O`ZBEKISTON RESPUBLIKASI

OLIY VA O`RTA MAXSUS TA`LIM VAZIRLIGI

FARG`ONA POLITEXNIKA INSTITUTI

“OLIY MATEMATIKA” KAFEDRASI

Oliy matematikaning

Matematik fizikaning asosiy tenglamalari

qismi bo`yicha

USLUBIY QO`LLANMA

Farg`ona – 2013 y.

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So`z boshi

Mazkur uslubiy qo`llanma mustaqil ta’limni bajarish uchun talabalarga mo`ljallangan bo`lib, unda har bir topshiriq uchun kerakli nazariy ma’lumotlar va formulalar keltirilgan. Mustaqil ta’lim bo`yicha bir nechta na’munaviy topshiriqlar yechib ko`rsatilgan, talabalar mustaqil bajarishi uchun topshiriq variantlari berilgan va shu bilan birga talabalar o`z bilimlarini sinashi nazorat ishlari variantlari keltirilgan. Mazkur uslubiy qo`llanma sodda va ravon tilda yozilgan bo`lib, talaba o`ziga berilgan topshiriqlarni mustaqil yecha oladi.

Mazkur uslubiy qo`llanma “Oliy matematika” kafedrasida ko`rib chiqilgan.

Bayonnoma №____________________2013 y.

Ishlab chiqarishda boshqaruv fakulteti uslubiy kengashi yig`ilishida ma`qullangan..

Bayonnoma №____________________2013 y.

Tuzuvchilar: katta o’qituvchi N.Mahmudova

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Kirish

Bozor iqtisodiyoti ilm-fan va texnikaning amalda tezroq va ko`proq qo`llanishini taqozo etadi. Bu masalani matematika sohasida hal etish o`ta muhim va dolzarbdir. Chunki matematika barcha fundamental fanlar uchun mustahkam poydevori va asosidir. Matematik amallarni amalda qo`llanilishi iqtisodiy samaraning natijasidir.

Talabalar matematikadan nazariy va amaliy bilimlarini mustahkamlashi, ularning matematik fikrlashi uchun ko`proq mustaqil masalalarni yechishi va ularni amaliyotdagi ahamiyatini tushunishi ularni bilim samaradorligini yuqori bo`lishiga olib keladi.

Tavsiya etilayotgan uslubiy qo’llanma “Oliy matematika” fanining “Matematik fizikaning asosiy tenglamalari va kompleks sonlar nazariyasi” qismi bo’yicha bajarilgan bo’lib, oliy o’quv yurti talabalari auditoriyasi uchun mo’ljallangan. Uslubiy qo’llanma ravon tilda ilmiy uslubiy bayon qilingan va davlat ta’lim standartlariga javob beradi.

Mavzu bo’yicha har bir temaga oid qisqacha nazariy qismining bayoni berilib, unga doir masalalar yechib ko’rsatilgan va mustaqil yechish uchun masalalar berilgan.

Uslubiy qo’llanmadan oliy texnika o’quv yurtlarining talabalari foydalanishi mumkin.

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Matematik fizikaning asosiy tenglamalari.

Matematik fizik tenglamalari ikki o'zgaruvchili noma`lum $u(x,y)$ funksiya uchun quyidagicha yoziladi:

$$A(x,y)\frac{\partial^2 u}{\partial x^2}+2B(x,y)\frac{\partial^2 u}{\partial x\partial y}+C(x,y)\frac{\partial^2 u}{\partial y^2}+D(x,y)\frac{\partial u}{\partial x}+E(x,y)\frac{\partial u}{\partial y}+G(x,y)u=F(x,y)(1)$$

Agar (1) tenglamada $F(x,y)=0$ bo'lsa, tenglama ikkinchi tartibli bir jinsli tenglama deyiladi, aks holda bir jinslimas deyiladi. O'zgarmas koeffisientli ikkinchi tartibli hususiy hosilali bir jinsli va chiziqli tenglamani quyidagi ko'rinishda qaraymiz:

$$A\frac{\partial^2 u}{\partial x^2}+2B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}+D\frac{\partial u}{\partial x}+E\frac{\partial u}{\partial y}+Gu=0 \qquad (2)$$

To'lqin tenglamasi.

$$\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$$

bo'lib har hil tebranma prosseslar unga keltiriladi.

Misol uchun:

a) torning ko'ndalang tebranishi;
b) simdagi elektr tebranishlar;
c) gazning tebranishi;
d) metall sterjenning uzunasiga tebranishi va hakozolar.

Issiqlik tarqalish tenglamasi.

$$\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2}$$

tenglama bo'lib unga:

a) issiqlikning tarqalish jarayoni;
b) g'ovak muhitda suyuqlik va gazning oqish masalasi;
c) ehtimollar nazariyasining ba`zi masalalari va boshqa masalalar keltiriladi.

Laplas tenglamasi.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial^2 y} = 0$$

tenglama bo'lib, unga:

a) diffuziya masalalari;
b) stasionar issiqlik haqidagi masalalar;
c) elektr va magnit maydon haqidagi masalalar keltiriladi.

Ikki o'zgaruvchili ikkinchi tartibli xususiy hosilali differensial tenglamalarni sinflarga ajratish va kanonik ko'rinishga keltirish

I. Asosiy tushunchalar

Ushbu

$$a_{11}\frac{\partial^2 u}{\partial x^2} + 2a_{12}\frac{\partial^2 u}{\partial x \partial y} + a_{22}\frac{\partial^2 u}{\partial y^2} + F\left(x, y, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, u\right) = 0 \qquad (1)$$

ko'rinishdagi ikki o'zgaruvchili ikkinchi tartibli xususiy hosilali differensial tenglamani qaraymiz. Bunda a_{11}, a_{12}, a_{22}koeffisientlar *x,y* ning funksiyalari. Bu yerda xususiy holda *F* funksiya $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, u$ larga nisbatan chiziqli bo'lishi ham mumkin.

(1) tenglamada quyidagi tengliklarga asosan x, y o'zgaruvchilarni ξ, η o'zgaruvchilarga almashtiramiz:

$$\xi = \varphi(x, y), \quad \eta = \psi(x, y) \quad (2)$$

bu yerda

$$\begin{vmatrix} \frac{\partial \xi}{\partial x} & \frac{\partial \xi}{\partial y} \\ \frac{\partial \eta}{\partial x} & \frac{\partial \eta}{\partial y} \end{vmatrix} \neq 0 \qquad (3)$$

Bu holda $u(x, y) = v(\xi, \eta)$ dan hosilalarni hisoblasak

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial v}{\partial \eta}\frac{\partial \eta}{\partial x}$$

,

$$\frac{\partial u}{\partial y}=\frac{\partial v}{\partial \xi}\frac{\partial \xi}{\partial y}+\frac{\partial v}{\partial \eta}\frac{\partial \eta}{\partial y}$$

,

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 v}{\partial \xi^2}\cdot\left(\frac{\partial \xi}{\partial x}\right)^2+2\frac{\partial^2 v}{\partial \xi\partial \eta}\cdot\frac{\partial \xi}{\partial x}\cdot\frac{\partial \eta}{\partial x}+\frac{\partial^2 v}{\partial \eta^2}\cdot\left(\frac{\partial \eta}{\partial x}\right)^2+\frac{\partial v}{\partial \xi}\cdot\frac{\partial^2 \xi}{\partial x^2}+\frac{\partial v}{\partial \eta}\cdot\frac{\partial^2 \eta}{\partial x^2}$$

$$\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^2 v}{\partial \xi^2}\cdot\frac{\partial \xi}{\partial x}\cdot\frac{\partial \xi}{\partial y}+\frac{\partial^2 v}{\partial \xi\partial \eta}\left(\frac{\partial \xi}{\partial x}\cdot\frac{\partial \eta}{\partial y}+\frac{\partial \xi}{\partial y}\cdot\frac{\partial \eta}{\partial x}\right)+\frac{\partial^2 v}{\partial \eta^2}\frac{\partial \eta}{\partial x}\cdot\frac{\partial \eta}{\partial y}+\frac{\partial v}{\partial \xi}\cdot\frac{\partial^2 \xi}{\partial x\partial y}+\frac{\partial v}{\partial \eta}\cdot\frac{\partial^2 \eta}{\partial x\partial y}$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 v}{\partial \xi^2}\cdot\left(\frac{\partial \xi}{\partial y}\right)^2+2\frac{\partial^2 v}{\partial \xi\partial \eta}\cdot\frac{\partial \xi}{\partial y}\cdot\frac{\partial \eta}{\partial y}+\frac{\partial^2 v}{\partial \eta^2}\cdot\left(\frac{\partial \eta}{\partial y}\right)^2+\frac{\partial v}{\partial \xi}\cdot\frac{\partial^2 \xi}{\partial y^2}+\frac{\partial v}{\partial \eta}\cdot\frac{\partial^2 \eta}{\partial y^2}\quad(4)$$

bo‘lib, (1) tenglama

$$\bar{a}_{11}\frac{\partial^2 v}{\partial \xi^2}+2\bar{a}_{12}\frac{\partial^2 v}{\partial \xi\partial \eta}+\bar{a}_{22}\frac{\partial^2 v}{\partial \eta^2}+\bar{F}\left(\xi,\eta,\frac{\partial v}{\partial \xi},\frac{\partial v}{\partial \eta}\right)=0\quad(5)$$

ko‘rinishga keladi. Bunda

$$\bar{a}_{11}=a_{11}\left(\frac{\partial \xi}{\partial x}\right)^2+2a_{12}\frac{\partial \xi}{\partial x}\cdot\frac{\partial \xi}{\partial y}+a_{22}\left(\frac{\partial \xi}{\partial y}\right)^2$$

$$\bar{a}_{12}=a_{11}\frac{\partial \xi}{\partial x}\cdot\frac{\partial \eta}{\partial x}+a_{12}\left(\frac{\partial \xi}{\partial x}\cdot\frac{\partial \eta}{\partial y}+\frac{\partial \eta}{\partial x}\cdot\frac{\partial \xi}{\partial y}\right)+a_{22}\frac{\partial \xi}{\partial y}\cdot\frac{\partial \eta}{\partial y}$$

$$\bar{a}_{22}=a_{11}\left(\frac{\partial \eta}{\partial x}\right)^2+2a_{12}\frac{\partial \eta}{\partial x}\cdot\frac{\partial \eta}{\partial y}+a_{22}\left(\frac{\partial \eta}{\partial y}\right)^2\quad(6)\qquad)$$

<u>1–lemma</u>. Agar $z=\varphi(x,y)$ funksiya ushbu

$$a_{11}\left(\frac{\partial z}{\partial x}\right)^2+2a_{12}\frac{\partial z}{\partial x}\cdot\frac{\partial y}{\partial y}+a_{22}\left(\frac{\partial z}{\partial y}\right)^2=0\quad(7)$$

tenglamaning xususiy yechimi bo‘lsa, $\varphi(x,y)=C$ ifoda

$$a_{11}(dy)^2-2a_{12}dxdy+a_{22}(dx)^2=0\quad(8)$$

oddiy differensial tenglamaning umumiy integrali bo‘ladi

<u>2–lemma</u> (teskari). Agar $\varphi(x,y)=C$ ifoda (8) oddiy differensial tenglamaning umumiy integrali bo‘lsa, $z=\varphi(x,y)$ funksiya (7) tenglamaning xususiy yechimi bo‘ladi.

(8) tenglama (1) tenglamaning xarakteristik tenglamasi deyiladi. Xarakteristik tenglamaning umumiy yechimlari (1) tenglamaning xarakteristikalari deyiladi.

(8) xarakteristik tenglama $a_{11} \neq 0$ bo‘lganda quyidagi ikkita oddiy 1–tartibli differensial tenglamalarga ajraydi:

$$y_x = \frac{a_{12}}{a_{11}} + \frac{\sqrt{a_{12}{}^2 - a_{11}a_{22}}}{a_{11}} \qquad (9)$$

$$y_x = \frac{a_{12}}{a_{11}} - \frac{\sqrt{a_{12}{}^2 - a_{11}a_{22}}}{a_{11}} \qquad (10)$$

Bu tenglamalardagi radikal ostidagi $D = a_{12}{}^2 - a_{11}a_{22}$ifodaning ishorasiga qarab, (1) tenglama tiplarga ajraladi.

1) Agar M nuqtada $D = a_{12}{}^2 - a_{11}a_{22} > 0$ bo‘lsa, (1) tenglama M nuqtada giperbolik tipdagi tenglama deyiladi.

2) Agar M nuqtada $D = a_{12}{}^2 - a_{11}a_{22} = 0$ bo‘lsa, (1) tenglama M nuqtada parabolik tipdagi tenglama deyiladi.

3) Agar M nuqtada $D = a_{12}{}^2 - a_{11}a_{22} < 0$ bo‘lsa, (1) tenglama M nuqtada elliptik tipdagi tenglama deyiladi.

Agar qaralayotgan sohaning barcha nuqtalarida $D > 0, D = 0, D < 0$ bo‘lsa, (1) tenglama shu sohada giperbolik, parabolik va elliptik tipga tegishli deyiladi.

Agar sohaning turli nuqtalarida $D = a_{12}^2 - a_{11}a_{22}$ ifodaning ishorasi turlicha bo‘lsa, (1) tenglama sohada aralash tipdagi tenglama deyiladi.

(6) ga asosan $\qquad \bar{a}_{12}{}^2 - \bar{a}_{11}\bar{a}_{22} = (a_{12}{}^2 - a_{11}a_{22}) \left(\frac{\partial \xi}{\partial x}\frac{\partial \eta}{\partial y} - \frac{\partial \xi}{\partial y}\frac{\partial \eta}{\partial x}\right)^2$

bo‘lib, bundan o‘zgaruvchilarni almashtirish natijasida hosil bo‘lgan (5) tenglamaning tipi o‘zgarmasligi kelib chiqadi.

1. $D > 0$ bo‘lsin. (1) giperbolik tipdagi tenglama bo‘lib, (8) xarakteristik tenglamaning umumiy yechimlari haqiqiy va har xil $\varphi(x, y) = C_1$ $\psi(x, y) = C_2$ bo‘ladi. Yangi o‘zgaruvchilarni $\xi = \varphi(x, y), \eta = \psi(x, y)$ deb olsak, yuqoridagi lemmalarga ko‘ra $\bar{a}_{11} = \bar{a}_{22} = 0$ bo‘lib, (5) tenglamani $2\bar{a}_{12}$ga bo‘lib yuborilsa,

$$\frac{\partial^2 v}{\partial \xi \partial \eta} = Q\left(\xi, \eta, v, \frac{\partial v}{\partial \xi}, \frac{\partial v}{\partial \eta}\right) Q = -\frac{F}{2\bar{a}_{12}} \qquad (11)$$

ko‘rinishga keladi. Bu tenglama giperbolik tipdagi tenglamaning kanonik ko‘rinishi deyiladi. (11) tenglamada ξ, η o‘zgaruvchilardan yangi α, β o‘zgaruvchilarga $\xi = \alpha + \beta,\ \ \eta = \alpha - \beta$ tengliklar yordamida o‘tsak, $v(\xi, \eta) = w(\alpha, \beta)$

$$\frac{\partial v}{\partial \xi} = \frac{1}{2}\left(\frac{\partial w}{\partial \alpha} + \frac{\partial w}{\partial \beta}\right), \qquad \frac{\partial v}{\partial \eta} = \frac{1}{2}\left(\frac{\partial w}{\partial \alpha} - \frac{\partial w}{\partial \beta}\right) \frac{\partial^2 v}{\partial \xi \partial \eta} = \frac{1}{2}\left(\frac{\partial^2 w}{\partial \alpha^2} + \frac{\partial^2 w}{\partial \beta^2}\right)$$

bo‘lib, tenglama

$$\frac{\partial^2 w}{\partial \alpha^2} + \frac{\partial^2 w}{\partial \beta^2} = Q_1\left(\alpha, \beta, w, \frac{\partial w}{\partial \alpha}, \frac{\partial w}{\partial \beta}\right)(Q_1 = 4Q) \qquad (12)$$

ko‘rinishga keladi. Bu tenglama giperbolik tipdagi tenglamaning ikkinchi kanonik ko‘rinishi deyiladi.

2. $D = 0$ bo‘lsin. (1) parabolik tipdagi tenglama bo‘lib, (8) xarakteristik tenglama bitta $\psi(x, y) = C$ haqiqiy umumiy yechimga ega bo‘ladi. Yangi o‘zgaruvchilarni $\xi = \psi(x, y), \eta = \eta(x, y)\psi(x, y), \eta(x, y)$ unksiyaga bog‘liq bo‘lmagan ixtiyoriy funksiya) deb olsak, $\bar{a}_{11} = 0,\ \bar{a}_{12} = 0$ bo‘lib, (5) tenglama

$$\frac{\partial^2 v}{\partial \eta^2} = Q_3\left(\xi, \eta, v, \frac{\partial v}{\partial \xi}, \frac{\partial v}{\partial \eta}\right)\left(Q_3 = \frac{\bar{F}}{\bar{a}_{22}}\right) \qquad (13)$$

ko‘rinishga keladi. Bu parabolik tipdagi tenglamaning kanonik ko‘rinishi deyiladi.

3. $D < 0$ bo‘lsin. (1) elliptik tipdagi tenglama bo‘lib, (8) xarakteristik tenglama ikkita kompleks qo‘shma $\psi(x, y) = C_{1,}\ \bar{\psi}(x, y) = C_2$ yechimlarga ega bo‘ladi. Yangi o‘zgaruvchilarni $\xi = Re\psi(x, y),\ \eta = Im\psi(x, y)$ deb olsak, $\bar{a}_{11} = \bar{a}_{22},\ \ \bar{a}_{12} = 0$ bo‘lib, (5) tenglama teng koeffitsientlarga bo‘lib yuborilsa,

$$\frac{\partial^2 v}{\partial \xi^2} + \frac{\partial^2 v}{\partial \eta^2} = Q_4\left(\xi, \eta, v, \frac{\partial v}{\partial \xi}, \frac{\partial v}{\partial \eta}\right)\left(Q_4 = \frac{\bar{F}}{\bar{a}_{11}}\right) \qquad (14)$$

ko‘rinishga keladi. Bu elliptik tipdagi tenglamaning kanonik ko‘rinishi deyiladi.

Agar (1) tenglamadagi F funksiya chiziqli bo‘lib, tenglama koeffisientlari o‘zgarmas sonlar bo‘lsa, bu tenglamani kanonik ko‘rinishga keltirilgandan so‘ng

$$v(\xi, \eta) = e^{\lambda\xi + \mu\xi} w(\xi, \eta)$$

tenglik yordamida yangi W(ξ,η) noma’lum funksiyani kiritib, λ va μ koeffitsientlarni tanlash hisobiga olingan kanonik tenglamani yanada soddalashtirish mumkin.

Yuqorida keltirilgan tiplarga ajratishga asoslanib, to‘lqin tenglamasi giperbolik tipdagi, issiqlik tarqalish tenglamasi parabolik tipdagi, zaryadlarning muvozanatlashuvi tenglamasi elliptik tipdagi tenglama ekanligini aytish mumkin.

Misol 1: Quyidagi tenglamalarni tipini aniqlang

$$\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + 2u - x^2 y = 0$$

Yechish:

$a_{11} = 1, a_{12} = 2, a_{22} = 1;\quad D = {a_{12}}^2 - a_{11}a_{22} = 2^2 - 1 \cdot 1 = 3 > 0$ giperbolik tipga tegishli.

Misol 2:

$$\frac{\partial^2 u}{\partial x^2}+2\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+3u-x^2y=0$$

Yechish:

$a_{11}=1, a_{12}=1\,, a_{22}=1;\quad D={a_{12}}^2-a_{11}a_{22}=1^2-1\cdot 1=0=0$ parabolik tipga tegishli

Misol 3:

$$2\frac{\partial^2 u}{\partial x^2}+2\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}+2\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}-u=0$$

Yechish:

$a_{11}=2, a_{12}=1, a_{22}=1;\quad D={a_{12}}^2-a_{11}a_{22}=1^2-1\cdot 2=-1<0$ elliptik tipga tegishli

Misol 4:

$$x\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}-u=0$$

Yechish:

$a_{11}=x, a_{12}=0, a_{22}=-1;\quad D={a_{12}}^2-a_{11}a_{22}=x$ ekanligidan, berilgan tenglama $x>0$ da giperbolik tipga, $x=0$ da parabolik tipga, $x<0$ da elliptik tipga kiradi.

Misol 5. Quyidagi tenglamani kanonik ko'rinishga keltiring:

$$2\frac{\partial^2 u}{\partial x^2}+3\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}+7\frac{\partial u}{\partial x}+4\frac{\partial u}{\partial y}-2u=0$$

Yechish: $a_{11}=2, a_{12}=\frac{3}{2},\ a_{22}=1;\quad D={a_{12}}^2-a_{11}a_{22}=\frac{9}{4}-2=\frac{1}{4}>0$

Demak, tenglama giperbolik tipga tegishli ekan. U holda kanonik tenglama $\frac{\partial^2 v}{\partial\xi\partial\eta}=Q$ ko‘rinishga ega bo‘ladi. Bunda $Q\xi,\eta,v,\frac{\partial v}{\partial\xi},\frac{\partial v}{\partial\eta}$ larning funksiyasi. Berilgan tenglamaning xarakteristik tenglamasini yozamiz:

$$a_{11}(dy)^2-2a_{12}dxdy+a_{22}(dx)^2=0$$

yoki $$2(dy)^2-3dxdy+(dx)^2=0$$

Bundan $\frac{dy}{dx}=1\quad \frac{dy}{dx}=\frac{1}{2}$ tenglamalarga ega bo‘lamiz. Bu tenglamalarni integrallab,

$y-x=C_1\ \ 2y-x=C_2$ umumiy yechimlarni topamiz. Yangi$\xi=y-x,\ \eta=2y-x$ o‘zgaruvchilarga o‘tamiz. $u(x,y)=v(\xi,\eta)$ ekanligini e’tiborga olib, berilgan tenglamada qatnashuvchi xususiy hosilalarni hisoblaymiz

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial v}{\partial \eta}\frac{\partial \eta}{\partial x} = -\frac{\partial v}{\partial \xi} - \frac{\partial v}{\partial \eta}$$

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial \xi}\frac{\partial \xi}{\partial y} + \frac{\partial v}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{\partial v}{\partial \xi} + 2\frac{\partial v}{\partial \eta}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{\partial \xi^2}\cdot\left(\frac{\partial \xi}{\partial x}\right)^2 + 2\frac{\partial^2 v}{\partial \xi \partial \eta}\cdot\frac{\partial \xi}{\partial x}\cdot\frac{\partial \eta}{\partial x} + \frac{\partial^2 v}{\partial \eta^2}\cdot\left(\frac{\partial \eta}{\partial x}\right)^2 + \frac{\partial v}{\partial \xi}\cdot\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial v}{\partial \eta}\cdot\frac{\partial^2 \eta}{\partial x^2}$$
$$= \frac{\partial^2 v}{\partial \xi^2} + 2\frac{\partial^2 v}{\partial \xi \partial \eta} + \frac{\partial^2 v}{\partial \eta^2}$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 v}{\partial \xi^2}\cdot\frac{\partial \xi}{\partial x}\cdot\frac{\partial \xi}{\partial y} + \frac{\partial^2 v}{\partial \xi \partial \eta}\left(\frac{\partial \xi}{\partial x}\cdot\frac{\partial \eta}{\partial y} + \frac{\partial \xi}{\partial y}\cdot\frac{\partial \eta}{\partial x}\right) + \frac{\partial^2 v}{\partial \eta^2}\frac{\partial \eta}{\partial x}\cdot\frac{\partial \eta}{\partial y} + \frac{\partial v}{\partial \xi}\cdot\frac{\partial^2 \xi}{\partial x \partial y} + \frac{\partial v}{\partial \eta}\cdot\frac{\partial^2 \eta}{\partial x \partial y}$$
$$= -\frac{\partial^2 v}{\partial \xi^2} - 3\frac{\partial^2 v}{\partial \xi \partial \eta} - 2\frac{\partial^2 v}{\partial \xi \partial \eta}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 v}{\partial \xi^2}\cdot\left(\frac{\partial \xi}{\partial y}\right)^2 + 2\frac{\partial^2 v}{\partial \xi \partial \eta}\cdot\frac{\partial \xi}{\partial y}\cdot\frac{\partial \eta}{\partial y} + \frac{\partial^2 v}{\partial \eta^2}\cdot\left(\frac{\partial \eta}{\partial y}\right)^2 + \frac{\partial v}{\partial \xi}\cdot\frac{\partial^2 \xi}{\partial y^2} + \frac{\partial v}{\partial \eta}\cdot\frac{\partial^2 \eta}{\partial y^2}$$
$$= \frac{\partial^2 v}{\partial \xi^2} + 4\frac{\partial^2 v}{\partial \xi \partial \eta} + 4\frac{\partial^2 v}{\partial \eta^2}$$

Bularni tenglamaga qo‘yib, soddalashtirish natijasida

$$\frac{\partial^2 v}{\partial \xi \partial \eta} + 3\frac{\partial v}{\partial \xi} - \frac{\partial v}{\partial \eta} + 2v = 0$$

ko‘rinishdagi kanonik tenglamaga kelamiz. Oxirgi kanonik tenglamani quyidagicha hosil qilish ham mumkin. $u = v$ni (–2) ga, topilgan xususiy hosilalarning tengliklarini, ya’ni $\frac{\partial u}{\partial x}$ ni 7 ga, $\frac{\partial u}{\partial y}U_y$ ni 4 ga, $\frac{\partial^2 u}{\partial x^2}$ ni 2 ga, $\frac{\partial^2 u}{\partial x \partial y}$ ni 3 ga, $\frac{\partial^2 u}{\partial y^2}$ ni 1 ga ko‘paytirib, $v, \frac{\partial v}{\partial \xi}, \frac{\partial v}{\partial \eta}, \frac{\partial^2 v}{\partial \xi^2}, \frac{\partial^2 v}{\partial \eta^2}\eta$ larning oldilaridagi koeffitsientlarni yig‘amiz, natijada

$$(2-3+1)\frac{\partial^2 v}{\partial \xi^2} + (4-9+4)\frac{\partial^2 v}{\partial \xi \partial \eta} + (2-6+4)\frac{\partial^2 v}{\partial \eta^2} + (-7+4)\frac{\partial v}{\partial \xi} + (-7+8)\frac{\partial v}{\partial \eta} - 2v = 0$$

yoki

$$-\frac{\partial^2 v}{\partial \xi \partial \eta} - 3\frac{\partial v}{\partial \xi} + \frac{\partial v}{\partial \eta} - 2v = 0$$

tenglama hosil bo‘ladi. Oxirgi tenglamani (–1) ga ko‘paytirib, kanonik tenglamaga ega bo‘lamiz.

Misol 6: Quyidagi tenglamani kanonik ko‘rinishga keltiring va kanonik tenglamani soddalashtiring.

$$\frac{\partial^2 u}{\partial x^2} - 4\frac{\partial^2 u}{\partial x \partial y} + 5\frac{\partial^2 u}{\partial y^2} - 3\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + u = 0$$

Yechish: Tenglamaning tipini aniqlaymiz:

$$a_{11} = 1, a_{12} = -2, a_{22} = 5; \quad D = {a_{12}}^2 - a_{11}a_{22} = 4 - 1 \cdot 5 = -1 < 0$$

bo‘lganligi uchun tenglama elliptik tipga tegishli bo‘ladi va kanonik tenglamasi taxminan $\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2} = 0$ ko‘rinishga ega bo‘ladi. Bunda Q x, y, noma’lum funksiya va uning 1–tartibli hosilalarining funksiyasi bo‘lishi mumkin.

Xarakteristik tenglamasi

$$(dy)^2 + 4dxdy + 5(dx)^2 = 0$$

bo‘lib, ikkita qo‘shma kompleks

$$2x + y + xi = C_1, \quad 2x + y - xi = C_2$$

yechimlarga ega. Yangi o‘zgaruvchilar sifatida $\xi = 2x + y,\ \eta = x$ funksiyalarni belgilaymiz.

$u(x, y) = v(\xi, \eta)$ funksiyaning xususiy hosilalarini topamiz:

$$\frac{\partial u}{\partial x} = 2\frac{\partial v}{\partial \xi} + \frac{\partial v}{\partial \eta} \ ; \qquad \frac{\partial u}{\partial y} = \frac{\partial v}{\partial \xi} \ ; \qquad \frac{\partial^2 u}{\partial x^2} = 4\frac{\partial^2 v}{\partial \xi^2} + 4\frac{\partial^2 v}{\partial \xi \partial \eta} + \frac{\partial^2 v}{\partial \eta^2}$$

$$\frac{\partial^2 u}{\partial x \partial y} = 2\frac{\partial^2 v}{\partial \xi^2} + \frac{\partial^2 v}{\partial \xi \partial \eta}; \qquad \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 v}{\partial \xi^2}$$

Topilgan ifodalarni tenglamaga qo‘yib, kanonik tenglamaga ega bo‘lamiz:

$$\frac{\partial^2 v}{\partial \xi^2} + \frac{\partial^2 v}{\partial \eta^2} - 5\frac{\partial v}{\partial \xi} - 3\frac{\partial v}{\partial \eta} + v = 0$$

Bu tenglamani soddalashtirish uchun yangi $w(\xi, \eta)$noma’lum funksiyani kiritamiz:

$$(\xi, \eta) = e^{\lambda\xi + \mu\xi} w(\xi, \eta)$$

Hosilalarni hisoblaymiz:

$$\frac{\partial v}{\partial \xi} = \lambda e^{\lambda\xi + \mu\xi} w + e^{\lambda\xi + \mu\xi} \frac{\partial w}{\partial \xi}$$

$$\frac{\partial v}{\partial \eta} = \mu e^{\lambda\xi + \mu\xi} w + e^{\lambda\xi + \mu\xi} \frac{\partial w}{\partial \eta}$$

$$\frac{\partial^2 v}{\partial \xi^2} = \lambda^2 e^{\lambda\xi + \mu\xi} w + 2\lambda e^{\lambda\xi + \mu\xi} \frac{\partial w}{\partial \xi} + e^{\lambda\xi + \mu\xi} \frac{\partial^2 w}{\partial \xi^2}$$

$$\frac{\partial^2 v}{\partial \eta^2} = \mu^2 e^{\lambda\xi+\mu\xi} w + 2\mu e^{\lambda\xi+\mu\xi} \frac{\partial w}{\partial \xi} + e^{\lambda\xi+\mu\xi} \frac{\partial^2 w}{\partial \eta^2}$$

Bu ifodalarni kanonik tenglamaga qo‘yib, soddalashtirish natijasida

$$\frac{\partial^2 w}{\partial \xi^2} + \frac{\partial^2 w}{\partial \eta^2} + (2\lambda - 5)\frac{\partial w}{\partial \xi} + (2\mu - 3)\frac{\partial w}{\partial \eta} + (\lambda^2 + \mu^2 - 5\lambda - 3\mu + 1)w = 0$$

tenglamaga ega bo‘lamiz. λ va μ sonlarni $2\lambda - 5 = 0,\ \ 2\mu - 3 = 0$ bo‘ladigan qilib tanlaymiz. U holda $\lambda = \frac{5}{2}, \quad \mu = \frac{3}{2}$;

$$\lambda^2 + \mu^2 - 5\lambda - 3\mu + 1 = \frac{25}{4} + \frac{9}{4} - \frac{25}{2} - \frac{9}{2} + 1 = -\frac{15}{2}$$

bo‘lib, soddalashtirilgan kanonik tenglama

$$\frac{\partial^2 w}{\partial \xi^2} + \frac{\partial^2 w}{\partial \eta^2} - \frac{15}{2} w = 0$$

ko‘rinishga ega bo‘ladi.

Auditoriya topshirig’i.

1. Quyidagi tenglamalarning tipini aniqlang:

1. $$\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} + 1 = 0$$

2. $$\frac{\partial^2 u}{\partial x^2} + 6\frac{\partial^2 u}{\partial x \partial y} + 16\frac{\partial^2 u}{\partial y^2} + u = 0$$

3. $$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} = 0$$

4. $$\frac{\partial^2 u}{\partial x^2} + y^{m-1}\frac{\partial^2 u}{\partial y^2} - u = 0 m - butunmanfiybo'lmaganson$$

5. $$y\frac{\partial^2 u}{\partial x^2} + x\frac{\partial^2 u}{\partial y^2} - u = 0$$

2. Quyidagi tenglamalarni tipi o‘zgarmaydigan sohada kanonik ko‘rinishga keltiring:

1. $$x^2\frac{\partial^2 u}{\partial x^2} - y^2\frac{\partial^2 u}{\partial y^2} = 0$$

2. $\dfrac{\partial^2 u}{\partial x^2} - 2\dfrac{\partial^2 u}{\partial x \partial y} + 2\dfrac{\partial^2 u}{\partial y^2} = 0$

3. $\dfrac{\partial^2 u}{\partial x^2} sin^2 x - 2ysinx\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = 0$

4. $\dfrac{\partial^2 u}{\partial x^2} - 4\dfrac{\partial^2 u}{\partial x \partial y} - 3\dfrac{\partial^2 u}{\partial y^2} - 2\dfrac{\partial u}{\partial x} + 6\dfrac{\partial u}{\partial y} = 0$

5. $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = 0$

Mustaqil yechish uchun misollar

1. Quyidagi tenglamalarning tipini aniqlang:

1. $3\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial y^2} + 4\dfrac{\partial u}{\partial x} - \dfrac{\partial u}{\partial y} + 1 = 0$

2. $5\dfrac{\partial^2 u}{\partial x^2} + 16\dfrac{\partial^2 u}{\partial x \partial y} + 16\dfrac{\partial^2 u}{\partial y^2} + 64u = 0$

3. $\dfrac{\partial^2 u}{\partial x^2} - 6\dfrac{\partial^2 u}{\partial x \partial y} + 9\dfrac{\partial^2 u}{\partial y^2} + 2\dfrac{\partial u}{\partial y} = 0$

4. $y^{2m+1}\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} - u = 0 \quad m - butun\ manfiy\ bo'lmagan\ son$

5. $x\dfrac{\partial^2 u}{\partial x^2} + y\dfrac{\partial^2 u}{\partial y^2} - u = 0$

2. Quyidagi tenglamalarni tipi o‘zgarmaydigan sohada kanonik ko‘rinishga keltiring:

1. $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial x \partial y} - 2\dfrac{\partial^2 u}{\partial y^2} - 3\dfrac{\partial u}{\partial x} - 15\dfrac{\partial u}{\partial y} + 27x = 0$

2. $\dfrac{\partial^2 u}{\partial x^2} - 2\dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{\partial^2 u}{\partial y^2} + 9\dfrac{\partial u}{\partial x} + 9\dfrac{\partial u}{\partial y} - 9 = 0$

3. $\dfrac{\partial^2 u}{\partial x^2} + 2\dfrac{\partial^2 u}{\partial x \partial y} + 5\dfrac{\partial^2 u}{\partial y^2} - 32u = 0$

4. $\dfrac{\partial^2 u}{\partial x^2} - (1+y^2)^2\dfrac{\partial^2 u}{\partial y^2} - 2y(1+y^2)\dfrac{\partial u}{\partial y} = 0$

5. $xy^2\dfrac{\partial^2 u}{\partial x^2} - 2x^2y\dfrac{\partial^2 u}{\partial x \partial y} + x^3\dfrac{\partial^2 u}{\partial y^2} - y^2\dfrac{\partial u}{\partial x} = 0$

Torning tebranish tenglamasini Dalamber usulida yechish.

Tor deganda elastik ip tushiniladi. Torning elastikligi unda bo'lgan zo'riqish (kuchlanish yoki taranglik) urinma bo'ylab yo'nalganligini anglatadi.

$T -$ urinma bo'yicha yo'nalgan taranglik kuchi, $v -$ torning chiziqli zichligi, $\frac{T}{v} = a^2$ deb belgilasak, tor tabranish tenglamasi

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} \qquad (1)$$

ko'rinishda bo'ladi.

Tor tebranish tenglamasining quyidagi

$$\begin{cases} u(x,0) = f(x) \\ u_t'(x,0) = F(x) \end{cases} \qquad (2')$$

boshlang'ich shartlarini qanoatlantiruvchi yechimni topish talab qilinsin. Bu yerda

$0 < x < +\infty \quad t > 0.$ Bu masala Koshi masalasi deyiladi.

Yechish:

(1)–(2) masalani Dalamber (xarakteristikalar) usuli bilan yechamiz. (1) tenglamaning xarakteristik tenglamasi

$$(dx)^2 - a^2(dt)^2 = 0$$

bo'lib, bu tenglama ikkita har xil

$$x - at = C_{1,} \qquad x + at = C_2$$

yechimlarga ega bo'ladi. (1) tenglamadagi *x* va *t* o'zgaruvchilarni

$$\xi = x - at, \qquad \eta = x + at \qquad u(x,t) = v(\xi,\eta)$$

tengliklarga asosan almashtiramiz. U holda

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 v}{\partial \xi^2} - 2a^2 \frac{\partial^2 v}{\partial \xi \partial \eta} + a^2 \frac{\partial^2 v}{\partial \eta^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{\partial \xi^2} + 2 \frac{\partial^2 v}{\partial \xi \partial \eta} + \frac{\partial^2 v}{\partial \eta^2}$$

bo'lib, (1) tenglama ushbu

$$\frac{\partial^2 v}{\partial\xi\partial\eta} = 0 \qquad (3)$$

kanonik ko‘rinishga keladi. (3) tenglamani

$$\frac{\partial}{\partial\xi}\left(\frac{\partial v}{\partial\eta}\right) = 0$$

ko‘rinishda yozib, ξ bo‘yicha integrallaymiz. Natijada birinchi tartibli

$$\frac{\partial v}{\partial\eta} = P(\eta)$$

((η)– ixtiyoriy funksiya) tenglama hosil bo‘ladi. Bu tenglamani η bo‘yicha integrallab,

$$v(\xi,\eta) = \int P(\eta)d\eta + \varphi(\xi)$$

ifodaga ega bo‘lamiz. Agar

$$\psi(\eta) = \int P(\eta)d\eta$$

deb belgilasak, u holda qaralayotgan kanonik tenglamaning umumiy yechimi

$$v(\xi,\eta) = \varphi(\xi) + \psi(\eta) \qquad (4)$$

ko‘rinishida yoziladi. Bu yerda $\varphi(\xi), \psi(\eta)$ ixtiyoriy funksiyalar. (4) ifodada ξ va $\eta\xi$ v o‘zgaruvchilardan eski x va t o‘zgaruvchilarga qaytib, berilgan (1) tenglamaning umumiy yechimini hosil qilamiz:

$$u(x,t) = \varphi(x - at) + \psi(x + at) \qquad (5)$$

Bunda φ va ψ funksiyalarni ixtiyoriy, ikkinchi tartibligacha uzluksiz hosilalarga ega deb qaraymiz.

U vaqtda ketma-ket hosila olsak,

$$u_x{}' = \varphi'(x - at) + \psi'(x + at)$$

$$u_{xx}{}'' = \varphi''(x - at) + \psi''(x + at)$$

$$u_t' = -a\varphi'(a - at) + a\psi'(x + at)$$

$$u''{}_{tt} = a^2\varphi''(x + at) + a^2\psi''(x + at)$$

(3) tenglamani qanoatlantiradi. Demak, (4) tenglamaning umumiy yechimi bo’ladi. $(2')$ boshlang’ich shartlardan foydalanib φ va ψ noma`lum funksiyalarni topamiz.

$t = 0$ da

$$\begin{cases} \varphi(x) + \psi(x) = f(x) \\ -a\varphi'(x) + a\psi'(x) = F(x) \end{cases}$$

Ikkinchi tenglamani 0 dan *x* gacha oraliqda integrallasak,

$$\int_0^x -a\varphi'(z)dz + \int_0^x a\psi'(z)dz = \int_0^x F(z)dz$$

$$-a[\varphi(x) - \varphi(0)] + a[\psi(x) - \psi(0)] = \int_0^x F(z)dz$$

$$-\varphi(x) + \psi(x) = \frac{1}{a}\int_0^x F(z)dz + [-\varphi(0) + \psi(0)]$$

yoki

$$-\varphi(x) + \psi(x) = \frac{1}{a}\int_0^x F(z)dz + C$$

Bu yerda, $C = -\varphi(0) + \psi(0)$ - o'zgarmas son.

$\varphi(x), \psi(x)$ noma`lum funksiyalarni aniqlash uchun,

$$\begin{cases} \varphi(x) + \psi(x) = f(x) \\ -\varphi(x) + \psi(x) = \dfrac{1}{a}\displaystyle\int_0^x F(z)dz + C \end{cases}$$

sistemani yechamiz. Natijada,

$$\begin{cases} \varphi(x) = \dfrac{1}{2}f(x) - \dfrac{1}{2a}\displaystyle\int_0^x F(z)dz - \dfrac{C}{2} \\ \psi(x) = \dfrac{1}{2}f(x) + \dfrac{1}{2a}\displaystyle\int_0^x F(z)dz + \dfrac{C}{2} \end{cases}$$

hosil bo'ladi.

Bu formulalarda x ni $x - at$ va $x + at$ larga almashtirib (4) ga qo'ysak,

$$u(x,t) = \frac{f(x-at) + f(x-at)}{2} + \frac{1}{2a}\int_{x-at}^{x+at} F(z)dz \quad (5)$$

formula kelib chiqadi.

(5) tor tebranish tenglamasi uchun Koshi masalasining Dalamber usulida yechilishi deyiladi va Dalamber formulasi deb yuritiladi.

Misol 1. $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ tenglamaning $u(0,x) = e^x u_t'(0,x) = Cx,\ C = const$ boshlang'ich shartni qanoatlantiruvchi yechimini toping.

Yechish:

$$u(x,t) = \frac{f(x-at) + f(x-at)}{2} + \frac{1}{2a}\int_{x-at}^{x+at} F(z)dz =$$

$$= \frac{e^{x-at} + e^{x+at}}{2} + \frac{1}{2a}\int_{x-at}^{x+at} Czdz = \frac{e^x(e^{-at} + e^{at})}{2} + \frac{1}{2a}\frac{Cz^2}{2}\Bigg|_{x-at}^{x+at} = e^x chat + Ctx$$

Bu yerda $\frac{e^{-at}+e^{at}}{2} = chat$

Misol 2. $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ tenglamani $u(0,x) = x^2 u_t'(0,x) = 0$ boshlang'ich shartlarni qanoatlantiruvchi yechimini toping.

Yechish: $a = 1$, $F(x) = 0$ bo'lganligi uchun, $u(x,t) = \frac{f(x-at)+f(x+at)}{2}$ ga teng. $f(x) = x^2$ ekanligini hisobga olsak,

$$u(x,t) = \frac{(x-t)^2 + (x+t)^2}{2} = x^2 + t^2$$

yechim bo'ladi.

Misol 3. $\frac{\partial^2 u}{\partial t^2} - a^2 \frac{\partial^2 u}{\partial x^2} = 0$ formula bilan berilgan torning $t = \frac{\pi}{2a}$ momentdagi formasini aniqlang, agar $u(x,0) = sinx,\ \ u_t'(0,x) = 1$ bo'lsa.

Yechish:

$$u = \frac{\sin(x+at) + \sin(x-at)}{2} + \frac{1}{2a}\int_{x-at}^{x+at} dz$$

$$u = sinxcosat + \frac{1}{2a} z|_{x-at}^{x+at} = sinxcosat + t$$

$t = \frac{\pi}{2a}$da $u = \frac{\pi}{2a}$ ya`ni tor absissalar o'qiga parallel bo'ladi.

Auditoriya topshirig'i.

Tor tebranish tenglamasi uchun Koshi masalasi yechimini Dalamber formulasidan foydalanib toping.

1. $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2};\quad u(x,0) = x,\ \ u_t'(0,x) = -x$

2. $\frac{\partial^2 u}{\partial t^2} - a^2\frac{\partial^2 u}{\partial x^2} = 0;\quad u(x,0) = 0,\ \ u_t'(0,x) = cosx$

$$3.\ \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0\ ; \quad u(x,0) = sinx,\ \ u_t'(0,x) = cosx$$

4. $\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0$ formula bilan berilgan torning $t = \pi$ momentdagi formasini aniqlang, agar $u(x,0) = sinx,\ \ u_t'(0,x) = cosx$ bo'lsa.

MUSTAQIL YECHISH UCHUN MISOLLAR

Tor tebranish tenglamasi uchun Koshi masalasi yechimini Dalamber formulasidan foydalanib toping

$$1.\ \frac{\partial^2 u}{\partial t^2} - a^2 \frac{\partial^2 u}{\partial x^2} = 0; \quad u(x,0) = 0,\ \ u_t'(0,x) = cosx$$

$$2.\ \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0; \ \ u(x,0) = x^2,\ \ u_t'(0,x) = 4x$$

$$3.\ \frac{\partial^2 u}{\partial t^2} - 4\frac{\partial^2 u}{\partial x^2} = 0; \quad u(x,0) = 0,\ \ u_t'(0,x) = x$$

Tor tebranish tenglamasi uchun aralash masalalarni o'zgaruvchilarni ajratish usuli bilan yechish

Tekislikdagi $D = \{(x,t): 0 < x < l, 0 < t < \infty\}$sohada bir jinsli

$$\frac{\partial^2 u}{\partial t^2} - a^2 \frac{\partial^2 u}{\partial x^2} = 0$$

tor tebranish tenglamasining

$$u(x,0) = \varphi(x),\ \ u_t'(x,0) = \psi(x)$$

boshlang'ich shartlarni va

$$u(0,t) = 0, \quad u(l,t) = 0$$

bir jinsli chegaraviy shartlarni qanoatlantiruvchi yechimi topilsin.

Tenglama yechimini $u(x,t) = X(x)T(t)$ ko'rinishda izlaymiz. Bunda $X(x)$ va $T(t)$ noma`lum funksiyalar.

Bu ifodani berilgan tenglamaga qo'yib,

$$X(x)T''(t) = a^2 X''(x)T(t)$$

ega bo'lamiz.

Bundan,

$$\frac{T''(t)}{a^2 T(t)} = \frac{X''(x)}{X(x)}$$

Bu tenglik faqat x va t larga bog'liq bo'lib, ikkala nisbat o'zgarmas $-\lambda(\lambda > 0)$ga teng bo'lgandagina o'rinlidir.

$$\frac{T''(t)}{a^2T(t)} = \frac{X''(x)}{X(x)} = -\lambda$$

$$X''(x) + \lambda X(x) = 0 \qquad\qquad T''(t) + \lambda a^2T(t) = 0$$

bu tenglamalarning umumiy yechimi

$$X(x) = Acos\sqrt{\lambda}x + Bsin\sqrt{\lambda}x \qquad T(t) = Ccosa\sqrt{\lambda}t + Dsin\sqrt{\lambda}t$$

ko'rinishda bo'lib, bu yerda $A, B, C, D -$ ihtioriy o'zgarmaslar. U holda

$$u(x,t) = \left(Acos\sqrt{\lambda}x + Bsin\sqrt{\lambda}x\right)\left(Ccosa\sqrt{\lambda}t + Dsin\sqrt{\lambda}t\right)$$

bo'ladi.

A, B o'zgarmaslarni chegaraviy shartlardan foydalanib topamiz: $T(t) \not\equiv 0 \implies X(0) = 0 \;\; X(l) = 0 \implies$

$$X(0) = A = 0, \quad X(l) = Acos\sqrt{\lambda}l + Bsin\sqrt{\lambda}l$$

Ya'ni, $A = 0$ va $Bsin\sqrt{\lambda}l = 0$ bo'lib, $B \neq 0$ ekanligidan, $Bsin\sqrt{\lambda}l = 0 \implies \sqrt{\lambda} = \frac{k\pi}{l}(k = 1,2, \dots)$

Demak, $X = Bsin\frac{k\pi}{l}x$

λ ning topilgan qiymatlari berilgan chegaraviy masalaning xos qiymatlari deyiladi, $X = Bsin\frac{k\pi}{l}x$ funksiya esa xos funksiyasi deb ataladi.

λ ning topilgan qiymatida

$$T(t) = Ccos\frac{ak\pi}{l}t + Dsin\frac{ak\pi}{l}t,$$

$$u_k(x,t) = sin\frac{k\pi}{l}x\left(a_kcos\frac{ak\pi}{l}t + b_ksin\frac{ak\pi}{l}t\right) \quad (k = 1,2,3 \dots)$$

k ning har bir qiymatiga C va D ning qiymati mos keladi, shuning uchun a_k, b_k deb yozib olamiz. B o'zgarmasni ham a_k, b_k larning ichida deb hisoblaymiz.

$$\frac{\partial^2u}{\partial t^2} - a^2\frac{\partial^2u}{\partial x^2} = 0$$

Tenglama chiziqli va bir jinsli bo'ganligi uchun, yechimlarining yig'indisi ham uning yechimi bo'ladi. Demak,

$$u(x,t)=\sum_{k=1}^{\infty}u_k(x,t)=\sum_{k=1}^{\infty}\left(a_k cos\frac{ak\pi}{l}t+b_k sin\frac{ak\pi}{l}t\right)sin\frac{k\pi}{l}x$$

Qator differensial tenglamaning yechimi bo'ladi, agar a_k va b_k koeffisientlarning topilgan qiymatlarida qator yaqinlashuvchi shuningdak, ikki marta x va t bo'yicha differensiallanishidan hosil bo'lgan qator ham yaqinlashuvchi bo'lsa. Bunda, a_k, b_k larning qiymatini boshlang'ich shartdan foydalanib topamiz:

$$u(x,0)=\sum_{k=1}^{\infty}a_k sin\frac{k\pi}{l}x=\varphi(x)$$

Agar $\varphi(x)$ funksiya Fur'e qatoriga $(0,l)$ oraliqda sinuslar bo'yicha yoyilsa, u holda

$$a_k=\frac{2}{l}\int_0^l \varphi(x)sin\frac{k\pi}{l}xdx \quad (*)$$

$u_t'(x,0)=\psi(x)$ shartga ko'ra,

$$u_t'(x,0)=\sum_{k=1}^{\infty}\frac{ak\pi}{l}b_k sin\frac{k\pi}{l}x=\psi(x)$$

Bundan, Fur'e qatorining koeffisientlarini topamiz:

$$b_k=\frac{2}{ak\pi}\int_0^l \psi(x)sin\frac{k\pi}{l}xdx \quad (**)$$

Shunday qilib, torning tebranish tenglamasining yechimi

$$u(x,t)=\sum_{k=1}^{\infty}\left(a_k cos\frac{ak\pi}{l}t+b_k sin\frac{ak\pi}{l}t\right)sin\frac{k\pi}{l}x \quad (1)$$

ko'rinishda bo'ladi, bunda a_k va b_k lar $(*)$ va $(**)$ formulalar yordamida topiladi.

Izoh: agar $\frac{T''(t)}{a^2T(t)}=\frac{X''(x)}{X(x)}=\lambda$ bo'lsa, $X''(x)-\lambda X(x)=0,\quad T''(t)-a^2\lambda T(t)=0$ bo'lib, ulardan birinchisining $X=Ae^{\sqrt{\lambda}x}+Be^{-\sqrt{\lambda}x}$ umumiy yechimi chegaraviy shartlarni qanoatlantirmaydi.

Misol 1: Chetlari $x=0,\ x=l$ mahkamlangan tor berilgan bo'lib, tor nuqtalarining boshlang'ich tezligi 0 ga teng. Boshlang'ich chetlanish parabola bo'lib, u tor o'rtasi $\frac{1}{2}$ ga nisbatan simmetrik va maksimal chetlanishi h ga teng.Tor tebrnishini aniqlang.

Yechish:

Masala shartiga ko'ra,

$$\varphi(x)=\frac{4h}{l^2}x(l-x),\qquad \psi(x)=0$$

tenglama yechimini aniqlovchi koeffisientlarni topamiz:

$$a_k = \frac{2}{l}\int_0^l \varphi(x) sin\frac{k\pi}{l}xdx = \frac{8h}{l^3}\int_0^l (lx - x^2) sin\frac{k\pi}{l}xdx; \qquad b_k = 0$$

a_k koeffisientni topish uchun bo’laklab intgrallash usulidan foydalanamiz:

$$u_1 = lx - x^2, \qquad dv_1 = sin\frac{k\pi x}{l}dx, \qquad du_1 = (l - 2x)dx, \quad v_1 = -\frac{1}{k\pi}cos\frac{k\pi x}{l};$$

$$a_k = -\frac{8h}{l^3}(lx - x^2)\frac{l}{k\pi}cos\frac{k\pi x}{l}\Big|_0^l + \frac{8}{k\pi l^2}\int_0^l (l - 2x)cos\frac{k\pi x}{l}dx$$

Ya`ni, $a_k = \frac{8}{k\pi l^2}\int_0^l (l - 2x)cos\frac{k\pi x}{l}dx$

Ikkinchi marta bo’laklaymiz: $u_2 = l - 2x,\ dv_2 = cos\frac{k\pi x}{l}dx,\ du_2 = -2dx,\ v_2 = \frac{l}{k\pi}sin\frac{k\pi x}{l}$

$$a_k = \frac{8}{k^2\pi^2 l}(l - 2x)sin\frac{k\pi x}{l}\Big|_0^l + \frac{16h}{k^2\pi^2 l}\int_0^l sin\frac{k\pi x}{l}dx = -\frac{16h}{k^3\pi^3}cos\frac{k\pi x}{l}\Big|_0^l = \frac{16h}{k^3\pi^3}[1 - (-1)^k]$$

Demak, yechim:

$$u(x,t) = \sum_{k=1}^{\infty}\frac{16h}{k^3\pi^3}[1 - (-1)^k]cos\frac{k\pi at}{l}sin\frac{k\pi x}{l}$$

ga teng.

Agar, $k = 2n$ bo’lsa, $1 - (-1)^k = 0$, agar, $k = 2n + 1$ bo’lsa, $1 - (-1)^k = 2$ bo’ladi. Shuning uchun umumiy yechim quyidagiga tengdir:

$$u(x,t) = \frac{32h}{\pi^3}\sum_{k=1}^{\infty}\frac{1}{(2n + 1)^3}cos\frac{(2n + 1)\pi at}{l}sin\frac{(2n + 1)\pi x}{l}$$

Misol 2

$$D = \{(x,t): 0 < x < l,\ \ 0 < t < +\infty\}$$

sohada

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2}, \qquad u(x,0) = 0,\ \ u_t(x,0) = sin\frac{5\pi}{l}x,\ \ u(0,t) = 0, \qquad u(l,t) = 0$$

aralash masalaning yechimi topilsin.

Yechish: (1) funksional qatorning koeffitsientlarini topamiz. $\varphi(x) = 0, \psi(x) = sin\frac{5\pi}{l}x$ ekanligidan,

$$a_k = 0, \quad b_k = \frac{2}{k\pi a}\int_0^l sin\frac{5\pi}{l}xsin\frac{k\pi}{l}xdx$$

bo‘ladi. $X_k(x) = \sqrt{\frac{2}{l}} sin\frac{k\pi}{l}x \quad (k = 1,2, \dots .)$ xos funksiyalar, $(0, l)$ oraliqda normallashgan ortogonal funksiyalar sistemasini tashkil qilganligi uchun

$$\frac{2}{l}\int_0^l sin\frac{k\pi x}{l} sin\frac{n\pi x}{l} dx = \begin{cases} 0, & agar\ \ k \neq n \\ 1, & agar,\ \ k = n \end{cases}$$

bo‘ladi. Bundan $k \neq 5$bo‘lganda $b_k = 0$,$k = 5$ bo‘lganda $b_5 = \frac{1}{5\pi a}$ ekanligi kelib chiqadi.

Demak, masalaning izlangan yechimi

$$u(x,t) = \frac{1}{5\pi a} sin\frac{5\pi at}{l} sin\frac{5\pi x}{l}$$

bo‘ladi.

Auditoriya topshirig’i.

$D = \{(x,t)\colon 0 < x < l,\ \ 0 < t < +\infty\}$ sohada bir jinsli $\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2}$ tor tebranish tenglamasi uchun quyidagi aralash masalalar yechilsin:

1. $u(0,t) = u(l,t) = 0,\ \ u(x,0) = 0,\ \ u_t(x,0) = sin\frac{2\pi}{l}x$

2. $u(0,t) = u(l,t) = 0,\ \ u(x,0) = x,\ \ u_t(x,0) = sin\frac{\pi}{2l}x + sin\frac{3\pi}{2l}x$

3. $u(0,t) = u(l,t) = 0,,\ \ u(x,0) = x,\ \ u_t(x,0) = 1$

Mustaqil yechish uchun misollar.

$D = \{(x,t)\colon 0 < x < l,\ \ 0 < t < +\infty\}$ sohada bir jinsli $\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2}$ tor tebranish tenglamasi uchun quyidagi aralash masalalar yechilsin:

$$1. u(0,t) = u(l,t) = 0,\ \ u(x,0) = sin\frac{5\pi x}{2l},\quad u_t(x,0) = sin\frac{\pi}{2l}x$$

$$2. u(0,t) = u(l,t) = 0,\ \ u(x,0) = x^2,\quad u_t(x,0) = sin\frac{\pi}{2l}x$$

$$3. u(0,t) = u(l,t) = 0,\ \ u(x,0) = cos\frac{\pi}{2l}x,\qquad u_t(x,0) = cos\frac{3\pi}{2l}x + cos\frac{5\pi}{2l}x$$

Issiqlik tarqalish tenglamasi uchun birinchi chegaraviy masalani Fur'e metodi bilan yechish.

$$\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2} \qquad (1)$$

Tenglamani $u(0,t)=0,\ u(l,t)=0$ (2) chegaraviy shartlarni va $u(x,0)=\varphi(x)$ (3) boshlang'ich shartni qanoatlantiruvchi yechimini topish talab qilinsin. Bu yerda boshlang'ich paytda sterjen temperaturasi $\varphi(x)$ funksiya orqali ifodalanadi. Butun tajriba davomida sterjen chekkalarida nol temperatura saqlanadi deb yuritiladi. (1) tenglamaning noldan farqli yechimini

$$u(x,t)=X(x)\cdot T(t) \ \ (4)$$

ko'rinishda izlaymiz. (2) chegaraviy shartida barcha $t>0$ uchun $X(0)\cdot T(t)=0,\ X(l)\cdot T(t)=0,$ ya`ni $X(0)=X(l)=0$ (5). Aks holda $T(t)=0$ bo'lib, $u(x,t)=0$ shartimizga zid bo'ladi. (4) ni (1) ga qo'yib, noma`lumlarni ajratib

$$\frac{1}{a^2}\cdot\frac{T'}{T}=\frac{X''}{X}=-\lambda \ \ (6)$$

ni hosil qilamiz, ya`ni $X''+\lambda X=0 \quad X(0)=X(\lambda)=0$ (7), $T'+a^2\lambda T=0$ (8)

bu oddiy differensial tenglamani umumiy yechimini topamiz

$$k^2+\lambda=0,\ \ k=\pm i\sqrt{\lambda}$$

$$X(x)=Acos\sqrt{\lambda}x+Bsin\sqrt{\lambda}x$$

A va B larni (5) shartdan foydalanib topamiz.

$$0=A\cdot 1+B\cdot 0$$

$$0=A\cdot cos\sqrt{\lambda}1+Bsin\sqrt{\lambda}1$$

Birinchisidan $A=0$, ikkinchisidan $Bsin\sqrt{\lambda}1=0$ ekanligi kelib chiqadi. $B\neq 0$ chunki, aks holda $X=0$ bo'lib, $u=0$ bo'lib qoladi. Bu shartga zid. Shuning uchun $sin\sqrt{\lambda}1=0$ bo'lishi kerak, bundan, $\sqrt{\lambda}=\frac{n\pi}{l}$ $(n=1,2,\dots)$ xos qiymatlarini topamiz. Ularga mos keladigan xos funksiyalar

$$X_n=B_n sin\frac{n\pi}{l}x \quad (9)$$

tenglik bilan ifodalanadi. Topilgan $\sqrt{\lambda}$ ning ifodasini (8) ga qo'ysak,

$$T'+a^2\frac{n^2\pi^2}{l^2}T=0$$

$$\frac{dT}{T}=-a^2\frac{n^2\pi^2}{l^2}$$

$$T_n=A_n e^{-a^2\frac{n^2\pi^2}{l^2}t}(10)$$

(9) va (10) larni (4) ga qo'yib (1) tenglamani (2) chegaraviy shartlarni qanoatlantiruvchi yechimini hosil qilamiz

$$u_n(x,t) = b_n e^{-a^2\frac{n^2\pi^2}{l^2}t} sin\frac{n\pi}{l}x \quad (11)$$

bu yerda $b_n = A_n B_n$

(1) tenglama chiziqli va bir jinsli bo'lganligi uchun yechimlarning yig'indisi ham yechim bo'ladi

$$u(x,t) = \sum_{n=1}^{\infty} b_n e^{-a^2\frac{n^2\pi^2}{l^2}t} \; sin\frac{n\pi}{l}x \quad (12)$$

b_n o'zgarmasni aniqlash uchun boshlang'ich (3) shartdan foydalanamiz.

$t = 0$ bo'lganda,

$$\varphi(x) = \sum_{n=1}^{\infty} b_n sin\frac{n\pi}{l}x \; (13)$$

bo'lib, $\varphi(x)$ funksiyani $(0;1)$ intervalda Fur`e qatoriga yoyilmasi mavjud deb faraz qilsak,

$$b_n = \frac{2}{l}\int_0^l \varphi(x) sin\frac{n\pi}{l}xdx \qquad (14)$$

ga teng bo'ladi.

Shunday qilib, qo'yilgan masalaning yechimi

$$u(x,t) = \sum_{n=1}^{\infty} b_n e^{-a^2\frac{n^2\pi^2}{l^2}t} \; sin\frac{n\pi}{l}x \quad (15)$$

ko'rinishda ekanligini aniqladik. Bu yerda b_n (14) formula yordamida topiladi.

Misol $\frac{\partial u}{\partial t} = a^2\frac{\partial^2 u}{\partial x^2}$ tenglamani $u(0,t) = u(l,t) = 0$ chegaraviy va $u(x,0) = \frac{x(l-x)}{l^2}$ boshlang'ich shartni qanoatlantiruvchi yechimini toping.

Yechish:

$$b_n = \frac{2}{l^3}\int_0^l x(l-x)sin\frac{n\pi}{l}xdx = \begin{cases} \frac{8}{n^3\pi^3}, & agar \;\; n - toq \\ 0, & agar \;\; n - juft \end{cases}$$

Misol: $\Omega = \{(x,t): 0 < x < l, 0 < t < +\infty\}$ sohada $\frac{\partial u}{\partial t} = a^2\frac{\partial^2 u}{\partial x^2}$ tenglamaning $u(x,0) = \begin{cases} x, & agar \;\; 0 < x < \frac{l}{2} \\ l - x, & agar \;\; \frac{l}{2} \le x < l \end{cases}$ boshlang'ich va $u(0,t) = u(l,t) = 0$ chrgaraviy shartlarni qanoatlantiruvchi yechimi topilsin.

Yechish:

$$b_n = \frac{2}{l}\int_0^l \varphi(x) sin\frac{n\pi}{l}xdx = \frac{2}{l}\left\{\int_0^{\frac{l}{2}} xsin\frac{n\pi}{l}xdx + \int_{\frac{l}{2}}^l (l-x)sin\frac{n\pi}{l}xdx\right\}$$

Ikkinchi integralda $l - x = y$ almashtirish bajarib, ba`zi hisob kitoblardan keyin, y ni x bilan almashtirib, ushbu tenglikka ega bo'lamiz

$$b_n = \frac{2}{l}[1-(-1)^n]\int_0^{\frac{l}{2}} xsin\frac{n\pi}{l}xdx$$

Bo'llaklab integrallash natijasida

$$b_n = 2[1-(-1)^n]\frac{l}{\pi n}\left\{\frac{-1}{2}cos\frac{\pi n}{2} + \frac{1}{\pi n}sin\frac{\pi n}{2}\right\}$$

Ga ega bo'lamiz. Topilgan b_n ni qiymatini (15) ga qo'yib, masala yechimini hosil qilamiz:

$$u(x,t) = \frac{2l}{\pi}\sum_{n=1}^{\infty}[1-(-1)^n]\left(-\frac{1}{2n}cos\frac{\pi n}{2} + \frac{1}{\pi n^2}sin\frac{\pi n}{2}\right)e^{-\left(\frac{\pi n}{l}a\right)^2 t}sin\frac{\pi n}{l}x$$

Agar, $n = 2k$ bo'lsa, $1-(-1)^n = 0$

Agar, $n = 2k+1$ bo'lsa, $1-(-1)^n = 2$ va $cos\frac{\pi n}{2} = \cos\left(\pi k + \frac{\pi}{2}\right) = 0,\ sin\frac{\pi n}{2} = \sin\left(\pi k + \frac{\pi}{2}\right) = (-1)^k$ bo'lganligi uchun, yechimni quyidagi ko'rinishda yozish mumkin:

$$u(x,t) = \frac{4l}{\pi^2}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)^2}e^{-\left(\frac{\pi(2k+1)}{l}a\right)^2 t}sin\frac{\pi(2k+1)}{l}x$$

Auditoriya topshirig'i

1. $u(x,t) = u(l,t) = 0,\ \ u(x,0) = Ax;$

2. $u_x(0,t) = u(l,t) = 0, \qquad u(x,0) = cos\frac{\pi}{2l}x$

3. $u_x(0,t) = u_x(l,t) = 0,\ \ u(x,0) = C,\ \ C = const$

Mustaqil ishlash uchun misollar

1. $u(0,t) = u_x(l,t) = 0,\ \ u(x,0) = sin\frac{\pi}{2l}x$

2. $u(x,t) = u(l,t) = 0,\ \ u(x,0) = x^2;$

$$3. u(0,t) = u_x(l,t) = \begin{cases} 0, & 0 < x < \dfrac{l}{2} \\ u_0, & \dfrac{l}{2}, x < l \end{cases}$$

Dirihle masalasini doira uchun Fur`e metodi bilan yechish.

Laplas tenglamasini qanoatlantiruvchi $u(x,y)$ funksiyalar garmonik funksiyalar deyiladi.

Dirihle masalasi: Oxy tekislikda markazi koordinatalar boshida bo'lgan R radiusli doira olingan bo'lib, uning aylanasida biror $f(\varphi)$ funksiya berilgan bo'lsin, bunda $\varphi -$ qutb burchagi. Doirada va uning chegarasida uzluksiz bo'lib, doira ichida Laplas tenglamasini

$$r^2 u''_{rr} + r u'_r + u''_{\varphi\varphi} = 0 \quad (1)$$

Qanoatlantiradigan hamda doira aylanasida berilgan $u|_{r=R} = f(\varphi)$ (2) qiymatni qabul qiladigan $u(r,\varphi)$ funksiyani topish masalasi Dirihle masalasi deyiladi.

Noldan farqli yechimni

$$u = F(\varphi)R(r) \qquad (3)$$

ko'rinishda izlab, Fur'e usulidan foydalanamiz. (3) dan hosilalar olib (1) tenglamaga qo'yamiz. O'zgartiruvchilarni ajratib quyidagi

$$\frac{F"(\varphi)}{F(\varphi)} = -\frac{r^2 R''(r) + rR'(r)}{R(r)}$$

tenglamani hosil qilamiz. Bu tenglik o'zgarmas songa teng bo'lgandagina o'rinli bo'ladi. Uni $-k^2$ deb belgilaymiz.

$$\frac{F"(\varphi)}{F(\varphi)} = -\frac{r^2 R''(r) + rR'(r)}{R(r)} = -k^2 \quad (4)$$

Bu tengliklardan ikkita tenglama hosil bo'ladi:

$$F''(\varphi) + k^2 F(\varphi) = 0 \quad (5)$$

$$r^2 R''(r) + rR'(r) - k^2 R(r) = 0 \quad (6)$$

Bu oddiy differensial tenglamalarning umumiy yechimlarini topamiz:

(5) ning umumiy yechimi:

$$\lambda^2 + k^2 = 0 \qquad \lambda = \pm ik$$

$$F(\varphi) = Acosk\varphi + Bsink\varphi \qquad (7)$$

(6) tenglamaning yechimini $R(r) = r^m$ ko'rinishda izlaymiz. Bu yerda m ni topish kerak. r^m ni (6) ga qo'yib quyidagini hosil qilamiz:

$$r^2m(m-1)r^{m-2}+rmr^{m-1}-k^2r^m=0$$

yoki

$$m^2-k^2=0 \qquad m=\pm k$$

Hususiy yechimlar r^k va r^{-k} bo'lib, umumiy yechim

$$R(r)=Cr^k+Dr^{-k} \qquad (8)$$

(7) va (8) ni (3) ga qo'ysak,

$$u_k(r,\varphi)=(A_k cosk\varphi+B_k sink\varphi)(C_kr^k+D_kr^{-k}) \qquad (9)$$

hosil bo'ladi.

Biz doirada uzluksiz va chekli yechimni izlaymiz. $r=0$ bo'lganda (9) formulada $D_k=0$ bo'lishi kerak. Agar $k=0$ bo'lsa, (5) va (6) tenglamalardan $F''(\varphi)=0$ $\quad rR''(r)+R'(r)=0$ hosil bo'ladi. Bularni integrallab $F(\varphi)=A_0\varphi+B_0$ $\quad R(r)=C_0+D_0lnr$ larni topamiz. $k=0$ da (9) bilan solishtirib $B_0=0,\ D_0=0$ ekanini topamiz. U vaqtda $u_0=\frac{a_0}{2}$ bo'ladi. Bu yerda $\frac{a_0}{2}=A_0B_0$ deb belgiladik. $k=1,2,...$ musbat qiymatlar bilan chegaralanamiz.

Yechimlar yig'indisi yana o'z navbatida yechim bo'lganligi uchun

$$u(r,\varphi)=\sum_{n=1}^{n}(a_n cosn\varphi+b_n sinn\varphi)\,r^n \qquad (10)$$

Bu yerda $a_n=C_nA_n,\ b_n=C_nB_n$ deb belgilash kiritdik. Endi ihtiyoriy a_n va b_n larni chetki (2) shartdan topamiz. $r=R$ da (10) dan

$$f(\varphi)=\frac{a_0}{2}+\sum_{n=1}^{n}(a_n cosn\varphi+b_n sinn\varphi)\,R^n(11)$$

Bu tenglikdan,

$$\begin{cases} a_n=\dfrac{1}{\pi R^n}\displaystyle\int_{-\pi}^{\pi}f(t)cosntdt \\ b_n=\dfrac{1}{\pi R^n}\displaystyle\int_{-\pi}^{\pi}f(t)sinntdt \end{cases} \qquad (12)$$

Koeffisientlarni aniqlab, (10) ga qo'yamiz. Trigonometrik almashtirishlr bajarib, ushbuni hosil qilamiz:

$$u(r,\varphi)=\frac{1}{2\pi}\int_{-\pi}^{\pi}f(t)dt+\frac{1}{\pi}\sum_{n=1}^{\infty}\int_{-\pi}^{\pi}f(t)cosn(t-\varphi)dt\left(\frac{r}{R}\right)^n=$$

$$=\frac{1}{2\pi}\int_{-\pi}^{\pi}f(t)\left[1+2\sum_{n=1}^{\infty}\left(\frac{r}{R}\right)^n cosn(t-\varphi)\right]dt \qquad (13)$$

Kvadrat qavs ichidagi ifodani soddalashtiramiz:

$$1+\sum_{n=1}^{\infty}\left(\frac{r}{R}\right)^{n}\left[e^{in(t-\varphi)}+e^{-in(t-\varphi)}\right]=$$

$$=1+\sum_{n=1}^{\infty}\left[\left(\frac{r}{R}e^{i(t-\varphi)}\right)^{n}+\left(\frac{r}{R}e^{-i(t-\varphi)}\right)^{n}\right]=1+\frac{\frac{r}{R}e^{i(t-\varphi)}}{1-\frac{r}{R}e^{i(t-\varphi)}}+\frac{\frac{r}{R}e^{-i(t-\varphi)}}{1-\frac{r}{R}e^{-i(t-\varphi)}}=$$

$$=\frac{1-\left(\frac{r}{R}\right)^{2}}{1-2\frac{r}{R}\cos(t-\varphi)+\left(\frac{r}{R}\right)^{2}}=\frac{R^2-r^2}{R^2-2Rrcos(t-\varphi)+r^2}$$

Hosil bo’lgan ifodani (13) ga qo’yamiz:

$$u(r,\varphi)=\frac{1}{2\pi}\int_{-\pi}^{\pi}f(t)\frac{R^2-r^2}{R^2-2Rrcos(t-\varphi)+r^2}dt \qquad (14)$$

Bu formula Puasson integrali deyiladi va Dirihle masalasini doira uchun yechimini ifodalaydi.

Misol : Radiusi R ga teng bo’lgan yupqa bir jinsli plastinkaning yuqori yarim qismining temperaturasi 1^0 ni saqlaydi, quyi yarim qismida temperatura 0^0 ga teng bo’lsa, issiqlikning stasionar tarqalish taqsimotini toping.

Yechish:

$-\pi<t<0$ uchun $f(t)=0$ va $0<t<\pi$ uchun $f(t)=1$. Issiqlikning tarqalishi

$$u(r,\varphi)=\frac{1}{2\pi}\int_{0}^{\pi}\frac{R^2-r^2}{R^2-2Rrcos(t-\varphi)+r^2}dt$$

Integral bilan aniqlanadi. (r,φ) nuqta yuqori yarim aylanada joylashgan bo’lsin, ya`ni $0<\varphi<\pi$. U holda $t-\varphi$ $-\varphi$ dan $\pi-\varphi$ gacha o’zgaradi va bu uzunligi π ga teng interval $\pm\pi$ nuqtalarni o’z ichiga olmaydi. Shuning uchun $tg\frac{t-\varphi}{2}=z,\ \cos(t-\varphi)=\frac{1-z^2}{1+z^2},\ dt=\frac{dz}{1+z^2}$ almashtirish bajaraylik.

U holda

$$u(r,\varphi)=\frac{1}{\pi}\int_{-tg\frac{\varphi}{2}}^{ctg\frac{\varphi}{2}}\frac{R^2-r^2}{(R-r)^2+(R+r)^2\cdot z^2}dz=\frac{1}{\pi}arctg(\frac{R+r}{R-r}z)\Bigg|_{-tg\frac{\varphi}{2}}^{ctg\frac{\varphi}{2}}=-\frac{1}{\pi}arctg\frac{R^2-r^2}{2Rrsin\varphi}$$

Yoki

$$tg(u\pi)=-\frac{R^2-r^2}{2Rrsin\varphi}$$

Ifodaning o’ng qismi manfiy, demak, $0<\varphi<\pi$ da $u\frac{1}{2}<u<1$ tengsizlikni qanoatlantiradi. Bu hol uchun yechim:

$$tg(\pi - u\pi) = \frac{R^2 - r^2}{2Rrsin\varphi}$$

yoki

$$u = 1 - \frac{1}{\pi} arcgtg \frac{R^2 - r^2}{2Rrsin\varphi} \qquad 0 < \varphi < \pi$$

ga teng.

Agar nuqta quyi yarim aylanada joylashgan bo'lsa: $\pi < \varphi < 2\pi$, u hoda $t - \varphi(-\varphi, \pi - \varphi)$ intervalda o'zgaradi. Bu interval $-\pi$ nuqtani o'zichiga olib, 0 nuqtani esa bu intervalda yotmaydi. Shuning uchun bu yerda

$$ctg\frac{t-\varphi}{2} = z, \qquad \cos(t-\varphi) = \frac{z^2-1}{z^2+1} \quad dt = -\frac{2dz}{1+t^2}$$

U holda φ ning bu qiymatlari uchun:

$$u(r,\varphi) = -\frac{1}{\pi}\int_{-ctg\frac{\varphi}{2}}^{tg\frac{\varphi}{2}} \frac{R^2 - r^2}{(R-r)^2 + (R+r)^2 \cdot z^2} dz$$
$$= -\frac{1}{\pi}\left\{arctg\left(\frac{R-r}{R+r} \cdot tg\frac{\varphi}{2}\right) + arctg\left(\frac{R-r}{R+r} \cdot ctg\frac{\varphi}{2}\right)\right\}$$

Yuqoridagidek almashtirish bajarib

$$u = -\frac{1}{\pi} arcgtg \frac{R^2 - r^2}{2Rrsin\varphi} \qquad \pi < \varphi < 2\pi$$

Ni topamiz. Bu yerda o'ng tomon musbat bo'lganligi uchun $sin\varphi < 0$ dan $0 < u < \frac{1}{2}$

Auditoriya topshirig'i

1. $D = \{(x,y): x^2 + y^2 < R^2\}\}$ doirada ushbu $\Delta u(x,y) = 0,\ u|_\sigma = g(x,y)$ Dirixlening ichki masalasini yeching, bu yerda

$$\sigma = \{(x,y): x^2 + y^2 = R^2,\ \ g(x,y) = x^2 - 2y^2$$

2. $u(x,y) = e^{-y}sinx$ funksiya $u''_{xx} + u''_{yy} = 0$ tenglamaning $0 \le x \le 1,\ \ 0 \le y \le 1$ kvadratda $u(0,y) = 0,\ u(1,y) = e^{-y}sin1u(x,0) = sinx,\ u(x,1) = e^{-1}sinx$ shartlarni qanoatlantiruvchi yechimi ekanini isbot qiling.

3. $u|_{r=1} = 0,\ u|_{r=2} = y$ chegaraviy shartlarni qanoatlantiruvchi $1 \le r \le 2$ halqaning ichki nuqtalari uchun Laplas tenglamasining yechimini toping.

Mustaqil yechish uchun misollar

1) Markazi koordinatalar boshida radiusi a ga teng bo‘lgan doira tashqarisida Laplas tenglamasining$u|_{r=a} = f(\varphi)$ chegaraviy shartni qanoatlantiruvchi, ya’ni Dirixle tashqi masalasi yechimini toping, bunda

1. $u|_{r=a} = Asin\varphi$
2. $u|_{r=a} = Tsin\frac{\varphi}{2}$
3. $u|_{r=a} = \begin{cases} Asin\varphi, & 0 \le \varphi \le \pi \\ \frac{1}{3}Asin^3\varphi, & \pi \le \varphi \le 2\pi \end{cases}$

bu yerda A, T –berilgan o‘zgarmas sonlar.

Kompleks o’zgaruvchili funksiyalar.
Asosiy elementar funksiyalar.

Aytaylik D- **C** kompleks tekislikning to’plam qismi bo’lsin. D sohada aniqlangan $f(z)$ kompleks funksiya deb, $\forall z \in D$ uchun ωning $f(z)$kompleks sonni mos keltiruvchi akslantirish tushuniladi. Agar $z = x + iy,\ \ \omega = u + iv$ bo’lsa, u holda $f(z)$ni

$$f(z) = \mathrm{u(x,y) + iv(x,y)}$$

Kabi ifodalash mumkin, bu yerda $u(x, y) = Re\, f(z)$ - yani $f(z)$ funksiyaning haqiqiy qismi, $\omega(x, y) = Im(z)$- $f(z)$ funksiyaning mavhum qismi.

Kompleks o’zgaruvchili ba’zi bir funksiyalarni keltiraylik:

$$e^{iz} = cosz + isinz, \qquad z \in R\ (Eyler\ funksiyasi)$$

$$sinz = \frac{e^{iz} - e^{-iz}}{2}, \quad cosz = \frac{e^{iz} + e^{-iz}}{2}$$

$$shz = \frac{e^{z} - e^{-z}}{2}, \quad chz = \frac{e^{z} + e^{-z}}{2}$$

$$lnz = ln|z| + iargz \qquad argz \in (-\pi, \pi)$$

$$lnz = ln|z| + iargz = ln|z| + iargz + i2\pi k = lnz + i2\pi k, k \in Z$$

lnz ko’p ma’noli funksiya, unda $\forall z \neq 0$son uchun $\{lnz\}$ to’plamdan mos ravishda cheksiz ko’p qiymat mos keladi.

Auditoriya topshirig’i

1.1.

$f(z) = \mathrm{u(x,y) + iv(x,y)},\ z = x + iy$bo’lsa, funksiya uchun haqiqiy va mavhum qismlarni ajrating.

$a) f(z) = z^2$ $\qquad$ $b) f(z) = e^z$

c) $f(z) = \frac{1}{\bar{z}}$ $\qquad$ $d) f(z) = sinz$

yechish:

$a) z^2 = (x+iy)^2 = x^2 + 2xiy + (iy)^2 = (x^2 - y^2) + i2xy$

$u(x,y) = x^2 - y^2, \quad v(x,y) = 2xy$

$b) \frac{1}{\bar{z}} = \frac{1}{\overline{(x+\imath y)}} = \frac{1}{(x-iy)} = \frac{x+iy}{(x-iy)(x+iy} = \frac{x+iy}{x^2+y^2} = \frac{x}{x^2+y^2} + i\frac{y}{x^2+y^2}, \qquad ya'ni$

$u(x,y) = \frac{x}{x^2+y^2}, \qquad v(x,y) = \frac{y}{x^2+y^2}.$

$c)\ e^z = e^{x+iy} = e^x \cdot e^{iy} = e^x(cosy + isiny) = e^x cosy + ie^x siny, \qquad ya'ni \quad u(x,y)$
$= e^x cosy, \ \ v(x,y) = e^x siny$

$$\begin{aligned} sinz &= \frac{e^{iz} - e^{-iz}}{2i} = \frac{1}{2i}\left(e^{i(x+iy)} - e^{-i(x+iy)}\right) \\ &= \frac{1}{2i}\Big(e^{-y}(cosx + isinx) - e^{y}\big(\cos(-x) + isin(-x)\big)\Big) \\ &= -\frac{i}{2}\big(cosx(e^{-y} - e^{y}) + isinx(e^{-y} + e^{y})\big) = icosx\frac{e^{-y} - e^{y}}{2} + sinx\frac{e^{-y} + e^{y}}{2} \\ &= sinxchy + icosxshy, \end{aligned}$$

$$u(x,y) == sinxch, v(x,y) = cosxshy$$

Mustaqil yechish uchun misollar

Berilgan funksiyalar uchun haqiqiy va mavhum qismlarni ajrating

1.2. z

1.3. iz

1.4. $(\bar{z})^2$

1.5. $z^2 - 2z + i$

1.6. z^3

1.7 $Re\bar{z} + iImz$

1.8 $z + \bar{z}$

1.9. $\frac{1+i}{z-i}$

1.10. $\frac{1}{z^2}$

Funktsiyaning differentsiallanuvchanligi.
Koshi- riman shartlari.

$f(z)$ funksiyaning z_0 nuqtadagi hosilasi deb, quyidagi limitga aytiladi:

$$\lim_{\Delta z\to 0}\frac{f(z_0+\Delta z)-f(z_0)}{\Delta z}=f'(z_0)$$

Teorema $f(z)=u(x,y)+iv(x,y)$ funksiya $z_0=x_0+iy_0$ nuqtada differensiallanuvchi bo'lishi uchun quyidagi shart bajarilashi yetarli:

1) $\frac{\partial u(x,y)}{\partial x}$, $\frac{\partial u(x,y)}{\partial y}$, $\frac{\partial v(x,y)}{\partial x}$, $\frac{\partial u(x,y)}{\partial x}$

Xususiy hosilalar $z_0=x_0+iy_0$ nuqta atrofida mavjud va uzluksiz bo'lishi kerak;

2) $z_0=x_0+iy_0$ nuqtada Koshi –Riman shartlari bajarilishi kerak:

$$\frac{\partial u(x,y)}{\partial x}=\frac{\partial v(x,y)}{\partial y},\qquad \frac{\partial u(x,y)}{\partial y}=-\frac{\partial v(x,y)}{\partial x}$$

Hosila quyidagi teng kuchli formulalar orqali topiladi:

$$f'(z)=\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x}=\frac{\partial v}{\partial y}-i\frac{\partial u}{\partial y}=\frac{\partial u}{\partial x}-i\frac{\partial u}{\partial y}=\frac{\partial v}{\partial y}+i\frac{\partial v}{\partial x}$$

Auditoriya topshirig'i

Misol1

e^zfunksiyaning xosila mavjud bo'lgan nuqtalarini toping.

Yechish:

Ma`lumki $e^z=e^x cosy+ie^x siny$

Ya`ni $u(x,y)=e^x cosy,\ \ v(x,y)=e^x siny$

Xususiy hosilalarni topamiz va ularni Koshi-Riman shartlarini bajarilishini tekshiramiz

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(e^x cosy) = e^x cosy, \quad \frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(e^x siny) = e^x cosy$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(e^x cosy) = -e^x siny, \quad \frac{\partial v}{\partial x} = \frac{\partial}{\partial x}(e^x siny) = e^x siny$$

Ya'ni Koshi-Riman shartlari barcha nuqtalar uchun bajariladi, shu hosilani topamiz:

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = e^x cosy + ie^x siny = e^x(cosy + isiny) = e^z$$

2-misol

$\bar{z}$funksiya hosilasini toping

Yechish: $\bar{z} = x - iy \quad u = x, \; v = -y$

$$\frac{\partial u}{\partial x} = \frac{\partial x}{\partial x} = 1, \; \frac{\partial v}{\partial y} = \frac{\partial(-y)}{\partial y} = -1$$

Ko'rinib turibdiki,

$$\frac{\partial u}{\partial x} = 1 \neq -1 = \frac{\partial v}{\partial y}$$

Ya'ni Koshi-Riman shartlari hech qaysi nuqtalar uchun bajarilmadi, demak hosila mavjud emas.

3-misol

$\frac{1}{z}$ funksiya hosilasini toping.

Yechish:

$f(z) = \frac{1}{z} = \frac{1}{x+iy} = \frac{x-iy}{x^2+y^2}$

Ya'ni$u(x,y) = \frac{x}{x^2+y^2}, \; v(x,y) = \frac{-y}{x^2+y^2}$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right) = \frac{y^2 - x^2}{(x^2+y^2)^2}$$

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial x}\left(\frac{-y}{x^2 + y^2}\right) = \frac{y^2 - x^2}{(x^2 + y^2)^2}$$

$$\frac{\partial u(x,y)}{\partial x} = \frac{\partial v(x,y)}{\partial y},$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\frac{x}{x^2 + y^2}\right) = \frac{-2xy}{(x^2 + y^2)^2}$$

$$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}\left(\frac{-y}{x^2 + y^2}\right) = \frac{2xy}{(x^2 + y^2)^2}$$

$$\frac{\partial u(x,y)}{\partial y} = -\frac{\partial v(x,y)}{\partial x}$$

Bundan berilgan funksiya ihtiyoriy nuqtada differensiallanuvchi ekanligini ko’rsatadi.

Hosilani topaylik:

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{y^2 - x^2}{(x^2 + y^2)^2} + i\frac{2xy}{(x^2 + y^2)^2} = -\frac{1}{(x^2 + y^2)^2} = -\frac{1}{z^2}$$

$$f'(z) = f'\left(\frac{1}{z}\right) = -\frac{1}{z^2}$$

Mustaqil yechish uchun misollar.

Berilgan funksiyalar uchun hosila mavjud bo’lgan nuqtalarni ko’rsating va bu nuqtalarda hosilani toping.

1.$f(Z) = i\bar{z}$

2. $f(Z) = iz^2 - 3z + 1$

3. $f(Z) = z^6$

4. $f(Z) = ln(z^2)$

5. $f(Z) = z + 2i$

6. $f(Z) = zRez$

7. $f(Z) = \frac{1}{z^3}$

8. $f(Z) = sinz$

Kompleks o’zgaruvchili funktsiyaning integrali va uning xossalari.

Kompleks o’zgaruvchili funktsiyaning integrali deb quyidagi limitga aytiladi:

$$\int_l f(z)dz$$

Kabi belgilanadi . Demak

$$\int_\gamma f(z)dz = \lim_{\lambda \to 0} \sum_{k=1}^{n} f(c_k)(z_k - z_{k-1}) \textbf{(1)}$$

$$\int_l f(z)dz = \int_l (u(x,y) + iv(x,y))\, d(x+iy) = \int_l (u(x,y) + iv(x,y))(dx + idy)$$

$$= \int_l (u(x,y)dx - v(x,y)dy) + i\int_l (v(x,y)dx + u(x,y)dy)$$

Teorema

1. Shunday f(z) uchun F(z) boshlang`ich funksiya mavjudki, unda Nyuton-Leybnits formulasi o’rinli

$$\int_l f(z)dz = F(z_2) - F(z_1)$$

Ya’ni, bu integral L chiziqning berilishiga emas, balki boshlang’ich va quyi nuqtalarning joylashuviga bog’liq.

2. Agar L- yopiq bo’lsa u holda Koshi teoremasi o’rinli va

$$\oint_l f(z)dz = 0$$

3. Agar z_0 nuqta L egri chiziqning ichida bo'lsa, Koshi integral formulasi o'rinli

$$f(z_0) = \frac{1}{2\pi i}\oint_l \frac{f(z)}{z - z_0}dz$$

va

$$f^n(z_0) = \frac{n!}{2\pi i}\oint_l \frac{f(z)}{(z - z_0)^{n+1}}dz\,, n = 1,2\,...$$

(egri chiziq bo'ylab soat strelkasiga teskari yo'nalishda o'tilgan).

Auditoriya topshirig'i

1–Misol. Quyidagi integralni hisoblang

$$J = \int_l Imzdz,$$

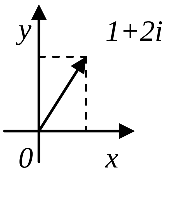
Bu yerda L chiziq

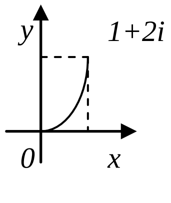
a) 0 nuqtadan $1 + 2i$nuqtagacha bo'lgan to'g'ri chiziq.
b) 0 nuqtadan $1 + 2i$nuqtagacha bo'lgan $y = 2x^2$ parabola yoyi

Yechish:

a)L- $y = 2x$ to'g'ri chiziq kesmasi va $Imz = y$ bo'lganligini e'tiborga olsak, u holda

$$J = \int_l yd(x + iy) = \int_0^1 2xd(x + i2x) = \int_0^1 2x(dx + i2dx) = \int_0^1 2xdx + \int_0^1 2x2idx$$
$$= (2 + 4i)\cdot\frac{x^2}{2}\bigg|_0^1 = 1 + 2i$$





b)L uchu quyidagiga egamiz:

$y = 2x^2$

$$J = \int_l yd(x+iy) = \int_0^1 2x^2 d(x+i2x^2) = \int_0^1 2x^2\big(dx + i2d(x^2)\big) = \int_0^1 2x^2 dx + 6i\int_0^1 x^3 dx$$
$$= 2\cdot\frac{x^3}{3}\Big|_0^1 + 8i\cdot\frac{x^4}{4}\Big|_0^1 = \frac{2}{3} + 2i$$

2–**Misol.**

$I = \int_{-\infty}^{+\infty} \frac{(x-1)\cos 5x}{x^2-2x+5}dx$ xisoblansin.

$$I = -2\pi\,\mathrm{Im}\Big(\sum_{\mathrm{Im}\, z_k>0} \underset{z=z_k}{res}(e^{i\lambda z}R(z))\Big) =$$

$$R(z) = \frac{z-1}{z^2-2z+5}$$
$$z^2 - 2z + 5 = 0$$

bunda, $z_{1,2} = 1 \pm 2i$

$$z_1 = 1+2i, \mathrm{Im}\, z_1 = 2 > 0$$
$$z_2 = 1-2i, \mathrm{Im}\, z_2 = -2 < 0$$

$$= 2\pi\,\mathrm{Im}\Big(\underset{z=1+2i}{res}\frac{e^{5iz}(z-1)}{z^2-2z+5}\Big) = \underset{z=1+2i}{res}\left[\frac{e^{5iz}(z-1)}{(z^2-2z+5)'}\right]_{z=1+2i} = \left[\frac{e^{5iz}(z-1)}{2z-2'}\right]_{z=1+2i} = \frac{e^{-10}}{2}\mathrm{Im}(\cos 5 + i\sin 5) =$$
$$-\pi e^{-10}\sin 5$$

3–**Misol.**

$I = \int_0^{+\infty} \frac{\cos x}{x^2+a^2}dx$ xisoblansin.

$$R(z) = \frac{1}{z^2+a^2}$$

$$\int_{\gamma_R} e^{iz}R(z)dz \underset{R\to\infty}{\to} 0$$

$$\int_{-R}^{+R} e^{ix}R(x)dx + \int_{\gamma_R} e^{iz}R(z)dz = 2\pi i\,\underset{z=ai}{res}(e^{iz}R(z))$$

$$\int_{-\infty}^{+\infty} e^{iz} R(x)dx = 2\pi i \underset{z=ai}{res}(e^{iz} R(z))$$

$$I = \int_{0}^{+\infty} \frac{\cos x}{x^2 + a^2} dx = \frac{1}{2}\int_{-\infty}^{+\infty} \frac{\cos x}{x^2 + a^2} dx = \frac{1}{2}(-2\pi)\,\mathrm{Im}(\underset{z=ai}{res} \frac{e^{iz}}{x^2 + a^2}) = -\pi\,\mathrm{Im}(\frac{e^{-a}}{2ai}) =$$

$$= -\pi\,\mathrm{Im}(-i\frac{e^{-a}}{2a}) = \frac{\pi e^{-a}}{2a} = \frac{\pi}{2ae^{a}}$$

$$\underset{z=ai}{res}\left[\frac{e^{iz}}{(z^2 + a^2)'}\right]_{z=ai} = \left[\frac{e^{iz}}{2z}\right]_{z=ai} = \frac{e^{-a}}{2ai}$$

Mustaqil yechish uchun misollar.

1.

$$J = \int_{l} Imzdz,$$

By yerda:

a) z_1=2 nuqtadan $z_2 = 3$ nuqtagacha bo’lgan to’g’ri chiziq kesmasi.
b) 0 nuqtadan $1 + i$nuqtagacha bo’lgan to’g’ri chiziq kesmasi
c) 0 nuqtadan $1 + i$nuqtagacha bo’lgan $y = x^2$ parabola yoyi

2.

$$J = \int_{l} (z + \bar{z})dz,$$

By yerda:

a) z_1=0 nuqtadan $z_2 = -1 + \mathrm{i}$ nuqtagacha bo’lgan to’g’ri chiziq kesmasi.
b) 0 nuqtadan $-1 + i$nuqtagacha bo’lgan $y = x^2$ parabola yoyi

3.

$$J = \int_{l} |z|dz,$$

By yerda:

a) $|z|=1$ aylananing $z_1=1$ nuqtadan $z_2=-1$ nuqtagacha bo'lgan qismi.
b) 1 nuqtadan -1 nuqtagacha bo'lgan to'g'ri chiziq kesmasi
c) 0 nuqtadan $2-2i$ nuqtagacha bo'lgan to'g'ri chiziq kesmasi

4.

$$J=\int_l \frac{dz}{z},$$

Bu yerda L chiziq $|z|=2$ aylananing $z_1=2$ nuqtadan $z_2=2e^{2\pi i}$ nuqtagacha bo'lgan qismi

Loran qatori

Loran qatori deb, quyidagi qatorga aytiladi:

$$\sum_{n=-\infty}^{+\infty} c_n(z-z_0)^n=\cdots+\frac{c_{-k}}{(z-z_0)^k}+\cdots\frac{c_{-1}}{z-z_0}+c_0+c_1(z-z_0)+\cdots c_n(z-z_0)^n+\cdots$$

Loran qatori 2ta qator yig'indisidan iborat:

$$\sum_{n=-\infty}^{-1} c_n(z-z_0)^n=\cdots+\frac{c_{-k}}{(z-z_0)^k}+\cdots\frac{c_{-1}}{z-z_0}$$

va

$$\sum_{n=0}^{+\infty} c_n(z-z_0)^n=c_0+c_1(z-z_0)+\cdots c_n(z-z_0)^n+\cdots$$

Teorema

Agar f(z) funksiya $0\le r<|z-z_0|<R$ xalqada aniqlangan bo'lsa, uni yaqinlashuvchi Loran qatori ko'rinishida berish mumkin,

$$f(z)=\sum_{n=-\infty}^{+\infty} c_n(z-z_0)^n$$

Shu bilan birga koeffisiyentlar quyidagi ko'rinishga ega

$$c_n=\frac{1}{2\pi i}\int_{|z-z_0|<\rho}\frac{f(z)dz}{(z-z_0)^{n+1}},\quad r<\rho<R, n=0,\pm1,\pm2,\ldots$$

Mustaqil yechish uchun misollar

Funksiyalarni Loran qatoriga yoying va asosiy va to'g'ri qismlarini ajrating, yaqinlashish sohasini aniqlang.

1.$\frac{1}{z}cosz$

a)$z_0 = 0$

b) $z_0 = \infty$

2. $\boldsymbol{\frac{1}{z}sinz}$

a)$z_0 = 0$

b) $z_0 = \infty$

3. $e^{\frac{1}{z+1}}$

a)$z_0 = 0$

b) $z_0 = \infty$

4. $\sin(2 + z)$

a)$z_0 = 0$

b) $z_0 = \infty$

5.$\frac{3+2i}{(z+i)^2}$

a)$z_0 = 0$

b) $z_0 = \infty$

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Mundarija