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TOSHKENT DAVLAT PEDAGOGIKA UNIVERSITETI

Matematik tahlil kafedrasi

M A T E M A T I K T A H L I L

MUSTAQIL TA'LIM UCHUN METODIK KO'RSATMALAR

I QISM

Tuzuvchilar:
Islomov B., Turgunbayev R.M.

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ANNOTATSIYA

Ushbu metodik ko`rsatma pedagogika universiteti «Matematika va informatika» bakalavriat yo`nalishi bo`yicha «Matematik tahlil» fanidan tahsil olayotgan I kurs talabalariga rejalashtirilgan mustaqil ta'lim uchun tuzilgan bo`lib, haqiqiy sonlar to`plami; funksiya va uning berilish usullari; sonli ketma-ketlik va uning limiti; funksiyaning limiti, uzluksizligi; kesmada uzluksiz bo`lgan funksiyaning xossalari; hosila, uning geometrik va mexanik ma'nolari; differensial va differensiallanuvchanlik; yuqori tartibli hosila va differensiallar; differensial hisobning asosiy teoremalari; funksiyani to`la tekshirish va grafigini chizishga oid mavzularni mustaqil o'rganish uchun savollar va individual vazifalardan iborat.

Tuzuvchilar: fizika-matematika fanlari doktori, professor v/b
Islomov B.

fizika-matematika fanlari nomzodi, dotsent v/b
Turgunbayev R.

Taqrizchilar: fizika-matematika fanlari nomzodi, professor
Toshmetov O`.
fizika-matematika fanlari nomzodi, dotsent
Sharifova T.
fizika-matematika fanlari nomzodi, dotsent
Жабборов H.

Metodik ko`rsatma Nizomiy nomidagi Toshkent Davlat Pedagogika Universiteti kengashida ko`rib chiqilgan va nashrga tavsiya qilingan.

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K I R I S H

Talaba mustaqil ishining asosiy maqsadlaridan biri o`qituvchining rahbarligi va nazorati ostida talabada muayyan o`quv ishlarini mustaqil ravishda bajarish uchun zarur bo`lgan bilim va ko`nikmalarni shakllantirish va rivojlantirishdan iborat. Ushbu metodik ko`rsatma bakalavriatning «matematika va informatika» yo`nalishida «Matematik tahlil» fanidan tahsil olayotgan I kurs talabalari uchun mo`ljallangan bo`lib, bunda har bir mavzu bo`yicha ma’ruzada qaralmaydigan, mustaqil o`rganishga rejalashtirilgan ma’lumotlar (ta’rif, teorema, masalalar)ni mustaqil o`rganish uchun yordamchi savollar, mustaqil isbot qilish uchun tasdiqlar hamda masala-misollar berilgan, adabiyotlar ko`rsatilgan.

Ma’ruza mashg`ulotlarida, darslik yoki o`quv qo`llanmalarda biror teorema isbotlanib, aynan shunga o`xshash teorema isbotsiz keltiriladi, talabaga taklif qilinadi. Matematik tahlilni o`rganish, xususan matematik tasdiqni isbotlash ko`nikmasini shakllantirish uchun isbotlanmagan teoremani avvalgi teorema isbotidan o`rganib isbotlash muhimdir. Shu sababli mustaqil ta’limga rejalashtirilgan topshiriqlar uchun isbotsiz keltirilgan teoremalarni isbotlash taklif qilingan. Ma’ruzada ko`pgina talabalar keltirilgan teorema shartlarining teorema xulosasi uchun etarli yoki zaruriy shartlar ekanligiga kam e’tibor beradi. Shularni e’tiborga olgan holda mustaqil ta’lim uchun berilgan topshiriqlarga nazariy masalalar ham kiritildi. Bunday topshiriqlarni bajarish natijasida talabalarda mantiqiy tafakkuri rivojlanib boradi.

Mustaqil ta’lim uchun individual vazifalar berilgan. Bu vazifalar amaliy xarakterga ega. Talabadan vazifani hal etishda nazariy bilimlarni, qoidalarni anglagan holda ishlatish talab qilinadi.

Mustaqil ishni talaba alohida daftarda bajarib boradi, kerakli konsultatsiyalarni oladi va bajarilgan ishni belgilagan vaqtda himoya qiladi.

I SEMESTR. MATEMATIK ANALIZGA KIRISH

1-mavzu. Matematik tahlil predmeti. Ratsional sonlar to`plami va uning xossalari. Ratsional sonlar to`plamining kesimlari. Irratsional son tushunchasi. (2 soat)

1. Ratsional sonlar to`plamida arifmetik amallarning qanday xossalari mavjud?
2. Arximed aksiomasi va uning ma'nosi nimadan iborat?
3. Ratsional sonlar to`plamida chegaralangan va chegaralanmagan to`plamlar qanday ta'riflanadi? Ularga misollar keltiring.
4. Ratsional sonlarni geometrik tasvirlash deganda nimani tushinasiz va u qanday amalga oshiriladi?
5. Ratsional va irratsional kesimlarni ta'riflang, misollarda tushintiring.
6. n, $\sqrt{n}$, $\sqrt[3]{n}$ (bu erda n gurux jurnalidagi talaba familiyasining tartib nomeri) ni aniqlaydigan kesimlarning tavsiflang.

Adabiyot: [1]. 21-32b.;

2-mavzu. Haqiqiy sonlar to`plamining xossalari (zichlik, tartiblanganlik, uzluksizlik). Haqiqiy sonlarni sonlar o`qida tasvirlash. (2 soat)

1. Quyidagi tasdiqni isbotlang: Agar $x<y$ va $y<z$ bo`lsa, u holda $x<z$ bo`ladi.
2. Haqiqiy sonlar to`plamida qo`shish amali qanday aniqlanadi?
3. Haqiqiy sonning absolyut qiymati qanday aniqlanadi va uning qanday xossalari mavjud?
4. $|x|$, $|x-y|$ ning geometrik ma'nosi nimadan iborat?
5. Quyidagi tasdiqlarni isbotlang: 1) $||x|-|y||\le|x-y|$; 2) $|x-y|\le|x|+|y|$;
   3) $|x_1+x_2+...x_n|\le|x_1|+|x_2|+...+|x_n|$ (Matematik induksiya metodi yordamida)
6. Irratsional sonni taqribiy hisoblash qanday amalga oshiriladi?
7. Haqiqiy sonni cheksiz o`nli kasr ko`rinishda ifodalanishini tushintiring.
8. Cheksiz davriy o`nli kasrning ratsional son ekanligi qanday isbotlanadi?
9. Cheksiz davriy bo`lmagan o`nli kasrning irratsional son ekanligi qanday isbotlanadi?
10. To`g`ri chiziqning uzluksizlik xossasi nimadan iborat?
11. Haqiqiy sonlar to`plami bilan to`g`ri chiziq nuqtalari to`plami orasida o`zaro bir qiymatli moslik qanday o`rnatiladi?

Adabiyot: [1]. 32-35; 52-61; [2], 4-10b.;

3-mavzu. Chegaralangan sonli to`plamlar. Quyidan va yuqoridan chegaralangan to`plamlar, ularning chegaralari. Chegaralarning mavjudligi haqidagi teorema. (2 soat)

1. Faqat quyidan chegaralangan, faqat yuqoridan chegaralangan va chegaralangan to`plamlarga misollar keltiring. Misollarni asoslang.
2. Quyidan chegaralanmagan, yuqoridan chegaralanmagan, chegaralanmagan to`plamlar qanday ta'riflanadi?
3. Aniq yuqori, aniq quyi chegaralari o`ziga tegishli, o`ziga tegishli bo`lmagan to`plamlarga misollar keltiring.
4. Quyidagi tasdiqni isbotlang: Har qanday quyidan chegaralangan to`plam uchun uning quyi chegaralari orasida eng kattasi mavjud.
5. Aniq yuqori va aniq quyi chegaralarning qanday xossalari bor?
6. Quyidagi tasdiqni isbotlang: Agar E to`plam quyidan chegaralangan bo`lib, $E_1\subset E$ bo`lsa, u holda $E_1$ to`plam ham quyidan chegaralangan va $infE_1\ge infE$ bo`ladi.

2) Agar E to`plam quyidan chegaralangan va $b=infE$ bo`lsa, u holda $\forall\varepsilon>0$ uchun shunday $x'\in E$ mavjudki, $x'<b+\varepsilon$ bo`ladi.

Adabiyot: [1]. 38-41; [2], 10-15b.;

4-mavzu. Funksiyaning ta'rifi va berilish usullari. Funksiyalar ustida arifmetik amallar. Funksiyalarning kompozitsiyasi. (2 soat)

1. 1-vazifa.

2. Qanday funksiyalar aynan teng deyiladi? Aynan teng funksiyalarga misollar keltiring.
3. Agar D_1 to`plam $f(x)$ funksiyaning $D_2$ to`plam $g(x)$ funksiyaning aniqlanish sohalari bo`lsa, u holda $f+g/f$, $f\cdot g+g^2$, $f/g + g/f$, $1/(fg)$; $f/(f-g)$ funksiyalarning aniqlanish sohalarini tavsiflang.
4. 2-vazifa

Adabiyot: [1]. 109-112; [2], 17-19b.

5-mavzu. Monoton funksiyalar. Teskari funksiya. (2 soat)

1. Agar $f(x)$ va $g(x)$ funksiyalar D to`plamda berilgan hamda o`suvchi bo`lsa, u holda $f(x) + g(x)$ o`suvchi ekanligini isbotlang.
2. Agar $f(x)$ va $g(x)$ funksiyalar D to`plamda nomanfiy o`suvchi (kamayuvchi) funksiyalar bo`lsa, u holda $f(x)\cdot g(x)$ o`suvchi (kamayuvchi) ekanligini isbotlang.
3. Agar $f(x)$ va $g(x)$ funksiyalar D to`plamda manfiy o`suvchi (kamayuvchi) funksiyalar bo`lsa, u holda $f(x)\cdot g(x)$ kamayuvchi (o`suvchi) ekanligini isbotlang.
4. Agar $f(x)$ funksiya D to`plamda o`suvchi va $f(x)>0$ bo`lsa, u holda $1/f(x)$ kamayuvchi ekanligini isbotlang.
5. Agar $f(x)$ funksiya D to`plamda o`suvchi va c o`zgarmas son bo`lsa, u holda $c\cdot f(x)$ funksiyaning monotonligi haqida nima deyish mumkin? Javobingizni asoslang.
6. Agar $f(x)$ va $g(x)$ funksiyalar D to`plamda berilgan hamda o`suvchi bo`lsa, u holda $f(x) - g(x)$ funksiyaning monotonligi haqida nima deyish mumkin? Javobingizni asoslang.
7. Teskari funksiya mavjud bo`lishining etarli sharti nimadan iborat?
8. Monoton bo`lmagan, lekin teskari funksiyasi mavjud bo`lgan funksiyaga misol keltiring.
9. Funksiya va unga teskari funksiya grafiklari $y=x$ to`g`ri chiziqqa nisbatan simmetrik bo`lishini isbotlang.
10. 3-vazifa.

Adabiyot: [1]. 119-121; [2], 23-35b.;

6-mavzu. Juft, toq, chegaralangan funksiyalar. Davriy funksiyalar. (2 soat)

1. Agar $f(x)$ va $g(x)$ X to`plamda aniqlangan juft funksiyalar bo`lsa, u holda $f(x)+g(x)$, $f(x)-g(x)$, $f(x)\cdot g(x)$, $f(x)/g(x)$, $g(x)\neq 0$ funksiyalar ham juft funksiya ekanligini ko`rsating.
2. Agar $f(x)$ va $g(x)$ X to`plamda aniqlangan toq funksiyalar bo`lsa, u holda $f(x)+g(x)$, $f(x)-g(x)$ funksiyalar toq, $f(x)\cdot g(x)$, $f(x)/g(x)$, $g(x)\neq 0$ funksiyalar juft funksiya ekanligini ko`rsating.
3. Quyidagi tasdiqni isbotlang: Koordinatalar boshiga nisbatan simmetrik bo`lgan $X$ to`plamda aniqlangan har qanday funksiyani toq va juft funksiyalarning yig`indisi ko`rinishda ifodalash mumkin.
4. Agar $f(x)$ va $g(x)$ X to`plamda aniqlangan chegaralangan funksiyalar bo`lsa, u holda $f(x)+g(x)$, $f(x)-g(x)$, $f(x)\cdot g(x)$, $|f(x)|$ funksiyalar ham chegaralangan funksiya ekanligini ko`rsating.
5. Quyidan chegaralanmagan, yuqoridan chegaralanmagan, chegaralanmagan funksiyalarni ta'riflang.
6. 4-vazifa.
7. Davriy funksiyaning aniqlanish sohasi haqida nima deyish mumkin?
8. Ikkita davriy funksiyalarning yig`indisi haqida nima deyish mumkin?
9. Davriy funksiya qat'iy monoton bo`lishi mumkinmi? Javobingizni asoslang.
10. Trigonometrik funksiyalardan farqli bo`lgan davriy funksiyalarga misol keltiring.

Adabiyot: [1]. 112-119; [2], 23-35b.;

7-mavzu. Sonli ketma-ketlik. Ketma-ketlikning limiti. Yaqinlashuvchi ketma-ketliklarning xossalari. Oraliq o`zgaruvchining limiti. (3 soat)

1. 5-vazifa;
2. Ketma-ketlik limitiga berilgan ta'rifda $|a_n-a|<\varepsilon$ tengsizlik o`rniga $|a_n-a|\leq\varepsilon$ tengsizlikni ishlatish mumkinmi, javobingizni asoslang.
3. $\lim\limits_{n\to\infty} a_n \neq a$ ni geometrik ma'nosi nimadan iborat?

4. «Ketma-ketlikning chekli limiti mavjud emas» degan jumlaning geometrik ma'nosi nimadan iborat?
5. Qo`yidagi jumlani isbotlang: Agar ketma-ketlik yaqinlashuvchi bo`lsa, u holda undan chekli sondagi hadlarini tashlab yuborishdan hosil bo`lgan ketma-ketlik yaqinlashuvchi bo`ladi.
6. Qo`yidagi jumla to`g`rimi «agar ketma-ketlikning faqat chekli sondagi hadlarigina manfiy bo`lsa, u holda uning limiti musbat bo`ladi» ?
7. Agar $\forall n \in N$ uchun $0 \le x_n \le y_n$ va $\lim\limits_{n\to\infty} y_n=0$ bo`lsa, u holda $\lim\limits_{n\to\infty} x_n=0$ ekanligini isbotlang.
8. Tenglik va tengsizliklarda limitga o`tish haqidagi teoremalar ([1], 79-80 b. $1^0$, $2^0$, $3^0$ xossalar) «$\forall n \in N$» o`rniga «biror nomerdan boshlab (ya'ni $\exists n_0$, $\forall n > n_0$)» ishlatsak ham o`rinli ekanligini ko`rsating.

Adabiyot: [1], 64-74 b.; [2], 39-42b.;

8-mavzu. Cheksiz kichik ketma-ketliklar va ularning xossalari. Cheksiz katta ketma-ketliklar. Yig`indi, ko`paytma va bo`linmaning limiti. (2 soat)

1. $\lim\limits_{n\to\infty} a_n = +\infty$ va $\lim\limits_{n\to\infty} a_n = -\infty$ larni ta'riflang.
2. Cheksiz katta ketma-ketlik chegaralanmagan bo`lishini ko`rsating, chegaralanmagan, lekin cheksiz katta bo`lmagan ketma-ketlikka misol ko`rsating.
3. Aniqmaslik deganda nimani tushunasiz?
4. Aniqmaslik turlarga misollar keltiring.
5. 6-9 vazifalar.

Adabiyot: [1], 70-71, 74-78, 81-85 b. [2], 43-47b.;

9-mavzu. Monoton ketma-ketlik, e soni. Ichma-ich joylashgan segmentlar prinsipi. (2 soat)

1. Chegaralangan, yuqoridan chegaralangan, quyidan chegaralangan, chegaralanmagan ketma-ketliklarga ta'rif bering. Misollar keltiring.
2. Quyidan chegaralangan kamayuvchi ketma-ketlik limiti mavjudligi haqidagi teoremani isbotlang.
3. Ichma-ich joylashgan segmentlar prinsipi haqidagi teoremada segmentni interval, yariminterval bilan almashtirib bo`lmasligini misollarda ko`rsating.
4. Agar ketma-ketlik biror hadidan boshlab o`suvchi va yuqoridan chegaralangan bo`lsa, u holda bu ketma-ketlik yaqinlashuvchi bo`ladimi?
5. 10-vazifa.

Adabiyot: [1], 86-95 b.; [2], 51-54b.;

10-mavzu. Qism ketma-ketlik. Bolsano-Veyershtrass teoremasi. Koshi kriteriyasi. (1 soat)

1. Qism ketma-ketlikka berilgan ta'rifdagi shartlarni sanang. Qism ketma-ketlikka misollar keltiring.
2. Biror qism ketma-ketligi yaqinlashuvchi, lekin o`zi yaqinlashuvchi bo`lmagan ketma-ketlikka misol keltiring.
3. Barcha qism ketma-ketliklari yaqinlashuvchi bo`lgan ketma-ketlikka misol keltiring.
4. Agar ketma-ketlikning biror qism ketma-ketligi uzoqlashuvchi bo`lsa, u holda bu ketma-ketlik chegaralanmagan bo`ladi. Jumla to`g`rimi?
5. Bolsano-Veyershtrass teoremasi isbotida chegaralangan ketma-ketlikni o`z ichida saqlaydigan segmentning mavjudligi qanday asoslanadi?
6. «Ketma-ketlik fundamental emas» degan iborani qanday tushuntirish mumkin?
7. 11-vazifa.

Adabiyot: [1], 98-103 b.; [2], 56-58b.;

11-mavzu. Funksiyaning nuqtadagi limiti. Limitga ega bo`lgan funksiyalarning sodda xossalari. Limitning yagonaligi. Cheksiz kichik funksiyalar va ularning xossalari. (2 soat)

1. To`plamning limit nuqtasi xossalarini sanang.
2. Chekli nuqta, ∞, +∞, -∞ «nuqta» atroflarini tengsizliklar yordamida bering, ularni sonlar o`qida tasvirlang.
3. (*a,b*) intervalning har bir nuqtasi va uchlari shu to`plamning limit nuqtasi bo`lishini ko`rsating.
3. Funksiyaning nuqtadagi limitiga berilgan Geyne va Koshi ta'riflarining ekvivalentligini isbotlang.
4. Funksiyaning +∞ «nuqta»dagi limitiga Geyne va Koshi ta'riflarini bering. Ularning ekvivalentligini qanday isbotlash mumkin?
5. $\lim\limits_{x\to a} f(x)=\infty$, $\lim\limits_{x\to a} f(x)=-\infty$, $\lim\limits_{x\to a} f(x)=+\infty$ larni ta'riflang. Ularning geometrik ma'nolari haqida nima deyish mumkin?
6. $\lim\limits_{x\to\infty} f(x)=a$, $\lim\limits_{x\to-\infty} f(x)=a$, $\lim\limits_{x\to+\infty} f(x)=a$ larni ta'riflang. Ularning geometrik ma'nolari haqida nima deyish mumkin?
7. $\lim\limits_{x\to\infty} f(x)=\infty$, $\lim\limits_{x\to-\infty} f(x)=\infty$, $\lim\limits_{x\to+\infty} f(x)=\infty$ larni ta'riflang.
8. Aniqmasliklarning har bir turiga misollar keltiring.
9. II.1-vazifa.

Adabiyot: [1], 127-133, 136-138, 145 b.; [2], 59-65b.;

12-mavzu. Ikki funksiya yig`indisi, ko`paytmasi va bo`linmasining limiti. Monoton funksiya limitining mavjudligi. Funksiyaning chekli limitga ega bo`lish sharti. (3 soat)

1. II. 2-, 3- vazifalar.
2. Monoton funksiyaning limiti haqidagi teoremada $\forall x\in X$ uchun $x\le a$ $(x\ge a)$ shart talab qilinadi. Bu shartni qanday tushuntirasiz?
3. Agar *f(x)* funksiya (*a,b*) intervalda aniqlangan, kamayuvchi va quyidan chegaralanmagan bo`lsa, u holda $\lim\limits_{x\to a} f(x)=-\infty$ bo`lishini isbotlang.
4. Funksiyaning chekli limitga ega bo`lishining zaruriy va etarli (Koshi alomati) shartini isbotlang.
5. Koshi shartining geometrik ma'nosi nimadan iborat?

Adabiyot: [1], 127-133 b.; [2], 65-68, 72-73b.;

13-mavzu. Bir tomonli limitlar. Funksiyalar kompozitsiyasining limiti. Ba'zi bir ajoyib limitlar. (2 soat)

1. Nuqtaning bir tomonli atrofini tengsizliklar yordamida yozing.
2. II. 4-6-vazifalar.
3. Funksiyaning a nuqtadagi chap (o`ng) limitiga Geyne ta'rifini bering.
4. $\lim\limits_{x\to a-0} f(x)=\infty$, $\lim\limits_{x\to a+0} f(x)=-\infty$, $\lim\limits_{x\to a-0} f(x)=+\infty$ larni ta'riflang. Ularning geometrik ma'nolari haqida nima deyish mumkin?

Adabiyot: [1], 132-133, 134-136, 139-140, 162-163 b.; [2], 66-71b.;

14-mavzu. Uzluksiz funksiyalar. Funksiyaning nuqtadagi uzluksizligi. Bir tomonli uzluksizlik. Monoton funksiyaning uzluksizligi va uzilish nuqtalari. (2 soat)

1. Agar *f(x)* funksiya x_0 nuqtada uzluksiz bo`lsa, u holda funksiya $x_0$ nuqtaning biror atrofida chegaralangan bo`lishini isbotlang.
2. Agar *f(x)* funksiya x_0 nuqtada uzluksiz va $f(x_0)>p$ ($f(x_0)<q$) bo`lsa, u holda $x_0$ nuqtaning biror atrofidagi barcha nuqtalarda $f(x)>p$ ($f(x)<q$) bo`lishini isbotlang.

3. Agar $f(x)$ funksiya x_0 nuqtada uzluksiz va $f(x_0)=0$ bo`lsa, u holda x_0 nuqtaning etarlicha kichik atrofida funksiya qiymatlari haqida nima deyish mumkin? Misollarda tushintiring. (Grafiklardan foydalanish mumkin)
4. III.3-vazifa

Adabiyot: [1], 151-159 b.; [2], 78-80, 82-83, 88-89b.;

15-mavzu. Ikkita uzluksiz funksiya yig`indisi, ko`paytmasi va bo`linmasining uzluksizligi. Funksiyalar kompozitsiyasining uzluksizligi. (2 soat)

1. III. 1-,2-vazifalar.
2. Ikkita funksiya yig`indisi, ayirmasi, ko`paytmasi x_0 nuqtada uzluksiz bo`lsa, funksiyalarning o`zi haqida nima deyish mumkin? Misollar keltiring.
3. Ikkita $f(x)$ va $g(x)$ funksiya bo`linmasining x_0 nuqtadagi uzluksizligi haqidagi tasdiqda $g(x_0)\neq 0$ shart berilgan. Ushbu shartni $g(x)\neq 0$ shart bilan almashtirish mumkinmi? Javobingizni asoslang.
4. $f(x)$ va $g(x)$ funksiyalar x_0 nuqtada uzluksiz, $g(x_0)=0$ bo`lsa, $f(x)/g(x)$ funksiyaning x_0 nuqtadagi limiti haqida nima deyish mumkin? Javobingizni asoslang (misollar orqali tushintiring)
5. Funksiyalar kompozitsiyasining uzluksizligi qanday isbotlanadi?

Adabiyot: [1], 150-152 b.; [2], 86-87b.;

16-mavzu. Kesmada uzluksiz bo`lgan funksiyalarning chegaralanganligi, eng kichik va eng katta qiymatlari. Uzluksiz funksiyalarning oraliq qiymatlari. (2 soat)

1. Qanday funksiya kesmada uzluksiz deyiladi?
2. Kesmada uzluksiz bo`lmagan, lekin chegaralangan funksiyaga misol keltiring.
3. Agar $\forall x\in[a,b]$ da $f(x)<c$ bo`lsa, u holda $\varphi(x)=1/(c-f(x))$ funksiyaning $[a,b]$ da uzluksiz ekanligini asoslang.
4. Kesmada chegaralangan uzluksiz funksiya o`zining aniq quyi chegarasiga erishishini isbotlang.
5. III. 4-vazifa.
6. Agar $f(x)$ funksiya (a,b) intervalda aniqlangan, uzluksiz va $\lim\limits_{x\to a+0} f(x)=-\infty$, $\lim\limits_{x\to b-0} f(x)=+\infty$ bo`lsa, u holda $(a,b)$ intervaldagi kamida bir nuqtada funksiya nol qiymat qabul qilishini isbotlang. Shunday xossaga ega bo`lgan funksiyaga misol keltiring.
7. Tengsizliklarni yechishning intervallar usulini asoslang.

Adabiyot: [1], 164-169 b.; [2], 84-88b.;

17-mavzu. Teskari funksiyaning mavjudligi va uzluksizligi. Tekis uzluksizlik. Kantor teoremasi. (1 soat)

1. III. 5-vazifa.
2. Teskari funksiyaning mavjudligi va uzluksizligi haqidagi teoremani kamayuvchi funksiya uchun isbotlang.
3. Kantor teoremasi isbotida funksiya qaralayotgan to`plamning kesma ekanligidan qaerda foydalaniladi?
4. Funksiya intervalda (yarimintervalda) tekis uzluksiz bo`lishi mumkinmi?
5. Agar $f(x)$ funksiya $[a,b]$ kesmada tekis uzluksiz bo`lsa, u holda bu funksiya $(a,b)$ intervalda tekis uzluksiz bo`ladimi? Javobingizni asoslang.
6. Funksiyaning to`plamdagi tebranishi qanday aniqlanadi?

Adabiyot: [1], 169-173 b.; [2], 89-92b.;

18-mavzu. Asosiy elementar funksiyalar. Haqiqiy ko`rsatkichli daraja. Ko`rsatkichli, logarifmik va darajali funksiya. (2 soat)

1. Ko`rsatkichli funksiyaning chegaralanmaganligini isbotlang.
2. Ko`rsatkichli funksiyaning davriy emasligini qanday ko`rsatish mumkin?

3. Logarifmik funksiyaning chegaralanmaganligini isbotlang.
4. Logarifmik funksiyaning davriy emasligini qanday ko`rsatish mumkin?
5. Logarifmik funksiyaning uzluksizligi qanday isbotlanadi?
6. Darajali funksiyaning uzluksizligi qanday isbotlanadi?
7. Giperbolik funksiyalarning uzluksizligini asoslang.

Adabiyot: [1], 121-125 b.; [2], 92-95, 98b.;

19-mavzu. Trigonometrik va teskari trigonometrik funksiyalar. (2 soat)

1. *cosx, tgx, ctgx* funksiyalarning o`z aniqlanish sohasida uzluksiz ekanligini isbotlang.
2. *tgx, ctgx* funksiyalarning chegaralanmaganligini ko`rsating.
3. Chegaralanmagan funksiyaga teskari funksiyaning chegaralangan, chegaralanmaganligi haqida nima deyish mumkin?
4. Toq (juft) funksiyaga teskari funksiyaning toq-juftligi haqida nima deyish mumkin?
5. Teskari trigonometrik funksiyalarning uzluksizligi qanday isbotlanadi?

Adabiyot: [1], 114-117 b.; [2], 99-101b.;

INDIVIDUAL VAZIFALAR

I. Funksiya, ketma-ketlik limiti
1-vazifa

1. Radiusi R ga teng bo`lgan yarim aylanaga to`g`ri to`rtburchak ichki chizilgan. Shu to`g`ri to`rtburchakning perimetrini bir tomonining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
2. Silindrning sirti S ga teng. Silindrning hajmini uning asosining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
3. Silindrning hajmi V ga teng. Silindr sirtini asosining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
4. Perimetri 2p ga teng bo`lganteng yonli uchburchak asosi atrofida aylantirishdan hosil bo`lgan jism hajmini uchburchak asosining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
5. Teng yonli uchburchakni balandligi atrofida aylantirishdan konus hosil bo`lgan. Teng yonli uchburchakning perimetri 2p ga teng. Konus hajmini teng yonli uchburchak asosining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
6. Radiusi R bo`lgan yarimsharga konus shunday tashqi chizilgan-ki, ular asoslarining markazlari ustma-ust tushadi. Konus yon sirtini konus balandligi funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
7. Radiusi R bo`lgan sharga konus ichki chizilgan. Konus yon sirtini konus balandligi funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
8. Yarim o`qlari a va b bo`lgan ellipsga to`g`ri to`rtburchak ichki chizilgan. To`g`ri to`rtburchak yuzini uning bir tomonining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
9. Markaziy burchagi 90^0 ga teng, radiusi R bo`lgan doiraviy sektorga to`g`ri to`rtburchak shunday chizilgan-ki, uning bir uchi doiraning markazi bilan ustma-ust tushadi, qarama-qarshi uchi esa aylanada yotadi. To`g`ri to`rtburchak yuzini uning bir tomonining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
10. Markaziy burchagi 90^0 ga teng, radiusi R bo`lgan doiraviy sektorga to`g`ri to`rtburchak shunday chizilgan-ki, uning bir uchi doiraning markazi bilan ustma-ust tushadi, qarama-qarshi uchi esa aylanada yotadi. To`g`ri to`rtburchak perimetrini uning bir tomonining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
11. M(4;1) nuqtadan to`g`ri chiziq shunday o`tkazilgan-ki, u birinchi kvadrantdan to`g`ri burchakli uchburchak ajratadi. Shu to`g`ri burchakli uchburchakning yuzini to`g`ri burchakli uchburchak katetlaridan birining funksiyasi sifatida ifodalang. Hosil

bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.

12. Asosi kvadratdan iborat qopqoqsiz yashik berilgan. Uning hajmi V ga teng bo`lsa, yashik sirtini asosi tomonining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
13. M(1;3) nuqtadan to`g`ri chiziq shunday o`tkazilgan-ki, u birinchi kvadrantdan to`g`ri burchakli uchburchak ajratadi. Shu to`g`ri burchakli uchburchak gipotenuzasini to`g`ri burchakli uchburchak katetlaridan birining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
14. M(1;4) nuqtadan to`g`ri chiziq shunday o`tkazilgan-ki, u birinchi kvadrantdan to`g`ri burchakli uchburchak ajratadi. Shu to`g`ri burchakli uchburchakning katetlari uzunliklari yig`indisini katetlardan birining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
15. Silindr yuqoridan yarim shar bilan (yopilgan) tugallangan. Bu jismning hajmi V ga teng. Jism to`la sirtini asosini radiusining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
16. Asosining diametri d, balandligi h bo`lgan konusga silindr ichki chizilgan. Silindrning hajmini silindr balandligining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
17. Radiusi R ga teng bo`lgan yarimsharga asosi kvadratdan iborat bo`lgan to`g`ri parallelepiped ichki chizilgan. Parallelepiped hajmini uning balandligi funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
18. Radiusi R ga teng bo`lgan yarimsharga asosi kvadratdan iborat bo`lgan to`g`ri parallelepiped ichki chizilgan. Parallelepiped hajmini uning asosi uzunligining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
19. Radiusi R ga teng bo`lgan yarimsharga asosi teng tomonli uchburchakdan iborat bo`lgan to`g`ri parallelepiped ichki chizilgan. Parallelepiped hajmini uning asosi uzunligining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
20. Radiusi R ga teng bo`lgan yarimsharga asosi teng tomonli uchburchakdan iborat bo`lgan to`g`ri parallelepiped ichki chizilgan. Parallelepiped hajmini uning balandligi funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
21. Radiusi R ga teng bo`lgan yarimsharga asosi teng tomonli uchburchakdan iborat bo`lgan to`g`ri parallelepiped ichki chizilgan. Parallelepiped to`la sirtini uning balandligi funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
22. Radiusi R ga teng bo`lgan yarimsharga asosi kvadratdan iborat bo`lgan to`g`ri parallelepiped ichki chizilgan. Parallelepiped to`la sirtini uning asosi uzunligining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping,

uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.

23. Radiusi R ga teng bo`lgan sharga silindr ichki chizilgan. Silindr hajmini balandlik funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
24. Hajmi V ga teng bo`lgan konus yon sirtini konus balandligining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
25. Yasovchisi 30 sm ga teng bo`lgan konik voronka berilgan. Voronkaning hajmini balandlik funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
26. Radiusi R ga teng bo`lgan sharga konus ichki chizilgan. Konus hajmini balandlik funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
27. Balandligi h, asosining radiusi r bo`lgan silindrga konus tashqi chizilgan (silindr va konus asoslari bir tekislikda yotadi). Konus hajmini balandligining funksiyasi Radiusi R ga teng bo`lgan sharga silindr ichki chizilgan. Silindr hajmini balandlik funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
28. Radiusi R ga teng bo`lgan sharga to`g`ri doiraviy konus tashqi chizilgan. Konus hajmini balandligining funksiyasi Radiusi R ga teng bo`lgan sharga silindr ichki chizilgan. Silindr hajmini balandlik funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
29. Balandligi H, asosining radiusi R bo`lgan konusga silindr ichki chizilgan. Silindr hajmini silindr asosi radiusining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.
30. Hajmi V ga teng bo`lgan konus yon sirtini asosi radiusining funksiyasi sifatida ifodalang. Hosil bo`lgan funksiyaning aniqlanish sohasini toping, uni funksiya analitik ifodasining aniqlanish sohasi bilan solishtiring. Funksiya grafigi eskizini chizing.

2-vazifa. f(x) va g(x) funksivalarning aniqlanish sohalari va qiymatlar to'plamlarini toping. f(g(x) va g(f(x)) funksivalarning aniqlanish sohalarini toping.

1. $f(x)=x^2$; $g(x)=\sqrt{x+2}$;
2. $f(x)=\frac{1}{x}$; $g(x)=\cos x$;
3. $f(x)=tgx$; $g(x)=\sin x$;
4. $f(x)=\cos x$; $g(x)=\sqrt{x}$
5. $f(x)=\arcsin x$; $g(x)=1-3x$;
6. $f(x)=\log_2 x$; $g(x)=2x-1$;
7. $f(x)=e^x$; $g(x)=\frac{1}{x}$;
8. $f(x)=\sqrt{x}+2$; $g(x)=x^2+1$;
9. $f(x)=\arccos x$; $g(x)=2x-1$;
10. $f(x)=\sqrt{x}$; $g(x)=x^2-4$;
11. $f(x)=\sqrt{x}$; $g(x)=x^3+1$;
12. $f(x)=\sqrt{x}$; $g(x)=\lg x$;

13. f(x)=lnx; g(x)= $\frac{1}{x}$;

14. f(x)= $\sqrt[4]{x}$; g(x)=cosx;

15. f(x)= $\sqrt[3]{x}$; g(x)=tgx;

16. f(x)=lgx; g(x)=x^2+x;

17. f(x)=tgx; g(x)= $\sqrt{x+2}$;

18. f(x)=e^x; g(x)=x^2-5;

19. f(x)= $\log_3 x$; g(x)=cosx;

20. f(x)=arcsinx; g(x)=x^2;

21. f(x)=cosx; g(x)= $\frac{1}{x}$;

22. f(x)=tgx; g(x)=5x-4;

23. f(x)= $\frac{1}{x}$; g(x)=arccosx;

24. f(x)=x^2; g(x)=ctgx;

25. f(x)=arccosx; g(x)=sinx;

26. f(x)=lnx; g(x)= $\sqrt[3]{x}$.

27. f(x)= $\sqrt[6]{x}$; g(x)=x^3-x;

28. f(x)= $\sqrt[5]{x}$; g(x)=3^x;

29. f(x)=lgx; g(x)=tgx;

30. f(x)=tgx; g(x)= $\sqrt{x}$;

3- **vazifa. Berilgan y=f(x), x∈X funksiyaga teskari funksiyaning mavjudligini asoslang va teskari funksiyani toping.**

1. f(x)=cosx, x∈[-π;0];
2. f(x)=x^2+2x, x∈(-1;1);
3. f(x)= $\frac{1}{x}$ +x, x∈(0;1);
4. f(x)=sinx, x∈(0,5π;π);
5. f(x)=sin(x+π/4), x∈(-π/4;π/4);
6. f(x)=cosx, x∈[4;5]
7. f(x)=x^2+2x, (-∞;-1);
8. f(x)= $\frac{1}{x}$ +x, x∈(1;+∞)
9. f(x)=sinx, x∈[3;4]
10. f(x)= $\sqrt{x}$, x∈[1;4];
11. f(x)=- $\sqrt{x}$, x∈[4;16];
12. f(x)= $\sqrt{1-x}$, x∈[0;1];
13. f(x)=1- $\sqrt{x}$, x∈[0;+∞)
14. f(x)=x^2+2x, x∈[1;+∞);
15. f(x)= $\frac{1}{x}$ +x, x∈(-∞;-1]
16. f(x)= $\sqrt[3]{1-x^3}$, x∈(-∞;+∞);
17. f(x)= $(1-x)^3$; x∈(0;+∞);
18. f(x)= $(1-x)^2$; x∈(1;+∞);
19. f(x)=2-|x|, x∈(-4;0);
20. f(x)=x^2-2x-3, x∈(-8;1];
21. f(x)=x^2+4x-5, x∈[-2;6];
22. f(x)= $\sqrt{1-x^2}$, x∈[-1;0];
23. f(x)=2|x-1|+4, x∈[-4;1];
24. f(x)=1- $\sqrt{1-x^2}$, x∈[-1;0];
25. f(x)=-cosx, x∈[4;5]
26. f(x)=-x^2+2x, (-∞;1);
27. f(x)= $\frac{1}{x}$ -4x, x∈(1;+∞)
28. f(x)=sinx, x∈[-1;1]
29. f(x)= $\sqrt{x^2-1}$, x∈[1;+∞);
30. f(x)= $\sqrt{x^3+1}$, x∈[-4;6];

4-vazifa. Quyidagi funksiyalarni chegaralanganlikka tekshiring.

1. f(x)=x^2-x;
2. f(x)=4x-x^2-6;
3. f(x)= $\frac{1}{x+2}$;
4. f(x)=$\log_2(2x+3)$;
5. f(x)=|x-6|;
6. f(x)=x^4-$2x^2$+3;
7. f(x)= $\sqrt{x-9}$;
8. f(x)=3sinx+4cosx;
9. f(x)=x+sinx;
10. f(x)=$2^{-|x|}$;
11. f(x)= 2|x-1|+4;
12. f(x)= $\sqrt{x^2-1}$;
13. f(x)= $\sqrt{x^3+1}$;
14. f(x)=4cos2x-sinx;
15. f(x)= 1- $\sqrt{x}$;

16. f(x)= $\frac{1}{x}$ +x;
17. f(x)= $\sqrt{1-x}$;
18. f(x)= $\sqrt[3]{x}$ +2;
19. f(x)=(1-x)^2;
20. f(x)= $\sqrt[6]{x}$;
21. f(x)= $\frac{4}{1+x^2}$;
22. f(x)=$\log_2(4+x^2)$;
23. f(x)=sin(x^2-1);
24. f(x)=x^2+x+1;
25. f(x)=7-3x;
26. f(x)= $\sqrt[3]{4+x^2}$;
27. f(x)=ctgx^3;
28. f(x)=tgx^2;
29. f(x)=cos3x-x;
30. f(x)= $\sqrt[3]{1-x^2}$.

5-vazifa.

a) ketma-ketlikning birnechta hadini yozing; b) ketma-ketlik limiti ta'rifdan foydalanib $\lim\limits_{n\to\infty} a_n = a$ tenglikni isbotlang

1. $a_n = \frac{3n-2}{2n+1}$, $a = \frac{3}{2}$;
3. $a_n = \frac{4n-1}{3n+1}$, $a = \frac{4}{3}$;
5. $a_n = \frac{5-3n}{4n-1}$, $a = -\frac{3}{4}$;
7. $a_n = \frac{7n+12}{2n-1}$, $a = \frac{7}{2}$;
9. $a_n = \frac{4n^2-2}{3n^2+2}$, $a = \frac{4}{3}$;
11. $a_n = \frac{3n^3-2}{3-2n^3}$, $a = -\frac{3}{2}$;
13. $a_n = \frac{5n-7}{3n+11}$, $a = \frac{5}{3}$;
15. $a_n = \frac{3-2n^2}{2n^2+1}$, $a = -1$;
2. $a_n = \frac{3n}{6n+7}$, $a = \frac{1}{2}$;
4. $a_n = \frac{2n-12}{5n+10}$, $a = \frac{2}{5}$;
6. $a_n = \frac{3n-2}{n+1}$, $a = 3$;
8. $a_n = \frac{3-2n^2}{2n^2+13}$, $a = -1$;
10. $a_n = \frac{4-3n^2}{n^2+1}$, $a = -3$;
12. $a_n = \frac{31n-12}{21n+11}$, $a = \frac{31}{21}$;
14. $a_n = \frac{3n^3+7}{2n^3-5}$, $a = \frac{3}{2}$;
16. $a_n = \frac{4n-2}{2n+1}$, $a = 2$;
17. $a_n = \frac{3n-1}{4n+1}$, $a = \frac{3}{4}$;
19. $a_n = \frac{5+3n}{4n-11}$, $a = \frac{3}{4}$;
21. $a_n = \frac{2n+12}{7n-1}$, $a = \frac{2}{7}$;
23. $a_n = -\frac{3n^2-2}{4n^2+2}$, $a = -\frac{3}{4}$;
25. $a_n = \frac{2n^3-1}{3-3n^3}$, $a = -\frac{2}{3}$;
27. $a_n = \frac{4n-7}{3n+1}$, $a = \frac{4}{3}$;
29. $a_n = \frac{4-6n^2}{2n^2+1}$, $a = -3$;
18. $a_n = \frac{3n}{7n+5}$, $a = \frac{3}{7}$;
20. $a_n = \frac{5n-12}{5n+14}$, $a = 1$;
22. $a_n = \frac{n-2}{3n+1}$, $a = \frac{1}{3}$;
24. $a_n = \frac{3-4n^2}{2n^2+3}$, $a = -2$;
26. $a_n = \frac{5-3n^2}{n^2+7}$, $a = -3$;
28. $a_n = \frac{n-1}{2n+1}$, $a = \frac{1}{2}$;
30. $a_n = \frac{n^3+3}{2n^3-5}$, $a = \frac{1}{2}$.

6-vazifa. Ketma-ketlikning limitini toping

1. $\lim\limits_{n\to\infty}\dfrac{(3-n)^2+(3+n)^2}{(3-n)^2-(3+n)^2}$
2. $\lim\limits_{n\to\infty}\dfrac{(3-n)^4-(2-n)^4}{(1-n)^4-(1+n)^4}$
3. $\lim\limits_{n\to\infty}\dfrac{(3-n)^4-(2-n)^4}{(1-n)^3-(1+n)^3}$
4. $\lim\limits_{n\to\infty}\dfrac{(1-n)^4-(1+n)^4}{(1+n)^3-(1-n)^3}$
5. $\lim\limits_{n\to\infty}\dfrac{(6-n)^2-(6+n)^2}{(6+n)^2-(1-n)^2}$
6. $\lim\limits_{n\to\infty}\dfrac{(n+1)^3-(n+1)^2}{(n-1)^3-(n+1)^3}$
7. $\lim\limits_{n\to\infty}\dfrac{(1+2n)^3-8n^3}{(1+2n)^2+4n^2}$
8. $\lim\limits_{n\to\infty}\dfrac{(3-4n)^2}{(n-3)^3-(n+3)^3}$
9. $\lim\limits_{n\to\infty}\dfrac{(3-n)^3}{(n+1)^2-(n+1)^3}$
10. $\lim\limits_{n\to\infty}\dfrac{(n+1)^2+(n-1)^2-(n+2)^3}{(4-n)^3}$
11. $\lim\limits_{n\to\infty}\dfrac{2(n+1)^3-(n-2)^3}{n^2+2n-3}$
12. $\lim\limits_{n\to\infty}\dfrac{(n+1)^3+(n+2)^3}{(n+4)^3+(n+5)^3}$
13. $\lim\limits_{n\to\infty}\dfrac{(n+3)^3+(n+4)^3}{(n+3)^4-(n+4)^4}$
14. $\lim\limits_{n\to\infty}\dfrac{(n+1)^4-(n-1)^4}{(n+1)^3+(n-1)^3}$
15. $\lim\limits_{n\to\infty}\dfrac{8n^3-2n}{(n+1)^4-(n-1)^4}$
16. $\lim\limits_{n\to\infty}\dfrac{(n+6)^3-(n+1)^3}{(2n+3)^2+(n+4)^2}$
17. $\lim\limits_{n\to\infty}\dfrac{(2n-3)^3-(n+5)^3}{(3n-1)^3+(2n+3)^3}$
18. $\lim\limits_{n\to\infty}\dfrac{(n+10)^2+(3n+1)^2}{(n+6)^3-(n+1)^3}$
19. $\lim\limits_{n\to\infty}\dfrac{(2n+1)^3+(3n+2)^3}{(2n+3)^3-(n-7)^3}$
20. $\lim\limits_{n\to\infty}\dfrac{(n+7)^3-(n+2)^3}{(3n+2)^2+(4n+1)^2}$
21. $\lim\limits_{n\to\infty}\dfrac{(2n+1)^3-(2n+3)^3}{(2n+1)^2+(2n+3)^2}$
22. $\lim\limits_{n\to\infty}\dfrac{n^3-(n-1)^3}{(n+1)^4-n^4}$
23. $\lim\limits_{n\to\infty}\dfrac{(n+2)^4-(n-2)^4}{(n+5)^2+(n-5)^2}$
24. $\lim\limits_{n\to\infty}\dfrac{(n+1)^4-(n-1)^4}{(n+1)^3+(n-1)^3}$
25. $\lim\limits_{n\to\infty}\dfrac{(n+1)^3-(n-1)^3}{(n+1)^2-(n-1)^2}$
26. $\lim\limits_{n\to\infty}\dfrac{(n+1)^3-(n-1)^3}{(n+1)^2+(n-1)^2}$
27. $\lim\limits_{n\to\infty}\dfrac{(n+2)^3+(n-2)^3}{n^4+2n^2-1}$
28. $\lim\limits_{n\to\infty}\dfrac{(n+1)^3+(n-1)^3}{n^3-3n}$
29. $\lim\limits_{n\to\infty}\dfrac{(n+1)^3+(n-1)^3}{n^3+1}$
30. $\lim\limits_{n\to\infty}\dfrac{(n+2)^2-(n-2)^2}{(n+3)^2}$

7-vazifa. Ketma-ketlikning limitini toping

1. $\lim\limits_{n\to\infty} n\left(\sqrt{n^2+1}+\sqrt{n^2-1}\right)$

2. $\lim\limits_{n\to\infty} n\left(\sqrt{n(n-2)}-\sqrt{n^2-3}\right)$

3. $\lim\limits_{n\to\infty}\left(n-\sqrt[3]{n^3-5}\right)n\sqrt{n}$

4. $\lim\limits_{n\to\infty}\left[\sqrt{\left(n^2+1\right)\left(n^2-4\right)}-\sqrt{n^4-9}\right]$

5. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^5-8}-n\sqrt{n\left(n^2+5\right)}}{\sqrt{n}}$

6. $\lim\limits_{n\to\infty}\left(\sqrt{n^2-3n+2}-n\right)$

7. $\lim\limits_{n\to\infty}\left(n+\sqrt[3]{4-n^3}\right)$

8. $\lim\limits_{n\to\infty}\left[\sqrt{n(n+2)}-\sqrt{n^2-2n+3}\right]$

9. $\lim\limits_{n\to\infty}\left[\sqrt{(n+2)(n+1)}-\sqrt{(n-1)(n+3)}\right]$

10. $\lim\limits_{n\to\infty} n^2\left[\sqrt{n\left(n^4-1\right)}-\sqrt{n^5-8}\right]$

11. $\lim\limits_{n\to\infty} n\left(\sqrt[3]{5=8n^3}-2n\right)$

12. $\lim\limits_{n\to\infty} n^2\left(\sqrt[3]{5+n^3}-\sqrt[3]{3+n^3}\right)$

13. $\lim\limits_{n\to\infty}\left[\sqrt[3]{(n+2)^2}-\sqrt[3]{(n-3)^2}\right]$

14. $\lim\limits_{n\to\infty}\dfrac{\sqrt{(n+1)^3}-\sqrt{n(n-1)(n-3)}}{\sqrt{n}}$

15. $\lim\limits_{n\to\infty}\left(\sqrt{n^2+3n-2}-\sqrt{n^2-3}\right)$

16. $\lim\limits_{n\to\infty}\sqrt{n}\left(\sqrt{n+2}-\sqrt{n-3}\right)$

17. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n\left(n^5+9\right)-\sqrt{\left(n^4-1\right)\left(n^2+5\right)}}}{n}$

18. $\lim\limits_{n\to\infty}\left(\sqrt{n(n+5)}-n\right)$

19. $\lim\limits_{n\to\infty}\sqrt{n^3+8}\left(\sqrt{n^3+2}-\sqrt{n^3-1}\right)$

20. $\lim\limits_{n\to\infty}\dfrac{\sqrt{\left(n^3+1\right)\left(n^2+3\right)}-\sqrt{n\left(n^4+2\right)}}{2\sqrt{n}}$

21. $\lim\limits_{n\to\infty}\left[\sqrt{\left(n^2+1\right)\left(n^2+2\right)}-\sqrt{\left(n^2-1\right)\left(n^2-2\right)}\right]$

22. $\lim\limits_{n\to\infty}\dfrac{\sqrt{\left(n^5+1\right)\left(n^2-1\right)}-n\sqrt{n\left(n^4+1\right)}}{n}$

23. $\lim\limits_{n\to\infty}\dfrac{\sqrt{\left(n^4+1\right)\left(n^2-1\right)}-\sqrt{n^6-1}}{n}$

24. $\lim\limits_{n\to\infty}\left[n-\sqrt{n(n-1)}\right]$

25. $\lim\limits_{n\to\infty} n^3\left[\sqrt[3]{n^2\left(n^6+4\right)}-\sqrt[3]{\left(n^3-1\right)}\right]$

26. $\lim\limits_{n\to\infty}\left[n\sqrt{n}-\sqrt{n(n+1)(n+2)}\right]$

27. $\lim\limits_{n\to\infty}\sqrt[3]{n}\left[\sqrt[3]{n^2}-\sqrt[3]{n(n-1)}\right]$

28. $\lim\limits_{n\to\infty}\sqrt{n+2}\left(\sqrt{n+3}-\sqrt{n-4}\right)$

29. $\lim\limits_{n\to\infty} n\left(\sqrt{n^4+3}-\sqrt{n^4-2}\right)$

30. $\lim\limits_{n\to\infty}\sqrt{n(n+1)(n+2)}\left(\sqrt{n^3-3}-\sqrt{n^3-2}\right)$

8-vazifa. Ketma-ketlikning limitini toping

1. $\lim\limits_{n\to\infty}\dfrac{n\sqrt[3]{5n^2}+\sqrt[4]{9n^3+1}}{\left(n+\sqrt{n}\right)\sqrt{7-n+n^2}}$

2. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n-1}-\sqrt{n^2+1}}{\sqrt[3]{3n^3+3}+\sqrt[4]{n^5+1}}$

3. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^3+1}-\sqrt{n-1}}{\sqrt[3]{n^3+1}-\sqrt{n-1}}$

4. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^2-1}+7n^3}{\sqrt[4]{n^{12}+n+1}-n}$

5. $\lim\limits_{n\to\infty}\dfrac{\sqrt{3n-1}-\sqrt[3]{125n^3+n}}{\sqrt[5]{n}-n}$

6. $\lim\limits_{n\to\infty}\dfrac{n\sqrt[5]{n}-\sqrt[3]{27n^6+n^2}}{\left(n+\sqrt[4]{n}\right)\sqrt{9+n^2}}$

7. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n+2}-\sqrt{n^2+2}}{\sqrt[4]{4n^4+1}-\sqrt[3]{n^4-1}}$

8. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^4+2}+\sqrt{n-2}}{\sqrt[4]{n^4+2}+\sqrt{n-2}}$

9. $\lim\limits_{n\to\infty}\dfrac{6n^3-\sqrt{n^5+1}}{\sqrt{4n^6+3}-n}$

10. $\lim\limits_{n\to\infty}\dfrac{\sqrt{5n+2}-\sqrt[3]{8n^3+5}}{\sqrt[4]{n+7}-n}$

11. $\lim\limits_{n\to\infty}\dfrac{n\sqrt[4]{3n+1}+\sqrt{81n^4-n^2+1}}{\left(n+\sqrt[3]{n}\right)\sqrt{5-n+n^2}}$

12. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n+3}-\sqrt{n^2-3}}{\sqrt[3]{n^5-4}-\sqrt[4]{n^4+1}}$

13. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^5+3}-\sqrt{n-3}}{\sqrt[5]{n^5+3}+\sqrt{n-3}}$

14. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n}-9n^2}{3n-\sqrt[4]{9n^8+1}}$

15. $\lim\limits_{n\to\infty}\dfrac{\sqrt{4n+1}-\sqrt[3]{27n^3+4}}{\sqrt[4]{n}-\sqrt[3]{n^5+n}}$

16. $\lim\limits_{n\to\infty}\dfrac{n\sqrt[3]{7n}-\sqrt[4]{81n^8-1}}{\left(n+4\sqrt{n}\right)\sqrt{n^2-5}}$

17. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^3-7}+\sqrt[3]{n^2+4}}{\sqrt[4]{n^5+5}+\sqrt{n}}$

18. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^6+4}+\sqrt{n-4}}{\sqrt[5]{n^6+6}-\sqrt{n-6}}$

19. $\lim\limits_{n\to\infty}\dfrac{4n^2-\sqrt[4]{n^3}}{\sqrt[3]{n^6+n^3+1}-5n}$

20. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n+3}-\sqrt[3]{8n^3+3}}{\sqrt[4]{n+4}-\sqrt[5]{n^5+5}}$

21. $\lim\limits_{n\to\infty}\dfrac{n\sqrt[4]{11n}+\sqrt{25n^4-81}}{\left(n-7\sqrt{n}\right)\sqrt{n^2-n+1}}$

22. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^2}-\sqrt{n^2+5}}{\sqrt[5]{n^7}-\sqrt{n+1}}$

23. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^7+5}-\sqrt{n-5}}{\sqrt[7]{n^7+5}+\sqrt{n-5}}$

24. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^2+2}-5n^2}{n-\sqrt{n^4-n+1}}$

25. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n+2}-\sqrt[3]{n^3+2}}{\sqrt[7]{n+2}-\sqrt[5]{n^5+2}}$

26. $\lim\limits_{n\to\infty}\dfrac{n\sqrt{71n}-\sqrt[3]{64n^6+9}}{\left(n-\sqrt[3]{n}\right)\sqrt{11+n^2}}$

27. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n+6}-\sqrt{n^2-5}}{\sqrt[3]{n^3+3}+\sqrt[4]{n^3+1}}$

28. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^3+6}-\sqrt{n-6}}{\sqrt[8]{n^8+6}+\sqrt{n-6}}$

29. $\lim\limits_{n\to\infty}\dfrac{n^2-\sqrt{n^3+1}}{\sqrt[3]{n^6+2}-n}$

30. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n+1}-\sqrt[3]{n^3+1}}{\sqrt[4]{n+1}-\sqrt[5]{n^5+1}}$

9-vazifa. Ketma-ketlikning limitini toping

1. $\lim\limits_{n\to\infty}\left(\dfrac{1}{n^2}+\dfrac{2}{n^2}+\dfrac{3}{n^2}+\ldots+\dfrac{n-1}{n^2}\right)$

2. $\lim\limits_{n\to\infty}\dfrac{(2n+1)!+(2n+2)!}{(2n+3)!}$

3. $\lim\limits_{n\to\infty}\left[\dfrac{1+3+5+7+\ldots+(2n-1)}{n+1}-\dfrac{2n+1}{2}\right]$

4. $\lim\limits_{n\to\infty}\dfrac{2^{n+1}+3^{n+1}}{2^n+3^n}$

5. $\lim\limits_{n\to\infty}\dfrac{1+2+3+\ldots+n}{\sqrt{9n^4+1}}$

6. $\lim\limits_{n\to\infty}\dfrac{1+3+5+\ldots+(2n-1)}{1+2+3+\ldots+n}$

7. $\lim\limits_{n\to\infty}\left[\dfrac{1+3+5+\ldots+(2n-1)}{n+3}-n\right]$

8. $\lim\limits_{n\to\infty}\dfrac{1+4+7+(3n-2)}{\sqrt{5n^4+n+1}}$

9. $\lim\limits_{n\to\infty}\dfrac{(n+4)!-(n+2)!}{(n+3)!}$

10. $\lim\limits_{n\to\infty}\dfrac{(3n-1)!+(3n+1)!}{(3n!)(n-1)}$

11. $\lim\limits_{n\to\infty}\dfrac{2^n-5^{n+1}}{2^{n+1}+5^{n+2}}$

12. $\lim\limits_{n\to\infty}\dfrac{1+\frac{1}{3}+\frac{1}{3^2}+\ldots+\frac{1}{3^n}}{1+\frac{1}{5}+\frac{1}{5^2}+\ldots+\frac{1}{5^n}}$

13. $\lim\limits_{n\to\infty}\dfrac{1-3+5-7+9-11+\ldots+(4n-3)-(4n-1)}{\sqrt{n^2+1}+\sqrt{n^2+n+1}}$

14. $\lim\limits_{n\to\infty}\dfrac{1-2+3-4+\ldots+(2n-1)-2n}{n}$

15. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^3+5}-\sqrt{3n^4+2}}{1+3+5+\ldots+(2n-1)}$

16. $\lim\limits_{n\to\infty}\dfrac{3^n-2^n}{3^{n-1}+2^n}$

17. $\lim\limits_{n\to\infty}\left[\dfrac{n+2}{1+2+3+\ldots+n}-\dfrac{2}{3}\right]$

18. $\lim\limits_{n\to\infty}\left(\dfrac{5}{6}+\dfrac{13}{36}+\ldots+\dfrac{3^n+2^n}{6^n}\right)$

19. $\lim\limits_{n\to\infty}\dfrac{2-5+4-7+\ldots+2n-(2n+3)}{n+3}$

20. $\lim\limits_{n\to\infty}\dfrac{(2n+1)!+(2n+2)!}{(2n+3)!-(2n+2)!}$

21. $\lim\limits_{n\to\infty}\dfrac{1+2+\ldots+n}{n-n^2+3}$

22. $\lim\limits_{n\to\infty}\dfrac{n^2+\sqrt{n}-1}{2+7+12+\ldots+(5n-3)}$

23. $\lim\limits_{n\to\infty}\left(\dfrac{3}{4}+\dfrac{5}{16}+\dfrac{9}{64}+\ldots+\dfrac{1+2^n}{4^n}\right)$

24. $\lim\limits_{n\to\infty}\dfrac{2+4+6+\ldots+2n}{1+3+5+\ldots+(2n-1)}$

25. $\lim\limits_{n\to\infty}\left[\dfrac{1+5+9+13+(4n-3)}{n+1}-\dfrac{4n-1}{2}\right]$

26. $\lim\limits_{n\to\infty}\dfrac{1-2+3-4+\ldots-2n}{\sqrt[3]{n^3+2n}+2}$

27. $\lim\limits_{n\to\infty}\dfrac{2^n+7^n}{2^n-7^{n-1}}$

28. $\lim\limits_{n\to\infty}\dfrac{n!+(n+2)!}{(n-1)!+(n+2)!}$

29. $\lim\limits_{n\to\infty}\dfrac{3+6+9+\ldots+3n}{n^2+4}$

30. $\lim\limits_{n\to\infty}\left(\dfrac{7}{10}+\dfrac{29}{100}+\ldots+\dfrac{2^n+5^n}{10^n}\right)$

10-vazifa. Ketma-ketlikning limitini toping

1. $\lim\limits_{n\to\infty}\left(\dfrac{n+1}{n-1}\right)^n$

2. $\lim\limits_{n\to\infty}\left(\dfrac{3n^2-5n}{3n^2-5n+7}\right)^{n+1}$

3. $\lim\limits_{n\to\infty}\left(\dfrac{n^2-1}{n^2}\right)^{n^4}$

4. $\lim\limits_{n\to\infty}\left(\dfrac{n^2-6n+5}{n^2-5n+5}\right)^{3n+2}$

5. $\lim\limits_{n\to\infty}\left(\dfrac{2n^2+2}{2n^2+1}\right)^{\frac{n}{2}}$

6. $\lim\limits_{n\to\infty}\left(\dfrac{7n^2+18n-15}{7n^2+11n+15}\right)^{n+2}$

7. $\lim\limits_{n\to\infty}\left(\dfrac{n^2-3n+6}{n^2+5n+1}\right)^{\frac{n}{2}}$

8. $\lim\limits_{n\to\infty}\left(\dfrac{n^3+n+1}{n^3+2}\right)^{2n^2}$

9. $\lim\limits_{n\to\infty}\left(\dfrac{5n-7}{6n+1}\right)^{3n+2}$

10. $\lim\limits_{n\to\infty}\left(\dfrac{2n+3}{2n+1}\right)^{n+1}$

11. $\lim\limits_{n\to\infty}\left(\dfrac{n^2+n+1}{n^2+n-1}\right)^{-n^2}$

12. $\lim\limits_{n\to\infty}\left(\dfrac{n-1}{n+3}\right)^{n+2}$

13. $\lim\limits_{n\to\infty}\left(\frac{n-1}{n+1}\right)^{n^2}$

14. $\lim\limits_{n\to\infty}\left(\frac{3n^2-6n+7}{3n^2+20n-1}\right)^{-n+1}$

15. $\lim\limits_{n\to\infty}\left(\frac{3n+1}{3n-1}\right)^{2n+3}$

16. $\lim\limits_{n\to\infty}\left(\frac{n-10}{n+1}\right)^{3n+1}$

17. $\lim\limits_{n\to\infty}\left(\frac{n+3}{n+5}\right)^{n+4}$

18. $\lim\limits_{n\to\infty}\left(\frac{3n^2+4n-1}{3n^2+2n+7}\right)^{2n+5}$

19. $\lim\limits_{n\to\infty}\left(\frac{2n^2+21n-7}{2n^2+18n+9}\right)^{2n+1}$

20. $\lim\limits_{n\to\infty}\left(\frac{2n^2+5n+7}{2n+5n+3}\right)^{n}$

21. $\lim\limits_{n\to\infty}\left(\frac{5n^2+3n-1}{5n^2+3n+3}\right)^{n^3}$

22. $\lim\limits_{n\to\infty}\left(\frac{n+4}{n+2}\right)^{n}$

23. $\lim\limits_{n\to\infty}\left(\frac{2n^2+7n-1}{2n^2+3n-1}\right)^{-n^3}$

24. $\lim\limits_{n\to\infty}\left(\frac{2n-1}{2n+1}\right)^{n+1}$

25. $\lim\limits_{n\to\infty}\left(\frac{n^3+1}{n^3-1}\right)^{2n-n^3}$

26. $\lim\limits_{n\to\infty}\left(\frac{13n+3}{13n-10}\right)^{n-3}$

27. $\lim\limits_{n\to\infty}\left(\frac{10n-3}{10n-1}\right)^{5n}$

28. $\lim\limits_{n\to\infty}\left(\frac{n^3+1}{n^3-1}\right)^{2n-n^3}$

29. $\lim\limits_{n\to\infty}\left(\frac{n+3}{n+1}\right)^{-n^2}$

30. $\lim\limits_{n\to\infty}\left(\frac{n+5}{n-7}\right)^{\frac{n}{6}+1}$

11-vazifa

Koshi teoremasidan foydalanib, quyidagi ketma-ketliklarning yaqinlashuvchi bo`lishini ko`rsating:

1. $x_n = a_0 + a_1 q + \ldots + a_n q^n$, $|a_k| < M$ $(k = 0,1,2,\ldots)$ $|q| < 1$.
2. $x_n = \frac{\cos 1!}{1\cdot 2} + \frac{\cos 2!}{2\cdot 3} + \ldots + \frac{\cos n!}{n\cdot(n+1)}$.
3. $x_n = \frac{\sin a}{2} + \frac{\sin 2a}{2^2} + \frac{\sin 3a}{2^3} + \ldots + \frac{\sin na}{2^n}$, $a \in R$.
4. $x_n = \frac{\sin 1}{2} + \frac{\sin 2}{2^2} + \ldots + \frac{\sin n}{2^n}$
5. $x_n = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots + \frac{1}{n^2}$
6. $x_n = \frac{\cos 1}{3} + \frac{\cos 2}{3^2} + \ldots + \frac{\cos n}{3^n}$
7. $x_n = 1 + \frac{1}{2!} + \ldots + \frac{1}{n!}$

Koshi teoremasidan foydalanib, quyidagi ketma-ketliklarning uzoqlashuvchi bo`lishini ko`rsating:

8. $x_n = 0.2^{(-1)^n n}$

9. $x_n = (-1)^n\left(1+\frac{1}{n}\right)^n$

10. $x_n = \left(1+\frac{(-1)^n}{n}\right)^n$

11. $x_n = \frac{n\cos\pi n - 1}{2n}$

12. $x_n = \frac{1}{2^2}+\frac{2}{3^2}+\ldots+\frac{n}{(n+1)^2}$

Monoton va chegaralangan ketmaketlikning limiti haqidagi teoremadan foydalanib, quyidagi ketma- ketliklarning yaqinlashuvchi bo`lishini ko`rsating:

13. $x_n = \left(1+\frac{1}{2}\right)\left(1+\frac{1}{4}\right)\ldots\left(1+\frac{1}{2^n}\right)$

14. $x_{n} = \left(1-\frac{1}{2}\right)\left(1-\frac{1}{4}\right)\ldots\left(1-\frac{1}{2^n}\right)$

15. $x_n = \frac{10}{1}\cdot\frac{11}{3}\cdot\ldots\cdot\frac{n+9}{2n-1}$

16. $x_1 = \sqrt{2}$, $x_2 = \sqrt{2+\sqrt{2}}$, …, $x_n = \underbrace{\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}}_{n \quad ma}$, …

17. $x_n = p_0 + \frac{p_1}{10}+\ldots+\frac{p_n}{10^n}$, $n = 1,2,\ldots$, $p_i\,(i = 0,1,2,\ldots)$ butun sonlar, $p_i \geq 0$, $p_i \leq 9$.

Berilgan ketma-ketliklar uchun *inf x_n, sup x_n,* $\underline{\lim}_{n\to\infty} x_n$ *va* $\overline{\lim}_{n\to\infty} x_n$ topilsin :

18. $x_n = \frac{n^2}{1+n}\cos\frac{2n\pi}{3}$.

19. $x_n = 3\left(1-\frac{1}{n}\right)+2(-1)^n$

20. $x_n = \frac{n}{n+1}\sin^2\frac{n\pi}{4}$.

21. $x_n = \left(1+\frac{1}{n}\right)^n(-1)^n + \sin\frac{\pi n}{4}$

22. $x_n = \sin^n\frac{2n\pi}{3}$

23. $x_n = \frac{1+(-1)^n n}{n}$

24. $x_n = \frac{1}{n}+\sin\frac{n\pi}{3}$

25. $x_n = (-n)^{\sin\left(\frac{\pi n}{2}\right)}$

Qismiy limitlar topilsin

26. $\frac{1}{2},\frac{1}{2},\frac{1}{4},\frac{3}{4},\frac{1}{8},\frac{7}{8},\ldots,\frac{1}{2^n},\frac{2^n-1}{2^n},\ldots$

27. $\frac{1}{2},\frac{1}{3},\frac{2}{3},\frac{1}{4},\frac{2}{4},\frac{3}{4},\frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{4}{5},\ldots$

28. $x_n = \sqrt[n]{1+2^{n(-1)^n}}$

29. $x_n = \frac{1}{2}\left[(a+b)+(-1)^n(a-b)\right]$

30. $\left\{1,\frac{1}{2},\frac{2}{2},\frac{3}{2},\frac{1}{4},\frac{2}{4},\frac{3}{4},\frac{4}{4},\ldots,\frac{9}{4},\ldots,\frac{1}{2^n},\frac{2}{2^n},\ldots,\frac{3^n}{2^n},\ldots\right\}$

II. Funksiyaning limiti

1-vazifa. Funksiyaning limiti ta'rifidan foydalanib tenglikni isbotlang.

1. $\lim\limits_{x\to 1}\dfrac{3x+1}{7x-5}=2;$
2. $\lim\limits_{x\to 2}\dfrac{3}{x-2}=\infty;$
3. $\lim\limits_{x\to 1}(2x^2-5)=-3;$
4. $\lim\limits_{x\to 1}(\sqrt{3+x^2}-x)=1;$
5. $\lim\limits_{x\to 4}\sqrt{1+2x}=3;$
6. $\lim\limits_{x\to\infty}(x^2-3)=\infty;$
7. $\lim\limits_{x\to 0}\dfrac{4}{x-3}=-\dfrac{4}{3};$
8. $\lim\limits_{x\to 1}\dfrac{x+1}{x-5}=-\dfrac{1}{2};$
9. $\lim\limits_{x\to 2}\dfrac{x^2-1}{x^2+1}=\dfrac{3}{5};$
10. $\lim\limits_{x\to 0}(x^2+x+3)=3;$
11. $\lim\limits_{x\to 1}\dfrac{1}{x-1}=\infty;$
12. $\lim\limits_{x\to 1}\dfrac{1}{6x-5}=1;$
13. $\lim\limits_{x\to+\infty}(\sqrt{1+x^2}-x)=0;$
14. $\lim\limits_{x\to 1}(2x^2-3x)=-1;$
15. $\lim\limits_{x\to 1}(x^2-3)=-2;$
16. $\lim\limits_{x\to 4}\sqrt{17+2x}=5;$
17. $\lim\limits_{x\to 2}\dfrac{x+1}{x-5}=-1;$
18. $\lim\limits_{x\to 0}\dfrac{4x}{x-3}=0;$
19. $\lim\limits_{x\to 0}(x^2-5x+2)=2;$
20. $\lim\limits_{x\to 2}\dfrac{x^2-1}{x^2+1}=\dfrac{3}{5};$
21. $\lim\limits_{x\to 1}\dfrac{x+1}{7x-5}=1;$
22. $\lim\limits_{x\to 3}\dfrac{1}{x-1}=\dfrac{1}{2};$
23. $\lim\limits_{x\to 1}(x^2-15)=-14;$
24. $\lim\limits_{x\to+\infty}(3-x)=-\infty;$
25. $\lim\limits_{x\to 4}\sqrt{8+2x}=4;$
26. $\lim\limits_{x\to 1}(2x^2-3)=-1;$
27. $\lim\limits_{x\to 0}\dfrac{x+3}{x-3}=-1;$
28. $\lim\limits_{x\to 2}\dfrac{2x+1}{x-7}=-1;$
29. $\lim\limits_{x\to 1}\dfrac{x^2-1}{x^2+1}=0;$
30. $\lim\limits_{x\to 0}(x^2-5x+4)=4;$

2-vazifa. Funksiyalarning limitini toping

1. $\lim\limits_{x\to -1}\dfrac{(x^3-2x-1)(x+1)}{x^4+4x^2-5}$
2. $\lim\limits_{x\to -1}\dfrac{x^3-3x-2}{x+x^2}$
3. $\lim\limits_{x\to -1}\dfrac{(x^2+3x+2)^2}{x^3+2x^2-x-2}$
4. $\lim\limits_{x\to 1}\dfrac{(2x^2-x-1)^2}{x^3+2x^2-x-2}$
5. $\lim\limits_{x\to -3}\dfrac{(x^2+2x-3)^2}{x^3+4x^2+3x}$
6. $\lim\limits_{x\to -1}\dfrac{(x^3-2x-1)^2}{x^4+2x+1}$
7. $\lim\limits_{x\to 0}\dfrac{(1+x)^3-(1+3x)}{x+x^5}$
8. $\lim\limits_{x\to 1}\dfrac{x^2-2x+1}{2x^2-x-1}$
9. $\lim\limits_{x\to -1}\dfrac{x^3-3x-2}{x^2-x-2}$
10. $\lim\limits_{x\to -1}\dfrac{x^3+5x^2+7x+3}{x^3+4x^2+5x+2}$
11. $\lim\limits_{x\to 1}\dfrac{x^3-3x+2}{x^3-x^2-x+1}$
12. $\lim\limits_{x\to 1}\dfrac{x^3+x^2-5x+3}{x^3-x^2-x+1}$

13. $\lim\limits_{x\to -1}\dfrac{x^3+4x^2+5x+2}{x^3-3x-2}$

14. $\lim\limits_{x\to 1}\dfrac{x^4-1}{2x^4-x^2-1}$

15. $\lim\limits_{x\to -2}\dfrac{x^3+5x^2+8x+4}{x^3+3x^2-4}$

16. $\lim\limits_{x\to 2}\dfrac{x^3-5x^2+8x-4}{x^3-3x^2+4}$

17. $\lim\limits_{x\to 2}\dfrac{x^3-6x^2+12x-8}{x^3-3x^2+4}$

18. $\lim\limits_{x\to -2}\dfrac{x^3+5x^2+8x+4}{x^3+7x^2+16x+12}$

19. $\lim\limits_{x\to -1}\dfrac{x^3-3x-2}{\left(x^2-x-2\right)^2}$

20. $\lim\limits_{x\to 2}\dfrac{x^3-3x-2}{x-2}$

21. $\lim\limits_{x\to -1}\dfrac{x^3-3x-2}{x^2+2x+1}$

22. $\lim\limits_{x\to 1}\dfrac{x^2-2x+1}{x^3-x^2-x+1}$

23. $\lim\limits_{x\to 1}\dfrac{x^4-1}{2x^4-x^2-1}$

24. $\lim\limits_{x\to -1}\dfrac{x^2+3x+2}{x^3+2x^2-x-2}$

25. $\lim\limits_{x\to 1}\dfrac{2x^2-x-1}{x^3+2x^2-x-2}$

26. $\lim\limits_{x\to -3}\dfrac{x^2+2x-3}{x^3+4x^2+3x}$

27. $\lim\limits_{x\to -1}\dfrac{x^3-2x-1}{x^4+2x+1}$

28. $\lim\limits_{x\to 0}\dfrac{(1+x)^3-(1+3x)}{x^2+x^5}$

29. $\lim\limits_{x\to 1}\dfrac{x^2-1}{2x^2-x-1}$

30. $\lim\limits_{x\to -3}\dfrac{x^3+7x^2+15x+9}{x^3+8x^2+21x+18}$

3-vazifa. Funksiyalarning limitini toping

1. $\lim\limits_{x\to 4}\dfrac{\sqrt{1+2x}-3}{\sqrt{x}-2}$

2. $\lim\limits_{x\to -8}\dfrac{\sqrt{1-x}-3}{2+\sqrt[3]{x}}$

3. $\lim\limits_{x\to 1}\dfrac{\sqrt{x-1}}{\sqrt[3]{x^2-1}}$

4. $\lim\limits_{x\to 3}\dfrac{\sqrt{x+13}-2\sqrt{x+1}}{x^2-9}$

5. $\lim\limits_{x\to -2}\dfrac{\sqrt[3]{x-6}+2}{x^3+8}$

6. $\lim\limits_{x\to 16}\dfrac{\sqrt[4]{x}-2}{\sqrt{x}-4}$

7. $\lim\limits_{x\to 8}\dfrac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2}$

8. $\lim\limits_{x\to 0}\dfrac{\sqrt{1-2x+x^2}-(1+x)}{x}$

9. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{8+3x+x^2}-2}{x+x^2}$

10. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{27+x}-\sqrt[3]{27-x}}{x+2\sqrt[3]{x^4}}$

11. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt{1+x}-\sqrt{2x}}$

12. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt[3]{1+x}-\sqrt[3]{1-x}}$

13. $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{4x}-2}{\sqrt{2+x}-\sqrt{2x}}$

14. $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{x^2-1}$

15. $\lim\limits_{x\to 3}\dfrac{\sqrt[3]{9x}-3}{\sqrt{3+x}-\sqrt{2x}}$

16. $\lim\limits_{x\to -2}\dfrac{\sqrt[3]{x-6}+2}{x+2}$

17. $\lim\limits_{x\to 4}\dfrac{\sqrt[3]{16x}-4}{\sqrt{4+x}-\sqrt{2x}}$

18. $\lim\limits_{x\to 8}\dfrac{\sqrt{9+2x}-5}{\sqrt[3]{x^2}-4}$

19. $\lim\limits_{x\to 1/2}\dfrac{\sqrt[3]{x/4}-1/2}{\sqrt{1/2+x}-\sqrt{2x}}$

20. $\lim\limits_{x\to 1/3}\dfrac{\sqrt[3]{x/9}-1/3}{\sqrt{1/3+x}-\sqrt{2x}}$

21. $\lim\limits_{x\to 1/4}\dfrac{\sqrt[3]{x/16}-1/4}{\sqrt{1/4+x}-\sqrt{2x}}$

22. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt[7]{x}}$

23. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{27+x}-\sqrt[3]{27-x}}{\sqrt[3]{x^2}+\sqrt[5]{x}}$

24. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{8+3x-x^2}-2}{\sqrt[3]{x^2+x^3}}$

25. $\lim\limits_{x\to 0}\dfrac{\sqrt{1-2x+3x^2}-(1+x)}{\sqrt[3]{x}}$

26. $\lim\limits_{x\to 8}\dfrac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2}$

27. $\lim\limits_{x\to 16}\dfrac{\sqrt[4]{x}-2}{\sqrt[3]{\left(\sqrt{x}-4\right)^2}}$

28. $\lim\limits_{x\to -2}\dfrac{\sqrt[3]{x-6}+2}{\sqrt[3]{x^3+8}}$

29. $\lim\limits_{x\to 4}\dfrac{\sqrt{x}-2}{\sqrt[3]{x^2-16}}$

30. $\lim\limits_{x\to -8}\dfrac{10-x-6\sqrt{1-x}}{2+\sqrt[3]{x}}$

4-vazifa. Funksiyalarning limitini toping

1. $\lim\limits_{x\to 0}\dfrac{\ln(1+\sin x)}{\sin 4(x-\pi)}$

2. $\lim\limits_{x\to -0}\dfrac{1-\cos 10(x+\pi)}{e^{x^2}-1}$

3. $\lim\limits_{x\to 0}\dfrac{3x^2-5x}{\sin 3x}$

4. $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{\cos 7x-\cos 3x}$

5. $\lim\limits_{x\to 0}\dfrac{4x}{tg(\pi(2+x))}$

6. $\lim\limits_{x\to 0}\dfrac{2x}{tg(2\pi(x+1/2))}$

7. $\lim\limits_{x\to 0}\dfrac{1+\cos^3 x}{4x^2}$

8. $\lim\limits_{x\to 0}\dfrac{\arcsin 3x}{\sqrt{2+x}-\sqrt{2}}$

9. $\lim\limits_{x\to 0}\dfrac{2^{x+1}-2}{\ln(1+4x)}$

10. $\lim\limits_{x\to 0}\dfrac{arctg\,2x}{\sin(2\pi(x+110))}$

11. $\lim\limits_{x\to 0}\dfrac{\ln(1-7x)}{\sin(\pi(x+7))}$

12. $\lim\limits_{x\to 0}\dfrac{\cos(x+5\pi/2)tgx}{\arcsin 2x^2}$

13. $\lim\limits_{x\to 0}\dfrac{\ln(1-3x)}{\sqrt{8x+4}-2}$

14. $\lim\limits_{x\to 0}\dfrac{1-\sqrt{3x+1}}{\cos[\pi(x+1)/2]}$

15. $\lim\limits_{x\to 0}\dfrac{\sin 7x}{x^2+\pi x}$

16. $\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}-2}{3arctgx}$

17. $\lim\limits_{x\to 0}\dfrac{2\sin[\pi(x+1)]}{\ln(1+2x)}$

18. $\lim\limits_{x\to 0}\dfrac{\cos 2x-\cos x}{1-\cos x}$

19. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1}{\sin[\pi(x+2)]}$

20. $\lim\limits_{x\to 0}\dfrac{\sin[5(x+\pi)]}{e^{3x-1}}$

21. $\lim\limits_{x\to 0}\dfrac{1-\sqrt{\cos x}}{x\sin x}$

22. $\lim\limits_{x\to 0}\dfrac{\arcsin 2x}{\sin 3(x+\pi)}$

23. $\lim\limits_{x\to 0}\dfrac{e^{4x}-1}{\sin\left[\pi\left(x/2+1\right)\right]}$

24. $\lim\limits_{x\to 0}\dfrac{1+\cos\left(x-\pi\right)}{\left(e^{3x}-1\right)^2}$

25. $\lim\limits_{x\to 0}\dfrac{\sin^2 x-tg^2 x}{x^4}$

26. $\lim\limits_{x\to 0}\dfrac{\arcsin 2x}{\ln\left(e-x\right)-1}$

27. $\lim\limits_{x\to 0}\dfrac{tgx-\sin x}{x\left(1-\cos 2x\right)}$

28. $\lim\limits_{x\to 0}\dfrac{\ln\left(x^2+1\right)}{2-\sqrt{2x^2+4}}$

29. $\lim\limits_{x\to 0}\dfrac{tg\left(\pi\left(1+x/2\right)\right)}{\ln\left(x+1\right)}$

30. $\lim\limits_{x\to 0}\dfrac{e^{4\pi x}-1}{\sqrt[3]{8+24x}-2}$

5-vazifa. Funksiyalarning limitini toping

1. $\lim\limits_{x\to 1}\dfrac{x^2-1}{\ln x}$

2. $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2-x+1}-1}{\ln x}$

3. $\lim\limits_{x\to \pi}\dfrac{1+\cos 3x}{\sin^2 7x}$

4. $\lim\limits_{x\to \pi/4}\dfrac{1-\sin 2x}{\left(\pi-4x\right)^2}$

5. $\lim\limits_{x\to 1}\dfrac{1+\cos \pi x}{tg^2 \pi x}$

6. $\lim\limits_{x\to \pi/2}\dfrac{tg3x}{tgx}$

7. $\lim\limits_{x\to \pi}\dfrac{\sin^2 x-tg^2 x}{\left(x-\pi\right)^4}$

8. $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2-x+1}-1}{tg\pi x}$

9. $\lim\limits_{x\to \pi}\dfrac{\cos 5x-\cos 3x}{\sin^2 x}$

10. $\lim\limits_{x\to 2\pi}\dfrac{\sin 7x-\sin 3x}{e^{x^2}-e^{4\pi^2}}$

11. $\lim\limits_{x\to 2}\dfrac{\sin 7\pi x}{\sin 8\pi x}$

12. $\lim\limits_{x\to 2}\dfrac{\ln\left(5-2x\right)}{\sqrt{10-3x}-2}$

13. $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2-3x+3}-1}{\sin \pi x}$

14. $\lim\limits_{x\to \pi}\dfrac{x^2-\pi^2}{\sin x}$

15. $\lim\limits_{x\to 1}\dfrac{3^{5x-3}-3^{2x^2}}{tg\pi x}$

16. $\lim\limits_{x\to -4}\dfrac{2^x-16}{\sin \pi x}$

17. $\lim\limits_{x\to \pi/2}\dfrac{\ln 2x-\ln \pi}{\sin\left(5x/2\right)\cos x}$

18. $\lim\limits_{x\to \pi/4}\dfrac{\ln tgx}{\cos 2x}$

19. $\lim\limits_{x\to \pi}\dfrac{e^{\pi}-e^{x}}{\sin 5x-\sin 3x}$

20. $\lim\limits_{x\to 2}\dfrac{\ln\left(9-2x^2\right)}{\sin 2\pi x}$

21. $\lim\limits_{x\to 2}\dfrac{1-2^{4-x^2}}{2\left(\sqrt{2x}-\sqrt{3x^2-5x+2}\right)}$

22. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}$

23. $\lim\limits_{x\to -2}\dfrac{tg\pi x}{x+2}$

24. $\lim\limits_{x\to \pi}\dfrac{1-\sin\left(x/2\right)}{\pi-x}$

25. $\lim\limits_{x\to \pi/3}\dfrac{1-2\cos x}{\pi-3x}$

26. $\lim\limits_{x\to 2}\dfrac{arctg\left(x^2-2x\right)}{\sin 3\pi x}$

27. $\lim\limits_{x\to 1}\dfrac{1-x^2}{\sin \pi x}$

28. $\lim\limits_{x\to 1}\dfrac{\cos(\pi x/2)}{1-\sqrt{x}}$

29. $\lim\limits_{x\to 1}\dfrac{3-\sqrt{10-x}}{\sin 3\pi x}$

30. $\lim\limits_{x\to \pi}\dfrac{\sin 5x}{tg\,3x}$

6-vazifa. Funksiyalarning limitini toping

1. $\lim\limits_{x\to \pi/2}\dfrac{2^{\cos^2 x}-1}{\ln \sin x}$

2. $\lim\limits_{x\to 1/2}\dfrac{(2x-1)^2}{e^{\sin \pi x}-e^{-\sin 3\pi x}}$

3. $\lim\limits_{x\to 2}\dfrac{\ln\left(x-\sqrt[3]{2x-3}\right)}{\sin(\pi x/2)-\sin[(x-1)\pi]}$

4. $\lim\limits_{x\to 2}\dfrac{tgx-tg\,2}{\sin\ln(x-1)}$

5. $\lim\limits_{x\to \pi/2}\dfrac{e^{tg\,2x}-e^{-\sin 2x}}{\sin x-1}$

6. $\lim\limits_{x\to \pi/6}\dfrac{\ln \sin 3x}{(6x-\pi)^2}$

7. $\lim\limits_{x\to 3}\dfrac{\sin\left(\sqrt{2x^2-3x-5}-\sqrt{1+x}\right)}{\ln(x-1)-\ln(x+1)+\ln 2}$

8. $\lim\limits_{x\to 2\pi}\dfrac{(x-2\pi)^2}{tg(\cos x-1)}$

9. $\lim\limits_{x\to 1/2}\dfrac{\ln(4x-1)}{\sqrt{1-\cos \pi x}-1}$

10. $\lim\limits_{x\to -2}\dfrac{\arcsin(x+2)/2}{\sqrt{2+x+x^2}-9}$

11 $\lim\limits_{x\to 3}\dfrac{2\sin \pi x-1}{\ln\left(x^2-6x-8\right)}$

12. $\lim\limits_{x\to \pi}\dfrac{\ln\cos 2x}{(1-\pi/x)^2}$

13. $\lim\limits_{x\to 2}\dfrac{tg\ln(3x-5)}{e^{x+3}-e^{x^2+1}}$

14. $\lim\limits_{x\to 2\pi}\dfrac{\ln\cos x}{3^{\sin 2x}-1}$

15. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{1+\ln^2 x}-1}{1+\cos \pi x}$

16. $\lim\limits_{x\to \pi}\dfrac{\cos(x/2)}{e^{\sin x}-e^{\sin 4x}}$

17. $\lim\limits_{x\to 3}\dfrac{\ln(2x-5)}{e^{\sin \pi x}-1}$

18. $\lim\limits_{x\to \pi/3}\dfrac{e^{\sin^2 6x}-e^{\sin^2 3x}}{\log_3 \cos 6x}$

19. $\lim\limits_{x\to \pi/2}\dfrac{e^{\sin 2x}-e^{tg\,2x}}{\ln(2x/\pi)}$

20. $\lim\limits_{x\to -2}\dfrac{tg\left(e^{x+2}-e^{x^2-4}\right)}{tgx+tg\,2}$

21. $\lim\limits_{x\to 1}\dfrac{\sqrt{2^x+7}-\sqrt{2^{x+1}+5}}{x^3-1}$

22. $\lim\limits_{x\to \pi}\dfrac{\ln(2+\cos x)}{\left(3^{\sin x}-1\right)^2}$

23. $\lim\limits_{x\to \pi}\dfrac{\left(x^3-\pi^3\right)\sin 5x}{e^{\sin^2 x}-1}$

24. $\lim\limits_{x\to -1}\dfrac{tg(x+1)}{e^{\sqrt[3]{x^3-4x^2+6}}-e}$

25. $\lim\limits_{x\to \pi}\dfrac{\ln\cos 2x}{\ln\cos 4x}$

26. $\lim\limits_{x\to \pi/2}\dfrac{\ln \sin x}{(2x-\pi)^2}$

27. $\lim\limits_{x\to a}\dfrac{a^{x^2-a^2}-1}{tg\ln(x/a)}$

28. $\lim\limits_{x\to -3}\dfrac{\sin\left(e^{\sqrt[3]{1-x^2/2}}-e^{\sqrt[3]{x+2}}\right)}{arctg(x+3)}$

29. $\lim\limits_{x\to a\pi}\dfrac{\ln(\cos(x/a)+2)}{a^{a^2\pi^2/x^2-a\pi/x}-a^{a\pi/x}-1}$

30. $\lim\limits_{x\to \pi}\dfrac{tg\left(3^{\pi/x}-3\right)}{3\cos(3x/2)-1}$

III. Funksiyaning uzluksizligi

1- vazifa. Geyne, Koshi va orttirmalar tilidagi ta'riflardan foydalanib, berilgan funksiyaning ko'rsatilgan $x=x_0$ nuqtada va aniqlanish sohasida uzluksiz ekanligini isbotlang.

1. $f(x)=x^2+4x-5$, $x_0=1$;
2. $f(x)=x^3-x+3$, $x_0=2$;
3. $f(x)=x-x^2$, $x_0=-1$;
4. $f(x)=x\sqrt{x}+5$, $x_0=4$;
5. $f(x)=\sqrt{x-2}+1$, $x_0=6$;
6. $f(x)=\sqrt{15-x}+4$, $x_0=-10$;
7. $f(x)=2^x+6$, $x_0=2$;
8. $f(x)=\log_5(1-2x)$, $x_0=-2$;
9. $f(x)=2\sin(2x-1)$, $x_0=2$;
10. $f(x)=\sqrt[3]{2x+1}-1$, $x_0=-1$;
11. $f(x)=3-\cos\pi x$, $x=-2{,}5$;
12. $f(x)=\sqrt{x+2}-12$, $x_0=3$;
13. $f(x)=3^x+x$, $x_0=2$;
14. $f(x)=\sqrt{3x-1}-4$, $x_0=5/3$;
15. $f(x)=4^{-x}+5$; $x_0=-1$;
16. $f(x)=2x^2+x-3$, $x_0=1$;
17. $f(x)=x^3-2x+1$, $x_0=2$;
18. $f(x)=4x-x^2$, $x_0=1$;
19. $f(x)=3\sqrt{x+5}+7$, $x_0=-1$;
20. $f(x)=\sqrt{5x-1}-4$, $x_0=1$;
21. $f(x)=3^x+7$, $x_0=-2$;
22. $f(x)=\log_3(7-2x)$, $x_0=-1$;
23. $f(x)=2\sin(1-3x)$, $x_0=2$;
24. $f(x)=\sqrt[3]{2-x}-4$, $x_0=-6$;
25. $f(x)=3-\cos 4x$, $x=\pi$;
26. $f(x)=(x+2)\sqrt{x+2}-4$, $x_0=2$;
27. $f(x)=4^x-3x$, $x_0=0$;
28. $f(x)=\sqrt{7x+4}-4$, $x_0=2$;
29. $f(x)=x4^x+8$; $x_0=1$;
30. $f(x)=x\sqrt{x-1}+2$, $x_0=5$.

2- **vazifa. f(x) va g(x) funksiyalar berilgan. $f\circ g$ funksiya uchun murakkab funksiyaning uzluksizligi haqidagi teorema shartlarini tekshiring va hulosa chiqaring.**

1. $f(x)=\sqrt{x}$; $g(x)=x^2-4$;
2. $f(x)=\sqrt{x}+2$; $g(x)=x^2+1$;
3. $f(x)=e^x$; $g(x)=\frac{1}{x}$;
4. $f(x)=\log_2 x$; $g(x)=2x-1$;
5. $f(x)=\arcsin x$; $g(x)=1-3x$;
6. $f(x)=\cos x$; $g(x)=\sqrt{x}$
7. $f(x)=\operatorname{tg}x$; $g(x)=\sin x$;
8. $f(x)=\frac{1}{x}$; $g(x)=\cos x$;
9. $f(x)=x^2$; $g(x)=\sqrt{x+2}$;
10. $f(x)=\arccos x$; $g(x)=2x-1$;
16. $f(x)=\operatorname{tg}x$; $g(x)=\sqrt{x+2}$;
17. $f(x)=\lg x$; $g(x)=x^2+x$;
18. $f(x)=\sqrt[3]{x}$; $g(x)=\operatorname{tg}x$;
19. $f(x)=\sqrt[4]{x}$; $g(x)=\cos x$;
20. $f(x)=\ln x$; $g(x)=\frac{1}{x}$;
21. $f(x)=\sqrt{x}$; $g(x)=\lg x$;
22. $f(x)=\sqrt{x}$; $g(x)=x^3+1$;
23. $f(x)=e^x$; $g(x)=x^2-5$;
24. $f(x)=\log_3 x$; $g(x)=\cos x$;

25. f(x)=arcsinx; g(x)=x^2;

11. f(x)=cosx; g(x)= $\frac{1}{x}$;

12. f(x)=tgx; g(x)=5x-4;

13. f(x)=$\frac{1}{x}$; g(x)=arccosx;

14. f(x)=x^2; g(x)=ctgx;

15. f(x)=arccosx; g(x)=sinx;

26. f(x)=tgx; g(x)=$\sqrt{x}$;

27. f(x)=lgx; g(x)=tgx;

28. f(x)=$\sqrt[5]{x}$; g(x)=3^x;

29. f(x)=$\sqrt[6]{x}$; g(x)=x^3-x;

30. f(x)=lnx; g(x)=$\sqrt[3]{x}$.

3-vazifa. Quyidagi funksiyalarning uzilish nuqtalari, ularning turlarini aniqlang va grafigini chizing.

1. f(x)=$\begin{cases} \frac{1}{x^2-1}, & agar \quad x<-1, \\ x^2, & agar \quad -1\le x<4, \\ \sqrt{x-4}, & agar \quad x>4. \end{cases}$

2. f(x)=$\begin{cases} \frac{1}{x^2-1}, & agar \quad x<-1, \\ x^2-16, & agar \quad -1\le x<4, \\ \sqrt{x-4}, & agar \quad x>4. \end{cases}$

3. f(x)=$\begin{cases} \frac{1}{x-1}, & agar \quad x<1, \\ x+1, & agar \quad 1\le x\le 5, \\ \sqrt{x+4}, & agar \quad x>5. \end{cases}$

4. f(x)=$\begin{cases} 4, & agar \quad x\le 0, \\ lg\, x, & agar \quad 0<x<10, \\ cos\,\pi x, & agar \quad 10\le x<10{,}5 \\ 2x-1, & agar \quad x\ge 10{,}5. \end{cases}$

5. f(x)=$\begin{cases} 4-x, & agar \quad x\le 2, \\ lg(x-2), & agar \quad 2<x<12, \\ sin\,\pi x, & agar \quad 12\le x<10{,}5 \\ 1-2x, & agar \quad x\ge 10{,}5. \end{cases}$

6. f(x)=$\begin{cases} |3-x|, & agar \quad x\le 5, \\ \frac{2}{x-5}, & agar \quad 5<x<10, \\ 0{,}4, & agar \quad 10\le x<14 \\ 50-3x, & agar \quad x\ge 14. \end{cases}$

7. f(x)=$\begin{cases} \frac{2}{x^2-4}, & agar \quad x<-2, \\ x^2+2, & agar \quad -2\le x<3, \\ \sqrt{x}, & agar \quad x>3. \end{cases}$

8. f(x)=$\begin{cases} \frac{1}{x-1}, & agar \quad x<-1, \\ 0{,}5x^2-1, & agar \quad -1\le x<4, \\ \sqrt{x-4}, & agar \quad x>4. \end{cases}$

9. f(x)=$\begin{cases} \frac{1}{x+2}, & agar \quad x<-2, \\ x+1, & agar \quad -2\le x\le 0, \\ \sqrt{x+4}, & agar \quad x>0. \end{cases}$

10. f(x)=$\begin{cases} 4-3x, & agar \quad x\le 1, \\ lg\, x, & agar \quad 1<x<10, \\ cos\,\pi x, & agar \quad 10\le x<10{,}5 \\ -1, & agar \quad x\ge 10{,}5. \end{cases}$

11. $f(x)=\begin{cases} 4-2x, & agar \quad x \le 2, \\ lg(x-1), & agar \quad 2 < x < 4, \\ \sin \pi x, & agar \quad 4 \le x < 4{,}5 \\ 10-2x, & agar \quad x \ge 4{,}5. \end{cases}$

12. $f(x)=\begin{cases} |3-x|, & agar \quad x \le 5, \\ \dfrac{2}{x-5}, & agar \quad 5 < x < 7, \\ x, & agar \quad 7 \le x < 10 \\ 1-x, & agar \quad x \ge 10. \end{cases}$

13. $f(x)=\begin{cases} x+1, & agar \quad x \le 0, \\ 1-x, & agar \quad 0 < x \le 1, \\ \dfrac{2}{1-x}, & agar \quad x > 1. \end{cases}$

14. $f(x)=\begin{cases} \dfrac{1}{\sin x}, & agar \quad -\pi < x < 0, \\ x^2-1, & agar \quad 0 \le x \le 3, \\ \sqrt{x-2}, & agar \quad x > 3. \end{cases}$

15. $f(x)=\begin{cases} \dfrac{1}{(x-1)^2}, & agar \quad x < 1, \\ 2x-1, & agar \quad 1 \le x \le 3, \\ \sqrt{2x+1}, & agar \quad x > 3. \end{cases}$

16. $f(x)=\begin{cases} ln|x|, & agar \quad x < 0, \\ x, & agar \quad 0 \le x \le 1, \\ ctg \pi x, & agar \quad 1 < x < 2 \\ x-3, & agar \quad x \ge 2. \end{cases}$

17. $f(x)=\begin{cases} ln(4-x), & agar \quad x < 4, \\ tg \pi x, & agar \quad 4 \le x < 5, \\ \pi, & agar \quad 5 \le x < 7 \\ 7-0{,}5x, & agar \quad x \ge 7. \end{cases}$

18. $f(x)=\begin{cases} |3-x^2|, & agar \quad x \le 2, \\ \dfrac{12}{x-2}, & agar \quad 2 < x < 5, \\ 4, & agar \quad 5 \le x < 8 \\ -x, & agar \quad x \ge 8. \end{cases}$

19. $f(x)=\begin{cases} 2x-1, & agar \quad |x| \ge 5, \\ ln(x+5), & agar \quad |x| < 5. \end{cases}$

20. $f(x)=\begin{cases} x+5, & agar \quad x \le 5, \\ log_2(x+5), & agar \quad 5 < x < 7, \\ 7, & agar \quad x = 7 \\ \dfrac{1}{x-6}, & agar \quad x \ge 7. \end{cases}$

21. $f(x)=\begin{cases} -x-1, & agar \quad x < -2, \\ 2, & agar \quad x = -2, \\ 1, & agar \quad |x| < 2 \\ x^3-7, & agar \quad x \ge 2. \end{cases}$

22. $f(x)=\begin{cases} 7-x^2, & agar \quad x \le -1, \\ 3x+9, & agar \quad -1 < x \le 0, \\ log_2(x+2), & agar \quad x > 0. \end{cases}$

23. $f(x)=\begin{cases} ctgx, & agar \quad x < 0, \\ tgx, & agar \quad 0 \le x \le \dfrac{\pi}{4}, \\ 2x-\dfrac{\pi}{4}, & agar \quad x > \dfrac{\pi}{4}. \end{cases}$

25. $f(x)=\begin{cases} 2, & agar \quad x \le -2, \\ \dfrac{12}{x+2}, & agar \quad -2 < x < 0{,}1, \\ |lg\, x|, & agar \quad 0{,}1 \le x < 10 \\ 9-x, & agar \quad x \ge 10. \end{cases}$

26. $f(x)=\begin{cases} |x-1|, & agar \quad |x| \ge 2, \\ log_2(x+2), & agar \quad |x| < 2. \end{cases}$

27. $f(x)=\begin{cases} arctgx, & agar \quad x < 0, \\ 1, & agar \quad x = 0, \\ 2^x, & agar \quad 0 < x < 1, \\ 1-x, & agar \quad x \ge 1. \end{cases}$

28. $f(x)=\begin{cases} 2^{-x}-1, & agar \quad x < 0, \\ 2, & agar \quad 0 \le x < 1, \\ 1-x, & agar \quad 1 \le x \le 3 \\ \dfrac{2}{x-3}, & agar \quad x > 3. \end{cases}$

29. $f(x)=\begin{cases} 3^{-x}, & agar \quad x < -1, \\ |x-2|, & agar \quad -1 \le x \le 3, \\ 1-\sqrt{x+1}, & agar \quad x > 3. \end{cases}$

30. $f(x)=\begin{cases} 2x, & agar \quad x \le -2, \\ \dfrac{12}{x+3}, & agar \quad -2 < x \le 1, \\ lg(x-1) & agar \quad 1 < x < 11 \\ 1, & agar \quad x \ge 10. \end{cases}$

24. $f(x)=\begin{cases} log_{0,5}(1-x), & agar \quad x<1, \\ 1-x, & agar \quad 1\le x\le 3, \\ \sqrt{2x+1}, & agar \quad x>3. \end{cases}$

4-vazifa. Berilgan tenglamaning ko'rsatilgan kesmada ildizi mavjudligini isbotlang. Uni 0,01 aniqlikda hisoblang. Shu tenglamaning haqiqiy ildizlar sonini grafik usulda aniqlang.

1. $\sqrt{x-1}+x-2=0$, [1;2];
2. x^3-3x+1=0, [-2;-1];
3. x^3+x^2-1=0, [0;1];
4. x^3+2x-6=0, [1;2];
5. x^3+3x-1=0, [0;1];
6. x-$\sqrt{2-x}$ +1=0, [0;1];
7. x^3-x^2+3x-2=0, [0;1];
8. x^3-5x^2+7x-1=0, [0;1];
9. x^3-7x^2-5x+1=0, [0;1];
10. x^4-2x^3-5=0, [2;3];
11. x^3+5x-15=0, [1;2];
12. x-2+$\sqrt{3-x}$ =0, [0;2];
13. x^3-2x-1=0, [1;2];
14. $2x^3-x^2$+1=0, [-1,0];
15. 1+x-x^3=0, [1;2];
16. $3x^3-2x^2$+x+10=0, [-2;-1];
17. $7x-x^2-x^4$=0, [1;2];
18. $3x^3-x^2$-2x+0,5=0, [-1;0];
19. $2x^3+x^2$-15x+5=0, [0;1];
20. $2x^3+x^2$-45x+5=0, [0;1];
21. $-2+2x^2-x^3$=0, [-2;-1];
22. $\sqrt{3+2x}+2x=0$, [0;1];
23. $\sqrt{3-x}$ +5x-4=0, [0;1];
24. $3x^3$+6x-5=0, [0;1];
25. x^3-x^2-3=0, [0;2];
26. x^3-15x+12=0, [0;1];
27. $2x^3+3x^2$-9x+3=0, [0;1];
28. $-x^3+2x^2$-1=0, [-2;-1];
29. $\sqrt{4+5x}$ +x-2=0, [0;1];
30. $\sqrt{4-x}$ -5x+3=0, [0;1].

5-fazifa. Berilgan f(x) funksiyani ko'rsatilgan A to'plamda tekis uzluksizlikka tekshiring.

1. $f(x)=\sqrt{x}$, A=[0;+∞);
2. $f(x)=x^2$-4, A=(0;3);
3. f(x)= $\sqrt{x}$, A=[1;5);
4. $f(x)=e^x$, A=[0;+∞);
5. f(x)= $\frac{1}{x}$, A=(0;+∞);
6. f(x)= $\log_2 x$, A=(0;1);
7. f(x)=2x-1, A=(3;9);
8. f(x)=cosx, A=(-∞;+∞);
9. f(x)=sinx, A=(-∞;+∞);
10. f(x)=$\frac{1}{x}$, A=(1;+∞);
11. $f(x)=x^2$, A=(0;+∞);
12. f(x)=$\sqrt{x+2}$, A=(-2;2);
13. f(x)=$\sqrt[3]{x}$, A=(-∞;+∞);
14. f(x)=$\sqrt[3]{x}$, A=(0;8);
19. f(x)=$\sqrt[4]{x}$, A=[1;5)
20. f(x)=lnx, A=(1;4)
21. f(x)= $\frac{1}{x-4}$, A=(4;5]
22. $f(x)=x^3$+1, A=(-∞;+∞);
23. $f(x)=x^2$-5x+3, A=(-1;1);
24. f(x)= $\log_3 x$, A=(3;9)
25. f(x)=arcsinx, A=[-1;1];
26. f(x)=tgx, A=[-π/4;π/4]
27. f(x)=5x-4, A=(-2;8];
28. f(x)=tgx, A=(-π/4;π/4)
29. f(x)=lgx, A=[10,+∞);
30. f(x)=$\sqrt[5]{x+4}$, A=(-4;4)

II SEMESTR. DIFFERENSIYAL HISOB

1-mavzu. Hosilaning ta'rifi, uning geometrik va mexanik ma'nolari. Egri chiziq urinmasi va normalining tenglamalari. Differensiallanuvchi funksiyaning uzluksizligi.(3 soat)

1. 1-vazifa; 2-vazifa, 3-vazifa
2. R da uzluksiz, lekin chekli sondagi $x_1, x_2, ..., x_n$ nuqtalarda hosilasi mavjud bo`lmagan funksiyalarga misollar keltiring.
3. Ikki funksiya grafiklari kesishish nuqtasida grafiklar orasidagi burchak qanday aniqlanadi?
4. Agar $f(x)$ va $g(x)$ funksiyalar $(a;b)$ intervalda differensiallanuvchi va $f(x)\equiv g(x)$ bo`lsa, u holda $f'(x)\equiv g'(x)$ bo`lishini isbotlang.

Adabiyotlar: [1], 1-t., 182-188 b.; [3], 109-113 b.

2-mavzu. Yig`indi, ko`paytma va bo`linmaning hosilasi. Murakkab funksiyaning hosilasi (2 soat)

1. 1-vazifa
2. Faraz qilaylik $y=f(x)$ va $y=g(x)$ funksiyalar $x=a$ nuqtada hosilaga ega bo`lmasin. U holda $y=f(x)+g(x), y=f(x)-g(x), y=f(x)\cdot g(x), y=f(x)/g(x)$ funksiyalar $x=a$ nuqtada hosilaga ega emas deb ta'kidlash mumkinmi? Javobingizni asoslang.
3. Faraz qilaylik $y=f(x)$ funksiya $x=a$ nuqtada hosilaga ega emas, $y=g(x)$ funksiya $x=a$ nuqtada hosilaga ega bo`lsin. U holda $y=f(x)+g(x), y=f(x)-g(x), y=f(x)\cdot g(x), y=f(x)/g(x)$ funksiyalar $x=a$ nuqtada hosilaga ega emas deb ta'kidlash mumkinmi? Javobingizni asoslang.
4. Quyidagi shartni qanoatlantiruvchi murakkab funksiyaga misollar keltiring:

a) $f'(g(x_0))$ mavjud emas, $g'(x_0)$ mavjud.
b) $f'(g(x_0))$mavjud, $g'(x_0)$ mavjud emas.
c) $f'(g(x_0))$ mavjud emas, $g'(x_0)$ mavjud.

Adabiyotlar: [1], 1-t., 182-184 b.; [3], 1-t., 162-164 b.

3-mavzu. Teskari funksiyaning hosilasi. Asosiy elementar funksiyalarning hosilalari (3 soat)

1. 1-vazifa
2. *arccosx, arctgx, arcctgx* funksiya hosilalarini keltirib chiqaring.
3. Giperbolik funksiyalarning hosilalari
4. Teskari giperbolik funksiya hosilalarini keltirib chiqaring.
5. Davriy funksiyaning hosilasi davriy funksiya bo`lishini isbotlang.
6. Toq funksiyaning hosilasi juft funksiya bo`lishini isbotlang.
7. Juft funksiyaning hosilasi toq funksiya bo`lishini isbotlang.

Adabiyotlar: [1], 1-t., 188-196 b.; [3], 115-120 b.

4-mavzu. Differensial. Differensialning geometrik ma'nosi. Differensial formasining invariantligi. (3 soat)

1. 4-vazifa, 5-vazifa
2. Differensialni hisoblash qoidalarini isbotlang.
3. Differensial yordamida taqribiy hisoblashda nima ishlar bajariladi?

Adabiyotlar: [1], 1-t., 196-201 b.; [3], 111-113 b.

5-mavzu. Yuqori tartibli hosilalar va differensiallar. Ikkinchi tartibli hosilaning mexanik ma'nosi. Leybnits formulasi (3 soat)

1. 4-vazifa, 8-vazifa
2. $\cos x$, a^x funksiyalar uchun n-tartibli hosila formulasini keltirib chiqaring.
3. Agar $f'(x_0)$ mavjud bo`lmasa, $f''(x_0)$ mavjud bo`ladimi?
4. $f'(x_0)$ mavjud, lekin $f''(x_0)$ mavjud bo`lmagan funksiyaga misol keltiring.

5. Agar $f(x)$ funksiya n marta differensiallanuvchi bo`lsa, u holda $f^{(n)}(ax+b)=a^n f^{(n)}(t)|_{t=ax+b}$ formula o`rinli ekanligini isbotlang.
Adabiyotlar: [1], 1-t., 194-202 b.; [3], 1-t., 179-186 b.

6-mavzu. Logarifmik hosila. Daraja ko`rsatkichli funksiyaning hosilasi. Funksiyaning parametrik berilishi va uni differensiallash.(3 soat)

1. 6-,7-vazifalar
2. Daraja ko`rsatkichli funksiyadan hosila olish qoidasini ayting.
3. Parametrik tenglama bilan berilgan funksiya uchun ikkinchi va uchinchi tartibli hosilalarni hisoblash qoidasini keltirib chiqaring.

Adabiyotlar: [1], 1-t., 196 b.; [3], 128 b.

7-mavzu. Haqiqiy o`zgaruvchining vektor funksiyasi va uni differensiallash. (2 soat)

1. Agar $\lim\limits_{t\to t_0}\vec{r}(t)=\vec{a}$ bo`lsa, u holda $\lim\limits_{t\to t_0}|\vec{r}(t)|=|\vec{a}|$ bo`lishini isbotlang.
2. Agar $\lim\limits_{t\to t_0}\vec{r}_1(t)=\vec{a}$, $\lim\limits_{t\to t_0}\vec{r}_2(t)=\vec{b}$, $\lim\limits_{t\to t_0}\vec{r}_3(t)=\vec{c}$ bo`lsa, u holda

1) $\lim\limits_{t\to t_0}(\vec{r}_1(t),\vec{r}_2(t))=(\vec{a},\vec{b})$, 2) $\lim\limits_{t\to t_0}[\vec{r}_1(t),\vec{r}_2(t)]=[\vec{a},\vec{b}]$, 3) $\lim\limits_{t\to t_0}(\vec{r}_1(t),\vec{r}_2(t),\vec{r}_3(t))=(\vec{a},\vec{b},\vec{c})$ tengliklarning to`g`riligini isbotlang.

3. Vektor-funksiya hosilasi ta'rifidan foydalanib, quyidagilarni isbotlang:

1) $(f\cdot\vec{r}\,)'=f\cdot\vec{r}\,'+f\cdot\vec{r}\,'$, bu erda f –skalyar funksiya;

2) $(\vec{r}_1,\vec{r}_2)'=(\vec{r}_1',\vec{r}_2)+(\vec{r}_1,\vec{r}_2')$; 3) $[\vec{r}_1,\vec{r}_2]'=[\vec{r}_1',\vec{r}_2]+[\vec{r}_1,\vec{r}_2']$.

Adabiyotlar: [3], 132-133 b.

8-mavzu. Roll, Lagranj teoremalari. Koshi teoremasi. Lopital qoidasi. (3 soat)

1. Agar $f(x)$ va $g(x)$ funksiyalar (a,b) intervalda uzluksiz, differensiallanuvchi, $\forall x\in(a,b)$ uchun $f'(x)=g'(x)$ bo`lsa, u holda $\forall x\in(a,b)$ uchun $f(x)=g(x)+C$, bu erda $C$ biror o`zgarmas son, bo`lishini isbotlang.
2. Agar $f(x)$ va $g(x)$ funksiyalar $[a,b]$ kesmada uzluksiz, (a,b) intervalda differensiallanuvchi, $\forall x\in(a,b)$ uchun $f'(x)\le g'(x)$ va $f(a)=g(b)$ bo`lsa, u holda $\forall x\in(a,b)$ uchun $f(x)\le g(x)$ bo`lishini isbotlang.
3. Agar $g(x)$ monoton funksiya bo`lsa, u holda uning hosilasi ham monoton bo`ladi deb ta'kidlash mumkinmi? Javobingizni asoslang.
4. Agar $\forall x\in(a,b)$ uchun $f(x)\le g(x)$ bo`lsa, $\forall x\in(a,b)$ uchun $f'(x)\le g'(x)$ bo`ladi deb ta'kidlash mumkinmi? Javobingizni asoslang.
5. Agar f(x) funksiya $[0;+\infty)$ da n marta differensiallanuvchi, $f(0)=f'(0)=\ldots=f^{(n-1)}(0)=0$, $f^{(n)}(0)>0$ bo`lsa, u holda $\forall x\in(0,+\infty)$ da $f(x)>0$ bo`lishini isbotlang.
6. $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}=c$ ekanligidan $\lim\limits_{x\to a}\dfrac{f'(x)}{g'(x)}=c$ kelib chiqadimi? Javobingizni asoslang.
7. 10-vazifa.

Adabiyotlar: [1], 1-t., 211-214 b.; [3], 135-146 b.

9-mavzu. Teylor formulasi. Ba'zi bir elementar funksiyalar uchun Teylor formulalari. (2 soat)

1. $\cos x$, $\ln(1+x)$, $(1+x)^\alpha$ funksiyalar uchun Lagranj ko`rinishdagi qoldiq hadli Makloren formulalarini yozing.
2. $\sin x\approx x-x^3/6$ taqribiy formulaning $|x|\le 1/2$ dagi absolyut xatoligini baholang.
3. Juft, toq funksiyalar uchun Makloren formulasi qanday xususiyatga ega?

4. $(1+x)^n$ funksiya uchun Makloren formulasini yozing. Uni Nьyuton binomi formulasi bilan solishtiring.
5. Teylor formulasi yordamida taqribiy hisoblashda xatolik qanday baholanadi?
6. e ni 10^{-7} aniqlikda hisoblang.
7. $\sqrt{10}$ ni 10^{-3} aniqlikda hisoblang.

Adabiyotlar: [1], 1-t., 214-226 b.; [3], 146-150 b.

10-mavzu. Funksiyaning doimiylik sharti. Funksiyaning nuqtadagi va to`plamdagi monotonlik sharti. Maksimum va minimumlar. Ekstremumning zaruriy sharti.(3 soat)

1. Funksiyaning doimiylik shartidan foydalanib $arcsinx=\pi/2-arccosx$ ayniyatni isbotlang.
2. Funksiyaning nuqtadagi monotonligi deganda nimani tushinasiz?
3. Agar $f'(x_0)=0$ bo`lsa, $x_0$ nuqtada $f(x)$ funksiyaning o`sishi yoki kamayishi haqida nima deyish mumkin? ($y=x^2$, $y=x^3$, $y=-x^3$ funksiyalarni $x_0=0$ nuqtada tekshiring, xulosa chiqaring)
4. Agar $f(x)$ funksiya $[a;b]$ kesmada uzluksiz va uning $f'(x)$ hosilasi $(a;b)$ intervalda manfiy bo`lsa, u holda bu funksiya $[a;b]$ da qat'iy kamayuvchi ekanligini isbotlang.
5. Parametrik tenglama bilan berilgan funksiya uchun to`plamdagi monotonlik sharti qanday bo`ladi?
6. Parametrik tenglama bilan berilgan funksiya uchun ekstremumning zaruriy shartini yozing.
7. 10-vazifa.

Adabiyotlar: [1], 1-t., 219-223 b.; [3], 210-214 b.

11-mavzu. Ekstremumning etarli shartlari. Eng kichik va eng katta qiymatlarini izlash.(3 soat)

1. 12-vazifa
2. Interval, yarimintervalda berilgan funksiyalarning eng katta va eng kichik qiymatlari haqida nima deyish mumkin?
3. $f(x)=2x^3+3x^2-120x+100$ funksiyaning $(-4;5]$ yarimintervalda eng katta va eng kichik qiymatlari mavjudmi? Mavjud bo`lsa, nimaga teng?
4. $f(x)=lnx-x$ funksiyaning $(0;+\infty)$ da $inff$ va $supf$ ni toping.
5. Parametrik tenglama bilan berilgan funksiya uchun ekstremumning etarli sharti nimadan iborat?

Adabiyotlar: [1], 1-t., 224-228 b.; [3], 1-t., 215-225 b.

12-mavzu. Funksiyaning qavariqligi, burilish nuqtasi. Asimptotalar. Hosilaning funksiya grafigini yasashga tatbiqi.(3 soat)

1. 11-vazifa,
2. Funksiyaning burilish nuqtasi uning ekstremum nuqtasi bo`lishi mumkinmi?
3. $(-\infty,+\infty)$ da qavariq (botiq) funksiya bittadan ortiq ekstremumga ega bo`lishi mumkinmi?
4. Har qanday ikki marta differensiallanuvchi funksiya uchun quyidagi tasdiqlarning o`rinli ekanligini isbotlang:

1) Ikki ekstremum nuqtasi orasida kamida bitta burilish nuqtasi mavjud.
2) Funksiyaning burilish nuqtalari orasida ekstremum nuqtasi bo`lmasligi ham mumkin.

5. Har qanday toq darajali (chiziqli bo`lmagan) ko`phad kamida bitta burilish nuqtasiga ega. Isbotlang.
6. Musbat koeffitsientli juft darajali ko`phad burilish nuqtasiga ega emas.
7. Parametrik tenglama bilan berilgan funksiyaning asimptotasi qanday izlanadi?
8. Parametrik tenglama bilan berilgan funksiyaning burilish nuqtasi qanday izlanadi?

Adabiyotlar: [1], 1-t., 238-245 b.; [3], 158-167 b.

INDIVIDUAL VASIFALAR

1-vazifa. Hosila ta'rifidan foydalanib, quyidagi funksiyalarning hosilalarini toping.

1. $f(x)=5^{4x-3}$;
2. $f(x)=4\sin(2-3x)$;
3. $f(x)=\ln(2+5x)$;
4. $f(x)=ctg(1-3x)$;
5. $f(x)=tg(3x-4)$;
6. $f(x)=2x^3-4x-6$;
7. $f(x)=\log_2(4x-3)$;
8. $f(x)=\sin^2 2x$;
9. $f(x)=\dfrac{1}{x^2+4}$
10. $f(x)=\sqrt{\sin x}$
11. $f(x)=\dfrac{1}{x^2-5}$
12. $f(x)=\dfrac{2}{3x-x^2}$
13. $f(x)=\sqrt[3]{x-4}$
14. $f(x)=\sqrt{x^2+5}$
15. $f(x)=\sqrt[3]{5-9x}$
16. $f(x)=x^4+3x^2+6$
17. $f(x)=tg^2 x$
18. $f(x)=4\sin 3x+5x$
19. $f(x)=e^{8x}-x^2$
20. $f(x)=\cos^2 x+1$
21. $f(x)=ctg^2 x$
22. $f(x)=\ln^2 x+x$
23. $f(x)=x^3-4x^2+5x-1$
24. $f(x)=1-3\sin^2 x$
25. $f(x)=4^{2x-7}$
26. $f(x)=\sqrt{4x-3}+x^2$
27. $f(x)=\dfrac{3}{x^2-6}$
28. $f(x)=\sqrt{ctgx}$
29. $f(x)=(1+2x)^2$;
30. $f(x)=x+\lg(1-x)$.

2-vazifa. Hosilalar jadvali va differentsiallash qoidalaridan foydalanib, berilgan funksiyalarning hosilalarini toping.

1. a) $y=\dfrac{2\left(3x^3+4x^2-x-2\right)}{15\sqrt{1+x}}$,

b) $y=x-\ln\left(2+e^x+2\sqrt{e^{2x}+e^x+1}\right)$

c) $y=\sqrt{x}\ln\left(\sqrt{x}+\sqrt{x+a}\right)-\sqrt{x+a}$,

d) $y=\sin\sqrt{3}+\dfrac{1}{3}\dfrac{\sin^2 3x}{\cos 6x}$,

e) $y=\operatorname{arctg}\dfrac{tgx-ctgx}{\sqrt{2}}$

e) $y=\dfrac{1}{4\sqrt{5}}\ln\dfrac{2+\sqrt{5}thx}{2-\sqrt{5}thx}$,

i) $y=(arctgx)^{\left(\frac{1}{2}\right)\ln(arctgx)}$,

j) $y=\dfrac{1}{24}\left(x^2+8\right)\sqrt{x^2-4}+\dfrac{x^4}{16}\arcsin\dfrac{2}{x}$, $x>0$,

k) $y=\dfrac{x\arcsin x}{\sqrt{1-x^2}}+\ln\sqrt{1-x^2}$,

l) $y=\dfrac{1}{\sin\alpha}\ln(tgx+ctg\alpha)$.

2. a) $y=\dfrac{\left(2x^2-1\right)\sqrt{1+x^2}}{3x^3}$,

b) $y=e^{2x}\dfrac{2-\sin 2x-\cos 2x}{8}$,

c) $y=\ln\left(x+\sqrt{a^2+x^2}\right)$,

d) $y=\cos\ln 2+\dfrac{1}{3}\dfrac{\cos^2 3x}{\sin 6x}$,

e) $y=\arcsin\dfrac{\sqrt{x}-2}{\sqrt{5x}}$,

e) $y=\dfrac{shx}{4ch^4 x}+\dfrac{3chx}{8ch^2 x}+\dfrac{3}{8}\operatorname{arctg}(shx)$,

i) $y=\left(\sin\sqrt{x}\right)^{\ln\left(\sin\sqrt{x}\right)}$,

j) $y=\dfrac{4x+1}{16x^2+8x+3}+\dfrac{1}{\sqrt{2}}\operatorname{arctg}\dfrac{4x+1}{\sqrt{2}}$,

k) $y=4\ln\dfrac{x}{1+\sqrt{1-4x^2}}-\dfrac{\sqrt{1-4x^2}}{x^2}$,

l) $y=x\cos\alpha+\sin\alpha\ln\sin(x-\alpha)$.

3. a) $y=\dfrac{x^4-8x^2}{2\left(x^2-4\right)}$, e) $y=\dfrac{2}{3}\operatorname{cth}x-\dfrac{\operatorname{ch}x}{3\operatorname{sh}^3x}$,

b) $y=\dfrac{-e^{3x}}{3\operatorname{sh}^3x}$, i) $y=(\sin x)^{5e^x}$,

c) $y=2\sqrt{x-4}\ln\left(2+\sqrt{x}\right)$, j) $y=ln\left(2x-3+\sqrt{4x^2-12x+10}\right)-\sqrt{4x^2-12x+10}\,arctg(2x-3)$,

d) $y=\sin^3\cos 2-\dfrac{\cos^2 30x}{60\sin 60x}$, k) $y=x\left(2x^2+5\right)\sqrt{x^2+1}+3ln\left(x+\sqrt{x^2+1}\right)$,

e) $y=\operatorname{arctg}\dfrac{\sqrt{1+x^2}-1}{x}$, l) $y=\operatorname{arctg}\left(\dfrac{\cos x}{\sqrt[4]{\cos 2x}}\right)$.

4. a) $y=\dfrac{2x^2-x-1}{3\sqrt{2+4x}}$, e) $y=-\dfrac{\operatorname{ch}x}{2\operatorname{sh}^2x}-\dfrac{1}{2}\ln\operatorname{th}\dfrac{x}{2}$,

b) $y=\operatorname{arctg}\left(e^x-e^{-x}\right)$, i) $y=(\arcsin x)^{e^x}$,

c) $y=\ln\dfrac{x^2}{\sqrt{1-ax^4}}$, j) $y=(3x+1)^4\arcsin\dfrac{1}{3x+1}+\left(3x^2+2x+1\right)\sqrt{9x^2+6x}$,

$3x+1>0$

d) $y=\cos^2\sin 3+\dfrac{\sin^2 29x}{29\cos 58x}$, k) $y=x^3\arcsin x+\dfrac{x^2+2}{3}\sqrt{1-x^2}$,

e) $y=\arccos\dfrac{x^2-4}{\sqrt{x^4+16}}$, l) $y=\operatorname{arctg}\dfrac{\sqrt{\sqrt{x^4+1}-x^2}}{x}$, $x>0$.

5. a) $y=\dfrac{\left(1+x^8\right)\sqrt{1+x^8}}{12x^{12}}$, e) $y=\dfrac{1}{2}\left[\dfrac{\operatorname{sh}x}{\operatorname{ch}^2x}+\operatorname{arctg}(\operatorname{sh}x)\right]$,

b) $y=\dfrac{e^x}{2}\left[\left(x^2+1\right)\cos x+(x-1)^2\sin x\right]$, i) $y=(\ln x)^{3^x}$,

c) $y=\ln\left(\sqrt{x}+\sqrt{x+1}\right)$, j) $y=\arcsin e^{-2x}+\ln\left(e^{2x}+\sqrt{e^{4x}-1}\right)$,

d) $y=\sin\ln\dfrac{1}{2}+\dfrac{\sin^2 25x}{25\cos 50x}$, k) $y=3\arcsin\dfrac{3}{4x+1}+2\sqrt{4x^2+2x-2}$, $4x+1>0$

e) $y=\sqrt{\dfrac{2}{3}}\operatorname{arctg}\dfrac{3x-1}{\sqrt{6x}}$, l) $y=\dfrac{1}{a\left(1+a^2\right)}\left[\operatorname{arctg}(a\cos x)+a\ln\operatorname{tg}\dfrac{x}{2}\right]$.

6. a) $y=\dfrac{x^2}{2\sqrt{1-3x^4}}$, e) $y=\dfrac{\operatorname{sh}x}{2\operatorname{ch}^2x}+\dfrac{1}{2}\operatorname{arctg}(\operatorname{sh}x)$,

b) $y=e^{\sin x}\left(x-\dfrac{1}{\cos x}\right)$, i) $y=x^{\arcsin x}$,

c) $y=\ln\dfrac{a^2+x^2}{a^2-x^2}$, j) $y=\dfrac{1}{x}\sqrt{1-4x^2}+\ln\dfrac{1+\sqrt{1-4x^2}}{2x}$,

d) $y=\operatorname{ctg}\sin\dfrac{1}{13}-\dfrac{1}{48}\dfrac{\cos^2 24x}{\sin 48x}$, k) $y=\sqrt{1+x^2}\operatorname{arctg}x-\ln\left(x+\sqrt{1+x^2}\right)$,

e) $y=\dfrac{1}{4}\ln\dfrac{x-1}{x+1}-\dfrac{1}{2}\operatorname{arctg}x$, l) $y=\dfrac{1}{2}\ln\dfrac{1+\cos x}{1-\cos x}-\dfrac{1}{\cos x}-\dfrac{1}{3\cos^3 x}$.

7. a) $y=\dfrac{\left(x^2-6\right)\sqrt{\left(4+x^2\right)^3}}{120x^5}$, e) $y=-\dfrac{shx}{2ch^2x}-\dfrac{1}{shx}-\dfrac{3}{2}\operatorname{arctg}shx$,

b) $y=\dfrac{1}{m\sqrt{ab}}\operatorname{arctg}\left(e^{mx}\sqrt{\dfrac{a}{b}}\right)$, i) $y=(\operatorname{ctg}3x)^{2e^x}$,

c) $y=\ln^2(x+\cos x)$, j) $y=\sqrt{49x^2+1}\operatorname{arctg}7x-\ln\left(7x+\sqrt{49x^2+1}\right)$,

d) $y=\ln\cos\dfrac{1}{3}+\dfrac{\sin^2 23x}{23\cos 46x}$, k) $y=2\arcsin\dfrac{2}{3x+4}+\sqrt{9x^2+24x+12}$, $3x+4>0$

e) $y=\dfrac{(1+x)\operatorname{arctg}\sqrt{x}}{x^2}+\dfrac{1}{3x\sqrt{x}}$, l) $y=\dfrac{3^x\left((\ln 3)\sin 2x-2\cos 2x\right)}{\ln^2 3+4}$.

8. a) $y=\dfrac{\left(x^2-8\right)\sqrt{x^2-8}}{6x^3}$, e) $y=\dfrac{3}{2}\ln th\dfrac{x}{2}+chx-\dfrac{chx}{2sh^2x}$,

b) $y=\dfrac{e^{x^3}}{1+x^3}$, i) $y=x^{e^{tgx}}$,

c) $y=\ln^3(1+\cos x)$, j) $y=\ln\left(e^{3x}+\sqrt{e^{6x}-1}\right)+\arcsin e^{-3x}$,

d) $y=\cos\ln 13-\dfrac{1}{44}\dfrac{\cos^2 22x}{\sin 44x}$, k) $y=x\left(2x^2+1\right)\sqrt{x^2+1}-\ln\left(x+\sqrt{x^2+1}\right)$,

e) $y=\dfrac{x^3}{3}\arccos x-\dfrac{2+x^2}{9}\sqrt{1-x^2}$, l) $y=2\dfrac{\cos x}{\sin^4 x}+3\dfrac{\cos x}{\sin^2 x}$.

9. a) $y=\dfrac{4+3x^3}{x^3\sqrt{\left(2+x^3\right)^2}}$, e) $y=\dfrac{1}{2}\operatorname{arctg}(shx)-\dfrac{shx}{2ch^2x}$,

b) $y=x-\ln\left(1+e^x\right)-2e^{\frac{x}{2}}\operatorname{arctge}^{\frac{x}{2}}-\left(\operatorname{arctge}^{\frac{x}{2}}\right)^2$, i) $y=(tgx)^{4e^x}$,

c) $y=\ln\dfrac{x^2}{1-x^2}$, j) $y=\dfrac{1}{\sqrt{2}}\operatorname{arctg}\dfrac{2x+1}{\sqrt{2}}+\dfrac{2x+1}{4x^2+4x+3}$,

d) $y=\sqrt{\operatorname{tg}4}+\dfrac{\sin^2 21x}{21\cos 42x}$, k) $y=\ln\left(x+\sqrt{x^2+1}\right)-\dfrac{\sqrt{1+x^2}}{x}$,

e) $y=\dfrac{1}{2\sqrt{x}}+\dfrac{1+x}{2x}\operatorname{arctg}\sqrt{x}$, l) $y=\dfrac{\ln(\operatorname{ctg}x+\operatorname{ctg}\alpha)}{\sin\alpha}$.

10. a) $y=\sqrt[3]{\dfrac{\left(1+x^{3/4}\right)^2}{x^{3/2}}}$, e) $y=\dfrac{8}{3}\operatorname{cth}2x-\dfrac{1}{3chx\cdot ch^3x}$,

b) $y=x-e^{-x}\arcsin e^{-x}-\ln\left(1+\sqrt{1-e^{2x}}\right)$, i) $y=(\cos 5x)^{e^x}$,

c) $y=\ln\operatorname{tg}\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)$, j) $y=\ln\left(5x+\sqrt{25x^2+1}\right)-\sqrt{25x^2+1}\cdot\operatorname{arctg}5x$,

d) $y=\operatorname{ctg}\cos 5-\dfrac{1}{40}\dfrac{\cos^2 20x}{\sin 40x}$, k) $y=\sqrt{1-3x-2x^2}+\dfrac{3}{2\sqrt{2}}\arcsin\dfrac{4x+3}{\sqrt{17}}$,

e) $y=\frac{3+x}{2}\sqrt{x(2-x)}+3\arccos\sqrt{\frac{x}{2}}$,

l) $y=\frac{2^x(\sin x+\cos x\ln 2)}{1+(\ln 2)^2}$.

11. a) $y=\frac{x^6+x^3-2}{\sqrt{1-x^3}}$,

e) $y=\frac{1-8\operatorname{ch}^2 x}{4\operatorname{ch}^4 x}$,

b) $y=\ln\left(e^x+\sqrt{e^{2x}-1}\right)+\arcsin e^{-x}$,

i) $y=\left(x^3+4\right)^{\operatorname{tg}x}$,

c) $y=\ln\sqrt[4]{\frac{1+2x}{1-2x}}$,

j) $y=\arcsin e^{-4x}+\ln\left(e^{4x}+\sqrt{e^{8x}-1}\right)$,

d) $y=\frac{\operatorname{tg}\ln 2\cdot\sin^2 19x}{19\cos 38x}$,

k) $y=\sqrt{(4+x)(1+x)}+3\ln\left(\sqrt{4+x}+\sqrt{1+x}\right)$,

e) $y=\frac{4+x^4}{x^3}\operatorname{arctg}\frac{x^2}{2}+\frac{4}{x}$,

l) $y=\frac{5^x(\sin 3x\ln 5-3\cos 3x)}{9+\ln^2 5}$.

12. a) $y=\frac{(x^2-2)\sqrt{4+x^2}}{24x^3}$,

e) $y=-\frac{1}{4}\arcsin\frac{5+3\operatorname{ch}x}{3+5\operatorname{ch}x}$,

b) $y=x+\frac{8}{1+e^{x/4}}$,

i) $y=x^{\sin x^3}$,

c) $y=\ln\sin\frac{2x+4}{x+1}$,

j) $y=\frac{2x-1}{4x^2-4x+3}+\frac{1}{\sqrt{2}}\operatorname{arctg}\frac{2x-1}{\sqrt{2}}$,

d) $y=\frac{\sin\operatorname{tg}\frac{1}{7}\cos^2 16x}{32\sin 32x}$,

k) $y=\ln\frac{\sqrt{x^2-x+1}}{x}+\sqrt{3}\operatorname{arctg}\frac{2x-1}{\sqrt{3}}$,

e) $y=\arcsin\sqrt{\frac{x}{x+1}}+\operatorname{arctg}\sqrt{x}$,

l) $y=\frac{\cos x}{\sin^2 x}-2\cos x-3\ln\operatorname{tg}\frac{x}{2}$.

13. a) $y=\frac{1+x^2}{2\sqrt{1+2x^2}}$,

e) $y=\frac{1}{\sqrt{8}}\arcsin\frac{3+\operatorname{ch}x}{1+3\operatorname{ch}x}$,

b) $y=x-3\ln\left[\left(1+e^{x/6}\right)\sqrt{1+e^{x/3}}\right]-3\operatorname{arctg}e^{x/6}$,

i) $y=\left(x^4+5\right)^{\operatorname{ctg}x}$,

c) $y=\log_{16}\log_5\operatorname{tg}x$,

j) $y=\ln\left(e^{5x}+\sqrt{e^{10x}-1}\right)+\arcsin\left(e^{-5x}\right)$,

d) $y=\frac{\cos\operatorname{tg}\frac{1}{3}\sin^2 15x}{15\cos 30x}$,

k) $y=4\arcsin\frac{4}{2x+3}+\sqrt{4x^2+12x-7}$,

$2x+3>0$

e) $y=\frac{1}{2}\sqrt{\frac{1}{x^2}-1}-\frac{\arccos x}{2x^2}$,

l) $y=\frac{4^x((\ln 4)\sin 4x-4\cos 4x)}{16+\ln^2 4}$.

14. a) $y=\frac{\sqrt{x-1}(3x+2)}{4x^2}$,

e) $y=-\frac{\operatorname{sh}x}{2\operatorname{ch}^2 x}+\frac{3}{2}\arcsin(\operatorname{th}x)$,

b) $y=x+\frac{1}{1+e^x}-\ln\left(1+e^x\right)$,

i) $y=(\sin x)^{5^{x/2}}$,

c) $y = \log_4 \log_2 tgx$,

d) $y = \dfrac{\cos ctg3 \cdot \cos^2 14x}{28 \sin 28x}$,

$3x + 1 > 0$,

e) $y = 6 \arcsin \dfrac{\sqrt{x}}{2} + \dfrac{6+x}{2}\sqrt{x(4-x)}$,

j) $y = \dfrac{1}{\sqrt{2}} arctg \dfrac{x-1}{\sqrt{2}} + \dfrac{x-1}{x^2 - 2x + 3}$,

k) $y = 2 \arcsin \dfrac{2}{3x+1} + \sqrt{9x^2 + 6x - 3}$,

l) $y = \dfrac{3^x (4 \sin 4x + kn3 \cos 4x)}{16 + \ln^2 3}$.

15. a) $y = \dfrac{\sqrt{(1+x^2)^3}}{3x^3}$,

b) $y = e^{\alpha x}\left[\dfrac{1}{2a} + \dfrac{a \cos 2bx + 2b \sin 2bx}{2(a^2 + 4b^2)}\right]$

c) $y = x \dfrac{\cos \ln x + \sin \ln x}{2}$

d) $y = 8 \sin ctg3 + \dfrac{1}{5}\dfrac{\sin^2 5x}{\cos 10x}$,

e) $y = \dfrac{x}{2\sqrt{1-4x^2}} \arcsin 2x + \dfrac{1}{8}\ln(1-4x^2)$,

e) $y = arctg \dfrac{\sqrt{sh2x}}{chx - shx}$,

i) $y = x^{29^x} 29^x$,

j) $y = \sqrt{9x^2 - 12x + 5} arctg(3x-2) - \ln\left(3x - 2 - \sqrt{9x^2 - 12x + 5}\right)$

k) $y = x^3 \arccos x - \dfrac{x^2 + 2}{3}\sqrt{1-x^2}$,

l) $y = x \cos\alpha + \sin\alpha \ln \sin(x - \alpha)$.

16. a) $y = \dfrac{2(3x^3 + 4x^2 - x - 2)}{15\sqrt{1+x}}$,

b) $y = e^{2x} \dfrac{2 - \sin 2x - \cos 2x}{8}$,

c) $y = 2\sqrt{x-4} \ln\left(2 + \sqrt{x}\right)$,

e) $y = \arccos \dfrac{x^2 - 4}{\sqrt{x^4 + 16}}$,

l) $y = \ln\left(x + \sqrt{x^2 + 1}\right) - \dfrac{\sqrt{1+x^2}}{x}$,

e) $y = \dfrac{1}{2}\left[\dfrac{shx}{ch^2 x} + arctg(shx)\right]$,

i) $y = x^{\arcsin x}$,

j) $y = \sqrt{49x^2 + 1} arctg7x - \ln\left(7x + \sqrt{49x^2 + 1}\right)$,

k) $y = \ln\left(e^{3x} + \sqrt{e^{6x} - 1}\right) + \arcsin e^{-3x}$,

l) $y = \dfrac{2^x (\sin x + \cos x \ln 2)}{1 + (\ln 2)^2}$

17. a) $y = \dfrac{\sqrt{2x+3}(x-2)}{x^2}$,

b) $y = e^{\alpha x} \dfrac{(\alpha \sin \beta x - \beta \cos \beta x)}{\alpha^2 + \beta^2}$,

c) $y = \lg \ln ctgx$,

d) $y = \dfrac{1}{3}\cos tg\dfrac{1}{2} + \dfrac{1}{10}\dfrac{\sin^2 10x}{\cos 20x}$,

e) $y = arctgx + \dfrac{5}{6}\ln\dfrac{x^2 + 1}{x^2 + 4}$,

e) $y = -\dfrac{sh3x}{\sqrt{ch6x}}$,

i) $y = x^{3^x} 2^x$,

j) $y = \dfrac{x+2}{x^2 + 4x + 6} + \dfrac{1}{\sqrt{2}} arxtg \dfrac{x+2}{\sqrt{2}}$,

k) $y = \sqrt{x^2 + 1} - \dfrac{1}{2}\ln\dfrac{\sqrt{x^2 + 1} - x}{\sqrt{x^2 + 1} + 1}$,

l) $y = \ln\dfrac{\sqrt{2} + thx}{\sqrt{2} - thx}$.

18. a) $y = (1 - x^2)\sqrt[5]{x3 + \dfrac{1}{x}}$,

e) $y = \dfrac{chx}{\sqrt{sh2x}}$,

b) $y = 2(x-2)\sqrt{1+e^x} - 2\ln\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right)$, i) $y = x^{e^{ctgx}}$,

c) $y = \ln \arcsin \sqrt{1-e^{2x}}$, j) $y = \ln\frac{1+2\sqrt{-x-x^2}}{2x+1} + \frac{4}{2x+1}\sqrt{-x-x^2}$,

d) $y = \sqrt[3]{ctg2} - \frac{1}{20}\frac{\cos^2 10x}{\sin 20x}$, k) $y = \ln\sqrt[3]{\frac{x-1}{x+1}} - \frac{1}{2}\left(\frac{1}{2} + \frac{1}{x^2-1}\right)arctgx$,

e) $y = \arcsin\frac{x-2}{(x-1)\sqrt{2}}$, l) $y = arctg\frac{\sqrt{2tgx}}{1-tgx}$.

19. a) $y = \frac{(2x^2+3)\sqrt{x^2-3}}{9x^3}$, e) $y = -\frac{shx}{1+chx}$,

b) $y = 2\frac{\sqrt{2^x-1} - arctg\sqrt{2^x-1}}{\ln 2}$, i) $y = x^{e^{\cos x}}$,

c) $y = \ln \arccos \sqrt{1-e^{4x}}$, j) $y = \ln\left(4x-1+\sqrt{16x^2-8x+2}\right) - \sqrt{16x^2-8x+2}\,arctg(4x-1)$,

d) $y = ctg\cos 2 + \frac{1}{6}\frac{\sin^2 6x}{\cos 12x}$, k) $y = x\ln\left(\sqrt{1-x}+\sqrt{1+x}\right) + \frac{1}{2}(\arcsin x - x)$,

e) $y = \sqrt{1-x^2} - x\arcsin\sqrt{1-x^2}$, l) $y = \frac{6^x(\sin 4x\ln 6 - 4\cos 4x)}{16+\ln^2 6}$.

20. a) $y = \frac{x-1}{(x^2+5)\sqrt{x^2+5}}$, e) $y = \sqrt[4]{\frac{1+thx}{1-thx}}$,

b) $y = \ln(e^x+1) + \frac{18e^{2x}+27e^x+11}{6(e^x+1)}$, i) $y = x^{2^x}5^x$,

c) $y = \ln\left(bx+\sqrt{a^2+b^2x^2}\right)$, j) $y = 3x - \ln\left(1+\sqrt{1-e^{6x}}\right) - e^{-3x}\arcsin(e^{3x})$,

d) $y = \cos ctg2 - \frac{1}{16}\frac{\cos^2 8x}{\sin 16x}$, k) $y = arctg\sqrt{x^2-1-\frac{\ln x}{\sqrt{x^2-1}}}$,

e) $y = arctg\frac{\sqrt{1-x}}{1-\sqrt{x}}$, l) $y = \frac{ctgx+x}{1-xctgx}$.

21. a) $y = \frac{(2x+1)\sqrt{x^2-x}}{x^2}$, e) $y = \frac{1}{6}\ln\frac{1-sh2x}{2+sh2x}$,

b) $y = \frac{1}{2}\ln(e^{2x}+1) - 2arctge^x$, i) $y = x^{e^{\sin x}}$,

c) $y = \ln\left(\arccos\frac{1}{\sqrt{x}}\right)$, j) $y = \frac{1}{\sqrt{2}}arctg\frac{3x-1}{\sqrt{2}} + \frac{1}{3}\frac{3x-1}{x^2-2x+1}$,

d) $y = \frac{\cos\ln 7\cdot\sin^2 7x}{7\cos 14x}$, k) $y = 3\arcsin\frac{3}{x+2} + \sqrt{x^2+4x-5}$,

e) $y = (2x^2+6x+5)arctg\frac{x+1}{x+2} - x$, l) $y = (1+x^2)e^{arctgx}$.

22. a) $y = 2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$, e) $y = arctg\frac{\sqrt{sh2x}}{chx-shx}$,

b) $y=\frac{2}{3}\sqrt{\left(\operatorname{arctge}^{x}\right)^{3}}$,

c) $y=\ln\left(e^{x}+\sqrt{1+e^{2x}}\right)$,

d) $y=\frac{\sin\cos 3\cos^{2}2x}{4\sin 4x}$,

e) $y=\operatorname{arctg}\frac{\operatorname{tg}\left(\frac{x}{2}\right)+1}{2}$,

i) $y=x^{e^{\operatorname{arctg}x}}$,

j) $y=\frac{x^{4}}{81}\arcsin\frac{3}{x}+\frac{1}{81}\left(x^{2}+18\right)\sqrt{x^{2}-9}$, $x>0$

k) $y=\sqrt{(3-x)(2+x)}+5\arcsin\sqrt{\frac{x+2}{5}}$,

l) $y=-\frac{1}{3\sin^{3}x}-\frac{1}{\sin x}+\frac{1}{2}\ln\frac{1+\sin x}{1-\sin x}$.

23. a) $y=\frac{1}{(x+2)\sqrt{x^{2}+4x+5}}$,

b) $y=2\sqrt{e^{x}+1}+\ln\frac{\sqrt{e^{x}+1}-1}{\sqrt{e^{x}+1}+1}$,

c) $y=\ln\frac{\ln x}{\sin\left(\frac{1}{x}\right)}$,

d) $y=\frac{\cos\sin 5\sin^{2}2x}{2\cos 4x}$,

e) $y=\frac{2x-1}{4}\sqrt{2+x-x^{2}}+\frac{9}{8}\arcsin\frac{2x-1}{3}$,

e) $y=-\frac{1}{2}\ln\operatorname{th}\frac{x}{2}-\frac{\operatorname{ch}x}{2\operatorname{sh}^{2}x}$,

i) $y=x^{29^{x}}29^{x}$,

j) $y=\frac{2}{x-1}\sqrt{2x-x^{2}}+\ln\frac{1+\sqrt{2x-x^{2}}}{x-1}$,

k) $y=x(\arcsin x)^{2}+2\sqrt{1-x^{2}}\arcsin x-2x$,

l) $y=\ln\frac{\sin x}{\cos x+\sqrt{\cos 2x}}$.

24. a) $y=3\frac{\sqrt[3]{x^{2}+x+1}}{x+1}$,

b) $y=\frac{1}{\ln 4}\ln\frac{1+2^{x}}{1-2^{x}}$,

c) $y=\ln\ln\sin\left(1+\frac{1}{x}\right)$,

d) $y=\operatorname{ctg}\sqrt[3]{5}-\frac{1}{8}\frac{\cos^{2}4x}{\sin 8x}$,

e) $y=\frac{x-3}{2}\sqrt{6x-x^{2}-8}+\arcsin\sqrt{\frac{x}{2}-1}$,

e) $y=\frac{3}{8\sqrt{2}}\ln\frac{\sqrt{2}+\operatorname{th}x}{\sqrt{2}-\operatorname{th}x}-\frac{\operatorname{th}x}{4\left(2-\operatorname{th}^{2}x\right)}$,

i) $y=x^{e^{x}}x^{9}$,

j) $y=\sqrt{9x^{2}-12x+5}\operatorname{arctg}(3x-2)-\ln\left(3x-2-\sqrt{9x^{2}-12x+5}\right)$,

k) $y=\frac{\sqrt{1-x^{2}}}{x}+\operatorname{src}\sin x$,

l) $y=\frac{7^{x}\left(3\sin 3x+\cos 3x\ln 7\right)}{9+\ln^{2}7}$.

25. a) $y=3\sqrt[3]{\frac{x+1}{(x-1)^{2}}}$,

b) $y=\frac{1}{2}\operatorname{arctg}\frac{e^{x}-3}{2}$,

c) $y=\ln\ln^{3}\ln^{2}x$,

d) $y=\operatorname{tg}\lg\frac{1}{3}+\frac{1}{4}\frac{\sin^{2}4x}{\cos 8x}$,

e) $y=\frac{x}{2\sqrt{1-4x^{2}}}\arcsin 2x+\frac{1}{8}\ln\left(1-4x^{2}\right)$,

e) $y=\frac{1}{2}\ln\frac{1+\sqrt{\operatorname{th}x}}{1-\sqrt{\operatorname{th}x}}-\operatorname{arctg}\sqrt{\operatorname{th}x}$,

i) $y=(\operatorname{tg}x)^{\ln\operatorname{tg}x/4}$,

j) $y=2x-\ln\left(1+\sqrt{1-e^{4x}}\right)-e^{-2x}\arcsin\left(e^{2x}\right)$,

k) $y=x^{3}\arccos x-\frac{x^{2}+2}{3}\sqrt{1-x^{2}}$,

l) $y=3\frac{\sin x}{\cos^{2}x}+2\frac{\sin x}{\cos^{4}x}$.

3-vazifa. Hosilaning ta'rifidan foydalanib, f'(0) ni toping:

1. $f(x)=\begin{cases}1-\cos\left(x\sin\dfrac{1}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$

2. $f(x)=\begin{cases}\operatorname{arctg}\left(x\cos\dfrac{1}{5x}\right), & x\neq 0\\ 0, & x=0\end{cases}$.

3. $f(x)=\begin{cases}\operatorname{tg}\left(x^3+x^2\sin\dfrac{2}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$.

4. $f(x)=\begin{cases}\arcsin\left(x^2\cos\dfrac{1}{9x}\right)+\dfrac{2}{3}x, & x\neq 0\\ 0, & x=0\end{cases}$.

5. $f(x)=\begin{cases}\ln\left(1-\sin\left(x^3\sin\dfrac{1}{x}\right)\right), & x\neq 0\\ 0, & x=0\end{cases}$.

6. $f(x)=\begin{cases}\sin\left(x\sin\dfrac{3}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$.

7. $f(x)=\begin{cases}\sin\left(e^{x^2\sin\frac{5}{x}}-1\right)+x, & x\neq 0\\ 0, & x=0\end{cases}$.

8. $f(x)=\begin{cases}x^2\cos\dfrac{4}{3x}+\dfrac{x^2}{2}, & x\neq 0\\ 0, & x=0\end{cases}$.

9. $f(x)=\begin{cases}\operatorname{arctg}\left(x^3-x^{3/2}\sin\dfrac{1}{3x}\right), & x\neq 0\\ 0, & x=0\end{cases}$.

10. $f(x)=\begin{cases}\sin x\cos\dfrac{5}{x}, & x\neq 0\\ 0, & x=0\end{cases}$.

11. $f(x)=\begin{cases}x+\arcsin\left(x^2\sin\dfrac{6}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$.

12. $f(x)=\begin{cases}\operatorname{arctgx}\sin\dfrac{7}{x}, & x\neq 0\\ 0, & x=0\end{cases}$.

13. $f(x)=\begin{cases}2x^2+x^2\cos\dfrac{1}{9x}, & x\neq 0\\ 0, & x=0\end{cases}$.

14. $f(x)=\begin{cases}x^2\cos^2\left(\dfrac{11}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$.

15. $f(x)=\begin{cases}2x^2+x^2\cos\dfrac{1}{x}, & x\neq 0\\ 0, & x=0\end{cases}$.

16. $f(x)=\begin{cases}\operatorname{arctg}\left(x\cos\dfrac{1}{5x}\right), & x\neq 0\\ 0, & x=0\end{cases}$.

17. $f(x)=\begin{cases}\operatorname{arctg}\left(x\cos\dfrac{1}{5x}\right), & x\neq 0\\ 0, & x=0\end{cases}$.

18. $f(x)=\begin{cases}\dfrac{e^{x^2}-\cos x}{x}, & x\neq 0\\ 0, & x=0\end{cases}$.

19. $f(x)=\begin{cases}e^{x\sin 5x}, & x\neq 0\\ 0, & x=0\end{cases}$.

20. $f(x)=\begin{cases}3^{x^2\sin\left(2/x\right)}, & x\neq 0\\ 0, & x=0\end{cases}$.

21. $f(x)=\begin{cases}e^{x\sin\left(\frac{3}{5}x\right)}-1, & x\neq 0\\ 0, & x=0\end{cases}$.

22. $f(x)=\begin{cases}\dfrac{2^{\operatorname{tg}x}-2^{\sin x}}{x^2}, & x\neq 0\\ 0, & x=0\end{cases}$.

23. $f(x)=\begin{cases}\operatorname{arctg}\left(\dfrac{3x}{2}-x^2\sin\dfrac{1}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$.

24. $f(x)=\begin{cases}\dfrac{\ln\left(1+2x^2+x^3\right)}{x}, & x\neq 0\\ 0, & x=0\end{cases}$.

25. $f(x)=\begin{cases}\dfrac{\cos x-\cos 3x}{x}, & x\neq 0\\ 0, & x=0\end{cases}$.

4-vazifa. y=f(x) funksiyaning grafigiga abstsissasi $x=x_0$ nuqtada o`tkazilgan urinma va normal tenglamalarini yozing.

1. $y = x - x^3,\ x_0 = -1.$
2. $y = \dfrac{x^{16}+9}{1-5x^2},\ x_0 = 1.$
3. $y = \dfrac{x^2-3x+6}{x^2},\ x_0 = 3.$
4. $y = 2x^2-3x+1,\ x_0 = 1.$
5. $y = x + \sqrt{x^3},\ x_0 = 1.$
6. $y = \dfrac{4x-x^2}{4},\ x_0 = 2.$
7. $y = 2x^2+3,\ x_0 = -1.$
8. $y = \dfrac{x^{29}+6}{x^4+1},\ x_0 = 1.$
9. $y = \sqrt{x} - 3\sqrt[3]{x},\ x_0 = 64.$
10. $y = \dfrac{x^2-2x-3}{4},\ x_0 = 4.$
11. $y = x^2 + 8\sqrt{x} - 32,\ x_0 = 4.$
12. $y = \dfrac{3x-2x^3}{3},\ x_0 = 1.$
13. $y = 8\sqrt[4]{x} - 70,\ x_0 = 16.$
14. $y = \sqrt[3]{x^2} - 20,\ x_0 = -8.$
15. $y = \dfrac{x^2}{10},\ x_0 = 2.$
16. $y = \dfrac{1+3x^2}{3+x^2},\ x_0 = 1.$
17. $y = \dfrac{x^2-3x+3}{3},\ x_0 = 3.$
18. $y = \dfrac{1+3x^2}{3+x^2},\ x_0 = 1.$
19. $y = \dfrac{1}{3x+2},\ x_0 = 2.$
20. $y = \dfrac{x}{x^2+1},\ x_0 = -2.$
21. $y = \dfrac{x^5+1}{x^4+1},\ x_0 = 1.$
22. $y = 2x + \dfrac{1}{x},\ x_0 = 1.$
23. $y = 2x^2+3x-1,\ x_0 = -2.$
24. $y = \dfrac{1+\sqrt{x}}{1-\sqrt{x}},\ x_0 = 4.$
25. $y = \dfrac{x^{16}+9}{1-5x^2},\ x_0 = 1.$

5-vazifa. Berilgan funksiyaning differensiali dy va ikkinchi tartibli differensiali d^2y -ni toping.

1. $y = x\arcsin\dfrac{1}{x} + \ln\left|x+\sqrt{x^2-1}\right|,\ x>0.$
2. $y = \ln\left(\cos^2 x + \sqrt{1+\cos^4 x}\right).$
3. $y = \arccos\dfrac{1}{\sqrt{1+2x^2}},\ x>0.$
4. $y = \sqrt{1+2x} - \ln\left(x+\sqrt{1+2x}\right).$
5. $y = \ln\left(x+\sqrt{1+x^2}\right) - \sqrt{1+x^2}\,\mathrm{arctg}x.$
6. $y = \dfrac{\ln|x|}{1+x^2} - \dfrac{1}{2}\ln\dfrac{x^2}{1+x^2}.$
7. $y = \ln\left(e^x + \sqrt{e^{2x}-1}\right) + \arcsin e^{-x}.$
8. $y = x\sqrt{4-x^2} + 4\arcsin\dfrac{x}{2}.$
9. $y = \ln\mathrm{tg}\dfrac{x}{2} - \dfrac{x}{\sin x}.$
10. $y = 2x + \ln\left|\sin x + 2\cos x\right|.$
11. $y = \sqrt{\mathrm{ctg}x} - \sqrt{\mathrm{tg}^3\dfrac{x}{3}}.$
12. $y = 2x + \ln\left|\sin x + 2\cos x\right|.$
13. $y = \mathrm{arcth}\dfrac{x^2-1}{x}.$
14. $y = \ln\left|x^2-1\right| - \dfrac{1}{x^2-1}.$
15. $y = \mathrm{arctg}\left(\mathrm{tg}\dfrac{x}{2}+1\right).$
16. $y = \ln\left|2x + 2\sqrt{x^2+x}+1\right|.$
17. $y = e^x\left(\cos 2x + 2\sin 2x\right).$
18. $y = x\left(\sin\ln x - \cos\ln x\right).$
19. $y = \sqrt{3+x^2} - x\ln\left|x+\sqrt{3+x^2}\right|.$
20. $y = \arccos\dfrac{1}{\sqrt{1+2x^2}},\ x>0.$

21. $y = \arccos\dfrac{x^2-1}{x^2\sqrt{2}}$.

22. $y = \operatorname{tg}\left(2\arccos\sqrt{1-x^2}\right),\ x>0$.

23. $y = \sqrt{x} - (1+x)\operatorname{arctg}\sqrt{x}$

24. $y = \cos x \ln \operatorname{tg} x - \ln \operatorname{tg}\dfrac{x}{2}$.

25. $y = x^2\operatorname{arctg}\sqrt{x^2-1} - \sqrt{x^2-1}$

6-vazifa. Differensial yordamida taqribiy hisoblang:

1. $y = \sqrt[3]{x^3+7x}$, $x = 1{,}012$.
2. $y = \sqrt[3]{x}$, $x = 7{,}76$.
3. $y = \sqrt[5]{x^2}$, $x = 1{,}03$.
4. $y = x^5$, $x = 2{,}997$.
5. $y = \sqrt{x^3}$, $x = 0{,}98$.
6. $y = \sqrt{4x-3}$, $x = 1{,}78$.
7. $y = x^7$, $x = 2{,}002$.
8. $y = \dfrac{1}{\sqrt{x}}$, $x = 4{,}16$.
9. $y = \sqrt[3]{x}$, $x = 8{,}36$.
10. $y = \dfrac{1}{\sqrt{2x^2+x+1}}$, $x = 1{,}016$.
11. $y = \sqrt{4x-1}$, $x = 2{,}56$.
12. $y = \sqrt[3]{x}$, $x = 7{,}64$.
13. $y = x^7$, $x = 1{,}996$.
14. $y = \sqrt[3]{x}$, $x = 8{,}24$.
15. $y = \sqrt{4x-3}$, $x = 1{,}78$.
16. $y = \sqrt[3]{x}$, $x = 8{,}24$.
17. $y = x^{21}$, $x = 0{,}998$.
18. $y = \sqrt[3]{x}$, $x = 1{,}21$.
19. $y = x^{11}$, $x = 1{,}021$.
20. $y = \sqrt{x^2+x+3}$, $x = 1{,}97$.
21. $y = \sqrt[3]{x}$, $x = 26{,}46$.
22. $y = \sqrt[3]{x^2+2x+5}$, $x = 0{,}97$.
23. $y = \sqrt[3]{x}$, $x = 27{,}54$.
24. $y = \dfrac{x+\sqrt{5-x^2}}{2}$, $x = 0{,}98$.
25. $y = \arcsin x$, $x = 0{,}08$.

7-vazifa. Parametrik tenglama bilan berilgan funksiyaning hosilasini toping y'_x:

1. $\begin{cases} x = \dfrac{t}{\sqrt{1-t^2}} \\ y = \ln\dfrac{1+\sqrt{1-t^2}}{t} \end{cases}$.

2. $\begin{cases} x = \dfrac{3t^2+1}{3t^3} \\ y = \sin\left(\dfrac{t^3}{3}+t\right) \end{cases}$.

3. $\begin{cases} x = \sqrt{1-t^2} \\ y = \operatorname{tg}\sqrt{1-t} \end{cases}$.

4. $\begin{cases} x = \sqrt{2t-t^2} \\ y = \dfrac{1}{\sqrt[3]{(t-1)^2}} \end{cases}$.

5. $\begin{cases} x = \arcsin(\sin t) \\ y = \arccos(\cos t) \end{cases}$.

6. $\begin{cases} x = \ln\left(t+\sqrt{t^2+1}\right) \\ y = t\sqrt{t^2+1} \end{cases}$.

7. $\begin{cases} x = \sqrt{2t-t^2} \\ y = \arcsin(t-1) \end{cases}$.

8. $\begin{cases} x = \operatorname{ctg}\left(2e^t\right) \\ y = \ln \operatorname{tg} e^t \end{cases}$.

9. $\begin{cases} x = \ln \operatorname{ctg} x \\ y = \dfrac{1}{\cos^2 t} \end{cases}$.

10. $\begin{cases} x = \operatorname{arctg} e^{\frac{t}{2}} \\ y = \sqrt{e^t+1} \end{cases}$.

11. $\begin{cases} x = \ln\sqrt{\dfrac{1-t}{1+t}} \\ y = \sqrt{1-t^2} \end{cases}$.

12. $\begin{cases} x = \ln \dfrac{1}{\sqrt{1-t^4}} \\ y = \arcsin \dfrac{1-t^2}{1+t^2} \end{cases}$.

13. $\begin{cases} x = \sqrt{1-t^2} \\ y = \dfrac{t}{\sqrt{1-t^2}} \end{cases}$.

14. $\begin{cases} x = \arcsin \sqrt{1-t^2} \\ y = (\arccos t)^2 \end{cases}$.

15. $\begin{cases} x = (1+\cos^2 t)^2 \\ y = \dfrac{\cos t}{\sin^2 t} \end{cases}$.

16. $\begin{cases} x = ctg(2e^t) \\ y = \ln tge^t \end{cases}$.

17. $\begin{cases} x = \dfrac{3t^2+1}{3t^3} \\ y = \sin\left(\dfrac{t^3}{3}+t\right) \end{cases}$.

18. $\begin{cases} x = \dfrac{3t^2+1}{3t^3} \\ y = \sin\left(\dfrac{t^3}{3}+t\right) \end{cases}$.

19. $\begin{cases} x = \arcsin \sqrt{t} \\ y = \sqrt{1+\sqrt{t}} \end{cases}$.

20. $\begin{cases} x = (\arcsin t)^2 \\ y = \dfrac{t}{\sqrt{1-t^2}} \end{cases}$.

21. $\begin{cases} x = t\sqrt{t^2+x} \\ y = \ln \dfrac{1+\sqrt{1+t^2}}{t} \end{cases}$.

22. $\begin{cases} x = arctgt \\ y = \ln \dfrac{\sqrt{1+t^2}}{t+1} \end{cases}$.

23. $\begin{cases} x = \ln(1-t^2) \\ y = \arcsin \sqrt{1-t^2} \end{cases}$.

24. $\begin{cases} x = arctg \dfrac{t+1}{t-1} \\ y = \arcsin \sqrt{1-t^2} \end{cases}$.

25. $\begin{cases} x = \ln tgt \\ y = \dfrac{1}{\sin^2 t} \end{cases}$.

8-vazifa. Parametrik tenglama bilan berilgan funksiyaning ikkinchi tartibli hosilasini toping.

1. $\begin{cases} x = \sqrt{1-t^2} \\ y = \dfrac{1}{t} \end{cases}$.

2. $\begin{cases} x = \sin t - t\cos t \\ y = \cos t + t\sin t \end{cases}$.

3. $\begin{cases} x = 2(t-\sin t) \\ y = 4(2+\cos t) \end{cases}$.

4. $\begin{cases} x = arctgt \\ y = \dfrac{t^2}{2} \end{cases}$.

5. $\begin{cases} x = \cos t \\ y = \sin^4 \dfrac{t}{2} \end{cases}$.

6. $\begin{cases} x = e^t \\ y = \arcsin t \end{cases}$.

7. $\begin{cases} x = \cos t + t\sin t \\ y = \sin t - t\cos t \end{cases}$.

8. $\begin{cases} x = \cos t \\ y = \ln \sin t \end{cases}$.

9. $\begin{cases} x = t - \sin t \\ y = 2 - \cos t \end{cases}$.

10. $\begin{cases} x = t + \sin t \\ y = 2 + \cos t \end{cases}$.

11. $\begin{cases} x = \sin t \\ y = \ln \cos t \end{cases}$.

12. $\begin{cases} x = \sqrt{t-3} \\ y = \ln(t-2) \end{cases}$.

13. $\begin{cases} x = \cos^2 t \\ y = tg^2 t \end{cases}$.

14. $\begin{cases} x = \cos t + t\sin t \\ y = \sin t - t\cos t \end{cases}$.

15. $\begin{cases} x = tgt \\ y = \dfrac{1}{\sin 2t} \end{cases}$.

16. $\begin{cases} x = \dfrac{\cos t}{1+2\cos t} \\ y = \dfrac{\sin t}{1+2\cos t} \end{cases}$.

17. $\begin{cases} x = \sqrt{t} \\ y = \sqrt[3]{t-1} \end{cases}$.

18. $\begin{cases} x = \sqrt{t-1} \\ y = \dfrac{t}{\sqrt{t-1}} \end{cases}$.

19. $\begin{cases} x = tgt \\ y = \dfrac{1}{\sin 2t} \end{cases}$.

20. $\begin{cases} x = \sqrt{t} \\ y = \dfrac{1}{\sqrt{1-t}} \end{cases}$.

21. $\begin{cases} x = \dfrac{1}{t} \\ y = \dfrac{1}{1+t^2} \end{cases}$.

22. $\begin{cases} x = \operatorname{arctg}\dfrac{t+1}{t-1} \\ y = \arcsin\sqrt{1-t^2} \end{cases}$.

23. $\begin{cases} x = sh^2 t \\ y = \dfrac{1}{ch^2 t} \end{cases}$.

24. $\begin{cases} x = e^t \cos t \\ y = e^t \sin t \end{cases}$.

25. $\begin{cases} x = cos\,2t \\ y = 2\,sec^2 t \end{cases}$.

9-vazifa. Funksiyaning n **-nchi tartibli hosilani toping**:

1. $y = \sin 2x + \cos(x+1)$.
2. $y = 3^{2x+5}$.
3. $y = \log_3(x+5)$.
4. $y = \dfrac{x}{x+1}$.
5. $y = 2^{kx}$.
6. $y = a^{2x+3}$.
7. $y = \dfrac{5x+1}{13(2x+3)}$.
8. $y = \dfrac{4}{x}$.
9. $y = \ln(1+x)$.
10. $y = \dfrac{x}{9(4x+9)}$.
11. $y = 7^{5x}$.
12. $y = \ln(3x+1)$.
13. $y = \dfrac{4+15x}{5x+1}$.
14. $y = \dfrac{2x+5}{13(3x+1)}$.
15. $y = 2^{3x+5}$.
16. $y = \dfrac{2x+5}{13(3x+1)}$.
17. $y = \sqrt{x}$.
18. $y = \lg(x+4)$.
19. $y = \dfrac{x}{2(3x+2)}$.
20. $y = a^{3x}$.
21. $y = \lg(5x+2)$.
22. $y = \dfrac{4x+7}{2x+3}$.
23. $y = \sqrt[5]{e^{7x-1}}$.
24. $y = xe^{\alpha x}$.
25. $y = \ln(3x+1)$.

10-vazifa.
Айниятларни исботланг. (1-13)

1. $arccosx+arcsinx=\pi/2;$

2. $arctgx+arctg\frac{1-x}{1+x}=\pi/4$, agar $x>-1;$

3. $arctgx=arcsin\frac{1}{\sqrt{1+x^2}};$

4. $arctgx+arctg\frac{1-x}{1+x}=-3\pi/4$, agar $x<-1;$

5. $arcsin\frac{2x}{1+x^2}=\pi-arctgx$, agar $x\geq 1;$

6. $arcsinx=arctg\frac{x}{1-x^2}$, agar $-1<x<1;$

7. $cos2x=cos^2x-sin^2x;$

8. $arcsin\frac{2x}{1+x^2}=2arctgx$, agar $-1\leq x\leq 1;$

9. $sin^6x+cos^6x=1/4+3/4(cos^2x-sin^2x)^2;$

10. $cos^4x-1/8cos4x=2cos^2x-1/2cos2x-5/8;$

11. $arcsin\frac{2x}{1+x^2}=-\pi-2arctgx$, agar $x\leq -1;$

12. $arctgx=arcctg\frac{1}{x}$, agar $x>0;$

13. $0,5arccos(2x^2-1)=arccosx,\ x\geq 0$

Tengsizliklarni isbotlang (14-25)

14. $cosx>1-x^2/2$, agar $x>0;$
15. $sinx>x-x^3/6$, agar $x>0;$
16. $2\sqrt{x}>3-1/x$, agar $x>1;$
17. $e^x\geq 1+x+x^2/2;$ agar $x\geq 0;$
18. $ln(1+x)\leq x$, agar $x\geq 0;$
19. $ln(1+x)\geq x-x^2/2$, agar $x\geq 0;$
20. $\frac{\sin x}{x}>\frac{\sin y}{y}$, agar $0<x<y<\pi/2;$
21. $\frac{tgx}{x}<\frac{tgy}{y}$, agar $0<x<y<\pi/2;$
22. $x+x^3/3<tgx$, agar $0<x<\pi/2;$
23. $2x^4-4x^3+3x^2+4x+1>0$, agar $x>0;$
24. $2x+1/x^2>5$, agar $0<x<0,5;$
25. $x^5+(1-x)^5\geq 1/16,\ x\in R$

11-vazifa. Funktsiyani to`la tekshiring va grafigini yasang:

1. $y=\frac{x^3+4}{x^2}.$
2. $y=\frac{x^2-x+1}{x-1}.$
3. $y=\frac{2}{x^2+2x}.$
4. $y=\frac{4x^2}{3+x^2}.$
5. $y=\frac{x^2-3x+3}{x-1}.$
6. $y=\frac{4-x^3}{x^2}.$

7. $y=\frac{x^2-4x+1}{x-4}$.

8. $y=\frac{2x^3+1}{x^2}$.

9. $y=\frac{(x-1)^2}{x^2}$.

10. $y=\frac{x^2}{(x-1)^2}$.

11. $y=\left(1+\frac{1}{x}\right)^2$.

12. $y=\left(\frac{x-1}{x+1}\right)^2$.

13. $y=\frac{3x^4+1}{x^3}$.

14. $y=\frac{2x^3+1}{x^2}$.

15. $y=\frac{x^3-4}{x^2}$.

16. $y=\frac{1}{x^4-1}$.

17. $y=-\left(\frac{x}{x+2}\right)^2$.

18. $y=\frac{x^3-32}{x^2}$.

19. $y=\frac{3x-2}{x^3}$.

20. $y=\frac{x^3-4}{x^2}$.

21. $y=\frac{-8x}{x^2+4}$.

22. $y=\frac{4}{3+2x-x^2}$.

23. $y=\frac{4}{x^2+2x-3}$.

24. $y=\frac{12-3x^2}{x^2+12}$

25. $y=\frac{12x}{9+x^2}$

12-vazifa. Berilgan oraliqda y=f(x) funksiyaning eng katta va eng kichik qiymatlarini toping:

1. *$y=x^4-8x^2-9$, [-1,3]*
2. *$y=x^2-4|x|+4$, [-3,4]*
3. $y=x^2+\frac{16}{x}-16, \quad [1,\ 4]$.
4. $y=4-x-\frac{4}{x^2}, \quad [1,\ 4]$.
5. $y=\sqrt[3]{2(x-2)^2(8-x)}-1, \quad [0,\ 6]$.
6. $y=\frac{2(x^2+3)}{x^2-2x+5}, \quad [-3,\ 3]$.
7. $y=2\sqrt{x}-x, \quad [0,\ 4]$.
8. $y=1+\sqrt[3]{2(x-1)^2(x-7)}, \quad [-1,\ 5]$.
9. $y=x-4\sqrt{x}+5, \quad [1,\ 9]$.
10. $y=\sqrt[3]{2x^2(x-3)}, \quad [-1,\ 6]$.
11. $y=\frac{2(-x^2+7x-7)}{x^2-2x+2}, \quad [1,\ 4]$.
12. $y=x-4\sqrt{x+2}+8, \quad [-1,\ 7]$.

13. $y=\sqrt[3]{2(x-2)^2(5-x)}, \quad [1,\ 5].$

14. $y=\sqrt[3]{2x^2(x-6)}, \quad [-2,\ 4].$

15. $y=-\frac{x^2}{2}+\frac{8}{x}+8, \quad [-4,\ -1].$

16. $y=3-x-\frac{4}{(x+2)^2}, \quad [-1,\ 2].$

17. $y=\frac{-2x(2x+3)}{x^2+4x+5}, \quad [1,\ 4].$

18. $y=-\frac{2(x^2+3)}{x^2+2x+5}, \quad [-5,\ 1].$

19. $y=\sqrt[3]{2(x-1)^2(x-4)}, \quad [0,\ 4].$

20. $y=x^2-2x+\frac{16}{x-1}-13, \quad [2,\ 5].$

21. $y=2\sqrt{x-1}-x+2, \quad [1,\ 5].$

22. $y=x^2+4x+\frac{16}{x+2}-9, \quad [-1,\ 2].$

23. $y=-\frac{x^2}{2}+2x+\frac{8}{x-2}+5, \quad [-2,\ 1].$

24. $y=8x+\frac{4}{x^2}-15, \quad \left[\frac{1}{2},\ 2\right].$

25. $y=\sqrt[3]{2(x+2)^2(x-4)}+3, \quad [-4,\ 2].$

Yakuniy nazorat uchun savollar

1-semestr

1. Ratsional sonlar to`plamining zichlik xossasi.
2. Ratsional sonlar to`plamining tartiblanganlik xossasi.
3. Ratsional sonlar to`plamining kesimi, uning turlari.
4. Ratsional sonlar to`plamining kesimi yordamida irratsional sonni kiritish.
5. Ratsional sonlarni son o`qida tasvirlash.
6. $\sqrt{2}$ ning ratsional son emasligining isboti.
7. Haqiqiy sonlar to`plamining kesimi.
8. Haqiqiy sonlar to`plamining tartiblanganlik xossasi.
9. Haqiqiy sonlar to`plamining zichlik xossasi.
10. Haqiqiy sonlar to`plamining to`laligi(Dedekind teoremasi).
11. Haqiqiy sonning absolyut qiymati va uning xossalari.
12. Yuqoridan chegaralangan to`plam va uning aniq yuqori chegarasi haqidagi teorema.
13. Quyidan chegaralangan to`plam va uning aniq quyi chegarasi haqidagi teorema.
14. Funksiya va uning berilish usullari.
15. Funksiyaning analitik usulda berilishi, misollar.
16. Funksiyaning grafigi va funksiyaning grafik usulda berilishi, misollar.
17. Funksiyaning jadval usulda berilishi, misollar.
18. Monoton funksiyalar.
19. Juft va toq funksiyalar. Misollar.
20. Davriy funksiyalar. Misollar.
21. Chegaralangan va chegaralanmagan funksiyalar. Misollar.
22. Teskari funksiya. Misollar.
23. Murakkab funksiya. Misollar.
24. Sonli ketma-ketlik va uning limiti. Yaqinlashuvchi ketma-ketlik.
25. Yaqinlashuvchi ketma-ketlikning xossalari.
26. Qismiy ketma-ketlik. Misollar.
27. Qismiy ketma-ketliklar haqidagi Boltsano-Veyershtrass teoremasi.
28. Monoton ketma-ketlik va uning limiti.
29. Ikkinchi ajoyib limiti. e soni.
30. Ichma-ich joylashgan segmentlar ketma-ketligi.
31. Ketma-ketlik yaqinlashishining zaruriy va etarli sharti (Koshi kriteriyasi).
32. Funksiyaning nuqtadagi limitining Koshi ta’rifi.
33. Funksiyaning nuqtadagi limitining Geyne ta’rifi.
34. Bir tomonli limitlar.
35. Funksiya limitlari ustida arifmetik amallar.
36. Chekli limitga ega bo`lgan funksiyaning xossalari.
37. Birinchi ajoyib limit($\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$)ning isboti.
38. $\lim\limits_{x\to a} f(x)=\infty$ holatni tushuntiring.
39. $\lim\limits_{x\to \infty} f(x)=b$ holatni tushuntiring.
40. Ikkinchi ajoyib limit($\lim\limits_{x\to \infty}\left(1+\dfrac{1}{x}\right)^{x}=e$)ning isboti.
41. Monoton funksiyaning limiti.
42. Funksiyaning chekli limitga ega bo`lishining zaruriy va etarli sharti (Koshi kriteriyasi).
43. Cheksiz kichik funksiyalar va ularni taqqoslash.

44. Funksiyaning nuqtadagi uzluksizligining Koshi ta’rifi.
45. Funksiyaning nuqtadagi uzluksizligining Geyne ta’rifi.
46. Funksiyaning nuqtadagi uzluksizligining orttirma yordamida ta’riflash.
47. Bir tomonli uzluksizliklar.
48. Funksiyaning uzilish nuqtalari va ularning turlari.
49. Uzluksiz funksiyalar ustida arifmetik amallar.
50. Monoton funksiyaning uzluksizlik sharti.
51. Murakkab funksiyaning uzluksizligi.
52. Teskari funksiyaning mavjudligi va uzluksizlik shartlari.
53. Sonning haqiqiy darajasi va uning xossalari.
54. Ko`rsatkichli funksiya va uning xossalari.
55. Logarifmik funtsiya va uning xossalari.
56. Trigonometrik funksiyalar va ularning xossalari.
57. Teskari trigonometrik funksiyalar va ularning xossalari.
58. Veyershtrassning 1-teoremasi.
59. Veyershtrassning 2-teoremasi.
60. Boltsano-Koshining 1-teoremasi.
61. Boltsano-Koshining 2-teoremasi.
62. Tekis uzluksiz funksiyalar. Misol.
63. Tekis uzluksiz funksiyaga oid Kantor teoremasi.
64. $\lim\limits_{x\to 0}\dfrac{\log_a(1+x)}{x}=\log_a e$ tenglikning isboti.
65. $\lim\limits_{x\to 0}\dfrac{a^x-1}{x}=\ln a$ tenglikning isboti.
66. $\lim\limits_{x\to 0}\dfrac{(1+x)^\mu-1}{x}=\mu$ tenglikning isboti.

2-semestr

1. Tekis egri chiziq urinmasi haqidagi masala.
2. Moddiy nuqtaning tezligi haqidagi masala.
3. Funksiya hosilasining ta’rifi.
4. Hosilaning mexanik va geometrik ma’nolari.
5. Bir tomonli hosilalari.
6. Cheksiz hosilalar.
7. Egri chiziq urinmasi va normalining tenglamalari.
8. Hosilaga ega bo`lgan funksiyaning uzluksizligi.
9. Ikki funksiya yig`indisi, ayirmasi, ko`paytmasi va bo`linmasining hosilalari.
10. Teskari funksiyaning hosilasi.
11. Murakkab funksiyaning hosilasi.
12. Darajali funksiyaning hosilasi.
13. Ko`rsatkichli funksiyaning hosilasi.
14. Logarifmik funksiyaning hosilasi.
15. Trigonometrik funksiyalarning hosilalari.
16. Teskari trigonometrik funksiyalarning hosilalari.
17. Logarifmik differensiallash. Daraja-ko`rsatkichli funksiyaning hosilasi.
18. Parametrik shaklda berilgan funksiyaning hosilasi.
19. Funksiyaning differensiali.
20. Funksiya differensiali bilan hosilasi orasidagi bog`lanish.
21. Murakkab funksiyaning differensiali. Differensial formasining invariantligi.
22. Vektor funksiya va uning hosilasi.

23. Yuqori tartibli hosilalar.
24. Yuqori tartibli differensiallar.
25. Ferma teoremasi.
26. Roll teoremasi.
27. Lagranj teormasi.
28. Koshi teoremasi.
29. Lopital qoidasi.
30. Ko`phad uchun Teylor formulasi.
31. Ixtiyoriy funksiya uchun Teylor formulasi.
32. $y = e^x$ funksiya uchun Makloren formulasi.
33. $y = \sin x$, $y = \cos x$ funksiyalar uchun Makloren formulalari.
34. $y = \ln(1 + x)$ funksiya uchun Makloren formulasi.
35. Funksiyaning doimiylik sharti.
36. Funksiyaning monotonlik sharti.
37. Funksiyaning kritik nuqtalari, misollar.
38. Ekstremumning zaruriy sharti.
39. Ekstremumning etarli shartlari.
40. Funksiyaning segmentdagi eng katta va eng kichik qiymatlari.
41. Qavariq, botiq egri chiziqlar, egri chiziqning burilish nuqtalari.
42. Egri chiziqning qavariqlik va botiqlik shartlari.
43. Egri chiziq asimptotasi. Vertikal asimptota.
44. Og`ma asimptota.
45. Funksiyani to`la tekshirish va grafigini yasash.

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