O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

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ABDUSAMATOVA HILOLA

TUB SONLAR QATNASHGAN BINAR ADDITIV MASALANING YECHIMLARI SONI HAQIDA

5A130101- Matematika ta'lim yo'nalishi bo'yicha
Magistr
akademik darajasini olish uchun yozilgan

DISSERTATSIYA

Ilmiy rahbar:

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MUNDARIJA

KIRISH

***a*). Mavzuning o'rganilganlik darajasi va dolzarbligi.** Yurtimiz istiqlolga erishgan ilk kunlardanoq, davlatimiz tomonidan amalga oshirilayotgan bunyodkorlik ishlari Vatanimiz mustaqilligi va ozodligi tufaylidir. Respublikamizda izchil suratda amalga oshirilib borilayotgan "Kadrlar tayyorlash milliy Dasturi" bugungi kunda jahon miqyosida e'tirof etildi va o'zining ijobiy natijalarini bermoqda.

O'zbekistonda ta'lim tizimini isloh qilishning dasturiy hujjatlarida [1,2] ta'kidlanganidek, mamlakatimiz ta'lim tizimi xodimlari oldiga raqobatbardosh kadrlar tayorlash, ta'l im tarbiya jarayonini jahon andozalari darajasiga etkazishni ta'minlash asosiy vazifa qilib qo'yilgan. Shu ma'noda olib qaraganda, yoshlarning yangi avlodi istiqbol masalalarini kun tartibiga dadil qoyadigan va uni yecha oladigan, fikr yuritishning yuksak madaniyatini egallagan, siyosiy hamda ijtimoiy-iqtisodiy hayotda o'ziga mustaqil yo'l topa oladigan qobiliyatga ega bo'lishi kerak.

Zero birinchi Prezidentimiz [3] ta'kidlaganlaridek: ―"Buyuk maqsadlarimizga, ezgu-niyatlarimizga erishishimiz, jamiyatimizning yangilanishi, hayotimiz taraqqiyoti va istiqboli, amalga oshirilayotgan islohatlarimiz va rejalarimizning samarali taqdiri, avvalambor davr talablariga javob beradigan yuqori malakali, ongli taffakurga ega bo'lgan mutaxassis kadrlar bilan bog'liq...".

Ushbu magistrlik dissertatsiyasi mavzusi ana shu talab va vazifalardan kelib chiqib tanlandi.

X.Goldbax va L.Eyler orasidagi 1742 yildagi yozishmalardan Eyler-Goldbax muammosi vujudga kelgan.U zamonaviy tilda quyidagicha ifodalanadi [4]:

I. Har qanday toq natural $n \geq 9$ sonni uchta toq tub sonlarning yig'indisi ko'rinishida yozish mumkin;

II.Har qanday juft natural $n \geq 6$ sonni ikkita toq tub sonlarning yig'indisi ko'rinishida yozish mumkin.

Bu tasdiqlarning birinchisiga Goldbaxning ternar problemasi, ikkinchisiga esa Goldbaxning binar problemasi ham deb yuritiladi.

I.M. Vinogradov [5] o’zi yaratgan trigonometrik yig’indilar metodi yordamida ternar problemani 1937 yilda yetarlicha katta $n \geq N_0$ lar uchun hal qildi. Bu problema yaqinda [6] to’la hal bo’ldi, ya’ni barcha $n \geq 9$ lar uchun isbotlandi.

Lekin binar problema hosirgacha t’ola hal etilgan emas. Bu sohada N.G.Chudakov [7], Van-der-Corput [8] va T. Esterman [9] lar Vinogradovning trigonometrik yig’indilar metodini qo’llab deyarli barcha juft sonlarning ikkita toq tub sonning yig’indisi ko’rinishida ifodalanishini ko’satdilar. Aniqroq qilib aytganda agar $E(X)$ bilan $[2, X)$ oraliqdagi ikkita tub son yig’indisi ko’rinishida ifodalanmaydigan juft sonlarning sonini belgilasak, yuqoridagi mualliflar belgilangan $A > 0$ soni uchun

$$E(X) \ll \frac{X}{ln^A x}$$

bahoning o’rinli ekanligini isbotladilar.

Keyinchalik bu natija bir necha marta yaxshilandi. Jumladan Montgomery H.L., Vaughan R.C. [10,11], I.Allakov [12], Chen J., Pan C.[13] lar tomonidan $E(X) < Xexp\left(-c\sqrt{lnX}\right)$, $E(X) < X^{1-\delta}$, $E(X) < X^{0,96}$ baholar olingan bo’lsada, muammo to’la hal etilgani yo’q.

Qaralayotgan problema yechimini topmagani uchun ham bu sohadagi har bir ilmiy izlanish soha uchun dolzarb hisoblanadi.

***b*). Ishning maqsadi**. Magistrlik dissertatsiyasining asosiy maqsadi $(1, X)$ oraliqdagi ikkita tub sonlarning yig’indisi

$$n = p_1 + p_2 \qquad (0.1)$$

ko’rinishida ifodalanadigan juft $n-$natural sonlar to’plamini o’rganib (0.1) ning tub sonlardagi yechimlari soni $R(n)$ uchun yangi baho olishdan iboratdir.

Ishning asosiy natijalari. Dissertatsiyaning asosiy ilmiy natijasi quyidagi:

1.Agar Dirixle $L-$ funktsiyasining maxsus haqiqiy no’li mavjud bo’lmasa yoki shunday maxsus haqiqiy no’li $\tilde{\beta}$ mavjud bo′lib, $(n, \tilde{r}) = 1$ bo’lsa, u holda X ning yetarlicha katta qiymatlarida $n, (n \leq X)$ning ko′pi bilan

$$E(X) < X^{0,9882}$$

tadan boshqa barcha qiymatlari uchun

$$R(n) > n^{0,991673}\left(0,0007262\cdot\frac{n^{1,008326}}{\varphi(n)} - 1\right) \qquad (0.2)$$

tengsizlik; qolgan qiymatlari uchun esa

$$R(n) \le n^{0,991673}\left(0,0007262\cdot\frac{n^{1,008326}}{\varphi(n)} - 1\right)$$

tengsizlik o'rinli.

2. Ishda Rimanning dzeta-funktsiyasi $\zeta(s), (s=\sigma+it)$ ning $0<\sigma<1$, $|t|\le T$ to'g'ri to'rtburchakdagi trivial bo'lmagan no'llarining soni *N(T)* uchun

$$N(T) = \frac{T}{2\pi}ln\frac{T}{2\pi} - \frac{T}{2\pi} + 6,512850\theta\ lnT \qquad (0.3)$$

formula isbotlangan.

Bu natija ilgari $N(T)$ bilan bog'liq mavjud natijalarning aniqlashtirilgani hisoblanadi. Taqqoslash uchun ilgarigi natija I. Allakov [14] tomonidan oligan bo'lib

$$N(T) = \frac{T}{2\pi}ln\frac{T}{2\pi} - \frac{T}{2\pi} + 32,22660\theta\ lnT$$

ko'rinishda edi.

Dissertatsiya ishining ilmiy-tadqiqot ishlari rejalari bilan bog'liqligi. Dissertatsiyaning mavzusi Termiz davlat universiteti ilmiy kengashi tomonidan tasdiqlangan va Termiz davlat universiteti matematika kafedrasida olib borilayotgan ilmiy tadqiqot ishlari bilan bevosita bog'liq.

Ishning nazariy va amaliy ahamiyati. Dissertatsiya ishi ilmiy-nazariy xarakterda bo'lib, matematikaning turli additiv masalalarini yechishda, xususan, qo'shiluvchilar soni chegaralangan bo'lgandagi Varing muammosini, Goldbax muammosini, Xardi-Littlvud muammosini, Xua-Lo-Ken muammosini hamda ularning umumlashmalarini yechishda foydalanish mumkin.

Natijalarning qo'llanishi. Dissertatsiya natijalaridan O'z RFA ning Matematika instituti va Qarshi va Termiz davlat universitetlarida ilmiy izlanishlar olib borayotgan mutaxassislar foydalanishlari mumkin. Shuningdek ishdan shu sohada ilmiy izlanishlar olib boruvchi mutaxassislar foydalanishlari hamda talabalarga maxsus kurs va seminarlar o'tishda foydalanish mumkin.

Ishning sinovdan o'tishi. Ishning asosiy natijalari " OTM larida tabiiy va aniq fanlarni o'qitish muammolari " respublika konferensiyasida (mart, 2017, Toshkent), shuningdek Termiz davlat universitetining yillik ilmiy konferensiyalarida (2016, 2017 yillar), Termiz davlat universiteti matematika kafedrasi seminarlarida ma'ruza qilinib muhokama etilgan.

Natijalarning e'lon qilinganligi. Ish yuzasidan 3ta maqola chop etildi [32-34].

Dissertatsiyaning tuzilishi va hajmi. Magistrlik dissertatsiyasi kirish, uch bob, xulosalar va foydalanilgan adabiyotlar ro'yxatidan iborat, umumiy hajmi kompyuter yozuvida 80 bet.

Dissertatsiyaning asosiy mazmuni

I-bob "Asosiy tushunchalar va yordamchi materiallar" –deb nomlangan bo'lib 3 ta paragrafdan iborat.

I.1-§ da Goldbaxning ternar problemasi (yuqoridagi I-tasdiq) va uning isboti haqhidagi materiallar bayon qilingan.

I.2-§da Goldbaxning binar problemasi (yuqoridagi II-tasdiq) va bu borada olingan keyingi natijalar o'rganilgan.

I.3- §. Binar additiv masalalarning maxsus t'oplami nima va uni qanday baholash haqida natijalar keltirilgan.

Ishning II-bobi "Dirixle L-fuksiyasining nollari haqida"—deb atalgan va 4ta paragrafni o'z ichiga oladi.

II.1- § da *L*-funksiyaning logarifmik yoyilmasini no'llari bo'yicha qatorga yoyish masalasi qaralgan.

II.2- § da $N(T,\chi)$ funksiya uchun formulaning qoldiq hadiga kiruvchi o’zgarmasning qiymati aniqlandan. Shu paragrafda $N(T,\chi_0) = N(T)$ ning qoldiq hadi uchun yangi baho (0.3) isbotlangan. Bunda χ_0 −bosh xarakter.

II.3- § da $\psi(x,\chi)$ funksiya uchun formulaning qoldiq hadiga kiruvchi o’zgarmasning qiymati aniqlandan. Bunda χ −Dirixle xarakteri.

II.4 -§ da zichlik teoremalari va ularning tub sonlar bo’yicha olingan

$$\sum_{p \leq x} \chi(p) lnp$$

yig’indini baholashda qo’llanishi bayon qilingan. P.X. Gallagher [15] teoremasi isbotlangan.

Ishning III-bobi “Sonlarni ikkita toq tub sonlarning yig’indisi ko’rinishida ifodalash”− deb nomlangan. Bu bobda yuqoridagi 2 ta bobning natijalaridan foydalanib ishning asosiy natijasi (0.2) isbotlangan.

III-bob 5 ta paragrafga bo’lingan.

III.1 -§ da asosiy belgilashlar va doiraviy metodni qo’llash uchun mos holda birlik intervalni kichik va katta yoylarga bo’lish raqalgan.

III.2- § da trigonometrik yig’indilar metodidan foydalanib kichik yoylar bo’yicha olingan integral, ya’ni qoldiq had baholangan.

III.3- § da katta yoylar qaralgan va Xardi-Litlvud-Ramanudjanning doiraviy metodi bilan katta yoylar bo’yicha olingan integrallar R_1 (L-funksiyaning maxsus no’li mavjud bo’lmasa) va $\tilde{R}_1$ (L-funksiyaning maxsus no’li mavjud bo’lsa) lar soddalashtirilgan.

III.4-§ da R_1 va $\tilde{R}_1$ integrallar baholangan.

III.5- § da asosiy teoremaning isboti, ya’ni (0.2) –natija isbotlangan.

ASOSIY QISM

I-BOB. ASOSIY TUSHUNCHALAR VA YORDAMCHI MATERIALLAR

I.1§. Goldbaxning ternar problemasi va uning isboti haqida

X.Goldbax va L.Eyler orasidagi 1742 yildagi yozishmalardan Eyler-Goldbax muammosi vujudga kelgan. U zamonaviy tilda quyidagicha ifodalanadi [4]:

I. Har qanday toq natural $n \geq 9$ sonni uchta toq tub sonlarning yig'indisi ko'rinishida yozish mumkin;

II.Har qanday juft natural $n \geq 6$ sonni ikkita toq tub sonlarning yig'indisi ko'rinishida yozish mumkin.

Bu tasdiqlarning birinchisiga Goldbaxning ternar problemasi, ikkinchisiga esa Goldbaxning binar problemasi ham deb yuritiladi.

Tushunarliki, Goldbaxning binar problemasining o'rinli ekanligidan ternar problemanig o'rinli ekanligi kelib chiqadi. Haqiqatan ham, $2n = p_1 + p_2$ bo'lsa, u holda $2n + 3 = p_1 + p_2 + 3$, tenglik barcha $n = 3,4, \ldots$ lar uchun bajariladi.

Bu muamolar o'z vaqtida matematikaning juda ham qiyin problemalardan hisoblangan. 1912 yilgacha Goldbax problemasini hozirgi zamon matematikasi metodlari bilan yechib bo'lmaydi degan fikr mavjud bo'lgan.

Faqat 1919 yilga kelib V.Brun mohiyati jihatidan Eratosfen g'alvirining takomillashtirilgani bo'lgan metodni ishlab chiqdi. U o'z metodi yordamida har qanday yetarlicha katta natural sonni har biri 9 tadan ortiq bo'lmagan tub sonlar ko'paytmasidan iborat bo'lgan ikkita qo'shiluvchining yig'indisi ko'rinishida ifodalash mumkin ekanligini ko'rsatdi.

Keyinchalik B. Brun natijasi bir necha bor yaxhilandi, lekin Goldbax problemasini bu metod bilan yechib bo'lmadi. Shunga qaramasdan B. Brun metodi, keyinchalik esa uning turli shakl o'zgartirilgan variantlari: A. Selberg g'alviri, Yu.Linnikning katta g'alvirlari tub sonlar taqsimoti nazariyasida tadbiq etilib, bu sohada salmoqli natijalar olish imkonini berdi.

Ingliz matematiklari G.Xardi va Dj. Littlvud (Hardy G.H., Littlewood J.E.) lar 1924 yilda Goldbaxning ternar problemaga doiraviy usulni qo'llab, hozircha isbotlanmagan Dirixle $L-$ funksiyaning no'llari haqidagi Rimanning umumlashgan gipotezasini(URG) (unga ko'ra Dirixle $L-$ funksiyasi

$$L(s,\chi)=\sum_{n=1}^{\infty}\frac{\chi(n)}{n^s},\quad Res=\sigma>1,\qquad (s=\sigma+it)$$

(bunda $\chi(n)$ $-$Dirixle xarakteri) ning barcha trivilal bo'lmagan no'llari $\sigma=\frac{1}{2}$ to'g'ri chiziqda yotadi) o'rinli deb qarab yetarlicha katta n toq sonining uchta tub son yig'indisi $n=p_1+p_2+p_3$ ko'rinishda ifodalashlar soni $\mathcal{R}\,(n)$ uchun asimptotik formula oladilar.

I.M. Vinogradov [5,16] o'zi yaratgan trigonometrik yig'indilar metodi yordamida 1937 yilda yetarlicha katta $n\geq N_0$ lar uchun bu masalani hal qildi. 1956 yilda K.G.Borozdkin bunda $N_0\leq expexpexp(41{,}96)$ bo'lishi kerak ekanlini ko'psatdi. Keyinchalik bu natija Chen Jing ren va A.Qosimovlar tomonidan bir necha bor yaxshilandi.

$(1,N_1)$ oraliqdagi toq sonlar uchun Goldbaxning ternar problemasining o'rinli ekanligi kompyuterlar yordamida tekshirib ko'rilgan. Shuning uchun ham faqat $(N_1,N_0)-$ oraliqda problemaning o'rinli ekanligini isbotlash qoldi. Bu sohadgi oxirgi natija J. –M.Deshouillers, G. Effinger, H.Te Riele va D. Zinoviev [6] larga tegishli. Ular agar URG o'rinli bo'lsa, ternar problemaning barcha toq $n\geq 6$ lar uchun o'rinli ekanligini ko'rsatdilar. Lekin URG esa hozircha to'la isbotlamagan.

Endi Xardi - Littlvud va I.M. Vinogradov metodlarining mohiyatiga to'xtalib o'tamiz.

Avvalo, *Xardi - Littlvud metodi* haqida. Bu metodning mohiyati quyidagidan iborat: faraz etaylik $A = \{a_m\}$ – manfiy bo'lmagan butun sonlarning qat'iy o'suvchi ketma-ketligi bo'lsin. Ushbu

$$F(z) = \sum_{m=1}^{\infty} z^{a_m}, \quad (|z|<1)$$

funksiyani qaraymiz. U holda uning s – darajasi

$$F^s(z) = \sum_{m_1=1}^{\infty} \dots \sum_{m_s=1}^{\infty} z^{a_{m_1}+\dots+a_{m_s}} = \sum_{n=0}^{\infty} R_s(n) z^n,$$

dan iborat bo'ladi. Bunda $R_s(n)$ bilan n sonining A dan olingan s ta hadning yig'indisi ko'rinishda ifodalashlar soni belgilangan. Masala hech bo'lmasa n ning katta qiymatlarida $R_s(n)$ ni baholashdan iborat.

Koshining integral formulasiga ko'ra

$$R_s(n) = \frac{1}{2\pi i} \int_{|z|=\rho} F^s(z) z^{-n-1} dz,$$

bunda $0<\rho<1$.

Xardi–Littlvud–Ramanudjan (X-L-R) metodi bo'yicha $R_s(n)$ ikkita I_1 va I_2 qo'shiluvchilarga ajratiladi. $R_s(n)$ uchun asimptotik formulada birinchi qo'shiluvchi I_1 bosh hadni, ikkinchi qo'shiluvchi I_2 esa qoldiq hadni beradi. Shunday qilib X–L–R ning doiraviy usuli, bu $R_s(n)$ dan taxmin qilinayotgan bosh hadni ajratish usulidir.

Endi I.M. Vinogradov metodining mohiyatiga to'xtalib o'tamiz.

Bu metodning mohiyati quyidagidan iborat: avvalo X–L–R metodidagi integral tagidagi funksiyani (cheksiz qatorni) chekli trigonometrik yig'indi bilan almashtirdi. Keyin I_1 ni X−L−R metodi bo'yicha tekshiradi, I_2 esa I.M. Vinogradovning trigonometrik yig'indilar metodi [5,16] bilan baholanadi.

I.M. Vinogradov metodi Goldbaxning ternar problemasini isbotlash va Varing problemasidagi qoldiq hadni yaxshilash imkonini beribgina qolmay, balki hozirgacha qiyin hisoblanib kelingan kasr qismlarining taqsimlanishi, kvadratik chegirmalarning soni singari ko'pchilik masalalarda ham muhim natijalar olish imkonini berdi.

I.2§. Goldbaxning binar problemasi va bu borada olingan keyingi natijalar.

Ikkita tub son yig'indisi haqidagi problemani esa Rimanning umumlashgan gipotezasiga tayanib ham hal etib bo'lmadi. G.Xardi va Dj. Littlvudlar faqatgina "deyarli barcha" juft sonlarning ikkita tub son yig'indisi ko'rinishida ifodalanishinigina ko'rsata oldilar xolos, ya'ni agar $E(X)$ bilan X dan katta bo'lmagan va ikkita tub son yig'indisi ko'rinishida ifodalanmaydigan deb gumon qilingan juft sonlar sonini belgilasak

$$\lim_{X\to\infty}\frac{E(X)}{X}=0$$

ekanligini isbotladilar.

1930 yilda L.G.Shnirelman sonlar nazariyasining additiv masalalarini yechish uchun yangi metodni taklif etdi. U o'zi taklif etgan metod bilan shunday bir r absolyut doimiysi mavjudki, har bir n natural sonini r tadan ortiq bo'lmagan tub sonlar yig'indisi ko'rinishida ifodalash mumkin ekanligini ko'rsatdi. Lekinda L.G.Shnirelman isbotidagi r soni ancha katta bo'lib chiqdi ($r \le 8\cdot 10^5$).

Keyinchalik r ning qiymati ketma-ket bir necha bor N.P.Romanov, X.Xeylbron, E.Landau, Sherka, D.Richchi, X.Shapiro, J.Varga, In Ven-Linya, N.I.Klimov, R.Von va boshqa matematiklar tomonidan yaxshilandi.

A.F.Lavrik faqatgina L.G.Shnirelman metodidan foydalanib $r = 8$ dan yaxshi natija olish mumkin emasligini ko'rsatdi. Shuning uchun ham ko'pchilik mualiflar o'z izlanishlarida L.G.Shnirelman metodining boshqa metodlar bilan kombinatsiyasidan foydalanganini aytib o'tish joizdir.

Lekin binar problema hosirgacha t'ola hal etilgan emas. Bu sohada N.G.Chudakov [7], T. Esterman [8] va Van-der-Corput [9] lar Vinogradovning trigonometrik yig'indilar metodini qo'llab deyarli barcha juft sonlarning ikkita toq tub sonning yig'indisi ko'rinishida ifodalanishini ko'satdilar. Aniqroq qilib aytganda agar $E(X)$ bilan $[2, X)$ oraliqdagi ikkita tub son yig'indisi ko'rinishida ifodalanmaydigan juft sonlarning sonini belgilasak, yuqoridagi mualliflar belgilangan $A > 0$ soni uchun

$$E(X) \ll \frac{X}{ln^A x}$$

bahoning o'rinli ekanligini isbotladilar. Bu natija boshqa metod bilan Yu.Linnik [17] tomonidan ham isbot qilingan.

A.F.Lavrik [18] n juft sonining ikkita tub son yig'indisi ko'rinishida ifodalashlar soni, ya'ni

$$n = p_1 + p_2 \qquad (0.1)$$

tenglamaning tub sonlardagi yechimlari soni $\mathcal{R}\,(n)$ uchun asimptotik formula oladi. Bu formula $(1, X)$ oraliqdagi n ning ko'pi bilan

$$\ll \frac{X}{ln^A x}$$

ta qiymatidan boshqa barcha qiymatlari uchun o'rinli.

Keyinchalik $E(X)$ ning yuqoridagi baholari bir necha bor yaxshilandi. Jumladan R.C.Vaughan [10]

$$E(X) < Xexp\left(-c\sqrt{lnX}\right), \qquad (0.2)$$

R.C. Vaughan va H.L.Montgomerylar [11]

$$E(X) < X^{1-\delta}, \tag{0.3}$$

bunda δ, $0 < \delta < 1$ shartni qanoatlantiruvchi effektiv konstanta. R.C. Vaughan va H.L. Montgomerylar [11] da, agar URG o'rinli bo'lsa, $\delta = \frac{1}{2} + \varepsilon$ deb olish mumkin ekaligini aytib o'tganlar. Bu yerda $\varepsilon > 0$ yetarlicha kichik o'zgarmas son. I.Allakov [12], J.Chen, C.Pan [13] lar tomonidan δ ning qiymati aniqlashtirilib yetarlicha katta X lar uchun

$$E(X) < X^{0,96}$$

baholar olingan.

Faraz etaylik χ - q ($q \le T$) moduli bo'yicha Dirixle xarakteri bo'lsin. Ma'lumki ([19], IX-bob, §2), shunday bir o'zgarmas c_1 soni mavjudki Dirixle L – funksiyasi $L(s,\chi)$, $(s = \sigma + it)$

$$\sigma > 1 - \frac{c_1}{\ln T}, \qquad |t| \le T$$

sohada faqat birta primitiv haqiqiy xarakter $\tilde{\chi}(\mathrm{mod}\,\tilde{r})\,(\tilde{r} \le T)$ uchun $\tilde{\delta} = 1 - \tilde{\beta}$ haqiqiy no'lga ega bo'lishi mumkin. Agar shu shartni qanoatlantiruvchi no'l $\tilde{\beta}$ mavjud bo'lsa, u

$$\frac{c_2}{\tilde{r}^{1/2}\ln^2\tilde{r}} \le 1 - \tilde{\beta} \le \frac{c_1}{\ln T}$$

tengsizlikni qanoatlantiradi. Bu no'l $\tilde{\beta}$ ni L – funksiyaning maxsus no'li deb ataymiz.

I.Allakov [20] da n juft sonining ikkita tub son yig'indisi ko'rinishida ifodalashlar soni, $\mathcal{R}\,(n)$ uchun asimptotik formula oldi. Bu formula $(1, X)$ oraliqdagi n ning ko'pi bilan

$$E(X) < Xexp\left(-c\sqrt{lnX}\right)$$

ta qiymatidan boshqa barcha qiymatlari uchun o'rinli. Lekin u Dirixle $L-$ funksiyasining yuqorida ko'rsatilgan maxsus noli mavjud bo'lsa, bu asimptotik formulada tartibi bosh hadning tartibi bilan bir xil bo'lgan had ham ishtirok etadi. Shuning uchun ham I.Allakov tomonidan [20] da isbotlangan formulalar Dirixle $L-$ funksiyasining maxsus no'li mavjud bo'lmagan holda oqatdagidek asimptotik formula bo'lsada shunday no'l mavjud bo'lgan holda oqatdagidan boshqacharoq formulani ifodalaydi. Ya'ni bu holda $\mathcal{R}\,(n)$ uchun quyidan baho olingan.

[21] da I.Allakov n juft sonining ikkita tub son yig'indisi ko'rinishida ifodalashlar soni, $\mathcal{R}\,(n)$ uchun quyidan baho olgan. Bu baho $(1,X)$ oraliqdagi n ning ko'pi bilan

$$E(X) < X^{0.96}$$

ta qiymatidan boshqa barcha qiymatlari uchun o'rinli.

I.3 §. Binar additiv masalalarning maxsus t'oplami va uni baholash haqida.

Sonlar nazariyasining tub sonlar ishtirok etgan binar masalalari:

a). G.Xardi va Dj. Littlvud masalasi "Har qanday natural n sonini tub son p va natural sonning darajasi m^k yig'indisi ko'rinishida ya'ni $n = p + m^k$ korinishda ifodalash".

b).Xya-Lo-Ken masalasi "Har qanday natural n sonini tub son p va tub sonning darajasi p^k yig'indisi ko'rinishida ya'ni $n = p + p^k$ korinishda ifodalash".

c). Goldbaxning binar masalasi va bularning turli umumlashmalari ([22] ga qarang).

Xardi – Littlvud–Ramanudjan doiraviy metodi nuqtai nazaridan dastavval bir xil sxemada qaraladi. Bunda n ni ko'rsatilgan ko'rinishda ifodalashlar sonini ifodalovchi $\mathcal{R}\,(n)-$ funksiya [0;1] oraliq boyicha olingan integral ko'rinishida ifodalab olinadi va L.Dirixlening approksimatsiya (haqiqiy sonlarni ratsional sonlar bilan almashtirish haqidagi) teoremasidan foydalanib bu integral ikkita integral yig'indisi

$$\mathcal{R}\,(n) = \mathcal{R}_1\,(n) + \mathcal{R}_2\,(n) \qquad (3.1)$$

ko'rinishida ifodalab olinadi. Bunda $\mathcal{R}_1\,(n)$ ga odatda katta yoylar bo'yicha, $\mathcal{R}_2\,(n)$ esa kichik yoylar bo'yicha olingan integral deyiladi.

Agar biz biror usul bilan qaralayotgan n ychun $\mathcal{R}\,(n) > 0$ ekanligini ko'rsatsak, u holda shu n uchun biz o'rganayotgan masala yechimga ega bo'ladi.

Odatda (3.1) dagi $\mathcal{R}_1\,(n)$ integral taxmin qilinayotgan asimptotik formula (yoki isbotlanayotgan bahoning) ning bosh hadini, $\mathcal{R}_2\,(n)$ esa qoldiq hadini beradi. $\mathcal{R}_2\,(n)$ irtegral I.M.Vinogradovning trigonometrik yig'indilar metodi bilan baholalanadi va hozirgi vaqtda trigonometrik yig'indilarning moduli uchun yetarlicha aniq baholari mavjud bo'lgani uchun uni baholshda katta muammolar kelib chiqmaydi. (3.1) dan

$$\mathcal{R}\,(n) \geq \mathcal{R}_1\,(n) - |\mathcal{R}_2\,(n)|$$

bo'lgani uchun $\mathcal{R}\,(n) > 0$ ekanligini ko'rsatish uchun

$$\mathcal{R}_1\,(n) > |\mathcal{R}_2\,(n)| \qquad (3.2)$$

tengsizlikning bajarilishini ko'psatishimiz kerak bo'ladi. Ana shunda qiyinchiliklarga duch kelinadi. $(1, X)$ oraliqdagi $(3.2) -$ tengsizlik o'rinli ekanligini ko'rsata olmagan n larni masalaning maxsus to'plami $M(X)$ ga kiritamiz. Shu maxsus to'plamdagi elementlar sonini $E(X)$ bilan belgilaymiz, ya'ni $E(X) = card\ M(X)$ va uni yuqoridan baholaymiz. Shunday qilib biz 2-paragrafda ko'p qo'llagan $\mathcal{R}\,(n) > f(n)(> 0)$ baho $(1, X)$ oraliqdagi n ning ko'pi bilan $E(X)$ ta qiymatidan boshqa barcha qiymatlari uchun o'rinli deganimiz bu qolgan n lar uchun masala yechimga ega emas degani emas, balki qolgan n lar uchun $\mathcal{R}\,(n) < f(n)$ bajariladi deb tushunilishi kerak.

Bu yerda shuni ham ta'kidlash kerakki, yuqorida eslatib o'tilgan barcha additiv masalalarda

$$E(X) \ll \frac{X}{ln^A x}$$

bo'lganda $\mathcal{R}\,(n)$ uchun turli mualliflar tomonidan asimptotik formulalar olingan, lekin

$$E(X) < Xexp\left(-c\sqrt{lnX}\right) \text{ va } E(X) < X^{0.96}$$

bo'lgan hollarda esa I. Allakov [20,22], V.I.Plaksin [23] va boshqalar tomonidan $\mathcal{R}\,(n)$ uchun quyidan baho olingan. Bu yerda asosiy qiyinchilik masalaning yechimining Dirixle $L-$ funksiyaning no'llari haqidagi URG ga bog'liq ekanligi va URG ning hozirgacha hal etilmaganidadir.

II-BOB. DIRIXLE L-FUKSIYASINING NO’LLARI HAQIDA

II.1- §. L-funksiyaning logarifmik hosilasini no’llari bo’yicha qatorga yoyish.

Bizga ma’lumki ([25], 12-§)

$$\xi(s,\chi)=\left(\frac{q}{\pi}\right)^{\frac{1}{2}s+\frac{1}{2}a}\Gamma\left(\frac{1}{2}s+\frac{1}{2}a\right)L(s,\chi)$$

tenglik bilan aniqlanuvchi $\xi(s,\chi)-$ funksiya uchun

$$\xi(s,\chi)=e^{A+Bs}\prod_{\rho}\left(1-\frac{s}{\rho}\right)e^{\frac{s}{\rho}} \qquad (1.1)$$

no’llar bo’yicha yoyilma orinli. Bunda $\Gamma(s)-$ Eylerning gamma funksiyasi, $a=0$ agar $\chi(n)$ haqiqiy xarakter bo’lib $\chi(-1)=1$ bo’lsa va $a=1$ agar $\chi(-1)=-1$ bo’lsa; $\rho=\beta+i\gamma$ bilan $L(s,\chi)$ning no’llari belgilangan. A,B lar χ bog’liq bo’lgan parametrlar. (1.1) ni logarifmlab keyin hosilasini olsak quyidagiga ega bo’lamiz:

$$\frac{L'}{L}(s,\chi)=-\frac{1}{2}ln\frac{q}{\pi}+B(\chi)+\sum_{\rho}\left(\frac{1}{s-\rho}+\frac{1}{\rho}\right)+\frac{\gamma_0}{2}-$$

$$-\frac{1}{s+a}+\sum_{n=1}^{\infty}\left(\frac{1}{s+a+2n}-\frac{1}{2n}\right) \qquad (1.2)$$

Bu yoyilmadam foydalanib keyinchalik kerak bo’ladigan quyidagi lemmani isbotlaymiz.

1.1-lemma. Agar $-1\le\sigma\le 2,\ s=2+it, 2\le|t|\le T$ bo’lsa, quyidagi tenglik o’rinli:

$$\frac{L'}{L}(s,\chi) = \sum_{|t-\gamma|\le 1} \frac{1}{s-\rho} + c_1\theta_1 lnqT, \quad |\theta_1| < 1, \tag{1.3}$$

bunda $\chi\ (modq)$ $-$primitiv xarakter va yig'indi L funksiya $L(s,\chi)$ ning $|t-\gamma| \le 1$ shartni qanoatlantiruvchi no'llari ρ bo'yicha olinadi;

$$c_1 = 10\left\{1 + \left(\gamma_0 + \frac{1}{2T_0} + \frac{1}{12T_0^2} + \frac{1}{2}\sqrt{1+9T^{-2}}\right)\mathcal{L}_0^{-1}\right\} + 3{,}5629 l_0^{-1},$$

$$\mathcal{L} = lnT, \qquad l = lnqT, \ \ T \ge T_0 \ge 2, \quad q \ge q_0 \ge 3, \qquad \mathcal{L} \ge \mathcal{L}_0, \ \ l \ge l_0 \,.$$

Isboti. Bu lemmaning turli isbotlari bor, masalan [19] ning 111-betiga qarang. $c_1$ning qiymatini aniqlash maqsadida biz isbotni takroran qarab chiqamiz. (1.2) dan $s = 2 + it$ bo'lgandagi xuddi shunday munosabatni ayirib quyidagini hosil qilamiz:

$$\left|\frac{L'}{L}(s,\chi)\right| \le \left|\sum_{\rho}\left(\frac{1}{s-\rho} - \frac{1}{2+it-\rho}\right)\right| + \left|\frac{1}{s+a} - \frac{1}{2+iT+a}\right| + \left|\frac{L'}{L}(2+iT,\chi)\right|$$
$$+ \sum_{n=1}^{\infty}\left|\frac{1}{s+a+2n} - \frac{1}{2+iT+2n}\right|. \tag{1.4}$$

Bu yerda

$$\left|\frac{L'}{L}(2+iT,\chi)\right| \le \sum_{n=1}^{\infty}\frac{\Lambda(n)}{n^2} = \sum_{n=1}^{41}\frac{\Lambda(n)}{n^2} + \sum_{n=43}^{\infty}\frac{\Lambda(n)}{n^2}$$
$$< 0.53088 + \int_{42}^{\infty}\frac{lnx}{x^2} = 0{,}64585; \tag{1.5}$$

$$\left|\frac{1}{s+a} - \frac{1}{2+iT+a}\right| = \frac{2-\sigma}{\sqrt{(\sigma+a)^2+t^2}\sqrt{(2+a)^2+t^2}} \le \frac{3}{2\sqrt{10}}; \tag{1.6}$$

agar $|t-\gamma|>1,\ 0<\beta<1, -1\leq\sigma\leq 2$ bo'lsa, u holda

$$\left|\frac{1}{s-\rho}-\frac{1}{2+it-\rho}\right|=\frac{2-\sigma}{\sqrt{(\sigma-\beta)^2+(t-\gamma)^2}\sqrt{(2-\beta)^2+(t-\gamma)^2}}$$
$$<\frac{3}{(t-\gamma)^2}; \tag{1.7}$$

$$\left|\sum_{|t-\gamma|\leq 1}\frac{1}{2+it-\rho}\right|\leq\sum_{|t-\gamma|\leq 1}\frac{1}{\sqrt{(2-\beta)^2+(t-\gamma)^2}}\leq\sum_{|t-\gamma|\leq 1}\frac{1}{2-\beta}\leq\sum_{|t-\gamma|\leq 1}1;$$

$$\sum_{n=1}^{\infty}\left|\frac{1}{s+a+2n}-\frac{1}{2+iT+2n}\right|\leq(2-\sigma)\sum_{n=1}^{\infty}\frac{((\sigma+a+2n)^2+\mathrm{t}^2)^{-\frac{1}{2}}}{((2+a+2n)^2+\mathrm{t}^2)^{\frac{1}{2}}}<$$
$$(2-\sigma)\frac{1}{4}\zeta(2)\leq\frac{\pi^2}{8} \tag{1.8}$$

ekanligini inobatga olib (1.4) dan

$$\left|\frac{L'}{L}(s,\chi)\right|\leq\left|\sum_{|t-\gamma|\leq 1}\frac{1}{s-\rho}\right|+\sum_{|t-\gamma|\leq 1}1+3\sum_{|t-\gamma|>1}\frac{1}{(t-\gamma)^2}+\ c_2, \tag{1.9}$$

bu yerda

$$c_2=0{,}65585+\frac{3}{2\sqrt{10}}+\frac{\pi^2}{8}.$$

Endi $|t|\leq T$ bo'lganda

$$\sum_{|t-\gamma|>1}\frac{1}{(t-\gamma)^2}\leq c_3\, l \tag{1.10}$$

bajarilishini ko'rsatamiz. Bu

$$\sum_{\gamma}\frac{1}{4+(t-\gamma)^2}\le c_4\, l \qquad (1.11)$$

bahodan kelib chiqadi. Haqiqatan ham (1.11) dan

$$\sum_{|t-\gamma|>1}\frac{1}{4+(t-\gamma)^2}\le c_4\, l.$$

Bundan

$$\sum_{|t-\gamma|>1}\frac{1}{4+(t-\gamma)^2}\ge\sum_{|t-\gamma|>1}\frac{1}{5(t-\gamma)^2}$$

ekanligini e'tiborga olib va $c_3=5c_4$ deb olib (1.10) ni hosil qilamiz. Endi (1.11) ni isbotlaymiz. (1.11) formulada $s=2+it$ deb olib va (1.12), (1.13) va (2.5) ga asosan

$$Re\sum_{\rho}\frac{1}{s-\rho}<\frac{1}{2}ln\frac{q}{\pi}-\frac{\gamma_0}{2}+Re\frac{1}{s+a}+0.64585+c_5\mathcal{L},$$

bunda

$$c_5=\frac{1}{2}\left\{1+\left(\gamma_0+\frac{1}{2T_0}+\frac{1}{12T_0^2}+\frac{1}{2}\sqrt{1+9T_0^{-2}}\right)\mathcal{L}_0^{-1}\right\},\qquad T\ge T_0.$$

Endi

$$Re\frac{1}{s+a}=\frac{2+a}{(2+a)^2+t^2}\le\frac{1}{4},$$

bo'lgani uchun

$$Re\sum_{\rho}\frac{1}{s-\rho}<(c_5+0{,}0348767\ l_0^{-2})lnqT$$

bo'ladi. Bundan tashqari

$$Re\sum_{\rho}\frac{1}{s-\rho}=Re\frac{1}{(2-\beta)+i(t-\gamma)}=\frac{2-\beta}{(2-\beta)^2+(t-\gamma)^2}\geq\frac{1}{4+(t-\gamma)^2}.$$

Demak,

$$\sum_{\gamma}\frac{1}{4+(t-\gamma)^2}\leq c_4\, l,$$

bunda

$$c_4=c_5+0{,}0348767\, l_0^{-2}=\frac{1}{2}\left\{1+\left(\gamma_0+\frac{1}{2T_0}+\frac{1}{12T_0^2}+\frac{1}{2}\sqrt{1+9T_0^{-2}}\right)\mathcal{L}_0^{-1}\right\}$$
$$+0{,}0348767\, l_0^{-2}.$$

Shuningdek (2.11) dan L funksiya $L(s,\chi)$ ning $|t-\gamma|\leq 1$ shartni qanoatlantiruvchi no'llari soni $c_3 l$ dan ko'p emas, ya'ni

$$\sum_{|t-\gamma|\leq 1} 1\leq c_3 l \qquad (2.12)$$

bajariladi. Endi hosil qilingan baholarni (2.9) ga qo'yib lemmadagi tasdiqqa ega bo'lamiz.

1-natija. Agar $\frac{1}{2}\leq\sigma\leq\frac{5}{2},\ s=\sigma+it,\ 2\leq|t|\leq T$ va $\chi\ (modq)-$ primitiv xarakter bo'lsa, u holda

$$\frac{L'}{L}(s,\chi)=\sum_{|t-\gamma|\leq 1}\frac{1}{s-\rho}+c_6\theta_2 lnqT$$

tenglik o'rinli. Bunda $|\theta_2|<1$ va

$$c_6=\frac{58}{9}\left\{1+\left(\gamma_0+\frac{1}{2T_0}+\frac{1}{12T_0^2}+\frac{1}{2}\sqrt{1+12{,}25T_0^{-2}}\right)\mathcal{L}_0^{-1}\right\}+1{,}9259\ l_0^{-2}.$$

2-natija. Agar $\frac{1}{2} \le \sigma \le \frac{5}{2},\ s = \sigma + it,\ 2 \le |t| \le T$ va $\chi\ (modq) -$ primitiv xarakter bo'lsa, u holda

$$\sum_{|t-\gamma|\le 1} 1 \le c_7 lnqT,$$

bu yerda

$$c_7 = \frac{29}{12}\left\{1 + \left(\gamma_0 + \frac{1}{2T_0} + \frac{1}{12T_0^2} + \frac{1}{2}\sqrt{1 + 12{,}25T_0^{-2}}\right)\mathcal{L}_0^{-1}\right\} + 0{,}0698\ l_0^{-2}.$$

Ikkala natija ham 2.1-lemmadan kelib chiqadi, agar uning isbotida $\frac{1}{2} \le \sigma \le \frac{5}{2}$ ekanligini e'tiborga olsak. Bu yerda [24] dagi natijadan foydalandik.

II.2- §. $N(T,\chi)$ funksiya uchun formulaning qoldiq hadiga kiruvchi o'zgarmaslarning qiymatini aniqlash.

Faraz etaylik $\chi(\mathrm{mod}q) -$Dirixle xarakteri bo'lsin. Ma'lumki, $L(s,\chi) -$ funksiya bilan bog'liq turli masalalarni yechishda ko'pincha $N(T,\chi)$va $\psi(x,\chi)$ funksiyalar uchun aniq formulalardan foydalaniladi ([25], 16-19§). Shuning uchun ham ko'pgina nazariy sonli o'zgarmaslarning qiymatlarini aniqlashda yuqorida eslatib o'tilgan formulalarda ishtiroq etuvchi o'zgarmaslarning qiymatlarini bilish talab etiladi. Xususan ular Montgomeri va Vo'nning teoremasidagi δ va $X_0(\delta)$ larning qiymatlarini aniqlash uchun kerak.

Ushbu paragrafda biz ana shularga doir quyidagi ikkita teoremani isbotlaymiz.

2.1-teorema. $T \ge T_0$ bo'lganda quyidagi formulalar o'rinli:

a) agar $q \ge 3$ bo'lib $\chi(modq) -$ primitiv xarakter bo'lsa, u holda

$$N(T,\chi) = \frac{T}{\pi} ln\frac{qT}{2\pi} - \frac{T}{\pi} + 33{,}3396\theta_1 logqT;$$

b) agar $q \ge 3$ bo'lib $\chi = \chi_0^* - 1$ moduli bo'yicha bosh xarakter bo'lsa, u holda

$$N(T,\chi_0^*) = \frac{T}{\pi} ln\frac{qT}{2\pi} - \frac{T}{\pi} + 32{,}2266\theta_2 logT;$$

c) agar $q \geq 3$ bo'lib $\chi(modq) -$ ixtiyoriy xarakter bo'lsa, u holda

$$N(T,\chi) = \frac{T}{\pi} ln\frac{qT}{2\pi} - \frac{T}{\pi} + 22{,}4621\theta_3 Tlogq$$

tenglik bajarildai. Bunda $|\theta_i| \leq 1,\ i = 1,2,3.$

2.1-teoremani isbotlash uchun bizga quyidagi lemma kerak bo;ladi.

2.1-lemma. Agar $-\pi < args < \pi$ bo'lsa quyidagi formula o'rinli:

$$ln\Gamma(s) = \left(s - \frac{1}{2}\right) lns - s + \frac{1}{2} ln2\pi + r\left(\frac{1}{s}\right),$$

bunda

$$\left|r\left(\frac{1}{s}\right)\right| \leq \frac{1}{12|s|}\left(1 + \frac{1}{30|s|^2} + \frac{1}{105|s|^4}\right).$$

Isboti. $|args| < \pi$ bo'lganda [26] dagi (6.1.42) formulaga asosan

$$r\left(\frac{1}{s}\right) = ln\Gamma(s) - \left(s - \frac{1}{2}\right) lns + s - \frac{1}{2} ln2\pi = R\left(\frac{1}{s}\right) + \sum_{m=1}^{n} \frac{B_{2m}}{2m(2m-1)s^{2m-1}},$$

bunda $B_{2m} - 2m$-Bernulli soni va

$$\left|R\left(\frac{1}{s}\right)\right| \leq \frac{|B_{2n+2}|\, K(s)}{(2n+1)(2n+2)|s|^{2n+1}},$$

$K(s)$ bilan $u \geq 0$ bo'lganda

$$\left|\frac{s^2}{u^2+s^2}\right|$$

ning yuqori chegarasi belgilangan. $K(s) \leq 1$ bo'lanligi sababli oxirgi tenglikda $n = 2$ deb olsak undan lemmadagi tasdiq kelib chiqadi.

1-natija. b o'zgarmas son ($|b| < |s|$) bo'lib $|args| < \pi$ bo'lsa, u holda quyidagi tenglik o'rinli:

$$ln\Gamma(s+b) = \left(s+b-\frac{1}{2}\right)lns - s + \frac{1}{2}ln2\pi + r_1\left(\frac{1}{s}\right),$$

bunda

$$\left|r_1\left(\frac{1}{s}\right)\right| \leq \frac{|3b^2-b|}{2(|s|-|b|)} + \left|r\left(\frac{1}{s+b}\right)\right|.$$

2-natija. Agar $|args| \leq \pi - \delta,\ (\delta > 0)$ bo'lsa, u holda quyidagi tenglik o'rinli:

$$\frac{\Gamma'}{\Gamma}(s) = lns - \frac{1}{2s} + r_\delta\left(\frac{1}{s}\right),$$

bunda

$$\left|r_\delta\left(\frac{1}{s}\right)\right| \leq \frac{\delta}{2|s|sin\delta} < \frac{\pi}{4|s|}.$$

Isboti. Ma'lumki, $ln\Gamma(s)$ uchun quyidagi munosabat o'rinli ([19] ning 28-beti):

$$ln\Gamma(s) = \left(s-\frac{1}{2}\right)lns - s + \frac{1}{2}ln2\pi + \int_0^\infty \frac{[u]-u+\frac{1}{2}}{u+s}\,du. \qquad (2.1)$$

Bundan

$$\left|\frac{\Gamma'}{\Gamma}(s) - lns + \frac{1}{2s}\right| = \left|\int_0^\infty \frac{[u] - u + \frac{1}{2}}{(u+s)^2} du\right| \le \frac{1}{2}\int_0^\infty \frac{du}{u^2 + |s|^2 - 2u|s|cos\delta} =$$

$$\frac{\delta}{2|s|sin\delta} < \frac{\pi}{4|s|}.$$

Biz bu (2.1) da integral ostida differensiallash qoidasidan foydalandik.

2.1-teoremaning isboti. Faraz etaylik $\chi(modq)$ primitiv xarakter va

$$\xi(s,\chi) = \left(\frac{q}{\pi}\right)^{\frac{1}{2}s+\frac{1}{2}a} \Gamma\left(\frac{1}{2}s + \frac{1}{2}a\right) L(s,\chi) \qquad (2.2)$$

bo’lsin. s uchlari $\frac{5}{2} \pm iT, -\frac{3}{2} \pm iT$ nuqtalarda bo’lgan to’g’ri to’rtburchakning tomonlari boyicha harakatlanganda $arg\xi(s,\chi)$ nind orttirmasini qaraymiz. Bu to’g’ri to’rtburchakga $L(s,\chi)$ ning faqat birta $s = 0$ yoki $s = 1$ bo’lgandagi trivial no’li tegishli va shuning uchun ham

$$2\pi(N(T,\chi) + 1) = \Delta_R arg\xi(s,\chi)$$

bajariladi. $\xi(s,\chi)$ning funksional tenglamasi [25]

$$\xi(1-s,\bar{\chi}) = \frac{i^a q^{\frac{1}{2}}}{\tau(\chi)}\xi(s,\chi)$$

dan $arg\xi(\sigma + it,\chi) = arg\overline{\xi(1 - \sigma - \imath t,\chi)} + c$, bunda

$$\tau(\chi) = \sum_{h=1}^{q} \chi(h) e\left(\frac{h}{q}\right)$$

Gauss yig'indisi va c soni s ga bog'liq emas, shuning uchun ham kontyrning chap tomoni bo'yicha harakatlangandagi argument orttirmasi uning o'ng tomoni bo'yicha harakatlangandagi argumenr orttirmasiga teng.

Shunday qilib

$$2\pi(N(T,\chi)+1)=\Delta_R arg\xi(s,\chi)=2\left(\Delta_{R_1} arg\xi(s,\chi)\right), \tag{2.3}$$

bu yerda Δ_{R_1}bilan s R ning o' ng yarmida o'zgargandagi $arg\xi(s,\chi)$ning orttirmasi belgilangan. Endu 1-natijadan foydalanib $\Delta_{R_1} arg\xi(s,\chi)$ ni hisoblaymiz:

$$\Delta_{R_1}\arg\left(\frac{q}{\pi}\right)^{\frac{1}{2}s+\frac{1}{2}a}=\Delta_{R_1}\left(\frac{t}{2}ln\frac{q}{\pi}\right)=Tln\frac{q}{\pi};$$

$$\begin{aligned}\Delta_{R_1}\arg\Gamma\left(\frac{1}{2}s+\frac{1}{2}a\right)&=\Delta_{R_1}\operatorname{Im}\log\Gamma\left(\frac{1}{2}s+\frac{1}{2}a\right)\\&=2\left\{\operatorname{Im}\log\Gamma\left(\frac{1}{2}\left(\frac{1}{2}+a+iT\right)\right)-\operatorname{Im}\log\Gamma\left(\frac{1}{2}\left(\frac{5}{2}+a\right)\right)\right\}\\&=2Im\left\{\left(-\frac{1}{4}+\frac{a}{2}+\frac{1}{2}iT\right)\ln\left(\frac{1}{2}iT\right)-\frac{1}{2}iT+\frac{1}{2}ln2\pi+r_1\left(\frac{1}{s}\right)\right\}\\&=Tln\frac{T}{2}-T+(2a-1)\frac{\pi}{4}+2\theta_4\left|r_1\left(\frac{1}{s}\right)\right|,\qquad|\theta_4|\le 1.\end{aligned}$$

1-natijadan $T\ge T_0$ bo'lganda

$$2\left|r_1\left(\frac{1}{s}\right)\right|<2\left\{\frac{15}{16(1-1{,}5T_0^{-1})}+\frac{1}{6}\left(1+\frac{2}{15T_0^2}+\frac{16}{105T_0^4}\right)\right\}\frac{1}{T}=c_8\frac{1}{T},$$

bunda

$$c_8=2\left\{\frac{15}{16(1-1{,}5T_0^{-1})}+\frac{1}{6}\left(1+\frac{2}{15T_0^2}+\frac{16}{105T_0^4}\right)\right\}.$$

Endi $L(s,\chi)$ ning argumentini qaraymiz.

$$\pi S(\chi, T) = \Delta_{R_1} arg\, L(s, \chi)$$

deb belgilab olamiz. Bu tenglikning o'ng tomonini quyidagicha yozish mumkin:

$$\Delta_{R_1} arg\, L(s, \chi) = 2 \int_{\frac{5}{2}}^{\frac{5}{2}+iT} Im\left\{\frac{L'}{L}(s, \chi)\right\} ds - 2 \int_{\frac{1}{2}+iT}^{\frac{5}{2}+iT} Im\left\{\frac{L'}{L}(s, \chi)\right\} ds.$$

Birinchi integralni quyidagicha baholash mumkin:

$$\left|2 \int_0^T Im \sum_{n=1}^{\infty} \frac{\chi(n)\Lambda(n)}{n^{\frac{5}{2}+it}} dt\right| \leq 4\zeta\left(\frac{5}{2}\right) = 5{,}364.$$

Ikkinchi integralni baholash uchun 2.1-lemmaning natijalaridan foydalanamiz. Bu yerda

$$\left|\int_{\frac{1}{2}+iT}^{\frac{5}{2}+iT} Im\,(s-\rho)^{-1} ds\right| = |\Delta \arg(s-\rho)| \leq \pi$$

bo'lgani uchun 2.1-lemmaning 1va 2-natijalaridan $q \geq q_0$, $T \geq T_0$ bo'lganda quyidagiga ega bo'lamiz:

$$\pi S(\chi, T) \leq c_9 l,$$

bunda $c_9 = 4c_6 + 2\pi c_7 + 5{,}364 l_0^{-1}$. Shunday qilib (2.3)-tenglikdan quyidagi tenglikni hosil qilamiz:

$$N(T, \chi) = \frac{T}{\pi} ln \frac{qT}{2\pi} - \frac{T}{\pi} + \theta_1 c_{10} l,$$

bu yerda

$$c_{10} = \frac{1}{\pi}(\, c_9 + \, c_8(T_0 l_0)^{-1})$$

$$= 13{,}0385\left\{1 + \left(\gamma_0 + \frac{1}{2T_0} + \frac{1}{12T_0^2} + \frac{1}{2}\sqrt{1 + 12{,}25T_0^{-2}}\right) \mathcal{L}_0^{-1}\right\}$$

$$+\left(4{,}4969+\frac{0{,}5968}{(1-1{,}5T_0^{-1})}+\frac{0{,}1061}{T_0}+\frac{0{,}0142}{T_0^3}+\frac{0{,}01617}{T_0^5}\right)\frac{1}{l_{0.}}. \qquad (2.4)$$

Endi faraz etaylik $\chi=\chi_0^*-1$ moduli bo'yicha bosh xarakter bo'lsin u holda $L(s,\chi_0^*)=\zeta(s)$ va biz $\xi(s,\chi)$ ning o'rniga ([25], 15-§)

$$\xi(s)=s(s-1)\pi^{-\frac{1}{2}s}\,\Gamma\left(\frac{1}{2}s+1\right)\zeta(s)$$

funksiyani qaraymiz. Bu holda ham yuqoridagi sungari mulohazani takrorlab quyidagi formulaga ega bo'lamiz:

$$N(T,\chi_0^*)=\frac{T}{\pi}ln\frac{T}{2\pi}-\frac{T}{\pi}+\theta_2c_{11}\mathcal{L}, \qquad (2.5)$$

bu yerda

$$c_{11}=5{,}4843\left\{1+\left(\gamma_0+\frac{1}{2T_0}+\frac{1}{12T_0^2}+\frac{1}{2}\sqrt{1+4T_0^{-2}}\right)\mathcal{L}_0^{-1}\right\}$$

$$+\left(21{,}7472+\frac{0{,}3786}{T_0}+\frac{0{,}7162}{T_0^2}+\frac{0{,}0091}{T_0^3}+\frac{0{,}0103}{T_0^5}\right)\frac{1}{\mathcal{L}_0}.$$

Nihoyat χ primitiv bo'lmagan xarakter bo'lib $\chi_1(modq_1)$ primitiv xarakter bilan indutsirlangan bo'lsin. U holda

$$L(s,\chi)=L(s,\chi_1)\prod_{p\backslash q,p\nmid q_1}\left(1-\frac{\chi_1(p)}{p^s}\right)$$

formulaga asosan $L(s,\chi)$ funksiyaning $L(s,\chi_1)$ ning no'llaridan farqli no'llari $1-\chi_1(p)p^{-s}$ ko'paytuvchining chekli sondagi no'llaridan iborat bo'lib $1-\chi_1(p)p^{-s}=0$ bajarilishi kerak. Bu esa q/p va $q_1\nmid p$ bo'lgandagina, ya'ni

$$s=\frac{ln\chi_1(p)}{lnp}=i\frac{arg\chi_1(p)+2\pi n}{lnp}, \qquad (n-\text{butun son}).$$

$|t| \leq T$ sohadagi bunday nuqtalar soni

$$\frac{1}{2\pi}\sum_{\frac{q}{p},q_1\nmid p}(Tlnp+1) \leq \frac{1}{2\pi}\left(1+\frac{1}{T_0 ln2}\right)Tlnq$$

ga teng. Shuning uchun ham bu holda

$$N(T,\chi) = \frac{T}{\pi}ln\frac{T}{2\pi} - \frac{T}{\pi} + \theta_6\frac{1}{2\pi}\left(1+\frac{1}{T_0 ln2}\right)Tlnq + c_{10}\theta_1 l$$

yoki agar $T \geq T_0(\geq e), q \geq q_0$ bo'lsa, u holda

$$N(T,\chi) = \frac{T}{\pi}ln\frac{T}{2\pi} - \frac{T}{\pi} + \theta_3 c_{12}Tlnq$$

kelib chiqadi. Bunda

$$c_{12} = \frac{1}{2\pi}\left(1+\frac{1}{T_0 ln2}\right) + \frac{c_{10}}{T_0} + \frac{c_{10}}{lnq_0}\underbrace{max}_{T\geq T_0}\frac{lnT}{T}. \qquad (2.6)$$

(2.4)-(2.6) larda $q_0 = 3,\quad T_0 = 3$ deb olib 2.1-teoremaning tasdig'iga ega bo'lamiz.

3-natija. Agar $N(T)$ Rimanning dzeta-funktsiyasi $\zeta(s), (s = \sigma + it)$ ning $0 < \sigma < 1,\ |t| \leq T$ to'g'ri to'rtburchakdagi trivial bo'lmagan no'llarining soni bo'lsa, u holda

$$N(T) = \frac{T}{2\pi}ln\frac{T}{2\pi} - \frac{T}{2\pi} + +6{,}512850\theta\ lnT$$

формула o'rinli.

Bu natija ilgari $N(T)$ bilan bog'liq mavjud natijalarning aniqlashtirilgani hisoblanadi.

Isboti. Avvalo

$$N(T)=\frac{T}{2\pi}ln\frac{T}{2\pi}-\frac{T}{2\pi}+\theta c_1\,lnT \qquad (2.7)$$

,

ning bajarilishini ko'rsatamiz. Bunda

$$T\geq T_0\geq 3, |\theta|<1\;va\;\;2c_1$$

$$=5{,}4843\left\{1+\gamma_0+\frac{1}{2T_0}+\frac{1}{12T_0^2}+\frac{1}{2lnT_0}\sqrt{1+4T_0^{-2}}\right\}+$$

$$\left(21{,}7472+0{,}3786\mathrm{T}_0^{-1}+0{,}7162\mathrm{T}_0^{-2}+0{,}0091\mathrm{T}_0^{-3}+0{,}0103\mathrm{T}_0^{-5}\right)\frac{1}{lnT_0}.$$

$0<\sigma<1,\;|t|<14{,}135$ sohada $\zeta(s)$ (см.[31]) ning no'llari mavjud bo'lmagani uchun, $T_0\geq 14{,}135$ deb olishimiz mumkin.U holda (2.7) dan

$$N(T)=\frac{T}{2\pi}ln\frac{T}{2\pi}-\frac{T}{2\pi}+6{,}512850\theta\,lnT$$

ga ega bo'lamiz. Taqqoslash uchun yuqorida isbotlangan 2.1-teoremadan

$$N(T)=\frac{T}{2\pi}ln\frac{T}{2\pi}-\frac{T}{2\pi}+32{,}22660\theta\,lnT$$

to'g'ridan-to'g'ri kelib chiqadi. Agar biz yuqoridagi teoremaning isbotida $\chi=\chi_0^*$ va $T_0\geq 14{,}135$ deb olsak (2.7)-formulaga ega bo'lamiz.

II.3-§. $\psi(x,\chi)$ funksiya uchun aniqlashtirilgan formula.

3.1-teorema. Agar $\chi(modq)-$ ixtiyoriy xarakter va $3\leq T\leq x$ bo'lsa, u holda

$$\psi(x,\chi)=\delta_\chi x-E_{\tilde{\beta}}\frac{x^{\tilde{\beta}}}{\tilde{\beta}}-\sum\nolimits'_{|\gamma|<T}\frac{x^\rho}{\rho}+R(x,T), \qquad (3.1)$$

tenglik o'rinli. Bu yerda

$$\delta_\chi = \begin{cases} 1, & \text{agar } \chi = \chi_0 - \text{bosh xarakter bo'lsa;} \\ 0, & agar\ \chi = \chi_0 \ \ \text{bo'lsa} \end{cases};$$

$$E_{\tilde{\beta}} = \begin{cases} 1, & \text{agar } \chi = \tilde{\chi} - \text{maxsus haqiqiy xarakter bo'lsa;} \\ 0, & agar\ \chi \neq \tilde{\chi} \ \ \text{bo'lsa} \end{cases};$$

o'ng tomondagi yig'indi $L(s,\chi)$ning $0 < \sigma < 1$, $|t| \leq T$ to'g'ri to'rtburchakdagi maxsus haqiqiy no'ldan boshqa barcha no'llari bo'yicha olinadi va

$$|R(x,T)| < 1445{,}9163\ \frac{x}{T} ln^2 qx + E_{\tilde{\beta}} x^{\frac{1}{4}} lnx$$
$$+ 2{,}6721(lnx) \min\left(1; \frac{x}{\pi\langle x\rangle T}\right), \qquad (3.2)$$

bunda $\langle x\rangle$ bilan x dan unga eng yaqin turgan tub sonning darajasigacha bo'lgan masofa belgilangan.

Isboti. Avvalo quyidagi belgilashlarni kiritamiz: $\delta(y)$ bilan

$$\frac{1}{2\pi i}\int_{\varkappa - i\infty}^{\varkappa + i\infty} y^s \frac{ds}{s} = \begin{cases} 0, & \text{agar } 0 < y < 1 \text{ bo'lsa;} \\ \frac{1}{2}, & \text{agar } y = 1 \text{ bo'lsa;} \\ 1, & \text{agar } y > 1 \text{ bo'lsa} \end{cases}$$

tenglikning o'ng tomoni bilan berilgan funksiyani belgilaymiz va

$$I(y,T) = \frac{1}{2\pi i}\int_{\varkappa - iT}^{\varkappa + iT} y^s \frac{ds}{s}$$

bo'lsin. 3.1-teoremani isbotlashda quyidagi lemmalardan foydalanamiz.

3.1-lemma. Agar $y > 0$, $\varkappa > 0\ va\ T > 0$ bo'lsa, quyidagi munosabat o'rinli:

$$|I(y,T)-\delta(y)| \leq \begin{cases} y^{\varkappa}\min(1;(\pi T)^{-1}|lny|^{-1}), & \text{agar } y \neq 1 \text{ bo'lsa;} \\ \varkappa(\pi T)^{-1}, & \text{agar } y = 1 \text{ bo'lsa.} \end{cases}$$

Bu lemmaning isboti [25] ning 17-§ da keltirilgan.

$\chi(modq)-$ ixtiyoriy primitiv xarakter bo'lsin. $\sigma \leq -1$ yarim tekislikdan

$$|s+a+2m| \leq \frac{1}{4}, \qquad m=0,1,2,\dots$$

ko'rinishdagi doiradagi nuktalarni chiqarib tashlaymiz. Bunda $a=\frac{1}{2}\{1-\chi(-1)\}$. Qolgan sohani G bilan belgilaymiz.

3.2-lemma. Agar $|t| \geq t_0, q \geq q_0$ bo'lsa, G sohada quyidagi tengsizlik o'rinli:

$$\left|\frac{L'}{L}(s,\chi)\right| \leq c_{13}lnq|s|,$$

bunda

$$c_{13}=\left(1+\frac{2{,}5708}{\sqrt{4+t_0^2\,ln(4+t_0^2)}}\right)\left(1+\frac{2\pi}{ln(4+t_0^2)}\right)\left(1+\frac{ln\left(1+\frac{1}{\sqrt{1+t_0^2}}\right)}{lnq_0\sqrt{1+t_0^2}}\right)$$

$$+\frac{6{,}2759}{lnq_0\sqrt{1+t_0^2}}.$$

Isboti. Bu lemma K.Praxar [27] dagi 4.3-teoremaning shakl o'zgarishidan iboratdir. U yerdagi isbotdan foydalanib c_{13} ning qiymatini aniqlaymiz. [27] ning VII-bob dagi (4.14)-formulaga ko'ra

$$\frac{L'}{L}(s,\chi)=ln\frac{2\pi}{q}+\frac{\pi}{2}ctg\frac{\pi}{2}(s+a)-\frac{\Gamma'}{\Gamma}(1-s)-\frac{L'}{L}(1-s,\bar{\chi}). \qquad (3.3)$$

Bu yerda

$$\left|\frac{L'}{L}(1-s,\bar{\chi})\right| \le \sum_{n=1}^{\infty}\frac{\Lambda(n)}{n^2} < 0{,}64585,$$

va $|t| \ge 1$ bo'lsa, u holda

$$\left|ctg\frac{\pi}{2}(s+a)\right| \le \frac{1+e^{-\pi}}{1-e^{-\pi}} < 1{,}0904.$$

$|t| < 1$ bo'lganda $\left|ctg\frac{\pi}{2}(s+a)\right|$ funksiyani ekstremumga tekshirib G sohada $\left|ctg\frac{\pi}{2}(s+a)\right| \le 2{,}4142$ ekanligini ko'ramiz. Shuning uchun ham $\left|ctg\frac{\pi}{2}(s+a)\right| \le c_{14}$, bunda

$$c_{14} = \begin{cases} 2{,}4142, \text{agar } |t| < 1 \text{ bo'lsa;} \\ 1{,}0904, \text{agar } |t| \ge 1 \text{ bo'lsa.} \end{cases}$$

3.1-lemmaning 2-natijasida s ni $1-s$ bilan almashtirib va $|t| \ge t_0$ bo'lganda

$$|\ln(1-s)| \le (1+\pi(ln|1-s|)^{-1})(ln(1+|s|^{-1})+ln|s|),$$

va $|t| \ge t_0$ bo'lganda

$$|1-s| \ge \sqrt{4+t_0^2}$$

ekanliklarini e'tiborga olib

$$\left|\frac{\Gamma'}{\Gamma}(1-s)\right| \le c_{15}(ln(1+|s|^{-1})+ln|s|),$$

ni hosil qilamiz. Bunda

$$c_{15} = \left(1+\frac{2{,}5708}{\sqrt{4+t_0^2}\,\ln(4+t_0^2)}\right)\left(1+\frac{2\pi}{ln(4+t_0^2)}\right).$$

Hosil qilingan baholarni yig'ib (3.3) dan 3.2-lemmaning tasdig'iga ega bo'lamiz va bunda c_{13} quyidagiga teng bo'ladi:

$$c_{13} = c_{15} + \left\{\frac{\pi}{2}c_{14} + 2{,}4837 + c_{15}\, ln\left(1 + \frac{1}{\sqrt{1+t_0^2}}\right)\frac{1}{lnq_0\sqrt{1+t_0^2}}\right\} =$$

$$= \left(1 + \frac{2{,}5708}{\sqrt{4+t_0^2}\, ln(4+t_0^2)}\right)\left(1 + \frac{2\pi}{ln(4+t_0^2)}\right)\left(1 + \frac{ln\left(1+\frac{1}{\sqrt{1+t_0^2}}\right)}{lnq_0\sqrt{1+t_0^2}}\right)$$

$$+ \frac{6{,}2759}{lnq_0\sqrt{1+t_0^2}}.$$

3.3-lemma. Agar $|s| \geq |s_0| > 2$ bo'lsa, u holda

$$\sum_{\rho} |2-\rho|^{-2} \leq c_{16}\, lnq$$

tengsizlik o'rinli bo'ladi. Bunda $\rho -\ L(s,\chi)$ning barcha no'llarini qabul qiladi, $\chi(modq) -$ ixtiyoriy primitiv xarakter.

Bu lemmaning isboti G. Davenport [25] ning 108-betida keltirilgan. Davenportning ana shu isbotidan foydalanib c_{16} ning son qiymatini aniqlaymiz. 3.1-lemmaga asosan

$$ln\left|\Gamma\left(\frac{s+a}{2}\right)\right| \leq c_{17}|s|ln|s|,$$

bu yerda

$$c_{17} = \frac{1}{2}\left(1 + \frac{a}{|s_0|}\right)\left(1 + \frac{1}{|s_0+a|} + \frac{1}{ln\frac{|s_0+a|}{2}} + \frac{ln2\pi}{|s_0+a|ln\frac{|s_0+a|}{2}}\right.$$

$$\left. + \frac{2\left|r\left(\frac{1}{s+a}\right)\right|}{|s_0+a|ln\frac{|s_0+a|}{2}}\right), \quad |s+a| \geq |s_0+a|,$$

va $\sigma \geq \frac{1}{2}$ bo'lganda [25] ning 14-betidagi (14)- formulaga k'ora

$$|L(s,\chi)| \leq 2q|s|$$

bo'lgani uchun (3.4) dan

$$|\xi(s,\chi)| \leq q^{\frac{1}{2}(\sigma+3)}\, exp(c_{18}|s|ln|s|)\,,$$

bunda

$$c_{18} = \frac{ln\frac{2}{\sqrt[4]{\pi}}}{|s_0|ln|s_0|} + |s_0|^{-1} + c_{17}, \qquad |s| \geq |s_0|, \qquad \sigma \geq \frac{1}{2}.$$

Shunday qilib $|s-2| = R$ aylanada quyidagiga ega bo'lamiz:

$$ln|\xi(s,\chi)| < Rlnq + c_{19}RlnR,$$

bunda

$$c_{19} = c_{18}(1-2|s_0|^{-1})^{-1}\{1 + (ln(1-2|s_0|^{-1})^{-1})\,(lnR_0)^{-1}\}\,,$$
$$R \geq R_0 \geq 3, |s_0| > 2.$$

$L-$ funksiyaning funksional tenglamasiga asosan bu tengsizlik $\sigma \leq \frac{1}{2}$ bo'lganda ham o'rinli. $s = 2$ da

$$|L(s,\chi)|^{-1} \leq \frac{\pi^2}{6} \qquad \text{va} \qquad \left|\Gamma\left(\frac{s+a}{2}\right)\right|^{-1} \leq 1{,}1281$$

bo'lgani uchun $\xi(s,\chi)$ ning ta'rifidan ((3.4) ga qarang) $-ln|\xi(s,\chi)| \leq 0{,}6877$ kelib chiqadi. Bu baholardan va Iyensen formulasi

$$\int_0^R r^{-1}n(r)dr = \frac{1}{2\pi}\int_0^{2\pi} ln|\xi(s,\chi)|ds - ln|\xi(s,\chi)|$$

dan

$$\int_0^R r^{-1}n(r)dr = Rlnq + c_{20}RlnR \qquad (3.4)$$

kelib chiqadi. Bunda $n(r)$ bilan $\xi(s,\chi)$ ning $|s-2|<r$ doiradagi no'llari soni, $r \le R$ va $c_{20} = c_{19} + 0,6877(R_0 lnR_0)^{-1}$ (3.4) va

$$\int_R^{2R} r^{-1}n(r)dr \ge n(R)\int_R^{2R} r^{-1}dr \ge n(R)ln2$$

tengsizlikdan

$$n(r) \le 2(ln2)^{-1}(rlnq + 1,6309c_{20}rlnr)$$

ga ega bo'lamiz. Bu yerdan

$$\sum_\rho |2-\rho|^{-2} = \sum_{\rho\in|s-2|\le 3} |2-\rho|^{-2} + \sum_{\rho\notin|s-2|\le 3} |2-\rho|^{-2} \le c_{21}lnq$$
$$+\int_3^\infty r^{-2}dn(r) < c_{16}lnq,$$

bu yerda

$$c_{16} = \frac{4}{3}c_{21} + 1,9236 + 6,5834c_{20}.$$

Lemma isbotini yakunlash uchun

$$\sum_{\rho\in|s-2|\le 3} |2-\rho|^{-2} \le c_{21}lnq \qquad (3.5)$$

ning o'rinli ekanligini ko'psatish qoldi. Bu yerda

$$\sum_{\rho\in|s-2|\le 3} |2-\rho|^{-2} \le \sum_{\rho\in|s-2|\le 3} 1 \le N(3,\chi)$$

bo'lganligi va 2.1-teoremaga asosan

$$N(3,\chi) \le 35,0715lnq$$

bajarilgani uchun biz (3.5) da $c_{21} = 35,0715$ deb olishimiz mumkin. Shuni ham ta'kidlab o'tamizki, agar $R_0 = 3$ va $Im\rho \ge 1$ bo'lsa, c_{21} ni $\frac{c_{21}}{2}$ bilan almashtirish mumkin.

3.1-teoremaning isboti. Faraz etaylik $\chi(modq)$ −ixtiyoriy primitiv xarakter bo'lsin.3.2-lemmani

$$\psi_0(x,\chi) = {\sum_{n\le x}}' \chi(n)\Lambda(n) = \begin{cases} \psi(x,\chi), & \text{agar } x \ne p^\alpha \text{ bo'lsa;} \\ \psi(x,\chi) - \frac{1}{2}\Lambda(x), & \text{agar } x \ne p^\alpha \text{ bo'lsa} \end{cases}$$

tenglik bilan aniqlanuvchi $\psi_0(x,\chi)$ − funksiyaga qo'llab quyidagiga ega bo'lamiz:

$$|\psi_0(x,\chi) - J(x,\chi,T)| < \sum_{\substack{n=1 \\ n\ne x}}^{\infty} \Lambda(n)\left(\frac{x}{n}\right)^{\varkappa} \min\left(1, \frac{(\pi T)^{-1}}{\left|ln\frac{x}{n}\right|}\right) + \varkappa\frac{\Lambda(x)}{\pi T}, \quad (3.6)$$

bunda $\varkappa > 1$ va

$$J(x,\chi,T) = \frac{1}{2\pi i}\int_{\varkappa-iT}^{\varkappa+iT}\left\{-\frac{L'}{L}(s,\chi)\right\}\frac{x^s}{s}ds. \qquad (3.7)$$

Biz

$$\varkappa = 1 + \frac{1}{lnx}$$

deb olib (3.12) ning o'ng tomonidagi qatorni baholaymiz. Avvalo, uning $n \le \frac{3}{4}x$ va $n \ge \frac{5}{4}x$ shartlarni qanoatlantiruvci hadlarini qaraymiz. [21] dagi 1.1-lemmaga asosan agar $1 < \sigma < 1{,}03$ bo'lsa,

$$\frac{\zeta'}{\zeta}(\sigma) < \frac{1}{\sigma-1} - \gamma_0 + 0{,}1856352(\sigma-1) < \frac{1}{\sigma-1}$$

bajariladi. Bunda $\gamma_0 = 0{,}57721566\ldots$ −Eyler doimiysi. Bundan va $x^{\varkappa} = ex$ ekanligidan (3.12) ning o'ng tomonidagi $n \le \frac{3}{4}x$ shartni qanoatlantiruvchi hadlarining hissasi

$$\frac{ex}{\pi T ln\frac{4}{3}}\left\{-\frac{\zeta'}{\zeta}(\varkappa)\right\} < \frac{e}{\pi ln\frac{4}{3}}\frac{x}{T}lnx, \qquad x \ge x_0$$

dan ko'p emas. Shuningdek, $n \geq \frac{5}{4}x$ shartni qanoatlantiruvchi hadlarining hissasi

$$\frac{e}{\pi ln\frac{5}{4}}\frac{x}{T}lnx, \qquad x \geq x_0$$

dan k'op emas. Endi $\frac{3}{4}x < n < x$ shartni qanoatlantiruvchi hadlarini qaraymiz. Faraz etaylik x_1 bilan x dan katta bo'lmagan tub sonning eng katta darajasi bo'lsin (agar $x = p^{\alpha}$ bo'lsa, u holda $x_1 \neq x$). $\frac{3}{4}x < x_1 < x$ deb hisoblashimiz mumkin. $n = x_1$ bo'lganda $ln\frac{x}{n} \geq \frac{x-x_1}{x}$ bajariladi. Shuning uchun ham $n = x_1$ bo'lganda (3.7) dan quyidagiga ega bo'lamiz:

$$\Lambda(x_1)\left(\frac{x}{x_1}\right)^{\varkappa}\min\left(1,\frac{x}{\pi T|x-x_1|}\right) < \left(\frac{4}{3}\right)^{\varkappa_0}(lnx)\min\left(1,\frac{x}{\pi T|x-x_1|}\right).$$

Bu yerda

$$\varkappa_0 = 1 + \frac{1}{lnx_0}, \qquad x \geq x_0.$$

Qaralayotgan qatorning qolgan hadlari uchun $n = x_1 - \nu,\ 0 < \nu < \frac{x}{4}$ deb olamiz. U holda $ln\frac{x}{n} \geq \frac{\nu}{x_1}$ bajariladi. Shuning uchun ham bu hadlarning yig'indiga qo'shadigan hissasi

$$\left(\frac{4}{3}\right)^{\varkappa}\frac{x}{\pi T}(lnx)\sum_{0<\nu<\frac{x}{4}}\frac{1}{\nu} < \frac{1}{\pi}\left(\frac{4}{3}\right)^{\varkappa_0}\left(1+\frac{2}{x_0 lnx_0}+\frac{1}{3x_0^2 lnx_0}\right)\frac{x}{T}ln^2x$$

dan ko'p emas. Bu yerda $x \geq 1$ bo'lsa,

$$\sum_{n\leq x}\frac{1}{n} < lnx + \gamma_0 + \frac{1}{2x} + \frac{1}{12x^2}$$

([21] dagi (1.8)-formula) ekanligidan foydalandik.

Endi (3.7) ning o'ng tomonidagi $x < n < \frac{5}{4}x$ shartni qanoatlantiruvchi hadlarning hissasini qaraymiz. x dan katta bo'lgan tub sonning eng kichik darajasini x_2 bilan belgilaymiz. U holda yuqorida qaralgan $\frac{3}{4}x < n < x$ holdagi singari mulohaza yuritib qaralayotgan hadlarning (3.7) ning o'ng tomoniga qo'shagigan hissasi

$$\left(1+\frac{0{,}2232}{lnx_0}\right)(lnx)\min\left(1,\frac{x}{\pi T|x-x_1|}\right)+$$

$$\frac{1}{\pi}\left(1+\frac{0{,}2232}{lnx_0}\right)\left(1+\frac{2+(3x_0)^{-1}}{x_0 lnx_0}\right)\frac{x}{T}ln^2x,\quad x\geq x_0.$$

Barcha hosil qilingan baholarni yig'ib (3.7) dan

$$|\psi_0(x,\chi)-J(x,\chi,T)|<M_1\frac{x}{T}ln^2x+M_2(lnx)\min\left(1,\frac{x}{\pi T\langle x\rangle}\right), \tag{3.8}$$

bu yerda

$$M_1=\frac{1}{\pi}\left\{\left(\left(\frac{4}{3}\right)^{\varkappa_0}+1+\frac{0{,}2232}{lnx_0}\right)\left(1+\frac{2+(3x_0)^{-1}}{x_0 lnx_0}\right)\right.$$
$$\left.+\left(8{,}0506\mathrm{e}+\frac{1}{x_0}+\frac{1}{x_0 lnx_0}\right)\frac{1}{lnx_0}\right\},$$

$$M_2=1+\frac{0{,}2232}{lnx_0}+\left(\frac{4}{3}\right)^{\varkappa_0}.$$

Endi (3.7) dagi integlashning vertikal yo'lini uchlari $\varkappa\pm iT, U\pm iT$ nuqtalarda bo'lgan to'g'ri to'rtburchakning qolgan uchta tomoni bilan almashtiramiz. Bunda $U-$ yetarlicha katta toq son agar $a=0$ bo'lsa va $U-$ yetarlicha katta juft son agar $a=1$ bo'lsa. Shunday qilib chap vertikal tomon $L(s,\chi)$ning ikkita trivial no'llari orasidan o'tadi. Integral tagidagi funksiyaning to'g'ri to'rtburchakning ichida yotuvchi qutb nuqtalaridagi chegirmalari yig'indisi quyidagiga teng bo'ladi:

$$-\sum_{|\gamma|<T}\frac{x^{\rho}}{\rho}-(1-a)lnx-b(\chi)+\sum_{m\leq U}\frac{x^{a-2m}}{2m-a}. \qquad (3.9)$$

$-1\leq\sigma\leq 2$ va ixtiyoriy q,T lar uchun (1.12) asosan $L(s,\chi)$ning $|T-\gamma|<1$ shartni qanoatlantiruvchi no'llari soni $\theta_4 c_3 lnqT$ ga teng va no'llar orasidagi masofa $(c_3 lnqT)^{-1}$ dan kam bo'lmasligi kerak. Bunda

$$c_3=\frac{5}{2}\left\{1+\left(\gamma_0+\frac{1}{2T_0}+\frac{1}{12T_0^2}+\frac{1}{2}\sqrt{1+9T_0^{-2}}\right)\mathcal{L}_0^{-1}+0{,}06972\ l_0^{-2}\right\}.$$

T ni $|T-\gamma|\geq(c_3 lnqT)^{-1}$tengsizlik bajariladigan qilib tanlaymiz ([27] ning 258-betiga qarang). U holda 2.1-lemmaga asosan $-1\leq\sigma\leq 2, s=\sigma+iT, T\geq T_0(\geq 3)$ bo'lganda

$$\left|\frac{L'}{L}(s,\chi)\right|\leq c_{22}lnqT$$

ni hosil qilamiz. Bu yerda

$$c_{22}=c_3^2+\left\{10\left[1+\left(\gamma_0+\frac{1}{2T_0}+\frac{1}{12T_0^2}+\frac{1}{2}\sqrt{1+9T^{-2}}\right)\mathcal{L}_0^{-1}\right]+3{,}5629 l_0^{-1}\right\}l_0^{-1}.$$

Shuning uchun ham σ ning bu qiymatlaridagi gorizontal kesma bo'yicha integralning qiymati

$$c_{22}\int_{-1}^{\varkappa}\left|\frac{x^s}{s}\right|(ln^2qT)d\sigma<c_{22}T^{-1}l^2\int_{-\infty}^{\varkappa}x^{\sigma}d\sigma=ec_{22}xl^2\,(Tlnx)^{-1}, \qquad (3.16)$$

dan katta emas. Endi $-U\leq\sigma\leq 1$ gorizontal kesma va $\sigma=-U$ vertikal to'g'ri chiziq bo'yicha integrallarni baholash qoldi. 3.3-lemmaga ko'ra gorizontal to'g'ri chiziqning qolgan qismi bo'yicha olingan integralni quyidagicha baholash mumkin:

$$c_{13}\int_{-U}^{-1}\frac{lnq|s|}{|s|}x^{\sigma}d\sigma\leq c_{13}\frac{l}{T}\int_{-\infty}^{-1}x^{\sigma}d\sigma=c_{13}\frac{lnqT}{xTlnx}. \qquad (3.17)$$

Vertikal to'g'ri chiziq bo'yicha integral $U \to \infty$ da no'lga intiladi. Shuning uchun ham (3.14)-(3.17) lar va

$$\sum_{m=1}^{\infty} \frac{x^{-(2m-a)}}{2m-a} < \sum_{m=1}^{\infty} x^{-n} = \frac{1}{x-1}, \qquad x > 1$$

dan quyidagiga ega bo'lamiz:

$$\psi_0(x,\chi) = -\sum_{|\gamma|<T} \frac{x^\rho}{\rho} - b(\chi) + \theta_5 M_3 \frac{x}{T} ln^2 qx$$
$$+ \theta_6 M_2 (lnx) \min\left(1, \frac{x}{\pi T\langle x\rangle}\right), \qquad (3.18)$$

bunda $|\theta_i| < 1, i = 1,2,3,4,5,6$ va

$$M_3 = \frac{ec_{22}}{\pi lnx_0} + \frac{c_{13}}{\pi x_0^2 l_0 \, lnx_0} + \frac{1}{(x_0-1)\, l_0^2} + \frac{1}{logx_0} + M_1;$$

$$M_2 = 1 + \frac{0{,}2232}{lnx_0} + \left(\frac{4}{3}\right)^{\varkappa_0}.$$

Endi $b(\chi)$ni boshqacha ko'rinishda ifodalaymiz. (2.2)- formulani

$$\frac{L'}{L}(s,\chi) = -\frac{1}{2} ln \frac{q}{\pi} - \frac{1}{2}\frac{\Gamma'}{\Gamma}\left(\frac{s+a}{2}\right) + B(\chi) + \sum_\rho \left(\frac{1}{s-\rho} + \frac{1}{\rho}\right) \quad (3.19)$$

ko'rinishda yozish mumkin. Bu yerda $s = 2$ deb olib hosil bo'lgan tenglikni (3.19) dan ayirsak quyidagi tenglikga ega bo'lamiz:

$$\frac{L'}{L}(s,\chi) = \frac{L'}{L}(2,\chi) + \frac{1}{2}\frac{\Gamma'}{\Gamma}\left(1 + \frac{a}{2}\right) - \frac{1}{2}\frac{\Gamma'}{\Gamma}\left(\frac{s+a}{2}\right) + \sum_\rho \left(\frac{1}{s-\rho} + \frac{1}{2-\rho}\right). (3.20)$$

Bu yerdan $a = \frac{1}{2}\{1 - \chi(-1)\} = 1$ bo'lganda

$$\left|\frac{L'}{L}(2,\chi)\right| < 0{,}65585, \qquad -\frac{\Gamma'}{\Gamma}\left(\frac{1}{2}\right) = \gamma_0 + 2ln2;$$

$$\frac{\Gamma'}{\Gamma}\left(1+\frac{a}{2}\right) = \begin{cases} -\gamma_0, & \text{agar } \; a = 0 \text{ bo'lsa;} \\ -\gamma_0 + 2 - ln2, \text{agar} & a = 1 \text{ bo'lsa} \end{cases}$$

bo’lganligi uchun

$$B(\chi) = \frac{L'}{L}(0,\chi) = 1{,}64585\theta_7 - \sum_{\rho}\left(\frac{1}{s-\rho}+\frac{1}{\rho}\right) \qquad (3.21)$$

kelib chiqadi. $a = \frac{1}{2}\{1-\chi(-1)\} = 0$, quyidagi yoyilmani qaraymiz:

$$\frac{L'}{L}(s,\chi) = s^{-1} + B(\chi) + sb_1 + \cdots.$$

Bu yerdan (3.20) va

$$-\frac{1}{2}\frac{\Gamma'}{\Gamma}\left(\frac{s+a}{2}\right) = \frac{\gamma_0}{2} + \frac{1}{s+a} + \sum_{n=1}^{\infty}\left(\frac{1}{s+a+2n} - \frac{1}{2n}\right)$$

dan

$$B(\chi) = 0{,}64585\theta_8 - \sum_{\rho}\left(\frac{1}{2-\rho}+\frac{1}{\rho}\right), \qquad |\theta_8| < 1. \qquad (3.22)$$

(3.21) va (3.22) lardan agar $0 < \theta_7 < 1$ ni mos ravishda tanlab olsak,(3.21) ning doimo o’rinli ekanligiga ishonch hosil qilamiz.

(3.21) ning o’ng tomonidagi qatorni qaraymiz. Bu qatorning $|\gamma| \geq 1$ shartni qanoatlantiruvchi hadlari uchun quyidagi tengsizlik o’rinli:

$$\sum_{|\gamma|\geq 1}\left|\frac{1}{2-\rho}+\frac{1}{\rho}\right| = 2\sum_{|\gamma|\geq 1}\left|\frac{1}{\rho(2-\rho)}\right| \leq 6\sum_{|\gamma|\geq 1}\frac{1}{|2-\rho|^2}\,.$$

Agar $|\gamma| < 1$ bo’lsa, u holda

$$\sum_{|\gamma|<1}\frac{1}{|2-\rho|} \leq \sqrt{5}\sum_{|\gamma|<1}\frac{1}{|2-\rho|^2}\,.$$

Shuning uchun ham 3.4-lemmani qo'llab (3.21) dan

$$B(\chi) = c_{23}\theta_9 lnq + \sum_{|\gamma|<1} \frac{1}{\rho},$$

bunda $|\theta_9| < 1$ va $c_{23} = (6+\sqrt{5})c_{16} + 1{,}64585(lnq_0)^{-1}$. Shunday qilb (3.18) ni quyidagicha yozish mumkin:

$$\psi_0(x,\chi) = -\sum\nolimits'_{|\gamma|<T} \frac{x^\rho}{\rho} + \sum\nolimits'_{|\gamma|<1} \frac{1}{\rho} - \frac{x^{1-\tilde{\beta}}-1}{1-\tilde{\beta}} - \frac{x^{\tilde{\beta}}-1}{\tilde{\beta}} + R_1(x,T),$$

bunda yig'indidagi shtrix yig'indining maxsus no'llar $1-\tilde{\beta}$ va $\tilde{\beta}$ dan (agar ular mavjud bo'lsa) boshqa barcha no'llari bo'yicha olinishini bildiradi va

$$|R_1(x,T)| < \left(M_3 + \frac{c_{23}}{lnq_0x_0}\right)\frac{x}{T} ln^2 qx + M_2(lnx)\min\left(1, \frac{x}{\pi T\langle x\rangle}\right).$$

$\frac{3}{4} \le \tilde{\beta} < 1$ bo'lgani uchun $\tilde{\beta}^{-1} \le \frac{4}{3}$. Shuningdek

$$\frac{x^{1-\tilde{\beta}}-1}{1-\tilde{\beta}} \le x^{\frac{1}{4}} lnx.$$

Shuning uchun ham $(x^{1-\tilde{\beta}}-1)(1-\tilde{\beta})^{-1}$ va $\tilde{\beta}^{-1}$ larni qoldiq hadga kiritishimiz mumkin.

$$\sum\nolimits'_{|\gamma|<1} \frac{1}{\rho}$$

hadni ham qoldiq hadga kiritish mumkin. Haqiqatan ham [21] 1.3-lemmaga ko'ra agar $\chi(modq)$, $(q \ge 3)$ haqiqiy bosh bo'lmagan xarakter bo'lsa, u holda unga mos Dirixle $L-$ funksiyasi $L(s,\chi)$ ning

$$\sigma \geq 1 - \frac{c_{24}}{lnq(|t|+3)}, \quad q \geq 3, \qquad t - \text{ixtiyoriy}$$

sohada ko'pi bilan birta oddiy haqiqiy no'lga ega bo'lishi mumkin. Bunda $c_{24} = 0{,}0109986$. $|t| \leq 1$ bo'lsa bundan $\sigma \geq 1 - c_{25}\,(lnq)^{-1}$kelib chiqadi. Bu yerda $c_{25} = c_{24}\,(1+(lnq_0)^{-1}ln4)^{-1}$. No'llar $\sigma = \frac{1}{2}$ to'g'ri chiziqqa nisbatan simmetrik joylashgani uchun ular $\beta > c_{26}\,(lnq)^{-1}$ shartni qanoatlantirishi kerak. $|\gamma| < 1$ shartni qanoatlantiruvchi no'llar soni 3.1-teoremaga ko'ra ($T = 3$), $c_{21}lnq$ dan ko'p emas ($c_{21} = 35{,}0715$). Shunday qilib

$$\left|\sum\nolimits'_{|\gamma|<1} \frac{1}{\rho}\right| \leq \sum\nolimits'_{|\gamma|<1} \frac{1}{\beta} \leq c_{26}\, ln^2 q, \quad c_{26} = \frac{c_{21}}{c_{25}}.$$

Endi $\psi_0(x,\chi)$ ning ta'rifiga asosan

$$\psi(x,\chi) = -E_{\tilde{\beta}}\frac{x^{\tilde{\beta}}}{\tilde{\beta}} - \sum\nolimits'_{|\gamma|<T} \frac{x^{\rho}}{\rho} + R_2(x,T), \tag{3.23}$$

$$|R_2(x,T)| < \left(M_3 + \frac{c_{23}}{lnq_0x_0} + \frac{4}{3(lnq_0x_0)^2} + c_{26} + \frac{1}{2lnq_0x_0}\right)\frac{x}{T}ln^2qx$$
$$+ M_2(lnx)\min\left(1,\frac{x}{\pi T\langle x\rangle}\right) + E_{\tilde{\beta}}x^{\frac{1}{4}}\,lnx. \tag{3.24}$$

Hozirgacha biz primitiv xarakterlarni qaradik. Endi $\chi = \chi_0^* - 1$ moduli bo'yicha bosh xarakter bo'lsin u holda $\psi(x,\chi_0^*) = \psi(x)$. Shuning uchunham bu holda $\psi_0(x,\chi)$ va $L(s,\chi)$ larning o'rniga mos ravishda $\psi_0(x)$ va $\zeta(s)$ larni qarab quyidagiga ega bo'lamiz:

$$\psi(x,\chi_0^*) = x - \sum_{|\gamma|<T} \frac{x^{\rho}}{\rho} + \bar{R}_2(x,T), \tag{3.25}$$

bunda

$$|\bar{R}_2(x,T)| < \left\{\frac{1}{lnx_0} + \frac{1}{2(x_0^2-1)ln^2x_0} + \frac{ln2\pi}{ln^2x_0} + \frac{1}{\pi}\left(\frac{c_{27}}{lnx_0} + \frac{c_{28}}{x_0^2ln^2x_0}\right) + M_1\right\}\frac{x}{T}ln^2qx + M_2(lnx)\min\left(1,\frac{x}{\pi T\langle x\rangle}\right) \qquad (3.26)$$

va

$$c_{27} = 25e\left\{\left(\frac{1}{T_0^2} + 0{,}64585\right)\frac{1}{\mathcal{L}_0} + \frac{1}{2}\left[1 + \left(\gamma_0 + \frac{1}{2T_0} + \frac{1}{12T_0^2} + \frac{1}{2}\sqrt{1+4T^{-2}}\right)\mathcal{L}_0^{-1}\right]\right\}^2$$

$$+e\left(\frac{23}{T_0^2} + 0{,}64585\right)\frac{1}{\mathcal{L}_0^2} + 11e\left[1 + \left(\gamma_0 + \frac{1}{2T_0} + \frac{1}{12T_0^2} + \frac{1}{2}\sqrt{1+4T^{-2}}\right)\mathcal{L}_0^{-1}\right]\mathcal{L}_0^{-1};$$

$$c_{28} = \left\{2 + \frac{2}{\sqrt{4+T_0^2}\,ln(4+T_0^2)} + \frac{1{,}2418}{ln(1+T_0^2)}\right\}\left(1 + \frac{ln(1+T_0^{-2})}{2\mathcal{L}_0}\right).$$

(3.24) va (3.25) lardan ko'ramizki, $\bar{R}_2(x,T)$ (3.24) ning o'ng tomonidagi miqdordan kichik.

Nihoyat, $\chi(modq)-$ primitiv bo'lmagan xarakter bo'lib, $\chi_1(modq_1)-$ primitiv xarakter bilan indutsirlangan bo'lsin. U holda

$$|\psi(x,\chi) - \psi(x,\chi_1)| \le \sum_{\substack{p^\nu \le x, p\backslash q \\ p\nmid q_1}} lnp \le \frac{lnx}{ln2}\sum_{p\backslash q, p\nmid q_1} lnp < \frac{lnx}{ln2}\,lnq.$$

Shuning uchun ham (3.23) va (3.24) lardan diomo

$$\psi(x,\chi) = \delta_\chi x - E_{\widetilde{\beta}}\frac{x^{\widetilde{\beta}}}{\widetilde{\beta}} - {\sum_{|\gamma|<T}}'\frac{x^\rho}{\rho} + R_3(x,T)$$

bajariladi degan hulosaga kelamiz. Bunda

$$|R_3(x,T)| < \left\{\frac{ec_{22}}{\pi lnx_0} + \frac{c_{13}}{\pi x_0^2 l_0\ lnx_0} + \frac{1}{(x_0-1)\ l_0^2} + \frac{1}{lnx_0} + M_1 + \frac{c_{23}}{lnq_0x_0}\right.$$
$$\left. + \frac{4}{3(lnq_0x_0)^2} + c_{26} + \frac{1}{2lnq_0x_0} + \frac{1}{ln2}\right\}\frac{x}{T}ln^2qx$$
$$+M_2(lnx)\min\left(1, \frac{x}{\pi T\langle x\rangle}\right) + E_{\tilde{\beta}}x^{\frac{1}{4}}\,lnx.$$

Bu yerdan $T_0 = 3,\ q_0 = 3,\ x_0 = 3$ deb olsak, 3.1-teoremadagi tasdiq kelib chiqadi.

Natija. Agar $q_0 \le q \le P,\ 3 \le T_0 \le T\left(= P^{\frac{19}{4}}\right),\ T^{2c} \le x$ va $x -$ butun son bo'lsa, quyidagi formula o'rinli:

$$\psi(x,\chi) = \delta_\chi x - E_{\tilde{\beta}}\frac{x^{\tilde{\beta}}}{\tilde{\beta}} - \sum\nolimits'_{|\gamma|<T}\frac{x^\rho}{\rho} + R(x,T),$$

bunda

$$|R(x,T)| < \left\{x_0^{-\eta}\left(\left(\frac{4}{3} + \frac{1}{x_0-1}\right)\frac{1}{ln^2x_0} + \frac{2}{19cln2} + \frac{4c_{26}}{361c^2} + \frac{2c_{23}}{19clnx_0}\right) + \frac{x_0^{-\eta+\frac{1}{4}}}{lnx_0}\right.$$
$$\left. + \frac{1}{2\pi}\left(\frac{c_{13}}{(cx_0lnx_0)^2} + \frac{ec_{22}}{2c^2lnx_0}\right) + \frac{1}{\sqrt{x_0}lnx_0} + M_1\right\}\frac{x}{T}ln^2x,$$
$$\eta = 1 - \frac{1}{2c}.$$

Agar natijaning shartida keltilgan parametrlarning qiymatlarini 3.1-teorema isboti inobatga olsak undan isbotlanishi talab etilgan natija kelib chiqadi.

III-BOB. SONLARNI IKKITA TOQ TUB SONLARNING YIG'INDISI KO'RINISHIDA IFODALASH

III.1- §. Asosiy belgilashlar va birlik intervalni bo'lish.

Quyidagicha belgilashlar kiritamiz:

$$S(\alpha) = \sum_{P<p\leq X} (lnp)\, e(p\alpha) \qquad (1.1)$$

va

$$S(\chi,\eta) = \sum_{P<p\leq X} (lnp)\chi(p)\, e(p\alpha). \qquad (1.2)$$

Biz

$$P = X^{3\delta} \quad \text{va} \quad Q = XP^{-1} \qquad (1.3)$$

deb olib [0;1] kesmani asosiy va qo'shimcha intervallarga bo'lamiz. $1 \leq a \leq q \leq P$, $(a,q) = 1$ lar uchun $\mathfrak{M}(q,a)$ bilan $\left[\frac{a}{q} - \frac{1}{qQ}, \frac{a}{q} + \frac{1}{qQ}\right] -$ asosiy intervalni belgilaymiz. Tushunarliki, bu asosiy intervallar kesishmaydi. Asosiy intervallarning birlashmasini $\mathfrak{M}$ bilan belgilaymiz. $Q^{-1} < \alpha < 1 + Q^{-1}, \alpha \notin \mathfrak{M}$ shartni qanoatlantiruvchi α lar to'plamini $\mathfrak{N}$ bilan belgilaymiz. Shunday qilib $\mathfrak{N} -$ qo'shimcha intervallarning birlashmasidan iborat.

Faraz qilaylik

$$\mathcal{R}(n) = \sum_{\substack{n=p_1+p_2 \\ P<p_1+p_2\leq X}} lnp_1 \cdot lnp_2$$

bo'lsin u holda

$$\mathcal{R}(n) = \mathcal{R}_1(n) + \mathcal{R}_2(n) = \int_{Q^{-1}}^{1+Q^{-1}} S^2(\alpha)\, e(-n\alpha) d\alpha, \qquad (1.4)$$

bunda

$$\mathcal{R}_1(n) = \int\limits_{\mathfrak{M}} S^2(\alpha)\, e(-n\alpha) d\alpha, \quad \mathcal{R}_2(n) = \int\limits_{\mathfrak{N}} S^2(\alpha)\, e(-n\alpha) d\alpha.$$

III.2- §. Qo'shimcha intervallar bo'yicha olingan integralni baholash.

Bu paragrafda avvalo X ning yetarlicha katta qiymatlari uchun

$$\sum_{n \le X} \mathcal{R}_2^2(n) < X^3 P^{-1} ln^{12} X \qquad (2.1)$$

ekanligini isbotlaymiz. Parseval ayniyatiga asosan

$$\sum_{n \le X} \mathcal{R}_2^2(n) = \int\limits_{\mathfrak{N}} |S(\alpha)|^4 \, d\alpha \le \Big(\underbrace{max}_{\mathfrak{N}} |S(\alpha)|\Big)^2 \int\limits_{\mathfrak{N}} |S(\alpha)|^2 \, d\alpha$$

deb yoza olamiz. Bu yerda

$$\int\limits_{\mathfrak{N}} |S(\alpha)|^2 \, d\alpha \le \int\limits_{Q^{-1}}^{1+Q^{-1}} |S(\alpha)|^2 \, d\alpha \le \sum_{p \le X} ln^2 p \le \pi(X) ln^2 X.$$

Buning o'ng tomonida [21] dagi I.4.2-lemmaning birinchi qismidan foydalanamiz. Unga ko'ra agar $x > 1$ bo'lsa,

$$\pi(x) < \frac{x}{lnx} + \frac{3x}{2ln^2 x}$$

tengsizlik o'rinli. Demak,

$$\int\limits_{\mathfrak{N}} |S(\alpha)|^2 \, d\alpha < \left(1 + \frac{3}{2lnX}\right) XlnX.$$

bajariladi. $\alpha \in \mathfrak{N}$ bo'lsin. Dirixle teoremasiga asosan shunday $q \le Q$ va $a, 1 \le a \le q$

$(a,q)=1$ sonlari mavjudki, ular uchun $\left|\alpha-\frac{a}{q}\right|\leq\frac{1}{qQ}$ bajariladi. Bu esa agar $q\leq P$ bo'lsa, $\alpha\in\mathfrak{M}(q,a)\subseteq\mathfrak{M}$ ekanligini bildiradi. Demak, $\alpha\in\mathfrak{N}$ bo'lsa, $q>P$ ekan. [28] dagi 2.1.1-natijaga asosan, agar $P<q<XP^{-1}$, $1\leq P\leq X^{\frac{1}{3}}$, $(a,q)=1$, $\left|\alpha-\frac{a}{q}\right|\leq\frac{2P}{qX}$ bo'lsa, u holda

$$|S(\alpha)|\leq c_{29}XP^{-\frac{1}{2}}ln^4X$$

bajariladi. Bundan yetarlicha katta X lar uchun

$$|S(\alpha)|<XP^{-\frac{1}{2}}ln^5X$$

ning bajarilishi kelib chiqadi.

Shunday qilib

$$\sum_{n\leq X}\mathcal{R}_2^2(n)<\left(1+\frac{3}{2lnX_0}\right)X^3P^{-1}ln^{11}X<X^3P^{-1}ln^{12}X,$$

ya'ni (2.1) isbot bo'ldi. (2.1) dan

$$|\mathcal{R}_2(n)|>XP^{-\frac{7}{24}}$$

tengsizlikni qanoatlantiruvchi n, $(n\leq X)$ lar soni

$$XP^{-\frac{5}{12}}ln^{12}X \tag{2.2}$$

dan ko'p emas. Qolgan n lar uchun

$$|\mathcal{R}_2(n)|\leq XP^{-\frac{7}{24}} \tag{2.3}$$

bajariladi.

III.3-§. Katta yoylar. R_1 va $\widetilde{R}_1$ larni soddalashtirish.

Endi faraz etaylik $\alpha \in \mathfrak{M}(q,a)$ bo'lsin. Biz $\alpha = \frac{a}{q} + \eta$ deb olamiz. $\mathfrak{M}$ da $q \leq P$ bo'lgani uchun agar $p > P$ bo'lsa, u holda $(p,q) = 1$ va demak,

$$e(p\alpha) = \frac{1}{\varphi(q)} \sum_{\chi} \chi(pa)\tau(\bar{\chi})e(p\eta).$$

Bunda $\varphi(q)$ $-$Eyler funksiyasi,

$$\tau(\chi) = \sum_{h=1}^{q} \chi(h) e\left(\frac{h}{q}\right)$$

-Gauss yig'indisi. Shunday qilib (1.1) va (1.2)- tengliklardan foydalanib

$$S(\alpha) = \frac{1}{\varphi(q)} \sum_{\chi} \chi(a)\tau(\bar{\chi})S(\chi,\eta) \tag{3.1}$$

deb yoza olamiz. Agar $\chi = \chi_0$ $-$bosh xarakter bo'lsa, $S(\chi_0,\eta) = T(\eta) + W(\chi_0,\eta)$; $E_{\widetilde{\beta}} = 1$ bo'lsa, u holda $S(\tilde{\chi}\chi_0,\eta) = \widetilde{T}(\eta) + W(\tilde{\chi}\chi_0,\eta)$; qolgan hollarda, ya'ni $\chi \neq \chi_0$ va $E_{\widetilde{\beta}} = 0$ bo'lsa, u holda $S(\chi,\eta) = W(\chi,\eta)$ deb olamiz.

Quyidagi iki holni qaraymiz.

a). Faraz etaylik $E_{\widetilde{\beta}} = 0$ bo'lsin. $\tau(\chi_0) = \mu(q)$ ([21], 2.2-lemma) bo'lgani uchun quyidagiga ega bo'lamiz:

$$\sum\nolimits_a' \int\limits_{\mathfrak{M}(q,a)} S(\alpha)^2\, e(-n\alpha)d\alpha = \frac{\mu^2(q)}{\varphi^2(q)}\, C_q(-n) \int\limits_{-\frac{1}{qQ}}^{\frac{1}{qQ}} T^2(\eta)\, e(-n\eta)d\eta +$$

$$+2\frac{\mu(q)}{\varphi^2(q)}\sum_{\chi}\tau(\bar{\chi})\,C_{\chi}(-n)\int\limits_{-\frac{1}{qQ}}^{\frac{1}{qQ}}T(\eta)W(\chi,\eta)\,e(-n\eta)d\eta+$$

$$\frac{1}{\varphi^2(q)}\sum_{\chi,\chi'}\tau(\bar{\chi})\tau(\bar{\chi}')\,C_{\chi\chi'}(-n)\int\limits_{-\frac{1}{qQ}}^{\frac{1}{qQ}}W(\chi,\eta)W(\chi',\eta)\,e(-n\eta)d\eta\,, \qquad (3.2)$$

bu yerda

$$C_{\chi}(m)=\sum_{h=1}^{q}\chi(h)e\left(\frac{hm}{q}\right)-\text{Ramanudjan yig'indisi};,C_q(m\,)=\sum\nolimits_h' e\left(\frac{hm}{q}\right),$$

oxirgi yig'indidagi shtrix yig'indining q moduli bo'yicha chegirmalarning keltirilgan sistemasi bo'yicha olinishuni bildiradi. Xususan $C_{\chi}(1)=\tau(\chi)$;

$$T(\eta)=T_X(\eta)=\sum_{P<n\le X}e(n\eta).$$

Faraz qilaylik $\chi(mod q)$, primitiv xarakter $\chi^*(mod r)$ bilan indutsirlangan xarakter bo'lsin. Quyidagicha belgilash kiritamiz:

$$W(\chi)=\left(\int\limits_{-\frac{1}{rQ}}^{\frac{1}{rQ}}|W(\chi,\eta)|^2\,d\eta\right)^{\frac{1}{2}}. \qquad (3.3)$$

Asosiy intervallar uchun $W(\chi)=W(\chi^*)$ bajariladi. (3.2) ning ikkala tomonini barcha q lar bo'yicha yig'ib, qulaylik uchun ikkinchi va uchunchi hadlarning yig'indisini R_1 bilan belgilab olamiz, u holda integrallarga Koshi tengsizligini qo'llab va [21] dagi II. 2.5-lemmadan foydalanib

$$R_1 \leq \frac{n}{\varphi(n)}\left(2K(r,1,n)X^{\frac{1}{2}}W + K(r,r,n)W^2\right) \tag{3.4}$$

ni hosil qilamiz. Bu yerda

$$W = \sum_{q\leq P}\sum_{\chi}{}^{*} W(\chi);$$

$$K(r_1,r_2,n) = \prod_{p\nmid r_3, p\nmid n}\left(1+\frac{1}{(p-1)^2}\right) \prod_{p\backslash r_3, p\nmid n}\frac{p^{\frac{3}{2}}}{(p-1)^2}, \quad r_3 = [r_1,r_2]$$

([A] dagi II. 2.5-lemmaga qarang). Shuningdek (3.4) da

$$\int_{-\frac{1}{qQ}}^{\frac{1}{qQ}} |T(\eta)|^2\, d\eta \leq \int_0^1 |T(\eta)|^2\, d\eta = \sum_{P<n_1,n_2\leq X}\int_0^1 e\big((n_1-n_2)\eta\big)d\eta < X$$

dan foydalandik.

(3.2) dagi birinchi hadni qaraymiz. [21] dagi II. 2.6-lemmaga asosan $|T_X(\eta)| \leq (2\|\eta\|)^{-1}$ bo'lgani uchun

$$\int_{\frac{1}{qQ}}^{\frac{1}{2}} |T(\eta)|^2\, d\eta \leq \frac{1}{4}\int_{\frac{1}{qQ}}^{\frac{1}{2}} \eta^{-2}\, d\eta < \frac{1}{4}qQ.$$

Shuning uchun ham

$$\int_0^1 T^2(\eta)\, e(-n\eta) d\eta = \sum_{P<n_1,n_2\leq X}\int_0^1 e\big((n_1-n_2)\eta\big)d\eta = \sum_{\substack{P<n_1,n_2\leq X\\ n=n_1+n_2}} 1 < n$$

ekanligini e'tiborga olib barcha $n, (n \leq X)$ lar uchun

$$\int_{-\frac{1}{qQ}}^{\frac{1}{qQ}} T^2(\eta)e(-n\eta)\,d\eta = n + \theta_{10}\frac{qQ}{2}, \quad |\theta_{10}| < 1$$

ni hosil qilamiz. Shunday qilib (3.2) dagi birinchi hadni

$$\sum_{q\leq P}\frac{\mu^2(q)}{\varphi^2(q)}\,C_q(-n)\left(n+\theta_{10}\frac{qQ}{2}\right) \tag{3.5}$$

ko'rinishda yozishimiz mumkin. θ_{10} qatnashgan hadlar yig'indisini R_2 bilan belgilasak [21] dagi (II.2.2)-tenglikga asosan

$$C_q(m) = \mu(q_1)\frac{\varphi(q)}{\varphi(q_1)}, \quad q_1 = \frac{q}{(q,m)} \tag{3.6}$$

bo'lgani uchun quyidagi bahoni hosil qilamiz:

$$R_2 < \frac{1}{2}Q\sum_{q\leq P}\frac{q\cdot\mu^2(q)}{\varphi(q)\cdot\varphi\left(\frac{q}{(q,n)}\right)}.$$

Agar $d = (q,n)$ desak, $d\cdot r = q, (d,r) = 1$ bo'ladi. Shuning uchun ham

$$R_2 < \frac{1}{2}Q\sum_{\substack{d\cdot r\leq P\\ d\backslash n}}\frac{d\cdot r}{\varphi(d)\cdot\varphi^2(r)}$$

$$\leq \frac{1}{2}Q\sum_{d\backslash n}\frac{d}{\varphi(d)}\sum_{r\leq Pd^{-1}}\frac{r}{\varphi^2(r)} \leq \frac{1}{2}Q\frac{n}{\varphi(n)}d(n)\sum_{\substack{r\leq P\\ r-kvadratsiz}}\frac{r}{\varphi^2(r)}.$$

R_2ni baholashni davom ettirish uchun [29] da keltirilgan

$$\frac{n}{\varphi(n)} < e^{\gamma_0} lnlnn + \frac{2{,}507}{lnlnn} = b(n) lnlnn, \quad (n \geq 3) \tag{3.7}$$

tengsizlikdan foydalanamiz. U holda

$$\sum_{\substack{r \leq P \\ r-kvadratsiz}} \frac{r}{\varphi^2(r)} = \sum_{\substack{r \leq 200 \\ r-kvadratsiz}} \frac{r}{\varphi^2(r)} + \sum_{\substack{201 \leq r \leq P \\ r-kvadratsiz}} \frac{r}{\varphi^2(r)} =$$

$$A_1(200) + b^2(201) \sum_{\substack{201 \leq r \leq P \\ r-kvadratsiz}} \frac{(lnlnr)^2}{r} < A_1(200) + b^2(P_0)(lnlnP)^2 \cdot lnP$$

$$\leq c_{30}(P_0)(lnlnP)^2 \cdot lnP,$$

bu yerda

$$c_{30}(P_0) = A_1(200)(lnlnP)^{-2}(lnP)^{-1} + b^2(P_0).$$

Shunday qilib $(1-\varepsilon)X < n \leq X, \quad (0 < \varepsilon < 1)$ bo'lganda

$$R_2 < K_1(\varepsilon_1, P_0, X_0) X^{1+\varepsilon_1} P^{-1} \, (lnlnX)(lnlnP)^2 \cdot lnP \tag{3.8}$$

ga ega bo'lamiz. Bunda

$$K_1(\varepsilon_1, P_0, X_0) = \tfrac{1}{2} a(\varepsilon_1) \cdot c_{30}(P_0) \cdot b\big((1-\varepsilon)X_0\big); \quad d(n) \leq a(\varepsilon_1) n^{\varepsilon_1}.$$

(3.5) dagi qolgan hadlarni barcha $q \geq 1$ lar boyicha yig'indi qilib yozsak, buning natijasida

$$R_3 = \left| n \sum_{d \backslash n} \frac{\mu^2(d)}{\varphi(d)} \sum_{r > Pd^{-1}} \frac{\mu^3(r)}{\varphi^2(r)} \right|$$

xatolik paydo bo'ladi. Bu yerda

$$\left|\sum_{r>Pd^{-1}} \frac{\mu^3(r)}{\varphi^2(r)}\right| \leq \left|\sum_{Pd^{-1}<r\leq 40} \frac{\mu^3(r)}{\varphi^2(r)}\right| + \left|\sum_{r>Pd^{-1}(41)} \frac{\mu^3(r)}{\varphi^2(r)}\right| <$$

$$3{,}2 + b^2(41)\frac{(lnlnPd^{-1})^2}{\sqrt{Pd^{-1}}} \sum_{r>Pd^{-1}(41)} \frac{1}{r^{\frac{3}{2}}} \leq c_{31}(P_0)\frac{d}{P}(lnlnP)^2$$

va $c_{31}(P_0) = 120(lnlnP_0)^{-1} + 20{,}9632$ bo'lganligi uchun

$$R_3 < K_2(\varepsilon_1, P_0, X_0)X^{1+\varepsilon_1}P^{-1}\,(lnlnX)\cdot(lnlnP)^2$$

bajariladi. Bunda $K_2(\varepsilon_1, P_0, X_0) = a(\varepsilon_1)\cdot c_{31}(P_0)\cdot b\big((1-\varepsilon)X_0\big)$.

Endi (3.2), (3.4) va (3.5) lardan Quyidagiga ega bo'lamiz:

$$\mathcal{R}_1(n) = n\sigma(n) + \theta_{11}K_3(\varepsilon_1, P_0, X_0)X^{1+\varepsilon_1}P^{-1}\,(lnlnX)\cdot(lnlnP)^2\cdot lnP$$

$$+\frac{n}{\varphi(n)}\Big(2K(r,1,n)X^{\frac{1}{2}}W + K(r,r,n)W^2\Big), \qquad (3.9)$$

bunda

$$K_3(\varepsilon_1, P_0, X_0) = a(\varepsilon_1)\cdot b\big((1-\varepsilon)X_0\big)\cdot\left(\frac{1}{2}c_{30}(P_0) + \frac{c_{31}(P_0)}{lnP_0}\right), \qquad (3.10)$$

$$\sigma(n) = \sum_{q=1}^{\infty}\frac{\mu^2(q)}{\varphi^2(q)}C_q(-n) = \prod_{p\nmid n}\left(1-\frac{1}{(p-1)^2}\right)\cdot\prod_{p\backslash n}\left(1+\frac{1}{p-1}\right). \quad (3.11)$$

b). Agar $E_{\widetilde{\beta}} = 1$ bo'lsa, u holda (3.2) ning o'ng tomonida yana uchta had

$$\frac{\tau^2(\tilde{\chi}\chi_0)}{\varphi^2(q)}\, C_q(-n) \int\limits_{-\frac{1}{qQ}}^{\frac{1}{qQ}} \tilde{T}^2(\eta)\, e(-n\eta)d\eta +$$

$$+2\frac{\mu(q)}{\varphi^2(q)} \sum_{\chi} \tau(\tilde{\chi}\chi_0)\, C_{\tilde{\chi}\chi_0}(-n) \int\limits_{-\frac{1}{qQ}}^{\frac{1}{qQ}} T(\eta)\tilde{T}(\eta)\, e(-n\eta)d\eta +$$

$$\frac{1}{\varphi^2(q)} \sum_{\chi,\chi'} \tau(\bar{\chi})\tau(\tilde{\chi}\chi_0)\, C_{\tilde{\chi}\chi}(-n) \int\limits_{-\frac{1}{qQ}}^{\frac{1}{qQ}} W(\chi,\eta)\tilde{T}(\eta)\, e(-n\eta)d\eta \qquad (3.\tilde{2})$$

paydo bo'ladi. Bunda

$$\tilde{T}(\eta) = \tilde{T}_X(\eta) = \sum_{P<n\le X} n^{\tilde{\beta}-1} e(n\eta)$$

va

$$\int\limits_{-\frac{1}{qQ}}^{\frac{1}{qQ}} \left|\tilde{T}(\eta)\right|^2 d\eta \le \int\limits_0^1 \left|\tilde{T}(\eta)\right|^2 d\eta = \sum_{P<n_1,n_2\le X} (n_1 n_2)^{\tilde{\beta}-1} \int\limits_0^1 e\big((n_1-n_2)\eta\big)d\eta =$$

$$\sum_{P<n_1,n_2\le X} n^{2(\tilde{\beta}-1)} < X$$

bo'lgani uchun yuorida R_1 ni baholagandagi singari mulohaza yuritib $(3.\tilde{2})$ dagi oxirgi had uchun (3.4) bahoning o'rniga quyidagi bahoga ega bo'lamiz:

$$\tilde{R}_1 \le \frac{n}{\varphi(n)}\Big(2\big(K(r,1,n) + K(r,\tilde{r},n)\big)X^{\frac{1}{2}}W + K(r,r,n)W^2\Big). \qquad (3.\tilde{4})$$

[21] dagi II. 2.6-lemmaning ikkinchi qismiga asosan

$$\tilde{T}_X(\eta) \le min\left(c_{32}X^{\tilde{\beta}};\ \frac{c_{33}}{\|\eta\|^{\tilde{\beta}}}\right)$$

bo'lgani uchun

$$\tilde{T}_X(\eta) \le \frac{c_{33}}{\|\eta\|^{\tilde{\beta}}}$$

deb olishimiz mumkin. Bunda

$$c_{32} = \left(1 - \frac{0{,}0019128}{lnP_0}\right)^{-1}; \qquad c_{33} = \frac{3}{2} + \frac{0{,}0019128}{lnP_0 - 0{,}0019128}.$$

Shuning uchun ham

$$\int_{-\frac{1}{qQ}}^{\frac{1}{2}} \left|\tilde{T}(\eta)\right|^2 d\eta < c_{33}^2 qQ; \qquad \int_{-\frac{1}{qQ}}^{\frac{1}{2}} \left|\tilde{T}(\eta)T(\eta)\right| d\eta < \frac{1}{2}c_{33}qQ$$

tengsizliklar o'rinli bo'ladi. Endi $(3.\tilde{2})$ integrallarni quyidagicha yozishimiz mumkin:

$$\int_{-\frac{1}{qQ}}^{\frac{1}{qQ}} \tilde{T}^2(\eta)\, e(-n\eta)d\eta = \tilde{I}(n) + 2\theta_{12}c_{33}^2 qQ, \qquad (0 < \theta_{12} < 1);$$

$$\int_{-\frac{1}{qQ}}^{\frac{1}{qQ}} \left|\tilde{T}(\eta)T(\eta)\right| d\eta = \tilde{J}(n) + \theta_{13}c_{33}qQ, \quad (0 < \ \theta_{13} < 1),$$

bunda

$$\left|\tilde{I}(n)\right| = \left|\int_0^1 \tilde{T}^2(\eta)\, e(-n\eta)d\eta\right| \le X;\ \left|\tilde{J}(n)\right| = \left|\int_0^1 \left|\tilde{T}(\eta)T(\eta)\right| d\eta\right| \le X. \quad (3.12)$$

Shunday qilib $(3.\tilde{2})$ dagi birinchi va ikkinchi hadlarni barcha asosiy intervallar bo'yi olib quydagicha yozib olish mumkin:

$$\sum_{\substack{q\le P\\ \tilde{r}\backslash q}}\frac{\tau^2(\tilde{\chi}\chi_0)}{\varphi^2(q)}\,C_q(-n)(\tilde{I}(n)+2\theta_{12}c_{33}^2qQ)+$$

$$2\sum_{\substack{q\le P\\ \tilde{r}\backslash q}}\frac{\mu(q)}{\varphi^2(q)}C_{\tilde{\chi}\chi_0}(-n)\tau(\tilde{\chi}\chi_0)(\tilde{J}(n)+\theta_{13}c_{33}qQ). \qquad (3.\tilde{5})$$

$(3.\tilde{5})$ da θ_{12} va θ_{13} lar qatnashgan hadlar yig'indisini mos ravishda R_4 va R_5 lar bilan belgilab, ularni baholaymiz.

[21] dagi II. 2.1-lemmaga asosan agar $\chi(modq)$, $q\ge 3-$ primitiv xarakter bo'lsa, u holda $|\tau(\chi)|=\sqrt{q}$; agarda $\chi(modq)$, $q\ge 3-$ haqiqiy primitiv xarakter bo'lsa, u holda $\tau^2(\chi)=\chi(-1)q$ va $\frac{q}{(4,q)}-$ kvadratsiz (ya'ni birorta ham tub sonning kvadratiga bo'linmaydigan) son bo'ladi. Bundan va (3.6) formuladan foydalanib R_4 ni baholaymiz, u holda

$$R_4\le 2c_{33}^2Q\tilde{r}\sum_{\substack{q\le P\\ \tilde{r}\backslash q}}\left|\frac{\mu\left(\frac{q}{(q,n)}\right)}{\varphi\left(\frac{q}{(q,n)}\right)}\frac{q}{\varphi(q)}\right|$$

ga ega bo'lamiz. $q=\tilde{r}\cdot l$ desak, $(\tilde{r},l)=1$ va $\mu^2(l)=1$ ([A] dagi 2.5-lemmaning isbotiga qarang) bo'ladi. Shuning uchun ham

$$R_4\le\frac{2c_{33}^2Q\tilde{r}^2}{\varphi(\tilde{r})\varphi\left(\frac{\tilde{r}}{(\tilde{r},n)}\right)}\sum_{l\le P\tilde{r}^{-1}}\left|\frac{\mu^2(l)\mu\left(\frac{l}{(l,n)}\right)}{\varphi\left(\frac{l}{(l,n)}\right)}\frac{l}{\varphi(l)}\right|$$

bajariladi. Endi agar $(l,n)=d$ deb belgilab olsak, $l=d\cdot k$, $(d,k)=1$ bo'ladi va

$$R_4 \leq 2c_{33}^2 \frac{\tilde{r}}{\varphi(\tilde{r})} \cdot \frac{\tilde{r}}{(\tilde{r},n)} \cdot \frac{Q}{\varphi\left(\frac{\tilde{r}}{(\tilde{r},n)}\right)} \cdot \sum_{d \backslash n} \frac{d}{\varphi(d)} \sum_{k \leq P(\tilde{r}d)^{-1}} \frac{k}{\varphi^2(k)} <$$

$$2c_{33}^2 Q \cdot \left(\frac{\tilde{r}}{\varphi(\tilde{r})}\right)^2 \cdot \frac{n}{\varphi(n)} \cdot d(n) \cdot (\tilde{r},n) \cdot \sum_{k \leq P(\tilde{r}d)^{-1}} \frac{k}{\varphi^2(k)} \leq$$

$$K_4(\varepsilon_1, P_0, X_0) X^{1+\varepsilon_1} P^{-1} \, (\tilde{r},n)(lnlnP)^4 \cdot (lnlnX) \cdot lnP,$$

bajariladi. Bunda

$$K_4(\varepsilon_1, P_0, X_0) = 2c_{33}^2 \cdot a(\varepsilon_1) \cdot b^2(P_0) \cdot b^2\big((1-\varepsilon)X_0\big) \cdot c_{30}(P_0).$$

Bu yerda $b^2(\tilde{r}_0)$ ni $b^2(P_0)$ bilan almashtirdik, binday qilish mumkin, chunki $\tilde{r}_i$ ning dastlabki 200ta qiymati uchun

$$\underbrace{max}_{1 \leq i \leq 200} \frac{\tilde{r}}{\varphi(\tilde{r})} \leq 3{,}75 \, \frac{lnlnP}{lnlnP_0} = c_{34} lnlnP$$

bo'ladi. $\tilde{r}_i$ ning qo'lgan qiymatlari uchun

$$\frac{\tilde{r}}{\varphi(\tilde{r})} < b(\tilde{r}_0) lnln\tilde{r} < b(P_0) lnlnP$$

tengsizluk o'rinli. Shuning uchun ham $P \geq P_0$ bo'lganda

$$b(P_0) > \frac{3{,}75}{lnlnP_0} = c_{34}$$

bajariladi.

Endi faraz etaylik $\chi(modq) -$ xarakter $\chi^*(modr)$ primitiv xarakter bilan indutsirlangan xarakter bo'lsin va ixtiyoriy $m -$ butun soni uchun $q_1 = \frac{q}{(q,m)}$ deb belgilab olsak, [21] dagi 2.4-lemmaga asosan: agar $r \nmid q_1$ bo'lsa, $C_\chi(m) = 0$; agarda $r \backslash q_1$ bo'lsa,

$$C_\chi(m) \stackrel{\text{def}}{=} \sum_{h=1}^{q} \chi(h) e\left(\frac{hm}{q}\right) = \bar{\chi}^*\left(\frac{m}{(q,|m|)}\right) \cdot \frac{\varphi(q)}{\varphi(q_1)} \cdot \mu\left(\frac{q_1}{r}\right) \cdot \chi^*\left(\frac{q_1}{r}\right) \cdot \tau(\chi^*)$$

bajariladi. Bundan foydalanib R_5 uchun

$$R_5 \leq 2c_{33} Q\tilde{r} \sum_{\substack{q \leq P \\ \tilde{r} \backslash q}} \left| \frac{\mu\left(\frac{\frac{q}{(q,n)}}{r}\right)}{\varphi\left(\frac{q}{(q,n)}\right)} \frac{\mu(q)q}{\varphi(q)} \right|$$

tengsizlikka ega bo'lamiz. Bu tengsizlikning o'ng tomonida R_4 ni baholagandagi singari mulohaza yuritib quyidagiga ega bo'lamiz:

$$R_5 < K_5(\varepsilon_1, P_0, X_0) \cdot X^{1+\varepsilon_1} \cdot P^{-1} \cdot (n, \tilde{r}) \cdot (lnlnP)^4 \cdot (lnlnX) \cdot lnP.$$

Bunda

$$K_5(\varepsilon_1, P_0, X_0) = \frac{1}{c_{33}} \cdot K_4(\varepsilon_1, P_0, X_0).$$

$(3.\tilde{5})$ dagi qolgan hadlarda yig'indini barcha $q \geq 1$ lar bo'yicha olamiz. Bu natijasida qo'shimcha ravishda R_6 va R_7 xatoliklar paydo bo'ladi. Endi R_6 va R_7 larni yuqoridan baholaymiz. U holda

$$R_6 \leq X\tilde{r} \sum_{\substack{q > P \\ \tilde{r} \backslash q}} \left| \frac{\mu\left(\frac{q}{(q,n)}\right)}{\varphi\left(\frac{q}{(q,n)}\right) \varphi(q)} \right| \leq X \frac{\tilde{r}}{\varphi(\tilde{r}) \cdot \varphi\left(\frac{\tilde{r}}{(\tilde{r},n)}\right)} \sum_{l > P\tilde{r}^{-1}} \frac{1}{\varphi(l) \cdot \varphi\left(\frac{l}{(l,n)}\right)} \leq$$

$$\frac{X\tilde{r}}{\varphi(\tilde{r})\cdot\varphi\left(\frac{\tilde{r}}{(\tilde{r},n)}\right)}\sum_{d\backslash n}\frac{1}{\varphi(d)}\sum_{k\leq P(\tilde{r}d)^{-1}}\frac{1}{\varphi^2(k)}$$

$$\leq K_6(\varepsilon_1,P_0,X_0)\cdot X^{1+\varepsilon_1}\cdot P^{-1}\cdot(n,\tilde{r})\cdot(lnlnP)^4\cdot lnlnX$$

bahoga ega bo'lamiz. Bunda

$$K_6(\varepsilon_1,P_0,X_0)=a(\varepsilon_1)\cdot b^2(P_0)\cdot b\big((1-\varepsilon)X_0\big)\cdot c_{31}(P_0).$$

Shunga o'xshash

$$R_7\leq 2X\tilde{r}\sum_{\substack{q>P\\ \tilde{r}\backslash q}}\left|\frac{\mu(q)\mu\left(\frac{\frac{q}{(q,n)}}{\tilde{r}}\right)}{\varphi\left(\frac{q}{(q,n)}\right)\varphi(q)}\right|.$$

Buning o'ng tomonini R_6 ni baholagandagi singari baholasak quyidagiga ega bo'lamiz:

$$R_7<K_6(\varepsilon_1,P_0,X_0)\cdot X^{1+\varepsilon_1}\cdot P^{-1}\cdot(n,\tilde{r})\cdot(lnlnP)^4\cdot lnlnX.$$

(3.$\tilde{5}$) dagi birinchi cheksiz yig'indi

$$\tilde{\sigma}(n)=\sum_{\substack{q=1\\ \tilde{r}\backslash q}}^{\infty}\frac{\tau^2(\tilde{\chi}\chi_0)}{\varphi^2(q)}\,C_q(-n)$$

va ikkinchi cheksiz yig'indi

$$\sum_{\substack{q=1\\ \tilde{r}\backslash q}}^{\infty}\frac{\mu(q)}{\varphi^2(q)}C_{\tilde{\chi}\chi_0}(-n)\tau(\tilde{\chi}\chi_0)$$

ga teng bo'ladi. Bu yig'indilarni [21] dagi 2.1-lemma: agar $\chi(modq),\ q\geq 3-$ primitiv xarakter bo'lsa, u holda $|\tau(\chi)|=\sqrt{q}$; agarda $\chi(modq),\ q\geq 3-$ haqiqiy primitiv xarakter bo'lsa, u holda $\tau^2(\chi)=\chi(-1)q$ va $\frac{q}{(4,q)}-$ kvadratsiz (ya'ni birorta ham tub sonning kvadratiga bo'linmaydigan) son bo'ladi. Hamda 2.2-lemma: agar $\chi(modk)$ xarakter $\chi^*(modr)$ primitiv xarakter bilan indutsirlangan xarakter bo'lsa, u holda $r\backslash k$ va

$$\tau(\chi)=\mu\left(\frac{k}{r}\right)\chi^*\left(\frac{k}{r}\right)\tau(\chi^*)$$

tenglik o'rinli. Bulardan foydalanib yuqoridagi birinchi cheksiz yig'indini quyidagicha yozish mumkin:

$$\tilde{\sigma}(n)=\tilde{\chi}(-1)\mu\left(\frac{\tilde{r}}{(\tilde{r},n)}\right)\frac{\tilde{r}}{\varphi(\tilde{r})}\varphi^{-1}\left(\frac{\tilde{r}}{(\tilde{r},n)}\right)\prod_{p\nmid\tilde{r},p\nmid n}\left(1-\frac{1}{(p-1)^2}\right)$$
$$\cdot\prod_{p\nmid\tilde{r},p\backslash n}\left(1+\frac{1}{p-1}\right). \tag{3.13}$$

Ikkinchi cheksiz yig'indini uchun esa quyidagi tengsizlik o'rinli:

$$\left|\sum_{\substack{q=1\\ \tilde{r}\backslash q}}^{\infty}\frac{\mu(q)}{\varphi^2(q)}C_{\tilde{\chi}\chi_0}(-n)\tau(\tilde{\chi}\chi_0)\right|$$
$$=\left|\mu(\tilde{r})\tilde{\chi}(n)\frac{\tilde{r}}{\varphi^2(\tilde{r})}\prod_{p\nmid\tilde{r},p\nmid n}\left(1-\frac{1}{(p-1)^2}\right)\cdot\prod_{p\nmid\tilde{r},p\backslash n}\left(1+\frac{1}{p-1}\right)\right|\leq$$

$$\frac{3}{4}\tilde{\chi}^2(n)\frac{\tilde{r}}{\varphi^2(\tilde{r})}\cdot\frac{n}{\varphi(n)}.$$

Shunday qilib olingan barcha baholarni yig'ib, $E_{\tilde{\beta}}=1$ bo'lgan holda

$$\mathcal{R}_1(n) = n\sigma(n) + \tilde{\sigma}(n)\tilde{I}(n) + \frac{3}{2}\tilde{\chi}^2(n)\frac{\tilde{r}}{\varphi^2(\tilde{r})}\cdot\frac{n}{\varphi(n)}\cdot X\cdot\theta_{14}$$

$$+ K_7(\varepsilon_1, P_0, X_0)X^{1+\varepsilon_1}P^{-1}\,(n,\tilde{r})(lnlnX)\cdot(lnlnP)^4\cdot lnP$$

$$+\frac{n}{\varphi(n)}\left(2(K(r,1,n+K(r,\tilde{r},n))\,)X^{\frac{1}{2}}W + K(r,r,n)W^2\right), \qquad (3.14)$$

ifodani hosil qilamiz. Bu yerda

$$K_7(\varepsilon_1, P_0, X_0) = a(\varepsilon_1)\cdot b\big((1-\varepsilon)X_0\big)\times$$

$$\times\left\{\frac{\left(\frac{c_{31}(P_0)}{lnP_0}+\frac{1}{2}c_{30}(P_0)\right)}{(n,\tilde{r})(lnlnP_0)^2}+\left(2(c_{33}+1)\cdot c_{33}\cdot c_{30}(P_0)+3\frac{c_{31}(P_0)}{lnP_0}\right)\right\}. \quad (3.15)$$

III.4- §. R_1 va $\widetilde{R}_1$ larni baholash.

Avvalo quyidagi lemmani isbotlaymiz.

Faraz etaylik $\delta = \theta T^{-1},\ 0 < \theta < 1$ va

$$S(t) = \sum_{\nu} c(\nu)e(\nu t)$$

absolyut yaqinlashuvchi trigonometrik qator bo’lsin. Bunda $\nu -$ haqiqiy sonlarning biror ketma-ketligida o’zgaradi va $c(\nu) -$ koeffisientlar kompleks sonlar.

4.1-lemma. Yuqoridagi shartlarda quyidagi munosabat o’rinli:

$$\int_{-T}^{T}|S(t)|^2\,dt \le c_{35}(\theta)\int_{-\infty}^{\infty}\left|\delta^{-1}\sum_{x}^{x+\delta}c(\nu)\right|^2\,dx,$$

bunda

$$c_{35}(\theta) = \left\{\min\left(\frac{2}{\pi},\qquad \frac{2(1-\theta)}{\pi\theta}\right)\right\}^{-2}.$$

Isboti. [30] dagi (1.25)- tenglikka asosan

$$\int_{-\infty}^{\infty} \left|S(t)\hat{F}_\delta(t)\right|^2 dt = \int_{-\infty}^{\infty} |C_\delta(x)|^2\, dt, \qquad (4.1)$$

bu yerda

$$C_\delta(x) = \delta^{-1} \sum_{|\nu - x| \le \frac{\delta}{2}} c(\nu)$$

va $\hat{F}_\delta(t)$ bilan

$$F_\delta(t) = \begin{cases} \delta^{-1}, \text{agar } |t| \le \dfrac{\delta}{2} \text{ bo'lsa;} \\ \quad 0, \quad \text{qolgan hollarda} \end{cases}$$

tenglik bilan aniqlanuvchi $F_\delta(t) -$ funksiyaning Furye almashtirishi belgilangan. $\hat{F}_\delta(t)$ ni qaraymiz. Bu yerda

$$\hat{F}_\delta(t) = \int_{-\infty}^{\infty} F_\delta(x - \nu)\, e(-(x - \nu)t)\, dx = \int_{|x-\nu| \le \frac{\delta}{2}} \delta^{-1}\, e(-(x - \nu)t)\, dx$$
$$= \frac{\sin\pi\delta t}{\pi\delta t}.$$

$|t| \le T$ va $\delta T = \theta$ lar uchun $0 < \theta < \frac{1}{2}$ bo'lganda

$$\frac{\sin\pi\delta t}{\pi\delta t} \ge \frac{\sin\pi\theta}{\pi\theta} \ge \frac{2}{\pi}$$

tengsizlik o'rinli. Agarda $\frac{1}{2} < \theta < 1$ bo'lsa, u holda $\theta = 1 - \varepsilon$, $\left(0 < \varepsilon < \frac{1}{2}\right)$ deb olib quyidagiga ega bo'lamiz:

$$\frac{\sin\frac{\pi(1-\varepsilon)t}{T}}{\frac{\pi(1-\varepsilon)t}{T}} \ge \frac{\sin\pi(1-\varepsilon)}{\pi(1-\varepsilon)} > \frac{2\varepsilon}{\pi(1-\varepsilon)} = \frac{2(1-\theta)}{\pi\theta}.$$

Shunday qilib,

$$\hat{F}_{\delta}(t)=\frac{sin\pi\delta t}{\pi\delta t}\geq \min\left(\frac{2}{\pi},\frac{2(1-\theta)}{\pi\theta}\right).$$

Bu tengsizlikdan foydalanib (4.1) dan lemmaning isbotlanishi talab etilgan tasdig’iga kelamiz.

Endi

$$W=\sum_{q\leq P}\sum_{\chi}^{*}W(\chi)$$

ni qaraymiz. Yuqoridagi 4.1-lemmaga asosan (3.3)-tenglikdan

$$W(\chi)\leq\left(\frac{\pi^2}{2}\int\limits_{P}^{X+\frac{1}{2}qQ}\left|\frac{1}{2}qQ\sum_{x-\frac{1}{2}qQ}^{x}{}^{\#}\chi(p)lnp\right|^2 dx)\right)^{\frac{1}{2}}<$$

$$\frac{\pi}{\sqrt{2}}\left(X+\frac{1}{2}qQ\right)^{\frac{1}{2}}\left(\underbrace{max}_{x\leq\frac{3}{2}X}\ \underbrace{max}_{h\leq\frac{1}{2}X}\left(h+\frac{X}{P}\right)^{-1}\left|\sum_{x-\frac{1}{2}qQ}^{x}{}^{\#}\chi(p)lnp\right|\right)\quad (4.2)$$

ni hosil qilamiz. Buning o’ng tomoniga [21] dagi I.4.1- teoremani qo’llaymiz. Unga ko’ra agar $\varkappa_1=\frac{1+16\varepsilon}{115}$, $0<\varepsilon<0{,}01$ va $exp\left(\sqrt{lnX}\right)\leq P\leq X^{\varkappa_1}$, $XP^{-1}\leq h\leq\frac{1}{2}X$ lar bajarilsa, u holda

$$\sum_{q\leq P}\sum_{\chi}^{*}\underbrace{max}_{x\leq\frac{3}{2}X}\ \underbrace{max}_{h\leq\frac{1}{2}X}\left(h+\frac{X}{P}\right)^{-1}\left|\sum_{x-h}^{x}{}^{\#}\chi(p)lnp\right|<$$

$$\begin{cases} c_{36}\,exp\left(-c_{37}\dfrac{lnX}{lnP}\right), & \text{agar } E_{\tilde{\beta}}=0 \text{ bo'lsa;}\\ c_{38}\,exp\left(-c_{39}\dfrac{lnX}{lnP}\right)(1-\tilde{\beta})lnP, & \text{agar } E_{\tilde{\beta}}=1 \text{ bo'lsa,}\end{cases}$$

tengsizlik o’rinli.

Bunda $c_{36} \leq 2{,}002$; $\quad c_{37} \geq 95{,}64 \cdot 10^{-5}$; $\quad c_{38} \leq exp(6{,}991466)$;

$$c_{39} \geq 72{,}637 \cdot 10^{-4} \qquad (4.3)$$

va yig'indidagi # belgi agar $q = 1$ yig'indining

$$\sum_{x-h}^{x} \tilde{\chi}(p) lnp + \sum_{x-h<n\leq x} 1$$

ga tengligini; $E_{\tilde{\beta}} = 1$ bo'lgan holda esa uning

$$\sum_{x-h}^{x} lnp - \sum_{\substack{x-h<n\leq x \\ n>0}} n^{\tilde{\beta}-1}$$

ga teng ekanligini bildiradi.

Agar $E_{\tilde{\beta}} = 0$ bo'lsa, (4.2) dan quyidagiga ega bo'lamiz:

$$W < \frac{\sqrt{3}}{2} \pi X^{\frac{1}{2}} c_{36}\, exp\left(-\frac{c_{37}}{3\delta}\right).$$

Buni va

$$K(r, r, n) = K(r, \tilde{r}, n) < 1{,}8371023,$$

$$K(r, 1, n) = \prod_{p\nmid\tilde{r}, p\nmid n} \left(1 + \frac{1}{(p-1)^2}\right)$$
$$\cdot \prod_{p\nmid\tilde{r}, p\backslash n} p^{-1} \left(1 - \frac{1}{p}\right)^{-2} \leq 1{,}4142 \cdot \frac{3}{4} = 1{,}06065$$

([21] dagi II.2.5-lemmaga qarang) ekanliklarini e'tiborga olib (3.4) dan quyidagiga ega bo'lamiz:

$$R_1 \leq \frac{n}{\varphi(n)} \left(2K(r, 1, n) X^{\frac{1}{2}} W + K(r, r, n) W^2\right) <$$

$$\frac{n}{\varphi(n)} X c_{36} \left(5{,}7714186 + 13{,}598604 c_{36} \, exp\left(-\frac{c_{37}}{3\delta}\right)\right) \cdot exp\left(-\frac{c_{37}}{3\delta}\right). \quad (4.4)$$

$Agar\ E_{\tilde{\beta}} = 1$ bo'lsa, (3.$\tilde{4}$) dan quyidagiga ega bo'lamiz:

$$\tilde{R}_1 < \frac{n}{\varphi(n)} X c_{38} \left(15{,}767823 \ + 13{,}598604 c_{38} \, exp\left(-\frac{c_{39}}{3\delta}\right)\right) \times$$

$$\cdot \left(1 - \tilde{\beta}\right)(lnP \, exp\left(-\frac{c_{39}}{3\delta}\right). \qquad (4.\tilde{3})$$

Endi biz asosiy natijani isbotlashimiz mumkin.

III.5- §. Asosiy teoremaning isboti.

Biz $\mathcal{R}(n)$>0 ekanligini isbotlashimiz kerak. (1.4) ga asosan

$$\mathcal{R}(n) = \mathcal{R}_1(n) + \mathcal{R}_2(n)$$

bo'lgani uchun

$$\mathcal{R}(n) > \mathcal{R}_1(n) - |\mathcal{R}_2(n)| > 0 \qquad (5.1)$$

ya'ni

$$\mathcal{R}_1(n) > |\mathcal{R}_2(n)|$$

ekanligini ko'rsatamiz. III.2-§ da $(1, X)$ oraliqdagi ko'pi bilan

$$E_1(X) < XP^{-\frac{5}{12}} ln^{12} X \qquad (2.2)$$

ta n, $(n \leq X)$ lar lardan boshqa barcha n, $(n \leq X)$ lar uchun

$$|\mathcal{R}_2(n)| \leq XP^{-\frac{7}{24}} \qquad (2.3)$$

munosabatning bajarilishini ko'rsatdik.

Endi $(1, X)$ oraliqdagi n ko'pi bilan $E_2(X)$ ta qiymatlaridan boshqa barcha qiymatlari qiymatlari uchun

$$\mathcal{R}_1(n) > XP^{-\frac{7}{24}} \qquad (5.2)$$

tengsizlikning bajarilishini ko'rsatamiz va $E_2(X)$ ni yuqoridan baholaymiz.

Avvalo $E_{\tilde{\beta}} = 0$ bo'lgan holni qaraymiz. U holda (3.9) dan

$$\mathcal{R}_1(n) > n\sigma(n) - K_3(\varepsilon_1, P_0, X_0)X^{1+\varepsilon_1}P^{-1}\,(lnlnX)\cdot(lnlnP)^2\cdot lnP$$
$$-\frac{n}{\varphi(n)}\left(2K(r,1,n)X^{\frac{1}{2}}W + K(r,r,n)W^2\right)$$
$$> n\sigma(n) - K_3(\varepsilon_1, P_0, X_0)X^{1+\varepsilon_1}P^{-1}\,(lnlnX)\cdot(lnlnP)^2\cdot lnP$$
$$-\frac{n}{\varphi(n)}Xc_{36}\left(5{,}7714186 + 13{,}598604c_{36}\,exp\left(-\frac{c_{37}}{3\delta}\right)\right)$$
$$\cdot\, exp\left(-\frac{c_{37}}{3\delta}\right).$$

Bunda

$$\sigma(n) \geq \prod_{p\geq 3}\left(1 - \frac{1}{(p-1)^2}\right)\frac{n}{\varphi(n)} > 0{,}65445\frac{n}{\varphi(n)}$$

ekaligini va (4.3) ni e'tiborga olsak $\frac{1}{2}X < n \leq X$ bo'lganda

$$\mathcal{R}_1(n) > \frac{n}{\varphi(n)}X\left\{\frac{0{,}65445}{2} - K_3(\varepsilon_1, P_0, X_0)X^{1+\varepsilon_1}P^{-1}\,(lnlnX)\cdot(lnlnP)^2\cdot lnP\right.$$
$$- exp(2{,}447064 - 31{,}88\cdot 10^{-5}\delta)$$
$$\left. - -ex\,p(3{,}998261 - 63{,}7610^{-5}\delta^{-1})\right\}$$

bajariladi. Endi bu yerda $\delta = 10{,}336\cdot 10^{-5}, \varepsilon_1 = \frac{5}{8}\delta$ desak, yetarlicha katta X lar uchun

$$\mathcal{R}_1(n) > 0{,}00072\frac{n}{\varphi(n)}X > 0{,}0007X > XP^{-\frac{7}{24}} \qquad (5.3)$$

$Agar\ E_{\tilde{\beta}} = 1$ bo'lsa, (3.14) va (4.$\tilde{3}$) larga asosan quyidagilarga ega bo'lamiz:

$$\mathcal{R}_1(n) = n\sigma(n) + \tilde{\sigma}(n)\tilde{I}(n) + \frac{3}{2}\tilde{\chi}^2(n)\frac{\tilde{r}}{\varphi^2(\tilde{r})}\cdot\frac{n}{\varphi(n)}\cdot X\cdot\theta_{14}$$

$$+ K_7(\varepsilon_1, P_0, X_0)X^{1+\varepsilon_1}P^{-1}\,(n,\tilde{r})(lnlnX)\cdot(lnlnP)^4\cdot lnP$$

$$+ \frac{n}{\varphi(n)}Xc_{38}\theta_{15}\left(15{,}767823 + 13{,}598604c_{38}\,exp\left(-\frac{c_{39}}{3\delta}\right)\right)\times$$

$$\cdot(1-\tilde{\beta})lnP\,exp\left(-\frac{c_{39}}{3\delta}\right). \qquad (5.4)$$

Agar $(n,\tilde{r}) = 1$ bo'lsa,

$$|\tilde{\sigma}(n)| \le \frac{\tilde{r}}{\varphi^2(\tilde{r})}\cdot\frac{n}{\varphi(n)}\prod_{p\nmid\tilde{r},p\nmid n}\left(1-\frac{1}{(p-1)^2}\right) \le \frac{3}{4}\frac{\tilde{r}}{\varphi^2(\tilde{r})}\cdot\frac{n}{\varphi(n)}.$$

Shuining uchun ham (5.4) dan

$$\mathcal{R}_1(n) > \frac{n}{\varphi(n)}X\times$$

$$\times\left\{0{,}65445 - \frac{9}{4}\frac{\tilde{r}}{\varphi^2(\tilde{r})} - K_7(\varepsilon_1,P_0,X_0)X^{\varepsilon_1-3\delta}\,(lnlnX)\cdot(lnlnP)^4\cdot lnP\right.$$
$$- c_{38}\theta_{15}\left(15{,}767823\right.$$
$$\left.\left. + 0{,}0260114c_{38}\,exp\left(-\frac{c_{39}}{3\delta}\right)\right)(0{,}0019128)exp\left(-\frac{c_{39}}{3\delta}\right)\right\}.$$

Bu yerda [21] dagi (2.1.6) ga asosan $\left(1-\tilde{\beta}\right)lnP \le 0{,}0019128$ ekanligidan foydalandik. Shuningdek [21] dagi (I.1.3) munosabatdan $\tilde{r} \ge 9{,}69\ lnP$ ekanligi kelib chiqadi.

Bu yerdan yetarlicha katta X lar uchun

$$\mathcal{R}_1(n) > 0{,}6\frac{n}{\varphi(n)}X \ge 0{,}6X$$

ning bajarilishi kelib chiqadi, ya'ni bu holda ham (5.3)- munosabat bu holda ham o'rinli bo'lib qoladi.

Agar $(n,\tilde{r})>1$ bo'lsa, u holda (5.4) dagi uchunchi had no'lga aylanadi, lekin to'rtinchi had $(n,\tilde{r})$ ning hisobidan katta bo'lishi mumkin. Shuning uchun ham n ning $(n,\tilde{r})>P^{\frac{1}{2}}$ shartni qanoatlantiruvchi juft qiymatlarini tashlab yuboramiz. U holda qolgan n lar uchun bu had

$$K_7(\varepsilon_1,P_0,X_0)X^{1+\varepsilon_1}P^{-\frac{1}{2}}(lnlnX)\cdot(lnlnP)^4\cdot lnP$$

dan katta bo'la olmaydi. Bu yerda tashlab yuborilgan n larning soni

$$\sum_{d\backslash\tilde{r},\ d>P^{\frac{1}{2}}}\ \sum_{\substack{d\backslash n\\ n\leq X}}1\leq\sum_{d\backslash\tilde{r},\ d>P^{\frac{1}{2}}}X\leq XP^{-\frac{1}{2}}d(\tilde{r})\leq a(\varepsilon_1)XP^{-\frac{1}{2}+\varepsilon_1}$$

dan ko'p emas. Shuning uchun ham X ning yetarlicha katta qiymatlarida

$$E_3(X)\leq a(\varepsilon_1)XP^{-\frac{1}{2}+\varepsilon_1}=a(\varepsilon_1)XP^{-\frac{5}{12}-\frac{1}{12}+\varepsilon_1}<\left(\frac{a(\varepsilon_1)}{P^{\frac{1}{12}-\varepsilon_1}}\right)XP^{-\frac{5}{12}}$$
$$<XP^{-\frac{5}{12}}ln^{12}X \qquad (5.5)$$

bajariladi. Endi $\frac{X}{2}<n\leq X$ va $1<(n,\tilde{r})\leq P^{\frac{1}{2}}$ shartlarni qanoatlantiruvchi juft n larni qarash qoldi.

$$|\tilde{\sigma}(n)|\leq\sigma(n)\prod_{\substack{p\backslash\tilde{r},p\nmid n\\ p>3}}\frac{1}{p-2} \qquad (5.6)$$

bo'lgani uchun agar (5.6) dagi ko'paytma bo'sh bo'lmasa (5.4) dan

$$\mathcal{R}_1(n)>\frac{n}{\varphi(n)}X\times$$

$$\times\left\{0{,}65445-\frac{1}{3}\cdot 0{,}65445-K_7(\varepsilon_1,P_0,X_0)X^{\varepsilon_1-1{,}5\delta}\ (lnlnX)\cdot(lnlnP)^4\cdot lnP\right.$$

$$-\,c_{38}\theta_{15}\left(0{,}00301606+0{,}49755\cdot 10^{-4}\ \cdot c_{38}\right.$$

$$\left.\left.\cdot\, exp\left(-\frac{c_{39}}{3\delta}\right)\right)\ \cdot\ exp\left(-\frac{c_{39}}{3\delta}\right)\ \right\}\geq$$

$$\frac{n}{\varphi(n)}X\left\{0{,}435-K_7(\varepsilon_1,P_0,X_0)X^{-\frac{7}{8}\delta}\ (lnlnX)\cdot(lnlnP)^4\cdot lnP\right\}$$

$$\geq 0{,}4\frac{n}{\varphi(n)}X\geq 0{,}4\,X. \qquad (5.7)$$

Agar (5.6) dagi ko'paytma bo'sh bo'lsa, [21] dagi II.2.1- lemmaga asosan

$$(n,\tilde{r})\geq\frac{\tilde{r}}{24}$$

bajariladi va qaralayotgan n lar $(n,\tilde{r})\leq P^{\frac{1}{2}}$ shartni qanoatlantirgani uchun ham

$$\frac{\tilde{r}}{24}\leq(n,\tilde{r})\leq P^{\frac{1}{2}}\qquad\rightarrow\qquad \tilde{r}\leq 24P^{\frac{1}{2}}. \qquad (5.8)$$

Shuningdek

$$n\sigma(n)+\tilde{\sigma}(n)\tilde{I}(n)\geq n\sigma(n)-\sigma(n)\left|\tilde{I}(n)\right|, \qquad (5.9)$$

bu yerda

$$\tilde{I}(n)=\sum_{P<k<n-P}\left(k(n-k)\right)^{\tilde{\beta}-1}\leq n\cdot n^{\tilde{\beta}-1}=n^{\tilde{\beta}}.$$

Chekli ayirmalar haqidagi Lagranj teoremasini qo'llab quyidagiga ega bo'lamiz:

$$n-n^{\tilde{\beta}}=\left(1-\tilde{\beta}\right)n^{\theta}lnn\geq\left(1-\tilde{\beta}\right)n^{\tilde{\beta}}lnn=\left(1-\tilde{\beta}\right)\cdot n\cdot(lnn)\cdot n^{\tilde{\beta}-1}.$$

[21] dagi I.1.1- teoremaga asosan

$$\frac{0{,}4941}{\tilde{q}^{\frac{1}{2}}\ ln^2\tilde{q}}<1-\tilde{\beta}<\frac{0{,}0019128}{lnP}. \qquad (5.10)$$

(5.10) dan

$$\tilde{\beta}-1 \geq -\frac{0,0019128}{lnP} \geq -\frac{0,0019128}{3\delta lnn}$$

kelib chiqadi. Shunday qilib

$$n-n^{\tilde{\beta}}=\left(1-\tilde{\beta}\right)\cdot n\cdot(lnn)\cdot exp\left(\left(\tilde{\beta}-1\right)lnn\right)\geq$$

$$\left(1-\tilde{\beta}\right)\cdot n\cdot(lnn)\cdot exp\left(-\frac{0,0019128}{3\delta}\right)\geq$$

$$\left(1-\tilde{\beta}\right)\cdot n\cdot(lnP)\cdot exp\left(-\frac{0,0019128}{3\delta}\right)>6,5274\left(1-\tilde{\beta}\right)\cdot n\cdot(lnP).$$

Endi (5.9) da bu tengsizlikdan foydalansak, quyidagiga ega bo'lamiz:

$$n\sigma(n)+\tilde{\sigma}(n)\tilde{I}(n)\geq\sigma(n)\left(n-n^{\tilde{\beta}}\right)\geq 0,65445\frac{n}{\varphi(n)}\left[6,5274\left(1-\tilde{\beta}\right)\cdot n\cdot(lnP)\right]$$

$$>2,1359\cdot\left(1-\tilde{\beta}\right)\cdot X\cdot\frac{n}{\varphi(n)}\cdot lnP.$$

Bularni inobatga olib (5.4) dan

$$\mathcal{R}_1(n)$$
$$>\left\{2,1359\cdot\left(1-\tilde{\beta}\right)\cdot X\cdot\frac{n}{\varphi(n)}\cdot lnP-K_7(\varepsilon_1,P_0,X_0)X^{1+\varepsilon_1-\frac{3}{2}\delta}\,(lnlnX)(lnlnP)^4\right.$$
$$\cdot\, lnP$$
$$-\ \frac{n}{\varphi(n)}Xc_{38}\left(15,767823\ +0,0260114c_{38}\,exp\left(-\frac{c_{39}}{3\delta}\right)\right)$$
$$\left.\times\left(1-\tilde{\beta}\right)lnP\,exp\left(-\frac{c_{39}}{3\delta}\right)\right\}$$
$$=\frac{n}{\varphi(n)}X\left\{\left(2,1359-c_{38}\left(15,767823\ +0,0260114c_{38}\,exp\left(-\frac{c_{39}}{3\delta}\right)\right)\right)(1\right.$$
$$-\tilde{\beta})lnP\,exp\left(-\frac{c_{39}}{3\delta}\right)-\left(K_7(\varepsilon_1,P_0,X_0)X^{-\frac{3}{32}\delta}\right)\ X^{-\frac{3}{4}\delta}\left[X^{-\frac{\delta}{32}}(lnlnX)(lnlnP)^4\right.$$
$$\left.\left.\cdot\, lnP\right]\right\}$$

ni hosil qilamiz. (5.10) dan

$$1-\tilde{\beta} \geq \frac{0{,}4941}{\tilde{r}^{\frac{1}{2}}\, ln^2\tilde{r}} \geq \frac{0{,}4941}{\sqrt{24}\left(\frac{ln24}{lnP_0}+\frac{1}{2}\right)^2 P^{\frac{1}{4}}\, ln^2P} > \frac{0{,}4034}{P^{\frac{1}{4}}\, ln^2P}$$

kelib chiqadi.

Endi bu yerda $\delta = 10{,}336 \cdot 10^{-5}, \varepsilon_1 = \frac{5}{8}\delta$ desak, yetarlicha katta X lar uchun

$$\mathcal{R}_1(n) > \frac{n}{\varphi(n)} X \left\{ \frac{0{,}8605}{P^{\frac{1}{4}}\, lnP} - \left(K_7(\varepsilon_1, P_0, X_0) X^{-\frac{3}{32}\delta}\right)\; X^{-\frac{3}{4}\delta}\left[X^{-\frac{\delta}{32}}(lnlnX)(lnlnP)^4 \cdot lnP\right]\right\} \geq$$

$$\frac{X}{P^{\frac{1}{4}}\, lnP}\left\{0{,}8605 - \left(K_7(\varepsilon_1, P_0, X_0) X^{-\frac{3}{32}\delta}\right)\; \left[X^{-\frac{\delta}{32}}(lnlnX)(lnlnP)^4 \cdot ln^2P\right]\right\}$$

$$\geq \frac{0{,}8X}{P^{\frac{1}{4}}\, lnP} > \; XP^{-\frac{7}{24}}\;. \tag{5.11}$$

(5.3),(5.7) va (5.11) lardan ko'pi bilan n ning

$$E_2(X) = E_3(X) < XP^{-\frac{5}{12}} ln^{12}X$$

ta qiymatlaridan boshqa barcha $\frac{X}{2} < n \leq X$ qiymatlari uchun

$$\mathcal{R}_1(n) > XP^{-\frac{7}{24}}$$

ning bajarilishi kelib chiqadi. Bundan va (5.1), (2.3) lardan ko'pi bilan n ning

$$E_2(X) = E_3(X) + E_3(X) < 2XP^{-\frac{5}{12}} ln^{12}X < XP^{-\frac{5}{12}} ln^{13}X$$

ta qiymatlaridan boshqa barcha $\frac{X}{2} < n \leq X$ qiymatlari uchun

$$\mathcal{R}(n) > \mathcal{R}_1(n) - |\mathcal{R}_2(n)| > 0$$

ga ega bo'lamiz.

Natija. a). Agar Dirixle $L-$ funktsiyasining maxsus haqiqiy no'li mavjud bo'lmasa, u holda X ning yetarlicha katta qiymatlarida $n, (n \leq X)$ ning ko'pi bilan $E(X) < X^{0,9882}$ tadan boshqa barcha qiymatlari uchun

$$R(n) > n^{0,991673}\left(0,0007262 \cdot \frac{n^{1,008326}}{\varphi(n)} - 1\right)$$

tengsizlik; qolgan qiymatlari uchun esa

$$R(n) \leq n^{0,991673}\left(0,0007262 \cdot \frac{n^{1,008326}}{\varphi(n)} - 1\right)$$

tengsizlik o'rinli.

b). Agar Dirixle $L-$ funktsiyasining maxsus haqiqiy no'li $\tilde{\beta}$ mavjud bo'lib, $(n, \tilde{r}) = 1$ bo'lsa u holda X ning yetarlicha katta qiymatlarida $n, (n \leq X)$ ning ko'pi bilan $E(X) < X^{0,9882}$ tadan boshqa barcha qiymatlari uchun

$$R(n) > n^{0,991673}\left(0,6\frac{n^{1,008326}}{\varphi(n)} - 1\right)$$

tengsizlik; qolgan qiymatlari uchun esa

$$R(n) \leq n^{0,991673}\left(0,6\frac{n^{1,008326}}{\varphi(n)} - 1\right)$$

tengsizlik o'rinli bo'ladi.

Natijaning o'rinli ekanligiga ishonch hosil qilish uchun uning shartini e'tiborga olgan holda yuqoridagi asosiy teoremaning isbotini takrorlash kifoya bo'ladi.

Xulosalar.

Ishda sonlar nazariyasida muhim ahamiyatga ega bo'lgan additiv masala qaralib quyidagi natija isbotlangan:

1.Agar Dirixle $L-$ funktsiyasining maxsus haqiqiy no'li mavjud bo'lmasa yoki shunday maxsus haqiqiy no'li $\tilde{\beta}$ mavjud bo′lib, $(n,\tilde{r})=1$ bo'lsa, u holda X ning yetarlicha katta qiymatlarida $n, (n \leq X)$ning ko′pi bilan

$$E(X) < X^{0,9882}$$

tadan boshqa barcha qiymatlari uchun

$$R(n) > n^{0,991673}\left(0,0007262 \cdot \frac{n^{1,008326}}{\varphi(n)} - 1\right) \qquad (0.2)$$

tengsizlik; qolgan qiymatlari uchun esa

$$R(n) \leq n^{0,991673}\left(0,0007262 \cdot \frac{n^{1,008326}}{\varphi(n)} - 1\right)$$

tengsizlik o'rinli.

Additiv masalarni yechishda muhim ahamiyatga ega bo'lgan Rimanning dzeta-funktsiyasi $\zeta(s)$ no'llarining soni haqida natija aniqlashtirilib quyidagi natija olingan:

2. Rimanning dzeta-funktsiyasi $\zeta(s), (s=\sigma+it)$ ning $0<\sigma<1,\ |t|\leq T$ to'g'ri to'rtburchakdagi trivial bo'lmagan no'llarining soni *N(T)* uchun

$$N(T) = \frac{T}{2\pi} ln\frac{T}{2\pi} - \frac{T}{2\pi} + +6,512850\theta\ lnT$$

formula isbotlangan.

Bu natija ilgari $N(T)$ bilan bog'liq mavjud natijalarning aniqlashtirilgani hisoblanadi. Taqqoslash uchun ilgarigi natija I. Allakov tomonidan olingan bo'lib

$$N(T) = \frac{T}{2\pi} ln \frac{T}{2\pi} - \frac{T}{2\pi} + 32{,}2266\theta \; lnT$$

ko'rinishda edi.

МИНИСТЕРСТВО ВЫСШЕГО И СПЕЦИАЛЬНОГО ОБРАЗОВАНИЕ РЕСПУБЛИКИ УЗБЕКИСТАН

ТЕРМЕЗСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

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Учебный год: 2015-2017

Специальность: 5А130101-Математика (по направлениям)

АННОТАЦИЯ

Магистерской диссертации на теме: «О РЕШЕНИИ БИНАРНОЙ АДДИТИВНОЙ ЗАДАЧИ С ПРОСТЫМИ ЧИСЛАМИ».

Актуальность темы. В 1742 году из переписки Х.Гольдбаха с Л.Эйлером возникла проблема Гольдбаха, представляющая собою гипотезу, согласно которой всякое четное число ≥ 6 есть сумма двух нечетных простых чисел (бинарная проблема Гольдбаха), а всякое нечетное число ≥ 9 есть сумма трех нечетных простых чисел (тернарная проблема Гольдбаха). Очевидно из справедливости бинарной проблемы Гольдбаха, тривиально следует справедливость тернарной проблемы Гольдбаха.

В 1937 году И.М.Виноградов, используя свой метод тригонометрических сумм, доказал, тернарную проблему Гольдбаха для всех достаточно больших $n>n_0$. Недавно в 1997 году эта проблема была полностью решена для всех $n \geq 9$. Но бинарная проблема до сих пор полностью не решена. Основываясь на методе Виноградова, Н.Г.Чудаков, Ван-дер-Корпут, Т.Эстерман показали, что почти все четные числа представимы в виде суммы двух простых чисел, точнее была доказана оценка $E(X) \ll X/\ln^A X$, где $E(X)$ – число четных натуральных чисел не представимых в виде суммы двух простых чисел. Этот результат другим методом также доказан Ю.Линником.

Дальнейшие улучшение оценки исключительного множества E(X) в этой задаче не давно получили Монтгомери и Вон, И. Аллаков, Ж.Чен, С.Ран. Они соответственно получили $E(X) < Xexp\left(-c\sqrt{lnX}\right)$, $E(X) < X^{1-\delta}$, $E(X) < X^{0,96}$. Несмотря этому пока бинарная проблема Гольдбаха остаётся нерешенным. Поэтому исследовании в этом направлении являются актуальными.

Цель и задачи исследования. Целью магистерской диссертации является изучение множество четных чисел в промежутке $(1, \mathrm{X})$, которые представима ввиде $n = p_1 + p_2$ и получение новую оценку снизу для $R(n)$ −числа представлении n, в указанном виде.

Объект и предмет исследования: Объектом исследования являются простые числа, числовые последовательности, числовые функции, тригонометрические суммы. Предметом исследования является задача о представлении чисел суммой двух простых чисел.

Методы исследования. При доказательстве результатов используются различные варианты метода тригонометрических сумм, аналитический метод оценок тригонометрических сумм А.Ф.Лаврика, круговой метод Харди – Литтлвуда – Рамануджана.

Научная новизна. В диссертационной работе получены следующие новые научные результаты:

1. Получена новая оценка снизу для $R(n)$ −числа представлении n ввиде $n = p_1 + p_2$.
2. Уточнен остаточный член формулы Манголдта.

1.Если несуществует исключительный вещественный нул $\tilde{\beta}$ $L-$ функции Дирихле или существует такой нул $\tilde{\beta}$ и $(n, \tilde{r}) = 1$, тогда при достаточно большом X, для всех $n, (n \leq X)$ исключая не более чем $E(X) < X^{0,9882}$ значении из них справедливо неравенство

$$R(n) > n^{0,991673}\left(0,0007262 \cdot \frac{n^{1,008326}}{\varphi(n)} - 1\right).$$

Для остальных $n, (n \leq X)$ справедливо неравенство

$$R(n) \leq n^{0,991673}\left(0,0007262 \cdot \frac{n^{1,008326}}{\varphi(n)} - 1\right).$$

2. Пусть $N(T)$ —количество нетривиальных нулей дзета функции Римана $\zeta(s), (s = \sigma + it)$ в прямоугольнике $0 < \sigma < 1,\ |t| \leq T$. В работе доказано, формула

$$N(T) = \frac{T}{2\pi} ln \frac{T}{2\pi} - \frac{T}{2\pi} + 6,512850\theta\ lnT\ ,$$

которое является уточнением формулы Мангольдта.

Практическая значимость результатов исследования. Работа носит теоретический характер, её результаты могут быть использованы в исследованиях различных аддитивных задач теории чисел, в частности в проблеме Варинга с ограниченными числами слагаемых, в проблеме Гольдбаха, в проблеме Хуа-Ло-Кена о представлении чисел суммой простого и фиксированной степени простого числа, в задаче об одновременном представлении чисел суммой простых чисел и в других задачах.

Реализация результатов. Результаты диссертации могут быть реализованы в научных исследованиях проводимых специалистами по теории чисел работающими в институтах математики АН Белоруссии, АН Р.Уз., Самаркандском и Термезском госуниверситетах.

Структура и объём работы. Работа изложена на 80 машинописных страницах, состоит из введения, трех глав, разбитых на девять параграфов, заключение и списка использованной литературы из 34 наименований.

Основные результаты работы. Основными результатами работы являются следующие результаты:

1.Если несуществует исключительный вещественный нул $\tilde{\beta}$ $L-$ функции Дирихле или существует такой нул $\tilde{\beta}$ и $(n, \tilde{r}) = 1$, тогда при достаточно большом X, для всех $n, (n \leq X)$ исключая не более чем $E(X) < X^{0,9882}$ значении из них справедливо неравенство

$$R(n) > n^{0,991673}\left(0,0007262 \cdot \frac{n^{1,008326}}{\varphi(n)} - 1\right).$$

Для остальных $n, (n \leq X)$ справедливо неравенство

$$R(n) \leq n^{0,991673}\left(0,0007262 \cdot \frac{n^{1,008326}}{\varphi(n)} - 1\right).$$

2. Пусть $N(T)$ —количество нетривиальных нулей дзета функции Римана $\zeta(s), (s = \sigma + it)$ в прямоугольнике $0 < \sigma < 1,\ |t| \leq T$. В работе доказано, формула

$$N(T) = \frac{T}{2\pi} ln \frac{T}{2\pi} - \frac{T}{2\pi} + 6{,}512850\theta\, lnT\ ,$$

которые является уточнением формулы Манголдта.

Краткое и обобщенные изложение заключении и предложении. В магистерском диссертации получена новая оценка снизу для $R(n)$ —числа представлении n ввиде $n = p_1 + p_2$ и уточнен остаточный член формулы Манголдта.

Научный руководитель:

доктор физико-математических наук Аллаков И. ______________

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Термиз давлат университети физика – математика факультети “Математика” кафедраси 2 –курс магистранти Абдусаматова Ҳилола-нинг “Туб сонлар қатнашган бинар аддитив масаланинг ечимлари сони ҳақида” мавзусида 5А130101 – математика таълим йўналиши бўйича магистр академик даражасини олиш учун ёзган диссертациясига раҳбарнинг

ХУЛОСАСИ

Ушбу магистрлик диссертацияси сонлар назариясининг долзарб масалаларидан бири Голдбахнинг бинар аддитив масаласига бағишланган.

Бу масала билан кўплаб сонлар назарияси соҳасининг таниқли математиклари Харди, Литлвуд, И.Виноградов, А.Карацуба, Г.Монтгомери, Р.Вон, А.Лаврик ва бошқалар шуғулланишган бўлсаларда, лекин масала ҳозиргача тўла ўз ечимини топган эмас. Шунинг учун ҳам танлаган мавзу соҳанинг долзарб масалаларидан ҳисобланади.

Ишнинг асосий мақсади $(1, X)$ораликдаги иккита туб сонларнинг йиғиндиси

$$n = p_1 + p_2 \qquad (1)$$

кўринишида ифодаланадиган жуфт $n -$натурал сонлар тўпламини ўрганиб (1) нинг туб сонлардаги ечимлари сони $R(n)$ учун янги баҳо олишдан иборат бўлиб диссертацияда бу мақсадга эришилган, яъни ишда $R(n)$ учун қуйидан янги баҳо олинган. Бундан ташқари ишда Риманнинг дзета-функтсияси $\zeta(s), (s = \sigma + it)$ нинг $0 < \sigma < 1,\ |t| \leq T$ тўғри тўртбурчакдаги тривиал бўлмаган ноллариниг сони *N(T)* учун

$$N(T) = \frac{T}{2\pi} ln \frac{T}{2\pi} - \frac{T}{2\pi} + 6{,}512850\theta\, lnT$$

формула исботланган. Бу формула илгари мавжуд формулалардан қолдиқ ҳаддаги ўзгармаснинг қиймати билан фарқ қилади.

Бу натижаларни олишда А.Карацуба, Г.Монтгомери, Р.Вон, А.Лавриклрнинг методлари ҳамда И.Аллаковнинг шу соҳадаги натижаларидан самарали фойдаланилган. Олинган натижаларнинг асосий қисми илмий мақола ва тезислар шаклида эълон қилинган.

Иш муаллифнинг соҳани яхши тушуниши ва бу соҳада мустақил масалаларни ҳал эта олишини кўрсатади.

Умуман бажарилган иш 5А130101 – математика таълим йўналиши бўйича магистрлик диссертацияларига қўйиладиган талабларга жавоб беради ва унинг муалифи Абдусаматова Ҳилолани математика магистри академик даражасини олишга лойиқ деб ҳисоблайман.

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10.06.2017 йил

Foydalanilgan adabiyotlar ro'yxati

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