O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

SAMARQAND DAVLAT UNIVERSITETI

MEXANIKA - MATEMATIKA FAKULTETI

"Differinsial tenglamalar" kafedrasi

Normurodov Hojiniurod Normuminovich

"Oddiy differinsial tenglamaga qo'yilgan Koshi hamda chegaraviy masalalarning yechimini topishda asimptotik usullardan foydalanish"

"5130100- matematika" taMim yo'nalishi bo'yicha

bakalavr dai ajasini olish uchun

BITIRUV MALAKAVIY ISHI

Ilmiy rahbar<: ________^ prof. A. B. Hasanov

Bitiruv maakaviy ishi "Differinsial tenglamalar" kafedrasida bajarildi.

Kafedraning 2018-yil majlisida muhokama etildi va himoyaga tavsiya etildi (_ bayonnoma).

Fakultet dekani. ________ prof. A. XT&egmatov

Kafedra mudiri: ________ prof. A. B. Hasanov

Bitiruv malakaviy ishi YaDAK 2018 - yil "____" iyundagi majlisida himoya va ball bilan baholandi (____- Ьаурштота).

YaDAK raisi: ________ ____________________

A'zolari: ________ ____________________

Samarqand 2018

SAMARQAND DAVLAT UNIVERSITETI

Fakultet: Mexanika- matematika
Yo'nalish: 5130100- matematika

«TASDIQLAYMAN»
O'quv ^J^m^boshlig'i

" 2017_yil

Bitiruv malakaviy ishi bo'yicha
TOPSHIRIQ VARAQASI

1. Ijrochi: Normurodov Hojimurod Normuminovich

2. Mavzu: Oddiy differensial tenglamaga qo'yilgan Koshi hamda chegaraviy masalalarning yechimini topishda asimptotik usullardan foydalanish

Mavzu universitet Ilmiy kengashining 31.10.2017 №3 sonli qarori bilan tasdiqlangan.

3. Mavzuning dolzarbligi, nazariy va amaliy (iqtisodiy,ijtimoiy9 ekologik, ilmiy-texnik, mehnat muhofazasi, hayot xavfsizligi va h.k.) ahamiyati

Amaliy matematikava fizikaning bir qator masalalari. Oddiy differensial tenglamaga qo yilgan Koshi yoki chegaraviy masalalarning taqribiy yechimini topishga keltiriladi. Odatda taqribiy yechimini topishda assimptotik usullardan keng foydalaniladi. Mazkur bitiruv ishida shunday usullardan ayrimlari, jumladan kichikparametrlar usuli bayon qilinadi

4. Bitiruv malakaviy ishini bajarish uchun tavsiya qilinadigan ilmiy, o'quv-uslubiy va boshqa axborot manbalari (darslik,o'quv qo'llanmalari, ma'ruzalar matni, monografiya, ilmiy maqolalar va h.k.)

1. М.В Федорюк Обыкновенные дифференциальные уравнения.

2. А.Ф. Филлипов. Введение в теории дифференциальных уравнений.

3. Эльсгольц Л.Е. Дифференциальные уравнения и вариационное исчиление. М.: Наука.. 1965.

4. Филиппов А.Ф. Сборник задач по дифференциальным уравнениям. М: *Наука,* ***1979*** *(**5-е** издание).*

5. Bitiruv malakaviy ishi bo'yicha ma'lumotlar to'plash hamda tadgigot *iMfon'otib* 6onsfi man6afarf va joyfari (o'quv zali va xonalari, ilmiy kutubxona, laboratoriya, tashkilot, korxona, ilmiy yoki ta'lim muassasasi)

Sam DU o 'quv zali

6. Bitiruv malakaviy ishini tayyorlash bo'yicha amalga oshiriladigan ishlar rejasi

№	**Ishning ma/muni**	**Taxminiy hajmi (bet)**	Ijro muddati	**Izoh**
i.	Masalaning qo'yilishi. Mavzuning dolzarbligi,yechilishi yoki o'rganilishi lozim bo'lgan masalaning mohiyatini va maqsadini yoritib berish (kirish qismi)	*3*	*2017 Sentyabr*	
2.	Mavzu bo'yicha ma'lumotlarni to'plash va tahlil qilish (yordamchi mulohaza va fakilar)	*8*	*2017 Dekabr*	
3.	Olib borilgan tajribalar tadqiqot ishlari natijalarini tahlil qilish, tartibga solish (paragraf,bob,bo'lim yoki rasmlar bo'yicha)	*30*	*2018 Yanvar* *Г 1*	
4.	Olingan natijalarni nazariy va amaliy ahamyati bo'yicha xulosa berish hamda tatbiq sohalari va usullariga oid takliflar tayyorlash	*5*	*Mart*	
5.	Bitiruv ishini rasmiylashtirish va uning himoyasi uchun zarur ko'rgazmali vositalarini (jadvallar, rasmlar,maket,grafiklar,diagramma,stend va h.k.)	*6*	*20 IS* *Л pre/*	
6.	Dastlabki himoyaga tayyorgarlik ko'rish va himoyaga chiqish matnini tayyorlash	*4*	**J** *2018 May*	
7.	Bitruv ishi bo'yicha qo'shimcha maslahatchilar.			

Ilmiy rah bar: ***prof.*** *A.B. Hasanov*

Kafedra mudiri: ***prof*** ***A.B.*** *Hasanov*

«Differensial tenglamalar» kafedrasi yig'ilishining
10-bayonnomasidan

KO CHIRMA

Samarqand shahri. 2018 yil 24-may

Qatnashdilar: Kafedra a'zolari: Kafedra mudiri prof. A.B. Xasanov, fakultet dekani prof. A. Begmatov, dots. Xaydarov A., dots. Ishankulov T., dots. Muxtorov Ya., dots. Xodizoda P.D., dots. Malikov Z., dots. Xoliqulov S.I., dots. Ochilov Z.H., ass. Shodiyev D.S., ass. Tursunov F.R. , ass. Ismoilov A.S., ass Usmanov A.V., ass Burxonov Sh.M. kabinet mudiri XujayevaM.

K U N T A R T I B I :

"Differensial tenglamalar" kafedrasi bo'yicha bitiruv malakaviy ishlarining dastlabki himoyasi.

ESHITILDI: 1. 4-kurs talabasi *Normurodov Hojlnn Normuminovichning "Oddiy differensial tenglamaga qoyilgan Koshi hu' chegaraviy masalalarning yechimini topishda asimptotik usullardan foyda!c ;* mavzusidagi malakaviy bitiruv ishi haqidagi ma'ruzasi.

Bitiruv malakaviy ishi rahbari: *prof A.B. Hasanov*

Bitiruvchi o'z ishining mazmuni va olingan natijalar haqida gapirib berdi.

Muhokamada savollar berildi:

1. Yechimning boshlang'ich shartlarga va parametrlarga bogMiqligi haqidagi teoremani tushuntiring?
2. Ushbu
$$\varepsilon y` = f(x,y), \quad y(x_0,\varepsilon) = y_0, \text{ va } \quad y` = f(x,y,\varepsilon), \quad y(x_0,\varepsilon) = y_0$$

Kichik parametrli Koshi masalalarining $y(x,\varepsilon)$ yechimini $\varepsilon \to +0$ dagi limiti haqida nima deyish mumkin.

3. Chegaraviy qatlam tushunchasi nima?

va tegishli javoblar olindi.

Bitiruv malakaviy bitiruv ishi haqida dots. Ochilov Z.1L, ass. SI D.S.lar o'z fikrlarini bildirib, uni Yakuniy Davlat Attestasiya Nazorati komissiyasida himoya qilish uchun tavsiya etish taklifini kiritdilar.

QAROR QILINDI:1. 4-kurs talabasi *Normurodov Hojimurod Normuminovichning "Oddiy differensial tenglamaga qo yilgan Koshi hamda chegaraviy masalalarning yechimini topishda asimptotik usullardan foydalanish"*"mavzusidagi malakaviy bitiruv ishi Yakuniy Davlat Attestasiya komissiyasida himoya qilishga tavsiya etilsin.

2. Mazkur bitiruv malakaviy ishga taqrizchi qilib Matematik analiz kafedrasi dotsenti A.B. Nematov tayinlansin.

Kafedra mudiri: [rof. A.B. Hasanov

Kabinet mudiri: Xujayeva M

Mexanika - matematika fakulteti matematika yo'nalishi 4-01 - guruh talabasi Normurodov Hojimurodning "Oddiy differinsial tenglamaga qo'yilgan Koshi hamda chegaraviy masalalarning yechimini topishda asimptotik usullardan foydalanish" mavzusidagi bitiruv malakaviy ishiga

MULOHAZA

Tabiatdagi biror fizik jarayonlarni tavsiflovchi differensial tenglama parametrlarga (jumladan massa, elastiklik koeffitsiyentlari va hakoza fizik kattaliklar) bog'liq bo'ladi. Bu parametrlarning qiymatlarini real masalalarda aniq o'lchamini hisoblashning imkoni yo'q, odatda taqribiy hisoblanadi. Ma'lum jarayonni tavsiflovchi differensial tenglamani keltirib chiqarish jarayonida ham xatolikka yoЧ qo'yiladi. Shu maqsadda differensial tenglama real jarayonni tavsiflashi uchun, uning yechimi parametrlarga uzluksiz ravishda bog'liq bo'lishi kerak, ya'ni parametrlarning kichik o'zgarishiga differensial tenglamaning yechimi ham mos ravishda kichik o'zgarishi lozim.

Ushbu bitiruv malakaviy ishi beshta paragrafni o'z ichiga olgan bo'lib, birinchi paragrafda oddiy differinsial tenglamaga qo'yilgan Koshi masalasi haqida umumiy tushunchalar hamda misollar keltirilgan. Ikkinchi paragrafda oddiy differinsial tenglamaga qo'yilgan Koshi masalasining korrektligi haqida ma'lumotlar berilgan. Uchinchi paragrafda differinsial tenglama yechimining parametrlarga va boshlang'ich shartlarga uzluksiz bog'liqligi haqidagi teoremalar keltirilgan bo'lib, misollar yechib ko'rsatilgan. To'rtinchi paragraf "Kichik parametrlar usuli" deb nomlanib, unda kichik parametrga bog'liq differinsial tenglama yechimining parametming cheksiz kichik qiymatlarida, parametr nol bo'lgan vaqtdagi differinsial tenglama yechimiga yaqinlash masalasi qaralgan va misollar yechib ko'rsatilgan. Beshinchi paragraf esa, kichik parametrli Shturm - Liuvill chegaraviy masalasiga bag'ishlangan bo'lib, unda kichik paramertga bog'liq Shutrm - Liuvill chegaraviy masalasining xos qiymatlari va ortanormal xos funksiyalari Relay - Shredinger usuli bilan asimptotasi $o(r^3)$ aniqlikda keltirilib chqarilgan.

Normurodov Hojimurod tomonidan tayyorlangan bitiruv malakaviy ishi qo'yilgan barcha talablarga to'liq javob beradi deb hisoblayman. Bitiruv malakaviy ishini "a'lo" baho bilan baholash mumkin.

Ilmiy rahbar: **prof*. A. 13. Hasanov**

SamDU Mexanika - matematika fakulteti matematika yo'nalishi 4-01 - guruh talabasi Normurodov Hojimurodning "Oddiy differinsial tenglamaga qo'yilgan Koshi hamda chegaraviy masalalarning yechimini topishda asimptotik usullardan foydalanish" mavzusidagi bitiruv malakaviy ishiga

TAQRIZ

Muhim ahamiyatga ega bo'lgan ko'pgina amaliy masalalar differinsial tenglamalar yordamida yechiladi. Ayrim hollarda bunday masalalarning aniq yechimini topishning imkoni yo'q.

Ushbu bitiruv malakaviy ishida asimptotik usullardan foydalangan holda oddiy differinsial tenglamaga qo'yilgan Koshi hamda chegaraviy masalalarning yechimini toppish algoritimi o'rganilgan.

Bitiruv malakaviy ishida differinsial tenglama yechimining parametrlarga va boshlang'ich shartlarga uzluksiz bog'liqligi haqidagi tasdiqlaming isbotlari to'liq keltirilib, ularga doir misollar yechib ko'rsatilgan. Bundan tashqari kichik parametrga bog'liq differnsial tenglama yechimining parameter darajalari bo'yicha Teylor formulasiga yoyish haqidagi teorema keltirilgan va misollar yechib ko'rsatilgan. So'ngra parametr aynan nol bo'lgan holdagi differinsial tenglama yechimiga yaqinlashish masalasi ham qaralgan. Shu bilan bir qatorda, ushbu bitiruv malakaviy ishida, kichik parametrli Shturm - Liuvill chegaraviy masalasining \os qiymatlari va ortanormal xos funksiyalari Relay - Shredinger usuli orqali $o(r^3)$ aniqlikda topilgan.

Normurodov Hojimurod tomonidan tayyorlangan bitiruv malakaviy ishi, qo'yilgan barcha talablarga to'liq javob beradi va talaba Normurodov Hojimurod "Matematika" bakalavr darajasini olishga loyiq deb hisoblayman.

Matematik analiz kafedrasi

katta o'qitvchisi: **f-m. f-n. A. B. Ne'matov**

MUNDARIJA

Kirish

Mavzuning dolzarbligi. Biror fizik jarayonni tavsiflovchi differensial tenglama parametrlarga (jumladan massa, elastiklik koeffitsiyentlari va hakoza fizik kattaliklar) bog'liq bo'ladi. Bu parametrlarning qiymatlarini real masalalarda aniq o'lchamini hisoblashning imkoni yo'q, odatda taqribiy hisoblanadi. Ma'lum jarayonni tavsiflovchi differensial tenglamani keltirib chiqarish jarayonida ham xatolikka yo'l qo'yiladi. Shu maqsadda differensial tenglama real jarayonni tavsiflashi uchun, uning yechimi parametrlarga uzluksiz ravishda bog'liq bo'lishi kerak, ya'ni parametrlarning kichik o'zgarishiga differensial tenglamaning yechimi ham mos ravishda kichik o'zgarishi lozim.

Mazkur bitiruv malakaviy ishida avvalo, hosilaga nisbatan yechilgan birinchi tartibli oddiy differensial tenglamaga qo'yilgan Koshi masalasining korrektligi ko'rsatilgan. So'ngra u masala yechimining parametrga va boshlang'ich shartga nisbatan silliqligi o'rganilgan. Bundan keyin kichik parametrli birinchi tartibli differinsial tenglamaga qo'yilgan Koshi masalasini yechish usullari bayon qilingan. Shu bilan bir qatorda kichik parametrli Shturm – Liuvill chegaraviy masalasi ham o'rganilgan bo'lib, yuqorida zikir etilgan masalalarga doir misollar ham keltirilgan.

Ishning maqsad va vazifalari. Mazkur bitiruv malakaviy ishida oddiy differinsial tenglamalarga qo'yilgan Koshi va chegaraviy masalalarni yechishda kichik parametrlar usulidan foydalaniladi. Bu usulning afzalliklari bir nechta misollarni yechish jarayonida namoyon qilinadi.

Ishning predmeti va obyekti. Bitiruv maakaviy ishida "Oddiy differinsial tenglamalar" va "Matematik analiz" fanlari predmetlaridan foydalanildi.

Ishning ilmiy ahamiyati. Mazkur bitiruv malakaviy ishida bayon qilingan tasdiqlar "amaliy matematika" va " mexanika" ning bir qator masalalarini o'rganishda keng qo'llaniladi.

1-§. Koshi masalasi yechimining mavjudligi va yagonaligi.

Hosilaga nisbatan yechilgan

$$y' = f(x,y) \qquad (1)$$

differensial tenglamaning

$$y(x_0) = y_0 \qquad (2)$$

boshlang'ich shartni qanoatlantiruvchi $y = y(x)$ yechimini topishga Koshi masalasi deyiladi.

Teorema-1 (Koshi). Agar (1) differensial tenglamadagi $f(x,y)$ funksiya

$$P = \{(x,y) \in R^2 : |x - x_0| \le a, \ |y - y_0| \le b\}$$

to'g'ri to'rtburchakda aniqlangan va uzluksiz bo'lib, $y -$o'zgaruvchi bo'yicha Lipshits shartini, ya'ni $\forall (x, y_j) \in P, \ j = 1,2$ nuqtalar uchun shunday $\exists N > 0$ soni topilib

$$|f(x,y_1) - f(x,y_2)| \le N|y_1 - y_2| \qquad (3)$$

tengsizlikni qanoatlantirsa, u holda shunday h soni topilib, (1)-(2) Koshi masalasining $[x_0 - h, x_0 + h]$ oraliqda aniqlangan va (2) boshlang'ich shartni qanoatlantiruvchi yagona $y = \varphi(x)$ yechimi mavjud. Bu yerda

$$h = \min\left(a, \frac{b}{M}\right), \ M = \max_{(x,y)\in P} |f(x,y)|. \qquad (4)$$

Izoh-1. Agar $f(x,y)$ funksiya P sohaning har bir nuqtasida $f'_y(x,y)$ xususiy hosilaga ega bo'lib,

$$|f'_y(x,y)| \le C, \ C = const$$

shartni qanoatlantirsa, u holda $f(x,y)$ funksiya P to'g'ri to'rtburchakda $y -$ o'zgaruvchi bo'yicha Lipshits shartini qanoatlantiradi.

Haqiqatan ham ixtiyoriy ikki $(x,y_1), (x,y_2) \in P$ nuqtalar uchun Lagranj teoremasiga asosan quyidagi

$$|f(x,y_1) - f(x,y_2)| = f'_y(x, y_1 + \theta(y_2 - y_1)) \cdot (y_2 - y_1)$$

munosabat bajariladi. Bu yerda $0 < \theta < 1$.

Oxirgi munosabatdan va $f'_y(x,y)$ xususiy hosilaning chegaralanganligidan (3) tengsizlik kelib chiqadi.

Ammo, ba'zi hollarda hosilaga ega bo'lmagan funksiyalar ham (3) Lipshits shartini qanoatlantiradi.

Masalan. Ushbu $f(x,y) = |y|$ funksiya $y = 0$ nuqtada $((x, 0))$ hosilaga ega emas, lekin

$$|f(x,y_1)-f(x,y_2)|=||y_1|-|y_2||\le|y_1-y_2|$$

o'rinli. Bunda Lipshits o'zgarmasi $N=1$ bo'ladi.

Misol-1. Ushbu

$$y'=3y^{\frac{2}{3}},\ y(1)=0$$

Koshi masalasining yechimini toping.

Yechish. Berilgan differensial tenglamada o'zgaruvchilarni ajratib quyidagi

$$\frac{1}{3}y^{-\frac{2}{3}}dy=dx,\ \frac{1}{3}\int y^{-\frac{2}{3}}dy=\int dx,\ y^{\frac{1}{3}}=x+C,\ \ y(x)=(x+C)^3,\ C=const$$

yechimni topamiz. Boshlang'ich shartdan foydalanib,

$$y(1)=0,\ (1+C)^3=0,\ C=-1$$

berilgan Koshi masalasining

$$y(x)=(x-1)^3$$

yechimini topamiz. Bundan tashqari, qaralayotgan Koshi masalasi $y(x)=0$ yechimga ega. Demak, berilgan Koshi masalasi ikkita

$$y(x)=(x-1)^3,\ y(x)=0$$

yechimga ega ekan. Bundan ko'rinadiki, berilgan differensial tenglamaning o'ng tomonidagi

$$f(x,y)=3y^{\frac{2}{3}}$$

funksiya (3) - Lipshits shartini qanoatlantirmaydi. Chunki

$$f'_y\Big|_{y=0}=\frac{2}{\sqrt[3]{y}}\Big|_{y=0}=+\infty.$$

Shuning uchun ham berilgan Koshi masalasining yechimi yagona emas.

Misol-2. Ushbu

$$y'=\sqrt[3]{y},\ y(0)=0$$

Koshi masalasining yechimini toping.

Yechish. Berilgan differensial tenglamada o'zgaruvchilarni ajratib, uning umumiy yechimini topamiz:

$$y^{-\frac{1}{3}}dy=dx,\ \int y^{-\frac{1}{3}}dy=\int dx,\ y^{\frac{2}{3}}(x)=\frac{2}{3}(x+C),\ C=const.$$

Endi boshlang'ich shartdan foydalanib $C-$o'zgarmasning qiymatini topamiz:

$$y(0)=0,\ 0=\frac{2}{3}(0+C),\ C=0,$$

$$y^{\frac{2}{3}}(x)=\frac{2}{3}x,\ y(x)=\left(\frac{2x}{3}\right)^{\frac{3}{2}},\ x>0\,.$$

Ushbu

$$y(x)=\left(\frac{2x}{3}\right)^{\frac{3}{2}},\ x>0$$

funksiya berilgan Koshi masalasining yechimidan iborat bo’lar ekan. Bundan tashqari $y(x)=0$ funksiya ham berilgan Koshi masalasining yechimi bo’ladi. Demak, berilgan Koshi masalasi ikkita yechimga ega ekan. Chunki, $f(x,y)=\sqrt[3]{y}$ funksiya $(x,0)$ nuqtaning atrofida Lipshits shartini qanoatlantirmaydi. Shuning uchun yechimning yagonaligi buziladi.

Quyidagi misolga e’tibor qarataylik.

Misol-3. Ushbu

$$y'=\frac{1}{y^2},\ y(x_0)=0$$

Koshi masalasining yechimini toping.

Yechish. Bu misolda ham, berilgan differensial tenglamada o’zgaruvchilarni ajratib, uning umumiy yechimini topamiz:

$$y^2dy=dx,\ \int y^2dy=\int dx,\ y^3(x)=3(x+C),\ y(x)=\sqrt[3]{3(x+C)}.$$

Boshlang’ich shartdan foydalanib $C-$o’zgarmasning qiymatini aniqlaymiz:

$$y(x_0)=0,\ x_0+C=0,\ C=-x_0.$$

Bundan ko’rinadiki, ushbu

$$y(x)=\sqrt[3]{3(x-x_0)}$$

funksiya berilgan Koshi masalasining yagona yechimidan iborat bo’ladi.

Shuni alohida qayd qilish lozimki, berilgan Koshi masalasidagi

$$f(x,y)=\frac{1}{y^2},\ f'_y(x,y)=-\frac{2}{y^3}$$

funksiyalarning $(x_0,0)$ nuqtada uzluksizligi buziladi. Ammo, berilgan Koshi masalasi yagona yechimga ega. Demak, Koshi teoremasidagi shartlar Koshi masalasi yechimi mavjud va yagona bo’lishi uchun yetarli shartlardir. Koshi masalasi yechimining yagonaligidan $f(x,y)$ unksiyaning uzluksizligi va y o’zgaruvchi bo’yicha Lipshits shartini qanoatlantirishi kelib chiqmaydi.

Lemma-1 (Gronuolla). Faraz qilaylik, $[x_0,x]$ kesmada $u(x),\ v(x)$ funksiyalar uzluksiz va manfiy bo’lmasin. Agar ular uchun, ushbu

$$u(x) \le A + \left| \int_{x_0}^{x} u(t)v(t)dt \right|, \quad A \ge 0 \tag{5}$$

baho o'rinli bo'lsa, u holda

$$u(x) \le A \exp\left\{ \left| \int_{x_0}^{x} v(t)dt \right| \right\} \tag{6}$$

tengsizlik bajariladi.

Isbot. Aytaylik, $A > 0,\ x \ge x_0$ bo'lsin. U holda (5) tengsizlikda modul ishorasini tashlab va uni $v(x)$ ga ko'paytirsak,

$$\frac{u(x)v(x)}{A + \int_{x_0}^{x} u(t)v(t)dt} \le v(x) \tag{7}$$

hosil bo'ladi. Oxirgi (7) tengsizlikni ushbu

$$\frac{d}{dx}\left(A + \int_{x_0}^{x} u(t)v(t)dt \right) = u(x)v(x)$$

munosabatdan foydalanib

$$\frac{d\left(A + \int_{x_0}^{x} u(t)v(t)dt \right)}{A + \int_{x_0}^{x} u(t)v(t)dt} \le v(x)dx$$

ko'rinishda yozish mumkin. Bu tengsizlikning ikkala tomonini integrallab

$$\ln\left(A + \int_{x_0}^{x} u(t)v(t)dt \right) - \ln A \le \int_{x_0}^{x} v(t)dt$$

munosabatni hosil qilamiz. Bundan

$$A + \int_{x_0}^{x} u(t)v(t)dt \le A \exp\left\{ \int_{x_0}^{x} v(t)dt \right\} = A \exp\left\{ \left| \int_{x_0}^{x} v(t)dt \right| \right\}$$

kelib chiqadi, lemma shartidagi (13) tengsizlikka asosan

$$u(x) \le A + \left| \int_{x_0}^{x} u(t)v(t)dt \right| = A + \int_{x_0}^{x} u(t)v(t)dt \le A \exp\left\{ \left| \int_{x_0}^{x} v(t)dt \right| \right\}$$

baho hosil bo'ladi. Bu baho $A > 0,\ x < x_0$ larda ham o'rinli. Chunki $x < x_0$ larda (13) tengsizlikni quyidagi

$$u(x) \leq A - \int_{x_0}^{x} u(t)v(t)dt = A + \int_{x}^{x_0} u(t)v(t)dt$$

ko'rinishda yozish mumkin. Bundan ham

$$u(x) \leq A \exp\left\{\left|\int_{x}^{x_0} v(t)dt\right|\right\} = A \exp\left\{\left|\int_{x_0}^{x} v(t)dt\right|\right\}$$

kelib chiqadi.

Agar $A = 0$ bo'lsa, u holda $u(x) \equiv 0$ bo'ladi. Haqiqatan ham

$$u(x) \leq \varepsilon + \left|\int_{x_0}^{x} u(t)v(t)dt\right|, \ \forall \varepsilon > 0$$

bo'lsa, (14) dan

$$u(x) \leq \varepsilon \exp\left\{\left|\int_{x_0}^{x} v(t)dt\right|\right\}$$

bahoga ega bo'lamiz. Bundan $\varepsilon \to +0$ da $u(x) \leq 0$ tengsizlikni olamiz, bu esa $u(x) \geq 0$ shartga zid. Shuning uchun $u(x) \equiv 0$. ■

2-§. Koshi masalasining korrektligi.

Quyidagi

$$\frac{dy}{dx}=f_1(x,y),\ y(x_0)=y_0^{(1)}, \tag{1}$$

$$\frac{dy}{dx}=f_2(x,y),\ y(x_0)=y_0^{(2)} \tag{2}$$

Koshi masalalarini qaraylik. Aytaylik, $y_j(x),\ j=1,2,\quad x\in[x_0-h,x_0+h]$ funksiyalar bu Koshi masalalarining yechimlaridan iborat bo'lsin. Bu yerda

$$h=\min\left(a;\frac{b}{M}\right),\ M_j=\max_P\left|f_j(x,y)\right|,\ j=1,2,\ M=\max(M_1;M_2).$$

Ta'rif-1. Agar $\forall\varepsilon>0$ soni uchun $\exists\delta>0$ soni topilib ushbu

$$|f_1(x,y)-f_2(x,y)|<\delta,\qquad \left|y_0^{(1)}-y_0^{(2)}\right|<\delta \tag{3}$$

tengsizliklari bajarilganda

$$|y_1(x)-y_2(x)|<\varepsilon,\ |x-x_0|\le h \tag{4}$$

baho o'rinli bo'lsa, Koshi masalasi korrekt deyiladi.

Teorema-1. Aytaylik $P=\{(x,y)\in R^2:|x-x_0|\le a,\ |y-y_0|\le b\}$ sohada $f_1(x,y)$ va $f_2(x,y)$ funksiyalar uzluksiz bo'lib, y o'zgaruvchi bo'yicha Lipshits shartini qanoatlantirsin. U holda $\frac{dy}{dx}=f(x,y),\ y(x_0)=y_0$ Koshi masalasi korrekt bo'ladi.

Endi, ushbu

$$\frac{dy}{dx}=f(x,y),\ y(x_0)=y_0 \tag{5}$$

Koshi masalasining $y=y(x,x_0,y_0)$ yechimini boshlang'ich shartga uzluksiz bog'liqligini o'rganamiz. Buning uchun quyidagi Koshi masalasini ham qaraymiz:

$$\frac{dy}{dx}=f(x,y),\ y(x_0)=\bar{y}_0. \tag{6}$$

Ta'rif-2. Agar $\forall\varepsilon>0$ soni uchun $\exists\delta>0$ soni topilib $|y_0-\bar{y}_0|<\delta$ tengsizligi bajarilganda

$$|y(x)-\bar{y}(x)|<\varepsilon,\ \forall x\in[x_0-h,\ x_0+h]$$

baho o'rinli bo'lsa, (5) Koshi masalasining yechimi boshlang'ich shartga uzluksiz ravishda bog'liq deyiladi.

Teorema-1 dan quyidagi natijalar kelib chiqadi.

Natija-1. Agar $f(x,y)$ funksiya Koshi teoremasining shartlarini qanoatlantirsa, u holda (5) masalaning yechimi boshlang'ich shartga uzluksiz ravishda bog'liq bo'ladi.

Aytaylik, (1), (2) masalalarda $y_0^{(1)} = y_0^{(2)}$ bo'lsin, u holda

$$\frac{dy}{dx} = f_1(x,y), \;\; y(x_0) = y_0,$$

$$\frac{dy}{dx} = f_2(x,y), \;\; y(x_0) = y_0$$

munosabatlarga ega bo'lamiz.

Natija-2. Agar $f_j(x,y),\ j=1,2$ funksiyalar Koshi teoremasining shartlarini qanoatlantirsa, u holda Koshi masalasining yechimi differensial tenglamaning o'ng tomoniga nisbatan uzluksiz ravishda bog'liq bo'ladi.

3-§. Differensial tenglama yechimining parametrlarga va boshlang'ich shartlarga bog'liqligi.

Biror fizik jarayonni tavsiflovchi differensial tenglama parametrlarga (jumladan massa, elastiklik koeffitsiyentlari va hakoza fizik kattaliklar) bog'liq bo'ladi. Bu parametrlarning qiymatlarini real masalalarda aniq o'lchamini hisoblashning imkoni yo'q, odatda taqribiy hisoblanadi. Ma'lum jarayonni tavsiflovchi differensial tenglamani keltirib chiqarish jarayonida ham xatolikka yo'l qo'yiladi.

Shuning uchun differensial tenglama real jarayonni tavsiflashi uchun, uning yechimi parametrlarga uzluksiz ravishda bog'liq bo'lishi kerak, ya'ni parametrlarning kichik o'zgarishiga differensial tenglamaning yechimi ham mos ravishda kichik o'zgarishi lozim.

Teorema-1. Agar $f(x,y,\lambda)$ funksiya

$$P=\left\{(x,y,\lambda)\in R^3 : |x-x_0|\le a, |y-y_0|\le b \ |\lambda-\lambda_0|\le c\right\}$$

sohada aniqlangan uzluksiz bo'lib, uzluksiz $f'_y(x,y,\lambda)$ va $f'_\lambda(x,y,\lambda)$ hosilalarga ega bo'lsa, u holda ushbu

$$y'=f(x,y,\lambda),\ y(x_0,\lambda)=y_0 \tag{1}$$

Koshi masalasining $y=y(x,\lambda)$ yechimi uchun quyidagi tasdiqlar o'rinli:

1. (x,λ) -o'zgaruvchilarning uzluksiz funksiyasidan iborat bo'ladi.

2. $\dfrac{\partial y(x,\lambda)}{\partial\lambda}=u(x,\lambda)$ uzluksiz funksiya bo'lib,

$$\frac{du}{dx}=\frac{\partial f(x,y,\lambda)}{\partial y}u+\frac{\partial f(x,y,\lambda)}{\partial\lambda},\ u(x_0,\lambda)=0$$

Chiziqli differinsial tenglamani qanoatlantiradi. Bunda

$$|x-x_0|\le h, h=\min\left(a,\frac{b}{M}\right),\ M=\max_P|f(x,y,\lambda)|.$$

Isbot. 1. Ixtiyoriy $\forall\lambda_1,\lambda_2\in[\lambda_0-c,\lambda_0+c]$ nuqtalarni olib, quyidagi

$$y'=f(x,y,\lambda_1),\ y(x_0,\lambda_1)=y_0 \tag{2}$$

$$y'=f(x,y,\lambda_2),\ y(x_0,\lambda_2)=y_0 \tag{3}$$

Koshi masalalarini qaraylik. Shu bilan bir qatorda, ularning yechimlarini mos ravishda $y(x,\lambda_1)$ va $y(x,\lambda_2)$ orqali belgilaylik.

Teorema shartiga ko'ra, $f'_y(x,y,\lambda)$ va $f'_\lambda(x,y,\lambda)$ funksiyalar P sohada uzluksiz bo'lganliklari uchun shunday $\exists N_1>0,\ N_2>0$ sonlari topilib,

$$|f'_y(x,y,\lambda)|\le N_1,\ |f'_\lambda(x,y,\lambda)|\le N_2,$$

tengsizliklar o'rinli bo'ladi. Bu munosabatlardan foydalanib quyidagi

$$\left|f(x,y_1,\lambda)-f(x,y_2,\lambda)\right|=\left|f'_y(x,y_1+\theta(y_2-y_1),\lambda)\right|\cdot\left|y_1-y_2\right|\le N_1\left|y_1-y_2\right|,$$

$$0<\theta<1,\ \ \forall(x,y_1,\lambda),(x,y_2,\lambda)\in P$$

$$\left|f(x,y,\lambda_1)-f(x,y,\lambda_2)\right|=\left|f'_\lambda(x,y,\lambda_1+\theta(\lambda_2-\lambda_1))\right|\cdot\left|\lambda_1-\lambda_2\right|\le N_2\left|\lambda_1-\lambda_2\right|,$$

$$\forall(x,y,\lambda_1),(x,y,\lambda_2)\in P$$

baholarni olamiz. Ushbu

$$y(x,\lambda_1)=y_0+\int_{x_0}^{x}f(t,y(t,\lambda_1),\lambda_1)dt$$

$$y(x,\lambda_2)=y_0+\int_{x_0}^{x}f(t,y(t,\lambda_2),\lambda_2)dt$$

integral tenglamalardan foydalanib

$$\left|y(x,\lambda_1)-y(x,\lambda_2)\right|$$

ayirmani baholaymiz:

$$\left|y(x,\lambda_1)-y(x,\lambda_2)\right|=\left|\int_{x_0}^{x}f(t,y(t,\lambda_1),\lambda_1)dt-\int_{x_0}^{x}f(t,y(t,\lambda_2),\lambda_2)dt\right|\le$$

$$\le\left|\int_{x_0}^{x}\left|f(t,y(t,\lambda_1),\lambda_1)-f(t,y(t,\lambda_2),\lambda_2)\right|dt\right|\le\left|\int_{x_0}^{x}\left|f(t,y(t,\lambda_1),\lambda_1)-f(t,y(t,\lambda_2),\lambda_1)\right|dt\right|+$$

$$+\left|\int_{x_0}^{x}\left|f(t,y(t,\lambda_2),\lambda_1)-f(t,y(t,\lambda_2),\lambda_2)\right|dt\right|\le N_1\left|\int_{x_0}^{x}\left|y(t,\lambda_1)-y(t,\lambda_2)\right|dt\right|+$$

$$+N_2\left|\lambda_1-\lambda_2\right|\cdot\left|x-x_0\right|\le N_2\left|\lambda_1-\lambda_2\right|\cdot h+N_1\left|\int_{x_0}^{x}\left|y(t,\lambda_1)-y(t,\lambda_2)\right|dt\right|$$

Demak $\left|y(x,\lambda_1)-y(x,\lambda_2)\right|$ funksiya quyidagi

$$\left|y(x,\lambda_1)-y(x,\lambda_2)\right|\le N_2\left|\lambda_1-\lambda_2\right|\cdot h+N_1\left|\int_{x_0}^{x}\left|y(t,\lambda_1)-y(t,\lambda_2)\right|dt\right|$$

tengsizlikni qanoatlantirar ekan. Bunda, quyidagi

$$u(x)=\left|y(x,\lambda_1)-y(x,\lambda_2)\right|,\ v(x)=N_1,\ A=N_2\cdot\left|\lambda_1-\lambda_2\right|\cdot h$$

belgilashni olib, Gronuolla tengsizligidan foydalansak

$$\left|y(x,\lambda_1)-y(x,\lambda_2)\right|\le N_2\left|\lambda_1-\lambda_2\right|\cdot he^{N_1h}$$

baho hosil bo'ladi. Agar ixtiyoriy $\forall\varepsilon>0$ soni uchun $\delta(\varepsilon)>0$ sonini ushbu

$$\delta=\frac{\varepsilon e^{-N_1h}}{N_2h}$$

ko'rinishda tanlasak, u holda $|\lambda_1 - \lambda_2| < \delta$ tengsizligi bajarilganda

$$|y(x,\lambda_1) - y(x,\lambda_2)| < N_2 \delta h e^{N_1 h} = \varepsilon,\ \forall x \in [x_0 - h, x_0 + h],\ \forall \lambda_1, \lambda_2 \in [\lambda_0 - c, \lambda_0 + c]$$

bahoning o'rinli bo'lishi kelib chiqadi. Bu esa $y(x,\lambda)$ yechimning (x,λ) o'zgaruvchilarga nisbatan uzluksiz ekanligini bildiradi. Teoremaning birinchi qismi isbotlandi.

2. Aytaylik $y = y(x,\lambda)$ (1) masalaning yechimi bo'lsin. U holda $y = y(x, \lambda + \Delta\lambda)$ funksiya ushbu

$$\frac{dy(x,\lambda+\Delta\lambda)}{dx} = f(x, y(x,\lambda+\Delta\lambda), \lambda+\Delta\lambda),\ y(x_0, \lambda+\Delta\lambda) = y_0 \tag{4}$$

Koshi masalasining yechimi bo'ladi. $y = y(x,\lambda)$-yechimning orttirmasi $\Delta y(x,\lambda) = y(x,\lambda+\Delta\lambda) - y(x,\lambda)$ bo'lgani uchun hamda

$$\frac{dy(x,\lambda)}{dx} = f(x, y(x,\lambda), \lambda),\ \ y(x_0,\lambda) = y_0 \tag{5}$$

o'rinli ekanligini inobatga olib ushbu

$$\frac{d[y(x,\lambda+\Delta\lambda) - y(x,\lambda)]}{dx} = f(x, y(x,\lambda+\Delta\lambda), \lambda+\Delta\lambda) - f(x, y(x,\lambda), \lambda)$$

tenglikka ega bo'lamiz. Bu tenglikni

$$\frac{d\Delta y(x,\lambda)}{dx} = f(x, y(x,\lambda+\Delta\lambda), \lambda+\Delta\lambda) - f(x, y(x,\lambda), \lambda) \tag{6}$$

$$\Delta y(x_0, \lambda) = 0 \tag{6'}$$

ko'rinishda yozish mumkin. Adamar lemmasiga (M.V. Fidaryuk "Обыкновенные дифференсиальные уравнения" kitobining 106-108 betlari) ko'ra (6) tenglamaning o'ng tomonini quyidagicha yozish mumkin:

$$\frac{d\Delta y(x,\lambda)}{dx} = F\Delta y(x,\lambda) + G\Delta\lambda$$

ya'ni

$$\frac{d}{dx}\left(\frac{\Delta y(x,\lambda)}{\Delta\lambda}\right) = F\frac{\Delta y(x,\lambda)}{\Delta\lambda} + G. \tag{7}$$

Ushbu $y(x,\lambda)$ va $y(x,\lambda+\Delta\lambda)$ funksiyalar bir xil (bitta) boshlang'ich shartlarni qanoatlantirgani uchun

$$\left.\frac{\Delta y(x,\lambda)}{\Delta\lambda}\right|_{x=x_0} = 0 \tag{7'}$$

shartga ega bo'lamiz. (7) tenglamaning o'ng tomoni x, $\Delta\lambda$ o'zgaruvchilar bo'yicha uzluksiz va $\dfrac{\Delta y}{\Delta\lambda}$ o'zgaruvchiga nisbatan uzluksiz differensiallanuvchi bo'lgani uchun Adamar lemmasiga asosan F, G funksiyalar ushbu $\frac{\partial f}{\partial y}, \frac{\partial f}{\partial \lambda}$ uzluksiz funksiyalarning integralidan iborat. Yechimning parametrlarga uzluksiz bog'liqligidan $\frac{\Delta y}{\Delta \lambda}$ funksiya kichik $|\Delta\lambda|$ larda uzluksiz. Shuning uchun quyidagi chekli limit mavjud:

$$\lim_{\Delta\lambda\to 0}\frac{\Delta y(x,\lambda)}{\Delta\lambda}=\frac{\partial y(x,\lambda)}{\partial\lambda}\equiv u(x,\lambda)$$

Yana Adamar lemmasiga asosan

$$\lim_{\Delta\lambda\to 0}F=\frac{\partial f(x,y,\lambda)}{\partial y},\ \lim_{\Delta\lambda\to 0}G=\frac{\partial f(x,y,\lambda)}{\partial\lambda}$$

munosabatlarga ega bo'lamiz. Demak, $u(x,\lambda)=\frac{\partial y(x,\lambda)}{\partial\lambda}$ hosila quyidagi

$$\frac{du}{dx}=\frac{\partial f(x,y,\lambda)}{\partial y}u+\frac{\partial f(x,y,\lambda)}{\partial\lambda} \tag{8}$$

differensial tenglamani va

$$u(x_0,\lambda)=\left.\frac{\partial y(x,\lambda)}{\partial\lambda}\right|_{x=x_0}=0 \tag{8'}$$

boshlang'ich shartni qanoatlantirar ekan.

Misol-1. Quyidagi

$$\frac{dy}{dx}=y^2+4\lambda x+\lambda^2,\ \ y(1,\lambda)=2\lambda-1 \tag{9}$$

masala yechimining $y'_\lambda(x,\lambda)$ hosilasini $\lambda=0$ nuqtadagi qiymatini toping.

Yechish. Bu holda (8) tenglama ushbu

$$\frac{du}{dx}=2y\cdot u+4x+2\lambda,\ u(1)=2 \tag{10}$$

ko'rinishni oladi. Bu yerda $u(x,\lambda)=y'_\lambda(x,\lambda)$. Agar $\lambda=0$ bo'lsa, (9) masala quyidagi

$$\frac{dy}{dx}=y^2,\ \ y(1,0)=-1$$

ko'rinishni oladi. Bu Koshi masalasini yechib $y=-\frac{1}{x}$ funksiyani topamiz. Bundan foydalanib (10) masalani $\lambda=0$ da

$$\frac{du}{dx}=-\frac{2}{x}u+4x,\ u(1)=2$$

ko'rinishda yozish mumkin. Hosil bo'lgan chiziqli tenglamani yechib

$$u=x^2+x^{-2}$$

ya'ni

$$y'_\lambda(x,\lambda)\big|_{\lambda=0} = x^2 + x^{-2}$$

funksiyani topamiz.

Misol-2. Quyidagi

$$y' = \lambda(1-x) + y - y^2,\ y(0,\lambda) = 0 \tag{11}$$

Koshi masalasi yechimining $y'_\lambda(x,\lambda)$ hosilasini $\lambda = 0$ nuqtadagi qiymatini toping.

Yechish. Qaralayotgan holda (8) tenglama

$$\frac{du}{dx} = [1-2y]u + 1 - x,\ u\big|_{x=0} = 0 \tag{12}$$

ko'rinishni oladi. $\lambda = 0$ holda (11) masala

$$y' = y - y^2,\ y(0) = 0$$

ko'rinishda bo'lgani uchun $y = y(x,0) = 0$ bo'ladi. Bundan foydalanib (12) tenglama

$$\frac{du(x,0)}{dx} = u(x,0) + 1 - x,\ u\big|_{x=0} = 0$$

ko'rinishga keladi. Chiziqli tenglamani yechib

$u(x,0) = x$, ya'ni $y'_\lambda(x,0) = u(x,0) = x$

ekanligini topamiz.

Endi, ushbu

$$y' = f(x,y),\ \ y(x_0) = y_0 \tag{13}$$

Koshi masalasining $y = \varphi(x,x_0,y_0)$ yechimini x_0, y_0 boshlang'ich shartga nisbatan silliqligini o'rganamiz.

Teorema-2. Aytaylik $f(x,y)$ va $f'_y(x,y)$ funksiyalar

$$G = \left\{(x,y)\in R^2 : |x-x_0| \le a, |y-y_0| \le b\right\}$$

sohada uzluksiz bo'lsin. U holda, shunday $\exists h > 0$ soni topilib, (13) Koshi masalasining ushbu $[x_0 - h, x_0 + h]$ oraliqda aniqlangan $y = \varphi(x,x_0,y_0)$ yechimi uchun quyidagi tasdiqlar o'rinli:

1. $\dfrac{\partial\varphi(x,x_0,y_0)}{\partial x_0}, \dfrac{\partial\varphi(x,x_0,y_0)}{\partial y_0}$ –xususiy hosilalar uzluksiz funksiyalardan iborat bo'lib, mos ravishda ushbu

$$\begin{aligned} &\frac{du}{dx} = \frac{\partial f(x,\varphi(x,x_0,y_0))}{\partial y}\cdot u,\ u\big|_{x=x_0} = 0, \\ &\frac{dv}{dx} = \frac{\partial f(x,\varphi(x,x_0,y_0))}{\partial y}\cdot v,\ v\big|_{x=x_0} = 1 \end{aligned} \tag{14}$$

tenglamalarni qanoatlantiradi. Bu yerda

$$u = u(x, x_0, y_0) \equiv \frac{\partial \varphi(x, x_0, y_0)}{\partial x_0},$$
$$v = v(x, x_0, y_0) \equiv \frac{\partial \varphi(x, x_0, y_0)}{\partial y_0} \tag{15}$$

2. $\varphi''_{xx_0}, \varphi''_{xy_0}$ –aralash hosilalar uzluksiz.

Isbot. Avvalo ushbu

$$\frac{\partial y}{\partial x_0} = \frac{\partial \varphi(x, x_0, y_0)}{\partial x_0} = \frac{\partial y(x, x_0, y_0)}{\partial x_0}$$

xususiy hosilani mavjudligini ko'rsatamiz. Buning uchun quyidagi yordamchi

$$y' = f(x, y), \ y(x_0) = y_1 \tag{16}$$

Koshi masalasini ham qaraymiz. Berilgan (13) Koshi masalasining $y = y(x, x_0, y_0)$ yechimi $I = [x_0 - h, x_0 + h]$ oraliqda mavjud. Shu bilan bir qatorda (16) Koshi masalasining yechimi $y = y(x, x_0, y_1)$, $x \in [x_0 - h_1, x_0 + h_1] = I_1$ oraliqda mavjud. Bu yechimlar boshlang'ich shartlarga nisbatan uzluksiz bo'lgani uchun, ushbu

$$|y(x, x_0, y_0) - y(x, x_0, y_1)| \le |y_0 - y_1| e^{L|x - x_0|}, \ x \in I \cap I_1$$

baho o'rinli, ya'ni $|y_0 - y_1| \to 0$ da $|y(x, x_0, y_0) - y(x, x_0, y_1)| \to 0$ munosabat o'rinli bo'ladi.

Bundan tashqari (13) va (16) Koshi masalalari quyidagi

$$y(x, x_0, y_0) = y_0 + \int_{x_0}^{x} f(t, y(t, x_0, y_0))dt$$
$$y(x, x_0, y_1) = y_1 + \int_{x_0}^{x} f(t, y(t, x_0, y_1))dt \tag{16'}$$

integral tenglamalarga ekvivalent. Shu bilan bir qatorda $z(x)$ funksiyaga nisbatan ushbu

$$z(x) = 1 + \int_{x_0}^{x} \frac{\partial f(t, y(t, x_0, y_0))}{\partial y} z(t)dt$$

integral tenglamani ham qaraylik. Yuqoridagi integral tenglamalardan foydalanib quyidagi ayirmani hisoblaymiz:

$$y(x, x_0, y_0) - y(x, x_0, y_1) - z(x)(y_0 - y_1) =$$
$$= (y_0 - y_1) + \int_{x_0}^{x} \left[f(t, y(t, x_0, y_0)) - f(t, y(t, x_0, y_1)) \right] dt -$$
$$-(y_0 - y_1) - \int_{x_0}^{x} \frac{\partial f(t, y(t, x_0, y_0))}{\partial y} z(t)dt(y_0 - y_1) =$$

$$=\int_{x_0}^{x}\left[f(t,y(t,x_0,y_0))-f(t,y(t,x_0,y_1))-\frac{\partial f(t,y(t,x_0,y_0))}{\partial y}z(t)(y_0-y_1)\right]dt=$$

$$=\int_{x_0}^{x}\frac{\partial f(t,y(t,x_0,y_0))}{\partial y}\left(y(t,x_0,y_0)-y(t,x_0,y_1)\right)+\alpha\left(y(x,x_0,y_0),y(x,x_0,y_1)\right)dt-$$

$$-\int_{x_0}^{x}\frac{\partial f(t,y(t,x_0,y_0))}{\partial y}z(t)(y_0-y_1)dt$$

ya’ni

$$y(x,x_0,y_0)-y(x,x_0,y_1)-z(x)(y_0-y_1)=$$

$$=\int_{x_0}^{x}\frac{\partial f(t,y(t,x_0,y_0))}{\partial y}\left[y(t,x_0,y_0)-y(t,x_0,y_1)-z(t)(y_0-y_1)\right]dt+$$

$$+\int_{x_0}^{x}\alpha\left[y(t,x_0,y_0),y(t,x_0,y_1)\right]dt$$

Bu yerda α cheksiz kichik miqdor, ya’ni $\alpha\left[y(t,x_0,y_0),y(t,x_0,y_1)\right]\to 0$ munosabat o’rinli bo’ladi, qachonki $|y(x,x_0,y_0)-y(x,x_0,y_1)|\to 0$ bo’lsa, bu esa $|y_0-y_1|\to 0$ da o’rinli. Oxirgi (17) tenglikni $\left|f'_y(x,y(x,x_0,y_0))\right|\le L,\ L>0$ tengsizlikdan foydalanib, baholaymiz:

$$|y(x,x_0,y_0)-y(x,x_0,y_1)-z(x)(y_0-y_1)|\le$$

$$\le L\left|\int_{x_0}^{x}|y(t,x_0,y_0)-y(t,x_0,y_1)-z(t)(y_0-y_1)|dt\right|+\bar{\bar{0}}\left(|y_0-y_1|\right) \tag{18}$$

Bu yerda $|y_0-y_1|\to 0$ da yechimning boshlangich shartga uzluksiz bog’liqligidan

$$y(x,x_0,y_0)-y(x,x_0,y_1)\to 0$$

bo’lishi, bundan esa o’z navbatida

$$\alpha\left[y(t,x_0,y_0),y(t,x_0,y_1)\right]\to 0$$

kelib chiqadi. Oxirgi (18) munosabatga Gronuolla tengsizligini qo’llasak quyidagi baho kelib chiqadi:

$$u(x)=|y(t,x_0,y_0)-y(t,x_0,y_1)-z(t)(y_0-y_1)|$$

$$v(x)=L,\ A=\bar{\bar{o}}(|y_0-y_1|)$$

$$|y(t,x_0,y_0)-y(t,x_0,y_1)-z(t)(y_0-y_1)|\le\bar{\bar{o}}(|y_0-y_1|)e^{L|x-x_0|}$$

Bu bahodan $|y_0-y_1|\to 0$ da

$$|y(t,x_0,y_0)-y(t,x_0,y_1)-z(t)(y_0-y_1)|\to 0$$

bo’lishi kelib chiqadi. Bu esa, o’z navbatida

$$\frac{|y(t,x_0,y_0)-y(t,x_0,y_1)|}{y_0-y_1}=z(x)+\overset{=}{o}(1)$$

ekanligini bildiradi. Bundan, ushbu $\frac{\partial y(x,x_0,y_0)}{\partial y_0}$ xususiy hosilaning mavjudligi va

$$z(x)=\frac{\partial y(x,x_0,y_0)}{\partial y_0}=\frac{\partial \varphi(x,x_0,y_0)}{\partial y_0}$$

tenglik kelib chiqadi. Teoremaning qolgan bandlari ham xuddi shunday isbotlanadi.

Misol-3. Ushbu

$$y'=\lambda y^2+1,\ \ y(0)=0$$

Koshi masalasi yechimining $z=y'_\lambda\big|_{\lambda=0}$ qiymatini toping.

Yechish. Avvalo $\lambda=0$ bo'lgan holda

$$y'=1,\ \ y(0)=0$$

masalaning $y(x,0)=x$ yechimini topamiz. So'ngra $f(x,y,\lambda)=\lambda y^2+1$ tenglikdan

$$\frac{\partial f}{\partial y}=2\lambda y,\ \ \frac{\partial f}{\partial \lambda}=y^2$$

munosabatlarni aniqlaymiz. Endi yuqoridagi munosabatlardan foydalanib quyidagi

$$\frac{dz(x,\lambda)}{dx}=\frac{\partial f}{\partial y}z(x,\lambda)+\frac{\partial f}{\partial \lambda},$$
$$z(0,\lambda)=0$$

differensial tenglamani tuzib olamiz:

$$\begin{cases} z'(x,\lambda)=2\lambda yz+y^2 \\ z(0,\lambda)=0 \end{cases}$$

Bu tenglamada y o'rniga $y=y(x,0)=x$ ni qo'yib quyidagi

$$\begin{cases} z'(x,\lambda)=2\lambda xz+x^2 \\ z(0,\lambda)=0 \end{cases}$$

masalani hosil qilamiz. Bu yerda $\lambda=0$ deb ushbu

$$\begin{cases} z'(x,0)=x^2 \\ z(0,0)=0 \end{cases}$$

masalaning yechimini topamiz:

$$z(x,0)=\frac{x^3}{3}$$

Bu funksiya $y'_\lambda(x,\lambda)\big|_{\lambda=0}=z(x,0)=\frac{x^3}{3}$ biz izlayotgan qiymatni beradi.

Ikkinchi tomondan berilgan tenglama o'zgaruvchilarga ajraladigan differensial tenglama bo'lgani uchun, uning yechimini topish mumkin:

$$\frac{dy}{(\sqrt{\lambda}y)^2+1}=dx,\ \frac{1}{\sqrt{\lambda}}\operatorname{arct}g\sqrt{\lambda}y=x+c$$

$$y(0)=0\rightarrow\frac{1}{\sqrt{\lambda}}\operatorname{arct}g\sqrt{\lambda}y=x,\ y=\frac{1}{\sqrt{\lambda}}tg\sqrt{\lambda}x$$

Agar $y=y(x,\lambda)$ ni quyidagi

$$y(x,\lambda)=y(x,0)+y'_{\lambda}(x,\lambda)\big|_{\lambda=0}\cdot\lambda+\overline{\overline{o}}(\lambda)$$

ko'rinishda yozsak, u holda

$$y(x,\lambda)=\frac{1}{\sqrt{\lambda}}tg(\sqrt{\lambda}x)=x+\lambda\frac{x^3}{3}+\overline{\overline{0}}(\lambda)$$

hosil bo'ladi.

Yuqoridagi teorema-1 da bayon qilingan tasdiqni quyidagicha umumlashtirish mumkin.

Teorema-3. Agar $f(x,y,\lambda)$ funksiya P sohada Koshi teoremasining shartlarini qanoatlantirib, x,y,λ o'zgaruvchilar bo'yicha $m\geq 2$ marta differensiallanuvchi bo'lsa, u holda (1) masalaning $y(x,\lambda)$ yechimi x,λ o'zgaruvchilar bo'yicha differensiallanuvchi bo'lib, λ o'zgaruvchi bo'yicha m marta differensiallanuvchi bo'ladi. Bundan tashqari $y(x,\lambda)$ yechimni λ^m ning darajalari bo'yicha Teylor formulasiga yoyish mumkin:

$$y(x,\lambda)=u_0(x)+\lambda u_1(x)+\lambda^2u_2(x)+\ldots+\lambda^m u_m(x)+\overline{\overline{o}}(\lambda^m) \qquad (19)$$

Misol-4. Ushbu

$$y'(x)=\frac{x}{y}-2\lambda x^2,\ \ y(1)=1 \qquad (20)$$

masala $y(x,\lambda)$ yechimining λ bo'yicha yoyilmasini λ^2 gacha aniqlikda toping.

Yechish. Berilgan tenglama o'ng tomoni

$$f(x,y,\lambda)=\frac{x}{y}-2\lambda x^2,$$

$y>0$ sohada barcha tartibli hosilalarga ega. $\lambda=0$ da berilgan masala ushbu

$$y'=\frac{x}{y},\ \ y(1)=1$$

ko'rinishni oladi. Bu masala $y(x,0)=x$ yechimga ega. Berilgan masalaning yechimini

$$y(x,\lambda)=u_0(x)+\lambda u_1(x)+\lambda^2u_2(x)+\overline{\overline{o}}(\lambda^2)$$

ko'rinishda izlaymiz. Bu yerda $u_0(x)=y(x,0)=x$. Shuning uchun yuqoridagi yoyilma ushbu

$$y(x,\lambda)=x+\lambda u_1(x)+\lambda^2 u_2(x)+\overset{=}{o}(\lambda^2) \tag{21}$$

ko'rinishni oladi. (21) yoyilmani (20) tenglamaga qo'yamiz:

$$1+\lambda u_1'+\lambda^2 u_2'+...=\frac{x}{x+\lambda u_1+\lambda^2 u_2+...}-2\lambda x^2$$

$$y(1,\lambda)=1\to 1+\lambda u_1(1)+\lambda^2 u_2(1)+...+=1\Rightarrow$$

$$u_1(1)=0,\ u_2(1)=0,... \ .$$

Endi ushbu

$$\frac{x}{x+\lambda u_1+\lambda^2 u_2+...}=\frac{1}{1+\lambda x^{-1}u_1+\lambda^2 x^{-1}u_2+..}=$$

$$=1-\left(\frac{\lambda}{x}u_1+\frac{\lambda^2}{x}u_2+...\right)+\left(\frac{\lambda}{x}u_1+...\right)^2-...=$$

$$=1-\frac{\lambda}{x}u_1-\frac{\lambda^2}{x}u_2+\frac{\lambda^2}{x^2}u_1^2+...$$

yoyilmadan foydalanib,

$$1+\lambda u_1'+\lambda^2 u_2'+...=1-\frac{\lambda}{x}u_1-\frac{\lambda^2}{x}u_2+\frac{\lambda^2}{x^2}u_1-2\lambda x^2$$

tenglikni hosil qilamiz. Bundan ushbu

$$u_1'(x)=-\frac{u_1(x)}{x}-2x^2,\ u_1(1)=0\ , \tag{22}$$

$$u_2'(x)=-\frac{u_2(x)}{x}+\frac{u_1^2(x)}{x^2},\ u_2(1)=0, \tag{23}$$

Koshi masalalarini topamiz. Avvalo (22) tenglamaning bir jinsli qismining umumiy yechimini topamiz:

1) $$\frac{du_1}{dx}=-\frac{u_1}{x}\Rightarrow\frac{du_1}{u_1}=-\frac{dx}{x}$$

$$\ln u_1=-\ln x+\ln c;\ u_1(x)=\frac{c}{x}.$$

So'ngra (22) tenglamaning xususiy yechimini topamiz:

$$\bar{u}_1(x)=ax^3;\ u_1'(x)=3ax^2,$$

$$3ax^2=-\frac{ax^3}{x}-2x^2,$$

$$3ax^2=-ax^2-2x^2,$$

$$3a=-a-2\ ,\ 4a=-2\ \ a=-\frac{1}{2}$$

Demak, biz izlagan xususiy yechim quyidagi

$$\overline{u_1}(x) = -\frac{x^3}{2}$$

ko'rinishda bo'lar ekan. Bundan va boshlang'ich shartdan foydalanib (22) Koshi masalasining yechimini topamiz:

$$u_1(x) = \frac{c}{x} - \frac{x^3}{2}; \quad u(1) = 0\ ,$$

$$c - \frac{1}{2} = 0,\ c = \frac{1}{2},$$

$$u_1(x) = \frac{1}{2x} - \frac{x^3}{2}.$$

Endi quyidagi

$$\frac{du_2}{dx} = -\frac{u_2}{x} + \frac{\left(\frac{1}{2x} - \frac{x^3}{2}\right)^2}{x^2},\ u_2(1) = 0$$

masalaning yechimini topamiz. Bu tenglamani yechish uchun, avvalo uning bir jinsli qismini yechamiz:

$$\frac{du_2}{dx} = -\frac{u_2}{x} \Rightarrow u_2(x) = \frac{c}{x}\ .$$

Endi, bir jinsli bo'lmagan differensial tenglmaning yechimini Lagranj usulidan foydalanib topamiz:

$$u_2(x) = \frac{c(x)}{x},\ u_2'(x) = \frac{c'(x)}{x} - \frac{1}{x^2}c(x),$$

$$\frac{c'(x)}{x} - \frac{1}{x^2}c(x) = -\frac{1}{x^2}c(x) + \frac{\left(\frac{1}{2x} - \frac{x^3}{2}\right)^2}{x^2},$$

$$c'(x) = \frac{\left(\frac{1}{2x} - \frac{x^3}{2}\right)^2}{x} = \frac{1}{x}\left(\frac{1}{4x^2} - \frac{1}{2}x^2 + \frac{x^6}{4}\right) = \frac{1}{4x^3} - \frac{1}{2}x + \frac{x^5}{4}\ ,$$

$$\frac{dc(x)}{dx} = \frac{1}{4x^3} - \frac{1}{2}x + \frac{x^5}{4},$$

$$c(x) = -\frac{1}{8x^2} - \frac{1}{4}x^2 + \frac{x^6}{24} + c_1,$$

$$u_2(x) = \frac{1}{x}\left(c_1 - \frac{1}{8x^2} - \frac{1}{4}x^2 + \frac{x^6}{24}\right),$$

$$u_2(1) = 0 \Rightarrow c_1 - \frac{1}{8} - \frac{1}{4} + \frac{1}{24} = 0,$$

$$c_1-\frac{1}{3}=0,\ c_1=\frac{1}{3},$$

$$u_2(x)=\frac{1}{x}\left(\frac{1}{3}-\frac{1}{8x^2}-\frac{1}{4}x^2+\frac{x^6}{24}\right).$$

Demak, (21) formulaga asosan berilgan (20) masalaning $y(x,\lambda)$ yechimi uchun quyidagi

$$y(x,\lambda)=x+\lambda\left(\frac{1}{2x}-\frac{x^3}{2}\right)+\lambda^2\left(\frac{1}{3x}-\frac{1}{8x^3}-\frac{1}{4}x+\frac{x^5}{24}\right)+\bar{\bar{o}}(\lambda^2)$$

asimptotik yoyilma o'rinli bo'lar ekan.

4-§. Kichik parametrlar usuli

1. Quyidagi

$$\frac{dy}{dx} = f(x, y, \varepsilon), \quad y(x_0) = y_0 \tag{1}$$

Koshi masalasining yechimini $y(x,\varepsilon)$ orqali belgilaylik.

Aytaylik, ushbu $(\varepsilon = 0)$

$$\frac{dy}{dx} = f(x, y, 0), \quad y|_{x=x_0} = y_0 \tag{2}$$

Koshi masalasining $y = y_0(x),\ x \in I = [0,l]$ yechimi mavjud va yagona bo'lsin. U holda quyidagi tasdiq o'rinli.

Teorema-1. Agar $\varepsilon_0 > 0$ yetarli kichik son bo'lsa, u holda $|\varepsilon| \le \varepsilon_0$ tengsizlikni qanoatlantiruvchi barcha ε lar uchun (1) masalaning $y(x,\varepsilon),\ x \in I = [0,l]$ yechimi mavjud bo'lib, ixtiyoriy $\forall N \ge 1$ larda quyidagi

$$y(x,\varepsilon) = y_0(x) + \varepsilon y_1(x) + \ldots + \varepsilon^{N-1} y_{N-1} + R_N(x,\varepsilon) \tag{3}$$

yoyilma o'rinli bo'ladi. Bu yerda qoldiq had ushbu

$$|R_N(x,\varepsilon)| \le c_N \varepsilon^N,\ x \in I,\ \varepsilon \le \varepsilon_0 \tag{4}$$

tengsizlikni qanoatlantiradi. c_N–o'zgarmas soni $x\ va\ \varepsilon$ larga bog'liq emas.

Isbot. Ushbu $y_0(x)$ funksiyani ma'lum deb qaraymiz. U holda (3) yoyilmani qolgan hadlarini qanday aniqlash mumkinligini ko'rsatamiz. Buning uchun (3) yoyilmani (1) differensial tenglamaga qo'yib

$$\sum_{j=0}^{N-1} \varepsilon^j \frac{dy_j(x)}{dx} + O(\varepsilon^N) = f\left(x, \sum_{j=0}^{N-1} \varepsilon^j y_j(x) + O(\varepsilon^N), \varepsilon\right) \tag{5}$$

munosabatni hosil qilamiz. Bu tenglikning o'ng tomonidagi funksiyani ε parametrning darajalari bo'yicha $O(\varepsilon^N)$ aniqlikgacha Teylor formulasiga yoysak,

$$f\left(x, \sum_{j=0}^{N-1} \varepsilon^j y_j(x) + O(\varepsilon^N), \varepsilon\right) = f(x, y_0(x), 0) + \frac{\partial f(x, y_0, 0)}{\partial y} y_1(x)\varepsilon + \frac{\partial f(x, y_0, 0)}{\partial \varepsilon}\varepsilon + \cdots$$

$$+\frac{1}{(N+1)!}\left(\varepsilon y_1 \frac{\partial}{\partial y} + \varepsilon \frac{\partial}{\partial \varepsilon}\right)^{N-1} f(x, y_0(x), 0) + \bar{\bar{0}}(\varepsilon^N)$$

munosabat hosil bo'ladi. U holda $y_0(x)$ ga nisbatan (2) Koshi masalasi hosil bo'ladi. $y_1(x)$–uchun esa ushbu

$$\frac{dy_1(x)}{dx} = \frac{\partial f(x, y_0, 0)}{\partial y} y_1(x)\varepsilon + \frac{\partial f(x, y_0, 0)}{\partial \varepsilon}, \quad y_1(x_0) = 0 \tag{6}$$

Koshi masalasiga ega bo'lamiz. Bu esa birinchi tartibli chiziqli differensial tenglamadir. Shuning uchun uning yechimi $I=[0,l]$ oraliqda mavjud va yagona. Qolgan barcha $y_2(x), y_3(x),...$ hadlar uchun ham quyidagi

$$\frac{dy_n(x)}{dx}=\frac{\partial f(x,y_0,0)}{\partial y}y_n(x)\varepsilon+F_n\big(x,y_0(x),\dots,y_{N-1}(x)\big),y_n(x_0)=0 \tag{7}$$

chiziqli differensial tenglamaga ega bo'lamiz. Bunda F_n –ma'lum funksiyalar. Bu differensial tenglamalar bir-biridan faqat o'ng tomoniga farq qiladi. (7) ko'rinishdagi Koshi masalalarining har birisining $x\in I=[0,l]$ oraliqda aniqlangan yechimi mavjud va yagona bo'lib, u $y_n(x)\in C^\infty(I)$-cheksiz differensiallanuvchi funksiyadan iborat bo'ladi.

Teorema-2. Faraz qilaylik:

1. $f(x,y:\varepsilon)$ va $f_y'(x,y,\varepsilon)$ funksiyalar $G=\{0\le x\le l, |y|\le b, |\varepsilon|<\varepsilon_0\}$ sohada uzluksiz va tekis chegaralangan, ya'ni

$$|f(x,y,\varepsilon)|\le M, \qquad |f_y'(x,y,\varepsilon)|\le L$$

bo'lsin.

2. Ushbu

$$\frac{dy}{dx}=f(x,y,0),\ y(0)=y_0$$

Koshi masalasining $y_0(x)$, $x\in[0,l]$ yechimi mavjud va yagona bo'lib, $D=\{(x,y)\in R^2: 0\le x\le l, |y|\le b\}$ sohaga tegishli bo'lsin. U holda, har bir yetarli kichik ε lar uchun

$$\frac{dy}{dx}=f(x,y,\varepsilon),\ y(0,\varepsilon)=y_0$$

Koshi masalasining $x\in[0,l]$ oraliqda aniqlangan yagona $y(x,\varepsilon)=y_\varepsilon(x)$ yechimi mavjud va u G sohaga tegishli bo'ladi hamda

$$\lim_{\varepsilon\to0}y_\varepsilon(x)=y_0(x), x\in[0,l]$$

munosabat x ga nisbatan tekis bajariladi.

Isbot: Quydagi

$$Z_\varepsilon(x)=y_\varepsilon(x)-y_0(x)$$

belgilashdan foydalansak, ushbu

$$\frac{dZ_\varepsilon(x)}{dx}=f(x,Z_\varepsilon(x)+y_0(x),\varepsilon)-f(x,y_0(x))$$
$$Z_\varepsilon(0)=0$$

Koshi masalasiga ega bo'lamiz. Bu masalaga ekvivalent bo'lgan integral tenglama tuzamiz:

$$Z_\varepsilon(x) = \int_0^x [f(t, Z_\varepsilon(x) + y_0(t), \varepsilon) - f(t, y_0(t), 0)]dt$$

Oxirgi tenglikning o'ng tomonini quyidagi ko'rinishda yozish mumkin:

$$Z_\varepsilon(x) = \int_0^x [f(t, Z_\varepsilon(x) + y_0(t), \varepsilon) - f(t, y_0(t), \varepsilon)]dt + F(x, \varepsilon)$$

bunda

$$F(x, \varepsilon) = \int_0^x [f(t, y_0(t), \varepsilon) - f(t, y_0(t), 0)]dt$$

ko'rnishda bo'lib, ushbu

$$|F(x,\varepsilon)| \le \alpha(\varepsilon)$$

tengsizlikni qanoatlantiradi, chunki $f(x,y,\varepsilon) -$ ε parametrga nisbatan uzluksiz. Bu yerda $\alpha(\varepsilon)$- cheksiz kichik miqdor, ya'ni

$$\lim_{\varepsilon\to 0} F(x,\varepsilon) = 0.$$

Endi $Z_\varepsilon(x)$ funksiyani baholaymiz:

$$Z_\varepsilon(x) \le \left| \int_0^x |[f(t, z_\varepsilon(t) + y_0(t), \varepsilon) - f(t, y_0(t), \varepsilon)]| dt \right| + |F(x,\varepsilon)| \le L \left| \int_0^x |Z_\varepsilon(x)| dt \right| + \alpha(x).$$

Bu tengsizlikka Gronuolla lemmasini qo'llasak

$$|Z_\varepsilon(x)| \le \alpha(\varepsilon) e^{Lx}, x \in [0, l]$$

baho kelib chiqadi. Bundan esa $Z_\varepsilon(x)$ funksiyaning nolga tekis yaqinlashishi kelib chiqadi.■

2. Fizikaning bir qator masalalari quyidagi

$$\varepsilon \frac{dy}{dx} = f(x,y) \tag{8}$$

$$y(x_0) = y_0 \tag{9}$$

kichik parametrli Koshi masalasi $y = y(x,\varepsilon)$ yechimning $\varepsilon \to 0$ dagi limiti mavjudligini o'rganishga keltiradi.

Agar (8) differensial tenglamani

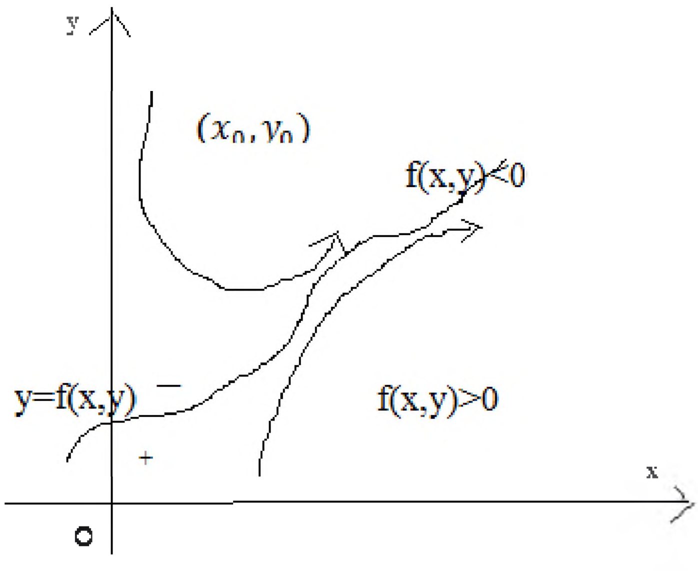
$$\frac{dy}{dx} = \frac{1}{\varepsilon} f(x,y) \tag{10}$$

ko'rinishda ifodalasak, u holda bu tenglamaning o'ng tomonidagi $F(x,y,\varepsilon)=\frac{1}{\varepsilon}f(x,y)$ funksiya $\varepsilon=0$ da uzilishga ega. Shuning uchun bu yerda $y(x,\varepsilon)$ yechimning ε parametrga nisbatan uzluksiz bog'liqligi haqidagi teoremadan foydalanib bo'lmaydi.

Bunday ko'rinishdagi differensial tenglamalarni o'rganishda $|\varepsilon|$ parametrning kichik qiymatlarida (8) tenglamada $\varepsilon y'$ hadni tashlab, ushbu $\varepsilon y'=f(x,y)$ differensial tenglamaning taqribiy yechimi sifatida $\varepsilon=0$ dagi

$$f(x,y)=0 \qquad (11)$$

tenglamaning ildizini (yechimini) olish mumkinmi degan savolning tug'ilishi tabiiy. Aytalik (11) tenglama faqat bitta $y=\phi(x)$ ildizga ega bo'lib, $\varepsilon>0$ bo'lsin. Ko'rinib turibdiki $\varepsilon\to 0$ da (10) differensial tenglama yechimining $\frac{dy}{dx}$ hosilasi, $f(x,y)\neq 0$ shartni qanoatlantiruvchi har qanday nuqtalarda absolyut qiymati bo'yicha cheksiz ortadi. Bundan kelib chiqadiki (10) tenglama integral chizig'ining $f(x,y)\neq 0$ bo'ladigan nuqtalariga o'tkazilgan urinmalar, $\varepsilon\to 0$ da OY o'q yo'nalishiga parallel ravishda intiladi, ya'ni agar $f(x,y)>0$ bo'lsa, u holda (10) tenglamaning $y(x,\varepsilon)$ yechim x o'sishiga mos o'sadi, chunki $\frac{dy}{dx}>0$. Agar $f(x,y)<0$ bo'lsa, u holda $y(x,\varepsilon)$ yechim x oshgan sari kamayadi, chunki $\frac{dy}{dx}<0$ (chizma-1 qarang):

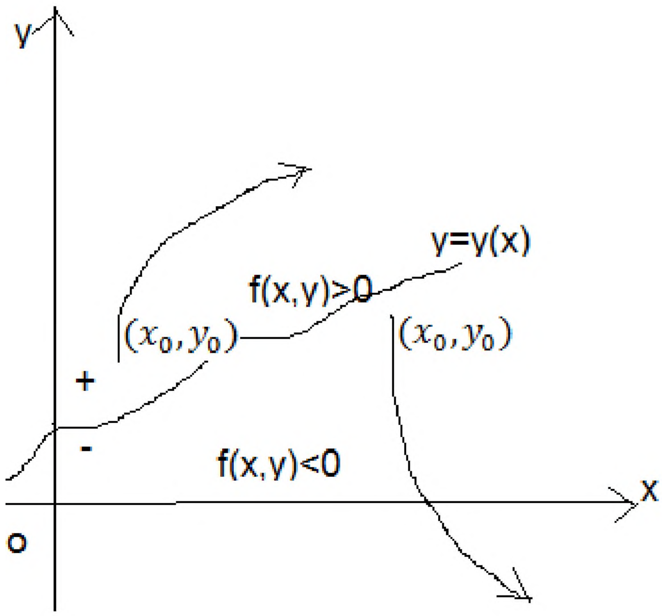


Chizma-1

Quydagi ikki holni qaraylik:

a) Aytaylik har bir tayinlangan x va y o'zgaruvchilarning ortishi natijasida $f(x,y)$ funksiya (11) tenglama $y = y(x)$ ildizining grafigidan o'tishda ishorasini + dan – ga o'zgartirsin. U holda (11) tenglamaning $y = y(x)$ildizi turg'un bo'lib (8) differensial tenglamaning $y(x,\varepsilon)$ yechimi $\varepsilon \to 0$ da $y = y(x)$ ga yaqinlashadi (chizma-1ga qarang).

b) Aytaylik har bir tayinangan $x$va y o'zgaruvchilarning ortishi natijasida $f(x,y)$ funksiya (11) tenglama $y = y(x)$ ildizining grafigidan o'tishda ishorasini – dan + ga o'zgartirsin. U holda (11) tenglamaning $y = y(x)$ ildizi noturg'un bo'lib $y(x,\varepsilon)$ yechimni taqriban $y(x)$ bilan almashtirib bo'lmaydi (chizma-2 ga qarang).



Chizma-2

Yuqoridagi (11) tenglamaning $y = y(x)$ ildizi turg'un yoki noturg'unligini tekshirishda quyidagi yetarlilik shartlaridan foydalanish maqsadga muvofiqdir.

1. Agar (11) tenglama $y = y(x)$ ildizi ustida

$$\frac{\partial f(x,y)}{\partial y}\Big|_{y=y(x)} < 0$$

shart bajarilsa, u holda $y = y(x)$ ildiz turg'un bo'ladi.

2. Agar (11) tenglama $y = y(x)$ ildizi ustida

$$\frac{\partial f(x,y)}{\partial y}\Big|_{y=y(x)} > 0$$

shart bajarilsa, u holda $y = y(x)$ ildiz noturg'un bo'ladi.

Shu jarayonlarni tavsiflovchi misollar qaraylik.

Misol-1. Ushbu

$$\varepsilon y' = x^2 - y,\ y(x_0) = y_0,\ \ x \in [x_0, \infty)$$

Koshi masalasining $y_\varepsilon(x) = y(x,\varepsilon),\ \varepsilon > 0$ yechimini $\varepsilon \to +0$ da $y = x^2$ funksiyaga yaqinlashishini ko'rsating.

Yechish: avvalo ushbu

$$f(x, y) = x^2 - y$$

funksiyani tuzib olamiz. Agar $\varepsilon = 0$ bo'lsa, u holda berilgan differensial tenglama quyidagi

$$f(x, y) = 0,\ x^2 - y = 0$$

ko'rinishni oladi. Bu tenglamani yechib, uning $y = x^2$ ildizini topamiz. So'ngra quyidagi shartdan:

$$\left.\frac{\partial f(x,y)}{\partial y}\right|_{y=x^2} = [\frac{\partial}{\partial y}(x^2 - y)]_{y=x^2} = -1 < 0.$$

$f(x, y) = 0$ tenglamaning $y = x^2$ ildizi turg'un bo'lishi kelib chiqadi. Shuning uchun

$$\lim_{\varepsilon \to +0} y_\varepsilon(x) = x^2, x \in (x_0, \infty)$$

munosabat o'rinli bo'ladi.

Bu fikrga, berilgan Koshi masalasining yechimini to'g'ridan-to'g'ri topish bilan ham kelish mumkin.

Haqiqatdan ham quydagi

$$y_\varepsilon(x) := y(x,\varepsilon) = (y_0 - x_0^2 + 2\varepsilon x_0 - 2\varepsilon^2)e^{-\frac{x-x_0}{\varepsilon}} + x^2 - 2\varepsilon x - 2\varepsilon^2$$

funksiya $y(x_0)=y_0$ boshlang'ich shartni va berilgan differensial tenglamani qanoatlantiradi. Ko'rinib turibdiki, agar $x\in(x_0,\infty)$ bo'lsa u holda

$$\lim_{\varepsilon\to+0} e^{-\frac{x-x_0}{\varepsilon}}=0, x\in(x_0,\infty)$$

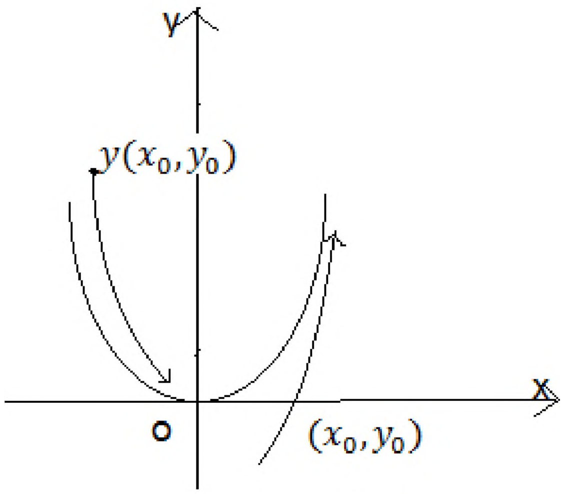
munosabat o'rinli bo'ladi. Shuning uchun ham $y_\varepsilon(x)$ yechimning ko'rinishidan

$$\lim_{\varepsilon\to+0} y_\varepsilon(x)=\lim_{\varepsilon\to+0}[(y_0-x^2+2\varepsilon x_0-2\varepsilon^2)e^{-\frac{x-x_0}{\varepsilon}}+x^2-2x\varepsilon+2\varepsilon^2]=x^2,\ x\in(x_0,\infty)$$

munosabat kelib chiqadi, ammo bu yaqinlashish tekis emas. Chunki agar δ sonini $x_0<\delta$ tengsizlikni qanoatlantiruvchi qilib tanlasak, u holda $\varepsilon\to+0$ da quyidagi funksiya

$$e^{-\frac{x-x_0}{\varepsilon}}\rightrightarrows 0,\ [x,\infty)$$

nolga tekis yaqinlashadi. Shuning uchun $\varepsilon\to+0$ da $y_\varepsilon(x)\rightrightarrows x^2,\ x\in[\delta,\infty)$ tekis yaqinlashadi. Bu holda $[x_0,\delta]$ kesma chegaraviy qatlam vazifasini o'taydi (chizma-3 ga qarang).



Chizma-3

Misol-2. Ushbu

$$\varepsilon y' = y - x,\ \ \varepsilon>0$$
$$y(x_0)=y_0$$

Koshi masalasining $y_\varepsilon=y(x,\varepsilon)$ yehimini $\varepsilon\to+0$ da $y=x$ funksiyaga yaqinlashishini ko'rsating.

Yechish: Avvalo ushbu

$$f(x,y)=y-x$$

funksiyani tuzib olamiz. Agar $\varepsilon=0$ bo'lsa, u holda berilgan tenglama

$$f(x,y)=0,\ y-x=0$$

ko'rinishni oladi. Bu tenglamani yechib uning $y=x$ ildizini topamiz. So'ngra quyidagi yetarlilik shartlarini tekshiramiz:

$$\frac{\partial f(x,y)}{\partial y}\Big|_{y=x}=\frac{\partial}{\partial y}[y-x]\Big|_{y=x}=+1>0.$$

Bundan $f(x,y)=0$ tenglamaning $y=x$ ildizi noturg'un bo'lishi kelib chiqadi. Shuning uchun $\varepsilon\to+0$ da $y_\varepsilon(x)\not\rightrightarrows x,\ x\in(x_0,\infty)$ yaqinlashish bo'lmaydi. Bu fikrga berilgan Koshi masalasi yechimini topish orqali ham kelish mumkin:

$$y_\varepsilon(x)=(y_0-x_0-\varepsilon)e^{\frac{x-x_0}{\varepsilon}}+x+\varepsilon$$

Ko'rinib turibdiki $\varepsilon\to+0$ da

$$e^{\frac{x-x_0}{\varepsilon}}\to\infty, x\epsilon(x_0,\infty)$$

munosabat o'rinli. Shuning uchun $\varepsilon\to+0$ da

$$y_\varepsilon(x)\not\rightrightarrows x,\ x\in(x_0,\infty)$$

yaqinlashish bo'maydi.

2. Endi, ushbu

$$\varepsilon y'+ay=f(x),\ x\in[0,l] \tag{12}$$
$$y(0)=y_0$$

ko'rinishdagi Koshi masalasini qaraylik. Bunda $\varepsilon>0,\ a\neq0$ va $f(x)-[0,l]$ oraliqda berilgan uzluksiz funksiya.

Berilgan (12) Koshi masalasining yechimini $y_\varepsilon(x)=y(x,\varepsilon)$ orqali, $\varepsilon=0$ bo'lgan holdagi yechimini esa $y_0(x)=y(x,0)$ orqali belgilaylik. U holda $y_\varepsilon(x)$ funksiyaning $\varepsilon\to+0$ dagi limiti $y_0(x)=a^{-1}f(x)$ funksiyaga $x\in[0,l]$ oraliqda yaqinlashishi mumkinligini o'rganamiz.

Teorema-3. Agar $a>0$ bo'lsa, u holda har bir $x\in(0,l]$ larda

$$\lim_{\varepsilon\to+0}y_\varepsilon(x)=y_0(x),\ \ y_0(x)=\frac{1}{a}f(x) \tag{13}$$

munosabat bajariladi.

Isbot. Berilgan (8) Koshi masalasining yechimini

$$y_\varepsilon(x)=y_0(x)+z_\varepsilon(x) \tag{14}$$

ko'rinishda izlaymiz. Bunda $z_\varepsilon(x)$ quyidagi

$$\varepsilon z'_\varepsilon(x)+az_\varepsilon(x)=-\frac{\varepsilon}{a}f'(x) \tag{15}$$

$$z_\varepsilon(0)=y_0-\frac{1}{a}f(0)$$

Koshi masalasining yechimidan iborat. Bu chiziqli tenglamani yechib

$$z_\varepsilon(x)=e^{-\frac{a}{\varepsilon}x}\left[y_0-\frac{1}{a}f(0)\right]-\frac{1}{a}\int_0^x e^{\frac{a}{\varepsilon}(t-x)}f'(t)dt$$

formulani topamiz. U holda

$$y_\varepsilon(x)=y_0(x)+\left[y_0-\frac{1}{a}f(0)\right]e^{-\frac{a}{\varepsilon}x}+O(\varepsilon),\ \ \varepsilon\to+0 \qquad (16)$$

baho o'rinli bo'ladi. Chunki $\varepsilon\to+0$ da

$$\left|\frac{1}{a}\int_0^x e^{\frac{a}{\varepsilon}(t-x)}f'(t)dt\right|\le\frac{1}{a}\max|f'(x)|\int_0^x e^{\frac{a}{\varepsilon}(t-x)}dt=$$

$$=\frac{\varepsilon}{a^2}\max|f'(x)|\left(1-e^{-\frac{a}{\varepsilon}x}\right)\le\frac{\varepsilon}{a^2}\max|f'(x)|$$

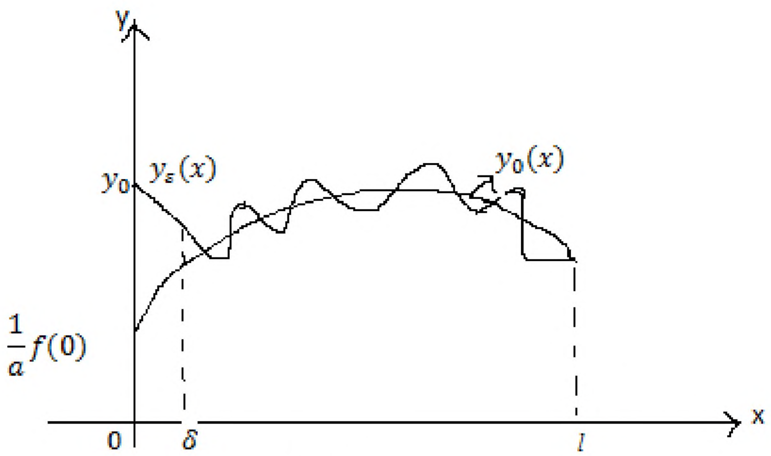
baho o'rinli. Agar (16) tenglikda $\varepsilon\to+0$ da ushbu

$$\lim_{\varepsilon\to+0}e^{-\frac{a}{\varepsilon}x}=0,\ \ \forall x\in(0,l],\ \ a>0$$

munosabatni inobatga olsak, undan (13) kelib chiqadi. Ammo $\varepsilon\to+0$ da $x\in(0,l]$ oraliqda $e^{-\frac{a}{\varepsilon}x}$ $(a>0)$ funksiya nolga tekis yaqinlashmaydi. Yuqoridagi (16) munosabatdan ko'rinadiki, agar $x\in[\delta,l],\ 0<\delta<l$ (δ - ixtiyoriy tayinlangan son) bo'lsa, u holda

$$y_\varepsilon(x)\rightrightarrows y_0(x),\ \ \varepsilon\to+0$$

tekis yaqinlashish o'rinli bo'ladi. Berilgan $[0,l]$ oraliqda $y_\varepsilon(x),\varepsilon\to+0$ da $y_0(x)=\frac{1}{a}f(x)$ funksiyaga tekis yaqinlashmaydi.



Chizma

Agar $a<0$ bo'lsa $y_\varepsilon(x)$ funksiya $\varepsilon\to+0$ da $y_0(x)$ ga yaqinlashmaydi.

Ushbu $[0,\delta]$, $0<\delta<l$ kesmaga chegaraviy qatlam deyiladi (пограничным слоем).

Endi bu holatni atroflicha tekshiraylik. Buning uchun berilgan (12) differensial tenglamani ushbu

$$\varepsilon y' = F(x,y), \quad F(x,y)=f(x)-ay$$

ko'rinishda yozib olamiz. U holda $y:=y_0(x)=\frac{1}{a}f(x)$ funksiya $F(x,y)=0$ tenglamaning ildizi bo'ladi. Bu ildizni turg'unlikka tekshiraylik:

$$\frac{\partial F(x,y)}{\partial y}=\frac{\partial}{\partial y}\big(f(x)-ay\big)=-a$$

Agar $a>0$ bo'lsa,

$$F'_y\Big|_{y=y_0(x)}=-a<0$$

munosabat bajariladi. Bu holda $y=y_0(x)$ funksiya $F(x,y)$ tenglamaning turg'un ildizi bo'ladi. Shuning uchun (13) munosabat bajariladi.

Agar $a<0$ bo'lsa, u holda

$$F'_y\Big|_{y=y_0(x)}=-a>0$$

munosabat o'rinli bo'ladi. Bu holda $y=y_0(x)$ funksiya $F(x,y)=0$ tenglamaning noturg'un ildizi bo'ladi. Shuning uchun $a<0$ holda (13) munosabat bajarilmaydi.

5-§. Kichik parametrli Shturm – Liuvill chegaraviy masalasi.

1. Quyidagi

$$u'' + [\lambda + \varepsilon f(x)]u = 0, \qquad (1)$$
$$u(0) = 0, \qquad u(1) = 0 \qquad (2)$$

chegaraviy masalani qaraylik. Bu yerda ε – kichik parametr. Agar $\varepsilon = 0$ bo'lsa, u holda biz ushbu

$$u'' + \lambda u = 0 \qquad (3)$$
$$u(0) = 0\,, u(1) = 0$$

chegaraviy masalaga ega bo'lamiz. Bu chegaraviy masalaning ortanormal xos funksiyalari va xos qiymatlari

$$u_n(x) = \sqrt{2}\sin n\pi x\,, \quad n = 1,2,3,\ldots \qquad (4)$$
$$\lambda_n = n^2\pi^2 \qquad (5)$$

ko'rinishda bo'lishi ma'lum. $u_n(x), n = 1,2,3,\ldots$ funksiyalar quydagi

$$\int_0^1 u_n(x)u_m(x)dx = \delta_{mn} \qquad (6)$$

ortonormallashtirish shartini qanoatlantiradi. Bu yerda δ_{mn} – Kroneker simvoli:

$$\delta_{mn} = \begin{cases} 1, & m = n \\ 0, & m \neq n \end{cases}$$

Juda kichik ε uchun (1), (2) chegaraviy masalaning $u_n(x,\varepsilon)$ – xos funksiyalarini va $\lambda_n(\varepsilon)$ – xos qiymatlarini quydagi

$$u_n(x,\varepsilon) = \sqrt{2}\sin n\pi x + \varepsilon u_{n1}(x) + \varepsilon^2 u_{n2}(x) + \varepsilon^3 u_{n3}(x) + \cdots \qquad (7)$$
$$\lambda_n(\varepsilon) = n^2\pi^2 + \varepsilon\lambda_{n1} + \varepsilon^2\lambda_{n2} + \varepsilon^3\lambda_{n3} + \cdots \qquad (8)$$

ko'rinishda izlaymiz. Bu (7) va (8) tengliklarni quydagi

$$u_n''(x,\varepsilon) + [\lambda_n(\varepsilon) + \varepsilon f(x)]u_n(x,\varepsilon) = 0,$$
$$u_n(0,\varepsilon) = 0, \qquad u_n(1,\varepsilon) = 0$$

chegaraviy masalaga qo'yamiz

$$-(n\pi)^2\sqrt{2}\sin n\pi x + \varepsilon u_{n1}''(x) + \cdots + [(n\pi)^2 + \varepsilon\lambda_{n1} + \cdots + \varepsilon f(x)] *$$
$$* \left[\sqrt{2}\sin n\pi x + \varepsilon u_{n1}(x) + \varepsilon^2 u_{n2}(x) + \varepsilon^3 u_{n3}(x) \ldots\right] = 0$$

Bu yerda ε – kichik parameter darajalari oldidagi koeffitsiyentlarni nolga tenglashtirib quyidagi

$$u_{n1}''(x) + (n\pi)^2 u_{n1}(x) = -f(x)u_{n0}(x) - \lambda_{n1}u_{n0}(x),$$
$$u_{n1}(0) = 0\,, \qquad u_{n1}(1) = 0 \qquad (9)$$

$$u_{n2}''(x) + (n\pi)^2 u_{n2}(x) = -f(x)u_{n1}(x) - \lambda_{n1}u_{n1}(x) - \lambda_{n2}u_{n0}(x)$$
$$u_{n2}(0) = 0\,, \qquad u_{n2}(1) = 0 \qquad (10)$$

$$u''_{n3}(x) + (n\pi)^2 u_{n3}(x) = -f(x)u_{n2}(x) - \lambda_{n1}u_{n2}(x) - \lambda_{n2}u_{n1}(x) - \lambda_{n2}u_{n0}(x),$$
$$u_{n3}(0) = 0\,, \qquad u_{n3}(1) = 0 \tag{11}$$
chegaraviy masalalarga ega bo'lamiz.
Bunda
$$u_{n0}(x) = \sqrt{2}\sin n\pi x.$$
Aytaylik $u_{n1}(x)$ – funksiyalar, ushbu $u_{n0}(x) = \sqrt{2}\sin n\pi x$ ortonormal xos funksiyalar orqali Furye qatoriga yoyilsin deylik. U holda
$$u_{n1}(x) = \sum_{m=1}^{\infty} a_{nm}\sqrt{2}\sin n\pi x \tag{12}$$
yoyilmaga ega bo'lamiz. Ravshanki $u_{n1}(x)$ berilgan $u_{n1}(0) = 0,\ u_{n1}(1) = 0$ chegaraviy shartlarni qanoatlantiradi. Endi (12) tenglikni (9) differensial tenglamaga qo'yib
$$\sum_{m=1}^{\infty} \sqrt{2}\pi^2(n^2 - m^2)a_{nm}\sin n\pi x = -\sqrt{2}f(x)\sin n\pi x - \sqrt{2}\lambda_{n1}\sin n\pi x$$
tenglikni keltirib chiqaramiz. Bu tenglikning ikki tomonini $\sqrt{2}\sin k\pi x$ funksiyaga ko'paytirib, so'ngra 0 dan 1 gacha integrallab , xos funksiyalarning ortonormallanganligidan foydalansak , ushbu
$$\pi^2(n^2 - k^2)a_{nk} = -F_{nk} - \lambda_{n1}\delta_{nk} \tag{13}$$
munosabat hosil bo'ladi. Bu yerda
$$F_{nk} = 2\int_0^1 f(x)\sin n\pi x \sin k\pi x\ dx. \tag{14}$$
Agar (13) tenglikda $n = k$ bo'lsa , u holda
$$\lambda_{n1} = -F_{nn} = -2\int_0^1 f(x)\sin^2 n\pi x\ dx. \tag{15}$$
Agar (13) tenglikda $n \neq k$ bo'lsa , u holda
$$a_{nk} = -\frac{F_{nk}}{\pi^2(n^2 - k^2)} \tag{16}$$
o'rinli bo'ladi. Qaralayotgan holda $u_{n1}(x)$ funksiay quydagi
$$u_{n1}(x) = -\sum_{k\neq n}\frac{F_{nk}}{\pi^2(n^2 - k^2)}\sqrt{2}\sin k\pi x + a_{nn}\sqrt{2}\sin n\pi x \tag{17}$$
formula bo'yicha hisoblanadi. Bu tenglikning o'ng tomonidagi a_{nn} hozircha noma'lum bo'lib, uning qiymatini $u_{n1}(x)$ ning normalashtirish shartda topish mumkin.

Endi (10) chegaraviy masalani qaraymiz. Faraz qilaylik

$$u_{n2}(x)=\sum_{r=1}^{\infty} b_{nr}\sqrt{2}\sin r\pi x \qquad (18)$$

ko'rinishda bo'lsin. Ravshanki $u_{n2}(x)$ funksiya aniqlanishiga ko'ra $u_{n2}(0)=0$, $u_{n2}(1)=0$ chegaraviy shartlarni qanoatlantiradi. Ushbu $u_{n2}(x), u_{n1}(x)$ va $u_{n0}(x)$ funksiyalarning ifodalarini (10) tenglamaga qo'yib

$$\pi^2\sum_{r=1}^{\infty}(n^2-r^2)\sqrt{2}b_{nr}\sin r\pi x=-\sum_{r=1}^{\infty}a_{nr}\sqrt{2}f(x)\sin r\pi x$$

$$-\sum_{r=1}^{\infty}a_{nr}\lambda_{n1}\sqrt{2}\sin r\pi x-\lambda_{n2}\sqrt{2}\sin n\pi x \qquad (19)$$

tenglikni hosil qilamiz. Bu (19) tenglikning ikki tomonini $\sqrt{2}\sin k\pi x$ funksiyaga ko'paytirib 0 dan 1 gacha integrallasak va ortonormallash shartini inobatga olib, ushbu

$$\pi^2(n^2-k^2)b_{nk}=-\sum_{r=1}^{\infty}a_{nr}F_{kr}-a_{nk}\lambda_{n1}-\lambda_{n2}\delta_{nk} \qquad (20)$$

munosabatni olamiz . Agar $k=n$ bo'lsa, u holda

$$\lambda_{n2}=-\sum_{r\neq n}a_{nr}F_{kr}=\sum_{r\neq n}\frac{F_{nr}}{\pi^2(n^2-r^2)}. \qquad (21)$$

formula o'rinli bo'ladi.

Agar $k\neq n$ bo'lsa, u holda (20) tenglikdan

$$b_{nk}=\sum_{r\neq n}\frac{F_{nk}F_{kr}}{\pi^2(n^2-k^2)(n^2-r^2)}-\frac{a_{nn}F_{nk}}{\pi^2(n^2-r^2)}-\frac{F_{nn}F_{nk}}{\pi^4(n^2-k^2)^2} \qquad (22)$$

formula kelib chiqadi. Bu yerda b_{nn} – hozircha noma'lum.

Endi yuqorida topilganlardan foydalanib (11) chegaraviy masalani yechamiz. Faraz qilaylik

$$u_{n3}(x)=\sum_{s=1}^{\infty}d_{ns}\sqrt{2}\sin s\pi x$$

ko'rinishda bo'lsin. Ravshanki $u_{n3}(x)$ funksiya aniqlanishiga ko'ra $u_{n3}(0)=0$, $u_{n3}(1)=0$ chegaraviy shartlarni qanoatlantiradi. U holda (11) tenglik quyidagi korinishni oladi:

$$\pi^2\sum_{s=1}^{\infty}(n^2-s^2)\sqrt{2}d_{ns}\sin s\pi x=-\sum_{s=1}^{\infty}b_{ns}\sqrt{2}f(x)\sin s\pi x-$$

$$-\sum_{s=1}^{\infty}b_{ns}\lambda_{n1}\sqrt{2}\sin s\pi x-\sum_{s=1}^{\infty}a_{ns}\lambda_{n2}\sqrt{2}\sin s\pi x-\lambda_{n3}\sqrt{2}\sin n\pi x \qquad (23)$$

Bu (23) tenglikning ikki tomonini $\sqrt{2}\sin k\pi x$ funksiyaga ko'paytirib 0 dan 1 gacha integrallasak va ortonormallash shartini inobatga olib, ushbu

$$\pi^2(n^2-k^2)d_{nk} = -\sum_{s=1}^{\infty} b_{ns}F_{ks} - b_{nk}\lambda_{n1} - a_{nk}\lambda_{n2} - \lambda_{n3}\delta_{nk} \tag{24}$$

tenglikni hosil bo'ladi. Agar $k=n$ bo'lsa, u holda

$$\lambda_{n3} = -\sum_{s=1}^{\infty} b_{ns}F_{ns} - b_{nn}\lambda_{n1} - a_{nn}\lambda_{n2} = -\sum_{s\neq n} b_{ns}F_{ns} - a_{nn}\lambda_{n2} =$$
$$-\sum_{s\neq n} F_{ns}\left(\sum_{r\neq n}\frac{F_{ns}F_{sr}}{\pi^2(n^2-s^2)(n^2-r^2)} - \frac{a_{nn}F_{ns}}{\pi^2(n^2-r^2)} - \frac{F_{nn}F_{ns}}{\pi^4(n^2-s^2)^2}\right)$$
$$-\sum_{r\neq n} a_{nn}\frac{F_{nr}}{\pi^2(n^2-r^2)}.$$

formula o'rinli bo'ladi. Agar $k\neq n$ bo'lsa, (24) tenglikdan

$$d_{nk} = \frac{1}{\pi^2(k^2-n^2)}\sum_{s=1}^{\infty} b_{ns}F_{ks} + \frac{b_{nk}\lambda_{n1}}{\pi^2(k^2-n^2)} + \frac{a_{nk}\lambda_{n2}}{\pi^2(k^2-n^2)} =$$
$$= \frac{1}{\pi^2(k^2-n^2)}\sum_{s\neq n} F_{ks}\left(\sum_{r\neq n}\frac{F_{ns}F_{sr}}{\pi^2(n^2-s^2)(n^2-r^2)} - \frac{a_{nn}F_{ns}}{\pi^2(n^2-r^2)} - \frac{F_{nn}F_{ns}}{\pi^4(n^2-s^2)^2}\right) +$$
$$+\frac{b_{nn}F_{kn}}{\pi^2(k^2-n^2)} - \frac{F_{nn}}{\pi^2(k^2-n^2)}\left(\sum_{r\neq n}\frac{F_{nk}F_{kr}}{\pi^2(n^2-k^2)(n^2-r^2)} - \frac{a_{nn}F_{nk}}{\pi^2(n^2-r^2)} - \frac{F_{nn}F_{nk}}{\pi^4(n^2-k^2)^2}\right)$$
$$-\frac{F_{nk}}{\pi^2(n^2-k^2)}\sum_{r\neq n}\frac{F_{nr}}{\pi^2(n^2-r^2)} \tag{25}$$

formula keib chiqadi. Bu yerda d_{nn}-hozircha noma'lum son.
Quyidagi

$$\int_0^1 (u_{n0} + \varepsilon u_{n1} + \varepsilon^2 u_{n2} + \varepsilon^3 u_{n3})^2 dx = 1 \tag{26}$$

normallashtirish shartidan foydalanib,

$$\int_0^1 u_{n0}u_{n1}dx = 0 \tag{27}$$

$$\int_0^1 (2u_{n0}u_{n2} + u_{n1}^2)dx = 0 \tag{28}$$

$$\int_0^1 (2u_{n1}u_{n2} + 2u_{n0}u_{n3})dx = 0 \tag{29}$$

tengliklarni topamiz. Bunda $u_{n0}(x)$ funksiyaning normallashgani inobatga olinadi. Yuqoridagi (27) shartdan $a_{nn}=0$ ekanligi kelib chiqadi. Bundan tashqari (28) tenglikdan foydalanib

$$b_{nn}=-\frac{1}{2}\sum_{k\neq n}a_{nk}^2=-\frac{1}{2}\sum_{k\neq n}\left(\frac{F_{nk}}{\pi^2(n^2-k^2)}\right)^2 \qquad (25)$$

formulani hosil qilamiz. (29) shartdan esa

$$d_{nn}=-\sum_{k\neq n}a_{nk}b_{nk}=\sum_{k\neq n}\frac{F_{nk}}{\pi^2(n^2-k^2)}\left(\sum_{r\neq n}\frac{F_{nk}F_{kr}}{\pi^2(n^2-k^2)(n^2-r^2)}-\frac{F_{nn}F_{nk}}{\pi^4(n^2-k^2)^2}\right)$$

formulani hosil qilamiz. Shuning uchun (7) va (8) munosabatlarni quydagicha yozish mumkin:

$$u_n(x,\varepsilon)=\sqrt{2}\sin n\pi x-\varepsilon\sum_{k\neq n}\frac{F_{nk}}{\pi^2(n^2-k^2)}\sqrt{2}\sin k\pi x+$$

$$+\varepsilon^2\sum_{k\neq n}\left\{\left[\sum_{r\neq n}\frac{F_{nk}F_{kr}}{\pi^2(n^2-k^2)(n^2-r^2)}-\frac{F_{nn}F_{nk}}{\pi^4(n^2-k^2)^2}\right]\sqrt{2}\sin k\pi x-\frac{1}{2}\frac{F_{nk}^2}{\pi^4(n^2-k^2)^2}\sqrt{2}\sin n\pi x\right\}$$

$$+\varepsilon^3\left[\sum_{k\neq n}\left\{\frac{1}{\pi^2(k^2-n^2)}\left(\sum_{s\neq n}F_{ks}\left(\sum_{r\neq n}\frac{F_{ns}F_{sr}}{\pi^2(n^2-s^2)(n^2-r^2)}-\frac{F_{nn}F_{ns}}{\pi^4(n^2-s^2)^2}\right)+F_{kn}\sum_{r\neq n}\frac{1}{2}\left(\frac{F_{nr}}{\pi^2(n^2-r^2)}\right)^2-\right.\right.\right.$$

$$\left.-F_{nn}\left(\sum_{r\neq n}\frac{F_{nk}F_{kr}}{\pi^2(n^2-k^2)(n^2-r^2)}-\frac{F_{nn}F_{nk}}{\pi^4(n^2-k^2)^2}\right)-\frac{F_{nk}}{\pi^2(n^2-k^2)}\sum_{r\neq n}\frac{F_{nr}}{\pi^2(n^2-r^2)}\right)\sqrt{2}\sin k\pi x$$

$$\left.\left.+\left(\frac{F_{nk}}{\pi^2(n^2-k^2)}\left(\sum_{r\neq n}\frac{F_{nk}F_{kr}}{\pi^2(n^2-k^2)(n^2-r^2)}-\frac{F_{nn}F_{nk}}{\pi^4(n^2-k^2)^2}\right)\right)\sqrt{2}\sin n\pi x\right\}\right]+o(\varepsilon^3).$$

$$\lambda_n(\varepsilon)=n^2\pi^2-\varepsilon F_{nn}+\varepsilon^2\frac{F_{nk}^2}{\pi^4(n^2-k^2)^2}$$

$$+\varepsilon^3\left[\sum_{s\neq n}F_{ns}\left(\sum_{r\neq n}\frac{F_{ns}F_{sr}}{\pi^2(n^2-s^2)(n^2-r^2)}-\frac{a_{nn}F_{ns}}{\pi^2(n^2-r^2)}-\frac{F_{nn}F_{ns}}{\pi^4(n^2-s^2)^2}\right)\right]+o(\varepsilon^3)$$

2. Endi yuqoridagi masaladan bir muncha umumiyroq bo’lgan holni qaraymiz.

Ushbu

$$-y''+\varepsilon q(x)y=\lambda y,\quad 0\leq x\leq\pi \qquad (1)$$

$$\begin{cases} y'(0)-hy(0)=0\\ y'(\pi)-Hy(\pi)=0\end{cases} \qquad (2)$$

chegaraviymasalani qaraylik. Bu yerda $q(x)\epsilon C^1[0,\pi]$ bi marta uzluksiz differinsiallanuvchi funksiya bo’lib, h va H haqiqiy sonlar, $\varepsilon>0$ kichik parameter. Bu bo’limda (1), (2) umumiy chegaraviy shartli Shturm-Liuvill masalasining $\lambda_n(\varepsilon)$ xos qiymatlari va $y_n(x,\varepsilon)$ ortonormal xos funksiyalarining kichik parametr ε ga nisbatan asimptotikalari $o(\varepsilon^3)$ aniqlikda Relay – Shredinger usuli yordamida keltirilib chiqarilgan.

Ushbu

$$-y'' = \lambda y, \quad 0 \le x \le \pi \tag{3}$$

$$\begin{cases} y'(0) - hy(0) = 0 \\ y'(\pi) - Hy(\pi) = 0 \end{cases} \tag{4}$$

chegaraviy masalaning, ya'ni ε=0 bo'lgn holidagi (1) +(2) masalaning xos qiymatlarini $\lambda_{n0} = \lambda_n(0)$ orqali, ortonormal xos funksiyalarini $y_{n0}(x) = y_n(x, 0)$ orqali belgilaymiz.

Ma'lumki, Shturm-Liuvill chegaraviy masalasining xos qiymatlari haqiqiy bo'lib uning har xil xos qiymatga mos keluvchi xos funksiyalar o'zaro ortoganal bo'ladi. Shu bilan bir qatorda (4) chegaraviy shartlarni qanoatlantiruvchi ixtiyoriy $f(x) \epsilon C^2[0,\pi]$ funksiya uchun quyidagi teorema o'rinli:

Teorema. (*Yoyilma haqida*). *Agar $f(x) \epsilon C^2[0,\pi]$ ushbu*

$$\begin{cases} f'(0) - hf(0) = 0 \\ f'(\pi) - Hf(\pi) = 0 \end{cases}$$

chegaraviy shartlarni qanoatlantiruvchi ixtiyoriy funksiya bo'lsa, u holda quyidagi

$$f(x) = \sum_{n=0}^{\infty} a_n y_n(x), \tag{*}$$

tasvir o'rinli bo'ladi. Bu yerda $y_n(x)$ funksiyalar (1), (2) chegaraviy masalaning ortanormallangan xos funksiyalari bo'lib, a_n koeffitsiyentlar ushbu

$$a_n = \int_0^\pi f(t)\, y_n(t) dt$$

tenglik bilan aniqlanadi. (*) qator tekis va absolyut yaqinlashuvchi bo'ladi[1].

Berilgan (1)+ (2) chegaraviy masalaning $y_n(x, \varepsilon)$ ortaganal xos funksiyalarni va $\lambda_n(x, \varepsilon)$ xos qiymatlarni mos ravishda ushbu

$$y_n(x, \varepsilon) = y_{n0}(x, \varepsilon) + \varepsilon y_{n1}(x) + \varepsilon^2 y_{n2}(x) + \varepsilon^3 y_{n3}(x) + o(\varepsilon^3) \tag{5}$$

$$\lambda_n(x, \varepsilon) = \lambda_{n0}(x, \varepsilon) + \varepsilon \lambda_{n1}(x) + \varepsilon^2 \lambda_{n2}(x) + \varepsilon^2 \lambda_{n2}(x) + o(\varepsilon^3) \tag{6}$$

ko'rinishida izlaymiz. Bu yerda $y_{n1}(x)$, $y_{n2}(x)$, $y_{n3}(x)$ hozircha noma'lum funksiyalar bo'lib, λ_{n1}, λ_{n2}, λ_{n3} miqdorlar noma'lum sonlar.

Yuqoridagi (1)+(2) masalada $y = y_n(x, \varepsilon)$ va $\lambda = \lambda_n(x, \varepsilon)$ deb olib, ularning o'rniga (5) va (6) ifodalarni qo'ysak, quyidagi chegaraviy masalalar ketma-ketligi hosil bo'ladi:

$$\begin{cases} -y''_{n0} = \lambda_{n0} y_{n0} \\ y'_{n0}(0) - hy_{n0}(0) = 0 \\ y'_{n0}(\pi) - Hy_{n0}(\pi) = 0 \end{cases} \tag{7}$$

[1] Teorema isbotini A.B. Hasanov "Shturm-Liuvill chegaraviy masalalari nazariyasiga kirish" 1-qism.{ "Fan" nashriyoti, Toshkent 2011} 135-betdan qarash mumkin.

$$\begin{cases} -y''_{n1} - \lambda_{n0} y_{n1} = -q(x) y_{n0} + \lambda_{n1} y_{n0} \\ y'_{n1}(0) - h y_{n1}(0) = 0 \\ y'_{n1}(\pi) - H y_{n1}(\pi) = 0 \end{cases} \tag{8}$$

$$\begin{cases} -y''_{n2} - \lambda_{n0} y_{n2} = -q(x) y_{n1} + \lambda_{n1} y_{n1} + \lambda_{n2} y_{n2} \\ y'_{n2}(0) - h y_{n2}(0) = 0 \\ y'_{n2}(\pi) - H y_{n2}(\pi) = 0 \end{cases} \tag{9}$$

$$\begin{cases} -y''_{n3} - \lambda_{n0} y_{n3} = -q(x) y_{n21} + \lambda_{n1} y_{n2} + \lambda_{n2} y_{n1} + \lambda_{n3} y_{n0} \\ y'_{n3}(0) - h y_{n3}(0) = 0 \\ y'_{n3}(\pi) - H y_{n3}(\pi) = 0 \end{cases} \tag{10}$$

Ko'rinib turibdiki (7) chegaraviy masala (3)+(4) bilan bir xil. Shuning uchun (8) chegaraviy masaladan, avvalo $y_{n1}(x)$ funksiyani va λ_{n1} sonni topamiz. So'ngra (9) chegaraviy masaladan $y_{n2}(x)$ funksiyani hamda λ_{n2} sonni aniqlaymiz va bu topilganardan foydalanib (10) chegaraviy masaladan $y_{n3}(x)$ funksiyani hamda λ_{n3} sonni topamiz.

Aniqlanishiga ko'ra $y_{n1}(x)$ funksiya (2) chegaraviy shartlarni qanoatlantiradi. Shuning uchun uni ushbu

$$y_{n1}(x) = \sum_{n=0}^{\infty} a_{nm} y_{m0}(x), \quad a_{nm} = (y_{n1}, y_{m0}) \tag{11}$$

ko'rinishida izlaymiz. Bu yerda $(f\ , g)$ orqali $f(x)$ va $g(x)$ funksiyalarning $L_2(0,\pi)$ fazodagi skalyar ko'paytmasi belgilangan. Bu yoyilman differensiallab, quyidagi

$$y''_{n1} = \sum_{m=0}^{\infty} a_{nm} y''_{m0}(x) \tag{12}$$

tenglikni hosil qilamiz. Agar (11) va (12) ifodalarni (8) chegaraviy masaladagi differinsial tenglamaga qo'ysak, undan

$$\sum_{m=0}^{\infty} a_{nm} [-y''_{m0}(x) - \lambda_{n0} y_{m0}(x)] = -\, q(x) y_{n0}(x) + \lambda_{n1} y_{n0}(x)$$

munosabatlar kelib chiqadi. Bu tenglikning ikki tomonini $y_{k0}(x)$ ga ko'paytirib, hosil bo'lgan ifodani $[0,\pi]$ oraliqda integallab, ushbu

$$-\int_0^{\pi} y''_{m0}(x) y_{k0}(x) dx = \lambda_{m0} \int_0^{\pi} y_{m0}(x) y_{k0}(x) dx = \lambda_{m0} \delta_{mk}$$

munosabatlardan foydalansak, quyidagi

$$\sum_{m=0}^{\infty} a_{nm}\left(\lambda_{m0}\int_0^{\pi} y_{m0}(x)y_{k0}(x)dx - \lambda_{n0}\int_0^{\pi} y_{m0}(x)y_{k0}(x)dx\right)$$
$$= -\int_0^{\pi} q(x)y_{n0}(x)y_{k0}(x)dx + \lambda_{n1}\int_0^{\pi} y_{n0}(x)y_{k0}(x)dx$$

tenglikka ega bo'lamiz. Bu yerda

$$\delta_{mk} = \begin{cases} 0, & m \neq k \\ 1, & m = k \end{cases}$$

Bunda $m = k$ deb $\{y_{k0}(x)\}$ funksiyalarrning ortonormal sistemani tashkil qilishini inobatga olsak, ushbu

$$a_{nk}(\lambda_{k0} - \lambda_{n0}) = -\int_0^{\pi} q(x)y_{n0}(x)y_{k0}(x)dx + \lambda_{n1}\int_0^{\pi} y_{n0}(x)y_{k0}(x)dx \tag{13}$$

formula hosil bo'ladi. Agar (13) da $k \neq n$ deb olsak, undan

$$a_{nk} = \frac{1}{\lambda_{n0} - \lambda_{k0}} q_{nk} \tag{14}$$

tenglik kelinb chiqadi. Bu yerda

$$q_{nk} = \int_0^{\pi} q(x)y_{n0}(x)y_{k0}(x)dx.$$

Yuqoridagi (13) tenglikda $n = k$ deb,

$$\lambda_{n1} = q_{nn}, \quad q_{nn} = \int_0^{\pi} q(x)(y_{n0}(x))^2 dx \tag{15}$$

ekanligini topamiz. (13) tenglikdan foydalanib, (11) yoyilmani quyidagi ko'rinishda yozish mumkin:

$$y_{n1}(x) = \sum_{n \neq k} \frac{1}{\lambda_{n0} - \lambda_{k0}} q_{nk}\, y_{k0}(x) + a_{nn}y_{n0}(x). \tag{16}$$

Bu tenglikdagi a_{nn} hozircha noma'lum son bo'lib, uning qiymati $y_{n1}(x)$ funksiyalarning normallashtirish shartidan foydalanib topishimiz mumkin.

Endi navbatdagi (9) chegaraviy masalani qaraylik. Bundagi $y_{n2}(x)$ funksiya ham (2) chegaraviy shartlarni qanoatlantiradi. Shuning uchun uni

$$y_{n2}(x) = \sum_{l=0}^{\infty} b_{nl} y_{l0}(x), \qquad b_{nl} = (y_{n2}, y_{l0}) \tag{17}$$

ko'rinishida izlaymiz. Yuqoridagi (11) va (17) yoyilmani (9) ga qo'ysak,

$$-\sum_{l=0}^{\infty} b_{nl} y_{l0}'' \text{ (x)} - \lambda_{n0} \sum_{l=0}^{\infty} b_{nl} y_{l0}(x)$$
$$= -q(x) \sum_{l=0}^{\infty} a_{nl} y_{l0}(x) + \lambda_{n1} \sum_{l=0}^{\infty} a_{nl} y_{l0}(x) + \lambda_{n2} y_{n0}(x)$$

munosabatlar kelib chiqadi. Bu tenglikning ikki tomonini $y_{k0}(x)$ ga ko'paytirib, hosil bo'lgan ifodani [0,π] oraliqda integallab, ushbu

$$-y_{l0}'' \text{ (x)} = \lambda_{l0} y_{l0}(x)$$

munosabatdan foydalanamiz. Natijada quyidagi

$$\sum_{l=0}^{\infty} b_{nl} \left(\lambda_{l0} \int_0^{\pi} y_{l0}(x) y_{k0}(x) dx - \lambda_{n0} \int_0^{\pi} y_{l0}(x) y_{k0}(x) dx \right)$$
$$= -\sum_{l=0}^{\infty} a_{nl} \int_0^{\pi} q(x) y_{l0}(x) y_{k0}(x) dx$$
$$+ \lambda_{n1} \sum_{l=0}^{\infty} a_{nl} \int_0^{\pi} y_{l0}(x) y_{k0}(x) dx + \lambda_{n2} \int_0^{\pi} y_{n0}(x) y_{k0}(x) dx$$

tenglikka ega bo'lamiz. Bunda $l = k$ deb, ushbu

$$b_{nk}(\lambda_{k0} - \lambda_{n0}) = -\sum_{l=0}^{\infty} a_{nl} q_{lk} + \lambda_{n1} a_{nk} + \lambda_{n2} \delta_{nk} \tag{18}$$

munosabatni olamiz. Oxirgi (17) tenglikda $n \neq k$ bo'lsa, u holda

$$b_{nk} = \frac{1}{\lambda_{n0} - \lambda_{s0}} \sum_{l=0}^{\infty} a_{nl} q_{lk} + \frac{\lambda_{n1} a_{nk}}{\lambda_{k0} - \lambda_{n0}} \tag{19}$$

formula hosil bo'ladi. Agar (14) va (15) formulalarni e'tiborga olsak, u holda (18) tenglik quyidagi ko'rinishni oladi:

$$b_{nk} = \sum_{l \neq n} \frac{q_{nl} q_{lk}}{(\lambda_{n0} - \lambda_{k0})(\lambda_{n0} - \lambda_{l0})} + \frac{a_{nn} q_{nk}}{\lambda_{n0} - \lambda_{k0}} - \frac{q_{nn} q_{nk}}{(\lambda_{n0} - \lambda_{k0})^2}. \tag{20}$$

Yuqoridagi (18) tenglikda $n = k$ bo’lsa, undan

$$\lambda_{n2} = \sum_{l=0}^{\infty} a_{kl} q_{lk} - \lambda_{k1} a_{kk} = \sum_{l \neq n} \frac{q_{kl}^2}{\lambda_{k0} - \lambda_{l0}}$$

kelib chiqadi.

Endi (20) formuladan foydalanib, (17) tasvirni quyidagicha yozamiz:

$$y_{n2}(x) = \sum_{n \neq k} \left[\sum_{l \neq n} \frac{q_{nl} q_{lk}}{(\lambda_{n0} - \lambda_{k0})(\lambda_{n0} - \lambda_{l0})} + \frac{a_{nn} q_{nk}}{\lambda_{n0} - \lambda_{k0}} - \frac{q_{nn} q_{nk}}{(\lambda_{n0} - \lambda_{k0})^2} \right] y_{k0}(x)$$
$$+ b_{nn} y_{n0}(x). \qquad (21)$$

Bunda b_{nn} hozircha noma’lum son.

Endi navbatdagi (10) chegaraviy masalani qaraylik. Bundagi $y_{n3}(x)$ funksiya ham (2) chegaraviy shartlarni qanoatlantiradi. Shuning uchun uni

$$y_{n3}(x) = \sum_{s=0}^{\infty} d_{ns} y_{s0}(x), \qquad d_{ns} = (y_{n3}, y_{s0}) \qquad (22)$$

ko’rinishida izlaymiz. Yuqoridagi (11), (17) va (22) shartlarni (10) tenglamaga qo’yib,

$$-\sum_{s=0}^{\infty} d_{ns} y_{s0}''(\mathrm{x}) - \lambda_{n0} \sum_{s=0}^{\infty} d_{ns} y_{s0}(x) = -q(x) \sum_{s=0}^{\infty} b_{ns} y_{s0}(x) + \lambda_{n1} \sum_{s=0}^{\infty} b_{ns} y_{s0}(x)$$

$$+\lambda_{n2} \sum_{s=0}^{\infty} a_{ns} y_{s0}(x) + \lambda_{n3} y_{n0}(x) \qquad (23)$$

munosabati hosil qilamiz. Ushbu

$$-y_{l0}''(\mathrm{x}) = \lambda_{l0} y_{l0}(x)$$

tenglikka ko’ra (23) munosabat quyidagi ko’rinishni oladi:

$$\sum_{s=0}^{\infty} d_{ns}(\lambda_{s0} - \lambda_{n0})\, y_{s0}(x) = -q(x) \sum_{s=0}^{\infty} b_{ns} y_{s0}(x) + \lambda_{n1} \sum_{s=0}^{\infty} b_{ns} y_{s0}(x)$$

$$+\lambda_{n2} \sum_{s=0}^{\infty} a_{ns} y_{s0}(x) + \lambda_{n3} y_{n0}(x)$$

Bu tenglikning har ikkala tomonini $y_{k0}(x)$ ga ko'paytirib $[0,\pi]$ oraliqda integrallaymiz. Natijada quyidagi

$$\sum_{s=0}^{\infty} d_{ns}\left(\lambda_{s0}\int_0^{\pi} y_{s0}(x)y_{k0}(x)dx - \lambda_{n0}\int_0^{\pi} y_{s0}(x)y_{k0}(x)dx\right)$$
$$= -\sum_{s=0}^{\infty} b_{ns}\int_0^{\pi} q(x)y_{s0}(x)y_{k0}(x)dx + \lambda_{n1}\sum_{s=0}^{\infty} b_{ns}\int_0^{\pi} y_{s0}(x)y_{k0}(x)dx$$

$$+\lambda_{n2}\sum_{s=0}^{\infty} a_{ns}\int_0^{\pi} y_{s0}(x)y_{k0}(x)dx + \lambda_{n3}\int_0^{\pi} y_{s0}(x)y_{k0}(x)dx$$

tenglikka ega bo'lamiz. Bunda $s = k$ deb, ushbu

$$d_{nk}(\lambda_{k0} - \lambda_{n0}) = -\sum_{s=0}^{\infty} b_{ns}q_{sk} + \lambda_{n1}b_{nk} + \lambda_{n2}a_{nk} + \lambda_{n3}\delta_{nk} \qquad (24)$$

munosabatni olamiz. Oxirgi (23) tenglikda $n = k$ bo'lsa, u holda

$$\lambda_{n3} = \sum_{s=0}^{\infty} b_{ns}q_{sn} - \lambda_{n1}b_{nn} - \lambda_{n2}a_{nn} = \sum_{s\neq n}^{\infty} b_{ns}q_{sn} - \lambda_{n2}a_{nn} \qquad (25)$$

formula hosil bo'ladi. Agar (20) formulani e'tiborga olsak, u holda (25) formula quyidagi ko'rinihni oladi:

$$\lambda_{n3} = \sum_{s\neq n}^{\infty}\left(\sum_{l\neq n}\frac{q_{nl}q_{ls}}{(\lambda_{n0} - \lambda_{s0})(\lambda_{n0} - \lambda_{l0})} + \frac{a_{nn}q_{ns}}{\lambda_{n0} - \lambda_{s0}} - \frac{q_{nn}q_{ns}}{(\lambda_{n0} - \lambda_{s0})^2}\right)q_{sn} - \lambda_{n2}a_{nn} \quad (26)$$

Agar (24) tenglikda $n \neq k$ bo'lsa, undan

$$d_{nk} = \frac{1}{\lambda_{n0} - \lambda_{k0}}\sum_{s=0}^{\infty} b_{ns}q_{sk} + \frac{\lambda_{n1}b_{nk}}{\lambda_{k0} - \lambda_{n0}} + \frac{\lambda_{n2}a_{nk}}{\lambda_{k0} - \lambda_{n0}} \qquad (27)$$

kelib chiqadi. Yuqorida topilgan b_{nk} va a_{nk} koeffetsiyentlarni hamda λ_{n1} va λ_{n2} sonlarni (27) qo'ysak, u quyidagi ko'rinishni oladi:

$$d_{nk}=\frac{1}{\lambda_{n0}-\lambda_{k0}}\sum_{s\neq n}^{\infty}\left(\sum_{l\neq n}\frac{q_{nl}q_{ls}}{(\lambda_{n0}-\lambda_{s0})(\lambda_{n0}-\lambda_{l0})}+\frac{a_{nn}q_{ns}}{\lambda_{n0}-\lambda_{s0}}-\frac{q_{nn}q_{ns}}{(\lambda_{n0}-\lambda_{s0})^2}\right)q_{sk}$$

$$+\frac{q_{nn}}{\lambda_{k0}-\lambda_{n0}}\left(\sum_{l\neq n}\frac{q_{nl}q_{lk}}{(\lambda_{n0}-\lambda_{k0})(\lambda_{n0}-\lambda_{l0})}+\frac{a_{nn}q_{nk}}{\lambda_{n0}-\lambda_{k0}}-\frac{q_{nn}q_{nk}}{(\lambda_{n0}-\lambda_{k0})^2}\right)$$

$$+\frac{q_{nk}}{(\lambda_{k0}-\lambda_{n0})^2}\sum_{l\neq n}\frac{q_{nl}^2}{\lambda_{n0}-\lambda_{l0}}+\frac{b_{nn}q_{nk}}{\lambda_{n0}-\lambda_{k0}} \qquad (28)$$

d_{nn}- hozircha noma'lum son.

Quyidagi

$$\int_0^{\pi}[y_{n0}(x,\varepsilon)+\varepsilon y_{n1}(x)+\varepsilon^2 y_{n2}(x)+\varepsilon^3 y_{n3}(x)+o(\varepsilon^3)]^2dx=1$$

normallashtirish shartidan foydalanib, ushbu

$$\int_0^{\pi} y_{n0}(x)y_{n1}(x)dx=0, \qquad (29)$$

$$\int_0^{\pi}[2y_{n0}(x)y_{n2}(x)+(y_{n1}(x))^2]dx=0 \qquad (30)$$

$$\int_0^{\pi}[2y_{n1}(x)y_{n2}(x)+2y_{n0}(x)y_{n3}(x)]dx=0 \qquad (31)$$

munosabatlarni topamiz. (29) shartdan

$$a_{nn}=0 \qquad (32)$$

kelib chiqadi. Bundan foydalanib, (16) tasvirni ushbu

$$y_{n1}(x)=\left(\sum_{n\neq k}\frac{1}{\lambda_{n0}-\lambda_{k0}}q_{nk}\,y_{k0}x\right) \qquad (33)$$

ko'rinishda yozib olamiz. Hosil bo'lgan (21) va (33) tasvirlarni (30) shartga qo'yib, quyidagi

$$b_{nn}=-\frac{1}{2}\sum_{k\neq n}\frac{{q_{nk}}^2}{(\lambda_{n0}-\lambda_{k0})^2}=-\frac{1}{2}\sum_{k\neq n}a_{nk}^2 \qquad (34)$$

tenglikka ega bo'lamiz. Demak (21) tasvirni (32) va (34) dan foydalanib, quyidagicha yozish mumkin:

$$y_{n2}(x) = \sum_{n\neq k}\left(\sum_{l\neq n}\frac{q_{nl}q_{lk}}{(\lambda_{n0}-\lambda_{k0})(\lambda_{n0}-\lambda_{l0})} - \frac{q_{nn}q_{nk}}{(\lambda_{n0}-\lambda_{k0})^2}\right)y_{k0}(x)$$
$$-\frac{1}{2}\sum_{k\neq n}\frac{{q_{nk}}^2}{(\lambda_{n0}-\lambda_{k0})^2}\, y_{n0}(x). \qquad (35)$$

Yuqorida topilgan (33), (35) formulalardan hamda (22) tasvirdan foydalanib d_{nn} sonni topamiz.

$$d_{nn} = -\sum_{k\neq n}\left\{\frac{1}{\lambda_{n0}-\lambda_{k0}}q_{nk}\left(\sum_{l\neq n}\frac{q_{nl}q_{lk}}{(\lambda_{n0}-\lambda_{k0})(\lambda_{n0}-\lambda_{l0})} - \frac{q_{nn}q_{nk}}{(\lambda_{n0}-\lambda_{k0})^2}\right)\right\}$$

Yuqoridagi topilgan ma'lumotlar asosida (5) va (6) yoyilmalar quyidagi ko'rinishni oladi:

$$y_n(x,\varepsilon) = y_{n0}(x,\varepsilon) + \varepsilon\sum_{n\neq k}\frac{1}{\lambda_{n0}-\lambda_{k0}}q_{nk}\, y_{k0}(x) +$$
$$+\varepsilon^2\sum_{k\neq n}\left\{\left[\sum_{l\neq n}\frac{q_{nl}q_{lk}}{(\lambda_{n0}-\lambda_{k0})(\lambda_{n0}-\lambda_{l0})} - \frac{q_{nn}q_{nk}}{(\lambda_{n0}-\lambda_{k0})^2}\right]y_{k0}(x) - \frac{1}{2}\frac{{q_{nk}}^2}{(\lambda_{n0}-\lambda_{k0})^2}\, y_{n0}(x)\right\} +$$
$$+\varepsilon^3\left\{\sum_{k\neq n}\left(\frac{1}{\lambda_{n0}-\lambda_{k0}}\left(\sum_{s\neq n}^{\infty}\left(\sum_{l\neq n}\frac{q_{nl}q_{ls}}{(\lambda_{n0}-\lambda_{s0})(\lambda_{n0}-\lambda_{l0})} - \frac{q_{nn}q_{ns}}{(\lambda_{n0}-\lambda_{s0})^2}\right)q_{sk} +\right.\right.\right.$$
$$+\, q_{nn}\left(\sum_{l\neq n}\frac{q_{nl}q_{lk}}{(\lambda_{n0}-\lambda_{k0})(\lambda_{n0}-\lambda_{l0})} - \frac{q_{nn}q_{nk}}{(\lambda_{n0}-\lambda_{k0})^2}\right) + \frac{q_{nk}}{(\lambda_{k0}-\lambda_{n0})}\sum_{l\neq n}\frac{q_{nl}^2}{\lambda_{n0}-\lambda_{l0}}$$
$$\left.\left.\left.+\frac{q_{nk}}{2}\sum_{s\neq n}a_{ns}^2\right)y_{k0}(x) - \left\{\frac{q_{nk}}{\lambda_{n0}-\lambda_{k0}}\left(\sum_{l\neq n}\frac{q_{nl}q_{lk}}{(\lambda_{n0}-\lambda_{k0})(\lambda_{n0}-\lambda_{l0})} - \frac{q_{nn}q_{nk}}{(\lambda_{n0}-\lambda_{k0})^2}\right)\right\} y_{n0}(x)\right)\right\}$$
$$+\ o(\varepsilon^3)$$

$$\lambda_n(x,\varepsilon) = \lambda_{n0}(x,\varepsilon) + \varepsilon q_{nn} + \varepsilon^2\sum_{l\neq n}\frac{q_{kl}^2}{\lambda_{k0}-\lambda_{l0}} +$$
$$+\varepsilon^3\sum_{s\neq n}^{\infty}\left(\sum_{l\neq n}\frac{q_{nl}q_{ls}}{(\lambda_{n0}-\lambda_{s0})(\lambda_{n0}-\lambda_{l0})} - \frac{q_{nn}q_{ns}}{(\lambda_{n0}-\lambda_{s0})^2}\right)q_{sn} + o(\varepsilon^3)$$

3. Yuqoridagi badlarda h va H sonlar chekli bo'lganda kichik parametrli Shturm-Liuvill chegaraviy masalasining $\lambda_n(\varepsilon)$ xos qiymatlari va $y_n(x,\varepsilon)$ ortanormal xos funksiyalarining kichik parametr ε ga nisbatan asimptotalari $o(\varepsilon^3)$ aniqlikda keltirib chiqarilgan edi. Quyida biz bu sonlarning ixtiyoriy bittasi cheksiz bo'lgan holni qaraymiz.

Agar (2) chegaraviy shartlarda $H=\infty\ va\ h=0$ deb tanlasak, u holda (1)+(2) chegaraviy masala quyidagi ko'rinishni oladi:

$$\begin{cases} -y'' + \varepsilon q(x)y = \lambda y, & 0 \le x \le \pi \\ y'(0) = 0, & y(\pi) = 0 \end{cases} \tag{36}$$

Bizga ma'lumki, ushbu

$$\begin{cases} -y'' = \lambda y, & 0 \le x \le \pi \\ y'(0) = 0, & y(\pi) = 0 \end{cases} \tag{37}$$

chegaraviy masalaning, ya'ni $\varepsilon = 0$ bo'lgan holdagi (36) masalaning xos qiymatlari va ortanormal xos funksiyalari

$$\lambda_{n0} = (n + \tfrac{1}{2})^2, \qquad y_{n0} = \sqrt{\frac{2}{\pi}} \cos\left(n + \frac{1}{2}\right)x, \qquad n = 0,1,2 \ldots \tag{38}$$

ko'rinishda bo'ladi. Berilgan (36) chegaraviy masalaning $y_n(x, \varepsilon)$ ortanormal xos funksiyalarni va $\lambda_n(x, \varepsilon)$ xos qiymatlarni mos ravishda ushbu

$$y_n(x, \varepsilon) = \sqrt{\frac{2}{\pi}} \cos\left(n + \frac{1}{2}\right)x + \varepsilon y_{n1}(x) + \varepsilon^2 y_{n2}(x) + \varepsilon^3 y_{n3}(x) + o(\varepsilon^3) \tag{39}$$

$$\lambda_n(x, \varepsilon) = (n + \tfrac{1}{2})^2 + \varepsilon \lambda_{n1}(x) + \varepsilon^2 \lambda_{n2}(x) + \varepsilon^2 \lambda_{n2}(x) + o(\varepsilon^3) \tag{40}$$

ko'rinishida izlaymiz. Bu yerda $y_{n1}(x)$, $y_{n2}(x)$, $y_{n3}(x)$ hozircha noma'lum funksiyalar bo'lib, λ_{n1}, λ_{n2}, λ_{n3} miqdorlar esa noma'lum sonlar.

Yuqoridagi (36) masalada $y = y_n(x, \varepsilon)$ va $\lambda = \lambda_n(x, \varepsilon)$ deb olib, ularning o'rniga (39) va (40) ifodalarni qo'ysak, quyidagi chegaraviy masalalar ketma-ketligi hosil bo'ladi:

$$\begin{cases} -y''_{n1} - (n + \frac{1}{2})^2 y_{n1} = -q(x)\sqrt{\frac{2}{\pi}} \cos\left(n + \frac{1}{2}\right)x + \lambda_{n1}\sqrt{\frac{2}{\pi}} \cos\left(n + \frac{1}{2}\right)x \\ y'_{n1}(0) = 0, \quad y_{n2}(\pi) = 0 \end{cases} \tag{41}$$

$$\begin{cases} -y''_{n2} - \left(n + \frac{1}{2}\right)^2 y_{n2} = -q(x)y_{n1} + \lambda_{n1}y_{n1} + \lambda_{n2}\sqrt{\frac{2}{\pi}} \cos\left(n + \frac{1}{2}\right)x \\ y'_{n2}(0) = 0, \quad y_{n2}(\pi) = 0 \end{cases} \tag{42}$$

$$\begin{cases} -y''_{n3} - \left(n + \frac{1}{2}\right)^2 y_{n3} = -q(x)y_{n2} + \lambda_{n1}y_{n2} + \lambda_{n2}y_{n1} + \lambda_{n3}\sqrt{\frac{2}{\pi}} \cos\left(n + \frac{1}{2}\right)x \\ y'_{n3}(0) = 0, \quad y_{n3}(\pi) = 0 \end{cases} \tag{43}$$

Avvalo, (41) chegaraviy masaladan, $y_{n1}(x)$ funksiyani va λ_{n1} sonni topamiz. So'ngra (42) chegaraviy masaladan $y_{n2}(x)$ funksiyani hamda λ_{n2} sonni aniqlaymiz va bu topilganardan foydalanib (43) chegaraviy masaladan $y_{n3}(x)$ funksiyani hamda λ_{n3} sonni topamiz.

Aniqlanishiga ko'ra $y_{n1}(x)$ funksiya (41) chegaraviy shartlarni qanoatlantiradi. Shuning uchun uni ushbu

$$y_{n1}(x)=\sum_{m=0}^{\infty} a_{nm}\sqrt{\frac{2}{\pi}}\cos\left(m+\frac{1}{2}\right)x,\quad a_{nm}=\left(y_{n1},\sqrt{\frac{2}{\pi}}\cos\left(n+\frac{1}{2}\right)x\right) \qquad (44)$$

ko'rinishida izlaymiz. Bu yoyilman differensiallab, quyidagi

$$y''_{n1}=-\sum_{m=0}^{\infty} a_{nm}(m+\frac{1}{2})^2\sqrt{\frac{2}{\pi}}\cos\left(m+\frac{1}{2}\right)x \qquad (45)$$

tenglikni hosil qilamiz. Agar (44) va (45) ifodalarni (8) chegaraviy masaladagi differinsial tenglamaga qo'ysak, undan

$$\sum_{m=0}^{\infty} a_{nm}\sqrt{\frac{2}{\pi}}\left[(m+\frac{1}{2})^2-(n+\frac{1}{2})^2\right]\cos\left(m+\frac{1}{2}\right)x=-\,q(x)\sqrt{\frac{2}{\pi}}\cos\left(n+\frac{1}{2}\right)x$$
$$+\lambda_{n1}\sqrt{\frac{2}{\pi}}\cos\left(n+\frac{1}{2}\right)x$$

munosabatlar kelib chiqadi. Bu tenglikning ikki tomonini $\sqrt{\frac{2}{\pi}}\cos\left(k+\frac{1}{2}\right)x$ga ko'paytirib, hosil bo'lgan ifodani [0,π] oraliqda integallab, xos funksiyalarning ortonormallanganligidan foydalansak , ushbu

$$\left((k+\frac{1}{2})^2-(n+\frac{1}{2})^2\right)a_{nk}=-F_{nk}+\lambda_{n1}\delta_{nk} \qquad (46)$$

munosabat hosil bo'ladi. Bu yerda

$$F_{nk}=\frac{2}{\pi}\int_0^1 q(x)\cos\left(k+\frac{1}{2}\right)x\cos\left(n+\frac{1}{2}\right)x\ dx. \qquad (47)$$

Agar (46) tenglikda $n=k$ bo'lsa , u holda

$$\lambda_{n1}=F_{nn}=\frac{2}{\pi}\int_0^1 q(x)\cos^2\left(n+\frac{1}{2}\right)x\ dx. \qquad (48)$$

Agar (46) tenglikda $n\neq k$ bo'lsa , u holda

$$a_{nk}=\frac{F_{nk}}{(n+\frac{1}{2})^2-(k+\frac{1}{2})^2} \qquad (49)$$

o'rinli bo'ladi. Qaralayotgan holda $y_{n1}(x)$ funksiay quydagi

$$y_{n1}(x)=\sum_{k\neq n}\frac{F_{nk}}{(k+\frac{1}{2})^2-(n+\frac{1}{2})^2}\sqrt{\frac{2}{\pi}}\cos\left(k+\frac{1}{2}\right)x+a_{nn}\sqrt{\frac{2}{\pi}}\cos\left(n+\frac{1}{2}\right)x \qquad (50)$$

formula bo'yicha hisoblanadi. Bu tenglikning o'ng tomonidagi a_{nn} hozircha noma'lum bo'lib, uning qiymatini $y_{n1}(x)$ ning normalashtirish shartda topish mumkin.

Endi (42) chegaraviy masalani qaraymiz . Faraz qilaylik

$$y_{n2}(x) = \sum_{r=1}^{\infty} b_{nr} \sqrt{\frac{2}{\pi}} \cos\left(r + \frac{1}{2}\right) x \qquad (51)$$

ko'rinishda bo'lsin. Ravshanki $y_{n2}(x)$ funksiya aniqlanishiga ko'ra (42) chegaraviy shartlarni qanoatlantiradi. Ushbu $y_{n2}(x), y_{n1}(x)$ va $y_{n0}(x)$ funksiyalarning ifodalarini (10) tenglamaga qo'yib

$$\sum_{r=0}^{\infty} b_{nr} \sqrt{\frac{2}{\pi}} \left[(r + \frac{1}{2})^2 - (n + \frac{1}{2})^2\right] \cos\left(r + \frac{1}{2}\right) x = = -\sum_{r=1}^{\infty} a_{nr}\, q(x) \sqrt{\frac{2}{\pi}} \cos\left(r + \frac{1}{2}\right) x$$

$$+ \sum_{r=1}^{\infty} a_{nr}\, \lambda_{n1} \sqrt{\frac{2}{\pi}} \cos\left(r + \frac{1}{2}\right) x + \lambda_{n2} \sqrt{\frac{2}{\pi}} \cos\left(r + \frac{1}{2}\right) x \qquad (52)$$

tenglikni hosil qilamiz. Bu tenglikning ikki tomonini $\sqrt{\frac{2}{\pi}} \cos\left(k + \frac{1}{2}\right) x$ funksiyaga ko'paytirib 0 dan 1 gacha integrallasak va ortonormallash shartini inobatga olib, ushbu

$$\left((k + \frac{1}{2})^2 - (n + \frac{1}{2})^2\right) b_{nk} = -\sum_{r=1}^{\infty} a_{nr}\, F_{kr} + a_{nk}\lambda_{n1} + \lambda_{n2}\delta_{nk} \qquad (53)$$

munosabatni olamiz . Agar $k = n$ bo'lsa, u holda

$$\lambda_{n2} = \sum_{r \neq n} a_{nr} F_{kr} = \sum_{r \neq n} \frac{{F_{nk}}^2}{(n + \frac{1}{2})^2 - (k + \frac{1}{2})^2}. \qquad (54)$$

formula o'rinli bo'ladi.

Agar $k \neq n$ bo'lsa, u holda (53) tenglikdan

$$b_{nk} = \sum_{r \neq n} \frac{F_{nk} F_{kr}}{\left((n + \frac{1}{2})^2 - (k + \frac{1}{2})^2\right)\left((n + \frac{1}{2})^2 - (n + \frac{1}{2})^2\right)} - \frac{a_{nn} F_{nk}}{\left(k + \frac{1}{2}\right)^2 - \left(n + \frac{1}{2}\right)^2}$$

$$- \frac{F_{nn} F_{nk}}{\left(\left(k + \frac{1}{2}\right)^2 - \left(n + \frac{1}{2}\right)^2\right)^2} \qquad (55)$$

formula kelib chiqadi. Bu yerda b_{nn} – hozircha noma'lum.

Endi yuqorida topilganlardan foydalanib (43) chegaraviy masalani yechamiz. Faraz qilaylik

$$y_{n3}(x) = \sum_{s=1}^{\infty} d_{ns} \sqrt{\frac{2}{\pi}} \cos\left(s + \frac{1}{2}\right) x \qquad (56)$$

ko'rinishda bo'lsin. U holda (43) tenglik quyidagi korinishni oladi:

$$\sum_{s=1}^{\infty}\left((s+\frac{1}{2})^2-(n+\frac{1}{2})^2\right)d_{ns}\sqrt{\frac{2}{\pi}}\cos\left(s+\frac{1}{2}\right)x=-\sum_{s=1}^{\infty}b_{ns}\,q(x)\sqrt{\frac{2}{\pi}}\cos\left(s+\frac{1}{2}\right)x+$$

$$+\sum_{s=1}^{\infty}b_{ns}\,\lambda_{n1}\sqrt{\frac{2}{\pi}}\cos\left(s+\frac{1}{2}\right)x+\sum_{s=1}^{\infty}a_{ns}\,\lambda_{n2}\sqrt{\frac{2}{\pi}}\cos\left(s+\frac{1}{2}\right)x+\lambda_{n3}\sqrt{\frac{2}{\pi}}\cos\left(n+\frac{1}{2}\right)x$$

Bu tenglikning ikki tomonini $\sqrt{\frac{2}{\pi}}\cos\left(k+\frac{1}{2}\right)x$ funksiyaga ko'paytirib 0 dan 1 gacha integrallasak va ortonormallash shartini inobatga olib, ushbu

$$\left((k+\frac{1}{2})^2-(n+\frac{1}{2})^2\right)d_{nk}=-\sum_{s=1}^{\infty}b_{ns}\,F_{ks}+b_{nk}\lambda_{n1}+\lambda_{n2}a_{nk}+\lambda_{n3}\delta_{nk} \tag{57}$$

tenglikni hosil bo'ladi. Agar $k=n$ bo'lsa, u holda

$$\lambda_{n3}=\sum_{s=1}^{\infty}b_{ns}\,F_{ns}-b_{nn}\lambda_{n1}-a_{nn}\lambda_{n2}=\sum_{s\neq n}b_{ns}F_{ns}-a_{nn}\lambda_{n2} \tag{58}$$

formula o'rinli bo'ladi. Agar $k\neq n$ bo'lsa, (57) tenglikdan

$$d_{nk}=\frac{1}{(n+\frac{1}{2})^2-(k+\frac{1}{2})^2}\sum_{s=1}^{\infty}b_{ns}\,F_{ks}+\frac{b_{nk}\lambda_{n1}}{(k+\frac{1}{2})^2-(n+\frac{1}{2})^2}+\frac{a_{nk}\lambda_{n2}}{(k+\frac{1}{2})^2-(n+\frac{1}{2})^2} \tag{59}$$

formula keib chiqadi. Bu yerda d_{nn}-hozircha noma'lum son.
Quyidagi

$$\int_0^1(y_{n0}+\varepsilon y_{n1}+\varepsilon^2y_{n2}+\varepsilon^3y_{n3})^2dx=1 \tag{60}$$

normallashtirish shartidan foydalanib,

$$\int_0^1 y_{n0}y_{n1}dx=0 \tag{61}$$

$$\int_0^1(2y_{n0}y_{n2}+y_{n1}^2)dx=0 \tag{62}$$

$$\int_0^1(2y_{n1}y_{n2}+2y_{n0}y_{n3})dx=0 \tag{63}$$

tengliklarni topamiz. Bunda $y_{n0}(x)$ funksiyaning normallashgani inobatga olinadi. Yuqoridagi (61) shartdan $a_{nn}=0$ ekanligi kelib chiqadi. Bundan tashqari (62) tenglikdan foydalanib

$$b_{nn} = -\frac{1}{2}\sum_{k\neq n} a_{nk}^2 = -\frac{1}{2}\sum_{k\neq n}\left(\frac{F_{nk}}{(n+\frac{1}{2})^2 - (k+\frac{1}{2})^2}\right)^2 \tag{64}$$

formulani hosil qilamiz. (63) shartdan esa

$$d_{nn} = -\sum_{k\neq n} a_{nk}b_{nk}$$

formulani hosil qilamiz.

$$y_{n1}(x) = \left(\sum_{n\neq k}\sqrt{\frac{2}{\pi}}\frac{F_{nk}\cos\left(k+\frac{1}{2}\right)x}{(n+\frac{1}{2})^2 - (k+\frac{1}{2})^2}\right) \tag{44}$$

$$\lambda_{n1} = F_{nn}, \quad F_{nk} = \frac{2}{\pi}\int_0^\pi q(x)\cos\left(n+\frac{1}{2}\right)x\cos\left(k+\frac{1}{2}\right)xdx \tag{45}$$

$$\lambda_{n2} = \sum_{l\neq n}\frac{F_{kl}^2}{(n+\frac{1}{2})^2 - (l+\frac{1}{2})^2} \tag{46}$$

$$y_{n2}(x) = \sum_{n\neq k}\left(\sum_{l\neq n}\frac{F_{nl}F_{lk}}{((n+\frac{1}{2})^2 - (k+\frac{1}{2})^2)((n+\frac{1}{2})^2 - (l+\frac{1}{2})^2)}\right.$$
$$\left. - \frac{F_{nn}F_{nk}}{((n+\frac{1}{2})^2 - (k+\frac{1}{2})^2)^2}\right)\sqrt{\frac{2}{\pi}}\cos\left(k+\frac{1}{2}\right)x$$
$$-\frac{1}{2}\sum_{k\neq n}\frac{{F_{nk}}^2}{((n+\frac{1}{2})^2 - (k+\frac{1}{2})^2)^2}\sqrt{\frac{2}{\pi}}\cos\left(n+\frac{1}{2}\right)x. \tag{47}$$

$$y_{n3}(x)=\sum_{k\neq n}\left\{\frac{1}{\left(n+\frac{1}{2}\right)^2-\left(k+\frac{1}{2}\right)^2}\left(\sum_{s\neq n}^{\infty}\left(\sum_{l\neq n}\frac{F_{nl}F_{ls}}{\left(\left(n+\frac{1}{2}\right)^2-\left(s+\frac{1}{2}\right)^2\right)\left(\left(n+\frac{1}{2}\right)^2-\left(k+\frac{1}{2}\right)^2\right)}\right.\right.\right.$$

$$\left.-\frac{F_{nn}F_{ns}}{\left(\left(n+\frac{1}{2}\right)^2-\left(s+\frac{1}{2}\right)^2\right)^2}\right)F_{sk}+$$

$$+F_{nn}\left(\sum_{l\neq n}\frac{F_{nl}F_{lk}}{\left(\left(n+\frac{1}{2}\right)^2-\left(k+\frac{1}{2}\right)^2\right)\left(\left(n+\frac{1}{2}\right)^2-\left(l+\frac{1}{2}\right)^2\right)}-\frac{F_{nn}F_{nk}}{\left(\left(n+\frac{1}{2}\right)^2-\left(k+\frac{1}{2}\right)^2\right)^2}\right)$$

$$\left.+\frac{F_{nk}}{\left(n+\frac{1}{2}\right)^2-\left(k+\frac{1}{2}\right)^2}\sum_{l\neq n}\frac{F_{nl}^2}{\left(n+\frac{1}{2}\right)^2-\left(l+\frac{1}{2}\right)^2}+\frac{F_{nk}}{2}\sum_{s\neq n}a_{ns}^2\right)\sqrt{\frac{2}{\pi}}\cos\left(k+\frac{1}{2}\right)x$$

$$-\left\{\frac{F_{nk}}{\left(n+\frac{1}{2}\right)^2-\left(k+\frac{1}{2}\right)^2}\left(\sum_{l\neq n}\frac{F_{nl}F_{lk}}{\left(\left(n+\frac{1}{2}\right)^2-\left(k+\frac{1}{2}\right)^2\right)\left(\left(n+\frac{1}{2}\right)^2-\left(l+\frac{1}{2}\right)^2\right)}\right.\right.$$

$$\left.\left.\left.-\frac{F_{nn}F_{nk}}{\left(\left(n+\frac{1}{2}\right)^2-\left(k+\frac{1}{2}\right)^2\right)^2}\right)\right\}\sqrt{\frac{2}{\pi}}\cos\left(n+\frac{1}{2}\right)x\right\} \tag{48}$$

$$\lambda_{n3}=\sum_{s\neq n}\left(\sum_{l\neq n}\frac{F_{nl}F_{ls}}{\left(\left(n+\frac{1}{2}\right)^2-\left(s+\frac{1}{2}\right)^2\right)\left(\left(n+\frac{1}{2}\right)^2-\left(k+\frac{1}{2}\right)^2\right)}-\frac{F_{nn}F_{ns}}{\left(\left(n+\frac{1}{2}\right)^2-\left(s+\frac{1}{2}\right)^2\right)^2}\right)F_{sn} \tag{49}$$

Bu topilganlarni (39) va (40) olib borib qo'ysak, (36) masalaning $o(\varepsilon^3)$ aniqlikdagi ortanoral xos fuksiyasi va xos qiymati hosil bo'ladi

3. Quyidagi

$$\varepsilon y''+ay'+by=0\,, \qquad \varepsilon>0\,,ab\neq 0 \tag{*}$$

$$y(0)=y_1\,,\ \ y(0)=y_2\,,\ \ |y_1|+|y_2|>0 \tag{**}$$

chegaraviy masala yechimini $\varepsilon\to+0$ dagi holatini o'rganamiz. Berilgan (*)+(**) chegaraviy masala yechimini $y_\varepsilon(x)$ orqali, ushbu

$$ay'+by=0$$

$$y(0)=y_1$$

Koshi masalasi yechimini esa $y_0(x)$orqali belgilaymiz.

Teorema-1. Aytaylik $a < 0$ bo'lsin. U holda (*) + (**) chegaraviy masalaning $y_\varepsilon(x)$ yechimi mavjud va yagona bo'lib, har bir $x \in [0,1-\delta]$ larda

$$y_\varepsilon(x) \to y_0(x), \ \ \varepsilon \to +0$$

munosabat o'rinli bo'ladi. Bundan tashqari, ixtiyoriy $x \in [0,1-\delta]$, $0 < \delta < 1$ da quydagi yaqinlashtirish

$$y_\varepsilon(x) \to y_0(x), \ \ \varepsilon \to +0$$

tekis bajariladi.

Isbot. Ushbu

$$\varepsilon\lambda^2 + a\lambda + b = 0$$

xarakteristik tenglamaning $\lambda_1 = \lambda_1(x)$, $\lambda_2 = \lambda_2(x)$ildizlarni yetarli kichik$\varepsilon > 0 \left(\varepsilon < \frac{a^2}{4b}\right)$ uchun haqiqiy va har xil bo'ladi. Shuning uchun (1) tenglamaning umumiy yechim

$$y_\varepsilon(x) = c_1(\varepsilon)e^{\lambda_1 x} + c_2(\varepsilon)e^{\lambda_2 x}$$

ko'rinishda bo'ladi. Qaralayotgan holda $e^{\lambda_1} \neq e^{\lambda_2}$ bo'lganligi uchun yetarli kichik $\varepsilon > 0$ da (*) + (**) chegaraviy masalaning yechimi mavjud va yagona bo'lib, u quyidagi

$$y_\varepsilon(x) = \frac{1}{e^{\lambda_1} - e^{\lambda_2}}\left[\left(\alpha e^{\lambda_2} - \beta\right)e^{\lambda_1 x} + \left(\alpha e^{\lambda_1} - \beta\right)e^{\lambda_2 x}\right]$$

ko'rinishda bo'ladi.

Ushbu

$$(1+x)^m = 1 + \frac{m}{1!}x + o(x^2)$$

formuladan foydalanib $\varepsilon \to +0$ da

$$\lambda_1 = \frac{-a + \sqrt{a^2 - 4\varepsilon b}}{2\varepsilon} = -\frac{b}{a} + o(\varepsilon)$$

$$\lambda_2 = \frac{-a - \sqrt{a^2 - 4\varepsilon b}}{2\varepsilon} = -\frac{a}{\varepsilon} + \frac{b}{a} + o(\varepsilon)$$

munosabatlarni topamiz. Endi quydagi ayirmani baholaymiz:

$$|y_\varepsilon(x)-y_0(x)|=\left|\frac{1}{e^{\lambda_1}-e^{\lambda_2}}\left[\left(\alpha e^{\lambda_2}-\beta\right)e^{\lambda_1 x}+\left(\alpha e^{\lambda_1}-\beta\right)e^{\lambda_2 x}\right]-\alpha e^{-\frac{b}{a}x}\right|\le$$

$$\le\frac{\left|\beta e^{\lambda_1 x}-\alpha e^{-\frac{b}{a}x}e^{\lambda_1}\right|}{e^{\lambda_2}-e^{\lambda_1}}+\frac{e^{\lambda_2}}{|e^{\lambda_2}-e^{\lambda_1}|}\left\{\left|-\alpha e^{\lambda_1 x}+\alpha e^{-\frac{b}{a}x}\right|+\left|\beta-\alpha e^{\lambda_1}\right|e^{\lambda_2(x-1)}\right\}$$

Ko'rinib turibdiki, shunday $A_1>0,\ A_2>0$ sonlar topilib, ixtiyoriy $x\in[0,1]$ va barcha $\varepsilon>0$ lar uchun

$$\left|\beta e^{\lambda_1 x}-\alpha e^{-\frac{b}{a}x}e^{\lambda_1}\right|\le A_1,\qquad \left|\beta-\alpha e^{\lambda_1}\right|\le A_2$$

tengsizliklar bajariladi. Bu tengsizliklardan foydalanib $\varepsilon\to+0$ da quyidagi

$$e^{\lambda_2}-e^{\lambda_1}=e^{\lambda_2}[1+o(\varepsilon)]=e^{\left(-\frac{a}{\varepsilon}+\frac{b}{a}\right)}[1+o(\varepsilon)],$$

$$\left|\alpha e^{\lambda_1 x}-\alpha e^{-\frac{b}{a}x}\right|=|\alpha|e^{-\frac{b}{a}x}\left|1+e^{xo(\varepsilon)}\right|=$$

$$=|\alpha|e^{-\frac{b}{a}x}xo(\varepsilon)\le A_3 o(\varepsilon)$$

baholarni olamiz. Bu yerda $A_3>0$.

Endi , ushbu

$$|y_\varepsilon(x)-y_0(x)|\le A_1 e^{\frac{a}{\varepsilon}-\frac{b}{a}}[1+o(\varepsilon)]+$$

$$+[1+o(\varepsilon)]\left\{A_3 o(\varepsilon)+A_2 e^{\left(-\frac{a}{\varepsilon}+\frac{b}{a}\right)(x-1)}[1+o(\varepsilon)]\right\}$$

bahoga ega bo'lamiz.

Agar $a<0$ bo'lsa, u holda $|y_\varepsilon(x)-y_0(x)|$ ayirma $\varepsilon\to+0$ da yuqoridagi tengsizliklarning oxirgi hadiga bog'liq holda o'zgaradi. Aniqrog'I $\varepsilon\to+0$ da

$$|y_\varepsilon(x)-y_0(x)|\to 0$$

munosabat har bir $0\le x<1$ da bajariladi. Bundan tashqari

$$\max_x|y_\varepsilon(x)-y_0(x)|\to 0$$

munosabat esa, har bir $0\le x\le 1-\delta,\quad 0<x<1$ da o'rinli bo'ladi.

Bu teoremadan kelib chiqadiki $a < 0$ holda (*) + (**) chegaraviy masala uchun har bir $[1-\delta, 1]$, $0 < x < 1$ kesma chegaraviy qatlam vazifasini o'taydi.

Xulosa

Biror fizik jarayonni tavsiflovchi differensial tenglama parametrlarga (jumladan massa, elastiklik koeffitsiyentlari va hakoza fizik kattaliklar) bog'liq bo'ladi. Bu parametrlarning qiymatlarini real masalalarda aniq o'lchamini hisoblashning imkoni yo'q, odatda taqribiy hisoblanadi. Ma'lum jarayonni tavsiflovchi differensial tenglamani keltirib chiqarish jarayonida ham xatolikka yo'l qo'yiladi. Shu maqsadda differensial tenglama real jarayonni tavsiflashi uchun, uning yechimi parametrlarga uzluksiz ravishda bog'liq bo'lishi kerak, ya'ni parametrlarning kichik o'zgarishiga differensial tenglamaning yechimi ham mos ravishda kichik o'zgarishi lozim.

Mazkur bitiruv malakaviy ishida avvalo, hosilaga nisbatan yechilgan birinchi tartibli oddiy differensial tenglamaga qo'yilgan Koshi masalasining korrektligi ko'rsatilgan. Bundan keyin kichik parametrli birinchi tartibli differinsial tenglamaga qo'yilgan Koshi masalasini yechish usullari bayon qilingan. Shu bilan bir qatorda kichik parametrli Shturm – Liuvill chegaraviy masalasi ham o'rganilgan.

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