**V. I. ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI
HUZURIDAGI ILMIY DARAJA BERUVCHI
DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH**

O‘ZBEKISTON MILLIY UNIVERSITETI

FAYZIYEV YUSUF ERGASHEVICH

ELLIPTIK QISMI YUQORI TARTIBLI BO‘LGAN KASR TARTIBLI XUSUSIY HOSILALI DIFFERENSIAL TENGLAMALAR UCHUN TO‘G‘RI VA TESKARI MASALALAR

**01.01.02 – Differensial tenglamalar va matematik fizika
(fizika – matematika fanlari)**

**FIZIKA – MATEMATIKA FANLARI DOKTORI (DSc)
DISSERTATSIYA AVTOREFERATI**

ТОШКЕНТ – 2022

УДК: 517.95

Doktorlik dissertatsiyasi avtoreferati mundarijasi

Content of the abstract of doctoral dissertation

Оглавление автореферата докторской диссертации

**V. I. ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI
HUZURIDAGI ILMIY DARAJA BERUVCHI
DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH**

O‘ZBEKISTON MILLIY UNIVERSITETI

FAYZIYEV YUSUF ERGASHEVICH

ELLIPTIK QISMI YUQORI TARTIBLI BO‘LGAN KASR TARTIBLI XUSUSIY HOSILALI DIFFERENSIAL TENGLAMALAR UCHUN TO‘G‘RI VA TESKARI MASALALAR

**01.01.02 – Differensial tenglamalar va matematik fizika
(fizika – matematika fanlari)**

**FIZIKA – MATEMATIKA FANLARI DOKTORI (DSc)
DISSERTATSIYA AVTOREFERATI**

ТОШКЕНТ – 2022

Fan doktori (DSc) dissertatsiya mavzusi O‘zbekiston Respublikasi Vazirlar Mahkamasi huzuridagi Oliy attestatsiya kommissiyasida B2022.3.DSc/FM198 raqam bilan ro‘yxatga olingan.

Dissertatsiya Mirzo Ulug‘bek nomidagi O‘zbekiston Milliy universitetida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o‘zbek, ingliz, rus (rezyume)) Ilmiy kengash web-sahifasi (http://kengash.mathinst.uz) va «Ziyonet» Axborot ta‘lim portalida (www.ziyonet.uz) joylashtirilgan.

Ilmiy maslahatchi:	**Ashurov Ravshan Radjabovich** fizika - matematika fanlari doktori, professor
Rasmiy opponentlar:	**Turmetov Botirxon Xudoybergenovich** fizika - matematika fanlari doktori, professor
	Taxirov Jozil Ostanovich fizika - matematika fanlari doktori, professor
	Karimov Erkinjon To‘lqinovich fizika - matematika fanlari doktori, katta ilmiy xodim
Yetakchi tashkilot:	**Kosmofizik tadqiqotlar va radioto‘lqinlar tarqalishi instituti (Rossiya)**

Dissertatsiya himoyasi V. I. Romanovskiy nomidagi Matematika Instituti huzuridagi Dsc.02/30.12.2019.FM.86.01 raqamli Ilmiy Kengashning 2022 yil « 29 » noyabr soat 16:00 dagi majlisida bo‘lib o‘tadi. (Manzil: 100174, Toshkent sh. Olmazor tumani, Universitet ko‘chasi, 9-uy. Tel.: (+99871)-207-91-40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz.)

Dissertatsiya bilan V. I. Romanovskiy nomidagi Matematika Institutining Axborot-resurs markazida tanishish mumkin (148-raqami bilan ro‘yxatga olingan). (Manzil: 100174, Toshkent sh. Olmazor tumani, Universitet ko‘chasi, 9-uy. Tel.: (+99871)-207-91-40).

Dissertatsiya avtoreferati 2022 yil « 11 » noyabr kuni tarqatildi.
(2022 yil « 11 » noyabrdagi 2- raqamli reestr bayonnomasi).

U.A. Rozikov
Ilmiy darajalar beruvchi Ilmiy kengash raisi f.-m.f.d., professor

J.K. Adashev
Ilmiy darajalar beruvchi Ilmiy kengash ilmiy kotibi f.-m.f.d., katta ilmiy xodim

A. Азамов
Ilmiy darajalar beruvchi Ilmiy kengash huzuridagi Ilmiy seminar raisi f.-m.f.d., akademik

KIRISH (doktorlik dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. So‘nggi yillarda ko‘pgina hayotiy jarayonlarning matematik modelini tuzib, uni matematik usullar bilan yechish matematiklar ichida keng tarqaldi. Bu jarayonlar meditsina va texnikalar rivojlanishi bilan uzviy bog‘liqdir. Kasr tartibli integral va hosilalarning fizika, biologiya, meditsina va texnika sohalariga tadbiqi juda ko‘p bo‘lib, u bu sohalarning rivojlanishida muhim ahamiyatga ega. Shu sababli so‘nggi yillarda matematiklar orasida kasr tartibli hosila qatnashgan differensial va xususiy hosilali differensial tenglamalarni o‘rganishga bo‘lgan qiziqish ortib bormoqda. Kasr tartibli tenglamalar bir vaqtda diffuziya va to‘lqin tarqalish jarayonlarini ifodalaydi. Bunday jarayonlar tabiatda juda ko‘p uchraydi. Shuning uchun ham kasr tartibli tenglamalarni o‘rganish juda muhim ahamiyatga egadir. Bunga misol sifatida virus tarqalish jarayonlarini olish mumkin. 2020 yil Differensial tenglamalar va uning tadbiqlari laboratoriyasi va New Haven universiteti (USA) professorlari tomonidan virus tarqalish tezligini aniqlash, tashqi va ichki ta‘sir manbasini va chegaraviy ta‘sir kuchini aniqlash modellari ishlab chiqildi. Shu bilan birga olingan natijalarni Jon Xopkins universiteti bilan birgalikda COVID-19 tarqalish jarayonlarini tahlil qilishga tadbiq qilishdi.

Xususiy hosilali kasr tartibli tenglamalarni yechishni o‘rganish bilan bir qatorda tenglamaning koeffitsiyentlarini, uning o‘ng tomonini, chegaraviy funksiyani va kasr tartibli hosilaning tartibini aniqlashga oid teskari masalalarni yechishni o‘rganish juda muhim ahamiyatga ega. Teskari masalalarni o‘rganish bizga kasr tartibli tenglama bilan berilgan jarayonlarni o‘rganish, tahlil qilish va boshqarish imkoniyatlarini beradi. Masalan, issiqlik tarqalish jarayonlarini olsak, chegaraviy funksiyani bilishimiz bizga issiqlik tarqalishini chegaradan boshqarish imkoniyatini beradi. Bunga o‘xshash ko‘plab misollar keltirish mumkin. Biz agar vaqt bo‘yicha kasr tartibli tenglamani qarasak, agar kasr tartibli hosilaning tartibi 1 va 2 oralig‘ida bo‘lsa, bir vaqtda diffuziya va to‘lqin tarqalish jarayonlarini aks ettiradi, agar 0 va 1 oralig‘ida bo‘lsa, sekin sodir bo‘luvchi diffuziya jarayonlarini ifodalaydi. Masalan, tabiatda juda ko‘plab uchraydigan virus tarqalish jarayonlarini olish mumkin. Shuning uchun ham kasr tartibli tenglamalarni o‘rganish muhim ahamiyatga egadir.

Mamlakatimizda fundamental fanlarning ilmiy va amaliy tadbiqiga ega bo‘lgan differensial tenglamalar va matematik fizikaning dolzarb yo‘nalishlariga e’tibor kuchaytirildi. Matematik fanlarning ustuvor yo‘nalishlari bo‘yicha, ayniqsa, «algebra va funksional analiz, differentsial tenglamalar va matematik fizika, dinamik sistemalar nazariyasi» bo‘yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish Fanlar Akademiyasi V.I. Romanovskiy nomidagi Matematika instituti faoliyatining asosiy vazifasi va yo‘nalishi etib belgilangan[1]. Qaror ijrosini taъminlashda matematik fizikaning yangi yo‘nalishlarini rivojlantirish muhim ahamiyatga ega. O‘zR Prezidentining 2018 yil 27 apreldagi

[1] O‘zbekiston Respublikasi Vazirlar Maxkamasining 2017 yil 18 maydagi “O‘zbekiston Respublikasi Fanlar Akademiyasining yangidan tashkil etilgan ilmiy tadqiqot muassasalari faoliyatini tashkil etish to‘g‘risida”gi 292-sonli qarori.

№ PQ-3682 “Innovatsion g‘oyalar, texnologiyalar va loyihalarni amaliyotga joriy qilish tizimini yanada takomillashtirish chora-tadbirlari to‘g‘risida” gi va 2019 yil 9 iyuldagi № PQ-4387 “Matematika ta‘limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek O‘zR FA V.I.Romanovskiy nomidagi matematika institute faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida” gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi. Mazkur tadqiqot ishi O‘zbekiston Respublikasida fan va texnikani rivojlantirishning ustuvor yo‘nalishiga muvofiq amalga oshirildi IV. “Matematika, mexanika va informatika”.

Dissertatsiya mavzusi bo‘yicha xorijiy ilmiy tadqiqotlar sharhi. Kasr tartibli hosilali differensial va xususiy hosilali differensial tenglamalar uchun to‘g‘ri va teskari masalalar bilan dunyoning deyarli barcha universitetlarida, ilmiy tekshirish institutlarida va ilmiy markazlarida izlanishlar olib borilmoqda. Bunga misol sifatida ba’zilarini sanab o‘tamiz. Yuqorida aytganimizdek, Jon Xopkins universiteti (AQSh), Tokyo universiteti (Yaponiya), Berlin amaliy ilmlar texnika universiteti (Germaniya), New Haven universiteti (AQSh), Berlin universiteti (Germaniya), Belorussiya davlat universiteti (Belorussiya), Xokkaydo universiteti (Yaponiya), La Rochelle universitetida (Fransiya), Amaliy matematika va avtomatlashtirish instituti (Rossiya), Yaqin Sharq universiteti (Turkiya), Xalqaro qozoq-turk universiteti (Qozog‘iston) va boshqalar.

Kasr tartibli hosilaning tartibini aniqlash bo‘yicha teskari masalalar Tokyo universiteti professori M. Yamamoto (Yaponiya) va boshqalar tomonidan o‘rganilgan. Ular tomonidan ishlab chiqilgan usul teskari masalaning yechimi yagonaligini ta‘minlab berdi xolos. Amerika qo‘shma shtatlari New Haven universiteti professori S. Umarov va O‘zbekiston Respublikasi FA qoshidagi Matematika instituti professori R. Ashurovlar tomonidan tenglama mos spectral masalaning birinchi xos soni nolga teng, birinchi xos funksiyasi o‘zgarmas son bo‘lsa, kasr tartibli hosilaning tartibini aniqlash bo‘yicha teskari masalaning yechimini bir qiymatli aniqlashning yangi shartlari olingan.

Kasr tartibli differensial va xususiy hosilali differensial tenglamalar uchun turli chegaraviy masalalar ko‘pgina olimlar tomonidan o‘rganib kelinmoqda. Diffuziya tenglamalari uchun boshlang‘ich vaqt va tayinlangan vaqtdagi qiymatlarini bog‘lovchi nolokal shart bilan A. Ashyaraliyev (Turkiya) va boshqalarning ishlarida o‘rganilgan. Elliptik tenglamalar uchun chegaraviy shart kasr tartibli hosilalar bilan berilgan masalalar B. Turmetov (Qozog‘iston) va boshqalar tomonidan o‘rganilgan.

Muammoning o‘rganilganlik darajasi. Hozirgi kunda kasr tartibli xususiy hosilali differensial tenglamalarni va unga bog‘liq teskari masalalarni yechish bilan juda ko‘plab matematiklar shug‘ullanishmoqda. Bu yo‘nalishda olingan natijalarga misol qilib Sh.O.Alimov, R.R.Ashurov, S.R.Umarov, M.Yamomota, Z.Li, A.Ashyaraliyev, B.Turmetov, Y.Zhang, H.T.Nguyn, A.V.Pskhu, A.S.Malik, E.Karimov va hokazolarning ishlarini aytish mumkin. 2019 yil Z. Li va boshqalar tomonidan chop etilgan maqolada “Ochiq muammolar” bo‘limida quyidagicha ta’kidlangan edi: “Kasr tartiblarini tiklashning teskari muammolari bo‘yicha tadqiqotlar qoniqarli emas, chunki barcha maqolalar muammoni t vaqtning qandaydir oraliqqa tegishli bo‘lgan holi uchun o‘rganilgan. Teskari masalani kuzatish ma‘lumotlari sifatida belgilangan tayin vaqtda yechimning qiymati bo‘yicha o‘rganish qiziqarli bo‘lar edi”. Shu sababli o‘rganilgan barcha ishlarda, J.Jannoning ishidan tashqari, faqat yechimning yagonaligi ko‘rsatilgan. J.Jannoning ishida esa, yechimning mavjudligi ham ko‘rsatilgan, lekin u qo‘shimcha shart sifatida boshqa shartdan foydalangan. R.R.Ashurov va S.R.Umarovlar agar birinchi xos son nolga teng bo‘lsa, unga mos xos element o‘zgarmas son bo‘lsa, yechimning mavjud va yagonaligini ta‘minlash uchun yangi shart ishlab chiqishdi. Ushbu dissertatsiya ishida ushbu shartdan foydalanib turli hollar uchun kasr tartibli hosilaning tartibini aniqlash masalalari o‘rganildi, shu bilan birga agar birinchi xos soni nolga teng bo‘lmasa ham unga mos xos funksiyaga proyeksiyasi biror tayinlangan vaqtda qandaydir songa teng bo‘lish shartidan foydalanish mumkin ekanligi ko‘rsatilgan.

Ushbu dissertatsiya ishida elliptik qismi o‘z-o‘ziga qo‘shma, musbat, chegaralanmagan, ixtiyoriy A operatordan iborat kasr tartibli subdiffuziya $d^{\rho}u(t)+Au(t)=f(t)$ ko‘rinishdagi kasr tartibli tenglamalarning vaqt bo‘yicha $u(\xi)=\alpha u(0)+\varphi$ (α o‘zgarmas son, ξ fiksirlangan nuqta va $0<\xi<T$) nolokal chegaraviy shartni qanoatlantiruvchi yechimini topish masalasini o‘rganamiz, bu yerda tenglamaning chap tomonidagi d^{ρ} operator orqali Kaputo ma’nosidagi yoki Riman-Liuvill ma’nosidagi kasr tartibli hosila ifodalangan. Eslatib o‘tamiz, kasr tartibli tenglamalar uchun bunday $u(\xi)=\alpha u(0)+\varphi$ nolokal shart ilk bor ushbu ishda o‘rganilgan va qaralayotgan masalaning yechimi mavjud va yagonaligi shartlari keltirib chiqarilgan, qo‘shimcha ravishda α parametrning qaralayotgan masalalarning yechimining mavjudligi va yagonaligiga ta‘siri ham o‘rganiladi. Agar $\alpha=0$ (va $\xi=T$) bo‘lsa, bu muammoga *orqaga qaytish masalasi deb ataladi*. Kaputo ma’nosida kasr tartibli hosila qatnashgan tenglamalar uchun orqaga qaytish masalasi, masalan, Y. Liu, M. Yamamoto, K. Sakamoto, G. Floridia, Z. Li, ishlarida batafsil o‘rganilgan. Riman-Luivill ma’nosida kasr tartibli hosilali tenglamalar uchun orqaga qaytish masalasi, Sh. Alimov, R. Ashurov ishida o‘rganilgan. Klassik tenglamalar uchun orqaga qaytish masalalari bilan S. Kabanikhinning kitobida tanishish mumkin. Shuning uchun, biz faqat nolokal shartda α nolga teng bo‘lgan holni o‘rganib chiqamiz. Diffuziya tenglamalari uchun $\alpha=1$ bo‘lgan holda A.O. Ashyralyev va boshqalarning ishlarida o‘rganilgan. Bundan tashqari koersitiv (coercive, коэрцитив) tipidagi tengsizliklar olingan. Tenglamaning o‘ng tomonini topish bo‘yicha ko‘plab izlanishlar olib borilgan. Bizga ma’lumbo‘lishicha, φ funksiyani topish bo‘yicha teskari masala faqat T. Yuldashev, B. Kadirkulov maqolasida yoritilgan. Mualliflar bu muammoni elliptik qismi o‘zgarmas koeffitsiyentli bir o‘zgaruvchili differensial ifodadan iborat Hilfer ma’nosida kasr tartibli tenglamalar uchun o‘rganib chiqganlar. Elliptik qismi ko‘p o‘zgaruvchili yuqori tartibli bo‘lgan butun va kasr hosilali tenglamalar uchun Koshi masalasi ilk bor R.R. Ashurov va A.T. Muhiddinovalarning ishida o‘rganilgan. Nolokal - chegaraviy shart bilan berilgan Riman - Liuvill ma’nosida

kasr tartibli subdiffuziya tenglamalari uchun to‘g‘ri va teskari masalalar esa ilk bor ushbu ishda o‘rganilgan. Elliptik tenglamalar uchun chegaraviy shart kasr tartibli hosila bilan berilgan masalalar S. Umarov, B. Turmetov va boshqalarning ishlarida o‘rganilgan. Ushbu ishlarda kasr tartibli hosilaning tartibi 0 va ½ oralig‘ida bo‘lsa, chegaraviy funksiyadan hech qanday qo‘shimcha shart talab qilinmasligi va ½ va 1 oralig‘ida bo‘lsa, qo‘shimcha shartlar talab qilinishi ko‘rsatilgan. R. Gorenflo, Yu. Luchko, S. Umarovlarning ishida elliptik psevdodifferensial operatorli tenglamalar uchun chegaraviy shart kasr tartibli hosila bilan berilgan hol uchun yechim mavjud bo‘lishligi uchun chegaraviy funksiyaga qo‘shimcha shartlar olingan. Barenblatt-Jeltov-Kochina tenglamasi bir jinsli suyuqliklarni filtrlashning asosiy tenglamalari hisoblanadi. Bu haqida ilk bora G. I. Barenblatt, Yu. P. Jeltov va I. N. Kochina ishida e‘lon qilingan. Keyinchalik boshqa ishlarida ham o‘rganilgan. Barenblatt-Jeltov-Kochina tipidagi kasr tartibli tenglamalar ushbu ishda birinchi marta o‘rganilgan.

Optimal boshqaruv masalalari yo‘nalishida ko‘pgina matematiklar izlanishlar olib borishgan. Bunga misol sifatida N.Yu. Satimov, Sh.O. Alimov, V.A. Il’in, A. A‘zamov, E.I. Moiseev, A.G. Butkovskiy, F.R. Vasilev, J. Lions, R. Varga, A.I. Yegorov, F.Y. Chernousko, S.A. Avdonin, S.V. Ivanov, H.G. Guseynova, V.V.Tixomirov, K.A. Lurelar, M. To‘xtasinov, G‘. Ibragimov, N. Mamadaliyevning ishlarini aytish mumkin. Ilk bor xususiy hosilali differensial tenglamalar bilan tasvirlangan jarayonlarning boshqaruv masalalari J.L. Lions ishida batafsil bayon qilib berilgan. So‘nggi yillarda turli xil differensial tenglamalar bilan tasvirlangan jarayonlarni boshqarish masalalarini o‘rganishga qiziqish ortib bormoqda. V. A. Il'in va E. I. Moiseev ishlarida to‘lqin tarqalishi bilan bog‘liq bo‘lgan jarayonlar uchun chegaradan boshqarish masalalari o‘rganilgan. Parabolik tipdagi tenglamalar bilan tavsiflangan jarayonlarni, xususan, issiqlik uzatish jarayonini boshqarish masalasi bilan bog‘liq muammolar V. Barbu, A. Rascanu va H.O. Fattorini ishlarida qayd etilgan. Bir jinsli issiqlik uzatish tenglamalari bilan ifodalangan jarayonlarni boshqarish masalasi akademik Sh.O. Alimov ishlarida uchratish mumkin. Kasr tartibli tenglamalar uchun manba funksiyasi yordamida boshqarish masalalari ilk bor ushbu ishda o‘rganilgan.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta‘lim yoki ilmiy-tadqiqot muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya ishi F-FA-2021-424 Matematika institutining “Butun va kasr tartibli xususiy hosilali differensial tenglamalar uchun chegaraviy masalalarning yechimi” rejalashtirilgan tadqiqot mavzusiga muvofiq amalga oshirildi. V.I. Romanovskiy nomidagi O‘zbekiston Respublikasi Fanlar akademiyasi va M. Ulug‘bek nomidagi O‘zbekiston Milliy universitetining “Differensial tenglamalar va matematik fizika” kafedrasida bajarildi.

Tadqiqot maqsadi ixtiyoriy elliptik operatorli xususiy hosilali va kasr tartibli xususiy hosilali tenglamalar uchun to‘g‘ri va teskari masalalarning yechimi mavjud va yagonaligini ko‘rsatishdan iborat.

Tadqiqot vazifalari quyidagilardan iborat:

Yuqori kasr tartibli tenglamaning yechimini qurish;

Kasr tartibli subdiffuziya tenglamalari uchun vaqt bo‘yicha nolokal shartli chegaraviy masalalarni yechish;

Kasr tartibli subdiffuziya tenglamalari uchun nolokal-chegaraviy masalalarni yechish;

Kasr tartibli xususiy hosilali differensial tenglamalarda hosilaning tartibi va tenglamaning o‘ng tomonini topish bo‘yicha to‘g‘ri va teskari masalalarni yechish;

Chegaraviy shart kasr tartibli hosila bilan berilgan giperbolik va parabolik tenglamalarni yechish;

Barenblatt-Jeltov-Kochina tipidagi kasr tartibli tenglamalar uchun to‘g‘ri va teskari masalalarni yechish;

Chegaraviy shart kasr tartibli hosila bilan berilgan Barenblatt-Jeltov-Kochina tipidagi tenglamalar uchun to‘g‘ri va teskari masalalarni yechish;

Kasr tartibli xususiy hosilali differensial tenglamalar uchun birinchi chegaraviy masalaning yechimini qurish;

Kasr tartibli xususiy hosilali differensial tenglamalar uchun boshqaruv masalalarini yechish;

Tadqiqotning ob’ekti. Kaputo, Riman-Liuvill, Gryunvold-Letnikov, Wayl kasr tartibli hosilalari, Mittag-Leffler funksiyalari, vaqt bo‘yicha kasr tartibli tenglamalar.

Tadqiqotning predmeti. Dissertatsiyaning tadqiqot predmeti xususiy hosilalali va kasr tartibli ixtiyoriy elliptik operatorli differensial tenglamalar uchun to‘g‘ri va teskari masalalardan iborat.

Tadqiqotning usullari. Dissertatsiya ishida matematik analiz, funksional analiz, differensial tenglamalar va matematik fizika usullaridan foydalanilgan. Matematik fizika usullaridan o‘zgaruvchilarni ajratish usuli qo‘llaniladi hamda Hilbert fazosida xos funksiyalar sistemasini to‘la ekanligi tadbiq qilinadi.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

kasr tartibli subdiffuziya tenglamalarining nolokal shartni qanoatlantiruvchi yechimi, tenglamaning o‘ng tomoni va chegaraviy funksiyani topish bo‘yicha teskari masalalarning yechimi mavjud va yagonaligi isbotlangan;

kasr tartibli xususiy hosilali differensial tenglamalarda hosilaning tartibi va tenglamaning o‘ng tomonini bir vaqtda topish bo‘yicha to‘g‘ri va teskari masalalarning yechimi mavjud va yagonaligi isbotlangan;

chegaraviy shart kasr tartibli hosila bilan berilgan giperbolik va parabolik tenglamalarning yechimi mavjud va yagonaligi isbotlangan;

Barenblatt-Jeltov-Kochina tipidagi kasr tartibli tenglamalar va chegaraviy shart kasr tartibli hosila bilan berilgan Barenblatt-Jeltov-Kochina tipidagi differensial tenglamalar uchun to‘g‘ri va teskari masalalarning yechimi mavjud va yagona bo‘lishligi isbotlangan;

kasr tartibli xususiy hosilali differensial tenglamalar uchun birinchi chegaraviy masalaning yechimi qurilgan va vaqtga bog‘liq holda to‘g‘ri to‘rtburchakli sohada jarayon oldindan berilgan funksiya kabi bo‘lishligi uchun manba funksiyasini qanday tanlash shartlari topilgan.

Tadqiqotning amaliy natijalari. Tadqiqot ishida kasr tartibli differensial va xususiy hosilali differensial tenglamalarni yechish usullari taklif etiladi va ixtiyoriy elliptik operatorli kasr tartibli tenglamalarni yechish usullari taqdim etiladi. Shu bilan birga tenglamaning o‘ng tomonini topish va kasr hosilaning tartibini aniqlash bo‘yicha teskari masalalarni yechish usuli bayon qilinadi.

Tadqiqot natijalarining ishonchliligi matematik tahlil, differensial tenglamalar va matematik fizika, fundamental yechimlarni qurish uchun maxsus funksiyalar nazariyasi, chegaraviy muammolarning aniq yechimlarini topish va elliptik tenglamalarni yechish nazariya masalalarini o‘rganish usullaridan qat’iy foydalanish bilan tasdiqlanadi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati kasr tartibli differensial va xususiy hosilali differensial tenglamalarni yechishda foydalanish mumkinligi bilan izohlanadi.

Tadqiqot natijalarning amaliy ahamiyati ularning kasr tartibli tenglamalar bilan berilgan jarayonlarni o‘rganishda, nazorat qilishda va boshqarishda qo‘llanilishi bilan belgilanadi.

Tadqiqot natijalarining joriy qilinishi. Eliptik qismi yuqori tartibli bo‘lgan kasr tartibli xususiy hosilali differensial tenglamalar uchun to‘g‘ri va teskari masalalar bo‘yicha olingan natijalar asosida:

chegaraviy shart kasr tartibli hosila bilan berilgan giperbolik va parabolik tenglamalarning yechimi mavjud va yagona bo‘lishlik shartlaridan OT-Ф-4-28 raqamli “Giperbolik sistemalar uchun adekvat hisoblash modellarini qurish” mavzusidagi fundamental loyihada sistemalarning modellarini qurishda foydalanilgan (O‘zbekiston Milliy universitetining 2022 yil 30 sentyabrdagi №04/11-5960-sonli ma‘lumotnomasi). Ilmiy natijaning qo‘llanilishi giperbolik tipidagi differensial tenglamalar sistemalarning hisoblash modellarini qurish imkonini bergan;

kasr tartibli xususiy hosilali differensial tenglamalar uchun birinchi chegaraviy masalaning yechimidan va manba funsiyasi yordamida boshqarishdan МД-758.2022.1.1 raqamli “Rossiya Prezidentining rossiyalik yosh olimlarni qullab-quvvatlash” mavzusidagi xorijiy grant loyihasida atmosferadagi radonlarini taqsimlanishini tekshirishda foydalanilgan (Kamchatka davlat universitetining 2022 yil 27 sentyabrdagi № 39-12-sonli ma‘lumotnomasi, Rossiya federatsiyasi). Ilmiy natijalarning qo‘llanilishi atmosfera grunt sistemasidagi radonlarni qayta taqsimlash imkonini bergan;

Barenblatt-Jeltov-Kochina tipidagi kasr tartibli tenglamalar va chegaraviy shart kasr tartibli hosila bilan berilgan Barenblatt-Jeltov-Kochina tipidagi differensial tenglamalar uchun to‘g‘ri va teskari masalalarning yechimlaridan MRU-OT-30/2017 raqamli “Zamonaviy superkompyuterlardan foydalanib, gazodinamika, filtratsiya va transport oqimi dinamikasi masalalarini yechish uchun yuqori aniqlikdagi hisoblash algoritmlari” mavzusidagi fundamental loyihada chegaraviy masalalarni yechishda foydalanilgan (O‘zbekiston Milliy universitetining 2022 yil 4 oktyabrdagi №04/11-6042-sonli ma‘lumotnomasi). Ilmiy natijalarining qo’llanishi chegaraviy masalalarni, filtratsiya va boshqaruv masalalarini hisoblash modellarini qurish imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Ushbu tadqiqot ishining natijalari 28 ta ilmiy-amaliy konferensiyalarda shulardan 13 tasi xalqaro, 15 tasi Respublika miqyosidagi ilmiy konferensiyalarda muhokama qilingan.

Tadqiqot natijalarining e‘lon qilinganligi. Dissertatsiya mavzusi bo‘yicha jami 42 ta ilmiy ish chop etilgan, shulardan O‘zbekiston Respublikasi Oliy attestatsiyasi komissiyasining fan doktorlik dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nasrlarda 13 ta maqola, jumladan 9 tasi xorijiy va 4 tasi mahalliy jurnallarda, hamda 1 ta maqola xorijiy jurnalda va 28 ta tezis nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish, 5 ta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan. Dissertatsiya 222 betdan iborat.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zaturati asoslangan, tadqiqotning respublika fan va texnologiyalarining ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, mavzu bo‘yicha xorijiy ilmiy-tadqiqotlar sharhi, muammoning o‘rganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, obyekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo‘yicha ma‘lumotlar berilgan.

Dissertatsiya birinchi bobining asosiy maqsadi kasr tartibli tenglamalar uchun vaqt bo‘yicha nolokal chegaraviy masalalarni o‘rganishdan iborat. Dissertatsiyaning birinchi paragrafida dissertatsiya natijalarini bayon qilishda zarur bo‘ladigan yordamchi ma‘lumotlar keltirilgan. Ikkinchi paragrafida yuqori kasr tartibli differensial tenglamalarning yechimini qurish masalasi o‘rganilgan. Yechimni qurishda R. Ashurov, A. Kabada va B. Turmetovlarning tomonidan Kaputo ma’nosidagi kasr tartibli hosilali chiziqli differensial tenglamalarining yechimini qurish usulidan foydalanilgan. Uchinchi paragrafida esa, Kaputo va Riman-Liuvill ma’nosida kasr tartibli tenglamalar uchun vaqt bo‘yicha nolokal chegaraviy shartli masalalar o‘rganilgan.

Aytaylik, H separabl Hilbert fazosi bo‘lsin. Unda skalyar ko‘paytma va norma aniqlangan bo‘lib, ularni mos ravishda $(\cdot,\cdot)$ va $\|\cdot\|$ kabi belgilaylik. $A: H \to H$ perator H Hilbert fazosida aniqlangan, o‘z-o‘ziga qo‘shma, musbat, chegaralanmagan ixtiyoriy operator bo‘lsin. Faraz qilaylik, A operator H fazoda to‘la ortonormal $\{v_k\}$ xos funksiyalar sistemasiga va $\{\lambda_k\}$ musbat xos qiymatlar to‘plamiga ega bo‘lsin. Xos qiymatlarni qayta nomerlash yordamida ularni kamaymaydigan qilib nomerlab olamiz, ya’ni $0 < \lambda_1 \le \lambda_2 \cdots \to +\infty$ kabi yozib olamiz.

Kuchli integral va kuchli hosila ta’riflaridan foydalanib, $h: \mathrm{R}_+ \to H$ vektor qiymatli funksiyalar (yoki oddiygina funksiyalar) uchun integral va hosilalarning kasr analoglari aniqlanishi mumkin, aniqlangan ta'riflar bir o'zgaruvchi bo'lgan holdagi formulalar va xossalarni saqlanib qoladi.

Aytaylik, $h(t)$ funksiya $[0,\infty)$ da aniqlangan bo'lsin. Agar

$$J_t^{\sigma} h(t) = \frac{1}{\Gamma(\sigma)} \int_0^t \frac{h(\xi)}{(t-\xi)^{1-\sigma}} d\xi, \quad t > 0 \tag{1}$$

tenglikning o'ng tomonidagi integral chekli bo'lsa, u holda ushbu integralga $h(t)$ funksiyaning σ kasr tartibli Riman-Liuvill integrali deyiladi, bunda $\Gamma(\sigma)$ Eylerning gamma funksiyasi. Eslatib o'tamiz, $J_t^0 : h(t) = h(t)$ deb qabul qilingan. Ushbu ta'rifdan foydalanib, $0 < \rho < 1$ bo'lsa, Riman-Liuvill ma'nosida ρ - kasr tartibli hosilaning ta'rifini quyidagicha aniqlash mumkin:

$$\partial_t^{\rho} h(t) = \frac{d}{dt} J_t^{1-\rho} h(t).$$

Agar bu ta'rifda biz differensial va kasr tartibli integralning o'rinlarini almashtiradigan bo'lsak, unda biz Kaputo ma'nosida kasr tartibli hosilasining ta'rifini olamiz:

$$D_t^{\rho} h(t) = J_t^{1-\rho} \frac{d}{dt} h(t).$$

Aytaylik, $\rho \in (0,1)$ bo'lsin. $C((a,b);H)$ orqali H dagi qiymatlari $t \in (a,b)$ oraliqda uzluksiz $u(t)$ funksiyalar to'plamini belgilaylik.

Ushbu paragrafda biz quyidagi masalalarni qaraymiz:

$$\begin{cases} D_t^{\rho} u(t) + Au(t) = f(t), & 0 < t \le T; \\ u(\xi) = \alpha u(0) + \varphi, & 0 < \xi \le T \end{cases} \tag{2}$$

va

$$\begin{cases} \partial_t^{\rho} u(t) + Au(t) = g(t), & 0 < t \le T; \\ J_t^{1-\rho} u(t)|_{t=\xi} = \alpha \lim\limits_{t\to 0} J_t^{1-\rho} u(t) + \phi, & 0 < \xi \le T, \end{cases} \tag{3}$$

bu yerda $f(t), g(t) \in C((0,T];H)$, $\varphi, \phi \in H$ va α – o'zgarmas son, ξ-qo'zg'almas nuqta. Bu muammolarga *to'g'ri masala* deb ataladi .

1-ta'rif. *Agar* $u(t) \in C([0,T];H)$ *funksiya* $D_t^{\rho} u(t), Au(t) \in C((0,T);H)$ *xossalarga ega bo'lib,* (2) *ning barcha shartlarini qanoatlantirsa, u holda bu funksiyaga* (2) *nolokal masalaning* ***yechimi*** *deb ataladi .*

(3) nolokal masala yechimining ta'rifi ham xuddi shunga o'xshash tarzda kiritiladi.

Agar $\alpha = 0$ (va $\xi = T$) bo'lsa, bu muammoga *orqaga qaytish masalasi deb ataladi*. (2) holidagi orqaga qaytish masalasi, masalan, M. Yamamoto va boshqalarning ishlarida batafsil o'rganilgan. (3) holidagi orqaga qaytish masalasi, Sh. Alimov va R. Ashurov ishida o'rganilgan. Shuning uchun, biz faqat nolokal shartda $\alpha \ne 0$ bo'lgan holni o'rganib chiqamiz.

$\rho = 1$ bo'lgan holda, bu muammolar vaqtga bog'liq teskari issiqlik o'tkazuvchanlik masalasi (*retrospektiv teskari masala*) deyiladi. Shuni ham ta'kidlash kerakki, bu holda $u(T)$ funksiyaning silliqligi ham yechimning barqarorligini kafolatlamaydi.

$\rho=1$, $\alpha=1$ bo'lgan holda, A operator Banax fazosida aniqlangan hol uchun A.O. Ashyralyev va boshqlarning ishlarida o'rganilgan.

Kasr tartibli tenglamalar uchun $u(\xi)=\alpha u(0)+\varphi$ $(J_t^{1-\rho}u(t)|_{t=\xi}=\alpha\lim\limits_{t\to 0}J_t^{1-\rho}u(t)+\phi)$ nolokal shart bilan berilgan masalalar ushbu ishda ilk bor o'rganilmoqda.

Ushbu paragrafda biz (2), (3) masalalarning yechimi mavjud va yagonaligi haqidagi teoremalarini isbotlaymiz. Shu bilan birga yechim mavjudligining α parametr qiymatiga bog'liqligini o'rganamiz. Biz, shuningdek, orqaga qaytish masalalaridan farqli ravishda, (2), (3) masalaning yechimi tenglamaning o'ng tomoniga va φ funksiyaga bog'liqligini isbotlaymiz. Bundan tashqari koersativ tipidagi tengsizliklar olinadi va bu tengsizliklar kasr hosilalarining ko'rib chiqilayotgan turiga qarab farqlanishi ko'rsatilgan. Tenglamaning o'ng tomonini va nolokal shartdagi φ funksiyani aniqlash bo'yicha teskari masalalar o'rganiladi.

(2) masalani yechish uchun biz uni quyidagi ikkita yordamchi masalaga keltirib olamiz:

$$\begin{cases} D_t^\rho \omega(t)+A\omega(t)=f(t), & 0<t\le T; \\ \omega(0)=0, \end{cases} \tag{4}$$

va

$$\begin{cases} D_t^\rho w(t)+Aw(t)=0, & 0<t\le T; \\ w(\xi)=\alpha w(0)+\psi, & 0<\xi\le T, \end{cases} \tag{5}$$

bu yerda $\psi\in H$ berilgan funksiya.

Agar $\psi=\varphi-\omega(\xi)$ va $\omega(t)$ va $w(t)$ mos ravishda (4) va (5) masalalarning yechimlari bo'lsa, $u(t)=\omega(t)+w(t)$ funksiya (2) masalaning yechimi ekanligini tekshirish qiyin emas. Shuning uchun (2) masalani yechish uchun (4) va (5) yordamchi masalalarni yechish kifoya qiladi.

Faraz qilaylik, $\lambda_0>0$ soni $E_\rho(-\lambda_0\xi^\rho)=\alpha$ tenglikni qanoatlantirsin, bu yerda $E_\rho(t)$ Mittag-Leffler funksiyasi. $\lambda_k=\lambda_0$ bo'lib, bu xos son p_0 karrali bo'lsin. Biz bunday k nomerlarni $K_0=\{k_0,k_0+1,...,k_0+p_0-1\}$ deb belgilab olamiz.

τ ixtiyoriy haqiqiy son bo'lsin. Biz H da A operatorning darajasini quyidagicha kiritamiz:

$$A^\tau h=\sum_{k=1}^{\infty}\lambda_k^\tau h_k v_k,$$

bu yerda $h_k=(h,v_k)$ lar $h\in H$ elementning Furye koeffitsiyentlari. Shubhasiz, ushbu operatorning aniqlanish sohasi quyidagi ko'rinishga ega bo'ladi:

$$D(A^\tau)=\{h\in H:\sum_{k=1}^{\infty}\lambda_k^{2\tau}|h_k|^2<\infty\}.$$

(4) masalani yechish uchun biz quyidagi teoremani isbotlaymiz.

1-teorema. *Aytaylik, qandaydir* $\varepsilon \in (0,1)$ *sonlar uchun* $f(t) \in C([0,T]; D(A^{\varepsilon}))$ *bo'lsin. U holda* (4) *masalaning yagona yechimi mavjud va u yechim quyidagi ko'rinishga ega*

$$\omega(t) = \sum_{k=1}^{\infty} \left[\int_0^t \eta^{\rho-1} E_{\rho,\rho}(-\lambda_k \eta^{\rho}) f_k(t-\eta) d\eta \right] v_k. \qquad (6)$$

Bundan tashqari quyidagi koersitiv turdagi tengsizlik o'rinli bo'ladi:

$$\| D_t^{\rho} \omega(t) \|^2 + \| \omega(t) \|_1^2 \le C_{\varepsilon} \max_{t \in [0,T]} \| f \|_{\varepsilon}^2, \quad 0 < t \le T, \qquad (7)$$

bu yerda $C_{\varepsilon} > 0$ *o'zgarmas son.*

Agar $f \in H$ funksiya t ga bog'liq bo'lmasa ham, 1 - teoremaning tasdig'i to'g'ri bo'ladi.

1-natija. *Aytaylik,* $f \in H$ *bo'lsin. U holda* (4) *masala yagona yechimga ega va bu yechim quyidagi ko'rinishda bo'ladi*

$$\omega(t) = \sum_{k=1}^{\infty} f_k t^{\rho} E_{\rho,\rho+1}(-\lambda_k t^{\rho}) v_k. \qquad (8)$$

Bundan tashqari, shunday C musbat son topiladiki, quyidagi koersitiv turdagi tengsizlik o'rinli bo'ladi:

$$\| D_t^{\rho} \omega(t) \|^2 + \| \omega(t) \|_1^2 \le C \| f \|^2, \quad 0 < t \le T. \qquad (9)$$

(5) masalani yechish uchun esa quyidagi teoremani isbotlaymiz.

2-teorema. *Aytaylik,* $\psi \in H$ *bo'lsin. Agar* $\alpha \notin (0,1)$ *yoki* $\alpha \in (0,1)$ *bo'lib, lekin barcha* $k \ge 1$ *lar uchun* $\lambda_k \ne \lambda_0$ *bo'lsa,* (5) *masala yagona yechimga ega va bu yechim quyidagi ko'rinishga ega*

$$w(t) = \sum_{k=1}^{\infty} \frac{\psi_k}{E_{\rho}(-\lambda_k \xi^{\rho}) - \alpha} E_{\rho}(-\lambda_k t^{\rho}) v_k. \qquad (10)$$

Agar $\alpha \in (0,1)$ *va* $\lambda_k = \lambda_0$, $(k \in K_0)$ *bo'lsa,*

$$\psi_k = (\psi, v_k) = 0, \quad k \in K_0; \quad K_0 = \{k_0, k_0+1, ..., k_0 + p_0 - 1\}, \qquad (11)$$

ortogonallik shartlari bajarilgan deb faraz qilamiz. U holda (5) *masalaning yechimi* b_k, $k \in K_0$ *ixtiyoriy koeffitsiyentlar bilan quyidagi ko'rinishga ega bo'ladi*

$$w(t) = \sum_{k \notin K_0} \frac{\psi_k}{E_{\rho}(-\lambda_k \xi^{\rho}) - \alpha} E_{\rho}(-\lambda_k t^{\rho}) v_k + \sum_{k \in K_0} b_k E_{\rho}(-\lambda_k t^{\rho}) v_k. \qquad (12)$$

Bundan tashqari quyidagi koersitiv turdagi tengsizlik o'rinli bo'ladi:

$$\| D_t^{\rho} w(t) \|^2 + \| w(t) \|_1^2 \le C_{\alpha} t^{-2\rho} \| \psi \|^2, \quad 0 < t \le T, \qquad (13)$$

bu yerda $C_{\alpha} > 0$ *o'zgarmas son.*

(2) masalaning yechimi quyidagi teoremada aks etgan.

3-teorema. *Aytaylik,* $\varphi \in H$ *va ba'zi* $\varepsilon \in (0,1)$ *lar uchun* $f(t) \in C([0,T]; D(A^{\varepsilon}))$ *bo'lsin. Agar* $\alpha \notin (0,1)$ *yoki* $\alpha \in (0,1)$ *bo'lib, lekin barcha*

$k \geq 1$ lar uchun $\lambda_k \neq \lambda_0$ bo'lsa, u holda (2) *masala yagona yechimga ega va bu yechim quyidagi ko'rinishga ega*

$$u(t) = \sum_{k=1}^{\infty} \left[\frac{\varphi_k - \omega_k(\xi)}{E_\rho(-\lambda_k \xi^\rho) - \alpha} E_\rho(-\lambda_k t^\rho) + \omega_k(t) \right] v_k, \qquad (14)$$

Agar $\alpha \in (0,1)$ va $k \in K_0$ lar uchun $\lambda_k = \lambda_0$ mavjud bo'lsa, hamda

$$(\varphi, v_k) = (\omega(\xi), v_k), \ k \in K_0; \ K_0 = \{k_0, k_0 + 1, ..., k_0 + p_0 - 1\}. \qquad (15)$$

ortogonallik shartlari bajarilgan bo'lsa, u holda (2) *masala yechimi b_k, $k \in K_0$ ixtiyoriy koeffitsiyentlar bilan quyidagi ko'rinishga ega*

$$u(t) = \sum_{k \notin K_0} \left[\frac{\varphi_k - \omega_k(\xi)}{E_\rho(-\lambda_k \xi^\rho) - \alpha} E_\rho(-\lambda_k t^\rho) + \omega_k(t) \right] v_k + \sum_{k \in K_0} b_k E_\rho(-\lambda_k t^\rho) v_k. \qquad (16)$$

Bundan tashqari, $C_\alpha > 0$ va $C_\varepsilon > 0$ sonlar mavjud bo'lib, quyidagi koersitiv turdagi tengsizlik o'rinli bo'ladi:

$$\| D_t^\rho u(t) \|^2 + \| u(t) \|_1^2 \leq C_\alpha t^{-2\rho} \| \varphi \|^2 + C_\varepsilon \max_{t \in [0,T]} \| f \|_\varepsilon^2, \quad 0 < t \leq T. \qquad (17)$$

Teskari masalalarni yechishda bizga qo'shimcha shart kerak bo'ladi. Biz ikki hol uchun ham qo'shimcha shart sifatida quyidagi shartdan foydalanamiz:

$$u(\tau) = \Psi, \quad 0 < \tau \leq T, \quad \tau \neq \xi. \qquad (18)$$

Teskari masalalarni o'rganishda biz $f \in H$ fnksiyani t ga bog'liq emas deb qaraymiz. E'tibor bering, agar $\tau = \xi$ bo'lsa, (2) dagi nolokal shart Koshi shartiga to'g'ri keladi $u(0) = \varphi_1$ ($\alpha \neq 0$). Bunday teskari masalalar oldingi ishlarda o'rganilgan.

2-ta'rif. *Agar $u(t) \in C([0,T]; H)$ va $f \in H$ funksiyalar $D_t^\rho u(t), Au(t) \in C((0,T]; H)$ xossalarga ega bo'lib,* (2), (18) *shartlarni qanoatlantirsa, u holda $\{u(t), f\}$ funksiyalar juftiga* (2), (18) *teskari masalaning* ***yechimi*** *deyiladi .*

Tenglamaning o'ng tomonini topish bo'yicha teskari masalani $\alpha \geq 1$ va $0 < \alpha < 1$ hollar uchun alohida-alohida o'rganamiz. $\alpha \geq 1$ bo'lgan hol uchun olingan natijani keltiramiz.

4-teorema. *Aytaylik, φ, $\Psi \in D(A)$ va $\alpha \geq 1$ bo'lsin. U holda* (2), (18) *teskari masalaning $\{u(t), f\}$ yagona yechimi mavjud va bu yechim quyidagi ko'rinishga ega bo'ladi*

$$f = \sum_{k=1}^{\infty} \left[\frac{\alpha - E_\rho(-\lambda_k \xi^\rho)}{E_\rho(-\lambda_k \tau^\rho) \xi^\rho E_{\rho,\rho+1}(-\lambda_k \xi^\rho) + \tau^\rho E_{\rho,\rho+1}(-\lambda_k \tau^\rho)[\alpha - E_\rho(-\lambda_k \xi^\rho)]} \Psi_k + \right.$$
$$\left. + \frac{E_\rho(-\lambda_k \tau^\rho)}{E_\rho(-\lambda_k \tau^\rho) \xi^\rho E_{\rho,\rho+1}(-\lambda_k \xi^\rho) + \tau^\rho E_{\rho,\rho+1}(-\lambda_k \tau^\rho)[\alpha - E_\rho(-\lambda_k \xi^\rho)]} \varphi_k \right] v_k, \qquad (19)$$

$$u(t)=\sum_{k=1}^{\infty}\left[\frac{E_\rho(-\lambda_k t^\rho)}{E_\rho(-\lambda_k\xi^\rho)-\alpha}[\varphi_k-f_k\xi^\rho E_{\rho,\rho+1}(-\lambda_k\xi^\rho)]+f_k t^\rho E_{\rho,\rho+1}(-\lambda_k t^\rho)\right]v_k. \quad (20)$$

Endi yuqorida aytganimizdek, $0<\alpha<1$ bo‘lgan hol uchun olingan natijalarni alohida keltiramz. Bu holda teskari masalaning yechimi mavjud va yagonaligi ξ va τ nuqtalarning joylashishiga ham bog‘liq bo‘ladi.

5-teorema. *Aytaylik,* $0<\alpha<1$, $\varphi,\Psi\in D(A)$ *va* (15) *ortogonallik shartlari bajarilsin. Agar* $0<\tau<\xi$ bo‘lsa, *u holda* (2), (18) *teskari masalaning* $\{u(t),f\}$ *yagona yechimi mavjud va bu yechim quyidagi ko‘rinishga ega bo‘ladi*

$$f=\sum_{k\notin K_0}\left[\frac{\alpha-E_\rho(-\lambda_k\xi^\rho)}{E_\rho(-\lambda_k\tau^\rho)\xi^\rho E_{\rho,\rho+1}(-\lambda_k\xi^\rho)+\tau^\rho E_{\rho,\rho+1}(-\lambda_k\tau^\rho)[\alpha-E_\rho(-\lambda_k\xi^\rho)]}\Psi_k+\right.$$

$$\left.+\frac{E_\rho(-\lambda_k\tau^\rho)}{E_\rho(-\lambda_k\tau^\rho)\xi^\rho E_{\rho,\rho+1}(-\lambda_k\xi^\rho)+\tau^\rho E_{\rho,\rho+1}(-\lambda_k\tau^\rho)[\alpha-E_\rho(-\lambda_k\xi^\rho)]}\varphi_k\right]v_k, \quad (21)$$

$$u(t)=\sum_{k\notin K_0}\left[\frac{E_\rho(-\lambda_k t^\rho)}{E_\rho(-\lambda_k\xi^\rho)-\alpha}[\varphi_k-f_k\xi^\rho E_{\rho,\rho+1}(-\lambda_k\xi^\rho)]+f_k t^\rho E_{\rho,\rho+1}(-\lambda_k t^\rho)\right]v_k.$$

$$+\sum_{k\in K_0}\frac{E_\rho(-\lambda_k t^\rho)\Psi_k}{E_\rho(-\lambda_k\tau^\rho)}v_k. \quad (22)$$

(2) masalada $u(t)$ va $u(\xi)=\alpha u(0)+\varphi$ nolokal shartdagi φ funksiya ham noma’lum deb faraz qilamiz. Ushbu teskari masalaga chegaraviy funksiyani topish bo‘yicha *teskari masala* deb ataladi. Ushbu teskari masalani hal qilish uchun bizga qo‘shimcha shart kerak bo‘ladi. Shuning uchun biz qo‘shimcha shart sifatida yana oldingi (18) shartdan foydalanamiz.

3-ta’rif. *Agar* $u(t)\in C([0,T];H)$ *funksiya* $D_t^\rho u(t), Au(t)\in C((0,T);H)$ *xossalarga ega bo‘lib,* $u(t)$ *va* $\varphi\in H$ *funksiyalar* (2), (18) *shartlarni qanoatlantiruvchi bo‘lsa, u holda* $\{u(t),\varphi\}$ *juftlik* (2), (18) *teskari* ***masalaning yechimi*** *deb ataladi .*

Yuqorida ta’kidlanganidek, α parametrga qo‘shimcha shart qo‘yish orqali yechimning yagonaligini isbotlashni soddalashtirish mumkin, shuning uchun bu masalani ikki hol $E_\rho(-\lambda_k\xi^\rho)\neq\alpha$ va $E_\rho(-\lambda_0\xi^\rho)=\alpha$ bo‘lgan hollar uchun o‘rganib chiqamiz.

6-teorema. *Aytaylik,* $\Psi\in D(A)$, *qandaydir* $\varepsilon\in(0,1)$ *lar uchun* $f\in C([0,T];D(A^\varepsilon))$ *bo‘lsin. Agar* $\alpha\notin(0,1)$ *yoki* $\alpha\in(0,1)$ *bo‘lib, lekin barcha* $k\geq 1$ *lar uchun* $\lambda_k\neq\lambda_0$ *bo‘lsa, u holda* (2), (18) *teskari masalaning* $\{u(t),\varphi\}$ *yagona yechimi mavjud va u quyidagi ko‘rinishga ega*

$$\varphi=\sum_{k=1}^{\infty}\left[\frac{E_\rho(-\lambda_k\xi^\rho)-\alpha}{E_\rho(-\lambda_k\tau^\rho)}[\Psi_k-\omega_k(\tau)]+\omega_k(\xi)\right]v_k, \quad (23)$$

$$u(t)=\sum_{k=1}^{\infty}\left[\frac{\varphi_k-\omega_k(\xi)}{E_\rho(-\lambda_k\xi^\rho)-\alpha}E_\rho(-\lambda_k t^\rho)+\omega_k(t)\right]v_k, \tag{24}$$

bu yerda $\omega_k(t)=\int_0^t \eta^{\rho-1}E_{\rho,\rho}(-\lambda_k\eta^\rho)f_k(t-\eta)d\eta.$

Endi $\alpha\in(0,1)$ bo‘lgan hol uchun quyidagi teoremani keltiramiz.

7-teorema. *Aytaylik,* $\alpha\in(0,1)$, $\Psi\in D(A)$, *qandaydir* $\varepsilon\in(0,1)$ *lar uchun* $f\in C([0,T];D(A^\varepsilon))$ *bo‘lsin, hamda* (15) *ortogonallik shartlari bajarilsa, u holda* (2), (18) *teskari masalaning* $\{u(t),\varphi\}$ *yagona yechimi mavjud va bu yechim quyidagi ko‘rinishga ega*

$$\varphi=\sum_{k\notin K_0}\left[\frac{E_\rho(-\lambda_k\xi^\rho)-\alpha}{E_\rho(-\lambda_k\tau^\rho)}[\Psi_k-\omega_k(\tau)]+\omega_k(\xi)\right]v_k, \tag{25}$$

$$u(t)=\sum_{k\notin K_0}\left[\frac{\varphi_k-\omega_k(\xi)}{E_\rho(-\lambda_k\xi^\rho)-\alpha}E_\rho(-\lambda_k t^\rho)+\omega_k(t)\right]v_k+\sum_{k\in K_0}\frac{E_\rho(-\lambda_k t^\rho)\Psi_k}{E_\rho(-\lambda_k\tau^\rho)}v_k, \tag{26}$$

bu yerda $\omega_k(t)=\int_0^t \eta^{\rho-1}E_{\rho,\rho}(-\lambda_k\eta^\rho)f_k(t-\eta)d\eta.$

Yuqorida Kaputo ma’nosidagi kasr tartibli hosilali subdiffuziya tenglamalari uchun olingan barcha natijalar Riman-Liuvill ma’nosida kasr tartibli hosilali tenglamalar uchun ham o‘rinli bo‘ladi, albatta natijalar ozgina o‘zgaradi. Shuning uchun biz Riman-Liuvill ma’nosida kasr hosilali bir jinsli subdiffuziya tenglamasi uchun yuqoridagi natijalarni keltirish bilan kifoyalanamiz.

Quyidagi masalani qaraylik:

$$\begin{cases}\partial_t^\rho u(t)+Au(t)=0, & 0<t\le T;\\ J_t^{1-\rho}u(t)|_{t=\xi}=\alpha\lim\limits_{t\to0}J_t^{1-\rho}u(t)+\phi, & 0<\xi\le T,\end{cases} \tag{27}$$

bu yerda $\phi\in H$ va α ixtiyoriy soni.

Yuqorida aytib o‘tilganidek (oldingi bo‘limga qarang) biz faqat barcha $k\ge1$ lar uchun $E_\rho(-\lambda_k\xi^\rho)\ne\alpha$ bo‘lgan holni ko‘rib chiqamiz.

8-teorema. *Aytaylik,* $\phi\in H$ *va barcha* $k\ge1$ *lar uchun* $E_\rho(-\lambda_k\xi^\rho)\ne\alpha$ *bo‘lsin. U holda* (27) *masalaning yagona yechimi mavjud va bu yechim quyidagi ko‘rinishga ega*

$$u(t)=\sum_{k=1}^{\infty}\frac{\phi_k}{E_\rho(-\lambda_k\xi^\rho)-\alpha}t^{\rho-1}E_{\rho,\rho}(-\lambda_k t^\rho)v_k. \tag{28}$$

Shu bilan birga shunday o‘zgarmas $C_\xi>0$ *son mavjud bo‘lib, quyidagi koersitiv tengsizlik o‘rinli bo‘ladi:*

$$\|\partial_t^\rho u(t)\|_1^2+\|u(t)\|_2^2\le C_\xi t^{-2\rho-2}\|\phi\|^2,\quad 0<t\le T. \tag{29}$$

Birinchi bobning to‘rtinchi paragrafda nolokal-chegaraviy masala o‘rganilgan bo‘lib, Riman-Liuvill ma’nosida kasr hosilali tenglamalar uchun klassik yechimni topish, tenglamaning o‘ng tomoni va nolokal shartdagi φ funksiyani topish masalalari o‘rganilgan.

Aytaylik, Ω yetarli darajada silliq $\partial\Omega$ chegaraga ega bo‘lgan, ixtiyoriy N o‘lchovli soha bo‘lsin. Quyidagi nolokal-chegaraviy masalani qaraymiz:

$$\partial_t^\rho u(x,t) - \Delta u(x,t) = f(x,t), \quad x \in \Omega, \quad 0 < t \le T; \tag{30}$$

$$u(x,t)|_{\partial\Omega} = 0; \tag{31}$$

$$J_t^{1-\rho} u(x,t)|_{t=\xi} = \alpha \lim_{t\to 0} J_t^{1-\rho} u(x,t) + \varphi(x), \quad 0 < \xi \le T, \quad x \in \overline{\Omega}, \tag{32}$$

bu yerda $f(x,t)$, $\varphi(x)$ berilgan funksiyalar, α o‘zgarmas son, ξ – tayinlangan nuqta va Δ – Laplas operatori. (30) – (32) masalaga *to‘g‘ri masala* deb ataladi.

4-ta’rif. *Quyidagi xossalarga ega $u(x,t)$ funksiya*

1. $t^{1-\rho} u(x,t) \in C(\overline{\Omega} \times [0,T])$,

2. $\partial_t^\rho u(x,t), \Delta u(x,t) \in C(\overline{\Omega} \times (0,T])$,

(30) - (32) *to‘g‘ri masalaning* ***klassik yechimi*** *deyiladi* .

Krasnoselskiy va boshqalarning quyidagi lemmasi ushbu paragrafda muhim rol o‘ynaydi.

1-lemma. *Aytaylik, $\tau > \frac{|\alpha|}{m} + \frac{N}{2m}$ bo‘lsin. U holda barcha $|\alpha| \le m$ lar uchun $D^\alpha (A+I)^{-\tau}$ operator $L_2(\Omega)$ ni $C(\overline{\Omega})$ ga uzluksiz (to‘laligicha) akslantiradi va quyidagi baholash o‘rinli bo‘ladi:*

$$\| D^\alpha (A+I)^{-\tau} g \|_{C(\Omega)} \le C \| g \|_{L_2(\Omega)} .$$

To‘g‘ri va teskari masalalarning yechimlari mavjudligini isbotlashda

$$\sum_{k=1}^{\infty} \lambda_k^\tau |h_k|^2, \quad \tau > \frac{N}{2}, \tag{33}$$

ko‘rinishdagi qatorlarning yaqinlashuvchi ekanligini o‘rganish kerak bo‘ladi, bunda h_k lar $h(x)$ funksiyaning Furye koeffitsiyentlari. τ sonining butun qiymatlarida (33) qatorning yaqinlashishi uchun $h(x)$ funksiyaning qaysi $W_2^k(\Omega)$ klassik Sobolev fazolariga tegishligi bo‘lishlik shartlari V.A. Il'inning fundamental ishida ko‘rsatilgan. Ushbu shartlarni keltirish uchun biz $\dot{W}_2^1(\Omega)$ sinfni kiritamiz. Ω sohada uzluksiz differensiallanuvchi va $\partial\Omega$ chegaraning atrofida nolga teng barcha funksiyalar to‘plamini $W_2^1(\Omega)$ ning normasi bo‘yicha yopilmasini belgilaymiz.

Demak, agar $h(x)$ funksiya quyidagi

$$h(x) \in W_2^{[\frac{N}{2}]+1}(\Omega) \ \text{ va } h(x), \Delta h(x), \cdots, \Delta^{[\frac{N}{4}]} h(x) \in \dot{W}_2^1(\Omega), \tag{34}$$

shartlarni qanoatlantirsa, u holda (33) qator yaqinlashuvchi bo‘ladi (agar N juft bo‘lsa, $\tau = \frac{N}{2} + 1$, agar N toq $\tau = \frac{N+1}{2}$ deb olish mumkin).

Xuddi shunday, agar (33) da τ ni $\tau + 2$ ga almashtirsak, yaqinlashish shartlari quyidagi ko‘rinishga ega bo‘ladi:

$$h(x) \in W_2^{[\frac{N}{2}]+3}(\Omega), \quad \text{va} \quad h(x),\ \Delta h(x), \cdots,\ \Delta^{[\frac{N}{4}]+1} h(x) \in \dot{W}_2^1(\Omega). \qquad (35)$$

To‘rtinchi paragrafning asosiy natijasini keltiramiz:

9-teorema. *Aytaylik,* $\varphi(x)$ *va* $t^{1-\rho} f(x,t)$ (*barcha* $t \in [0,T]$ *uchun*) *funksiyalar* (34) *shartlarni qanoatlantirsin. Agar* $\alpha \notin [0,1)$ *yoki* $\alpha \in (0,1)$, *lekin barcha* k *lar uchun* $\lambda_k \neq \lambda_0$ *bo‘lsa, u holda* (30) - (32) *masalaning yagona yechimi mavjud va bu yechim quyidagi*

$$u(x,t) = \sum_{k=1}^{\infty} \left[\frac{\varphi_k - \omega_k(\xi)}{E_\rho(-\lambda_k \xi^\rho) - \alpha} t^{\rho-1} E_{\rho,\rho}(-\lambda_k t^\rho) + \omega_k(t) \right] v_k(x), \qquad (36)$$

ko‘rinishda bo‘ladi, bu yerda $\omega_k(t) = \int_0^t \eta^{\rho-1} E_{\rho,\rho}(-\lambda_k \eta^\rho) f_k(t-\eta) d\eta.$

Agar $\alpha \in (0,1)$ *va* $\lambda_k = \lambda_0, k \in K_0$ *bo‘lsa,* (15) *ortogonallik shartlari bajarilgan deb faraz qilamiz. U holda* (30) – (32) *masala yechimi ixtiyoriy* b_k, $k \in K_0$ *koeffitsiyentlar bilan quyidagi ko‘rinishga ega bo‘ladi:*

$$u(x,t) = \sum_{k \notin K_0} \left[\frac{\varphi_k - \omega_k(\xi)}{E_\rho(-\lambda_k \xi^\rho) - \alpha} t^{\rho-1} E_{\rho,\rho}(-\lambda_k t^\rho) + \omega_k(t) \right] v_k(x) +$$
$$+ \sum_{k \in K_0} b_k t^{\rho-1} E_{\rho,\rho}(-\lambda_k t^\rho) v_k(x) \qquad (37)$$

Ushbu paragrafda biz tenglamaning o‘ng tomonini va chegaraviy funksiyani topish bo‘yicha teskari masalalarni ham o‘rganamiz. Teskari masalani o‘rganish uchun bizga qo‘shimcha shart kerak bo‘ladi. Qo‘shimcha shart sifatida

$$u(x,\theta) = \Psi(x), \quad 0 < \theta \le T, \quad \theta \neq \xi, \quad x \in \overline{\Omega}, \qquad (38)$$

shartni olamiz. Bu yerda manba funksiyasini ifodalovchi $f(x)$ noma’lum funksiya t ga bog‘liq emas va $\Psi(x)$ berilgan funksiya.

10-teorema. *Aytaylik,* $\varphi(x)$, $\Psi(x)$ *funksiyalar* (35) *shartlarni qanoatlantirsin. U holda* (30) - (32), (38) *teskari masalaning* $\{u(x,t), f(x)\}$ *yagona yechimi mavjud va bu yechim quyidagi ko‘rinishga ega bo‘ladi*

$$f(x) =$$
$$= \sum_{k=1}^{\infty} \left[\frac{\alpha - E_\rho(-\lambda_k \xi^\rho)}{\theta^{\rho-1} E_{\rho,\rho}(-\lambda_k \theta^\rho) \xi^\rho E_{\rho,\rho+1}(-\lambda_k \xi^\rho) + \theta^\rho E_{\rho,\rho+1}(-\lambda_k \theta^\rho)[\alpha - E_\rho(-\lambda_k \xi^\rho)]} \Psi_k + \right.$$
$$\left. + \frac{\theta^{\rho-1} E_{\rho,\rho}(-\lambda_k \theta^\rho)}{\theta^{\rho-1} E_{\rho,\rho}(-\lambda_k \theta^\rho) \xi^\rho E_{\rho,\rho+1}(-\lambda_k \xi^\rho) + \theta^\rho E_{\rho,\rho+1}(-\lambda_k \theta^\rho)[\alpha - E_\rho(-\lambda_k \xi^\rho)]} \varphi_k \right] v_k(x), \qquad (39)$$

$$u(x,t)=$$

$$=\sum_{k=1}^{\infty}\left[\frac{E_{\rho,\rho}(-\lambda_k t^{\rho})}{E_{\rho}(-\lambda_k \xi^{\rho})-\alpha}t^{\rho-1}[\varphi_k - f_k\xi^{\rho}E_{\rho,\rho+1}(-\lambda_k\xi^{\rho})]+f_k t^{\rho}E_{\rho,\rho+1}(-\lambda_k t^{\rho})\right]v_k(x). \tag{40}$$

Faraz qilaylik, (30) - (32) masalada nafaqat $u(x,t)$ funksiya, balki nolokal shartdagi $\varphi(x)$ funksiya ham noma'lum bo'lsin. $\{u(x,t),\varphi(x)\}$ funksiyalar juftini topish masalasiga φ funksiyani aniqlashning teskari masalasi deb ataladi. Ushbu teskari masalani yechish uchun qo'shimcha shart sifatida biz yana (38) shartni olamiz.

11-teorema. *Aytaylik,* $t^{1-\rho}f(x,t)$ *ifoda* x *ning funksiyasi sifatida barcha* $t\in[0,T]$ *larda* (34) *shartlarni qanoatlantirsin va* $\Psi(x)$ *funksiya* (38) *shartlarni qanoatlantirsin. U holda* (30) - (32), (38) *teskari masala* $\{u(x,t),\varphi(x)\}$ *yagona yechimga ega va bu yechim quyidagi ko'rinishda bo'ladi*

$$\varphi(x)=\sum_{k=1}^{\infty}\left[\frac{E_{\rho}(-\lambda_k\xi^{\rho})-\alpha}{\theta^{\rho-1}E_{\rho,\rho}(-\lambda_k\theta^{\rho})}[\Psi_k-\omega_k(\theta)]+\omega_k(\xi)\right]v_k(x), \tag{41}$$

$$u(x,t)=\sum_{k=1}^{\infty}\left[\frac{\varphi_k-\omega_k(\xi)}{E_{\rho}(-\lambda_k\xi^{\rho})-\alpha}t^{\rho-1}E_{\rho,\rho}(-\lambda_k t^{\rho})+\omega_k(t)\right]v_k(x), \tag{42}$$

bu yerda $\omega_k(t)=\int_0^t \eta^{\rho-1}E_{\rho,\rho}(-\lambda_k\eta^{\rho})f_k(t-\eta)d\eta.$

Dissertatsiyaning ikkinchi bobi vaqt bo'yicha hosilaning tartibi va tenglamaning o'ng tomonini topish bo'yicha teskari masalalarni yechishga bag'ishlangan.

Hosilaning tartibini aniqlash bo'yicha ko'plab olimlar izlanishlar olib borishgan. Shuni ta'kidlash kerakki, ushbu ishlarning barchasida quyidagi munosabat qo'shimcha shart sifatida qabul qilingan

$$u(x_0,t)=h(t),\ \ 0<t<T,$$

bu yerda $x_0\in\Omega$ kuzatuv nuqtasi. Mutaxassislar, asosan, kasr hosilasining tartibini aniqlashning teskari masalasida yechimning yagonaligini o'rgandilar.

Faqat estoniyalik matematik Jaan Jannoning maqolasi mavjudlik va yagonalik masalasiga bag'ishlangan. J. Janno o'zining maqolasida Kaputo hosilali bir o'zgaruvchili vaqt bo'yicha subdiffuziya tenglamasini ko'rib chiqdi. Muallif qo'shimcha chegaraviy shart sifatida $Bu(\cdot,t)=h(t),\ \ 0<t<T$ ni oldi, bunda B biror operator, u tenglamada hosila tartibini va integral operator yadrosining aniqlash uchun mavjudlik teoremasini isbotlashga muvaffaq bo'ldi.

2019 yil Z. Li va boshqalar tomonidan chop etilgan maqolada "Ochiq muammolar" bo'limida to'g'ri ta'kidlangan edi: "Kasr hosilaning tartiblarini tiklashning teskari muammolari bo'yicha tadqiqotlar qoniqarli emas, chunki barcha maqolalar ... muammoni $t\in(0,\infty)$ da o'rganishgan. Teskari masalani kuzatish ma‘lumotlari sifatida belgilangan tayin vaqtda yechimning qiymati bo‘yicha o‘rganish qiziqarli bo‘lar edi”.

Ushbu bobda yuqorida ta’kidlangan muammo qisman hal qilingan bo‘lib, biz kasr tartibli xususiy hosilali differensial tenglamalarda tenglamaning tartibini aniqlash bo‘yicha teskari masalaning yechimi mavjud va yagonaligini isbotlaymiz. Buning uchun qo‘shimcha shart sifatida “Ochiq muammolar” bo‘limida ta’kidlanganidek, kuzatuv nuqtasi sifatida yechimning tayin vaqtdagi qiymatini olamiz. Buning uchun biz t_0 tayinlangan vaqtda berilgan quyidagi qo‘shimcha shartdan foydalanamiz:

$$U(\rho;t_0) \equiv \int_{\mathrm{T}^N} u(x,t_0)dx = d_0, \quad t_0 \geq T_0, \tag{43}$$

bu yerda T_0 o‘zgarmas son.

Ikkinchi bobning birinchi paragrafida N - o‘lchovli torda o‘zgarmas koeffitsiyentli ixtiyoriy elliptik differensial operatorli, bir jinsli bo‘lmagan subdiffuziya tenglamasida vaqt bo‘yicha kasr tartibli hosilaning tartibini aniqlash uchun teskari masala ko‘rib chiqilgan. Klassik Furye usulidan foydalanib, tayinlangan vaqtdagi yechimning qiymati haqidagi ma‘lumotlarga tayangan holda kasr hosilaning tartibini qayta tiklash masalasi o‘rganilgan. Shu bilan birga ixtiyoriy N o‘lchovli sohaga va o‘zgaruvchan koeffitsiyentli elliptik operatorlar holi uchun umumlashtirish masalasi ham qarab chiqilgan.

Faraz qilaylik, $A(D) = \sum_{|\alpha|=m} a_\alpha D^\alpha$ o‘zgarmas koeffitsiyentli, bir jinsli, simmetrik elliptik differensial operator bo‘lsin, ya’ni barcha $\xi \neq 0$ lar uchun $A(\xi) > 0$ bo‘lsin, bu yerda $\alpha = (\alpha_1, \alpha_2, ..., \alpha_N)$ - multi-indeks va $D = (D_1, D_2, ..., D_N)$, $D_j = \frac{\partial}{\partial x_j}$. Biz $C^m(\mathrm{T}^N)$ orqali, har bir x_j o‘zgaruvchi bo‘yicha 2π davrli $C^m(\mathrm{R}^N)$ dan olingan $v(x)$ funksiyalar sinfini belgilaymiz. Aytaylik, A operator $C^m(\mathrm{T}^N)$ da aniqlangan $Av(x) = A(D)v(x)$ ko‘rinishdagi standart operator bo‘lsin. Faraz qilaylik, $\rho \in (0,1)$ bo‘lsin.

Quyidagi boshlang‘ich-chegaraviy masalani qaraylik

$$\partial_t^\rho u(x,t) + Au(x,t) = f(x,t), \quad x \in \mathrm{T}^N, \quad 0 < t \leq T, \tag{44}$$

$$\lim_{t\to 0} J_t^{1-\rho} u(x,t) = \varphi(x), \quad x \in \mathrm{T}^N. \tag{45}$$

Quyidagi

$$f_0 = const, \quad \varphi_0^2 + f_0^2 \neq 0, \tag{46}$$

shartlar bajarilganda kasr tartibli hosilaning tartibini topish uchun klassik Furye usuliga asoslangan usuldan foydalanamiz. Agar (46) shartlar bajarilmasa bu usul murakkab bo‘lib qoladi. (43) dagi T_0 parametrni quyidagicha aniqlaymiz

$$T_0=\begin{cases} 2, & \varphi_0 f_0 \ge 0,\\ \max\left\{5, 4\dfrac{|\varphi_0|}{|f_0|}\right\}, & \varphi_0 f_0 < 0.\end{cases}$$

Endi ushbu paragrafning asosiy natijasini keltiramiz.

12-teorema. *Aytaylik* (46) *shartlar bajarilsin,* $\tau > \dfrac{N}{2m}$ *va*

(*a*) $\varphi \in D(\hat{A}^\tau)$;

(*b*) $f(x,t) \in D(\hat{A}^\tau)$ *barcha* $t \in [0,T]$ *da;*

(*c*) $F(t) = \hat{A}^\tau f(x,t)$ *barcha* $t \in [0,T]$ *lar uchun* $L_2(\mathrm{T}^N)$ *fazoning normasida uzluksiz bo'lsin.*

U holda (43) – (45) *teskari masala,* $\{u(x,t), \rho\}$ *yagona yechimga ega bo'ladi, faqat va faqat quyidagi qo'sh tengsizlik o'rinli bo'lsa*

$$\min\{f_0, \varphi_0 + t_0 f_0\} < d_0 < \max\{f_0, \varphi_0 + t_0 f_0\}.$$

2-lemma. *Aytaylik,* (46) *shartlar bajarilsin va* $t_0 \ge T_0$ *bo'lsin. U holda* $U(\rho; t_0)$ *funksiya,* $\rho \in (0,1)$ *funksiyasi sifatida qat'iy monoton va*

$$\lim_{\rho\to 0} U(\rho; t_0) = f_0, \quad U(1; t_0) = \varphi_0 + t_0 f_0 \tag{47}$$

tengliklar bajariladi.

13-teorema. *Aytaylik,* $\tau > \dfrac{N}{2m}$ *va* 12-*teoremadagi* (*a*) – (*c*) *shartlar bajarilgan bo'lsin. U holda* (43) - (45) *boshlang'ich-chegaraviy masalaning yagona yechimi mavjud va u quyidagi ko'rinishga ega*

$$u(x,t) = \sum_{n\in \mathrm{Z}^N} [\varphi_n t^{\rho-1} E_{\rho,\rho}(-A(n)t^\rho) + \int_0^t f_n(t-\xi)\xi^{\rho-1} E_{\rho,\rho}(-A(n)\xi^\rho)d\xi] e^{inx}, \tag{48}$$

har bir $t \in (0,T]$ *da* $x \in \mathrm{T}^N$ *bo'yicha absolyut va tekis yaqinlashadi.*

Ikkinchi bobning ikkinchi paragrafida biz vaqt bo'yicha kasr tartibli to'lqin tenglamalarida kasr tartibli hosilaning tartibini aniqlash masalasi bilan tanishamiz.

Bu paragrafda Gerasimov-Kaputo ma'nosida kasr hosilasining tartibini aniqlash bo'yicha teskari masalasi o'rganilgan. O'rganilayotgan masalada tenglamaning elliptik qismi musbat, o'z-o'ziga qo'shma, chegaralanmagan A diskret spektrga ega bo'lgan ixtiyoriy operatoridan iborat. Klassik Furye usulidan foydalanib, tayinlangan vaqtda yechimning biror xos funksiyaga proyeksiyasining qiymati hosilaning tartibini aniqlab beradi. A operatorga misol sifatida jumladan, kasr tartibli differensial tenglamalarning chiziqli sistemasi, Shturm-Liuvill kasr operatorlari va boshqalar ko'rib chiqiladi.

Aytaylik, $\beta \in (1,2)$ berilgan son bo'lsin. $C((a,b); H)$ orqali $t \in (a,b)$ larda H dagi qiymati uzluksiz $u(t)$ funksiyalar to'plamini belgilaymiz.

Quyidagi Koshi masalasini qaraylik:

$$D_t^\beta u(t) + Au(t) = f, \quad 0 < t \le T, \tag{49}$$

$$u(0)=\varphi, \quad u'(0)=\psi, \tag{50}$$

bu yerda f, φ va ψ vektorlar H da berilgan.

5-ta'rif. *Quyidagi xossalarga ega*

1. $u(t),\ u'(t)\in C([0,T];H)$,

2. $D_t^{\beta}u(t),\ Au(t)\in C((0,T];H)$

va (49), (50) *shartlatni qanoatlantiruvchi* $u(t)$ *funksiya* (49), (50) *Koshi masalasining yechimi deb ataladi.*

Ikkinchi paragrafning asosiy natijasi quyidagi teoremalardan iborat.

14-teorema. *Aytaylik,* $\psi, f\in H$ *va* $\varphi\in D(A^{\frac{1}{\beta}})$ *bo'lsin. U holda* (49), (50) *Koshi masalasi yagona yechimga ega va u quyidagi ko'rinishda bo'ladi:*

$$u(t)=\sum_{k=1}^{\infty}[\varphi_k E_{\beta,1}(-\lambda_k t^{\beta})+\psi_k t E_{\beta,2}(-\lambda_k t^{\beta})+f_k t^{\beta} E_{\beta,\beta+1}(-\lambda_k t^{\beta})]v_k, \tag{51}$$

bu yerda qator barcha $t\ge 0$ *larda* H *da yaqinlashuvchidir*.

Endi teskari masalani keltiramiz. Faraz qilaylik, $\beta\in(1,2)$ noma'lum parametr bo'lsin. Ushbu paragrafning asosiy maqsadi ushbu parametrni aniqlashning teskari masalasini o'rganishdir. Tabiiyki, β parametri aniqlash uchun qo'shimcha shart kerak bo'ladi. Biz qo'shimcha shart sifatida quyidagi shartni olamiz:

$$U(\beta;t_0)\equiv(u(t_0),v_{k_0})=d_0, \quad t_0\ge T_0, \tag{52}$$

bu yerda d_0 berilgan son, T_0 quyida aniqlangan musbat son va $k_0\ge 1$ ixtiyoriy butun son bo'lib, shundayki $f_{k_0}^2+\varphi_{k_0}^2+\psi_{k_0}^2\ne 0$ (aniqki, bunday son mavjud, chunki f, φ va ψ bir vaqtda nolga teng emas), v_{k_0} xos funksiya.

(49) - (50) masalaning (52) qo'shimcha shartni qanoatlantiruvchi yechimini topishga *teskari masala* deb ataladi.

6-ta'rif. *Agar* $u(t)$ *funksiya* (49) - (50) *masalaning* (52) *shartni qanoatlantiradigan yechimi bo'lsa,* $\{u(x,t),\beta\}$, $\beta\in[\beta_1,\beta_2]$ *juftligi teskari masala yechimi deb ataladi.*

3-lemma. *Aytaylik,*

$$\psi, f\in H;\ \ \varphi\in D(A^{\frac{1}{\beta_1}}), \tag{53}$$

$$\|\psi\|+\|\varphi-A^{-1}f\|\ne 0 \tag{54}$$

bo'lsin. U holda shunday $T_0=T_0(k_0)$ *son mavjudki, barcha* $t_0\ge T_0$ *lar uchun* $U(\beta;t_0)$ *funksiya* $[\beta_1,\beta_2]$ *oraliqda qat'iy monoton bo'ladi.*

Bu lemma, shubhasiz, ushbu maqolaning quyidagi asosiy natijasini nazarda tutadi.

15-teorema. *Aytaylik,* (53) *va* (54) *shartlar bajarilgan bo'lsin. U holda barcha* $t_0\ge T_0$ *lar uchun teskari masalaning yechimi mavjud va yagonadir.*

Eslatma. *Agar teoremaning* (54) *sharti bajarilmasa, ya'ni* $\psi=0$ *va* $\varphi=A^{-1}f$ *bo'lsa, u holda barcha* k *lar uchun* $\psi_k=0$ *va* $\varphi_k=f_k/\lambda_k$ *bo'ladi. Bu holda Koshi masalasining yagona yechimi quyidagi shaklga ega*

$$u(t)=\sum_{k=1}^{\infty}\frac{f_k}{\lambda_k}v_k,$$

ya'ni ifoda β *ga bog'liq emas. Shuning uchun,* $U(\beta;t_0)$ *funksiya ham* β *ga bog'liq emas. Shunday qilib, bu holda, o'rganilayotgan teskari masala ma'noga ega emas.*

Ikkinchi bobning uchinchi paragrafida, kasr tartibli hosilasining tartibini va tenglamaning o'ng tomoni bir vaqtning o'zida aniqlash bo'yicha teskari masalani o'rganamiz.

Aytaylik, Ω yetarlicha silliq $\partial\Omega$ chegaraga ega bo'lgan ixtiyoriy N-o'lchovli soha bo'lsin. Faraz qilaylik, $\partial\Omega$ chegaraning berilgan koordinatalar sistemasidagi tenglamasini aniqlaydigan funksiya ikki marta uzluksiz differensiallanuvchi funksiyadan iborat.

Quyidagi boshlang'ich-chegaraviy masalani qaraylik:

$$\begin{cases}\partial_t^{\rho}u(x,t)-\Delta u(x,t)=f(x),\, x\in\Omega, \quad 0<t<T;\\ Bu(x,t)=\dfrac{\partial u(x,t)}{\partial n}=0, \quad x\in\partial\Omega, \quad 0<t<T;\\ \lim\limits_{t\to 0}J_t^{1-\rho}u(x,t)=\varphi(x), \quad x\in\overline{\Omega},\end{cases} \tag{55}$$

bu yerda Δ – Laplas operatori, $\varphi(x)$ oldindan berilgan funksiya va n vektor $\partial\Omega$ ning tashqi normali.

Ushbu ishda, shuningdek, kasr tartibli hosilaning tartibini aniqlash uchun qo'shimcha shart sifatida quyidagi

$$U(\rho,t_0)\equiv\frac{1}{|\Omega|^{1/2}}\int_{\Omega}u(x,t_0)dx=d_0 \tag{56}$$

tenglikdan foydalanamiz, bu yerda t_0 tayinlangan vaqt.

Manba funksiyasini aniqlash uchun, yuqorida sanab o'tilgan ishlarda bo'lgani kabi, biz

$$u(x,T)=\psi(x), \quad x\in\overline{\Omega}, \tag{57}$$

qo'shimcha ma'lumotdan foydalanamiz. Shubhasiz, agar (56) shartda $t_0=T$ qeb olsak, u hech qanday yangi ma'lumot bermaydi va shuning uchun biz $t_0<T$ deb olamiz. Ushbu ishning asosiy natijasi shuni ko'rsatadiki, (56) va (57) qo'shimcha shartlar (55) masala uchun bir vaqtning o'zida kasr hosilaning tartibi va manba funksiyasining mavjudli va yagonaligi kafolatlaydi.

7 - ta'rif . *Agar* $u(x,t)$, $f(x)$ *funksiyalar va* ρ *parametr quyidagi*

1. $\rho\in(0,1)$,

2. $f(x)\in C(\overline{\Omega})$,

3. $\partial_t^{\rho} u(x,t), \quad \Delta u(x,t) \in C(\overline{\Omega} \times (0,T))$,

4. $J_t^{1-\rho} u(x,t) \in C(\overline{\Omega} \times [0,T])$,

shartlarni va (55), (56) *va* (57) *masalaning barcha shartlarini qanoatlantirsa, u holda bu* $\{u(x,t), f(x), \rho\}$ *uchlikka* (55) - (57) *teskari masalaning klassik ma'nodagi yechimi deyiladi. Yuqoridagi xossalarga ega bo'lgan* $u(x,t)$ *funksiya to'g'ri masalaning yechimi deb ataladi.*

Agar biz Furye usulidan foydalansak, quyidagi spektral masalaga kelamiz:

$$-\Delta v(x) = \lambda v(x), \quad x \in \Omega;$$

$$Bu(x,t) \equiv \frac{\partial u(x,t)}{\partial n} = 0, \quad x \in \partial\Omega.$$

Chegara $\partial\Omega$ ikki marta differensiallanadigan bo'lganligi sababli, bu masala $L_2(\Omega)$ da to'la ortonormal $\{v_k(x)\}$, $k \geq 1$ xos funksiyalar sistemasiga va manfiy bo'lmagan $\{\lambda_k\}$ xos qiymatlar to'plamiga ega bo'ladi. Shu narsani ta'kidlab o'tamizki, bu yerda $\lambda_1 = 0$ va $v_1(x) = |\Omega|^{-1/2}$ bo'ladi.

Faraz qilaylik, berilgan $\varphi(x)$ va $\psi(x)$ funksiyalar quyidagi shartlarni qanoatlantirsin:

a) $\varphi(x) \in C^{[\frac{N}{2}]}(\Omega)$, $D^{\alpha}\varphi(x) \in L_2(\Omega), \quad |\alpha| = [\frac{N}{2}] + 1$;

b) $B\varphi(x) = B(\Delta\varphi(x)) = ... = B(\Delta^{[\frac{N}{2}]}\varphi(x)) = 0, \quad x \in \partial\Omega$;

c) $\psi(x) \in C^{[\frac{N}{2}]+1}(\Omega)$, $D^{\alpha}\psi(x) \in L_2(\Omega), \quad |\alpha| = [\frac{N}{2}] + 2$;

d) $B\psi(x) = B(\Delta\psi(x)) = ... = B(\Delta^{[\frac{N}{2}]+1}\psi(x)) = 0, \quad x \in \partial\Omega$.

Shuni ta'kidlash kerakki, (a) va (b) shartlarning bajarilishi to'g'ri masalaning yechimi mavjud va yagonaligini ta'minlaydi, (57) qo'shimcha shartdagi $\psi(x)$ funksiya (c) va (d) shartlarni qanoatlantirsa, u holda, manba funksiya $f(x)$ mavjud va yagonadir.

Aytaylik, quyidagi shart bajarilgan bo'lsin

$$\varphi_1^2 + \psi_1^2 \neq 0. \tag{58}$$

(56) qo'shimcha shartdagi t_0 parametrni quyidagi tarzda tanlaymiz. Agar $\varphi_1 \cdot \psi_1 \leq 0$ bo'lsa, u holda $t_0 \in (1,T)$ va boshqa hollarda

$$t_0 \in (1,T) \cap \begin{cases} \left(1, \dfrac{\varphi_1}{\psi_1} \cdot T\right) & agar \quad \dfrac{\varphi_1}{\psi_1} \cdot T > 1; \\ (2(\ln T + 1)\dfrac{\varphi_1}{\psi_1} \cdot T, T) & agar \quad \dfrac{\varphi_1}{\psi_1} \cdot T \leq 1. \end{cases} \tag{59}$$

Bu paragrafning asosiy natijasi.

16-teorema. *Aytaylik,* (a) - (d) *va* (58) - (59) *shartlar bajarilsin. Agar*

$$\min\left\{\psi_1,\varphi_1\left[1-\frac{t_0}{T}\right]+\frac{t_0\psi_1}{T}\right\}<d_0<\max\left\{\psi_1,\varphi_1\left[1-\frac{t_0}{T}\right]+\frac{t_0\psi_1}{T}\right\}$$

shart bajarilsa, u holda (55) - (57) *teskari masala* $\{u(x,t), f(x), \rho\}$ *yagona yechimga ega bo'ladi va bu yechimlar quyidagi ko'rinishga ega*

$$u(x,t)=\sum_{k=1}^{\infty}[\varphi_k t^{\rho-1}E_{\rho,\rho}(-\lambda_k t^{\rho})+f_k t^{\rho}E_{\rho,\rho+1}(-\lambda_k t^{\rho})]v_k(x), \tag{60}$$

$$f(x)=\sum_{k=1}^{\infty}\frac{\psi_k}{T^{\rho}E_{\rho,\rho+1}(-\lambda_k T^{\rho})}v_k(x)-\sum_{k=1}^{\infty}\frac{\varphi_k E_{\rho,\rho}(-\lambda_k T^{\rho})}{TE_{\rho,\rho+1}(-\lambda_k T^{\rho})}v_k(x), \tag{61}$$

bu yerda qatorlar tekis va absolyut yaqinlashadi.

Uchinchi bobda biz $\frac{\partial^k u}{\partial y^k}=(-1)^k A(x,D)u$, ($k=1,2$) tenglamalar uchun chegaraviy shart kasr tartibli hosila bilan berilgan chegaraviy masalalar bilan shug'ullanamiz, bu yerda $A(x,D)$ manfiy bo'lmagan elliptik differensial operator va chegaraviy shart musbat haqiqiy ρ kasr tartibli hosila bilan aniqlangan B_y^{ρ} operator yordamida berilgan. Xususan, B_y^{ρ} chegaraviy operator Marchaud, Grunwald-Letnikov yoki Liuvill-Weyl kasr tartibli hosilalar orqali berilishi mumkin.

Quyidagi chegaraviy masalani qaraylik:

$$u_{yy}(x,y)=A(x,D)u(x,y),\quad x\in\Omega,\quad y>0, \tag{62}$$

$$B_j u(x,y)=\sum_{|\alpha|\le m_j}b_{\alpha,j}(x)D^{\alpha}u(x,y)=0,\quad 0\le m_j\le m-1,\ j=1,2,...,l;\ x\in\partial\Omega, \tag{63}$$

$$B_y^{\rho}u(x,+0)=\varphi(x),\quad \rho>0, x\in\Omega, \tag{64}$$

$$|u(x,y)|\to 0,\quad y\to\infty,\quad x\in\Omega, \tag{65}$$

bu yerda $\varphi(x)$ va $b_{\alpha,j}(x)$ koeffitsiyentlar berilgan funksiyalar.

7-ta'rif. *Agar* $u(x,y)$ *funksiya quyidagi* $u_{yy}(x,y)$, $A(x,D)u(x,y)\in C(\overline{\Omega}\times(0,\infty))$, $u(x,y)$, $B_y^{\rho}u(x,y)\in C(\overline{\Omega}\times[0,\infty))$ *xossalariga ega bo'lib,* (62) - (65) *masalaning barcha shartlarini qanoatlantirsa bu funksiya* (62) - (65) *chegaraviy masalaning* ***(klassik) yechimi*** *deyiladi.*

Agar (62) - (65) masalani yechish uchun Furye usulini qo'llasak, biz quyidagi spektral masalaga kalamiz:

$$A(x,D)v(x)=\lambda v(x)\quad x\in\Omega; \tag{66}$$

$$B_j v(x)=0,\quad j=1,2,...,l;\quad x\in\partial\Omega.. \tag{67}$$

S. Agmon ishida teskari operatorning kompaktligini, ya'ni Ω sohaning $\partial\Omega$ chegarasi hamda A va B_j operatorlarning koeffitsiyentlari uchun (66) - (67) spektral masala $L_2(\Omega)$ da $\{v_k(x)\}$, $k\ge 1$ xos funksiyalar ortonormal to'la sistemasi va manfiy bo'lmagan λ_k xos qiymatlar to'plamiga ega bo'lishini kafolatlovchi zaruriy shartlarni topdi. Bundan keyin biz ushbu shartlar bajarilgan deb faraz qilamiz. Aytaylik, $\lambda = 0$ xos qiymat k_0 karrali ildiz bo‘lsin, ya’ni $\lambda_k = 0, \quad k = 1,2,...,k_0$. Faraz qilaylik, $\varphi(x)$ chegaraviy funksiya quyidagi ortogonallik shartlarini qanoatlantirsin

$$\varphi_k = \int_\Omega \varphi(x) v_k(x) dx = 0, k = 1,2,...,k_0. \qquad (68)$$

17-teorema. *Ayatylik ,* $\varphi(x) \in D(\hat{A}^\tau)$ *,* $\tau > \frac{N}{2m}$ *va* (68) *shartlar bajarilsin . U holda* (62) - (65) *to‘g‘ri masala quyidagi ko‘rinishda yagona yechimga ega*

$$u(x,y) = \sum_{k=k_0+1}^{\infty} \frac{1}{(\sqrt{\lambda_k})^\rho} \varphi_k v_k(x) e^{-\sqrt{\lambda_k} y}, \qquad (69)$$

bu yerda qator barcha $y \in [0,\infty)$ *lar uchun* $x \in \overline{\Omega}$ *da absolyut va tekis yaqinlashadi.*

Uchinchi bobning ikkinchi paragrafida quyidagicha masala qaralgan.

$$u_t(x,t) + A(x,D)u(x,t) = 0, \quad x \in \Omega, \quad t > 0, \qquad (70)$$

$$B_j u(x,t) = \sum_{|\alpha| \le m_j} b_{\alpha,j}(x) D^\alpha u(x,t) = 0, \quad 0 \le m_j \le m-1, \ j = 1,2,...,l; \ x \in \partial\Omega, \qquad (71)$$

$$B_t^\rho u(x,+0) = \varphi(x), \quad \rho > 0, \ x \in \overline{\Omega}, \qquad (72)$$

bu yerda $\varphi(x)$ va koeffitsiyentlar $b_{\alpha,j}(x)$ funksiyalar berilgan.

18-teorema. *Aytaylik,* (68) *va* $\varphi(x) \in D(\hat{A}^\tau), \tau > \frac{N}{2m}$ *shartlar bajarilsin. U holda* (70) - (72) *to‘g‘ri masala quyidagi ko‘rinishda yagona yechimga ega*

$$u(x,t) = \sum_{k=k_0+1}^{\infty} \frac{1}{\lambda_k^\rho} \varphi_k v_k(x) e^{-\lambda_k t}, \qquad (73)$$

bu yerda qator barcha $t \in [0,\infty)$ *lar uchun* $x \in \overline{\Omega}$ *da absolyut va tekis yaqinlashadi.*

To’rtinchi bob Barenblatt-Jeltov-Kochina tipidagi butun va kasr tartibli tenglamalar uchun to‘g‘ri va teskari masalalarni o‘rganishga bag‘ishlangan. To‘rtinchi bobning birinchi paragrafida Barenblatt-Jeltov-Kochina tipidagi kasr tartibli tenglamalar uchun Koshi masalasining yechimi mavjud va yagonaligi ko‘rsatilgan. Shu bilan birga tenglamaning o‘ng tomonini topish bo‘yicha teskari masalaning yechimi mavjud va yagonaligi ham isbotlangan.

To‘rtinchi bobning ikkinchi paragrafida esa, chegaraviy shart kasr tartibli hosila bilan berilgan Barenblatt-Jeltov-Kochina tenglamasi uchun to‘g‘ri va teskari masalalarning yechimi mavjud va yagonaligi isbotlangan.

Quyidagi Koshi masalasini qaraymiz:

$$\begin{cases} D_t^\rho u(t) + A(D_t^\rho u(t)) + Au(t) = f, & 0 < t \le T; \\ u(+0) = \varphi, \end{cases} \qquad (74)$$

bu yerda $\varphi, f \in H$.

8-ta'rif. *Agar* $u(t) \in C((0,T];H)$ *funksiya* $D_t^\rho u(t), A(D_t^\rho u(t)),$ $Au(t) \in C((0,T);H)$ *xossalarga ega bo'lib,* (74) *masalaning barcha shartlarini qanoatlantirsa, u holda* $u(t)$ *funksiyaga* (74) Koshi masalasining ***yechimi*** deyiladi.

19-teorema. *Aytaylik,* $\varphi \in D(A)$, $f \in H$ *bo'lsin. U holda* (74) *Koshi masalasining yagona yechimi mavjud va u quyidagicha ko'rinishga ega*

$$u(t) = \sum_{k=1}^{\infty} \left[\varphi_k E_{\rho,1}(-\mu_k t^\rho) + \frac{f_k}{1+\lambda_k} t^\rho E_{\rho,\rho+1}(-\mu_k t^\rho) \right] v_k \ , \tag{75}$$

bu yerda f_k, φ_k *lar Furye koeffitsiyentlari,* $\mu_k = \dfrac{\lambda_k}{1+\lambda_k}$.

Teskari masalani o'rganish uchun bizga qo'shimcha shart kerak bo'ladi. Biz qo'shimcha shart sifatida quyidagi shartdan foydalanamiz:

$$u(\tau) = \Psi, \quad 0 < \tau \le T \,, \tag{76}$$

bu yerda τ fiksirlangan nuqta.

(74), (76) masalada $u(t)$ va f funksiyalarning $\{u(t), f\}$ juftligini topish masalasiga tenglamaning o'ng tomonini topish bo'yicha ***teskari masala*** deyiladi.

Endi ushbu punktning asosiy natijasini keltiramiz.

20-teorema. *Aytaylik,* φ, $\Psi \in D(A)$ *bo'lsin. U holda* (74), (76) *teskari masalaning* $\{u(t), f\}$ *yagona yechimi mavjud va u quyidagi ko'rinishga ega*

$$u(t) = \sum_{k=1}^{\infty} \left[\varphi_k E_{\rho,1}(-\mu_k t^\rho) + \frac{f_k}{1+\lambda_k} t^\rho E_{\rho,\rho+1}(-\mu_k t^\rho) \right] v_k \,, \tag{77}$$

va $f = \sum\limits_{k=1}^{\infty} f_k v_k$, bu yerda $f_k = \left(\dfrac{\psi_k}{\tau^\rho E_{\rho,\rho+1}(-\mu_k \tau^\rho)} - \dfrac{\varphi_k E_{\rho,1}(-\mu_k \tau^\rho)}{\tau^\rho E_{\rho,\rho+1}(-\mu_k \tau^\rho)} \right)(1+\lambda_k)$.

To'rtinchi bobning ikkinchi paragrafida chegaraviy shart kasr tartibli hosila bilan berilgan Barenblatt-Jeltov-Kochina tipidagi tenglamalar uchun to'g'ri va teskari masalalar o'rganiladi.

Quyidagi masalani qaraylik:

$$\begin{cases} u_t(t) + A(u_t(t)) + Au(t) = 0, & 0 < t \le T; \\ B_t^\rho u(+0) = \varphi, \end{cases} \tag{79}$$

bu yerda chegaraviy shart musbat haqiqiy ρ kasr tartibli hosila bilan aniqlangan B_t^ρ operator yordamida berilgan. Xususan, B_t^ρ chegaraviy operator Marchaud, Grunwald-Letnikov, Liuvill-Weyl kasr tartibli hosilalar orqali berilishi mumkin.

21-teorema. *Aytaylik,* $\varphi \in D(A)$ *bo'lsin. U holda* (79) *masala yagona yechimga ega va u quyidagicha ko'rinishda bo'ladi*

$$u(t) = \sum_{k=1}^{\infty} \frac{\varphi_k}{\mu_k^\rho} e^{-\mu_k t} v_k \,, \tag{80}$$

bu yerda $\varphi_k = (\varphi, v_k)$ *lar* φ *funksiyaning Furye koeffitsiyentlari.*

Bu paragrafda bundan tashqari chegaraviy funksiyani topish bo'yicha teskari masala ham o'rganilgan. Buning uchun qo'shimcha shart sifatida quyidagi shartdan foydalanamiz:

$$u(\tau)=\Psi, \quad 0<\tau\le T, \tag{81}$$

bu yerda τ fiksirlangan nuqta.

22-teorema. *Aytaylik,* $\Psi\in D(A)$ *bo'lsin. U holda* (79), (81) *teskari masalaning* $\{u(t),\varphi\}$ *yagona yechimi mavjud va u quyidagi ko'rinishda ega*

$$u(t)=\sum_{k=1}^{\infty}\frac{\varphi_k}{\mu_k^{\rho}}e^{-\mu_k t}v_k, \tag{82}$$

va $\varphi=\sum_{k=1}^{\infty}\varphi_k v_k$, bu yerda $\varphi_k=\Psi_k\mu_k^{\rho}e^{\mu_k\tau}$.

Be bobda to'g'ri to'rtburchakli sohada kasr tartibli tenglamalar bilan berilgan jarayonlarni manba funksiyasi yordamida boshqarish masalalari o'rganilgan.

Quyidagi Kaputo ma'nosida kasr tartibli

$$D_t^{\alpha}u=u_{xx}+u_{yy}+f(x,y,t), \quad 0<x<l_1, \quad 0<y<l_2, \ 0<t<T \tag{83}$$

tenglamaning

$$u(0,y,t)=0, u(l_1,y,t)=0, u(x,0,t)=\mu(t), u(x,l_2,t)=0, \mu(0)=0, \ 0\le t\le T \tag{84}$$

chegaraviy va

$$u(x,y,0)=\varphi(x,y), \qquad 0\le x\le l_1, 0\le y\le l_2, \tag{85}$$

boshlang'ich shartni qanoatlantiruvchi yechimini topish masalasini qaraylik, bu yerda $\varphi(x,y)$ oldindan berilgan funksiya, $\mu(t)$ chegaralangan, monoton funksiya, $\mu'(t)\le M_1$, $\mu(0)=0$ va $T>0$ – fiksirlangan son. $\Omega=(0,l_1)\times(0,l_2)$ deb belgilash kiritaylik.

23-teorema. *Aytaylik,* $f(x,y,t)\in C_{x,y,t}^{2,2,1}(\bar{\Omega}\times[0,T])$, $\varphi(x,y)\in C_{x,y}^{2,2}(\bar{\Omega})$ *bo'lsin. Agar* $f(x,y,t)$ *va* $\varphi(x,y)$ *funksiyalar ularning* x,y *bo'yicha ikkinchi tartibli hosilalari va* t *ga nisbatan birinchi tartibli hosilasi* Ω *sohaning chegarasida nolga teng va* f_{xxy}''', f_{xyy}''', φ_{xxy}''', φ_{xyy}''' *lar chegaralangan funksiyalar bo'lib,* $f_{xxyy}^{(4)}$, $\varphi_{xxyy}^{(4)}$, f_{txy}''' *funksiyalar* $\bar{\Omega}$ *da integrallanuvchi bo'lsa, u holda* (83) - (85) *masalaning* Ω *sohada yechimi mavjud va u quyidagi ko'rinishga ega*

$$u(x,y,t)=U(x,y,t)+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\bar{f}_{nm}(t)+\varphi_{nm}E_{\alpha,1}(-\lambda_{nm}^2t^{\alpha})\right]sin\alpha_n x\cdot sin\beta_m y, \tag{86}$$

bu yerda

$$U(x,y,t)=\begin{cases}\dfrac{l_2-y}{l_2}\mu(t), & agar \quad 0<x<l_1;\\ 0, & agar \quad x=0 \ va \ x=l_1.\end{cases}, \quad \lambda_{nm}=\sqrt{\left(\frac{\pi n}{l_1}\right)^2+\left(\frac{\pi m}{l_2}\right)^2},$$

$$\bar{f}_{nm}(t)=\frac{4}{l_1l_2}\int_0^t\int_0^{l_1}\int_0^{l_2}\left[f(\xi,\eta,\tau)+\frac{\eta-l_2}{l_2}D^{\alpha}\mu(t)\right]\sin\alpha_n\xi\sin\beta_m\eta\times$$

$$\times(t-\tau)^{\alpha-1}E_{\alpha,\alpha}(-\lambda_{nm}^2(t-\tau)^{\alpha})d\eta d\xi d\tau,$$

$$\varphi_{nm}=\frac{4}{l_1 l_2}\int_0^{l_1}\int_0^{l_2}\varphi(\xi,\eta)\sin\frac{\pi n\xi}{l_1}\sin\frac{\pi m\eta}{l_2}d\eta d\xi,\quad \alpha_n=\frac{\pi n}{l_1},\ \beta_m=\frac{\pi m}{l_2}.$$

Boshqaruv masalalarini yechishda f funksiya t ga bog‘liq emas va $\mu(t)=0$ deb faraz qilamiz. Aytaylik, $\Psi(x,y)$ funksiya berilgan bo‘lsin. Shunday $f(x,y)$ manba funksiyasini topish kerakki, natijada (83) - (85) masalaning yechimi quyidagi shartni qanoatlantirsin:

1-masala. $t=\theta$ daqiqada quyidagi tenglikni qanoatlantirsin

$$u(x,y,\theta)=\Psi(x,y). \tag{87}$$

Boshqacha qilib aytganda, biz manba funksiyasi $f(x,y)$ ni shunday tanlashimiz kerakki, natijada $t=\theta$ vaqtda Ω muhitdagi harorat $\Psi(x,y)$ kabi taqsimlangan bo‘lishi kerak.

2-masala. $[0,T]$ vaqt oralig‘idagi quyidagi tenglikni qanoatlantirsin

$$\int_0^T u(x,y,t)dt=\Psi(x,y). \tag{88}$$

3-masala. t_1 va t_2 daqiqadagi qiymatlari yig‘indisi quyidagi tenglikni qanoatlantirsin

$$u(x,y,t_1)+u(x,y,t_2)=\Psi(x,y). \tag{89}$$

Aytaylik, $\varphi(x,y)$ va $\Psi(x,y)$ funksiyalar quyidagi shartlarni qanoatlantirsin.

a) $\varphi(x,y)\in C_{x,y}^{2,2}(\bar{\Omega})$ funksiya va uning ikkinchi tartibli hosilalari Ω sohaning chegarasida nolga teng va φ_{xxy}''', φ_{xyy}''' funksiyalar chegaralangan bo‘lib, $\varphi_{xxyy}^{(4)}$ funksiya Ω sohada integrallanuvchi bo‘lsin.

b) $\Psi(x,y)\in C_{x,y}^{4,4}(\bar{\Omega})$ funksiya va $0\le i+j\le 7$ bo‘lib, agar $i+j$ juft bo‘lsa, $\dfrac{\partial^{i+j}}{\partial x^i\partial y^j}\Psi(x,y)$ hosilalar Ω sohaning chegarasida nolga teng va agar $i+j$ toq bo‘lsa, $\dfrac{\partial^{i+j}}{\partial x^i\partial y^j}\Psi(x,y)$ hosilalar Ω sohaning chegarasida chekli bo‘lsin, hamda $\dfrac{\partial^8}{\partial x^4\partial y^4}\Psi(x,y)$ funksiya Ω sohada integrallanuvchi bo‘lsin.

24-teorama. *Aytaylik, φ va Ψ funksiyalar mos ravishda* a) *va* b) *shartlarini qanoatlantirsin. U holda* (83) - (85) *masalaning yechimi* (87) *shartni qanoatlantirishi uchun $f(x,y)$ funksiya quyidagi formula bilan aniqlanishi yetarli*

$$f(x,y)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{\Psi_{nm}}{\theta^\alpha E_{\alpha,\alpha+1}(-\lambda_{nm}^2\theta^\alpha)}-\frac{\varphi_{nm}E_{\alpha,1}(-\lambda_{nm}^2\theta^\alpha)}{\theta^\alpha E_{\alpha,\alpha+1}(-\lambda_{nm}^2\theta^\alpha)}\right]sin\alpha_n x\cdot sin\beta_m y\ , \tag{90}$$

bu yerda φ_{nm} va Ψ_{nm} lar Furye koeffitsiyentlari.

25-teorema. *Aytaylik, φ va Ψ funksiyalar mos ravishda* a) *va* b) *shartlarini qanoatlantirsin. U holda* (83) - (85) *masalaning yechimi* (88) *shartni qanoatlantirishi uchun $f(x,y)$ funksiya quyidagi formula bilan aniqlanishi yetarli*

$$f(x,y)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{\Psi_{nm}}{T^{\alpha+1}E_{\alpha,\alpha+2}(-\lambda_{nm}^{2}T^{\alpha})}-\frac{\varphi_{nm}E_{\alpha,2}(-\lambda_{nm}^{2}T^{\alpha})}{T^{\alpha}E_{\alpha,\alpha+2}(-\lambda_{nm}^{2}T^{\alpha})}\right]sin\alpha_{n}x\cdot sin\beta_{m}y\,. \quad (91)$$

26-teorema. *Aytaylik,* φ *va* Ψ *funksiyalar mos ravishda* a) *va* b) *shartlarini qanoatlantirsin. U holda* (83) - (85) *masalaning yechimi* (89) *shartni qanoatlantirishi uchun* $f(x,y)$ *funksiya quyidagi formula bilan aniqlanishi yetarli*

$$f(x,y)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{\Psi_{nm}}{t_{2}^{\alpha}E_{\alpha,\alpha+1}(-\lambda_{nm}^{2}t_{2}^{\alpha})+t_{1}^{\alpha}E_{\alpha,\alpha+1}(-\lambda_{nm}^{2}t_{1}^{\alpha})}-\right.$$

$$\left.-\frac{\varphi_{nm}\left[E_{\alpha,1}(-\lambda_{nm}^{2}t_{2}^{\alpha})+E_{\alpha,1}(-\lambda_{nm}^{2}t_{1}^{\alpha})\right]}{t_{2}^{\alpha}E_{\alpha,\alpha+1}(-\lambda_{nm}^{2}t_{2}^{\alpha})+t_{1}^{\alpha}E_{\alpha,\alpha+1}(-\lambda_{nm}^{2}t_{1}^{\alpha})}\right]sin\alpha_{n}x\cdot sin\beta_{m}y\;. \quad (92)$$

XULOSA

Dissertatsiya ishi kasr tartibli differensial va xususiy hosilali differensial tenglamalar uchun turli to‘g‘ri va teskari masalalarni yechishga va kasr tartibli tenglamalar uchun boshqaruv masalalarni yechishga bag‘ishlangan. Tadqiqot ishlarining natijalari xususida quyidagi xulosalarni qilish mumkin.

Oldin o‘rganilgan ishlardan asosiy farqi bu ishda qaralayotgan vaqt bo‘yicha nolokal shartlar boshqa ishlarda o‘rganilmagan, ya’ni kasr hosilalari bilan berilgan xususiy hosilali differensial tenglamalar uchun bunday nolokal shartlarga ega masalalar oldin o‘rganilmagan. Ushbu masalalarni o‘rganishda nolokal shartda qatnashgan α parametrning o‘rganilayotgan masalalarning yechimi yagonaligiga ta‘siri aniqlangan. To‘g‘ri masalani o‘rganishda biz α parametrning ba’zi qiymatlarida yechimning yagona emasligi va yechimning mavjudligini ta‘minlash uchun berilgan φ va f funksiyalar uchun ortogonallik shartlarini talab qilish kerak bo‘lishi ko‘rsatilgan.

Kasr hosilaning tartibini aniqlashda oldin foydalanilgan shartlardan boshqa shart ya’ni biror xos funksiyaga proyeksiyalashdan foydalanilgan. Oldingi ishlarda foydalanilgan shartdan izlanuvchilar faqat yechimning yagonaligini ko‘rsatishda foydalanishgan. Bu yerda foydalanilgan shart nafaqat yechimning yagonaligini, balki uning mavjudligini ham ta‘minlashga kifoya qiladi.

Tenglamaning o‘ng tomoni va chegaraviy funksiyani topish bo‘yicha teskari masalani topish shartlari keltirib chiqarilgan.

Elliptik qismi yuqori tartibli va ko‘p o‘zgaruvchili kasr tartibli tenglamalarning klassik yechimni olish uchun Krasnoselskiy lemmasidan foydalanish usuli ishlab chiqilgan.

Chegaraviy shart kasr tartibli hosila bilan berilgan tenglamalarning yechimi mavjud va yagona bo‘lishligi uchun chegaraviy funksiya uchun ortogonallik shartlari kelib chiqishi tushuntirilgan.

Barenblatt-Jeltov-Kochina va boshqa tipidagi kasr tartibli tenglamalar uchun to‘g‘ri va teskari masalalarni yechish usullari ishlab chiqilgan.

Kasr tarttibli xususiy hosilali differensial tenglamalar uchun ushbu kasr tartibli tenglama bilan berilgan turli jarayonlarni manba funksiyasi yordamida boshqarish usullari ko‘rsatilgan.

SCIENTIFIC COUNCIL AWARDING OF THE SCIENTIFIC DEGREES DSc.02/30.12.2019.FM.86.01 AT V.I. ROMANOVSKIY INSTITUTE OF MATHEMATICS

NATIONAL UNIVERSITY OF UZBEKISTAN

FAYZIEV YUSUF ERGASHEVICH

FORWARD AND INVERSE PROBLEMS FOR FRACTIONAL PARTICULAR DIFFERENTIAL EQUATIONS WITH HIGHER-ORDER ELLIPTIC PART

01.01.02-Differential equations and mathematical physics

DISSERTATION ABSTRACT OF DOCTORAL DISSERTATION (DSc) ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT-2022

The theme of dissertation of doctor of science (DSc) on physical and mathematical sciences was registered at the Supreme Attestation Commission at the Cabinet of Ministers of the Republic of Uzbekistan under number B2022.3.DSc/FM198.

Dissertation has been prepared at National University of Uzbekistan.

The abstract of the dissertation is posted in three languages (uzbek, russian, english (resume)) on the website (http://kengash.mathinst.uz) and the "Ziyonet" Information and educational portal (www.ziyonet.uz).

Scientific consultant:	**Ashurov Ravshan Radjabovich** Doctor of Sciences in Physics and Mathematics, Professor
Official opponents:	**Turmetov Botirxan Xudaybergenovich** Doctor of Sciences in Physics and Mathematics, Professor
	Taxirov Jozil Ostanovich Doctor of Sciences in Physics and Mathematics, Professor
	Karimov Erkinjon Tulqinovich Doctor of Sciences in Physics and Mathematics, senior researcher
Leading organization:	**Institute of Cosmophysical Research and Radio Wave Propagation (Russian)**

Defense will take place « 29 » November 2022 at 16:00 at the meeting of Scientific council number DSc.02/30.12.2019.FM.86.01 at V.I. Romanovskiy Institute of Mathematics (Address: University str. 9, Almazar area, Tashkent, 100174, Uzbekistan, Ph.: (+99871)-207-91-40). E-mail: uzbmath@umail.uz, Website: www.mathinst.uz

Doctoral dissertation is possible to review in Information-resource centre of V.I. Romanovskiy Institute of Mathematics (is registered № 148) (Address: University str. 9, Almazar area, Tashkent, 100174, Uzbekistan, Ph.: (+99871)-207-91-40). E-mail: uzbmath@umail.uz, Website: www.mathinst.uz).

Abstract of dissertation sent out on « 11 » November 2022.
(mailing report № 2 on « 11 » November 2022).

U.A. Rozikov
Chairman of Scientific Council on award of scientific degrees, DSc, Professor

J.K. Adashev
Scientific secretary of Scientific Council on award of scientific degrees, DSc, senior researcher

A. Azamov
Chairman of Scientific Seminar under Scientific Council on award of scientific degrees, DSc, Academician

INTRODUCTION (Abstract of DSc thesis)

Actuality and necessity of the theme of the thesis. In recent years, creating a mathematical model of many life processes and solving it with mathematical methods has become widespread among mathematicians. These processes are inextricably linked with the development of medicine and techniques. Fractional integrals and derivatives have many applications in the fields of physics, biology, medicine and technology, and it is important in the development of these specialities. For this reason, in recent years, mathematicians are increasingly interested in studying differential equations with fractional derivatives and partial differential equations. Fractional equations represent diffusion and wave propagation processes simultaneously. Such processes are very common in nature; Therefore it is very important to study fractional equations. Take an example the process of virus spread. In 2020, Professors of “the Laboratory of Differential Equations and Its Applications” and “the University of New Haven” developed models for determining the rate of virus spread, the source of enternal and internal influence and the boundary influence. At the same time, the obtained results were applied to the analysis of the spread of COVID-19 in association with Johns Hopkins University.

In addition to learning how to solve partial differential equations, it is very important to learn how to solve inverse problems related to determining the coefficients of the equation, its right side, the boundary function, and the order of the fractional derivative. Studying inverse problems gives us the opportunity to study, analyze and control the processes given by the fractional equation. For example, if we take heat diffusion processes, our knowledge of the boundary function allows us to control the heat diffusion from the boundary. There are many similar examples. If we consider the fractional equation if the order of the fractional derivative $1<\rho<2$, it represents simultaneous diffusion and wave propagation processes, if $0<\rho<1$, it indicates slow diffusion processes. For example, one can take the processes of virus propagation. There are many such processes in our life. That is why it is important to study fractional equations.

In our country, attention has been paid to the current directions of differential equations and mathematical physics, which have scientific and practical application of fundamental research[1]. Conducting scientific research at the level of international standards in priority areas of mathematical sciences, especially “algebra and functional analysis, differential equations and mathematical physics, theory of dynamic systems” is defined as the main task and direction of the activity of the V.I. Romanovsky Institute of Mathematics named after Academy of Sciences. It is important to develop new directions of mathematical physics in ensuring the implementation of the decision. Decree and Resolutions of the President of the Republic of Uzbekistan No. PQ-3682 of April 27, 2018 “On

[1] Decree of the Cabinet of Ministers of the Republic of Uzbekistan at the 2017 year 18 May , "On measures on the organization of activities of the first created scientific research institutions of the Academy of Sciences of the Republic of Uzbekistan" No. 292

measures to further improve the system of implementation of innovative ideas, technologies and projects” and No. PQ-4387 of July 9, 2019 “Mathematics education and state support for the further development of sciences, as well as on measures to fundamentally improve the activity of the Mathematical Institute named after V.I. Romanovsky of the AS of UzR, as well as in the implementation of the tasks specified in other regulatory legal documents related to this activity dissertation research serves a certain level.

Connection of research to priority directions of development of science and technologies of the Republic. This research work was carried out in accordance with the priority direction of development of science and technology in the Republic of Uzbekistan IV. “Mathematics, mechanics and informatics”.

Review of foreign scientific researches on the topic the dissertation. Almost all universities, research institutes and scientific centers of the world are conducting researches on direct and inverse problems for fractional differential and partial differential equations. We will list some of them as an example. As we said above, they are Johns Hopkins University (USA), Tokyo University (Japan), Berlin University of Applied Sciences (Germany), New Haven University (USA), Berlin University (Germany), Belorussian State University (Belarus), Hokkaido University (Japan), at La Rochelle University (France), Institute of Applied Mathematics and Automation (Russia), Middle East University (Turkey), International Kazakh-Turkish University (Kazakhstan) and others.

Inverse problems on determining the order of the fractional derivative were studied by Professor M. Yamamoto of Tokyo University (Japan) and others. Their method ensured the uniqueness of the solution of the inverse problem only. Professor S. Umarov of the New Haven University of the United States of America and Professor R. Ashurov of the Mathematical Institute of the FA of the Republic of Uzbekistan obtained new conditions for single-valued determination of the solution of the inverse problem for determining the order of the fractional derivative, if the first eigenvalue of the spectral problem corresponding to the equation is zero and the first eigenfunction is a constant number.

Various boundary problems for fractional differential and partial differential equations are studied by many scientists. The nonlocal condition connecting the values of the initial time and the assigned time for the diffusion equations was studied in the works of A. Ashyaraliyev (Turkey) and others. Boundary conditions for elliptic equations, which given with fractional derivatives were studied by B.Turmetov (Kazakhstan) and others.

The degree of scrutiny of the problem. Nowadays, many mathematicians are engaged in solving fractional partial differential equations and related inverse problems. As an example of the results obtained in this direction, we can choose following: Sh.O. Alimov, R.R. Ashurov, S.R. Umarov, M. Yamamota, Z. Li, A.Ashyaraliyev, B. Turmetov, Y. Zhang, H.T. Nguyen, A.V. We can mention the works of Pskhu, A.S. Malik, E. Karimov and etc. An article, which was published by Z. Li and others, in 2019, was stated in “Open Problems” section: “The research on the inverse problem of restoring fractional orders is not satisfactory, because all articles were studied the problem in $t \in (0,\infty)$. It would be interesting to study the value of the fixed-time solution as observation data for the inverse problem". Therefore, in all studied works, except for the work of J. Janno, only the uniqueness of the solution is indicated. In the work of J. Janno, the existence of the solution is also shown, but he used another condition as an additional condition. R.R. Ashurov and S.R. Umarov developed a new condition to ensure the existence and uniqueness of the solution, if the first eigenvalue is equal to zero, and the corresponding eigenfunction is a constant number. In this dissertation work, using this condition, the problems of determining the order of the derivative with fractional order for different cases were studied. Additionally, even if the first eigenvalue is not equal to zero, the condition that its projection to the corresponding eigenfunction is equal to some number at a given time and one can use this case

In this dissertation work, the elliptic part consists of a self-adjoint, positive, unbounded, arbitrary operator A of fractional equations of the form $d^{\rho}u(t)+Au(t)=f(t)$ in time $u(\xi)=\alpha u(0)+\varphi$ (α is a constant number, ξ is a fixed point) we study the problem of finding a satisfying solution of the nonlocal boundary condition, where the fractional derivative in the Caputo sense or in the Riemann-Liouville sense is represented by the operator d_t on the left side of the equation. We remind you that such $u(\xi)=\alpha u(0)+\varphi$ nonlocal condition for fractional equations was studied for the first time in this work, and the conditions for the parametr α existence and uniqueness of the solution of the problem under consideration were derived. If α =0 (and $\xi=T$), this is called a *backward problem*. Backward problem for equations involving fractional derivatives in the Caputo sense, e.g. Y. Liu, K. Sakamoto, M. Yamamoto, G. Floridia, Z. Li, studied in detail in his work. Backward problem for fractional derivative equations in the Riemann-Louisville sense was studied in the work of Sh. Alimov, R. Ashurov. You can be familiarized with the regression problems for classical equations in the book of S. Kabanikhin. Therefore, we will only study the case of $\alpha \neq 0$ in the nonlocal condition. For diffusion equations, where α =1, it was studied in the works of A.O. Ashyralyev and others. In addition, coercive type inequalities are obtained. Many researches has been done to find the function f on the right side of the equation. As far as we know, the inverse problem of finding the φ function is covered only in the article by T. Yuldashev, B. Kadirkulov. The authors studied this problem for fractional equations in the sense of Hilfer, consisting of a one-variable differential expression with constant coefficients in the elliptic part. The Cauchy problem for integral and fractional derivative equations with an elliptic part of high order with multiple variables was studied for the first time in R.R. Ashurov, A.T. Mukhiddinova. The direct and inverse problems for the subdiffusion equations of fractional order in the Riemann-Liouville sense given by the nonlocal boundary condition were studied for the first time in this work. The boundary conditions for elliptic equations are given by the fractional derivative, studied by S. Umarov, B. Turmetov and other works. In these works ρ is shown that if the order of the fractional derivative is $0<\rho<1/2$, then non additional conditions are required for the boundary function, and if it is $1/2<\rho<1$, additional conditions are required. R. Gorenflo, Yu. In the work of Luchko, S.Umarov, additional conditions were obtained for the boundary function for the existence of a solution for the case where the boundary condition for equations with elliptic pseudo-differential operators is given by a fractional derivative. The Barenblatt-Zheltov-Kochina equation is the basic equation for the filtration of homogeneous fluids. Published in the work of G. I. Barenblatt, Yu. P. Zheltov and I. N. Kochina. Later it was studied in other works. Fractional equations of the Barenblatt-Zheltov-Kochina type were studied for the first time in this work.

Many mathematicians have conducted researches in the direction of optimal control issues. As an example N.Yu. Satimov, Sh.O. Alimov, V.A. Il'in, A.Azamov, E.I. Moiseev, A.G. Butkovsky, F.R. Vasiliev, J. Lyons, R. Varga, A.I.Yegorov, F.Y. Chernousko, S.A. Avdonin, S.V. Ivanov, H.G. Guseynova, V.V.Tikhomirov, K.A. Lurelar, M. Tokhtasinov, G‘. Ibragimov, N. Mamadaliyev's works can be mentioned. For the first time, the control issues of the processes described by differential equations are described in detail in book of J.L. Lions. In recent years, there has been an increasing interest in the study of process control problems described by various differential equations. In the works of V. A. Il'in and E.I. Moiseev, boundary control issues for processes related to wave propagation were studied. The problems related to the management of the processes described by parabolic type equations, in particular, the heat transfer process, were noted in works Barbu V., Rascanu A., Tessitore G. and Fattorini H.O. The problem of control of processes represented by homogeneous heat transfer equations was discussed by academic Sh.A. Alimov. It can be found in the works of Sh. A. Alimov. For the first time in this work, control issues using the source function for fractional equations were studied.

The connection of the theme of the thesis with the research plans of the higher education institute, where the research on the thesis is carried out. The dissertation work was carried out in accordance with the planned research topic F-FA-2021-424 of the Institute of Mathematics “Solution of boundary value problems for partial differential equations of integer and fractional order”. It was carried out at the Department of “Differential Equations and Mathematical Physics” of the Academy of Sciences of the Republic of Uzbekistan named after V.I. Romanovsky and the National University of Uzbekistan named after M. Ulugbek.

The aim of the research work is to indicate that there is a unique solution to the direct and inverse problems for partial differential and fractional partial differential equations with an arbitrary elliptic operator.

Research problems:

Construction of the solution of the equation of higher fractional order;

Solving nonlocal conditional boundary value problems in time for subdiffusion equations of fractional order;

Solving nonlocal boundary-value problems for subdiffusion equations of fractional order;

Solving direct and inverse problems on finding the order of the derivative and the right side of the equation in fractional partial differential equations;

Solving hyperbolic and parabolic equations given by the boundary condition fractional derivative;

Solving direct and inverse problems for fractional equations of the Barenblatt-Zheltov-Kochina type;

Solving direct and inverse problems for equations of the Barenblatt-Jeltov-Kochina type, where the boundary condition is given by a fractional derivative;

Construction of the solution of the first boundary value problem for fractional partial differential equations;

Solving control problems for fractional partial differential equations;

The research object. Caputo, Riemann-Liuville, Grünvold-Letnikov, Weyl fractional derivatives, Mittag-Leffler functions, fractional time equations.

The research subject. The research subject of the dissertation consists of direct and inverse problems for differential equations with arbitrary elliptic operators with partial derivatives and fractional derivatives.

Research methods. Mathematical analysis, functional analysis, differential equations and mathematical physics methods were used in the thesis work. The method of separation of variables from the methods of mathematical physics is used, and the completeness of the system of eigenfunctions in the Hilbert space is applied.

The scientific novelty of the research work consists of the following:

the solution of fractional subdiffusion equations satisfying the nonlocal condition, the solution of inverse problems on finding the right side of the equation and the boundary function exists and it is proven to be unique;

in fractional partial differential equations, there is a solution to the correct and inverse problems of finding the order of the derivative and the right side of the equation at the same time, and it has been proven that it is unique;

it is proved that the solution of hyperbolic and parabolic equations, given by the boundary condition with the fractional derivative, is exist and unique;

it has been proven that the solution of the direct and inverse problems for fractional equations of the Barenblatt-Zheltov-Kochina type exists and it is unique. Additionally, it has indicated the existence and uniqueness of the solution of the direct and inverse problems for differential equations of Barenblatt-Zheltov-Kochina type, which boundary condition is given by a fractional derivative;

the solution of the first boundary value problem for fractional partial differential equations was constructed, it was found selection conditions of the source function, what is for the temperature, depends on time, in a rectangular area is distributed as a predefined function.

Practical results of the research. In the research paper, methods for solving fractional differential and partial differential equations are proposed, and methods for solving fractional equations with an arbitrary elliptic operator are presented. At the same time, the method of solving inverse problems on finding the right side of the equation and determining the order of the fractional derivative is described.

The reliability of the results of the study. The results have been obtained by using the methods of modern methods of mathematical physics, mathematical analysis, differential equations, the theory of special functions for the construction of fundamental solutions, finding exact solutions of boundary value problems and solving the theoretical problems of elliptic equations.

Scientific and practical significance of the research results. The scientific significance of the research results is that the scientific results obtained in the work can be used in the theory of fractional differential and partial differential equations.

The practical importance of the results obtained in the dissertation work is determined by their application in the study, control and management of processes given by fractional equations.

Implementation of the research results. Based on the results obtained for the direct and inverse problems for fractional partial differential equations with a high-order elliptic part:

the uniqueness and existence conditions of the solution of hyperbolic and parabolic equations, whose boundary condition given by the fractional derivative, were used to build the models of the systems in the fundamental project on the topic “Building adequate computational models for hyperbolic systems” numbered OT-F-4-28 (reference of the National University of Uzbekistan, dated September 30, 2022, under the number 04/11-5960). The application of the scientific results of hyperbolic type differential equations made it possible to build computational models of systems;

controlling with the help of the source function of the processes given by fractional derivative equations was used in the transfer of radon in the atmospheric soil system in the project “Support of the Russian President of Young Scientists” No. MD-758.2022.1.1 (reference of Vitus Bering Kamchatka State University dated September 27, 2022, under the number 39-12). The application of the scientific results of hyperbolic type differential equations made it possible to build computational models of systems;

from the solutions of the Barenblatt-Zheltov-Kochina differential equations, which are the basic equation of process control and filtration given by various equations, number MRU-OT-30/2017 on the topic “High-precision computational algorithms for solving problems of gas dynamics, filtration and transport flow dynamics using modern supercomputers” used to solve boundary issues in a scientific project (reference of the National University of Uzbekistan, dated October 4, 2022, under the number 04/11-6042). Equations of Barenblatt-Zeltov-Kochina type are the basic equations for filtering homogeneous fluids. The usage of the results of scientific work made it possible to build calculation models of boundary problems, filtration and control problems.

Approbation of the research results. The results of this research were discussed at 28 scientific-practical conferences, 13 of them international and 15 national scientific conferences.

Publications of the research results. On the topic of the dissertation, 42 scientific papers were published, 13 of which are included in the list of scientific publications proposed by the Higher Attestation Commission of the Republic of Uzbekistan for the defense of thesis of the DSc, including 9 of them published in international foreign journals and 4 in national scientific journals, 1 in a foreign journal and 28 abstracts.

The structure and volume of the thesis. The dissertation consists of an introduction, 5 chapters, a conclusion and a list of used literature. The dissertation consists of 222 pages.

THE MAIN CONTENT OF THE DISSERTATION

In the introduction, the relevance and necessity of the dissertation topic is based, the compatibility of the research with the priority directions of the republic's science and technology is shown, the review of foreign scientific researches on the topic, the level of research of the problem is presented, the purpose, tasks, object and subject of the research are described. The scientific novelty and practical results of the research are described, the theoretical and practical significance of the obtained results is disclosed, information is given on the implementation of the research results, published works and the structure of the dissertation.

The main goal of the first chapter of the dissertation is to study nonlocal boundary value problems in time for fractional equations. The first paragraph of the dissertation contains auxiliary information,it is necessary for the presentation of the results of the dissertation. In the second paragraph, the problem of constructing the solution of differential equations of higher fractional order is studied. R. Ashurov, A. Cabada and B. Turmetov used the method of constructing the solution of fractional derivative linear differential equations in the sense of Caputo. In the third paragraph, nonlocal boundary condition problems in time for fractional equations in the sense of Caputo and Riemann-Liouville are studied.

Let H be a separable Hilbert space with the scalar product $(\cdot,\cdot)$ and the norm $\|\cdot\|$ and $A: H \to H$ be an arbitrary unbounded positive selfadjoint operator in H. Suppose that A has a complete in H system of orthonormal eigenfunctions $\{v_k\}$ and a countable set of positive eigenvalues $\{\lambda_k\}$. It is convenient to assume that the eigenvalues do not decrease as their number increases, i.e., $0 < \lambda_k \le \lambda_k \cdots \to \infty$.

Using the definitions of a strong integral and a strong derivative, fractional analogues of integrals and derivatives can be determined for vector-valued functions (or simply functions) $h: \mathrm{R}_+ \to H$, while the well-known formulae and properties are preserved.

Recall that the fractional integration of order σ of the function $h(t)$ defined on $[0,\infty)$ da has the form

$$J_t^{\sigma} h(t) = \frac{1}{\Gamma(\sigma)} \int_0^t \frac{h(\xi)}{(t-\xi)^{1-\sigma}} d\xi, \quad t > 0 \tag{1}$$

provided the right-hand side exists. Here $\Gamma(\sigma)$ is Euler's gamma function. Using this definition one can define the Riemann-Liouville fractional derivative of order ρ, $0<\rho<1$, as

$$\partial_t^\rho h(t) = \frac{d}{dt} J_t^{1-\rho} h(t).$$

If in this definition we interchange differentiation and fractional integration, then we obtain the definition of the regularized derivative, that is, the definition of the fractional derivative in the sense of Caputo:

$$D_t^\rho h(t) = J_t^{1-\rho} \frac{d}{dt} h(t).$$

Let $\rho \in (0,1)$ be a fixed number and let $C((a,b);H)$ stand for a set of continuous functions $u(t)$ of $t \in (a,b)$ with values in H.

The subject of this work is the following two nonlocal boundary value problems:

$$\begin{cases} D_t^\rho u(t) + Au(t) = f(t), & 0<t\le T; \\ u(\xi) = \alpha u(0) + \varphi, & 0<\xi \le T \end{cases} \tag{2}$$

and

$$\begin{cases} \partial_t^\rho u(t) + Au(t) = g(t), & 0<t\le T; \\ J_t^{1-\rho} u(t)\,|_{t=\xi} = \alpha \lim\limits_{t\to 0} J_t^{1-\rho} u(t) + \phi, & 0<\xi \le T, \end{cases} \tag{3}$$

where $f(t), g(t) \in C((0,T];H)$, $\varphi, \phi \in H$ and α – is a constant, ξ– fixed point. These problems are also called *the forward problems*.

Definition 1. *A function* $u(t) \in C([0,T];H)$ *with the properties* $D_t^\rho u(t), Au(t) \in C((0,T);H)$ *and satisfying conditions* (2) *is called* **the solution** *of the nonlocal problem* (2).

The definition of the solution to the nonlocal problem (3) is introduced in a similar way.

If $\alpha=0$ (and $\xi=T$), then these problems are called *the backward problems*. The backward problems in case (2) were studied in detail, for example, it was studied in detail in the works of M. Yamamoto and others. (3) the issue of going back in the case of Sh. Alimov and R. Ashurov. Therefore, in what follows we only consider the case

$$\alpha \neq 0.$$

In the case $\rho=1$ these problems are also called the inverse heat conduction problem with inverse time (*retrospective inverse problem*). It should also be noted that, in this case, even the smoothness of the function $u(T)$ does not guarantee the stability of the solution.

In the case $\rho=1$, $\alpha=1$, the operator *A* is for the case defined in the Banach space. It was studied in the works of A.O. Ashyralyev and others.

The problems given by the nonlocal condition for fractional equations $u(\xi)=\alpha u(0)+\varphi$ ($J_t^{1-\rho}u(t)|_{t=\xi}=\alpha\lim\limits_{t\to 0}J_t^{1-\rho}u(t)+\phi$) are studied for the first time in this work.

In the present paper we prove the existence and uniqueness theorems for solutions of problems (2) and (3). Next, we will study the dependence of the existence of a solution on the value of the parameter α. We will also prove, in contrast to the backward problems, that the solutions of problems (2) and (3) continuously depend on the right-hand side of the equation and on the function φ. Inequalities of coercivity type are obtained and it is shown that these inequalities differ depending on the considered type of fractional derivatives. The inverse problems of determining the right-hand side of the equation and function φ in the boundary conditions are investigated.

To solve problem (2), we divide it into two auxiliary problems:

$$\begin{cases} D_t^\rho \omega(t)+A\omega(t)=f(t), & 0<t\le T; \\ \omega(0)=0, \end{cases} \tag{4}$$

and

$$\begin{cases} D_t^\rho w(t)+Aw(t)=0, & 0<t\le T; \\ w(\xi)=\alpha w(0)+\psi, & 0<\xi\le T, \end{cases} \tag{5}$$

where $\psi\in H$ is a given function.

If $\psi=\varphi-\omega(\xi)$ and $\omega(t)$ and $w(t)$ are the corresponding solutions, then it is easy to verify that function $u(t)=\omega(t)+w(t)$ is a solution to problem (2). Therefore, it is sufficient to solve the auxiliary problems.

Let $\lambda_0>0$ such that $E_\rho(-\lambda_0\xi^\rho)=\alpha$, where $E_\rho(t)$ is Mittag-Leffler function. Let $\lambda_k=\lambda_0$ and let this eigenvalue be a multiple of p_0. We denote such *k* numbers as $K_0=\{k_0,k_0+1,...,k_0+p_0-1\}$.

Let τ be an arbitrary real number. We introduce the power of operator A, acting in H according to the rule:

$$A^\tau h=\sum_{k=1}^{\infty}\lambda_k^\tau h_k v_k,$$

where $h_k=(h,v_k)$ are the Fourier coefficients of a function $h\in H$. Obviously, the domain of this operator has the form:

$$D(A^\tau)=\{h\in H:\sum_{k=1}^{\infty}\lambda_k^{2\tau}|h_k|^2<\infty\}.$$

To solve problem (4), we prove the following theorem.

Theorem 1. *Let* $f(t)\in C([0,T];D(A^\varepsilon))$ *for some* $\varepsilon\in(0,1)$. *Then problem* (4) *has a unique solution and this solution has the representation*

$$\omega(t)=\sum_{k=1}^{\infty}\left[\int_0^t\eta^{\rho-1}E_{\rho,\rho}(-\lambda_k\eta^{\rho})f_k(t-\eta)d\eta\right]v_k. \quad (6)$$

Moreover, there is a constant $C_\varepsilon>0$ *such that the following coercive type inequality holds:*

$$\|D_t^{\rho}\omega(t)\|^2+\|\omega(t)\|_1^2\le C_\varepsilon\max_{t\in[0,T]}\|f\|_\varepsilon^2,\quad 0<t\le T, \quad (7)$$

If $f\in H$ does not depend on *t*, then the statement of Theorem 1 is true.

Corollary 1. *Let* $f\in H$. *Then problem* (4) *has a unique solution and this solution has the representation*

$$\omega(t)=\sum_{k=1}^{\infty}f_kt^{\rho}E_{\rho,\rho+1}(-\lambda_kt^{\rho})v_k. \quad (8)$$

Moreover, there is a positive constant C such that the following coercive type inequality holds:

$$\|D_t^{\rho}\omega(t)\|^2+\|\omega(t)\|_1^2\le C\|f\|^2,\quad 0<t\le T. \quad (9)$$

To solve problem (5), we prove the following theorem.

Theorem 2. *Let* $\psi\in H$.

If $\alpha\notin(0,1)$ *or* $\alpha\in(0,1)$, *but* $\lambda_k\ne\lambda_0$ *for all* $k\ge1$, *then problem* (5) *has a unique solution and this solution has the form:*

$$w(t)=\sum_{k=1}^{\infty}\frac{\psi_k}{E_\rho(-\lambda_k\xi^{\rho})-\alpha}E_\rho(-\lambda_kt^{\rho})v_k. \quad (10)$$

If $\alpha\in(0,1)$ *and* $\lambda_k=\lambda_0$, *(*$k\in K_0$*)*

$$\psi_k=(\psi,v_k)=0,\quad k\in K_0;\quad K_0=\{k_0,k_0+1,....,k_0+p_0-1\}, \quad (11)$$

then we assume that the orthogonality conditions are satisfied. The solution of problem (5) with arbitrary coefficients b_k, $k\in K_0$ *has the form*

$$w(t)=\sum_{k\notin K_0}\frac{\psi_k}{E_\rho(-\lambda_k\xi^{\rho})-\alpha}E_\rho(-\lambda_kt^{\rho})v_k+\sum_{k\in K_0}b_kE_\rho(-\lambda_kt^{\rho})v_k. \quad (12)$$

Moreover, there is a constant $C_\alpha>0$ *such that the following coercive type inequality holds:*

$$\|D_t^{\rho}w(t)\|^2+\|w(t)\|_1^2\le C_\alpha t^{-2\rho}\|\psi\|^2,\quad 0<t\le T. \quad (13)$$

The solution to problem (2) is reflected in the following theorem.

Theorem 3. *Let* $\varphi\in H$ *and* $f(t)\in C([0,T];D(A^{\varepsilon}))$ *for some* $\varepsilon\in(0,1)$.

If $\alpha\notin(0,1)$ *or* $\alpha\in(0,1)$, *but* $\lambda_k\ne\lambda_0$ *for all* $k\ge1$ *then problem* (2) *has a unique solution and this solution has the form*

$$u(t)=\sum_{k=1}^{\infty}\left[\frac{\varphi_k-\omega_k(\xi)}{E_\rho(-\lambda_k\xi^{\rho})-\alpha}E_\rho(-\lambda_kt^{\rho})+\omega_k(t)\right]v_k, \quad (14)$$

If $\alpha\in(0,1)$ *and ,* $\lambda_k=\lambda_0$ $k\in K_0$

$$(\varphi,v_k)=(\omega(\xi),v_k),\quad k\in K_0;\quad K_0=\{k_0,k_0+1,....,k_0+p_0-1\}. \quad (15)$$

then we assume that the orthogonality conditions (15) *are satisfied. The solution of problem* (2) *with arbitrary coefficients* b_k, $k \in K_0$ *has the form*

$$u(t) = \sum_{k \notin K_0} \left[\frac{\varphi_k - \omega_k(\xi)}{E_\rho(-\lambda_k \xi^\rho) - \alpha} E_\rho(-\lambda_k t^\rho) + \omega_k(t) \right] v_k + \sum_{k \in K_0} b_k E_\rho(-\lambda_k t^\rho) v_k. \quad (16)$$

Moreover, there are constants $C_\alpha > 0$ *and* $C_\varepsilon > 0$ *such that the following coercive type inequality holds:*

$$\| D_t^\rho u(t) \|^2 + \| u(t) \|_1^2 \le C_\alpha t^{-2\rho} \| \varphi \|^2 + C_\varepsilon \max_{t \in [0,T]} \| f \|_\varepsilon^2, \quad 0 < t \le T. \quad (17)$$

When solving inverse problems, we need an additional condition. We use the following condition as an additional condition for both cases:

$$u(\tau) = \Psi, \quad 0 < \tau \le T, \quad \tau \ne \xi. \quad (18)$$

In the study of inverse problems, we treat the function $f \in H$ not depend on t. Note that if $\tau = \xi$, then the nonlocal condition in (2) coincides with the Cauchy condition $u(0) = \varphi_1$ ($\alpha \ne 0$). Such inverse problems have been studied in previous works.

Definition 2. *A pair* $\{u(t), f\}$ *of functions* $u(t) \in C([0,T]; H)$ *and* $f \in H$ *with the properties* $D_t^\rho u(t), Au(t) \in C((0,T]; H)$ *and satisfying conditions* (2), (18) *is called* ***the solution*** *of the inverse problem* (2), (18).

We study the inverse problem of finding the right side of the equation separately for the cases $\alpha \ge 1$ and $0 < \alpha < 1$.

At first, we present the result obtained for the case of $\alpha \ge 1$.

Theorem 4. *Let* $\varphi, \Psi \in D(A)$ *and* $\alpha \ge 1$. *Then the inverse problem* (2), (18) *has a unique solution* $\{u(t), f\}$ *and this solution has the following form:*

$$f = \sum_{k=1}^{\infty} \left[\frac{\alpha - E_\rho(-\lambda_k \xi^\rho)}{E_\rho(-\lambda_k \tau^\rho) \xi^\rho E_{\rho,\rho+1}(-\lambda_k \xi^\rho) + \tau^\rho E_{\rho,\rho+1}(-\lambda_k \tau^\rho)[\alpha - E_\rho(-\lambda_k \xi^\rho)]} \Psi_k + \right.$$

$$\left. + \frac{E_\rho(-\lambda_k \tau^\rho)}{E_\rho(-\lambda_k \tau^\rho) \xi^\rho E_{\rho,\rho+1}(-\lambda_k \xi^\rho) + \tau^\rho E_{\rho,\rho+1}(-\lambda_k \tau^\rho)[\alpha - E_\rho(-\lambda_k \xi^\rho)]} \varphi_k \right] v_k, \quad (19)$$

and

$$u(t) =$$

$$= \sum_{k=1}^{\infty} \left[\frac{E_\rho(-\lambda_k t^\rho)}{E_\rho(-\lambda_k \xi^\rho) - \alpha} [\varphi_k - f_k \xi^\rho E_{\rho,\rho+1}(-\lambda_k \xi^\rho)] + f_k t^\rho E_{\rho,\rho+1}(-\lambda_k t^\rho) \right] v_k. \quad (20)$$

Now, as we said above, we present the results obtained for the case of $0 < \alpha < 1$ separately. In this case, there is a solution to the inverse problem, and its uniqueness also depends on the location of points ξ and τ.

Theorem 5. *Let* $0 < \alpha < 1$, $\varphi, \Psi \in D(A)$ *and we assume that the orthogonality conditions* (15) *are satisfied. If* $0 < \tau < \xi$, *the inverse problem* (2), (18) *has a unique solution* $\{u(t), f\}$ *and this solution has the following form:*

$$f=\sum_{k\notin K_0}\left[\frac{\alpha-E_\rho(-\lambda_k\xi^\rho)}{E_\rho(-\lambda_k\tau^\rho)\xi^\rho E_{\rho,\rho+1}(-\lambda_k\xi^\rho)+\tau^\rho E_{\rho,\rho+1}(-\lambda_k\tau^\rho)[\alpha-E_\rho(-\lambda_k\xi^\rho)]}\Psi_k+\right.$$

$$\left.+\frac{E_\rho(-\lambda_k\tau^\rho)}{E_\rho(-\lambda_k\tau^\rho)\xi^\rho E_{\rho,\rho+1}(-\lambda_k\xi^\rho)+\tau^\rho E_{\rho,\rho+1}(-\lambda_k\tau^\rho)[\alpha-E_\rho(-\lambda_k\xi^\rho)]}\varphi_k\right]v_k, \qquad (21)$$

and

$$u(t)=\sum_{k\notin K_0}\left[\frac{E_\rho(-\lambda_k t^\rho)}{E_\rho(-\lambda_k\xi^\rho)-\alpha}[\varphi_k-f_k\xi^\rho E_{\rho,\rho+1}(-\lambda_k\xi^\rho)]+f_k t^\rho E_{\rho,\rho+1}(-\lambda_k t^\rho)\right]v_k.$$

$$+\sum_{k\in K_0}\frac{E_\rho(-\lambda_k t^\rho)\Psi_k}{E_\rho(-\lambda_k\tau^\rho)}v_k. \qquad (22)$$

Consider the problem (2) and assume that, together with function $u(t)$, function φ in the nonlocal condition $u(\xi)=\alpha u(0)+\varphi$ is also unknown. To solve *this inverse problem*, we need an additional condition, and as such we again take the condition that was used in the previous inverse problem (18).

Definition 3. *A pair* $\{u(t),\varphi\}$ *of function* $u(t)\in C([0,T];H)$ *and* $\varphi\in H$ *with the properties* $D_t^\rho u(t), Au(t)\in C((0,T);H)$ *and satisfying conditions* (2),(18) *are called* ***the solution*** *of* ***the inverse problem*** (2),(18).

As mentioned above, it is possible to simplify the proof of the uniqueness of the solution by placing an additional condition on the α parameter, so we will study this problem for two cases $E_\rho(-\lambda_k\xi^\rho)\neq\alpha$ and $E_\rho(-\lambda_0\xi^\rho)=\alpha$.

Theorem 6. *Let* $\Psi\in D(A)$, $f\in C([0,T];D(A^\varepsilon))$ *for some* $\varepsilon\in(0,1)$. *If is* $\alpha\notin(0,1)$ *or* $\alpha\in(0,1)$, *but for all* $k\geq 1$ *is* $\lambda_k\neq\lambda_0$, *then the inverse problem* (2), (18) *has a unique solution and it has the following form*

$$\varphi=\sum_{k=1}^{\infty}\left[\frac{E_\rho(-\lambda_k\xi^\rho)-\alpha}{E_\rho(-\lambda_k\tau^\rho)}[\Psi_k-\omega_k(\tau)]+\omega_k(\xi)\right]v_k, \qquad (23)$$

and

$$u(t)=\sum_{k=1}^{\infty}\left[\frac{\varphi_k-\omega_k(\xi)}{E_\rho(-\lambda_k\xi^\rho)-\alpha}E_\rho(-\lambda_k t^\rho)+\omega_k(t)\right]v_k, \qquad (24)$$

where

$$\omega_k(t)=\int_0^t\eta^{\rho-1}E_{\rho,\rho}(-\lambda_k\eta^\rho)f_k(t-\eta)d\eta.$$

Now we present the following theorem for the case of $\alpha\in(0,1)$.

Theorem 7. *Let* $\alpha\in(0,1)$, $\Psi\in D(A)$, *and* $f\in C([0,T];D(A^\varepsilon))$ *are for some* $\varepsilon\in(0,1)$, *and if the orthogonality conditions (15) are satisfied, then the* *inverse problem (2), (18) has a unique solution* $\{u(t),\varphi\}$*, and this solution has the following form*

$$\varphi=\sum_{k\notin K_0}\left[\frac{E_\rho(-\lambda_k\xi^\rho)-\alpha}{E_\rho(-\lambda_k\tau^\rho)}[\Psi_k-\omega_k(\tau)]+\omega_k(\xi)\right]v_k, \tag{25}$$

and

$$u(t)=\sum_{k\notin K_0}\left[\frac{\varphi_k-\omega_k(\xi)}{E_\rho(-\lambda_k\xi^\rho)-\alpha}E_\rho(-\lambda_k t^\rho)+\omega_k(t)\right]v_k+\sum_{k\in K_0}\frac{E_\rho(-\lambda_k t^\rho)\Psi_k}{E_\rho(-\lambda_k\tau^\rho)}v_k, \tag{26}$$

where

$$\omega_k(t)=\int_0^t\eta^{\rho-1}E_{\rho,\rho}(-\lambda_k\eta^\rho)f_k(t-\eta)d\eta.$$

In the case of fractional Riemann-Liouville derivatives, we consider only the forward problem for the homogeneous subdiffusion equation. The inhomogeneous equations and inverse problems considered above are studied in exactly the same way as in the case of the Caputo derivatives.

Consider the forward problem:

$$\begin{cases}\partial_t^\rho u(t)+Au(t)=0, & 0<t\le T;\\ J_t^{1-\rho}u(t)|_{t=\xi}=\alpha\lim\limits_{t\to0}J_t^{1-\rho}u(t)+\phi, & 0<\xi\le T,\end{cases} \tag{27}$$

where $\phi\in H$ and the number α are given.

As mentioned above (see the previous section), we only consider the case where all $k\ge1$ are $E_\rho(-\lambda_k\xi^\rho)\ne\alpha$.

Theorem 8. *Let* $\phi\in H$ *and* $E_\rho(-\lambda_k\xi^\rho)\ne\alpha$ *for all* $k\ge1$.. *Then problem (27) has a unique solution and this solution has the form*

$$u(t)=\sum_{k=1}^{\infty}\frac{\phi_k}{E_\rho(-\lambda_k\xi^\rho)-\alpha}t^{\rho-1}E_{\rho,\rho}(-\lambda_k t^\rho)v_k. \tag{28}$$

Moreover, there is a constant $C_\xi>0$ *such that the following coercive type inequality holds:*

$$\|\partial_t^\rho u(t)\|_1^2+\|u(t)\|_2^2\le C_\xi t^{-2\rho-2}\|\phi\|^2,\quad 0<t\le T. \tag{29}$$

In the fourth paragraph of Chapter I, the nonlocal-boundary value problem is studied, the problems of finding the classical solution for fractional derivative equations in the Riemann-Liouville sense, the right side of the equation and the function φ in the nonlocal condition are studied.

Let Ω be an arbitrary N – dimensional domain with sufficiently smooth $\partial\Omega$. boundary. We consider the following nonlocal boundary-value problem:

$$\partial_t^\rho u(x,t)-\Delta u(x,t)=f(x,t),\quad x\in\Omega,\quad 0<t\le T; \tag{30}$$

$$u(x,t)|_{\partial\Omega}=0; \tag{31}$$

$$J_t^{1-\rho}u(x,t)|_{t=\xi}=\alpha\lim_{t\to0}J_t^{1-\rho}u(x,t)+\varphi(x),\quad 0<\xi\le T,\quad x\in\overline{\Omega}, \tag{32}$$

where $f(x,t)$, $\varphi(x)$ are given functions, α is a constant, ξ is a fixed point and $\Delta=\sum_{k=1}^{N}\frac{\partial^2}{\partial x_k^2}$ Laplace operator. This problem is also called *the forward problem*.

Definition 4. *A function $u(x,t)$ with the properties*

1. $t^{1-\rho}u(x,t)\in C(\overline{\Omega}\times[0,T])$,
2. $\partial_t^{\rho}u(x,t),\Delta u(x,t)\in C(\overline{\Omega}\times(0,T])$,

and satisfying conditions (30) - (32) is called *the solution* to the forward problem.

The following lemma of Krasnoselsky et al plays an important role in this paragraph.

Lemma 1. *Let $\tau>\frac{|\alpha|}{m}+\frac{N}{2m}$. Then, for all $|\alpha|\le m$, the operator $D^{\alpha}(A+I)^{-\tau}$ maps $L_2(\Omega)$ to $C(\overline{\Omega})$ continuously (completely), and the following evaluation holds:*

$$\|D^{\alpha}(A+I)^{-\tau}g\|_{C(\Omega)}\le C\|g\|_{L_2(\Omega)}.$$

In proving the existence of solutions of forward and inverse problems

$$\sum_{k=1}^{\infty}\lambda_k^{\tau}|h_k|^2,\quad \tau>\frac{N}{2}, \tag{33}$$

it is necessary to learn whether the series in the form are convergent, where h_k are the Fourier coefficients of the $h(x)$ function. For the convergence of the series at integer values of the number τ in (33), depends on belong to which classical Sobolev spaces $W_2^k(\Omega)$ of the function $h(x)$. It is shown in the fundamental work of V.A. Il'in. To introduce these conditions, we introduce the class $\dot{W}_2^1(\Omega)$. We define the closure of the set of all functions continuously differentiable in the field Ω and equal to zero around the boundary $\partial\Omega$ according to the norm of $W_2^1(\Omega)$.

So if $h(x)$ is the following function

$$h(x)\in W_2^{[\frac{N}{2}]+1}(\Omega)\ \text{ va } h(x),\ \Delta h(x),\cdots,\ \Delta^{[\frac{N}{4}]}h(x)\in\dot{W}_2^1(\Omega), \tag{34}$$

satisfies the conditions, then the series (33) is convergent (if N is even, $\tau=\frac{N}{2}+1$, if N is odd, it can be taken as $\tau=\frac{N+1}{2}$).

Similarly, if we replace τ with $\tau+2$ in (33), the approximation conditions will have the following form:

$$h(x)\in W_2^{[\frac{N}{2}]+3}(\Omega)\ \text{and}\ h(x),\ \Delta h(x),\cdots,\ \Delta^{[\frac{N}{4}]+1}h(x)\in\dot{W}_2^1(\Omega). \tag{35}$$

We present the main result of the fourth paragraph:

Theorem 9. *Let* $\varphi(x)$ *and* $t^{1-\rho}f(x,t)$ (*for all* $t\in[0,T]$) *functions satisfy conditions (34). If* $\alpha\notin[0,1)$ *or* $\alpha\in(0,1)$, *but* $\lambda_k\neq\lambda_0$ *for all* k , *then problem* (30) - (32) *has a unique solution, and this solution is the form*

$$u(x,t)=\sum_{k=1}^{\infty}\left[\frac{\varphi_k-\omega_k(\xi)}{E_\rho(-\lambda_k\xi^\rho)-\alpha}t^{\rho-1}E_{\rho,\rho}(-\lambda_k t^\rho)+\omega_k(t)\right]v_k(x), \tag{36}$$

where $\omega_k(t)=\int_0^t\eta^{\rho-1}E_{\rho,\rho}(-\lambda_k\eta^\rho)f_k(t-\eta)d\eta.$

If $\alpha\in(0,1)$ *and* $\lambda_k=\lambda_0, k\in K_0$, *we assume that the orthogonality conditions (15) are fulfilled. Then the solution of problem* (30) - (32) *is with arbitrary coefficients* b_k, $k\in K_0$ *has the form*

$$u(x,t)=\sum_{k\notin K_0}\left[\frac{\varphi_k-\omega_k(\xi)}{E_\rho(-\lambda_k\xi^\rho)-\alpha}t^{\rho-1}E_{\rho,\rho}(-\lambda_k t^\rho)+\omega_k(t)\right]v_k(x)+$$
$$+\sum_{k\in K_0}b_k t^{\rho-1}E_{\rho,\rho}(-\lambda_k t^\rho)v_k(x) \tag{37}$$

In this paragraph, we will also study inverse problems for finding the right-hand side of the equation and the marginal function. To study the inverse problem, we need an additional condition. As an additional condition

$$u(x,\theta)=\Psi(x),\quad 0<\theta\leq T,\quad \theta\neq\xi,\quad x\in\overline{\Omega}, \tag{38}$$

we get the condition. Here $f(x)$ representing the source function is an unknown function independent of *t* and $\Psi(x)$ is a given function.

Theorem 10. *Let the functions* $\varphi(x),\Psi(x)$ *satisfy conditions* (35). *Then there is a unique solution* $\{u(x,t),f(x)\}$ *of the inverse problem* (30)-(32), (38) *and this solution has the following form*

$$f(x)=$$
$$=\sum_{k=1}^{\infty}\left[\frac{\alpha-E_\rho(-\lambda_k\xi^\rho)}{\theta^{\rho-1}E_{\rho,\rho}(-\lambda_k\theta^\rho)\xi^\rho E_{\rho,\rho+1}(-\lambda_k\xi^\rho)+\theta^\rho E_{\rho,\rho+1}(-\lambda_k\theta^\rho)[\alpha-E_\rho(-\lambda_k\xi^\rho)]}\Psi_k+\right.$$
$$\left.+\frac{\theta^{\rho-1}E_{\rho,\rho}(-\lambda_k\theta^\rho)}{\theta^{\rho-1}E_{\rho,\rho}(-\lambda_k\theta^\rho)\xi^\rho E_{\rho,\rho+1}(-\lambda_k\xi^\rho)+\theta^\rho E_{\rho,\rho+1}(-\lambda_k\theta^\rho)[\alpha-E_\rho(-\lambda_k\xi^\rho)]}\varphi_k\right]v_k(x), \tag{39}$$

$$u(x,t)=$$
$$=\sum_{k=1}^{\infty}\left[\frac{E_{\rho,\rho}(-\lambda_k t^\rho)}{E_\rho(-\lambda_k\xi^\rho)-\alpha}t^{\rho-1}[\varphi_k-f_k\xi^\rho E_{\rho,\rho+1}(-\lambda_k\xi^\rho)]+f_k t^\rho E_{\rho,\rho+1}(-\lambda_k t^\rho)\right]v_k(x). \tag{40}$$

Let us assume that in the problem (30) - (32) not only the function $u(x,t)$, but also the function $\varphi(x)$ in the nonlocal condition is unknown. The problem of finding a pair of functions $\{u(x,t),\varphi(x)\}$ is called the inverse problem of finding a function φ. As an additional condition to solve this inverse problem, we again obtain condition (38).

Theorem 11. Let the expression $t^{1-\rho}f(x,t)$ as a function of x satisfies the conditions (34) on all $t\in[0,T]$ and the function $\Psi(x)$ satisfies the conditions (38). Then the inverse problem (30) - (32), (38) has a unique solution $\{u(x,t),\varphi(x)\}$ and this solution has the following form

$$\varphi(x)=\sum_{k=1}^{\infty}\left[\frac{E_{\rho}(-\lambda_k\xi^{\rho})-\alpha}{\theta^{\rho-1}E_{\rho,\rho}(-\lambda_k\theta^{\rho})}[\Psi_k-\omega_k(\theta)]+\omega_k(\xi)\right]v_k(x), \tag{41}$$

$$u(x,t)=\sum_{k=1}^{\infty}\left[\frac{\varphi_k-\omega_k(\xi)}{E_{\rho}(-\lambda_k\xi^{\rho})-\alpha}t^{\rho-1}E_{\rho,\rho}(-\lambda_k t^{\rho})+\omega_k(t)\right]v_k(x), \tag{42}$$

where $\omega_k(t)=\int\limits_0^t \eta^{\rho-1}E_{\rho,\rho}(-\lambda_k\eta^{\rho})f_k(t-\eta)d\eta.$

The second chapter of the dissertation is devoted to solving inverse problems on finding the order of the time derivative and the right side of the equation.

Many scientists have conducted research on determining the order of the product. It should be noted that in all these works the following relation is taken as an additional condition

$$u(x_0,t)=h(t),\ \ 0<t<T,$$

here is the tracking point $x_0\in\Omega$. Experts mainly studied the uniqueness of the solution in the inverse problem of determining the order of the derivative of a fraction.

Only Estonian mathematician Jaan Janno‘s article deals with existence and uniqueness. In his paper, J. Janno considered the one-variable time subdiffusion equation of the Caputo derivative. The author took $Bu(\cdot,t)=h(t),\quad 0<t<T$ as an additional boundary condition, where an operator, he was able to prove the order of the derivative in Eq. and the existence theorem to determine the kernel of the integral operator.

A 2019 paper by Z. Li et al correctly stated in the Open Problems section: “Research on the inverse problem of fractional derivative order recovery is not satisfactory because all the papers $t\in(0,\infty)$ the problem studied in. It would be interesting to study the inverse problem on the value of the fixed-time solution as observation data”.

In this chapter, the above-mentioned problem is partially solved, and we prove that the solution of the inverse problem of determining the order of the equation in fractional partial differential equations exists and is unique. For this, as an additional condition, as noted in the “Open problems” section, we take the value of the solution at the given time as the observation point. To do this, we use the following additional condition given at the time t_0 is assigned:

$$U(\rho;t_0) \equiv \int_{T^N} u(x,t_0)dx = d_0, \quad t_0 \ge T_0, \ (43)$$

here T_0 is a fixed number.

In the first paragraph of the second chapter, the inverse problem for determining the order of the fractional time derivative in the inhomogeneous subdiffusion equation with an arbitrary elliptic differential operator with constant coefficients on an N - dimensional string is considered. Using the classical Fourier method, the problem of restoring the order of the fractional derivative based on the information about the value of the solution at the appointed time is studied. At the same time, the problem of generalization to arbitrary N - dimensional domain and elliptic operators with variable coefficients was considered.

Suppose that $A(D) = \sum_{|\alpha|=m} a_\alpha D^\alpha$ is a homogeneous, symmetric elliptic differential operator with constant coefficients, that is, $A(\xi) > 0$ for all $\xi \ne 0$, where $\alpha = (\alpha_1, \alpha_2, ..., \alpha_N)$ is a multi-index and $D = (D_1, D_2, ..., D_N)$, $D_j = \frac{\partial}{\partial x_j}$. By $C^m(T^N)$, we define a class of functions derived from x_j with period 2π and $C^m(R^N)$ for each variable $v(x)$. Suppose operator A is a standard operator of the form $C^m(R^N)$ defined in $Av(x) = A(D)v(x)$. Let us assume that $\rho \in (0,1)$.

Consider the following initial boundary value problem

$$\partial_t^\rho u(x,t) + Au(x,t) = f(x,t), \quad x \in T^N, \quad 0 < t \le T, \qquad (44)$$

$$\lim_{t \to 0} J_t^{1-\rho} u(x,t) = \varphi(x), \quad x \in T^N. \qquad (45)$$

The following

$$f_0 = const, \quad \varphi_0^2 + f_0^2 \ne 0, \qquad (46)$$

we use a method based on the classical Fourier method to find the order of the fractional derivative when the conditions are met. If conditions (46) are not fulfilled, this method becomes complicated. We define the parameter T_0 in (43) as follows

$$T_0 = \begin{cases} 2, & \varphi_0 f_0 \ge 0, \\ \max\left\{5, 4\frac{|\varphi_0|}{|f_0|}\right\}, & \varphi_0 f_0 < 0. \end{cases}$$

Now we present the main result of this paragraph.

Theorem 12. *Let conditions* (46) *are satisfied,* $\tau > \frac{N}{2m}$ *and*

(a) $\varphi \in D(\hat{A}^\tau)$;

(*b*) $f(x,t)\in D(\hat{A}^{\tau})$ *for any* $t\in[0,T]$;

(*c*) $F(t)=\hat{A}^{\tau}f(x,t)$ *be continuous in the norm of space* $L_2(\mathrm{T}^N)$ *for all* $t\in[0,T]$.

Then the inverse problem (43) – (45) *has unique solution* $\{u(x,t),\rho\}$ *if and only if*

$$\min\{f_0,\varphi_0+t_0f_0\}<d_0<\max\{f_0,\varphi_0+t_0f_0\}.$$

Lemma 2. *Let conditions* (46) *be satisfied and* $t_0\geq T_0$. *Then functions* $U(\rho;t_0)$ *as a function of* $\rho\in(0,1)$ *is a strictly monotone and*

$$\lim_{\rho\to 0}U(\rho;t_0)=f_0,\quad U(1;t_0)=\varphi_0+t_0f_0 \tag{47}$$

Theorem 13. *Let* $\tau>\frac{N}{2m}$ *and conditions* (*a*) – (*c*) *o Theorem 12 be satisfied. Then it exists there a unique solution of initial-boundary value problem* (43) - (45) *and it has the form*

$$u(x,t)=\sum_{n\in Z^N}[\varphi_n t^{\rho-1}E_{\rho,\rho}(-A(n)t^{\rho})+\int_0^t f_n(t-\xi)\xi^{\rho-1}E_{\rho,\rho}(-A(n)\xi^{\rho})d\xi]e^{inx}, \tag{48}$$

which absolutely and uniformly converges on $x\in\mathrm{T}^N$ *and for each* $t\in(0,T]$.

In the second paragraph of the second chapter, we will get acquainted with the issue of determining the order of the fractional derivative in time-fractional wave equations.

In this paragraph, the inverse problem of determining the order of the derivative of a fraction in the sense of Gerasimov-Caputo is studied. In the studied problem, the elliptic part of the equation consists of an arbitrary operator with a positive, self-adjoint, unbounded discrete spectrum A. Using the classical Fourier method, the value of the projection of the solution to a characteristic function at a given time determines the order of the derivative. Examples of the A operator include linear systems of fractional differential equations, Sturm-Liouville fractional operators, and others.

Let $\beta\in(1,2)$ is a given number. By $C((a,b);H)$ we denote the set of $u(t)$ functions whose value in H is continuous in $t\in(a,b)$s.

Consider the following Cauchy problem:

$$D_t^{\beta}u(t)+Au(t)=f,\quad 0<t\leq T, \tag{49}$$

$$u(0)=\varphi,\quad u'(0)=\psi, \tag{50}$$

where f,φ and ψ vectors are given in H.

Definition 5. *A function* $u(t)$ *with the properties*

1. $u(t),\ u'(t)\in C([0,T];H)$,

2. $D_t^{\beta}u(t),\ Au(t)\in C((0,T];H)$

and satisfying conditions (49), (50) *is called* ***the solution*** *to the Cauchy problem.*

The main result of the second paragraph consists of the following theorems.

Theorem 14. *Let* $\psi, f \in H$ *and* $\varphi \in D(A^{\frac{1}{\beta}})$. *Then there exists a unique solution of Cauchy problem* (49), (50) *and it has the form:*

$$u(t) = \sum_{k=1}^{\infty}[\varphi_k E_{\beta,1}(-\lambda_k t^{\beta}) + \psi_k t E_{\beta,2}(-\lambda_k t^{\beta}) + f_k t^{\beta} E_{\beta,\beta+1}(-\lambda_k t^{\beta})]v_k, \qquad (51)$$

where the series is convergent in H *for all* $t \geq 0$.

We will consider inverse problem. Let $\beta \in (1,2)$ is unknown parametr. The main purpose of this paragraph is to study the inverse problem of determining this parameter. Naturally, an additional condition is needed to determine the β parameter. We get the following condition as an additional condition:

$$U(\beta;t_0) \equiv (u(t_0), v_{k_0}) = d_0, \quad t_0 \geq T_0, \qquad (52)$$

where d_0 is given number, T_0 is a positive number defined below and $k_0 \geq 1$ is an arbitrary integer such that $f_{k_0}^2 + \varphi_{k_0}^2 + \psi_{k_0}^2 \neq 0$ (obviously such a number exists because f, φ and ψ are both non-zero), v_{k_0} is a eigenfunction.

(49) - (50) is called *the inverse problem* of finding the solution of the problem (52) satisfying the additional condition.

Definition 6. *If the function* $u(t)$ *is a solution of the problem (52) satisfying the condition (52), the pair* $\{u(x,t), \beta\}$, $\beta \in [\beta_1, \beta_2]$, *is called the solution of the inverse problem.*

Lemma 3. *Let*

$$\psi, f \in H; \ \varphi \in D(A^{\frac{1}{\beta_1}}), \qquad (53)$$

$$\| \psi \| + \| \varphi - A^{-1} f \| \neq 0 \qquad (54)$$

Then there exists a number $T_0 = T_0(k_0)$ *such that for all* $t_0 \geq T_0$ *the function* $U(\beta;t_0)$ *is strictly monotone on the interval* $\beta \in [\beta_1, \beta_2]$.

This lemma obviously implies the following main result of this paper.

Theorem 15. *Let conditions* (53) *and* (54) *are satisfied.Then a unique solution exists* ***the inverse problem*** *for all* $t_0 \geq T_0$.

Reamark. *If condition (54) of theorem is not fulfilled,* $\psi = 0$ *and* $\varphi = A^{-1} f$, *then for all* k *there are* $\psi_k = 0$ *and* $\varphi_k = f_k / \lambda_k$. *In this case, the only solution of the Cauchy problem has the following form*

$$u(t) = \sum_{k=1}^{\infty} \frac{f_k}{\lambda_k} v_k$$

that is, the expression does not depend on β. *Therefore, the function* $U(\beta;t_0)$ *does not depend on* β *either. Thus, in this case, the reverse issue under study does not make sense.*

In the third paragraph of the second chapter, we study the order of the fractional derivative and the inverse problem of determining the right side of the equation at the same time.

Let Ω be an arbitrary N- dimensional domain with a sufficiently smooth boundary $\partial\Omega$. Suppose that the function that determines the equation of the limit $\partial\Omega$ in the given coordinate system consists of a twice continuously differentiable function.

Consider the initial-boundary value problem pf the form:

$$\begin{cases} \partial_t^\rho u(x,t) - \Delta u(x,t) = f(x), x \in \Omega, \quad 0 < t < T; \\ Bu(x,t) = \dfrac{\partial u(x,t)}{\partial n} = 0, \quad x \in \partial\Omega, \quad 0 < t < T; \\ \lim\limits_{t\to 0} J_t^{1-\rho} u(x,t) = \varphi(x), \quad x \in \overline{\Omega}, \end{cases} \tag{55}$$

where Δ – is the Laplace operator, n is the unit outward normal vector to $\partial\Omega$ and $\varphi(x)$ is a given function.

In this work we will consider the following additional information on the solution at a time instant t_0:

$$U(\rho, t_0) \equiv \frac{1}{|\Omega|^{1/2}} \int_\Omega u(x,t_0)dx = d_0 \tag{56}$$

To identify the source function as in the cases listed above the following equality

$$u(x,T) = \psi(x), \quad x \in \overline{\Omega}, \tag{57}$$

Is considered as an additional condition. Obviously, if we put $t_0 = T$ in condition (56), then it does not give any new information and,therefore,we assume that $t_0 < T$. The main result of this work shows that with this choice, conditions (56) and (57) guarantee in (55) both the existence and uniqueness of both the unknown order of the derivative and the source function.

Definition 7. *The triple* $\{u(x,t), f(x), \rho\}$ *if the functions* $u(x,t)$, $f(x)$ *and the parameter* ρ *with the properties*

1. $\rho \in (0,1)$,
2. $f(x) \in C(\overline{\Omega})$,
3. $\partial_t^\rho u(x,t), \quad \Delta u(x,t) \in C(\overline{\Omega} \times (0,T))$,
4. $J_t^{1-\rho} u(x,t) \in C(\overline{\Omega} \times [0,T])$,

Satisfying all the conditions of problem (55) and (56),(57) in the classical sense, is called a solution of the inverse problem. The function $u(x,t)$ *with these properties is called a solution of the forward problem.*

In accordance with the Fourier method, one should consider the following spectral problem:

$$-\Delta v(x) = \lambda v(x), \quad x \in \Omega;$$

$$Bu(x,t) \equiv \frac{\partial u(x,t)}{\partial n} = 0, \quad x \in \partial\Omega.$$

Since the boundary $\partial\Omega$ is twice differentiable, then this problem has a complete in $L_2(\Omega)$ set of orthonormal eigenfunctions $\{v_k(x)\}$, $k \ge 1$, and a countable set of nonnegative eigenvalues $\{\lambda_k\}$. Note, that $\lambda_1 = 0$, $v_1(x) = |\Omega|^{-1/2}$.

Futher, suppose, that given functions $\varphi(x)$ and $\psi(x)$ satisfy the following conditions:

a) $\varphi(x) \in C^{[\frac{N}{2}]}(\Omega)$, $D^{\alpha}\varphi(x) \in L_2(\Omega)$, $|\alpha| = [\frac{N}{2}] + 1$;

b) $B\varphi(x) = B(\Delta\varphi(x)) = ... = B(\Delta^{[\frac{N}{2}]}\varphi(x)) = 0, \quad x \in \partial\Omega$;

c) $\psi(x) \in C^{[\frac{N}{2}]+1}(\Omega)$, $D^{\alpha}\psi(x) \in L_2(\Omega)$, $|\alpha| = [\frac{N}{2}] + 2$;

d) $B\psi(x) = B(\Delta\psi(x)) = ... = B(\Delta^{[\frac{N}{2}]+1}\psi(x)) = 0, \quad x \in \partial\Omega$.

It should be noted that conditions (a) and (b) ensure the existence and uniqueness of the solution of the forward, and if the function $\psi(x)$ from additional condition (57) satisfies the conditions (c) and (d), then, as follows the source function $f(x)$ exists and is unique.

Let satisfied following condition

$$\varphi_1^2 + \psi_1^2 \ne 0. \tag{58}$$

we choose the parameter t_0 in extra condition (56) in the following way. If $\varphi_1 \cdot \psi_1 \le 0$then $t_0 \in (1,T)$ and otherwise

$$t_0 \in (1,T) \cap \begin{cases} \left(1, \dfrac{\varphi_1}{\psi_1} \cdot T\right) & agar \quad \dfrac{\varphi_1}{\psi_1} \cdot T > 1; \\ (2(\ln T + 1)\dfrac{\varphi_1}{\psi_1} \cdot T, T) & agar \quad \dfrac{\varphi_1}{\psi_1} \cdot T \le 1. \end{cases} \tag{59}$$

This is the main result of this paragraph.

Theorem 16. *Let conditions* (a) - (d) *and* (58) - (59) *be satisfied. Then the inverse problem has a unique solution* $\{u(x,t), f(x), \rho\}$ *if and only if*

$$\min\left\{\psi_1, \varphi_1\left[1 - \frac{t_0}{T}\right] + \frac{t_0\psi_1}{T}\right\} < d_0 < \max\left\{\psi_1, \varphi_1\left[1 - \frac{t_0}{T}\right] + \frac{t_0\psi_1}{T}\right\}$$

The unique solutions the inverse problem (55) - (57) *have the form*

$$u(x,t) = \sum_{k=1}^{\infty}[\varphi_k t^{\rho-1} E_{\rho,\rho}(-\lambda_k t^{\rho}) + f_k t^{\rho} E_{\rho,\rho+1}(-\lambda_k t^{\rho})]v_k(x), \tag{60}$$

$$f(x)=\sum_{k=1}^{\infty}\frac{\psi_k}{T^{\rho}E_{\rho,\rho+1}(-\lambda_k T^{\rho})}v_k(x)-\sum_{k=1}^{\infty}\frac{\varphi_k E_{\rho,\rho}(-\lambda_k T^{\rho})}{TE_{\rho,\rho+1}(-\lambda_k T^{\rho})}v_k(x), \qquad (61)$$

where the series converge uniformly and absolutely.

In the third chapter, we deal with boundary value problems for the equations $\frac{\partial^k u}{\partial y^k}=(-1)^k A(x,D)u$, ($k=1,2$) where the boundary condition is given by the fractional derivative, where $A(x,D)$ is a non-negative elliptic differential operator and the boundary condition is given by the B_y^{ρ} operator defined by the positive real ρ fractional derivative. In particular, the limit operator B_y^{ρ} can be given by Marchaud, Grunwald-Letnikov or Liouville-Weyl fractional derivatives.

Consider the following boundary problem:

$$u_{yy}(x,y)=A(x,D)u(x,y), \quad x\in\Omega, \quad y>0, \qquad (62)$$

$$B_j u(x,y)=\sum_{|\alpha|\le m_j} b_{\alpha,j}(x)D^{\alpha}u(x,y)=0, \quad 0\le m_j\le m-1,\ j=1,2,...,l;\ x\in\partial\Omega, (63)//////$$

$$|u(x,y)|\to 0, \quad y\to\infty, \quad x\in\Omega, \qquad (65)$$

where $\varphi(x)$ and $b_{\alpha,j}(x)$ coefficient are given functions.

Definition 8. *If the functions* $u(x,y)$ *with the following properties* $u_{yy}(x,y)$, $A(x,D)u(x,y)\in C(\bar{\Omega}\times(0,\infty))$, $u(x,y)$, $B_y^{\rho}u(x,y)\in C(\bar{\Omega}\times[0,\infty))$ *and satisfies all the conditions of the problem* (62) - (65), *this function is called a* ***(classical)*** *solution of the boundary value problem* (62) - (65).

If we use the Fourier method to solve the problem (62) - (65), we come to the following spectral problem:

$$A(x,D)v(x)=\lambda v(x) \quad x\in\Omega; \qquad (66)$$

$$B_j v(x)=0, \quad j=1,2,...,l; \quad x\in\partial\Omega.. \qquad (67)$$

In the work of S. Agmon, the compactness of the inverse operator, the $\partial\Omega$ limit of the domain Ω and the coefficients of the operators A and B_j (66) - (67) is shown in the spectral problem $L_2(\Omega)$, an orthonormal complete system of eigenfunctions $\{v_k(x)\}$, $k\ge 1$ and a set of non-negative eigenvalues λ_k found the necessary conditions to guarantee its existence. In what follows, we assume that these conditions are met. Let $\lambda=0$ eigenvalue is a multiple root of $k_0, \lambda_k=0$, $k=1,2,...,k_0$. Suppose that the boundary function $\varphi(x)$ satisfies the following orthogonality conditions

$$\varphi_k=\int_{\Omega}\varphi(x)v_k(x)dx=0, k=1,2,...,k_0. \qquad (68)$$

Theorem 17. *Let the conditions* $\varphi(x)\in D(\hat{A}^{\tau})$, $\tau>\frac{N}{2m}$ *and (68) be fulfilled. Then the correct problem (62) - (65) has a unique solution in the following form*

$$u(x,y)=\sum_{k=k_0+1}^{\infty}\frac{1}{(\sqrt{\lambda_k})^{\rho}}\varphi_k v_k(x)e^{-\sqrt{\lambda_k}y}, \tag{69}$$

where the series converges absolutely and uniformly in $x\in\overline{\Omega}$ *for all* $y\in[0,\infty)$.

The second paragraph of Chapter III deals with the following issue.

$$u_t(x,t)+A(x,D)u(x,t)=0,\quad x\in\Omega,\quad t>0, \tag{70}$$

$$B_j u(x,t)=\sum_{|\alpha|\le m_j}b_{\alpha,j}(x)D^{\alpha}u(x,t)=0,\quad 0\le m_j\le m-1,\ j=1,2,...,l;\ x\in\partial\Omega, \tag{71}$$

$$B_t^{\rho}u(x,+0)=\varphi(x),\quad \rho>0,\ x\in\overline{\Omega}, \tag{72}$$

where $\varphi(x)$ and coefficients $b_{\alpha,j}(x)$ are functions.

Theorem 18. *Suppose that conditions (68) and* $\varphi(x)\in D(\hat{A}^{\tau}),\tau>\frac{N}{2m}$. *Then the correct problem (70) - (72) has a unique solution in the following form*

$$u(x,t)=\sum_{k=k_0+1}^{\infty}\frac{1}{\lambda_k^{\rho}}\varphi_k v_k(x)e^{-\lambda_k t}, \tag{73}$$

where the series converges absolutely and uniformly in $x\in\overline{\Omega}$ *for all* $t\in[0,\infty)$.

Chapter IV is dedicated to the study of exact and inverse problems for integral and fractional equations of the Barenblatt-Jeltov-Kochina type. In the first paragraph of the fourth chapter, the existence and uniqueness of the solution of the Cauchy problem for fractional equations of the Barenblatt-Jeltov-Kochina type is shown. At the same time, the solution of the inverse problem of finding the right side of the equation exists and is also proven to be unique.

In the second paragraph of the fourth chapter, the solution of the correct and inverse problems for the Barenblatt-Zheltov-Kochina equation, where the boundary condition is given by a fractional derivative, is available and its uniqueness is proved.

Consider the following Cauchy problem:

$$\begin{cases} D_t^{\rho}u(t)+A(D_t^{\rho}u(t))+Au(t)=f, & 0<t\le T;\\ u(+0)=\varphi, \end{cases} \tag{74}$$

where $\varphi, f\in H$.

Definition 9. *If the function* $u(t)\in C((0,T];H)$ *with the following properties* $D_t^{\rho}u(t), A(D_t^{\rho}u(t)),\ Au(t)\in C((0,T);H)$ *and satisfies all the conditions of the problem* (74), *this function is called a solution of the Cauchy problem (74).*

Theorem 19. *Let* $\varphi\in D(A),\ f\in H$. *Then there exists a unique solution of the Cauchy problem*(74) *and has the form*

$$u(t)=\sum_{k=1}^{\infty}\left[\varphi_k E_{\rho,1}(-\mu_k t^{\rho})+\frac{f_k}{1+\lambda_k}t^{\rho}E_{\rho,\rho+1}(-\mu_k t^{\rho})\right]v_k\ , \qquad (75)$$

where f_k, φ_k *are Fourier coefficient,* $\mu_k=\dfrac{\lambda_k}{1+\lambda_k}$.

To study the inverse problem, we need an additional condition. We use the following condition as an additional condition:

$$u(\tau)=\Psi,\quad 0<\tau\le T\ , \qquad (76)$$

where τ is a fixed point.

In problem (74), (76), the problem of finding the pair $\{u(t),f\}$ of the functions $u(t)$ and f is called ***the inverse problem*** of finding the right side of the equation.

Now we present the main result of this point.

Theorem 20. *Let* $\varphi,\ \Psi\in D(A)$. *Then there exists a unique solution* $\{u(t),f\}$ *of the inverse problem* (74), (76) *have the forms*

$$u(t)=\sum_{k=1}^{\infty}\left[\varphi_k E_{\rho,1}(-\mu_k t^{\rho})+\frac{f_k}{1+\lambda_k}t^{\rho}E_{\rho,\rho+1}(-\mu_k t^{\rho})\right]v_k\ , \qquad (77)$$

and $f=\sum\limits_{k=1}^{\infty}f_k v_k$, *where* $f_k=\left(\dfrac{\psi_k}{\tau^{\rho}E_{\rho,\rho+1}(-\mu_k\tau^{\rho})}-\dfrac{\varphi_k E_{\rho,1}(-\mu_k\tau^{\rho})}{\tau^{\rho}E_{\rho,\rho+1}(-\mu_k\tau^{\rho})}\right)(1+\lambda_k)$.

In the second paragraph of the fourth chapter, correct and inverse problems for Barenblatt-Zheltov-Kochina type equations, where the boundary condition is given by a fractional derivative, are studied.

Consider the following problem:

$$\begin{cases} u_t(t)+A(u_t(t))+Au(t)=0, & 0<t\le T;\\ B_t^{\rho}u(+0)=\varphi, \end{cases} \qquad (79)$$

where the boundary condition is given using the B_t^{ρ} operator defined by the positive real ρ fractional derivative. In particular, the boundary operator B_t^{ρ} can be given by Marchaud, Grunwald-Letnikov, Liouville-Weyl fractional derivatives.

Theorem 21. *Let* $\varphi\in D(A)$. *Then the problem (79) has a unique solution and the form*

$$u(t)=\sum_{k=1}^{\infty}\frac{\varphi_k}{\mu_k^{\rho}}e^{-\mu_k t}v_k\ , \qquad (80)$$

where $\varphi_k=(\varphi,v_k)$ *are Fourir coefficient of the function* φ .

In this paragraph, it is also inverse to find the marginal function the issue has also been studied. For this, the following condition is an additional condition we use:

$$u(\tau)=\Psi,\quad 0<\tau\le T\ , \qquad (81)$$

where τ is fixed point.

Theorem 22. *Let $\Psi \in D(A)$. Then there is a unique solution $\{u(t),\varphi\}$ of the inverse problem (79), (81) and it has the following form*

$$u(t)=\sum_{k=1}^{\infty}\frac{\varphi_k}{\mu_k^{\rho}}e^{-\mu_k t}v_k \ , \tag{82}$$

and $\varphi=\sum_{k=1}^{\infty}\varphi_k v_k$, where $\varphi_k=\Psi_k\mu_k^{\rho}e^{\mu_k\tau}$.

In chapter V, the problems of controlling processes given by fractional equations in a rectangular area using the source function are studied.

In the following Caputo sense, the fraction is orderly

$$D_t^{\alpha}u=u_{xx}+u_{yy}+f(x,y,t),\quad 0<x<l_1,\quad 0<y<l_2,\ 0<t<T \tag{83}$$

of the equation

$$u(0,y,t)=0, u(l_1,y,t)=0, u(x,0,t)=\mu(t), u(x,l_2,t)=0, \mu(0)=0,\ 0\le t\le T \tag{84}$$

boundary and

$$u(x,y,0)=\varphi(x,y),\qquad 0\le x\le l_1, 0\le y\le l_2, \tag{85}$$

initial condition consider the problem of finding a satisfactory solution, where $\varphi(x,y)$ is a given function, a bounded, $\mu(t)$ monotone function, $\mu'(t)\le M_1$, $\mu(0)=0$ and $T>0$ are a fixed number. Let enter the mark $\Omega=(0,l_1)\times(0,l_2)$.

Theorem 23. *Let $f(x,y,t)\in C_{x,y,t}^{2,2,1}(\bar{\Omega}\times[0,T])$, $\varphi(x,y)\in C_{x,y}^{2,2}(\bar{\Omega})$. If the functions $f(x,y,t)$ and $\varphi(x,y)$ are their second-order derivatives with respect to x,y and their first-order derivative with respect to t is equal to zero at the boundary of the domain Ω, and f_{xxy}''', f_{xyy}''', φ_{xxy}''', φ_{xyy}''' are bounded functions, and the functions $f_{xxyy}^{(4)}$, $\varphi_{xxyy}^{(4)}$, f_{txy}''' are integrable at $\bar{\Omega}$, then (83) - (85) has a solution in the field Ω and it has the following form*

$$u(x,y,t)=U(x,y,t)+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\bar{f}_{nm}(t)+\varphi_{nm}E_{\alpha,1}(-\lambda_{nm}^2t^{\alpha})\right]sin\alpha_n x\cdot sin\beta_m y, \tag{86}$$

where

$$U(x,y,t)=\begin{cases}\dfrac{l_2-y}{l_2}\mu(t), & if\qquad 0<x<l_1;\\ 0, & if\quad x=0\ and\ x=l_1.\end{cases},\quad \lambda_{nm}=\sqrt{\left(\frac{\pi n}{l_1}\right)^2+\left(\frac{\pi m}{l_2}\right)^2},$$

$$\bar{f}_{nm}(t)=\frac{4}{l_1l_2}\int_0^t\int_0^{l_1}\int_0^{l_2}\left[f(\xi,\eta,\tau)+\frac{\eta-l_2}{l_2}D^{\alpha}\mu(t)\right]\sin\alpha_n\xi\sin\beta_m\eta\times$$
$$\times(t-\tau)^{\alpha-1}E_{\alpha,\alpha}(-\lambda_{nm}^2(t-\tau)^{\alpha})d\eta d\xi d\tau,$$

$$\varphi_{nm}=\frac{4}{l_1l_2}\int_0^{l_1}\int_0^{l_2}\varphi(\xi,\eta)\sin\frac{\pi n\xi}{l_1}\sin\frac{\pi m\eta}{l_2}d\eta d\xi,\quad \alpha_n=\frac{\pi n}{l_1},\ \beta_m=\frac{\pi m}{l_2}.$$

When solving control problems, we assume that the function f does not depend on t and $\mu(t)=0$. Suppose a function $\Psi(x,y)$ is given. It is necessary to find the source function $f(x,y)$ such that the solution of problem (83) - (85) satisfies the following condition:

Problem 1. Let $t=\theta$ satisfy the following equality in a minute

$$u(x,y,\theta)=\Psi(x,y). \tag{87}$$

In other words, we should choose the source function $f(x,y)$ in such a way that the resulting temperature in Ω environment at time $t=\theta$ should be distributed as $\Psi(x,y)$.

Problem 2. Let $[0,T]$ satisfy the following equality in the time interval

$$\int_0^T u(x,y,t)dt=\Psi(x,y). \tag{88}$$

Problem 3. Let the sum of the values of t_1 and t_2 in minutes satisfy the following equality

$$u(x,y,t_1)+u(x,y,t_2)=\Psi(x,y). \tag{89}$$

Suppose that the functions $\varphi(x,y)$ and $\Psi(x,y)$ satisfy the following conditions.

a) function $\varphi(x,y)\in C_{x,y}^{2,2}(\bar{\Omega})$ and its second-order derivatives are equal to zero at the boundary of the domain Ω and $\varphi_{xxy}^{'''}$, $\varphi_{xyy}^{'''}$ the functions are bounded, and the function $\varphi_{xxyy}^{(4)}$ is integrable in the domain Ω.

b) $\Psi(x,y)\in C_{x,y}^{4,4}(\bar{\Omega})$ is a function and $0\le i+j\le 7$, if $i+j$ is even, then the derivatives $\frac{\partial^{i+j}}{\partial x^i \partial y^j}\Psi(x,y)$ are equal to zero at the boundary of the domain Ω, and if $i+j$ is odd, then the derivatives $\frac{\partial^{i+j}}{\partial x^i \partial y^j}\Psi(x,y)$ are finite at the boundary of the domain Ω, and the $\frac{\partial^8}{\partial x^4 \partial y^4}\Psi(x,y)$ function is integrable in the domain Ω.

Theorem 24. *Let functions φ and Ψ satisfy conditions a) and b), respectively. Then, in order for the solution of the problem* (83) - (85) *to satisfy the condition* (87), *it is sufficient that the function $f(x,y)$ is determined by the following form*

$$f(x,y)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{\Psi_{nm}}{\theta^\alpha E_{\alpha,\alpha+1}(-\lambda_{nm}^2\theta^\alpha)}-\frac{\varphi_{nm}E_{\alpha,1}(-\lambda_{nm}^2\theta^\alpha)}{\theta^\alpha E_{\alpha,\alpha+1}(-\lambda_{nm}^2\theta^\alpha)}\right] sin\alpha_n x\cdot sin\beta_m y\ , \tag{90}$$

where φ_{nm} and Ψ_{nm} are Fourier coefficients.

Theorem 25. *Let functions φ and Ψ satisfy conditions a) and b), respectively. Then the solution of problem* (83) - (85) *is condition* (88) *it is sufficient that the function $f(x,y)$ is defined by the following formula to satisfy*

$$f(x,y)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{\Psi_{nm}}{T^{\alpha+1}E_{\alpha,\alpha+2}(-\lambda_{nm}^2T^{\alpha})}-\frac{\varphi_{nm}E_{\alpha,2}(-\lambda_{nm}^2T^{\alpha})}{T^{\alpha}E_{\alpha,\alpha+2}(-\lambda_{nm}^2T^{\alpha})}\right]sin\alpha_n x\cdot sin\beta_m y\,. \quad (91)$$

Theorem 26. *Let functions* φ *and* Ψ *satisfy conditions a) and b), respectively. Then, in order for the solution of the problem* (83) - (85) *to satisfy the condition* (89), *it is sufficient that the function* $f(x,y)$ *is determined by the following form*

$$f(x,y)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{\Psi_{nm}}{t_2^{\alpha}E_{\alpha,\alpha+1}(-\lambda_{nm}^2t_2^{\alpha})+t_1^{\alpha}E_{\alpha,\alpha+1}(-\lambda_{nm}^2t_1^{\alpha})}-\right.$$
$$\left.-\frac{\varphi_{nm}\left[E_{\alpha,1}(-\lambda_{nm}^2t_2^{\alpha})+E_{\alpha,1}(-\lambda_{nm}^2t_1^{\alpha})\right]}{t_2^{\alpha}E_{\alpha,\alpha+1}(-\lambda_{nm}^2t_2^{\alpha})+t_1^{\alpha}E_{\alpha,\alpha+1}(-\lambda_{nm}^2t_1^{\alpha})}\right]sin\alpha_n x\cdot sin\beta_m y\;. \quad (92)$$

CONCLUSION

The dissertation work is devoted to solving various direct and inverse problems for fractional differential and partial differential equations and solving control problems for fractional equations. The following conclusions can be made about the results of the research.

The main difference from the previously studied works is that the nonlocal conditions for the time under consideration in this work were not studied in other works, that is, the problems with such nonlocal conditions for partial differential equations given by fractional derivatives were not studied before. In the study of these problems, the influence of the α parameter, which was involved in the nonlocal condition, on the uniqueness of the solution of the studied problems was determined. In studying the exact problem, we show that the solution is not unique for some values of the parameter α and that orthogonality conditions for the given functions φ and f should be required to ensure the existence of the solution.

In order to determine the order of the fractional derivative, a different condition was used, that is, projection to a specific function. Researchers used the condition used in previous works only to show the uniqueness of the solution. The condition used here is sufficient to ensure not only the uniqueness of the solution, but also its existence.

Conditions for finding the inverse problem for finding the right side of the equation and the boundary function are derived.

A method of using Krasnoselsky's lemma has been developed to obtain the classical solution of high-order and multivariable fractional equations with an elliptic part.

The derivation of the orthogonality conditions for the boundary function is explained so that the solution of the equations given by the derivative of the fractional order of the boundary condition is unique and it exists

Barenblatt-Zheltov-Kochina and other types of fractional equations have been developed for solving direct and inverse problems.

For differential equations with fractional order differential equations, the methods of controlling various processes given by this fractional equation using the source function are shown.

НАУЧНЫЙ СОВЕТ DSc.02/30.12.2019.FM.86.01 ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ИНСТИТУТЕ МАТЕМАТИКИ ИМЕНИ В.И. РОМАНОВСКОГО

НАЦИОНАЛЬНЫЙ УНИВЕРСИТЕТ УЗБЕКИСТАНА

ФАЙЗИЕВ ЮСУФ ЭРГАШЕВИЧ

ПРЯМЫЕ И ОБРАТНЫЕ ЗАДАЧИ ДЛЯ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ В ЧАСТНЫХ ПРОИЗВОДНЫХ С ЭЛЛИПТИЧЕСКОЙ ЧАСТЬЮ ВЫСОКОГО ПОРЯДКА.

01.01.02- Дифференциальные уравнения и математическая физика

АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА (DSc) ФИЗИКО-МАТЕМАТИЧЕСКИХ НАУК

ТАШКЕНТ – 2022

Тема докторской (DSc) диссертации зарегистрирована в Высшей аттестационной комиссии при Кабинете Министров Республики Узбекистан B2022.3.DSc/FM198.

Диссертация выполнена в Национальном университете Узбекистана имени Мирзо Улугбека.

Автореферат диссертации на трех языках (узбекский, русский, английский (резюме)) размещен на веб-странице Научного совета (http://kengash.mathinst.uz) и на Информационно-образовательном портале «Ziyonet» (www.ziyonet.uz).

Научный консультанть:	**Ашуров Равшан Раджабович** доктор физико-математических наук, профессор
Официальные оппоненты:	**Турметов Ботирхан Худайбергенович** доктор физико-математических наук, профессор
	Тахиров Жозил Останович доктор физико-математических наук, профессор
	Каримов Эркинжон Тулкинович доктор физико-математических наук, старший научный сотрудник
Ведущая организация:	**Института космофизических исследований и распространения радиоволн (Россия).**

Защита диссертации состоится « 29 » ноября 2022 г. в 16:00 часов на заседании научного совета DSc.02/30.12.2019.FM.86.01 при Институте Математики имени В.И. Романовского (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 9. Тел.: (+99871)-207-91-40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz).

С диссертацией можно ознакомиться в Информационно-ресурсном центре Института Математики имени В.И. Романовского (регистрационный номер 148). (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 9. Тел.: (+99871)-207-91-40).

Автореферат диссертации разослан « 11 » ноября 2022 г.
(протокол рассылки № 2 от « 11 » ноября 2022 г.).

У.А. Розиков
Председатель Научного совета по присуждению ученых степеней, д.ф.-м.н., профессор

Ж.К. Адашев
Ученый секретарь Научного совета по присуждению ученых степеней, д.ф.-м.н., старший научный сотрудник

А. Азамов
Председатель Научного семинара при Научном совете по присуждению ученых степеней, д.ф.-м.н., академик

ВВЕДЕНИЕ (аннотация докторской диссертации)

Целью исследования является доказательство существования единственного решения прямых и обратных задач для дифференциальных уравнений в частных производных и дробных дифференциальных уравнений с произвольным эллиптическим оператором.

Объектом исследования являются дробные производные в смысле Капуто, Римана-Лиувилля, Грюнвольда-Летникова, Вейля, уравнения с дробными производными по времени, функции Миттаг-Леффлера.

Научная новизна исследования заключается в следующем:

доказаны, существование и единственность решение удовлетворяещее условие нелокальности уравнений субдиффузии дробного порядка, и по определению правой части уравнения и граничной функции решений обратных задач;

доказано существование и единственность решения прямых и обратных задач нахождения порядка производной и правой части уравнения в уравнениях в частных производных дробного порядка;

доказано существование и единственность решения гиперболических и параболических уравнений, краевое условие которое дается с производным дробным порядком;

доказано, существование и единственность решений прямых и обратных задач для уравнений дробного порядка типа Баренблатта-Желтова-Кочиной и для дифференциальных уравнений типа Баренблатта-Желтова-Кочиной, граничные условия которых содержат дробные производные;

построено решение первой краевой задачи для уравнений в частных производных дробного порядка и найдены условия как выбрать функцию источника, чтобы заранее по заданной функции в прямоугольной области процесс зависело от времени.

Внедрение результатов исследования на основании полученных результатов для прямой и обратной задач для уравнений в частных производных дробного порядка с эллиптической частью высшего порядка:

из условий существование и единственности решения гиперболических и параболических уравнений, при краевых условиях, заданных дробными производными использовались для построения моделей систем в фундаментальном проекте на тему «Построение адекватных вычислительных моделей гиперболических систем» под номером ОТ-Ф-4-28 (справка Национального университета Узбекистана от 30 сентября 2022 года за номером 04/11-5960). Применение научных результатов позволило построить расчетные модели систем дифференциальных уравнений гиперболического типа;

решение первой краевой задачи для уравнений в частных производных дробного порядка и управление с помощью функции источника были использованы в исследовании процесса распределении радона в атмосфере в рамках гранта Президента Российской Федерации по теме «Поддержка молодых российских ученых» под номером МД-758.2022.1.1 (справка

Камчатского государственного университета от 27 сентября 2022 года за номером 39-12). Применение научных результатов позволило перераспределить радоны в системе грунт атмосфера;

решения прямых и обратных задач уравнений дробного порядка типа Баренблатта-Желтова-Кочиной и решения дифференциальных уравнений типа Баренблатта-Желтова-Кочиной, заданные производной дробного порядка использованы для решения краевых задач в фундаментальном проекте «Высокоточные вычислительные алгоритмы для решения задач газовой динамики, фильтрации и динамики транспортных потоков с использованием современных суперкомпьютеров» под номером МРУ-ОТ-30/2017 (справка Национального университета Узбекистана от 4 октября 2022 года за номером 04/11-6042). Использование научных результатов позволило построить модели для расчета краевых задач, задач фильтрации и управления.

Объем и структура диссертации. Диссертация состоит из введения, пяти глав, заключения и списка использованной литературы. Объем диссертации составляет 222 страниц.

E‘LON QILINGAN ISHLAR RO‘YXATI
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ
LIST OF PUBLISHED WORKS

I bo‘lim (part 1; 1 часть)

1. Ashurov R., Fayziev Yu. On the nonlocal problems in time for time-fractional subdiffusion equations // Fractal and Fractional. 2022. V. 6 (41), (3. Scopus IF=0,644).
2. Ashurov R., Fayziev Yu. Determination of fractional order and source term in a fractional subdiffusion equation // Eurasian Mathematical Journal, 2022. V.13(1), p. 19-31, (3. Scopus IF=0,335).
3. Ashurov R., Fayziev Yu. On the uniqueness of solutions of two inverse problems for two inverse problems for the subdiffusion equation // Fractional Differential Calculus, 2022. V. 12(1), p. 77 - 90, (3. Scopus).
4. Fayziev Yu., Buvaev Q., Juraev D., Nuralieva N., Sadullaeva Sh. The inverse problem for determining the source function in the equation with the Riemann-Liouville fractional derivative // Global and Stochastic Analysis (GSA), 2022. V.9(2), p. 43-52, (3. Scopus IF=0,248).
5. Ashurov R.R. Fayziev Yu.E. Uniqueness and existence for inverse problem of determining an order of time-fractional derivative of subdiffusion equation // Lobachevskii Journal of Mathematics, 2021. V. 42(3), p. 508-516, (3. Scopus IF=0,378).
6. Ashurov R.R. Fayziev Yu.E. On some boundary value problems for equations with boundary operators of fractional order // International Journal of Applied Mathematics (IJAM), 2021. V. 34(2), p. 283-295, (3. Scopus IF=0,27).
7. Ashurov R.R. Fayziev Yu.E. Inverse Problem for Finding the Order of the Fractional Derivative in the Wave Equation // Mathematical Notes, 2022. V. 110(5), p. 842-852, (3. Scopus IF=0,580).
8. Fayziev Yu.E. On the control of heat conduction // IIUM Engineering Journal, 2018. V.19(1), p. 168-177, (3. Scopus IF =0,307)
9. Ashurov R., Fayziev Yu. On the nonlocal problems in time for subdiffusion equations with the Riemann-Liouville derivatives. Bulletin of the Karaganda University. Mathematics series, 2022. V. 106 (2), p. 18 – 37, (1. Web of Science).
10. Ашуров Р.Р., Файзиев Ю.Э. О построении решений линейного дифференциального уравнения с дробными производными и постоянными коэффициентами // Уз. Мат. Журнал, 2017. N 3, с. 3-21, (01.00.00; № 6)
11. Файзиев Ю.Э., Кучкаров А.Ф., Насирова Д., Об одной задаче управления процессом теплопроводности в прямоугольнике // Вестник НУУз, N 2, 2017, с. 239-244, (01.00.00; № 8)
12. Файзиев Ю.Э., Халилова Н. Об одной задаче управления процессом теплопроводности // Вестник НУУз, 2016. N 2/1, с. 49-54. (01.00.00; № 8)
13. Fayziev Yu. E. On a control problem for the subdiffusion equation in a

rectangular domain // Bull. Inst. Math., 2022. V. 5(4), p. 97-102. (01.00.00; № 17)

II bo‘lim (part 2; 2 часть)

14. Файзиев Ю.Э. Об одной задаче управления для уравнения субдиффузии с дробным производным в смысле Капуто // Вестник КРАУНЦ. Физ.-мат. науки. 2022. Т.39. № 2. с. 62-78, MathSciNet.
15. Ashurov R.R., Fayziev Yu.E., On the Cauchy problems for a Barenblatt-Zheltov-Kochina type time-fractional subdiffusion equations // Республиканская научная конференция “Операторные алгебры, неассоциативные структуры и смежные проблемы” 14–15 сентября 2022 года Ташкент, с. 164-165.
16. Fayziev Yu.E. On some initial - boundary value problems for the parabolic equations with the initial operators of fractional order // Республиканская научная конференция “Операторные алгебры, неассоциативные структуры и смежные проблемы” 14–15 сентября 2022 года Ташкент, с.175-177.
17. Ashurov R.R., Fayziev Yu.E., Tokhtaeva N., Khushvaktov N.Kh. On the non-local problems for a Barenblatt-Zheltov-Kochinatype time-fractional equations with hilfer derivative // Proceedings of the International Scientific Conference “Actual problems of mathematics, mechanics and informatics” dedicated to the 80th anniversary of Professor T.G. Mustafin, 2022 September, 8-9, p. 83-84.
18. Fayziev Yu.E. Forward and inverse problems for the Barenblatt-Zheltov-Kochina type equation with the initial condition given by a fractional derivative // Proceedings of the International Scientific Conference “Actual problems of mathematics, mechanics and informatics” dedicated to the 80th anniversary of Professor T.G. Mustafin, 2022, September, 8-9, p. 92-93.
19. Ашуров Р.Р., Файзиев Ю.Э., Обратная задача по определению порядка дробной производной в волновом уравнении // XVI Международная научная конференция “Теория операторов и дифференциальные уравнения” 20-25 сентября 2021 года г.Владикавказ, с. 68-69.
20. Файзиев Ю.Э., Машарипова Г.А., Об одной задаче управления для уравнений дробного порядка в смысле Капуто // Республиканская научная конференция с участием зарубежных ученых, Сарымсаковские чтения 16-18 сентября 2021 года Ташкент, с. 136 - 137.
21. Файзиев Ю.Э., Нуралиева Н.Ш., Единственность и существование для уравнений дробного порядка в смысле Римана-Лиувилля // Республиканская научная конференция с участием зарубежных ученых, Сарымсаковские чтения 16-18 сентября 2021 года Ташкент, с. 138 - 139.
22. Ashurov R.R., Fayziev Yu. E. On some boundary value problems for equations with boundary operators of fractional order // Республиканская научная конференция с участием зарубежных ученых, Сарымсаковские чтения 16-18 сентября 2021 года Ташкент, с. 173 - 175.

23. Ashurov R., Fayziyev Yu. On the non-local problems in time for time-fractional subdiffusion equation // Of the VII International scientific conference, “Modern problems of applied mathematics and information technologies Al-Khwarizmi” , 15-17 November 2021, p. 108.
24. Fayziev Yu., Buvaev Q., Nuralieva N., Sadullaeva Sh. The inverse problem for determining the source function in the equation with the Riemann-Liouville fractional derivative // International scientific conference, “Contemporary mathematics and its application”, 19-21 November 2021, Tashkent, p. 112-113
25. Файзиев Ю.Э., Машарипова Г.А., О существовании решения одного уравнения дробного порядка // Международная научная конференция “Современные проблемы математической физики и математического моделирования ”, 3-4 декабря 2021 года, Карши, с. 141 - 142.
26. Ashurov R., Fayziev Yu. On the non-local problems in time for time-fractional subdiffusion equations // International conference “Non-local boundary problems and related problems of mathematical biology, informatics and physics”, 5-9 December 2021, Nalchik, p.215.
27. Fayziyev Yu., Xudoyqulova M. O‘zgaruvchi koeffitsiyentli kasr tartibli tenglama uchun Koshi masalasi. “Математика, механика ва интеллектуал технологиялар” мавзусида ёш олимларнинг илмий-амалий конференцияси, 21-22 апреля 2022 года, Тошкент, б. 175-176
28. Fayziyev Yu., To‘xtayeva N., Kenjayeva G. Kasr tartibli hosilali tenglamada manba funksiyasini topish bo‘yicha teskari masala // “Математика, механика ва интеллектуал технологиялар” мавзусида ёш олимларнинг илмий-амалий конференцияси, 21-22 апрель 2022 йил, Тошкент, б. 171-172
29. Ashurov R., Fayziev Yu., Sulaymonov I. Inverse problem for determining a source function in the higher order equation with the Gerasimov-Caputo fractional derivative // Annual International April Mathematical Conference, 6 - 8 April 2022, Kazakhstan, p. 125-126
30. Ashurov R., Fayziyev Yu. On the non-local problems in time for time-fractional subdiffusion equation // Амалий математика ва ахборот технологияларининг замонавий муаммолари, Халқаро илмий-амалий анжуман материаллари, 11-12 май 2022 йил, Бухоро, б.151.
31. Ashurov R., Fayziyev Yu., Maxmasoatov M. Issiqlik tarqalishi tenglamasi uchun nolokal chegaraviy masala // Амалий математика ва ахборот технологияларининг замонавий муаммолари, Халқаро илмий-амалий анжуман материаллари, 11-12 май 2022 йил, Бухоро, б. 142-143.
32. Ashurov R., Fayziev Yu., Nosirova D., Amrullaeva D., Latipova Sh. On the non-local problems for a Boussinesq type time-fractional subdiffusion equations // Амалий математика ва ахборот технологияларининг замонавий муаммолари, Халқаро илмий-амалий анжуман материаллари, 11-12 май 2022 йил, Бухоро, б. 143.
33. Fayziyev Yu., Sulaymonov I. Hadamard kasr tartibli hosilali tenglamalar uchun Koshi masalasi // Амалий математика ва ахборот

технологияларининг замонавий муаммолари, Халқаро илмий-амалий анжуман материаллари, 11-12 май 2022 йил, Бухоро, б. 153-154.
34. Ashurov R., Fayziev Yu., Determination of fractional order and source term in a fractional subdiffusion equation // Fifth International Conference on Analysis and Applied Mathematics, Abstract book of the conference ICAAM, 23-30 September 2020, Mersin, p. 42.
35. Ashurov R., Fayziev Yu., Uniqueness and existence for inverse problem of determining // Computational models and technologies, August 24-25, 2020, p. 155-156.
36. Файзиев Ю.Э., Об одной задаче управления для уравнений дробного порядка // Неклассические уравнения математической физики и их приложения, 24 - 26 октября 2019 года, Ташкент, с. 174-175
37. Файзиев Ю.Э. Шакарова М. Д. Об одной задаче управления процессом теплопроводности в стержне // Актуальные проблемы и применения анализа, 4-5 октябрь 2019 года, Карши, с. 203-204
38. Файзиев Ю.Э., Носирова Д., Бутаева З. Об одной задаче управления процессом теплообмена // Республиканская научная конференция "Актуальные проблемы дифференциальных уравнений и их приложения", 15 - 17 декабрь 2017 года. Ташкент, с. 147-148.
39. Fayziev Yu., Nosirova D., On the control of the heat exchange process // Республиканская научная конференция "Актуальные проблемы динамических систем и их приложения", 1 - 3 мая 2017 г. Ташкент, с. 346-347.
40. Файзиев Ю.Э., Кучкаров А. Ф., Носирова Д., Об одной задаче управления процессом теплопроводности в прямоугольнике // Актуальные проблемы прикладной математике и информационных технологий - Аль-Хорезми, 9-10 ноября 2016 г. с. 346-347.
41. Файзиев Ю.Э., Халилова Н. Об одной задаче управления процессом теплопроводности // Республиканская научная конференция "Современные методы математической методы и их приложения". Ташкент, 15-17 апреля 2015 г. с. 126-128.
42. Ashurov R.R., Fayziev Yu.E., On construction of solution of linear fractional differentional equation with constant coefficients and the Riemann-Liouville derivatives // Республиканская научная конференция "Неклассические уравнения математической физики и их приложения", 23-25 октября 2014 г. Ташкент, с. 173-175.

Автореферат «_________________________________» журнали таҳририятида таҳрирдан ўтказилиб, ўзбек, рус ва инглиз тилларидаги матнлар ўзаро мувофиқлаштирилди.

Босмахона лицензияси:

9338

Бичими: 84x60 $^{1}/_{16}$. «Times New Roman» гарнитураси.
Рақамли босма усулда босилди.
Шартли босма табоғи: 4,5. Адади 100 дона. Буюртма № 59/22.

Гувоҳнома № 851684.
«Tipograff» МЧЖ босмахонасида чоп этилган.
Босмахона манзили: 100011, Тошкент ш., Беруний кўчаси, 83-уй.