlil qilish uchun (r, φ, z) silindrik koordinata sistemasini olamiz. Silindrik sistemasida kuch funksiyasini ushbu ko`rinishda yozish mumkin

$$U = \frac{\mu}{r_1} + kz \qquad (2.1)$$

bu yerda μ – birinchi tortishish markazining gravitatsion parametrik, $k > 0$ – o`zgarmas miqdor; $r_1 = \sqrt{r^2 + z^2}$; $r_1$ – qo`zg`almas 1- markazdan nuqtagacha bo`lgan masofa z – o`qini markazlarni tutashtiruvchi chiziq bo`ylab yo`naltiramiz

Endi silindrik koordinata sistemasida variatsion masalaning harakat differensial tenglamalarini yozamiz

$$\dot{\vec{v}} = \frac{cm}{M}\frac{\lambda_v}{\lambda} + \vec{v}$$

$$\dot{\vec{r}} = \vec{v}; \; \dot{M} = -m \tag{2.2}$$

$$\dot{\overrightarrow{\lambda_v}} = -\overrightarrow{\lambda_r}; \; \dot{\lambda}_r = -\frac{\partial \bar{g}}{\partial r}\overline{\lambda_v}; \; \dot{\lambda}_7 = \frac{cm}{M^2}$$

Bu tenglamalarda qatnashgan tezlik va tezlanishni silindrik koordinata o`qlariga proyeksiyasini kiritamiz:

$$\begin{cases} v_r = \dot{r} \\ v_\varphi = r\dot{\varphi} \\ v_z = \dot{z} \end{cases}$$

$$\begin{cases} \text{w}_r = \ddot{r} + r\dot{\varphi}^2 \\ \text{w}_\varphi = 2\dot{r}\dot{\varphi} + r\ddot{\varphi} \\ \text{w}_z = \ddot{z} \end{cases} \tag{2.3}$$

yoki

$$\begin{cases} v_1 = \dot{r} \\ v_2 = r\dot{\varphi} \\ v_3 = \dot{z} \\ \ddot{r} - r\dot{\varphi}^2 = \dfrac{cm}{M}\dfrac{\lambda_1}{\lambda} + g_r \\ 2\dot{r}\dot{\varphi} + r\ddot{\varphi} = \dfrac{cm}{M}\dfrac{\lambda_2}{\lambda} + g_\varphi \\ \ddot{z} = \dfrac{cm}{M}\dfrac{\lambda_3}{\lambda} + g_z \end{cases}$$

Olingan sistemaning birinchi uchta tenglamasidan vaqt bo`yicha hosila olib, ushbu formulalarni olamiz

$$\begin{cases} \dot{v}_1 = \ddot{r} \\ \dot{v}_2 = \dot{r}\dot{\varphi} + r\ddot{\varphi} \\ \dot{v}_3 = \ddot{z} \end{cases} \tag{2.4}$$

(2.4) sistemani hisobga olgan holda (2.3) sistemani qayta yozamiz

$$\begin{cases} \dot{v}_1 = \dfrac{cm}{M}\dfrac{\lambda_1}{\lambda} + g_r + r\dot{\varphi}^2 \\ \dot{v}_2 = \dfrac{cm}{M}\dfrac{\lambda_2}{\lambda} + g_\varphi - \dot{r}\dot{\varphi} \\ \dot{v}_3 = \dfrac{cm}{M}\dfrac{\lambda_3}{\lambda} + g_z \end{cases}$$

yoki

$$\begin{cases} \dot{v}_1 = \frac{cm}{M}\frac{\lambda_1}{\lambda} + g_r + \frac{{v_2}^2}{r} \\ \dot{v}_2 = \frac{cm}{M}\frac{\lambda_2}{\lambda} + g_\varphi - \frac{v_1 v_2}{r} \\ \dot{v}_3 = \frac{cm}{M}\frac{\lambda_3}{\lambda} + g_z \\ \dot{r} = v_1;\ \ \dot{\varphi} = \frac{v_2}{r};\ \ \dot{z} = v_3;\ \dot{M} = -m. \end{cases} \tag{2.5}$$

$\bar{g}$ vektorni tashkil etuvchilarini $g_i = \frac{1}{H_i}\frac{\partial U}{\partial x_i}$ formula yordamida topamiz, bu yerda H_i-Lyame koeffitsienti.

$$\begin{cases} g_r = \dfrac{\partial U}{\partial r} = (-\dfrac{1}{2}(r^2+z^2)^{-\frac{3}{2}} \times 2r) \\ g_\varphi = \dfrac{\partial U}{\partial \varphi} = 0 \\ g_z = \dfrac{\partial U}{\partial z} = \mu(-\dfrac{1}{2}\left(r^2+z^2\right)^{-\frac{3}{2}} \times 2z\right) + k \end{cases}$$

yoki

$$\begin{cases} g_r = -(\dfrac{\mu r}{(r^2+z^2)^{-\frac{3}{2}}}) \\ g_\varphi = 0 \\ g_z = -\left(\dfrac{\mu z}{(r^2+z^2)^{-\frac{3}{2}}}\right) + k \end{cases} \qquad (2.6)$$

Unda (2.6) ni (2.5) ga qo`yib OT qisimdagi nuqtani variatsion masalasini 1-guruh differensial tenglamalari silindirik koordinata sistemasida quyidagi ko`rinishini olamiz

$$\begin{cases} \dot{v}_1 = \dfrac{cm}{M}\dfrac{\lambda_1}{\lambda} - \dfrac{\mu r}{(r^2+z^2)^{-\frac{3}{2}}} + \dfrac{{v_2}^2}{r} \\ \dot{v}_2 = \dfrac{cm}{M}\dfrac{\lambda_2}{\lambda} - \dfrac{v_1 v_2}{r} \\ \dot{v}_3 = \dfrac{cm}{M}\dfrac{\lambda_3}{\lambda} - \left(\dfrac{\mu z}{(r^2+z^2)^{-\frac{3}{2}}}\right) + k \\ \dot{r} = v_1; \\ \dot{\varphi} = \dfrac{v_2}{r}; \\ \dot{z} = v_3; \\ \dot{M} = -m; \end{cases} \qquad (2.7)$$

bu yerda $(r^2+z^2)^{-\frac{3}{2}} = {r_1}^3$ deb olamiz

$$\begin{cases} \dot{v}_1 = \frac{cm}{M}\lambda_1 - \frac{\mu r}{{r_1}^3} + \frac{{v_2}^2}{r} \\ \dot{v}_2 = \frac{cm}{M}\lambda_2 + - \frac{v_1 v_2}{r} \\ \dot{v}_3 = \frac{cm}{M}\lambda_3 - \frac{\mu z}{{r_1}^3} + k \\ \dot{r} = v_1; \\ \dot{\varphi} = \frac{v_2}{r}; \\ \dot{z} = v_3; \\ \dot{M} = -m; \end{cases} \tag{2.8}$$

Bu sistema uchun Gamilton sistemasi o`rinli deb uni yozamiz.

$$H = \lambda_1 f_1 + \lambda_2 f_2 + \lambda_3 f_3 + \lambda_4 v_1 + \lambda_5 v_2 + \lambda_6 v_3 + \lambda_7 m$$

f_i larni o`rniga (2.7) sistemani mos ravishda qo`yamiz va quyidagi ifodaga kelamiz

$$H = \lambda_1 \frac{cm}{M}\frac{\lambda_1}{\lambda} - \frac{\mu r}{{r_1}^3} + \frac{{v_2}^2}{r} + \lambda_2 \frac{cm}{M}\frac{\lambda_2}{\lambda} + - \frac{v_1 v_2}{r} + \lambda_3 \frac{cm}{M}\frac{\lambda_3}{\lambda} - \left(\frac{\mu z}{{r_1}^3}\right) + k + \lambda_4 v_1 + \\ + \lambda_5 \frac{v_2}{r} + \lambda_6 v_3 - \lambda_7 m$$

.

Nihoyat (2.7) tenglamalar sistemasini tuzishimiz mumkin

$$\dot{v}_1 = \frac{cm}{M}\lambda_1 - \frac{\mu r}{{r_1}^3} + \frac{{v_2}^2}{r} \tag{2.9}$$

$$\dot{v}_2 = \frac{cm}{M}\lambda_2 - \frac{v_1 v_2}{r} \tag{2.10}$$

$$\dot{v}_3 = \frac{cm}{M}\lambda_3 - \frac{\mu z}{{r_1}^3} + k \tag{2.11}$$

$$\dot{r} = v_1 \tag{2.12}$$

$$\dot{\varphi} = \frac{v_2}{r} \tag{2.13}$$

$$\dot{z} = v_3 \tag{2.14}$$

$$\dot{M} = -m; \tag{2.15}$$

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial v_1} = \frac{v_2}{r}\lambda_2 - \lambda_4 \qquad (2.16)$$

$$\dot{\lambda}_2 = -\frac{\partial H}{\partial v_2} = \frac{1}{r}(-2v_2\lambda_1 + v_1\lambda_2 - \lambda_5) \qquad (2.17)$$

$$\dot{\lambda}_3 = -\frac{\partial H}{\partial v_3} = -\lambda_6 \qquad (2.18)$$

$$\dot{\lambda}_4 = -\frac{\partial H}{\partial r} = \lambda_1\left(\frac{{v_2}^2}{r^2} - \frac{\mu}{{r_1}^2} + \frac{3\mu r^2}{{r_1}^5}\right) - \frac{v_1 v_2}{r^2}\lambda_2 - \frac{3\mu z r}{{r_1}^5}\lambda_3 + \frac{v_2}{r^2}\lambda_5 \qquad (2.19)$$

$$\dot{\lambda}_5 = -\frac{\partial H}{\partial \varphi} = 0 \qquad (2.20)$$

$$\dot{\lambda}_6 = -\frac{\partial H}{\partial z} = -3\lambda_1\frac{\mu r z}{{r_1}^5} - \lambda_3\left(-\frac{\mu}{{r_1}^3} + \frac{3\mu z^2}{{r_1}^5}\right)$$

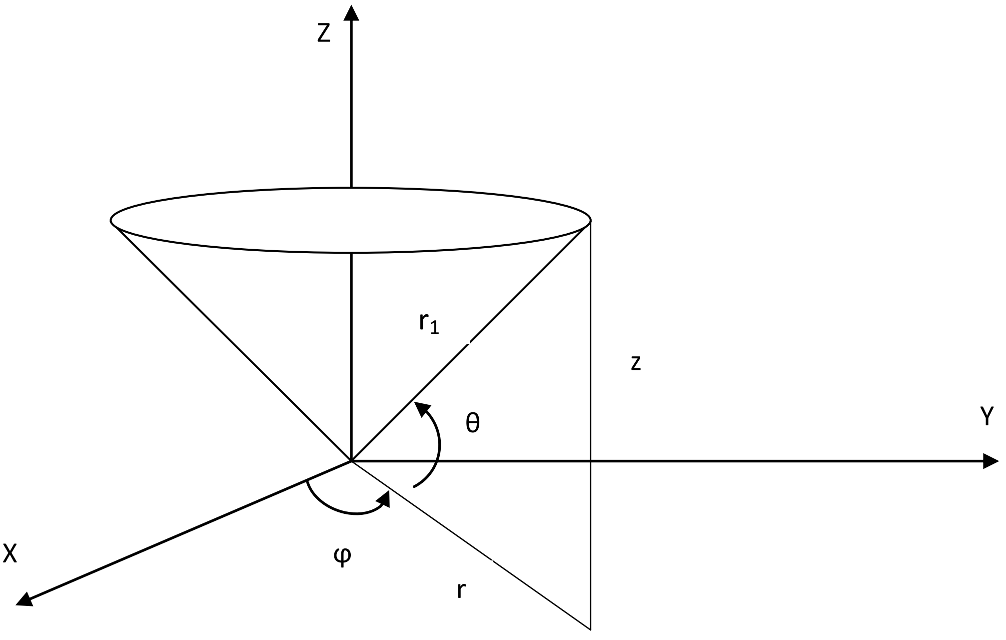
(2.21)

$$\dot{\lambda}_7 = -\frac{\partial H}{\partial M} = \frac{cm}{M^2}\lambda \qquad (2.22)$$

(2.20) tenglamadan ko`rish mumkinki, $\varphi$ siklik koordinata bo`ladi, demak, shu koordinataga mos siklik integral mavjud.

(2.1) maydon holida xarakteristik tezlikni minimallashtirish masalasini oraliq tortishish qismlari $0 < m < \widetilde{m}$ uchun ko`rib chiqamiz.

[2] manbada markaz chiziqlarga perpendikulyar tekisliklarda yotuvchi aylanma trayektoriyalarga mos keluvchi xususiy yechimlar sinfi topilgan. Qo`zg`almas markazga yaqinlashgan sari trayektoriyaning mavjud bo`lish sohasi torayadi. Simmetriya o'qi markazlar chizig'i bilan ustma-ust tushadigan, uchi qo'zg'almas markazda joylashgan konus sirtlarida yotuvchi aylanmadan farqli bo`lgan tekislikda trayektoriyalar mavjudmi degan savol tug`iladi. Bu holda ko'rilayotgan variatsion masalada qo`shimcha shart bajarilishi talab qilinadi.



1-rasm

$$tg\theta = const \tag{2.23}$$

bu yerda θ- konus yasovchining og`ish burchagi

(2.23) shartni quyidagicha yozamiz:

$$\frac{z}{r} = b > 0 \qquad z - br = 0 \tag{2.24}$$

bu yerda

$$b = tg(\theta) = const \tag{2.25}$$

konus yasovchining og`ishini xarakterlaydi.

U holda Gamilton funksiyasi quyidagi ko`rinishda yoziladi

$$\tilde{H} = H - \gamma_1({e_1}^2 + {e_2}^2 + {e_3}^2 - 1) - \gamma_2(z - br) - \gamma_3(m(\tilde{m} - m) - \alpha^2)$$

bu yerda γ_2 –Lagranj ko`paytuvchilari, $f_i$ (2.9), (2.22) sistemaning birinchi 7 ta tenglamasining o`ng qismlari. (2.24) shart Eyler –Logranj tenglamalariga (2.16) -(2.22) o`zgartirish kiritadi, aynan tenglamada $\dot{\lambda}_4$ , $\dot{\lambda}_6$ uchun quyidagi qo`shiluvchilar qo`shiladilar.

$$\dot{\lambda}_4 = -\frac{\partial H}{\partial r} - \gamma_2 b \tag{2.26}$$

$$\dot{\lambda}_6 = -\frac{\partial H}{\partial r} - \gamma_2 \tag{2.27}$$

(2.9) – (2.22) sistemaning qolgan tenglamalar o`zgarishsiz qoladi. (2.24) shartdan

$$\dot{z} = b\dot{r} \qquad \text{yoki} \quad v_3 = bv_1 \tag{2.28}$$

ekanligi kelib chiqadi. Ayrim o`zgartirishlar kiritamiz:

$r_1 = \sqrt{r^2 + z^2}$ hisobga olsak (2.24) quyidagi ko`rinishni oladi.

$$r_1 = \sqrt{z^2 + z^2} = \sqrt{r^2 + b^2 r^2} = r\sqrt{1 + b^2} = Br, \qquad r_1 = Br \tag{2.29}$$

bu yerda

$$B = \sqrt{1 + b^2} \ \text{yoki} \quad B = \frac{1}{\cos\theta} \tag{2.30}$$

2.2. Dokshevich usuli yordamida xususiy yechimlarni topish

(2.26), (2.27) o`zgartirishlarni e'tiborga olgan holda (2.9)-(2.22) Gamilton sistemasi uchun faqat 4 ta umumiy integrallar mavjud [2], bu umumiy yechim aniqlash uchun yetarli emas.

Xususiy integrallarni aniqlash uchun Dokshevich usulidan foydalanamiz. Masalaning xususiy integrali ushbu ko'rinishga ega bo'lsin

$$F(v_2, v_3, \lambda_1, \lambda_3, \lambda_4) = const \tag{2.31}$$

Undan vaqt bo`yicha to`liq hosila (2.9)-(2.22) variatsion masala differensial tenglamalari hisobiga aynan 0 ga teng. U holda F funksiya qanoatlantirishi kerak bo'lgan birinchi tartibli xususiy hosilali quyidagi chiziqli bir jinsli tenglamani olamiz

$$\frac{\partial F}{\partial v_2}\cdot\left(\frac{cm}{M}\frac{\lambda_2}{\lambda}-\frac{v_1 v_2}{r}\right)+\frac{\partial F}{\partial v_3}\cdot\left(\frac{cm}{M}\frac{\lambda_3}{\lambda}-\frac{\mu z}{{r_1}^2}+k\right)+\frac{\partial F}{\partial \lambda_1}\cdot\left(\frac{v_2}{r}\lambda_2+\lambda_4\right)-\frac{\partial F}{\partial \lambda_3}\cdot\lambda_6+$$

$$\frac{\partial F}{\partial \lambda_4}\cdot\left(\lambda_1\left(\frac{{v_2}^2}{r^2}-\frac{\mu}{{r_1}^2}+\frac{3\mu r^2}{{r_1}^5}\right)-\frac{v_1 v_2}{r^2}\lambda_2-\frac{3\mu z}{{r_1}^5}\lambda_3+\frac{v_2}{r^2}\lambda_5-\gamma_2 b\right)\equiv 0$$

F dan r, z bo`yicha hosilalar bo`lmaganligi sababli yuqorida olingan tenglamalar (2.24) va (2.29) ni hisobga olgan holda quyidagicha yozishimiz mumkin. $\left(\frac{{v_2}^2}{r^2}-\frac{\mu}{B^3 r^3}+\frac{3\mu}{B^5 r^3}\right)-\frac{v_1 v_2}{r^2}\lambda_2-\frac{3\mu b}{B^5 r^3}\lambda_3+\frac{v_2}{r^2}\lambda_5-\gamma b$

$$\frac{\partial F}{\partial v_2}\cdot\left(\frac{cm}{M}\frac{\lambda_2}{\lambda}-\frac{v_1 v_2}{r}\right)+\frac{\partial F}{\partial v_3}\cdot\left(\frac{cm}{M}\frac{\lambda_3}{\lambda}-\frac{\mu b}{B^3 r^2}+k\right)+$$

$$+\frac{\partial F}{\partial \lambda_1}\cdot\left(\frac{v_2}{r}\lambda_2-\lambda_4\right)-\frac{\partial F}{\partial \lambda_3}\cdot\lambda_6+\frac{\partial F}{\partial \lambda_4}\cdot[\lambda_1\left(\frac{{v_2}^2}{r^2}-\frac{\mu}{B^3r^3}+\frac{3\mu}{B^5r^3}\right)-\frac{v_1v_2}{r^2}\lambda_2-$$
$$\frac{3\mu b}{B^5r^3}\lambda_3+\frac{v_2}{r^2}\lambda_5-\gamma b]\equiv 0$$

(2.32)

(2.30) ga kirmaydigan kattaliklar koeffitsientlarni 0 ga tenglashtiramiz $\lambda_2,\frac{1}{r^2},\frac{1}{r^3}$ qolgan hadlarni birga yig`ib chiqamiz

$$\frac{\partial F}{\partial v_2}\cdot\frac{cm}{M}+\frac{\partial F}{\partial \lambda_1}\cdot\frac{v_2}{r}+\frac{\partial F}{\partial \lambda_4}\cdot\frac{v_1v_2}{r}\equiv 0 \qquad (2.33)$$

$$\frac{\partial F}{\partial \lambda_4}\cdot\left[\lambda_1\left(\frac{\mu}{B^3}-\frac{3\mu}{B^5}\right)-\lambda_3\frac{3\mu b}{B^5}\right]\equiv \qquad (2.34)$$

$$-\frac{\partial F}{\partial v_3}\cdot\frac{\mu b}{B^3}+\frac{\partial F}{\partial \lambda_4}\cdot\left(\lambda_1{v_2}^2+av_2\right)\equiv 0 \qquad (2.35)$$

$$-\frac{v_1v_2}{r}\cdot\frac{\partial F}{\partial v_2}+\frac{\partial F}{\partial v_3}\left(\frac{cm}{M}\lambda_3+k\right)-\frac{\partial F}{\partial \lambda_1}\cdot\lambda_4-\frac{\partial F}{\partial \lambda_3}\lambda_6-\frac{\partial F}{\partial \lambda_4}\cdot\gamma_2 b\equiv 0$$

(2.36)

(2.31) sababli $\frac{\partial F}{\partial \lambda_4}\neq 0,$ bo`lganligi uchun (2.34)dan olamiz.

$$\lambda_1\left(\frac{\mu}{B^3}-3\frac{\mu}{B^5}\right)-3\lambda_3\frac{\mu b}{B^5}=0$$

$$\lambda_1\left(1-3\frac{1}{B^2}\right)=3\lambda_3\frac{b}{B^2}$$

$\lambda_1(B^2-3)=3\lambda_3 b$ yoki (2.30) ni hisobga olgan holda invariant tenglikni olamiz

$$\lambda_1(b^2-3)=3\lambda_3 b \qquad (2.37)$$

(2.37) formula yordamida reaktiv kuchning markazlar chizig'idan o'tuvchi tekislikka proyeksiyasini XY tekislik bilan hosil qiluvchi α burchakni topish mumkin. Bu burchak vaqtga bog`liq emas.

$$\operatorname{tg}\alpha = \frac{\lambda_3}{\lambda_1} = \frac{b^2-2}{3b} \tag{2.38}$$

(2.33), (2.36) sistemaning qolgan munosabatlarini chap qismlar uchun $X_i(F)$, (i=1,2,3) operatorlarini kiritamiz [3].

$$X_1(F) = \frac{\partial F}{\partial v_2}\cdot\frac{cm}{M} + \frac{\partial F}{\partial \lambda_1}\cdot\frac{v_2}{r} + \frac{\partial F}{\partial \lambda_4}\cdot\frac{v_1 v_2}{r}$$

$$X_2(F) = \frac{\partial F}{\partial v_3}\frac{\mu b}{B^3} + \frac{\partial F}{\partial \lambda_4}\cdot(\lambda_1 {v_2}^2 + a v_2)$$

$$X_3 = -\frac{v_1 v_2}{r}\cdot\frac{\partial F}{\partial v_2} + \frac{\partial F}{\partial v_3}\left(\frac{cm}{M}\lambda_3 + k\right) - \frac{\partial F}{\partial \lambda_1}\cdot\lambda_4 - \frac{\partial F}{\partial \lambda_3}\lambda_6 - \frac{\partial F}{\partial \lambda_4}\cdot\gamma_2 b$$

(2.31) integralda o`zgaruvchilar 5 ta bo`lgani uchun $X_i(F) = 0$ tenglarmalar ham 5 ta bo`lishi kerak. Shu maqsadda, Puasson qavslaridan foydalanib yana ikkita operator kiritamiz

$$(X_1, X_2) = X_1(X_2) - X_2(X_1) = \frac{\partial F}{\partial \lambda_4}\left(2v_2\lambda_1\frac{cm}{M} + a\frac{cm}{M} + \frac{{v_2}^2}{r}\right) = 0$$

$\frac{\partial F}{\partial \lambda_4} \neq 0$, bo`lgan uchun $2v_2\lambda_1\frac{cm}{M} + a\frac{cm}{M} + \frac{{v_2}^2}{r} = 0$ invariant munosabatni topdik.

$$\frac{cm}{M}(2v_2\lambda_1 + a) + \frac{{v_2}^2}{r} = 0 \tag{2.39}$$

$$(X_2, X_3) = X_2(X_3) - X_3(X_2) = -\frac{\partial F}{\partial v_3{}^2}\frac{\mu b}{B^3}\left(\frac{cm}{M}\lambda_3 + k\right) + (\lambda_1 v_2{}^2 + av_2)\left(-\frac{\partial F}{\partial \lambda_1} - \frac{\partial^2 F}{\partial \lambda^2{}_4}\cdot\gamma_2 b\right) + \frac{v_1 v_2}{r}\frac{\partial F}{\partial \lambda_4}(2v_2\lambda_1 + a) + \frac{\mu b}{B^3}\cdot\frac{\partial^2 F}{\partial v^2{}_3}\left(\frac{cm}{M}\lambda_3 + k\right) + \lambda_4\frac{\partial F}{\partial \lambda_4}v_2{}^2 + \frac{\partial^2 F}{\partial \lambda^2{}_4}\gamma_2 b\cdot(\lambda_1 v_2{}^2 + av_2) = \frac{\partial F}{\partial \lambda_4}(2v_2\lambda_1 + a)\frac{v_1 v_2}{r} + \lambda_4\frac{\partial F}{\partial \lambda_4}v_2{}^2 - \frac{\partial F}{\partial \lambda_1}(\lambda_1 v_2{}^2 + av_2)$$

,

$$(X_2, X_3) = \frac{\partial F}{\partial \lambda_4}\left(2\frac{v_1 v_2{}^2}{r}\lambda_1 + a\frac{v_1 v_2}{r} + \lambda_4 v_2{}^2\right) - \frac{\partial F}{\partial \lambda_1}(\lambda_1 v_2{}^2 + av_2) = 0,$$

Quyidagicha belgilash kiritamiz:

$$X_4 = \frac{\partial F}{\partial \lambda_4}\left(2\frac{v_1 v_2}{r}\lambda_1 + a\frac{v_1}{r} + \lambda_4 v_2\right) - \frac{\partial F}{\partial \lambda_1}(\lambda_1 v_2 + a).$$

$$(X_1, X_3) = X_1(X_3) - X_3(X_1) = -\frac{cm}{M}\left(\frac{\partial^2 F}{\partial v_2{}^2}\cdot\frac{v_1 v_2}{r} + \frac{\partial F}{\partial v_2}\cdot\frac{v_1}{r}\right) - \frac{\partial^2 F}{\partial \lambda_1{}^2}\lambda_4\frac{v_2}{r} - \frac{v_1 v_2}{r^2}\left(-\frac{\partial F}{\partial \lambda_1} - \frac{\partial^2 F}{\partial \lambda_4{}^2}\gamma_2 b\right) + \frac{v_1 v_2}{r}\left(\frac{\partial^2 F}{\partial v_2{}^2}\cdot\frac{cm}{M} + \frac{\partial F}{\partial \lambda_1}\cdot\frac{1}{r} - \frac{\partial F}{\partial \lambda_4}\cdot\frac{v_1}{r}\right) + \lambda_4\frac{\partial^2 F}{\partial \lambda_1{}^2}\cdot\frac{v_2}{r} + \gamma_2 b\left(-\frac{\partial^2 F}{\partial \lambda_4{}^2}\cdot\frac{v_1 v_2}{r^2}\right) = \frac{\partial F}{\partial v_2}\cdot\frac{v_1}{r}\cdot\frac{cm}{M} + 2\frac{\partial F}{\partial \lambda_1}\cdot\frac{v_1 v_2}{r^2} - \frac{\partial F}{\partial \lambda_4}\cdot\frac{v_1{}^2 v_2}{r^3} = 0$$

.

$$X_5(F) = \frac{\partial F}{\partial v_2}\cdot\frac{cm}{M} + 2\frac{\partial F}{\partial \lambda_1}\cdot\frac{v_2}{r} - \frac{\partial F}{\partial \lambda_4}\frac{v_1 v_2}{r}.$$

Dokshevich usuliga ko’ra $X_i(F) = 0$ (i=1,2,3,4,5) sistema uchun determinant tuzamiz.

$$\begin{vmatrix} \frac{cm}{M} & 0 & \frac{v_2}{r} & 0 & -\frac{v_1 v_2}{r^2} \\ 0 & -\frac{\mu b}{B^3} & 0 & 0 & (\lambda_1 v_2{}^2 + av_2) \\ -\frac{v_1 v_2}{r} & \left(\frac{cm}{M}\lambda_3 + k\right) & -\lambda_4 & -\lambda_6 & -\gamma_2 b \\ 0 & 0 & -(\lambda_1 v_2 + a) & 0 & 2\frac{v_1 v_2}{r}\lambda_1 + a\frac{v_1}{r} + \lambda_4 v_2 \\ -\frac{cm}{M} & 0 & 2\frac{v_2}{r} & 0 & -\frac{v_1 v_2}{r^2} \end{vmatrix} = 0$$

Agar $\lambda_6 \neq 0$ bo`lsa, bu qiymat (2.9), (2.22) differensial tenglamalar sistemasiga qo`yib (2.18) dan

$$\dot{\lambda}_3 = 0, \qquad \dot{\lambda}_3 = const$$

va (2.21) dan

$$\dot{\lambda}_6 = 0$$

tenglikni olamiz.

λ_6 qo`shma ko`paytuvchi-siklik, demak,

$$\lambda_3 = const$$

siklik integral o`rinli

Bu holda masala o`ninchi darajali Gamilton sistemasini integrallashtirishga keltiriladi. Bazis vektorning tashkil etuvchisi $\lambda_3 = const$ bo'lgani uchun (2.31) xususiy integral mavjud bo`lmaydi. Demak, (2.32) ko`rinishdagi xususiy integral

$$F(v_2, v_3, \lambda_1, \lambda_3, \lambda_4) = const.$$

ko`rinishdan integralga o`tadi.

Faraz qilamiz, $\lambda_6 \neq 0$

$$\begin{vmatrix} \frac{cm}{M} & 0 & \frac{v_2}{r} & -\frac{v_1 v_2}{r^2} \\ 0 & -\frac{\mu b}{B^3} & 0 & (\lambda_1 {v_2}^2 + a v_2) \\ 0 & 0 & -(\lambda_1 v_2 + a) & 2\frac{v_1 v_2}{r}\lambda_1 + a\frac{v_1}{r} + \lambda_4 v_2 \\ -\frac{cm}{M} & 0 & 2\frac{v_2}{r} & -\frac{v_1 v_2}{r^2} \end{vmatrix} = 0$$

$\frac{\mu b}{B^3} \neq 0$ va $\frac{cm}{M} \neq 0$ bo`lgan uchun determinantni ikkinchi ustun bo`yicha yoyamiz.

$$\begin{vmatrix} \frac{cm}{M} & \frac{v_2}{r} & -\frac{v_1 v_2}{r^2} \\ 0 & -(\lambda_1 v_2 + a) & 2\frac{v_1 v_2}{r}\lambda_1 + a\frac{v_1}{r} + \lambda_4 v_2 \\ -\frac{cm}{M} & 2\frac{v_2}{r} & -\frac{v_1 v_2}{r^2} \end{vmatrix} = 0$$

$$\frac{cm}{M} \cdot \frac{v_1 v_2}{r^2} \cdot (\lambda_1 v_2 + a) - \frac{cm}{M}\frac{v_2}{r}\left(2\frac{v_1 v_2}{r}\lambda_1 + a\frac{v_1}{r} + \lambda_4 v_2\right) +$$
$$+\frac{cm}{M}\frac{v_1 v_2}{r^2} \cdot (\lambda_1 v_2 + a) - \frac{cm}{M} \cdot 2\frac{v_2}{r} \cdot \left(2\frac{v_1 v_2}{r}\lambda_1 + a\frac{v_1}{r} + \lambda_4 v_2\right) =$$
$$= 2\frac{cm}{M} \cdot \frac{v_1 v_2}{r^2} \cdot (\lambda_1 v_2 + a) - 3\frac{cm}{M} \cdot \frac{v_2}{r}\left(2\frac{v_1 v_2}{r}\lambda_1 + a\frac{v_1}{r} + \lambda_4 v_2\right) = 0.$$

$$2\frac{v_1}{r} \cdot (\lambda_1 v_2 + a) - 3\left(2\frac{v_1 v_2}{r}\lambda_1 + a\frac{v_1}{r} + \lambda_4 v_2\right) = 0$$

Bu yerda ikkita hol bo`lishi mumkin:

1. $4\,\frac{v_1 v_2}{r}\lambda_1 + a\frac{v_1}{r} + 3\lambda_4 v_2 = 0$

2. $\begin{cases} \lambda_1 v_2 + a = 0, \\ 2\frac{v_1 v_2}{r}\lambda_1 + a\frac{v_1}{r} + \lambda_4 v_2 = 0. \end{cases}$

(2.40)

Birinchi hol hech qanday yangi munosabatni bermaydi, shuning uchun ikkinchi holni qaraymiz

$$\begin{cases} \lambda_1 v_2 + a = 0, \\ \frac{v_1}{r}(2\lambda_1 v_2 + a) + \lambda_4 v_2 = 0 \end{cases}$$

(2.40) ni sistemaning birinchi tenglamasidan foydalanib ikkinchi tenglikni o`zgartiramiz.

$$\lambda_1 v_1 v_2 + \lambda_4 v_2 r = 0$$

$$\lambda_1 v_1 + \lambda_4 r = 0 \tag{2.41}$$

(2.28) shartni qarab chiqamiz.

$$v_3 = b v_1 \text{ yoki } \dot{v}_3 = b\dot{v}_1$$

(2.9), (2.11) dan $\dot{v}_3$, $\dot{v}_1$ qiymatni qo`yamiz:

$$\frac{cm}{M}\lambda_3 - \frac{\mu z}{{r_1}^3} + k = b\left(\frac{cm}{M}\lambda_1 - \frac{\mu r}{{r_1}^3} + \frac{{v_2}^2}{r}\right);$$

$$\frac{cm}{M}\lambda_3 - b\frac{cm}{M}\lambda_1 = b\left(-\frac{\mu r}{{r_1}^3} + \frac{{v_2}^2}{r}\right) + \frac{\mu z}{{r_1}^3} - k;$$

$$\frac{cm}{M}(\lambda_3 - b\lambda_1) = \frac{\mu z}{{r_1}^3}(z - br) + b\frac{{v_2}^2}{r} - k\,.$$

(2.24) ni hisobga olib ushbu tenglikka ega bo`lamiz

$$\frac{cm}{M}(\lambda_3 - b\lambda_1) = b\frac{{v_2}^2}{r} - k \tag{2.42}$$

(2.38) tenglamalardan λ_3 ni topamiz (2.42) ga qo`yamiz

$$\lambda_3 = \frac{\lambda_1(b^2-2)}{3b};$$

$$\lambda_3 - b\lambda_1 = \frac{\lambda_1(b^2-2)}{3b} - b\lambda_1 = \lambda_1\left(\frac{(b^2-2)}{3b} - b\right) = \lambda_1\left(\frac{b^2-2-3b^2}{3b}\right) = -2\lambda_1\frac{b^2+1}{3b}$$

;

$$2\lambda_1 \cdot \frac{cm}{M} \cdot \frac{b^2+1}{3b} = k - b\frac{{v_2}^2}{r} \qquad (2.43)$$

$2\frac{cm}{M} \cdot \frac{b^2+1}{3b} > 0$ bo`lgani uchun (2.43) invariant munosabatdan quyidagi xulosa kelib chiqadi:

Agar $\lambda_1 > 0$,bo`lsa $\quad k - b\frac{{v_2}^2}{r} > 0, \quad {v_2}^2 < \frac{kr}{b}$;

Agar $\lambda_1 < 0$, bo`lsa $\quad k - b\frac{{v_2}^2}{r} < 0, \quad {v_2}^2 > \frac{kr}{b}$

(2.44)

(2.40) sistemani birinchi tenglamasini , (2.39) qo`yib topamiz.

$$\frac{cm}{M}\lambda_1 + \frac{{v_2}^2}{r} = 0 \qquad (2.45)$$

$\frac{cm}{M} > 0$ va $\frac{{v_2}^2}{r} > 0$ bo`lgani uchun (2.45) tenglama bajarilishi uchun $\lambda_1$hamma vaqt manfiy bo`lishi kerak, yani (2.44) bajarilishi kerak.

Xulosa, bazis vektorning λ_1 – radial tashkil etuvchisi manfiy kattalik, shuning uchun reaktiv kuch konussimon sirtning ichkari tomoniga yo`nalgan bo`ladi. Nuqtaning tezligi pastdan chegaralangan .

(2.45) va (2.9) differensial tenglamani taqqoslab

$$\dot{v}_1 = -\frac{\mu r}{{r_1}^3}$$

munosabatni olamiz yoki (2.29) ni hisobga olgan holda olamiz.

$$\dot{v}_1 = -\frac{\mu r}{B^3 r^2} < 0.$$

Shunday qilib, v_1 ning tashkil etuvchisi kamayadi, shunday ekan, v_3 ham kamayadi.

Oxirgi tenglikning ikkala tomonini ham dr ga ko`paytiramiz.

$$dr\frac{dv_1}{dt} = -\frac{\mu}{B^3 r^2}dr.$$

$$v_1 dv_1 = -\frac{\mu}{B^3 r^2}dr.$$

hosil bo`lgan tenglamani integrallab

$$\frac{{v_1}^2}{r} = \frac{\mu}{B^3 r} + C_1$$

munosabatni olamiz, bu yerda $C_1 - \forall$ o`zgarmas. Belgilangan boshlang`ich shartlardan $r = r_0$, $v_1 = v_{10}$, C_1 integrallash doimiysini aniqlaymiz.

$$C_1 = \frac{{v_{10}}^2}{2} - \frac{\mu}{B^3 r_0}.$$

Natijada

$${v_1}^2 = {v_{10}}^2 + \frac{2\mu}{B^3}\left(\frac{1}{r} - \frac{1}{r_0}\right) \tag{2.46}$$

yechimga ega bo`lamiz $v_1$ kamaygani uchun ${v_{10}}^2 > {v_1}^2$, yani $\frac{1}{r} - \frac{1}{r_0} < 0$ yoki $r_0 < r$ bo`ladi.

Shunday qilib, nuqtaning vaqt o`tishi bilan qo`zg`almas tortishish markazidan uzoqlashadi.

(2.22) differensial tenglamani (2.46) ni hisobga olgan holda o`zgartiramiz.

$$\dot{r} = \sqrt{2\left(\frac{{v_{10}}^2}{2} + \frac{\mu}{B^3}\left(\frac{1}{r} - \frac{1}{r_0}\right)\right)} = \sqrt{{v_{10}}^2 + \frac{2\mu}{B^3}\left(\frac{1}{r} - \frac{1}{r_0}\right)}$$

θ burchakning ayni bir qiymati uchun o`zgarmas bo`ladigan quyidagi belgilashlar kiritamiz

$$S(\theta) = \frac{2\mu}{B^3} = 2\mu cos^2\theta \ ,$$

$$P(\theta) = {v_{10}}^2 - \frac{S}{r_0}$$

Shunda

$$\dot{r} = \sqrt{{v_{10}}^2 + \frac{S}{r} - \frac{S}{r_0}};\ \dot{r} = \sqrt{\frac{Pr+S}{r}}\ ;$$

$$v_1 = \sqrt{\frac{Pr+S}{r}}\ ; \tag{2.47}$$

$$\sqrt{\frac{Pr+S}{r}}\,dr = dt \tag{2.48}$$

Xususiy yechimlarni aniqlash

(2.48) differensial tenglamani integrallash P kattalik qiymatiga bog`liq.

1. $P > 0$ bo`lgan holni qaraymiz.

Shunda ${v_{10}}^2 > \frac{2\mu}{r_0}\frac{1}{B^3}$, yoki ${v_{10}}^2 > \frac{2\mu}{r_0}cos^2\theta$. Yani v_{10} mahalliy $\frac{2\mu}{r_0}$ “parabolik” tezlikdan katta kichik yoki teng bo`lishi mumkin.

(2.48) ni integrallab topamiz.

$$\int\sqrt{\frac{r}{Pr+S}}\,dr = \frac{\sqrt{r(Pr+S)}}{P} - \frac{S}{2P}\left[\frac{2}{\sqrt{P}}\ln(\sqrt{P(Pr+S)} + P\sqrt{r})\right] = \frac{\sqrt{r(Pr+S)}}{P} - \frac{S}{P\sqrt{P}}\ln(\sqrt{P(Pr+S)} + P\sqrt{r})$$

$$\frac{\sqrt{r(Pr+S)}}{P} - \frac{S}{P\sqrt{P}}\ln(\sqrt{P(Pr+S)} + P\sqrt{r}) = \mathrm{t} + C_2 \qquad (2.49)$$

bu yerda.

$$C_2 = \frac{\sqrt{r_0(Pr_0+S)}}{P} - \frac{S}{P\sqrt{P}}\ln(\sqrt{P(Pr_0+S)} + P\sqrt{r_0}).$$

(2.39) dagi $r(t)$ ni qiymatini (2.37) ga qo`yib nuqtaning v_1 tezligining tashkil etuvchisini topishimiz mumkin.

$$\frac{\sqrt{r(Pr+S)}}{P} = t + \frac{\sqrt{r_0(Pr_0+S)}}{P} - \frac{S}{P\sqrt{P}}\ln(\sqrt{P(Pr_0+S)} + P\sqrt{r_0}) + \frac{S}{P\sqrt{P}}\ln(\sqrt{P(Pr+S} + P\sqrt{r})$$

.

ifodaning har ikki qismiga $\frac{P}{r}$ ni ko`paytiramiz

$$\frac{\sqrt{r(Pr+S)}}{r} = \frac{Pt}{r} + \frac{\sqrt{r_0(Pr_0+S)}}{r} - \frac{S}{r\sqrt{P}}\ln(\sqrt{P(Pr_0+S)} + P\sqrt{r_0}) + \\ + \frac{S}{r\sqrt{P}}\ln(\sqrt{P(Pr+S)} + P\sqrt{r}).$$

Tezlikning v_1 radial tashkil etuvchisi uchun ifoda olamiz.

$$v_1 = \frac{Pt}{r} + \frac{\sqrt{r_0(Pr_0+S)}}{r} - \frac{S}{r\sqrt{P}}\ln(\sqrt{P(Pr_0+S)} + P\sqrt{r_0}) +$$

$$+\frac{S}{r\sqrt{P}}\ln(\sqrt{P(Pr+S}+P\sqrt{r}). \qquad (2.50)$$

$r(t)$ ni (2.49) o`zgarish qonunidan foydalanib (2.24),(2.28),(2.37),(2.40),(2.43),(2.45) tengliklardan reaktiv kuchni, trayektoriyani, nuqta tezligini va massani o`zgarishini vaqtning funksiyasi sifatida topish mumkin.

2. $P = 0$ bo`lgan holatni qaraymi..

bu holda $v_{10} = \sqrt{\frac{D}{r_0}}$ yoki $v_{10} = \sqrt{\frac{2\mu}{B^3 r_0}}$

$$v_{10} = \sqrt{\frac{2\mu}{B^3 r_0}cos^3\theta} < \sqrt{\frac{2\mu}{r_0}}.$$

Shunday qilib, boshlang`ich tezlikning v_{10} radial tashkil etuvchisi mahalliy $\frac{2\mu}{r_0}$ “parabolik” tezlikdan kichik ekanligini topamiz.

(2.48) ni integrallaymiz:

$$\sqrt{r}dr = \sqrt{S}dt; \qquad \frac{2}{3}r^{3/2} = \sqrt{S}\cdot t + C_3.$$

Berilgan $r = r_0$ boshlang`ich shartlarda C_3 integrallash doimiysini aniqlaymiz:

$C_3 = \frac{2}{3}(r_0)^{3/2}$ va uni hisobga olgan holda quyidagi tenglikka ega bo`lamiz.

$$\frac{2}{3}r^{3/2} = \sqrt{S}\cdot t + \frac{2}{3}(r_0)^{3/2} \qquad r^{3/2} = (r_0)^{\frac{3}{2}} + \frac{3}{2}\sqrt{S}\cdot t;$$

$$r(t,\theta) = ((r_0)^{\frac{3}{2}} + \frac{3}{2}\sqrt{2\mu cos^3\theta}\cdot t)^{2/3}.$$

Shunday qilib, nuqtaning harakati davomida r ortadi v_1 kamayadi (2.24) dan z ni aniqlaymiz

$$z = b \cdot \left((r_0)^{\frac{3}{2}} + \frac{3}{2}\sqrt{S} \cdot t\right)^{2/3}.$$

Nuqtadan markazlar to`g`ri chiziqlargacha bo`lgan $r$ masofa ortgani sababli mos ravishda nuqtaning konus o`qiga proyeksiyasidan qo`zg`almas tortishish markazigacha bo`lgan z masofa ortadi.

(2.50) ni (2.47) ga qo`yib nuqtaning tezligining v_1 radial tashkil etuvchisini topamiz.

$$v_1 = \sqrt{\frac{S}{r}} = \frac{\sqrt{S}}{\sqrt[3]{(r_0)^{\frac{3}{2}} + \frac{3}{2}\sqrt{S} \cdot t}}$$

yoki

$$v_1 = \frac{\sqrt{2\mu cos^3\theta}}{\sqrt[3]{(r_0)^{\frac{3}{2}} + \frac{3}{2}\sqrt{2\mu} \cdot t \cdot (\cos\theta)^{3/2}}}.$$

$\cos\theta < 1$ bo`lgani uchun

$$v_1 = \sqrt{\frac{D}{r}} = \sqrt{\frac{2\mu}{rB^3}} = \sqrt{\frac{2\mu}{r}cos^3\theta} < \sqrt{\frac{2\mu}{r}}$$

Nuqta tezligining v_1 radial tashkil etuvchisi mahalliy “parabolik” tezlikdan kichik bo`lishi va vaqt o`tishi bilan kamayishini oldik.

(2.28) dan nuqta tezligining v_3 tashkil etuvchisini olamiz.

$$v_3 = \frac{b\sqrt{S}}{\sqrt[3]{(r_0)^{\frac{3}{2}} + \frac{3}{2}\sqrt{S} \cdot t}}$$

yoki

$$v_3 = \frac{tg\theta \cdot \sqrt{2\mu cos^3\theta}}{\sqrt[3]{(r_0)^{\frac{3}{2}} + \frac{3}{2}\sqrt{2\mu} \cdot t \cdot (\cos\theta)^{3/2}}}.$$

Tezlikning v_1 radial tashkil etuvchisi kamayganligi sababli (2.28) ga ko`ra tezlikning v_3 tashkil etuvchisi ham kamayishi kelib chiqadi.

v_2 ,λ_1 va $M(t)$ massa o`zgarishi qonunini aniqlash uchun yuqorida olingan shartlarni qarab chiqamiz, bu shartlar (2.40), (2.43), (2.45) .

(2.45) ni (2.43) ga qo`yib quyidagini topamiz.

$$\frac{{v_2}^2}{r}\left(b - 2\frac{b^2+1}{3b}\right) = k \qquad (2.51)$$

$k > 0$, $\frac{{v_2}^2}{r} > 0$, bo`lgani uchun $b - 2\frac{b^2+1}{3b} > 0$ kelib chiqadi

$$b - 2\frac{b^2+1}{3b} = \frac{3b^2-2b^2-2}{3b} = \frac{b^2-2}{3b}.$$

(2.25) ni hisobga olib

$$\frac{b^2-2}{3b} = \frac{b}{3} - \frac{2}{3b} = \frac{\sin\theta}{3\cos\theta} - \frac{2\cos\theta}{3\sin\theta} = \frac{sin^2\theta - 2cos^2\theta}{3cos^2\theta \cdot tg\theta} = \frac{1-3cos^2\theta}{3\sin\theta \cdot \cos\theta} > 0.$$

topamiz. Bundan

$$1 - 3cos^2\theta >; \ \cos\theta < \frac{1}{\sqrt{3}} \approx 0.5774 .$$

kelib chiqadi. Burchakning ortib borishi bilan kosinus kamayishi sababli konus yasovchining og`ishi $\theta > 55°$ deb xulosa qilish mumkin. Demak, qidirilayotgan trayektoriyaning mavjud bo`lish sohasi:

$$55° < \theta < 90°$$

shartni qanoatlantiradi.

(2.51) dan tezlikning v_2 transversal tashkil etuvchisini topamiz.

$$v_2{}^2 = \frac{3bkr}{b^2-2},$$

$$v_2 = \pm\sqrt{kr\cdot\frac{3b}{b^2-2}}.$$

yoki (2.38) ni hisobga olib:

$$v_2 = \pm\sqrt{kr\cdot ctg\alpha}.$$

Vaqt o`tgan sari nuqta tortishish markazidan uzoqlashib borgani va $r_0 < r$ bo`lganligi sababli $|v_2|$ vaqt o`tishi bilan ortadi.

(2.40) dan bazis vektorning $\lambda_1(\theta, t)$ radial tashkil etuvchisini topamiz.

$$\lambda_1 = -\frac{a}{v_2} \quad \text{yoki} \qquad \lambda_1 = \mp\frac{a\cdot\sqrt{b^2-2}}{\sqrt{3bkr}} \tag{2.52}$$

(2.38) ni hisobga olib:

$$\lambda_1 = \mp\frac{a\cdot\sqrt{tg\alpha}}{\sqrt{kr}}$$

$\lambda_1 < 0$, bo`lgani uchun $\frac{a}{v_2} > 0$, ya'ni a va $v_2$ bir xil ishorali bo`lishi kelib chiqadi.

Bazis vektor $\lambda_3(\theta, t)$ tashkil etuvchisini (2.37) dan topamiz, topilgan radial tashkil etuvchining qiymatini (2.52) ga qo`yib quyidagini olamiz:

$$\lambda_3 = \lambda_1\frac{b^2-2}{3b}$$

$\lambda_1 <$ va $\frac{b^2-2}{3b} > 0$, bo`lgani uchun $\lambda_3 < 0$

$$\lambda_3 = \frac{|a| \cdot \sqrt{b^2-2}}{\sqrt{3bkr}} \cdot \frac{(b^2-2)}{3b},$$

$$\lambda_3 = \frac{|a|}{\sqrt{kr}} \cdot \sqrt{\left(\frac{(b^2-2)}{3b}\right)^3}$$

Bazis vektorning λ_2 transversal tashkil etuvchisini.

$$\lambda_2 = \pm\sqrt{1-{\lambda_1}^2-{\lambda_3}^2}\ (\lambda_2 > 0 \text{ da } v_2 > 0 \text{ va } \lambda_2 < 0 \text{ da } v_2 < 0)$$

munosabatni topamiz.

$$\lambda_2 = \pm\sqrt{1 - a^2 \cdot \frac{(b^2-2}{3bkr} - \frac{a^2}{kr} \cdot \left(\frac{(b^2-2)}{3b}\right)^3}.$$

yoki (2.38) ni hisobga olgan holda

$$\lambda_2 = \pm\sqrt{1 - \frac{a^2}{kr} \cdot tg\alpha - \frac{a^2}{kr} \cdot tg^3\alpha}.$$

(2.45) dan massani o`zgarish qonunini topamiz

$$\frac{cm}{M} = -\frac{{v_2}^2}{r\lambda_1} > 0.$$

(2.25) ga qo`ysak:

$$\frac{c}{M} \cdot \frac{dM}{dt} = \frac{{v_2}^2}{r\lambda_1} < 0.$$

u holda:

$$\frac{dM}{M} = -\frac{N(t)}{c\lambda_1(t)}dt \qquad (2.53)$$

bu yerda

$$N(t) = \frac{{v_2}^2}{r} = \frac{3bkr}{b^2 - 2} \cdot \frac{1}{r} = \frac{3bk}{b^2 - 2} = k \cdot ctg\alpha > 0$$

k vaqtga bog`liq bo`lmagani uchun N ham vaqtga bog`liq bo`lmaydi. Bundan (2.53) tenglamani integrallash quyidagi ifodani integrallashga keladi.

$$\int \frac{dt}{\lambda_1(t)} = \mp \frac{\sqrt{3bk}}{a \cdot \sqrt{b^2 - 2}} \int \sqrt{r} dt = \mp \frac{\sqrt{3bk}}{a \cdot \sqrt{b^2 - 2}} \int \sqrt{\left((r_0)^{\frac{3}{2}} + \frac{3}{2}\sqrt{S} \cdot t\right)^{2/3}} dt.$$

$$\int \frac{dt}{\lambda_1(t)} = \mp \frac{\sqrt{3bk}}{a \cdot \sqrt{b^2 - 2}} \int \left((r_0)^{\frac{3}{2}} + \frac{3}{2}\sqrt{S} \cdot t\right)^{1/3} dt = \mp \frac{\sqrt{3bk}}{a \cdot \sqrt{b^2 - 2}} \cdot \frac{8}{9\sqrt{S}} \cdot \sqrt[3]{\left((r_0)^{\frac{3}{2}} + \frac{3}{2}\sqrt{S} \cdot t\right)^4} + C_4$$

.

C_4 ni aniqlaymiz:

$$C_4 = \frac{\sqrt{3bk}}{a \cdot \sqrt{b^2 - 2}} \cdot \frac{8}{9\sqrt{S}} \cdot {r_0}^2$$

Unda

$$lnM = -\frac{1}{c} \cdot f(t) + lnM_0.$$

bu yerda

$$f(t) = N \cdot \int \frac{dt}{\lambda_1} = -\frac{N}{a} \cdot \int v_2 \, dt, \qquad (2.54)$$

$$f(t) = \mp \frac{8}{9a\sqrt{S}} \cdot \left(\frac{3bk}{b^2 - 2}\right)^{\frac{3}{2}} \cdot \sqrt[3]{\left((r_0)^{\frac{3}{2}} + \frac{3}{2}\sqrt{S} \cdot t\right)^4} \pm \frac{8{r_0}^2}{9a\sqrt{S}} \cdot \sqrt{\frac{3bk}{b^2 - 2}}..$$

Shunday qilib, reaktiv kuchning qiymati ko`rsatkichli qonuniyat bilan o`zgarar ekan.

$$M = M_0 e^{-f(t)/c}.$$

Xarakteristik tezlik quyidagi formula bo`yicha topiladi.

$$V = c\ln\frac{M_0}{M} = -f(t).$$

Massa kamayishi uchun $f(t) < 0$ bo`lishi kerak.

(2.54) da $N > 0$, a va v_2 esa bir xil ishorali bo'lgani uchun $f(t) < 0$ doim bajariladi.

Olingan invariant nisbatlar variatsion masalani differensial tenglamalarini qanoatlantirishi lozim, tekshirib ko`ramiz:

$$\frac{d}{dt}(\lambda_1 v_2 + a) = 0$$

$$\dot{\lambda}_1 v_2 + \lambda_1 \dot{v}_2 = 0$$

(2.10) va (2.16) dan

$$\lambda_1 \frac{{v_2}^2}{r} - \lambda_4 v_2 + \frac{cm}{M}\lambda_1\lambda_2 - \lambda_1 \frac{v_1 v_2}{r} = 0$$

munosabatni olamiz, (2.45) hisobga olgan holda quyidagini yozamiz.

$$\lambda_1 \frac{{v_2}^2}{r} - \lambda_4 v_2 - \lambda_2 \frac{{v_2}^2}{r} - \lambda_1 \frac{v_1 v_2}{r} = 0;$$

$v_2(\lambda_4 r + \lambda_1 v_2) = 0$. lekin (1.34) bajarilishida: $0 = 0$.

Shunday qilib, (2.41) (2.40) invariant munosabatini mavjudlik sharti bo`ladi:

$$\lambda_1(b^2 - 2) = 3\lambda_3 b;$$

$$\dot{\lambda}_1(b^2 - 2) = 3\dot{\lambda}_3 b;$$

(2.26) va (2.18) dan quydagini olamiz:

$$\left(\lambda_2 \frac{v_2}{r} - \lambda_4\right)(b^2 - 2) + 3b\lambda_6 = 0;$$

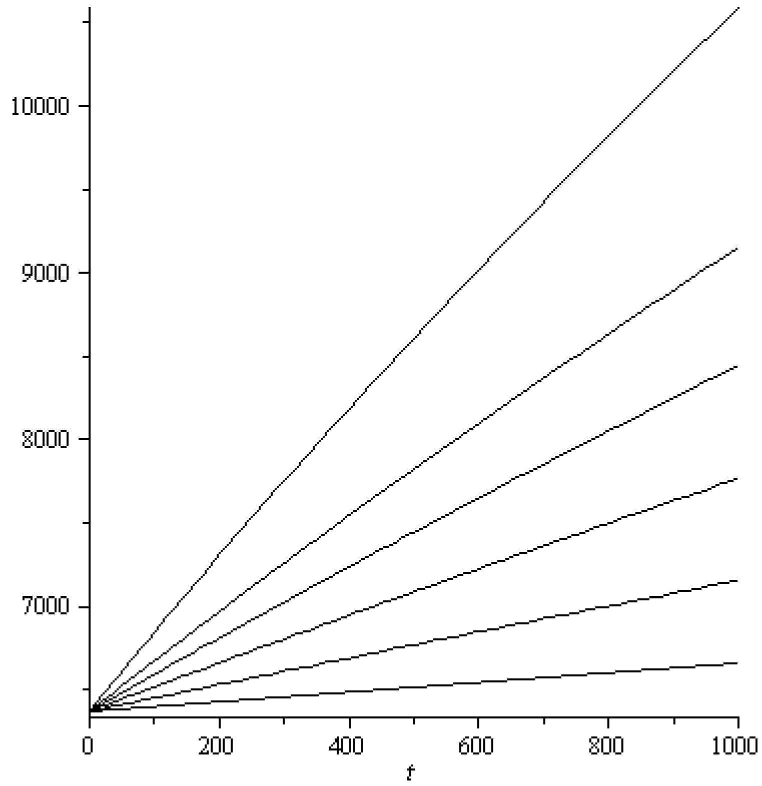
$$\lambda_6 = \left(\lambda_2 \frac{v_2}{r} - \lambda_4\right)\frac{(2-b^2)}{3b} \qquad (2.25)$$

(2.55) nisbat λ_6 ga qo`yiladigan shart hisoblanadi.

2.3 Olingan natijalarning sonli hisobi

Olingan natijalarni Yer atrofida harakatlanuvchi moddiy nuqta misolida tahlil qilamiz. Bu holda $r_0 = 6378km$, gravitatsion parametr esa $\mu = 398 \cdot 10^3 km^3/s^2$ ga teng.

θ burchakning turli qiymatlari uchun r ning o'zgarish grafigini chizamiz.



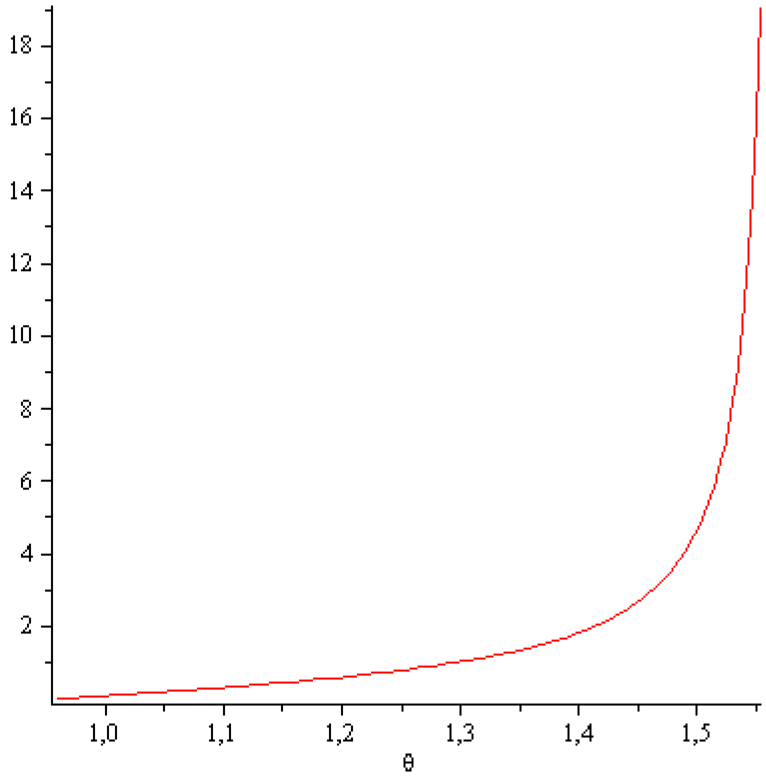
2-rasm

Yuqoridagi grafikdagi chiziqlar yuqoridan pastga qarab quyidagi burchak qiymatlariga mos keladi $\theta: 55°, 65°, 75°, 80°, 85$. Grafikdan nuqta qo`zg`almas tortishish markazidan uzoqlashgan sari $r(t)$ masofa konus yasovchining θ burchagi qancha kichkina bo`lsa, shuncha tez o`sishini ko`rish mumkin.

Bazis-vektorning $\lambda_3(\theta, t)$ tashkil etuvchisini (2.37) dan topamiz, topilgan radial tashkil etuvchining qiymatini (2.52) ga qo`yib quyidagini olamiz:

$$\lambda_3 = \lambda_1 \frac{b^2 - 2}{3b}$$

$$\lambda_3 = \frac{|a|}{\sqrt{kr}} \cdot \sqrt{\left(\frac{(b^2 - 2)}{3b}\right)^3}$$



3-rasm

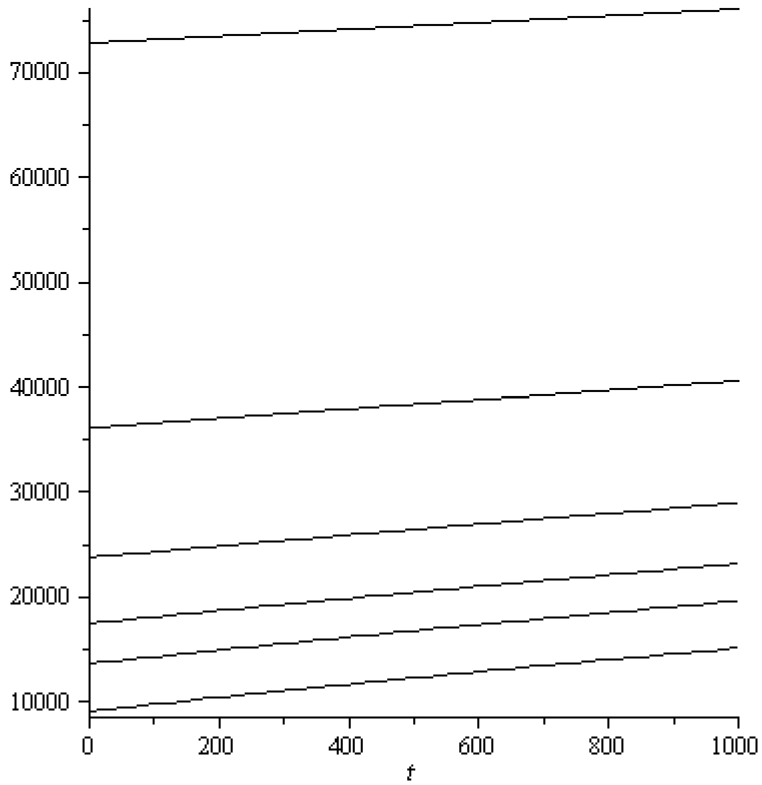
3-rasmda reaktiv kuchni markazlar chizig`iga perpendikulyar bo`lgan tekislikka og`ish burchagi tangensini grafigi keltirilgan $\theta$ burchak radianlarda o`lchanadi (ordinata o`qi).

Grafikdan ko`rinib turibdiki, $\theta$ burchak ortishi bilan reaktiv kuchning og`ish burchagi avvaliga keskin ortadi so`ng taxminan $80°$ dan boshlab bu o`sish sekinlashadi.

(2.24) dan z ni aniqlaymiz

$$z = b \cdot \left((r_0)^{\frac{3}{2}} + \frac{3}{2} \sqrt{S} \cdot t \right)^{2/3}.$$

u holda uning grafigi quyidagicha bo`ladi

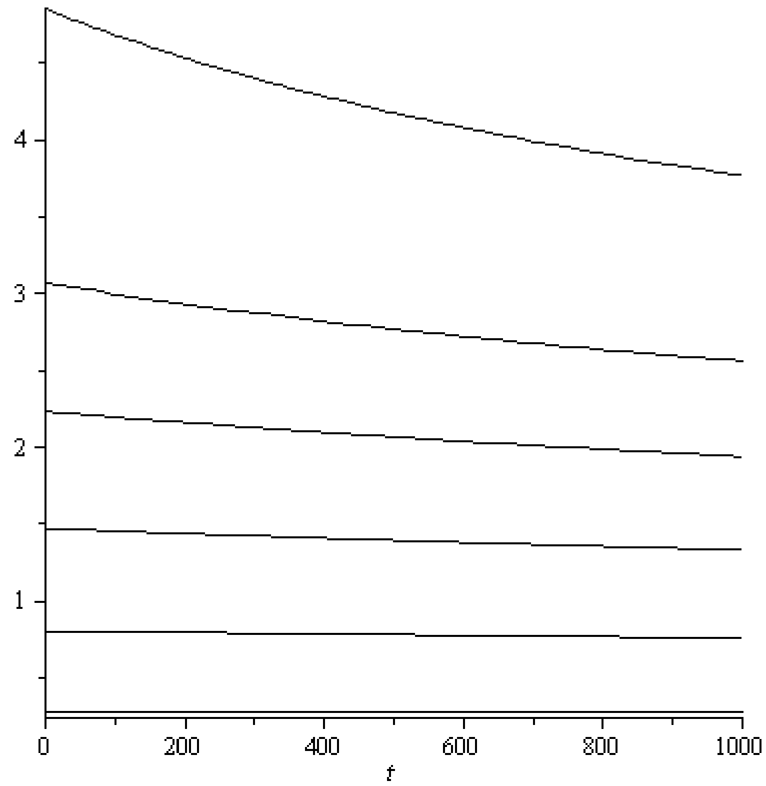


4-rasm

4-rasmda nuqtadan markazlar to`g`ri chizig'igacha bo`lgan $r$ masofa ortgani sababli mos ravishda nuqtaning konus o`qiga proyeksiyasidan qo`zg`almas tortishish markazigacha bo`lgan z masofa ham ortadi.

(2.50) ni (2.47) ga qo`yib nuqtaning tezligining v_1 radial tashkil etuvchisini topamiz

$$v_1 = \frac{\sqrt{2\mu cos^3\theta}}{\sqrt[3]{(r_0)^{\frac{3}{2}}+\frac{3}{2}\sqrt{2\mu}\cdot t\cdot(\cos\theta)^{3/2}}}.$$

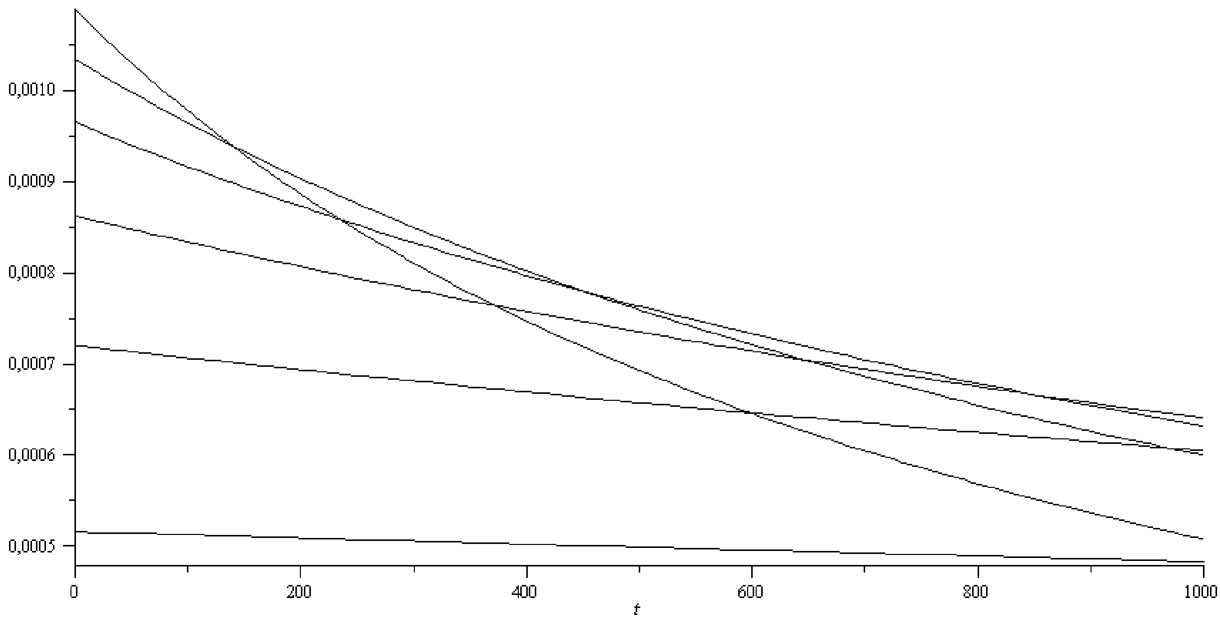


5-rasm

5-rasmda nuqtaning v_1 tezligi radial tashkil etuvchisi mahalliy “parabolik” tezlikdan kichik bo`lishi va vaqt o`tishi bilan kamayishini ko`rishimiz mumkin.

(2.28) dan nuqta tezligining v_3 tashkil etuvchisini olamiz

$$v_3 = \frac{tg\theta\cdot\sqrt{2\mu cos^3\theta}}{\sqrt[3]{(r_0)^{\frac{3}{2}}+\frac{3}{2}\sqrt{2\mu}\cdot t\cdot(\cos\theta)^{3/2}}}.$$



6-rasm

6-rasmda tezlikning v_1 radial tashkil etuvchisi kamayganligi sababli (2.28) ga ko`ra tezlikning v_3 tashkil etuvchisi ham kamayishi kelib chiqadi.

XULOSA

Shunday qilib, ushbu ishda ikkita qo`zg`almas markazlarning chegaraviy masalasi ko`rib chiqilgan, simmetriya o`qi markazlar chizig'i bilan mos keluvchi va uchi qo`zg`almas tortishish markazida bo`lgan konussimon sirtda yotuvchi trayektoriyalarning mavjud bo`lishi o`rganilgan.

Dokshevich usuli yordamida, aytib o'tilgan konussimon sirtda yotuvchi trayektoriya bo'ylab nuqta harakatiga mos keluvchi invariant nisbatlar va xususiy yechimlar sinfi topilgan. Massani saqlanish qonuni, reaktiv kuch yo`nalishini tavsiflaydigan bazis vektorning hamda tezlikning tashkil etuvchilari vaqt va konus yasovchining og`ish burchagining funksiyasi sifatida topilgan.

Ushbu ishimda olingan natijalarni Yer atrofida harakatlanuvchi moddiy nuqta misolida ko`rib chiqdim va quydagilarni oldim:

Trayektoriyaning mavjudlik sohasi chegaralari topilgan, reaktiv kuchning yo`nalishi butun harakat davomida trayektoriya ichiga yo`nalganligi ko`rsatilgan. Boshlang`ich tezlikning radial tashkil etuvchisi mahalliy parabolik tezlikdan kichik bo'lishi topilgan. Reaktiv kuchni markaziy to`g`ri chiziqdan o`tuvchi tekislikka proyeksiyasi markazlar chizig'i bilan hosil qilgan burchagi vaqtga bog`liq emasligi topilgan. Konus yasovchisining har xil og`ish burchaklarining qiymatlarida nuqtadan markazlar chizig'igacha masofani vaqtga bog`liqligi grafigi keltirilgan.

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