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*Keywords- SH waves, regularizing algorithm, inverse problem, direct problem, porosity, porous medium.*

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1.

 $z > 0$ 

SH

 $b(z)$ , [6,7]

$$\rho_s(z)u_{tt} = (\mu(z)u_z)_z - \rho_l(z)b(z)(u_t - v_t), \quad (1)$$

$$\rho_l(z)v_{tt} = \rho_l(z)b(z)(u_t - v_t), \quad (2)$$

 $u \quad v -$  $\rho_s(z) \quad \rho_l(z)$  $t < 0:$ 

$$u|_{t=0} = u_t|_{t=0} = 0, \quad (3)$$

$$v|_{t=0} = v_t|_{t=0} = 0. \quad (4)$$

 $z = 0$ 

:

$$\mu u_z|_{z=0} = \delta(t), \quad (5)$$

 $\delta(t) -$ 

(5)

 $\rho_s(z), \mu(z),$  $\rho_l(z), b(z)$  $u(t, z), v(t, z) \quad (1) \text{ ó } (4).$

[5]

1.

$$u|_{z=0} = \phi(t),$$

$$\mu(z) \quad (1) \text{ ó } (5) \quad (\rho_s(z), \rho_l(z), b(z)).$$

3.

(1) ó (5)

z

x:

$$x = \int_0^z \frac{d\xi}{c_t(\xi)},$$

$$c_t(z) = \sqrt{\frac{\mu(z)}{\rho_s(z)}}$$

x,

$$\frac{\partial}{\partial z} = \frac{1}{c_t} \frac{\partial}{\partial x},$$

(1), (2)

$$u_{tt} - u_{xx} = (\ln \sigma)' u_x - b(x) \frac{\rho_l(x)}{\rho_s(x)} (u_t - v_t), \quad x > 0, \quad (6)$$

$$v_t = b(x)(u - v), \quad x > 0, \quad (7)$$

$$u|_{t=0} = u_t|_{t=0} = 0, \quad (8)$$

$$v|_{t=0} = 0, \quad (9)$$

$$u_x|_{x=0} = \delta(t)/\sigma(0). \quad (10)$$

$$(6) \quad \sigma(x) = \sqrt{\mu(x)\rho_s(x)} - , \quad \sigma > 0.$$

$$0 < \rho_{0s} \leq \rho_s(x) \leq \rho_{00s} < \infty, 0 < \rho_{0l} \leq \rho_l(x) \leq \rho_{00l} < \infty, 0 < b_0 \leq b(x) \leq b_{00} < \infty. \quad (11)$$

$$1 \quad :$$

$$[0, T]$$

$$u|_{x=0} = \phi(t), \quad t \in [0, T], \quad (12)$$

$$\sigma(x), \quad x \in [0, T/2].$$

$$, \quad \phi = A \ln \sigma, \quad \sigma \in C^1[0, T/2]$$

$$C^1[0, T/2] \quad .$$

$$[1, 2, 3-5], \quad , \quad \acute{o}$$

$$C^1[0, T],$$

$$\Phi = \left\{ \phi \in C^1[0, T] : \phi(0) < 0, \|\psi\|_{L_2(0, T)}^2 + \int_{-T/2}^{T/2} \int_{-T/2}^{T/2} \phi'(|t-s|) \psi(t) \psi(s) dt ds \geq 0, \forall \psi \in L_2(0, T) \right\},$$

$$C^1[0, T] \quad \Phi.$$

4.

(6) ó (10).

$$\phi = A \ln \sigma$$

$$\tilde{\phi} \in C^1[0, T], \|\phi - \tilde{\phi}\| \leq \delta.$$

$$\phi \in \Phi,$$

$$A^{-1}$$

$$\sigma$$

$$\tilde{\sigma} = \exp[A^{-1} \tilde{\phi}].$$

$$\tilde{\phi} \in C^1[0, T], \tilde{\phi}(0) < 0, \quad ,$$

$$R : \tilde{\Phi} \rightarrow C^1[0, T/2], \tilde{\Phi} = \{\phi \in C^1[0, T] : \phi(0) < 0\},$$

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$$\tilde{\Phi} [1, 2, 3-5].$$

$$C^1[0, T/2] \quad C^1[0, T]$$

$$, \quad (A \ln \sigma)(0) = -1/\sigma(0) < 0. \quad \sigma \in C^1[0, T/2].$$

(6) ó (10)

$$u(x, t) = \theta(t-x) u_{\Delta}(x, t),$$

$$v(x, t) = \theta(t-x) v_{\Delta}(x, t).$$

$\theta(t) -$  ,  $u_\Delta, v_\Delta -$   $u, v$

$\Delta = \{(x, t): 0 \leq x \leq t\}$ ,  $u_\Delta, v_\Delta \in C^1(\Delta)$ .

(6) ó (10) ,  $T > 0$   $u, v$

$\Delta(T) = \{(x, t): 0 \leq x \leq t \leq T - x\}$ ,

$u, v$ ,

$$Lu \equiv u_{tt} - u_{xx} = (\ln \sigma)' u_x - b(x) \frac{\rho_l(x)}{\rho_s(x)} (u_t - v_t), \quad 0 < x < t < T - x, \quad (13)$$

$$v_t = b(x)(u - v), \quad 0 < x < t < T - x, \quad (14)$$

$$u_x|_{x=0} = 0, \quad 0 < t < T, \quad (15)$$

$$u(x, x) = -\frac{1}{\sqrt{\sigma(0)\sigma(x)}} e^{-\int_0^x \frac{b(y)\rho_l(y)}{2\rho_s(y)} dy}, \quad 0 \leq x \leq T/2, \quad (16)$$

$$v|_{t=0} = 0, \quad 0 \leq x \leq T/2 \quad (17)$$

, (13) ó (17)

$$\begin{aligned} u(x, t) = & w\left(\frac{t+x}{2}\right) + w\left(\frac{t-x}{2}\right) - w(0) - \frac{1}{2} \int_0^x (\ln \sigma)'(\xi) d\xi \int_{t-x+\xi}^{t+x-\xi} U(\xi, \zeta) d\zeta + \\ & + \frac{1}{2} \int_0^{(t+x)/2} (\ln \sigma)'(\xi) d\xi \int_\xi^{t+x-\xi} U(\xi, \zeta) d\zeta + \frac{1}{2} \int_0^{(t-x)/2} (\ln \sigma)'(\xi) d\xi \int_\xi^{t-x-\xi} U(\xi, \zeta) d\zeta + \\ & + \frac{1}{2} \int_0^{(t+x)/2} b(\xi) \frac{\rho_l(\xi)}{\rho_s(\xi)} d\xi \int_{t-x+\xi}^{t+x-\xi} \left( W(\xi, \zeta) - b(\xi) \int_0^\zeta e^{-b(\xi)(\zeta-s)} u(\xi, s) ds \right) d\zeta - \\ & - \frac{1}{2} \int_0^{(t+x)/2} b(\xi) \frac{\rho_l(\xi)}{\rho_s(\xi)} d\xi \int_\xi^{t+x-\xi} \left( W(\xi, \zeta) - b(\xi) \int_0^\zeta e^{-b(\xi)(\zeta-s)} u(\xi, s) ds \right) d\zeta - \\ & - \frac{1}{2} \int_0^{(t-x)/2} b(\xi) \frac{\rho_l(\xi)}{\rho_s(\xi)} d\xi \int_\xi^{t-x-\xi} \left( W(\xi, \zeta) - b(\xi) \int_0^\zeta e^{-b(\xi)(\zeta-s)} u(\xi, s) ds \right) d\zeta, \quad (18) \end{aligned}$$

$$u(x, t) = b(x) \int_0^t e^{-b(x)(t-s)} u(x, s) ds \quad (19)$$

$$U = u_x, \quad W = u_t, \quad \omega(x) = -1/\sqrt{\sigma(0)\sigma(x)} e^{-\int_0^x \frac{b(y)\rho_l(y)}{2\rho_s(y)} dy}. \quad (18) \quad x \quad t,$$

$U(x, t), W(x, t), u(x, t)$

$C(\Delta(T)).$   $U(x, t), W(x, t)$  (18),

$u(x, t)$  (  $C^1$ ) (13), (15), (16).

$$u(x, t) \quad (19),$$

$$v(x, t) \quad (14), \quad (17).$$

$$, \quad \phi(t) = u|_{x=0} \quad C^1[0, T] \quad \phi(0) = -1/\sigma(0) < 0$$

, . . .  $\phi \in \tilde{\Phi}$ .

$$A \ln \sigma = \phi, \phi \in \Phi,$$

$$Z \quad \acute{o}$$

$$z(x, t) = (z_1(x, t), z_2(x, t), z_3(x), z_4(x)),$$

$$\Delta(T),$$

$$(\Delta(T))$$

$$\|z\| = \max \{ \|z_1\|, \|z_2\|, \|z_3\|, \|z_4\| \}.$$

$$Z \quad \tilde{Z}_0, \quad \acute{o}$$

$$z_0(x, t) = (P\phi)(x, t) \equiv \left\{ \frac{1}{2}\phi'(t+x) - \frac{1}{2}\phi'(t-x), \frac{1}{2}\phi'(t+x) + \frac{1}{2}\phi'(t-x), \phi'(2x), 1/\phi(0) \right\}, \phi \in \tilde{\Phi} \quad (20)$$

$$, \quad \tilde{Z}_0 \quad \tilde{\Phi} \quad \acute{o} \quad .$$

$$(20) \quad \varphi \in \tilde{\Phi}, \quad z_0 \in \tilde{Z}_0.$$

$$M : Z \times \bar{R}_+ \rightarrow Z, \quad \bar{R}_+ = \{t : t \geq 0\}$$

$$\begin{aligned} (M(z, \alpha))_1(x, t) &= \int_0^x f(z, \alpha)(\xi) [z_1(\xi, t+x-\xi) + z_1(\xi, t-x+\xi)] d\xi + \\ &+ \frac{1}{2} \int_0^x b(\xi) \frac{\rho_l(\xi)}{\rho_s(\xi)} \{ z_1(\xi, t+x-\xi) + z_1(\xi, t-x+\xi) + z_2(\xi, t+x-\xi) + z_2(\xi, t-x+\xi) - \\ &- b(\xi) \int_0^{t+x-\xi} e^{-b(\xi)(t+x-\xi-s)} z_2(\xi, s) ds - b(\xi) \int_0^{t-x+\xi} e^{-b(\xi)(t-x+\xi-s)} z_2(\xi, s) ds \} d\xi \\ (M(z, \alpha))_2(x, t) &= \int_0^x f(z, \alpha)(\xi) [z_1(\xi, t+x-\xi) - z_1(\xi, t-x+\xi)] d\xi + \\ &+ \frac{1}{2} \int_0^x b(\xi) \frac{\rho_l(\xi)}{\rho_s(\xi)} \{ z_1(\xi, t+x-\xi) - z_1(\xi, t-x+\xi) + z_2(\xi, t+x-\xi) - z_2(\xi, t-x+\xi) - \\ &- b(\xi) \int_0^{t+x-\xi} e^{-b(\xi)(t+x-\xi-s)} z_2(\xi, s) ds + b(\xi) \int_0^{t-x+\xi} e^{-b(\xi)(t-x+\xi-s)} z_2(\xi, s) ds \} d\xi \\ (M(z, \alpha))_3(x) &= 2 \int_0^x f(z, \alpha)(\xi) z_1(\xi, 2x-\xi) d\xi + \int_0^x b(\xi) \frac{\rho_l(\xi)}{\rho_s(\xi)} \{ z_1(\xi, 2x-\xi) + \end{aligned}$$

$$+z_2(\xi, 2x-\xi) - b(\xi) \int_0^{2x-\xi} e^{-b(\xi)(2x-\xi-s)} z_2(\xi, s) ds \} d\xi$$

$$(M(z, \alpha))_4(x) = - \int_0^x f(z, \alpha)(\xi) z_4(\xi) d\xi, \quad (21)$$

$$f(z, \alpha) = z_3 z_4 / (1 + \alpha z_3^2)(1 + \alpha z_4^2).$$

$$1. \quad z = z_0 + M(z, 0), \quad z_0 \in \tilde{Z}_0, \quad Z$$

$$, \quad z_0 \in Z.$$

$$A \ln \sigma = \phi, \quad \phi \in \Phi,$$

$$z = z_0 + M(z, 0), \quad z_0 \in \tilde{Z}_0, \quad \phi \in \Phi$$

$$\tilde{\phi} \in \tilde{\Phi} \quad \|\phi - \tilde{\phi}\| \leq \delta,$$

$$, \quad \phi(0) = \tilde{\phi}(0) \quad ( \quad \phi(0) \neq \tilde{\phi}(0)$$

$$).$$

$$z_0 = P\phi \quad \tilde{z}_0 = P\tilde{\phi}$$

$$, \quad z_0 \in Z_0, \quad \tilde{z}_0 \in \tilde{Z}_0 \quad \|z_0 - \tilde{z}_0\| \leq \delta. \quad z_0 \quad Z_0,$$

$$1 \quad z = z_0 + M(z, 0) \quad .$$

$$z = z_0 + M(z, \alpha), \quad \alpha > 0. \quad B_r - \quad Z \quad r,$$

$$B_r = \{z \in Z : \|z\| \leq r\},$$

$$\|z\|(x) = \max \left\{ \sup_{x \leq t \leq T-x} |z_1(x, t)|, |z_2(x, t)|, |z_3(x)|, |z_4(x)| \right\}, \quad z \in Z.$$

$$2. \quad 1. \quad M \in C^1(Z \times \bar{R}_+; Z), \quad \dots \quad Z \times \bar{R}_+ \quad Z$$

$$M_z(z, \alpha), M_\alpha(z, \alpha).$$

$$2. \quad z \in Z, \quad \alpha > 0$$

$$\|M(z, \alpha)\|(x) \leq \frac{c_1}{2\alpha} \int_0^x \|z\|(\xi) d\xi, \quad x \in [0, T/2], \quad (22)$$

$$c_1(\alpha, T) = \left( 1 + 2b_{00} \frac{\rho_{00,l}}{\rho_{0,s}} \right) (1 + 2Tb_{00}).$$

$$3. \quad r > 0, \quad \alpha > 0, \quad z \in B_r, \quad y \in B_r$$

$$\|M(z, \alpha) - M(y, \alpha)\|(x) \leq c_2(r, \alpha, T) \int_0^x \|z - y\|(\xi) d\xi, \quad x \in [0, T/2] \quad (23)$$

$$c_2(r, \alpha, T) = \left( 1 + 4r\sqrt{\alpha} + b_{00} \frac{\rho_{00,l}}{\rho_{0,s}} \alpha \right) (1 + 2T b_{00}) / (2\alpha).$$

$$z = z_0 + M(z, \alpha). \quad (24)$$

$$1. \quad z_0 \in Z. \quad \alpha > 0 \quad Z$$

$$z(\alpha) \quad (24),$$

$$\alpha \quad R_+$$

$$\|z(\alpha)\| \leq \|z_0\| \exp(c_1 T / 4\alpha). \quad (25)$$

$$\cdot \quad (25).$$

$$z(\alpha) \in Z - \quad , \quad \alpha > 0. \quad (22) \quad ,$$

$$\|z(\alpha)\|(x) \leq \|z_0\| + \frac{c_1}{2\alpha} \int_0^x \|z(\alpha)\|(\xi) d\xi, \quad x \in [0, T/2], \quad (26)$$

$$(25) \quad (26).$$

$$\cdot \quad z(\alpha), y(\alpha) \in Z -$$

$$(24).$$

$$B_{r(\alpha)}, r(\alpha) = \|z_0\| \exp(c_1 T / 4\alpha),$$

$$2 \quad w(\alpha) = z(\alpha) - y(\alpha)$$

$$\|w(\alpha)\|(x) \leq c_2(r(\alpha), \alpha, T) \int_0^x \|z - y\| d\xi, \quad x \in [0, T/2],$$

$$\alpha > 0 \quad w(\alpha) = 0, \quad \dots \quad z(\alpha) = y(\alpha).$$

,

$$z^{(n+1)}(\alpha) = z_0 + M(z^{(n)}(\alpha), \alpha), \quad n > 0, \quad z^{(0)} = z_0 \quad (27)$$

$$(22)$$

,

,

$$n \geq 0$$

$$\|z^{(n)}(\alpha)\|(x) \leq \|z_0\| \sum_{k=0}^n \frac{1}{k!} \left( \frac{c_1 x}{2\alpha} \right)^k \leq \|z_0\| \exp(c_1 T / 4\alpha),$$

..

$$B_{r(\alpha)}.$$

$$w^{(n)}(\alpha) = z^{(n+1)}(\alpha) - z^{(n)}(\alpha).$$

$$\|w^{(0)}(\alpha)\| = \|M(z_0, \alpha)\| \leq c_3 \|z_{(0)}\|, \quad c_3 = \frac{c_1 T}{4\alpha},$$

$$\|w^{(n)}(\alpha)\| = \|M(z^n(\alpha), \alpha) - M(z^{n-1}(\alpha), \alpha)\| \leq c_2(r(\alpha), \alpha, T) \int_0^x \|w^{(n-1)}(\alpha)\|(\xi) d\xi, \quad n \geq 1,$$

$$, \quad n \geq 0$$

$$\|w^{(n)}(\alpha)\| \leq c_3 \|z_0\| \frac{1}{n!} \left(\frac{c_2 T}{2}\right)^n$$

$$, \quad z_0 + \sum_{n=0}^{\infty} w^{(n)}(\alpha)$$

$$\|z_0\| + c_3 \|z_{(0)}\| \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{c_2 T}{2}\right)^n = \|z_0\| (1 + c_3 e^{c_2 T/2})$$

, ,

$$z^{(n+1)}(\alpha) = z_0 + M(z^{(n)}(\alpha), \alpha) = z_0 + \sum_{m=0}^n w^{(m)}(\alpha)$$

$$Z. \quad z^{(n)}(\alpha) \in B_{r(\alpha)},$$

$$z(\alpha) = \lim_{n \rightarrow \infty} z^{(n)}(\alpha) \in B_{r(\alpha)}.$$

$$(27) \quad , \quad M,$$

$$z - (24).$$

$$z(\alpha)$$

$$[8]. \quad , \quad 2$$

$$G: Z \times R_+ \rightarrow Z, \quad G(z, \alpha) = z - z_0 - M(z, \alpha),$$

$$G_\alpha = -M_\alpha, \quad G_z = I - M_z. \quad (z, \alpha) \in Z \times R_+, \quad G_z(z, \alpha): Z \rightarrow Z$$

$$( \quad , \quad M_z(z, \alpha) -$$

$$).$$

$$z(\alpha) \in C^1(R_+, Z).$$

$$z_0 \in Z_0. \quad 1 \quad Z$$

$$z(0)$$

$$z = z_0 + M(z, 0)$$

( [3, 4]). ,  $z_0 \in Z_0$ , (24)  $Z$   
 $\alpha \geq 0$ . ,

$z(\alpha)$   
 $R_+$  , 1  $z(\alpha^2)$   
 $\alpha = 0$ .

,  $\tilde{G}(z, 0) = 0, \tilde{G} \in C^1(Z \times R_+, Z)$ ,  
 $\tilde{G}(z, \alpha) = G(z, \alpha^2)$ , ,  $\tilde{G}(z(0), 0)$  .

1.

,  $\tilde{\phi} \in \tilde{\Phi}$   
 $\phi(0) = \tilde{\phi}(0), \|\phi - \tilde{\phi}\| \leq \delta, \phi \in \Phi$  ,  $z_0 = P\phi \in Z_0$   
 $\tilde{z}_0 = P\tilde{\phi} \in \tilde{Z}_0, \|z_0 - \tilde{z}_0\| \leq \delta$ .

$$z = \tilde{z}_0 + M(z, \alpha).$$

1  $\alpha > 0$   $Z$ .

$$\tilde{z}(\alpha). \quad z = z_0 + M(z, \alpha), \alpha > 0 \quad z(\alpha).$$

,  $z(0)$  .

$$\tilde{z}(\alpha) \quad R: \tilde{Z}_0 \times R_+ \rightarrow Z, \tilde{R}(\tilde{z}_0, \alpha) = \tilde{z}(\alpha).$$

, ,

$$z = z_0 + M(z, 0).$$

**2.**  $\delta \leq \delta_0$ .

$$\alpha(\delta) \in C(0, \delta_0], \alpha > 0, \lim_{\delta \rightarrow 0} \alpha(\delta) = 0,$$

$$, \quad \lim_{\delta \rightarrow 0} \|\tilde{z}(\alpha) - z(0)\| = 0.$$

,  $R$

$Z_{0l}$   $Z_0$

$$Z_{0l} = \{z_0 \in Z_0 : l(z_0) < l\}.$$

