O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

URGANCH DAVLAT UNIVERSITETI

“FIZIKA-MATEMATIKA” FAKULTETI

142-matematika guruhi talabasi Qurbanov Kamronning

5130100-Matematika ta’lim yo‘nalishi bo‘yicha

bakalavr darajasini olish uchun

BITIRUV MALAKAVIY ISHI

Mavzu: $\widehat{m}-$ **subgarmonik funksiyalar va ularning xossalari**

Urganch 2018 yil

O’ZBEKISTON RESPUBLIKASI

OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

URGANCH DAVLAT UNIVERSITETI

“FIZIKA-MATEMATIKA” FAKULTETI

“MATEMATIKA” KAFEDRASI

Mavzu: $\widehat{m}-$ **subgarmonik funksiyalar va ularning xossalari**

Bajaruvchi: **Qurbanov Kamron Sanjar o‘g‘li**

Rahbar: **Abdullayev Bahrom Ismoilovich**

Urganch shahri

2018-yil

URGANCH DAVLAT UNIVERSITETI FIZIKA –MATEMATIKA FAKULTETI
MATEMATIKA KAFEDRASI
BITIRUV MALAKAVIY ISHNI BAJARISH BO‘YICHA
TOPSHIRIQLAR REJASI:

1. Talaba Qurbanov Kamron Universitet rektorining 2017 yil 1-noyabrdagi 187-T §4 buyrug‘i bilan bitiruv malakaviy ish bajarish uchun “$\widehat{m}-$ **subgarmonik funksiyalar va ularning xossalari**” mavzusi tasdiqlangan.

2. “Matematika” kafedrasining 201__ yil “__” _____ dagi “___”-sonli majlisining qaroriga binoan f.-m.f.d. B.I.Abdullayev bitiruv malakaviy ishini bajarishda Qurbanov Kamronga rahbar qilib tayinlangan.

3. Bitiruv malakaviy ishining tarkibiy tuzilmasi: Ushbu bitiruv malakaviy ishi referativ xarakterda bo‘lib, $\widehat{m}-$subgarmonik funksiyalar va ularning xossalari o‘ganishga bag‘ishlangan. Bitiruv malakaviy ishi kirish, ikkita bobdan iborat bo‘lib, kirish qismida mavzuning dolzarbligi, tadqiqot maqsadi va vazifalari keltirilgan. Birinchi bobda subgarmonik va plyurisubgarmonik funksiyalar va ularning xossalari, differensial formalar o‘rganilgan. Ikkinchi bobda Gessian tushunchasi, musbat Gessianlar, $m-$subgarmonik funksiyalar va ularning xossalari o‘rganilgan. Bitiruv malakaviy ishning so‘ngida xulosa, hamda foydalanilgan adabiyotlar ro‘yxati keltirilgan.

4. Bitiruv malakaviy ish uchun ma’lumotlar quyidagi adabiyotlardan olinadi:

1. Брело М., Основы классической теории потенциала, М., ИЛ, 1964
2. Ландкоф Н.С. Основы современной теории потенциала. М., НАУКА, 1966.
3. Садуллаев А., Абдуллаев Б., Теория потенциалов в классе m-субгармонических функций. Труды математического института им. В.А.Стеклова. 2012. т.279.с.166-192.
4. Садуллаев А. Теория плюрипотенциала. Применения. Palmarium Academic Publishing, Германия 2012г.
5. Садуллаев А. Кўп аргументли голоморф функциялар. Урганч 2004й.
6. Уэрмер Дж., Теория потенциала, М.,МИР.1980
7. Хейман У., Кеннеди П., Субгармонические функции. М.,МИР.1980
8. Чирка Е.М., Комплексные аналитические множества, М.,НАУКА,1985
9. Шабат Б.В., Введение в комплексный анализ. ч.II . М., НАУКА,1985.

5. Bitiruv malakaviy ishga PowerPoint dasturida ishlangan multimediali taqdimot slaydlari ilova qilinadi.

Bitiruv malakaviy ishni bajarish jadvali

№	***Bajarilgan ishning mazmuni***	***Bajarish muddati***
1.	Bitiruv malakaviy ishni mavzusi bo‘yicha adabiyotlarni to‘plash, o‘rganish va tahlil qilish	
2.	Bitiruv malakaviy ishning birinchi bobini tayyorlash	
3.	Ikkinchi bob bo‘yicha ma’lumotlarni o‘rganish va tahlil qilish	
4.	Bitiruv malakaviy ishning ikkinchi bobini tayyorlash	
5.	Bitiruv malakaviy ishning foydalanilgan adabiyotlar ro‘yxatini tuzish	
6.	Bitiruv malakaviy ishning loyhasini tayyorlash va kafedra muxokamasidan o‘tkazish	

Bitiruv malakaviy ish rahbari: f.-m.f.d. B.I. Abdullayev

Bajaruvchi talaba: Qurbanov Kamron

201_ yil “_____” ______________

Topshiriqlar rejasi va jadvali kafedra majlisida 201_ yil tasdiqlandi
(“____” - sonli bayonnoma)

Kafedra mudiri: dots. Atamuratov A.A.

Urganch davlat universiteti

Fizika-matematika fakulteti “Matematika” kafedrasi

5130100-matematika ta’lim yo‘nalishi

Tasdiqlayman

Fizika-matematika fakulteti dekani

___________dots. J.U.Xujamov

“______”_____________ 2018 y.

BITIRUV MALAKAVIY ISH BO‘YICHA TOPSHIRIQ

Talaba Qurbanov Kamron Sanjar o‘g‘li

1. Ishning mavzusi: “$\widehat{m}-$ subgarmonik funksiyalar va ularning xossalari” 2017 yil 1-noyabrda universitet rektorining 187-T §4 sonli buyrug‘i bilan tasdiqlangan.

2. Ishni topshirish muddati: “___”__________2018 y.

3. Mavzu bo‘yicha dastlabki ma’lumotlar beruvchi adabiyotlar ro‘yxati

1. Садуллаев А., Абдуллаев Б., Теория потенциалов в классе m-субгармонических функций. Труды математического института им. В.А.Стеклова. 2012. т.279.с.166-192.
2. Садуллаев А. Теория плюрипотенциала. Применения. Palmarium Academic Publishing, Германия 2012г.
3. Садуллаев А. Кўп аргументли голоморф функциялар. Урганч 2004й.
4. Хейман У., Кеннеди П., Субгармонические функции. М.,МИР.1980
5. Чирка Е.М., Комплексные аналитические множества, М., НАУКА, 1985
6. Шабат Б.В., Введение в комплексный анализ. ч. II . М., НАУКА,1985.

4. Ishning maqsadi: $\widehat{m}-$ subgarmonik funksiyalar va ularning muhim xossalarini o‘rganib, tahlil qilish.

5. Chizma materiallar ro‘yxati: - - - - -

6. Maslahatchilar:

Bo‘limlar	Maslahatchi F.I.Sh.	Imzo, sana	
		Topshiriq berdi	Topshiriq qabul qildi
Subgarmonik funksiyalar va ularning xossalari	B.I. Abdullayev		
Plyurisubgarmonik funksiyalar va ularning xossalari	B.I. Abdullayev		
Musbat aniqlangan differensial forma va oqimlar	B.I. Abdullayev		
Musbat gessianalar bilan differensial formalar orasidagi bog‘lanish	B.I. Abdullayev		
$\widehat{m}-$ subgarmonik funksiyalar va ularning xossalari	B.I. Abdullayev		

Ishga taqrizchi **TATU Urganch filiali, “Tabiiy va umumkasbiy fanlar” kafedrasi mudiri f.-m.f.n. Q. Mamedov**

Ilmiy rahbar: **f.-m.f.d. B.I.Abdullayev**

BMI bajaruvchi talaba: **Qurbanov Kamron**

Kafedra mudiri: **f.-m.f.n. A.A.Atamuratov**

Urganch davlat universiteti Fizika-matematika fakulteti

“Matematika” kafedrasi

Bitiruv malakaviy ish ____________ sonli tartib raqam bilan qayd qilindi.

Bitiruv malakaviy ishni bajaruvchining ismi-sharifi: Qurbanov Kamron Sanjar oʻgʻli

Bitiruv malakaviy ishning mavzusi: $\widehat{m}-$ subgarmonik funksiyalar va ularning xossalari.

Ilmiy rahbar (maslahatchi)ning ismi-sharifi: *B.I.Abdullayev*

Bitiruv malakaviy ish kafedraning 2018 yil «___»________da oʻtkazilgan majlisi qaroriga muvofiq DAK majlisida himoya qilishga tavsiya qilindi.

Bitiruv malakaviy ishga taqrizchi qilib TATU Urganch filiali, “Tabiiy va umumkasbiy fanlar” kafedrasi mudiri f.-m.f.n. Q. Mamedov tayinlandi.

Kafedra mudiri: **dots.AA.Atamuratov**

Kafedraning bitiruv malakaviy ishni DAK majlisida himoya qilish boʻyicha tavsiyasiga roziman.

Fakultet dekani: **dots. J.U.Xujamov**

Mavzu: $\widehat{m}-$ subgarmonik funksiyalar va ularning xossalari

REJA

KIRISH

Gessianlardagi tenglamalar uchun Dirixle masalasi asosida $\widehat{m}$-subgarmonik funksiyalar sinfi tushunchasi kiritilib, potentsiallar nazariyasiga to‘liq mos kelishi hamda plyurisubgarmonik funksiyalar Monj-Amper tenglamasi uchun qanchalik muhim bo‘lsa, $\widehat{m}$-subgarmonik funksiyalar sinfi ham gessianlardagi tenglamalar uchun shunchalik muhim bo‘lishi aniqlangan. Shu sababdan $\widehat{m}$-subgarmonik funksiyalar sinfini potentsial-sig‘im xossalarini keng tadqiq etilishi dolzarb muammolardan hisoblanadi.

Shular bilan bir qatorda mavzuning dolzarbligi klassik va kompleks potentsiallar nazariyasini o‘z ichiga olgan va gessian operatoriga asoslangan potentsiallar nazariyasini to‘liq asoslangan, $\widehat{m}$-subgarmonik funksiyalar sinfida Dirixle masalasini yechish usullarini ishlab chiqish, $\widehat{m}$-subgarmonik funksiyalar supremumining $\widehat{m}$-subgarmonik bo‘lishi hamda $\widehat{m}$-subgarmonik funksiyalar kompleks gipertekisliklar ustidagi qisqartmasining ($\widehat{m}$-1)-subgarmonik funksiya bo‘lishi muammolarini hal etish bilan xarakterlanadi.

Bitiruv malakaviy ishda $\widehat{m}$-subgarmonik funksiyalar va ularning ba’zi muhim xossalari o‘rganilgan:

2.2-ta’rif: Agar $u(z)$ funksiya yuqoridan yarim uzluksiz va ixtiyoriy silliq $v_1,...,v_{m-1}$ $\widehat{m}-$subgarmonik funksiyalar uchun $dd^c u\wedge dd^c v_1\wedge ...\wedge dd^c v_{m-1}\wedge \beta^{n-m}$, oqim

$$\left[dd^c u\wedge dd^c v_1\wedge ...\wedge dd^c v_{m-1}\wedge \beta^{n-m}\right](\omega)=$$

$$=\int udd^c v_1\wedge ...\wedge dd^c v_{m-1}\wedge \beta^{n-m}\wedge dd^c\omega,\quad \omega\in F^{0,0},$$

musbat aniqlangan bo‘lsa, u holda $u\in L^1_{loc}(D)$ funksiya $D\subset\mathbb{C}^n$ sohada $\widehat{m}-$subgarmonik deyiladi.

2.2-Teorema. Agar $u \in \widehat{m} - sh$ bo‘lsa, u holda ixtiyoriy $\mathbb{P} \subset \mathbb{C}^n$ kompleks gipertekislik uchun $u|_{\mathbb{P}} \in (\widehat{m} - 1) - sh$ bo‘ladi.

2.3-Teorema. Agar $u(z) - B = \{|z| < 1\}$ sharda ikki marta silliq $\widehat{m} - sh$ funksiya bo‘lsa, u holda ixtiyoriy $r < 1$ uchun

$$\int_0^r dt \int_{|z|^2 \le t} \left(dd^c u\right)^k \wedge \beta^{n-k} \le \left(M - m\right) \int_{|z|^2 \le r} \left(dd^c u\right)^{k-1} \wedge \beta^{n-k+1},$$

bo‘ladi. Bu yerda $1 \le k \le m$, $M = \sup_B u\left(z\right)$, $m = \inf_B u(z)$.

I BOB.
DASTLABKI MA'LUMOTLAR VA ASOSIY TUSHUNCHALAR

1.1. Subgarmonik funksiyalar va ularning xossalari.

1.1.-Ta'rif. *Faraz qilaylik,* $D \subset \mathbb{R}^n$ *sohada aniqlangan lokal integrallanuvchi*

$$u: \quad D \to [-\infty, +\infty)$$

funksiya berilgan bo'lsin, $u \in L^1_{loc}(D)$.

Agar bu funksiya quyidagi ikki shartni bajarsa:

1) $u(x)$ *yuqoridan yarim uzluksiz:* $\forall x^o \in D$ *uchun* $\overline{\lim_{x \to x^o}} u(x) \le u(x^o)$

(bundan $u(x)$ *funksiyaning* D *ga tegishli ixtiyoriy kompaktda yuqoridan chegaralanganligi kelib chiqadi);*

2) har bir $x^o \in D$ *nuqta uchun shunday* $r(x^o) > 0$ *topiladiki,* $r \le r(x^o)$ *uchun*

$$u(x^o) \le \frac{1}{\sigma_n r^{n-1}} \int_{S(x^o, r)} u(x) d\sigma ; \qquad (1.1)$$

u holda $u(x)$ *funksiya* D *sohada* ***subgarmonik funksiya*** *deyiladi.*

Subgarmonik funksiyalar sinfi $Sh\ D$ kabi belgilanadi. Kelgusida qulaylik uchun biz trivial $u(x) \equiv -\infty$ funksiyasi ham D da $Sh(D)$ ga tegishli deb qaraymiz. *Endi subgarmonik funksiyalarning xossalarini keltiramiz:*

1) subgarmonik funksiyalarning musbat koeffitsientli chiziqli kombinatsiyasi subgarmonik funksiya bo'ladi:

$$u_k(x) \in Sh(D), \quad a_k \in R_+ \quad (k = 1, 2, \ldots, m) \quad \Rightarrow$$
$$\Rightarrow a_1 u_1(x) + a_2 u_2(x) + \ldots + a_m u_m(x) \in Sh(D) ;$$

2) chekli sondagi subgarmonik funksiyalarning maksimumi subgarmonik funksiya bo'ladi:

$$u_1(x), u_2(x), ..., u_m(x) \in Sh(D) \;\Rightarrow$$
$$\Rightarrow \;\; \max \;\; u_1(x), u_2(x), ..., u_m(x) \;\; \in Sh(D)\ ;$$

3) monoton kamayuvchi subgarmonik funksiyalar ketma – ketligining limiti subgarmonik funksiya bo‘ladi:

$$u_j(x) \in Sh(D), \qquad u_j(x) \geq u_{j+1}(x) \quad (j = 1, 2, ...) \Rightarrow$$
$$\Rightarrow \lim_{j \to \infty} u_j(x) \in Sh(D)\ ;$$

4) tekis yaqinlashuvchi subgarmonik funksiyalar ketma – ketligi subgarmonik funksiyaga yaqinlashadi:

$$u_j(x) \in Sh(D) \;\; (j = 1, 2, ...), \qquad u_j(x) \rightrightarrows u(x) \Rightarrow u(x) \in Sh(D)\ ;$$

5) *Maksimumlar printsipi.* Agar $u(x) \in Sh(D)$ funksiya biror $x^o \in D$ nuqtada o‘zining maksimumiga erishsa, ya’ni

$$u(x^o) = \sup_{x \in D} u(x) \tag{1.2}$$

bo‘lsa, u holda $u(x) \equiv const$ bo‘ladi.

Isbot. Ushbu

$$M = \quad x \in D: \; u(x) = u(x^o) \qquad\qquad M \neq \varnothing$$

to‘plamni qaraylik. $u(x)$ funksiyaning yuqoridan yarim uzluksiz bo‘lganligidan va (1.5) shartdan M to‘plamning D da yopiqligi kelib chiqadi.

Ikkinchi tomondan, ixtiyoriy $p \in M$ uchun (5) formulaga ko‘ra

$$u(x^o) = u(p) \leq \frac{1}{\sigma_n r^{n-1}} \int\limits_{S(p,r)} u(x) d\sigma \leq u(x^o)\ , \qquad r \leq r(p)\ ,$$

bo‘ladi. Bu munosabatdan $u\Big|_{S(p,r)} \equiv u(x^o)$, ya’ni p ning $B(p, r(p))$ atrofida $u(x) = u(x^o)$ bo‘lishini ko‘ramiz. Bu esa M to‘plamning D da ochiq ekanligini ko‘rsatadi. Demak, $M = D$.

6) Agar $\vartheta(x) \in Sh(D), \quad u(x) \in h(D)$ funksiyalar uchun $G \subset\subset D$ sferada

$$\vartheta\Big|_{\partial G} \le u\Big|_{\partial G}$$

bo‘lsa, u holda G da $\vartheta(x) \le u(x)$, $x \in G$ tengsizlik o‘rinli bo‘ladi.

Bu xossadan quyidagi muhim xulosaga kelamiz. Faraz qilaylik,

$$\vartheta(x) \in Sh(D) \bigcap C(D)$$

berilgan bo‘lib, $B(x^o, r) \subset\subset D$ bo‘lsin. Ushbu

$$u(x) = \int_{\partial B} \vartheta(y) P(x,y) d\sigma(y) \ , \qquad x \in B(x^o, r),$$

Puasson integralini qaraylik. $u(x) \in h(B)$, $u\Big|_{\partial B} = \vartheta\Big|_{\partial B}$ bo‘lib, 6) – hossaga ko‘ra $B = B(x^o, r)$ da $\vartheta(x) \le u(x)$ bo‘ladi. Uzluksiz funksiyalar uchun isbotlangan bu xossa ixtiyoriy $\vartheta \in Sh\ D$ uchun ham o‘rinlidir: $\vartheta(x) \le u(x)$ va deyarli barcha $x \in S$ lar uchun $\vartheta(x) = u(x)$ o‘rinlidir;

7) aytaylik, $u(x) \in Sh(D)$ funksiya va $x^o \in D$ nuqta berilgan bo‘lsin. Ushbu

$$m(x^o, r) = \frac{1}{\sigma_n r^{n-1}} \int\limits_{S(x^0, r)} u(x) d\sigma$$

va

$$n(x^o, r) = \frac{1}{V_n r^n} \int\limits_{B(x^o, r)} u(x) dV \,,$$

bunda $V_n r^n$ miqdor $B(x^o, r)$ ning hajmi, integrallarni (o‘rta qiymatlarni) qaraylik.

1.1–Teorema. Faraz qilaylik, $u(x) \in Sh(D)$ va $u(x) \not\equiv -\infty$ bo‘lsin. U holda ixtiyoriy $x^o \in D$ uchun

$$u(x^0) \le n_u(x^o, r) \le m_u(x^o, r) \ , \ n_u(x^o, r) > -\infty \ , \ 0 < r < \rho(x^o, \partial D) \ ,$$

tengsizliklar o‘rinli bo‘ladi. Bundan tashqari, agar r monoton kamayib nolga intilsa, unda $n_u(x^o, r)$ va $m_u(x^o, r)$ o‘rta qiymatlar monoton kamayib, $u(x^o)$ ga intiladi.

1.1 – teoremadan ushbu muhim natijaga ega bo‘lamiz: $D \subset \mathbb{R}^n$ sohada subgarmonik $u(x) \neq -\infty$ funksiya D da lokal integrallanuvchidir, ya’ni ixtiyoriy $B \subset\subset D$ uchun $\int_B u(x)dV$ integral mavjuddir;

8) subgarmonik funksiya D sohaning har bir nuqtasida uzilishga ega bo‘lishi mumkin. Ayni paytda, quyidagi teorema o‘rinli bo‘ladi.

1.2 –Teorema. Agar $u(x) \in Sh(D)$ bo‘lsa, u holda shunday monoton to‘plamlar

$$D_j \subset D_{j+1} \subset D \ , \quad \bigcup_{j=1}^{\infty} D_j = D$$

ketma – ketligi va shunday monoton kamayuvchi funksiyalar

$$u_j(x) \in Sh(D_j) \cap C^{\infty}(D_j) \ , \qquad j = 1,2,3,\ldots$$

ketma – ketligi mavjudki,

$$u(x) = \lim_{j \to \infty} u_j(x) \qquad (x \in D)$$

bo‘ladi.

Ushbu

$$K(x) = \begin{cases} ce^{-\frac{1}{1-|x|^2}}, & agar \quad |x| \leq 1 \quad bo`lsa, \\ 0, \quad agar \quad |x| > 1 \quad bo`lsa, \end{cases}$$

funksiyani qaraylik. Bu funksiya $C^{\infty}(\mathbb{R}^n)$ sinfga tegishli bo‘lib, uning salmog‘i $\mathrm{supp} K(x) = B(0,1)$ dir. Endi $K(x)$ ning ifodasidagi o‘zgarmas c ni shunday tanlab olamizki,

$$\int_{\mathbb{R}^n} K(x)dV = 1$$

bo‘lsin. $u(x) \not\equiv -\infty$ deb, bu yadro yordamida ushbu

$$u_\delta(x) = \frac{1}{\delta^n} \int\limits_{|y-x|\le\delta} u(y)K\left(\frac{y-x}{\delta}\right)dV \ , \quad \delta > 0 \quad , \qquad (1.3)$$

integralni qaraylik. $u_\delta(x)$ funksiyasi

$$D_\delta = \ x \in D : \ \rho(x, \partial D) < \delta$$

ochiq to'plamda aniqlangan bo'lib, $x \in D_\delta$ da

$$u_\delta(x) = \frac{1}{\delta^n} \int\limits_{\mathbb{R}^n} u(y)\, K\left(\frac{y-x}{\delta}\right) dV = \int\limits_{\mathbb{R}^n} u(x+\delta y)\, K(y) dV$$

bo'ladi. Keyingi tenglikdagi birinchi integral $C^\infty(D_\delta)$ sinfga, ikkinchi integral esa $Sh(D_\delta)$ sinfga tegishli bo'ladi. Binobarin,

$$u_\delta(x) \in Sh(D_\delta) \bigcap C^\infty(D_\delta) .$$

$u(x)$ ning subgarmonik funksiya ekanligidan, u_δ ning $\delta \downarrow 0$ da, monoton kamayuvchi bo'lishligi va

$$\int\limits_{\mathbb{R}^n} K(x) dV = 1$$

tenglikdan, $u_\delta(x)$ ning $u(x)$ ga intilishini topamiz.

9) Subgarmonik funksiyalar umumiy holda C^2 sinfga tegishli bo'lmasligidan, Δu - Laplas operatorini faqat umumlashgan funksiya sifatida qarash mumkin bo'ladi:

$$\Delta u(\varphi) = \int u \Delta \phi, \qquad\qquad \varphi \in F(D).$$

1.3–Teorema. Har qanday subgarmonik funksiya $u(x) \in Sh(D)$, $u \not\equiv -\infty$, uchun umumlashgan ma'noda $\Delta\, u \ge 0$ bo'ladi, ya'ni ixtiyoriy $\varphi \in F(D)$, $\varphi \ge 0$ uchun $\int u\, \Delta\, \varphi \ge 0$ munosabat o'rinlidir.

Jumladan, $u(x)$ funksiya C^2 sinfga tegishli bo‘lib, u subgarmonik bo‘lsa, $u \in Sh(D) \bigcap C^2(D)$, u holda $\Delta u = \frac{\partial^2 u}{\partial x_1^2} + ... + \frac{\partial^2 u}{\partial x_n^2} \geq 0$ dir.

Yuqorida keltirilgan teoremaning aksi ham o‘rinli: agar umumlashgan funksiya u uchun

$$\Delta u(\varphi) = \int u \Delta \varphi \geq 0 \ , \quad \varphi \in F(D) \ , \quad \varphi \geq 0 \ ,$$

munosabat bajarilsa, u holda u subgarmonik funksiya bo‘ladi;

10) *F.Riss teoremasi.* Har qanday $u(x) \in Sh(D)$, $u \not\equiv -\infty$, funksiya olinganda ham D sohada shunday musbat o‘lchov μ mavjudki, ixtiyoriy $D^0 \subset\subset D$ soha uchun

$$u(x) = \int_{D^0} K(x-y) d\mu_y + \Phi_{D^0}(x) \tag{1.4}$$

bo‘ladi, bunda $\Phi_{D^0}(x) \in h(D^0)$ va

$$K(x) = \begin{cases} \frac{1}{2\pi} \ln |x|, & agar \quad n = 2 \quad bo`lsa, \\ -\frac{1}{(n-2)\sigma_n} \cdot \frac{1}{|x|^{n-2}}, & agar \quad n > 2 \quad bo`lsa, \end{cases}$$

Eslatib o‘tamiz, $K(x)$ funksiyasi Laplas tenglamasining fundamental yechimidir: $\Delta K(x) = \delta(x)$ bo‘lib, bu yerda $\delta(x)$ - Dirakning umumlashgan funksiyasi, $\delta \circ \varphi(x) = \varphi(0)$, $\varphi \in F(\mathbb{R}^n)$. δ ga massasi $x = 0$ da bo‘lgan birlik o‘lchov (zaryad) mos keladi.

11) Agar $u(x) \in Sh(D)$ bo‘lsa, u holda $e^{u(x)} \in Sh(D)$ dir.

8) – xossaga ko‘ra, $u(x)$ funksiyasini Sh va C^∞ funksiyalar bilan yaqinlashtirish mumkin. Shundan kelib chiqib, qaralayotgan xossani

$u(x) \in Sh(D) \bigcap C^{\infty}(D)$ funksiyalar uchun isbotlash yetarlidir. Bu holda

$$De^{u(x)} = e^{u(x)} \left[\Delta u + \sum_{j=1}^{n} \left(\frac{\partial u}{\partial x_j} \right)^2 \right] \geq 0$$ ekanligini ko‘rish qiyin emas.

Xuddi shunday quyidagi xossa ham o‘rinlidir.

12). Agar $u(x) \in Sh(D)$, $u|_D < 0$ bo‘lsa, u holda $-\ln[-u(x)] \in Sh(D)$ dir.

1.2. Plyurisubgarmonik funksiyalar va ularning xossalari.

Plyurisubgarmonik funksiya tushunchasini keltirish uchun yuqoridan yarim uzluksiz funksiya tushunchasini kiritishimiz kerak. $\varphi : D \to [-\infty; \infty)$ funksiya $D \subset \mathbb{C}^n$ sohada yuqoridan yarim uzluksiz deyiladi, agar $\forall z^0 \in D$ uchun

$$\overline{\lim_{z \to z^0}} \varphi(z) \leq \varphi(z^0)$$

tengsizlik bajarilsa. Boshqacha qilib aytadigan bo‘lsak, $\forall \varepsilon > 0$ soni uchun shunday $\delta(z^0, \varepsilon) > 0$ soni topilib,

$$\left| z - z^0 \right| < \delta \Rightarrow \begin{cases} \varphi(z) - \varphi(z^0) < \varepsilon, & \varphi(z^0) \neq -\infty \\ \varphi(z) < -\dfrac{1}{\varepsilon}, & \varphi(z^0) = -\infty \end{cases}$$

munosabat o‘rinli bo‘lsa.

1.2-Ta’rif. $\varphi : D \to [-\infty; \infty)$ funksiya $D \subset \mathbb{C}^n$ sohada plyurisubgarmonik deyiladi, agar quyidagi shartlar bajarilsa:

1) $\varphi(z)$ funksiya D da yuqoridan yarim uzluksiz;

2) Ixtiyoriy $z^0 \in D$ va ixtiyoriy $z = l(\xi) = z^0 + w \cdot \xi, \quad w \in \mathbb{C}^n, \xi \in \mathbb{C}$ kompleks to‘g‘ri chiziqda φ subgarmonik, ya’ni $\varphi(l(\xi))$ funksiya $\xi \in \mathbb{C}, l(\xi) \in D$ ochiq to‘plamda subgarmonik.

Plyurisubgarmonik funksiyaga misol qilib golomorf funksiya modulining logorifmini misol qilib keltirishimiz mumkin, ya’ni agar $f \in \theta(D)$ bo‘lsa, u holda $u(z) = \ln\left|f(z)\right|$ funksiya D da plyurisubgarmonik bo‘ladi. Bizga ma’lumki,

$$\Delta u = \frac{\partial^2 u}{\partial \xi \partial \overline{\xi}} \geq 0$$

bo‘lishi kerak. Agar $D \subset \mathbb{C}^n \ (n > 1)$ bo‘lsa, murakkab funksiyani differensiallash qoidasiga ko‘ra har qanday $z = z^0 + w\xi$ uchun

$$\frac{\partial^2 u(z^0 + w \cdot \xi)}{\partial \xi \partial \overline{\xi}} = \sum_{i,j=1}^{n} \frac{\partial^2 u}{\partial z_i \partial \overline{z}_j} w_i \cdot \overline{w}_j \quad \text{bo‘ladi.}$$

Demak, $u \in \mathbb{C}^2(D)$ funksiya D da plyurisubgarmonik bo‘lishi uchun har qanday $z \in D$ da

$$H_z(u, w) = \sum_{i,j=1}^{n} \frac{\partial^2 u}{\partial z_i \partial \overline{z}_j} w_i \cdot \overline{w}_j \geq 0, \ \forall w \in \mathbb{C}^n.$$

bo‘lishi zarur va yetarli.

Endi plyurisubgarmonik funksiyalarning ba’zi xossalarini keltiramiz.

1^0. Agar $u(z)$ funksiya D sohada plyurisubgarmonik va biror $z_0 \in D$ nuqtada lokal eng katta qiymatiga erishsa, u holda u o‘zgarmasdir.

Faraz qilaylik $u(z)$ funksiyamiz $z_0 \in D$ nuqtada lokal eng katta qiymatiga erishadi. U holda $\varphi(\xi) = u(z_0 + w\xi)$ funksiya $\xi = 0$ nuqtaning biror atrofida subgarmonik va $\xi = 0$ da lokal eng katta qiymatiga erishadi. Subgarmonik funksiyaning maksimum prinsipiga asosan $\varphi = c\ w$ o‘zgarmasdir. $c\ w\ = u\ z_0 + w \cdot 0\ = \varphi\ 0$, ya’ni o‘zgarmas son $w \in \mathbb{C}^n$ ga bog‘lik emas. w ixtiyoriy bo‘lgani uchun $u\ z$ funksiya D sohada o‘zgarmasdir.

2^0. Agar $u\ z$ funksiya har bir $z_0 \in D$ nuqtaning biror atrofida subgarmonik bo‘lsa, u holda D da subgarmonikdir.

3^0. Agar u_α, $\alpha \in A -$ plyurisubgarmonik funksiyalar ketma-ketligi bo‘lib, $u\ z\ = \sup\limits_{\alpha \in A} u_\alpha\ z$ funksiya yuqoridan yarim uzluksiz bo‘lsa, u holda u plyurisubgarmonikdir.

Ixtiyoriy $z_0 \in D$ nuqtada u funksiya plyurisubgarmonik ekanligini ko‘rsatamiz. z_0 nuqtadan $\forall z = z_0 + w\xi,\ w \in \mathbb{C}^n -$ kompleks to‘g‘ri chiziqni o‘tkazamiz. u_α funksiya z_0 nuqtada plyurisubgarmonik bo‘lgani uchun $u_\alpha\ z_0 + w \cdot \xi\ = \varphi_\alpha\ \xi$ funksiya $\xi = 0$ nuqtaning $\xi \in \mathbb{C} : z_0 + w \cdot \xi \in D$ atrofida subgarmonik. Subgarmonik funksiyaning xossasiga ko‘ra $\varphi\ \xi\ = \sup\limits_{\alpha \in A} \varphi_\alpha\ \xi$ funksiya $\xi \in \mathbb{C} : z_0 + w \cdot \xi \in D$ sohada subgarmonik. Demak, $u\ z$ funksiya D sohada plyurisubgarmonik.

Quyidagi xossalar ham subgarmonik funksiyaning xossasidan kelib chiqadi.

4^0. $u\ z$ funksiya D sohada yuqoridan yarim uzluksiz bo‘lsin. $u\ z$ funksiya D sohada plyurisubgarmonik bo‘lishi uchun ixtiyoriy $z \in D$ va har qanday $w \in \mathbb{C}^n$ vektor olganda ham shunday $r_0 = r_0\ r, w$ son topilib, $\forall r < r_0$ larda

$$u\ z\ \leq \frac{1}{2\pi} \int_0^{2\pi} u\ z + w \cdot re^{it}\ dt$$

tengsizlikning bajarilishi zarur va yetarli.

Bu xossa plyurisubgarmonik kriteriyasi deb yuritiladi.

5^0. $u(z)$ funksiya $z_0 \in \mathbb{C}^n$ nuqtaning biror atrofida plyurisubgarmonik bo‘lsin. U holda yetarlicha kichik $r > 0$ larda

$$u(z_0) \le \frac{1}{\sigma(r)} \int\limits_{|z-z_0|=r} u(z)\, d\sigma$$

tengsizlik o‘rinlidir. Bunda $\sigma(r)$ sfera yuzi, $d\sigma$ yuza elementi.

6^0. $u(z)$ funksiyamiz $z_0 \in \mathbb{C}^n$ nuqtaning atrofida plyurisubgarmonik va $S(r) = \int\limits_{|z-z_0|=r} u(z)\, d\sigma$ bo‘lsin. U holda $S(r)$ o‘suvchi funksiyadir.

7^0. $u(z)$ funksiya $D \subset \mathbb{C}^n$ sohada plyurisubgarmonik bo‘lsin. U holda shunday ochiq $G_k \left(k = 1, 2, \ldots, \bigcup\limits_{k=1}^{\infty} G_k = D \right)$ to‘plamlar va monoton kamayuvchi $u_k \in P(G_k) \cap C^\infty(G_k)$ funksiyalar mavjudki, ixtiyoriy $z \in D$ larda $u_k(z) \to u(z)$ bo‘ladi.

8^0. $u(z)$ funksiya $D \setminus A$ ($A \in D -$ analitik to‘plam) to‘plamda plyurisubgarmonik va chegaralangan bo‘lsa, u holda u butun D ga plyurisubgarmonik davom qiladi.

1.3. Musbat aniqlangan differensial forma va oqimlar

1. Differensial formalar. $f \in C^1(G), G \subset \mathbb{C}^n$ funksiyaning differensiali

$$df = \sum_{j=1}^{n} \frac{\partial f}{\partial x_j} dx_j$$

1-darajali differensial formaga misol bo‘ladi. Umuman aytganda,

$$\omega=\sum_{j=1}^{n}a_j\left(x\right)dx_j$$

ko‘rinishdagi ifodaga 1-darajali differensial forma deyiladi va $\deg\omega=1$ kabi belgilanadi. Yuqori darajali differensial formalarni ta’riflash uchun biz quyidagi shartlarni qanoatlantiruvchi $\ll\wedge\gg$ —tashqi ko‘paytma belgisini kiritamiz:

a) $a\left(x\right)\wedge dx_j=dx_j\wedge a\left(x\right)=a\left(x\right)dx_j$

b) $dx_k\wedge dx_j=-dx_j\wedge dx_k,\Rightarrow dx_j\wedge dx_j=0$

1.3.1-ta’rif. Ushbu

$$\omega=\sum_{0\le j_1<j_2<...<j_p\le n}a_{j_1j_2...j_p}dx_{j_1}\wedge dx_{j_2}\wedge...\wedge dx_{j_p}$$

ko‘rinishdagi ifodaga p - darajali differensial forma (bunda $a_{j_1j_2...j_p}$-G sohada aniqlangan funksiya) deyiladi va $\deg\omega=p$ kabi belgilanadi. p - darajali differensial formalar to‘plami $F^p\left(G\right)$ orqali belgilanadi. $\omega\in F^p\left(G\right)$ differensial formani quyidagi qulay ko‘rinishda ham yozishimiz mumkin:

$$\omega=\sum_{0\le j_1<j_2<...<j_p\le n}a_{j_1j_2...j_p}dx_{j_1}\wedge dx_{j_2}\wedge...\wedge dx_{j_p}=\sum_{J}a_Jdx_J$$

bunda $dx_J=dx_{j_1}\wedge dx_{j_2}\wedge...\wedge dx_{j_p}$, $J=\left(j_1,j_2,...,j_p\right),1\le j_1,j_2,...,j_p\le n$ — multiindekslar. Ravshanki, 0 - darajali differensial forma bu - $a\left(x\right)$ skalyar funksiya.

$\mathbb{C}^n\simeq\mathbb{R}^{2n}$ kompleks fazoda differensial formani dz_j *va* $d\overline{z_j}$ lardan foydalanib yozish mumkin. $z=\left(z_1,z_2,...,z_n\right)$ koordinatalar $\mathbb{C}^n\simeq\mathbb{R}^{2n}$, $z_j=x_j+ix_{n+j}$, $j=1,2,...$ dagi kompleks koordinatalar bo‘lsin. Unda

$$dx_j=\frac{dz_j+d\overline{z_j}}{2},\quad dx_{n+j}=\frac{dz-d\overline{z_j}}{2i}$$

bo‘lganligidan $\omega=\sum_J a_J dx_J$ differensial formani dz_j va $d\overline{z_k}$ larning chiziqli kombinatsiyasi ko‘rinishida yozish mumkin. Har bir hadda dz_j va $d\overline{z_k}$ lar turlicha bo‘lishi mumkin. Masalan, $\mathbb{C}^2 \simeq \mathbb{R}^4$ da

$$dx_1 \wedge dx_2 \wedge dx_3 = \frac{i}{4}\left(-dz_1 \wedge dz_2 \wedge d\overline{z_1} + dz_1 \wedge d\overline{z_1} \wedge d\overline{z_2}\right)$$

Ammo ba’zi hollarda ω dagi barcha qo‘shiluvchilarda dz_j larning darajasi p ga, $d\overline{z_k}$ larniki esa q ga teng bo‘lib qolishi mumkin. Bunday hollarda biz ω differensial formani (p,q) bidarajali differensial forma deb aytamiz va $\omega \in F^{p,q}$ deb belgilaymiz.

2. Musbat aniqlangan differensial formalar.

1.3.2-ta’rif. Ushbu

$$\omega = \mathop{\wedge}_{j=1}^{p} \frac{i}{2} dl_j \wedge d\overline{l_j}$$

ko‘rinishdagi differensial forma (p,p) bidarajali bosh kuchli musbat differensial forma deyiladi, bu yerda $l_j = a_{j_1} dz_1 + \ldots + a_{j_n} dz_n$- chiziqli kombinatsiya $(dz_1,\ldots,dz_n)$ bazis bo‘yicha $a_{jk} \in \mathbb{C},\ j=1,2,\ldots,p, k=1,2,\ldots,n$. Bunday formalarning ixtiyoriy chiziqli kombinatsiyasi kuchli musbat differensial forma deb ataladi, ya’ni kuchli musbat differensial forma

$$\omega = \sum_s \lambda_s(z) \mathop{\wedge}_{j=1}^{p} \frac{i}{2} dl_{j_s} \wedge d\bar{l}_{j_k}$$

ko‘rinishdagi formadir, bu yerda $\lambda_s(z)$ - qaralayotgan $G \subset \mathbb{C}^n$ sohadagi manfiy bo‘lmagan funksiya. Agar $z_j = x_j + i x_{n+j}$ kompleks koordinatalardan x_j, x_{n+j}, $j=1,2,\ldots,n$ haqiqiy koordinatalarga o‘tsak, unda ushbu

$$\frac{i}{2} dz_1 \wedge d\overline{z_1} \wedge \ldots \wedge dz_p \wedge d\overline{z_p} = dx_1 \wedge dx_{n+1} \wedge \ldots \wedge dx_p \wedge dx_{n+p}$$

ifoda $\mathbb{R}^{2n}$ fazoda $2p$ darajali musbat forma bo‘ladi.

(n,n) bidarajali differensial forma (kuchli) musbat bo‘ladi, faqat va faqat shu holdaki, qachonki u

$$\omega = \lambda(z) \mathop{\wedge}_{j=1}^{n} \frac{i}{2} dz_j \wedge d\overline{z_j} = \lambda(z) dx_1 \wedge dx_{n+1} \wedge \dots \wedge dx_n \wedge dx_{2n}$$

ko‘rinishga ega bo‘lsa, bu yerda $\lambda(z)-$ musbat funksiya. (n,n) bidarajali $\Omega = \mathop{\wedge}_{j=1}^{n} \frac{i}{2} dz_j \wedge d\overline{z_j} = dV$ differensial forma $\mathbb{C}^n$ da hajm formasi rolini o‘ynaydi.

$p=1$ da $\omega = \sum \lambda_{jk} dz_j \wedge d\overline{z_k}$ – differensial forma musbat bo‘ladi, faqat va faqat shu holdaki, qachonki $\sum\limits_{jk} \lambda_{jk} \xi_j \xi_k$ – ermit kvadratik formasi musbat bo‘lsa. Shuningdek, ixtiyoriy $z^o \in G$ tayinlangan nuqtada unitar almashtirish yordamida uni

$$\omega = \frac{i}{2} \sum_{j=1}^{n} \mu_j dz_j \wedge d\overline{z_j}, \ \mu_j \ge 0$$

ko‘rinishda yozish mumkin.

1.3.3-ta’rif. $\omega \in F^{p,p}$ differensial forma kuchsiz musbat yoki musbat deyiladi, agar ixtiyoriy kuchli musbat $\nu \in F^{n-p,n-p}$ forma uchun $\omega \wedge \nu \in F^{n,n}$ forma musbat bo‘lsa. $p = 0,1,n-1,n$ da kuchli musbatlik kuchsiz musbatlikka ekvivalent. Biroq boshqa p lar uchun bu sinflar ustma-ust tushmaydi. Buni aniq tushunish uchun kuchli musbat formani quyidagi ko‘rinishda yozamiz:

$$\omega = \sum_s \lambda_s(z) \mathop{\wedge}_{j=1}^{p} \frac{i}{2} dl_{j_s} \wedge d\bar{l}_{j_s} = \frac{i^{p^2}}{2^p} \sum_s \lambda_s(z) \mathop{\wedge}_{j=1}^{p} \frac{i}{2} dl_{j_s} \mathop{\wedge}_{j=1}^{p} d\bar{l}_{j_s} = \frac{i^{p^2}}{2^p} \sum_s \lambda_s(z) dl_{j_s} \wedge d\bar{l}_{j_s} \qquad (*)$$

Agar $\theta = \sum_s \mu_s(z) dl_s$ formani qarasak , bunda $\mu_s -$ kompleks qiymatli funksiya, unda $\frac{i^{p^2}}{2^p}\theta \wedge \overline{\theta}$ ko‘rinishdagi forma (kuchsiz) musbat bo‘ladi, biroq $2 \le p \le n-2$ da u har doim ham $(*)$ ko‘rinishga ega bo‘lavermaydi.

3.Oqimlar. Qo‘pol qilib aytganda, M ko‘pxillikdagi oqim bu koeffitsiyentlari umumlashgan funksiyalardan iborat differensial formadir.

Tayinlangan $G \subset \mathbb{R}^n$ soha uchun asosiy differensial formalar fazosi qaraladi:

$$F^p = F^p(G) = \left\{\omega \in F^p(G) \cap C^\infty(G) : \operatorname{supp}\omega \subset\subset G\right\}$$

F^p da topologiya $F(G)$ fazodagi topologiyadan aniqlanadi.

1.3.4-ta’rif. F^p fazodagi uzluksiz chiziqli (kompleks qiymatli) T funksional $n-p=q$ darajali oqim deyiladi. Barcha $q-$ darajali oqimlar fazosini F^q orqali belgilaymiz. F^q oqimlar fazosi umumlashgan funksiyalarning barcha xossalarini qanoatlantiradi. Xususan, ular uchun kuchsiz yaqinlashish, kuchsiz chegaralanganlik tushunchalarini kiritish mumkin va $T_\delta \in F^q \cap C^\infty$ o‘ramani aniqlash mumkin, ya’ni $\delta \to 0$ da oqim sifatida $T_\delta \circ \omega \to T \circ \omega,\ \forall \omega \in F^p$ undan tashqari T oqim differensial formadek differensiallanadi: $dT \circ \omega = (-1)^p T \circ d\omega$.

1.3.5-ta’rif. Agar $F^p = \left\{\omega \in F^p(G) \cap C^\infty(G) : \operatorname{supp}\omega \subset\subset G\right\}$ fazodan olingan T oqim (funksional) $\left\{\omega \in F^p(G) \cap C^m(G) : \operatorname{supp}\omega \subset\subset G\right\}$fazoga uzluksiz davom qilsa, unda T oqim m dan ko‘p bo‘lmagan singulyarlikka ega deyiladi, bunda $0 \le m \le \infty$.

Agar undan tashqari Toqim $\left\{\omega \in F^p(G) \cap C^{m-1}(G) : \operatorname{supp}\omega \subset\subset G\right\}$fazoga davom qilmasa, unda T oqim aniq m singulyarlikka ega deyiladi.

$T = \sum_{|k|=q} \beta_k dx_k$ koeffitsiyentlari umumlashgan funksiya bo‘lgan differensial forma oqim deyiladi.

1.3.1-teorema. Ixtiyoriy $q-$ darajali oqim koeffitsiyentlari umumlashgan funksiya bo‘lgan differensial forma bo‘ladi, shu bilan birga singulyarligi nolga teng bo‘lgan oqim o ‘lchov tipidagi oqim bo‘ladi va uni barcha koeffitsiyentlari kompleks qiymatli o‘lchov bo‘ladi.

O‘lchov tipidagi oqimlar fazosini F_0^q orqali belgilaymiz. F_0^q nafaqat $G \subset \mathbb{R}^n$ soha uchun, balki ixtiyoriy $E \subset \mathbb{R}^n$ borel to‘plami uchun ham aniqlangan. Xususan, $K \subset \mathbb{R}^n$ kompakt uchun u Banax fazosi bo‘ladi, xuddi vektorlarni tekis normasi bilan differensial forma koeffitsiyentlaridan tashkil topgan $F_0^q(K) = F_0^q(K) \cap C(K)$ fazoga qo‘shma fazo kabi. $T \circ \omega = \mu(z)\Omega$ O‘lchov tipidagi $T \in F_0^q$ oqim uchun

$$T \circ \omega = \int_G d\mu$$

o‘rinli. Bu yerda $\mu-$ kompleks qiymatli o‘lchov bo ‘lib, u $\omega \in F_0^p$ ga bog`liq va

$\Omega = dx_1 \wedge ... \wedge dx_n$

Xuddi o‘lchovlar fazosi kabi quyidagi xossalar o ‘rinli:

a) $\{T_j\} \subset F_0^q$ oqimlar ketma-ketligi T ga kuchsiz yaqinlashadi, ya’ni $\lim_{j\to\infty} T_j \circ \omega = T \circ \omega,\ \forall \omega \in F_0^p$ faqat va faqat shu holdaki, qachon mos o‘lchovlar ketma-ketligini yaqinlashishi T_j ni koeffitsiyentlaridan bo‘lsa.

b) $\{T_j\} \subset F_0^q$ oqimlar ketma-ketligi T ga kuchsiz yaqinlashadi, faqat va faqat shu holdaki, qachon T_j $\|T_j\|_k \le c,\ \forall K \subset\subset G,\ j = 1, 2, ...$ norma bo‘yicha chegaralangan bo‘lsa va $j \to \infty$ da $T_j \circ \omega = T \circ \omega$ limit biror hamma yerda zich $E \subset F_0^p(K)$ to‘plamdan olingan barcha ω lar uchun o‘rinli bo‘lsa, masalan barcha cheksiz differensiallanuvchi $\omega \in F^p \cap C^\infty(K)$ lar uchun

d) ixtiyoriy kuchsiz chegaralangan $E \subset F_0^q(K)$ to‘plam osti kuchsiz kompakt ya’ni uni ixtiyoriy ketma-ketligi kuchsiz yaqinlashuvchi qismiy ketma-ketligini o‘zida saqlaydi.

1.3.6-ta’rif. $q-$ darajali differensial forma (oqim) yopiq deyiladi, agar uni differensiali nolga teng bo‘lsa; u aniq deyiladi, agar u biror $q-1$ darajali formani(oqimni)differensiali bo‘lsa. Aniq formalar(oqimlar) yopiq bo‘ladi, teskari tasdiq uchun G sohaga qo‘shimcha geometrik shartlarni qo‘yish zarur.

Endi $\mathbb{C}^n$ da asosiy differensial formalar fazosini qaraymiz:

$F^{p,p} = F^{p,p}(G)\left\{\omega \in F^{p,p}(G) \cap C^{\infty}(G) : \operatorname{supp}\omega \subset\subset G\right\}$

1.3.7-ta’rif. $F^{p,p}$ fazoda uzluksiz chiziqli(kompleks qiymatli) T funksionalga $(n-p, n-p) = (q,q)$ bidarajali oqim deyiladi.

1.3.2-teorema. Musbat oqimlar o‘lchov tipidagi oqimlar bo‘ladi.

1-misol. Koeffitsiyentlari L'_{loc} sinfdan olingan ixtiyoriy $T \in F^{q,q}$ differensial forma (q,q) bidarajali oqim bo‘ladi. $T \circ \omega$ funksional $T \circ \omega = \int\limits_G T \wedge \omega, \quad \forall \omega \in F^{p,p}$ kabi aniqlanadi. Bunday oqimlar regulyar oqim deyiladi.

2-misol. $A \subset \mathbb{C}^n - p$ o‘lchamli analitik to‘plam bo‘lsin. U holda

$$[A] \circ \omega = \int\limits_A \omega$$

integral (q,q) bidarajali oqimni aniqlaydi. Ko‘rish qiyin emaski, $[A]$ musbat oqim bo‘ladi.

II BOB.
$\widehat{m}-$SUBGARMONIK FUNKSIYALAR
VA ULARNING XOSSALARI

2.1. Musbat gessianlar bilan differensial formalar orasidagi bog‘lanish.

1. Gessianlar. Ikki marta uzluksiz differensiallanuvchi $u \in C^2(D)$ funksiya uchun uning ikkinchi tartibli differensiali

$$dd^c u = \frac{i}{2}\sum_{j,k} u_{j,\bar{k}} dz_j \wedge d\overline{z_k} \tag{2.1}$$

(tayinlangan $z^0 \in D$ nuqtada) Ermit kvadratik formasi bo‘ladi.

Bunda dd^c operator Monj-Amper operatori deyiladi. $d = \partial + \bar{\partial}$, $d^c = \frac{1}{4i}\left(\partial - \bar{\partial}\right)$

$$du = \partial u + \bar{\partial} u = \sum_{k=1}^{n} \frac{\partial u}{\partial z_k} dz_k + \sum_{k=1}^{n} \frac{\partial u}{\partial \overline{z_k}} d\overline{z_k}, \quad d^c u = \frac{1}{4i}\left(\sum_{k=1}^{n} \frac{\partial u}{\partial z_k} dz_k - \sum_{k=1}^{n} \frac{\partial u}{\partial \overline{z_k}} d\overline{z_k}\right).$$

Xususan, $n=1$ bo‘lgan hol uchun ko‘ramiz:

$$du = \frac{\partial u}{\partial z} dz + \frac{\partial u}{\partial \bar{z}} d\bar{z}, \qquad d^c u = \frac{1}{4i}\left(\frac{\partial u}{\partial z} dz - \frac{\partial u}{\partial \bar{z}} d\bar{z}\right)$$

$$dd^c u = \frac{1}{4i}\left(\partial\left(\frac{\partial u}{\partial z} dz - \frac{\partial u}{\partial \bar{z}} d\bar{z}\right) + \bar{\partial}\left(\frac{\partial u}{\partial z} dz - \frac{\partial u}{\partial \bar{z}} d\bar{z}\right)\right) =$$

$$= \frac{1}{4i}\left(\frac{\partial^2 u}{\partial z^2} dz \wedge dz - \frac{\partial^2 u}{\partial z \partial \bar{z}} d\bar{z} \wedge dz + \frac{\partial^2 u}{\partial \bar{z} \partial z} dz \wedge d\bar{z} - \frac{\partial^2 u}{\partial^2 \bar{z}} d\bar{z} \wedge d\bar{z}\right) = \frac{1}{2i}\frac{\partial^2 u}{\partial \bar{z} \partial z} dz \wedge d\bar{z}$$

Demak, $u \in C^2(D),\ D \subset \mathbb{C}$ uchun $dd^c u = \frac{1}{2i}\partial\bar{\partial} u$ bo‘ladi. $n=1$ da $dd^c u = \frac{i}{2}\cdot\frac{\partial^2 u}{\partial \bar{z} \partial z} dz \wedge d\bar{z}$ bo‘lar ekan.

Endi (2.1) formulani $n=3$ bo‘lgan hol uchun ko‘rib chiqamiz.

$$du = \partial u + \bar{\partial} u = \frac{\partial u}{\partial z_1} dz_1 + \frac{\partial u}{\partial z_2} \partial z_2 + \frac{\partial u}{\partial z_3} \partial z_3 + \frac{\partial u}{\partial \overline{z_1}} \partial \overline{z_1} + \frac{\partial u}{\partial \overline{z_2}} \partial \overline{z_2} + \frac{\partial u}{\partial \overline{z_3}} \partial \overline{z_3}$$

$$d^c u = \frac{1}{4i} \left(\frac{\partial u}{\partial z_1} dz_1 + \frac{\partial u}{\partial z_2} dz_2 + \frac{\partial u}{\partial z_3} dz_3 - \frac{\partial u}{\partial \bar{z}_1} d\bar{z}_1 - \frac{\partial u}{\partial z_2} d\overline{z_2} - \frac{\partial u}{\partial z_3} d\overline{z_3} \right)$$

Yuqorida $dz_k \wedge dz_k = 0$, $d\overline{z_k} \wedge d\overline{z_k} = 0$, $dz_k \wedge dz_j = -dz_j \wedge dz_k$ ligidan ba’zi yig‘indilar yo‘qoladi, chunki $u \in C^2(D)$ ekanligidan uning 2-tartibli aralash hosilalar uzluksiz va shuning uchun

$$\frac{\partial^2 u}{\partial z_j \partial z_k} = \frac{\partial^2 u}{\partial z_k \partial z_j}$$

tenglik o‘rinli. Ba’zi yig‘indilar esa bir-biriga teng bo‘lib qoladi.

$$\frac{1}{4i}\left(2\frac{\partial^2 u}{\partial \overline{z}_1 \partial z_1} d\, z_1 \wedge d\overline{z_1} + 2\frac{\partial^2 u}{\partial \overline{z}_2 \partial z_1} dz_1 \wedge d\overline{z_2} + 2\frac{\partial^2 u}{\partial \overline{z}_3 \partial z_1} dz_1 \wedge d\overline{z_3} + \right.$$

$$+2\frac{\partial^2 u}{\partial \overline{z}_2 \partial z_1} dz_1 \wedge d\overline{z_2} + 2\frac{\partial^2 u}{\partial \overline{z}_2 \partial z_3} dz_3 \wedge d\overline{z_2} + 2\frac{\partial^2 u}{\partial \overline{z}_3 \partial z_3} dz_3 \wedge d\overline{z_3} +$$

$$\left. +2\frac{\partial^2 u}{\partial z_2 \partial \overline{z}_1} d\overline{z_1} \wedge dz_2 + 2\frac{\partial^2 u}{\partial \overline{z}_3 \partial z_1} dz_2 \wedge d\overline{z_3} + 2\frac{\partial^2 u}{\partial \overline{z}_1 \partial z_3} dz_3 \wedge d\overline{z_1} \right) =$$

$$= \frac{1}{2i}\left(\frac{\partial^2 u}{\partial z_1 \partial \overline{z}_1} dz_1 \wedge d\bar{z}_1 + \frac{\partial^2 u}{\partial z_1 \partial \overline{z}_2} dz_1 \wedge d\bar{z}_2 + \frac{\partial^2 u}{\partial z_1 \partial \overline{z}_3} dz_1 \wedge d\bar{z}_3 + \frac{\partial^2 u}{\partial z_2 \partial \overline{z}_1} dz_2 \wedge d\bar{z}_1 + \right.$$

$$+ \frac{\partial^2 u}{\partial z_2 \partial \overline{z}_2} dz_2 \wedge d\bar{z}_2 + \frac{\partial^2 u}{\partial z_2 \partial \overline{z}_3} dz_2 \wedge d\bar{z}_3 + \frac{\partial^2 u}{\partial z_3 \partial \overline{z}_1} dz_3 \wedge d\bar{z}_1 + \frac{\partial^2 u}{\partial z_3 \partial \overline{z}_2} dz_3 \wedge d\bar{z}_2 +$$

$$+ \frac{\partial^2 u}{\partial z_3 \partial \overline{z}_3} dz_3 \wedge d\bar{z}_3 = \frac{1}{2i} \sum_{j,k=1}^{3} \frac{\partial^2 u}{\partial z_j \partial \overline{z}_k} dz_j \wedge d\bar{z}_k = \frac{1}{2i} \sum_{j,k=1}^{3} u_{j\bar{k}} dz_j \wedge d\bar{z}_k .$$

Demak, $dd^c u = \frac{i}{2} \partial \bar{\partial} u$ ekan.

Unitar almashtirishlardan keyin (2.1) quyidagi dioganal formaga keltiriladi:

$$dd^c u = \frac{i}{2}\left[\lambda_1 dz_1 \wedge d\overline{z_1} + \ldots + \lambda_n dz_n \wedge d\overline{z_n} \right],$$

Bu yerda $\lambda_1(u),...,\lambda_n(u)$ - $(u_{j,\bar{k}})$ Ermit matritsani xos qiymatlari, ya’ni $\lambda=(\lambda_1,...\lambda_n)\in\mathbb{R}^n$.

Haqiqiy qiymatlardan iborat xos vektorlar va xos qiymatga ta’rif beramiz: *agar noldan farqli* $x\in u$ *vektor uchun* $Ax=\lambda x$ *tenglik* λ *son uchun o‘rinli bo‘lsa, u holda* x *vektorga* A *chiziqli almashtirishni xos vektori deyiladi.*
Unga mos bo‘lgan λ *songa esa xos qiymat deyiladi.*

$$\left(dd^c u\right)^n=0$$

tenglamaga Manj-Amper tenglamasi deyiladi.

$$\left(dd^c u\right)^n=\underbrace{dd^c u\wedge dd^c u\wedge...\wedge dd^c u}_{n\ marta}$$

$n=1$ bo‘lganda $dd^c u=0 \;\Rightarrow\; \Delta u=0$.

Endi bu tasdiqni isbotini keltirib o‘tamiz

$$dd^c u=\frac{i}{2}\partial\bar{\partial}u.$$

Faraz qilaylik, $f(z)=u(x,y)+iv(x,y)$ funksiya $z_0=x_0+iy_0\in D,\ (D\in\mathbb{C})$ nuqtada $\mathbb{R}^2$ ma’noda differensiallanuvchi bo‘lsin.

Ushbu $du(x_0,y_0)+id(x_0,y_0)$ ifoda $f(z)$ funksiyaning z_0 nuqtadagi differensiali deyiladi va $df(z_0)$ kabi belgilanadi:

$$df(z_0)=du(x_0,y_0)+idv(x_0,y_0)$$

$$du=\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy,\quad dv=\frac{\partial v}{\partial x}dx+\frac{\partial v}{\partial y}dy$$

Shularni e’tiborga olib,

$$\begin{aligned} df&=\left(\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy\right)+i\left(\frac{\partial v}{\partial x}dx+\frac{\partial v}{\partial y}dy\right)=\left(\frac{\partial v}{\partial x}+i\frac{\partial v}{\partial y}\right)dx+\\ &+\left(\frac{\partial u}{\partial x}+i\frac{\partial u}{\partial y}\right)dy=\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy \end{aligned} \tag{2.2}$$

$$z = x + iy\,; \quad \bar{z} = x - iy\,; \quad dx = \frac{1}{2}\left(dz + d\bar{z}\right); \quad dy = \frac{1}{2i}\left(dz - d\bar{z}\right) \tag{2.3}$$

(2.2) va (2.3) dan

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial u}{\partial y}dy = \frac{\partial f}{\partial x}\cdot\frac{1}{2}\left(dz + d\bar{z}\right) + \frac{\partial f}{\partial y}\cdot\frac{1}{2i}\left(dz - d\bar{z}\right) =$$

$$= \frac{1}{2}\left(\frac{\partial f}{\partial x} - i\frac{\partial u}{\partial y}\right)dz + \frac{1}{2}\left(\frac{\partial f}{\partial x} + i\frac{\partial u}{\partial y}\right)d\bar{z}$$

$$\frac{\partial f}{\partial z} = \frac{1}{2}\left(\frac{\partial f}{\partial x} - i\frac{\partial f}{\partial y}\right) = \frac{1}{2}\left(\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial y}\right) + \frac{i}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \tag{2.4}$$

$$\frac{\partial f}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y}\right) = \frac{1}{2}\left(\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial y}\right) + \frac{i}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \tag{2.5}$$

$$df = \frac{\partial f}{\partial z}dz + \frac{\partial f}{\partial \bar{z}}d\bar{z}$$

bo‘ladi, bundan esa

$$dd^c u = \frac{i}{2}\partial\bar{\partial}u = \frac{i}{2}\cdot\frac{1}{4}\left(\frac{\partial^2 f}{\partial x^2} + i\frac{\partial^2 f}{\partial x\partial y} - i\frac{\partial^2 f}{\partial y\partial x} + \frac{\partial^2 f}{\partial y^2}\right) = 0 \quad \Rightarrow \quad \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0\,.$$

ekanligi kelib chiqadi.

Ko‘rish qiyin emaski,

$$\left(dd^c u\right)^m \wedge \beta^{n-m} = m!\left(n-m\right)!H_m\left(u\right)\beta^n, \tag{2.6}$$

Bu yerda $\beta = dd^c|z|^2$ hajm formasi, $H_m(u) = \sum\limits_{1\le j_1<\ldots<j_m\le n} \lambda_{j_1}\ldots\lambda_{j_m}$ esa $\lambda(u)\in\mathbb{R}^n$ vektorning gessiani deyiladi.

Shunday qilib, $u\in C^2\left(D\right)$ funksiya uchun

$$\left(dd^c u\right)^m \wedge \beta^{n-m}, \;\; 1\le m\le n$$

operator $\lambda = (\lambda_1,\ldots,\lambda_n)$ vektorning gessiani bilan bog‘liq ekan.

2. Musbat gessianlar.

Berilgan $\lambda = (\lambda_1,\ldots,\lambda_n)\in\mathbb{R}^n$ uchun

$$H_m(\lambda)=\sum_{1\le j_1<...<j_m\le n}\lambda_{j_1}...\lambda_{j_m} \tag{2.7}$$

o‘rinli.

Gessianni tushunish uchun quyidagi yordamchi ifodadan foydalanamiz:

$$(t+\lambda_1)(t+\lambda_2)...(t+\lambda_n)=t^n+H_1(\lambda)t^{n-1}+...+H_n(\lambda) \quad t\in\mathbb{R},$$

$n=2$ uchun

$$(t+\lambda_1)(t+\lambda_2)=t^2+(\lambda_1+\lambda_2)t+\lambda_1\lambda_2 \quad\Rightarrow\quad H_1(\lambda)=(\lambda_1+\lambda_2),\ H_2(\lambda)=\lambda_1\lambda_2$$

(2.7) formula bo‘yicha yozadigan bo‘lsak,

$$H_1(\lambda)=\sum_{1\le j_1\le 2}\lambda_{j_1}=\lambda_1+\lambda_2,\qquad H_2(\lambda)=\sum_{1\le j_1<j_2\le 2}\lambda_{j_1}\lambda_{j_2}=\lambda_1\lambda_2$$

$n=3$ uchun

$$(t+\lambda_1)(t+\lambda_2)(t+\lambda_3)=t^3+(\lambda_1+\lambda_2+\lambda_3)t^2+(\lambda_1\lambda_2+\lambda_3\lambda_2+\lambda_1\lambda_3)t+\lambda_1\lambda_2\lambda_3$$

(2.7) formula bo‘yicha yozadigan bo‘lsak,

$$H_1(\lambda)=\sum_{1\le j_1\le 3}\lambda_{j_1}=\lambda_1+\lambda_2+\lambda_3,$$

$$H_2(\lambda)=\sum_{1\le j_1<j_2\le 3}\lambda_{j_1}\lambda_{j_2}=\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3$$

$$H_3(\lambda)=\sum_{1\le j_1<j_2<j_3\le 3}\lambda_{j_1}\lambda_{j_2}\lambda_{j_3}$$

Endi biz $H_m(\lambda)\ge 0$ bo‘ladigan $\lambda\in\mathbb{R}^n$ vektorlar bilan qiziqamiz. Biroq, barcha bunday vektorlarni birlashmasi

$$S_m=\{\lambda\in\mathbb{R}^n : H_m(\lambda)\ge 0\},$$

$m>1$ bo‘lganda bu to‘plam qavariq konusni hosil qilmaydi, ya’ni $\lambda',\lambda''\in S_m$ ekanligidan $\lambda'+\lambda''\in S_m$ ekanligi kelib chiqmaydi. Shuning uchun bu to‘plamni toraytiramiz, ya’ni $\lambda\in S_m$ vektorlar uchun shunday $\Gamma_m\subset S_m$, to‘plamni olamizki,

$$\{\lambda+tI=(\lambda_1+t_1,...,\lambda_n+t_n), t\in\mathbb{R}^n_+\}\subset S_m$$

bo‘ladi. Unda Γ_m ning ko‘rinishi

$$\Gamma_m = \left\{ H_1(\lambda) \ge 0, ..., H_m(\lambda) \ge 0 \right\}$$

bo‘ladi. Undan tashqari Γ_m to‘plam $(1,1,...,1)$ vektorni o‘z ichiga oluvchi S_m to‘plamni $S_m^0 = \left\{ \lambda \in \mathbb{R}^n : H_m(\lambda) > 0 \right\}$ ochiq yadrosining bog‘lamli komponentasini yopig‘i bilan ustma-ust tushadi.

Γ_m **quyidagi xossalarga ega:**

1) $\Gamma_n = \left\{ \lambda_1 \ge 0, ..., \lambda_n \ge 0 \right\} = \mathbb{R}_+^n$, $\Gamma_n \subset \Gamma_{n-1} \subset ... \subset \Gamma_1$, Γ_m uchi 0 nuqtada bo‘lgan yopiq konus bo‘lsa, u holda Γ_1, $\mathbb{R}_+^n \setminus \{0\}$ ga tegishli bo‘ladi, lekin $m > 1$ da koordinata yarim o‘qida $\left\{ \lambda_j \ge 0, \lambda_k = 0, k \ne j \right\} \subset \partial \Gamma_m$ bo‘ladi.

2) Γ_m qavariq konusdan iborat, ya’ni agar $\lambda, \mu \in \Gamma_m$ bo‘lsa, u holda $\forall\, a, b \in \mathbb{R}^+$ uchun $a\lambda + b\mu \in \Gamma_m$ o‘rinli bo‘ladi.

3) $\left\{ H_m^{1/m}(\lambda), \lambda \in \Gamma_m \right\}$ o‘suvchi va botiq konusdan iborat, ya’ni

$$H_m^{1/m}(\mu) \le H_m^{1/m}(\lambda) + \sum_j \left(\mu_j - \lambda_j \right) \frac{\partial}{\partial \lambda_j} H_m^{1/m}(\lambda), \forall \lambda, \mu \in \Gamma_m;$$

4) $\displaystyle\sum_j \mu_j \frac{\partial}{\partial \lambda_j} H_m(\lambda) \ge m H_m^{1/m}(\mu) H_m^{1-1/m}(\lambda), \forall \lambda, \mu \in \Gamma_m;$

5) $\left[\dfrac{H_m}{\binom{n}{m}} \right]^{1/m} \le \left[\dfrac{H_m}{\binom{n}{k}} \right]^{1/k}$, $\forall\, 1 \le k \le m \le n.$ Bu tengsizlik Makloren tengsizligi deyiladi.

3. Musbat gessianlarning differensial formasi.

Endi bizga $\alpha = \dfrac{i}{2} \sum\limits_{j,k} a_{j\bar{k}} dz_j \wedge d\overline{z_k}$ - (1,1) bidarajali haqiqiy differensial forma berilgan bo‘lsin. Bunda $a_{j\bar{k}}$ - ermit matritsa. Agar $\lambda(\alpha) = \left(\lambda_1(\alpha), ..., \lambda_n(\alpha) \right)$ bu matritsaning xos qiymati bo‘lsa, u holda $H_m(\alpha) = H_m\left(\lambda(\alpha) \right)$ gessian aniqlangan. Bu esa (1,1) differensial forma va $\lambda \in \mathbb{R}^n$ vektor orasidagi bog‘liqlikni aniqlashga yordam beradi

$$\Gamma_m = \left\{ \alpha : \lambda(\alpha) \in \Gamma_m \right\} =$$

$$= \left\{ H_m \left(\alpha + t_1 dd^c |z_1|^2 + \ldots + t_n dd^c |z_n|^2 \right) \geq 0, \quad \forall t = (t_1, \ldots, t_n) \in \mathbb{R}_n^+ \right\} =$$

$$= \left\{ \alpha \wedge \beta^{n-1} \geq 0, \ldots, \alpha^m \wedge \beta^{n-m} \geq 0 \right\}.$$

2.1-Teorema. (q. [Bl]) Agar $\alpha_1, \ldots, \alpha_k \in \Gamma_m$, $1 \leq k \leq m$ bo‘lsa, u holda

$$\alpha_1 \wedge \ldots \wedge \alpha_k \wedge \beta^{n-m} \geq 0$$

o‘rinli.

Xususan, $k = m$ bo‘lganda

$$\alpha_1 \wedge \ldots \wedge \alpha_m \wedge \beta^{n-m} \geq 0 \qquad (2.8)$$

tengsizlikga ega bo‘lamiz.

2.2. $\widehat{m}$ - subgarmonik funksiyalar va ularning xossalari.

2.1-Ta’rif. *Ikki marta silliq* $u \in C^2(D)$ *funksiya (bu yerda* $D \subset \mathbb{C}^n$ *)* $z^0 \in D$ *nuqtada* $\widehat{m}-$*subgarmonik* $(1 \leq m \leq n)$ *deyiladi, agar* $\left(u_{j\bar{k}}\right)|_{z=z^0}$ *matritsani* $\lambda(u) = (\lambda_1(u), \ldots, \lambda_n(u))$ *xos qiymatlari* Γ_m *ga tegishli bo‘lsa yoki* $dd^c u \in \Gamma_m$ *bo‘lsa.*

Boshqacha aytganda $u \in \mathbb{C}^2(D)$ funksiya $\widehat{m}-$subgarmonik $(1 \leq m \leq n)$ deyiladi, agar $dd^c u \in \Gamma_m$ bo‘lsa yoki unga quyidagi tengsizlik teng kuchli:

$$\left(dd^c u\right)^k \wedge \beta^{n-k} \geq 0 \quad \forall\, k = 1, 2, \ldots, m. \qquad (2.9)$$

(2.9) ga quyidagi tengsizlik ham ekvivalent:

$$\left[dd^c \left(u + t_1 dd^c |z_1|^2 + \ldots + t_n dd^c |z_n|^2 \right) \right]^m \wedge \beta^{n-m} \geq 0 \quad \forall t = (t_1, \ldots, t_n) \in \mathbb{R}_+^n.$$

Bundan tashqari quyidagi tasdiq ham o‘rinli:

$$dd^c u\wedge\alpha_1\wedge\ldots\wedge\alpha_{k-1}\wedge\beta^{n-m}\geq 0\quad \forall\alpha_1,\ldots,\alpha_{k-1}\in\Gamma_m,\ 1\leq k\leq m. \tag{2.10}$$

Bu tasdiq ikki xil harakterga ega: agar u ikki marta silliq bo‘lib, $\alpha_1,\ldots,\alpha_{m-1}\in\Gamma_m$ uchun (2.10) shartni qanoatlantirsa, u holda u $\widehat{m}-$subgarmonik bo‘ladi. u funksiyani $\widehat{m}-$subgarmonikligini tekshirish uchun (2.10) da faqat $\alpha_j=dd^c u_j,\ u_j\in C^2(D),\ j=1,2,\ldots,m-1$ differensial formani chegaralangan bo‘lishi yoki $\widehat{m}-$subgarmonik ko‘phad bo‘lishi yetarli (q.[8]).

Bu esa bizga L^1_{loc} funksiyalar sinfida $\widehat{m}-$subgarmonik funksiyalarni aniqlashga yordam beradi.

2.2-Ta’rif. $u\in L^1_{loc}(D)$ funksiya $D\subset\mathbb{C}^n$ sohada $\widehat{m}-sh$ deyiladi, agar u yuqoridan yarim uzluksiz va ixtiyoriy silliq $v_1,\ldots,v_{m-1}$ $\widehat{m}-sh$ funksiyalar uchun $dd^c u\wedge dd^c v_1\wedge\ldots\wedge dd^c v_{m-1}\wedge\beta^{n-m}$, oqim

$$\left[dd^c u\wedge dd^c v_1\wedge\ldots\wedge dd^c v_{m-1}\wedge\beta^{n-m}\right](\omega)=$$

$$=\int u dd^c v_1\wedge\ldots\wedge dd^c v_{m-1}\wedge\beta^{n-m}\wedge dd^c\omega,\qquad \omega\in F^{0,0}, \tag{2.11}$$

musbat aniqlangan bo‘lsa.

Biz bilamizki, (q. [15], [20]), (p,p) bidarajali finit differensial formalar fazosidagi chiziqli, uzluksiz T funksional (kompleks qiymatli)

$$F^{p,p}=F^{p,p}(D)=\left\{\omega\in F^{p,p}(D)\cap C^\infty(D):\operatorname{supp}\omega\subset\subset D\right\}$$

$(n-p,n-p)=(q,q)$ bidarajali oqim deyilar edi.

$\widehat{m}-sh$ funksiyalarning xossalari:

1) $u\in C^2(D)$ funksiya, $\widehat{m}-sh$ bo‘ladi, faqat va faqat shu holdaki, $dd^c u\in\Gamma,\ \forall z^0\in D$ bo‘lsa.

2) agar $u\in\widehat{m}-sh(D)$ bo‘lsa, u holda $u_j(z)=u*K_j(z-w)$ standart approksimatsiya (o‘rama) ham $\widehat{m}-sh$ bo‘ladi, ($j\to\infty$ da $u_j\downarrow u$ bo‘ladi)

3) agar $u,v\in\widehat{m}-sh$ bo‘lsa, u holda ixtiyoriy $a,b\geq 0$ uchun $au+bv\in\widehat{m}-sh$ kelib chiqadi.

4) $psh=n-sh\subset...\subset 1-sh=sh$

5) agar $\gamma(t)-t\in\mathbb{R}$ parametrning o‘suvchi botiq funksiyasi va $u\in\widehat{m}-sh$ bo‘lsa, u holda $\gamma\circ u\in\widehat{m}-sh$ bo‘ladi.

6) tekis yaqinlashuvchi yoki kamayuvchi $\widehat{m}-sh$ funksiyalarning limiti yana $\widehat{m}-sh$ bo‘ladi.

2.1-Teorema. Chekli sondagi $\widehat{m}-sh$ funksiyalarning maksimumi $\widehat{m}-sh$ funksiyadan iborat bo‘ladi; ixtiyoriy lokal tekis chegaralangan $\{u_\theta\}\subset\widehat{m}-sh$, $\widehat{m}-$ subgarmonik funksiyalar oilasining supremumi $u(z)=\left\{\sup\limits_{\theta}u_\theta(z)\right\}$ regulyarizatsiyasi $u*(z)$ ham $\widehat{m}-sh$ bo‘ladi. $\widehat{m}-sh\subset sh$ ekanligidan $\{u(z)<u*(z)\}-$ to‘plam $\mathbb{C}^n\approx\mathbb{R}^{2n}$ da polyar to‘plam bo‘ladi. Qisqacha aytganda uning Lebeg o‘lchovi nolga teng.

Ixtiyoriy lokal tekis chegaralangan $\{u_j\}\subset\widehat{m}-sh$ funksiyalar ketma-ketligining limiti $u(z)=\overline{\lim\limits_{j\to\infty}}u_j(z)$ regulyarizatsiyasi $u*(z)$ ham $\widehat{m}-sh$ va $\{u(z)<u*(z)\}-$ polyar to‘plam bo‘ladi.

2.2-Teorema. Agar $u\in\widehat{m}-sh$ bo‘lsa, u holda ixtiyoriy $\mathbb{P}\subset\mathbb{C}^n$ kompleks gipertekislik uchun $u|_{\mathbb{P}}\in(\widehat{m}-1)-sh$ bo‘ladi.

Isbot. Gessianning 4-xossasidan foydalanib, $\lambda,\mu\in\Gamma_m$ vektor uchun

$$\sum_j\mu_j\frac{\partial}{\partial\lambda_j}H_m(\lambda)\geq mH_m^{1/m}(\mu)H_m^{1-1/m}(\lambda).$$

$\mu = (0,...,0,1)$ deb oladigan bo‘lsak, $\frac{\partial}{\partial \lambda_n} H_m(\lambda) = H_{m-1}('\lambda) \geq 0.$ Bu shuni anglatadiki, agar $\lambda = (\lambda_1,...,\lambda_{n-1},\lambda_n) \in \Gamma_m$ bo‘lsa, u holda $'\lambda = (\lambda_1,...,\lambda_{n-1}) \in \Gamma_{m-1}$ bo‘ladi. Endi $u \in sh_m(D)$ va $P \subset \mathbb{C}^n$ gipertekislik bo‘lsin. Umumiylikka zid qilmagan holda biz $u \in C^2(D)$, $P = \{z_n = 0\}$ deb faraz qilamiz va $u|_P = u('z,0) \in (\widehat{m}-1)-sh$ ekanini isbotlaymiz. $\lambda = (\lambda_1,...,\lambda_n) - U = (u_{j\bar{k}})$, $j,k = 1,2,...,n$ matrisani xos qiymatlari bo‘lsin. $\mu = (\mu_1,...,\mu_{n-1})$ esa $'U = (u_{j\bar{k}})$, $j,k = 1,2,...,n-1$ matrisani xos qiymatlari bo‘lsin. Unitar almashtirishlardan keyin (tayinlangan $('z^0,0) \in D$ nuqtada) U quyidagi ko‘rinishga keladi:

$$U = \begin{pmatrix} \mu_1 & 0 & ... & 0 & u_{1\bar{n}} \\ ... & ... & ... & ... & ... \\ 0 & 0 & ... & \mu_{n-1} & u_{n-1\bar{n}} \\ u_{n\bar{1}} & u_{n\bar{2}} & ... & u_{n\overline{n-1}} & u_{n\bar{n}} \end{pmatrix}$$

$u(z) \in \widehat{m}-sh$ ekan, u holda (ixtiyoriy tayinlangan $t \in \mathbb{R}_+$ parametr uchun) $u_t(z) = u(z) + t|z_n|^2 \in \widehat{m}-sh$ bo‘ladi.

Natijada, $\lambda(t) = \lambda(u_t) \in \Gamma_m$ vektor yuqorida isbotlanganiga ko‘ra $'\lambda(t) = (\lambda_1(t),...,\lambda_{n-1}(t)) \in \Gamma_{m-1}$. Boshqa tomondan $\lambda(t)$ quyidagi

$$\begin{vmatrix} \lambda-\mu_1 & 0 & ... & 0 & u_{1\bar{n}} \\ ... & ... & ... & ... & ... \\ 0 & 0 & ... & \mu_{n-1} & u_{n-1\bar{n}} \\ u_{n\bar{1}} & u_{n\bar{2}} & ... & u_{n\overline{n-1}} & u_{n\bar{n}} \end{vmatrix} = 0$$

tenglamaning ildizi. Yana almashtirishlar bajarsak,

$$\begin{vmatrix} \lambda-\mu_1 & 0 & \dots & 0 & \dfrac{u_{1\bar{n}}}{t} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \lambda-\mu_{n-1} & \dfrac{u_{n-1\bar{n}}}{t} \\ u_{n\bar{1}} & u_{n\bar{2}} & \dots & u_{n\overline{n-1}} & \dfrac{\lambda-u_{n\bar{n}}}{t}-1 \end{vmatrix}=0$$

$t\to\infty$ da $\lambda_j(t)\to\mu_j,\ j=1,2,...,n-1,\ '\lambda(t)\in\Gamma_{m-1}$ ekan, u holda Γ_{m-1} ning yopiqligidan $\mu\in\Gamma_{m-1}$, ya'ni $u|_P=u('z,0)\in(m-1)-sh$ ekanligi kelib chiqadi. Teorema isbot bo'ldi.

Quyidagi integral formula $\widehat{m}-sh$ funksiyalar uchun $\left(dd^c u\right)^k\wedge\beta^{n-m}$, $k=1,2,...,m$ operatorni baholashda muhim ahamiyat kasb etadi:

2.3-Teorema. Agar $u(z)-B=\{|z|<1\}$ sharda ikki marta silliq $\widehat{m}-sh$ funksiya bo'lsa, u holda ixtiyoriy $r<1$ uchun

$$\int_0^r dt\int_{|z|^2\le t}\left(dd^c u\right)^k\wedge\beta^{n-k}\le(M-m)\int_{|z|^2\le r}\left(dd^c u\right)^{k-1}\wedge\beta^{n-k+1}, \tag{2.12}$$

bo'ladi. Bu yerda $1\le k\le m,\quad M=\sup\limits_B u(z),\quad m=\inf\limits_B u(z)$.

Isbot. Bu integral formulani ixtiyoriy $1\le k\le n$ uchun k marta tadbiq qilsak,

$$\int_0^1 dt_1\int_0^{t_1} dt_2 ...\int_0^{t_{k-1}} dt_k\int_{|z|^2\le t_k}\left(dd^c u\right)^k\wedge\beta^{n-k}\le(M-m)^k\int_B\beta^n=C\cdot(M-m)^k, \tag{2.13}$$

ko'rinishda bo'ladi. Bu yerda $C=Vol(B)$.

Tayinlangan $0<r<1$ uchun (2.13) ning chap qismini quyidan baholashimiz mumkin:

$$\int_0^1 dt_1\int_0^{t_1} dt_2 ...\int_0^{t_{k-1}} dt_k\int_{|z|^2\le t_k}\left(dd^c u\right)^k\wedge\beta^{n-k}\ge\int_r^1 dt_1\int_r^{t_1} dt_2 ...\int_r^{t_{k-1}} dt_k\int_{|z|^2\le r}\left(dd^c u\right)^k\wedge\beta^{n-k}=$$

$$=\frac{(1-r)^k}{k!}\int\limits_{|z|^2\le r}\left(dd^c u\right)^k\wedge\beta^{n-k}.$$

Natijada, ixtiyoriy $r<0$ uchun

$$\int\limits_{|z|^2\le r}\left(dd^c u\right)^k\wedge\beta^{n-k}\le\frac{C(M-m)^k k!}{(1-r)^k},\ \ 1\le k\le m. \tag{2.14}$$

(2.14) baholash bizga tekis chegaralangan $\widehat{m}-sh$ funksiyalar sinfida lokal tekis chegaralangan $\left(dd^c u\right)^k\wedge\beta^{n-m}$ oqimni beradi. Teorema isbot bo‘ldi.

XULOSA

Ushbu bitiruv malakaviy ishi $\widehat{m}-$ subgarmonik funksiyalar va uning xossalarini o‘rganishga bag‘ishlangan bo‘lib, musbat gessianlar bilan differensial formalar orasidagi bog’lanish va $\widehat{m}$-subgarmonik funksiyalar supremumining $\widehat{m}$-subgarmonik bo‘lishi hamda $\widehat{m}$-subgarmonik funksiyalar kompleks gipertekisliklar ustidagi qisqartmasining ($\widehat{m}$-1)-subgarmonik funksiya bo‘lishi o‘rganildi.

FOYDALANILGAN ADABIYOTLAR

1. Брело М., Основы классической теории потенциала, М., ИЛ, 1964.
2. Садуллаев А., Абдуллаев Б., Теория потенциалов в классе m-субгармонических функций. Труды математического института им. В.А.Стеклова. 2012. т.279.с.166-192.
3. Садуллаев А. Теория плюрипотенциала. Применения. Palmarium Academic Publishing, Германия 2012 г.
4. Садуллаев А. Кўп аргументли голоморф функциялар. Урганч 2004 й.
5. Ландкоф Н.С. Основы современной теории потенциала. М., НАУКА, 1966.
6. Blocki Z., Weak solutions to the complex Hessian equation. Ann.Inst.Fourier (Grenoble), V.55:5, (2005), 1735-1756.
7. Chern S.S., Levine H., Nirenberg L., Intrinsic norms on a complex manifold. Global analysis paper in honor of Kodaira, University of Tokyo Press, (1969), 119-139.
8. Dinev S., Kolodziej S., A priori estimates for the complex Hessian equations. arxive math.,1112.3063V1, 1-18.
9. Уэрмер Дж., Теория потенциала, М.,МИР.1980
10. Харви Р., Голоморфные цепи и их границы. М., МИР, 1979.
11. Хейман У., Кеннеди П., Субгармонические функции. М.,МИР.1980
12. Чирка Е.М., Комплексные аналитические множества, М.,НАУКА,1985
13. Шабат Б.В., Распределение значений голоморфных отображений. М., АУКА,1982.
14. Шабат Б.В., Введение в комплексный анализ. ч.II . М., НАУКА,1985.

MUNDARIJA