O’ZBEKISTON RESPUBLIKASI OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

TOSHKENT TO’QIMACHILIK VA YENGIL SANOAT INSTITUTI

“MATEMATIKA VA INFORMATIKA” KAFEDRASI

«OLIY MATEMATIKA» FANINING

«DIFFERENSIAL TENGLAMALAR» BO’LIMI BO’YICHA

METODIK QO’LLANMA

(Barcha yo’nalishdagi bakalavr talabalari uchun)

Toshkent 2018

Annotatsiya

Ushbu metodik qo'llanmada differensial tenglamalar haqida tushuncha, ularga olib kelinadigan ba'zi masalalar, birinchi va yuqori tartibli differensial tenglamalar va ularning yechilish usullari keltirilgan.

Tuzuvchi: Texnika fanlari doktori, professor A.Z.Mamatov
Asssistent G'.A.Roziqov

Taqrizchilar: N. Jabborov – O'zMU, проф., f.m.f.d.
M. Saydamatov - TTYeSI, dots., f.m.f.n.

Toshkent to'qimachilik va yengil sanoat institutining "Matematika va informatika" kafedrasi yig'ilishida ma'qullandi va o'quv-uslubiy kengashiga tasdiqlash uchun tavsiya qulindi. Bayonnoma № ___ «___» ________2018 yil

Toshkent to'qimachilik va yengil sanoat instituti o'quv-uslubiy kengashida tasdiqlangan. Bayonnoma № _____ «___» ____________2018 yil

Mundarija

Differensial tenglamalar haqida tushuncha

Reja

1. Differensial tenglamalar haqida boshlangich tushunchalar.
2. Differensial tenglamalarga olib kelinadigan ba'zi masalalar.
3. Asosiy ta'riflar va tushunchalar.
4. Birinchi tartibli differensial tenglamalar (umumiy tushunchalar).

A D A B I YO T L A R

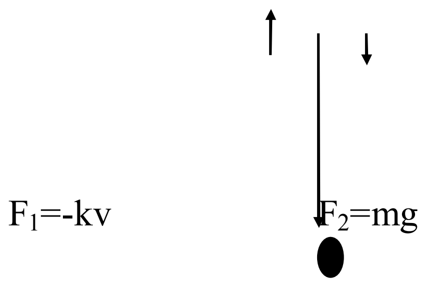
1. A.S. Piskunov. ifferensial va integral hisob. T. «O'qituvchi»,1974 y ,7 – 17 betlar
2. M.S.Salohitdinov, O'.N.Nasritdinov. Oddiy differensial tenglamalar. T. «Uzbekiston» , 1994 y., 6 – 9 betlar.
3. V.P.Minorskiy. Oliy matematikadan masalalar to'plami. T.«O'qituvchi», 1977, 224-228 betlar.
4. Е.У. Соатов «Олий математика» 5-жилд «Укитувчи», 1988.
5. http://docs.titli/uz/

Tabiatda uchraydigan turli jarayonlar (fizik, ximik, mexanik, biologik va boshqalar) o'z harakat qonunlariga ega. Ba'zi jarayonlar bir xil qonun bo'yicha sodir bo'lishi mumkin, bunday hollarda ularni o'rganish ancha yengillashadi. Ammo jarayonlarni tavsiflaydigan qonunlarni to'g'ridan-to'g'ri topish har doim ham mumkin bo'lavermaydi. Xarakterli miqdorlar va ularning hosilalari orasidagi munosabatlarni topish tabiatan yengil bo'ladi. Ko'pgina tabiiy va texnika masalalarini yechish shunday noma'lum funksiyalarni izlashga keltiriladiki, bunda bu funksiya berilgan hodisa yoki jarayonni ifodalab, ma'lum munosabatlar va bog'lanish esa shu noma'lum funksiya va uning hosilalari orasida beriladi. Mana shunday munosabat va qonunlar asosida bog'langan ifodalar differensial tenglamalarga misol bo'ladi.

1 - masala. Massasi m bo'lgan jism $V(0)=V_0$ boshlang'ich tezlik bilan biror balandlikdan tashlab yuborilgan. Jism tezligining o'zgarish qonunini toping. (1 - rasm)

Nyutonning ikkinchi qonuniga ko'ra $mdv/dt=F$

bu erda F - jismga ta'sir etayotgan kuchlarning yig'indisi (teng ta'sir etuvchi). Jismga faqat 2 ta kuch ta'sir etsin deb hisoblaylik: havoning qarshilik kuchi $F_1=-kv$, $k>0$; yerning tortish kuchi $F_2=mg$.



1-rasm

Demak, matematik nuqtai nazardan F kuch a) F_2 ga; b) F_1 ga; v) F_1+F_2 ga teng bo'lishi mumkin.

a)Agar $F=F_1$ bo'lsa, $mdv/dt=-kv$ tenglamaga ega bo'lamiz. Bunda $V(t)=V_0e^{-kt/m}$ bo'ladi.

b) $F=F_2$ bo'lsa, U holda birinchi tartibli $mdv/dt=mg$ differentsial tenglamaga egamiz. Bu tenglamani yechimini $V(t)=gt+c$ (c - ixtiyoriy o'zgarmas son) ko'rinishda ekanligini oddiy hisoblarda tekshirish mumkin. $V(0)=V_0$ bo'lgani uchun $c=V_0$ bo'lib, u holda izlangan qonun $V_1=gt+V_0$ ko'rinishida bo'ladi.

v) $F=F_1+F_2$ bo'lsin. Bu holda mdv/dt=mg-kv (k>0) tenglamaga kelamiz. Noma'lum funksiya

$$\vartheta(t) = Ce^{-\frac{k}{m}t} + \frac{mg}{k} \qquad \vartheta(0) = \vartheta_0$$

$$\vartheta(t) = (\vartheta_0 - \frac{mg}{k})e^{-\frac{mg}{k}t} + \frac{mg}{k}$$

ko'rinishida bo'ladi.

<u>1 – ta'rif</u>. Differensial tenglama deb erkli o'zgaruvchi x, noma'lum y=f(x) funksiya va uning u', u'',.....,$u^{(n)}$ hosilalari orasidagi bog'lanishni ifodalaydigan tenglamaga aytiladi.

Agar izlangan funksiya y=f(x) bitta erkli o'zgaruvchining funksiyasi bo'lsa, u holda differensial tenglama oddiy differentsial tenglama, bir nechta o'zgaruvchilarning funksiyasi bo'lsa $u=U(x_1, x_2,...., x_n)$ xususiy hosilali differensial tenglama deyiladi.

<u>2-ta'rif</u>. Differensial tenglamaning tartibi deb tenglamaga kirgan hosilaning eng yuqori tartibiga aytiladi.

<u>3-ta'rif</u>. Differensial tenglamaning yechimi yoki integrali deb differensial tenglamaga qo'yganda uni ayniyatga aylantiradigan har qanday y=f(x) funksiyaga aytiladi.

Birinchi tartibli differentsial tenglama umumiy holda quyidagi ko'rinishda bo'ladi.

$$F(x,y,y')=0 \qquad (1.1)$$

Agar bu tenglamani birinchi tartibli xosilaga nisbatan yechish mumkin bo'lsa, u holda

$$y'=f(x,y) \qquad (1.2)$$

tenglamaga ega bo'lamiz. Odatda, (1.2) tenglama hosilaga nisbatan yechilgan tenglama deyiladi. (1.2) tenglama uchun yechimning mavjudligi va yagonaligi haqidagi teorema o'rinli :

Teorema. Agar (1.2) tenglamada $f(x,y)$ funksiya va undan y bo'yicha olingan df/dy xususiy hosila X0Y tekisligidagi (x_0,y_0) nuqtani o'z ichiga oluvchi biror sohada uzluksiz funksiyalar bo'lsa, u holda berilgan tenglamaning $y(x_0)=y_0$ shartnii qanoatlantiruvchi birgina $y=\varphi(x)$ yechimi mavjud.

$x=x_0$ da $y(x)$ funksiya y_0 songa teng bo'lishi kerak degan shart boshlang'ich shart deyiladi:

$$y(x_0)=y_0$$

4 – ta'rif. Birinchi tartibli differensial tenglamaning umumiy yechimi deb bitta ixtiyoriy C o'zgarmas miqdorga bog'liq quyidagi shartlarni qanoatlantiruvchi

$$y=\varphi(x,c)$$

funksiyaga aytiladi:

a) bu funksiya differensial tenglamani ixtiyoriy c da qanoatlantiradi;

b) $x=x_0$ da $y=y_0$ boshlang'ich shart har qanday bo'lganda ham shunday $c=c_0$ qiymat topiladiki, $y=\varphi(x,c_0)$ funksiya berilgan boshlang'ich shartni qanoatlantiradi.

5 – ta'rif. Umumiy yechimni oshkormas holda ifodalovchi $F(x,y,c)=0$ tenglik (1.1) differentsial tenglamaning umumiy integrali deyiladi.

6 – ta'rif. Ixtiyoriy c - o'zgarmas miqdorda $c=c_0$ ma'lum qiymat berish natijasida $y=\varphi(x,c)$ umumiy yechimdan hosil bo'ladigan har qanday $y=\varphi(x,c_0)$ funksiya xususiy yechim deyiladi. $F(x,y,c_0)$ - xususiy integral deyiladi.

7-ta'rif. (1.1) differensial tenglama uchun $dy/dx=c=const$ munosabat bajariladigan nuqtalarning geometrik o'rni berilgan differensial tenglamaning izoklinasi deyiladi.

Misol. Ushbu $x^2 + y^2 - 2cx = 0$ $(-\infty, +\infty)$ intervalda aniqlangan funksiya, quyidagi $x^2 - y^2 + 2xyy' = 0$ differensial tenglama yechimi ekanini ko‘rsatamiz.

Yechish: haqiqatan dastlabki tenglikni differensiallab, hosil qilamiz:

$$2x - 2yy' - 2c = 0\ ,\ y' = \frac{c - x}{y}.$$

Endi hosilaning topilgan ifodasini berilgan differensial tenglamaga qo‘ysak, unda quyidagicha bo‘ladi:

$$x^2 - y^2 + 2cx - 2x^2 = -(x^2 + y^2 - 2cx) \equiv 0.$$

Demak, $x^2 + y^2 - 2cx = 0$ funksiya berilgan differensial tenglama yechimi ekan.

Nazorat savollari:

1. Differensial tenglama deb qanday tenglamaga aytiladi?
2. Differensial tenglamalarga olib kelinadigan masalalar.
3. Differensial tenglamaning tartibi deb nimaga aytiladi?
4. Differensial tenglamaning yechimi deb nimaga aytiladi?
5. Differensial tenglamaning umumiy yechimi, xususiy yechimi deb qanday yechimga aytiladi?

O’zgaruvchilari ajraladigan tenglamalar.
Birinchi tartibli tenglamalar

Reja

1. O’zgaruvchilari ajralgan tenglamalar.
2. O’zgaruvchilari ajraladigan tenglamalar**.**
3. Birinchi tartibli tenglamalar:
   a) bir jinsli;
   b) chiziqli;
   v) Bernulli.

A D A B I YO T L A R

1. A.S.Piskunov. Differensial va integral hisob.
T. «O’qituvchi», 1974 y ,17 – 31 betlar.
2. L.E.Elsgolts. Differensialnie uravneniya i variatsionnoe ischislenie.
M., “Nauka”, 1969 g., s . 24 – 31.
3. L.S.Pontryagin. Differensialnie uravneniya i ix prilojeniya.
M., Nauka, 1965 g., s.15 – 20.
4. M.S.Salohitdinov, O’.N.Nasritdinov. Oddiy differensial tenglamalar. T. «Uzbekiston», 1994 y., 19 – 31 betlar .
6. V.P.Minorskiy. Oliy matematikadan masalalar to’plami. T. «O’qituvchi», 1977, 224-230 betlar.

O’ng tomoni faqat x hamda faqat y o’zgaruvchilarning funksiyalari ko’paytmasidan iborat tenglama o’zgaruvchilari ajraladigan differensial tenglama deyiladi, ya’ni

$$\frac{dy}{dx} = f(x)g(y) \qquad (2.1)$$

bu tenglikni dx ga ko'paytirib va $g(y) \neq 0$ ga bo'lib

$$\frac{dy}{g(y)} = f(x)dx$$

tenglikni hosil qilamiz.

Uni integrallab yechimni topish mumkin:

$$\int \frac{dy}{g(y)} = \int f(x)dx + C$$

Misol. $dy/dx = -\,{}^{y}\!/_{x}$

tenglama yechilsin.

Yechish. O'zgaruvchilarni ajratib

$$dy/y = -dx/x$$

integrallaymiz:

$$\int \frac{dy}{y} = -\int \frac{dx}{x} + C,\ yani\ \ln|y| = -\ln|x| + \ln|c|, \quad y = \frac{c}{x}.$$

<u>1 – ta'rif.</u> Agar λ ning har qanday qiymatida

$$f(\lambda x, \lambda y) = \lambda^k f(x,y)$$

tenglik bajarilsa, f(x,y) funksiya x va y o'zgaruvchilarga nisbatan k - tartibli bir jinsli funksiya deyiladi.

<u>2 – ta'rif.</u> Agar birinchi tartibli

$$\frac{dy}{dx} = f(x,y) \qquad (1.2)$$

differensial tenglamaning o'ng tomoni - f(x,y) 0-tartibli bir jinsli funksiya bo'lsa, u holda (1.2) tenglama bir jinsli tenglama deyiladi.

f(x,y) nolinchi tartibli bir jinsli bo'lsa, u holda ixtiyoriy λ uchun f(λx,λy)=f(x,y) bo'ladi. Xususan,

$$f(1,\frac{y}{x}) = f(x,y)$$

u holda (1.2) tenglama

$$y^1 = f(1,\frac{y}{x}) = \varphi(\frac{y}{x}) \qquad (2.2)$$

Bu tenglamani yechish uchun y/x=U deb olamiz.

U holda y=Ux, y’=U’x+U.

Bularni (2.2) ga qo’yib

$$U'x + U = \varphi(U)$$

$$x\frac{dU}{dx} = \varphi(U) - U$$

o’zgaruvchilari ajraladigan tenglamaga kelamiz.

$$\frac{dU}{\varphi(U)-U} = \frac{dx}{x}$$

Uni integrallaymiz

$$\int\frac{dU}{\varphi(U)-U} = \int\frac{dx}{x} + \ln C, \int\frac{dU}{\varphi(U)-U} = \ln Cx$$

Integrallagandan so’ng U ni o’rniga u/x ni qo’ysak, (2.2) tenglamaning umumiy integrali hosil bo’ladi.

Misol.

$$\frac{dy}{dx} = \frac{y}{x+y} \qquad tenglama \qquad \cdot echil\text{sin}.$$

Echish.

$$f(x,y) = \frac{y}{x+y} \quad f(\lambda x,\lambda y) = \frac{\lambda y}{\lambda x+\lambda y} = \frac{\lambda y}{\lambda(x+y)} = \frac{y}{x+y} = f(x,y)$$

- 0-tartibli bir jinsli funksiya.

Tenglamani quyidagicha yozib olamiz:

$$\frac{dy}{dx}=\frac{\frac{y}{x}}{\frac{x+y}{x}},\quad \frac{dy}{dx}=\frac{\frac{y}{x}}{1+\frac{y}{x}}$$

$\frac{y}{x}=U$ *deb*, $y=Ux$, $y'=U'x+U$ *larni* xisobga olib

$$U'x+U=\frac{U}{1+U} \quad \text{yoki} \quad U'x=-\frac{U^2}{1+U}$$

uzgaruvchilari ajraladigan differentsial tenglamani hosil qilamiz.

Natijada

$-\frac{1+U}{U^2}du=\frac{dx}{x}$ tenglamalarni xosil qilamiz. Uni integrallab

$$\int\left(-\frac{1+U}{U^2}\right)du=\int\frac{dx}{x}+\ln|C| \ , \quad \frac{1}{U}-\ln U=\ln Cx \ , \quad x=y\ln Cy.$$

<u>3–ta’rif.</u> Birinchi tartibli chiziqli tenglama deb hosilaga nisbatan chiziqli bo’lgan ushbu

$$y'+\rho(x)y+q(x)=0$$

ko’rinishdagi tenglamaga aytiladi, bunda $\rho(x),q(x)\in C\ (R^1)$.

(2.3) tenglama yechimini

$$y=U(x)v(x)=Uv$$

ko’rinishida izlaymiz.

y’=U’v+Uv’ ni tenglamaga qo’yib

$$U'V+V'U+\rho(x)UV+q(x)=0$$

$$U'V+(V'+\rho(x)V)U+q(x)=0$$

V(x) funksiyani

$$V'(x)+\rho(x)V(x)=0$$

tenglama o'rinli bo'ladigan qilib tanlab olamiz.

Bu tenglamani yechamiz:

$$\frac{dV}{V}=-\rho(x)dx\ , \quad \ln(V)=-\int\rho(x)dx+\ln|C|\ , \ V(x)=C_1e^{-\int\rho(x)dx}\ ,$$

$$V(x)=e^{-\int\rho(x)dx}.$$

bo'lsin. Topilgan V(x) ni (2.4) tenglamaga qo'yamiz va hosil bo'lgan tenglamani yechamiz:

$$U^{э}e^{-\int\rho(x)dx}+q=0 \quad \frac{dU}{dx}=-qe^{\int\rho(x)dx}$$

$$U(x)=-\int q(x)e^{\int\rho(x)dx}dx+C$$

Natijada

$$y(x)=UV\ =e^{-\int\rho(x)dx}(C-\int qe^{\int\rho(x)dx}dx)$$

berilgan tenglamaning umumiy yechimini hosil qilamiz.

Misol.

$y'+xy-x^3=0$ *tenglama echil* sin .

Echish ..Bu erda

$\rho(x)=x,\ q(x)=-x^3$.

U xolda echim

$$y=e^{-\int xdx}(C-\int(-x^3)e^{\int xdx}dx)=e^{-\frac{x^2}{2}}(C+\int x^3e^{\frac{x^2}{2}}dx)=$$

$$=e^{-\frac{x^2}{2}}(C+(x^2-2)e^{\frac{x^2}{2}})=Ce^{-\frac{x^2}{2}}+x^2-2.$$

<u>4 – ta'rif.</u>

$$\frac{dy}{dx}+\rho(x)y=q(x)y^n \qquad (2.5)$$

ko'rinishdagi tenglamaga, bunda n≠0, n≠1, Bernulli tenglamasi deyiladi.

Bu tenglama quyidagicha almashtirish yordamida yechiladi. Tenglamaning barcha hadlarini $y^n \neq 0$ ga bo'lib

$$y^{-n}\frac{dy}{dx}+\rho(x)y^{1-n}=q(x) \qquad (2.6)$$

tenglamaga ega bo'lamiz.

$$z=y^{1-n}$$

almashtirish bajaramiz. U holda

$$\frac{dz}{dx}=(1-n)y^{-n}\frac{dy}{dx},$$
$$\frac{dy}{dx}=\frac{1}{1-n}y^n\frac{dz}{dx}.$$

Bu qiymatlarni (2.6) ga qo'yib

$$\frac{dz}{dx}+(1-n)\rho z=(1-n)Q$$

chiziqli tenglamani hosil qilamiz. Buning umumiy integralini topib hamda z o'rniga y^{1-n} ifodani qo'yib, Bernulli tenglamasining umumiy yechimini hosil qilamiz.

Misollar:

Quyidagi differensial tenglamalarni yeching:

a) $yy'=\frac{-2x}{\cos y}$. b) $y'=y^{2/3}$.

Yechish. a) $yy'=\frac{-2x}{\cos y}$ tenglamani soddalashtiramiz:

$$y\cos y\cdot\frac{dy}{dx}=-2x \Leftrightarrow y\cos y dy=-2xdx$$

Oxirgi tenglama o'zgaruvchilari ajralgan, uni integrallaymiz:

$$\int y\cos y dy=-2\int xdx$$

Chap tarafdagi integral bo'laklab integrallash usuli yordamida hisoblanadi:

$$\int y\cos y\,dy = \begin{Bmatrix} u = y; & dv = \cos y\,dy; \\ du = dy; & v = \sin y \end{Bmatrix} = y\sin y - \int \sin y\,dy = y\sin y + \cos y$$

Natijada

$$y\sin y + \cos y + x^2 = C$$

umumiy integralni hosil qilamiz.

Javob: $y\sin y + \cos y + x^2 = C$.

b) Berilgan $y' = y^{2/3}$ tenglamadan o'zgaruvchilari ajralgan

$$y^{-2/3}dy = dx$$

tenglamani hosil qilamiz.

Bu tenglamani integrallaymiz:

$$\int y^{-2/3}dy = \int dx$$

Bundan $3y^{1/3} - x = C$ ko'rinishdagi umumiy integralga ega bo'lamiz.

Natijada $y = \frac{1}{27}(x+C)^3$ umumiy yechimni topamiz. $y^{2/3} = 0$ algebraik tenglamaning $y = 0$ yechimi berilgan tenglamaning maxsus yechimi bo'lishini qayd etamiz.

Javob: $y = \frac{1}{27}(x+C)^3$, $y = 0$.

Nazorat savollari:

1. Qanday differensial tenglamalarga o'zgaruvchilari ajraladigan differensial tenglama deyiladi?
2. Qanday differensial tenglamalarga bir jinsli differentsial tenglamalar deyiladi?
3. Qanday differensial tenglamalarga birinchi tartibli chiziqli differensial tenglamalar deyiladi?
4. Qanday differensial tenglamaga Bernulli tenglamasi deyiladi?

To'la differensial tenglama. Hosilaga nisbatan yechilmagan birinchi tartibli tenglamalar

Reja

1. To'la differensial tenglama.
2. Integrallovchi ko'paytuvchi.
3. Hosilaga nisbatan yechilmagan birinchi tartibli tenglamalar.

a. Lagranj tenglamasi.

b. Klero tenglamasi.

A D A B I YO T L A R

1. A.S.Piskunov, Differensial va integral hisob. T. «Uqituvchi», 1974 y, 31 – 49 betlar.
2. L.E.Elsgolts, Differensialnie uravneniya i variatsionnoe ischislenie. M., "Nauka", 1969 g., s .32–38, 68–82.
3. L.S.Pontryagin, Differensialnie uravneniya i ix prilojeniya. M., Nauka, 1965 g., s. 13–25.
4. M.S.Salohitdinov, O'.N.Nasritdinov. Oddiy differensial tenglamalar. T. «Uzbekiston», 1994 y., 32 - 42 betlar.
5. V.P.Minorskiy, Oliy matematikadan masalalar to'plami. T. «O'qituvchi», 1977, 230-234 betlar.

To'la differensial tenglama

<u>1- ta'rif</u> Agar

$$M(x,y)dy+N(x,y)dy=0 \qquad (3.1)$$

tenglamada M(x,y), N(x,y) funksiyalar uzluksiz, differensiallanuvchi bo'lsa, va

$$\partial M/\partial y=\partial N/\partial x \qquad (3.2)$$

munosabat bajarilsa, (3.1) to'la differensial tenglama deyiladi, bunda $\partial M/\partial y$, $\partial N/\partial x$ - uzluksiz funksiyalar.

(3.1) tenglamani integrallashga o'tamiz.

(3.1) tenglamaning chap tomoni biror U(x,y) funksiyaning to'la differensiali bo'lsin deb faraz qilamiz, ya'ni

$$M(x,y)dx+N(x,y)dy=dU(x,y), \quad dU/dx=(\partial U/\partial x)dx+(\partial U/\partial y)dy$$

u holda

$$M=\partial U/\partial x, \qquad N=\partial U/\partial y \qquad (3.3)$$

$\partial U/\partial x=M$ munosabatdan

$$U(x,y)=\int_{x_0}^{x} M(x,y)dx+\varphi(y) \text{ ni}$$

topamiz. Bu tenglikni har ikki tomonini *u* bo'yicha differensiallab natijani N (x,y) ga tenglaymiz:

$$\frac{dU}{dy}=\int_{x_0}^{x}\frac{\partial M}{\partial y}dx+\varphi'(y)=N(x,y)$$

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

bo'lgani uchun

$$\int_{x_0}^{x}\frac{\partial N}{\partial x}dx+\varphi^{э}(y)=N(x,y)\ ,yani \quad N(x,y)\Big|_{x_0}^{x}+\varphi(x)=N(x,y).$$

yoki

$$N(x,y)-N(x_0,y)+\varphi'(y)=N(x,y).$$

Demak

$$\varphi'(y) = N(x_0, y)$$

$$\text{yoki } \varphi(y) = \int_{y_0}^{y} N(x_0, y)dy + C_1$$

Shunday qilib

$$U(x,y) = \int_{x_0}^{x} M(x,y)dx + \int_{y_0}^{y} N(x_0,y)dy + C_1$$ ko'rinishda bo'ladi.

dU=0 bo'lganda , U(x,y)=C.

Demak, umumiy integral

$$\int_{x_0}^{x} M(x,y)dx + \int_{y_0}^{y} N(x_0,y)dy = C \qquad (3.4)$$

Integrallovchi ko'paytuvchi

(3.1) tenglamada (3.2) munosabat bajarilmasin. Ba'zan shunday $\mu(x,y)$ funksiyani tanlab olish mumkinki, (3.1) tenglamani shu funksiyaga ko'paytirganda tenglamaning chap tomoni biror funksiyaning to'la differensialini ifodalaydi. Bunday tanlangan μ(x,y) funksiyaga (3.1) tenglamaning integrallovchi ko'paytuvchisi deyiladi.

μ(x,y) ni topish usuli: (3.1) ni μ(x,y) ga ko'paytiramiz

$$\mu Mdx+\mu Ndy=0$$

Keyingi tenglama to'la differensialli tenglama bo'lishi uchun (3.2) munosabat bajarilishi zarur va etarli:

$$\frac{\partial(\mu M)}{\partial y} = \frac{\partial(\mu N)}{\partial x}$$

$$\mu\frac{\partial M}{\partial y} + M\frac{\partial \mu}{\partial y} = \mu\frac{\partial N}{\partial x} + N\frac{\partial \mu}{\partial x}$$

$$M\frac{\partial \mu}{\partial y} - N\frac{\partial \mu}{\partial x} = \mu\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right).$$

Oxirgi tenglamaning har ikki qismini μ ga bo'lib

$$M\frac{\partial \ln \mu}{\partial y} - N\frac{\partial \ln \mu}{\partial x} = \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \qquad (3.5)$$

munosabatni hosil qilamiz. (3.5) tenglamani qanoatlantiruvchi har qanday μ(x,y) funksiya (3.1) tenglamaning integrallovchi ko'paytuvchisi bo'ladi. (3.5) tenglama μ(x,y) funksiyaga nisbatan xususiy hosilali tanglama.

Ma'lum shartlar bajarilganda bu tenglama yechimga ega. Lekin umumiy holda (3.5) ni yechish (3.1) ni integrallashga qaraganda ancha murakkab. Ba'zi bir xususiy hollardagina μ(x,y) ni topish mumkin:

1) μ(x,y) faqat y o'zgaruvchiga bog'liq bo'lsin: μ=μ(y)

U holda

$$\frac{d\ln\mu(y)}{dx} = 0 \quad \text{va} \quad (3.5) \text{ dan } \frac{d\ln\mu}{dy} = \frac{dN/dx - dM/dy}{M}$$

oddiy differensial tenglama hosil bo'ladi.

Bu tenglamani yechib $\mu(y) = e^{-\int\left(\frac{dM}{dy} - \frac{dN}{dx}\right)/Mdy}$ ni topamiz.

2) μ=μ(x) bo'lsa

$$\mu(x) = e^{\int\left(\frac{dM}{dy}-\frac{dN}{dx}\right)/N dx}$$

bo’ladi.

Misol. Ushbu tenglama integrallansin.

$$(2x-y)dx+(4y-x)dy=0$$

Yechish. Bu tenglamani quyidagicha yozamiz:

$$2xdy+4ydy-(ydx+xdx)=0$$

ravshanki tenglamaning chap tomoni $u(x,y)=x^2+2y^2-xy$ funksiyaning to‘liq differensiali. Shuning uchun tenglamani

$$d(x^2+2y^2-xy)=0$$

ko‘rinishda yozish mumkin, bundan

$$x^2+2y^2-xy=c$$

umumiy integralni topamiz, c - ixtiyoriy o‘zgarmas.

Misol. $y(1+xy)dx-xdy=0$ tenglamani yeching.

Yechish. Bu tenglamani quyidagicha yozib olamiz:

$$ydx-xdy+xy^2dx=0$$

Tenglamaning ikkala tomonini $y^2\neq 0$ ga bo‘lib olsak, chap tomoni to‘la differensial bo‘ladi

$$\frac{ydx-xdy}{y^2}+xdx=0,\ d\left(\frac{x}{y}+\frac{x^2}{2}\right)=0\ \Rightarrow \frac{x}{y}+\frac{x^2}{2}=c$$

Demak, tenglamaning umumiy integrali $2x+x^2y=cy$ bo‘ladi.

Misol. $y(1+xy)dx-xdy=0$ tenglamaning $u(1)=1$ boshlang‘ich shartni qanoatlantiruvchi yechimi topilsin.

Yechish. Tenglamani quyidagicha yozib olamiz:

$$ydx-xdy+xy^2dx=0.$$

Endi buning ikkala tomonini $y^2\neq 0$ ga bo‘lib olamiz

$$\frac{ydx-xdy}{y^2}+xdx=0\ \Rightarrow\quad d\left(\frac{x}{y}+\frac{x^2}{2}\right)=0$$

to‘la differensial tenglama hosil bo‘ldi.

Bundan: $\frac{x}{y}+\frac{x^2}{2}=c,\quad\Rightarrow\ 2x+x^2y=cy$ umumiy integralni yozib olamiz, bunda c - ixtiyoriy o‘zgarmas.

Endi x=1 da u=1 deb olsak, c=3 bo‘ladi va izlanayotgan xususiy yechim hosil bo‘ladi

$$2x + x^2 y = 3y\,.$$

Hosilaga nisbatan yechilmagan birinchi tartibli tenglamalar

Hosilaga nisbatan yechilmagan birinchi tartibli tenglama umumiy holda quyidagi ko’rinishda bo’ladi:

$$F(x,y,y')=0 \qquad (3.6)$$

Agar bu tenglamani y’ ga nisbatan yechish mumkin bo’lsa, u holda bir yoki bir necha tenglama hosil bo’ladi.

$$y'=f(x,y) \qquad (i=1,2...)$$

Bu tenglamalarni integrallab, (3.6) tenglama yechimlarini hosil qilamiz.

Lekin (3.6) tenglamani har doim y' ga nisbatan oson yechilmaydi va y' ga nisbatan tenglamalar sodda integrallanmasligi mumkin. Shuning uchun (3.6) tenglamani boshqa usullarda integrallash qulay bo’ladi. Quyidagi hollarni qaraymiz.

1. $F(y')=0$, bunda hech bo’lmaganda tenglamaning bitta $y'=k_i$ yechimi mavjud bo’lsin. Tenglama x va y o’zgaruvchilarga bog’liq bo’lmaganligi sababli, k_i=const. $y'=k_i$ ni integrallab $y=k_ix+C$ yoki $k_i=(y-C)/x$. k_i berilgan tenglama yechimi ekanligidan

$$F((y-C)/x)=0$$

qaralayotgan tenglama yechimi bo’ladi.

Misol $(y')^7 - (y')^5 + y' + 3 = 0$ tenglama integrali

$$((y-C)/x)^7-((y-C)/x)^5+(y-C)/x+3=0$$

2. (3.6) tenglama quyidagi ko'rinishda bo'lsin.

$$F(x,\ y')=0 \qquad (3.7)$$

Agar tenglamani y' ga nisbatan yechish qiyin bo'lsa, u holda t parametr kiritish bilan (3.7) ikkita tenglamaga keltiriladi:

$x=\varphi(t)$ va $y'=\Psi(t)$

$dy=y'dx$ ekanligidan, $dy=\psi(t)\,\varphi'(t)dt$,

bundan $y=\int\psi(t)\varphi'(t)dt+C.$

Demak, (3.7) tenglama yechimlari parametrik holda quyidagi ko'rinishda bo'ladi

$$x=\varphi(t)$$

$$y=\int\psi(t)\varphi'(t)dt+C.$$

Agar (3.7) x ga nisbatan yechilsa, ya'ni $x=\varphi(y')$, u holda deyarli har doim $y'=t$ deb parametr kiritish qulay. U holda

$x=\varphi(t)$ $\quad dy=y'dx,$ $\quad y=\int t\varphi'(t)dt+C.$

Misol. $x=(y')^3-y'-1$ tenglamani yechish uchun

$y'=t$ deb belgilash kiritamiz. Natijada

$$x=t^3-t-1$$

Bu erdan $dy=y'dx=t(3t^2-1)$, $\quad y=3t^4/4-t^2/2+C_1$

Demak,

$$\begin{cases} x=t^3-t-1 \\ y=\dfrac{3t^4}{4}-\dfrac{t^2}{2}+C_1 \end{cases}$$

sistema izlanayotgan integral chiziqning parametrik formasini ifodalaydi.

3. (3.6) quyidagi ko'rinishda bo'lsin.

$$F(y,\ y')=0 \qquad (3.8)$$

Agar tenglamani y' ga nisbatan yechish qiyin bo'lsa, quyidagicha parametr kiritamiz: ó=φ(t), y'=Ψ(t)

Bu erdan dy=y'dx bo'lganligidan dx=dy/y'=φ'(t)dt/Ψ(t), b demak,

$$x = \int \frac{\varphi'(t)dt}{\psi(t)} + C$$

$$\begin{cases} y = \varphi(t) \\ x = \int \frac{\varphi'(t)}{\psi(t)} dt + C \end{cases}$$

izlanayotgan integral chiziqning parametrik tenglamasidir.

Xususiy holda, (3.8) tenglamani y ga nisbatan yechish mumkin bo'lsa, parametr deb y' olish qulay:

y=φ(y') da y'=t belgilash kiritsak y=φ(t),

dx=dy/y'=φ'(t)/t dt

$$x = \int \frac{\varphi'(t)}{t} dt + C$$

Lagranj tenglamasi

Lagranj tenglamasi deb

$$y = x\,\varphi(y') + \psi(y') \qquad (3.9)$$

ko'rinishdagi tenglamaga aytiladi.

Bu tenglama ham parametr kiritish bilan sodda integrallanadi:

y' =ρ deb,

y=xφ(ρ)+ψ(ρ)

tenglamani hosil qilamiz. Bu tenglamani x ga nisbatan differensiallab

$$\rho = \varphi(\rho) + [x\varphi'(\rho) + \psi'(\rho)]\frac{d\rho}{dx}, \text{ yoki}$$
$$[\rho - \varphi(\rho)]\frac{dx}{d\rho} = x\varphi'(\rho) + \psi'(\rho) \qquad (3.10)$$

Hosil bo'lgan tenglama x(ρ) va dx/dρ ga nisbatan chiziqli tenglamadir. Uni yechib F(x, ρ ,c)=0 ni hosil qilamiz. Demak, Lagranj tenglamasini yechimi

$$\begin{cases} y = x\varphi(\rho) + \psi(\rho) \\ \Phi(x, \rho, c) = 0 \end{cases}$$

parametrik ko'rinishda bo'ladi.

(3.10) tenglamani hosil qilishda $d\rho / dx \neq 0$ deb qaralgan edi. Demak, bunda ρ=const yechimlar, agar ular mavjud bo'lsa, yo'qotilgan edi. ρ=const bo'lsa, u holda (3.10_1) tenglama faqat $\rho - \varphi(\rho) = 0$, bo'lganda bajariladi.

Demak, agar $\rho - \varphi(\rho) = 0$ tenglama haqiqiy $r=r_i$ ildizlarga ega bo'lsa, yuqoridagi yechimlarga yana

$$y = x\,\varphi(\rho) + \Psi(\rho), \; \rho = \rho_i$$

yechimlarni ham qo'shish kerak bo'ladi.

Misol: Lagranj - o'zgarmasni variasiyalash usuli bilan ushbu

$$y' - y\sin x = \sin x \cos x$$

chiziqli differensial tenglamaning umumiy yechimini toping.

Yechish. Avvalo bu tenglamaga mos bir jinsli tenglamani yechamiz:

$$y' - y\sin x = 0$$
$$\frac{dy}{dy} = y\sin x \Rightarrow \frac{dy}{y} = \sin x dx$$

integrallaymiz:

$$\ln|y| = -\cos x + \ln|c| \Rightarrow y = C \cdot e^{-\cos x}$$

bu bir jinsli tenglamaning umumiy yechimi, bunda *C* – ixtiyoriy o'zgarmas.

Endi bu tenglikda *C=C(x)* deb berilgan differensial tenglamaning yechimini quyidagi ko'rinishda izlaymiz:

$$y = C(x) \cdot e^{-\cos x} \Rightarrow y' = C'(x)e^{-\cos x} + C(x) \cdot \sin x \cdot e^{-\cos x}$$

Endi berilgan tenglamaga *u* va y' ifodalarni qo‘yamiz:

$$C'(x) \cdot e^{-\cos x} + C(x)\sin x \cdot e^{-\cos x} - C(x)\sin x \cdot e^{-\cos x} = \sin x \cos x$$

yoki

$$C'(x) \cdot e^{-\cos x} = \sin x \cos x \Rightarrow C'(x) = \sin x \cos x e^{\cos x}.$$

Keyingi tenglikdan topamiz:

$$C(x) = \int \sin x \cos x \cdot e^{\cos x} dx + C_1$$

O‘ng tomondagi integralda *t=cosx* deb oson toppish mumkin

$$C(x) = -\cos x e^{\cos x} + e^{\cos x} + C.$$

Demak, berilgan chiziqli differensial tenglamaning umumiy yechimi

$$y = C(x) \cdot e^{-\cos x} = e^{-\cos x}(-\cos x e^{\cos x} + e^{\cos x} + C)$$

yoki

$$y = Ce^{-\cos x} - \cos x + 1$$

bo‘ladi, bunda *C* - ixtiyoriy o‘zgarmas.

Klero tenglamasi

ρ-φ(ρ)≡0 bo’lsin. dρ/dx ga bo’lishdan ρ=c, c=const yechimlar yo’qotilgan bo’ladi. Bu holda φ(y')=y' bo’lib, (3.9) tenglama

$$y = xy' + \Psi(y') \qquad (3.11)$$

ko’rinishiga keladi va bu tenglama- Klero tenglamasi deyiladi.

Bu teglamani yechish uchun y'=ρ deb belgilash kiritamiz.

Natijada y=xρ+Ψ(ρ) ni hosil qilamiz.

Bu tenglamani x bo’yicha differensiallab

$$\rho = \rho + x d\rho/dx + \Psi'(\rho) d\rho/dx$$

yoki

$$[x + \psi'(\rho)]\frac{d\rho}{dx} = 0$$

tenglamani hosil qilamiz. Bundan dρ/dx=0, demak ρ=C yoki x+Ψ’(ρ)=0.

$\rho=c$ da yechimdan

$$y=Cx+\Psi(c)$$

ikkinchi holda esa yechim

$$\begin{cases} y = x\rho + \psi(\rho) \\ x + \psi'(\rho) = 0 \end{cases}$$

ko'rinishda bo'ladi.

Nazorat savollari:

1. To'la differensial tenglama deb qanday differensial tenglamalarga aytiladi?
2. Integrallovchi ku'paytuvchi deb kanday funksiyaga aytiladi?
3. Xosilaga nisbatan yechilmagan differensial tenglamalar, umumiy ko'rinishi.
4. Xosilaga nisbatan yechilmagan differensial tenglamalar, xususiy xollari.
5. Lagranj tenglamasi.
6. Klero tenglamasi.

Yuqori tartibli differensial tenglamalar

Reja

1. Yuqori tartibli differensial tenglamalar.
2. Yuqori tartibli tartibi pasayadigan differensial tenglamalar.
3. O'zgarmas koeffitsientli bir jinsli chiziqli differensial tenglamalar.
4. O'zgarmas koeffitsientli bir jinsli bo'lmagan chiziqli differensial tenglamalar.

A D A B I YO T L A R

1. A.S.Piskunov. Differensial va integral hisob.
   T. «O'qituvchi», 1974 y, 54 – 100 betlar.
2. L.E.Elsgolts. Differensialnie uravneniya i variatsionnoe ischislenie.
   M., "Nauka", 1969 g., s. 85 – 124.
3. L.S.Pontryagin. Differentsialnie uravneniya i ix prilojeniya.
   M., Nauka, 1965 g., s. 41 – 66.
4. M.S.Salohitdinov, O'.N.Nasritdinov. Oddiy differensial tenglamalar.
   T. «Uzbekiston», 1994 y., 103 – 155 betlar.
5. V.P. Minorskiy. Oliy matematikadan masalalar to'plami.
   T. «O'qituvchi», 1977, 234-240 betlar.

Yuqori tartibli differensial tenglamalar

Ta'rif. $F(x,y,y',....,y^{(n)})=0$ ko'rinishdagi tenglamaga n - tartibli differensial tenglama deyiladi.

Ta'rif. n - tartibli differensial tenglamaning umumiy yechimi deb n ta c_1, c_2, ... c_n - ixtiyoriy o'zgarmas miqdorlarga bog'liq bo'lgan

$$y=\varphi\ (x, c_1, c_2, ... c_n)$$

funksiyaga aytiladi. Bu funksiya:

1) $c_1,...,c_n$ larning ixtiyoriy qiymatlarida tenglamani qanoatlantiradi;

2) berilgan $y(x_0)=y_0$, $y'(x_0)=y_1, \ldots, y^{(n-1)}(x_0)=y_{n-1}$ boshlang'ich shartda $c_1, c_2, \ldots, c_n$ larni shunday tanlash mumkinki,

$y= \varphi(x, c_1, c_2, \ldots c_n)$ funksiya bu boshlang'ich shartni qanoatlantiradi.

<u>Ta'rif.</u> Umumiy yechimdan $c_1, c_2, \ldots c_n$ miqdorlarning tayin qiymatlarida hosil bo'ladigan funksiya xususiy yechim deyiladi.

Yuqori tartibli tartibi pasayadigan differensial tenglamalar

1. **$y^{(n)}$=f(x) ko'rinishidagi tenglama**.

$y^{(n)}=(y^{(n-1)})'$ ni e'tiborga olib

$$y^{(n-1)} = \int_{x_0}^{x} f(x)dx + C_1,$$

ni hosil qilamiz, bunda x_0 x ning tayinlangan qiymati, c_1 - o'zgarmas miqdor.

Integrallashni shunday davom ettirib

$$y = \int_{x_0}^{x} \ldots \int_{x_0}^{x} f(x)dx \ldots dx + \frac{(x-x_0)^{n-1}}{(n-1)!}C_1 + \frac{(x-x_0)^{n-2}}{(n-2)!}C_2 + \ldots + C_n$$

ifodani hosil qilamiz.

Boshlang'ich shartlarni

$$y(x_0) = y_0,\; y'(x_0) = y_1,\; \ldots,\quad y^{(n-1)}(x_0) = y_{n-1}$$

qanoatlantiruvchi xususiy yechimni topish uchun

$$C_n=y_0,\quad C_{n-1}=y_1,\quad \ldots,\quad C_1=y_{n-1}$$

deb olish etarli.

2. y''=f(x,y) ko'rinishidagi tenglama.

y'=p deb, y''=p' ni xosil qilamiz.

Demak,

$$p' = f(x,y)$$

Bu tenglamani integrallab

$p = p(x, C_1)$ - umumiy yechimni topamiz.

$\frac{dy}{dx} = p$ munosabatdan esa $y = \int p(x, C_1)dx + C_2$ - umumiy yechimni xosil qilamiz.

3. $y^{(n)}(x) = f(x, y^{(n-1)})$ ko'rinishidagi tenglama ham $y^{(n-1)} = p$ deb parametr kiritish bilan

$$(\, y^{(n)} = p' \, ekanligidan, p' = f(x, p) - \,)$$

yuqorida o'rganilgan tenglamaga keltiriladi.

$y^{(n-1)} = p$ munosabatdan y ni topib, yechim xosil qilinadi.

4. $y'' = f(y, y')$ ko'rinishidagi tenglama.

Bu tenglamani yechish uchun $y' = p$ deb olamiz.

Ammo p ni y ning funksiyasi deb qaraymiz: p=p(y)

U xolda,

$$y'' = \frac{dp}{dx} = \frac{dp}{dy}\frac{dy}{dx} = p\frac{dp}{dy}.$$

y' va y'' larni berilgan tenglamaga qo'yib

$$p\frac{dp}{dy} = f(y, p)$$

birinchi tartibli differensial tenglamani xosil qilamiz. Bu tenglamani integrallab $p=p(y,c_1)$ yechimni va

$\frac{dy}{dx}=p$ munosabatdan

$$\frac{dy}{p(y,C_1)}=dx$$

tenglamani olamiz.

Bu tenglamani integrallab, dastlabki tenglamaning

$$F(x,y,c_1,c_2)=0$$

umumiy yechimini xosil qilamiz.

O'zgarmas koeffitsientli bir jinsli chiziqli differensial tenglamalar

Ta'rif.

$$a_0y^{(n)}+a_1y^{(n-1)}+..+a_{n-1}y'+a_ny=f(x) \qquad (4.2)$$

ko'rinishdagi tenglama n-tartibli chiziqli , o'zgarmas koeffitsientli differensial tenglama deyiladi, bunda

$a_0, a_1,..,a_{n-1},a_n$ – o'zgarmas miqdorlar, $a_0 \neq 0$.

Agar $f(x) \neq 0$ bo'lsa, bir jinsli bo'lmagan tenglama,

$$f(x) \equiv 0$$

bo'lsa, bir jinsli tenglama deyiladi.

1-teorema

y_1 va y_2 2- tartibli bir jinsli chiziqli

$$y''+a_1y'+a_2y=0 \qquad (4.3)$$

tenglamaning xususiy yechimlari bo'lsa, u xolda $y=y_1+y_2$ ham shu tenglamaning yechimi bo'ladi.

<u>2- teorema</u>

Agar y (4.3) tenglamaning yechimi bulsa , u xolda ***cy*** ham shu tenglamaning yechimi bo'ladi.

<u>Ta'rif</u>

Agar $x \in [a,b]$ da (4.3) tenglamaning 2 ta yechimining nisbati o'zgarmas miqdorga teng , ya'ni

$$\frac{y_1}{y_2} \neq const$$

bo'lsa y_1 va y_2 yechimlar $x \in [a,b]$ da chiziqli erkli yechimlar deyiladi, aks xolda chiziqli bog'lik yechimlar deyiladi .

<u>Ta'rif</u>

$$W(y_1 , y_2)=\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y'_2 - y'_1 y_2$$

- ko'rinishdagi determinant Vronskiy determinanti deyiladi.

<u>3- teorema</u>

Agar y_1 va y_2 yechimlar $x \in [a,b]$ da chiziqli bog'liq bo'lsa,u xolda bu kesmada Vronskiy determinanti nolga teng.

<u>4- teorema</u>

Agar (4.3) tenglama yechimlaridan tuzilgan $W(y_1 , y_2)$ - Vronskiy determinanti tenglama koeffitsientlari uzluksiz bo'lgan [a,b] kesmadagi biror $x=x_0$ qiymatida nolga teng bo'lmasa ,u xolda $W(y_1,y_2)$ bu kesmada nolga aylanmaydi.

<u>Isbot</u>

y_1 va y_2 (4.3) tenglamaning yechimlari bo'lsin. U xolda

$y_1'' + a_1 y_1' + a_2 y_1 = 0$, $y_2'' + a_1 y_2' + a_2 y_2 = 0$.

Birinchi tenglikni y_2 ga, ikkinchi tenglikni y_1 ga kupaytirib, ayiramiz:

$$(y_1 y_2'' - y_2 y_1'') + a_1(y_1 y_2' - y_2 y_1') = 0 \qquad (4.4)$$

$W(y_1, y_2) = y_1 y_2' - y_1 y'_2$ dan $W_x(y_1, y_2) = y_1 y_2'' - y_1'' y_2$ xosil bo'ladi. Demak, (4.4) tenglama

$$W_x + a_1 W = 0$$

ko'rinishni oladi. Bu tenglamaning $W|_{x=x_0} = W_0$ shartni qanoatlantiruvchi yechimini topamiz:

$$\frac{dW}{dx} = -a_1 W, \qquad \frac{dW}{W} = -a_1 dx,$$

$$\ln W = -\int_{x_0}^{x} a_1 dx + \ln C,$$

$$W = Ce^{-\int_{x_0}^{x} a_1(x)dx} \qquad (4.6).$$

(4.6) formula Livuill formulasi deyiladi.

$W|_{x=x_0} = W_0$ boshlang'ich shartdan $C = W_0$ ni topamiz. Demak,

$$W = W_0 e^{-\int_{x_0}^{x} a_1(x)dx} \qquad (4.7)$$

$W_0 \neq 0$, bu xolda (4.7) dan x ning xech bir qiymatida $W \neq 0$ kelib chiqadi.

5- teorema.

Agar (4.3) tenglamaning y_1 va y_2 yechimlari chiziqli erkli bo'lsa , bu yechimlardan tuzilgan $W(y_1,y_2)$ - Vronskiy determinanti xech bir nuktada nolga aylanmaydi.

(4.3) tenglamani integrallashga kirishamiz. Yuqoridagi 1-teoremaga ko'ra bu tenglama umumiy yechimi uning
2ta chiziqli erkli xususiy yechimlari yig'indisidan iborat.
Xususiy yechimni

$$y = e^{kx} \quad ,k\text{-const}$$

ko'rinishda izlaymiz: $y' = ke^{kx}, y'' = k^2 e^{kx}$.

Xosilalarni (4.3) ga qo'yib

$$(k^2 +a_1k+a_2)e^{kx}=0$$

yoki

$$k^2 +a_1k+a_2=0$$

tenglamani xosil qilamiz.

Bu tenglama (4.3) tenglamaning xarakteristik tenglamasi deyiladi.

$$k_1 = -\frac{a_1}{2} + \sqrt{\frac{a_1^2}{4} - a_2},$$
$$k_2 = -\frac{a_1}{2} - \sqrt{\frac{a_1^2}{4} - a_2}$$

berilgan (4.8) xarakteristik tenglamaning ildizlari bo'lsin.

1. Xarakteristik tenglamaning ildizlari k_1 va k_2 haqiqiy va xar xil sonlar bo'lsin. Bu xolda

$y_1 = e^{k_1 x}$ va $y_2 = e^{k_2 x}$

funksiyalar xususiy yechimlar bo'ladi.

$$\frac{y_1}{y_2}=\frac{e^{k_1x}}{e^{k_2x}}=e^{(k_1-k_2)x}\neq const$$

bo'lgani uchun ular chiziqli bog'liq emas.

Demak, umumiy yechim

$$\mathbf{y=c_1e^{k_1x}+c_2e^{k_2x}}\quad .$$

ko'rinishda bo'ladi.

Misol.

$y''+y'-2y=0$ tenglamaning umumiy yechimi topilsin.

Yechish.

Bu tenglamaning xarakteristik tenglamasini yozamiz:

$$k^2+k-2=0$$

Uni yechib, $k_1=1$ va $k_2=-2$ topib, quyidagi umumiy yechimni hosil qilamiz:

$$y=c_1e^x+c_2e^{-2x}\quad .$$

2. Xarakteristik tenglamaning ildizlari k_1 va k_2 haqiqiy va teng sonlar bo'lsin: $k_1=k_2$.

Bu xolda $k_1=k_2=-\frac{a_1}{2}$.

Bitta hususiy yechim ma'lum

$$y_1=e^{k_1x}=e^{-\frac{a_1}{2}x}$$

Ikkinchi xususiy yechimni $y_2=u(x)e^{k_1x}$ shaklda izlaymiz:

$$y_2'=(u'(x)+k_1u(x))e^{k_1x},$$

$$y_2''=(u''(x)+2k_1u'(x)+k_1^2u(x))e^{k_1x}.$$

Bularni (4.3) ga qo'yib va soddalashtirib

$$(u''(x) + (2k_1+a_1)\, u'(x) + (k_1^2+k_1a_1+a_2)\, u(x))e^{k_1x} = 0$$

xosil qilamiz.

$k_1 = -\frac{a_1}{2}$ bo'lganda $2k_1+a_1 = 0$ va k_1- xarakteristik tenglama karrali ildizi bo'lganidan

$$u''(x)\, e^{k_1x} = 0 \quad \text{yoki} \quad u''(x) = 0.$$

Uni integrallab $u(x)=Ax+B$ ni xosil qilamiz.

Xususiy xolda, A=1 va B=0 deb olish mumkin: $u(x)=x$.

Demak, ikkinchi xususiy yechim $y_2 = xe^{k_1x}$ ko'rinishda buladi.

Demak, bu xolda umumiy yechim

$$\mathbf{y = (c_1 + c_2x)e^{k_1x}}$$

ko'rinishida bo'ladi.

3. Xarakteristik tenglamaning ildizlari k_1 va k_2 kompleks sonlar bo'lsin:

$$k_1 = \alpha + i\beta \quad , k_2 = \alpha + i\beta \,,$$

$$\alpha = -\frac{a_1}{2}, \beta = \sqrt{\frac{a_1^{\,2}}{4} - a_2}\ .$$

Xususiy yechimlarni

$$y_1 = e^{(\alpha + i\beta)x} \quad \text{va} \quad y_2 = e^{(\alpha - i\beta)x}$$

shaklida yozish mumkin.

Quyidagi natijadan foydalanamiz: agar xaqiqiy koeffitsentli bir jinsli chiziqli tenglamaning hususiy yechimi kompleks funksiyalardan iborat bo'lsa, u xolda uning haqiqiy va mavxum qismlari xam shu tenglamaning yechimi bo'ladi.

Demak, xususiy yechim

$$e^{(\alpha+i\beta)x} = e^{\alpha x}\cos(\beta x) + ie^{\alpha x}\sin(\beta x)$$

bo'lgani uchun $e^{\alpha x}\cos(\beta x)$, $e^{\alpha x}\sin(\beta x)$ lar (4.3) tenglamaning yechimlari buladi.

Umumiy yechim esa

$$\mathbf{y = e^{\alpha x}(c_1 \cos(\beta x) + c_2 \sin(\beta x))}$$

ko'rinishda bo'ladi.

<u>Misol.</u>

$y'' - 4y' + 7y = 0$ tenglamaning umumiy yechimi topilsin.

<u>Yechish.</u>

Bu tenglamaning xarakteristik tenglamasini yozamiz:

$$k^2 - 4k + 7 = 0.$$

Uni yechib, $k_1 = 2 + i\sqrt{3}$ va $k_2 = 2 - i\sqrt{3}$ topib, umumiy yechimni xosil kilamiz:

$$y = e^{2x}(c_1 \cos(\sqrt{3}x) + c_2 \sin(\sqrt{3}x)).$$

Bir jinsli bo'lmagan ikkinchi tartibli chiziqli, o'zgarmas koeffitsientli differensial tenglama

Bir jinslimas ikkinchi tartibli chiziqli, o'zgarmas koeffitsientli differensial tenglama

$$y'' + a_1 y' + a_2 y = f(x) \qquad (4.8)$$

berilgan bo'lsin.

Agar $x \in [a,в]$ da (4.8) a_1 ,a_2 tenglamaning koeffitsientlari va o'ng tomoni - f(x) uzluksiz bo'lsa, u xolda shu oraliqdagi har qanday $x_0 \in [a,в]$ uchun

$$y(x_0) = y_0, y'(x_0) = y_0^{(1)}$$

shartni qanoatlantiruvchi yagona yechim mavjuddir.

CHiziqli differentsial tenglamaning yechimlarining xossalarini ifodalovchi 1 va 2- teoremalarga ko'ra (4.8) tenglamaning umumiy yechimi quyidagi teorema orqali ifodalanadi:

<u>6- teorema.</u>

Bir jinsli bo'lmagan (4.8) chiziqli ,o'zgarmas koeffitsientli differensial tenglamaning umumiy yechimi bu tenglamaning $y^{\times}$- xususiy yechimi bilan mos bir jinsli

$$u'' + a_1 y' + a_2 y = 0$$

tenglamaning $\overline{y}$ - umumiy yechimi yig'indisidan iboratdir, ya'ni $y_{ум} = y^* + \overline{y}$.

<u>Isboti.</u> $$y_{ум} = y^* + \overline{y}. \qquad (4.9)$$

(4.8) tenglamaning yechimi ekanligini ko'rsatamiz.

Buni (4.8) ga qo'yib

$$(y^{\times} + \overline{y})'' + a_1 (y^{\times} + \overline{y})' + a_2 (y^{\times} + \overline{y}) = f(x)$$

yoki

$$(\overline{y}'' + a_1 \overline{y}' + a_2 \overline{y}) + (y^{\times''} + a_1 y^{\times'} + a_2 y^{\times} = f(x) \qquad (4.10)$$

tenglikka ega bo'lamiz.

Birinchi qavsdagi ifoda nolga teng, chunki $\overline{y}$ - bir jinsli

$$y'' + a_1 y' + a_2 y = 0$$

tenglamaning umumiy yechimi, ikkinchi qavsdagi ifoda esa f(x) ga teng, chunki tenglamaning $y^{\times}$- (4.8) tenglamaning xususiy yechimlaridan biri. Demak, (4.10) ayniyat.

Yechimdagi o'zgarmaslarni shunday tanlash mumkinki,_ $x_0, y_0, y_0^{(1)}$- sonlar qanday bo'lmasin

$$y(x_0) = y_0, y'(x_0) = y_0^{(1)} \qquad (4.11)$$

boshlang'ich shartni qanoatlantiradigan qilib tanlash mumkin.

$$\overline{y} = C_1 y_1 + C_2 y_2$$

ekanligini xisobga olib

$$\overline{y} = C_1 y_1 + C_2 y_2 + y^{\times}$$

ni xosil qilamiz. (4.11) ga ko'ra

$$\begin{cases} C_1 y_{10} + C_2 y_{20} + y^{\times}{}_0 = y_0 \\ C_1 y_{10}' + C_2 y'_{20} + y_0^{\times} = y_0^{(1)} \end{cases}$$

Bu sistemadan c_1 va c_2 ni topish uchun uni quyidagi ko'rinishga keltiramiz

$$\begin{cases} C_1 y_{10} + C_2 y_{20} = y_0 - y^{\times}{}_0 \\ C_1 y_{10}' + C_2 y'_{20} = y_0^{(1)} - y_0^{\times} \end{cases} \qquad (4.12)$$

Bu sistemaning determinanti $x=x_0$ nuqtada Vronskiy determinantidir. y_1 va y_2 lar chiziqli erkli yechimlar bo'lganligi uchun Vronskiy determinanti nolga teng emas, ya'ni (4.12) aniq sistema. Teorema isbotlandi.

Demak, agar chiziqli bir jinsli differensial tenglamaning yechimi - $\overline{y}$ ma'lum bo'lsa, u xolda bir jinslimas (4.8) tenglamaning yechimini topish uning biror $y^{\times}$ - xususiy yechimini topishdan iborat bo'lar ekan.

Xususiy yechimni tanlash usuli.

1. (4.8) tenglamaning o'ng tomoni ko'rsatkichli funksiya va ko'pxad ko'paytmasidan, ya'ni

$$f(x) = P_n(x)e^{\alpha x}$$

ko'rinishida bo'lsin, $P_n(x)$ – n-darajali ko'pxad.

Quyidagi hollar bo'lishi mumkin:

A) α soni

$$k^2 + a_1k + a_2 = 0$$

xarakteristik tenglamani ildizi emas. Bu holda xususiy yechimni

$$y^{\times} = (A_0x^n + A_1x^{n-1} + \ldots + A_n)e^{\alpha x} = Q_n(x)e^{\alpha x}$$

ko'rinishida izlaymiz.

Misol.

$$y'' - y = x$$

$$k^2 - 1 = 0, \qquad k_{1,2} = \pm 1$$

$$\alpha = 0, \quad \alpha \neq k_{1,2}$$

$$y^{\times} = (Ax + B)e^{kx} = Ax + B, \qquad y^{\times'} = A, \qquad y^{\times''} = 0$$

$$-Ax - B = x, \quad A = -1, \qquad B = 0, \qquad y^{\times} = -x$$

Demak

$$\overline{y} = C_1e^{-x} + C_2e^{x},$$

$$y = C_1e^{-x} + C_2e^{x} - x.$$

B) α soni

$$k^2 + a_1k + a_2 = 0$$

xarakteristik tenglamaning bir karrali ildizi. Bu holda xususiy yechimni

$$y^{\times} = xQ_n(x)e^{\alpha x}$$

ko'rinishida izlaymiz.

C) α soni

$$k^2+a_1k+a_2=0$$

xarakteristik tenglamaning ikki karrali ildizi. Bu holda xususiy yechimni

$$y^{\times} = x^2 Q_n(x)e^{\alpha x}$$

ko'rinishida izlaymiz.

Misol.

$$y'' + y' = x - 2$$

$$\alpha = 0, \quad k^2 + k = 0, \quad k_1 = 0, \; k_2 = -1.$$

$$\alpha = k_1, \quad y^{\times} = x(Ax + B) = Ax^2 + Bx, \quad y^{\times'} = 2Ax + B, \quad y^{\times''} = 2A.$$

Bularni tenglamaga qo'yib, A=1/2, V=-3 ekanligini topamiz. U xolda

$$y^{\times} = \frac{1}{2}x - 3x, \qquad \bar{y} = C_1 + C_2e^{-x}$$

$$y = C_1 + C_2e^{-x} + \frac{1}{2}x - 3x.$$

2. (4.8) tenglamaning o'ng tomoni

$$f(x) = (P(x)\cos\beta x + Q(x)\sin\beta x)e^{\alpha x}$$

ko'rinishida bo'lsin.

Quyidagi hollar bo'lishi mumkin:

A) $\alpha + i\beta$ soni

$$k^2+a_1k+a_2=0$$

xarakteristik tenglama ildizi emas. Bu holda xususiy yechimni

$$y^{\times} = (U(x)\cos\beta x + V(x)\sin\beta x)e^{\alpha x}$$

ko'rinishida izlaymiz.

B) $\alpha + i\beta$ soni

$$k^2+a_1k+a_2=0$$

xarakteristik tenglamaning bir karrali ildizi. Bu holda xususiy yechimni

$$y^{\times} = x(U(x)\cos \beta x + V(x)\sin \beta x)e^{\alpha x}$$

ko'rinishida izlaymiz.

Agar

$$f(x)=M\cos \beta x+N\sin \beta x$$

ko'rinishida bo'lsa (M,N-o'zgarmas sonlar), tenglamaning xususiy yechimini :

c) βi soni

$$k^2+a_1k+a_2=0$$

xarakteristik tenglama ildizi emas. Bu holda xususiy yechimni

$$y^{\times} = A\cos \beta x + B\sin \beta x$$

ko'rinishida izlaymiz.

d) βi soni

$$k^2+a_1k+a_2=0$$

xarakteristik tenglamaning bir karrali ildizi. Bu holda xususiy yechimni

$$y^{\times} = x(A\cos \beta x + B\sin \beta x)$$

ko'rinishida izlaymiz.

Misol.

Tenglamani yeching.

$$y'' + 4y = \cos 2x$$

Yechish.

$$k^2 + 4 = 0, k_{1,2} = \pm 2i$$
$$\overline{y} = C_1\cos 2x + C_2\sin 2x.$$

Xususiy yechimni

$$y^{\times} = x(A\cos 2x + B\sin 2x)$$

ko'rinishida izlaymiz.

$y^{\times}$ ni tenglamaga qo'yib, tenglikning o'ng va chap tomonidagi $\cos 2x$ va $\sin 2x$ oldidagi koeffitsentlarni tenglab, A=0 va V=1/4 ekanligini topamiz. Demak,

$$y^{\times} = \frac{x}{4}\sin 2x,$$

$$y = C_1 \cos 2x + C_2 \sin 2x + \frac{1}{4} x \sin 2x.$$

Nazorat savollari

1. n - tartibli differensial tenglama deb qanday differensial tenglamalarga aytiladi?
2. n - tartibli differensial tenglamaning yechim deb qanday funksiyaga aytiladi?
3. n - tartibli differensial tenglamaning xususiy yechim deb qanday funksiyaga aytiladi?
4. Yuqori tartibli differentsial tenglamaning tartibini pasaytirish usullari.
5. n - tartibli chiziqli, o'zgarmas koeffitsientli differensial tenglama deb qanday differensial tenglamalarga aytiladi?
6. Chiziqli differensial tenglamaning yechimlarining xossalari.
7. 2 - tartibli chiziqli, o'zgarmas koeffitsientli differentsial tenglama uchun Vronskiy determinanti.
8. 2 - tartibli chiziqli, o'zgarmas koeffitsentli differensial tenglama yechimlarining Vronskiy determinanti orqali ifodalanuvchi xossalari.
9. Xarakteristik tenglama.
10. Xarakteristik tenglama ildizlariga qarab bir jinsli tenglama umumiy yechimining ifodalanishi.
11. Bir jinsli bo'lmagan chiziqli differensial, o'zgarmas koeffitsientli tenglama.
12. Bir jinsli bo'lmagan chiziqli differensial, o'zgarmas koeffitsientli tenglamaning xususiy yechimini tanlash usuli.

Tayanch iboralar

Funksiya, argument, o'zgaruvchi, hosila, differensial, tenglama, integral, xarakteristik tenglama, oddiy differensial tenglama, uzluksiz funksiya, chiziqli tenglama, bir jinsli, umumiy yechim, xususiy yechim, o'zgarmas koeffitsientli tenglama, Bernulli tenglamasi, Lagranj tenglamasi, yuqori tartibli tenglama.

MUSTAQIL YECHISH UCHUN MASHQLAR:

1-Variant

1. $x^3dy - y^3dx = 0$.

2. $(3x^2 + 2xy - y^2)\, dx + (x^2 - 2xy - 3y^2)dy = 0$.

3. $y' + 2y = e^{-x}$.

4. $y' + 2xy = 2xe^{-x^2}$, $\qquad y(0) = 0$.

5. $x = cy + \cos y$.

6. $y - y' = +xy'$.

7. $\frac{2x}{y^3}dx + \frac{y^2-3x^2}{y^4}dy = 0$.

8. $(x^2 + y^2 + 2x)dx + 2ydy = 0$

2-Variant

1. $tgx \sin^2 ydx + \cos^2 xctgydy = 0$.

2. $xdx+(x+y)dy=0$.

3. $y' - 2xy = 2xe^{x^2}$.

4. $y' + \frac{3}{x}y = \frac{2}{x^3}$, $\qquad y(1) = 1$.

5. $x = cy^2 - \sin y$.

6. $\dfrac{dx}{x} = \left(\dfrac{1}{y} - 2x\right)dy.$

7. $e^y dx + (xe^y - 2y)dy = 0.$

8. $\dfrac{y}{x}dx + (y^2 - \ln x)dy = 0\cdot$

3-Variant

1. $(xy^2 + x)dx + (y - x^2 y)dy = 0\,.$

2. $(x^2+y^2\ )\ dx-2xydy=0.$

3. $y' + 2xy = e^{-x^2}.$

4. $y' - 2xy = 1\,,\qquad\qquad y(0) = 0\,.$

5. $x = \dfrac{c}{y^3} + 5y\,.$

6. $2x^2 y = y^2(2xy' - y).$

7. $(4x^3 + 5x^4y^2)dx + (2x^5y + 6y^3)dy = 0.$

8. $(x^2 + y)dx + xdy = 0\,.$

4-Variant

1. $(xy^2 + x)dx + (x^2 y - y)dy = 0\,.$

2. $(x^3 - 3x^2 y)dx + (y^3 - x^3)dy = 0\,.$

3. $xy' - 2y = x^3 \cos x.$

4. $xy' - 2y = x\,.\qquad\qquad y(0) = 0\,.$

5. $x = cy^3 + \ln y\,.$

6. $y = (xy' + 2y)^2.$

7. $(5x + y - 7)dx + (8y + x - 9)dy = 0.$

8. $\left(\frac{x}{y}+1\right)dx+\left(\frac{x}{y}-1\right)dy=0$.

5-Variant

1. $x\cdot\frac{dy}{dx}-y=y^3$.

2. $(xye^{\frac{x}{y}}+y^2)dx-x^2e^{\frac{x}{y}}dy=0$.

3. $y'x\ln x-y=3x^3\ln^2 x$.

4. $xy'=x+\frac{1}{2}y$, $\qquad y(0)=0$.

5. $x=y(c+\cos y)$.

6. $xy^2y''+x^2+y^3=0$.

7. $\frac{y-7}{x^3}dx-\frac{1}{2x^2}dy=0$.

8. $(1-y\sin x)dx-\cos xdy=0$.

6-Variant

1. $x\cdot\frac{dy}{dx}-y=y^3$.

2. (x+y-1)dy+(2x+2y-3)dy=0.

3. $(2x-y^2)y'=2y$.

4. $xy'=x+y$, $\qquad y(0)=0$.

5. $x=ctgy-\frac{1}{\sin y}$.

6. $xy^2y''+x^2+y^3=0$.

7. $\left(\frac{1}{x^2}-y\right)dx+(y-x)dy=0$.

8. $x^2y^3+y+(x^3y^2-x)y^1=0$.

7-Variant

1. $tgx \cdot \frac{dy}{dx} - y = a$.

2. $y' = \frac{x}{y} + \frac{y}{x}$.

3. $y' = \frac{y}{2y \ln y + y - x}$.

4. $x^2 + xy' = y$, $\qquad y(1) = 0$.

5. $x = ce^{y^2} + y$.

6. $(x+1)(y' + y^2) = -y$.

7. $(x+y)dx+(x+\frac{1}{y})dy=0$.

8. $(1 - y^2)dx + (3 - 4xy)dy = 0$.

8-Variant

1. $\frac{dy}{dx} = \cos(x+y)$.

2. $2xydx+(y^2 - x^2)dy = 0$.

3. $\left(e^{\frac{y^2}{2}} - xy\right)dy - dx = 0$.

4. $y' - y = -2e^{-x}$, $\qquad y \to 0,\ agar\ \ x \to +\infty$.

5. $x = c\cos y + \frac{\cos y}{x}$.

6. $xy \cdot dy = (y^2 + x)dx$.

7. $(1+\frac{2}{x} + \frac{y}{x^2})dx+(3y - \frac{1}{y})dy=0$,

8. $(1 - y\sin x)dx - \cos x dy = 0$.

9-Variant

1. $\frac{dy}{dx} = (1+y^2)x$.

2. $\frac{dy}{dx} = \frac{y}{x} + \sqrt{1\frac{y}{x}}$.

3. $y' - y.e^{x} = 2xe^{ex}$.

4. $x^2 y' \cos\frac{1}{x} - y\sin\frac{1}{x} = -1$, $\qquad y \to 1,\ agar\ \ x \to \infty$

5. $x = c\ln^2 y - \ln y$.

6. $xy' - 2x^2\sqrt{y} = 4y$.

7. $(1 - \frac{y^2}{x^2} + \frac{y}{x^2})dx + \frac{2y-1}{x}dy = 0.$

8. $(2x^3y^2 - y)dx + (2x^2y^3 - x)dy = 0$.

10-Variant

1. $\dfrac{dy}{dx} = y \cdot \sin x$.

2. $2xydx - (x^2 - y^2)dy = 0$.

3. $(x^2 + 2x + 1)y' - (x+1)y = x - 1$.

4. $y' \cdot \sin x - y\cos x = -\dfrac{\sin^2 x}{x^2}$, $\qquad y \to 0,\ agar\ \ x \to \infty$

5. $x = c \cdot e^{y^2} + y^2 - 1$.

6. $xy' + 2y + x^5y^3e^x = 0$.

7. $\left(2x - \frac{1}{x^2\, y}\right)dx + \left(2y - \frac{1}{xy^2}\right)dy = 0.$

8. $(x^2 + y)dy + (x - xy)dx = 0$.

11-Variant

1. $y \cdot \cos xdx - \sin x \cdot dy = 0$.

2. $xy' = y(\ln y - \ln x)$.

3. $y'x - y = x\cos x - \sin x.$

4. $2xy' - y = 1 - \dfrac{2}{\sqrt{x}}$, $\qquad .$

5. $x = cctgy - \dfrac{1}{\cos y}$.

6. $2y' - \frac{x}{y} = \frac{x \cdot y}{x^2 - 1}$.

7. $\left(xy^2 + \frac{1}{x}\right)dx + \left(x^2y - \frac{1}{y}\right)dy = 0.$

8. $(1 - 2xy)dy - y(y-1)dx = 0$.

12-Variant

1. $xy' = 1 + y^2$.

2. $(y^2 - 3x^2)y' + 2xy = 0$.

3. $2(x - y^2)dy = y \cdot dx$.

4. $2xy' + y = 2x$,

5. $x = ce^y + \sin y$.

6. $x(x-1) \cdot y' + y^3 = x \cdot y$.

7. $(xy^2 + x)dx + \left(x^2y - \frac{1}{y}\right)dy = 0.$

8. $(1 - x^2y)dx + x^2(y - x)dy = 0$.

13-Variant

1. $1 + y' = e^y$.

2. $(2xy + 3y^2)dx + (x^2 + 6xy - 3y^2)dy = 0$.

3. $xy' + 3y = 15$.

4. $y' \sin x + y \cos x = 1$.

5. $y \cdot y' + y^2 ctgx = \cos x$.

6. $(x^2 + y^3 + 7)dx + 3xy^2dy = 0.$

7. $(\sqrt{x^2 - y} + 2x)dx - dy = 0$.

8. $3y'^3 - xy + 1 = 0.$

14-Variant

1. $y' = 3^{x+y}$.

2. $(y^2 + 2xy)dx + (2x^2 + 3xy)dy = 0$.

3. $xy' - 3y = 3\ln x - 1$.

4. $y'\cos x - y\sin x = -\sin 2x$, $y \to 0$ agar $x \to \frac{\pi}{2}$

5. $x = \frac{c}{y} + tgy$.

6. $y' + y = x \cdot y^3$.

7. $(x^2 cosy - ysin)dy + xsinydx = 0.$

8. $(x^2+y^2+x)dx+ydy=0$.

15-Variant

1. $2x \cdot \sqrt{1-y^2}dx = (1+x^2)dy$.

2. $xy' - y = \sqrt{x^2 + y^2}$.

3. $(1\text{-}2xy)y' = y(y-1)$.

4. $y'\cos x - y\sin x = 2x$, .

5. $x \cdot y' = 2\sqrt{y}\cos x - 2y$.

6. $(x^2 + y)dx + (x+7)dy = 0.$

7. $y\sqrt{1-y^2}dx+(x\sqrt{1-y^2}+y)dy=0$ $\mu=\mu(y)$.

16-Variant

1. $(1+2y-y^2)dx + x(1-y)dy = 0$.

2. $(x - \sqrt{x^2+y^2})dx + ydy = 0$.

3. $y' \cdot \sin 2x = 2(y + \cos x)$.

4. $y' + y\cos x = \cos x$,

5. $x = \frac{1}{ce^y + y + 5}$.

6. $\frac{x \cdot y'}{y} + 2xy\ln x + 1 = 0$.

7. $(y^2 + x + y)dx + (2y+1)xdy = 0.$

8. $(x^2-y)dx+xdy=0 \quad \mu=\mu(x)$.

17-Variant

1. $y(y^2+1)dx+x(y^2-1)dy=0$.

2. $(x^3-3x^2y)dx+(y^3-x^3)dy=0$.

3. $x-\dfrac{y}{y'}=\dfrac{2}{y}$.

4. $y'-y=\sin x-\cos x$,

5. $x=c\ln^2 y-\ln y$.

6. $(y'-x\sqrt{y})(x^2-1)=x\cdot y$.

7. $(2xlny-7x^2)dx+\left(\frac{x^2}{y}+4\right)dy=0$.

8. $(x^2\sin^2 y)dx+x\sin 2ydy=0;\ \sin y=z$.

18-Variant

1. $(xy^2+x)dx+y(y-x^2y)dy=0$.

2. $x^2y^2-2xyy'=x^2+3y^2$.

3. $y'+y=4x^2+8x$.

4. $y'-ytgx=y^2\sin x\cos x$,

5. $x=y(c+\sin x)$.

6. $xy\cdot dy=(y^2+x)dx$.

7. $\left(\frac{1}{2}x^2y+x+1\right)dx+\left(\frac{1}{6}x^3-y^2\right)dy=0$.

8. $(y-\dfrac{1}{x})dx+\dfrac{dy}{y}=0$.

19-Variant

1. $(y^2+1)dx-x(y+1)dy=0$.

2. $yy'=4x+3y-2$.

3. $(2x+y)dy = ydx + 4\ln y dy$.

4. $y^{'}\cos x + y\sin x = 2\cos^{2} x$.

5. $x^{-2} = y^{4}(2e^{4} + c)$.

6. $2y^{'} - \frac{x}{y} = \frac{x \cdot y}{x^{2}-1}$.

7. $2y sin 2x dx - (9 + 2cos^{2}x)dy = 0.$

8. $(x\cos y - y\sin y)dy + (x\sin y + y\cos y - \sin y)dx = 0, \;\; \mu = \mu(x)$

20-Variant

1. $y^{'} = \frac{1}{1-x}$.

2. $(x - y\cos\frac{y}{x})dx + x \cdot \cos\frac{y}{x}dx = 0$.

3. $x(-1)y^{'} + 2xy = 1$.

4. $xy^{'} - 2y = x^{2}\sqrt{y}$.

5. $x(e^{y} + ce^{2x}) = 1$.

6. $y^{'} = y^{4}\cos x + ytgx$.

7. $(e^{y} - e^{x})dx + xe^{y}dy = 0.$

8. $(3y^{2} - x)dx + (2y^{3} - 6xy)dy = 0, \;\; \mu = \mu(x + y^{2})$ ·

Uy ishlari, auditoriyada oddiy differensial tenglamalar bo'yicha joriy nazoratlar o'tkazish uchun toshiriqlar variantlari:

I. O'zgaruvchilari ajraladigan differensial tenglamalarni yeching.

1. $x^3 dy - y^3 dx = 0$
2. $tgx \sin^2 ydx + \cos^2 xctgydy = 0$
3. $(xy^2 + x)dx + (y - x^2 y)dy = 0$
4. $(xy^2 + x)dx + (x^2 y - y)dy = 0$
5. $x \cdot \dfrac{dy}{dx} - y = y^3$
6. $x \cdot \dfrac{dy}{dx} + y = y^2$
7. $tgx \cdot \dfrac{dy}{dx} - y = a$
8. $\dfrac{dy}{dx} = \cos(x + y)$ $\quad 1 + sosz = 0\ z = (2\pi + 1)\pi$
9. $\dfrac{dy}{dx} = (1 + y^2)x$
10. $\dfrac{dy}{dx} = y \cdot \sin x$
11. $y \cdot \cos dx - \sin x \cdot dy = 0 \quad y\left(\dfrac{\pi}{2}\right) = 1$
12. $xy' = 1 + y^2$
13. $1 + y' = e^y$
14. $y' = 3^{x+y}$
15. $2x \cdot \sqrt{1 - y^2} dx = (1 + x^2)dy$
16. $(1 + 2y - y^2)dx + x(1 - y)dy = 0$
17. $y(y^2 + 1)dx + x(y^2 - 1)dy = 0$
18. $(xy^2 + x)dx + y(y - x^2 y)dy = 0$
19. $(y^2 + 1)dx - x(y + 1)dy = 0$
20. $y' = \dfrac{1}{1 - x}$

II. Bir jinsli va unga keltiriladigan differensial tenglamalar.

1. $(3x^2+2xy-y^2)\,dx+(x^2-2xy-3y^2)dy=0$

2. $xdx+(x+y)dy=0$

3. $(x^2+y^2)\,dx-2xydy=0$

4. $(x^3-3x^2y)dx+(y^3-x^3)dy=0$

5. $(xye^{\frac{x}{y}}+y^2)dx-x^2e^{\frac{x}{y}}dy=0$

6. $(x+y-1)dy+(2x+2y-3)dy=0$

7. $y'=\frac{x}{y}+\frac{y}{x}$

8. $2xydx+(y^2-x^2)dy=0$

9. $\frac{dy}{dx}=\frac{y}{x}+\sqrt{1\frac{y}{x}}$

10. $2xydx-(x^2-y^2)dy=0$

11. $xy'=y(\ln y-\ln x)$

12. $(y^2-3x^2)y'+2xy=0$

13. $(2xy+3y^2)dx+(x^2+6xy-3y^2)dy=0$

14. $(y^2+2xy)dx+(2x^2+3xy)dy=0$

15. $xy'-y=\sqrt{x^2+y^2}$

16. $(x-\sqrt{x^2+y^2})dx+ydy=0$

17. $(x^3-3x^2y)dx+(y^3-x^3)dy=0$

18. $x^2y^2-2xyy'=x^2+3y^2$

19. $yy'=4x+3y-2$

20. $(x-y\cos\frac{y}{x})dx+x\cdot\cos\frac{y}{x}dx=0$

III. Chiziqli differensial tenglamani quyidagi usullar bilan yeching.

a) Oʻzgarmasni variasiyalash usuli.

b) Bernulli almashtirish orqali.

c) Umumiy yechim formulasidan foydalanib.

d) Integrallovchi koʻpaytuvchi orqali.

1. $y'+2y = e^{-x}$
2. $y'-2xy = 2xe^{x^2}$
3. $y'+2xy = e^{-x^2}$.
4. $xy'-2y = x^3 \cos x$.
5. $y' x \ln x - y = 3x^3 \ln^2 x$.
6. $(2x - y^2)y' = 2y$.
7. $y' = \dfrac{y}{2y \ln y + y - x}$.
8. $\left(e^{\frac{y^2}{2}} - xy\right)dy - dx = 0$
9. $y' - ye^x = 2xe^{ex}$
10. $(x^2 + 2x + 1)y' - (x+1)y = x - 1$
11. $y' x - y = x \cos x - \sin x$.
12. $2(x - y^2)dy = y \cdot dx$.
13. $xy'+3y = 15$
14. $xy'-3y = 3\ln x - 1$.
15. $(1 - 2xy)y' = y(y-1)$.
16. $y' \cdot \sin 2x = 2(y + \cos x)$.
17. $x - \dfrac{y}{y'} = \dfrac{2}{y}$
18. $y'+y = 4x^2 + 8x$
19. $(2x + y)dy = ydx + 4\ln ydy$
20. $x(-1)y'+2xy = 1$

IV. Berilgan differensial teglamaning ko‘rsatilgan boshlang‘ich shartini qanotlantiruvchi yechimini toping. Chiziqli tenglamaning umumiy yechim formulasidan foydalaning.

1. $y' + 2xy = 2xe^{-x^2}$ $\quad y(0) = 0$

2. $y' + \frac{3}{x}y = \frac{2}{x^3}$ $\quad y(1) = 1$

3. $y' - 2xy = 1$ $\quad y(0) = 0$

4. $xy' - 2y = x$ $\quad y(0) = 0$

5. $xy' = x + \frac{1}{2}y$ $\quad y(0) = 0$

6. $xy' = x + y$ $\quad y(0) = 0$

7. $x^2 + xy' = y$ $\quad y(1) = 0$

8. $y' - y = -2e^{-x}$ $\quad y \to 0$ agar $x \to +\infty$

9. $x^2 y' \cos\frac{1}{x} - y\sin\frac{1}{x} = -1$ $\quad y \to 1$ agar $x \to \infty$

10. $y' \cdot \sin x - y\cos x = -\frac{\sin^2 x}{x^2}$ $\quad y \to 0$ agar $x \to \infty$

11. $2xy' - y = 1 - \frac{2}{\sqrt{x}}$ $\quad y \to -1$ agar $x \to \infty$

12. $2xy' + y = 2x$ $\quad y$ chegaralangan agar $x \to 0$

13. $y'\sin x + y\cos x = 1$ $\quad y$ chegaralangan agar $x \to 0$

14. $y'\cos x - y\sin x = -\sin 2x$ $\quad y \to 0$ agar $x \to \frac{\pi}{2}$

15. $y'\cos x - y\sin x = 2x$ $\quad y(0) = 0$

16. $y' + y\cos x = \cos x$ $\quad y(0) = 1$

17. $y' - y = \sin x - \cos x$ $\quad y$ chegaralangan agar $x \to +\infty$

18. $y' - ytgx = y^2 \sin x \cos x$ $\quad y\left(\frac{\pi}{3}\right) = \frac{4}{3}$

19. $y'\cos x + y\sin x = 2\cos^2 x$ $\quad y(0) = 0$

20. $xy' - 2y = x^2\sqrt{y}$ $\quad y(0) = 0$

V. Egri chiziqlar oilasining differensial tenglamasini tuzing va turini aniqlang.

1. $x = cy + \cos y$
2. $x = cy^2 - \sin y$
3. $x = \frac{c}{y^3} + 5y$
4. $x = cy^3 + \ln y$
5. $x = y(c + \cos y)$
6. $x = ctgy - \frac{1}{\sin y}$
7. $x = ce^{y^2} + y$
8. $x = c\cos y + \frac{\cos y}{x}$
9. $x = c\ln^2 y - \ln y$
10. $x = c \cdot e^{y^2} + y^2 - 1$
11. $x = cctgy - \frac{1}{\cos y}$
12. $x = ce^{y} + \sin y$
13. $x = cy^3 + y$
14. $xy = (x^3 + c)e^{x}$
15. $x = \frac{1}{ce^{y} + y + 5}$
16. $x = c\ln^2 y - \ln y$
17. $x = y(c + \sin x)$
18. $x^{-2} = y^4(2e^4 + c)$
19. $x(e^{y} + ce^{2x}) = 1$

1. $y = c \cdot \sin x + \frac{\sin x}{x}$
2. $y = c \cdot tgx - \frac{1}{\cos x}$
3. $y = c \cdot e^{x^2} + x^3$
4. $y = x \cdot (c + \sin x)$
5. $y = c \cdot \ln^2 x - \ln x$
6. $y = x^4 \ln^2 cx$
7. $x = e^{y} + c \cdot e^{-y}$
8. $x = 2\ln y - y + 1 + cy^2$
9. $xy = (x^3 + c)e^{-x}$
10. $y^{-2} = x^4(2e^{x} + c)$
11. $y(e^{x} + c \cdot e^{2x}) = 1$
12. $y = x(ce^{-x} - 1)$
13. $x = \frac{c}{y} + tgy$
14. $x = (c - \cos y) \cdot \sin y$
15. $y = \frac{1}{ce^{x} + x + 2}$
16. $x = cy + y^3$
17. $y^2 = c \cdot (xy - 1)$
18. $x = y^2(c - 2\ln y)$
19. $y(xy - 1) = c \cdot x$

VI. Bernulli tenglamasini quyidagi usullar bilan yeching.

a) Chiziqli tenglamaga keltirib;

b) O‘zgarmasni variatsiyalash usuli bilan

1. $y - y' = xy'.$

2. $\frac{dx}{x} = \left(\frac{1}{y} - 2x\right)dy.$

3. $2x^2 y = y^2(2xy' - y).$

4. $y = (xy' + 2y)^2.$

5. $(1 - x^2)y' - 2xy^2 = xy.$

6. $xy^2 y' + x^2 + y^3 = 0.$

7. $(x+1)(y' + y^2) = -y$

8. $xy \cdot dy = (y^2 + x)dx.$

9. $xy' - 2x^2\sqrt{y} = 4y$

10. $xy' + 2y + x^5 y^3 e^x = 0$

11. $2y' - \frac{x}{y} = \frac{x \cdot y}{x^2 - 1}$

12. $x(x-1) \cdot y' + y^3 = x \cdot y.$

13. $y \cdot y' + y^2 ctgx = \cos x.$

14. $y' + y = x \cdot y^3$

15. $x \cdot y' = 2\sqrt{y}\cos x - 2y$

16. $\frac{xy'}{y} + 2xy\ln x + 1 = 0$

17. $(y' - x\sqrt{y})(x^2 - 1) = x \cdot y$

18. $xy \cdot dy = (y^2 + x)dx$

19. $2y' - \frac{x}{y} = \frac{x \cdot y}{x^2 - 1}$

20. $y' = y^4 \cos x + ytgx$

VII. Quyidagi tenglamalar to'liq differensial tenglamalar ekanligini tekshiring va integrallang.

1. $\frac{2x}{y^3}dx + \frac{y^2-3x^2}{y^4}dy = 0$
2. $e^y dx + (xe^y - 2y)dy = 0$
3. $(4x^3 + 5x^4y^2)dx + (2x^5y + 6y^3)dy = 0$
4. $(5x + y - 7)dx + (8y + x - 9)dy = 0$
5. $\frac{y-7}{x^3}dx - \frac{1}{2x^2}dy = 0$
6. $\left(\frac{1}{x^2} - y\right)dx + (y - x)dy = 0$
7. $(x + y)dx + \left(x + \frac{1}{y}\right)dy = 0$
8. $1 - \frac{2}{x} + \frac{y}{x^2})dx + \left(3y - \frac{1}{y}\right)dy = 0$
9. $(1 - \frac{y^2}{x^2} + \frac{y}{x^2})dx + \frac{2y-1}{x}dy = 0$
10. $\left(2x - \frac{1}{x^2y}\right)dx + \left(2y - \frac{1}{xy^2}\right)dy = 0$
11. $\left(xy^2 + \frac{1}{x}\right)dx + \left(x^2y - \frac{1}{y}\right)dy = 0$
12. $(xy^2 + x)dx + \left(x^2y - \frac{1}{y}\right)dy = 0$
13. $(x^2 + y^3 + 7)dx + 3xy^2dy = 0$
14. $(x^2cosy - ysin)dy + xsinydx = 0$
15. $(x^2 + y)dx + (x + 7)dy = 0$
16. $(y^2 + x + y)dx + (2y + 1)xdy = 0$
17. $(2xlny - 7x^2)dx + \left(\frac{x^2}{y} + 4\right)dy = 0$
18. $\left(\frac{1}{2}x^2y + x + 1\right)dx + \left(\frac{1}{6}x^3 - y^2\right)dy = 0$
19. $2ysin2xdx - (9 + 2cos^2x)dy = 0$
20. $(e^y - e^x)dx + xe^y dy = 0$

VIII. Qulay almashtirish yoki integrallovchi ko‘paytuvchi yordamida quyidagi differensial tenglamalarni integrallang.

1. $(x^2+y^2+2x)dx+2ydy=0$
2. $\frac{y}{x}dx+(y^2-\ln x)dy=0$
3. $(x^2+y)dx+xdy=0$
4. $\left(\frac{x}{y}+1\right)dx+\left(\frac{x}{y}-1\right)dy=0$
5. $(2xy^2-y)dx+(y^2+x+y)dy=0$
6. $x^2y^2+y+(x^2y^2-x)y'=0$
7. $(1-y^2)dx+(3-4xy)dy=0$
8. $(1-y\sin x)dx-\cos xdy=0$
9. $(2x^3y^2-y)dx+(2x^2y^3-x)dy=0$
10. $(x^2+y)dy+(x-xy)dx=0$
11. $(1-2xy)dy-y(y-1)dx=0$
12. $(1-x^2y)dx+x^2(y-x)dy=0$

Test topshiriqlari

Birinchi tartibli differensial tenglamalar mavzulari bo’yicha testlar

1. $e^{x+3y}dy = xdx$ ning yechimini toping.

A) $e^{3y} = 3(C - xe^{-x} - e^{-x})$, B) $\ln y = Ctg\frac{x}{2}$, C) $\ln|\cos y| = x - x^2 + C$, D) $C = tgxtgy$.

2. $y'\sin x = y\ln y$ ning yechimini toping.

A) $y = C\sin x - 2$, B) $\ln y = Ctg\frac{x}{2}$, C) $C = \frac{\cos x}{\cos y}$, D) $tgy = \frac{C}{e^x - 1}$.

3. $y' = (2x-1)ctgy$ ning yechimini toping.

A) $e^{3y} = 3(C - xe^{-x} - e^{-x})$, B) $\ln y = Ctg\frac{x}{2}$, C) $\ln|\cos y| = x - x^2 + C$, D) $tgy = \frac{C}{e^x - 1}$.

4. $\sec^2 x \cdot tgydy + \sec^2 y \cdot tgxdx = 0$ ning yechimini toping.

A) $tgy = \frac{C}{e^x - 1}$, B) $\ln y = Ctg\frac{x}{2}$, C) $arctgy = C + \frac{1}{2}e^{x^2}$, D) $C = tgxtgy$.

5. $(1+e^x)ydy - e^y dx = 0$ ning yechimini toping.

A) $-e^{-y}(y+1) = \ln\frac{e^x}{e^x + 1} + C$, B) $\ln y = Ctg\frac{x}{2}$, C) $arctgy = C + \frac{1}{2}e^{x^2}$, D) $C = \frac{\cos x}{\cos y}$.

6. $(y^2+3)dx - \frac{e^x}{x}ydy = 0$ ning yechimini toping.

A) $tgy = \frac{C}{e^x - 1}$, B) $\ln(y^2+3) = 2(C - xe^{-x} - e^{-x})$, C) $C = tgxtgy$, D) $tgy = \frac{C}{e^x - 1}$.

7. $\sin y\cos xdy = \cos y\sin xdx$ ning yechimini toping.

A) $tgy = \frac{C}{e^x - 1}$, B) $arctgy = C + \frac{1}{2}e^{x^2}$, C) $C = \frac{\cos x}{\cos y}$, D) $\sin y = C(e^x - 1)^3$.

8. $y' = (2y+1)tgx$ ning yechimini toping.

A) $C = tgxtgy$, B) $\ln y = Ctg\frac{x}{2}$, C) $\sin 2y = tgx + C$, D) $\sqrt{2y+1} = \frac{C}{\cos x}$.

9. $(\sin(x+y) + \sin(x-y))dx + \frac{dy}{\cos y} = 0$ ning yechimini toping.

A) $tgy = C + \cos 2x$, B) $\ln y = Ctg\frac{x}{2}$, C) $\sqrt{2y+1} = \frac{C}{\cos x}$, D) $C = \frac{\cos x}{\cos y}$.

10. $((1+e^x)yy' = e^x$ ning yechimini toping.

A) $C(e^y - 1) = e^{-x}$, B) $y^2 = 2\ln C(e^x + 1)$, C) $\ln\left|tg\left(\frac{\pi}{4} + \frac{y}{2}\right)\right| = C - e^x$, D) $tgy = \frac{C}{e^x - 1}$.

11. $(xy + x^3 y)y' = 1 + y^2$ ning yechimini toping.

A) $\sqrt[3]{y^3+1} = \frac{C}{\sqrt{x^2-1}}$, B) $y = C\sqrt{x^2-1}$, C) $Cx = \sqrt{(1+x^2)(1+y^2)}$, D) $\frac{C\sqrt[3]{x}}{\sqrt[3]{x+3}} + 3$.

12. $\frac{y'}{7^{y-x}} = 3$ ning yechimini toping.

A) $\frac{C\sqrt[3]{x}}{\sqrt[3]{x+3}} + 3$, B) $y = C\sqrt{x^2-1}$, C) $y = \frac{Cx}{x+1} + 1$, D) $7^{-y} = 3 \cdot 7^{-x} + C\ln 7$.

13. $y - xy' = 2(1 + x^2 y')$ ning yechimini toping.

A) $y = \frac{Cx}{\sqrt{1+2x^2}} + 2$, B) $\sqrt{\frac{y-2}{y}} = Ce^x$, C) $y = \frac{Cx}{x+1} + 1$, D) $\frac{y}{y+1} = C - x$.

14. $y - xy' = 1 + x^2 y'$ ning yechimini toping.

A) $\frac{y}{y+1} = C - x$, B) $y = \frac{Cx}{x+1} + 1$, C) $\sqrt{\frac{y-2}{y}} = Ce^x$, D) $\frac{y}{y+1} = Cx$.

15. $(x+4)dy - xydx = 0$ ning yechimini toping.

A) $y = C\sqrt{x^2-1}$, B) $\sqrt{y^2+1} = \ln Cx$, C) $y = \dfrac{Ce^x}{(x+4)^4}$, D) $\dfrac{C\sqrt[3]{x}}{\sqrt[3]{x+3}} + 3$.

16. $y' + y + y^2 = 0$ ning yechimini toping.

A) $\dfrac{1}{y} + \ln y = C + \dfrac{1}{2}\ln^2 x$, B) $y = \dfrac{Cx}{\sqrt{1+2x^2}} + 2$, C) $y = C\sqrt{x^2-1}$, D) $\dfrac{y}{y+1} = C - x$.

17. $y^2 \ln x dx - (y-1)xdy = 0$ ning yechimini toping.

A) $y + \ln\dfrac{(y-1)^2}{y} = C + \ln x$, B) $\dfrac{C\sqrt[3]{x}}{\sqrt[3]{x+3}} + 3$, C) $C(e^y - 1) = e^{-x}$, D) $\ln\left|\dfrac{y}{y+2}\right| = C + x^2$.

18. $(x + xy^2)dy + ydx - y^2 dx = 0$ ning yechimini toping.

A) $y = C\sqrt{x^2-1}$, B) $y + \ln\dfrac{(y-1)^2}{y} = C + \ln x$, C) $arctgy = C + arctgx$, D) $\sin\dfrac{y}{x} = \ln\dfrac{C}{|x|}$.

19. $y' + 2y - y^2 = 0$ ning yechimini toping.

A) $\dfrac{C\sqrt[3]{x}}{\sqrt[3]{x+3}} + 3$, B) $\dfrac{y}{y+1} = C - x$, C) $\sqrt{\dfrac{y-2}{y}} = Ce^x$, D) $y^3 = 3(C - x + \ln|x+1|$.

20. $(x^2 + x)ydx + (y^2 + 1)dy = 0$ ning yechimini toping.

A) $\dfrac{y}{y+1} = C - x$, B) $y = \dfrac{Cx}{x+1} + 1$, C) $C = \dfrac{\cos x}{\cos y}$, D) $\dfrac{y^2}{2} + \ln y = C - \dfrac{x^3}{3} - \dfrac{x^2}{2}$.

21. $y - xy' = x\sec\dfrac{y}{x}$ ning yechimini toping.

A) $\sin\dfrac{y}{x} = \ln\dfrac{C}{|x|}$, B) $y = -\dfrac{x}{\ln(Cx)}$, C) $y^2 = x^2\ln(Cx)^2$, D) $y = xe^{\frac{C}{x}}$.

22. $(y^2-3x^2)dy+2xydx=0$ ning yechimini toping.

A) $\frac{y}{y+1}=C-x$, B) $(y^2-x^2)^2=Cx^2y^3$, C) $C=\frac{\cos x}{\cos y}$, D) $\sqrt{\frac{y}{x}}-\frac{y}{x}=\ln Cx$.

23. $(x+2y)dx-xdy=0$ ning yechimini toping.

A) $\frac{y}{y+1}=C-x$, B) $y=xe^{\frac{C}{x}}$, C) $y=Cx^2-x$, D) $y^2=x^2\ln(Cx)^2$.

24. $(x-y)dx+(x+y)dy=0$ ning yechimini toping.

A) $y=xe^{\frac{C}{x}}$, B) $y=\frac{Cx}{x+1}+1$, C) $y=-\frac{x}{\ln(Cx)}$, D) $arctg\frac{y}{x}+\frac{1}{2}\ln\frac{y^2+x^2}{x^2}=\ln\frac{C}{x}$.

25. $(y^2-2xy)dx+x^2dy=0$ ning yechimini toping.

A) $\frac{y}{x-y}=Cx$, B) $y=\frac{Cx}{x+1}+1$, C) $C=\frac{\cos x}{\cos y}$, D) $y=-\frac{x}{\ln(Cx)}$.

26. $y^2+x^2y'=xyy'$ ning yechimini toping.

A) $\ln\left|1+\frac{y}{x}\right|=Cx$, B) $e^{\frac{y}{x}}=Cy$, C) $y=\frac{C}{x}-\frac{x}{2}$, D) $\frac{y^2}{2}+\ln y=C-\frac{x^3}{3}-\frac{x^2}{2}$.

27. $xy'-y=xtg\frac{y}{x}$ ning yechimini toping.

A) $\frac{y}{y+1}=C-x$, B) $\ln\left|1+\frac{y}{x}\right|=Cx$, C) $\sin\left(\frac{y}{x}\right)=Cx$, D) $y=\frac{C}{x}-\frac{x}{2}$.

28. $xy'=y-xe^{\frac{y}{x}}$ ning yechimini toping.

A) $y=\frac{C}{x}-\frac{x}{2}$, B) $y=\frac{Cx}{x+1}+1$, C) $\sqrt{\frac{y}{x}}-\frac{y}{x}=\ln Cx$, D) $e^{-\frac{y}{x}}=\ln Cx$.

29. $xy'-y=(x+y)\ln\left(\frac{x+y}{x}\right)$ ning yechimini toping.

A) $\ln\left|1+\frac{y}{x}\right|=Cx$, B) $\ln\left|1+\frac{y}{x}\right|=Cx$, C) $-e^{-\frac{y}{x}}=\ln Cx$, D) $y=x\ln\left(\frac{C}{x}\right)$.

30. $xy'=y\cos\ln\frac{y}{x}$ ning yechimini toping.

A) $y=\frac{x}{4}\ln^2 Cx$, B) $ctg\left(\frac{1}{2}\ln\frac{y}{x}\right)=\ln Cx$, C) $arc\sin\frac{y}{x}=\ln Cx$, D) $-e^{-\frac{y}{x}}=\ln Cx$.

31. $(x^2+1)y'+4xy=3, y(0)=0.$ ning yechimini toping.

A) $y=\frac{x^3+3x}{(x^2+1)^2}$, B) $y=x^2-1$, C) $y=(\sin x-1)x$, D) $x=y^2-y$.

32. $y'+ytgx=\sec x, y(0)=0.$ ning yechimini toping.

A) $y=(\sin x-1)x$, B) $y=\sin x$, C) $x=y^2-y$, D) $-e^{-\frac{y}{x}}=\ln x$.

33. $(1-x)(y'+y)=e^{-x}, y(0)=0.$ ning yechimini toping.

A) $x=y^2-y$, B) $ctg\left(\frac{1}{2}\ln\frac{y}{x}\right)=\ln x$, C) $y=e^{-x}\ln\frac{1}{1-x}$, D) $y=(\sin x-1)x$.

34. $xy'-2y=2x^4, y(1)=0.$ ning yechimini toping.

A) $y=x^2-1$, B) $y=e^x\ln x$, C) $arc\sin\frac{y}{x}=\ln x$, D) $y=x^4-x^2$.

35. $y'=2x(x^2+y), y(0)=0.$ ning yechimini toping.

A) $y=x^2+1-e^{x^2}$, B) $y=\ln x$, C) $x=e^y-e^{-y}$, D) $-e^{-\frac{y}{x}}=\ln x$.

36. $y'-y=e^x, y(0)=1.$ ning yechimini toping.

A) $y=\frac{x}{4}\ln^2 x$, B) $y=(x+1)e^x$, C) $arc\sin\frac{y}{x}=\ln x$, D) $y=\ln x$.

Test javoblari

Birinchi tartibli differensial tenglamalar											
1	A	7	A	13	C	19	D	25	C	31	B
2	D	8	A	14	A	20	D	26	C	32	B
3	D	9	C	15	B	21	D	27	D	33	D
4	B	10	B	16	D	22	A	28	B	34	D
5	A	11	B	17	D	23	C	29	D	35	C
6	C	12	B	18	D	24	D	30	C	36	A

Sinov savollari:

1. Differensial tenglamaning xususiy va umumiy yechimlari deb nimaga aytiladi?
2. Koshi masalasi qanday qo'yiladi?
3. O'zgaruvchilari ajraladigan differensial tenglama ta'rifi.
4. Umumiy integral deb nimaga aytiladi?
5. Qanday funksiya bir jinsli funksiya deyiladi?
6. Bir jinsli differensial tenglama deb nimaga aytiladi?
7. Bir jinsli differensial tenglama qanday integrallanadi?
8. Qanday tenglamaga umumlashgan bir jinsli differensial tenglama deyiladi?
9. Qanday tenglamaga chiziqli differensial tenglama deyiladi?
10. Chiziqli differensial tenglamalarni qanday usullarda yechish mumkin?
11. Bernulli tenglamasini ko'rinishi qanday?
12. Rikkati tenglamasining ko'rinishi qanday?
13. To'liq differensialli tenglama deb qanday tenglamaga aytiladi?
14. Integrallovchi ko'paytuvchi deb qanday funksiyaga aytiladi?

Adabiyotlar:

1. Salohiddinov M.S., Nasretdinov G‘.N. Oddiy differenisial tenglamalar, Toshkent, «O‘qituvchi». 1982.

2. Sharipov Sh.R., Mo‘minov N.S. Oddiy differensial tenglamalar. Toshkent, «O‘qituvchi», 1992.

3. Hodizoda P.D. Differensial tenglamalar ma’ruzalar matni. Samarqand – 2000.

4. Hodizoda P.D. Differensial tenglamalar, metodik qo’llanma. Samarqand –2013. 158 bet.

5. Guter R.S., Yanpolskiy A.R. Differensial tenglamalar, «O‘qituvchi», Toshkent, 1978.