3-§. CHIZIQLI ALGEBRAIK TENGLAMALAR SISTEMASI.
KRAMER FORMULASI VA GAUSS USULI.

Kramer formulasi.

Faraz qilaylik birinchi darajali, ikkita noma'lumli ikkita algebraik tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases} \qquad \textbf{(1)}$$

(1) sistemaning 1-tenglamasini a_{22} ga, 2-tenglamasini $-a_{12}$ ga ko'paytirib qo'shsak

$$(a_{11}a_{22}-a_{12}a_{21})x_1 = b_1a_{22}-b_2a_{12} \quad ⌡ \quad x_1 = \frac{b_1a_{22}-b_2a_{12}}{a_{11}a_{22}-a_{12}a_{21}} \qquad (2)$$

Agar (1) sistemaning 1-tenglamasini $-a_{21}$ ga, 2-tenglamasini a_{11} ga ko'paytirib qo'shsak

$$(a_{11}a_{22}-a_{12}a_{21})x_2 = b_2a_{11}-b_1a_{21} \quad ⌡ \quad x_2 = \frac{b_2a_{11}-b_1a_{21}}{a_{11}a_{22}-a_{12}a_{21}} \qquad (3)$$

(2) va (3) larga e'tibor bersak ikkinchi tartibli determinantning ta'rifiga ko'ra

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{\Delta_1}{\Delta}; \qquad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{\Delta_2}{\Delta};$$

(4)

(4) ga Kramer formulasi deyiladi.

Misol.

1) $\begin{cases} 2x+5y=8 \\ 3x+y=-1 \end{cases}$ (x=-1; u=2), **2)** $\begin{cases} 5x-3y+2z=9 \\ 2x+2y-5z=3 \\ 2x-y-3z=7 \end{cases}$, (x=1; y=-2; z=-1).

Agar uch noma'lumli bir jinsli ikkita tenglamalar sistemasi

$$\begin{cases} a_1x+b_1y+c_1z=0 \\ a_2x+b_2y+c_2z=0 \end{cases}$$

berilgan bo'lib,

$$\Delta_1=\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}, \quad \Delta_2=\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}, \quad \Delta_3=\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

determinantning loaqal bittasi noldan farqli bo'lsa, u holda sistemaning barcha yechimlari

$$x=\Delta_1 t, \quad y=\Delta_2 t, \quad z=\Delta_3 t$$

formula bilan aniqlanadi. (t-ixtiyoriy son).

$$\begin{cases} a_1x + b_1y + c_1z = 0 \\ a_2x + b_2y + c_2z = 0 \\ a_1x + d_2y + c_3z = 0 \end{cases}$$

Bu sistemada $\Delta\neq0$ bo'lsa, x=0 ,u=0 ,z=0 lar sistemaning yagona yechimi bo'ladi.

Agar $\Delta=0$ bo'lsa, cheksiz ko'p yechimi bo'ladi.

Misol.

1) $\begin{cases} 3x-5y+z=0 \\ x+2y-z=0 \end{cases}$ (x=3t; u=4t;z=11t),

2) $\begin{cases} x-y-z=0 \\ x+4y+2z=0 \\ 3x+7y+3z=0 \end{cases}$ (x=2t;y=-3t; z=5t).

Chiziqli algebraik tenglamalar sistemasini Gauss usuli bilan yechish.

Quyidagi n ta noma'lumli m ta chiziqli tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m \end{cases} \qquad (1)$$

Endi (1) sistemani Gauss usuli bilan yechishga o'taylik. Bu usulning mohiyati shundan iboratki noma'lumlarni ketma-ket yo'qotib ,berilgan sistemaga teng kuchli bo'lgan uchburchak (yoki pog'onasimon) ko'rinishdagi sistemaga keltiriladi. $a_{11}\neq0$ deb (1) ning birinchi tenglamasini a_{11} ga bo'lib, so'ngra uni $-a_{21}$ ga ko'paytirib, ikkinchi tenglamaga qo'shamiz.

Keyin $-a_{31}$ ga ko'paytirib, uchinchi tenglamaga qo'shamiz va shu jarayonni davom ettiraversak natijada shunday sistema hosil bo'ladiki, u sistemaning faqat birinchi tenglamasida x_1 qatnashib qolganlarida qatnashmaydi.

Shu jarayonni (1) sistemaning qolgan tenglamalariga ketma-ket tatbiq etsak, qo'yidagi ikkita sistemaning bittasiga kelamiz.

$$\left.\begin{aligned} x_1 + c_{12}x_2 + c_{13}x_3 + \ldots + c_{1n}x_n &= d_1 \\ x_2 + c_{23}x_3 + \ldots + c_{2n}x_n &= d_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \\ x_n &= d_m \end{aligned}\right\}$$

(2) yoki

$$\left.\begin{aligned} x_1 + c_{12}x_2 + c_{13}x_3 + \ldots + c_{1n}x_n &= d_1 \\ x_2 + c_{23}x_3 + \ldots + c_{2n}x_n &= d_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \\ x_p + \ldots + c_{pn}x_n &= d_p \\ p < n & \end{aligned}\right\}$$

(3)

(2) sistemaga uchburchak sistema , (3) ga esa pog'onali sistema deyiladi.

Agar (1) sistema (2) ko'rinishdagi sistemaga keltirilsa, u holda (1)sistema birgalikda bo'lgan sistema bo'lib yechimi yagona bo'ladi. Agar(1)sistema (3) ko'rinishdagi sistemaga keltirilsa u holda (1) sistema birgalikda bo'lib, yechimi cheksiz ko'p bo'ladi.

Misol. **1)**

$$\left.\begin{aligned} 2x_1 + 7x_2 + 13x_3 &= 0 \\ 3x_1 + 14x_2 + 12x_3 &= 18 \\ 5x_1 + 25x_2 + 16x_3 &= 39 \end{aligned}\right\}$$

Yechish. $a_{11}=2\neq0$ bo'lgani uchun birinchi tenglamani 2 ga bo'lamiz.

$$\left.\begin{aligned} x_1 + 7/2x_2 + 13/2x_3 &= 0 \\ 3x_1 + 14x_2 + 12x_3 &= 18 \\ 5x_1 + 25x_2 + 16x_3 &= 39 \end{aligned}\right\}$$

Bu sistemaning 1-tenglamasini (-3) ga ko'paytirib 2-tenglamaga, (-5)ga ko'paytirib 3-tenglamaga qo'shsak

$$\left.\begin{aligned} x_1 + 7/2x_2 + 13/2x_3 &= 0 \\ 7/2x_2 - 15/2x_3 &= 18 \\ 15/2x_2 - 33/2x_3 &= 39 \end{aligned}\right\}$$

Endi $a_{22} = \frac{7}{2} \neq 0$ bo'lgani uchun 2-tenglamani $\frac{7}{2}$ ga bo'lib , so'ngra uni $\frac{15}{2}$ ga ko'paytirib 3- tenglamadan ayirsak:

$$\left.\begin{aligned} x_1 + 7/\ 2x_2 + 13/\ 2x_3 &= 0 \\ x_2 - 15/\ 7x_3 &= 36/\ 7 \\ -3/\ 7x_3 &= 3/\ 7 \end{aligned}\right\}$$

$x_1=-4; x_2=3; x_3=-1.$

2)

$$\left.\begin{aligned} x_1 + 2x_2 - 4x_3 &= 1 \\ 2x_1 + x_2 - 5x_3 &= -1 \\ x_1 - x_2 - x_3 &= -2 \end{aligned}\right\}$$

1-tenglamani (-2) ga ko'paytirib 2-tenglamaga ,(-1) ga ko'paytirib

3-tenglamaga qo’shsak $\left.\begin{aligned} x_1+2x_2-4x_3&=1\\ -3x_2+3x_3&=-3\\ -3x_2+3x_3&=-3\end{aligned}\right\}$

$$\left.\begin{aligned} x_1+2x_2-4x_3&=1\\ x_2-x_3&=1\end{aligned}\right\}$$

$x_2=1+x_3$; $x_1=1-2-2x_3+ 4x_3-= 2x_3-1$.
Shunday qilib $x_1=2x_3-1$; $x_2=1+x_3$.
Demak berilgan sistema cheksiz ko’p yechimga ega ekan, chunki x_3 ga ihtiyoriy son berib, x_1, x_2 larning cheksiz ko’p qiymatlarini hosil qilamiz.

Misollar.

51. $\begin{cases} 3x-5y=13\\ 2x+7y=81\end{cases}$

52. $\begin{cases} 3x-4y=-6\\ 3x+4y=18\end{cases}$

53. $\begin{cases} 3x+2y=7\\ 4x-5y=40\end{cases}$

54. $\begin{cases} 2ax-3by=0\\ 3ax-6by=ab\end{cases}$

55. $\begin{cases} ax-3y=1\\ ax-2y=2\end{cases}$

56. $\begin{cases} mx-ny=(m-n)^2\\ 2x-y=n\end{cases}$

57. $\begin{cases} 2x+y=5\\ x+3z=16\\ 5y-z=10\end{cases}$

58. $\begin{cases} x+y-2z=6\\ 2x+3y-7z=16\\ 5x+2y+z=16\end{cases}$

59. $\begin{cases} 2x-3y+z-2=0\\ x+5y-4z+5=0\\ 4x+y-3z+4=0\end{cases}$

60. $\begin{cases} 2x-4y+3z=1\\ x-2y+4z=3\\ 3x-y+5z=2\end{cases}$

61. $\begin{cases} x+2y+3z=4\\ 2x+y-z=3\\ 3x+3y+2z=7\end{cases}$

62. $\begin{cases} x+2y+3z=4\\ 2x+y-z=3\\ 3x+3y+2z=10\end{cases}$

63. $\begin{cases} x+2y+3y=4\\ 2x+4y+6z=3\\ 3x+y-z=1\end{cases}$

64. $\begin{cases} x-2y+z=4\\ 2x-3y-z=3\\ 4x-y+z=11\end{cases}$

65. $\begin{cases} 3x-y+2z=0\\ 2x+3y-5z=0\\ x+y+z=0\end{cases}$

66. $\begin{cases} 7x-y=8\\ 3y-3z=0\\ 5x+z=6\end{cases}$

67. $\begin{cases} 2x-y+z=2\\ 3x+2y+2z=-2\\ x-2y+z=1\end{cases}$

Matrisalar yordamida chiziqli algebraik tenglamalar sistemasini yechish.

Uchta noma'lumli uchta chiziqli tenglamalar sistemasini ko'raylik.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = c_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = c_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = c_3 \end{cases} \qquad \textbf{(1)}$$

Elementlari noma'lumlarning koeffi**s**iyentlaridan, noma'lumlardan va ozod hadlardan tuzilgan quyidagi matri**s**alarni ko'raylik.

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \qquad X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \qquad C=\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

Bu holda (1) sistemani qo'yidagicha yozish mumkin.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \Rightarrow AX=C \qquad (2).$$

(2) ning har ikkala tomonini chapdan A^{-1} ga ko'paytirsak

$$A^{-1}(AX)= A^{-1}C \Rightarrow (A^{-1}A)X= A^{-1}C$$

Agar $A^{-1}A=AA^{-1}=E$ va $EA=AE=A$ tengliklarni e'tiborga olsak

$$(A^{-1}A)X= A^{-1}C \Rightarrow EX= A^{-1}C \Rightarrow X= A^{-1}C \qquad (3),$$

(3) (1)-sistemaning yechimini ifodalaydi.

Misol. Quyidagi tenglamalar sistemasini matrisaviy usulda yeching:

$$\begin{cases} 3x_1 - x_2 = 5 \\ -2x_1 + x_2 + x_3 = 0 \\ 2x_1 - x_2 + 4x_3 = 15 \end{cases}$$

Yechish. Sistemani matrisa ko'rinishida yozaylik:

$$\begin{pmatrix} 3 & -1 & 0 \\ -2 & 1 & 1 \\ 2 & -1 & 4 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 15 \end{pmatrix}$$

$$A=\begin{pmatrix} 3 & -1 & 0 \\ -2 & 1 & 1 \\ 2 & -1 & 4 \end{pmatrix}, \qquad \det A=|A|=\begin{vmatrix} 3 & -1 & 0 \\ -2 & 1 & 1 \\ 2 & -1 & 4 \end{vmatrix}=5$$

Demak A matrisa uchun A^{-1} matrisa mavjud . Berilgan A matrisa elementlarining algebraik to'ldiruvchilarini hisoblab teskari matrisani topamiz

$$A^{-1} = \begin{pmatrix} 1 & 4/5 & -1/5 \\ 2 & 12/5 & -5/3 \\ 0 & 1/5 & 1/5 \end{pmatrix}$$

Endi (3) formulaga asosan

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 & 4/5 & -1/5 \\ 2 & 12/5 & -5/3 \\ 0 & 1/5 & 1/5 \end{pmatrix} \begin{pmatrix} 5 \\ 0 \\ 15 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

$x_1=2$; $x_2=1$; $x_3=3$.

Misollar.

Quyidagi tenglamalar sistemalarini matrisalar hislbi yordamida eching (68-71).

68. $\begin{cases} 2x_1 + 3x_2 + 2x_3 = 9 \\ x_1 + 2x_2 - 3x_3 = 14 \\ 3x_1 + 4x_2 + x_3 = 16 \end{cases}$ 69. $\begin{cases} 2x_1 + x_2 - x_3 = 5 \\ x_1 - 2x_2 + 2x_3 = -5 \\ 7x_1 + x_2 - x_3 = 10 \end{cases}$

70. $\begin{cases} 3x_1 - 2x_2 - 5x_3 + x_4 = 3 \\ 2x_1 - 3x_2 + x_3 + 5x_4 = -3 \\ x_1 + 2x_2 - 4x_4 = -3 \\ x_1 - x_2 - 4x_3 + 9x_4 = 22 \end{cases}$ 71. $\begin{cases} x_1 + x_2 - 6x_3 - 4x_4 = 6 \\ 3x_1 - x_2 - 6x_3 - 4x_4 = 2 \\ 2x_1 + 3x_2 + 9x_3 + 2x_4 = 6 \\ 3x_1 + 2x_2 + 3x_3 + 8x_4 = -7 \end{cases}$

Quyidagi tenglamalar sistemalarini noma'lumlarni ketma-ket yo'qotish usulida eching (72-77).

72. $\begin{cases} 2x_1 + x_2 - x_3 = 5 \\ 5x_1 + 2x_2 + 4x_3 = 1 \\ 7x_1 + 3x_2 + 2x_3 = 4 \end{cases}$ 73. $\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ 2x_1 - x_2 + 2x_3 = 6 \\ x_1 + x_2 - 3x_3 = 7 \end{cases}$

74. $\begin{cases} x_1 - 1x_2 + 2x_3 = 1 \\ x_1 + 3x_2 - 1x_3 = 2 \\ x_1 - 5x_2 + 5x_3 = 1 \end{cases}$ 75. $\begin{cases} x_1 + 2x_2 + 3x_3 = 7 \\ 2x_1 + 3x_2 + x_3 = 5 \\ 2x_1 + 4x_2 + 6x_3 = 14 \end{cases}$

76. $\begin{cases} 2x_1 + 5x_2 + 4x_3 + x_4 = 20 \\ x_1 + 3x_2 + 2x_3 + x_4 = 11 \\ 2x_1 + 10x_2 + 9x_3 + 9x_4 = 40 \\ 3x_1 + 8x_2 + 9x_3 + 2x_4 = 37 \end{cases}$ 77. $\begin{cases} 3x_1 + 4x_2 + x_3 + 2x_4 = -3 \\ 3x_1 + 5x_2 + 3x_3 + 5x_4 = -6 \\ 6x_1 + 8x_2 + x_3 + 5x_4 = -8 \\ 3x_1 + 5x_2 + 3x_3 + 7x_4 = -8 \end{cases}$

4-§. VEKTORIAL ALGEBRA

Aniq yo'nalishga ega bo'lgan chekli kesmaga vektor deyiladi.
A nuqtani vektorning boshi, B nuqtani esa vektorning ohiri yoki uchi deyiladi. Odatda vektor $\overline{AB}$ yoki $\vec{a}$ ko'rinishda yoziladi. Kesmaning uzunligi $\overline{AB}$ vektorning modulini ya'ni son qiymatini ifodalaydi va $|\overline{AB}|$ yoki $|\vec{a}|$ ko'rinishda yoziladi. Vektor degan so'z asli lotincha bo'lib, ko'chiruvchi, siljituvchi yoki tortuvchi degan ma'noni bildiradi.

Vektorlarni qo'shish, ayirish amallari o'rta maktab dasturidan ma'lum bo'lgan uchburchak va parallelogramm qoidalariga asosan amalga oshiriladi.

Vektorni songa ko'paytirish. $\vec{a}$ vektorni biror α haqiqiy songa ko'paytirganda shu $\vec{a}$ ga kollinear bo'lgan $\vec{b}$ vektor hosil bo'lib, uning uzunligi $|\vec{b}|= |\alpha||\vec{a}|$ ga teng bo'lib, yo'nalishi esa $\alpha>0$ bo'lsa, $\vec{a}$ vektor yo'nalishi bilan bir hil , $\alpha<0$ bo'lsa, $\vec{a}$ yo'nalishiga qarshi bo'ladi. Vektorlarni songa ko'paytirish qoidasidan ko'rinadiki $\vec{b}=\alpha\vec{a}$ bo'lsa $\vec{a}$ va $\vec{b}$ vektorlar kollinear vektorlar va aksincha.

Vektorlarning o'qqa proyeksiyasi.

$\vec{a}$ vektorning u o'qdagi proyeksiyasi shu vektor uzunligini, shu vektor bilan u o'q orasidagi φ burchak kosinus ko'paytmasiga teng bo'ladi:

$$pr_u\vec{a}=|\vec{a}|\cos\varphi$$

Vektor koordinatalari deganda vektorning uchi bilan boshining bir hil koordinatalari ayirmalariga shu vektorning koordinatalari deyiladi va qo'yidagicha yoziladi

$$\vec{a}=\{x_2-x_1;\ y_2-y_1\}$$

Vektor koordinatalar kvadratlarining yisindisidan olingan kvadrat ildizga vektor uzunligi deyiladi.

$$|\vec{a}|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Vektorni bazislar bo'yicha yoyish.

Tekislikdagi bazis deb ikkita kollinear bo'lmagan, ya'ni chiziqli bog'liqsiz $\vec{a}_1$, $\vec{a}_2$ vektorlarga aytiladi.

Fazodagi biror $\vec{a}$ vektorning $\vec{a}_1, \vec{a}_2, \vec{a}_3$ bazislar orqali yoyilmasi $\vec{a} = \lambda_1 \vec{a}_1 + \lambda_2 \vec{a}_2 + \lambda_3 \vec{a}_3$ (2)
ko'rinishda bo'lib, yagona bo'ladi.

Agar $\vec{a}$ vektorning koordinata o'qlaridagi proyeksiyalarini x,y,z desak,

$$\vec{a} = x\vec{i} + y\vec{j} + z\vec{k} \quad \text{yoki} \quad \vec{a} = \{x, y, z\},$$

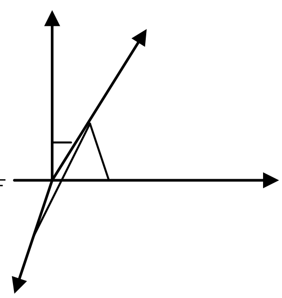
$\vec{a} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}$ yoki $\vec{a} = \{x_2 - x_1, y_2 - y_1, z_2 - z_1\}$
ko'rinishlarda ham yozish mumkin.

$\vec{a} = \{x, y, z\}$ vektor Ox,Ou,Oz koordinata o'qlari bilan mos ravishda α, β, γ burchaklar tashkil qilsin.

$\vec{a}$ vektorning koordinata o'qlari bilan hosil qilgan burchaklar kosinuslariga ya'ni $\cos\alpha$, $\cos\beta$, $\cos\gamma$ larga $\vec{a}$ vektorning yo'naltiruvchi kosinuslari deyiladi.
Proyeksiyalash qoidalaridan foydalansak chizmadan ko'rinadiki

$x = a_x = pr_{Ox}\vec{a} = |\vec{a}|\cos\alpha$,

$$\cos\alpha = \frac{x}{|\vec{a}|} = \frac{x}{\sqrt{x^2 + y^2 + z^2}}$$



z $\vec{a}$

$y = a_y = pr_{OU}\vec{a} = |\vec{a}|\cos\beta$

$$\cos\beta = \frac{y}{|\vec{a}|} = \frac{y}{\sqrt{x^2 + y^2 + z^2}}$$

γ β

x α y

$x = a_z = pr_{Oz}\vec{a} = |\vec{a}|\cos\gamma$

$$\cos\gamma = \frac{z}{|\vec{a}|} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

Misol. A(1,2,3) B(2,4,5) bo'lsa, $\vec{a} = \overrightarrow{AB}$ vektorning yo'naltiruvchi kosinuslarini toping.

Yechish. $\overrightarrow{AB} = \{1;2;2\}$, $|\overrightarrow{AB}| = 3$, $\cos\alpha = 1/3$; $\cos\beta = 2/3$; $\cos\gamma = 2/3$.

Kesmani berilgan nisbatda bo'lish.

A(x_1, y_1, z_1) N(x,y,z)
B(x_2, y_2, z_2)

x=q ; y= q ;z=q

$$\frac{\overrightarrow{AN}}{\overrightarrow{NB}} = \lambda \quad => \quad \overrightarrow{AN} = \lambda \overrightarrow{NB}.$$

$\overrightarrow{AN}$ *ва* $\overrightarrow{NB}$ vektorlarning kollinearlik shartidan

$\overrightarrow{AN} = \lambda \overrightarrow{NB} \Rightarrow (x-x_1)\vec{i} + (y-y_1)\vec{j} + (z-z_1)\vec{k} = \lambda \cdot [(x_2-x)\vec{i} + (y_2-y)\vec{j} + (z_2-z)\vec{k}]$

$$x = \frac{x_1 + \lambda x_2}{1+\lambda} \quad ; \quad y = \frac{y_1 + \lambda y_2}{1+\lambda} \quad ; \quad z = \frac{z_1 + \lambda z_2}{1+\lambda}$$

xususiy holda λ=1 bo'lsa, $x = \frac{x_1 + x_2}{2}$; $y = \frac{y_1 + y_2}{2}$; $z = \frac{z_1 + z_2}{2}$

Misollar.

78. $\vec{a} = \{6;3;-2\}$ vektorning moduli hisoblansin.
79. $\vec{a}$ vektorning ikkita $x = 4,\ y = -12$ koordinatasi berilgan. Agar $|\vec{a}| = 13$ bo'lsa, uning uchinchi z koordinatasi topilsin.
80. Agar ;-1;4} vektorning boshi M(1;2;-3) nuqta bilan ustma-ust tushsa uning oxiri bilan ustma-ust tushuvchi nuqta aniqlansin.
81. Agar $\vec{a} = \{2;\ -3;\ -1\}$ vektorning uchi $(1, -1,\ 2)$ nuqta bilan ustma-ust tushsa, uning boshi aniqlansin.
82. $\vec{a} = \{12;\ -15;\ -16\}$ vektorning yo'nltiruvchi kosinuslari hisoblansin.
83. $\vec{a} = \left\{\frac{3}{13}; \frac{4}{13}; \frac{12}{13}\right\}$ vektorning yunaltiruvchi kosinuslari hisoblansin.
84. Vektor Ox va Oz uklari bilan mos ravishda $\alpha = 120^\circ, \gamma = 45^\circ$ burchak tashkil etadi. Vektor Oy o'q bilan qanday burchak tashkil etadi?
85. $\vec{a}$ vektor Ox va Ou uklari bilan mos ravishda $\alpha = 60^\circ, \beta = 60^\circ$ burchak tashkil etadi. $|\vec{a}| = 2$ deb, uning koordinatalari hisoblansin.
86. Quyidagilar berilgan $|\vec{a}| = 13$, $|\vec{b}| = 19$ va $|\vec{a} + \vec{b}| = 24.$ $|\vec{a} - \vec{b}|$ hisoblansin.

87. Quyidagilar berilgan $|\vec{a}|=11$, $|\vec{b}|=23$ va $|\vec{a}-\vec{b}|=23.$ $|\vec{a}+\vec{b}|$ aniqlansin.

88. $\vec{a}$ va $\vec{b}$ vertorlar o'zaro perpendikulyar va $|\vec{a}|=5$, $|\vec{b}|=12$. $|\vec{a}+\vec{b}|$ va $|\vec{a}-\vec{b}|$ lar aniqlansin.

89. $\vec{a}$ va $\vec{b}$ vertorlar o'zaro $\varphi=60^\circ$ burchak tashkil etadi, shu bilan birga $|\vec{a}|=5$ va $|\vec{b}|=8$, $|\vec{a}+\vec{b}|$ va $|\vec{a}-\vec{b}|$ lar aniqlansin.

90. Berilgan $\vec{a}$ va $\vec{b}$ vertorlar yordamida quyidagi vektorlarni yasang:
1) $3\vec{a}$; 2) $-\frac{1}{2}\vec{b}$; 3) $\overrightarrow{2a}+\frac{1}{3}\vec{b}$; 4) $\frac{1}{2}\vec{a}-3\vec{b}$.

91. ABS uchburchakda $\overline{AB}=\vec{m}$ va $\overline{AC}=\vec{n}$ bo'lsin.
Quyidagi vektorlarni yasang: 1) $\frac{\vec{m}+\vec{n}}{2}$; 2) $\frac{\vec{m}-\vec{n}}{2}$; 3) $\frac{\vec{n}-\vec{m}}{2}$;
4) $-\frac{\vec{m}+\vec{n}}{2}$. Masshtab birligi sifatida $\frac{1}{2}|\vec{n}|$ ni olib, quyidagi vektor yasalsin:
5) $|\vec{n}|\vec{m}+|\vec{m}|\vec{n}$; 6) $|\vec{n}|\vec{m}-|\vec{m}|\vec{n}$.

92. $ABCDA^1B^1C^1D^1$ parallepipedda uning qirralari bilan ustma-ust tushuvchi vektorlar berilgan: $\overrightarrow{AB}=\vec{m}$, $\overline{AD}=\vec{n}$ $AA^1=\vec{p}$. Quyidagi vektorlarni yasang:

1) $\vec{m}+\vec{n}+\vec{p}$; 2) $\vec{m}+\vec{n}+\frac{1}{2}\vec{p}$

3) $\frac{1}{2}\vec{m}+\frac{1}{2}\vec{n}+\vec{p}$; 4) $\vec{m}+\vec{n}-\vec{p}$

5) $-\vec{m}-\vec{n}+\frac{1}{2}\vec{p}$.

93. α va β koeffisientlarning qanday qiymatlarida $\vec{a}=-2\vec{i}+3\vec{j}+\beta\vec{k}$ va $\vec{b}=\alpha\vec{i}-6\vec{j}+2\vec{k}$ vektorlar kollinear bo'ladi?

94. Quyidagi to'rtta nuqtani trapesiyaning uchlari ekanligi tekshirilsin:
$A(3;-1;2)$, $B(1;-2;-3)$, $C(1;-1;-3)$, $D(3;-5;3)$

95. $\vec{a}=\{6;\ -2;\ -3\}$ vektorning orti topilsin

96. $\vec{a}=\{3;\ 4;\ -12\}$ vektorning orti topilsin

97. $\vec{a}=\{3;\ -5;\ 8\}$ va $\vec{b}=\{-1;\ 1;\ -4\}$ vektor yig'indisi va ayirmasining modullari aniqlansin.

98. $\overline{AB}=\{2;\ 6;\ -4\}$ va $\overline{AC}=\{4;\ -2;\ 2\}$ vektor ABC uchbukchakning tomonlari bilan ustma-ust tushidi. Shu uchburchakning uchlariga qo'yilgan va uning AM, BN, CP, medianalari bilan ustma-ust tushuvchi vektorlarning koordinatalari aniqlansin.

99. Tekislikda uchta $\vec{a}=\{3;-2\}$, $\vec{b}=\{-2;1\}$ va $\vec{c}=\{7;-4\}$ vektor berilgan. Bu vektorlarning har birining, qolgan ikkita vektorni bazis sifatida qabul qilib, yoyilmasi aniqlansin.

100. Uchta $\vec{a}=\{3;-1\}$, $\vec{b}=\{1;-2\}$ va $\vec{c}=\{-1;7\}$ vektor berilgan. $\vec{p}=\vec{a}+\vec{b}+\vec{c}$ vektorning $\vec{a}$, $\vec{b}$ bazis bo'yicha yoyilmasi topilsin

Skalyar ko'paytma.

$\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko'paytmasi deb, shunday songa aytiladiki, bu son shu vektorlar uzunliklari bilan ular orasidagi burchak kosinusi ko'paytmasiga teng bo'ladi va odatda $\vec{a}\,\vec{b}$ yoki $(\vec{a}\,\vec{b})$ ko'rinishda yoziladi.

Demak ta'rifga ko'ra $\vec{a}\,\vec{b}=|\vec{a}||\vec{b}|\cos\varphi$; $\varphi=\vec{a}\wedge\vec{b}$

Misol. $|\vec{a}|=3$, $|\vec{b}|=2$, $\varphi=60°$ bo'lsa $(\vec{a}\,\vec{b})=3\cdot 2\cdot\frac{1}{2}=3$

Ikki vektorning skalyar ko'paytmasi deb, ihtiyoriy bittasining uzunligini ikkinchisining birinchi vektor yo'nalishidagi proyeksiyasi bilan ko'paytmasiga aytiladi.

$pr_a\vec{b}=|\vec{b}|\cos\varphi$ yoki $pr_b\vec{a}=|\vec{a}|\cos\varphi$ tengliklardan foydalansak

$\vec{a}\,\vec{b}=|\vec{a}||\vec{b}|\cos\varphi=|\vec{a}|pr_a\vec{b}=|\vec{b}|pr_b\vec{a}$; $pr_a\vec{b}=\dfrac{\vec{a}\,\vec{b}}{|\vec{a}|}$; $pr_b\vec{a}=\dfrac{\vec{a}\,\vec{b}}{|\vec{b}|}$

Skalyar ko'paytmaning fizik ma'nosi: $\vec{F}$ kuchning moddiy nuqtani s masofaga ko'chirgandagi bajargan ishdir. $A=\vec{F}\cdot\vec{s}$ yoki $A=|\vec{F}||\vec{s}|\cos\varphi$.

Agar $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchakni φ desak, bu vektorlarning skalyar ko'paytmasidan

$$\vec{a}\,\vec{b}=|\vec{a}||\vec{b}|\cos\varphi \Rightarrow \cos\varphi=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} \qquad (1)$$

ikki vektor orasidagi burchak kosinusini hisoblash formulasi kelib chiqadi.

Agar $\vec{a}=\{x_1, y_1, z_1\}$, $\vec{b}=\{x_2, y_2, z_2\}$ koordinatalari bilan berilgan bo'lsa,

$$\cos\varphi = \frac{x_1x_2+y_1y_2+z_1z_2}{\sqrt{x_1^2+y_1^2+z_1^2}\cdot\sqrt{x_2^2+y_2^2+z_2^2}} \qquad (2)$$

Agar $\vec{a}\perp\vec{b}$ bo'lsa, $\varphi=\frac{\pi}{2}$ bo'lib $\cos\varphi=0$ bo'ladi va (2) dan

$$x_1x_2+y_1y_2+y_1y_2+z_1z_2=0 \qquad (3)$$

(3) ikki vektorning perpendikulyarlik sharti. Agar $\vec{a}$ va $\vec{b}$ vektorlar parallel bo'lsa, u holda bu vektorlarning kollinearlik shartidan ya'ni $\vec{\boldsymbol{a}}=\lambda\vec{b}$ dan

$x_1\vec{i}+y_1\vec{j}+z_1\vec{k}=\lambda(x_2\vec{i}+y_2\vec{j}+z_2\vec{k})\Rightarrow x_1=\lambda x_2$; $y_1=\lambda y_2$; $z_1=\lambda z_2$.

$$\frac{x_1}{x_2}=\frac{y_1}{y_2}=\frac{z_1}{z_2} \qquad (5)$$

(5) ikki vektorning parallelik sharti.

Misol. $|\vec{a}|=3$, $|\vec{b}|=4$, $\varphi = \hat{\vec{a}\,\vec{b}}=\frac{2\pi}{3}$ bo'lsa $(\vec{a}+\vec{b})^2=q$,

$$(\vec{a}+\vec{b})^2=\vec{a}^{\,2}+2(\vec{a}\,\vec{b})+\vec{b}^{\,2} = 9-12+16=13$$

MISOLLAR.

101. $\vec{a}$ $\vec{b}$ va $\vec{b}$ vektorlar $\varphi=\frac{2}{3}\pi$ burchakni tashkil etadi. $|\vec{a}|=3,$ $|\vec{b}|=4,$ qiymatlarni bilgan holda quyidagilar hisoblansin:
1) $\vec{a}\vec{b}$; 2) $\vec{a}^{\,2}$, 3) $\vec{b}^{\,2}$; 4) $(\vec{a}+\vec{b})^2$; 5) $(3\vec{a}-2\vec{b})(\vec{a}+2\vec{b})$;
6) $(\vec{a}-\vec{b})^2$; 7) $(3\vec{a}+2\vec{b})^2$.

102. $\vec{a}$ va $\vec{b}$ vektorlar o'zaro perpendikulyar; $\vec{c}$ vektor ularning har biri bilan $\frac{\pi}{3}$ burchak tashkil etadi; $|\vec{a}|=3,$ $|\vec{b}|=5,$ $|\vec{c}|=8$ ekani ma'lum bo'lsa, quyidagilar hisoblansin: 1) $(3\vec{a}-2\vec{b})(\vec{b}+3\vec{c})$; 2) $(\vec{a}+\vec{b}+\vec{c})^2$; 3) $(\vec{a}+2\vec{b}-3\vec{c})^2$.

103. $\vec{a}+\vec{b}+\vec{c}=0$ shartni qanoatlantiradigan $\vec{a}$, $\vec{b}$ va $\vec{c}$ birlik vektor berilgan. $\vec{a}\vec{b}+\vec{b}\vec{c}+\vec{c}\vec{a}$ hisoblansin.

104. $\vec{a}+\vec{b}+\vec{c}=0$ shartni qanoatlantiradigan uchta $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektor berilgan. $|\vec{a}|=3,$ $|\vec{b}|=1,$ $|\vec{c}|=4$ tengliklarni bilgan holda $\vec{a}\vec{b}+\vec{b}\vec{c}+\vec{c}\vec{a}$ hisoblansin.

105. $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar juft-jufti bilan $60°$ burchak tashkil etadi. $|\vec{a}| = 4$, $|\vec{b}| = 2$, $|\vec{c}| = 6$ tengliklarni bilgan holda $\vec{p} = \vec{a} + \vec{b} + \vec{c}$ vektorning moduli aniqlansin.
106. $|\vec{a}| = 3$, $|\vec{b}| = 5$ tengliklar berilgan α ning qanday qiymatida $\vec{a} + a\vec{b}$, $\vec{a} - a\vec{b}$ vektorlar o'zaro perpendikulyar bo'lishi aniqlansin.

107. $\vec{p} = \vec{b}(\vec{a}\vec{c}) - \vec{c}(\vec{a}\vec{b})$ vektorning $\vec{c}$ vektorga perpendikulyar ekanligi isbotlansin.

108. $\vec{p} = \vec{b} - \dfrac{\vec{a}(\vec{a}\vec{b})}{\vec{a}^2}$ vektorning $\vec{a}$ vektorga perpendikulyar ekanligi isbotlansin.
109. $\vec{a}$ va $\vec{b}$ vektorlar $\varphi = \dfrac{\pi}{6}$ burchak tashkil etadi; $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 1$ ekanligini bilgan holda, $\vec{p} = \vec{a} + \vec{b}$ va $\vec{q} = \vec{a} - \vec{b}$ vektorlar orasidagi a burchak hisoblansin.
110. Teng yonli, to'g'ri burchakli uchburchakning o'tkir burchaklaridan o'tkazilgan medianalari orasidagi o'tmas burchak hisoblansin.
111. $A(-1;3;-7)$, $B(2;-1;5)$ va $C(0;1;-5)$ nuqtalar berilgan.
Quyidagilar hisoblansin: 1) $(2\overline{AB} - \overline{CB})\ (2\overline{CB} - \overline{BA})$; 2) $\sqrt{\overline{AB}^2}$; 3) $\sqrt{\overline{AC}^2}$;
4) $(2\overline{CB}\ \overline{BA})$ va $\overline{AB}(\overline{AC}\ \overline{BC})$ vektorlarning koordinatalari topilsin.
112. $\vec{f} = \{3;\ -2;\ -5\}$ kuch qo'yilgan nuqta to'g'ri chiziq bo'ylab harakat qilib, $A(2;\ -3;\ 5;)$ nuqtadan $B(3;\ -2;\ -1)$ nuqtaga siljidi. $\vec{f}$ kuchning bajargan ishi hisoblansin.
113. Bir nuqtaga qo'yilgan uchta kuch berilgan: $\vec{M} = \{3;\ -4;\ 2\}$, $\vec{N} = \{2;\ 3;\ -5\}$ va $\vec{P} = \{-3;\ -2;\ 4\}$. Shu kuchlarning teng ta'sir etuvchisining qo'yilish nuqtasi to'g'ri chizik bo'ylab harakatlanib, $M_1(5;\ 3;\ -7)$ holatdan $M_2(4;\ -1;\ -4)$ holatga ko'chganda, teng ta'sir etuvchi bajargan ish hisoblansin.
114. To'rtburchakning uchlari A(1; -2; 2), B (1; 4; 0), C (-4; 1; 1) va D(-5; -5; 3) nuqtalarda yotadi. Uning AC va BD diagonallari o'zaro perpendikulyar ekanligi isbotlansin.

115. α ning qanday qiymatida $\vec{a} = \alpha\vec{i} - 3\vec{j} + 2\vec{k}$ va $\vec{b} = \vec{i} + 2\vec{j} - \alpha\vec{k}$ vektorlar o'zaro perpendikulyar bo'lishi aniqlansin.
116. $\vec{a} = \{2;\ -4;\ 4\}$ va $\vec{b} = \{-3;\ 2;\ 6\}$ vektorlar tashkil etgan burchakning kosinusi hisoblansin.
117. Uchburchakning uchlari A(-1; -2; 4), B(-4; -2; 0) va C (3; -2; 1) nuqtalarga yotadi. Uning B uchidagi ichki burchagi aniqlansin.
118. Uchburchakning uchlari A(-1; -2; 4), B(-4; -2; 0) va C(3; -2; 1) nuqtalarga yotadi. Uning A uchidagi tashqi burchagi aniqlansin.

119. Uchlari A(1; 2; 1), B(3; -1; 7), C(7; 4; -2) bo'lgan uchbarchakning ichki burchakni hisoblash yordamida uchburchakning teng yonli ekanini isbotlang.

120. $\vec{a}=\{6;\ \ -8;\ \ -7.5\}$ vektorga kollinear bo'lgan $\vec{x}$ vektor Oz o'q bilan o'tkir burchak tashkil etadi. $|\vec{x}|=50$ ekanligini bilgan holda, uning koordinatalari aniqlansin.

121. $\vec{a}=\{2;\ \ 1;\ \ -1\}$ vektorga kollinear bo'lgan hamda $\vec{x}\vec{a}=3$ shartni qanoatlantiradigan $\vec{x}$ vektor topilsin.

122. $\vec{a}=3\vec{i}+2\vec{j}+2\vec{k}$ va $\vec{b}=18\vec{i}-22\vec{j}-5\vec{k}$ vektorlarga perpendikulyar bo'lgan $\vec{x}$ vektor Oy o'q bilan o'tmas burchak tashkil qiladi. $|\vec{x}|=14$ ekanligini bilgan holda, uning koordinatalari aniqlansin.

123. $\vec{a}=\{2;\ \ 3;\ \ -1\}$ va $\vec{b}=\{1;\ \ -2;\ \ 3\}$ vektorlarga perpendikulyar bo'lgan va $\vec{x}\left(2\vec{i}-\vec{j}+\vec{k}\right)=-6$ shartni qanoatlantiradigan $\vec{x}$ vektor topilsin.

124. $\vec{a}=\{3;\ \ -1;\ \ 5\}$ va $\vec{b}=\{1;\ \ 2;\ \ -3\}$ vektorlar berilgan. OZ o'qqa perpendikulyar bo'lgan va $\vec{x}\vec{a}=9, \vec{x}\vec{b}=-4$ shartlarni qanoatlantiruvchi $\vec{x}$ vektor topilsin.

125. Uchta $\vec{a}=2\vec{i}-\vec{j}+3\vec{k}, \vec{b}=\vec{i}-3\vec{j}+2\vec{k}$ va $\vec{c}=3\vec{i}+2\vec{j}-4\vec{k}$ vektor berilgan. $\vec{x}\vec{a}=-5, \vec{x}\vec{b}=-11, \vec{x}\vec{c}=20$ shartlarni qanoatlantiruvchi $\vec{x}$ vektor topilsin