O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

TOSHKENT KIMYO-TEXNOLOGIYA INSTITUTI

«OZIQ-OVQAT SANOATI MASHINA VA JIHOZLARI – MEXANIKA ASOSLARI»KAFEDRASI

NAZARIY MEXANIKA

fanidan hisoblash loyihalash ishlari va ularni bajarish bo'yicha uslubiy qo'llanma

Toshkent – 2017

ANNOTATSIYA

Texnika oliy o’quv yurtlarida o’qitiladigan «Nazariy mexanika» fanini o’zlashtirishda, bu fan bo’yicha talabalar bajarishi lozim bo’lgan hisoblash loyihalash ishlari, ushbu fan asoslarini o’rganishda alohida o’rin tutadi. Chunki hisoblash loyihalash ishlarini bajarish jarayonida talabalar nazariy bilimlarini amaliy masalalarni yechishga tadbiq etishni hamda hisoblash loyihalash ishlarini bajarish ko’nikmalarini hosil qiladilar.

Ushbu uslubiy qo’llanmada texnika oliy o’quv yurtlarida bakalavriat ta’lim yo’nalishi bo’yisha ta’lim olayotgan talabalarning nazariy mexanikaning «Statika» «Kinematika» «Dinamika» bo’limlari bo’yicha bajarishi lozim bo’lgan hisoblash loyihalash ishlarining variantlari va ularni yechishning namunalari keltirilgan.

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Mazkur uslubiy qo’llanma “Oziq-ovqat sanoati mashina va jihozlari – mexanika asoslari” kafedrasi majlisida muhokama qilindi va “Oziq-ovqat mahsulotlari texnologiyasi” fakulteti “Ilmiy uslubiy kengashi” ga tavsiya etildi.

Bayonnoma №____ ___ _____________ 2017 y.

Uslubiy qo’llanma TKTI “Ilmiy uslubiy kengashi” da muhokama qilindi va ko’p nusxada nashr etilishga ruxsat berildi.

Bayonnoma № _____ _____-______________ 2017 yil.

KIRISH

«Nazariy mexanika» fani texnika oliy o'quv yurtlarida o'qitiladigan umum-texnika fanlarining boshlang'ichi bo'lib, bu fandan olingan chuqur bilim texnikada, sanoatda va ishlab chiqarishda uchraydigan turli texnikaviy muammolarni hal qilishda (yechishda) hamda talabalarga kelgusida (keyingi kurslarda) o'qitiladigan boshqa texnika fanlarini (materiallar qarshiligi, mashina va mexanizmlar nazariyasi, mashina detallari) o'zlashtirishlarida muhim ahamiyat kasb etadi.

Bu fandan bakalavriat ta'lim yo'nalishi o'quv rejasida belgilangan HLI (hisoblash loyihalash ishlari) bajarish – bu fan asoslarini chuqur o'rganishni ta'minlashdan tashqari talabalarda bu fandan olingan nazariy bilimlarni amaliy masalalarni echishga tadbiq etishni hamda HLI bajarish ko'nikmalarini hosil qilishni ta'minlaydi.

Hozirgi kunda «Nazariy mexanika»dan HLI bajarish bo'yicha o'quv adabiyotlari va uslubiy qo'llanmalarning kamligi, borlarining ham juda eskirganligini e'tiborga olib yozilgan ushbu uslubiy qo'llanmada nazariy mexanikaning «Statika», «Kinematika», «Dinamika» bo'limlaridan texnika oliy o'quv yurtlarining bakalavriat ta'lim yo'nalishi bo'yicha ta'lim olayotgan talabalarning o'quv rejasi asosida bajarishi lozim bo'lgan HLI turli mavzular bo'yicha variantlari (har bir mavzuga 30 tadan) berilgan.

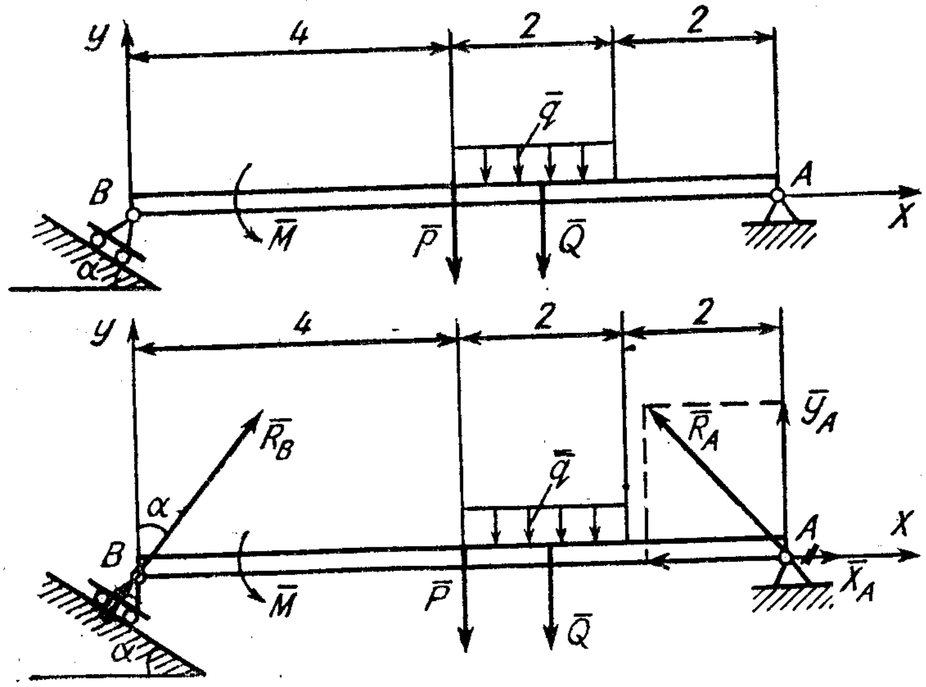
Topshiriqlar (HLI)dagi shakllarning o'lchamlari variantlarga tegishli shakllarda ko'rsatilgan hamda variantlarni yechishda kerak bo'ladigan kattaliklarning qiymatlari, variantlarning shakllaridan oldin berilgan jadvallarda keltirilgan. Berilgan variantlarning har bir mavzu bo'yicha namunalari atroflicha mulohazalar yordamida yechib ko'rsatilgan.

Uslubiy qo'llanma mualliflarining texnika oliy o'quv yurtlaridagi ko'p yillik ish tajribalari asosida yozilgan bo'lib, ta'lim standartlari va o'quv rejasiga mos keladi. Qo'llanmadan texnika oliy o'quv yurtlarining magistrlari ham qo'shimcha adabiyotlardan biri sifatida foydalanishlari mumkin.

1-§. QATTIQ JISM TAYANCH REAKSIYALARINI ANIQLASH

Bunda qattiq jism tayanib turgan tayanch nuqtalardagi tayanch reaksiyalarini topish talab qilinadi. Bu tipdagi masalalar quyidagicha echiladi.

1. Dekart koordinata o'qlari o'tkaziladi.
2. Tayanch reaksiyalari yo'naltiriladi.
3. Agar teng taqsimlangan kuchlar berilgan bo'lsa, ularni bitta kuch bilan almashtiriladi.
4. Tekislikda ixtiyoriy joylashgan kuchlar sistemasining muvozanat sharti tenglamalari yoziladi.
5. Agar tayanchlarning birortasi qo'zg'almas sharnirli yoki qistirib mahkamlangan tayanch bo'lsa, bu tayanch reaksiyasining o'qlardagi tashkil etuvchilarining haqiqiy yo'nalishlariga parallelogramm qurib, tayanch reaksiyasi shaklda ko'rsatiladi.



1- sh a k l.

1-masala. Uzunligi 8 metr bo'lgan *AB* balka *A* va *B* nuqtalarda qo'zg'almas va qo'zg'aluvchan tayanchlarga tayanib turadi. Balkaga vertikal bo'yicha $\overline{P}$ va intensivligi $\overline{q}$ bo'lgan teng taqsimlangan kuchlar hamda *M* burovchi moment ta'sir qiladi. *A* va *B* tayanch reaksiyalari topilsin (1-shakl).

Berilgan: M=12 kNm; P=6 kN, q= 2 kN/m; $\alpha = 45^0$, topish kerak: $\overline{R}_A$, $\overline{R}_B$,

Yechish. Koordinata o'qlarini o'tkazib, topilishi lozim bo'lgan tayanch reaksiyalarini shaklda ko'rsatamiz. *B* tayanch qo'zg'aluvchan tayanch bo'lgani uchun bu tayanch reaksiyasi $\overline{R}_B$ shu tayanch harakatlanishi mumkin bo'lgan qiya tekislikka perpendikulyar bo'lib, yuqoriga yo'naladi. *A* tayanch esa qo'zg'almas sharnirli tayanch bo'lgani uchun bu tayanch reaksiyasini oldindan ko'rsata olmaymiz, shuning uchun bu tayanch reaksiyasining o'qlar bo'yicha tashkil etuvchilari $\overline{X}_A$, $\overline{Y}_A$ larni o'qlar bo'yicha boshda ixtiyoriy yo'naltiramiz. Shakldagi intensivligi $\overline{q}$ bo'lgan teng taqsimlangan kuchlarni bitta kuch $\overline{Q}$ ga almashtiramiz. Bu kuchning kattaligi teng taqsimlangan kuch intensivligining ta'sir etish masofasiga ko'paytmasiga teng, ya'ni *Q=q·2=4kN*. Shakldagi tekislikda ixtiyoriy yo'nalgan 5 ta kuchlar ta'sirida biz tekshirayotgan jism *(AB* balka) muvozanatda bo'lgani uchun tekislikda ixtiyoriy joylashgan kuchlar sistemasining asosiy muvozanat shartini tatbiq qilamiz:

$$\sum_{k=1}^{5} X_k = 0, \quad \sum_{k=1}^{5} Y_k = 0, \quad \sum_{k=1}^{5} M_B(\overline{F}_k) = 0.$$

Ma'lumki bu tenglamalarning birinchi ikkitasi ta'sir etuvchi kuchlarning o'qlardagi proekstiyalarining yig'indisi nolga tengligini, uchinchi tenglama esa shu kuchlardan ixtiyoriy *B* nuqtaga nisbatan olingan momentlarning yig'indisi nolga tengligini bildiradi. Bizning misolimiz uchun bu tenglamalar quyidagicha yoziladi:

$$\sum_{k=1}^{5} X_k = 0, \quad X_A + R_B \sin\alpha = 0, \tag{1.1}$$

$$\sum_{k=1}^{5} Y_k = 0, \quad Y_A - Q - P + R_B \cos\alpha = 0, \tag{1.2}$$

$$\sum_{k=1}^{5} M_B(\overline{F}_k) = 0, \quad Y_A \cdot 8 - Q \cdot 5 - P \cdot 4 + M = 0. \tag{1.3}$$

(1.3) dan $Y_A = \dfrac{Q \cdot 5 + P \cdot 4 - M}{8} = \dfrac{20 + 24 - 12}{8} = 4\kappa N$

(1.2) dan $R_B = \dfrac{-Y_A + Q + P}{\cos\alpha} = \dfrac{-4 + 4 + 6}{\frac{\sqrt{2}}{2}} = 8{,}6\kappa N$

(1.1) dan $X_A = -R_B \ \sin\alpha = -8{,}6 \cdot \dfrac{\sqrt{2}}{2} = -6\kappa N.$

$\overline{X}_A$ va $\overline{Y}_A$ larga ko'ra *A* tayanch reaksiyasining moduli quyidagicha topiladi:

$$R_A = \sqrt{X_A^2 + Y_A^2} = \sqrt{(-6)^2 + (4)^2} = \sqrt{52} = 7{,}2\kappa N.$$

$\overline{R}_A$ kuchni shaklda ko'rsatish uchun masshtab tanlaymiz va masshtab tanlashda $\overline{X}_A$ va $\overline{Y}_A$ larning son qiymatiga ahamiyat beramiz. Bizning misolimizda $X_A = |-6\,\kappa N|$, $Y_A = 4\kappa N$ bo'lgani uchun masshtabni 1 sm da 2 kN deb olishimiz mumkin. $\overline{X}_A$ ning oldidagi manfiy ishora, *A* tayanch reaksiyasining *x* o'qidagi tashkil etuvchisining biz olgan yo'nalish noto'g'ri ekanligini, ya'ni, bu kuch teskari tomonga yo'nalganligini bildiradi (shaklda $\overline{X}_A$ ning oldingi yo'nalishiga ikki chiziq tortib qo'yamiz). $\overline{R}_A$ kuch $\overline{X}_A$ va $\overline{Y}_A$ larga qurilgan parallelogrammning diagonali bo'ylab yo'nalgan bo'ladi.

2-masala. *AB, CD, EK* sterjenlardan tashkil topgan konstruksiyaga gorizont bo'ylab *P=6* kN kuch, *M=10* kNm moment va gorizontga α=30^0 burchak ostida intensivligi q=1 kN/m bo'lgan teng taqsimlangan kuchlar ta'sir qiladi. Sterjenlardagi zo'riqishlar topilsin. O'lchamlar 4-shaklda ko'rsatilgan.

Berilgan: *M* = 10 kN ·m, q=1 kN/m, *P* = 6 kN, α=30^0.

Topish kerak: $\overline{S}_{AB}$, $\overline{S}_{CD}$, $\overline{S}_{EK}$.

Yechish: Koordinata o'qlarini shakldagiday yo'naltiramiz va q kuchlarni bitta kuch $\overline{Q}$ bilan almashtiramiz. Teng taqsimlangan kuchlarning teng ta'sir etuvchisi $Q = q \cdot EK = 1\kappa N/m \cdot 4m = 4\kappa N.$ Bu misolda *AB, CD, EK* larning har ikkala uchida sharnir bo'lgani uchun ularni sterjen' deb qaraymiz va shu sterjenlardagi reaksiya kuchlarini (zo'riqishlarni) aniqlaymiz. Ma'lumki sterjenlardagi zo'riqishlar ularga ta'sir etuvchi kuchlarga teskari yo'nalgan bo'ladi (agarda ta'sir etuvchi kuchlar birqancha bo'lib, natijaviy ta'sir etuvchi kuchning yo'nalishi aniq bo'lmasa, zo'riqish yo'nalishi ixtiyoriy yo'naltiriladi, so'ngra ishorasiga qarab haqiqiy yo'nalishi topiladi).

Muvozanat tenglamalarini tuzamiz:

$$\sum_{k=1}^{5} X_k = 0, \qquad P - Q\sin\alpha - S_{EK}\cos\alpha = 0, \qquad (2.1)$$

$$\sum_{k=1}^{5} Y_k = 0, \qquad -S_{AB} + S_{CD} + S_{EK}\sin\alpha - Q\cos\alpha = 0, \qquad (2.2)$$

$$\sum_{k=1}^{5} M_E(\overline{F}_k) = 0, \qquad S_{AB}\cdot 6 - S_{CD}\cdot 3 - Q\cdot\frac{EK}{2} + M = 0. \qquad (2.3)$$

shakldan $EK = \dfrac{EL}{\sin\alpha} = \dfrac{2}{0{,}5} = 4\,\text{m}.$

(2.1) dan

$$S_{EK} = \frac{P - Q\sin\alpha}{\cos\alpha} = \frac{6 - 4\cdot\frac{1}{2}}{\frac{\sqrt{3}}{2}} = 4{,}6\kappa N.$$

(2.2) dan

$$S_{AB} = S_{CD} + S_{EK}\sin\alpha - Q\cos\alpha = S_{CD} + 4{,}6\cdot\frac{1}{2} - 4\cdot\frac{\sqrt{3}}{2} = S_{CD} - 1{,}16. \qquad (2.3)$$

(2.3) ni (2.22) ga qo'yamiz.

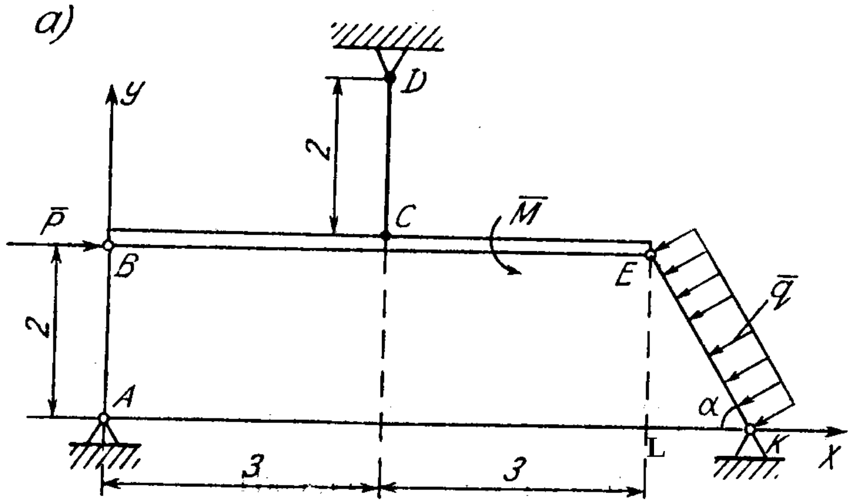
$$(S_{CD} - 1{,}16)\cdot 6 - S_{CD}\cdot 3 - Q\cdot\frac{4}{2} + 10 = 0,$$

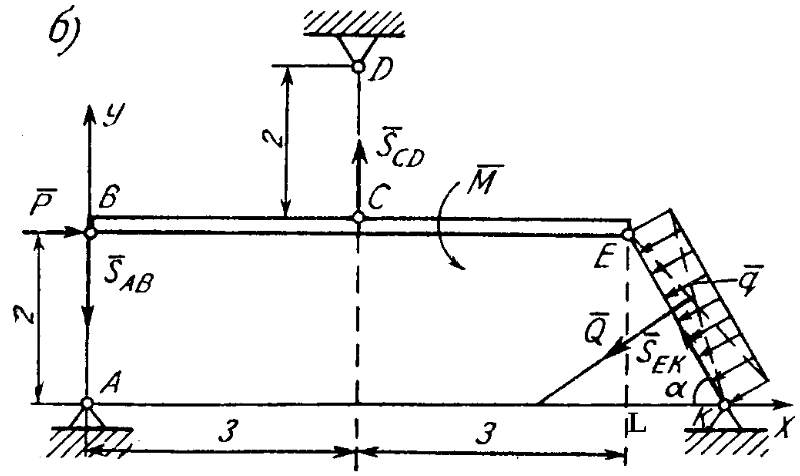
$$6S_{CD} - 6{,}96 - S_{CD}\cdot 3 - 8 + 10 = 0,$$

$$S_{CD} = \frac{4{,}96}{3} = 1{,}65\ \kappa N.$$

(2.3) dan $S_{AB} = S_{CD} - 1{,}16 = 1{,}65 - 1{,}16 = 0{,}49\ \kappa N$.

Uchala sterjen' zo'riqishlari ishoralari musbat chiqqani uchun bu sterjenlardagi zo'riqish yo'nalishlari to'g'ri tanlanganligini bildiradi.





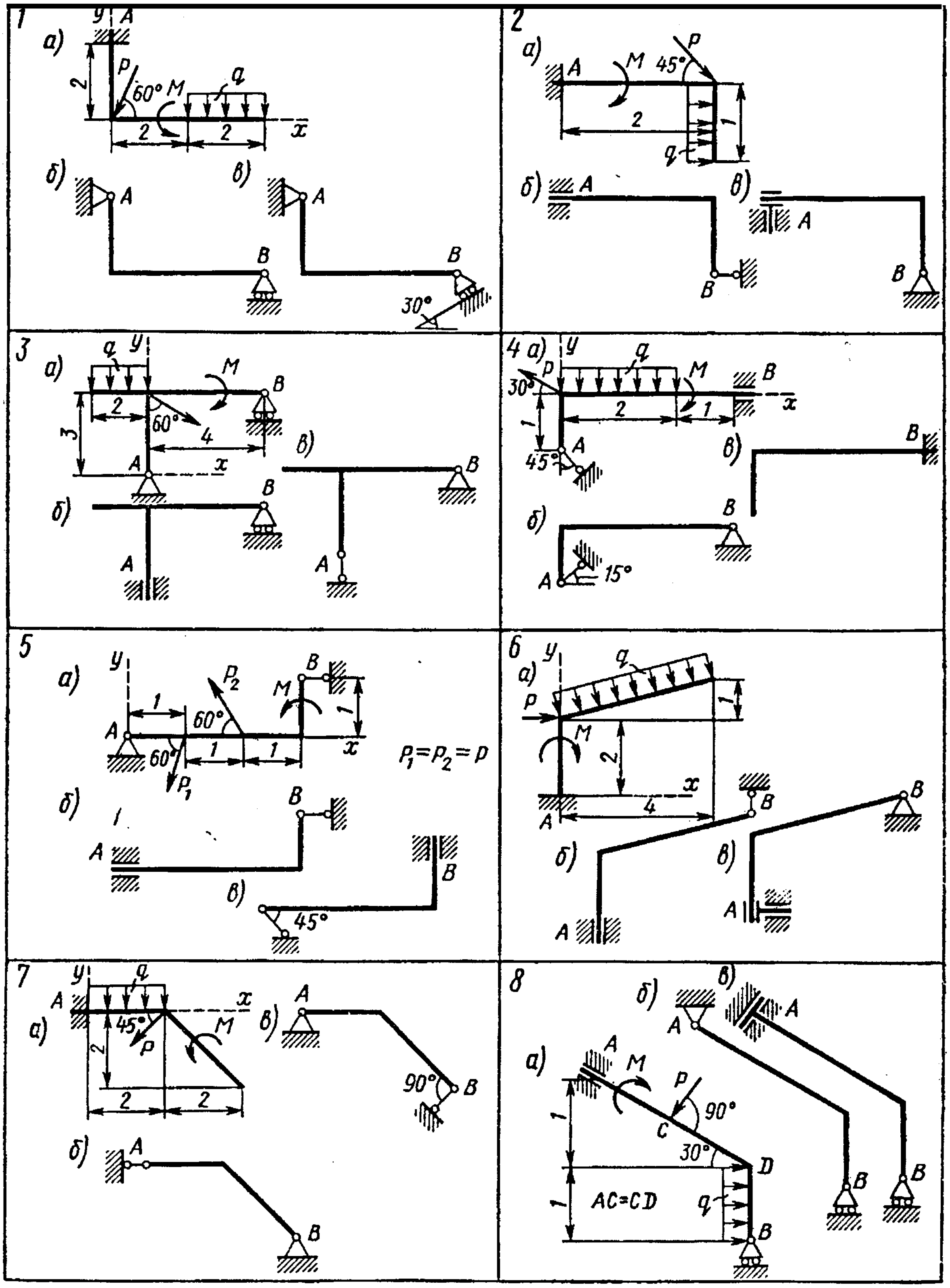
2-sh a k l.

S-1. Qattiq jismning tayanch reakstiya kuchlarini aniqlash

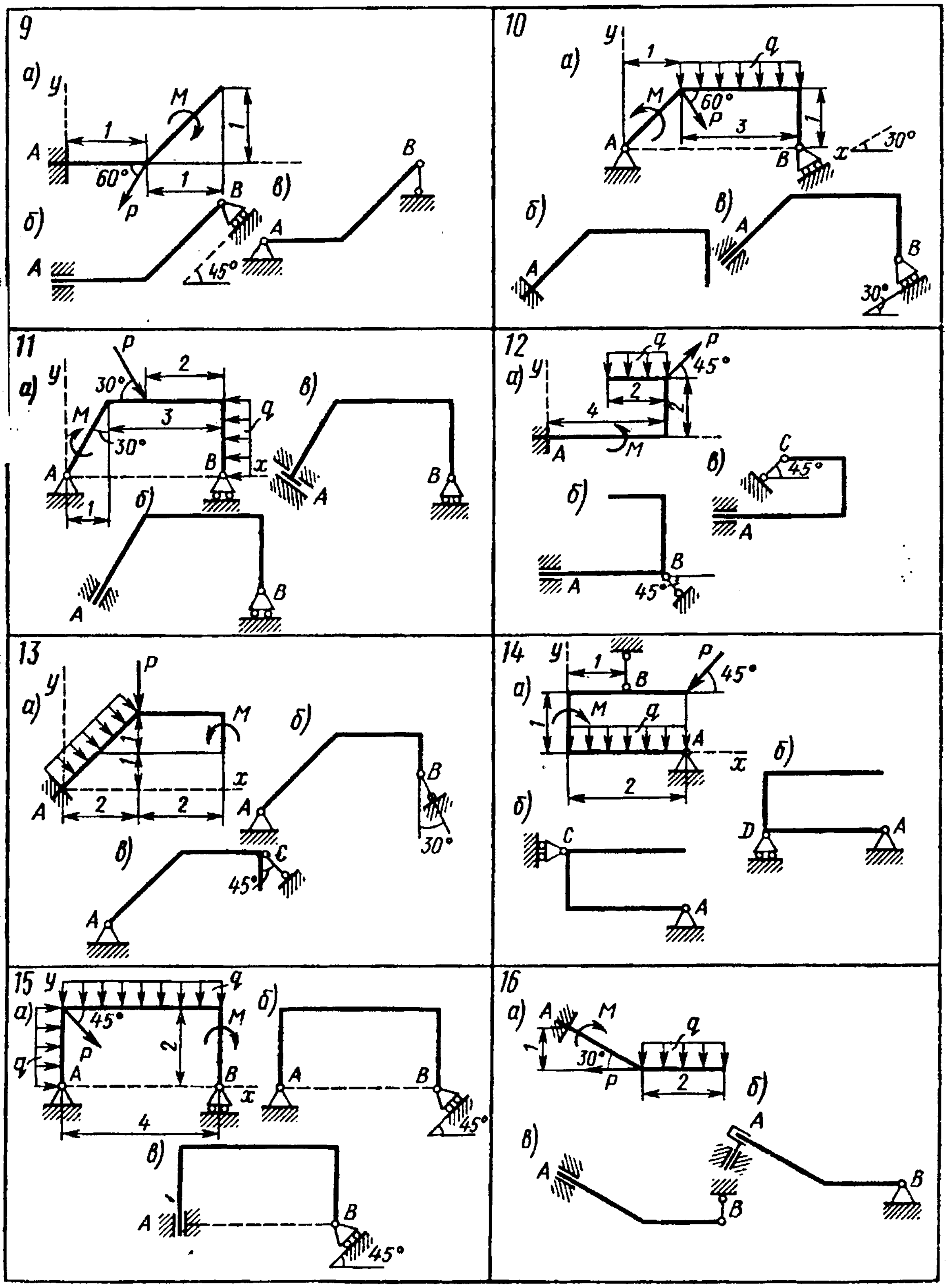
Jadval-1.

Variant nomerlari	R, kN	M, kN m	q, kN/m
1	10	6	2
2	20	5	4
3	15	8	1
4	5	2	1
5	10	4	-
6	6	2	1
7	2	4	2
8	20	10	4
9	10	6	-
10	2	4	2
11	4	10	1
12	10	5	2
13	20	12	2
14	15	4	3
15	10	5	2
16	12	6	2
17	20	4	3
18	14	4	2
19	16	6	1
20	10	-	4
21	20	10	2
22	6	6	1
23	10	4	2
24	4	3	1
25	10	10	2
26	20	5	2
27	10	6	1
28	20	10	2
29	25	-	1
30	20	10	2

S-1 vazifa (HLI) rasmlari



Rasm-1.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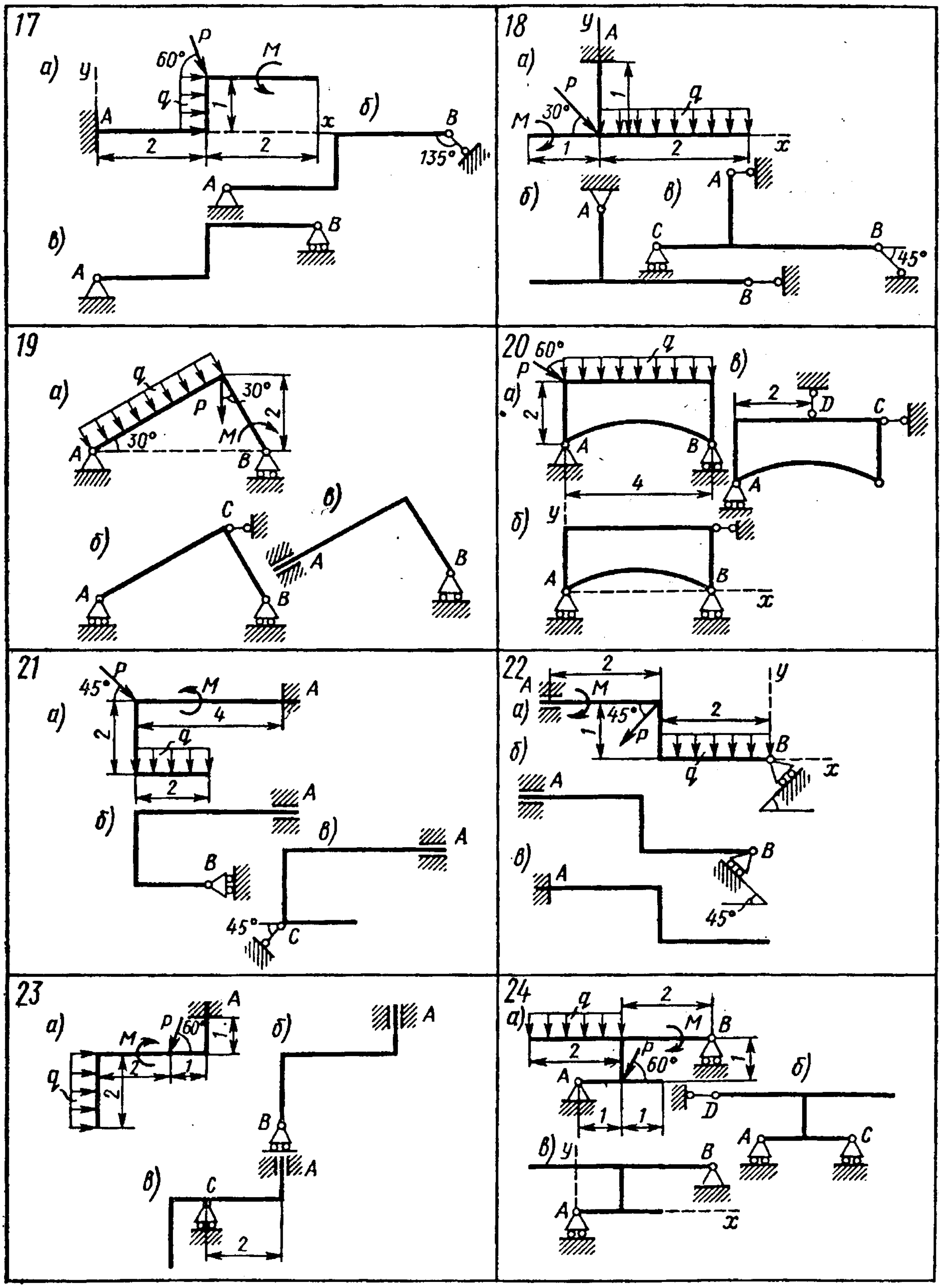

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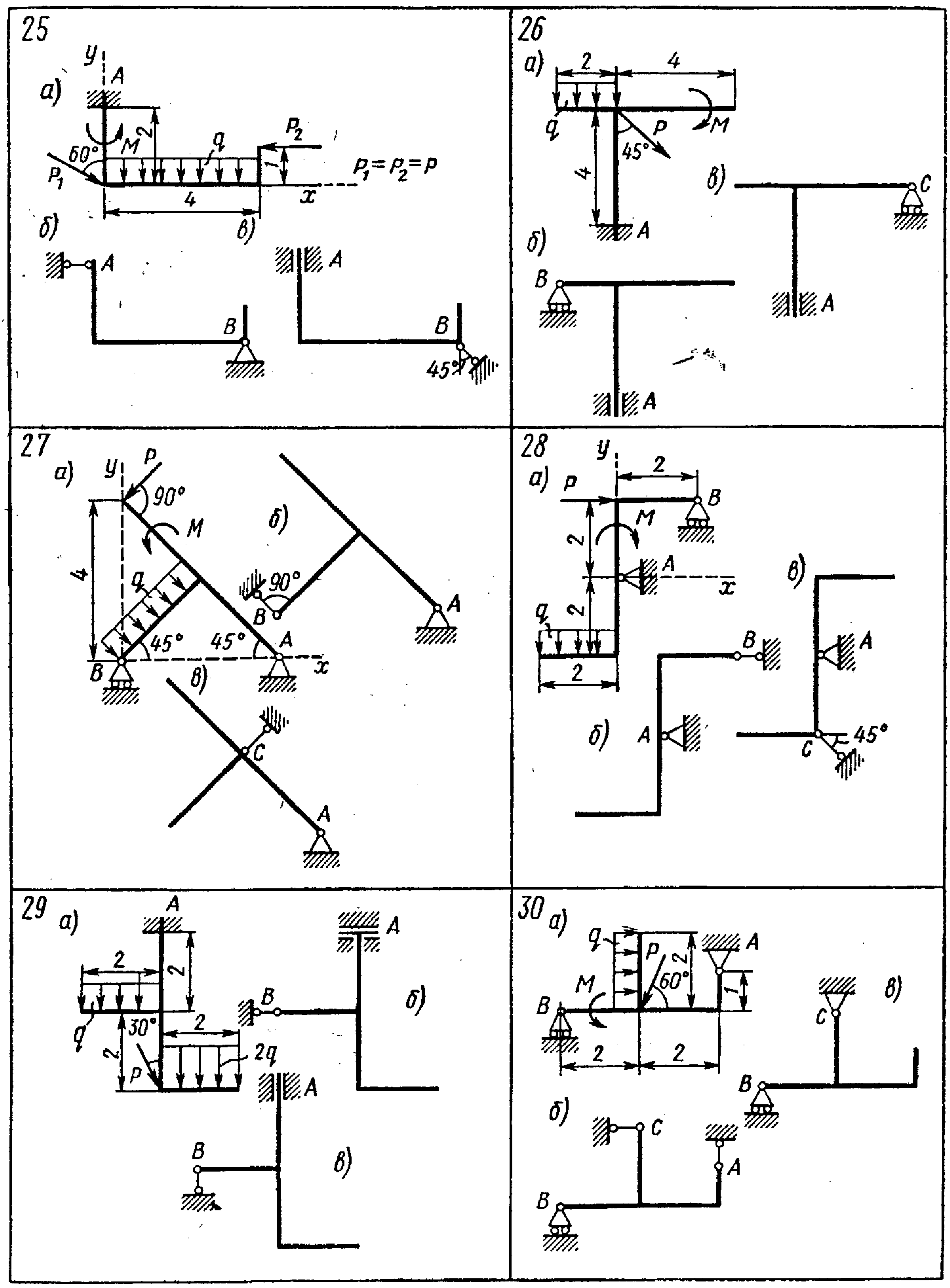
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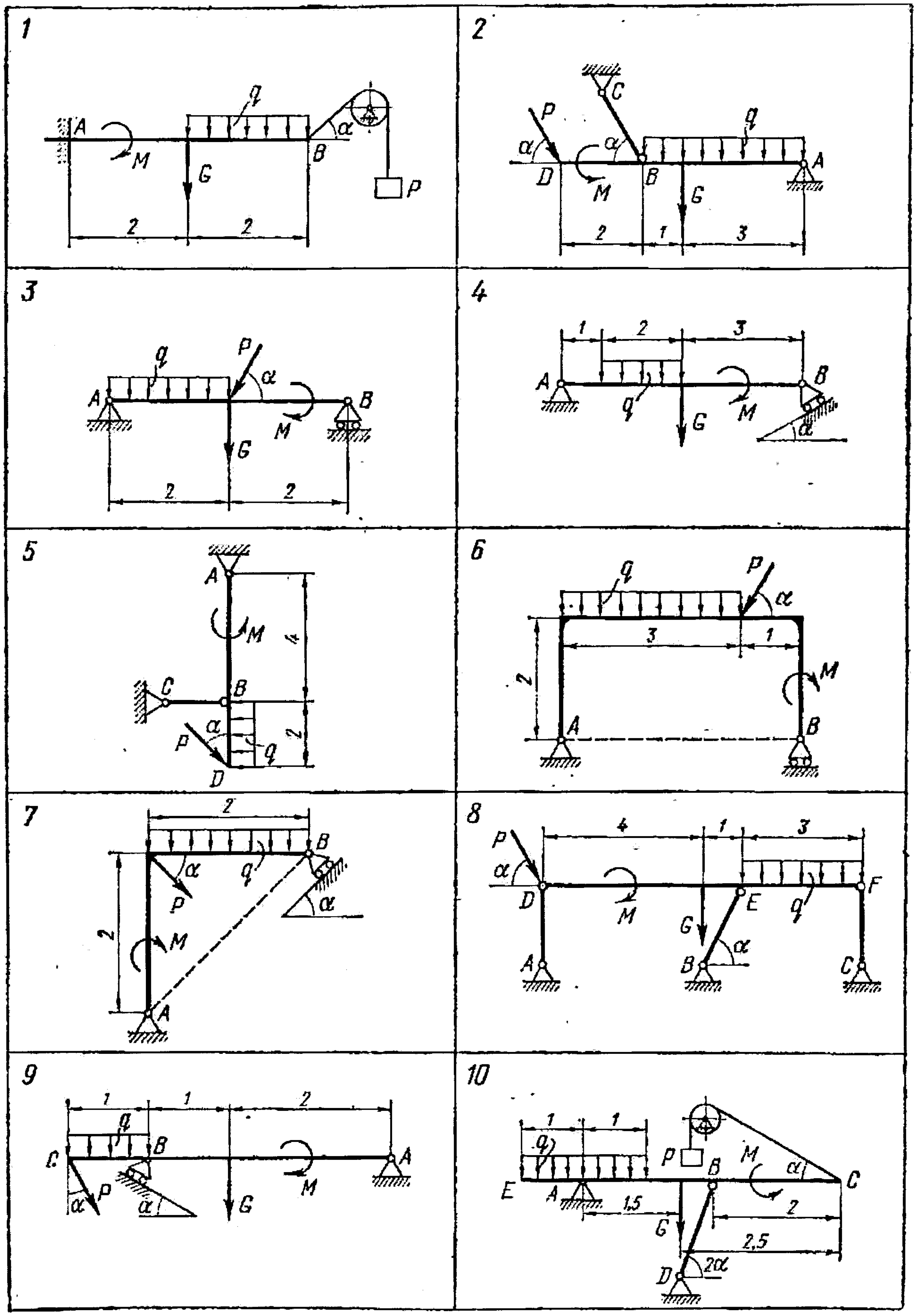
S-2. Qattiq jismning tayanch

reakstiya kuchlarini aniqlash

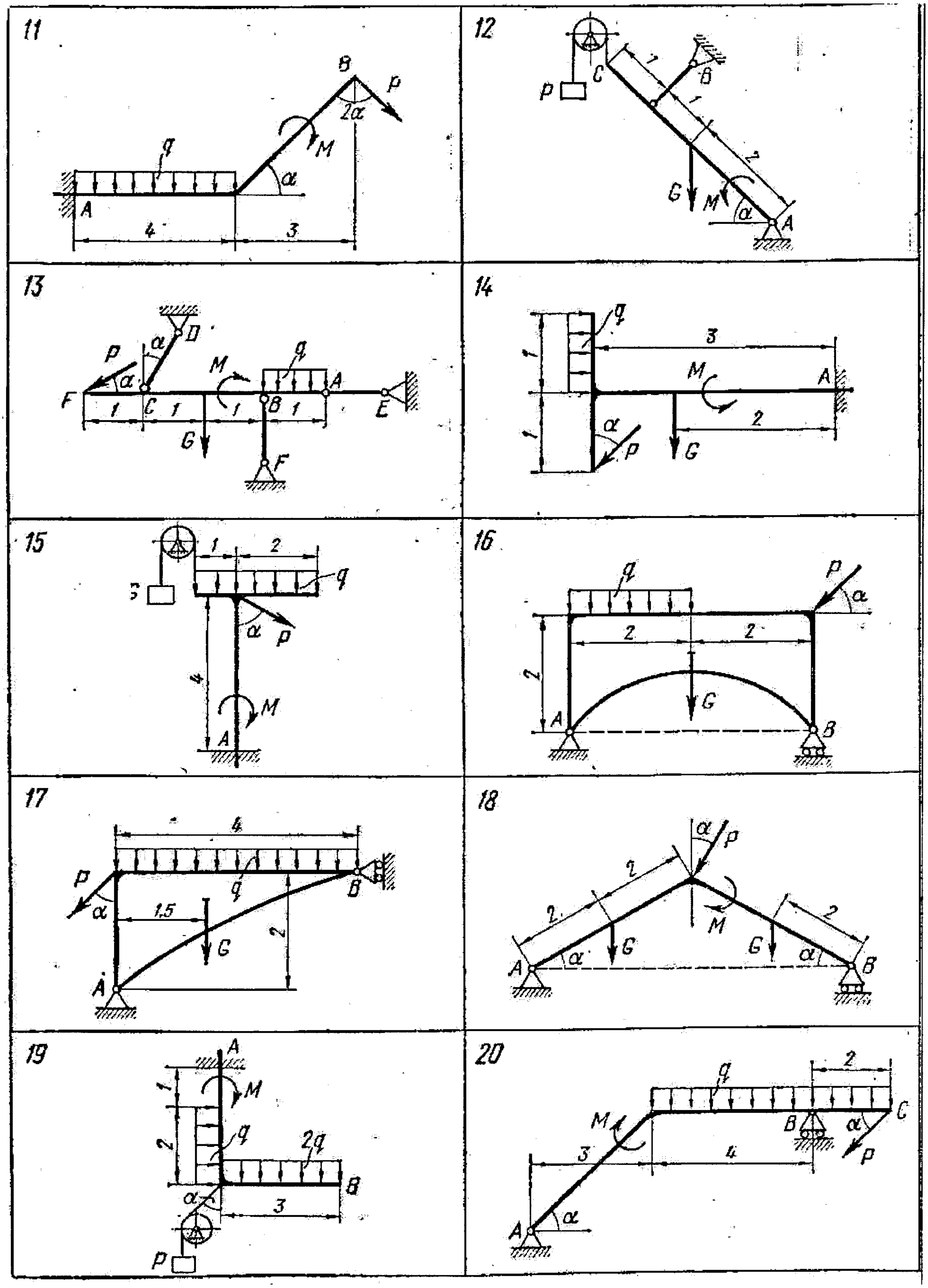
Jadval-2

Variant nomerlari	G	P	M kNm	q, kN/m	α, grad
	kN				
1	10	5	20	1	30
2	12	8	10	4	60
3	8	4	5	2	60
4	14	-	8	3	30
5	-	6	7	1	45
6	-	10	4	2	60
7	-	6	5	1	45
8	10	7	6	2	60
9	6	6	4	2	30
10	10	8	9	1	30
11	-	4	7	0,5	45
12	10	6	8	-	45
13	12	10	6	2	30
14	10	6	10	1	45
15	4	4	4	2	60
16	20	10	-	2	45
17	25	5	-	0,5	45
18	20	10	10	-	30
19	-	4	8	1	45
20	-	10	6	0,5	45
21	-	8	7	0,5	30
22	-	10	8	1	30
23	-	7	10	2	30
24	-	6	7	1,5	30
25	-	14	20	0,5	45
26	-	16	14	1	30
27	5	4	8	2,5	45
28	-	10	7	3	30
29	-	6	8	1	15
30	15	10	14	-	30

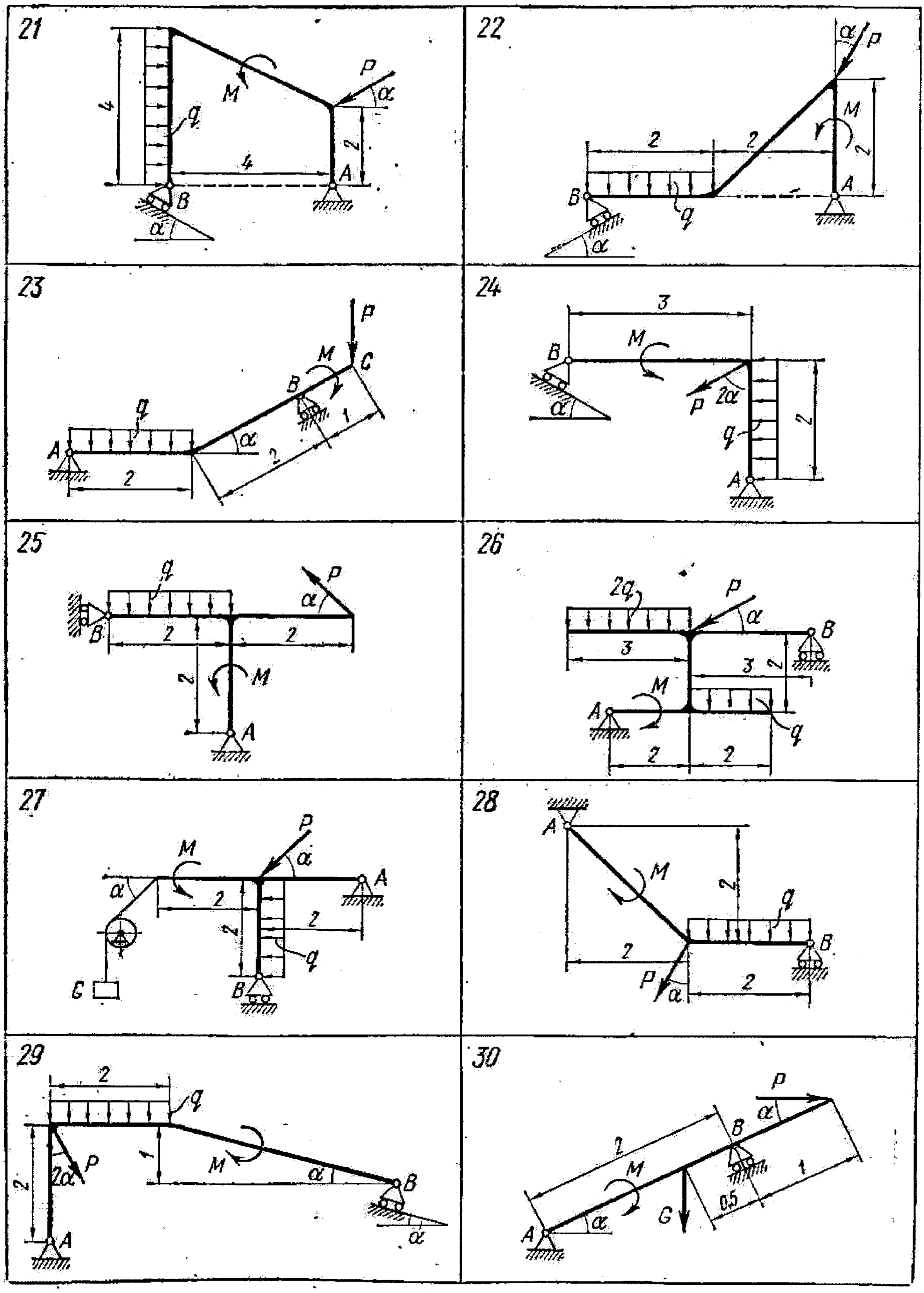
S-2 vazifa (HLI) rasmlari



Rasm-5.



Rasm-6



Rasm-7

2-§. MODDIY NUQTA HARAKAT TENGLAMALARIGA KO'RA UNING TEZLIK VA TEZLANISHLARINI TOPISH

1-masala. Harakat tenglamalari $x=2t(m)$; $u=t^2(m)$ bo'lgan moddiy nuqtaning $t=2$ sekunddagi tezlik va tezlanishlari qiymati topilsin va shaklda ko'rsatilsin.

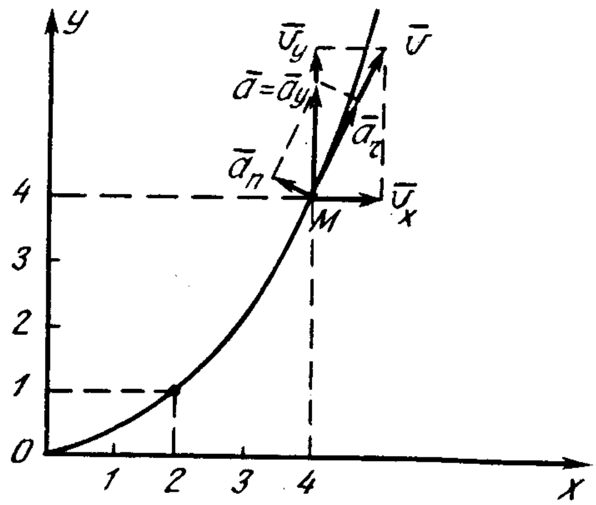
Yechish. Bu vazifa quyidagi etaplarga bo'lib yechiladi.

1) Trayektoriya tenglamasi topiladi va bu tenglama bilan ifodalanuvchi shakl chiziladi.

2) Tezlik vektori va uning berilgan vaqtdagi qiymati topiladi hamda shaklda ko'rsatiladi.

3) Nuqtaning urinma, normal va to'la tezlanishlari, ularning berilgan vaqtdagi qiymati topiladi va shaklda ko'rsatiladi.

4) Trayektoriya egrilik radiusi topiladi.



1-shakl.

1) Nuqtaning berilgan harakat tenglamalariga ko'ra trayektoriya tenglamasini topish uchun harakat tenglamalaridagi o'zgaruvchi t ni har xil matematik amallarni bajarish yo'li bilan yo'qotish kerak. Berilgan tenglamalarning birinchisidan t ni topib, $t=\frac{x}{2}$ ni ikkinchisiga qo'yamiz:

$$y=\frac{x^2}{4}. \qquad (1.1)$$

Bu tenglama parabolaning tenglamasi bo'lgani uchun nuqtaning trayektoriyasi paraboladan iborat ekan, degan xulosaga kelamiz va (1.1) tenglamaning grafigini chizish uchun x ga qiymatlar berib, unga mos y ning qiymatlarini topamiz (1-shakl).

x	0	2	4
y	0	1	4

So'ngra bu egri chiziqda $t=2$ sekunddagi nuqtaning o'rnini topamiz, buning uchun berilgan harakat tenglamalaridagi t ning berilgan qiymatini qo'yib, nuqtaning koordinatalarini topamiz. $x=2t=2\cdot2=4$, $y=t^2=2^2=4$. Demak, $t=2$ sekundda M nuqtaning koordinatalari (4; 4) bo'lar ekan.

2) Nuqtaning tezlik vektori $\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2}$ formula yordamida topiladi, υ_x, υ_y larni topish uchun, nuqtaning harakat tenglamalaridan vaqt bo'yicha birinchi tartibli hosila olamiz:

$$\upsilon_x = \frac{dx}{dt} = x' = (2t)' = 2;$$

$$\upsilon_y = \frac{dy}{dt} = y' = (t^2)' = 2t;$$

$$\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2} = \sqrt{2^2 + (2t)^2} = \sqrt{4 + 4t^2};$$

$t=2$ sekundda $\upsilon_x=2$ *m/s*, $\upsilon_y=2t=2\cdot2=4$ *m/s*;

$$\upsilon == \sqrt{4 + 4\cdot 2^2} = \sqrt{4+16} = \sqrt{20} = 4{,}5 \text{ m/s.}$$

Tezlik uchun masshtabni 1 sm da 2 m/s deb tanlaymiz va shakldа ko'rsatamiz.

3) Nuqtaning tezlanishlarini topish.

a) Nuqtannng *urinma tezlanishini* topish uchun uning tezlik vektoridan vaqt bo'yicha birinchi tartibli hosila olamiz.

$$a_\tau = \frac{d\upsilon}{dt} = \upsilon' = \left(\sqrt{4+4t^2}\right)' = \frac{8t}{2\sqrt{4+4t^2}} = \frac{2t}{\sqrt{1+t^2}};$$

$t=2$ sekundda $a_\tau = \dfrac{2\cdot 2}{\sqrt{1+2^2}} = \dfrac{4}{2{,}25} = 1{,}77$ m/s^2.

b) *To'la tezlanish* esa quyidagi formulalar yordamida hisoblanadi:

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{yoki} \quad a = \sqrt{a_\tau^2 + a_n^2}.$$

a_x, a_u larni topish uchun υ_x, υ_y lardan vaqt bo'yicha birinchi tartib hosila olamiz:

$$a_x = \frac{d^2x}{dt^2} = \frac{d\upsilon_x}{dx} = (\upsilon_x)' = (2)' = 0;$$

$$a_y = \frac{d^2y}{dt^2} = \frac{d\upsilon_y}{dt} = (\upsilon_y)' = (2t)' = 2;$$

$$a = \sqrt{0^2 + 2^2} = 2;$$

$t=2$ sekundda $a_x=0$; $a_y=2m/s^2$; $a=2$ m/s^2.

v) Nuqtaning *normal tezlanishi* esa, uning to'la tezlanishi kvadratidan urinma tezlanish kvadrati ayirmasidan chiqarilgan kvadrat ildiz tariqasida topiladi:

$$a_n = \sqrt{a^2 - a_\tau^2} = \sqrt{2^2 - \frac{4t^2}{1+t^2}} = \sqrt{4 - \frac{4t^2}{1+t^2}};$$

$t=2$ sekundda $a_n = \sqrt{4 - \dfrac{4\cdot 2^2}{1+2^2}} = \sqrt{4 - \dfrac{16}{5}} = 0{,}9$ m/s^2.

Tezlanishlar uchun masshtabni 1 sm da 2 m/s^2 deb tanlaymiz va shaklda ko'rsatamiz. Nuqtaning to'la tezlanishi a_τ, a_n va a_x, a_y larga qurilgan parallelogrammning diagonali bo'ylab yo'naladi va a_x nolga teng bo'lgani uchun bu diagonal a_u dan iborat bo'ladi.

4) Nuqta trayektoriyasining egrilik radiusi quyidagi formuladan hisoblanadi:

$$\rho = \frac{\upsilon^2}{a_n} = \frac{(4{,}5)^2}{0{,}9} = \frac{20}{0{,}9} = 22{,}2 \text{ m.}$$

2- masala. Harakat tenglamalari $x = 3\sin\frac{\pi}{3}t + 3$ (m); $y = -1 - 2\cos\frac{\pi}{3}t$ (m) bo'lgan moddiy nuqtaning *t=1* sekunddagi tezlik, tezlanishlari qiymatlari topilsin va shaklda ko'rsatilsin.

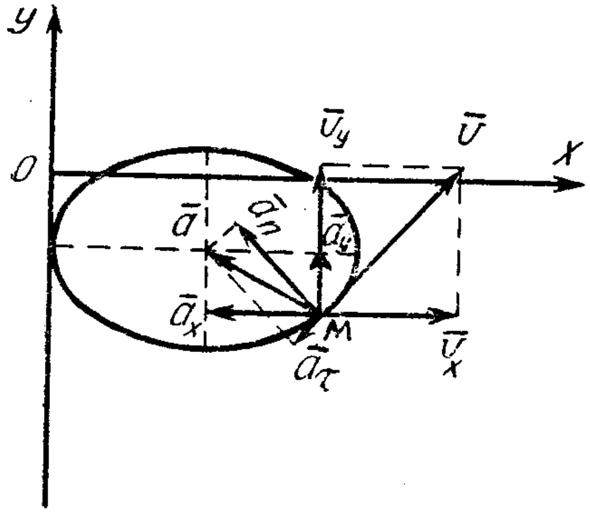
1) Trayektoriya tenglamasini topish. Harakat tenglamalarini quyidagi ko'rinishda yozamiz:

$$\frac{x-3}{3} = \sin\frac{\pi}{3}t, \qquad \frac{y+1}{2} = -\cos\frac{\pi}{3}t.$$

Bu tenglamalarning har ikki tomonini kvadratga ko'taramiz:

$$\frac{(x-3)^2}{3^2} = \sin^2\frac{\pi}{3}t, \qquad \frac{(y+1)^2}{2^2} = \cos^2\frac{\pi}{3}t.$$

Oxirgi tengliklarni o'ng va chap tomonlarini alohida qo'shsak, $\frac{(x-3)^2}{3^2} + \frac{(y+1)^2}{2^2} = 1$ ni hosil qilamiz. Bu tenglama ellipsni ifodalaganligi uchun material nuqtaning traaektoriyasi, markazi (3; -1) nuqtada va yarim o'qlari 3 va 2 ga teng bo'lgan ellipsdan iborat degan xulosaga kelamiz (4-shakl). t=1 sekundda nuqtaning koordinatalarini berilgan harakat tenglamalari yordamida topamiz:



4-shakl.

$$x = 3\sin\frac{\pi}{3}\cdot 1 + 3 = 3\frac{\sqrt{3}}{2} + 3 = 5{,}6;$$

$$y = -1 - 2\cos\frac{\pi}{3}\cdot 1 = -1 - 2\cdot\frac{1}{2} = -2.$$

Demak, *t=1* sekundda *M* nuqtaning koordinatalari (5, 6; -2) bo'lar ekan.

2) Nuqtaning tezlik vektorini $\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2}$ formula yordamida topamiz (4-shakl). υ_x, υ_y larni topish uchun nuqtaning harakat tenglamalaridan vaqt bo'yicha birinchi tartibli hosila olamiz:

$$\upsilon_x = \frac{dx}{dt} = x' = \left(3\sin\frac{\pi}{3}t + 3\right)' = 3\cos\frac{\pi}{3}t \cdot \frac{\pi}{3} = \pi\cos\frac{\pi}{3}t;$$

$$\upsilon_y = \frac{dy}{dt} = y' = \left(-1 - 2\cos\frac{\pi}{3}t\right)' = 2\sin\frac{\pi}{3}t \cdot \frac{\pi}{3} = \frac{2\pi}{3}\sin\frac{\pi}{3}t;$$

$$\upsilon = \sqrt{\pi^2\cos^2\frac{\pi}{3}t + \frac{4\pi^2}{9}\sin^2\frac{\pi}{3}t}.$$

Nuqta tezliklarining *t=1* sekunddagi qiymati:

$$\upsilon_x = \pi\cos\frac{\pi}{3}\cdot 1 = 3{,}14\cdot\frac{1}{2} = 1{,}57 \text{ m/s};$$

$$\upsilon_y = \frac{2\pi}{3}\sin\frac{\pi}{3}\cdot 1 = \frac{2\cdot 3{,}14}{3}\cdot\frac{\sqrt{3}}{2} = 1{,}8 \ m\text{/s};$$

$$\upsilon = \sqrt{\pi^2\cos^2\frac{\pi}{3}\cdot 1 + \frac{4\pi^2}{9}\sin^2\frac{\pi}{3}\cdot 1} = \sqrt{9{,}8\cdot\frac{1}{4} + \frac{4\cdot 9{,}8}{9}\cdot\frac{3}{4}} = 2{,}4\, m\text{/s}.$$

Tezlik uchun masshtabni 1 sm da 1 m/s deb tanlaymiz va shaklda ko'rsatamiz. Nuqtaning tezlik vektori *M* nuqtadan trayektoriyaga o'tkazilgan urinma bo'ylab υ_x, υ_y larga qurilgan parallelogrammning diagonali bo'yicha yo'nalgan bo'ladi.

2) Nuqta tezlanishlarini topamiz.

a) Urinma tezlanishni topish uchun uning tezlik vektoridan vaqt bo'yicha hosila olamiz:

$$a_\tau = \frac{d\upsilon}{dt} = \upsilon' = \left(\sqrt{\pi^2\cos^2\frac{\pi}{3}t + \frac{4\pi^2}{3}\sin^2\frac{\pi}{3}t}\right)' =$$

$$= -\frac{\pi^2\cdot 2\cos\frac{\pi}{3}t\cdot\sin\frac{\pi}{3}t\cdot\frac{\pi}{3} + \frac{4\pi^2}{9}\cdot 2\sin\frac{\pi}{3}t\cdot\cos\frac{\pi}{3}t\cdot\frac{\pi}{3}}{2\sqrt{\pi^2\cos^2\frac{\pi}{3}t + \frac{4\pi^2}{9}\cdot\sin^2\frac{\pi}{3}t}}.$$

b) To'la tezlanishni $a = \sqrt{a_x^2 + a_y^2}$ formuladan topamiz. a_x, a_y larni topish uchun υ_x, υ_y lardan vaqt bo'yicha birinchi tartibli hosila olamiz:

$$a_x = \frac{d^2x}{dt^2} = \frac{d\upsilon_x}{dt} = \left(\pi\cos\frac{\pi}{3}t\right)' = -\pi\sin\frac{\pi}{3}t\cdot\frac{\pi}{3} = -\frac{\pi^2}{3}\sin\frac{\pi}{3}t;$$

$$a_y = \frac{d^2y}{dt^2} = \frac{d\upsilon_y}{dt} = \left(\frac{2\pi}{3}\sin\frac{\pi}{3}t\right)' = \frac{2\pi}{3}\cos\frac{\pi}{3}t\cdot\frac{\pi}{3} = \frac{2\pi^2}{9}\cos\frac{\pi}{3}t;$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\frac{\pi^4}{9}\sin^2\frac{\pi}{3}t + \frac{4\pi^4}{81}\cos^2\frac{\pi}{3}t}.$$

v) Normal tezlanishni topamiz:

$$a_n = \sqrt{a^2 - a_\tau^2} = \sqrt{\left(\frac{\pi^4}{9}\sin^2\frac{\pi}{3}t + \frac{4\pi^4}{81}\cos^2\frac{\pi}{3}t\right)^2 - \frac{\frac{25\pi^6}{729}\sin^2\frac{2\pi t}{3}}{4\left(\pi^2\cos^2\frac{\pi}{3}t + \frac{4\pi^2}{9}\sin^2\frac{\pi}{3}t\right)}}.$$

Nuqta tezlanishlarining *t=1* sekunddagi qiymatini topamiz:

$$a_x = -\frac{\pi^2}{3}\sin\frac{\pi}{3}\cdot 1 = -\frac{9{,}8}{3}\cdot\frac{\sqrt{3}}{2} = -2{,}825\ m/s^2;$$

$$a_y = \frac{2\pi^2}{9}\cos\frac{\pi}{3}\cdot 1 = \frac{2\cdot 9{,}8}{9}\cdot\frac{1}{2} = 1{,}09\ m/s^2;$$

$$a = \sqrt{\frac{\pi^4}{9}\sin^2\frac{\pi}{3}\cdot 1 + \frac{4\pi^4}{81}\cos^2\frac{\pi}{3}\cdot 1} = \sqrt{8 + 1{,}18} = 3{,}09\ m/s^2;$$

$$a_\tau = \frac{-\frac{5\pi^3}{27}\sin\frac{2\pi}{3}\cdot 1}{2\sqrt{\pi^2\cos^2\frac{\pi}{3}\cdot 1 + \frac{4\pi^2}{9}\sin^2\frac{\pi}{3}\cdot 1}} = -\frac{266{,}15}{259{,}4} = -1{,}02\ m/s^2;$$

$$a_n = \sqrt{(3{,}09)^2 - (-1{,}02)^2} = \sqrt{8{,}14} = 2{,}85\ m/s^2.$$

Tezlanishlar uchun masshtabni 1,5 sm da 2,8 m/s^2 deb tanlaymiz va shaklda ko'rsatamiz. a_x ning manfiy ishorali ekanligi *y x* o'qining manfiy tomoniga yo'nalganligini, a_τ ning manfiyligi esa uning tezlik vektoriga teskari yo'nalganligini bildiradi.

4) Trayektoriyaning egrilik radiusini topamiz:

$$\rho = \frac{\upsilon^2}{a_n} = \frac{5{,}76}{2{,}85} = 2\ m.$$

K-1

Harakat tenglamalariga ko'ra material nuqta tezlik va tezlanishlarini topish

Jadval-1

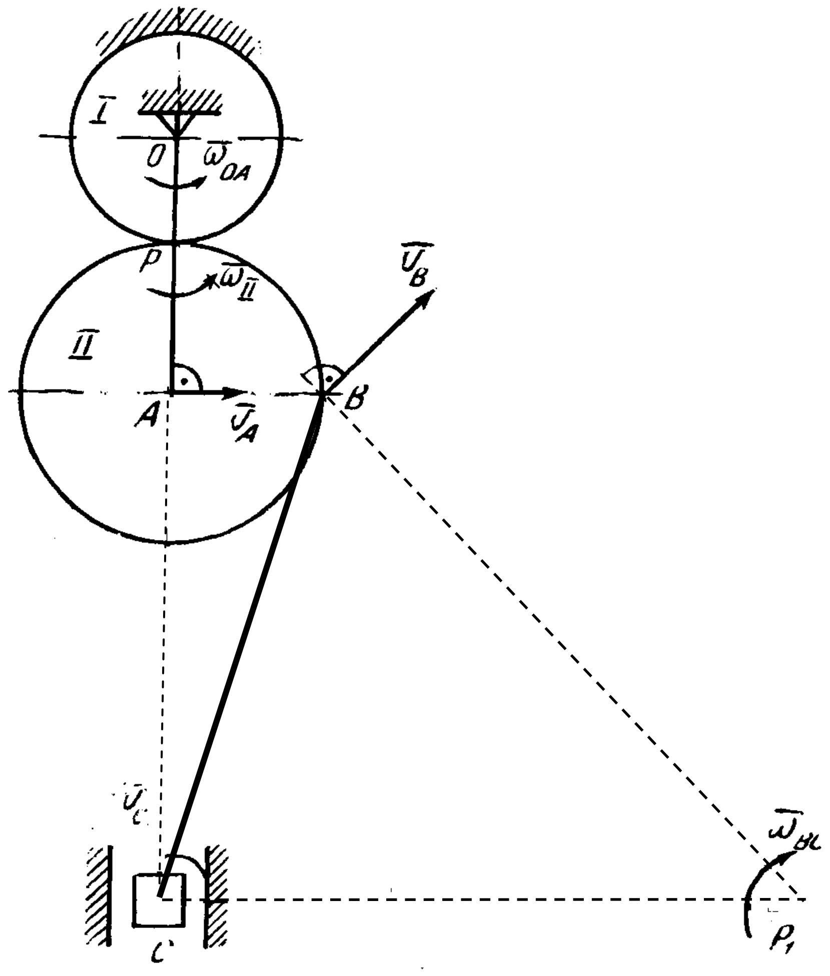
Variant nomerlari	Harakat tenglamalari		t_1, sek
	x=x(t), sm	y=y(t), sm	
1	$-2t^2+3$	$-5t$	1/2
2	$4\cos^2\frac{\pi}{3}t+2$	$4\sin^2\frac{\pi}{3}t$	1
3	$-\cos\frac{\pi}{3}t^2+3$	$\sin\frac{\pi}{3}t^2-1$	1
4	$4t+4$	$-\frac{4}{t+1}$	2
5	$2\sin\frac{\pi}{3}t$	$-3\cos\frac{\pi}{3}t+4$	1
6	$3t^2+2$	$-4t$	1/2

7	$3t^2 - t + 1$	$5t^2 - \frac{5}{3}t - 2$	1
8	$7\sin\frac{\pi}{6}t^2 + 3$	$2 - 7\cos\frac{\pi}{6}t^2$	1
9	$-\frac{3}{t+2}$	$3t + 6$	2
10	$-4\cos\frac{\pi}{3}t$	$-2\sin\frac{\pi}{3}t - 3$	1
11	$-4t^2 + 1$	$-3t$	1/2
12	$5\sin^2\frac{\pi}{6}t$	$-5\cos^2\frac{\pi}{6}t - 3$	1
13	$5\cos\frac{\pi}{3}t^2$	$-5\sin\frac{\pi}{3}t^2$	1
14	$-2t - 2$	$-\frac{2}{t+1}$	2
15	$4\cos\frac{\pi}{3}t$	$-3\sin\frac{\pi}{3}t$	1
16	$3t$	$4t^2 + 1$	1/2
17	$7\sin^2\frac{\pi}{6}t - 5$	$-7\cos^2\frac{\pi}{6}t$	1
18	$1 + 3\cos\frac{\pi}{3}t^2$	$3\sin\frac{\pi}{3}t^2 + 3$	1
19	$-5t^2 - 4$	$3t$	1
20	$2 - 3t - 6t^2$	$3 - \frac{3}{2}t - 3t^2$	0
21	$6\sin\frac{\pi}{6}t^2 - 2$	$6\cos\frac{\pi}{6}t^2 + 3$	1
22	$7t^2 - 3$	$5t$	1/4
23	$3 - 3t^2 + t$	$4 - 5t^2 + \frac{5}{3}t$	1
24	$-4\cos\frac{\pi}{3}t - 1$	$-4\sin\frac{\pi}{3}t$	1
25	$-6t$	$-2t^2 - 4$	1
26	$8\cos^2\frac{\pi}{6}t + 2$	$-8\sin^2\frac{\pi}{6}t - 7$	1
27	$-3 - 9\sin\frac{\pi}{6}t^2$	$-9\cos\frac{\pi}{6}t^2 + 5$	1
28	$-4t^2 + 1$	$-3t$	1
29	$5t^2 + \frac{5}{3}t - 3$	$3t^2 + t + 3$	1
30	$2\cos\frac{\pi}{3}t^2 - 2$	$-2\sin\frac{\pi}{3}t^2 + 3$	1

3-§. TEKIS PARALLEL HARAKATDAGI QATTIQ JISM NUQTALARINING TEZLIKLARINI ANIQLASH

Bu vazifani yechish uchun avval bo’g’inlar soni aniqlanadi va har bir bo’g’in uchun oniy markaz (tezliklar oniy markazi) topiladi. So’ngra har bir bo’g’inning burchak tezligi va berilgan nuqtalarning tezliklari topiladi. Quyida biz ushbu vazifaga oid bir necha misollar bilan tanishib chiqamiz.

1-masala (I.V. Meshcherskiy, 538). Uzunligi 30 sm bo’lgan *OA* krivoship O o’q atrofida $\omega_{OA} = 0{,}5\frac{1}{s}$ burchak tezlik bilan aylanadi. Radiusi r_{11}=20 sm bo’lgan tishli g’ildirak radiusi r_1= 10 sm bo’lgan qo’zg’almas g’ildirak bo’ylab sirpanmay g’ildiraydi va unga biriktirilgan uzunligi $20\sqrt{26}$ sm bo’lgan *BC* shatunni harakatga keltiradi (12-shakl). *AB* radius *OA* krivoshipga tik bo’lgan paytdagi shatunning burchak tezligi hamda B va C nuqtalarning tezliklari aniqlansin (12-shakl).



12-shakl.

Berilgan: r_1=10 sm, r_{11}=20 sm, ω_{OA}=0,5 s^{-1}, *OA*=30 sm, $BC = 20\sqrt{26}$ cm.
Topish kerak ω_{BC}, υ_A, υ_B, υ_c.

Yechish. Ushbu masaladagi mexanizm bo’g’inlari soni 4 ta (*I* va *II*, disklar, *OA* krivoship va *BC* shatun). Bu bo’g’inlarning har biri uchun masalani alohida-alohida echamiz.

1) *I* disk qo'zg'almas bo'lgani uchun, uning hamma nuqtalarining tezliklari nolga teng, ya'ni bu bo'g'in uchun masala yechilmaydi.

2) *OA* krivoship uchun oniy markaz *O* nuqtada, chunki bu nuqta tezligi nolga teng (oniy markaz bo'lishning asosiy sharti). *A* nuqta tezligi yo'nalishini topish uchun, nuqtani oniy markaz bilan birlashtiramiz, nuqta tezligi shu birlashtiruvchi chiziqqa tik yo'naladi (burchak tezlik yo'nalishi tomonga). Nuqta tezligining qiymati esa

$\upsilon_A = \omega_{OA} \cdot OA$ formula yordamida aniqlanadi:

$\upsilon_A = 0{,}5s^{-1} \cdot (r_1 + r_{11}) = 0{,}5s^{-1} \cdot 30sm = 15sm/s,\ \upsilon_A = 15sm/s.$

3) *II* disk uchun oniy markaz bu diskning qo'zg'almas 1 disk bilan tegishib turgan *P* nuqtasida bo'ladi. B nuqta tezligi yo'nalishini topish uchun bu nuqtani oniy markaz bilan birlashtiramiz va tezlik vektorini shu chiziqqa tik qilib, diskning burchak tezligi yo'nalgan tomonga yo'naltiramiz. So'ngra bu bo'g'in burchak tezligini uning ixtiyoriy nuqtasi tezligini, shu nuqtadan bo'g'inning oniy markazigacha bo'lgan masofaga bo'lib topamiz:

$$\omega_{11} = \frac{\upsilon_A}{AP} = \frac{\upsilon_B}{BP}. \qquad (3.1)$$

Bu tenglikning birinchi nisbatidan $\omega_{11} = \frac{15sm/s}{20sm} = \frac{3}{4}s^{-1}$, ikkinchisidan esa $\upsilon_B = \omega_{11} \cdot BP$. Noma'lum masofa BP ni *ΔABP* dan topamiz. Pifagor teoremasiga asosan $BP = \sqrt{AB^2 + AP^2} = 20\sqrt{2}$ sm. BP ning qiymatini yuqoridagi formulaga qo'ysak

$$\upsilon_B = \frac{3}{4}s^{-1} \cdot 20\sqrt{2}sm = 15\sqrt{2}sm/s;\ \upsilon_B = 15\sqrt{2}sm/s.$$

4) BC shatunga tegishli va tezligi nolga teng bo'lgan nuqtani shaklda ko'rishi mumkin emas. Shuning uchun bu erda oniy markaz topishning asosiy usulidan foydalanamiz. BC bo'g'inning B va *C* nuqtalari tezliklariga perpendikulyar chiziqlar o'tkazamiz. Bu chiziqlar kesishgan P_1 nuqta biz tekshirayotgan onda BC bo'g'in hamma nuqtalarining tezliklari uchun oniy markaz bo'ladi. So'ngra bo'g'in burchak tezligi ifodasini yozamiz:

$$\omega_{BC} = \frac{\upsilon_B}{BP_1} = \frac{\upsilon_C}{CP_1}. \qquad (3.2)$$

BP_1 va CP_1 masofalarni shakldan topamiz:

ΔABC dan $AC = \sqrt{BC^2 - AB^2} = \sqrt{20^2 \cdot 26 - 20^2} = 100$ sm.

To'g'ri burchakli uchburchak PCP_1 ning CPP_1 burchagi 45^0 bo'lgani uchun, uch burchak teng yonli uchburchak bo'lib, CP=CP_1 dir.

$$CP = CP_1 = AC + AP = 100\ sm + 20\ sm = 120\ sm;$$

ΔPCP_1 dan $PP_1 = \sqrt{CP^2 + CP_1^2} = \sqrt{120^2 + 120^2} = 120\sqrt{2}$ sm.

Shakldan

$$BP_1 = PP_1 - BP = 120\sqrt{2}sm - 20\sqrt{2}sm = 100\sqrt{2}\ \text{sm}.$$

(3.2) ning birinchi nisbatidan $\omega_{BC} = \frac{15\sqrt{2}sm/s}{100\sqrt{2}sm} = 0{,}15s^{-1}$, ikkinchisidan $\upsilon_C = \omega_{BC} \cdot CP_1 = 0{,}15s^{-1} \cdot 120\ sm = 18\ sm/s,\quad \upsilon_C = 18\ sm/s.$

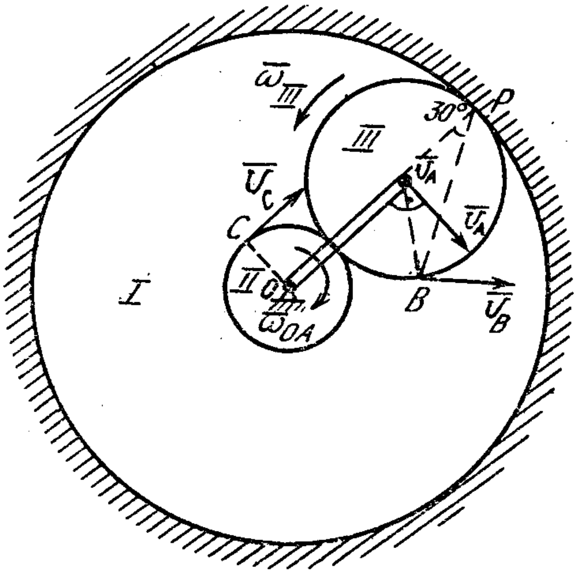
2- masala. Berilganlarga ko'ra 14-shakldagi mexanizm *A, B, C* nuqtalarning tezliklari qiymatlari topilsin.

Berilgan: ω_{OA}=1,5 s^{-1}, r_1=40 sm, r_{11}=10 sm.

Topish kerak: υ_A, υ_B, υ_c.

Yechish: Mexanizm bo'g'inlari sonini aniqlaymiz, ularning soni 4 ta: (*I, II, III* disklar va *OA* krivoship). Masalani har bir bo'g'in uchun alohida yechamiz.

1) I disk uchun tezliklar oniy markazi yo'q, chunki bu disk qo'zg'almas bo'lgani uchun bu bo'g'in hamma nuqtalarining tezliklari nolga teng.



14-shakl.

2) *OA* mexanizm uchun oniy markaz *O* nuqtada bo'ladi, chunki bu nuqtaning tezligi nolga teng. *A* nuqta tezligi nuqtani oniy markaz bilan birlashtiruvchi chiziqqa tik (burchak tezlik yo'nalishi bo'yicha) yo'naladi, qiymati esa *OA* bo'g'in burchak tezligini nuqtadan bo'g'inning oniy markazigacha bo'lgan masofa ko'paytmasiga teng:

$$\upsilon_A = \omega_{OA} \cdot OA = 1{,}5s^{-1} \cdot 30sm = 45sm/s, \ \upsilon_A = 45sm/s.$$

3) *II* disk uchun oniy markaz *O* nuqtada, chunki bu diskning hamma nuqtalari, shu qo'zg'almas nuqta atrofida aylanma harakat qiladi. *C* nuqta tezligi *II* diskning *C* nuqtasidan o'tkazilgan urinma bo'ylab ω_{11} yo'nalishi bo'yicha yo'naladi, qiymati esa $\upsilon_C = \omega_{11} \cdot OC$ formula yordamida topiladi.

4) *III* disk uchun oniy markaz *I* va *III* disklarning tegib turgan qo'zg'almas *P* nuqtada bo'ladi. *III* diskning burchak tezligi:

$$\omega_{III} = \frac{\upsilon_A}{AP} = \frac{45sm/s}{30sm} = 1{,}5s^{-1}.$$

II disk burchak tezligini topish uchun *II* va *III* disklar burchak tezliklari nisbati, ularning radiuslarining nisbatiga teskari proporstionalligidan foydalanamiz:

$$\frac{\omega_{II}}{\omega_{III}} = \frac{r_{III}}{r_{II}}; \qquad \omega_{II} = \frac{r_{II}}{r_{II}} \cdot \omega_{III} = \frac{30sm}{10sm} \cdot 1{,}5s^{-1} = 4{,}5s^{-1}.$$

B nuqta tezligining qiymati *III* disk burchak tezligini, nuqtadan shu bo'g'in oniy markazigacha bo'lgan masofa ko'paytmasiga teng:

$$\upsilon_B = \omega_{III} \cdot BP. \qquad (3.4)$$

$\overline{\upsilon}_B$ ning yo'nalishi B nuqtani *I* diskning oniy markazi bilan tutashtiruvchi BP chiziqqa tik bo'lib, ω_{III} yo'nalishi bo'ylab yo'naladi. Shakldan kosinuslar teoremasiga asosan:

$$BP = \sqrt{AB^2 + AP^2 - 2AB \cdot AP\cos 120^0} = \sqrt{225 + 225 + 2 \cdot 15 \cdot 15\frac{1}{2}} = \sqrt{675} = 26 \text{ см}.$$

(3.4) dan $\upsilon_B = \omega_{III} \cdot BP = 1{,}5c^{-1} \cdot 26\ см = 39\ см/с; \qquad \upsilon_B = 39 см/с.$

S nuqta tezligi: $\upsilon_C = \omega_{II} \cdot OC = 4{,}5c^{-1} \cdot 10\ см = 45\ см/с; \qquad \upsilon_C = 45 см/с.$

K-4

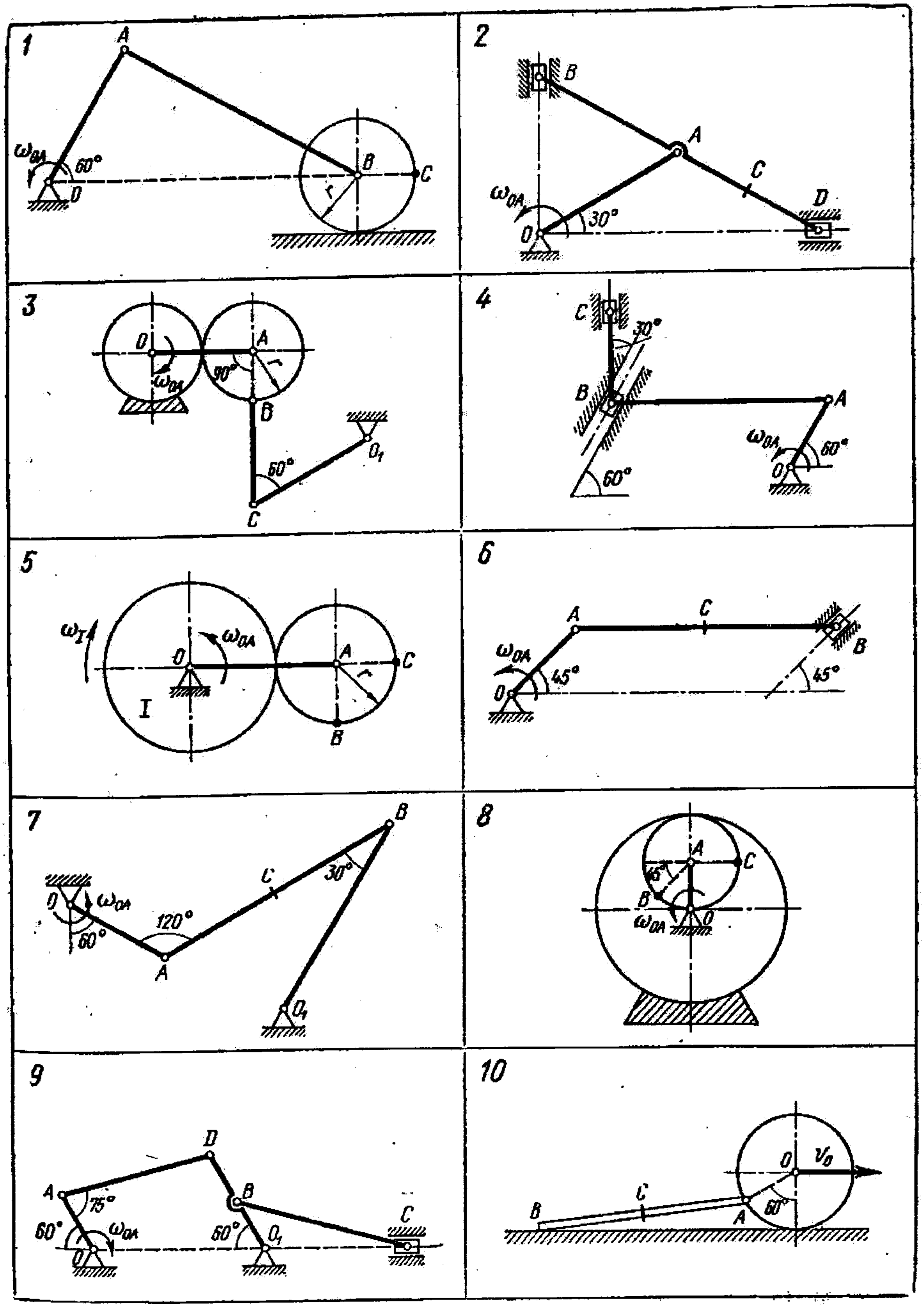
Ilgarilanma va aylanma harakat qilayotgan qattiq jism nuqtasining tezlik va tezlanishlarini aniqlang.

Jadval-3

Vari-antlar raqa-mi	O'lchami, sm					ω*OA, s^{-1}	Qo'shimcha ma'lumotlar
	OA	*AV*	AD	*BC*	*r*		
1	35	65	-	-	15	2	
2	40	40	40	60	-	1,5	
3	22	-	-	24	11	3	O_1S=30 sm
4	20	50	-	24	-	1	
5	35	-	-	-	15	4	ω_1=1,5 s^{-1}
6	20	60	-	30	-	1,2	
7	30	60	-	30	-	2	O_1V=50 sm
8	12	-	-	-	-	1,5	
9	14	-	40	45	-	1	BD=BO_1
10	15	50	-	25	-	-	v_0=80 sm/s
11	27	-	-	34	12	2,5	
12	20	25	50	35	-	2	
13	22	44	-	-	15	-	v_0=100 sm/s
14	60	25	-	35	-	1,4	
15	25	60	-	-	15	1,6	
16	27	-	-	-	12	1,2	ω_1=3 s^{-1}
17	16	-	-	-	8	0,6	
18	22	36	72	25	-	2,4	
19	23	57	-	-	14	1,5	
20	23	56	-	-	-	4	
21	24	24	24	35	-	3	
22	25	-	-	40	10	2	
23	26	-	-	36	12	1	
24	17	12	32	15	-	2,1	
25	28	75	-	15	10	2,5	
26	12	54	25	42	-	2,2	
27	55	-	-	-	10	1,8	
28	25	-	-	30	10	2,3	O_1S=36 sm
29	16	25	50	35	-	2	
30	16	60	-	14	10	1,5	

K-4

Vazifa rasmlari



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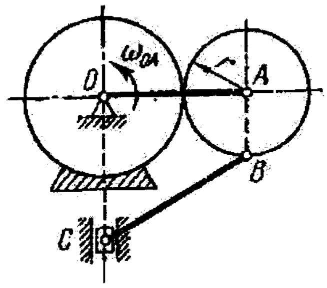

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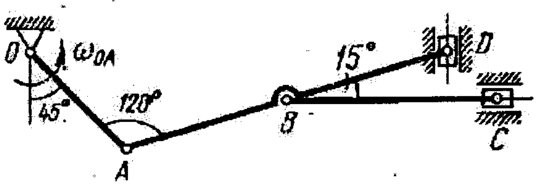

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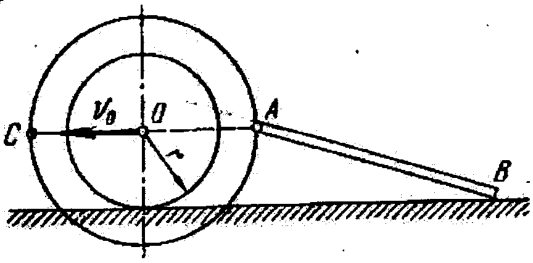

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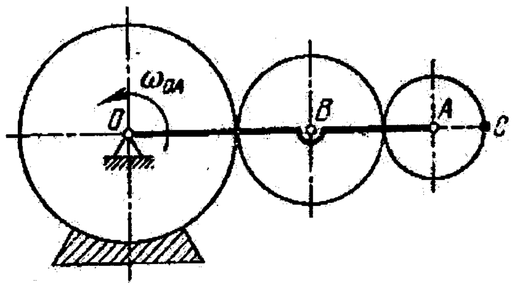

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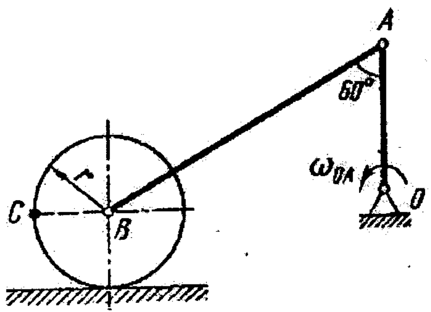

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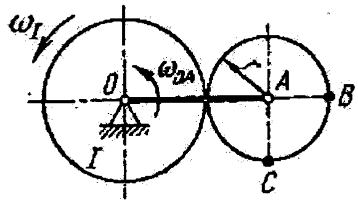

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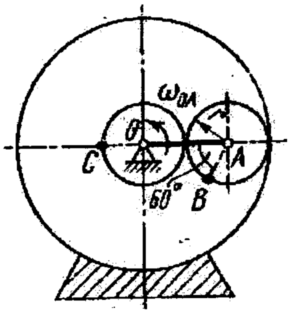

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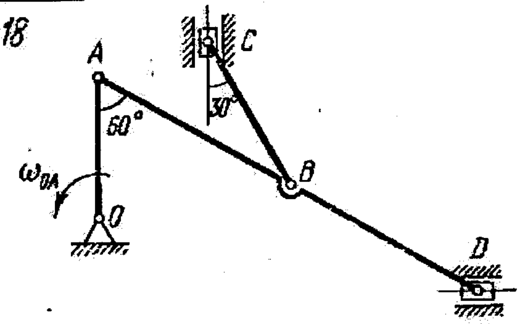

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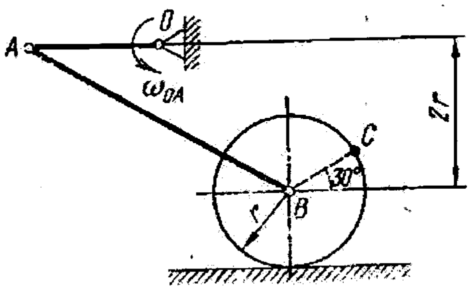

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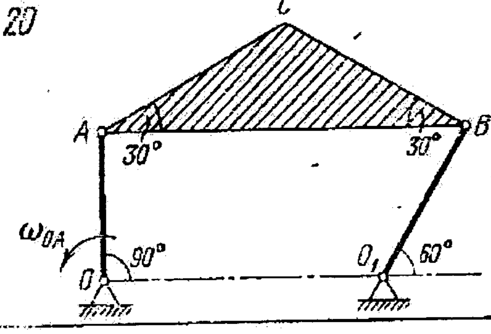

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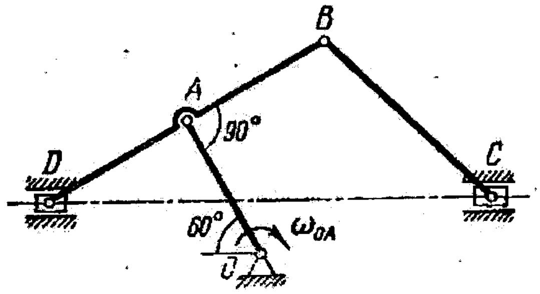

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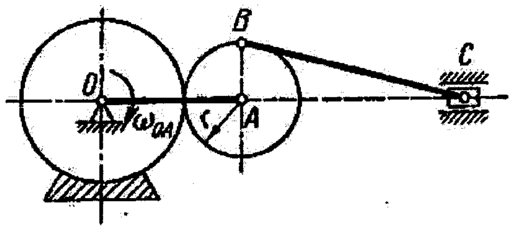

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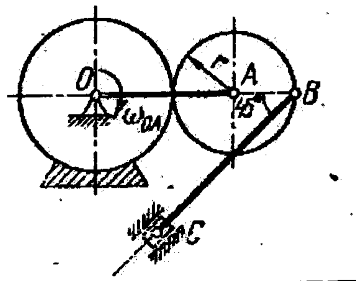

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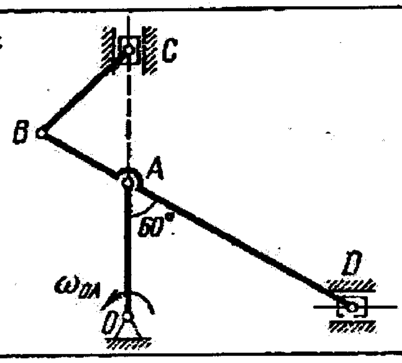

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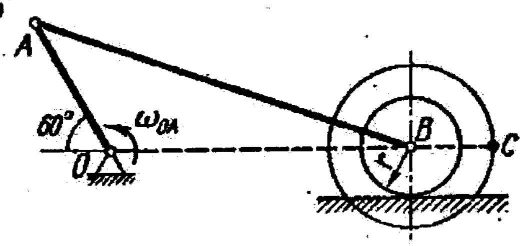

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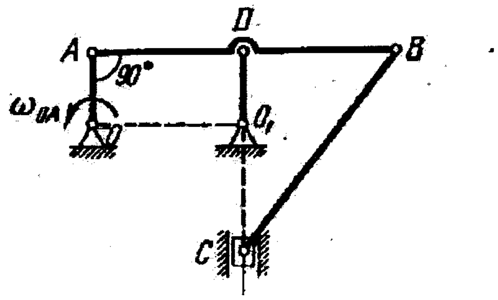

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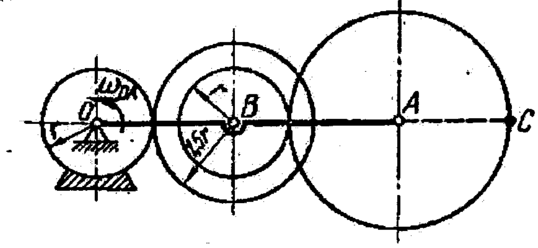

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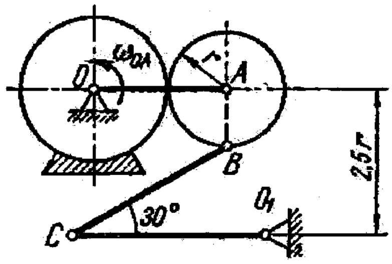

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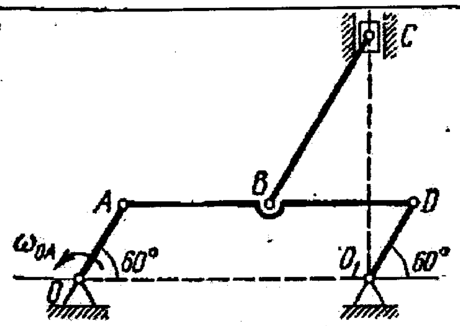

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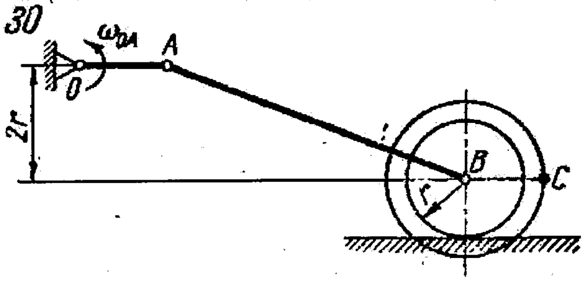
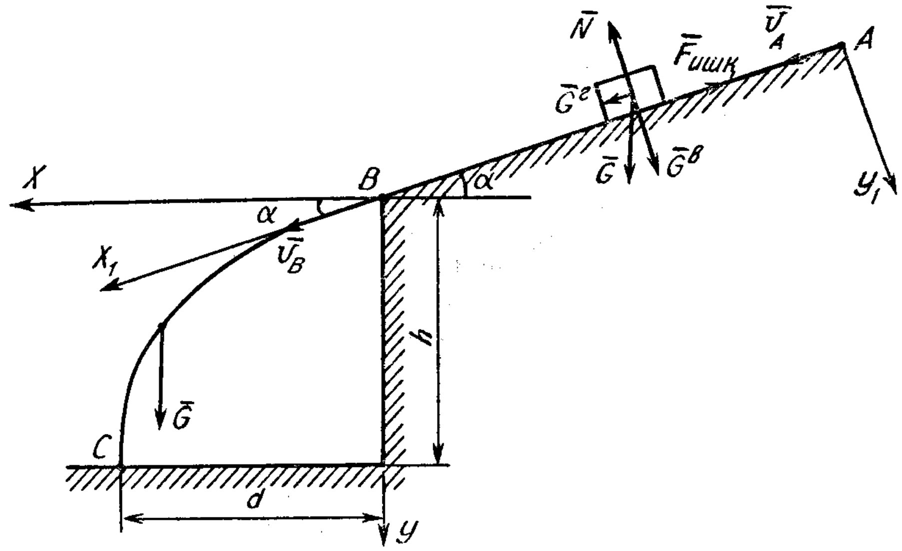


4-§. DOIMIY KUCHLAR TA’SIRIDAGI QATTIQ JISM (MODDIY NUQTA) HARAKAT DIFFERENSIAL TENGLAMALARINI INTEGRALLASH

Bu vazifa to‘g‘ri chiziqli va egri chiziqli uchastkalarga bo‘lib ishlanadi. Har bir uchastka uchun alohida differensial tenglama tuziladi va bu tenglamalar integrallanadi. Integrallashda hosil bo‘lgan integral doimiylari boshlang‘ich shartdan foydalanib topiladi.

Jismning harakat differensial tenglamalarining echimlaridan foydalanib, vazifada so‘ralgan kattaliklarning qiymatlari topiladi.



1-shakl.

1-masala. Jism *AV=l* uchastkaning *A* nuqtasidan $\overline{\upsilon}_A$ tezlik bilan harakatlana boshlab, *V* nuqtada $\overline{\upsilon}_B$ tezlikka va *VS* uchastkaning *S* nuqtasida $\overline{\upsilon}_C$ tezlikka ega bo‘ladi. Jism *AV* uchastkani o‘tish uchun τ vaqt, *VS* uchastkani o‘tish uchun esa *T* vaqt sarf qiladi. To‘g‘ri chiziqli uchastkada ishqalanish koeffisienti *f* ga teng. Masalani echishda jism nuqta deb qabul qilinsin va *VS* uchastkada havoning qarshilik kuchi hisobga olinmasin (1-shakl).

Berilgan: $\alpha=30^0$, f=0,1, τ=2s, l=10m, d=7,8m.

Topish kerak: *VS* uchastka traektoriya tenglamasi va *h*.

Echish: 1) ***AV* uchastka uchun.** Bu uchastka uchun koordinata o‘qlari x_1, u_1 bo‘lib, koordinata boshi *A* nuqtada. *AV* uchastkada harakatlanayotgan jismning harakat differensial tenglamasini yozamiz. Jism bu uchastkada to‘g‘ri chiziqli, ya’ni bitta o‘q bo‘ylab harakatlangani uchun jismning harakat differensial tenglamasi bitta bo‘ladi:

$$m=\frac{d^2x_1}{dt^2}=\sum_{k=1}^{3}X_{1k}.$$

Ma’lumki bu tenglikning o‘ng tomonidagi ifoda jismga ta’sir etuvchi kuchlarning x_1 o‘qidagi proeksiyalarining yig‘indisidir:

$$\sum_{k=1}^{3} X_{1k} = (\overline{G})_{x_1} + (\overline{N})_{x_1} + (\overline{F}^{иши.})_{x_1} = G\sin\alpha + 0 - F^{иши.} =$$

$$= G\sin\alpha - fN = G\sin\alpha - fG\cos\alpha = mg\sin\alpha - fmg\cos\alpha.$$

Bularni jismning harakat differensial tenglamasiga qo‘ysak,

$$m\frac{d^2x_1}{dt^2} = mg\sin\alpha - fmg\cos\alpha.$$

Bu tenglikning har ikki tomonini *t* ga bo‘lib, chap tomonini o‘zgartirib yozamiz:

$$\frac{d}{dt}\left(\frac{dx_1}{dt}\right) = g\sin\alpha - fg\cos\alpha.$$

Tenglikning har ikki tomonini *dt* ko‘paytiramiz va uni integrallaymiz:

$$\int d\left(\frac{dx_1}{dt}\right) = \int g\sin\alpha \cdot dt - \int fg\cos\alpha \cdot dt.$$

$$\frac{dx_1}{dt} = g\sin\alpha \cdot t - fg\cos\alpha \cdot t + C_1.$$

Integral doimiysi S_1 ni boshlang‘ich shartdan foydalanib topamiz. *t=0*da, yuqoridagi tenglikdagi *t* o‘rniga nol’ qo‘ysak,

$$c_1 = \left(\frac{dx_1}{dt}\right)_{t=0} = (\overline{\upsilon}_0)_{x_1} = (\overline{\upsilon}_A)_{x_1} = \upsilon_A, \; C_1 = \upsilon_A.$$

Bu erda $\overline{\upsilon}_0$ o‘rniga $\overline{\upsilon}_A$ ni oldik, chunki *AB* uchastka uchun boshlang‘ich tezlik harakat boshlanadigan *A* nuqtaning tezligiga tengdir. S_1 ni o‘rniga qo‘ysak:

$$\frac{dx_1}{dt} = g\sin\alpha \cdot t - fg\cos\alpha \cdot t + \upsilon_A. \qquad (1.1)$$

Bu tenglikning har ikki tomonini *dt* ga ko‘paytirib, uni integrallaymiz:

$$\int dx_1 = \int g\sin\alpha \cdot t \cdot dt - \int fg\cos\alpha \cdot t \cdot dt + \int \upsilon_A \cdot dt,$$

$$x_1 = (g\sin\alpha - fg\cos\alpha)\frac{t^2}{2} + \upsilon_A t + C_2.$$

S_2 integral doimiysini boshlang‘ich shartdan foydalanib topamiz:

t=0 da, oxirgi tenglikdan $C_2 = (x_1)_{t=0} = 0$;

$$x_1 = (g\sin\alpha - fg\cos\alpha)\frac{t^2}{2} + \upsilon_A \cdot t. \qquad (1.2)$$

Bu tenglama *AV* uchastka bo‘ylab harakatlanayotgan jismning harakat tenglamasi.

2) ***VS* uchastka uchun.** *VS* uchastka uchun koordinata o‘qlari *x* va *u* bo‘lib, koordinata boshi *V* nuqtada. Jismning *VS* uchastka bo‘ylab harakat differensial tenglamalarini yozamiz. Bu uchastkada jism egri chiziqli, ya’ni har ikkala o‘q bo‘ylab harakatlangani uchun, jismning harakat differensial tenglamalari ikkita yoziladi:

$$m\frac{d^2x}{dt^2} = \sum_{k=1}^{n} X_k, \; m\frac{d^2y}{dt^2} = \sum_{k=1}^{n} Y_k.$$

VS uchastkada harakatlanayotgan jismga havoning qarshilik kuchini e’tiborga olmasak faqatgina bitta og‘irlik kuchi ta’sir qiladi. SHuning uchun

$$\sum_{k-1}^{1} X_k = G_x = 0; \ \sum_{k=1}^{1} Y_k = G_y = G = mg.$$

Bularni e'tiborga olib, jismning harakat differensial tenglamalarini yozamiz:

$$m\frac{d^2x}{dt^2} = 0; \qquad m\frac{d^2y}{dt^2} = mg. \qquad (1.3)$$

Bu tenglamalarning har birini alohida-alohida echamiz. (1.3) ni echish uchun uning har ikki tomonini *t* ga bo'lamiz va chap tomonini o'zgartirib yozamiz:

$$\frac{d}{dt}\cdot\left(\frac{dx}{dt}\right) = 0.$$

Bu tenglikning har ikki tomonini *dt* ga ko'paytirib, uni integrallaymiz: $\int d\left(\frac{dx}{dt}\right) = \int 0\cdot dt,$ bundan $\frac{dx}{dt} = 0 + C_3$. C_3 integral doimiysini boshlang'ich shartdan foydalanib topamiz.

$$t{=}0 \text{ da } C_3 = \left(\frac{dx}{dt}\right)_{t=0} = (\overline{\upsilon}_0)_x = (\overline{\upsilon}_B)_x = \upsilon_B\cos\alpha.$$

Bu erda $\overline{\upsilon}_0$ o'rniga $\overline{\upsilon}_B$ ni oldik, chunki *VS* uchastka uchun boshlang'ich tezlik, bu uchastka uchun koordinata boshi deb olingan *V* nuqta tezligiga teng. S_3 ning qiymatini qo'ysak:

$$\frac{dx}{dt} = \upsilon_B\cos\alpha.$$

Tenglikning har ikki tomonini *dt* ga ko'paytiramiz va uni integrallaymiz:

$$\int dx = \int \upsilon_B\cos\alpha\cdot dt; \text{ bundan } x = \upsilon_B\cos\alpha\cdot t + C_4.$$

S_4 integral doimiysini boshlang'ich shartdan foydalanib topamiz. *t=0* da, oxirgi tenglikdan $C_4 = (x)_{t=0} = 0$.

$$x = \upsilon_B\cos\alpha\cdot t. \qquad (1.4)$$

(1.4) tenglikka *VS* uchastka bo'ylab harakatlanayotgan jismning *x* o'q bo'yicha harakat tenglamasi deyiladi.

Endi (1.3) tenglamani echamiz. Tenglamaning har ikki tomonini *t* ga bo'lib, uning chap tomonini o'zgartirib yozamiz:

$$\frac{d}{dt}\cdot\left(\frac{dy}{dt}\right) = g.$$

Bu tenglikning har ikki tomonini *dt* ga ko'paytiramiz va integrallaymiz.

$$\int d\left(\frac{dy}{dt}\right) = \int gdt; \ \frac{dy}{dt} = gt + C_5.$$

S_5 integral doimiysini boshlang'ich shartdan foydalanib topamiz. *t=0*da oxirgi tenglikdan

$$C_5 = \left(\frac{dy}{dt}\right)_{i-0} = (\overline{\upsilon}_0)_y = (\overline{\upsilon}_B)_y = \upsilon_B\sin\alpha.$$

C_5 ni o'rniga qo'ysak

$$\frac{dy}{dt} = gt + \upsilon_B\sin\alpha.$$

Bu tenglikning har ikki tomonini *dt* ga ko‘paytiramiz va integrallaymiz:

$$\int dy = \int gt \cdot dt + \int \upsilon_B \sin\alpha \cdot dt;$$

$$y = \upsilon_B \sin\alpha \cdot t + \frac{gt^2}{2} + C_6.$$

S_6 integral doimiysini boshlang‘ich shartdan foydalanib topamiz, *t=0* da, oxirgi tenglikdan $C_6 = (y)_{t=0} = 0$.

$$y = \upsilon_B \sin\alpha \cdot t + \frac{gt^2}{2}. \qquad (1.5)$$

(1.5) tenglikka *VS* uchastka bo‘ylab harakatlanayotgan jismniyag *u* o‘qi bo‘yicha harakat tenglamasi deyiladi.

SHunday qilib, jism harakat differensial tenglamalari echib bo‘lingandan so‘ng, vazifada so‘ralgan kattaliklarni topishga kirishamiz. Buning uchun bizga kerakli bo‘lgan tenglamalarni alohida yozib olamiz:

$$\frac{dx_1}{dt} = (g\sin\alpha - fg\cos\alpha) \cdot t + \upsilon_A; \qquad (1.1)$$

$$x_1 = (g\sin\alpha - fg\cos\alpha)\frac{t^2}{2} + \upsilon_A \cdot t; \qquad (1.2)$$

$$x = \upsilon_B \cos\alpha \cdot t; \qquad (1.4)$$

$$y = \upsilon_B \sin\alpha \cdot t + \frac{gt^2}{2} \qquad (1.5)$$

$t=\tau$ bo‘lganda τ vaqt o‘tgan bo‘lib, *A* nuqtadagi jism *V* nuqtada bo‘ladi.

(1.1) dagi $\left(\frac{dx_1}{dt}\right)_{t=\tau} = (\overline{\upsilon}_B)_{x_1} = \overline{\upsilon}_B$ bo‘ladi.

(1.2) dagi $(x_1)_{t=\tau} = AB = l$ bo‘ladi.

t=T bo‘lganda esa, *T* vaqt o‘tgan bo‘lib *V* nuqtadagi jism *S* nuqtada bo‘ladi.

(1.4) dagi $(x)_{t=T} = d$ bo‘ladi.

(1.5) dagi $(y)_{t=T} = h$ bo‘ladi.

YUqoridagilarni e’tiborga olsak

$$\upsilon_B = (g\sin\alpha - fg\cos\alpha)\tau + \upsilon_A; \qquad (1.6)$$

$$l = (g\sin\alpha - fg\cos\alpha)\frac{\tau^2}{2} + \upsilon_A \cdot \tau; \qquad (1.7)$$

$$d = \upsilon_B \cos\alpha \cdot T \qquad (1.8)$$

$$h = \upsilon_B \sin\alpha \cdot T + \frac{gT^2}{2} \qquad (1.9)$$

(1.7) dan

$$\upsilon_A = \frac{l - (g\sin\alpha - fg\cos\alpha)\frac{\tau^2}{2}}{\tau} = \frac{10 - \left(9{,}8 \cdot \frac{1}{2} - 0{,}1 \cdot 9{,}8 \cdot \frac{\sqrt{3}}{2}\right) \cdot \frac{4}{2}}{2} = \frac{10 - 8{,}11}{2} = 0{,}945 \ м/с.$$

(1.6) dan

$$\upsilon_B = (g\sin\alpha - fg\cos\alpha)\tau + \upsilon_A = \left(9{,}8 \cdot \frac{1}{2} - 0{,}1 \cdot 9{,}8 \cdot \frac{\sqrt{3}}{2}\right) \cdot 2 + 0{,}945 = 8{,}11 + 0{,}945 = 9{,}055 \ м/с.$$

(1.8) dan:

$$T = \frac{d}{\upsilon_B \cos\alpha} = \frac{7{,}8}{9{,}055 \cdot \frac{\sqrt{3}}{2}} = 1\, c.$$

(1.9) dan:

$$h = \upsilon_B \sin\alpha \cdot T + \frac{gT^2}{2} = 9{,}055 \cdot \frac{1}{2} \cdot 1 + \frac{9{,}8^2 \cdot 1^2}{2} = 4{,}527 + 4{,}9 = 9{,}4275\, м,\ h = 9{,}4275\ м.$$

VS uchastka traektoriya tenglamasini topish uchun esa, bu uchastka bo‘ylab jism harakatining tenglamalaridan foydalanamiz. YA’ni, (1.4) va (1.5) tenglamalarini birgalikda echib, ulardagi parametr *t* ni har xil matematik amallarni bajarib yo‘qotamiz.

(1.4) dan: $t = \frac{x}{\upsilon_B \cos\alpha}$, buni (1.5) ga qo‘yamiz.

$$y = \upsilon_B \sin\alpha \cdot \frac{x}{\upsilon_B \cos\alpha} + \frac{g \cdot \frac{x^2}{\upsilon_B^2 \cos^2\alpha}}{2} = xtg\alpha + \frac{gx^2}{2\upsilon_B^2 \cos^2\alpha};$$

$$y = xtg\alpha + \frac{gx^2}{2\upsilon_B^2 \cos^2\alpha} \qquad \text{yoki} \qquad y = 0{,}08x^2 + 0{,}58x.$$

Bu tenglama parabolani ifodalagani uchun *VS* uchastkadagi traektoriya paraboladan iborat degan xulosaga kelamiz. (1.4—1.9) tenglamalar yordamida ushbu vazifada istalgan boshqa kattaliklarni ham topish mumkin. Quyida biz bir necha variantlarni ko‘rib chiqamiz.

1-variant.

Berilgan: $f = 0$, $\alpha = 45^0$, $d = 6$ m, $l = AV = 10$m, $h = 8$ m.

Topish kerak: υ_A va υ_B.

Echish:

(1.8) dan: $T = \frac{d}{\upsilon_B \cos\alpha}$; buni (1.9) ga qo‘yamiz:

$$h = \upsilon_B \sin\alpha \cdot \frac{d}{\upsilon_B \cos\alpha} + \frac{gd^2}{2\upsilon_B^2 \cos^2\alpha}$$

yoki

$$h = dtg\alpha + \frac{gd^2}{2\upsilon_B^2 \cos^2\alpha},\ \text{bundan}\ 8 = 6 \cdot 1 + \frac{9{,}8 \cdot 36}{2\upsilon_B^2 \cdot \frac{2}{4}}.$$

$$2\upsilon_B^2 = 352{,}8; \quad \upsilon_B^2 = 176{,}4, \quad \upsilon_B = 13{,}3\ м/c.$$

(1.6) dan $f = 0$ bo‘lgani uchun $\tau = -\frac{\upsilon_B - \upsilon_A}{g \sin\alpha}$; ; buni

(1.7) ga qo‘ysak

$$l = g\sin\alpha \cdot \frac{(\upsilon_B-\upsilon_A)^2}{2g^2\sin^2\alpha} + \upsilon_A \cdot \frac{\upsilon_B-\upsilon_A}{g\sin\alpha};$$

$$10 = \frac{\upsilon_B^2 - 2\upsilon_B\cdot\upsilon_A + \upsilon_A^2}{2\cdot 9{,}8\cdot\frac{\sqrt{2}}{2}} + \frac{\upsilon_A\cdot\upsilon_B - \upsilon_A^2}{9{,}8\cdot\frac{\sqrt{2}}{2}};$$

$$1{,}4\cdot 98 = \upsilon_B^2 - 2\upsilon_B\upsilon_A + \upsilon_A^2 + 2\upsilon_A\upsilon_B - 2\upsilon_A^2;$$

$$137{,}2 = 176{,}4 - \upsilon_A^2;\ \upsilon_A^2 = 39{,}2;\ \upsilon_A = 6{,}3\ м/с.$$

Jismning S nuqtadagi tezligini topish uchun kinematika bo‘limidan ma’lum bo‘lgan $\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2}$ formuladan foydalanamiz. υ_x va υ_y larni topish uchun (1,4), (1,5) tenglamalardan vaqt bo‘yicha birinchi tartibli hosila olamiz:

$$\upsilon_x = \frac{dx}{dt} = x' = (\upsilon_B\cos\alpha\cdot t)' = \upsilon_B\cos\alpha;$$

$$\upsilon_y = \frac{dy}{dt} = y' = \left(\upsilon_B\sin\alpha\cdot t + \frac{gt^2}{2}\right)' = \upsilon_B\sin\alpha + gt;$$

$$\upsilon = \sqrt{\upsilon_B^2\cos^2\alpha + (\upsilon_B\sin\alpha + gt)^2}.$$

Bu formuladaga t o‘rniga T qo‘ysak, jismning T vaqt o‘tgandan keyingi, ya’ni S nuqtadagi tezligi topiladi:

$$\upsilon_0 = \sqrt{\upsilon_B^2\cos^2\alpha + (\upsilon_B\sin\alpha + gT)^2} = \sqrt{\upsilon_B^2\cos^2\alpha + \upsilon_B^2\sin^2\alpha + 2\upsilon_B\sin\alpha\cdot g\cdot T + g^2T^2} =$$

$$= \sqrt{\upsilon_B^2 + 2\upsilon_B\sin\alpha\cdot\frac{g\cdot d}{\upsilon_B\cdot\cos\alpha} + g^2\cdot\frac{d^2}{\upsilon_B^2\cos^2\alpha}} = \sqrt{176{,}4 + 2\cdot 13{,}3\cdot\frac{\sqrt{2}}{2}\cdot 9{,}8\cdot\frac{6}{13{,}3\cdot\frac{\sqrt{2}}{2}} + 9{,}8^2\cdot\frac{36}{176{,}4\cdot\frac{2}{4}}} =$$

$$= \sqrt{333{,}2} = 18{,}3\ м/с;\quad \upsilon_C = 18{,}3\ м/с.$$

2-variant. Berilgan: υ_A = 10 m/s, f = 0, τ = 2 s, υ_B = 19,8 m/s, d = 8m.

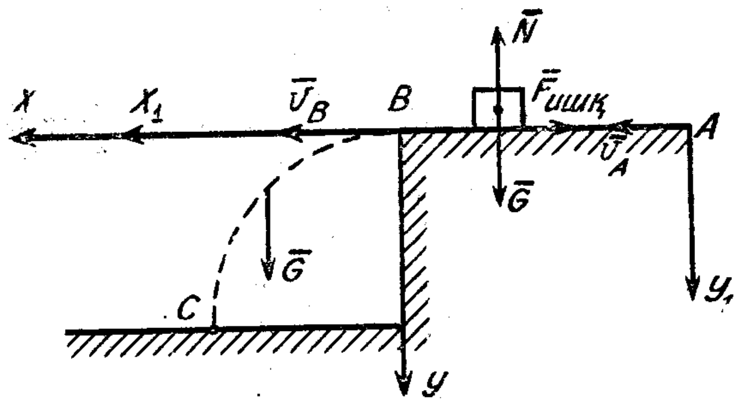
Topish kerak: α va h.

Yechish.

(1.6) dan $\sin\alpha = \frac{\upsilon_B-\upsilon_A}{g\tau} = \frac{19{,}8-10}{9{,}8\cdot 2}\cdot\frac{9{,}8}{9{,}8\cdot 2} = \frac{1}{2};$ bundan α=30^0,

(1.8) dan $T = \frac{d}{\upsilon_B\cos\alpha} = \frac{8}{19{,}8\cdot\frac{\sqrt{3}}{2}} = \frac{8}{17{,}1} = 0{,}46\ с.$

(1.9) dan $h = 19{,}8\cdot\frac{1}{2}\cdot 0{,}46 + \frac{9{,}8\cdot 0{,}46^2}{2} = 5{,}5\ м,\quad h = 5{,}5\ м.$



2-shakl.

Endi AV uchastka qiya tekislik bo‘lmasdan gorizontal uchastkadan iborat bo‘lgan holni ko‘raylik (2-shakl).

Bunda masalani echish yanada osonlashadi va bu uchastka bo‘ylab jismning harakat differensial tenglamasining o‘ng tomonida faqatgina ishqalanish kuchining x_1 o‘qidagi proeksiyasigina qatnashadi, chunki og‘irlik va normal reaksiya kuchlarining gorizontal o‘qdagi proeksiyalari nolga teng:

$$m\frac{d^2x_1}{dt^2} = \sum_{k=1}^{3} X_{1k} = G_{x_1} + N_{x_1} + F_{x_1}^{ишк} = 0 + 0 - F^{ишк} = -fN = -fG = -fmg$$

yoki

$$m\frac{d^2x_1}{dt^2} = -fmg. \qquad (1.10)$$

Bu tenglama AB uchastka qiya tekislik bo‘lgan holda differensial tenglamani echilgani kabi echiladi. (1.10) tenglamaning har ikki tomonini t ga bo‘lamiz va chap tomonini o‘zgartirib yozamiz:

$$\frac{d}{dt}\cdot\left(\frac{dx_1}{dt}\right) = -fg.$$

Bu tenglikning har ikki tomonini dt ga ko‘paytirib, uni integrallaymiz:

$$\int d\left(\frac{dx_1}{dt}\right) = \int -fgdt, \quad \frac{dx_1}{dt} = -fgt + C_1.$$

C_1 integral doimiysi boshlang‘ich shartdan foydalanib topiladi. $t=0$ da, oxirgi tenglikdan $C_1 = \left(\frac{dx_1}{dt}\right)_{t=0} = (\overline{\upsilon}_0)_{x_1} = (\overline{\upsilon}_A)_{x_1} = \upsilon_A$, C_1 ning o‘rniga υ_A qo‘ysak

$$\frac{dx_1}{dt} = -fgt + \upsilon_A. \qquad (1.11)$$

Bu tenglikdan jismning AV uchastka bo‘ylab harakati sekinlanuvchan hamda $t = \frac{\upsilon A}{fg}$ vaqtdan keyin uning tezligi nolga teng bo‘lishini ko‘rish mumkin.

(1.11) tenglikning har ikki tomonini dt ga ko‘paytiramiz va uni integrallaymiz:

$$\int dx_1 = \int -fgtdt + \int \upsilon_A dt, \quad x_1 = \upsilon_A\cdot t - \frac{fgt^2}{2} + C_2.$$

S_2 integral doimiysi boshlang‘ich shartdan foydalanib topiladi.

t = 0 da, oxirgi tenglikdan $C_2 = (x_1)_{t=0} = 0.$ uchastka qiya tekislik bo‘lmasdan, gorizontal uchastkadan iborat bo‘lganda, jismning bu uchastkadagi harakat tenglamasi:

$$x_1 = \upsilon_A \cdot t - \frac{fgt^2}{2}. \qquad (1.12)$$

t=τ bo‘lganda (1.11) va (1.12) lar o‘rniga

$$\upsilon_B = -fg\tau + \upsilon_A \quad (1.13) \text{ va } l = \upsilon_A \cdot \tau - fg\frac{\tau^2}{2} \quad (1.14)$$

tenglamalarni hosil qilamiz.

Bu tenglamalarni (1.6) va (1.7) tenglamalardagi α o‘rniga nol’ gradus (*AB* gorizontal uchastka) qo‘yib ham hosil qilish mumkin. *AB* gorizontal uchastkadan iborat bo‘lgan holda jismning BC uchastka bo‘ylab harakati differensial tenglamalari o‘zgarmaydi, ya’ni:

$$m\frac{d^2x}{dt^2} = G_x = 0; \quad (1.15) \quad m\frac{d^2y}{dt^2} = G_y = mg. \quad (1.16)$$

Bu tenglamalarning echimlari esa *AB* uchastka qiya tekislik bo‘lgandagiga nisbatan oddiyroq bo‘ladi. (1.15) va (1.16) tenglamalar xuddi (1.3). (1.3) tenglamalar kabi echiladi. Quyida shu echimlarni qisqacha takrorlaymiz:

$$\frac{d}{dt}\left(\frac{dx}{dt}\right) = 0, \int d\left(\frac{dx}{dt}\right) = \int 0, \frac{dx}{dt} = 0 + C_3;$$

$$t=0 \text{ da } C_3 = \left(\frac{dx}{dt}\right)_{t=0} = (\overline{\upsilon}_0)_x = (\overline{\upsilon}_B)_x = \upsilon_B;$$

$$\frac{dx}{dt} = \upsilon_B,\ dx = \upsilon_B \cdot dt,\ \int dx = \int \upsilon_B dt,\ x = \upsilon_B \cdot t + C_4;$$

$$t=0 \text{ da } C_4 = (x)_{t=0} = 0,\ x = \upsilon_B \cdot t; \qquad (1.17)$$

$$\frac{d}{dt}\left(\frac{dy}{dt}\right) = g,\ d\left(\frac{dy}{dt}\right) = gdt,\ \int d\left(\frac{dy}{dt}\right) = \int gdt;\ \frac{dy}{dt} = gt + C_5,$$

$$t=0 \text{ da } C_5 = \left(\frac{dy}{dt}\right)_{t=0} = (\overline{\upsilon}_0)_y = (\overline{\upsilon}_B)_y = 0.$$

$$\frac{dy}{dt} = gt,\ dy = gt \cdot dt,\ \int dy = \int gtdt,\ y = \frac{gt^2}{2} + C_6,$$

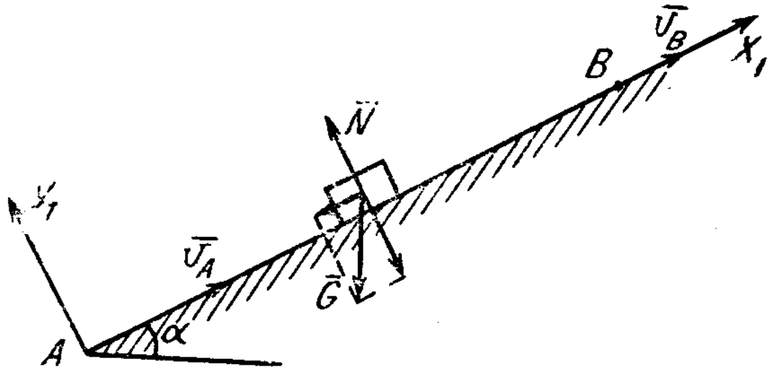
$$t=0 \text{ da } C_6 = (y)_{t=0} = 0,\ y = \frac{gt^2}{2}. \qquad (1.18)$$

t=T bo‘lganda (1.17), (1.18) tenglamalar o‘rniga

$$d = \upsilon_B \cdot T, \qquad (1.19)$$

$$h = \frac{gT^2}{2} \qquad (1.20)$$

tenglamalarni hosil qilamiz.



3-shakl.

AB uchastka qiya tekislikdan iborat bo‘lib, bu uchastka bo‘ylab, jism pastga emas, yuqoriga ko‘tarilayotgan bo‘lishi ham mumkin (3-shakl). Bunday holda $\overline{G}$ og‘irlik kuchining x_1 o‘qidagi proeksiyasi manfiy ishorali bo‘lib, jism harakat differensial tenglamasi va uning echimidagi og‘irlik kuchiga bog‘liq bo‘lgan hadning oldida manfiy ishora bo‘ladi:

$$m\frac{d^2x_1}{dt^2} = -mg\sin\alpha - fmg\cos\alpha;$$

$$\frac{dx_1}{dt^2} = (-g\sin\alpha - fg\cos\alpha)t + \upsilon_A;$$

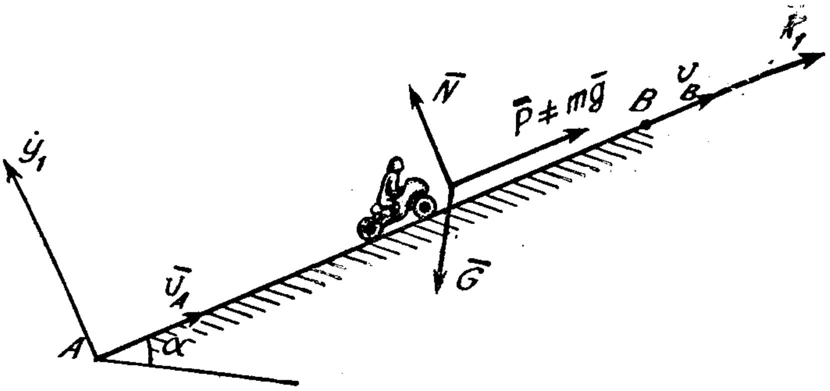
$$x_1 = (-g\sin\alpha - fg\cos\alpha)\frac{t^2}{2} + \upsilon_A \cdot t,$$

$t=\tau$ bo‘lganda esa (1.6), (1.7) tenglamalar o‘rniga

$$\upsilon_B = (-g\sin\alpha - fg\cos\alpha)\tau + \upsilon_A;$$

$l = (-g\sin\alpha - fg\cos\alpha)\frac{\tau^2}{2} + \upsilon_A\tau$ tenglamalarni hosil qilamiz.

Agar *AB* uchastka qiya tekislikdan iborat bo‘lib, bu uchastka bo‘ylab ko‘tarilayotgan jism o‘rniga motosikl bo‘lsa, bunda motosikl motorining tortish kuchi ham e’tiborga olinadi (4-shakl). Bu erda $\overline{P}$ - motosikl motorining tortish kuchi. Havoning qarshilik kuchi va tekislik yuzasidagi ishqalanish kuchlarini e’tiborga olmasak, bunday jism harakatining differensial tenglamasi va uni echimi quyidagicha bo‘ladi:



4-shakl.

$$m\frac{d^2x_1}{dt^2} = -mg\sin\alpha + P;$$

$$\frac{dx_1}{dt} = \left(-g\sin\alpha + \frac{P}{m}\right)t + \upsilon_A; \qquad (1.21)$$

$$x_1 = \left(-g\sin\alpha + \frac{P}{m}\right)\frac{t^2}{2} + \upsilon_A \cdot t. \qquad (1.22)$$

$t=\tau$ bo‘lganda (1.21) dagi $\left(\frac{dx_1}{dt}\right)_{t=\tau} = (\overline{\upsilon}_B)_{x_1} = \overline{\upsilon}_B$. (1.22) dagi $(x_1)_{t=\tau} = AB = l$ bo‘ladi va

$$\upsilon_B = \left(-g\sin\alpha + \frac{P}{m}\right)\tau + \upsilon_A; \qquad (1.23)$$

$$l = \left(-g\sin\alpha + \frac{P}{m}\right)\frac{\tau^2}{2} + \upsilon_A \cdot \tau. \qquad (1.24)$$

tenglamalarni hosil qilamiz.

Jism *AB* uchastka bo‘ylab yuqoriga ko‘tarilayotgan hol uchun, jismning BC uchastka bo‘ylab harakat differensial tenglamalari va ularning echimlari o‘zgarmasdan qoladi va (1.3), (1.4), (1.5) bo‘ladi. Quyida biz *AB* uchastka gorizontal bo‘lgan ($\alpha=0^0$) va jism gorizont bilan ($\alpha\neq0^0$) burchak tashkil qilgan qiya tekislik bo‘ylab yuqoriga ko‘tarilayotgan hollar uchun bu vazifaning echilishi bilan tanishib chiqamiz.

3-variant. Berilgan: $\alpha=0^0$, f= 0,1, τ = 5 s, d = 2 m, h =4,9 m.

Topish kerak: υ_A, l.

Echish: (1.20) dan $T = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\cdot 4{,}9}{9{,}8}} = 1c.$

(1.19) dan $\upsilon_B = \frac{d}{T} = \frac{2}{1} = 2\,м/c;$ (1.13) dan $\upsilon_A = \upsilon_B + fg\tau = 6{,}9\,м/c,$

(1.14) dan $l = \upsilon_A\tau - fg\frac{\tau^2}{2} = 6{,}9\cdot 5 - 0{,}1\cdot 9{,}8\cdot\frac{25}{2} = 22{,}25\ м.$

4-variant. Berilgan: $\alpha=45^0$, t=200 kg, υ_A=0, T=1 s, l=10m, h=6m.

Topish kerak: *R,d*.

Echish: (1.9) dan $\upsilon_B = \dfrac{h - \frac{gT^2}{2}}{\sin\alpha\cdot T} = \dfrac{6 - 9{,}8\cdot\frac{1}{2}}{0{,}7\cdot 1} = \dfrac{1{,}1}{0{,}7} = 1{,}6\ м/c.$

(1.8) dan $d = \upsilon_B\cos\alpha\cdot T = 1{,}6\cdot 0{,}7\cdot 1 = 1{,}12\,м.$

(1.23) dan $\upsilon_A = 0$ bo‘lganda $\left(-g\sin\alpha + \frac{P}{m}\right) = \frac{\upsilon_B}{\tau};$ (1.25)

buni (1.24) ga qo‘ysak $l = \frac{\upsilon_B}{\tau}\cdot\frac{\tau^2}{2},\ \tau = \frac{2l}{\upsilon_B} = \frac{20}{1{,}6} = 12{,}5\,c.$ (1.25) dan $\frac{P}{m} = \frac{\upsilon_B}{\tau} + g\sin\alpha = \frac{1{,}6}{12{,}5} + 9{,}8\cdot 0{,}7 = 7\ м/c^2;$ bundan $P = m\cdot 7м/c^2 = 200кг\times 7м/c^2 =$ $= 1400H = 14кH.$

D-1
Doimiy kuchlar ta'siri ostidagi material nuqta harakat differensial tenglamalarini integrallash

Variant 1-5.

Jism gorizont bilan α burchak tashkil etgan qiya tekislik bo'ylab τ sek mobaynida *A* nuqtadan boshlab $\bar{\upsilon}_A$ boshlang'ich tezlik bilan harakatlanadi.

Jism bilan qiya tekislik orasidagi ishqalanish koeffisienti *f* ga teng.

B nuqtada jism qiya tekislikni $\bar{\upsilon}_B$ tezlik bilan tark etadi va havoda *T* vaqt bo'lib gorizont bilan β burchak tashkil etgan *VD* tekislikning C nuqtasiga $\bar{\upsilon}_C$ tezlik bilan kelib tushadi. Masalani echishda jismni material nuqta deb olinsin va havoning qarshilik kuchi e'tiborga olinmasin.

Variant 1.

Berilgan: $\alpha=30^0$; $V_A=0$; f=0,2; l=10 m; $\beta=60^0$

Topish kerak: τ va h.

Variant 2.

Berilgan: $\alpha=15^0$; V_A=2 m/s; f=0,2; h=4 m; $\beta=45^0$

Topish kerak: *l* va *BC* uchastka trametriya tenglamasi.

Variant 3.

Berilgan: $\alpha=30^0$; V_A=2,5 m/s; f≠0; l=8 m; d=10 m; $\beta=60^0$

Topish kerak: $\bar{\upsilon}_B$ va τ.

Variant 4.

Berilgan: $V_A=0$; τ=2 sek; l=9,8 m; $\beta=60^0$; f=0

Topish kerak: α va T.

Variant 5.

Berilgan: $\alpha=30^0$; $V_A=0$; f=0,2; l=10 m; $\beta=60^0$

Topish kerak: f va V_c.

Variant 6-10.

CHang'ida uchuvchi gorizont bilan α burchak tashkil etgan uzunligi *l* bo'lgan qiya tekislikning A nuqtasiga V_A tezlik bilan etib kelib, AB uchastkani τ vaqtda bosib o'tadi. AB uchastkadagi ishqalanish koeffisienti *f* ga teng. AB uchastkani $\bar{V}_B$ tezlik bilan tark etgan chang'ichi T vaqtdan so'ng, gorizont bilan β burchak tashkil etgan qiyalikning C nuqtasiga V_C tezlik bilan kelib tushadi.

Masalani echishda chang'ichini minimal nuqta deb olinsin va havoning qarshilik kuchi e'tiborga olinmasin.

Variant 6.

Berilgan: $\alpha=20^0$; f=0,1; τ=0,2 sek; h=4sm; $\beta=30^0$

Topish kerak: *l* va V_c.

Variant 7.

Berilgan: $\alpha=15^0$; f=0,1; V_A=16 m/s; l=5m; $\beta=45^0$

Topish kerak: V_B va T.

Variant 8.

Berilgan: V_A=21 m/s; f=0; τ=0,3 sek; V_B=20m/s; $\beta=60^0$

Topish kerak: α va d.

Variant 9.

Berilgan: $\alpha=15^0$; τ=0,3 sek; *f*=0,1; $h = 30\sqrt{2}$; $\beta=45^0$

Topish kerak: V_B va V_A.

Variant 10.

Berilgan: $\alpha=15^0$; f=0; V_A=12m/s; d=50 m; $\beta=60^0$

Topish kerak: τ va chang'ichining VS uchastka bo'ylab harakat traektoriyasini.

<u>Variant 11-15.</u>

A nuqtada V_A tezlikka ega bo'lgan motosikl gorizont bilan α burchak tashkil etgan, uzunligi *l* bo'lgan qiya tekislik bo'ylab τ vaqt ichida harakatlanmoqda. AB uchastka bo'ylab doimiy $\overline{P}$ harakatlantiruvchi kuchga ega bo'lgan motosiklning B nuqtadagi tezligi V_B bo'lib, motosiklist T vaqt ichida kengligi d bo'lgan jarlikdan sakrab o'tib, C nuqtaga V_c tezlik bilan kelib tushadi. Motosiklning motosiklist bilan birgalikdagi massasi m ga teng.

Masalani echishda motosiklistni material nuqta deb hisoblansin va havoning qarshilik kuchi e'tiborga olinmasin.

Variant 11.

Berilgan: $\alpha=30^0$; $\rho\neq0$; *l*=40 m; V_A=0; V_B=4,5m/s; d=3 m

Topish kerak: τ va h.

Variant 12.

Berilgan: $\alpha=30^0$; $\rho\neq0$; *l*=40 m; V_B=4,5m/s; h=1,5 m

Topish kerak: V_A va d.

Variant 13.

Berilgan: $\alpha=30^0$; m=400 m^2; V_A=0; τ=20 sek; d=3m; h=1,5 m

Topish kerak: ρ va *l*.

Variant 14.

Berilgan: $\alpha=30^0$; m=400 m^2; ρ=2,2kN; V_A=0; *l*=40m; d=5m

Topish kerak: V_B va V_S.

Variant 15.

Berilgan: $\alpha=30^0$; V_A=0; ρ=2kN; *l*=50m; h=2m; d=4m

Topish kerak: T va m.

<u>Variant 16-20.</u>

Tosh (jism) gorizont bilan α burchak tashkil etgan uzunligi *l* bo'lgan qiya tekislik bo'ylab τ vaqt mobaynida harakatlanadi. AB uchastka bo'ylab ishqalanish koeffisient *f* ga teng.

B nuqtada V_B tezlikga ega bo'lgan jism vertikal devorning C nuqtasiga T vaqtdan so'ng kelib uriladi.

Masalani echishni toshni (jismni) material nuqta deb olinsin va havoning qarshilik kuchi otosiklistni material nuqta deb hisoblansin va havoning qarshilik kuchi e’tiborga olinmasin.

Variant 16.

Berilgan: $\alpha=30^0$; $V_A=1$ m/c; $l=3$ m; f=0,2; d=2,5 m

Topish kerak: h va T.

Variant 17.

Berilgan: $\alpha=45^0$; $l=6$ m; $V_B=2V_A$; $\tau=1$ sek; h=6 m.

Topish kerak: *d* va f.

Variant 18.

Berilgan: $\alpha=30^0$; $l=2$ m; $V_A=0$; f=0,1; d=3 m.

Topish kerak: h va τ.

Variant 19.

Berilgan: $\alpha=15^0$; $l=3$ m; $V_B=3$ m/s; $f\neq 0$; $\tau=1,5$ c; d=2 m.

Topish kerak: V_A va h.

Variant 20.

Berilgan: $\alpha=45^0$; $V_A=0$; f=0,3; d=2 m; h=4m.

Topish kerak: *l* va τ.

<u>Variant 21-25.</u>

Jism gorizont bilan α burchak tashkil qilgan *l* uzunlikdagi qiya tekislik bo‘ylab A nuqtadan boshlab V_A tezlik bilan harakatlanadi. Tekislikning sirpanib ishqalanish koeffisienti f ga teng.

Jism τ vaqtdan so‘ng qiya tekislikni V_B tezlik bilan tark etadi va T vaqtdan so‘ng gorizontal tekislikning C nuqtasiga V_C tezlik bilan kelib tushadi.

Masalani echishda jismni material nuqta deb hisoblansin va havoning qarshilik kuchi e’tiborga olinmasin.

Variant 21.

Berilgan: $\alpha=30^0$; f=0,1; $V_A=1$ m/c; $\tau=1,5$ sek; h=10 m.

Topish kerak: V_A va d.

Variant 22.

Berilgan: $V_A=0$; $\alpha=45^0$; $l=10$ m; $\tau=2$ sek.

Topish kerak: *f* va BC uchastka troektoriya tenglamasi.

Variant 23.

Berilgan: f=0; $V_A=0$; $l=9,81$ m; $\tau=2$ sek; h=20 m.

Topish kerak: α va T.

Variant 24.

Berilgan: $V_A=0$; $\alpha=30^0$; f=0,2; $l=10$ m; d=12 m.

Topish kerak: τ va h.

Variant 25.

Berilgan: $V_A=0$; $\alpha=30^0$; f=0,2; $l=6$ m; h=4,5 m.

Topish kerak: τ va V_C.

Variant 26-30.

Jism A nuqtadan V_A tezlik bilan uzunligi *l* bo‘lgan gorizontal tekislik bo‘ylab τ vaqt mobaynida harakatlanmoqda. Gorizontal tekislik bo‘ylab harakatdagi ishqalanish koeffisienti f ga teng.

Jism B nuqtadan V_B tezlik bilan gorizontal tekislikni tark etadi va havoda T vaqt bo‘lib S nuqtaga V_C tezlik bilan kelib tushadi.

Masalani echishda jismni material nuqta deb qaralsin va havoning qarshilik kuchi e’tiborga olinmasin.

Variant 26.

Berilgan: V_A=7 m/s; f=0,2; *l*=8 m; h=20 m.

Topish kerak: d va V_C.

Variant 27.

Berilgan: V_A=4 m/s; f=0,1; τ=2 sek; d=2 m.

Topish kerak: V_B va h.

Variant 28.

Berilgan: V_B=3 m/c; f=0,3; *l*=0,3 m; h=5 m.

Topish kerak: V_A va T.

Variant 29.

Berilgan: V_A=3 m/s; V_B=1 m/s; *l*=2,5 m; h=20 m.

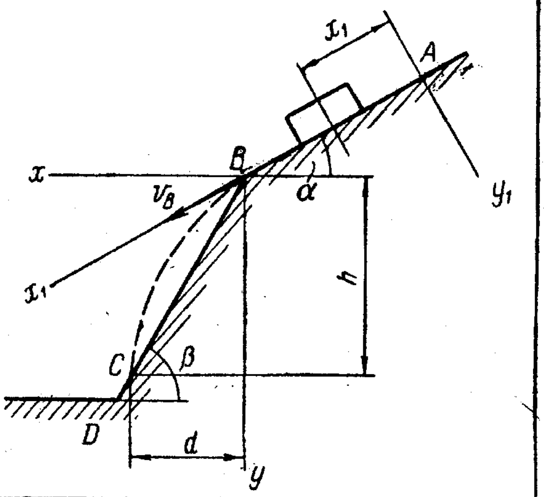
Topish kerak: f va d.

Variant 30.

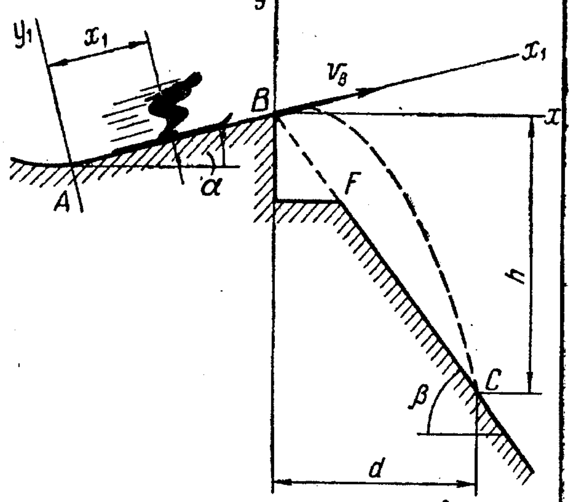
Berilgan: f=0,25; *l*=4 m; d=3 m; h=5 m.

Topish kerak: V_A va τ.

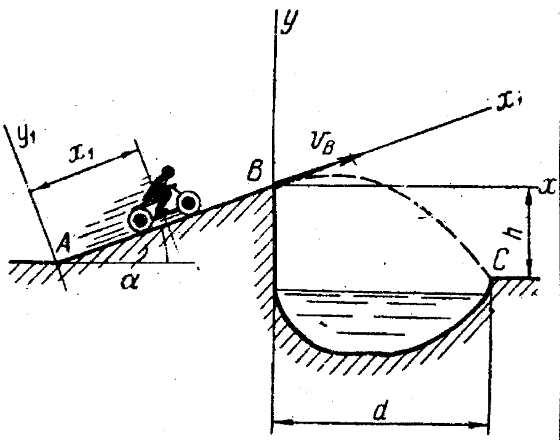
1-5



6-10



11-15



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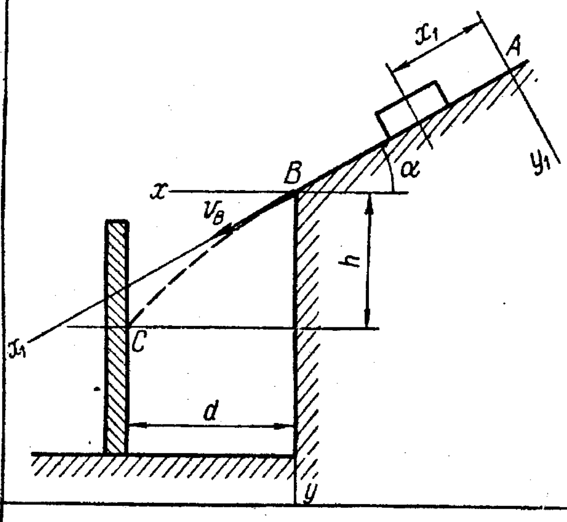

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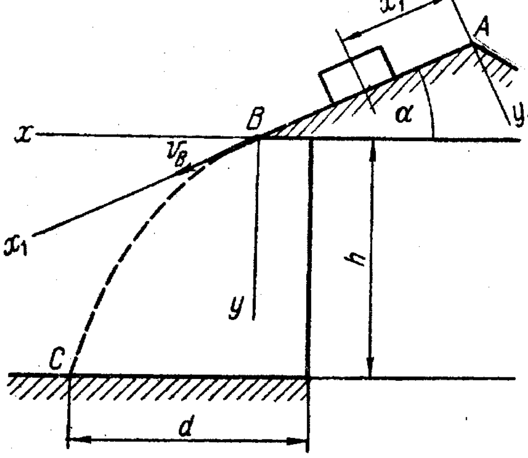

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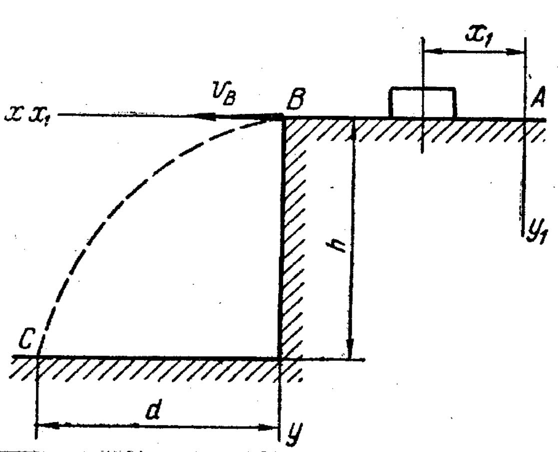

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USLUBIY QO'LLANMADAN FOYDALANISH QOIDALARI

Texnika oliy o'quv yurtida ta'lim olayotgan talaba nazariy mexanikaning «Statika» bo'limidan kurs ishlarini bajarish uchun quyidagilarga e'tibor qaratishi lozim.

1. Ushbu fandan seminar darsini olib boruvchi o'qituvchidan har bir mavzu (kurs ishi) bo'yisha o'z variantini aniqlab olishi.
2. Aniqlab olingan o'z variantining shaklini daftariga ko'chirishi.
3. Shu variantni yechish zarur bo'lgan qiymatlarni mavzuga tegishli shakllardan oldin berilgan jadvallardan olishi.
4. Har bir talaba uslubiy ko'rsatmada har bir mavzuga tegishli yechim namunalaridan foydalangan holda o'z variantini yechishga kirishishi lozim.

«Uslubiy ko'rsatmadagi jadvallarga, shakllarga yozish, chizish, nusxa olish yoki belgilar qo'yish qati'yan man etiladi, chunki ushbu uslubiy qo'llanmadan Sizdan keyin ham foydalanishlarini UNUTMANG!!!»

M U N D A R I J A