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Kompleks sonlar nazariyasining ba`zi bir tatbiqlari

mavzusida yozgan

BITIRUV MALAKAVIY ISHI

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M u n d a r i j a.

Kirish.

Elementar algebra kursini o`rganish davomida sonlar sohasini kengaytira bordik.Butun musbat sonlar $\rightarrow$butun musbat va manfiy sonlar $\rightarrow$rasional sonlar$\rightarrow$rasional va irrasional sonlardan iborat haqiqiy sonlar.O`rta maktab kursidan ham ma`lumki har qanday haqiqiy koeffisientli kvadrat tenglama ham haqiqiy sonlar sohasida yechimga ega bo`lavermaydi.Shu sababli ham haqiqiy sonlar sistemasini kengaytirishga to`g`ri keladi.

Bitiruv malakaviy ishning dolzarbligi: Har qanday haqiqiy koeffisientli ko`phad haqiqiy sonlar sohasida ildizga ega bo`lavermasligi bu sonlar sistemasini kengaytishga to`g`ri keladi.Kompleks sonlar to`plamining yopiqligi ya`ni har qanday darajasi birdan kichik bo`lmagan ko`phad albatta bitta kompleks ildizga ega bo`lishi bu to`plamni o`rganishimizga asos bo`ladi.

Bitiruv malaraiy ishning maqsadi:Kompleks sonlar xossalarini chuqurroq o`rganib, u asosida trigonometriyadagi ba`zi bir muhim ayniyatlarni isbotlash va trigonometriyadagi ba`zi bir muhim yig`indilarni hisoblashdan iborat.

Bitiruv malakaviy ishning ilmiy va ilmiy ahamiyati: Bitiruv ishi mavzusiga oid barcha muhim bo`lgan adabiyotlarni to`plash va ular asosida kompleks sonlar ustida bajariladigan amallar va ularning xossalarini o`rganish hamda Muavr formulasidan foydalanib, trigonometriyada juda muhim bo`lgan ayniyatlarni isbotlash, yig`indilarni topishdan iborat.

Ushbu bitiruv malakaviy ish referatif xarakterga ega bo`lib, ikkita bob va oltita paragrafdan iborat.

Birinchi bob birinchi paragrafda kompleks sonlar haqida boshlang`ich tushunchalar ular ustida bajariladigan asosiy amallar, kompleks sonlar sistemasining kiritishning asosiy sababi keltirilgan.

Ikkinchi paragrafda esa kompleks sonnig trigonometrik ko`rinishi, ushbu ko`rinishda berilgan sonlar ustida amallar xossalari o`rganilgan.

Uchinchi paragrafda algebraik ko`rinishda berilgan sondan kvadrat ildiz chiqarish, trigonometrik ko`rinishda berilgan kompleks sondan ixtiyoriy darajali ildiz chiqarish formulalari ko`rsatilgan.

To`rtinchi paragrafda birning n- darajali ildizlarni hisoblash formulalari keltirilib chiqarilib, ularni xossalari isbotlab berilgan.

Ikkinchi bob beshinchi paragrafda kompleks sonlar xossalaridan foydalanib, Muavr va N`yuton formulalarini qo`llash natijasida isbotlash mumkin bo`lgan ba`zi bir trigonometrik ayniyatlar keltirilgan.

Oxirgi paragrafda kompleks sonlarni trigonometrik yig`indilarni hisoblashdagi tatbiqlari ko`rsatilgan.

I-Bob. Kompleks sonlar haqida asosiy tushunchalar.

1.1.Kompleks sonlar sistemasi haqida boshlang`ich tushunchalar.

Elementar algebra kursini o`rganish davomida sonlar sohasini kengaytira borgan edik: Butun musbat sonlar→butun musbat va manfiy sonlar →rasional sonlar→ rasional va irrasional sonlardan iborat haqiqiy sonlar sistemalarini o`rgandik.

Algebrani o`rganayotgan maktab o`quvchisi butun musbat va kasrlar haqidagi bilimini algebraga arifmetikadan olib kiradi.Aslida algebra manfiy sonlarni kiritishdan, ya`ni eng muhim sonlar sistemalari ichida birinchi sistema barcha butun musbat va manfiy sonlardan hamda noldan iborat butun sonlar sistemasini tayin etishdan va musbat, shuningdek manfiy bo`lgan barcha butun va kasr sonlardan iborat ancha kengroq sistema- rasional sonlar sistemasini tayin etishdan boshlanadi.

Sonlar zapasining bundan keyingi kengaytirilishi muhokamalariga irrasional sonlarni kiritishda sodir bo`ladi.Barcha rasional va barcha irrasional sonlardan iborat sistema haqiqiy sonlar sistemasi deyiladi.haqiqiy sonlar sistemasining asosiy nazariyasi universitetning matematik analiz kursida o`rganiladi. Elementar matematika kursining oxirida haqiqiy sonlar sistemasi kompleks sonlar sistemasigacha kengaytiriladi.Sonlarning bu sistemasi o`quvchi uchun haqiqiy sonlar sistemasiga qaraganda ancha notanish bo`lib ko`rinadi, lekin bu sistema ko`plab ajoyib xossalarga ega.

Endi haqiqiy sonlar sistemasini kompleks sonlar sistemasigacha kengaytiramiz. Kompleks sonlar ushbu masala munosabati bilan kiritiladi. Ma`lumki haqiqiy koeffisientli istalgan kvadrat tenglamani echish uchun haqiqiy sonlarni o`zi etarli emas.

$$x^2 + 1 = 0 \qquad (1)$$

tenglama haqiqiy sonlar ichida ildizi bo`lmagan eng sodda tenglamadir. Olishimizga ko`ra quyidagicha masala qo`yamiz. Haqiqiy sonlar sistemasini shunday sonlar sistemasigacha kengaytiraylikki (1) tenglama yechimga ega bo`lsin.

Sonlarning bu sistemasini qurish uchun ko`rgazmali material sifatida tekislik nuqtalarini olamiz.Haqiqiy sonlarni to`g`ri chiziq nuqtalari orqali ifodalash bizga juda tanish( bunda koordinata boshi va masshab birligi berilganda to`g`ri chiziqning ixtiyoriy nuqtasiga uning absissasini mos qo`ysak, to`g`ri chiziqdagi barcha nuqtalar to`plami bilan barcha haqiqiy sonlar to`plami orasida o`zaro bir qiymatli moslik o`rnatiladi) .Bu mos qo`yishlik matematikaning turli bo`limlariga ishlatiladi va biz unga shunchalik o`rganib qolganmizki,asosan haqiqiy sonlar bilan uni tasvirlovchi nuqtani bir biridan farqlamaymiz.

Endi tekislikning barcha nuqtalari bilan tasvirlanuvchi sonlar sistemasini ta`riflaylik.Shu maqsadda tekislik nuqtalarini qo`shish yoki ko`paytirish amallarini kiritaylik.Yangi amallar kiritayotganimiz sababli,biz uni qaysi maqsad uchun tuzayotgan bo`lsak,o`sha xossalarga ega bo`lishini ta`minlashimiz lozim.Bu ta`riflar ayniqsa ko`paytirish amali uchun ancha sun`iy bo`lib ko`rinadi.

Tekislikda to`g`ri burchakli koordinatalar sistemasi tanlangan bo`lsin. Tekislik nuqtalarini $\alpha, \beta, \gamma, ...$ harflar bilan belgilashni hamda absissasi $a$ , ordinatasi $b$ bo`lgan α nuqtani (a,b) orqali belgilashga ya`ni analitik geometriyada qabul qilinganidan bir oz chetga chiqib , $\alpha = (a,b)$ deb yozishga kelishib olamiz.

Agar $\alpha = (a,b)$ va $\beta = (c,d)$ nuqtalar berilgan bo`lsa, bu nuqtalarning yig`indisi deb absissasi $a + c$ va ordinatasi $b + d$ bo`lgan nuqtani ataymiz, yani

$$(a,b) + (c,d) = (a + c, b + d) \qquad (2)$$

$\alpha = (a, b)$ va $\beta = (c, d)$ nuqtalarning ko`paytmasi deb, absissasi $ac - bd$ va ordinatasi $ad + bc$ bo`lgan nuqtalarni ataymiz, ya`ni

$$(a, b)(c, d) = (ac - bd, ad + bc) \quad (3)$$

Ana shunday yo`l bilan tekislikning barcha nuqtala to`plamida ikkita arifmetik amallarni aniqladik.Quyida bu kiritilgan amallar haqiqiy sonlar sistemasida yoki rasional sonlar sistemasida amallar qanday asosiy xossalarga ega bo`lsa, bu amallar ham shunday asosiy xossalarga egadik; ularning har ikkalasi ham kommutativ va assosiativdir hamda distributivlik qonuni bilan bog`langan va ular uchun teskari amallar-ayirish va bo`lish (nolga bo`lishdan tashqari) amallari mavjud.

Qo`shishning komutativligi va assosiativligi ravshandir,aniqroq aytadigan bo`lsak,haqiqiy sonlarni qo`shishning tegishli xossalaridan kelib chiqadi, chunki tekislikning nuqtalarini qo`shishda ularning absissalarini alohida va ordinatalarini alohida qo`shiladi.Ko`paytirishning kommutativligi ko`payuvchi nuqtalar ko`paytirish ta`rifiga simmetrik ravishda kirishiga asoslanadi. Haqiqatan ham,

$$(a, b)(c, d) = (ac - bd, ad + bc),$$

$$(c, d)(a, b) = (ac - bd, ad + bc) \ ,$$

demak, $(a, b)(c, d) = (c, d)(a, b)$ ya`ni ko`paytirish komutativdir.

Yuqorida aniqlangan aniqlangan ko`paytirish assosiativdir.

Isboti.

$$[(a, b)(c, d)](e, f) = (ac - bd, ad + bc)(e, f) = (ace - bde - adf - bcf, ade + bce + acf - bdf),$$

$$(a, b)[(c, d)(e, f)] = (a, b)(ce - df, cf + de) = (ace - adf - bcf - bde, acf + ade + bce - bdf),$$

demak,

$$[(a, b)(c, d)](e, f) = (a, b)[(c, d)(e, f)]$$

Distributivlik qonuni o`rinli ekanligi quyidagi tenglikdan kelib chiqadi.

$$[(a, b) + (c, d)](e, f) = (a + c, b + d)(e, f) = (ae + ce - bf - df, af + cf + be + de),$$

$$[(a, b) + (c, d)](e, f) = (a, b)(e, f) + (c, d)(e, f) = (ae - bf, af + be) + (ce - df, cf + de) = (ae - bf + ce - df, af + be + cf + de).$$

Endi qo`shish va ko`paytirish amallariga teskari amallarni qaraylik.Agar $\alpha=(a,b)$ va $\beta=(c,d)$ nuqtalar berilgan bo`lsa, u holda ularning ayirmasi shunday $(x,y)$ nuqtalar bo`ladiki, uning uchu $(c,d)+(x,y)=(a,b)$ bo`ladi. Bundan (2) ko`ra

$$\begin{aligned} c+x&=a,\\ d+y&=b. \end{aligned}$$

bo`ladi.

Demak ,$\alpha=(a,b)$ va $\beta=(c,d)$ nuqtalar ayirmasi

$$\alpha-\beta=(a-c,b-d) \qquad (4)$$

nuqta bo`ladi va bu ayirma bir qiymatli aniqlangandir.Xususan nol bo`lib koordinatalar boshi $(0,0)$ nuqta va $\alpha=(a,b)$ nuqta uchun qarama-qarshi nuqta bo`lib esa

$$-\alpha=(-a,-b) \qquad (5)$$

nuqta xizmat qiladi.

$\alpha=(a,b)$ va $\beta=(c,d)$ nuqtalar berilgan bo`lsin va $\beta=(c,d)$ nuqta noldan farqli bo`lsin (ya`ni $c$ va $d$ koordinatalardan hech bo`lmaganda biri noldan farqli) demak, $c^2+d^2\neq 0$.$\alpha=(a,b)$ ni $\beta=(c,d)$ ga bo`lishdan chiqqan bo`linma shunday (x,y) nuqta bo`lshi kerakki, uning uchun

$$(c,d)(x,y)=(a,b)$$

bo`ladi.Bundan (3) ga ko`ra

$$\begin{aligned} cx-dy&=a,\\ dx+cy&=b \end{aligned}$$

bo`ladi.

Bu sistemanini yechib quyidagilarni topamiz.

$$x=\frac{ac+bd}{c^2+d^2}, \qquad x=\frac{bc-ad}{c^2+d^2}$$

Demak, $\beta\neq 0$ bo`lganda $\frac{\alpha}{\beta}$ bo`linma mavjud va bir qiymatli niqlangan.

$$\frac{\alpha}{\beta}=\left(\frac{ac+bd}{c^2+d^2},\frac{bc-ad}{c^2+d^2}\right) \qquad (6)$$

Ushbu tenglikda $\alpha=\beta$ deb olsak, bizning bu ko`paytirishimizda bir bo`lib,absissalar o`qida koordinatalar boshidan $1$ masofada o`tuvchi $(1,0)$ nuqta xizmat qilishini ko`ramiz.Agar (6) da $\alpha=1=(1,0)$ deb olsak, u holda $\beta\neq 0$ uchun teskari nuqta

$$\beta^{-1}=\frac{1}{\beta}=\left(\frac{c}{c^2+d^2},\frac{-d}{c^2+d^2}\right) \qquad (7)$$

ekanligini hosil qilamiz.

Shunday qilib, tekislik nuqtalari bilan tasvirlanadigan sonlar sistemasini tuzdik, shu bilan birga bu sonlar ustida bajariladigan amallar (2) va (3) formulalar bilan aniqlanadi. Bu sonlar sistemasi kompleks sonlar sistemasi deyiladi.

Tasdiq[2]. Kompleks sonlar sistemasi haqiqiy sonlar sistemasining kengaytmasidir.

Isboti.Ushbu tasdiqni isbotlash uchun absissalar o`qida yotuvchi nuqtalar ya`ni $(a,0)$ ko`rinishdagi nuqtalarni qaraymiz. $(a,0)$ nuqtaga $a$ haqiqiy sonni mos keltirib, ko`ramizki, qalayotgan nuqtalar to`plami va barcha haqiqiy sonlar to`plami orasida o`zaro bir qiymatli moslik hosil qilamiz. Ushbu nuqtalarga (2) va (3) formulalarni qo`llasak

$$(a,0)+(b,0)=(a+b,0),$$
$$(a.0)-(b,0)=(a-b,0),$$

kelib chiqadi, ya`ni $(a,0)$ nuqtalar bir-biri bilan mos haqiqiy sonlar kabi qo`shiladi va ko`paytiruiladi.Bundan buyon $(a,0)$ nuqtani $a$ haqiqiy sondan farqlamaymiz, ya`ni

$$(a,0)=a$$

deb olamiz.Shunday qilib,absissalar o`qida yotuvchi va kompleks sonlar sistemasining bir qismi sifatida qaraluvchi nuqtalar to`plami o`zining algebraic xossalari bo`yicha to`g`ri chiziqning nuqtalari kabi odatdagi usulda tasvirlanadigan haqiqiy sonlar sistemasidan hech bir farq qilmaydi.Bu esa yuqorida aytgandek, $(a,0)$ nuqtani a haqiqiy sondan farqlamaslikka imkon beradi.Xususan, kompleks sonlar sistemasidagi nol $(0,0)$ va $(1,0)$ odatdagi haqiqiy sonlar 0 va 1 lardir.

Endi kompleks sonlar ichida (1) tenglamani ildizi bor ekanligini, ya`ni kvadrati haqiqiy son $-1$ ga teng bo`lgan son bor ekanligini ko`rsatamiz. Bu son $(0,1)$ nuqta, ya`ni ordinatalar o`qida koordinata boshidan $1$ birlik masofa yuqorida joylashgan nuqta bo`ladi. Haqiqatan ham (3) ni, ya`ni ko`paytirish amalini qo`llab $(0,1)(0,1)=(-1,0)=-1$ tenglikni hosil qilamiz.Bu nuqtani i deb belgilashga kelishib olaylik, demak $i^2=-1$.

Endi tuzilgan kompleks sonlar uchun ularning odatdagi yozuvini hosil qilish mumkin ekanligini ko`rsataylik.Buning uchun abbalo $b$ haqiqiy sonni $i$ nuqtaga ko`paytmasini topaylik:

$$bi=(b,0)(0,1)=(0,b),$$

demak bu nuqtalar , ordinatalar o`qida yotuvchi va ordinatasi $b$ ga teng bo`lgan nuqtalardir, shu bilan birga ordinatalar o`qining barcha nuqtalari shunday ko`paytmalar ko`rinishda ifodalanadi..Agar $(a,b)$ ixtiyoriy nuqta bo`lsa, u holda $(a,b)=(a,0)+(0,b)$ tenglikka ko`ra

$$(a,b)=a+bi$$

tenglikni hosil qilamiz,ya`ni biz haqiqatan ham kompleks sonlarning odatdagi yozivuga kelamiz. Bu $a+bi$ kompleks sonning odatdagi yozuvidir.Ushbu ifodadagi ko`paytma va yig`indini biz qurgan kompleks sonlar sistemasida aniqlangan ma`noda tushunmoq lozim.

Kompleks sonlar nazariyasining biz amalgam oshirgan qurilishi quyidagi savolni keltirib chiqarishi tabiiy.

Uch o`lchovli fazo nuqtalarini qo`shishni va ko`paytirishni bu nuqtalar to`plami kompleks sonlar sistemasini yoki, hech bo`lmasa , haqiqiy sonlar sistemasini o`z ichiga oladigan qilib aniqlash mumkin emasmikin?

Bu savol ushbu bitiruv ishimiz mavzusidan chetga chiqadi, faqat shuni aytish mumkinki,bu savolga beriladigan javob salbiydir.

Ikkinchi tomondan,kompleks sonlarni yuqorida aniqlangan ma`noda qo`shish,umuman olganda ,tekislikda koordinatalar boshidan chiqqan vektorlarni qo`shish bilan bir xilda ekanligini nazarga olsak,quyidagi savolni qo`yilishi tabiiy:biron-bir n lar uchun n o`lchovli haqiqiy vektor fazoda vektorlarni ko`paytirishni shunday aniqlash mumkinki, vektorlarni bunday ko`paytirishga va odatdagiqo`shishga nisbatan bizning fazo haqiqiy sonlar sistemasini o`z ichiga olgan sonlar sistemasi bo`lib qolsin.Agar amallarning rasional,haqiqiy va kompleks sonlar sistemasiga ega bo`lgan barcha xossalarning bajarilishini talab qiladigan bo`lsak, buni bajarib bo`lmasligini ko`rsatish mumkin.Agar ko`paytirishning kommutativligidan voz kechadigan bo`lsak, u holda bunday yasashni to`rt o`lchovli fazoda bajarish mumkin;sonlarning hosil bo`ladigan sistemasi kvaternionlar sistemasi deyiladi.Shunga o`xshash yasash sakkiz o`lchovli fazoda ham mumkin-unga Keli sonlar sistemasi deb ataluvchi sistema hosil bo`ladi.Shuni ham aytish mumkinki, bu yerda ko`paytirishning faqat kommutativligi emas, balki assosiativligidan ham (uni ancha kuchzis talab bilan almashtirib) voz kechishga to`g`ri keldi.

Tarixiy an`analarga aylanib qolgan kelishuvga asosan,kompleks son $i$ ni mavhum birlik, $bi$ ko`rinishdagi sonlarni sof mavhum sonlar deb ataladi.Ammo bizda bu sonlarning mavjud ekanligi hech qanday shubha uyg`otmaydi va tekislikning bu sonlar bilan ifodalanadigan nuqtalarini ordinate o`qi nuqtalarini ko`rsatishimiz mumkin. $\alpha = a + bi$ kompleks sondagi a son α sonning haqiqiy qismi , bi esa unig mavhum qismi deyiladi.

Nuqtalari kompleks sonlar bilan o`zaro mos qo`yilgan tekislik kompleks tekislik deyiladi. Bu tekislikdagi absissa o`qi haqiqiy o`q va ordinatalar o`qi esa mavhum o`q deyiladi. $a + bi$ ko`rinishdagi kompleks sonlar ustida algebraik amallar yuqoridagi (2)-(4) va (6) formulalarga ko`ra quyidagicha ko`rinishda bajariladi:

$$(a+bi)+(c+di)=(a+c)\_(b+d)i,$$
$$(a+bi)-(c+di)=(a-c)+(b-d)i,$$
$$(a+bi)(c+di)=(ac=bd)+(ad+bc)i,$$
$$\frac{a+bi}{c+di}=\frac{ac+bd}{c^2+d^2}+\frac{bc-ad}{c^2+d^2}i.$$

Kompleks sonlarni qo`shishda ularning haqiqiy qismlarini alohida va mavhum qismlarini alohida qo`shiladi deb ayta olamiz;shunga o`xshash ayirish amali uchun ham aytish o`rinlidir.Ko`paytirsh va bo`lish amallari uchun qoidalar so`zlari ancha uzun bo`lib, ularni bu yerda keltirmaymiz. Odatdagi ko`rinishda berilgan kompleks sonlarni bo`lish amalini yodda saqlab qolish uchun quyidagini eslab qolish etarli; berilgan kasrni surat va maxrajini uning maxrajini qo`shmasiga ko`paytirib, so`ngra soddalashtirishlar qilish lozim ekanligini ko`rish mumkin.

Haqiqatan ham,yuqoridagi fikrlardan

$$\frac{a+bi}{c+di}=\frac{(a+bi)(c-di)}{(c+di)(c-di)}=\frac{(ac+bd)+i(bc-ad)}{c^2+d^2}=\frac{ac+bd}{c^2+d^2}+\frac{bc-ad}{c^2+d^2}i.$$

tenglikni hosil qilamiz.

Kompleks sonlarni tekislikni nuqtalari bilan tasvirlash kompleks sonlar uchun aniqlangan amallarni geometric talqin etilishini taqozo qilishi tabiiy.Qo`shish uchun bunday qalqin qilish hech qanday qiyinchilik tug`dirmaydi.

$\alpha=a+bi$ va $\beta=c+di$ kompleks sonlar berilgan bo`lsin.Ularga mos nuqtalar bilan koordinatalar boshini tutashtiramiz.Tomonlari bu kesmalardan iborat bo`lgan parallelogramm yasaymiz , u holda bu parallelogrammning to`rtinchi uchi, ravshanki $(a+c,b+d)$ nuqta bo`ladi.Demak, geometrik nuqtai nazaridan, kompleks sonlarni qo`shish parallelogramm qoidasi bo`yicha qo`shiladi.

$\alpha = a + bi$ songa qarama-qarshi bo`lgan son kompleks tekislikdagi nuqta bo`lib, u α songa koordinatalar boshiga nisbatan simmetrik nuqta bo`ladi. Bu yerdan ayirishning geometric talqinini hech qiyinchiliksiz hosil qilish mumkin.Kompleks sonlarni ko`paytirish va bo`lishning geometric ma`nosi kompleks sonlarning shu paytga qadar foydalanib kelingan odatdagi yozivudan farqli trigonometric ko`rishdagi yozuvini kiringandan keyingina tushunarli bo`ladi.

α sonning $\alpha = a + bi$ ko`rinishdagi yozivuda bu songa mos keluvchi nuqtaning dekart koordinatalaridan foydalaniladi. Biroq nuqtaning tekislikdagi vaziyati uning qutb koordinatalari:koordinatalar boshidan nuqtagacha bo`lgan masofa r va absissalar o`qining musbat yo`nalishi bilan koordinatalar boshidan bu nuqta tomon yo`nalish orasidagi $\varphi$ burchakning berilishi bilan to`la aniqlanadi.

1.2. **Kompleks sonnig trigonometrik ko`rinishi.**

Analitik geometriya kursidan ma`lumki, biror nuqtaning tekislikdagi vaziyati uning qutb koordinatalari: koordinalar boshidan nuqtagacha bo`lgan masofa r absissalar o`qining musbat yo`nalishi bilan koordinatalar boshidan shu nuqta tomon musbat yo`nalish orasidadi $\varphi$ burchakning berilishi bilan to`la aniqlanadi. Bunda r manfiy bo`lmagan haqiqiy son va faqat nol nuqta uchun nolga teng .

$\alpha = (a, b)$ nuqta qutb koordinatalarda berilgan bo`lsin, u holda unga aniq bir r va φ mos keladi. Bu r son α kompleks sonning moduli deyiladi va $|\alpha|$ deb belgilanadi.

φ burchak α sonning argumenti deyiladi va $\arg\alpha$ deb belgilanadi. φ burchak ichtiyoriy qiymatlarni qabul qilishi mumkin. Bunda musbat burchaklar soat strelkasiga qarama-qarshi yo`nalishda hisoblanadi. Argumenti

2π ga karrali burchakka farq qiluvchi moduli teng bo`lgan sonlar teng deb hisoblanadi.

Shunday qilib, α kompleks sonning argumenti bir-biridan 2π ga karrali bo`lgan sonlarga farq qiladigan cheksiz ko`p qiymatlarga ega;binobarin,moduli va argumentlari bilan berilgan ikkita kompleks sonning tengligidan,ularning modullari teng bo`lib,argumentlari $2\pi$ ga karrali butun songagina farq qilishi to`g`risida xulosa chiqarish mumkin.

Argument faqat 0 son uchun aniqlanmagan, lekin u $|0| = 0$ tenglikdan to`la aniqlanadi.

Kompleks sonning argumenti haqiqiy son ishorasining tabiiy umumlashmasidir. Haqiqatan ham,musbat haqiqiy sonning argumenti 0 ga teng, manfiy haqiqiy sonning argumenti π gat eng; haqiqiy o`qda koordinatalar boshidan faqat ikkita yo`nalish chiqadi va ularni ikkita simvol "+" va - orqali farqlash mumkin, kompleks tekislikda esa 0 nuqtadan chiquvchi yo`nalishlar cheksiz ko`p va ular endi o`zlarining haqiqiy o`qning yo`nalishi bilan hosil qilgan burchaklari bilan farq qiladilar.

Ma`lumki, Dekart va qutb koordinatalar orasida ushbu munosabatlar mavjud:

$$a = r\cos\varphi \ , \ \ b = r\sin\varphi \qquad (1)$$

bundan

$$a^2 + b^2 = r^2 \qquad (2)$$

yoki

$$r = +\sqrt{a^2 + b^2} \qquad (2`)$$

u holda $\alpha = a + bi$ ko`rinishdagi kompleks son quyidagi ko`rinishga keladi.

$$\alpha = a + bi = r\cos\varphi + (r\sin\varphi)i = r(\cos\varphi + i\sin\varphi) \qquad (3)$$

Har qanday komplers sonni (3) ko`rinishda yozish yagonadir.

Faraz qilaylik, $\alpha = a + bi$ kompleks sonni yana bir $\alpha = r_0(\cos\varphi_0 + i\sin\varphi_0)$ ko`rinishda yozish mumkin bo`lsin, bunda r_0 va φ_0 - biror haqiqiy sonlar va $r_0 \geq 0$.U holda $r_0 \cos\varphi_0 = a, r_0 \sin\varphi_0 = b$, bundan $r_9 = +\sqrt{a^2 + b^2}$, ya`ni (2) ko`ra $r_0 = |\alpha|$.Bu yerdan (1) dan foydalanib, $\cos\varphi_0 = \cos\varphi, \sin\varphi_0 = \sin\varphi$ ni hosil qilamiz, ya`ni $\varphi_0 = \arg\alpha$.

Demak, $r = r_0, \varphi = \varphi_0$.

$\alpha = r(\cos\varphi + i\sin\varphi)$ ko`rinishdagi yozuv $\alpha$ kompleks sonning trigonometrik shakli deyiladi va undan kelgusida ko`p marta foydalanamiz.

$\alpha = r(\cos\varphi + i\sin\varphi)$ va $\beta = r_1(\cos\varphi_1 + i\sin\varphi_1)$ kompleks sonlar berilgan bo`lsin.Bu sonlarni ko`paytiraylik:

$$\alpha\beta = [r(\cos\varphi + i\sin\varphi)][r_1(\cos\varphi_1 + i\sin\varphi_1)] = rr_1(\cos\varphi\cos\varphi_1 + i\sin\varphi\cos\varphi_1 +$$
$$+ i\cos\varphi\sin\varphi_1 - \sin\varphi\sin\varphi_1) = rr_1[\cos(\varphi + \varphi_1) + i\sin(\varphi + \varphi_1)]$$

yoki

$$\alpha\beta = rr_1[\cos(\varphi + \varphi_1) + i\sin(\varphi + \varphi_1)] \tag{4}$$

kelib chiqadi. Demak, kompleks sonlar ko`paytmasisining moduli ko`paytuvchilar modullarining ko`paytmasiga teng, argumenti esa ko`paytuvchilar argumentlari yig`indisiga teng,ya`ni

$$|\alpha\beta| = |\alpha||\beta|, \arg(\alpha\beta) = \arg\alpha + \arg\beta \ . \qquad (4`)$$

$\alpha = r(\cos\varphi + i\sin\varphi)$ va $\beta = r_1(\cos\varphi_1 + i\sin\varphi_1)$ kompleks sonlar berilgan bo`lsin va $\beta \neq 0$ bo`lsin. Demak, $r_1 \neq 0$, u holda

$$\frac{\alpha}{\beta} = \frac{r(\cos\varphi + i\sin\varphi)}{r_1(\cos\varphi_1 + \sin\varphi_1)} = \frac{r(\cos\varphi + i\sin\varphi)(\cos\varphi_1 - i\sin\varphi_1)}{r_1(\cos\varphi_1 + i\sin\varphi_1)(\cos\varphi_1 - i\sin\varphi_1)} = \frac{r}{r_1}(\cos(\varphi - \varphi_1) + i\sin(\varphi - \varphi_1)).$$

yoki

$$\frac{\alpha}{\beta} = \frac{r}{r_1}(\cos(\varphi - \varphi_1) + i\sin(\varphi - \varphi_1)). \tag{5}$$

Demak, kompleks sonning bo`linmasining moduli bo`linuvchining modulini bo`luvchining moduliga bo`linganiga, argumenti esa bo`linuvchini argumentidan bo`luvchini argumentini ayrilganiga teng, ya`ni

$$\left|\frac{\alpha}{\beta}\right| = \frac{|\alpha|}{|\beta|}, \arg\left(\frac{\alpha}{\beta}\right) = \arg\alpha - \arg\beta \ .$$

Bu qoidalar , ravshanki,istalgan chekli sondagi kompleks sonlar uchunham o`rinli.Haqiqiy sonlar bo`lgan holga tadbiq etganda,(4`) formulaning birinchisi bu sonlar absolyut qiymatlarining ma`lum xossalarini beradi, ikkinchisi esa haqiqiy sonlarni ko`paytirishdasi ishoralar qoidasiga aylanadi.

Endi ko`paytirish va bo`lishning geometrik ma`nosini aniqlaylik. ( 4 ) formuladan ko`rinadiki, α sonni $\beta = r_1(\cos\varphi_1 + i\sin\varphi_1)$ songa ko`paytmasini tasvirlovchi nuqtani quyidagicha topish mumkin: O nuqtadan $\alpha$ nuqtaga tomon yo`nalgan $|\alpha|$ ga teng vektorni $\varphi_1 = \arg\beta$ burchakka burish, so`ngra esa uni $r_1 = |\beta|$ marta cho`zishdan hosil bo`lgan vektorni uchini koordinatasi $\alpha\beta$ ko`paytmaga mos nuqtani koordinatsini ifodalaydi.

(5) ifodadan $\alpha = r(\cos\varphi + i\sin\varphi) \neq 0$ uchun

$$\alpha^{-1} = r^{-1}[\cos(-\varphi) + i\sin(-\varphi)] \qquad (6)$$

kelib chiqadi.

$\alpha = r(\cos\varphi + i\sin\varphi)$ va $\beta = r_1(\cos\varphi_1 + i\sin\varphi_1)$ kompleks sonlar berilgan bo`lsin. Trigonometrik shaklda berilgan kompleks sonlarning yig`indisini va ayirmasini (4) va (5) ga o`xshash formulalar bilan ifodalsh mumkin emas.Biroq yig`indini moduli uchun quyidagi muhim tengsizliklar mavjud.

Tasdiq. Ikkita kompleks sonning yig`indisining moduli qo`shiluvchlar modullari yig`indisidan katta emas, bu modullar ayirmasidan kichik emas:

$$|\alpha| - |\beta| \leq |\alpha + \beta| \leq |\alpha| + |\beta| \qquad (7)$$

Ushbu tasdiqning isboti elementar geometriyadagi ma`lum uch burchak tomonlari haqidagi teoremadan kelib chiqadi(tomonlari $|\alpha|$ va $|\beta|$gat eng bo`lgan parallelogmmning dioganali $|\alpha + \beta|$ ga tengligi ma`lum). $\alpha, \beta$ va $0$ nuqtalar bitta to`g`ri chiziqda yotgan hol alohida diqqarga sazavordir; faqat shu holdagina (7) formulalar tenglikka aylanadi. (7) ga $\beta$ o`rniga $-\beta$ qo`ysak va $|-\beta| \leq |\beta|$ ekanligini hisobga olsak, u holda

$$|\alpha| - |\beta| \leq |\alpha - \beta| \leq |\alpha| + |\beta| \qquad (8)$$

kelib chiqadi,ya`ni ayirmaning moduli uchun yig`indining modulidagidek o`xshash tengsizliklar hosil bo`ladi.

(7) tengsizlikni quyidagi yo`l bilan ham chiqarish mumkin.Faraz qilaylik, $\alpha = r(\cos\varphi + i\sin\varphi)$ va $\beta = r_1(\cos\varphi_1 + i\sin\varphi_1)$ kompleks sonlar berilgan va $\alpha + \beta$ sonning trigonometric shakli $\alpha + \beta = R(\cos\psi + i\sin\psi)$ bo`lsin.Haqiqiy qismlarini alohida va mavhum qismlarini alohida qo`shib,

$$r\cos\varphi + r_1\cos\varphi_1 = R\cos\psi,$$
$$r\sin\varphi + r_1\sin\varphi_1 = R\sin\psi$$

ifodalarni hosil qilamiz;birinchi tenglikning har ikkala tomonini $\cos\psi$ ga,ikkinchi tenglikni har ikkala tomonini $\sin\psi$ ga ko`paytirib, ularni qo`shsak,quyidagi tenglikni hosil qilamiz:

$$r(\cos\varphi\cos\psi + \sin\varphi\sin\psi) + r_1(\cos\varphi_1\cos\psi + \sin\varphi_1\sin\psi) = R(\cos^2\psi + \sin^2\psi)$$

ya`ni

$$r\cos(\varphi - \psi) + r_1\cos(\varphi - \psi) = R$$

Bu yerdan kosinus hech qachon birdan katta bo`la olmasligi sababli, $r + r_1 \geq R$ tengsizlik kelib chiqadi, ya`ni

$$|\alpha| + |\beta| \geq |\alpha + \beta|.$$

Ikkinchi tomondan, $\alpha = (\alpha + \beta) - \beta = (\alpha + \beta) + (-\beta)$. Bu yerdan hozir isbotlanganiga ko`ra,

$$|\alpha| \leq |\alpha + \beta| + |-\beta| = |\alpha + \beta| + |\beta|$$

tengsizlikni hosil qilamiz, bundan esa

$$|\alpha| - |\beta| \leq |\alpha + \beta|$$

tengsizlikni hosil qilamiz.

Kompleks sonlar uchun “katta” va “kichik” tushunchalarini ma`noga ega bo`ladigan qilib aniqlab bo`lmaydi, chunki bu sonlar, nuqtalari tabiiy ravishda tartiblangan to`g`ri chiziqda yotgan haqiqiy sonlardan farqli o`laroq, to`g`ri chiziqda yotmasdan, balki tekislikda yotadi.Shuning uchun kompleks sonlarning o`zini (ularning modullarini emas) hech qachon tengsizlik belgisi bilan solishtirib bo`lmaydi.

$\alpha = a + bi$ kompleks son berilgan bo`lsin, u holda $\overline{\alpha} = a - bi$ kompleks son $\alpha$ songa qo`shma son deyiladi.

Ravshanki , α son $\overline{\alpha}$ songa qo`shma son bo`ladi. Haqiqiy sonni qo`shmasi shu sonnig o`ziga teng bo`ladi.

Geometrik nuqtai nazaridan o`zaro qo`shma sonlar haqiqiy o`qqa nisbatan simmetrik bo`lgan nuqtalardan iborat bo`ladi. Bu yerdan

$$|\overline{\alpha}| = |\alpha|, \arg \overline{\alpha} = -\arg \alpha$$

tengliklar kelib chiqadi.

2-Tasdiq. Qo`shma kompleks sonlar yig`indisi va ko`paytmasi haqiqiy sonlar bo`ladi.

Isboti. Haqiqatan, ham

$$\alpha + \overline{\alpha} = 2a \ , \quad \alpha\overline{\alpha} = a^2 + b^2 = |\alpha|^2 .$$

Oxirgi tenglik $\alpha\overline{\alpha}$ son $\alpha \neq 0$ bo`lganda , hatto musbat bo`lishini ko`rsatadi.

$$(a - bi) + (c - di) = (a + c) - (b + d)i$$

tenglik ikkita sonnning yig`indisi bilan qo`shma bo`lgan son qo`shiluvchilar bilan qo`shma bo`lgan sonlarning yig`indisiga teng ekanligini ko`rsatadi.

Shunga o`xshash,

$$(a - bi)(c - di) = (ac - bd) - (ad + bc)i$$

tenglikdan ko`paytmaga qo`shma bo`lgan son ko`paytuvchilar bilan qo`shma bo`lgan sonlarning ko`paytmasiga teng ekanligi kelib chiqadi.

Bevosita tekshirish yo`li bilan

$$\overline{\alpha - \beta} = \overline{\alpha} - \overline{\beta},$$

$$\overline{\left(\frac{\alpha}{\beta}\right)} = \frac{\overline{\alpha}}{\overline{\beta}}.$$

formulalarni to`g`riligini tekshirish mumkin.Shunday qilib, quyidagi tasdiq isbotlandi.

3-Tasdiq.

$$\overline{\alpha+\beta}=\overline{\alpha}+\overline{\beta},$$
$$\overline{\alpha\beta}=\overline{\alpha}\overline{\beta},$$
$$\overline{\alpha-\beta}=\overline{\alpha}-\overline{\beta},$$
$$\overline{\left(\frac{\alpha}{\beta}\right)}=\frac{\overline{\alpha}}{\overline{\beta}}.$$

4-Tasdiq[2].

Agar α son biron-bir usul bilan $\beta_1,\beta_2,....,\beta_n$ kompleks sonlar orqali qo`shish,ko`paytirish,ayirish va bo`lish yordamida ifodalangan bo`lsa, u holda bu ifodada barcha β_k sonlarni ularning qo`shmalari bilan almashtirsak, u holda $\alpha$ bilan qo`shma bo`lgan sonni hosil qilamiz; xuxusan,agar $\alpha$ son haqiqiy bo`lsa,u barcha β_k sonlarni ularning qo`shmalari bilan almashtish natijasida o`zgarmaydi.

Bu tasdiqni n bo`yicha matematik induksiya metodidan foydalanib isbotlaymiz. $n=2$ da ushbu tasdiqni to`g`riligi yuqoridagi 3-tasdiqdan kelib chiqadi.faraz qilaylik, $\alpha$ son biron-bir usul bilan $\beta_1,\beta_2,....,\beta_n$ kompleks sonlar orqali qo`shish,ko`paytirish,ayirish va bo`lish yordamida ifodalangan bo`lsin.Bu ifodada qo`shish,ayirish,ko`paytirish va bo`lish amallari qay tartibda bajarilishi aniq ko`rsatilgan.Oxirgi bajaradigan ishimiz bu amallarni birortasini$\beta_1,\beta_2,....,\beta_k$ (bu yerda $1\le k\le n-1$) sonlar orqali ifodalangan $\gamma_1$ songa va $\beta_{k+1},\beta_{k+2},....,\beta_n$ sonlar orqali ifodalangan$\gamma_2$ songa tadbiq qilish bo`ladi.Induktiv faraz bo`yicha $\beta_1,\beta_2,....,\beta_k$ sonlarni ularning qo`shmalariga almashtirish γ_1 ni $\bar{\gamma}_1$ ga, $\beta_{k+1},\beta_{k+2},....,\beta_n$ sonlarni ularni qo`shmasi bilan almashtirish esa$\gamma_2$ ni $\bar{\gamma}_2$ ga almashtirishga olib keladi.Endi $\alpha$ ni ikkita $\gamma_1,\gamma_2$ larga bog`liqligini hisobga olib, 3-tasdiqni qo`llasak, ya`ni $\beta_1,\beta_2,....,\beta_n$ larni qoshmalariga almashtirsak, u holda γ_1,γ_2 lar qo`shmalariga almashadi , α esa $\overline{\alpha}$ ga aylanadi.

Tasdiq isbotlandi.

1.3.Kompleks sondan ildiz chiqarish.

Kompleks sonlarni darajaga ko`tarish.

i mavhum birlikning aniqlanishiga ko`ra:

$$i^2 = -1 \quad , \quad i^3 = -i \quad , \quad i^4 = 1,$$

bo`ladi, umuman bulardan

$$i^{4k} = 1, i^{4k+1} = i. i^{4k+2} = -1, i^{4k+3} = -i \qquad (1)$$

kelib chiqadi.

$\alpha = a + bi$ kompleks sonni butun musbat n-darajaga ko`tarish kerak bo`lsin. Buning uchun $(a+bi)^n$ ifodaga N`yuton formulasini tadbiq qilish va (1) tenglikdan foydalanish etarli.

Trigonometrik ko`rinishdagi kompleks son berilgan bo`lsin.

$\alpha = r(\cos\varphi + i\sin\varphi)$ trigonometrik ko`rinishdagi kompleks sonni butun musbat $n$- darajaga ko`tarish uchun trigonometrik ko`rinishdagi kompleks sonlarni ko`paytirish formulasi

$$\alpha\beta = rr_1[\cos(\varphi + \varphi_1) + i\sin(\varphi + \varphi_1)]$$

dan foydalansak Muavr formulasi[2] deb ataluvchi quyidagi formulani hosil qilamiz:

$$[r(\cos\varphi + i\sin\varphi)]^n = r^n(\cos n\varphi + i\sin n\varphi) \qquad (2)$$

Demak, berilgan kompleks sonni n-darajaga ko`tarish uchun, shu sonni modulini $n$-darajaga ko`tarish, argumentini esa n marta ortirish kerak. Bu (2) formula n manfiy butun son bo`lgan hol uchun ham o`rinli.Bu fakt to`g`riligi

$$\alpha^{-n} = (\alpha^{-1})^n$$

tenglikdan kelib chiqadi.

Muavr formulasining xususiy holi , ya`ni

$$(\cos\varphi + i\sin\varphi)^n = \cos n\varphi + i\sin n\varphi$$

tenglik karrali burchakning sinusi va kosinusi uchun formulalarni osongina hosil qilishga imkon beradi. Haqiqatan ham, bu tenglikning chap tomonini N`yuton binom formulasi bo`yicha ochib chiqib va tenglikning har ikkala tomonining haqiqiy va mavhum qismlarini alohida alohida tenglab, quyidagilarni hosil qilamiz:

$$\cos n\varphi = \cos^n \varphi - C_n^2 \cos^{n-2} \varphi \sin^2 \varphi + C_n^4 \cos^{n-4} \varphi \sin^4 \varphi - .... ,$$

$$\sin n\varphi = C_n^1 \cos^{n-1} \varphi \sin \varphi - C_n^3 \cos^{n-3} \varphi \sin^3 \varphi + C_n^5 \cos^{n-5} \varphi \sin^5 \varphi - ....$$

Bu yerda $C_n^k = \dfrac{n(n-1)(n-2)....(n-k+1)}{1 \cdot 2 \cdot 3 \cdot ... \cdot k}$,

$n = 2$ bo`lganda bizga elementar matematikadan ma`lum bo`lgan quyidagi formulalarni hosil qilamiz:

$$\cos 2\varphi = \cos^2 \varphi - \sin^2 \varphi,$$
$$\sin 2\varphi = 2\sin \varphi \cdot \cos \varphi.$$

$n = 3$ bo`lganda esa quyidagi formularni hosil qilamiz:

$$\cos 3\varphi = \cos^3 \varphi - 3\cos \varphi \sin^2 \varphi,$$
$$\sin 3\varphi = 3\cos^2 \varphi \sin \varphi - \sin^3 \varphi.$$

Kompleks sonlardan ildiz chiqarish.

Kompleks sondan ildiz chiqarish ko`pgina qiyinchiklar bilan bog`liq. Avvalo $\alpha = a + bi$ ko`rinishdagi sondan kvadrat ildiz chiqarishdan boshlaylik. Faraz qikaylik kvadrati $\alpha$ ga teng son mavjud va u $u + vi$ ko`rinishdagi son bo`lsin. Bizning maqsadimiz ushbu $u, v$ larni topishdan iborat. Olishimizga ko`ra

$$\sqrt{a + bi} = u + vi$$

bo`ladi.Bundan

$$(u + vi)^2 = a + bi$$

bu tenglikdan esa

$$u^2 - v^2 = a,$$
$$2uv = b,$$

kelib chiqadi. Bu har ikki tenglikni tomonlarini kvadratga ko`tarib, so`hgra ularni qo`shamiz:

$$(u^2 - v^2)^2 + 4u^2v^2 = a^2 + b^2$$

Bundan

$$u^2+v^2=\sqrt{a^2+b^2} \qquad (4)$$

kelib chiqadi.(3) tenglikni birinchisidan va bu (4) tenglikdan quyidagilarni hosil qilamiz:

$$u^2=\frac{1}{2}(a+\sqrt{a^2+b^2}),$$
$$v^2=\frac{1}{2}(-a+\sqrt{a^2+b^2}),$$

bulardan kvadrat ildiz chiqarib u va v ular uchun ikkitadan qiymatga ega bo`lamiz. Bu sonlarni ixtiyoriy olish mumkin emas. Ularni $uv$ ko`paytma ishorasi b ni ishorasi bilan bir xil bo`ladigan qilib tanlab olish kerak.Natijada faqat $u$ va $v$ larni bir biriga bog`liq 2 ta qiymatini olish mumkin bo`ladi, hosil bo`lgan $u+vi$ sonlar 2 ta bo`ladi va ular faqat ishorasi bilan farqlanadi. Demak, kompleks sondan har doim kvadrat ildiz chiqarish mumkin va bu ildizlar bir-biridan faqat ishorasi bilan farq qiladi.

Misol.

$\sqrt{-15+8i}$ ni hisoblang.

Yechish:

Quyidagi formuladan foydalanamiz

$$\sqrt{a+bi}=\pm(u+vi),$$

bunda , $u=\pm\sqrt{\frac{a+\sqrt{a^2+b^2}}{2}}$, $v=\pm\sqrt{\frac{-a+\sqrt{a^2+b^2}}{2}}$ .Agar $b>0$ bo`lsa u va v bir xil ishorada ,

$b<0$ bo`lsa , u holda u va v lar qarama-qarshi ishorada olinadi.

Berilgan misolda : $a=-15, b=8>0$.

$$u=\pm\sqrt{\frac{\sqrt{225+64}-15}{2}}=\pm1, \quad v=\pm\sqrt{\frac{\sqrt{225+64}+15}{2}}=\pm4.$$

Bundan

$$\sqrt{-15+8i}=\pm(1-4i)$$

tenglikni hosil qilamiz.

Endi $\alpha = r(\cos\varphi + i\sin\varphi)$ sondan n -darajali ildiz chiqaraylik. Faraz qilaylik, natijada $\rho(\cos\psi + i\sin\psi)$ son hosil bo`lsin. U holda

$$[\rho(\cos\psi + i\sin\psi)]^n = r(\cos\varphi + i\sin\varphi) \qquad (5)$$

bundan Muavr formulasiga ko`ra $\rho^n = r$ yoki $\rho = \sqrt[n]{r}$ Ikkinchidan, ( 5 ) tehglikni chap tomonida turgan kompleks son argumenti $n\psi$ ga teng. Shu sababli , $n\psi = \varphi + 2k\pi$ (bu yerda $k$ - butun son) bo`ladi. Bundan $\psi = \dfrac{\varphi + 2k\pi}{n}$ bo`ladi.

Endi ko`rish qiyin emaski, agar $\sqrt[n]{r}(\cos\dfrac{\varphi + 2k\pi}{n} + i\sin\dfrac{\varphi + 2k\pi}{n})$ sonni olsak , uni n -darajasi $\alpha = r(\cos\varphi + i\sin\varphi)$ songa tehg. Demak,

$$\sqrt[n]{r(\cos\varphi + i\sin\varphi)} = \sqrt[n]{r}(\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi + 2k\pi}{n}) \qquad (6)$$

Agar $k = 0,1,2,3,...,n-1$ (7) qiymatlar bersak har xil ildizlarni hosil qilamiz.

Endi $k \geq n$ ixtiyoriy butun son bo`lsin, u holda $k = nq + r$ (bunda $0 \leq r \leq n-1$, q -biror butun son) deb olish mumkin. Bundan,

$$\frac{\varphi + 2k\pi}{n} = \frac{\varphi + 2(nq + r)\pi}{n} = \frac{\varphi + 2r\pi}{n} + 2q\pi$$

Demak , $k = nq + r$ bo`lganda kosinus va sinuslarni davri $2\pi$ bo`lgani uchun yana (7) sistemaga kiruvchi $k = r$ bo`lgandagi ildizni qiymatini hosil qilamiz. Demak, $\alpha$ kompleks sondan har doim $n$ -darajali ildiz chiqarish mumkin, natijada $n$ ta har xil qiymatlar hosil bo`ladi. Bu barcha ildizlarni moduli $\sqrt[n]{r}$ ga teng. Ular markazi nol nuqtaga bo`lgan aylanada yotadi va uni teng $n$ ta bo`lakka bo`ladi.

Misollar.

1. $$\sqrt[6]{8(\cos\frac{3}{4}\pi + i\sin\frac{3}{4}\pi)} = \sqrt[6]{2^3}(\cos\frac{\frac{3}{4}\pi + 2k\pi}{6} + i\sin\frac{\frac{3}{4}\pi + 2k\pi}{6}) \;,$$

bunda , $k = 0,1,2,3,4,5.$ 2

yoki

$$\beta_0=\sqrt{2}(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}),\qquad \beta_1=\sqrt{2}(\cos\frac{11\pi}{24}+i\sin\frac{11\pi}{24}),$$

$$\beta_2=\sqrt{2}(\cos\frac{19\pi}{24}+i\sin\frac{19\pi}{24}),\quad \beta_3=\sqrt{2}(\cos\frac{9\pi}{8}+i\sin\frac{9\pi}{8}),$$

$$\beta_4=\sqrt{2}(\cos\frac{35\pi}{24}+i\sin\frac{35\pi}{24}),\quad \beta_5=\sqrt{2}(\cos\frac{43\pi}{24}+i\sin\frac{43\pi}{24})..$$

2. $(\frac{1+i\sqrt{3}}{1-i})^{20}$ ni hisoblang.

Yechish:

Avvalo $1+i\sqrt{3}$ va $1-i$ sonlarni trigonometrik ko`rinishga keltiramiz:

$$r_1=\sqrt{1+3}=2\ .$$

$$\cos\varphi_1=\frac{1}{2}\ ,\ \sin\varphi_1=\frac{\sqrt{3}}{2}\ .\ \text{ Bundan }\ \varphi_1=\frac{\pi}{3}\ .$$

Demak, $1+i\sqrt{3}=2(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})$.

$$r_2=\sqrt{1+1}=\sqrt{2}\ ,\ \cos\varphi_2=\frac{1}{\sqrt{2}}\ ,\ \sin\varphi_2=-\frac{1}{\sqrt{2}}\ .\text{Bundan }\ \varphi_2=\frac{7\pi}{4}\ .$$

ya`ni $1-i=\sqrt{2}(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4})$.

U holda,

$$\frac{1+i\sqrt{3}}{1-i}=\frac{2(-"s\frac{\pi}{3}+i\sin\frac{\pi}{3})}{\sqrt{2}(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4})}=\sqrt{2}(cos(\frac{\pi}{3}-\frac{7\pi}{4})+i\sin(\frac{\pi}{3}-\frac{7\pi}{4})=$$

$$=\sqrt{2}(\cos(2\pi+\frac{\pi}{3}-\frac{7\pi}{4})+i\sin(2\pi+\frac{\pi}{3}-\frac{7\pi}{4})=\sqrt{2}(\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12}).$$

Nihoyat,

$$\left(\frac{1+i\sqrt{3}}{1-i}\right)^{20}=[\sqrt{2}(\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12})]^{20}=\left(\sqrt{2}\right)^{20}[\cos\frac{20\cdot 7\pi}{12}+i\sin\frac{20\cdot 7\pi}{12}]=$$

$$=2^{10}(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3})=2^{9}(1-i\sqrt{3}).$$

1.4. Birning ildizlari.

1 sonidan n- darajali ildiz chiqarish holi ayniqsa muhimdir.Bu ildiz n ta har xil qiymatga ega bolib ular quyidagicha topiladi.

$1=\cos 0^0+\sin 0^0$ sondan n- darajali ildiz chiqaraylik. Oldingi mavzudan ma`lumki,bu ildizlar quyidagi formula orqali topiladi:

$$\sqrt[n]{1}=\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}\ ,\quad k=0,1,2,...,n-1 \qquad (1)$$

1 sonidan chiqarilgan n- darajali ildizning haqiqiy qiymatlari (1) formuladan, agar n juft bo`lsa, $k=0$ va $k=\frac{n}{2}$ bo`lganda, agar n toq bo`lsa, $k=0$ bo`lganda hosil bo`ladi.Kompleks tekislikda birning $n$- darajali ildizlari birlik aylanada joylashgan bo`lib, uni bir biriga teng bo`lgan $n$ ta yoyga ajratadi; ana shunday nuqtalardan biri $1$ sonidir . Bu yerdan, birning $n$- darajali ildizlari ichida haqiqiy bo`lmaganlari haqiqiy o`qqa nisbatan simmetrik joylashganligi, ya`ni juft-jufti bilan qo`shma ekanligi kelib chiqadi.Birning kvadrat ildizi ikkita qiymatga ega, ya`ni $n=2$ bo`lsa birning kvadrat ildizi $2$ ta: $1$ va $-1$ sonlari bo`ladi.

$n=3$ da birning uchta ildizi bo`lib, ular quyidagi sonlar bo`ladi:

$$\varepsilon_0=1,\ \varepsilon_1=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}=-\frac{1}{2}+i\frac{\sqrt{3}}{2}\ ,$$

$$\varepsilon_2=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}=-\frac{1}{2}-i\frac{\sqrt{3}}{2}\ . \qquad (2)$$

$n=4$ da esa birning to`rta ildizi bo`lib, ular $1,i,-1,-i$ sonlardan iborat bo`ladi.

1-Tasdiq. α kompleks sonning n- darajali barcha ildizlarini bu ildizlardan birortasini birning n- darajali barcha ildizlariga ko`paytirib chiqish bilan hosil qilish mumkin.

Isboti. β son α sonni n- darajali ilizlaridan biri bo`lsin, yani $\beta^n = \alpha$ bo`lsin. γ son esa birning n- darajali ildizlaridan biri bo`lsin, ya`ni $\gamma^n = 1$ bo`lsin.Bundan

$$(\beta\gamma)^n = \beta^n \gamma^n = \alpha$$

bo`ladi. Demak, $\beta\gamma$ ham $\alpha$ ni $n$-darajali ildizlaridan biri bo`ladi.Bundan esa β ni birni n- darajali ildizlarini har biriga ko`paytirib, n ta turli qiymatlarni hosil qilamiz.

Misollar.

1. -27 sondan kub ildiz chiqaraylik, ravshanki uni ildizlaridan biri -3 ga teng (2) formulaga ko`ra , uning qolgan ikkita ildizi quyidagi sonlarga teng bo`ladi:

$$\beta_1 = -3\varepsilon_1 = \frac{3}{2} - i\frac{3\sqrt{3}}{2}, \beta_2 = -3\varepsilon_2 = -\frac{3}{2} + i\frac{3\sqrt{3}}{2}.$$

2. $\sqrt[4]{256}$ ildizni quymatlarini hisoblaylik. Bizga bu 256 sonini bitta ildizi 4 ga teng ekanligi ma`lum , uning qolgan ildizlarini topish uchun ushbu ildizni birning to`rtinchi darajali ildizlariga ketma-ket ko`paytirib chiqamiz, natijada quyidagi quymatlarni topamiz: $4, -4, 4i, -4i$.

2- Tasdiq. Birning n-darajali ikkita ildizining ko`paytmasi yana birning $n$- darajali ildizi bo`ladi.

Isboti. Agar $\varepsilon^n = 1$ va $\gamma^n = 1$ bo`lsin, u holda,

$$(\varepsilon\gamma)^n = \varepsilon^n \gamma^n = 1$$

Demak , $\varepsilon\gamma$ ham birning n- darajali ildizi bo`ladi.

3-Tasdiq. Birning n- darajali ildiziga tekari son ham birning n-darajali ildizi bo`ladi, Umuman, birning $n$- darajali ildizining har qanday darajasi yana birning $n$- darajali ildizi bo`ladi.

Isboti. $\varepsilon^n = 1$ bo`lsin, ya`ni ε birning biror n- darajali ildizi bo`lsin , u holda $\varepsilon \cdot \varepsilon^{-1} = 1$ ekanligidan $\varepsilon^n \cdot (\varepsilon^{-1})^n = 1$ kelib chiqadi. Bundan esa $(\varepsilon^{-1})^n = 1$

kelib chiqadi, ya`ni $\varepsilon^{-1}$ soni ham birning $n$- darajali ildizi bo`ladi.

Birning k- darajali har qanday ildizi k ga karrali bo`lgan har qanday $l$ uchun ham birning $l$- darajali ildizi bo`ladi.Bu erdan, agar birning n- darajali barcha ildizlari to`plamini qarab chiqadigan bo`lsak, u holda bu ildizlarning ba`zilari $n$ ning bo`luvchilari bo`lgan biror $n`$ lar uchun birning $n`$-darajali ildizlari bo`lishligi kelib chiqadi.biroq har qanday n uchun birning n- darajali shunday ildizlari mavjudki, ular birning n dan kichik darajali hech qanday ildizi bo`la olmaydi.Bunday ildizlar birning $n$- darajali boshlang`ich ildizlari deyiladi.Ularning mavjud ekanligi (1) formuladan kelib chiqadi:agar uldizning berilgan k ning qiymatiga mos keluvchi qiymatini ε_k orqali (demak, $\varepsilon_0 = 1$ bo`ladi) belgilasak, u holda Muavr formulasiga ko`ra,

$$\varepsilon_1^k = \varepsilon_k .$$

Demak, ε_1 ning hech qanday n dan darajasi 1 ga teng bo`la olmaydi, ya`ni $\varepsilon_1 = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$ boshlang`ich ildiz bo`ladi.

4-Tasdiq. Birning n- darajali ε ildizining $\varepsilon^k, k = 0,1,...,n-1$ darajalari har xil, ya`ni birning $n$- darajali barcha ildizlarini tashkil etganda va faqat shundagina $\varepsilon$ boshlang`ich ildiz bo`ladi.

Isboti.haqiqatan ham, birning tasdiqda ko`rsatilgan barcha darajalari har xil bo`lsa, u holda ravshanki ε birning n-darajali boshlang`ich ildizi bo`ladi. Agar,masalan , $0 \le k < l \le n-1$ bo`lganda $\varepsilon^k = \varepsilon^l$ bo`lsa, $\varepsilon^{l-k} = 1$ bo`lib, $1 \le l-k \le n-1$ tengsizliklarga binoan $\varepsilon$ boshlang`ich ildiz bo`lmaydi.

Yuqorida topilgan ε_1 son, umumiy holda, yagona n-darajali boshlang`ich ildiz emas.barcha bunday ildizlarni topish uchun quyidagi teorema xizmat qiladi.

Teorema.Agar ε birning n-darajali boshlang`ich ildizi bo`lsa, u holda k son n bilan o`zaro tub bo`lganda va faqat shundagina ε^k son n-darajali boshlang`ich ildiz bo`ladi.

Isboti.Haqiqatan ham, k va n sonlarning eng katta umumiy bo`luvchisi $d$ bo`lsin. Agar $d > 1$ va $k = dk`, n = dn`$ bo`lsa, u holda

$$(\varepsilon^k)^{n`} = \varepsilon^{kn`} = (\varepsilon^n)^{k`} = 1,$$

ya`ni $\varepsilon^k$ ildiz birning $n`$-darajali ildizi ekan.

Ikkinchi tomondan,aytaylik, $d = 1$va shu bilan birga ε^k son birning m-darajali ($1 \le m < n$) ildizi bo`lsin.Demak,

$$(\varepsilon^k)^m = \varepsilon^{km} = 1.$$

ε son birning n-darajali boshlang`ich ildizi bo`lgani sababli, ya`ni faqatgina uning $n$ ga karrali bo`lgan darajalarigina 1 gat eng bo`lgani uchun $km$ son $n$ ga karrali bo`ladi.Biroq, $1 \le m < n$ bo`lgani uchun $k$va $n$ sonlar o`zaro tub bo`la olmaydi.Bu esa shartimizga ziddir.

Shunday qilib, birning n-darajali boshlang`ich ildizlari soni $n$ dan kichik va u bilan o`zaro tub bo`lgan musbat butun k larning soniga teng. Odatda $\varphi(n)$ orqali belgilanadigan bu sonning ifodasini sonlar nazariyasi kursidan topish mumkin.

Agar p tub son bo`lsa, u holda birning o`zidan tashqari ana shu ildizlar birning p-darajali boshlang`ich ildizlari bo`ladi.Shuning ham aytish kerakki, birning to`rtinchi darajali ildizlari ichida boshlang`ich ildizlar faqat i va $-i$ lar bo`ladi.

II-bob.Kompleks sonlar nazariyasining ba`zi bir tadbiqlari.

2.1.Trigonometrik ayniyatlarni isbotlash.

1.Trigonometrik ko`rinishda berilan kompleks sonni darajaga ko`tarishning ushbu

$$[r(\cos\varphi + i\sin\varphi)]^n = r^n(\cos n\varphi + i\sin n\varphi) \qquad (1)$$

formulasida $r = 1$ deb olsak, u holda Muavr formulasining xususiy holi , ya`ni

$$(\cos\varphi + i\sin\varphi)^n = \cos n\varphi + i\sin n\varphi \qquad (*)$$

tenglikni hosil qilamiz.Bu formulaning chap tomonidagi qavsni N`yuton binom formulasi bo`yicha ochib chiqib ,so`ngra $i^2 = -1, i^3 = -i, i^4 = 1$ va umuman ixtiyoriy musbat butun $k$son uchun

$$i^{4k} = 1, i^{4k+1} = i, i^{4k+2} = -1, i^{4k+3} = -i \qquad (2)$$

ekanligini hisobga olsak va (*) tenglikning ikkala tomonidagi kompleks sonlarning haqiqiy va mavhum qismlarini tenglashtirib quyidagi formulalarni hosil qilamiz:

$$\cos n\varphi = \cos^n \varphi - C_n^2 \cos^{n-2} \varphi \sin^2 \varphi + C_n^4 \cos^{n-4} \varphi \sin^4 \varphi - .... ,$$

$$\sin n\varphi = C_n^1 \cos^{n-1} \varphi \sin \varphi - C_n^3 \cos^{n-3} \varphi \sin^3 \varphi + C_n^5 \cos^{n-5} \varphi \sin^5 \varphi - ....$$

Bu yerda $C_n^k = \dfrac{n(n-1)(n-2)....(n-k+1)}{1 \cdot 2 \cdot 3 \cdot ... \cdot k}$,

Shunday qilib, biz karrali burchakning kosinus va sinuslarini oddiy burchak kosinus va sinuslari orqali ifodalovchi formulalarni hosil qildik.Xususiy hollarni qaraylik.

$n = 2$ bo`lganda bizga elementar matematikadan ma`lum bo`lgan quyidagi formulalarni hosil qilamiz:

$$\cos 2\varphi = \cos^2 \varphi - \sin^2 \varphi,$$
$$\sin 2\varphi = 2 \sin \varphi \cdot \cos \varphi.$$

$n = 3$ bo`lganda esa quyidagi formularni hosil qilamiz:

$$\cos 3\varphi = \cos^3 \varphi - 3 \cos \varphi \sin^2 \varphi,$$
$$\sin 3\varphi = 3 \cos^2 \varphi \sin \varphi - \sin^3 \varphi.$$

2. $\sin^3 x$ ni birinchi darajali kosinus va sinuslarning karralisi orqali ifodalang.
Yechish.

$$\alpha = \cos x + i \sin x$$

deb belgilash kiritaylik.U holda ushu belgilashdan

$$\alpha^{-1} = \frac{1}{\alpha} = \frac{1}{\cos x + i \sin x} = \frac{\cos x - i \sin x}{(\cos x + i \sin x)(\cos x - i \sin x)} =$$

$$= \frac{\cos x - i \sin x}{\cos^2 x + \sin^2 x} = \cos x - i \sin x.$$

U holda

$$\alpha^k = (\cos x + i \sin x)^k = \cos kx + i \sin kx,$$

$$\alpha^{-k} = (\cos x + i \sin x)^{-k} = [(\cos x + i \sin x)^{-1}]^k = [\cos(-x) + i \sin(-x)]^k =$$

$$= \cos(-kx) + i \sin(-kx) = \cos kx - \sin kx.$$

Shunday qilib,

$$\alpha^k = \cos kx + i \sin kx,$$
$$\alpha^{-k} = \cos kx - i \sin kx.$$

Bu erdan

$$\cos kx = \frac{\alpha^k + \alpha^{-k}}{2},$$
$$\sin kx = \frac{\alpha^k - \alpha^{-k}}{2i}. \qquad (3)$$

Formulalarni hosil qilamiz.Ushbu formulada $k=1$ deb olsak, u holda

$$(\sin x)^3 = \left(\frac{\alpha - \alpha^{-1}}{2i}\right)^3 = -\frac{1}{8i}(\alpha^3 - 3\alpha + 3\alpha^{-1} - \alpha^{-3}) = -\frac{1}{8i}[(\alpha^3 - \alpha^{-3}) - 3(\alpha - \alpha^{-1})] =$$

$$= -\frac{1}{8i}[2i\sin 3x - 3\cdot 2i\sin x] = \frac{1}{4}(3\sin x - \sin 3x).$$

Shunday qilib,

$$\sin^3 x = \frac{3\sin x - \sin 3x}{4}.$$

3. $\cos^8 x$ ni birinchi darajali kosinus va sinuslarning karralisi orqali ifodalang.
Yechish.
(2) tenglikda $k=1$ deb quyidagi tenglikni hosil qilamiz:

$$(\cos x)^8 = \left(\frac{\alpha + \alpha^{-1}}{2}\right)^8 = \frac{1}{256}(\alpha^8 + 8\alpha^6 + 28\alpha^4 + 56\alpha^2 + 70 +$$

$$+56\alpha^{-2} + 28\alpha^{-4} + 8\alpha^{-6} + \alpha^{-8}) = \frac{1}{256}[(\alpha^8 + \alpha^{-8}) + 8(\alpha^6 + \alpha^{-6}) +$$

$$+28(\alpha^4 + \alpha^{-4}) + 56(\alpha^2 + \alpha^{-2}) + 70] = \frac{1}{256}(2\cos 8x + 16\cos 6x +$$

$$+56\cos 4x + 112\cos 2x + 70) = \frac{\cos 8x + 8\cos 6x + 28\cos 4x + 56\cos 2x + 35}{128}.$$

Demak,

$$\cos^8 x = \frac{\cos 8x + 8\cos 6x + 28\cos 4x + 56\cos 2x + 35}{128}$$

4. Ushbu

$$1 - C_n^2 + C_n^4 - C_n^6 + C_n^8 - C_n^{10} + \dots$$

ig`indini hisoblang.
Yechish. $(1+i)^n$ ni N`yuton formulasidan foydalanib yoyib yozamiz:

$$(1+i)^n = 1 + C_n^1\cdot i + C_n^2\cdot i^2 + C_n^3\cdot i^3 + \dots + C_n^{n-1}\cdot i^{n-1} + i^n$$

Bizga ma`lum (*) tengliklardan foydalanib ushbu tenglikni quyidagicha o`zgartiramiz:

$$(1+i)^n = 1 - C_n^2 + C_n^4 - C_n^6 + C_n^8 - \dots) + i(C_n^1 - C_n^3 + C_n^5 - C_n^7 + \dots).$$

Ikkinchi tomondan,

$$1 + i = \sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$$

ekanligini hisobga olsak, u holda

$$(1+i)^n = 2^{\frac{n}{2}}(\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4})$$

tenglikka kelamiz.Demak,

$$1 - C_n^2 + C_n^4 - C_n^6 + C_n^8 - C_n^{10} + ... = 2^{\frac{n}{2}}\cos\frac{n\pi}{4},$$

$$C_n^1 - C_n^3 + C_n^5 - C_n^7 + C_n^9 - ... = 2^{\frac{n}{2}}\sin\frac{n\pi}{4},$$

tengliklarni hosil qilamiz, Xususan, $n=14$ deb olsak, u holda

$$1 - C_{14}^2 + C_{14}^4 - C_{14}^6 + C_{14}^8 - C_{14}^{10} + C_{14}^{12} - C_{14}^{14} = 2^7\cos\frac{14\pi}{4} = 0,$$

$$C_{14}^1 - C_{14}^3 + C_{14}^5 - C_{14}^7 + C_{14}^9 - C_{14}^{11} + C_{14}^{13} = 2^7\sin\frac{14\pi}{4} = 128.$$

5. $tg6x$ ni tgx orqali ifodalang.

Yechish. Ma`lumki,

$$tg6x = \frac{\sin 6x}{\cos 6x},$$

shu sababli avvalo $\sin 6x$ va $\cos 6x$ larni $\sin x$ va $\cos x$ lar orqali ifodasini topamiz:

$$\sin 6x = 6\cos^5 x\sin x - 20\cos^3 x\sin^3 x + 6\cos x\sin^5 x,$$

$$\cos 6x = \cos^6 x - 15\cos^4 x\sin^2 x + 15\cos^2 x\sin^4 x - \sin^6 x.$$

Bulardan,

$$tg6x = \frac{6\cos^5 x\sin x - 20\cos^3 x\sin^3 x + 6\cos x\sin^5 x}{\cos^6 x - 15\cos^4 x\sin^2 x + 15\cos^2 x\sin^4 x - \sin^6 x} =$$

$$= \frac{6tgx - 20tg^3x + 6tg^5x}{1 - 15tg^2x + 15tg^4x - tg^6x}.$$

Ushbu oxirgi tenglikni hosil qilish uchun yuqoridagi kasrni surati va maxrajini $\cos^6 x$ ga bo`ldik.

6.Quyidagi tenglikni isbotlang.

$$2\cos mx = (2\cos x)^m - \frac{m}{1}(2\cos x)^{m-2} + \frac{m(m-3)}{1\cdot 2}(2\cos x)^{m-4} - ... +$$
$$+ (-1)^p\frac{m(m-p-1)(m-p-2)...(m-2p+1)}{p!}(2\cos x)^{m-2p} + ... \qquad (3)$$

Yechish.

$$C_{m-p}^{p}+C_{m-p-1}^{p-1}=\frac{(m-p)(m-p-1)(m-p-2)...(m-2p+1)}{p!}+$$
$$+\frac{(m-p-1)(m-p-2)...(m-2p+1)}{(p-1)!}=$$
$$\frac{m(m-p-1)(m-p-2)...(m-2p+1)}{p!},$$

bo`ladi.

$$2\cos mx=S_m, 2\cos x=a$$

belgilash kiritamiz, u holda

$$S_m=a^m-ma^{m-2}+(C_{m-2}^{2}+C_{m-3}^{1})a^{m-4}-...-$$
$$-(-1)^p(C_{m-p}^{p}+C_{m-p-1}^{p-1})a^{m-2p}+...$$

tenglikni hosil qilamiz. Ko`rsatish qiyin emaski

$$2\cos mx=2\cos x\cdot 2\cos(m-1)x-2\cos(m-2)x,$$

yoki yuqoridagi belgilashimizga ko`ra

$$S_m=aS_{m-1}-S_{m-2}.$$

Ko`rish qiyin emaski,

$m=1$ da isbotlanishi lozim bo`lgan tenglik o`rinli:

$$2\cos x=2\cos x.$$

$m=2$ bo`lsin ,u holda

$$2\cos 2x=(2\cos x)^2-\frac{2}{1}(2\cos x)^0=4\cos^2 x-2$$

Bu esa bizga elementar matematikadan ma`lum bo`lgan ayniyatdir:

$$1+\cos 2x=2\cos^2 x.$$

Faraz qilaylik,

$$S_{m-1}=a^{m-1}-(m-1)a^{m-3}+(C_{m-3}^{2}+C_{m-4}^{1})a^{m-5}-...-$$
$$-(-1)^p(C_{m-p-1}^{p}+C_{m-p-2}^{p-1})a^{m-2p-1}+...,$$
$$S_{m-2}=a^{m-2}-(m-2)a^{m-4}+(C_{m-4}^{2}+C_{m-5}^{1})a^{m-6}-...-$$
$$-(-1)^{p-1}(C_{m-p-1}^{p-1}+C_{m-p-2}^{p-1})a^{m-2p}+....$$

U holda

$$S_m=aS_{m-1}-S_{m-2}=a^m-ma^{m-2}+..+$$
$$+(-1)^p(C_{m-p-1}^{p}+C_{m-p-2}^{p-1}+C_{m-p-1}^{p-1}+C_{m-p-2}^{p-2})a^{m-2p}+...,$$

ushbu tenglikda ma`lum

$$C_n^k=C_{n-1}^{k}+C_{n-1}^{k-1}$$

formulani qo`llab, isbotlanishi lozim bo`lgan (3) tenglikka kelamiz.

7.Quyidgi tenglikni isbotlang.

$$\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\cos\frac{7\pi}{11}+\cos\frac{9\pi}{11}=\frac{1}{2}.$$

Isboti.

$$S=\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\cos\frac{7\pi}{11}+\cos\frac{9\pi}{11},$$

$$T=\sin\frac{\pi}{11}+\sin\frac{3\pi}{11}+\sin\frac{5\pi}{11}+\sin\frac{7\pi}{11}+\sin\frac{9\pi}{11},$$

deb belgilash kiritib, $V=S+Ti$ ifodani qaraylik. $\alpha=\cos\frac{\pi}{11}+i\sin\frac{\pi}{11}$ deb olsak, u holda

$$V=S+Ti=\alpha+\alpha^3+\alpha^5+\alpha^7+\alpha^9=\frac{\alpha^{11}-\alpha}{\alpha^2-1}=$$

$$\frac{\alpha(\alpha^{10}-1)}{\alpha(\alpha-\alpha^{-1})}=\frac{\alpha^{10}-1}{\alpha-\alpha^{-1}}=\frac{\alpha^5(\alpha^5-\alpha^{-5})}{\alpha-\alpha^{-1}}.$$

Ushbu ifodada $\alpha^5=\cos\frac{5\pi}{11}+i\sin\frac{5\pi}{11},\alpha^{-5}=\cos\frac{5\pi}{11}-i\sin\frac{5\pi}{11}$ tenglilardan foydalansak, u holda

$$V=S+Ti=\frac{\alpha^5(\alpha^5-\alpha^{-5})}{\alpha-\alpha^{-1}}=\frac{(\cos\frac{5\pi}{11}+i\sin\frac{5\pi}{11})\cdot 2i\sin\frac{5\pi}{11}}{2i\sin\frac{\pi}{11}}=$$

$$=\frac{\cos\frac{5\pi}{11}\sin\frac{5\pi}{11}}{\sin\frac{\pi}{11}}+i\frac{\sin^2\frac{5\pi}{11}}{\sin\frac{\pi}{11}}=\frac{\sin\frac{10\pi}{11}}{2\sin\frac{\pi}{11}}+i\frac{\sin^2\frac{5\pi}{11}}{\sin\frac{\pi}{11}}$$

Shunday qilib,

$$S=\frac{\sin\frac{5\pi}{11}\cos\frac{5\pi}{11}}{\sin\frac{\pi}{11}}=\frac{\sin\frac{10\pi}{11}}{2\sin\frac{\pi}{11}}=\frac{\sin(\pi-\frac{\pi}{11})}{2\sin\frac{\pi}{11}}=\frac{1}{2}$$

Yuqoridagi isbotlanishi lozim bo`lgan tenglik isbotlandi.

8.Quyidgi tenglikni isbotlang.

$$\cos\frac{2\pi}{11}+\cos\frac{4\pi}{11}+\cos\frac{6\pi}{11}+\cos\frac{8\pi}{11}+\cos\frac{10\pi}{11}=-\frac{1}{2}.$$

Isboti.

$$S=\cos\frac{2\pi}{11}+\cos\frac{4\pi}{11}+\cos\frac{6\pi}{11}+\cos\frac{8\pi}{11}+\cos\frac{10\pi}{11},$$

$$T=\sin\frac{2\pi}{11}+\sin\frac{4\pi}{11}+\sin\frac{6\pi}{11}+\sin\frac{8\pi}{11}+\sin\frac{10\pi}{11},$$

deb belgilash kiritib, $V=S+Ti$ ifodani qaraylik. $\alpha=\cos\frac{\pi}{11}+i\sin\frac{\pi}{11}$ deb olsak, u holda

$$V=S+Ti=\alpha^2+\alpha^4+\alpha^6+\alpha^8+\alpha^{10}=\frac{\alpha^{12}-\alpha^2}{\alpha^2-1}=$$

$$\frac{\alpha^2(\alpha^{10}-1)}{\alpha(\alpha-\alpha^{-1})}=\frac{\alpha(\alpha^{10}-1)}{\alpha-\alpha^{-1}}=\frac{\alpha^6(\alpha^5-\alpha^{-5})}{\alpha-\alpha^{-1}}.$$

Ushbu ifodada $\alpha^6=\cos\frac{6\pi}{11}+i\sin\frac{6\pi}{11},\alpha^{-5}=\cos\frac{5\pi}{11}-i\sin\frac{5\pi}{11}$ tenglilardan foydalansak, u holda

$$V=S+Ti=\frac{\alpha^6(\alpha^5-\alpha^{-5})}{\alpha-\alpha^{-1}}=\frac{(\cos\frac{6\pi}{11}+i\sin\frac{6\pi}{11})\cdot 2i\sin\frac{5\pi}{11}}{2i\sin\frac{\pi}{11}}=$$

$$=\frac{\cos\frac{6\pi}{11}\sin\frac{5\pi}{11}}{\sin\frac{\pi}{11}}+i\frac{\sin\frac{6\pi}{11}\sin\frac{5\pi}{11}}{\sin\frac{\pi}{11}}.$$

Shunday qilib,

$$S=\frac{\sin\frac{5\pi}{11}\cos\frac{6\pi}{11}}{\sin\frac{\pi}{11}}=\frac{\sin(\frac{5\pi}{11}+\frac{6\pi}{11})+\sin(\frac{5\pi}{11}-\frac{6\pi}{11})}{2\sin\frac{\pi}{11}}=\frac{\sin\pi-\sin\frac{\pi}{11}}{2\sin\frac{\pi}{11}}=-\frac{1}{2}.$$

Yuqoridagi isbotlanishi lozim bo`lgan tenglik isbotlandi.

9.Quyidgi tenglikni isbotlang.

$$\cos\frac{\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}+\cos\frac{7\pi}{13}+\cos\frac{9\pi}{13}+\cos\frac{11\pi}{13}=\frac{1}{2}.$$

Isboti.

$$S=\cos\frac{\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}+\cos\frac{7\pi}{13}+\cos\frac{9\pi}{13}+\cos\frac{11\pi}{13},$$

$$T=\sin\frac{\pi}{13}+\sin\frac{3\pi}{13}+\sin\frac{5\pi}{13}+\sin\frac{7\pi}{13}+\sin\frac{9\pi}{13}+\sin\frac{11\pi}{13},$$

deb belgilash kiritib, $V=S+Ti$ ifodani qaraylik. $\alpha=\cos\frac{\pi}{13}+i\sin\frac{\pi}{13}$ deb olsak, u holda

$$V=S+Ti=\alpha+\alpha^3+\alpha^5+\alpha^7+\alpha^9+\alpha^{11}=\frac{\alpha^{13}-\alpha}{\alpha^2-1}=$$

$$\frac{\alpha(\alpha^{12}-1)}{\alpha(\alpha-\alpha^{-1})}=\frac{\alpha^{12}-1}{\alpha-\alpha^{-1}}=\frac{\alpha^6(\alpha^6-\alpha^{-6})}{\alpha-\alpha^{-1}}.$$

Ushbu ifodada $\alpha^6=\cos\frac{6\pi}{13}+i\sin\frac{6\pi}{13},\alpha^{-6}=\cos\frac{6\pi}{13}-i\sin\frac{6\pi}{13}$ tenglilardan foydalansak, u holda

$$V=S+Ti=\frac{\alpha^6(\alpha^6-\alpha^{-6})}{\alpha-\alpha^{-1}}=\frac{(\cos\frac{6\pi}{13}+i\sin\frac{6\pi}{13})\cdot 2i\sin\frac{6\pi}{13}}{2i\sin\frac{\pi}{13}}=$$

$$=\frac{\cos\frac{6\pi}{13}\sin\frac{6\pi}{13}}{\sin\frac{\pi}{13}}+i\frac{\sin^2\frac{6\pi}{13}}{\sin\frac{\pi}{13}}=\frac{\sin\frac{12\pi}{13}}{2\sin\frac{\pi}{13}}+i\frac{\sin^2\frac{6\pi}{13}}{\sin\frac{\pi}{13}}$$

Shunday qilib,

$$S=\frac{\sin\frac{6\pi}{13}\cos\frac{6\pi}{13}}{\sin\frac{\pi}{13}}=\frac{\sin\frac{12\pi}{13}}{2\sin\frac{\pi}{13}}=\frac{\sin(\pi-\frac{\pi}{13})}{2\sin\frac{\pi}{13}}=\frac{1}{2}$$

Yuqoridagi isbotlanishi lozim bo`lgan tenglik isbotlandi.

2.2.Trigonometrik yig`indilarni hisoblash.

1.Quyidagi yig`indini hisoblang.

$S=1+a\cos x+a^2\cos 2x+a^3\cos 3x+...+a^k\cos kx$.

Yechish.

$T=a\sin x+a^2\sin 2x+a^3\sin 3x+...+a^k\sin kx$

deb belgilash kiritaylik. U holda

$$S+Ti=1+a(\cos x+i\sin x)+a^2(\cos 2x+i\sin 2x)+$$
$$+a^3(\cos 3x+i\sin 3x)+...+a^k(\cos kx+i\sin kx)$$

bo`ladi. $\alpha=\cos x+i\sin x$ deb olsak, u holda

$$S+Ti=1+a\alpha+a^2\alpha^2+a^3\alpha^3+...+a^k\alpha^k=$$
$$=\frac{a^{k+1}\alpha^{k+1}-1}{a\alpha-1}.$$

S hosil qilingan yig`indining haqiqiy qismiga teng, shu sababli

$$S+Ti=\frac{a^{k+1}\alpha^{k+1}-1}{a\alpha-1}\cdot\frac{a\alpha^{-1}-1}{a\alpha^{-1}-1}=\frac{a^{k+2}\alpha^{k}-a^{k+1}\alpha^{k+1}+a\alpha^{-1}+1}{a^2-a(\alpha+\alpha^{-1})+1}.$$

Bu yerdan

$$S+Ti=1+a\alpha+a^2\alpha^2+a^3\alpha^3+...+a^k\alpha^k=$$

$$=\frac{a^{k+1}\alpha^{k+1}-1}{a\alpha-1}.$$

$$S=\frac{a^{k+2}\cos kx-a^{k+1}\cos(k+1)x-a\cos x+1}{a^2-2a\cos x+1}.$$

2.Quyidagi tenglikni isbotlang.

$$\sin x+\sin 2x+\sin 3x+...+\sin nx=\frac{\sin\frac{n+1}{2}x\sin\frac{nx}{2}}{\sin\frac{x}{2}}.$$

Yechish.

$$T=\sin x+\sin 2x+\sin 3x+...+\sin nx,$$
$$S=\cos x+\cos 2x+\cos 3x+...+\cos nx,$$
$$\alpha=\cos\frac{x}{2}+i\sin\frac{x}{2},$$

bo`lsin.U holda

$$S+Ti=1+\alpha^2+\alpha^4+\alpha^6+...+\alpha^{2n}=\alpha^2\frac{\alpha^{2n}-1}{\alpha^2-1}=$$

$$=\alpha^2\frac{\alpha^n(\alpha^n-\alpha^{-n})}{\alpha(\alpha-\alpha^{-1})}=(\cos\frac{n+1}{2}x+i\sin\frac{n+1}{2}x)\frac{\sin\frac{n}{2}x}{\sin\frac{x}{2}}.$$

Bu yerdan quyidagi tengliklarni hosil qilamiz:

$$T=\sin x+\sin 2x+\sin 3x+...+\sin nx=\frac{\sin\frac{n+1}{2}x\sin\frac{nx}{2}}{\sin\frac{x}{2}}.$$

$$S=\cos x+\cos 2x+\cos 3x+...+\cos nx=\frac{\cos\frac{n+1}{2}x\sin\frac{nx}{2}}{\sin\frac{x}{2}}.$$

3.Quyidagi limitni hisoblang.

$$\lim_{n\to\infty}(1+\frac{1}{2}\cos x+\frac{1}{4}\cos 2x+...+\frac{1}{2^n}\cos nx)$$

Yechish. 7-misolda $a=\frac{1}{2}$ deb olib, undan foydalanamiz.

$$\lim_{n\to\infty}(1+\frac{1}{2}\cos x+\frac{1}{4}\cos 2x+...+\frac{1}{2^n}\cos nx)=$$

$$\lim_{n\to\infty}\left(\frac{\left(\frac{1}{2}\right)^{n+2}\cos nx-\left(\frac{1}{2}\right)^{n+1}\cos(n+1)x-\frac{1}{2}\cos x+1}{\left(\frac{1}{2}\right)^{2}-2\left(\frac{1}{2}\right)\cos x+1}\right)=$$

$$=\frac{-\frac{1}{2}\cos x+1}{\frac{1}{4}-\cos x+1}=\frac{2(2=\cos x)}{5-4\cos x}.$$

4.Quyidagi yig`indilarni hisoblang.

$$S=\cos a-\cos(a+h)+\cos(a+2h)-...+(-1)^{n-1}\cos(a+(n-1)h),$$

$$T=\sin a-\sin(a+h)+\sin(a+2h)-...+(-1)^{n-1}\sin(a+(n-1)h).$$

Yechish .Quyidagi belgilashlar kiritaylik.

$$\alpha=\cos a+i\sin a,$$

$$\beta=\cos h+i\sin h.$$

U holda

$$\alpha\cdot\beta=\cos(a+h)+i\sin(a+h),$$

$$\alpha\cdot\beta^2=\cos(a+2h)+i\sin(a+2h),$$

....

$$\alpha\cdot\beta^{n-1}=\cos(a+(n-1)h+i\sin(a+(n-1)h).$$

Ushbu tengliklardan

$$V=S+iT=\alpha-\alpha\beta+\alpha\beta^2-...+(-1)^{n-1}\alpha\beta^{n-1},$$

uni hisoblaylik.Bu hisoblashda geometric prosressiyaning hadlari yig`indisi formulasidanfoydalanamiz.

$$V=\alpha(1-\beta+\beta^2-\beta^3+...(-)^{n-1}\beta^{n-1})=\alpha\cdot\frac{(-1)^n\beta^n-1}{-\beta-1}=\alpha\cdot\frac{(-1)^{n+1}\beta^n+1}{\beta+1}.$$

$n-$juft son bo`lsin. U holda

$$V=\alpha\cdot\frac{-\beta^n+1}{\beta+1}=\alpha\cdot\frac{(-\beta^n+1)(\beta-1)}{\beta^2-1}=\alpha\cdot\frac{\beta^{\frac{n}{2}}(\beta^{\frac{n}{2}}-\beta^{-\frac{n}{2}})(1-\beta)}{\beta(\beta-\beta^{-1})}=$$

$$\alpha\cdot\frac{(\beta^{\frac{n}{2}-1}-\beta^{\frac{n}{2}})(\beta^{\frac{n}{2}}-\beta^{-\frac{n}{2}})}{\beta-\beta^{-1}}=\frac{(\alpha\beta^{\frac{n}{2}-1}-\alpha\beta^{\frac{n}{2}})(\beta^{\frac{n}{2}}-\beta^{-\frac{n}{2}})}{\beta-\beta^{-1}}$$

Endi α va β larni yuqorida belgilaganimizga asosan V ni ifodasiga qo`yaylik:

$$V=\frac{\{[\cos(a+(\frac{n}{2}-1)h)+i\sin(a+(\frac{n}{2}-1)h]-[\cos(a+\frac{n}{2}h)+i\sin(a+\frac{n}{2}h)]\}\cdot 2i\sin\frac{n}{2}h}{2i\sinh}$$

Bu yerdan

Endi α va β larni yuqorida belgilaganimizga asosan V ni ifodasiga qo\`yaylik:

$$V=\frac{\{[\cos(a+(\frac{n}{2}-1)h)+i\sin(a+(\frac{n}{2}-1)h]-[\cos(a+\frac{n}{2}h)+i\sin(a+\frac{n}{2}h)]\}\cdot 2i\sin\frac{n}{2}h}{2i\sinh}$$

Bu yerdan

$$S=\frac{[\cos(a+(\frac{n}{2}-1)h)-\cos(a+\frac{n}{2}h)]\sin\frac{n}{2}h}{\sinh}=\frac{-2\sin(a+\frac{n-1}{2}h)\sin(-\frac{h}{2})\sin\frac{n}{2}h}{2\sin\frac{h}{2}\cos\frac{h}{2}}=$$

$$=\frac{\sin(a+\frac{n-1}{2}h)\sin\frac{n}{2}h}{\cos\frac{h}{2}}.$$

$$T=\frac{-[\sin(a+(\frac{n}{2}-1)h)-\sin(a+\frac{n}{2}h)]\sin\frac{n}{2}h}{\sinh}=\frac{2\cos(a+\frac{n-1}{2}h)\sin\frac{h}{2}\cdot\sin\frac{n}{2}h}{2\sin\frac{h}{2}\cos\frac{h}{2}}=$$

$$=\frac{\cos(a+\frac{n-1}{2}h)\sin\frac{n}{2}h}{\cos\frac{h}{2}}.$$

$n-$ toq son bo\`lsin. U holda

$$V=\alpha\cdot\frac{-\beta^n-1}{-\beta-1}=\alpha\cdot\frac{(\beta^n+1)(\beta-1)}{\beta^2-1}=\alpha\cdot\frac{\beta^{\frac{n}{2}}(\beta^{\frac{n}{2}}+\beta^{-\frac{n}{2}})(\beta-1)}{\beta(\beta-\beta^{-1})}=$$

$$\alpha\cdot\frac{(\beta^{\frac{n}{2}}-\beta^{\frac{n}{2}-1})(\beta^{\frac{n}{2}}+\beta^{-\frac{n}{2}})}{\beta-\beta^{-1}}=\frac{(\alpha\beta^{\frac{n}{2}}-\alpha\beta^{\frac{n}{2}-1})(\beta^{\frac{n}{2}}+\beta^{-\frac{n}{2}})}{\beta-\beta^{-1}}$$

α va β larni V ni ifodasiga qo\`yaylik:

$$V=\frac{\{[\cos(a+\frac{n}{2}h)+i\sin(a+\frac{n}{2}h)]-[\cos(a+\frac{n-2}{2}h)+i\sin(a+\frac{n-2}{2}h)]\}\cdot 2\cos\frac{n}{2}h}{2i\sinh}$$

Bu yerdan

$$S=\frac{[\sin(a+\frac{n}{2}h)-\sin(a+\frac{n-2}{2}h)]\cos\frac{n}{2}h}{\sinh}=\frac{2\cos(a+\frac{n-1}{2}h)\sin\frac{h}{2}\cos\frac{n}{2}h}{2\sin\frac{h}{2}\cos\frac{h}{2}}=$$

$$=\frac{\cos(a+\frac{n-1}{2}h)\cos\frac{n}{2}h}{\cos\frac{h}{2}}.$$

$$T=\frac{-[\cos(a+\frac{n}{2}h)-\cos(a+\frac{n-2}{2}h)]\cos\frac{n}{2}h}{\sinh}=\frac{2\sin(a+\frac{n-1}{2}h)\sin\frac{h}{2}\cdot\cos\frac{n}{2}h}{2\sin\frac{h}{2}\cos\frac{h}{2}}=$$

$$=\frac{\sin(a+\frac{n-1}{2}h)\cos\frac{n}{2}h}{\cos\frac{h}{2}}.$$

Shundy qilib,

$$S=\frac{\sin(a+\frac{n-1}{2}h)\sin\frac{n}{2}h}{\cos\frac{h}{2}},\qquad n\text{-juft bo`lsa,}$$

$$S=\frac{\cos(a+\frac{n-1}{2}h)\cos\frac{n}{2}h}{\cos\frac{h}{2}},\qquad n\text{-toq bo`1sa;}$$

$$T=\frac{\cos(a+\frac{n-1}{2}h)\sin\frac{n}{2}h}{\cos\frac{h}{2}},\qquad n\text{-juft bo`lsa,}$$

$$T=\frac{\sin(a+\frac{n-1}{2}h)\cos\frac{n}{2}h}{\cos\frac{h}{2}},\qquad n\text{-toq bo`lsa.}$$

5.Agar x absolyut qiymati jihatidan birdan kichik bo`lsa, u holda

a) $S=\cos a+x\cos(a+b)+x^2\cos(a+2b)+...+x^n\cos(a+nb)+...$

b) $T=\sin a+x\sin(a+b)+x^2\sin(a+2b)+...+x^n\sin(a+nb)+...$

qatorlar yaqinlashuvchi ekanligini va ularning yig`indisi mos ravishda

$$\frac{\cos a-x\cos(a-b)}{1-2x\cos b+x^2},\qquad \frac{\sin a-x\sin(a-b)}{1-2x\cos b+x^2}$$

teng ekanligini isbotlang.

Isboti.

$$S_k=\cos a+x\cos(a+b)+x^2\cos(a+2b)+...+x^k\cos(a+kb),$$

$$T_k=\sin a+x\sin(a+b)+x^2\sin(a+2b)+...+x^k\sin(a+kb),$$

deb belgilash kiritaylik. U holda

$$S_k+iT_k=(\cos a+i\sin a)+x[\cos(a+b)+i\sin(a+b)]+$$

$$+x^2[\cos(a+2b)+i\sin(a+2b)]+...+x^k[\cos(a+kb)+i\sin(a+kb)]$$

bo`ladi.

$$\alpha=\cos a+i\sin a,\qquad \beta=(\cos b+i\sin b)$$

deb olsak, u holda

$$S_k+iT_k=\alpha(1+x\beta+x^2\beta^2+x^3\beta^3+...+x^k\beta^k)=$$

$$=\frac{\alpha(x^{k+1}\beta^{k+1}-1)}{x\beta-1}\cdot\frac{x\beta^{-1}-1}{x\beta^{-1}-1}=\frac{\alpha(x^{k+2}\beta^k-x^{k+1}\beta^{k+1}+x\beta^{-1}+1)}{x^2-x(\beta+\beta^{-1})+1}$$

Demak,

$$S_k=\frac{x^{k+2}\cos(a+kb)-x^{k+1}\cos[a+(k+1)b]-x\cos(a-b)+\cos a}{x^2-2x\cos b+1},$$

$$T_k=\frac{x^{k+2}\sin(a+kb)-x^{k+1}\sin[a+(k+1)b]-x\sin(a-b)+\sin a}{x^2-2x\cos b+1}.$$

Ushbu yig`indilarda $k\to\infty$ da limitga o`tsak, u holda $|x|<1$ bo`lgani uchun

$$\lim_{k\to\infty}|x|=0$$

bo`ladi.Shunday qilib,

$$S=\lim_{k\to\infty}S_k=\frac{\cos a-x\cos(a-b)}{1-2x\cos b+x^2},$$

$$T=\lim_{k\to\infty}T_k=\frac{\sin a-x\sin(a-b)}{1-2x\cos b+x^2}.$$

6.Yig`indini toping.

$$S=\sin^2 x+\sin^2 3x+\sin^2 5x+...+\sin^2(2n-1)x.$$

Yechish.

Elementar matematikadan ma`lum bo`lgan

$$\sin^2\alpha=\frac{1-\cos 2\alpha}{2}$$

formuladan foydalanamiz.U holda

$$S=\frac{1-\cos 2x}{2}+\frac{1-\cos 6x}{2}+\frac{1-\cos 10x}{2}+...+\frac{1-\cos[4n-2]x}{2}=$$

$$=\frac{n}{2}-\frac{1}{2}\left[\cos 2x+\cos 6x+\cos 10x+...+\cos(4n-2)x\right]$$

Endi

$$s=\cos 2x+\cos 6x+\cos 10x+...+\cos(4n-2)x,$$

$$t=\sin 2x+\cos 6x+\cos 10x+...+\cos(4n-2)x$$

yig`indilarni hisoblaylik.

$$\alpha = \cos 2x + i\sin 2x$$

deb belgilash kiritaylik, u holda

$$s + it = \alpha \ \ + \alpha^3 + ... + \alpha^{2n-1} = \frac{\alpha^{2n+1} - \alpha}{\alpha^2 - 1} = \frac{\alpha^n(\alpha^n - \alpha^{-n})}{\alpha - \alpha^{-1}} =$$

$$= \frac{(\cos 2nx + i\sin 2nx)\cdot 2i\sin 2nx}{2i\sin 2x} = \frac{\cos 2nx \sin 2nx}{\sin 2x} + i\frac{\sin^2 2nx}{\sin 2x}.$$

Demak,

$$S = \frac{n}{2} - \frac{1}{2}\cdot\frac{\cos 2nx \sin 2nx}{\sin 2x} = \frac{n}{2} - \frac{\sin 4nx}{4\sin 2x}.$$

7.Quyidagi tengliklarni isbotlang.

$$\cos^2 x + \cos^2 2x + \cos^2 3x + ... + \cos^2 nx = \frac{n}{2} + \frac{\cos(n+1)x \sin nx}{2\sin x};$$

$$\sin^2 x + \sin^2 2x + \sin^2 3x + ... + \sin^2 nx = \frac{n}{2} - \frac{\cos(n+1)x \sin nx}{2\sin x}.$$

Isboti.

6-misolga o`xshash $\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$ va $\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$ formulalardan foydalanamiz. U holda

$$\cos^2 x + \cos^2 2x + ... + \cos^2 nx = \frac{1+\cos 2x}{2} + \frac{1+\cos 4x}{2} + ... + \frac{1+\cos 2nx}{2} =$$

$$= \frac{n}{2} + \frac{1}{2}[\cos 2x + \cos 4x + ... + \cos 2nx];$$

$$\sin^2 x + \sin^2 2x + ... + \sin^2 nx = \frac{1-\cos 2x}{2} + \frac{1-\cos 4x}{2} + ... + \frac{1-\cos 2nx}{2} =$$

$$= \frac{n}{2} - \frac{1}{2}[\cos 2x + \cos 4x + ... + \cos 2nx].$$

Endi

$$s = \cos 2x + \cos 4x + ... + \cos 2nx, \quad t = \sin 2x + \sin 4x + ... + \sin 2nx$$

yig`indilarni hisoblaylik.

$$s + it = (\cos 2x + i\sin 2x) + (\cos 4x + i\sin 4x) + ... + (\cos 2nx + i\sin 2nx) = \alpha^2 + \alpha^4 +$$

$$... + \alpha^{2n} = \frac{\alpha^{2n+2} - \alpha^2}{\alpha^2 - 1} = \frac{\alpha^{n+1}(\alpha^n - \alpha^{-n})}{\alpha - \alpha^{-1}} = \frac{[\cos(n+1)x + i\sin(n+1)x]\cdot 2i\sin nx}{2i\sin x} =$$

$$= \frac{\cos(n+1)x \sin nx}{\sin x} + i\frac{\sin(n+1)x \sin nx}{\sin x}.$$

Ushbu tenglikdan esa isbotlanishi lozim bo`lgan tenglik kelib chiqadi:

$$\cos^2 x+\cos^2 2x+...+\cos^2 nx=\frac{n}{2}+\frac{1}{2}\left[\cos 2x+\cos 4x+...+\cos 2nx\right]=$$
$$=\frac{n}{2}+\frac{\cos(n+1)x\sin nx}{2\sin x};$$
$$\sin^2 x+\sin^2 2x+...+\sin^2 nx=\frac{n}{2}-\frac{1}{2}\left[\cos 2x+\cos 4x+...+\cos 2nx\right]=$$
$$=\frac{n}{2}-\frac{\cos(n+1)x\sin nx}{2\sin x}.$$

8.Yig`indilarni toping.

$$S=\cos^3 x+\cos^3 2x+....+\cos^3 nx,\quad T=\sin^3 x+\sin^3 2x+....+\sin^3 nx.$$

Yechish. Yuqorida isbotlangan

$$\cos^3\alpha=\frac{3\cos\alpha-\cos 3\alpha}{4},\quad \sin^3\alpha=\frac{3\sin\alpha-\sin 3\alpha}{4}$$

tengliklardan foydalanamiz. U holda

$$S=\frac{1}{4}\left[3(\cos x+\cos 2x+...+\cos nx)+(\cos 3x+\cos 6x+...+\cos 3nx)\right];$$
$$T=\frac{1}{4}\left[3(\sin x+\sin 2x+...+\sin nx)-(\sin 3x+\sin 6x+...+\sin 3nx)\right].$$

Quyidagicha belgilashlar kiritaylik:

$$\alpha=\cos\frac{x}{2}+i\sin\frac{x}{2},\quad \beta=\cos\frac{3x}{2}+i\sin\frac{3x}{2}.$$

U holda

$$S_1+iT_1=(\cos x+\sin x)+(\cos 2x+i\sin 2x)+...+(\cos nx+i\sin nx)=\alpha^2+\alpha^4+$$
$$+...+\alpha^{2n}=\frac{\alpha^{2n+2}-\alpha^2}{\alpha^2-1}=\frac{\alpha^{n+1}(\alpha^n-\alpha^{-n})}{\alpha-\alpha^{-1}}=$$
$$=\frac{(\cos\frac{n+1}{2}x+i\sin\frac{n+1}{2}x)\cdot 2i\sin\frac{n}{2}x}{2i\sin\frac{x}{2}}=\frac{\cos\frac{n+1}{2}x\sin\frac{n}{2}x}{\sin\frac{x}{2}}+i\frac{\sin\frac{n+1}{2}x\sin\frac{n}{2}x}{\sin\frac{x}{2}},$$

$$S_2+iT_2=(\cos 3x+\sin 3x)+(\cos 6x+i\sin 6x)+...+(\cos 3nx+i\sin 3nx)=\beta^2+\beta^4+$$
$$+...+\beta^{2n}=\frac{\beta^{2n+2}-\beta^2}{\beta^2-1}=\frac{\beta^{n+1}(\beta^n-\beta^{-n})}{\beta-\beta^{-1}}=$$
$$=\frac{(\cos\frac{3(n+1)}{2}x+i\sin\frac{3(n+1)}{2}x)\cdot 2i\sin\frac{3n}{2}x}{2i\sin\frac{3x}{2}}=\frac{\cos\frac{3(n+1)}{2}x\sin\frac{3n}{2}x}{\sin\frac{3x}{2}}+i\frac{\sin\frac{3(n+1)}{2}x\sin\frac{3n}{2}x}{\sin\frac{3x}{2}},$$

Demak,

$$S=\frac{3\cos\frac{n+1}{2}x\sin\frac{n}{2}x}{4\sin\frac{x}{2}}+\frac{\cos\frac{3(n+1)}{2}x\sin\frac{3n}{2}x}{4\sin\frac{3x}{2}};$$

$$T=\frac{3\sin\frac{n+1}{2}x\sin\frac{n}{2}x}{4\sin\frac{x}{2}}-\frac{\sin\frac{3(n+1)}{2}x\sin\frac{3n}{2}x}{4\sin\frac{3x}{2}}.$$

X u l o s a .

Ma`lumki, har qanday haqiqiy koeffisientli ko`phadlar ham haqiqiy sonlar sohasida ildizga ega bo`lavermaydi, lekin bu hol kompleks sonlar to`plami uchun o`rinli emas ekan, ya`ni kompleks koeffisientli har ko`phad kompleks sonlar sohasida kompleks ildizlarga ega bo`ladi.Ushbu bitiruv malakaviy ishda kompleks sonlar ustida amallar bajarish orqali hosil bo`ladigan sonlarni haqiqiy qismini haqiqiy qismiga mavhum qismini mavhum qismiga tenglashtirib muhim natijalar erishildi.

Shunday qilib,ushbu bitiruv malakaviy ishni tayorlash davomida quyidagi muhim xulosalarga kelindi.

1.Kompleks sonlar sistemasi haqiqiy sonlar sistemasining kengaytmasidir.

2.Kompleks sonlar ichida kvadrati minus birga teng bo`lgan son mavjud.

3.Kompleks sonlar ustida qo`shish, ayirish,ko`paytirish, darajaga ko`tarish va ildiz chiqarish amallarini bajarish mumkin.

4.Har qanday kompleks sonni trigonometrik ko`rinishga keltirish mumkin.

5.Ko`mpleks sonlar uchun katta va kichik tushunchalari mavjud emas, ularni modullari bo`yicha solishtirishi mumkin.

6.Kompleks sonlar vektorlar kabi ya`ni parallelogram qoidasi bo`yicha qo`shiladi.

7.Trigonometrik ko`rinishda berilgan kompleks sonni darajaga ko`tarish uchun ushbu sonni modulini shu darajaga ko`tarish , argumentini esa shuncha marta ortirish lozim.

8.Kompleks sonlar xossalaridan foydalanib ko`plab trigonometrik ayniyatlarni isbotlash va yig`indilarni hisoblash mumkin.

9.Birning har qanday darajali ildizlari birlik aylana yoyini teng bo`laklarga bo`ladi.

F o y d a l a n i l g a n a d b i y o t l a r r o` y x a t i.

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