O’ZBEKISTON RESPUBLIKASI OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

Toshkent Moliya instituti

1-KURS SIRTQI BO’LIM TALABALARI UCHUN “IQTISODCHILAR UCHUN MATEMATIKA” FANIDAN

(O’quv-uslubiy qo’llanma)

Toshkent-2019

Matritsalar. Matritsalar ustida amallar

Tayanch so'z va iboralar: *matritsa, satr matritsa, ustun matritsa, satr-vektor, ustun-vektor, vektor komponenti, nol matritsa, teng matritsa, zanjirlangan matritsalar,kvadrat matritsaning bosh diagonali, diagonal matritsa, skalyar matritsa, birlik matritsa, transponirlangan matritsa, simmetrik matritsa, qiya simmetrik matritsa, trapetsiyasimon (zinapoyasimon) matritsa, kanonik matritsa.*

Reja

1. Kirish.
2. Matritsagad oir asosiy tushunchalar.
3. Matritsa ustida chiziqli amallar.
4. Matritsalarni ko'paytirish.

Matritsa tushunchasi va unga asoslangan matematikaning “Matritsalar algebrasi” bo'limi amaliyotda, jumladan, iqtisodiyotda katta ahamiyat kasb etadi.Bu shu bilan tushuntiriladiki, aksariyat iqtisodiy ob’ekt va jarayonlarning matematik modellari matritsalar yordamida sodda va kompakt ko'rinishida tasvirlanadi.

Matritsa tushunchasi birinchi marta ingliz matematiklari U.Gamilton (1805-1865 y.y.) va A.Kel (1821-1895 y.y.) ishlarida uchraydi. Hozirgi kunda matritsa tushunchasi tabiiy va amaliy jarayonlarni matematik modellashtirishda muhim apparat sifatida qo'llaniladi.

1-ta’rif. Matritsa deb m ta satr va n ta ustunga ega bo'lgan to'rtburchakli sonlar jadvaliga aytiladi va quyidagicha belgilanadi:

$$(a_{ij})_{mn} = \|a_{ij}\|_{mn} = \left[a_{ij}\right]_{mn} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}.$$

Bu matritsa $(m \times n)$ o‘lchamli bo‘lib, a_{ik} - haqiqiy songa (bunda birinchi indeks satr nomerini, ikkinchisi esa ustun nomerini ko‘rsatib, bularning kesishgan joyida a_{ik} element turadi, hamda $i = 1,2,3,...,m;\ k = 1,2,3,...,n)$ uning elementi deb ataladi.

Matritsalar lotin grafikasining bosh harflari bilan belgilanadi. Masalan, *A*, *B*,*C*,…. Quyida keltirilgan jadvallar matritsa bo‘la olmaydi, chunki ular to‘g‘ri to‘rtburchakli emas:

$$B = \begin{pmatrix} & 3 & \\ 2 & & 7 \\ & 8 & \end{pmatrix}, C = \begin{pmatrix} & 3 & 4 & \\ 2 & 5 & 7 & \\ & 3 & 9 & 7 \end{pmatrix}, D = \begin{pmatrix} 5 & 10 & 32 \\ & 0 & \end{pmatrix},$$

Matritsalar yordamida iqtisodiy jarayonlarning o’zaro bog’liqligini kompakt formada ifodalash qulay.

1-misol. Quyidagi jadvalda iqtisodiyotning tarmoqlari bo’yicha resurslarning taqsimlanishi berilgan bo’lsin.

Resurslar	Iqtisodiyot tarmoqlari	
	Sanoat	Qishloq xo’jaligi
Elektren ergiyasiresurslari	7.3	5.2
Mehnat resurslari	4.6	3.1
Suvresurslari	4.8	6.1

Bu resurslar taqsimotini kompakt formada matritsaviy ko’rinishda quyidagicha yozish mumkin: $A_{3x2} = \begin{pmatrix} 7.3 & 5.1 \\ 4.6 & 3.1 \\ 4.8 & 6.1 \end{pmatrix}$.

2-misol. Ishlab chiqarishni tashkil etish va rejalashtirish masalasi

Faraz qilaylik, korxonada *n* xil mahsulot ishlab chiqarilsin; ulardan ixtiyoriy birini *i (i=1,…,n)*bilan belgilaymiz. Bu mahsulotlarni ishlab chiqarish uchun *m* xil ishlab chiqarish faktorlari zarur bo'lsin.Ulardan ixtiyoriy birini*j* (*j=1,…,m*) bilan belgilaymiz. Har bir ishlab chiqarish faktorining umumiy miqdori va bir birlik mahsulotni ishlab chiqarish uchun sarf qilinadigan normasi quyidagi jadvalda berilgan.

Mahsulot / Ish./ch. turla / Factorlari	1	2	…	m	Ish./factor lari zaxirasi
1	a_{11}	a_{12}	…	a_{1m}	b_1
2	a_{21}	a_{22}	…	a_{2m}	b_2
…	…	…	…		
n	a_{n1}	a_{n2}		a_{n4}	b_n
Daromad (sh.p.b)	c_1	c_2		c_m	

Jadvaldagi har bir b_j –*j*-ishlab chiqarish faktorining umumiy miqdori (zahirasi); a_{ij} –*j*-mahsulotning bir birligini ishlab chiqarish uchun sarf qilinadigan *i*-faktorning miqdori; c_j –korxonaning *j*-mahsulotning bir birligini sotishdan oladigan daromadi.

Masalaning iqtisodiy ma'nosi: korxonaning ishini shunday rejalashtirish kerakki: a)hamma mahsulotlarni ishlab chiqarish uchun sarf qilinadigan har bir ishlab chiqarish faktorining miqdori ularning zahirasidan oshmasin; b) mahsulotlarni sotishdan korxonaning oladigan umumiydaromadi maksimal bo'lsin.

Bu ishlab chiqarishni tashkil etish va rejalashtirish masalasining texnologik matritsasini yozing.

Yechish. $A = (a_{ij})_{mn} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$

3-misol. Firma o'zining faoliyatini amalgam oshirish uchun 5 turdagi avtomobillarni ijaraga oladi va haftasiga spark uchun -500 ming so'm, neksiya uchun -600 ming so'm, lasetti uchun -700 mingso'm, orlanda uchun 900 ming so'm, kaptiva uchun -1000 ming so'm pul to'laydi. Firmaning 3 hafta davomida avtomobillarga bo'lgan talabi quyidagi jadvalda berilgan

Avtomobilmarkalari	1-hafta	2-hafta	3-hafta
spark	4	7	2
neksiya	3	5	5
lasetti	12	9	5
orlanda	2	1	3
kaptiva	1	1	2

Firmaning 3 hafta davomida avtomobillarga bo'lgan talabini ifodalovchi texnologik matritsasini yozing.

Yechish.

Firmaning 3 hafta davomida avtomobillarga bo'lgan ehtiyojini quyidagi matrisa yordamida kompakt ko'rinishda yozish mumkin:

$$A = \begin{pmatrix} 4 & 7 & 2 \\ 3 & 5 & 5 \\ 12 & 9 & 5 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{pmatrix}$$

Masqlarni bajaring.

1)Firma bir kvartal davomida uch xildagi xom-ashyoni ishlatib uch turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish xarakteristikalari 1-jadvalda berilgan.

1-jadval

Xom ashyo turlari	Mahsulot turlari bo'yicha xom ashyo sarflari		
	1	2	3
S_1	285	523	264
S_2	242	361	251
S_3	353	622	252

Firmaning bir oy davomida xom ashyoga bo'lgan talabini ifodalovchi texnologik matritsani yozing.

2) Firma bir kvartal davomida uch xildagi xom-ashyoni ishlatib uch turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish xarakteristikalari 2-jadvalda berilgan.

2-jadval

Xom ashyo turlari	Mahsulot turlari bo'yicha xom-ashyo sarflari		
	1	2	3
S_1	122	142	127
S_2	172	72	72
S_3	214	321	35

Firmaning bir oy davomida xom-ashyoga bo'lgan talabini ifodalovchi texnologik matritsani yozing.

3) Firma bir kvartal davomida uch xildagi xom-ashyoni ishlatib uch turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish xarakteristikalari 3-jadvalda berilgan.

3-jadval

Xom-ashyo turlari	Mahsulot turlari bo'yicha xom-ashyo sarflari			Xom-ashyo zahirasi
	1	2	3	
S_1	332	254	136	565
S_2	221	332	125	136
S_3	265	245	334	119

Firmaning bir oy davomida xom-ashyoga bo'lgan talabini ifodalovchi texnologik matritsani yozing.

(1x*n*) o'lchamli matritsaga **satr matritsa**, (*m*x1) o'lchamli matritsaga esa**ustun matritsa** deyiladi, ya'ni

$$K = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \end{pmatrix},\ L = \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix}$$

Bundan tashqari ba'zida bu matritsalar mos ravishda **satr-vektor** va **ustun-vektor** deb ham ataladi. Vektorlarning elementlari esa ularning **komponentlari** deyiladi.

Har bir elementi nolga teng bo'lgan, ixtiyoriy o'lchamli matritsaga **nol matritsa** deb aytiladi va quyidagi ko'rinishda bo'ladi:

$$\theta = \begin{pmatrix} 0 & 0 & ... & 0 \\ 0 & 0 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & 0 \end{pmatrix}$$

2-ta'rif. Agar A va B matritsalarning barcha mos elementlari o'zaro teng bo'lsa, bunday matritsalar **tengmatritsalar** deyiladi va $A=B$ ko'rinishda yoziladi.

3-ta'rif. Agar A matritsaning ustunlar soni B matritsaning satrlari soniga teng bo'lsa, A matritsa B matritsa bilan **zanjirlangan matritsa** deyiladi.

4-misol. $A=\begin{pmatrix} 2 & 3 & 4 \\ 4 & 5 & 2 \\ 9 & 8 & 2 \end{pmatrix}$ va $B=\begin{pmatrix} 5 & 8 \\ 1 & 4 \\ 4 & 3 \end{pmatrix}$ matritsalar zanjirlangan matritsalar bo'ladi.

4-ta'rif. Ham satrlar soni,ham ustunlar soni n ga teng bo'lgan, ya'ni($n\times n$) o'lchamli matritsaga n - tartibli **kvadrat matritsa** deyiladi.

5-misol. $A=\begin{pmatrix} 1 & 8 & 6 & 1 \\ -2 & 5 & 7 & 3 \\ 1 & 0 & 11 & 15 \\ 0 & 5 & 3 & -9 \end{pmatrix}$ 4-tartibli kvadrat matrisaga misol bo'ladi.

Kvadrat matritsaning **bosh diagonali** deb, indekslari bir xil bo'lgan elementlarni xayolan birlashtirish natijasida hosil bo'lgan to'g'ri chiziqqa aytiladi. Boshqacha aytganda, $a_{11}\quad a_{22}\quad ...\quad a_{nn}$ elementlarga diagonal elementlari deyiladi.

Agar $A=(a_{ij})$ kvadrat matritsada $i>j$ $(i<j)$munosabat bajarilganda $a_{ij}=0$ bo'lsa, u holda A matritsaga yuqori (quyi)***uchburchak matritsa*** deyiladi.

6-misol. $A=\begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ 0 & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ 0 & 0 & ... & a_{nn} \end{pmatrix}$ -yuqori $B=\begin{pmatrix} a_{11} & 0 & ... & 0 \\ a_{21} & a_{22} & ... & 0 \\ ... & ... & ... & ... \\ a_{n1} & a_{n2} & ... & a_{nn} \end{pmatrix}$ -quyi uchburchakli matritsa bo'ladi.

$A=(a_{ij})$ kvadrat matritsada $i\neq j$ bo'lganda, $a_{ij}=0$ bo'lsa, u holda A matritsaga ***diagonal matritsa***deyiladi.

$$A = \begin{pmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{nn} \end{pmatrix}$$

Agar diagonal matritsaning barcha a_{ii} elementlari o‘zaro teng bo‘lsa, u holda bunday matritsaga skalyar matritsa deyiladi, ya’ni

$$A = \begin{pmatrix} a & 0 & \dots & 0 \\ 0 & a & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a \end{pmatrix}$$

Agar skalyar matritsada $a = 1$ bo‘lsa, u holdabunday matritsaga ***birlik matritsa*** deyiladi va odatda E harfi bilan belgilanadi, ya’ni

$$E = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

Agar A matritsada barcha satrlar mos ustunlar bilan almashtirilsa, u holda hosil bo‘lgan A^T matritsaga A matritsaga **transponirlangan matritsa** deyiladi.

7-misol. $A = \begin{pmatrix} 2 & -1 \\ 3 & 4 \\ 5 & 0 \end{pmatrix}$ $\quad A^T = \begin{pmatrix} 2 & 3 & 5 \\ -1 & 4 & 0 \end{pmatrix}$

Agar A kvadrat matritsada $A = A^T$ munosabat o‘rinli bo‘lsa, u holda bunday matritsaga **simmetrik matritsa**deb ataladi.

8-misol.

$$A = \begin{pmatrix} 4 & 5 & 2 \\ 5 & 8 & 3 \\ 2 & 3 & 7 \end{pmatrix}$$

Misoldan ko‘rinib turibdiki matritsaning elementlari bosh diagonalga nisbatan simmetrik joylashgan.

n–tartibli simmetrik matritsaning turli elementlari soni ko‘pi bilan $\frac{n(n+1)}{2}$ ga teng, bunda n-natural son.

Agar A kvadrat matritsada $A=-A^T$ munosabat o‘rinli bo‘lsa, boshqacha aytganda bosh diagonalga nisbatan simmetrik bo‘lgan elementlari bir-biridan faqat ishoralari bilangina farqlansa, ya’ni barcha i va j larda $a_{ij}=-a_{ji}$ bo‘lsa (bundan barcha i lar uchun $a_{ii}=-a_{ii}=0$ ekanligi kelib chiqadi), bunday matritsaga qiya **simmetrik** matritsa deb ataladi.

$$A=\begin{pmatrix} 0 & 5 & -2 \\ -5 & 0 & 3 \\ 2 & -3 & 0 \end{pmatrix}$$

n–tartibli qiya simmetrik matritsaning turli elementlari soni ko‘pi bilan n^2-n+1 formula yordamida topiladi, bunda n-natural son.

O‘lchamlari aynan teng bo‘lgan matritsalar ustidagina algebraik qo‘shish amali bajariladi.

O‘lchamlari aynan teng bo‘lgan A va B matritsalarni qo‘shish uchun, ularning mos elementlari qo‘shiladi:

$$A+B=C=\begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \dots & a_{1j}+b_{1j} & \dots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \dots & a_{2j}+b_{2j} & \dots & a_{2n}+b_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{i1}+b_{i1} & a_{i2}+b_{i2} & \dots & a_{ij}+b_{ij} & \dots & a_{in}+b_{in} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \dots & a_{mj}+b_{mj} & \dots & a_{mn}+b_{mn} \end{pmatrix}$$

Shuningdek, ikkita matritsa ayirmasi, ya’ni A - B = C ham xuddi shunday topiladi:

$$A-B=C=\begin{pmatrix} a_{11}-b_{11} & a_{12}-b_{12} & \dots & a_{1j}-b_{1j} & \dots & a_{1n}-b_{1n} \\ a_{21}-b_{21} & a_{22}-b_{22} & \dots & a_{2j}-b_{2j} & \dots & a_{2n}-b_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{i1}-b_{i1} & a_{i2}-b_{i2} & \dots & a_{ij}-b_{ij} & \dots & a_{in}-b_{in} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{m1}-b_{m1} & a_{m2}-b_{m2} & \dots & a_{mj}-b_{mj} & \dots & a_{mn}-b_{mn} \end{pmatrix}$$

9-misol. Quyidagi matritsalarning yig‘indisi va ayirmasini toping:

$$A=\begin{pmatrix} 3 & 1 & 0 & 2 \\ 1 & 4 & 3 & 1 \end{pmatrix}, B=\begin{pmatrix} 4 & -1 & 2 & -2 \\ -3 & 0 & 4 & 0 \end{pmatrix}$$

Yechish. Ta’rifga asosan

$$A+B=\begin{pmatrix} 3+4 & 1-1 & 0+2 & 2-2 \\ 1-3 & 4+0 & 3+4 & 1+0 \end{pmatrix}=\begin{pmatrix} 7 & 0 & 2 & 0 \\ -2 & 4 & 7 & 1 \end{pmatrix};$$

$$A-B=\begin{pmatrix} 3-4 & 1+1 & 0-2 & 2+2 \\ 1+3 & 4-0 & 3-4 & 1-0 \end{pmatrix}=\begin{pmatrix} -1 & 2 & -2 & 4 \\ 4 & 4 & -1 & 1 \end{pmatrix}.$$

10-misol. Quyidagi matritsalarning yig‘indisi va ayirmasini toping:

$$A=\begin{pmatrix} -2 & 1 & 0 & 2 \\ 3 & -6 & 3 & 6 \end{pmatrix}, B=\begin{pmatrix} 4 & -1 & 2 & -8 \\ -3 & 5 & 4 & 0 \end{pmatrix}$$

Yechish. $A+B=\begin{pmatrix} -2+4 & 1-1 & 0+2 & 2-8 \\ 3-3 & -6+5 & 3+4 & 6+0 \end{pmatrix}=\begin{pmatrix} 2 & 0 & 2 & -6 \\ 0 & -1 & 7 & 6 \end{pmatrix}$

$$A-B=\begin{pmatrix} -2-4 & 1+1 & 0-2 & 2+8 \\ 3+3 & -6-5 & 3-4 & 6-0 \end{pmatrix}=\begin{pmatrix} -6 & 2 & -2 & 10 \\ 6 & -11 & -1 & 6 \end{pmatrix}$$

Quyidagi mashqlarni bajaring.

11-misol. Quyidagi matritsalarning yig‘indi va ayirmasini toping:

$$A=\begin{pmatrix}1 & -3 & 2\\ -5 & 3 & 3\\ 2 & -3 & 7\end{pmatrix},\ B=\begin{pmatrix}5 & -7 & 2\\ 5 & 9 & 3\\ -2 & -3 & 2\end{pmatrix}$$

12-misol. Quyidagi matritsalarning yig‘indi va ayirmasini toping:

$$A=\begin{pmatrix}2 & 6 & -26 & 9\\ 8 & -14 & 1 & 14\\ -9 & -14 & -15 & 23\\ 10 & 25 & 24 & 32\end{pmatrix},\ B=\begin{pmatrix}14 & -12 & -18 & 21\\ 15 & 41 & 20 & -56\\ 10 & 15 & -35 & 32\\ 16 & 18 & 14 & 81\end{pmatrix}$$

13-misol. Quyidagi matritsalarning yig‘indi va ayirmasini toping:

$$A=\begin{pmatrix}1.5 & 3.5 & -5.6 & 3.2 & 2.5\\ -12 & -48 & 25 & 12 & 3.9\\ 2.4 & -9.3 & -3.4 & 10 & 2.5\end{pmatrix},\ B=\begin{pmatrix}5 & 3.5 & 6 & 3.2 & 2.5\\ -1.2 & 8 & 25 & 1.2 & 9\\ 2.4 & 3 & -4 & 10 & 2.5\end{pmatrix}.$$

Biror haqiqiy λ sonni matritsaga ko‘paytirish uchun shu son matritsaning har bir elementiga ko‘paytiriladi: $\lambda(a_{in})=(\lambda a_{in})$.

$$\lambda A=\begin{pmatrix}\lambda a_{11} & \lambda a_{12} & \dots & \lambda a_{1j} & \dots & \lambda a_{1n}\\ \lambda a_{21} & \lambda a_{22} & \dots & \lambda a_{2j} & \dots & \lambda a_{2n}\\ \dots & \dots & \dots & \dots & \dots & \dots\\ \lambda a_{i1} & \lambda a_{i2} & \dots & \lambda a_{ij} & \dots & \lambda a_{in}\\ \dots & \dots & \dots & \dots & \dots & \dots\\ \lambda a_{m1} & \lambda a_{m2} & \dots & \lambda a_{mj} & \dots & \lambda a_{mn}\end{pmatrix}$$

14-misol. Quyidagi A matritsani $\lambda=2$ soniga ko’paytiring

$$A=\begin{pmatrix}2 & 3\\ 8 & 2\\ 7 & 6\end{pmatrix}$$

Yechish.

$$\lambda A = 2 \cdot \begin{pmatrix} 2 & 3 \\ 8 & 2 \\ 7 & 6 \end{pmatrix} = \begin{pmatrix} 2\cdot 2 & 2\cdot 3 \\ 2\cdot 8 & 2\cdot 2 \\ 2\cdot 7 & 2\cdot 6 \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 16 & 4 \\ 14 & 12 \end{pmatrix}$$

15-misol. Quyidagi A matritsani $\lambda = 3$sonigako'paytiring

$$A = \begin{pmatrix} 1.5 & 3.5 & -5.6 & 3.2 & 2.5 \\ -12 & -48 & 25 & 12 & 3.9 \\ 2.4 & -9.3 & -3.4 & 10 & 2.5 \end{pmatrix}$$

Yechish.

$$\lambda \cdot A = 3 \cdot \begin{pmatrix} 1.5 & 3.5 & -5.6 & 3.2 & 2.5 \\ -12 & -48 & 25 & 12 & 3.9 \\ 2.4 & -9.3 & -3.4 & 10 & 2.5 \end{pmatrix} =$$

$$= \begin{pmatrix} 3\cdot 1.5 & 3\cdot 3.5 & 3.(-5.6) & 3\cdot 3.2 & 3\cdot 2.5 \\ 3\cdot(-12) & 3\cdot(-48) & 3\cdot 25 & 3\cdot 12 & 3\cdot 3.9 \\ 3\cdot 2.4 & 3\cdot(-9.3) & 3\cdot(-3.4) & 3\cdot 10 & 3\cdot 2.5 \end{pmatrix} =$$

$$= \begin{pmatrix} 4.5 & 10.5 & -16.8 & 9.6 & 7.5 \\ -36 & -144 & 75 & 36 & 11.7 \\ 7.2 & -27.9 & -10.2 & 30 & 7.5 \end{pmatrix}$$

16-misol. Quyidagi A matritsani $\lambda = 4$sonigako'paytiring

$$A = \begin{pmatrix} 5 & 3.5 & -5.2 & 3.9 & 6.5 \\ 2 & 8.3 & 25 & 2 & 3.2 \\ 3.4 & -2.3 & -3.4 & 12 & 2.4 \end{pmatrix}$$

17-misol. Quyidagi A matritsani $\lambda = -2$sonigako'paytiring

$$A = \begin{pmatrix} 2 & 6 & -26 & 9 \\ 8 & -14 & 1 & 14 \\ -9 & -14 & -15 & 23 \\ 10 & 25 & 24 & 32 \end{pmatrix}$$

18-misol. Quyidagi A matritsani $\lambda = 5$sonigako'paytiring

$$A = \begin{pmatrix} 14 & -12 & -18 & 21 \\ 15 & 41 & 20 & -56 \\ 10 & 15 & -35 & 32 \\ 16 & 18 & 14 & 81 \end{pmatrix}$$

19-misol. Firma 5 turdagi maxsulotni ikkita sexda ishlab chiqaradi.Firmaning bir oyda ishlab chiqargan maxsulotlari taqsimoti quyidagi jadvalda berilgan;

Maxsulot turlari	1	2	3	4	5
1-sexda bir oyda ish./chiqan maxsulotlar miqdori	139	160	205	340	430
2-sexda bir oyda ish./chiqan maxsulotlar miqdori	122	130	145	162	152

Firma ishlab chiqarish uskunalarini yangilash natijasida ishlab chiqarishni 17% ga oshirdi.Firma ishlab chiqarish uskunalarini yangilagandan keyin,firmaning bir oyda ishlab chiqargan maxsulotlari taqsimoti qanday bo'ladi?

Yechish. Firmaning ishlab chiqarish uskunalarini yangilamasdan oldingi ishlab chiqargan maxsulotlari taqsimotini quyidagi satr matritsa ko'rinishda yozish mumkin

$$P = \begin{pmatrix} 139 & 160 & 205 & 340 & 430 \\ 122 & 130 & 145 & 162 & 152 \end{pmatrix}$$

Firma ishlab chiqarish uskunalarini yangilagandan keyin, firmaning bir oyda ishlab chiqargan maxsulotlari taqsimotini topish uchun, bu ishlab chiqarish matritsasini 1.17 ga ko'paytirish zarur bo'ladi

$$1.17\cdot P = 1.17\cdot\begin{pmatrix}139 & 160 & 205 & 340 & 430\\ 122 & 130 & 145 & 162 & 152\end{pmatrix} =$$

$$= \begin{pmatrix}162.63 & 187.2 & 239.85 & 397.8 & 503.1\\ 142.74 & 152.1 & 169.65 & 189.54 & 177.84\end{pmatrix}$$

Matritsalarni qo'shish, ayirish, ya'ni algebraik qo'shish va matritsani songa ko'paytirish amallariga matritsalar ustida ***chiziqli amallar*** deyiladi.

Matritsalarni qo'shish va songa ko'paytirish amallari quyidagi xossalarga bo'ysinadi:

1) $A + B = B + A$
2) $A + (B + C) = (A + B) + C$
3) $k(A + B) = kA + kB$
4) $k(nA) = (kn)A$
5) $(k + n)A = kA + nA$

Faqat va faqat zanjirlangan matritsalar ustida ko'paytirish amali bajariladi. Zanjirlangan, boshqacha aytganda $m\times p$ o'lchamli $A = (a_{ij})$ matritsaning $p\times n$ o'lchamli $B = (b_{jk})$ matritsaga ***ko'paytmasi*** deb, elementlari $c_{ik} = a_{i1}b_{1k} + a_{i2}b_{2k} + ... + a_{ip}b_{pk}$ kabi aniqlanadigan $m\times n$ o'lchamli $C = (c_{ij})$ matritsaga aytiladi.

Masalan, bizga umumiy holda $A = \begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\\ a_{31} & a_{32}\end{pmatrix}$ va $B = \begin{pmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{pmatrix}$ ko'rinishdagi matritsalar berilgan bo'lsin. Bu matritsalarni ko'paytirish quyidagicha amalga oshiriladi:

$$AB=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}=\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \\ a_{31}b_{11}+a_{32}b_{21} & a_{31}b_{12}+a_{32}b_{22} \end{pmatrix}.$$

Endi buni aniq misolda k o‘rib chiqamiz.

20-misol. Quyidagi A matritsani B matritsaga ko‘paytiring:

$$A=\begin{pmatrix} 3 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix}, B=\begin{pmatrix} 1 & 1 & -1 \\ 2 & -1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

Yechish. 1.Izlanayotgan $C=AB$ matritsaning birinchi satr va birinchi ustunining elementi A matritsaning birinchi satr elementlarini mosravishda $\boldsymbol{B}$ matritsaning birinchi ustun elementlari bilan ko‘payganini yig‘indisiga teng, ya’ni

$$\boldsymbol{c}_{11}=3\cdot 1+1\cdot 2+1\cdot 1=6.$$

2. Izlanayotgan $C=AB$ matritsaning birinchi satr va ikkinchi ustunining elementi A matritsaning birinchi satr elementlarini B matritsaning ikkinchi ustun elementlari bilan mos ravishda ko‘payganini yig‘indisiga teng:

$$\boldsymbol{c}_{12}=3\cdot 1+1\cdot(-1)+1\cdot 0=2.$$

3. Birinchi satr va uchinchi ustun elementi

$\boldsymbol{c}_{13}=3\cdot(-1)+1\cdot 1+1\cdot 1=-1$ kabi aniqlanadi.

4. Izlanayotgan matritsaning ikinchi satr elementlari A matritsaning ikkinchi satr elementlarini B matritsaning mos ravishda 1-,2-,3- ustun elementlari bilan ko‘paytmasini yig‘indisi sifatida topiladi:

$$\begin{aligned} c_{21} &= 2\cdot 1+1\cdot 2+2\cdot 1=6; \\ c_{22} &= 2\cdot 1+1\cdot(-1)+2\cdot 0=1; \\ c_{23} &= 2\cdot(-1)+1\cdot 1+2\cdot 1=1; \end{aligned}$$

5. C matritsaning uchinchi satr elementlari ham shunga o‘xshash topiladi:

$$c_{31} = 1 \cdot 1 + 2 \cdot 2 + 3 \cdot 1 = 8;$$
$$c_{32} = 1 \cdot 1 + 2 \cdot (-1) + 3 \cdot 0 = -1;$$
$$c_{33} = 1 \cdot (-1) + 2 \cdot 1 + 3 \cdot 1 = 4.$$

Shunday qilib,

$$C = AB = \begin{pmatrix} 6 & 2 & -1 \\ 6 & 1 & 1 \\ 8 & -1 & 4 \end{pmatrix}.$$

Agar A va B bir xil tartibli kvadrat matritsalar bo‘lsa, AB va BA ko‘paytmalarini topish mumkin. Agar $AB = BA$tenglik o‘rinli bo‘lsa, uholda A va B matritsalarga ***kommutativ matritsalar*** deyiladi. Masalan, E birlik matritsa ixtiyoriy A kvadrat matritsa bilan kommutativdir. Haqiqatan ham

$$AE = EA = A.$$

Aytaylik bizga $A = \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}$ va $B = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$ matritsalar berilgan bo‘lsin.

Bu matritsalar zanjirlangan bo‘lganligi sababli ular ustida ko‘paytirish amali bajariladi.

$$AB = \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = 1 + 4 + 9 + 16 = 30$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{pmatrix}$$

Keltirilgan misoldan ko‘rinib turibdiki, A va B matritsalarning ko‘paytmasi o‘rin almashtirish qonuniga bo‘ysunmaydi, ya’ni $AB \neq BA$.

Matritsalarning ko`paytmasi ko`paytuvchi matritsalar nolmas bo`lishiga qaramasdan, nol matritsani berishi xam mumkin.

Misol. $\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

A va B matritsalarning ko`paytmasi har doim o`rin almashtirish qonuniga bo`ysunavermaydi, ya'ni umuman olganda $AB \neq BA$. $AB = BA$ tenglikni qanoatlantiruvchi $A$ va $B$ matritsalarga **o`rin almashinuvchi** matritsalar deyiladi.

Misol. $\begin{pmatrix} 2 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 2 & 2 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 8 & 14 \\ 14 & 22 \end{pmatrix}$

Chiziqli algebra elementlarini iqtisodiyotda qo'llash

Juda ko'p iqtisodiy masalalarni echishda matritsalar algebrasidan foydalaniladi. Matritsalar chiziqli algebraning asosiy quroli hisoblanadi. Matritsa matematik tushuncha sifatida 19-asrning o'rtalarida irlandiyalik olim U.Gamel'ton, ingliz olimi A.Keli va Dj. Sil'vestrlarning ilmiy ishlarida uchraydi. Matritsalar nazariyasining asoslari nemis olimlari K. Veyershtras va G. Frobeniuslar tomonidan 19- asrning oxiri 20-asrning boshlarida yaratilgan. Aksariyat iqtisodiy ob'ekt va jarayonlarning matematik modellari matritsalar yordamida sodda va kompakt ko'rinishda tasvirlanadi. Hozirgi kunda matritsalar tabiiy va amaliy jarayonlarning matematik modellarini tuzishda muhim apparat sifatida qo'llanilmoqda. Ma'lumotlar asosan matritsa ko'rinishida saqlanadi va qayta ishlanadi. Shuning uchun ma'lumotlar bazasi bilan ishlash aktual masalalardan hisoblanadi. Matritsalar algebrasining ba'zi tadbiqlarini quyidagi masalada ko'rishimiz mumkin.

1. Beshta korxona to'rt xil mahsulot ishlab chiqaradi. Bu mahsulotlarni ishlab chiqarish uchun uch turdagi xom-ashyodan foydalaniladi. Korxonalarning bir yildagi ish kunlari va har bir xom- ashyoning bahosi haqidagi ma'lumotlar quyidagi jadvalda keltirilgan.

Mahsulot turi	Korxonalarning mehnat unumdorligi (bir kunda ishlab chiqarilgan mahsulot miqdori)					Xom-ashyo sarfi (bir birlik mahsulot uchun)		
	1	2	3	4	5	1	2	3

1	4	5	3	6	7	2	3	4
2	0	2	4	3	0	3	5	6
3	8	15	0	4	6	4	4	5
4	3	10	7	5	4	5	8	6
1 yildagi ish kunlari soni						Xom-ashyo bahosi		
	1	2	3	4	5	1	2	3
	200	150	170	120	140	40	50	60

Berilgan malumotlar asosida quyidagilarni toping.

1) Korxonalarning har bir mahsulot turi bo'icha bir yillik mehnat unumdorligi.

2) Korxonalarning har bir turdagi xom-ashyodan bir yillik talabi.

3) Rejada ko'rsatilgan mahsulot miqdorini ishlab chiqarish uchun har bir korxona bir yilda barcha turdagi xom-ashyoga qancha mablag' sarflaydi ?

Yechimi:

1) Iqtisodiy jarayonning bizni qiziqtirgan barcha savollariga javob berish uchun kerak bo'ladigangan matritsalarni tuzamiz va ular ustida amallar orqali o'zimizga kerak bo'lgan savollarga javob olamiz. Birinchi navbatda korxonalarning mehnat unumdorligini tavsiflovchi A matritsani tuzamiz.

$$A = \begin{pmatrix} 4 & 5 & 3 & 6 & 7 \\ 0 & 2 & 4 & 3 & 0 \\ 8 & 15 & 0 & 4 & 6 \\ 3 & 10 & 7 & 5 & 4 \end{pmatrix}$$

Matritsadagi ustunlar malum bir korxonaning har bir mahsulot turi bo'icha bir kunlik ishlab chiqarish miqdorini ifodalaydi. Bundan, j-korxonaning har bir mahsulot turi bo'yicha bir yilda ishlab chiqargan mahsulot miqdorini topish uchun, A matritsaning j – ustunini shu korxonaning bir yillik ish kunlari soniga ko'paytirish kerak.

$$A_{yillik} = \begin{pmatrix} 800 & 750 & 510 & 720 & 980 \\ 0 & 300 & 680 & 360 & 0 \\ 1600 & 2250 & 0 & 480 & 140 \\ 600 & 1500 & 1190 & 600 & 560 \end{pmatrix}$$

Xom-ashyo sarfi.

2) Mahsulot turi

$$B=\begin{pmatrix} 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 8 \\ 4 & 6 & 5 & 6 \end{pmatrix}$$

Bir kunlik xom- ashyo sarfi $B\times A=\begin{pmatrix} 55 & 126 & 53 & 62 & 58 \\ 68 & 165 & 85 & 89 & 77 \\ 74 & 167 & 78 & 92 & 82 \end{pmatrix}$

3) Bir yillik xom-ashyo sarfini aniqlash uchun j – ustunni j– korxonaning bir yildagi ish kunlari soniga ko'paytirish kerak.

$$B\times A_{yillik} = \begin{pmatrix} 11000 & 18900 & 9010 & 7440 & 8120 \\ 13600 & 24750 & 14450 & 10680 & 10780 \\ 14800 & 25050 & 13260 & 11040 & 11480 \end{pmatrix}$$

Baho vektorini kiritamiz $\vec{r}$ (40,50,60). U holda korxonalarning uch tudagi xom-ashyoga sarflagan bir yillik mablag'ini topish uchun

R=$\vec{r}$ x(B × A_{yillik}) = (2008000 3496500 1878500 1494000 15526000)

Vektorning har bir koordinatasi mos korxonaning bir yillik xom-ashyoga sarf qiladigan mablag'ini ifodalaydi.

2.Korxona bir kunda to'rt turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish haqidagi ma'lumotlar quyidagi 1-jadvalda keltirilgan.

1-jadval

Mahsulot turi	Ishlabchiqar ishmiqdori, dona	Xom-ashyo sarfi,kg	Ishlab chiqarishga ketgan vaqt ,mah/soat	Mahsulot bahosi
1	20	5	10	30
2	50	2	5	15

3	30	7	15	45
4	40	4	8	20

Quyidagi bir kunlik ko'rsatkichlar: S-xom-ashyo sarfi ,T-sarflangan ish vaqti, hamda P-ishlab chiqarilgan mahsulot bahosini topish talab etiladi.

Yechimi: jadval asosida ishlab chiqarish jarayonini aks ettiradigan to'rtta vektorni tuzib olamiz.

$\vec{q}$= (20,50,30,40)-ishlab chiqarilgan mahsulot hajmi

$\vec{s}$= (5,2,7,4) – xom-ashyo sarfi

$\vec{t}$= (10,5,15,8) – ish vaqti sarfi

$\vec{p}$= (30,15,45,20) – baho vektori

U holda izlanayotgan S, T, P miqdorlar $\vec{q}$ vektorni $\vec{s},\ \vec{t},\ \vec{p}$ vektorlar bilan skalyar ko'paytmasidan iborat.

S=$\vec{q}\vec{s}$= 100 + 100 +210 +160 = 570 kg

T=$\vec{q}\vec{t}$ = 1220 soat,

P=$\vec{q}\vec{t}$=3500 (pul birligi)

2. Korxona to'rt xil xom-ashyodan foydalangan holda 4 turdagi mahsulot ishlab chiqaradi. Xom-ashyo sarfi quyidagi matritsa ko'rinishda berilgan.

xom-ashyoturi

$$A=\begin{array}{c}\begin{pmatrix} 2 & 3 & 4 & 5 \\ 1 & 2 & 5 & 6 \\ 7 & 2 & 3 & 2 \\ 4 & 5 & 6 & 8 \end{pmatrix}\end{array}\begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \end{array}\text{mahsulot turi}$$

Berilgan ushbu $\vec{q}$ = (60,50,35,40) ishlab chiqarish rejasi asosida har bir mahsulot turi bo'icha xom-ashyo sarfini hisoblang.

Echimi:$\vec{q}$= (60,50,35,40)

Har bir mahsulot turi bo'yicha xom-ashyo sarfi

$\vec{q}$×A=(60,50,35,40) $\begin{pmatrix} 2 & 3 & 4 & 5 \\ 1 & 2 & 5 & 6 \\ 7 & 2 & 3 & 2 \\ 4 & 5 & 6 & 8 \end{pmatrix}$=(120 + 50 +245 + 160   180 + 100 +70 + 200   160 + 250 + 105 + 240   300 + 300 + 70 + 320) =(575 550 835990)

3. To'rtxil turdagi mahsulot ishlab chiqarish uchun kerak bo'ladigan xom-ashyo sarfi yuqoridagi A matritsa shaklida bo'lsin. $\vec{m}$(4,6,5,8) vektor - har bir turdagi xom-ashyo tan narxi, $\vec{n}$(2,1,3,2) vektor – xom-ashyoni yetkazib berish uchun sarflangan transport xarajati. Berilganlar asosida quyidagilarni toping.

1) Xar bir mahsulot turi bo'yicha sarf bo'lgan xom-ashyo va transport xarajatini hisoblang.

2) A= $\begin{pmatrix} 2 & 3 & 4 & 5 \\ 1 & 2 & 5 & 6 \\ 7 & 2 & 3 & 2 \\ 4 & 5 & 6 & 8 \end{pmatrix}$ matritsa bilan berilgan ishlab chiqarish uchun jami xom-ashyo sarfi va transport xarajati topilsin.

Yechimi: Xom-ashyo tan narxi va uni yetkazishga ketadigan transport xarajati quyidagi matritsa ko'rinishda ifodalanadi: $C = \begin{pmatrix} 4 & 6 & 58 \\ 2 & 1 & 32 \end{pmatrix}$

U holda birinchi shart bo'yicha masalaning echimi

$A \times \ S^t$= $\begin{pmatrix} 2 & 3 & 4 & 5 \\ 1 & 2 & 5 & 6 \\ 7 & 2 & 3 & 2 \\ 4 & 5 & 6 & 8 \end{pmatrix}$ × $\begin{pmatrix} 4 & 2 \\ 6 & 1 \\ 5 & 3 \\ 8 & 2 \end{pmatrix}$ = $\begin{pmatrix} 86 & 29 \\ 89 & 31 \\ 71 & 29 \\ 140 & 47 \end{pmatrix}$(bir turdagi mahsulot uchun) ko'rinishda bo'ladi.

2)Ishlab chiqarish $\vec{q}$(60,50,35,40) reja asosida tashkil etilgan bo'lsa, sarflangan umumiy xarajat quyidagi matritsalarning ko'paytmasidan iborat bo'ladi.

$\vec{q}(A \times \ S^t)$ = (60 503540) $\begin{pmatrix} 86 & 29 \\ 89 & 31 \\ 71 & 29 \\ 140 & 47 \end{pmatrix}$ = (176956185)

Matritsani matritsaga ko‘paytirish quyidagi xossalarga bo‘ysinadi:

$$1)(kA)B = k(AB)$$
$$2)(A+B)C = AC + BC$$
$$3)A(B+C) = AB + AC$$
$$4)A(BC) = (AB)C$$

Keltirilgan xossalardan to‘rtinchisini isbotini aniq misollar yordamida keltiramiz.

Faraz qilamiz bizga $A = \begin{pmatrix} 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}$ va $C = \begin{pmatrix} 3 & 0 & 2 \\ 5 & 1 & 0 \end{pmatrix}$ matritsalar berilgan bo‘lsin:

$$a)\, AB = \begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 6 \end{pmatrix},$$

$$(AB)C = \begin{pmatrix} 7 & 6 \end{pmatrix}\begin{pmatrix} 3 & 0 & 2 \\ 5 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 51 & 6 & 14 \end{pmatrix},$$

$$b)\ BC = \begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 & 2 \\ 5 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 29 & 4 & 6 \\ 11 & 1 & 4 \end{pmatrix},$$

$$A(BC) = \begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 29 & 4 & 6 \\ 11 & 1 & 4 \end{pmatrix} = \begin{pmatrix} 51 & 6 & 14 \end{pmatrix}.$$

Xossa isbotlandi.

Matritsalar ustida bajarilgan transponirlash amali quyidagi xossalarga bo‘ysinadi:

$$1.\left(A^T\right)^T = A;$$
$$2.(\lambda A)^T = \lambda A^T;$$
$$3.(A+B)^T = A^T + B^T;$$
$$4.(AB)^T = B^T A^T.$$

Trapetsiyasimon (zinapoyasimon) matritsa deb biror satrini noldan farqli elementi k – o‘rinda hamda qolgan barcha satrlarining birinchi k ta o‘rnida nollar turgan matritsaga aytiladi. Masalan,

$$\begin{pmatrix} 3 & 4 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 3 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Kanonik matritsa deb uning bosh diagonal elementlarining dastlabki bir nechtasi birlar qolgan elementlari nollardan iborat matritsaga aytiladi. Masalan,

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Amaliy mashg`ulot uchun masalalar va ularni yechishga tavsiyalar

Matritsalar ustida quyidagi amallarni bajarish mumkin:

21-misol . Amallarni bajaring

a) $A = \begin{pmatrix} 2 & 4 & 1 \\ -1 & 0 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ bo‘lsa, $A + B$ matritsani toping.

$$A + B = \begin{pmatrix} 2+0 & 4+2 & 1+1 \\ -1+1 & 0+1 & 2+2 \end{pmatrix} = \begin{pmatrix} 2 & 6 & 2 \\ 0 & 1 & 4 \end{pmatrix}$$

b) $A = \begin{pmatrix} 7 & -12 \\ -4 & 7 \end{pmatrix}, B = \begin{pmatrix} 26 & 45 \\ 15 & 26 \end{pmatrix}$ $A \cdot B$ bo‘lsamatritsani toping.

$$A \cdot B = \begin{pmatrix} 7 \cdot 26 + (-12) \cdot 15 & 7 \cdot 45 + (-12) \cdot 26 \\ -4 \cdot 26 + 7 \cdot 15 & -4 \cdot 45 + 7 \cdot 26 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$$

22-misol. Kopxona 3 xil mahsulot ishlab chiqarish uchun 2 xil hom-ashyudan foydalanadi. Hom-ashyo harajatlari $A = \begin{pmatrix} 2 & 3 \\ 5 & 2 \\ 1 & 4 \end{pmatrix}$ matritsa bilan berilgan.Maxsulot ishlab chiqarish rejasi $C = \begin{pmatrix} 100 & 80 & 130 \end{pmatrix}$ - satr-matritsa ko‘rinishida berilgan.Har bir hom-ashyo turning bir birligi bahosi (pul.birl.) $B=\begin{pmatrix}30\\50\end{pmatrix}$ - ustun-matritsa ko‘rinishida berilgan. Rejani bajarish uchun sarflanadigan hom-ashyo miqdorini va hom-ashyoning umumiy bahosini aniqlang:

1-usul.Har bir xom-ashyo sarfi

$$S=C\cdot A=\begin{pmatrix}100 & 80 & 130\end{pmatrix}\cdot\begin{pmatrix}2 & 3\\5 & 2\\1 & 4\end{pmatrix}=\begin{pmatrix}730 & 980\end{pmatrix}$$

bo‘lsa, hom-ashyoning umumiy bahosi

$$Q=S\cdot B=(C\cdot A)\cdot B=\begin{pmatrix}730 & 980\end{pmatrix}\cdot\begin{pmatrix}30\\50\end{pmatrix}=\begin{pmatrix}70900\end{pmatrix}$$

bo‘ladi.

2-usul.Avval har bir mahsulot turiga sarflanuvchi hom-ashyo miqdori

$$R=A\cdot B=\begin{pmatrix}2 & 3\\5 & 2\\1 & 4\end{pmatrix}\cdot\begin{pmatrix}30\\50\end{pmatrix}=\begin{pmatrix}210\\250\\230\end{pmatrix}$$

So‘ngra, hom-ashyoning umumiy bahosini aniqlaymiz

$$Q=C\cdot R=\begin{pmatrix}100 & 80 & 130\end{pmatrix}\cdot\begin{pmatrix}210\\250\\230\end{pmatrix}=\begin{pmatrix}70900\end{pmatrix}$$

Berilgan matritsalar ustida chiziqli amallarni bajaring:

23. $A=\begin{pmatrix}1 & 5\\2 & -4\end{pmatrix}, B=\begin{pmatrix}3 & 2\\4 & 1\end{pmatrix}, 2A-B=?$

24. $A=\begin{pmatrix} 1 & -1 & -3 \\ 2 & 1 & 5 \end{pmatrix}, B=\begin{pmatrix} 0 & 3 & 2 \\ -1 & 4 & 1 \end{pmatrix}, 3A-2B=?$

25. $\begin{pmatrix} 7 & 0 \\ 3 & 1 \\ -1 & 2 \end{pmatrix} - 3\cdot\begin{pmatrix} 2 & \sqrt{2} \\ 1 & -1 \\ -1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & \sqrt{18} \\ 4 & -5 \\ 3 & 1 \end{pmatrix}$

26. $A=\begin{pmatrix} 3 & 5 \\ 4 & 1 \end{pmatrix}, B=\begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix}, 2A+5B=?$

27. $A=\begin{pmatrix} 3 & 5 & 7 \\ 2 & -1 & 0 \\ 4 & 3 & 2 \end{pmatrix}, B=\begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & -2 \\ -1 & 0 & 1 \end{pmatrix}, A+B=?$

Berilgan matritsalar ustida talab qilingan amallani bajaring:

28. $C=\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}, F=\begin{pmatrix} 4 & -3 \\ 1 & 2 \\ 0 & 2 \end{pmatrix} C\cdot F=?$

29. $A=\begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, B=\begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}, A\cdot B=?$

30. $A=\begin{pmatrix} 1 & -1 & 2 \\ 2 & 3 & 4 \\ -4 & 5 & 1 \end{pmatrix}, B=\begin{pmatrix} 3 & 4 & 1 \\ 0 & 2 & 5 \\ 1 & -1 & 4 \end{pmatrix}, A\cdot B=?$

31. $A=\begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 5 \end{pmatrix}, C=\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, A\cdot C=?$

32. $A=\begin{pmatrix} 1 & 3 & -1 \\ 2 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix}, F=\begin{pmatrix} 1 & 1 \\ 2 & 3 \\ 1 & 0 \end{pmatrix}, A\cdot F=?$

33. $A=\begin{pmatrix} 2 & 0 & 1 \\ -2 & 3 & 2 \\ 4 & -1 & 5 \end{pmatrix}, B=\begin{pmatrix} -3 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 3 \end{pmatrix}, A\cdot B=?$

34. $\begin{pmatrix} 1 & -3 & 2 \\ 3 & -4 & 1 \\ 2 & -5 & 3 \end{pmatrix}\cdot\begin{pmatrix} 2 & 5 & 6 \\ 1 & 2 & 5 \\ 1 & 3 & 2 \end{pmatrix}=?$

35. $\begin{pmatrix} 2 & -1 & 3 & -4 \\ 3 & -2 & 4 & -3 \\ 5 & -3 & -2 & 1 \\ 3 & -3 & -1 & 2 \end{pmatrix}\begin{pmatrix} 7 & 8 & 6 & 9 \\ 5 & 7 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 2 & 1 & 1 & 2 \end{pmatrix}=?$

36. $\begin{pmatrix} 5 & 7 & -3 & -4 \\ 7 & 6 & -4 & -5 \\ 6 & 4 & -3 & -2 \\ 8 & 5 & -6 & -1 \end{pmatrix}\cdot\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \end{pmatrix}=?$

37. $A= A=\begin{pmatrix} 1 & -3 & 0 \\ 2 & 5 & 1 \end{pmatrix}, B=\begin{pmatrix} 0 & -1 & 3 \\ 3 & 5 & 2 \\ 4 & -2 & 1 \end{pmatrix}, A\cdot B=?,$

38. $\begin{pmatrix} 1 & 3 & 1 \\ 2 & 0 & 4 \\ 1 & 2 & 3 \end{pmatrix}\cdot\begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & 2 \\ 3 & 2 & 1 \end{pmatrix}=?$

39. $\begin{pmatrix} 5 & 8 & -4 \\ 6 & 9 & -5 \\ 4 & 7 & -3 \end{pmatrix}\cdot\begin{pmatrix} 3 & 2 & 5 \\ 4 & -1 & 3 \\ 9 & 6 & 5 \end{pmatrix}=?$

40. $\begin{pmatrix} 5 & 2 & -2 & 3 \\ 6 & 4 & -3 & 5 \\ 9 & 2 & -3 & 4 \\ 7 & 6 & -4 & 7 \end{pmatrix}\cdot\begin{pmatrix} 2 & 2 & 2 & 2 \\ -1 & -5 & 3 & 11 \\ 16 & 24 & 8 & -8 \\ 8 & 16 & 0 & -16 \end{pmatrix}=?$

41. $\begin{pmatrix} 1 & 1 & 1 & -1 \\ -5 & -3 & -4 & 4 \\ 5 & 1 & 4 & -3 \\ -16 & -11 & -15 & 14 \end{pmatrix} \cdot \begin{pmatrix} 7 & -2 & 3 & 4 \\ 11 & 0 & 3 & 4 \\ 5 & 4 & 3 & 0 \\ 22 & 2 & 9 & 8 \end{pmatrix} = ?$

42. $\begin{pmatrix} 83 & -29 & -52 & 46 \\ -15 & 97 & 78 & -112 \\ 38 & -4 & 69 & 85 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = ?$

43. $\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 83 & -29 & -52 & 46 \\ -15 & 97 & 78 & -112 \\ 38 & -4 & 69 & 85 \end{pmatrix} = ?$

44. $\begin{pmatrix} 5 & 2 & -3 & -3 \\ -7 & -2 & 4 & 2 \\ -1 & 2 & 1 & 1 \\ 2 & -2 & -3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = ?$

45. $\begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 5 & 2 & -3 & -3 \\ -7 & -2 & 4 & 2 \\ -1 & 2 & 1 & 1 \\ 2 & -2 & -3 & 4 \end{pmatrix} = ?$

46. $\begin{pmatrix} 923 & 2115 & 0 & 0 \\ 1097 & 518 & 0 & 0 \\ 652 & 769 & 0 & 0 \\ 841 & 134 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 476 & 372 & 1505 & 882 \\ 549 & 795 & 999 & 400 \end{pmatrix} = ?$

47.

$$A = \begin{pmatrix} 1 & 2 & -3 \\ 1 & 0 & 2 \\ 4 & 5 & 3 \end{pmatrix}, B = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 2 & 0 & 5 \end{pmatrix}, - \textit{birlik matritsa bo‘lsa}, A \cdot B \cdot C - 3E = ?$$

48. $A=\begin{pmatrix}4 & 3\\ 7 & 5\end{pmatrix}, B=\begin{pmatrix}-28 & 93\\ 38 & -126\end{pmatrix}, C=\begin{pmatrix}7 & 3\\ 2 & 1\end{pmatrix}, A\cdot B\cdot C=?$

49. $\begin{pmatrix}1 & 3\\ 2 & 0\\ 1 & -1\end{pmatrix}\cdot\begin{pmatrix}1 & -2 & 3\\ 5 & 4 & 0\end{pmatrix}+\begin{pmatrix}-10 & -9 & 7\\ 1 & 5 & 8\\ -1 & -3 & 6\end{pmatrix}=?$

50. $\begin{pmatrix}991 & 992 & 993\\ 994 & 995 & 996\\ 997 & 998 & 999\\ 1000 & 1001 & 1002\end{pmatrix}\cdot\begin{pmatrix}12 & -6 & -2\\ 18 & -9 & -3\\ 24 & -12 & -4\end{pmatrix}\cdot\begin{pmatrix}1 & 1\\ 1 & 2\\ 3 & 0\end{pmatrix}=?$

51. $\begin{pmatrix}3\\ 5\\ 7\end{pmatrix}\cdot\begin{pmatrix}213 & 510 & 128\end{pmatrix}\cdot\begin{pmatrix}3\\ -1\\ -1\end{pmatrix}\cdot\begin{pmatrix}1 & -2 & 1\end{pmatrix}=?$

52. $A=\begin{pmatrix}1 & 0 & -1\\ 2 & 3 & -2\end{pmatrix}, B=\begin{pmatrix}1 & 2\\ 1 & 0\\ 3 & 1\end{pmatrix}, AB=?, A^tB^t=?$

53. $A=\begin{pmatrix}3 & 2\\ 1 & 4\end{pmatrix}, A^2=?$

54. $A=\begin{pmatrix}21 & 21 & 2\\ 11 & 3 & 1\\ 4 & 21 & 1\end{pmatrix}, E-birlik\ matritsa \quad 2A^2+3A+5E=?$

55 $A=\begin{pmatrix}3 & 4 & 2\\ 1 & 0 & 5\end{pmatrix}, B=\begin{pmatrix}2 & 0\\ 1 & 3\\ 0 & 5\end{pmatrix}, C=\begin{pmatrix}1 & 3\\ 0 & 4\end{pmatrix}, A\cdot B-C^2=?$

56. $A=\begin{pmatrix}4 & 3\\ 2 & 1\end{pmatrix}, B=\begin{pmatrix}5 & 7\\ -1 & 2\end{pmatrix}, A^2-A\cdot B+2BA=?\ \dfrac{\partial^2\Omega}{\partial u\partial v}$

57. $A=\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}, A^2+A+E=?$

58. $A=\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}, A^{20}=?$ **59.** $\begin{pmatrix} 4 & -1 \\ 5 & 2 \end{pmatrix}^5 =?$

60. $\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}^n =?$ **60.** $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^n =?$

61. Kopxona 3 xil mahsuloti shlab chiqarish uchun 2 xil hom-ashyodan foydalanadi. Hom-ashyo harajatlari $A=\begin{pmatrix} 2 & 1 & 3 \\ 1 & 3 & 4 \end{pmatrix}$matritsa bilan berilgan.Maxsulot ishlab chiqarish rejasi $C=\begin{pmatrix} 100 \\ 200 \\ 150 \end{pmatrix}$ - satr-matritsa ko‘rinishida berilgan.Har bir hom-ashyo turning bir birligi bahosi (pul.birl.) $B=\begin{pmatrix} 10 & 15 \end{pmatrix}$- ustun-matritsa ko‘rinishida berilgan. Rejani bajarish uchun sarflanadigan hom-ashyo miqdorini va hom ashyoning umumiy bahosini aniqlang.

62. Kopxona 3 xil mahsulot ishlab chiqarish uchun 2 xil hom-ashyodan foydalanadi. Hom-ashyo harajatlari $A=\begin{pmatrix} 3 & 1 \\ 2 & 2 \\ 5 & 4 \end{pmatrix}$matritsa bilan berilgan.Maxsulot ishlab chiqarish rejasi $C=\begin{pmatrix} 150 & 120 & 100 \end{pmatrix}$ - satr-matritsa ko‘rinishida berilgan. Har bir hom-ashyo turning bir birligi bahosi (pul.birl.) $B=\begin{pmatrix} 40 \\ 60 \end{pmatrix}$- ustun-matritsa ko‘rinishida berilgan. Rejani bajarish uchun sarflanadigan hom-ashyo miqdorini va hom-ashyoning umumiy bahosini aniqlang.

63. Kopxona 4 xil mahsulot ishlab chiqarish uchun 2 xil hom-ashyodan foydalanadi. Hom-ashyo harajatlari $A=\begin{pmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 4 \\ 3 & 2 \end{pmatrix}$ matritsa bilan berilgan.Maxsulot ishlab chiqarish rejasi $C=\begin{pmatrix} 120 & 80 & 150 & 130 \end{pmatrix}$ - satr-matritsa ko‘rinishida berilgan. Har bir hom ashyo turining bir birligi bahosi (pul.birl.) $B=\begin{pmatrix} 80 \\ 60 \end{pmatrix}$ - ustun-matritsa ko‘rinishida berilgan. Rejani bajarish uchun sarflanadigan hom-ashyo miqdorini va hom-ashyoning umumiy bahosini aniqlang:

O`z – o`zini tekshirish uchun savollar:

1. Matritsa deb nimaga aytiladi?
2. Satr matritsa, ustun matritsa deb qanday matritsaga aytiladi?
3. Nol matritsa deb-chi?
4. Qanday matritsalarga o`zaro teng matritsalar deyiladi?
5. n-tartibli kvadratik matritsa deb qanday matritsaga aytiladi?
6. Kvadrat matritsaning qanday xususiy ko`rinishlarini bilasiz?
7. Aynan bir xil o`lchamli matritsalar qanday qo`shiladi?
8. Sonni matritsaga ko`paytirish amali qanday bajariladi?
9. Matritsalarni qo`shish va matritsani songa ko`paytirish amallari bo`ysunadigan xossalarni sanab o`ting.
10. Matritsa satrlarini mos ustunlari bilan almashtirish amali qanday nomlanadi?
11. O`zaro zanjirlangan matritsalar qanday ko`paytiriladi?
12. Matritsalarni ko`paytirish amali qanday xossalarga bo`ysunadi?
13. Matritsalarni ko`paytirish amali o`rin almashtirish qonuniga bo`ysunadimi?

Determinantlar nazariyasi

Tayanch so'z va iboralar: *matritsa, determinant, kvadrat matritsa, aniqlovchi,* $n-$*tartibli determinant, Sarryus qoidasi.*

Reja

1. Kirish.
2. Ikkinchi tartibli determinantlar va ularni hisoblash.
3. Uchinchi tartibli determinantlar va ularni hisoblash usullari.
4. Determinantningxossalari.
5. Minor va algebraik to'ldiruvchi.
6. Determinantlarni hisoblash (Laplas teoremasi).

A kvadrat matritsani tavsiflovchi determinant (ba'zi adabiyotlarda A kvadrat matritsaninig determinanti o'rniga unga teng kuchli bo'lgan matritsaning aniqlovchisi termini qo'llaniladi) tushunchasining kiritilishi chiziqli tenglamalar sistemasini yechish bilan chambarchas bog'liq.

1-ta'rif. $2-$tartibli kvadrat matritsaning determinanti deb $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ belgi bilan belgilanuvchi $a_{11}a_{22}-a_{21}a_{12}$ ifodaga aytiladi va

$$|A|=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}=a_{11}a_{22}-a_{12}a_{21}.$$

ko'rinishda aniqlanadi. Bu yerda, a_{ij} $-$determinantning $i-$satr $j-$ustunda joylashgan elementini ifodalaydi. $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ ifoda ikkinchi tartibli determinant deb ham ataladi.

1-misol. Quyida berilganikkinchi tartibli matritsaning determinantini hisoblang.

$$A=\begin{pmatrix} 6 & 9 \\ 10 & 8 \end{pmatrix}$$

Yechish. $|A|=\begin{vmatrix} 6 & 9 \\ 10 & 8 \end{vmatrix}=6\cdot 8-10\cdot 9=48-90=42$.

2-misol. Quyida berilganikkinchi tartibli determinant $\begin{vmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{vmatrix}$ni hisoblang.

Yechish. Yuqoridagi ta'rifga asosan:

$$\begin{vmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{vmatrix}=\cos^2\beta+\sin^2\beta=1$$

3-misol. Ikkinchi tartibli determinantlarni hisoblang:

1) $\begin{vmatrix} 3 & -4 \\ 2 & 5 \end{vmatrix}=3\cdot 5-(-4)\cdot 2=15+8=23$

2) $\begin{vmatrix} \sqrt{a} & -1 \\ a & \sqrt{a} \end{vmatrix}=\sqrt{a}\cdot\sqrt{a}-(-1)\cdot a=a+a=2a$

4-misol. Quyidagi ikkinchi tartibli diterminabtlarni hisoblang.

$a)\begin{vmatrix} 7 & 5 \\ 2 & 3 \end{vmatrix},\ b)\begin{vmatrix} tg(x) & \sin(x) \\ \sin(x) & ctg(x) \end{vmatrix},\ c)\begin{vmatrix} \ln(x) & \cos(x) \\ \cos(x) & \ln^{-1}(x) \end{vmatrix},\ d)\begin{vmatrix} -5 & 10 \\ 3 & -6 \end{vmatrix}$

2-ta'rif. Uchinchi tartibli determinant deb $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ belgi bilan belgilanuvchi $a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32}-a_{13}a_{22}a_{31}-a_{12}a_{21}a_{33}-a_{11}a_{23}a_{32}$ ifodaga aytiladi.

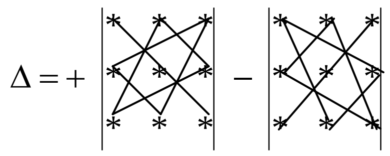
Boshqacha aytganda,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}$$

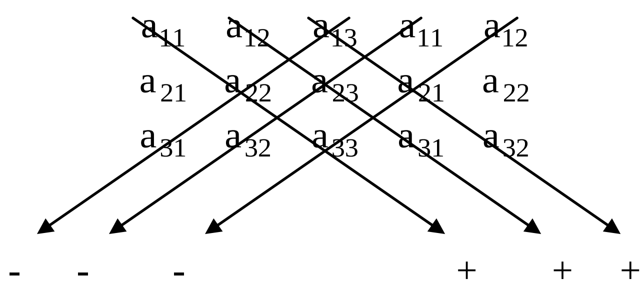
Uchinchi tartibli determinantni hisoblash usullari.

Uchinchi tartibli determinantni hisoblashning ikki xil usuli mavjud:

1.Uchburchak usuli.Uchinchi tartibli determinantni hisoblashning uchburchak usuli quyidagicha sxematik ko‘rinishda amalga oshiriladi:



2.Uchinchi tartibli determinantni hisoblashning Sarryus qoidasi.Uchinchi tartibli determinantni hisoblashning Sarryus qoidasi quyidagicha amalga oshiriladi. Determinant ustunlarining o‘ng yoniga chapdagi birinchi va ikkinchi ustunlar ko‘chirib yoziladi. Hosil bo‘lgan kengaytirilgan jadvalda bosh diagonal yo‘nalishida joylashgan elementlar ko‘paytirilib musbat ishora bilan, ikkilamchi diagonal yo‘nalishidagi elementlar ko‘paytirilib manfiy ishora bilan olinib yig‘indi tuziladi. Bu yig‘indi uchinchi tartibli determinantning qiymatidan iborat.Buni sxema ko‘rinishida quyidagicha tasvirlash mumkin:



5-misol Quyidagi uchinchi tartibli determinantni uchburchak usuli bilan hisoblang:

$$\begin{vmatrix} 2 & 1 & -1 \\ 3 & 4 & 0 \\ 5 & 1 & 2 \end{vmatrix}$$

Yechish.

$$\begin{vmatrix} 2 & 1 & -1 \\ 3 & 4 & 0 \\ 5 & 1 & 2 \end{vmatrix} = 2\cdot 4\cdot 2 + 1\cdot 0\cdot 5 + 3\cdot 1\cdot(-1) - (-1)\cdot 4\cdot 5 - 1\cdot 3\cdot 2 - 2\cdot 0\cdot 1 = 27$$

6-misol. YuqoridagiuchinchitartiblideterminantniSarryus qoidasiyordamidahisoblang:

$$\begin{vmatrix} 2 & 1 & 1 \\ 3 & 4 & 0 \\ 5 & 1 & 2 \end{vmatrix} = 2\cdot 4\cdot 2 + 1\cdot 3\cdot 1 + 1\cdot 0\cdot 5 - 1\cdot 4\cdot 5 - 1\cdot 3\cdot 2 - 2\cdot 0\cdot 1 =$$

$$= 16 + 3 + 0 - 20 - 6 - 0 = -7$$

7-misol. Quyidagi uchinchi tartibli determinantlarni uchburchak va Sarryus qoidasi yordamida hisoblang:

$$1)\ \begin{vmatrix} -1 & 2 & 3 \\ 4 & -4 & 5 \\ 7 & -7 & 8 \end{vmatrix},\ 2)\ \begin{vmatrix} 7 & 8 & 5 \\ 6 & 4 & 2 \\ 1 & 2 & 8 \end{vmatrix},\ 3)\ \begin{vmatrix} 2 & 4 & 3 \\ 2 & 5 & 9 \\ 4 & 8 & 6 \end{vmatrix}$$

Endi $n-$tartibli determinant ta'rifini beramiz.

Biz bundan oldin $n-$darajali o‘rniga qo‘yish tushunchasini kiritamiz. O'rinlashtirish sonini aniqlash uchun berilgan sonlarni o'sib borish tartibida yozilgan asosiy o'rinlashtirish bilan taqqoslab topiladi. n simvoldan iborat ikkita o‘rin almashtirishni birining ostiga ikkinchisini yozib jadval hosil qilamiz:

i	1	2	…	n
j_i	j_1	j_2	$\cdots$	j_n

Jadvaldagi j_i lar I sonidan avval o‘zidan kattalari soni. O‘rinlashtirish toq yoki juftligini $j=\sum_{i=1}^{n} j_i$ yig`indi bilan aniqlanadi. j - toq (juft) bo’lsa, o’rinlashtirish ham toq (juft) deyiladi.

8-misol . $(7,5,3,1,6,4,2)$ o’rinlashtirish sonini aniqlang.

i	1	2	3	4	5	6	7
j_i	3	5	2	3	1	1	0

$j_1=3$, chunki 1 sonidan oldin 1 dan katta sonlar 3 ta (7, 5, 3); $j_2=5$, chunki 2 sonidan oldin 2 dan katta sonlar 5 ta (7, 5, 3, 6, 4);

$$j=\sum_{i=1}^{7} j_i =3+5+2+3+1+1=15$$

Berilgan o’rinlashtirish soni 15 ga teng va o’rinlashtirish toq.

Bizga $n-$tartibli

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \qquad (1)$$

kvadrat matritsa berilgan bo‘lsin. Bu matritsaning turli satrlari va turli ustunlarida joylashgan n ta elementning mumkin bo‘lgan barcha ko‘paytmalarini, ya’ni

$$a_{1j_1}a_{2j_2}\dots a_{nj_n} \qquad (2)$$

ko‘rinishdagi ko‘paytmani qaraymiz, bu yerda $j_1, j_2, \dots, j_n$ indekslar $1,2,\dots,n$ sonlardan iborat biror o‘rin almashtirishni tashkil etadi. Bu ko‘paytmalar soni $n!$ga teng. Bu ko‘paytmalar (1) matritsaga mos keladigan kelajakda $n-$tartibli determinant deb ataluvchi ifodaning hadlari bo‘ladi.

(2) ko‘paytma $n-$tartibli determinantning tarkibiga qanday ishora bilan kirishini

aniqlash uchun bu ko‘paytmalarning indekslaridan o‘rniga qo‘yish tuzish mumkinligiga e’tibor beramiz,bu yerda agar (2) ko‘paytma tarkibiga $i-$satr va α_i-ustunda turgan element kirsa, i simvol α_i ga o‘tadi.Ikkinchi va uchinchi tartibli determinantlardagi kabi $n-$tartibli determinant ifodasiga indekslari juft o‘rniga qo‘yishlar musbat ishorali had sifatida indekslari toq o‘rniga qo‘yishlar esa manfiy ishorali had sifatida kiradi.

Shunday qilib, biz quyidagi ta’rifni keltirishimiz mumkin.

3-Ta’rif.(2) matritsaga mos keluvchi $n-$tartibli determinant deb $n!$ hadning quyidagi tartibda tuzilgan algebraik yig‘indisiga aytiladi:hadlari matritsaning har qaysi satridan va har qaysi ustunidan bittadan olingan n ta elementdan tuzilgan bo‘lib, mumkin bo‘lgan barcha ko‘paytmalar xizmat qiladi;shu bilan birga hadning indekslari juft o‘rniga qo‘yishni tashkil etsa, musbat ishora bilan,aks holda esa manfiy ishora bilan olinadi.

$n-$tartibli determinantni yozishda quyidagi belgidan foydalanamiz

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}.$$

Determinantning ba’zi eng sodda xossalarini keltiramiz:

1^o.Agar determinant biror satrining (ustunining) barcha elementlari nolga teng bo‘lsa, u holda uning qiymati nolga teng bo‘ladi;

9-misol.

$$\begin{vmatrix} 6 & 7 & 3 \\ 0 & 0 & 0 \\ 3 & 4 & 2 \end{vmatrix} = 6\cdot 0\cdot 2 + 0\cdot 4\cdot 3 + 7\cdot 0\cdot 3 - 3\cdot 0\cdot 3 - 6\cdot 4\cdot 0 - 7\cdot 0\cdot 2 = 0.$$

2^o.Dioganal matritsaning determinanti dioganal elementlarining ko’paytmasiga teng,ya’ni

$$\det(A) = a_{11} \cdot a_{22} \cdots a_{nn} = \prod_{i=1}^{n} a_{ii}$$

10-misol. $\begin{vmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{vmatrix} = 2 \cdot 5 \cdot 3 = 30$.

3°.Yuqori (quyi) uchburchakli,matritsalarning determinantlari uning bosh diagonal elementlari ko‘paytmasiga teng. $\det(A) = a_{11} \cdot a_{22} \cdots a_{nn} = \prod_{i=1}^{n} a_{ii}$.

11-misol.

$$\begin{vmatrix} 2 & 3 & 4 \\ 0 & 5 & 9 \\ 0 & 0 & 6 \end{vmatrix} = 2 \cdot 5 \cdot 6 = 60$$

4^{o}.Determinantning biror satri (ustuni) elementlarini $k \neq 0$ songa ko‘paytirish determinantni shu songa ko‘paytirishga teng kuchlidir yoki biror satr (ustun) elementlarining umumiy ko‘paytuvchisini determinant belgisidan tashqariga chiqarish mumkin,ya’ni

$$k \cdot \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} ka_{11} & ka_{12} & ka_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & ka_{13} \\ a_{21} & a_{22} & ka_{23} \\ a_{31} & a_{32} & ka_{33} \end{vmatrix}$$

12-misol.

$$3 \cdot \begin{vmatrix} 2 & 7 & 3 \\ 5 & 6 & 8 \\ 1 & 4 & 2 \end{vmatrix} = 3 \cdot (24 + 60 + 56 - 18 - 70 - 64) = -36$$

$$\begin{vmatrix} 3 \cdot 2 & 3 \cdot 7 & 3 \cdot 3 \\ 5 & 6 & 8 \\ 1 & 4 & 2 \end{vmatrix} = 72 + 180 + 168 - 54 - 192 - 210 = -36$$

$$\begin{vmatrix} 3\cdot 2 & 7 & 3 \\ 3\cdot 5 & 6 & 8 \\ 3\cdot 1 & 4 & 2 \end{vmatrix} = 72+180+168-54-192-210=-36$$

5^o **.** n-tartibli diterminant uchun quyidagi tenglik o’rinli:

$$\det(kA) = k^n \cdot \det(A)$$

6^o. Determinantda ikkita satr (yokiustun) o‘rinlari almashtirilsa, determinantning ishorasi o‘zgaradi;

13-misol.

$$\begin{vmatrix} 2 & -4 & 3 \\ 3 & 1 & 5 \\ -1 & -2 & 1 \end{vmatrix} = 39$$

Endi bu matritsada birinchi va uchinchi ustunlarini o‘rinlarini almashtiramiz, u holda

$$\begin{vmatrix} 3 & -4 & 2 \\ 5 & 1 & 3 \\ 1 & -2 & -1 \end{vmatrix} = -3-12-20-2-20+18=-39.$$

Bundan ko‘rinib turibdiki, determinantlar faqat ishorasi bilan farq qiladi.

7^o**.** Agar determinant ikkita bir xil satrga (ustunga) ega bo‘lsa, u holda uning qiymati nolga teng bo‘ladi;

14-misol. $\begin{vmatrix} 5 & 3 & 6 \\ 10 & 6 & 12 \\ 2 & 5 & 3 \end{vmatrix} = 90+72+300-72-90-300=0$.

8^o**.** Agar determinantning biror satri (yoki ustuni) elementlariga boshqa satrning (yoki ustunning) mos elementlarini biror songa ko‘paytirib qo‘shilsa, determinantning qiymati o‘zgarmaydi;

Masalan,

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \cdots & \cdots & \cdots & \cdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{i1}+ka_{j1} & a_{i2}+ka_{j2} & \cdots & a_{in}+ka_{jn} \\ \cdots & \cdots & \cdots & \cdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

15-misol.

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 2+1\cdot 2 & 4+2\cdot 2 & 5+3\cdot 2 \\ 3 & 5 & 1 \end{vmatrix}$$

Haqiqatdan ham,tenglikning chap tarafi

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 1 \end{vmatrix} = 4+30+30-36-4-25=-1$$

tenglikning o’ng tarafi

$$\begin{vmatrix} 1 & 2 & 3 \\ 2+1\cdot 2 & 4+2\cdot 2 & 5+3\cdot 2 \\ 3 & 5 & 1 \end{vmatrix} = 8+66+60-72-55-8=-1$$

Demak tenglik o’rinli.

9^o. Agar determinant ikki satrining (ustunining) mos elementlari proportsional bo‘lsa, u holda uning qiymati nolga teng bo‘ladi,ya’ni

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ ka_{11} & ka_{12} & ka_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & ka_{11} \\ a_{21} & a_{22} & ka_{21} \\ a_{31} & a_{32} & ka_{31} \end{vmatrix} = 0$$

16-misol

$$\begin{vmatrix} 2\cdot 3 & 2\cdot 4 & 2\cdot 2 \\ 12 & 6 & 8 \\ 3 & 4 & 2 \end{vmatrix} = \begin{vmatrix} 6 & 8 & 4 \\ 12 & 6 & 8 \\ 3 & 4 & 2 \end{vmatrix} = 72+192+192-72-192-192=0$$

$$\begin{vmatrix} 2\cdot 12 & 2\cdot 6 & 2\cdot 8 \\ 12 & 6 & 8 \\ 3 & 4 & 2 \end{vmatrix} = \begin{vmatrix} 24 & 12 & 16 \\ 12 & 6 & 8 \\ 3 & 4 & 2 \end{vmatrix} == 288+288+768-288-288-768=0$$

10^o. Transponirlash natijasida determinantning qiymati o‘zgarmaydi, ya’ni

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix} == \begin{vmatrix} a_{11} & a_{21} & \dots & a_{n1} \\ a_{12} & a_{22} & \dots & a_{n2} \\ \dots & \dots & \dots & \dots \\ a_{1n} & a_{2n} & \dots & a_{nn} \end{vmatrix}$$

17-misol

$$\begin{vmatrix} 2 & -4 & 3 \\ 3 & 1 & 5 \\ -1 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 3 & -1 \\ -4 & 1 & -2 \\ 3 & 5 & 1 \end{vmatrix} = 39$$

11^o. Agar determinant biror satrining (ustunining) har bir elementi ikki qo‘shiluvchi yig‘indisidan iborat bo‘lsa, u holda determinant ikki determinant yig‘indisiga teng bo‘lib, ulardan birining tegishli satri (ustuni) birinchi qo‘shiluvchilaridan, ikkinchisining tegishli satri (ustuni) ikkinchi qo‘shiluvchilaridan iborat bo‘ladi, ya’ni

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{i1}+b_{i1} & a_{i2}+b_{i2} & \cdots & a_{in}+b_{in} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} +$$

$$+ \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ b_{i1} & b_{i2} & \cdots & b_{in} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

18-misol.

$$\begin{vmatrix} 5 & 7 & 6 \\ 8 & 10 & 12 \\ 3 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 5 & 7 & 6 \\ 4 & 5 & 6 \\ 3 & 2 & 1 \end{vmatrix} + \begin{vmatrix} 5 & 7 & 6 \\ 4 & 5 & 6 \\ 3 & 2 & 1 \end{vmatrix}$$

Haqiqatdan ham,tenglikning chap tomonidagi diterminant

$$\begin{vmatrix} 5 & 7 & 6 \\ 8 & 10 & 12 \\ 3 & 2 & 1 \end{vmatrix} = 5\cdot 10\cdot 1 + 8\cdot 2\cdot 6 + 7\cdot 12\cdot 3 - 3\cdot 10\cdot 6 - 8\cdot 7\cdot 1 - 2\cdot 12\cdot 5 =$$

$$= 50 + 96 + 252 - 180 - 56 - 120 = 398 - 356 = 42$$

O’ng tomonidagi diterminantlar yigindisi

$$2\cdot\begin{vmatrix} 5 & 7 & 6 \\ 4 & 5 & 6 \\ 3 & 2 & 1 \end{vmatrix} = 2\cdot\left(5\cdot 5\cdot 1 + 4\cdot 2\cdot 6 + 7\cdot 6\cdot 3 - 3\cdot 5\cdot 6 - 2\cdot 6\cdot 5 - 4\cdot 7\cdot 1\right) =$$

$$= 2\cdot\left(25 + 48 + 126 - 90 - 60 - 28\right) = 2\cdot\left(199 - 178\right) = 42$$

12°. Agar determinant satrlaridan biri uning qolgan satrlarining chiziqli kombinatsiyasidan iborat bo‘lsa, determinant nolga teng.

19-misol.

$$\begin{vmatrix} 2 & 3 & 2\cdot 2 + 4\cdot 3 \\ -2 & 1 & -2\cdot 2 + 4\cdot 1 \\ 3 & -2 & 3\cdot 2 - 4\cdot 2 \end{vmatrix} = \begin{vmatrix} 2 & 3 & 16 \\ -2 & 1 & 0 \\ 3 & -2 & -2 \end{vmatrix} =$$

$$= 2\cdot 1\cdot\left(-2\right) + 3\cdot 0\cdot 3 + \left(-2\right)\cdot\left(-2\right)\cdot 16 - 3\cdot 1\cdot 16 - 3\cdot\left(-2\right)\cdot\left(-2\right) - \left(-2\right)\cdot 0\cdot 2 =$$

$$= -4 + 0 + 64 - 48 - 12 + 0 = 0$$

13°. Toq tartibli har qanday qiya simmetrik determinant nolga teng.

20-misol.

$$\begin{vmatrix} 0 & -3 & 4 \\ 3 & 0 & -5 \\ -4 & 5 & 0 \end{vmatrix} = 0 + 60 - 60 + 0 + 0 + 0 = 0$$

14^o. Bir xil tartibli ikkita matritsalarning ko'paytmasining diterminanti, bu matritsalar diterminantlarining ko'paytmasiga teng,ya'ni

$$\det(A\cdot B)=\det(A)\cdot\det(B)$$

21-misol. Faraz qilaylik $A=\begin{pmatrix}1 & 6\\ -1 & 3\end{pmatrix}$ *va* $B=\begin{pmatrix}3 & 4\\ 2 & 5\end{pmatrix}$ bo'lsin,u holda

$$\det(A\cdot B)=\det\begin{pmatrix}15 & 34\\ 3 & 11\end{pmatrix}=165-102=63;\ \det(A)\cdot\det(B)=(3+6)(15-8)=9\cdot 7=63$$

Demak yuqoridagi xossa o'rinli.

Biz $2-, 3-$va yuqori tartibli determinantlarni hisoblashning ba'zi sodda va zarur holatlarda qulay usullari bilan tanishib chiqdik. Bunda yuqori tartibli determinantlarni hisoblash uchun uning tartibini pasaytirib uni, asosan, $2-$va $3-$ tartibli determinantlarga keltirib,so‘ngra bu determinantlarni hisoblash usullaridan foydalandik. Determinantlarni hisoblashning umumiy usuli mavjud bo‘lib,bunda determinant quyi tartibli determinantlar orqali ifodalanadi.Bu usuldan foydalanish uchun determinantning minori va algebraik to‘ldiruvchisi tushunchalarini kiritishimiz kerak bo‘ladi.

Bizga $n-$tartibli determinant berilgan bo‘lsin.

3-Ta'rif.$n-$tartibli d determinantning $1\le k\le n-1$ shartni qanoatlantiruvchi ixtiyoriy k ta satrlari va k ta ustunlari kesishgan joyda turgan, ya'ni bu satrlardan biriga hamda ustunlardan biriga tegishli bo‘lgan elementlardan tashkil topgan $k-$tartibli matritsaning determinanti d determinantning $k-$tartibli minori deb ataladi.

$k-$tartibli minorini bu d determinantning $n-k$ ta satr va $n-k$ ta ustunini o‘chirishdan hosil bo‘lgan determinant deb ham aytish mumkin. Xususan, determinantda bitta $i-$satr va bitta $j-$ustunni o‘chirishdan keyin $(n-1)-$tartibli M_{ij} minor hosil qilamiz. Masalan, uchinchi tartibli determinantda biror elementi turgan satr va ustun chizib tashlansa, qolgan elementlardan tuzilgan $2-$tartibli determinantga uning $2-$tartibli ***minori*** deyiladi.

22-misol. $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ determinantning $2-$satrini va $3-$ ustuninio'chirish natijasida $M_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$ minor hosil bo'ladi.

Birinchi tartibli minorlar sifatida determinantning elementlari hizmat qiladi.

4-ta'rif. Determinantning a_{ii} elementi joylashgan satr va ustunni o'chirishdan hosil bo'lgan minor bosh minor deb ataladi.

Masalan, 1) misolda $M_{11}, \quad M_{22}, \quad M_{33}$ minorlar bosh minorlardir.

5-Ta'rif. $n-$tartibli determinantda k $(k<n)$ dan n gacha satr va ustunlarni o'chirishdan hosil bo'lgan minorlar burchak minorlar deb ataladi.

$n-$tartibli d determinantda $k-$tartibli M minor olingan bo'lsin.

6-Tar'if. $n-$tartibli d determinantda $k-$tartibli M minor turgan satrlar va ustunlar o'chirib tashlangandan so'ng qolgan $(n-k)-$tartibli M' minorga M minorning to'ldiruvchisi deyiladi va aksincha.

M minor va uning M' to'ldiruvchi minorini sxematik ravishda quyidagicha tasvirlash mumkin

$$d = \begin{vmatrix} a_{11} & \dots & a_{1k} & a_{1k+1} & \dots & a_{1n} \\ \dots & \boxed{M} & \dots & \dots & \dots & \dots \\ a_{k1} & \dots & a_{kk} & a_{kk+1} & \dots & a_{kn} \\ a_{k+11} & \dots & a_{k+1k} & a_{k+1k+1} & \dots & a_{k+1n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & \dots & a_{nk} & a_{nk+1} & \dots & a_{nn} \end{vmatrix}.$$

Shunday qilib, determinantning o‘zaro to‘ldiruvchi minorlar jufti haqida gapirish mumkin. Xususiy holda, a_{ij} element va determinantning $i-$satri va $j-$ustunini o‘chirishdan hosil bo‘lgan $(n-1)-$tartibli minor o‘zaro to‘ldiruvchi minorlar juftini hosil qiladi.

7-Tar’if. Agar $k-$tartibli M minor $i_1, i_2, ..., i_k$ nomerli satlar va $j_1, j_2, ..., j_k$ ustunlarda joylashgan bo‘lsa, u holda M minorning algebraik to‘ldiruvchisi deb uning to‘ldiruvchi M' minorini aytamiz.

Bu minorning ishorasi M minor joylashgan satrlar va ustunlarning nomerlari yig‘indisining, ya’ni

$$s_M = i_1 + i_2 + ... + i_k + j_1 + j_2 + ... + j_k$$

yig‘indining juft yoki toqligi bilan aniqlanadi.

Shunday qilib, M minorning algebraik to‘ldiruvchisi

$$A = (-1)^{s_M} M'$$

formula bilan aniqlanadi.

Xususan, a_{ij} minorning (elementning) algebraik to‘ldiruvchisi

$$A_{ij} = (-1)^{i+j} M_{ij}$$

formula bilan aniqlanadi.

Masalan, uchinchi tartibli determinantda ,mos holda a_{11} ,a_{21} *va* a_{32} elementlarning algebraik to‘ldiruvchilari,mos holda,

$$A_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}, A_{21} = -\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} \quad va \quad A_{32} = -\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$$

ko‘rinishda bo‘ladi.

Laplas teoremasi. Determinantning qiymati uning ixtiyoriy satr (ustun) elementlari bilan, shu elementlarga mos algebraik to‘ldiruvchilar ko‘paytmalari yig‘indisiga teng, ya’ni

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \dots & \dots & \dots & \dots \\ a_{i1} & a_{i2} & \dots & a_{in} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix} = a_{i1}A_{i1} + a_{i2}A_{i2} + \dots + a_{in}A_{in} == \sum_{j=1}^{n} (-1)^{i+j} a_{ij} M_{ij}$$

Bu formulaga $\boldsymbol{\Delta}$ determinantni $\boldsymbol{i}$**-satr** elementlari bo‘yicha **yoyish formulasi** deyiladi.

Shuningdek $\boldsymbol{\Delta}$ determinantni quyidagi $\boldsymbol{j}$ **-ustun** elementlari bo‘yicha **yoyish formulasi** ham o‘rinli: $\boldsymbol{\Delta = \sum_{i=1}^{n} (-1)^{i+j} a_{ij} M_{ij}}$.

Determinantning biror satr (ustun) elementlari bilan uning boshqa satri (ustuni) elementlari algebraik to‘ldiruvchilari ko‘paytmalarining yig‘indisi nolga teng.

23-misol. Quyidagi determinantniLaplasformulasibilan hisoblang:

$$\Delta = \begin{vmatrix} 2 & 1 & 3 \\ 5 & 3 & 2 \\ 1 & 4 & 3 \end{vmatrix}$$

Berilgan determinantni birinchi satr elementlari bo‘yicha yoysak

$$\Delta = \begin{vmatrix} 2 & 1 & 3 \\ 5 & 3 & 2 \\ 1 & 4 & 3 \end{vmatrix} = 2A_{11} + A_{12} + 3A_{13} = 2(-1)^{1+1} \cdot M_{11} + (-1)^{1+2} \cdot M_{12} + (-1)^{1+3} \cdot M_{13} =$$

$$= 2 \cdot \begin{vmatrix} 3 & 2 \\ 4 & 3 \end{vmatrix} - \begin{vmatrix} 5 & 2 \\ 1 & 3 \end{vmatrix} + 3 \cdot \begin{vmatrix} 5 & 3 \\ 1 & 4 \end{vmatrix} = 2(9-8) - (15-2) + 3(20-3) = 2 - 13 + 51 = 40.$$

ga ega bo‘lamiz.

24-misol . Quyidagi diterminantni Laplas formulasi bilan hisoblang:

$$\Delta = \begin{vmatrix} 1 & 0 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 3 & 2 & 1 & 5 \\ 1 & 0 & 6 & 2 \end{vmatrix}$$ determinantni hisoblang.

Berilgan determinantni ikkinchi ustun elementlari bo‘yicha yoyib chiqamiz. Bu ustunda 2 ta noldan farqli element bo‘lgani uchun natijada 2 ta 3-tartibli determinant hosil bo‘ladi.

$$\Delta = -1 \cdot (-1)^{2+2} \begin{vmatrix} 1 & 3 & 2 \\ 3 & 1 & 5 \\ 1 & 6 & 2 \end{vmatrix} + 2 \cdot (-1)^{3+2} \begin{vmatrix} 1 & 3 & 2 \\ 2 & 4 & 1 \\ 1 & 6 & 2 \end{vmatrix}$$

Yoki, avval $a_{32} = 2$ elementni nol bilan almashtiramiz. Buning uchun 2-satrni 2 ga ko‘paytirib 3-satrga qo‘shamiz va hosil bo‘lgan determinantni 2-ustun elementlariga nisbatan yoyamiz va hisoblaymiz:

$$\Delta = \begin{vmatrix} 1 & 0 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 7 & 0 & 9 & 7 \\ 1 & 0 & 6 & 2 \end{vmatrix} = -1 \cdot (-1)^{2+2} \cdot \begin{vmatrix} 1 & 3 & 2 \\ 7 & 9 & 7 \\ 1 & 6 & 2 \end{vmatrix} = -21$$

Amaliy mashg`ulot uchun masalalar

Quyidagii kkinchi tartibli determinantlarni hisoblang:

25. $\begin{vmatrix} -7 & 6 \\ 5 & -4 \end{vmatrix}$

26. $\begin{vmatrix} 10 & -5 \\ 9 & -8 \end{vmatrix}$

27. $\begin{vmatrix} \sqrt{a}+\sqrt{b} & \sqrt{a}-\sqrt{b} \\ \sqrt{a}-\sqrt{b} & \sqrt{a}+\sqrt{b} \end{vmatrix}$

28. $\begin{vmatrix} \sin 1^0 & \sin 89^0 \\ -\cos 1^0 & \cos 89^0 \end{vmatrix}$

29. $\begin{vmatrix} \frac{x+y}{x} & \frac{2x}{x-y} \\ \frac{y-x}{x^2-y^2} & \frac{y-x}{x^2-y^2} \end{vmatrix}$

30. $\begin{vmatrix} \sin^2\alpha & \cos^2\alpha \\ \sin^2\beta & \cos^2\beta \end{vmatrix}$

31. $\begin{vmatrix} \sqrt{5}-a^{\frac{1}{2}} & a^{\frac{1}{2}} \\ -a^{\frac{1}{2}} & \sqrt{5}+a^{\frac{1}{2}} \end{vmatrix}$ **32.** $\begin{vmatrix} 1,(3) & 2,25 \\ 23/3 & 6 \end{vmatrix}$

33. $\begin{vmatrix} \sin 60^0 & \cos 45^0 \\ \sin 45^0 & tg 30^0 \end{vmatrix}$ **34.** $\begin{vmatrix} tg\alpha & -1 \\ 4 & ctg\alpha \end{vmatrix}$

35. $\begin{vmatrix} \dfrac{a-1}{2\sqrt{a}} & \dfrac{a+\sqrt{a}}{\sqrt{a}-1} \\ \dfrac{a\sqrt{a}-\sqrt{a}}{2a} & \dfrac{a-\sqrt{a}}{\sqrt{a}+1} \end{vmatrix}$

Quyidagi uchinchi tartibli determinantlarni qulay usulda hisoblang:

36. $\begin{vmatrix} 2 & 3 & 4 \\ 5 & -2 & 1 \\ 1 & 2 & 3 \end{vmatrix}$ **37.** $\begin{vmatrix} a & 1 & a \\ -1 & a & 1 \\ a & -1 & a \end{vmatrix}$

38. $\begin{vmatrix} 5 & 3 & 2 \\ -1 & 2 & 4 \\ 7 & 3 & 6 \end{vmatrix}$ **39.** $\begin{vmatrix} 1 & 2 & 3 \\ 8 & 1 & 4 \\ 2 & 1 & 1 \end{vmatrix}$

40. $\begin{vmatrix} 3 & -1 & -2 \\ 1 & 2 & 5 \\ -4 & 1 & 6 \end{vmatrix}$ **41.** $\begin{vmatrix} 1 & 2 & -1 \\ 3 & 7 & 2 \\ 2 & 3 & -7 \end{vmatrix}$

42. $\begin{vmatrix} a & -a & a \\ a & a & -a \\ a & -a & -a \end{vmatrix}$ **43.** $\begin{vmatrix} a+x & x & x \\ x & b+x & x \\ x & x & c+x \end{vmatrix}$ **43.**

$\begin{vmatrix} \cos\alpha & \sin\alpha\cos\beta & \sin\alpha\sin\beta \\ -\sin\alpha & \cos\alpha\cos\beta & \cos\alpha\sin\beta \\ 0 & -\sin\beta & \cos\beta \end{vmatrix}$ **45.** $\begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix}$

46. $\begin{vmatrix} m+a & m-a & a \\ n+a & 2n-a & a \\ a & -a & a \end{vmatrix}$ **47.** $\begin{vmatrix} ax & a^2+x^2 & 1 \\ ay & a^2+y^2 & 1 \\ az & a^2+z^2 & 1 \end{vmatrix}$

48. $\begin{vmatrix} \sin 3\alpha & \cos 3\alpha & 1 \\ \sin 2\alpha & \cos 2\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \end{vmatrix}$ **49.** $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

50. $\begin{vmatrix} a & x & x \\ x & b & x \\ x & x & c \end{vmatrix}$

Tenglik bajarilishini tekshiring:

51. $\begin{vmatrix} a_1 & b_1 & a_1x+b_1y+c_1 \\ a_2 & b_2 & a_2x+b_2y+c_2 \\ a_3 & b_3 & a_3x+b_3y+c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

52. $\begin{vmatrix} a_1+b_1x & a_1-b_1x & c_1 \\ a_2+b_2x & a_2-b_2x & c_2 \\ a_3+b_3x & a_3-b_3x & c_3 \end{vmatrix} = -2x\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

53. $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (b-a)(c-a)(c-b)$

54. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (b-a)(c-a)(c-b)$

55. $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

56. $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

57. $\begin{vmatrix} (a+b)^2 & c^2 & c^2 \\ a^2 & (b+c)^2 & a^2 \\ b^2 & b^2 & (c+a)^2 \end{vmatrix} = 2bc(a+b+c)^3$

Determinantlarni 3-ustun elementlari bo‘yicha yoyib hisoblang:

58. $\begin{vmatrix} 1+\cos\alpha & 1+\sin\alpha & 1 \\ 1-\sin\alpha & 1+\cos\alpha & 1 \\ 1 & 1 & 1 \end{vmatrix}$ **59.** $\begin{vmatrix} 2\cos^2\dfrac{\alpha}{2} & \sin\alpha & 1 \\ 2\cos^2\dfrac{\beta}{2} & \sin\beta & 1 \\ 1 & 0 & 1 \end{vmatrix}$

60. $\begin{vmatrix} \sin\alpha & \cos\alpha & 1 \\ \sin\beta & \cos\beta & 1 \\ \sin\gamma & \cos\gamma & 1 \end{vmatrix}$

61. Qanday shart bajarilganda quyidagi tenglik o‘rinli bo‘ladi?

$$\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$$

Tenglamalarni yeching:

62. $\begin{vmatrix} x & 3 \\ 1 & 2x \end{vmatrix} + 3\begin{vmatrix} 0,(4) & x \\ 1 & 3 \end{vmatrix} = 0$

63. $(0.6)^x \cdot \left(\dfrac{25}{9}\right)^{\begin{vmatrix} x & 3 \\ 4 & x \end{vmatrix}} = \left(\dfrac{27}{125}\right)^3$

64. $log_3 \dfrac{2}{\begin{vmatrix} 2 & x \\ 1 & 2 \end{vmatrix}} = log_3 \begin{vmatrix} x & 2 \\ \dfrac{1}{2} & 1 \end{vmatrix}$

65. $\begin{vmatrix} \sin x & 0 & -3/2 \\ -2 & 1 & 4 \\ 0,5 & 0 & \cos x \end{vmatrix} = 1$

Tengsizliklarni yeching:

66. $\begin{vmatrix} x & 1 \\ -4 & x \end{vmatrix} \le \begin{vmatrix} 5 & 2 \\ 1 & x \end{vmatrix}$

67. $\dfrac{1}{\begin{vmatrix} x & 1 \\ 2 & 1 \end{vmatrix}} < \dfrac{1}{3}$

68. $\begin{vmatrix} 3^x & 2 & -1 \\ 9^x & 2^x & 0 \\ 2^x & 0 & 1 \end{vmatrix} > 0$

O`z – o`zini tekshirish uchun savollar

1. Ikkinchi tartibli determinant deb nimaga aytiladi?
2. Determinantning kattaligi uning satrlari mos ustunlari bilan almashtirilsa qanday o`zgaradi?
3. Determinantning satrlari (ustunlari) o`rinlari almashtirilsa-chi?
4. Determinant biror-bir satri (ustuni) elementlari umumiy ko`paytuvchisini uning ishorasidan tashqariga ko`paytuvchi sifatida chiqarish mumkinmi?
5. Qanday hollarda determinantning kattaligi nolga teng?
6. Uchinchi tartibli determinant deb nimaga aytiladi?
7. Uchinchi tartibli determinantni hisoblash Sarryus qoidasi nimadan iborat?
8. Uchinchi tartibli determinantni hisoblash uchburchak sxemasini yozing.
9. Transpozisiyalash deganda nimani tushunasiz?
10. Juft yoki toq o`rin almashtirish tizimi deb, qanday o`rin almashtirishga aytiladi?
11. n-tartibli determinant deb nimaga aytiladi?
12. n-tartibli determinant ixtiyoriy elementi minori deb nimaga aytiladi? Algebraik to`ldiruvchi yoki ad'yunkt deb nimaga aytiladi?

13 n-tartibli determinantni i-satr elementlari bo`yicha yoyib hisoblash formulasini yozing.

1. n-tartibli determinantni k-ustun elementlari bo`yicha yoyib hisoblash formulasini yozing.
2. Determinantni transponirlashdan tashqari uning ustida qanday almashtirishlar bajarganda kattaligi o`zgarmaydi?
3. Yuqori tartibli determinantlarni nollar yigib hisoblash usuli nimadan iborat?
4. Determinantlarni qo`shish mumkinmi va qanday?
5. Tartiblari teng determinantlarni ko`paytirish qoidasi nimadan iborat?

Matritsa rangi.Teskari matrisa.

Tayanch so'z va iboralar: *matritsa,matritsaostiminori, matritsa diogonali,matritsa rangi, determinant, trapetsiyasimon (zinapoyasimon) matritsa.*

REJA:

1.Matritsa osti minori.

2.Matritsa rangi.

3.Xos va xosmas matritsalar.

4.Ikki matritsa ko'paytmasining determinanti.

5.Teskari matritsa.

Ixtiyoriy o'lchamli matritsaning *matritsa osti minorlari* deb, uning bir necha satr yoki ustunlarini o'chirishdan kvadrat matritsa ko'rinishiga keltirilgan qismiga aytiladi. Agar berilgan matritsa kvadrat shaklda bo'lsa, uning eng katta tartibli minori o'ziga teng.

1-misol.Masalan $A=\begin{pmatrix} 4 & 5 & 7 \\ 2 & 1 & 4 \\ 3 & 7 & 0 \end{pmatrix}$ matritsaning 1-qator va 1-ustunini o'chirishdan 2-tartibli minor $M_2=\begin{vmatrix} 1 & 4 \\ 7 & 0 \end{vmatrix}$, 2-qator va 3-ustunini o'chirishdan 2-tartibli minor $M_2=\begin{vmatrix} 4 & 5 \\ 3 & 7 \end{vmatrix}$

1-ta'rif. A matritsaning **rangi** deb, noldan farqli matritsa osti minorlarining eng katta tartibiga aytiladi va $rang\left(A\right)=r(A)$ ko'rinishida ifodalanadi.

Shuni ta'kidlash kerakki, matritsa rangi uning o'lchamlarining kichigidan katta emas, ya'ni $rang\left(A\right)\le \min(m,n)$. Bunda m, n larberilgan matritsaning o'lchamlari.

2-misol . $\mathrm{A}=\begin{pmatrix} 1 & -2 \\ -2 & 4 \\ 3 & -7 \end{pmatrix}$ matritsa rangini aniqlang.

Yechish. Berilgan matritsa (3×2) o'lchamli bo'lgani uchun satrlari va ustunlari sonini taqqoslaymiz va kichigi 2 ni tanlaymiz. Matritsadan ikkinchi tartibli minorlar ajratamiz va ularning kattaligini hisoblaymiz. Bu jarayonni noldan farqli ikkinchi tartibli minor ajralmaguncha davom etamiz:

$$M_1 = \begin{vmatrix} 1 & -2 \\ -2 & 4 \end{vmatrix} = 0, \quad M_2 = \begin{vmatrix} 1 & -2 \\ 3 & -7 \end{vmatrix} = -1 \neq 0.$$

Berilgan matritsadan noldan farqli eng yuqori ikkinchi tartibli minor ajraldi. Demak, ta’rifga binoan, A matritsa rangi 2 ga teng, ya’ni $rang(A) = 2$.

3-misol. Matritsaning rangini toping:

$$\boldsymbol{A} = \begin{pmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{pmatrix}.$$

Yechish: Chap tarafdagi 2-tartibli minor nolga teng $M_1 = \begin{vmatrix} 2 & -4 \\ 1 & -2 \end{vmatrix} = 0.$ Boshqa 2-tartibli minor olamiz, masalan, $M_2 = \begin{vmatrix} -4 & 3 \\ -2 & 1 \end{vmatrix} = 2 \neq 0.$buminorni qamrab oluvchi uchinchi tartibli minorni olamiz:

$$M_3 = \begin{vmatrix} 2 & -4 & 3 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{vmatrix} = 1 \neq 0.$$

Endi buminorni qamrab oluvchi to‘rtinchi tartibli minorni hisoblaymiz. Bunda ikkita variant mavjud.

$$M_4 = \begin{vmatrix} 2 & -4 & 3 & 1 \\ 1 & -2 & 1 & -4 \\ 0 & 1 & -1 & 3 \\ 4 & -7 & 4 & -4 \end{vmatrix} = 0 \quad M_5 = \begin{vmatrix} 2 & -4 & 3 & 0 \\ 1 & -2 & 1 & 2 \\ 0 & 1 & -1 & 1 \\ 4 & -7 & 4 & 5 \end{vmatrix} = 0$$

Ikkala holda ham to‘rtinchi tartibli minorlar nolga teng. Shuning uchun berilgan matritsaning rangi $r(A) = 3$ ga teng.

Mashqlarni bajaring.

4-misol. Quyidagi matritsalarning rangini hisoblang:

1. $A=\begin{pmatrix}1 & 5\\ 3 & 4\end{pmatrix}, B=\begin{pmatrix}10 & 5\\ 3 & 4\end{pmatrix}, C=\begin{pmatrix}6 & 3\\ 6 & 4\end{pmatrix}, D=\begin{pmatrix}6 & 8\\ 3 & 4\end{pmatrix}, E=\begin{pmatrix}6 & 3\\ 8 & 4\end{pmatrix}$

2. $A=\begin{pmatrix}2 & 6 & 9\\ 1 & 3 & 5\\ 3 & 4 & 2\end{pmatrix}, B=\begin{pmatrix}2 & 6 & 9\\ 1 & 4 & 6\\ 3 & 4 & 2\end{pmatrix}, C=\begin{pmatrix}3 & 6 & 9\\ 1 & 3 & 5\\ 1 & 2 & 3\end{pmatrix}, D=\begin{pmatrix}1 & 6 & 3\\ 1 & 3 & 3\\ 1 & 2 & 3\end{pmatrix}$

3. $A=\begin{pmatrix}1 & 6 & 9 & 11\\ 12 & 3 & 7 & 10\\ 13 & 14 & 4 & 8\\ 15 & 16 & 17 & 5\end{pmatrix}, B=\begin{pmatrix}1 & 6 & 5 & 11\\ 2 & 12 & 10 & 10\\ 3 & 14 & 15 & 8\\ 15 & 16 & 17 & 5\end{pmatrix},$

Matritsa rangi uning ustida quyidagi elementar almashtirishlar bajarganda o‘zgarmaydi.

1.Matritsa biror satri (ustuni) har bir elementini biror noldan farqli songa ko‘paytirganda;

2.Matritsa satrlari (ustunlari) o‘rinlari almashtirilganda;

3.Matritsa biror satri (ustuni) elementlariga uning boshqa parallel satri (ustuni) mos elementlarini biror noldan farqli songa ko‘paytirib, so‘ngra qo‘sh-ganda;

4.Matritsatransponirlanganda.

*A***matritsaning rangi** deb, noldan farqli matritsa osti minorlarining eng katta tartibiga aytiladi va $rang(A)$ yoki $r(A)$ ko`rinishida ifodalanadi.

1-masala.$A=\begin{pmatrix}1 & -2\\ -2 & 4\\ 3 & -7\end{pmatrix}$ matritsa rangini aniqlang.

Berilgan matritsa 3×2 o`lchamli bo`lgani uchun satrlari va ustunlari sonini taqqoslaymiz va kichigi 2 ni tanlaymiz. Matritsadan ikkinchi tartibli minorlar ajratamiz va ularning kattaligini hisoblaymiz. Jarayonni noldan farqli 2- tartibli minor ajralmaguncha davom ettiramiz:

$$M_1=\begin{vmatrix}1 & -2\\ -2 & 4\end{vmatrix}=0,\ \ M_2=\begin{vmatrix}1 & -2\\ 3 & -7\end{vmatrix}=-1\neq 0.$$

Berilgan matritsadan noldan farqli eng yuqori ikkinchi tartibli minor ajraldi. Demak, ta'rifga binoan, A matritsa rangi 2 ga teng, ya'ni $rang\ (A) = 2$.

2-masala. $B = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \end{pmatrix}$ matritsa rangini aniqlang.

B matritsadan ajralishi mumkin bo`lgan eng yuqori - ikkinchi tartibli har qanday minor nolga teng:

$$M_1 = \begin{vmatrix} 1 & -2 \\ -2 & 4 \end{vmatrix} = 0, \qquad M_2 = \begin{vmatrix} 1 & 3 \\ -2 & -6 \end{vmatrix} = 0, \qquad M_3 = \begin{vmatrix} -2 & 3 \\ 4 & -6 \end{vmatrix} = 0.$$

Demak, matritsa rangi ikkiga teng bo`la olmaydi. $B$ matritsa nolmas matritsa bo`lgani uchun uning rangi 1 ga teng.

3-masala. $C = \begin{pmatrix} -1 & 1 & -2 \\ 2 & -3 & 4 \\ -1 & 0 & -2 \end{pmatrix}$ matritsa rangini aniqlang.

C matritsa uchinchi tartibli kvadratik matritsa. Undan yagona eng yuqori 3-tartibli minor ajraladi va minor kattaligi hisoblanadi:

$$M_1 = \begin{vmatrix} -1 & 1 & -2 \\ 2 & -3 & 4 \\ -1 & 0 & -2 \end{vmatrix} = 0$$

bo`lgani uchun $C$ matritsa rangi 3 ga teng bo`la olmaydi. $M_2 = \begin{vmatrix} -1 & 1 \\ 2 & -3 \end{vmatrix} = 1 \neq 0$ bo`lgani uchun $rang(C) = 2$.

Matritsa rangi uning ustida quyidagi **elementar almashtirishlar** bajarganda o`zgarmaydi:

1. Matritsa biror satri (ustuni) har bir elementini bir xil noldan farqli songa ko`paytirganda;
2. Matritsa satrlari (ustunlari) o`rinlari almashtirilganda;
3. Matritsa biror satri (ustuni) elementlariga uning boshqa parallel satri (ustuni) mos elementlarini bir xil songa ko`paytirib, so`ngra qo`shganda;
4. Matritsa transponirlanganda.

4-masala. $A = \begin{pmatrix} 1 & -2 & 1 & 3 \\ 3 & 1 & 0 & 7 \\ 2 & 3 & -1 & 4 \end{pmatrix}$ matritsaning rangini nollar yig`ish usulida aniqlang.

Berilgan dastlabki matritsa ustida quyidagicha elementar almashtirishlar bajaramiz va uning ko`rinishini trapetsiyasimon ko`rinishga keltiramiz:

$$\begin{pmatrix} 1 & -2 & 1 & 3 \\ 3 & 1 & 0 & 7 \\ 2 & 3 & -1 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 1 & 3 \\ 0 & 7 & -3 & -2 \\ 0 & 7 & -3 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 1 & 3 \\ 0 & 7 & -3 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Trapetsiyasimon matritsa bosh diagonal elementlaridan ikkitasi noldan farqli bo`lgani uchun uning rangi va shu bilan birga berilgan matritsa rangi ikkiga teng, ya’ni $rang(A) = 2$.

Matritsalar ko`paytmasining rangi haqida quyidagilarni aytish mumkin. Ko`paytuvchi matritsalar ranglari bilan ko`paytma matritsa rangi orasida aniq bir bog`lanish yo`q, buni quyidagi misollarda ko`rish mumkin:

$$\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Har ikkala holda ham rangi 1 bo`lgan matritsalar ko`paytirilgan, biroq ko`paytma matritsa ranglari birinchi misolda 1 ga teng bo`lsa, ikkinchi misolda 0 ga teng. Ixtiyoriy o`lchamli matritsalar uchun quyidagi teorema o`rinli:

Teorema. Matritsalar ko`paytmasining rangi ko`paytuvchi matritsa ranglarining kichigidan katta emas $r(AB) \leq min(r(A), r(B))$.

3. n–tartibli kvadratik $A = (a_{ik})$ matritsa berilgan bo`lsin. Agar $A$ matritsa determinanti noldan farq qilib, uning rangi tartibi $n$ ga teng, ya’ni $det(A) \neq 0, rang(A) = n$ bo`lsa, matritsaga **maxsusmas matritsa** deyiladi. Agarda $det(A) = 0$ bo`lib, rangi $n$ dan kichik bo`lsa, A matritsaga **maxsus matritsa** deyiladi.

Teorema. Ikki teng tartibli kvadrat matritsalarning ko`paytmasi, ko`paytuvchi matritsalarning har biri maxsusmas bo`lgandagina, maxsusmas matritsadan iborat bo`ladi.

Matritsa rangini aniqlashning ta’rif asosida biz yuqorida masalalarda ko‘rgan **«minorlar ajratib hisoblash»** usuli va nollar yig‘ib hisoblashga asoslangan **«Gauss algoritmi»** usullari mavjud.

Matritsarangi «Gauss algoritmi» yoki nollar yig‘ish usuli asosida quyidagicha aniqlanadi: dastlabki ko‘rinishdagi matritsa yuqorida sanab o‘tilgan elementar almashtirishlar yordamida «**trapetsiya simon matritsa**» ko‘rinishiga keltiriladi. Trapetsiyasimon matritsa deb, bosh diagonaldan yuqorida yoki quyida joylashgan har bir elementi nolga teng bo‘lgan matritsaga aytiladi. Trapetsiya simon matritsaning rangi yoki xuddi shuning o‘zi dastlabk I matritsaning rangi trapetsiya simon matritsaning noldan farqli bosh diagonal elementlari soniga teng.

5-misol. $A=\begin{pmatrix} 1 & -2 & 1 & 3 \\ 3 & 1 & 0 & 7 \\ 2 & 3 & -1 & 4 \end{pmatrix}$ matritsaning rangini nollar yig‘ish usulida aniqlang.

Berilgan dastlabki matritsa ustida quyidagicha elementar almashtirishlar bajaramiz va uning ko‘rinishini trapetsiya ko‘rinishiga keltiramiz:

$$\begin{pmatrix} 1 & -2 & 1 & 3 \\ 3 & 1 & 0 & 7 \\ 2 & 3 & -1 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 1 & 3 \\ 0 & 7 & -3 & -2 \\ 0 & 7 & -3 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 1 & 3 \\ 0 & 7 & -3 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Trapetsiya simon matritsa bosh diagonal elementlaridan ikkitasi noldan farqli bo‘lgan iuchun uning rangi va shu bilan birga berilgan matritsa rangi ikkiga teng.

Chiziqli tenglamalar sistemasi nazariyasida matritsalar va ular ustida bajariladigan amallar alohida o‘rin egallaydi. Shu sababli biz matritsalarga doir ba’zi bir tushunchalarni keltirib o‘tamiz.

3-ta’rif. Kvadrat matritsa elementlaridan tuzilgan determinant noldan farqli bo‘lsa, u holda bunday matritsa **xosmas** yoki **maxsusmas matritsa** deyiladi. Aks holda, ya’ni agar determinant nolga teng bo‘lsa, bu matritsa **xos** yoki **maxsus matritsa** deyiladi.

6-misol. $A=\begin{pmatrix} 1 & -2 & 3 \\ 0 & 4 & -1 \\ 5 & 0 & 0 \end{pmatrix}$, $\boldsymbol{B}=\begin{pmatrix} 1 & -2 \\ -5 & 10 \end{pmatrix}$ va $\boldsymbol{E}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

matritsalarning xos yoki xosmasligini aniqlang.

Yechish. Berilgan matritsalarningdeterminantlarini hisoblaymiz:

$$|\boldsymbol{A}|=\begin{vmatrix}1 & -2 & 3\\ 0 & 4 & -1\\ 5 & 0 & 0\end{vmatrix}=10-60=-50.\ |\boldsymbol{B}|=\begin{vmatrix}1 & -2\\ -5 & 10\end{vmatrix}=10-10=0.$$

$$|\boldsymbol{E}|=\begin{vmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix}=1.$$

Bundan ta'rifga asosan $\boldsymbol{A}$ va $\boldsymbol{E}$ matritsalar – xosmas, $\boldsymbol{B}$ matritsa esa xos matritsa ekanligi kelib chiqadi.

Determinantlarni ko'paytirish haqidagi quyidagi muhim teoremani keltiramiz.

Teorema. $n-$tartibli bir nechta matritsalar ko'paytmasining determinanti bu matritsalar determinantlarining ko'paytmasiga teng.

Bu teoremaning isboti odatda ikkita matritsa uchun keltiriladi. Shu sababdi biz ham teoremaning mohiyatini ikkita matritsa misolida tushuntirib o'tamiz.

Agar $\boldsymbol{A}$ va $\boldsymbol{B}$ - kvadrat matritsalar bir xil tartibli bo'lsa, u holda yuqoridagi teoremaga asosan $|\boldsymbol{AB}|=|\boldsymbol{BA}|=|\boldsymbol{A}|\cdot|\boldsymbol{B}|$.

7-misol. $A=\begin{pmatrix}2 & 1 & 3\\ 5 & 3 & 2\\ 1 & 4 & 3\end{pmatrix}$ va $B=\begin{pmatrix}3 & 2 & 1\\ 2 & 5 & 3\\ 3 & 4 & 2\end{pmatrix}$ matritsalar berilgan bo'lsin.

AB va ***BA*** larni hisoblaymiz:

$$\boldsymbol{AB}=\begin{pmatrix}17 & 21 & 11\\ 27 & 33 & 18\\ 20 & 34 & 19\end{pmatrix};\qquad \boldsymbol{BA}=\begin{pmatrix}17 & 13 & 16\\ 32 & 29 & 25\\ 28 & 23 & 23\end{pmatrix}.$$

Ko'paytirish natijasida hosil bo'lgan matritsalar turli xil bo'lsada, lekin ularning determinantlari qiymatlari o'zaro teng, ya'ni $|\boldsymbol{AB}|=|\boldsymbol{BA}|=-120$. Ikkinchi tomondan esa $|\boldsymbol{A}|\cdot|\boldsymbol{B}|=-3\cdot 40=-120$.

4-ta'rif. A kvadrat matritsaning har bir a_{ik} elementini unga mos algebraik to'ldiruvchisi bilan almashtirish natijasida olingan matritsa ustida transponirlash amalini bajarishdan hosil bo'lgan $\overline{A}$ matritsaga berilgan matritsaga qo'shma matritsa deyiladi.

Masalan, $A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{in} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nj} & \dots & a_{nn} \end{pmatrix}$ matritsaga transponirlangan matritsa $A^T=\begin{pmatrix} a_{11} & a_{21} & \dots & a_{i1} & \dots & a_{n1} \\ a_{12} & a_{22} & \dots & a_{i2} & \dots & a_{n2} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{1j} & a_{2j} & \dots & a_{ij} & \dots & a_{nj} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{1n} & a_{2n} & \dots & a_{in} & \dots & a_{nn} \end{pmatrix}$ ko‘rinishda bo‘lsa, A matritsaga qo‘shma matritsa $\overline{A}=\begin{pmatrix} A_{11} & A_{21} & \dots & A_{i1} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{i2} & \dots & A_{n2} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ A_{1j} & A_{2j} & \dots & A_{ij} & \dots & A_{nj} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ A_{1n} & A_{2n} & \dots & A_{in} & \dots & A_{nn} \end{pmatrix}$ ko‘rinishda bo‘ladi.

8-misol. Quyidagi $\boldsymbol{A}=\begin{pmatrix} 1 & -2 & 3 \\ 0 & 4 & -1 \\ 5 & 0 & 0 \end{pmatrix}$ matritsa uchun qo‘shma matritsa topilsin.

Yechish. $\boldsymbol{A}$ matritsaga nisbatan transponirlangan A^T matritsani topamiz:

$$A^T=\begin{pmatrix} 1 & 0 & 5 \\ -2 & 4 & 0 \\ 3 & -1 & 0 \end{pmatrix}$$

Endi qo‘shma matritsaning barcha elementlarini hisoblaymiz:

$$\boldsymbol{A}_{11}=(-1)^{1+1}\cdot\begin{vmatrix} 4 & 0 \\ -1 & 0 \end{vmatrix}=0,$$

$$\boldsymbol{A}_{12} = (-1)^{1+2} \cdot \begin{vmatrix} -2 & 0 \\ 3 & 0 \end{vmatrix} = 0,$$

$$\boldsymbol{A}_{13} = (-1)^{1+3} \cdot \begin{vmatrix} -2 & 4 \\ 3 & -1 \end{vmatrix} = -10,$$

$$\boldsymbol{A}_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 0 & 5 \\ -1 & 0 \end{vmatrix} = -5,$$

$$\boldsymbol{A}_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 1 & 5 \\ 3 & 0 \end{vmatrix} = -15,$$

$$\boldsymbol{A}_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 1 & 0 \\ 3 & -1 \end{vmatrix} = 1,$$

$$\boldsymbol{A}_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 0 & 5 \\ 4 & 0 \end{vmatrix} = -20$$

$$\boldsymbol{A}_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 1 & 5 \\ -2 & 0 \end{vmatrix} = -10,$$

$$\boldsymbol{A}_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 1 & 0 \\ -2 & 4 \end{vmatrix} = 4.$$

Shunday qilib, berilgan $\boldsymbol{A}$ kvadrat matritsaga qo‘shma bo‘lgan $\overline{\boldsymbol{A}}$ matritsa

$$\overline{\boldsymbol{A}} = \begin{pmatrix} 0 & 0 & -10 \\ -5 & -15 & 1 \\ -20 & -10 & 4 \end{pmatrix}$$

ko‘rinishda aniqlanadi.

5-ta’rif. Agar A kvadrat matritsa uchun $AA^{-1} = A^{-1}A = E$ tenglik bajarilsa, A^{-1} matritsa A matritsaga ***teskari matritsa*** deyiladi.

Matritsa teskarisini topishning 2 hil usuli bor:

1) Klassik yoki algebraik to‘ldiruvchilar usuli.

Agar A matritsa xosmas matritsa bo‘lsa, u holda uning uchun yagona A^{-1} matritsa mavjud bo‘ladi va u quyidagi tenglik bilan aniqlanadi:

$$A^{-1}=\frac{1}{\Delta}\begin{pmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \dots & \dots & \dots & \dots \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{pmatrix} (1)$$

9-misol. $A=\begin{pmatrix} 5 & -1 & -1 \\ 1 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix}$ matritsaga teskari matritsani algebraik to‘ldiruvchilar yordamida toping.

Yechish. Berilgan A matritsaning determinantini hisoblaymiz:

$$\Delta=|A|=\begin{vmatrix} 5 & -1 & -1 \\ 1 & 2 & 3 \\ 4 & 3 & 2 \end{vmatrix}=5(4-9)+1(2-12)-1(3-8)=-25-10+5=-30.$$

Determinant noldan farqli, demak, matritsaning teskarisi mavjud. Matritsaning har bir elementi algebraik to‘ldiruvchisini topamiz:

$$A_{11}=(-1)^{1+1}\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix}=-5;$$

$$A_{12}=(-1)^{1+2}\begin{vmatrix} 1 & 3 \\ 4 & 2 \end{vmatrix}=10;$$

$$A_{13}=(-1)^{1+3}\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix}=-5;$$

$$A_{21}=(-1)^{2+1}\begin{vmatrix} -1 & -1 \\ 3 & 2 \end{vmatrix}=-1\ ;$$

$$A_{22}=(-1)^{2+2}\begin{vmatrix} 5 & -1 \\ 4 & 2 \end{vmatrix}=14;$$

$$A_{31}=(-1)^{3+1}\begin{vmatrix} -1 & -1 \\ 2 & 3 \end{vmatrix}=\ -1$$

$$A_{32}=(-1)^{3+2}\begin{vmatrix} 5 & -1 \\ 1 & 3 \end{vmatrix}=-16;$$

$$A_{33}=(-1)^{3+3}\begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix}=11;$$

Natijada teskari matritsani quramiz

$$A^{-1}=\frac{1}{\Delta}\cdot\overline{A}=-\frac{1}{30}\begin{pmatrix}-5 & -1 & -1\\ 10 & 14 & -16\\ -5 & -19 & 11\end{pmatrix}=\begin{pmatrix}\frac{1}{6} & \frac{1}{30} & \frac{1}{30}\\ -\frac{1}{3} & -\frac{7}{15} & \frac{8}{15}\\ \frac{1}{6} & \frac{19}{30} & -\frac{11}{30}\end{pmatrix};$$

Tekshirish:

$$AA^{-1}=\begin{pmatrix}5 & -1 & -1\\ 1 & 2 & 3\\ 4 & 3 & 2\end{pmatrix}\begin{pmatrix}\frac{5}{30} & \frac{1}{30} & \frac{1}{30}\\ -\frac{10}{30} & -\frac{14}{30} & \frac{16}{30}\\ \frac{5}{30} & \frac{19}{30} & -\frac{11}{30}\end{pmatrix}=$$

$$=\frac{1}{30}\begin{pmatrix}25+10-5 & 5+14-19 & 5-16+11\\ 5-20+15 & 1-28+57 & 1+32-33\\ 20-30+10 & 4-42+38 & 4+48-22\end{pmatrix}=E.$$

2) Gauss-Jordan usuli. Matritsaning teskarisini topishning Gauss-Jordan usulida maxsusmas matritsani shu tartibdagi birlik matritsa bilan kengaytiriladi, kengaytirilgan matritsa satrlari orasida elementar almashtirish to kengaytirilgan matritsa birinchi qismida birlik matritsa hosil bo‘lguncha olib boriladi, natijada kengaytirilgan matritsaning ikkinchi qismida berilgan matritsaga teskari bo‘lgan matritsa hosil bo‘ladi. Bu jarayonni qisqacha qilib, Gauss-Jordan modifikatsiyasi (yoki formulasi) ko‘rinishida yozishimiz mumkin: $(A|E)\sim(E|A^{-1})$.

10-misol. $A=\begin{pmatrix}5 & -1 & -1\\ 1 & 2 & 3\\ 4 & 3 & 2\end{pmatrix}$ matritsaga teskari matritsani Gauss-Jordan usulida toping.

Yechish. $(A|E)=\left(\begin{array}{ccc|ccc}5 & -1 & -1 & 1 & 0 & 0\\ 1 & 2 & 3 & 0 & 1 & 0\\ 4 & 3 & 2 & 0 & 0 & 1\end{array}\right)\sim\left(\begin{array}{ccc|ccc}0 & -11 & -16 & 1 & -5 & 0\\ 1 & 2 & 3 & 0 & 1 & 0\\ 0 & -5 & -10 & 0 & -4 & 1\end{array}\right)\sim$

$$\sim\left(\begin{array}{ccc|ccc} 0 & 0 & -30 & -5 & -19 & 11 \\ 5 & 0 & -5 & 0 & -3 & 2 \\ 0 & -5 & -10 & 0 & -4 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 0 & 0 & -30 & -5 & -19 & 11 \\ -30 & 0 & 0 & -5 & -1 & -1 \\ 0 & 15 & 0 & -5 & -7 & 8 \end{array}\right) \sim$$

$$\sim\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/6 & -1/30 & -1/30 \\ 0 & 1 & 0 & -1/5 & -7/15 & 8/15 \\ 0 & 0 & 1 & 1/6 & 19/30 & -11/30 \end{array}\right)$$

Agar umumiy maxrajni matritsa belgisidan tashqariga chiqarsak, teskari matritsaning teskarisi quyidagicha bo‘ladi:

$$A^{-1} = \frac{1}{30}\begin{pmatrix} 5 & -1 & -1 \\ -10 & -14 & 16 \\ 5 & 19 & -11 \end{pmatrix}.$$

4-va5-misollardagi javoblarni taqqoslab, bir hil natijaga ega bo‘lganimizni ko‘rish mumkin.

Amaliy mashg`ulot uchun masalalar

13-misol. $A = \begin{pmatrix} 2 & 3 & 2 \\ 5 & 1 & 4 \\ 1 & -2 & -1 \end{pmatrix}$ matritsa uchun teskari A^{-1} matritsani klassik usulda toping.Klassik usulda teskari matritsa

$$A^{-1} = \frac{1}{|A|}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix} \qquad (2)$$

Formula bo‘yicha hisoblanadi. Bu yerda $|A|$ berilgan matritsa determinanti, $A_{ij}\ (i=1,2,3;\ j=1,2,3)$ A matritsa elementlarining algebraik to‘ldiruvchilari.

$$|A| = \begin{vmatrix} 2 & 3 & 2 \\ 5 & 1 & 4 \\ 1 & -2 & -1 \end{vmatrix} = -2+12-20-2+15+16 = 43-24 = 19 \neq 0$$

Demak A maxsusmas matritsa, va A^{-1} teskari matritsa mavjud. Algebraik to‘ldiruvchilarini hisoblaymiz:

$$A_{11} = \begin{vmatrix} 1 & 4 \\ -2 & -1 \end{vmatrix} = -1 + 8 = 7$$

$$A_{21} = -\begin{vmatrix} 3 & 2 \\ -2 & -1 \end{vmatrix} = -\left(-3 + 4\right) = -1$$

$$A_{31} = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 12 - 2 = 10$$

$$A_{12} = -\begin{vmatrix} 5 & 4 \\ 1 & -1 \end{vmatrix} = -\left(-5 - 4\right) = 9$$

$$A_{22} = \begin{vmatrix} 2 & 2 \\ 1 & -1 \end{vmatrix} = -2 - 2 = -4$$

$$A_{32} = -\begin{vmatrix} 2 & 2 \\ 5 & 4 \end{vmatrix} = -\left(8 - 10\right) = 2$$

$$A_{13} = \begin{vmatrix} 5 & 1 \\ 1 & -2 \end{vmatrix} = -10 - 1 = -11$$

$$A_{23} = -\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -\left(-4 - 3\right) = 7$$

$$A_{33} = \begin{vmatrix} 2 & 3 \\ 5 & 1 \end{vmatrix} = 2 - 15 = -13$$

A_{ij} larni (2) formulaga qo‘yamiz va teskari

$$A^{-1} = 1/19 \begin{pmatrix} 7 & -1 & 10 \\ 9 & -4 & 2 \\ -11 & 7 & -13 \end{pmatrix}$$

matritsani olamiz. Teskari matritsaning to’griligini tekshirish uchun quyidagi tenglikni tekshiramiz;

$$AA^{-1} = A^{-1}A = E \text{ (3)}$$

$$\begin{pmatrix} 2 & 3 & 2 \\ 5 & 1 & 4 \\ 1 & -2 & -1 \end{pmatrix} \cdot 1/19 \begin{pmatrix} 7 & -1 & 10 \\ 9 & -4 & 2 \\ -11 & 7 & -13 \end{pmatrix} =$$

$$=1/19 \cdot \begin{pmatrix} 14+27-22 & -2-12+14 & 20+6-26 \\ 35+9-44 & -5-4+28 & 50+2-52 \\ 7-18+11 & -1+8-7 & 10-4+13 \end{pmatrix} = 1/19 \cdot \begin{pmatrix} 19 & 0 & 0 \\ 0 & 19 & 0 \\ 0 & 0 & 19 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = F$$

Demak, A^{-1} to‘g‘ri topilgan.

14-misol. $A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & -1 & -3 \\ 4 & 3 & -2 \end{pmatrix}$ matritsa uchun A^{-1} matritsani Gauss-Jordan usulida toping:

$|A| = 16 \neq 0$ teskari matritsa mavjud. Berilgan matritsani birlik matritsa hisobida kengaytirib, elementar almashtirishlar bajaramiz, bu usulni to chap tomonda A matritsa o‘rnida birlik matritsa hosil bo‘lguncha davom ettiramiz, o‘ng tomonda hosil bo‘lgan matritsa berilgan matritsaga nisbatan teskari matritsa bo‘ladi.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ -1 & -1 & -3 & 0 & 1 & 0 \\ 4 & 3 & -2 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 1 & 0 \\ 0 & -5 & -6 & -4 & 0 & 1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 1 & 0 \\ 0 & 0 & -16 & 1 & 5 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1/16 & -5/16 & -1/16 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 5 & -1 & -2 & 0 \\ 0 & 1 & 0 & 14/16 & 6/16 & -2/16 \\ 0 & 0 & 1 & -1/16 & -5/16 & -1/16 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -11/16 & -7/16 & 5/16 \\ 0 & 1 & 0 & 14/16 & 6/16 & -2/16 \\ 0 & 0 & 1 & -1/16 & -5/16 & -1/16 \end{array}\right)$$

$$A^{-1} = 1/16 \begin{pmatrix} -11 & -7 & 5 \\ 14 & 6 & -2 \\ -1 & -5 & -1 \end{pmatrix}$$

teskari matritsa to‘g‘ri topilganini (3) formulaga qo‘yib tekshiramiz:

$$AA^{-1} = 1/16 \begin{pmatrix} 1 & 2 & 1 \\ -1 & -1 & -3 \\ 4 & 3 & -2 \end{pmatrix} \cdot \begin{pmatrix} -11 & -7 & -5 \\ 14 & 6 & -2 \\ -1 & -5 & -1 \end{pmatrix} =$$

$$=1/16\begin{pmatrix} -11+28-1 & -7+12-5 & 5-4-1 \\ 11-14+3 & 7-6+15 & -5+2+3 \\ -44+42+2 & -28+18+10 & 20-6+2 \end{pmatrix}=$$

$$=1/16\begin{pmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ demak, teskari matritsa to‘gri topilgan.

15-misol. Matritsa rangini ta’rifga asosan hisoblang:

$$A=\begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix}$$

A matritsa tartibli, demak uning rangi 3 dan yuqori bo‘lmaydi. Uchinchi tartibli minorlarni hisoblaymiz:

$$M_1=\begin{vmatrix} 2 & -1 & 3 \\ 4 & -2 & 5 \\ 2 & -1 & 1 \end{vmatrix}=-4-10-12+12+4+10=0$$

$$M_2=\begin{vmatrix} 2 & -1 & -2 \\ 4 & -2 & 1 \\ 2 & -1 & 8 \end{vmatrix}=-32-2+8-8+32+2=0$$

$$M_3=\begin{vmatrix} 2 & -1 & 4 \\ 4 & -2 & 7 \\ 2 & -1 & 2 \end{vmatrix}=-8-14-16+16+8-14=0$$

$$M_4=\begin{vmatrix} -1 & 3 & -2 \\ -2 & 5 & 1 \\ -1 & 1 & 8 \end{vmatrix}=-40-3+4-10+48+1=0$$

$$M_5=\begin{vmatrix} 3 & -2 & 4 \\ 5 & 1 & 7 \\ 1 & 8 & 2 \end{vmatrix}=6-14+160-4+20-168=0$$

Barcha uchinchi tartibli minorlar nolga teng. Ikkinchi tartibli minorlarni hisoblaymiz:

$$M_1^1=\begin{vmatrix} -1 & 3 \\ -2 & 5 \end{vmatrix}=-5+6=1 \qquad M_1^1\neq 0, r(A)=2$$

Bu usulda noldan farqli minor topilgunga qadar hisoblashlar davom etadi.Shuning uchun tartibi kattaroq matritsa rangini hisoblash bir muncha qiyinchiliklarga olib keladi.

16-misol. Matritsa rangini elementar almashtirish lar yordamida nollar yig‘ib hisoblang: $A=\begin{pmatrix} 25 & 31 & 17 & 43 \\ 75 & 94 & 53 & 132 \\ 75 & 94 & 54 & 134 \\ 25 & 32 & 20 & 48 \end{pmatrix}$

$$A=\begin{pmatrix} 25 & 31 & 17 & 43 \\ 75 & 94 & 53 & 132 \\ 75 & 94 & 54 & 134 \\ 25 & 32 & 20 & 48 \end{pmatrix} \sim \begin{pmatrix} 25 & 31 & 17 & 43 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 3 & 5 \\ 0 & 1 & 3 & 5 \end{pmatrix} \sim \begin{pmatrix} 25 & 31 & 17 & 43 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Bu matritsaning rangi $\begin{pmatrix} 25 & 31 & 17 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$ matritsa rangiga teng.

$$\begin{vmatrix} 25 & 31 & 17 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{vmatrix} = 25 \neq 0 \qquad\qquad r\begin{pmatrix} 25 & 31 & 17 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} = 3$$

Demak, berilganmatritsaningrangiham 3gateng. $r(A)=3$

Mashqlarni bajaring:

17-misol. Berilgan kvadrat matritsaning determinantva rangini toping. Maxsusmas matritsaning teskarisini toping:

$$a)A=\begin{pmatrix} 1 & 2 \\ -2 & 0 \end{pmatrix}; \qquad b)B=\begin{pmatrix} 2 & 5 \\ -4 & 2 \end{pmatrix};$$

$$c)C=\begin{pmatrix} -1 & 0 & 8 \\ 5 & 9 & 0 \\ 0 & 4 & 3 \end{pmatrix}; d)D=\begin{pmatrix} 2 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 2 \end{pmatrix}; e)E=\begin{pmatrix} 1 & 0 & 5 \\ 4 & -2 & -1 \\ 2 & 1 & 3 \end{pmatrix}$$

$$f)F=\begin{pmatrix} 2 & 3 & 4 & 0 \\ 1 & 5 & 7 & 0 \\ 3 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \qquad g)G=\begin{pmatrix} 1 & 2 & 1 & 0 \\ 1 & 1 & 3 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 3 & 0 \end{pmatrix}$$

18-misol.Quyidagi matritsalar rangini minorlar ajratish usuli bilan hisoblang:

$$a)\begin{pmatrix} 3 & 5 & 7 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix};b)\begin{pmatrix} 1 & 2 & 3 & 6 \\ 2 & 3 & 1 & 6 \\ 3 & 1 & 2 & 6 \end{pmatrix};\ c)\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{pmatrix};\ d)\begin{pmatrix} 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & 3 & 0 \end{pmatrix}$$

$$e)\begin{pmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{pmatrix};\ j)\begin{pmatrix} 1 & 2 & 1 & 3 \\ 4 & -1 & -5 & -6 \\ 1 & -3 & -4 & -7 \\ 2 & 1 & -1 & 0 \end{pmatrix};k)\begin{pmatrix} 0 & 2 & -4 \\ -1 & -4 & 5 \\ 3 & 1 & 7 \\ 0 & 5 & -10 \\ 2 & 3 & 0 \end{pmatrix}$$

23.-33-misollarda matritsalar rangini elementar almashtirish usuli bilan hisoblang:

$$23.\begin{pmatrix} 1 & 2 & 1 & 3 & 4 \\ 3 & 4 & 2 & 6 & 8 \\ 1 & 2 & 1 & 8 & 4 \end{pmatrix}; \qquad 24.\begin{pmatrix} 1 & 7 & 5 & 8 & 9 & 2 \\ 3 & 21 & 15 & 24 & 27 & 6 \\ 2 & 14 & 10 & 16 & 18 & 4 \end{pmatrix}$$

$$25.\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{pmatrix}; \qquad 26.\begin{pmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 2 & 0 & 4 & 0 & 0 \end{pmatrix}$$

$$27.\begin{pmatrix} 4 & 3 & -5 & 2 & 3 \\ 8 & 6 & -7 & 4 & 2 \\ 4 & 3 & -8 & 2 & 7 \\ 4 & 3 & 1 & 2 & -5 \\ 8 & 6 & -1 & 4 & -6 \end{pmatrix}; \qquad 28.\begin{pmatrix} 24 & 19 & 36 & 72 & -38 \\ 49 & 40 & 73 & 147 & -80 \\ 73 & 59 & 98 & 219 & -118 \\ 47 & 36 & 71 & 141 & -72 \end{pmatrix}$$

$$29.\begin{pmatrix} 17 & -28 & 45 & 11 & 39 \\ 24 & -37 & 61 & 13 & 50 \\ 25 & -7 & 32 & -18 & -11 \\ 31 & 12 & 19 & -43 & -55 \\ 42 & 13 & 29 & -55 & -68 \end{pmatrix}; \qquad 30.\begin{pmatrix} 47 & -67 & 35 & 201 & 155 \\ 26 & 98 & 23 & -294 & 6 \\ 16 & -428 & 1 & 1284 & 52 \end{pmatrix}$$

$$31.\begin{pmatrix}4 & 5 & 2 & 1 & -3\\ 0 & 2 & 1 & 1 & 2\\ 4 & 7 & 3 & 3 & -1\\ 8 & 12 & 5 & 3 & -4\end{pmatrix};\quad 32.\begin{pmatrix}1 & 3 & 5 & -1\\ 2 & -1 & -3 & 4\\ 5 & 1 & -1 & 7\\ 7 & 7 & 9 & 1\end{pmatrix};\quad 33.\begin{pmatrix}3 & -1 & 3 & 2 & 5\\ 5 & -3 & 2 & 3 & 4\\ 1 & -3 & -5 & 0 & -7\\ 7 & -5 & 1 & 4 & 1\end{pmatrix}$$

34-misol. Berilgan kvadrat matritsalar uchun teskari matritsani ikki usulda toping:

$$a)\begin{pmatrix}-1 & 1\\ 4 & 2\end{pmatrix};\quad b)\begin{pmatrix}1 & 3\\ 2 & 6\end{pmatrix};\quad c)\begin{pmatrix}tg\alpha & 1\\ 2 & ctg\alpha\end{pmatrix};\quad d)\begin{pmatrix}2 & 1 & 1\\ 1 & 0 & 2\\ 3 & 1 & 2\end{pmatrix}$$

$$e)\begin{pmatrix}1 & -1 & 1\\ -38 & 41 & -34\\ 27 & -29 & 24\end{pmatrix};\quad f)\begin{pmatrix}2 & 5 & 7\\ 6 & 3 & 4\\ 5 & -2 & -3\end{pmatrix};\quad j)\begin{pmatrix}3 & -4 & 5\\ 2 & -3 & 1\\ 3 & -5 & -1\end{pmatrix}$$

35-misol. Berilgan kvadrat matritsalar uchun teskari matritsani qulay usulda toping:

$$a)\begin{pmatrix}1 & 5 & 7\\ 3 & 1 & 1\\ 2 & 3 & 4\end{pmatrix};\quad b)\begin{pmatrix}2 & -1 & 7\\ 5 & 3 & 2\\ 1 & 4 & 3\end{pmatrix};\quad c)\begin{pmatrix}1 & 0 & -2\\ 3 & 1 & 0\\ -1 & 2 & 4\end{pmatrix};\quad e)\begin{pmatrix}3 & 2 & 2\\ 1 & 3 & 1\\ 5 & 3 & 4\end{pmatrix}$$

$$d)\begin{pmatrix}1 & 1 & 1 & \dots & 1\\ 0 & 1 & 1 & \dots & 1\\ 0 & 0 & 1 & \dots & 1\\ \dots & \dots & \dots & \dots & \dots\\ 0 & 0 & 0 & \dots & 1\end{pmatrix};\quad f)\begin{pmatrix}1 & 0 & 0 & 0 & \dots & 0 & 0\\ a & 1 & 0 & 0 & \dots & 0 & 0\\ 0 & a & 1 & 0 & \dots & 0 & 0\\ 0 & 0 & a & 1 & \dots & 0 & 0\\ \dots & \dots & \dots & \dots & \dots & \dots & \dots\\ 0 & 0 & 0 & 0 & \dots & a & 1\end{pmatrix};\quad g)\begin{pmatrix}1 & a & a^2 & a^3 & \dots & a^n\\ 0 & 1 & a & a^2 & \dots & a^{n-1}\\ 0 & 0 & 1 & a & \dots & a^{n-2}\\ \dots & \dots & \dots & \dots & \dots & \dots\\ 0 & 0 & 0 & 0 & \dots & 1\end{pmatrix}$$

<u>O`z- o`zini tekshirish uchun savollar</u>

1. Matritsaning rangi deb nimaga aytiladi?
2. Matritsa rangini hisoblashning qanday usullarini bilasiz?
3. Matritsa ustida qanday amallarni bajarganda uning rangi o`zgarmaydi?
4. Matritsa rangini nollar yig`ib aniqlash usuli nimadan iborat?
5. Maxsusmas matritsa deb qanday kvadratik matritsaga aytiladi?

6. Maxsus matritsa deb-chi?

7. Teng tartibli qanday kvadratik matritsalarni ko`paytirganda ko`paytma maxsusmas matritsadan iborat bo`ladi?

8. Maxsusmas matritsaning teskari matritsasi deb qanday matritsaga aytiladi?

12. Nima uchun maxsus matritsaning teskarisi mavjud emas?

9. Kvadratik matritsaning teskari matritsasini qurishning qanday usullarini bilasiz?

10. Teskari matritsa qurishning klassik usuli qanday jarayonlardan iborat? Formulasini yozing.

11. Uchinchi tartibli maxsusmas matritsa teskarisini klassik usulda qurish kengaytirilgan formulasini yozing.

12. Teskari matritsa qurish Jordan usuli algoritmi nimalardan iborat?

13.Teskari matritsaning qanday xossalarini bilasiz?

Chiziqli algebraik tenglamalar sistemasi. Asosiy tushunchalar

Tayanch so‘z va iboralar: *Chiziqli tenglamalar sistemasi(CHTS), tenglamalar sistemasi yechishning qo’shish usuli, o’rniga qo’yish usuli, grafik usuli, yagona yechim, Sistema birgalikda, aniq bo’lmagan sistema, ekvivalent sistema, birgalikda bo’lmagan Sistema, tenglamalar sistemasining iqtisodiyotda qo’llanilishi.*

Reja:

1. Chiziqli tenglamalar sistemasi haqida umumiy tushunchalar.
2. Chiziqli tenglamalar sistemasini iqtisodiyotda qo’llanilishi.

I. CHTS haqida umumiy tushunchalar. Ma’lumki bir necha tenglamalar birgalikda qaralsa, ularga tenglamalar sistemasi deyiladi.

Quyidagi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2, \\ \quad ... \quad ... \quad ... \quad ... \quad ... \quad ... \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m \end{cases} \qquad (1.1)$$

sistemaga n noma’lumli m ta chiziqli algebraik tenglamalar sistemasi (yoki soddalik uchun chiziqli tenglamalar sistemasi) deyiladi. Bu yerda $a_{11}, a_{12}, ..., a_{mn}$ sonlar sistemaning koeffitsientlari, $x_1,\ x_2, \ldots, x_n$ lar noma’lumlar, $b_1, b_2, ..., b_m$ sonlar esa ozod hadlar deyiladi.

1-ta’rif. Agar $\alpha_1, \alpha_2, \ldots, \alpha_n$ sonlar $x_1, x_2, \ldots, x_n$ larning o‘rniga qo‘yilganda (1) sistemadagi tenglamalarni to‘g‘ri tenglikka aylantirsa, bu sonlarga (1) sistemaning yechimlari tizimi deb aytiladiki va $(\alpha_1, \alpha_2, \ldots, \alpha_n)$ kabi belgilanadi.

Tenglamalar sistemasining yechishning bir nechta usullari.

a) Qo’shish usuli.

$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$ 1-tenglamani $\left(-\frac{a_{21}}{a_{11}}\right)$ ga ko’paytirib, 2-tenglamaga qo’shib quyidagini hosil qilamiz. $y = \frac{b_2 a_{11} - b_1 a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$, $x = \frac{b_1 a_{22} - b_2 a_{12}}{a_{11}a_{22} - a_{12}a_{21}}$.

1-misol. $\begin{cases} 5x + 3y = 24 \\ 3x - 2y = 3 \end{cases}$ $y = \frac{3 \cdot 5 - 24 \cdot 3}{5 \cdot (-2) - 3 \cdot 3} = 3$, $x = \frac{24 \cdot (-2) - 3 \cdot 3}{5 \cdot (-2) - 3 \cdot 3} = 3$

Javob: $(x, y) = (3, 3)$.

Mashqlarni bajaring.

Tenglamalar sistemasini qo’shish usulidan foydalanib yeching.

a) $\begin{cases} 7x+3y=10 \\ 3x-2y=1 \end{cases}$ b) $\begin{cases} 6x+3y=15 \\ 3x-2y=4 \end{cases}$ c) $\begin{cases} 2x+3y=13 \\ 5x-3y=1 \end{cases}$.

b) O'rniga qo'yish usuli.

$\begin{cases} a_{11}x+a_{12}y=b_1 \\ a_{21}x+a_{22}y=b_2 \end{cases}$ 1-tenglamadan $x=\frac{1}{a_{11}}(\mathrm{b}_1-\mathrm{a}_{12}\mathrm{y})$ noma'lumni topib, 2-tenglamaga olib borib qo'yamiz $\frac{a_{21}}{a_{11}}(\mathrm{b}_1-\mathrm{a}_{12}\mathrm{y})+\mathrm{a}_{22}\,y=b_2$ natijada $y=\frac{b_2a_{11}-b_1a_{21}}{a_{11}a_{22}-a_{12}a_{21}}$ hosil bo'ladi. Topilgan y ni ixtiyoriy bitta tenglamaga olib borib qo'ysak natijada $x=\frac{b_1a_{22}-b_2a_{12}}{a_{11}a_{22}-a_{12}a_{21}}$ hosil bo'ladi.

2-misol. $\begin{cases} x+3y=8 \\ 2x-y=2 \end{cases}$ 1-tenglamadan $x=8-3y$ noma'lumni topib, 2-tenglamaga olib borib qo'yamiz $2\cdot(8-3\mathrm{y})-\mathrm{y}=2$ natijada $y=2$ hosil bo'ladi. Topilgan y ni ixtiyoriy bitta tenglamaga olib borib qo'ysak natijada $x=2$ hosil bo'ladi. Javob: $(\mathrm{x},\mathrm{y})=(2,2)$.

Mashqlarni bajaring.

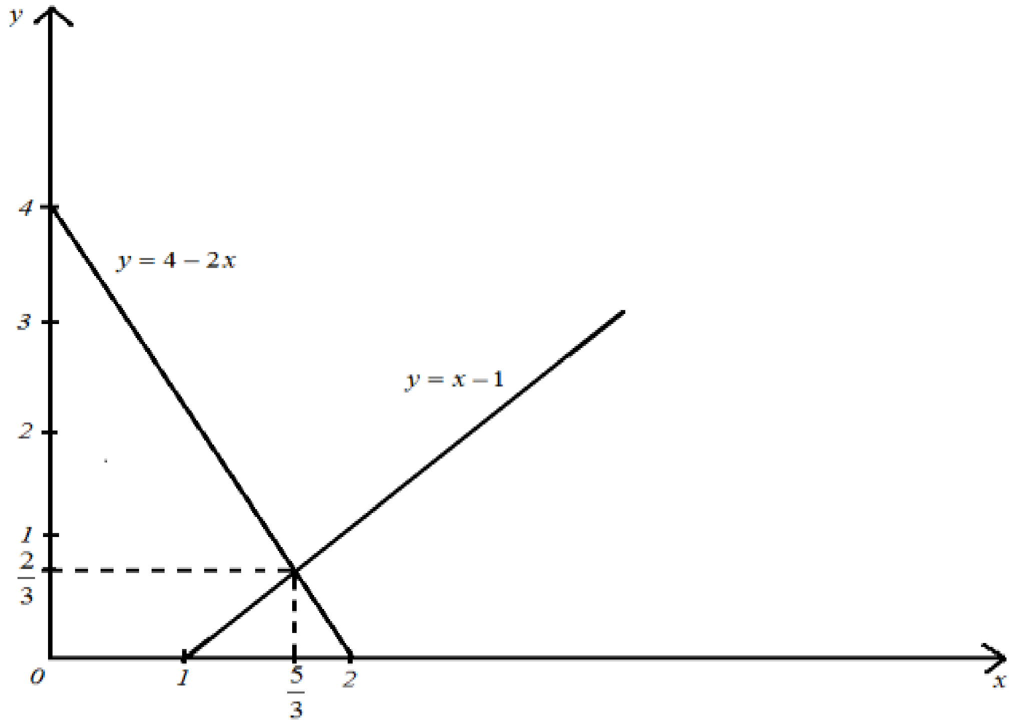
Tenglamalar sistemasini o'rniga qo'yish usulidan foydalanib yeching.

a) $\begin{cases} 6x+3y=9 \\ 3x-2y=1 \end{cases}$ b) $\begin{cases} 4x+3y=11 \\ 3x-2y=4 \end{cases}$ c) $\begin{cases} 8x+3y=19 \\ 5x-3y=7 \end{cases}$.

c) Grafik usuli.

3-misol. $\begin{cases} 2x+y=4 \\ x-y=1 \end{cases}$ sistema nechta yechimga ega?

Yechimni grafik yordamida ko'rsatamiz.

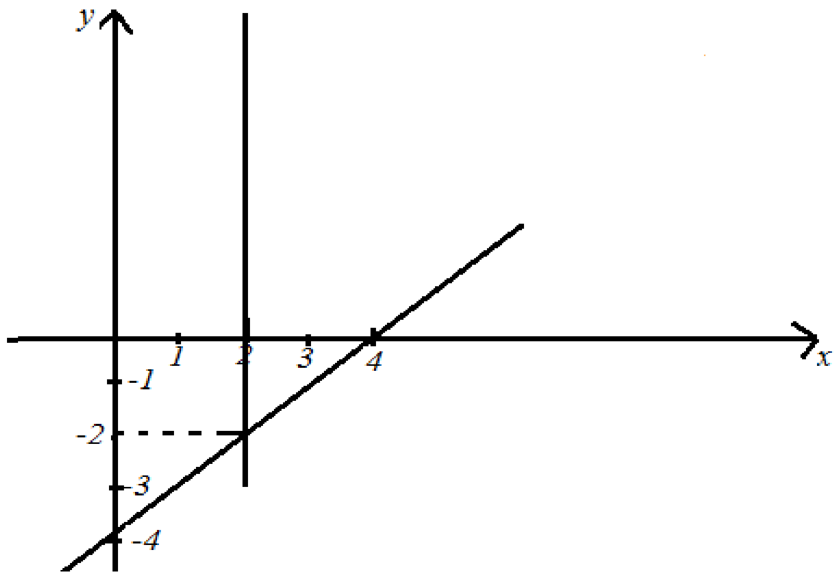


1-rasm.

funksiyalar grafiklari bitta nuqtada kesishdi, demak tenglamalar sistemasi yagona yechimga ega. Javob: $(x.y) = (\frac{5}{3}, \frac{2}{3})$

4-misol. Quyidagi tenglamalar juftligining chizmasini chizing va yechimni toping?

$$\begin{cases} 2x = 4 \\ x - y = 4 \end{cases}$$



Javob: $(x,y)=(2,-2)$.

Mashqlarni bajaring.

Tenglamalar sistemasini grafik usulidan foydalanib yeching.

a) $\begin{cases} 5x+3y=16 \\ 3x-2y=2 \end{cases}$ b) $\begin{cases} 5x+3y=13 \\ 3x-2y=4 \end{cases}$ c) $\begin{cases} 4x+3y=11 \\ 5x-3y=7 \end{cases}$.

Chiziqli tenglamalar sistemasikamida bitta yechimga ega bo‘lsa, u holda bundaysistema birgalikdadeyiladi.

5-misol. $\begin{cases} x-y=2, \\ 2x+y=7 \end{cases}$ sistemabirgalikda va aniq, chunki u yagona $x=3, y=1$ yechimga ega.

Mashqlarni bajaring.

Tenglamalar sistemasini yeching.

a) $\begin{cases} 5x+3y=16 \\ 3x-2y=2 \end{cases}$ b) $\begin{cases} 5x+3y=13 \\ 3x-2y=4 \end{cases}$ c) $\begin{cases} 4x+3y=11 \\ 5x-3y=7 \end{cases}$.

Bitta ham yechimga ega bo‘lmagan chiziqli tenglamalar sistemasi*birgalikda bo‘lmagan sistema*deyiladi.

6-misol. $\begin{cases} x+y+z=1, \\ 3x+3y+3z=5 \end{cases}$ sistema yechimga ega bo‘lmaganligi sababli birgalikda emas.

Mashqlarni bajaring.

Tenglamalar sistemasini yeching.

a) $\begin{cases} 5x+3y=6 \\ 10x+6y=32 \end{cases}$ b) $\begin{cases} 6x-4y=9 \\ 3x-2y=4 \end{cases}$ c) $\begin{cases} 10x-6y=15 \\ 5x-3y=7 \end{cases}$.

Birgalikda bo‘lgan sistema yagona yechimga ega bo‘lsa *aniqsistema* va cheksiz ko‘p yechimga ega bo‘lsa *aniqmas sistema* deyiladi.

7-misol. $\begin{cases} x-y=1, \\ 2x-2y=2, \\ 3x-3y=3 \end{cases}$ sistema esa birgalikda, ammo aniqmas, chunki bu sistema $x=\alpha$, $y=-1+\alpha$ ko‘rinishdagi cheksiz ko‘p yechimga ega, bunda α - ixtiyoriy haqiqiy son.

Mashqlarni bajaring.

Tenglamalar sistemasini yeching.

a) $\begin{cases} 5x+3y=16 \\ 10x+6y=32 \end{cases}$ b) $\begin{cases} 6x-4y=8 \\ 3x-2y=4 \end{cases}$ c) $\begin{cases} 10x-6y=14 \\ 5x-3y=7 \end{cases}$.

Birgalikda bo’lgan tenglamalar sistemasi bir xil yechimlar majmuiga ega bo’lsa, bunday sistemalar ekvivalent deyiladi.

8-misol.

$\begin{cases} 2x+3y=5 \\ x+2y=3 \end{cases}$ (a) tenglamalar sistemaning yechimi $(x,y)=(1,1)$

$\begin{cases} 3x-2y=1 \\ 3x+y=4 \end{cases}$ (b) tenglamalar sistemasining yechimi $(x,y)=(1,1)$

(a) va (b) tenglamalar sistemasi ekvivalent tenglamalar sistemasi deyiladi.

Mashqlarni bajaring.

Tenglamalar sistemalarini ekvivalaent ekanligini ko’rsating.

a) $\begin{cases} 5x+3y=8 \\ x+y=2 \end{cases}$ b) $\begin{cases} 6x-4y=2 \\ x-2y=-1 \end{cases}$ c) $\begin{cases} x-6y=-5 \\ x-3y=-2 \end{cases}$.

Berilgan tenglamalar sistemasining birorta tenglamasini 0 dan farqli songa ko'paytirib, boshqa tenglamasiga hadma-had qo'shish bilan hosil bo'lgan sistema berilgan sistemaga ekvivalent bo'ladi.

9-misol. $\begin{cases} x+3y=5 \\ 3x-y=5 \end{cases}$ (a) tenglamalar sistemadagi 1-tenglamani (-3) ga ko'paytirib 2-tenglamaga qo'shib quyidagini hosil qilamiz. $\begin{cases} x+3y=5 \\ -10y=-10 \end{cases}$ (b) natijada (a) va (b) tenglamalar sistemasi ekvivalent.

Fan va texnikadaning ko'p sohalarida bo'lganidek, iqtisodiyotning ham ko'p masalalarining matematik modellari chiziqli tenglamalar sistemasi orqali ifodalanadi.

II Chiziqli tenglamalar sistemasini iqtisodiyotda qo'llanilishi.

Chiziqli tenglamalar sistemasini tuzishga iqtisodiyotdan misol qaraymiz.

10-misol. Korxona uch xildagi xom-ashyoni ishlatib uch turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish xarakteristikalari 1-jadvalda berilgan.

1-jadval.

Xom-ashyo turlari	Mahsulot turlari bo'yicha xom-ashyo sifatlari			Xom-ashyo zahirasi
	1	2	3	
1	5	12	7	2000
2	10	6	8	1660
3	9	11	4	2070

Berilgan xom-ashyo zahirasini ishlatib, mahsulot turlari bo'yicha ishlab chiqarish hajmini aniqlang.

Yechish: Ishlab chiqarilishi kerak bo'lgan mahsulotlar hajmini mos ravishda x_1, x_2, x_3 lar bilan belgilaymiz. 1-tur mahsulotga, 1-xil xom-ashyo, bittasi uchun sarfi 5 birlik bo'lganligi uchun $5x_1$ 1-tur mahsulot ishlab chiqarish uchun ketgan 1-xil xom-ashyoning sarfini bildiradi. Xuddi shunday 2,3-tur mahsulotlarni ishlab chiqarish uchun ketgan 1-xil xom-ashyo sarflari mos ravishda $12x_2$, $7x_3$ bo'lib, uning uchun quyidagi tenglama o'rinli bo'ladi:

$5x_1 + 12x_2 + 7x_3 = 2000$. Yuqoridagiga o'xshash 2,3-xil xom-ashyolar uchun

$$10x_1 + 6x_2 + 8x_3 = 1660,$$

$$9x_1 + 11x_2 + 4x_3 = 2070$$

Tenglamalar hosil bo'ladi. Demak, masala shartlarida quyidagi uch noma'lumli uchta chiziqli tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} 5x_1 + 12x_2 + 7x_3 = 2000, \\ 10x_1 + 6x_2 + 8x_3 = 1660, \\ 9x_1 + 11x_2 + 4x_3 = 2070 \end{cases}$$

Bu masalaning matematik modeli uch noma'lumli chiziqli tenglamalar sistemasidan iborat bo'ladi.

Mashqlarni bajaring.

a) Korxona uch xildagi xom-ashyoni ishlatib uch turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish xarakteristikalari 2-jadvalda berilgan.

2-jadval.

Xom-ashyo turlari	Mahsulot turlari bo'yicha xom-ashyo sifatlari			Xom-ashyo zahirasi
	1	2	3	

1	2	3	7	20
2	1	6	8	19
3	3	11	4	29

Berilgan xom-ashyo zahirasini ishlatib, mahsulot turlari bo'yicha ishlab chiqarish hajmini aniqlang.

b) Korxona uch xildagi xom ashyoni ishlatib uch turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish xarakteristikalari 3-jadvalda berilgan.

3-jadval.

Xom-ashyo turlari	Mahsulot turlari bo'yicha xom-ashyo sifatlari			Xom-ashyo zahirasi
	1	2	3	
1	4	3	7	21
2	3	6	8	26
3	3	1	4	12

Berilgan xom-ashyo zahirasini ishlatib, mahsulot turlari bo'yicha ishlab chiqarish hajmini aniqlang.

c) Korxona uch xildagi xom-ashyoni ishlatib uch turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish xarakteristikalari 4-jadvalda berilgan.

4-jadval.

Xom-ashyo turlari	Mahsulot turlari bo'yicha xom-ashyo sifatlari			Xom-ashyo zahirasi
	1	2	3	

1	2	3	7	12
2	1	6	8	15
3	2	1	4	7

Berilgan xom-ashyo zahirasini ishlatib, mahsulot turlari bo'yicha ishlab chiqarish hajmini aniqlang.

Ikki bozordagi muvozanat.

Ko'p bozorli muvozanat modelida , tenglamalar sistemasi har bir bozordagi talab taklifni aniqlik bilan eslatadi ya'ni talab va taklif har bir bozorda , boshqa bozordagi narxlarga bog'liq. Masalan, kofega bo'lgan talab , faqat kofening narxiga bog'liq emas , shuningdek o'rin bosuvchi tovar bo'lgan choyning ham narxiga bog'liq. Mashinaga talab uning narxiga bog'liq va shuningdek to'ldiruvchi tovar bo'lgan uning yoqilg'isiga ham bog'liq . Korxonalarning taklifi turli ko'rinishdagi tovarlar narxiga bog'liq. Masalan, biror firma ishlab chiqargam mahsulot, boshqasi uchun xom-ashyo material bo'lishi mumkin.

Korxonalar taklifi ishlab chiqarishda aniqlangan narx orqali aniqlanadi hamda

Shuningdek boshqa korxona ishlab chiqargan material narxi orqali ham aniqlanadi.

Yalpi ikki Tovar bog'liqlik modeli.

$$\left.\begin{aligned} q_1^s &= \alpha_1 + \beta_{11}p_1 + \beta_{12}p_2 \\ q_2^s &= \alpha_2 + \beta_{21}p_1 + \beta_{22}p_2 \end{aligned}\right\} \text{taklif} \qquad (1.2)$$

$$\left.\begin{aligned} q_1^d &= a_1 + b_{11}p_1 + b_{12}p_2 \\ q_2^d &= a_2 + b_{21}p_1 + b_{22}p_2 \end{aligned}\right\} \text{talab} \qquad (1.3)$$

Natijada masalan $\beta_{12} < 0$ ikinchi firmagi materiallar narxi o'sishi, birinchi firmani material sarfini kamaytiradi, natijada esa birinchi firma ishlab chiqarishni kamaytiradi. Har bir bozordagi talab va taklifning tengligi ni o'rnatilishi muvozanat narxlar bo'lgan p_1 va p_2 larni aniqlash uchun ikki tenglamalar sistemasini beradi. $b_{ij} \cdot s$ va $\beta_{ij} \cdot s$ lar nolga teng ham bo'lishi mumkin.

Bu tenglamalar modelning asosini tashkil etadi va strukturali tenglik deb ataladi.

$$(b_{11} - \beta_{11}) \cdot p_1 + (b_{12} - \beta_{12}) \cdot p_2 = \alpha_2 - \alpha_1$$

$$(b_{21} - \beta_{21}) \cdot p_1 + (b_{22} - \beta_{22}) \cdot p_2 = \alpha_2 - \alpha_1$$

Ikkinchisidan $p_1$ni topsak

$$p_1 = \frac{(\alpha_2 - \alpha_1) - (b_{22} - \beta_{22}) \cdot p_2}{(b_{21} - \beta_{21})}$$

Endi buni birinchi tenglikka qo'yamiz;

$$p_2 = \frac{(b_{11} - \beta_{11}) \cdot (\alpha_2 - a_2) - (b_{21} - \beta_{21}) \cdot (\alpha_1 - a_1)}{(b_{11} - \beta_{11}) \cdot (b_{22} - \beta_{22}) - (b_{21} - \beta_{21}) \cdot (b_{12} - \beta_{12})} \qquad (1.4)$$

p_1 ni topsak

$$p_2 = \frac{(b_{22} - \beta_{22}) \cdot (\alpha_1 - a_1) - (b_{12} - \beta_{12}) \cdot (b_{12} - \beta_{12})}{(b_{11} - \beta_{11}) \cdot (b_{22} - \beta_{22}) - (b_{21} - \beta_{21}) \cdot (b_{12} - \beta_{12})} \qquad (1.5)$$

p_1 va p_2 larni bunday tariflash (kamaytirilgan forma, deb ataladi. Chunki ular faqat modelning ko'rsatkichlariga bo'g'liq. $a_i, \alpha_i, b_{ij}, \beta_{ij}$ $i, j = 1,2$ larning alohida parametrlari uchun biz p_i ning qiymatlarini topaolamiz. Keyinchalik bu qiymatni (1.2) va (1.3) ga qo'yib muvozanat miqdorni topaolamiz. Keyingi misollarda bu qimatni qanday toppish ko'rsatilgan.

To'ldiruvchi tovarlar uchun ikki bozor muvozanati

Faraz qilaylik iste'molchilar bozorida o'rin bosadigan tovarlarga talab taklif tengligi quyidagicha

$q_1^s = -1 + p_1,\ \ q_1^d = 20 - 2p_1 - p_2$ 1-tovar

$q_2^s = p_2,\ \ q_2^d = 40 - 2p_2 - p_1$ 2-tovar

Bu yerda q_i^s va q_i^d talab va taklif miqdori p_i Tovar narxlari.bu tovarlarning o'rin bosar ekanligidan agar birinchi Tovar ga talab kamaysa, ikkinchi Tovar narxi ko'tariladi . Endi muvozanat narxni toping (ikki Tovar uchun)

Yechim; $q_i^s = q_i^d$ tengligidan ikkita tenglik kelib chiqadi

$$3p_1 + p_2 = 21$$

$$p_1 + 3p_2 = 40$$

Ikkinchisidan $p_1 = 40 - 3p_2$ topib, birinchisiga qo'ysak

$$3\cdot(40 - 3p_2) + p_2 = 21 \Rightarrow 8p_2 = 99 \Rightarrow p_2 = 12{,}375$$

va $p_1 = 2{,}875$ ekanligi keladi. Natijada faqat va faqat bu narx bozordagi muvozanat narxni beradi.

Mashqlarni bajaring.

Tenglamalar sistemasini o'rniga qo'yish, qo'shish va grafik usullaridan foydalanib yeching.

1) $\begin{cases} 9x + 3y = 3 \\ 3x - 2y = -2 \end{cases}$ 2) $\begin{cases} 5x + 3y = 8 \\ 3x - 2y = 1 \end{cases}$ 3) $\begin{cases} 2x + y = 6 \\ 5x - 3y = 4 \end{cases}$.

4) $\begin{cases} 5x + 5y = 10 \\ 3x - 2y = 1 \end{cases}$ 5) $\begin{cases} 4x + 3y = 1 \\ 3x - 2y = 5 \end{cases}$ 6) $\begin{cases} 2x + 3y = 20 \\ 5x - 3y = 8 \end{cases}$.

7) $\begin{cases} x + 3y = 9 \\ 3x - 2y = 6 \end{cases}$ 8) $\begin{cases} 5x + 3y = 13 \\ 3x - 2y = 4 \end{cases}$ 9) $\begin{cases} 4x + 3y = 11 \\ 5x - 3y = 7 \end{cases}$.

10) $\begin{cases} 5x+3y=16 \\ 3x-2y=2 \end{cases}$ 11) $\begin{cases} 5x+3y=13 \\ 3x-2y=4 \end{cases}$ 12) $\begin{cases} 4x+3y=11 \\ 5x-3y=7 \end{cases}$.

13) $\begin{cases} 5x+3y=16 \\ 10x+6y=32 \end{cases}$ 14) $\begin{cases} 6x-4y=8 \\ 3x-2y=4 \end{cases}$ 15) $\begin{cases} 10x-6y=14 \\ 5x-3y=7 \end{cases}$.

c) Korxona uch xildagi xom ashyoni ishlatib uch turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish xarakteristikalari 5-jadvalda berilgan.

5-jadval.

Xom-ashyo turlari	Mahsulot turlari bo'yicha xom-ashyo sifatlari			Xom-ashyo zahirasi
	1	2	3	
1	3	3	7	13
2	5	6	8	19
3	2	2	4	8

Berilgan xom-ashyo zahirasini ishlatib, mahsulot turlari bo'yicha ishlab chiqarish hajmini aniqlang.

c) Korxona uch xildagi xom ashyoni ishlatib uch turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish xarakteristikalari 6-jadvalda berilgan.

6-jadval.

Xom-ashyo turlari	Mahsulot turlari bo'yicha xom-ashyo sifatlari			Xom-ashyo zahirasi
	1	2	3	
1	5	3	7	15

2	4	6	8	18
3	4	2	4	10

Berilgan xom-ashyo zahirasini ishlatib, mahsulot turlari bo'yicha ishlab chiqarish hajmini aniqlang.

c) Korxona uch xildagi xom ashyoni ishlatib uch turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish xarakteristikalari 7-jadvalda berilgan.

7-jadval.

Xom-ashyo turlari	Mahsulot turlari bo'yicha xom-ashyo sifatlari			Xom-ashyo zahirasi
	1	2	3	
1	5	3	8	16
2	6	6	8	20
3	2	5	3	10

Berilgan xom-ashyo zahirasini ishlatib, mahsulot turlari bo'yicha ishlab chiqarish hajmini aniqlang.

Xom-ashyo turlari	Mahsulot turlari bo'yicha xom-ashyo sarflari			Xom-ashyo zahirasi
	1	2	3	
S_1	3	2	2	9
S_2	2	1	5	9
S_3	5	3	6	17

Chiziqli tenglamalar sistemasini yechishning Kramer va teskari matritsalar usuli

Tayanch so'z va iboralar:*chiziqli tenglamalar sistemasini yechishning Kramer va matritsalarusuli,iqtisodiyotda qo'llanilishi..*

Reja:

1. Chiziqli tenglamalar sistemasini yechishningKramer qoidasi.
2. Chiziqli tenglamalar sistemasini yechishningteskari matritsalar qoidasi.
3. Chiziqli tenlamalar sistemasining yechishning Kramer va teskari matritsalar qoidasini iqtisodiyotda qo'llanilishi.

I. Chiziqli tenglamalar sistemasining yechishning Kramer qoidasi.

1-teorema. (Kramer teoremasi) Agar n ta noma'lumli n ta

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots. + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots. + a_{2n}x_n = b_2, \\ \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots. + a_{nn}x_n = b_n. \end{cases} \qquad (1.1)$$

chiziqli tenglamalar sistemaning Δ determinanti noldan farqli bo'lsa, u holda (1) sistema yagonayechimga ega bo'ladi va bu yechim quyidagi formulalar bilan topiladi:

$$\begin{cases} x_1 = \dfrac{\Delta x_1}{\Delta}, \\ x_1 = \dfrac{\Delta x_1}{\Delta}, \\ \ldots\ldots\ldots \\ x_n = \dfrac{\Delta x_n}{\Delta} \end{cases} \qquad (2)$$

bu yerda Δx_1, Δx_2, …, Δx_n determinantlar Δ determinantda noma'lum oldidagi koeffitsientlarni mos ravishda ozod hadlar bilan almashtirish orqali hosil qilinadi.

(2) formulalarga *Kramerformulalari*deyiladi.

1-Misol. Quyidagi

$$\begin{cases} -2x_1 + x_2 - x_3 = 7, \\ 4x_1 + 2x_2 + 3x_3 = -5, \\ x_1 + 3x_2 - 2x_3 = 1 \end{cases}$$

Chiziqlitenglamalar sistemasining yechimini Kramer formulalari yordamida

toping.

Yechish. Sistemaning asosiy determinanti Δ ni hisoblaymiz. Bunda

$\Delta = \begin{vmatrix} -2 & 1 & -1 \\ 4 & 2 & 3 \\ 1 & 3 & -2 \end{vmatrix} = 27$. $\Delta \neq 0$ bo'lganligi sababli berilgan sistema aniq sistemani tashkil qiladi va u yagona yechimga ega bo'ladi. Bu yechim Kramer formulalari yordamida quyidagicha topiladi:

$$x_1 = \frac{\Delta x_1}{\Delta} = \frac{\begin{vmatrix} 7 & 1 & -1 \\ -5 & 2 & 3 \\ 1 & 3 & -2 \end{vmatrix}}{27} = -\frac{81}{27} = -3,$$

$$x_2 = \frac{\Delta x_2}{\Delta} = \frac{\begin{vmatrix} -2 & 7 & -1 \\ 4 & -5 & 3 \\ 1 & 1 & -2 \end{vmatrix}}{27} = \frac{54}{27} = 2,$$

$$x_3 = \frac{\Delta x_3}{\Delta} = \frac{\begin{vmatrix} -2 & 1 & 7 \\ 4 & 2 & -5 \\ 1 & 3 & 1 \end{vmatrix}}{27} = \frac{27}{27} = 1.$$

Demak, tenglamalar sistemaning yechimi: (-3; 2; 1).

Mashqlarni bajaring.

a) $\begin{cases} 7x_1 - 5x_2 = 31 \\ 4x_1 + 11x_3 = -43 \\ 2x_1 + 3x_2 + 4x_3 = -20 \end{cases}$ b) $\begin{cases} x_1 + 2x_2 + 4x_3 = 31 \\ 5x_1 + x_2 + 2x_3 = 20 \\ 3x_1 - x_2 + x_3 = 9 \end{cases}$ c) $\begin{cases} x_1 - x_2 + x_3 = 6 \\ 2x_1 + x \ + x_3 = 3 \\ x_1 + x_2 + x_3 = 5 \end{cases}$

d) $\begin{cases} x_1 + x_2 - x_3 = 2 \\ -2x_1 + x_2 + x_3 = 0 \\ x \ + x_2 + x_3 = 6 \end{cases}$ e) $\begin{cases} x_1 + 2x_2 + x_3 = 10, \\ 2x_1 + x_2 + x_3 = 20, \\ x_1 + 3x_2 + x_3 = 30 \end{cases}$

Agar $\Delta = 0$ bo‘lib, Δx_1, Δx_2, …, Δx_n lardan birortasi noldan farqli bo‘lsa, u holda (1) sistema yechimga ega bo‘lmaydi.

2-Misol Quyidagi

$$\begin{cases} 2x_1 + x_2 + 3x_3 = 3, \\ 4x_1 + 2x_2 + 6x_3 = 5, \\ 3x_1 + 2x_2 = 2 \end{cases}$$

Chiziqli tenglamalar sistemasining yechimini Kramer formulalari yordamida toping.

Yechish. tenglamalar Sistemaning asosiy determinanti Δ ni hisoblaymiz. Bunda

$\Delta = \begin{vmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 3 & 2 & 0 \end{vmatrix} = 0$. $\Delta = 0$ bo‘lganligisababli berilgan sistemani Δx_1, Δx_2, Δx_3 lari hisoblaymiz. Bu yechim Kramer formulalari yordamida quyidagicha topiladi:

$$\Delta x_1 = \begin{vmatrix} 3 & 1 & 3 \\ 5 & 2 & 6 \\ 2 & 2 & 0 \end{vmatrix} = -6,$$

$$\Delta x_2 = \begin{vmatrix} 2 & 3 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 0 \end{vmatrix} = 9,$$

$$\Delta x_3 = \begin{vmatrix} 2 & 1 & 3 \\ 4 & 2 & 5 \\ 3 & 2 & 2 \end{vmatrix} = 1.$$

Demak, tenglamalar sistemaning yechimga ega emas chunki $\Delta = 0$ va $\Delta x_1 \neq 0$, $\Delta x_2 \neq 0$, $\Delta x_3 \neq 0$.:

Mashqlarni bajaring.

a) $\begin{cases} 2x_1 + x_2 + 3x_3 = 3, \\ 4x_1 + 2x_2 + 6x_3 = 5, \\ 3x_1 + 2x_2 + x_3 = 2. \end{cases}$ b) $\begin{cases} 4x_1 + x_2 + 3x_3 = 3, \\ 8x_1 + 2x_2 + 6x_3 = 5, \\ 3x_1 + 2x_2 + 2x_3 = 2. \end{cases}$ c) $\begin{cases} x_1 + x_2 + 3x_3 = 3, \\ 2x_1 + 2x_2 + 6x_3 = 5, \\ 3x_1 + 2x_2 + 3x_3 = 2. \end{cases}$

Agar $\Delta = 0$ bo‘lib, $\Delta x_1 = \Delta x_2 = ... = \Delta x_n = 0$ bo‘lsa, u holda (1) sistema cheksiz ko‘p yechimga ega bo‘ladi.

3-Misol Quyidagi

$$\begin{cases} 2x_1 + x_2 + 3x_3 = 3, \\ 4x_1 + 2x_2 + 6x_3 = 6, \\ 6x_1 + 3x_2 + 9x_3 = 9. \end{cases}$$

Chiziqli tenglamalar sistemasining yechimini Kramer formulalari yordamida toping.

Yechish. Sistemaning asosiy determinanti Δ ni hisoblaymiz. Bunda

$\Delta = \begin{vmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 6 & 3 & 9 \end{vmatrix} = 0$. $\Delta = 0$ bo‘lganligisababli berilgan sistemani Δx_1, Δx_2, Δx_3 lari hisoblaymiz. Bu yechim Kramer formulalari yordamida quyidagicha topiladi:

$$\Delta x_1 = \begin{vmatrix} 3 & 1 & 3 \\ 6 & 2 & 6 \\ 9 & 3 & 9 \end{vmatrix} = 0,$$

$$\Delta x_2 = \begin{vmatrix} 2 & 3 & 3 \\ 4 & 6 & 6 \\ 6 & 9 & 9 \end{vmatrix} = 0,$$

$$\Delta x_3 = \begin{vmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 6 & 3 & 9 \end{vmatrix} = 0.$$

Demak, tenglamalar sistemaning yechimga cheksiz ko'p yechimga ega chunki $\Delta = 0$ va $\Delta x_1 = 0$, $\Delta x_2 = 0$, $\Delta x_3 = 0$:

Mashqlarni bajaring.

a) $\begin{cases} x_1 + x_2 + 3x_3 = 3, \\ 4x_1 + 4x_2 + 12x_3 = 12, \\ 3x_1 + 3x_2 + 9x_3 = 6. \end{cases}$ b) $\begin{cases} 4x_1 + x_2 + 3x_3 = 3, \\ 8x_1 + 2x_2 + 6x_3 = 6, \\ 12x_1 + 3x_2 + 9x_3 = 9. \end{cases}$ c) $\begin{cases} x_1 + 2x_2 + 3x_3 = 4, \\ 2x_1 + 4x_2 + 6x_3 = 8, \\ 3x_1 + 6x_2 + 9x_3 = 12. \end{cases}$

Kramer formulalari asosan nazariy jihatdan ahamiyatga ega. Agar sistemada noma'lumlar soni ko‘p bo‘lsa, bu usulda yechilganda katta va og‘ir hisoblashlarni bajarishga olib keladi. Lekin, bu formulalar muhim afzallikka ega: ular barcha noma'lumlarning qiymatlarini aniq ifodalaydi.

II Chiziqli tenglamalar sistemasini yechishningteskari matritsalar qoidasi.

Ushbu n noma'lumli n ta chiziqli tenglamalar sistemasi berilgan bo‘lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots. + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots. + a_{2n}x_n = b_2, \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots. + a_{nn}x_n = b_n. \end{cases} \qquad (1.1)$$

(1.1) tenglamalar sistemada quyidagi belgilashlarni kiritamiz:

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix},\ X=\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix},\ B=\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}.$$

Bunda, A – noma'lumlar oldida turgan koeffisientlardan tuzilgan matrisa;
X – noma'lumlardan tuzilgan matrisa; B – ozod hadlardan tuzilgan matrisa.
U holda (1.1) tenglamalar sistemani

$$AX=B$$

ko'rinishda ifodalash mumkin.

Faraz qilamiz $\det|A|\neq 0$ bo'lsin. U holda A matrisa uchun A^{-1} teskari matrisa mavjud. $AX=B$ tenglikning har ikkala tomonini A^{-1} ga chapdan ko'paytiramiz:

$$A^{-1}AX=A^{-1}B,\ EX=A^{-1}B,\ X=A^{-1}B.$$

Hosil bo'lgan $X=A^{-1}B$ ifodachiziqli tenglamalar sistemasining **matrisalar qoidasi bilan yechish** formulasidan iborat.

4-Misol. Chiziqlitenglamalar sistemasini matrisalarusuli bilan yeching:

$$\begin{cases} 2x_1+2x_2-3x_3=5, \\ x_1-x_2+x_3=0, \\ 3x_1+x_2+x_3=2. \end{cases}$$

Yechish: A, X, B matrisalarni tuzib olamiz:

$$A=\begin{pmatrix} 2 & 2 & -3 \\ 1 & -1 & 1 \\ 3 & 1 & 1 \end{pmatrix},\ X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix},\ B=\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}.$$

Bundan, $\det|A|=-12\neq 0.$

Teskari matrisani topamiz:

$$A_{11}=\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}=-2,\ A_{12}=\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix}=2,\ A_{13}=\begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix}=4,$$

$$A_{21}=\begin{vmatrix}2 & -3\\ 1 & 1\end{vmatrix}=-5,\ A_{22}=\begin{vmatrix}2 & -3\\ 3 & 1\end{vmatrix}=11,\ A_{23}=\begin{vmatrix}2 & 2\\ 3 & 1\end{vmatrix}=4,$$

$$A_{31}=\begin{vmatrix}2 & -3\\ -1 & 1\end{vmatrix}=-1,\ A_{11}=\begin{vmatrix}2 & -3\\ 1 & 1\end{vmatrix}=-5,\ A_{33}=\begin{vmatrix}2 & 2\\ 1 & -1\end{vmatrix}=-4,$$

$$A^{-1}=-\frac{1}{12}\begin{pmatrix}-2 & -5 & -1\\ 2 & 11 & -5\\ 4 & 4 & -4\end{pmatrix}.$$

Bundan,

$$X=A^{-1}B=-\frac{1}{12}\begin{pmatrix}-2 & -5 & -1\\ 2 & 11 & -5\\ 4 & 4 & -4\end{pmatrix}\cdot\begin{pmatrix}5\\ 0\\ 2\end{pmatrix}=-\frac{1}{12}\begin{pmatrix}-10+0-2\\ 10+0-10\\ 20+0-8\end{pmatrix}=-\frac{1}{12}\begin{pmatrix}-12\\ 0\\ 12\end{pmatrix}=\begin{pmatrix}1\\ 0\\ -1\end{pmatrix}$$

Demak, $x_1=1$, $x_2=0$, $x_3=-1$ yoki $(1;0;-1)$.

Mashqlarni bajaring.

a) $\begin{cases}2x_1+x_2+2x_3=6\\ 3x_1+2x_3=8\\ x_1+x_2-x_3=1\end{cases}$ b) $\begin{cases}3x_2+2x_3=5\\ 2x_1+x_2-x_3=4\\ -x_2+2x_3=1\end{cases}$ c) $\begin{cases}x_1+2x_3=5\\ 3x_1-x_2=9\\ x_1+x_2-2x_3=1\end{cases}$

d) $\begin{cases}x_1+x_2+2x_3=4\\ 2x_2+x_3=3\\ x_1-x_2=2\end{cases}$

5-Misol. Ushbu chiziqli tenglamalar sistemasini teskari matritsa usulida yeching:

$$\begin{cases}x_1-2x_2+3x_3-5x_4=2,\\ 2x_1+x_2+4x_3+x_4=-3,\\ 3x_1-3x_2+8x_3-2x_4=-1,\\ 2x_1-2x_2+5x_3-12x_4=4\end{cases}$$

Yechish. tenglamalarSistema matrisasi$A$ va kengaytirilgan matrisasi $(A|B)$

$$A=\begin{pmatrix}1&-2&3&-5\\2&1&4&1\\3&-3&8&-2\\2&-2&5&-12\end{pmatrix},\ (A|B)=\left(\begin{array}{cccc|c}1&-2&3&-5&2\\2&1&4&1&-3\\3&-3&8&-2&-1\\2&-2&5&-12&4\end{array}\right)$$

larning rangini topib

$$\left(\begin{array}{cccc|c}1&-2&3&-5&2\\2&1&4&1&-3\\3&-3&8&-2&-1\\2&-2&5&-12&4\end{array}\right)\Rightarrow\left(\begin{array}{cccc|c}1&-2&3&-5&2\\0&5&-2&11&-7\\0&3&-1&13&-7\\0&2&-1&-2&0\end{array}\right)\Rightarrow$$

$$\Rightarrow\left(\begin{array}{cccc|c}1&-2&3&-5&2\\0&5&-2&11&-7\\0&0&1&32&-14\\0&0&-1&-32&14\end{array}\right)\Rightarrow\left(\begin{array}{cccc|c}1&-2&3&-5&2\\0&5&-2&11&-7\\0&0&1&32&-14\\0&0&0&0&0\end{array}\right)$$

$r(A)=r(A|B)=3$ ekanligini ko‘ramiz. Uning minori

$$\Delta=\begin{vmatrix}1&-2&3\\2&1&4\\3&-3&8\end{vmatrix}=8-18-24-9+32+12=1$$

noldan farqli. Shuning uchun to‘rtinchi tenglamani tashlab yuboramiz, qolgan tenglamalarda x_4 qatnashgan hadlarni o‘ng tomonga o‘tkazamiz

$$\begin{cases}x_1-2x_2+3x_3=2+5x_4,\\2x_1+x_2+4x_3=-3-x_4,\\3x_1-3x_2+8x_3=-1+2x_4.\end{cases}$$

Bu sistemani teskari matritsa usuli bilan yechamiz. Avval asosiy matritsa teskarisini Gauss – Jordan usulida topamiz:

$$\left(\begin{array}{ccc|ccc}1&-2&3&1&0&0\\2&1&4&0&1&0\\3&-3&8&0&0&1\end{array}\right)\Rightarrow\left(\begin{array}{ccc|ccc}1&-2&3&1&0&0\\0&5&-2&-2&1&0\\0&3&-1&-3&0&1\end{array}\right)\Rightarrow\left(\begin{array}{ccc|ccc}1&7&0&-8&0&3\\0&-1&0&4&1&-2\\0&-3&1&3&0&-1\end{array}\right)\Rightarrow$$

$$\Rightarrow\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 20 & 7 & -11 \\ 0 & 1 & 0 & -4 & -1 & 2 \\ 0 & 0 & 1 & -9 & -3 & 5 \end{array}\right), \quad A^{-1}=\begin{pmatrix} 20 & 7 & -11 \\ -4 & -1 & 2 \\ -9 & -3 & 5 \end{pmatrix}.$$

Tenglamalar sistemasini umumiy yechimni topaish uchun $X = A^{-1}\cdot B$ amalni bajaramiz:

$$X=\begin{pmatrix} 20 & 7 & -11 \\ -4 & -1 & 2 \\ -9 & -3 & 5 \end{pmatrix}\cdot\begin{pmatrix} 2+5x_4 \\ -3-x_4 \\ -1+2x_4 \end{pmatrix}=\begin{pmatrix} 10+71x_4 \\ -7-15x_4 \\ -14-32x_4 \end{pmatrix}$$

Javob: $\left(30+71x_4;\,-7-15x_4;\,-14-32x_4;\,x_4\right),\ \ x_4\in R$

x_4 ga ixtiyoriy qiymatlar berib $x_1,\ x_2,\ x_3$ noma’lumlarning mos qiymatlarini topamiz. Sistema cheksiz ko‘p yechimga ega.

Mashqlarni bajaring. Ushbu chiziqli tenglamalar sistemasini teskari matritsa usulida yeching:

a) $\begin{cases} x_1+x_2+3x_3=3, \\ 4x_1+4x_2+12x_3=12, \\ 2x_1+3x_2+4x_3=5. \end{cases}$ b) $\begin{cases} 4x_1+x_2+3x_3=3, \\ 8x_1+2x_2+6x_3=6, \\ 2x_1+x_2+5x_3=4. \end{cases}$ c) $\begin{cases} x_1+2x_2+3x_3=4, \\ 2x_1+4x_2+6x_3=8, \\ 2x_1+3x_2+5x_3=12. \end{cases}$

6-Misol. Quyidagi tenglamani yeching:

$$\begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix} X \begin{pmatrix} 1 & 3 \\ 3 & 6 \end{pmatrix}=\begin{pmatrix} 4 & 0 \\ 5 & 8 \end{pmatrix}$$

Yechish: Tenglamaga quyidagi belgilashlarni kiritamiz:

$$A=\begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix},\ B=\begin{pmatrix} 1 & 3 \\ 3 & 6 \end{pmatrix},\ C_1=\begin{pmatrix} 4 & 0 \\ 5 & 8 \end{pmatrix}.$$

Uholda berilgan tenglama

$$A\cdot X\cdot B=C$$

ko‘rinishni oladi.

Agar AXB ifodaning chap tomondan A^{-1} va o‘ng tomondan B^{-1} ga ko‘paytirsak, hamda $A^{-1}A = E$, $EX = X$, $BB^{-1} = E$ va $XE = X$ ekanligini hisobga olsak quyidagi yechimga ega bo‘lamiz:

$$X = A^{-1}CB^{-1} = -\frac{1}{2}\begin{pmatrix} 1 & -1 \\ -2 & 0 \end{pmatrix}\cdot\begin{pmatrix} 4 & 0 \\ 5 & 8 \end{pmatrix}\cdot\begin{pmatrix} -2 & 1 \\ 1 & -\frac{1}{3} \end{pmatrix} =$$

$$= -\frac{1}{2}\begin{pmatrix} -1 & -8 \\ -8 & 0 \end{pmatrix}\cdot\begin{pmatrix} -2 & 1 \\ 1 & -\frac{1}{3} \end{pmatrix} = \begin{pmatrix} 3 & -\frac{5}{6} \\ -8 & 4 \end{pmatrix}.$$

Mashqlarni bajaring.

Quyidagi tenglamani yeching:

a) $\begin{pmatrix} 0 & 1 \\ 3 & 1 \end{pmatrix} X \begin{pmatrix} 1 & 3 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 2 & 8 \end{pmatrix}$, b) $\begin{pmatrix} 0 & 1 \\ 5 & 1 \end{pmatrix} X \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 2 & 5 \end{pmatrix}$,

c) $\begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix} X \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 2 & 1 \end{pmatrix}$

Agar sistemada $m \neq n$ va $r(A) \neq m$ bo‘lib, $r(A) = r(A|B)$ bo‘lgan holda ham teskari matritsa usulidan foydalanib uning yechimini topsa bo‘ladi.

III Chiziqli tenlamalar sistemasining yechishning Kramer va teskari matritsalar usulini iqtisodiyotda qo’llanilishi.

Chiziqli tenglamalar sistemasining iqtisodiyotda qo’llanilishiga doir misollar keltiramiz.

Masala. A va B mahsulotlarni ishlab chiqarishuchun 2 turdagi xom-ashyodan foydalaniladi. Bitta A mahsulotni ishlab chiqarish uchun 5 birlik 1-tur va 4 birlik 2-tur xom-ashyo sarflanadi, bitta B mahsulotni ishlab chiqarish uchun esa, 3 birlik 1-tur va 5 birlik 2-tur xom-ashyo ishlatiladi. 1-tur xom-ashyo 62 birlik, 2-tur xom-ashyo 73 birlik berilgan bo’lsa, eng katta foyda olinadigan ishlab chiqarishni rejalashtirish uchun xom-ashyo sarfi modelini tuzing.

Bu masalaning matematik modelini tuzish maqsadida x_1 bilan ishlab chiqarilishi kerak bo'lgan A mahsulot miqdorini, x_2 bilan esa ishlab chiqarilishi kerak bo'lgan B mahsulot miqdorini belgilaylik. Bu holda $5x_1$ A mahsulotni ishlab chiqarish uchun sarflangan 1-tur xom-ashyo miqdorini, $3x_2$ esa B mahsulotni ishlab chiqarish uchun sarflangan 1-tur xom-ashyo miqdorini ifodalaydi. $5x_1+3x_2$ A va B mahsulotni ishlab ishlab chiqarish uchun sarflanadigan 1-tur xom-ashyo jami sarfi miqdorinni ifodalaydi, bu xom-ashyo chegaralangan bo'lib, 62 birlikda mavjud, demak $5x_1+3x_2=62$ tenglama kelib chiqadi. Xuddi shunday qilib, 2-tur xom-ashyo sarfi uchun $4x_1+5x_2=73$ tenglamani hosil qilamiz. Shunday qilib,

$$\begin{cases} 5x_1+3x_2=62, \\ 4x_1+5x_2=73 \end{cases}$$

Ikki noma'lumli ikkita chiziqli tenglamalar sistemasini hosil qildik. Bu tenglamalar sistemasi berilgan A va B mahsulotlarni ishlab chiqarishda, xom-ashyo sarfining matematik modelini ifodalaydi.

Yechish Kramer usulidan foydalanib yechimini topamiz.

$$A=\begin{pmatrix} 5 & 3 \\ 4 & 5 \end{pmatrix} \det|A|=25-12=13\neq 0$$. Bunda

$\det|A|\neq 0$ bo‘lganligi sababli berilgan sistemaaniq sistemani tashkil qiladi va u yagona yechimga ega bo‘ladi. Bu yechim Kramer formulalari yordamida quyidagicha topiladi:

$$x_1=\frac{\Delta x_1}{\det|A|}=\frac{\begin{vmatrix} 62 & 3 \\ 73 & 5 \end{vmatrix}}{13}=\frac{91}{13}=7,$$

$$x_2 = \frac{\Delta x_2}{\det|A|} = \frac{\begin{vmatrix} 5 & 62 \\ 4 & 73 \end{vmatrix}}{13} = \frac{117}{13} = 9,$$

Demak, tenglamalar sistemaning yechimi: $(x_1, x_2) = (7,9)$.

7-misol. Korxona uch xildagi xom-ashyoni ishlatib uch turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish xarakteristikalari 1-jadvalda berilgan.1-jadval.

Xom-ashyo turlari	Mahsulot turlari bo'yicha xom-ashyo sarflari			Xom-ashyo zahirasi (tonna)
	1	2	3	
1	5	12	3	20
2	2	6	8	16
3	9	7	4	20

Berilgan xom-ashyo zahirasini ishlatib, mahsulot turlari bo'yicha ishlab chiqarish hajmini aniqlang.

Yechish: Ishlab chiqarilishi kerak bo'lgan mahsulotlar hajmini mos ravishda x_1, x_2, x_3 lar bilan belgilaymiz. 1-tur mahsulotga, 1-xil xom-ashyo, bittasi uchun sarfi 5 birlik bo'lganligi uchun $5x_1$ 1-tur mahsulot ishlab chiqarish uchun ketgan 1-xil xom-ashyoning sarfini bildiradi. Xuddi shunday 2,3-tur mahsulotlarni ishlab chiqarish uchun ketgan 1-xil xom-ashyo sarflari mos ravishda $12x_2$, $3x_3$ bo'lib, uning uchun quyidagi tenglama o'rinli bo'ladi:

$5x_1 + 12x_2 + 3x_3 = 20$. Yuqoridagiga o'xshash 2,3-xil xom-ashyolar uchun

$$2x_1 + 6x_2 + 8x_3 = 16,$$

$$9x_1 + 7x_2 + 4x_3 = 20$$

Tenglamalar hosil bo'ladi. Demak, masala shartlarida quyidagi uch noma'lumli uchta chiziqli tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} 5x_1 + 12x_2 + 3x_3 = 20, \\ 2x_1 + 6x_2 + 8x_3 = 16, \\ 9x_1 + 7x_2 + 4x_3 = 20 \end{cases}$$

Bu masalaning matematik modeli uch noma'lumli uchta tenglamalar sistemasidan iborat bo'ladi. Bu masala tenglamalar sistemasining yechimini topish bilan yechiladi. Bunday tenglamalar sistemasini yechishda Teskari matritsalar usulidan foydalanamiz.

$$\begin{cases} 5x_1 + 12x_2 + 3x_3 = 20, \\ 2x_1 + 6x_2 + 8x_3 = 16, \\ 9x_1 + 7x_2 + 4x_3 = 20 \end{cases}$$

Yechish.

$$A = \begin{pmatrix} 5 & 12 & 3 \\ 2 & 6 & 8 \\ 9 & 7 & 4 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, B = \begin{pmatrix} 20 \\ 16 \\ 20 \end{pmatrix}.$$

Bundan, $\det|A| = 488 \neq 0.$

Teskari matrisani topamiz:

$$A_{11} = \begin{vmatrix} 6 & 8 \\ 7 & 4 \end{vmatrix} = -32, \quad A_{12} = -\begin{vmatrix} 2 & 8 \\ 9 & 4 \end{vmatrix} = 64, \quad A_{13} = \begin{vmatrix} 2 & 6 \\ 9 & 7 \end{vmatrix} = -40,$$

$$A_{21} = -\begin{vmatrix} 12 & 3 \\ 7 & 4 \end{vmatrix} = -27, \quad A_{22} = \begin{vmatrix} 5 & 3 \\ 9 & 4 \end{vmatrix} = -7 \quad A_{23} = -\begin{vmatrix} 5 & 12 \\ 9 & 7 \end{vmatrix} = 73,$$

$$A_{31} = \begin{vmatrix} 12 & 3 \\ 6 & 8 \end{vmatrix} = 78, \quad A_{32} = -\begin{vmatrix} 5 & 3 \\ 2 & 8 \end{vmatrix} = -34, \quad A_{33} = \begin{vmatrix} 5 & 12 \\ 2 & 6 \end{vmatrix} = 6$$

$$A^{-1} = \frac{1}{488}\begin{pmatrix} -32 & -27 & 78 \\ 64 & -7 & -34 \\ -40 & 73 & 6 \end{pmatrix}.$$

Bundan,

$$X = A^{-1}B = \frac{1}{495}\begin{pmatrix} -36 & -27 & 81 \\ 64 & -7 & -34 \\ -31 & 73 & 1 \end{pmatrix}\cdot\begin{pmatrix} 20 \\ 16 \\ 20 \end{pmatrix} = \frac{1}{495}\begin{pmatrix} -640-432+1560 \\ 1280-112-680 \\ -800+1168+120 \end{pmatrix} =$$

$$= \frac{1}{488}\begin{pmatrix} 488 \\ 488 \\ 488 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Demak, $x_1 = 1,\ x_2 = 1,\ x_3 = 1$ yoki $(1;1;1)$.

Mashqlarni bajaring.

Quyidagi tenglamalar sistemasini Kramer va teskari matritsa usullaridan foydalanib yeching:

a) Korxona uch xildagi xom-ashyoni ishlatib uch turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish xarakteristikalari 2-jadvalda berilgan.

2-jadval

Xom-ashyo turlari	Mahsulot turlari bo'yicha xom-ashyo sarflari			Xom-ashyo zahirasi
	1	2	3	
S_1	5	3	4	2700
S_2	2	1	1	800
S_3	3	2	2	1600

b) Korxona uch xildagi xom-ashyoni ishlatib uch turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish xarakteristikalari 3-jadvalda berilgan.

3-jadval

Xom-ashyo turlari	Mahsulot turlari bo'yicha xom-ashyo sarflari			Xom-ashyo zahirasi
	1	2	3	
S_1	1	1	1	1
S_2	1	2	2	2
S_3	2	3	3	3

c) Korxona uch xildagi xom-ashyoni ishlatib uch turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish xarakteristikalari 4-jadvalda berilgan.

4-jadval

Xom-ashyo turlari	Mahsulot turlari bo'yicha xom-ashyo sarflari			Xom-ashyo zahirasi
	1	2	3	
S_1	3	2	1	5
S_2	2	3	1	1
S_3	2	2	3	11

Mashqlarni bajaring.

Quyidagi tenglamalar sistemasini Kramer usuli bilan yeching:

1. $\begin{cases} 3x_1 + 2x_2 + x_3 = 5 \\ 2x_1 + 3x_2 + x_3 = 1 \\ 2x_1 + x_2 + 3x_3 = 11 \end{cases}$ 2. $\begin{cases} x_1 - 2x_2 + 3x_3 = 6 \\ 2x_1 - 3x_2 - 4x_3 = 20 \\ 3x_1 - 2x_2 - 5x_3 = 6 \end{cases}$

3. $\begin{cases} 4x_1 - 3x_2 + 2x_3 = 9 \\ 2x_1 + 5x_2 - 3x_3 = 4 \\ 5x_1 + 6x_2 - 2x_3 = 18 \end{cases}$ 4. $\begin{cases} x_1 + x_2 + 2x_3 = -1 \\ 2x_1 - x_2 + 2x_3 = -4 \\ 4x_1 + x_2 + 4x_3 = -2 \end{cases}$

5. $\begin{cases} 2x_1 - x_2 - x_3 = 4 \\ 3x_1 + 4x_2 - 2x_3 = 11 \\ 3x_1 - 2x_2 + 4x_3 = 11 \end{cases}$ 6. $\begin{cases} 3x_1 + 4x_2 + 2x_3 = 8 \\ 2x_1 - x_2 - 3x_3 = -4 \\ x_1 + 5x_2 + x_3 = 0 \end{cases}$

7. $\begin{cases} x_1 + x_2 + x_3 = 1 \\ 8x_1 + 3x_2 - 6x_3 = 2 \\ 4x_1 + x_2 - 3x_3 = 3 \end{cases}$ 8. $\begin{cases} x_1 - 4x_2 - 2x_3 = -3 \\ 3x_1 + x_2 + x_3 = 5 \\ 3x_1 - 5x_2 - 6x_3 = -9 \end{cases}$

9. $\begin{cases} 7x_1 - 5x_2 = 31 \\ 4x_1 + 11x_3 = -43 \\ 2x_1 + 3x_2 + 4x_3 = -20 \end{cases}$ 10. $\begin{cases} x_1 + 2x_2 + 4x_3 = 31 \\ 5x_1 + x_2 + 2x_3 = 20 \\ 3x_1 - x_2 + x_3 = 9 \end{cases}$

11. $\begin{cases} x_1 - x_2 + x_3 = 6 \\ 2x_1 + x_2 + x_3 = 3 \\ x_1 + x_2 + x_3 = 5 \end{cases}$ 12. $\begin{cases} x_1 + x_2 - x_3 = 2 \\ -2x_1 + x_2 + x_3 = 0 \\ x \ + x_2 + x_3 = 6 \end{cases}$

Quyidagi tenglamalar sistemasini teskari matritsalar usuli bilan yeching:

13. $\begin{cases} 3x_1 - x_2 = 5 \\ -2x_1 + x_2 + x_3 = 0 \\ 2x_1 - x_2 + 4x_3 = 15 \end{cases}$ 14. $\begin{cases} 5x_1 + 4x_3 = 1 \\ x_1 - x_2 + 2x_3 = 0 \\ 4x_1 + x_2 + 2x_3 = 1 \end{cases}$

15. $\begin{cases} 2x_1 + 2x_2 - x_3 = 4 \\ 3x_1 + x_2 - 3x_3 = 7 \\ x_1 + x_2 + 2x_3 = 3 \end{cases}$ 16. $\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 + 2x_2 + 2x_3 = 2 \\ 2x_1 + 3x_2 + 3x_3 = 3 \end{cases}$

17. $\begin{cases} 2x_1+3x_2+5x_3=10 \\ 3x_1+7x_2+4x_3=3 \\ x_1+2x_2+2x_3=3 \end{cases}$ 18. $\begin{cases} 5x_1-6x_2+4x_3=3 \\ 3x_1-3x_2+2x_3=2 \\ 4x_1-5x_2+2x_3=1 \end{cases}$

19. $\begin{cases} 4x_1-3x_2+2x_3=-4 \\ 6x_1-2x_2+3x_3=-1 \\ 5x_1-3x_2+2x_3=-3 \end{cases}$ 20. $\begin{cases} 5x_1+2x_2+3x_3=-2 \\ 2x_1-2x_2+5x_3=0 \\ 3x_1+4x_2+2x_3=-10 \end{cases}$

21. $\begin{cases} 2x_1+x_2=3 \\ 3x_1+4x_3=-1 \\ x_1-x_2+2x_3=-2 \end{cases}$ 22. $\begin{cases} 2x_1+3x_3=4 \\ 4x_1+x_2=6 \\ 2x_1-x_2-2x_3=0 \end{cases}$

23. $\begin{cases} 2x_1+x_2+2x_3=6 \\ 3x_1+2x_3=8 \\ x_1+x_2-x_3=1 \end{cases}$ 24. $\begin{cases} 3x_2+2x_3=5 \\ 2x_1+x_2-x_3=4 \\ -x_2+2x_3=1 \end{cases}$

25. $\begin{cases} x_1+2x_3=5 \\ 3x_1-x_2=9 \\ x_1+x_2-2x_3=1 \end{cases}$ 26. $\begin{cases} x_1+x_2+2x_3=4 \\ 2x_2+x_3=3 \\ x_1-x_2=2 \end{cases}$

27. Quyidagi tenglamalar sistemasini Kramer va teskari matritsalar usulidan foydalanib yeching:

a) Korxona uch xildagi xom-ashyoni ishlatib uch turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish xarakteristikalari 5-jadvalda berilgan.

5-jadval

Xom-ashyo turlari	Mahsulot turlari bo'yicha xom-ashyo sarflari			Xom-ashyo zahirasi
	1	2	3	

S_1	2	1	2	6
S_2	3	0	2	8
S_3	1	1	-1	1

b) Korxona uch xildagi xom-ashyoni ishlatib uch turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish xarakteristikalari 6-jadvalda berilgan.

6-jadval

Xom-ashyo turlari	Mahsulot turlari bo'yicha xom-ashyo sarflari			Xom-ashyo zahirasi
	1	2	3	
S_1	2	1	1	6
S_2	3	0	2	8
S_3	1	1	0	3

c) Korxona uch xildagi xom-ashyoni ishlatib uch turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish xarakteristikalari 7-jadvalda berilgan.

7-jadval

Xom-ashyo turlari	Mahsulot turlari bo'yicha xom-ashyo sarflari			Xom-ashyo zahirasi
	1	2	3	
S_1	1	1	2	4
S_2	3	0	2	5

S_3	1	4	1	6

d) Korxona uch xildagi xom-ashyoni ishlatib uch turdagi mahsulot ishlab chiqaradi. Ishlab chiqarish xarakteristikalari 8-jadvalda berilgan.

8-jadval

Xom-ashyo turlari	Mahsulot turlari bo'yicha xom-ashyo sarflari			Xom-ashyo zahirasi
	1	2	3	
S_1	3	2	2	9
S_2	2	1	5	9
S_3	5	3	6	17

IQTISODIY MASALALARNING CHIZIQLI MODELLARINI TUZISH

Tayanch so'z va iboralar:*Matematik model, chiziqli va chiziqsiz programmalashtirish, stoxastik programmalashtirish, dinamik programmalashtirish.*

REJA:

1. Chiziqli programmalashtirishning asosiy masalalari.

2. Iqtisodiy matematik model tushunchasi.

3. Eng sodda iqtisodiy masalalarning matematik modellari.

Chiziqli programmalashtirish matematik programmalashtirishning bir bo'limi bo'lib, u chegaralangan resurslar (xom-ashyo, texnika vositalari, kapital qo'yilmalar, yer, suv, mineral o'g'itlar va boshqalar)ni ratsional taqsimlab eng ko'p foyda olish yoki eng kam xarajat qilish yo'llarini o'rgatadi.

Chiziqli programmalashtirishning shakllanishi XX asrning ikkinchi yarmidagi iqtisodiy fikrlarning takomillashishiga katta ta'sir ko'rsatdi. 1975 yilda chiziqli programmalashtirish nazariyasini birinchi bor kashf qilgan rus olimi L.V.Kantorovichga va matematik iqtisodiyot bo'yicha mutaxassis, "Chiziqli programmalashtirish" terminining birinchi muallifi, amerika olimi T.Kupmansga Nobel mukofotining berilishi chiziqli programmalashtirishning iqtisodiy nazariyaga qo'shgan hissasini tan olishdan iborat deb hisoblash mumkin.

Chiziqli programmalashtirish chiziqli funksiyaning, uning tarkibiga kiruvchi noma'lumlarga chegaralovchi shartlar qo'yilganda, eng katta va eng kichik qiymatini izlash va topish uslubini o'rgatuvchi bo'limdir.

Noma'lumlarga chiziqli chegaralashlar qo'yilgan chiziqli funksiyaning ekstremumini topish chiziqli programmalashtirishning predmetini tashkil qiladi. Shunday qilib, chiziqli programmalashtirish chiziqli funksiyaning shartli ekstremumini topish masalalari turkumiga kiradi.

Iqtisodiy jarayonlarning o'ziga xos qonuniyatlarini o'rganish uchun, birinchi navbatda, bu jarayonlarni tavsiflovchi matematik modellarni tuzish kerak. O'rganilayotgan iqtisodiy jarayonning asosiy xossalarini matematik munosabatlar yordamida tavsiflash tegishli iqtisodiy jarayonning matematik modelini tuzish deb ataladi.

Iqtisodiy jarayonlarning (masalalarning) matematik modelini tuzish uchun quyidagi bosqichlardagi ishlarni bajarish kerak:

1) masalaning iqtisodiy ma'nosi bilan tanishib, undagi asosiy shartlar va maqsadni aniqlash;

2) masaladagi ma’lum parametrlarni belgilash;
3) masaladagi noma’lumlarni (boshqaruvchi o’zgaruvchilarni) belgilash;
4) masaladagi cheklamalarni, ya’ni boshqaruvchi o’zgaruvchilarning qanoatlantirishi kerak bo’lgan chegaraviy shartlarni chiziqli tenglamalar yoki tengsizliklar orqali ifodalash;
5) masalaning maqsadini chiziqli funksiya orqali ifodalash.

Boshqaruvchi o’zgaruvchilarning barcha cheklamalarni qanoatlantiruvchi shunday qiymatini topish kerakki, u maqsad funksiyaga eng katta (maksimum) yoki eng kichik (minimum) qiymat bersin. Bundan ko’rinadiki, maqsad funksiya boshqaruvchi noma’lumlarning barcha qiymatlari ichida eng yaxshisini (optimalini) topishga yordam beradi. Shuning uchun ham maqsad funksiyani foydalilik yoki optimallik mezoni deb ham ataladi.

Iqtisodiy masalalarning matematik modelini tuzish jarayonini amaliyotda nisbatan ko’p uchraydigan quyidagi iqtisodiy masalalar misolida o’rganamiz.

Ishlab chiqarishni tashkil qilish va rejalashtirish masalasi. Faraz qilaylik, korxonada m xil mahsulot ishlab chiqarilsin; ulardan ixtiyoriy birini i bilan belgilaymiz. Bu mahsulotlarni ishlab chiqarish uchun n xil ishlab chiqarish faktorlari zarur bo’lsin. Har bir xom-ashyoning umumiy miqdori va bir birlik mahsulotni ishlab chiqarish uchun sarf qilinadigan normasi haqidagi ma’lumotlar quyidagi jadvalda berilgan bo’lsin.

Xom-ashyolar / Mahsulot turlari	1	2	3	…	n	Daromad
1	a_{11}	a_{12}	a_{13}	…	a_{1n}	c_1
2	a_{21}	a_{22}	a_{23}	…	a_{2n}	c_2
…	…	…	…	…	…	…
m	a_{m1}	a_{m2}	a_{m3}	…	a_{mn}	c_m
Xom-ashyolar zahirasi	b_1	b_2	b_3	…	b_n	

Jadvaldagi har bir: $b_j - j$ xom-ashyoning umumiy miqdori (zahirasi); $a_{ij} - i$ mahsulotning bir birligini ishlab chiqarish uchun sarf qilinadigan j xom-ashyo miqdori; $c_j -$ korxonaning j mahsulotning bir birligini sotishdan oladigan daromadi.

Masalaning iqtisodiy ma'nosi: korxonaning ishini shunday rejalashtirish kerakki:

a) hamma mahsulotlarni ishlab chiqarish uchun sarf qilinadigan har bir xom-ashyoning miqdori ularning umumiy miqdoridan oshmasin;

b) mahsulotlarni sotishdan korxonaning oladigan daromadi maksimal bo'lsin.

Rejalashtirilgan davr ichida ishlab chiqariladigan i mahsulotning miqdorini x_i bilan belgilaymiz. U holda masaladagi a) shart quyidagi tengsizliklar sistemasi orqali ifodalanadi:

$$\begin{cases} a_{11}x_1 + a_{21}x_2 + \cdots + a_{m1}x_m \le b_1, \\ a_{12}x_1 + a_{22}x_2 + \cdots + a_{m2}x_m \le b_2, \\ \dots\dots\dots\dots\dots\dots\dots\dots, \\ a_{1n}x_1 + a_{2n}x_2 + \cdots + a_{mn}x_m \le b_n, \end{cases}$$

Masalaning iqtisodiy ma'nosiga ko'ra noma'lumlar manfiy bo'lmasligi kerak, ya'ni: $x_i \ge 0, \quad (i = \overline{1,m})$.

Masaladagi b) shart uning maqsadini aniqlaydi. Demak, masalaning maqsadi mahsulotlarni sotishdan korxonaning oladigan umumiy daromadini maksimallashtirishdan iborat bo'lib, uni $y = c_1x_1 + c_2x_2 + \cdots + c_mx_m$ funksiya orqali ifodalash mumkin. Shunday qilib, ishlab chiqarishni rejalashtirish masalasining matematik modeli quyidagi ko'rinishda bo'ladi:

$$\begin{cases} a_{11}x_1 + a_{21}x_2 + \cdots + a_{m1}x_m \le b_1, \\ a_{12}x_1 + a_{22}x_2 + \cdots + a_{m2}x_m \le b_2, \\ \dots\dots\dots\dots\dots\dots\dots\dots, \\ a_{1n}x_1 + a_{2n}x_2 + \cdots + a_{mn}x_m \le b_n, \end{cases}$$

$$x_i \ge 0, \quad i = \overline{1,m},$$

$$y = c_1 x_1 + c_2 x_2 + ... + c_m x_m \to \max .$$

Iste'mol savati masalasi. Faraz qilaylik, kishi organizmi uchun bir sutkada$n$xil $A_1,\ A_2,\ ...,\ A_n$ oziqa moddalari kerak bo'lsin, jumladan bir sutkadaA_1 oziqa moddasidan kamida b_1 miqdorda, A_2 oziqa moddasidan b_2 miqdorda, A_3 oziqa moddasidan $b_3$miqdorda va hokazo, A_n ozuqadan $b_n$miqdorda zarur bo'lsin va ularni m ta$B_1, B_2, ..., B_m$mahsulotlar tarkibidan olish mumkin bo'lsin. Har bir B_i mahsulot tarkibidagi A_j oziqa moddasining miqdori a_{ij} birlikni tashkil qilsin.

Ozuqa moddalari / **Mahsulot turlari**	A_1	A_2	A_3	**...**	A_n	**Mahsulot bahosi**
B_1	a_{11}	a_{12}	a_{13}	$\cdots$	a_{1n}	c_1
B_2	a_{21}	a_{22}	a_{23}	$\cdots$	a_{2n}	c_2
...	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
B_m	a_{m1}	a_{m2}	a_{m3}	$\cdots$	a_{mn}	c_m
Ozuqa moddasining minimal normasi	b_1	b_2	b_3	$\cdots$	b_n	

Masalaning iqtisodiy ma'nosi: iste'mol savatiga qanday mahsulotlardan qancha miqdorda kiritish kerakki, natijada:

a) odam organizmi qabul qiladigan turli oziqa moddasining miqdori belgilangan minimal miqdordan kam bo'lmasin;

b) iste'mol savatining umumiy bahosi minimal bo'lsin.

Iste'mol savatiga kiritiladigan i-mahsulotning miqdorini x_i bilan belgilaymiz. U holda masalaning

a) sharti quyidagi tengsizliklar sistemasi orqali ifodalanadi:

$$\begin{cases} a_{11}x_1 + a_{21}x_2 + \cdots + a_{m1}x_m \ge b_1, \\ a_{12}x_1 + a_{22}x_2 + \cdots + a_{m2}x_m \ge b_2, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots, \\ a_{1n}x_1 + a_{2n}x_2 + \cdots + a_{mn}x_m \ge b_n. \end{cases}$$

Masalaning iqtisodiy ma’nosiga ko’ra, undagi noma’lumlar manfiy bo’laolmaydi, ya’ni: $x_i \geq 0, \quad (i = \overline{1,m})$.

Masalaning b) sharti uning maqsadini ifodalaydi. Demak, masalaning maqsadi iste’mol savatiga kiritiladigan mahsulotlarning umumiy bahosini minimallashtirishdan iborat bo’lib, uni quyidagicha ifodalash mumkin:

$$y = c_1x_1 + c_2x_2 + \cdots + c_mx_m \to \min.$$

Shunday qilib, iste’mol savati masalasining matematik modeli ko’rinishda bo’ladi.

$$\begin{cases} a_{11}x_1 + a_{21}x_2 + \cdots + a_{m1}x_m \geq b_1, \\ a_{12}x_1 + a_{22}x_2 + \cdots + a_{m2}x_m \geq b_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \\ a_{1n}x_1 + a_{2n}x_2 + \cdots + a_{mn}x_m \geq b_n, \end{cases}$$

$$x_1 \geq 0,\ x_2 \geq 0,\ \ldots,\ x_m \geq 0,$$

$$Y = c_1x_1 + c_2x_2 + \ldots + c_mx_m \to \min.$$

Optimal bichish masalasi. Optimal bichish masalasining eng sodda holi bilan tanishamiz. Faraz qilamiz, uzunligi L bo’lgan xomaki materiallardan uzunliklari $\Delta_i \ (i = \overline{1,m})$ bo’lgan m xil detallarning har biridan c_i miqdorda tayyorlash kerak bo’lsin. Bundan tashqari xomaki materiallarni n usul bilan kesish mumkin, hamda har bir j usul bilan kesilgan xomaki materialdan a_{ij} miqdorda*i* detal tayyorlash va b_j miqdorda chiqindi hosil qilish mumkin ekanligi aniqlangan bo’lsin. Xomaki materiallardan qanchasini qaysi usul bilan kesganda tayyorlangan detallar miqdori rejadagiga teng bo’ladi va hosil bo’lgan chiqindilarning umumiy miqdori eng kam (minimal) bo’ladi.

Tayyorlanadigan detallarning uzunliklari	**Kesish usullari**				**Detallar ishlab chiqarish rejasi**
	1	2	…	n	
Δ_1	a_{11}	a_{12}	…	a_{1n}	c_1
Δ_2	a_{21}	a_{22}	…	a_{1n}	c_2

$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
Δ_m	a_{m1}	a_{m2}	$\dots$	a_{mn}	c_m
Chiqindilar	b_1	b_2	$\dots$	b_n	

j usulbilankesiladigan xomakimateriallarmiqdorini x_j bilanbelgilaymiz. Uholda masalaningmatematikmodeliquyidagiko'rinishda yoiziladi:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = c_1, \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = c_2, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots, \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = c_m, \end{cases}$$

$$x_1 \ge 0,\ x_2 \ge 0,\ \dots,\ x_n \ge 0,$$

$$Y = b_1x_1 + b_2x_2 + \dots + b_nx_n \to \min.$$

1-misol. Uzunligi 110 sm. bo'lgan po'lat xipchinlardan uzunliklari 45 sm, 35 sm va 50 sm bo'lgan xomaki mahsulotlar tayyorlash kerak bo'lsin. Talab qilingan xomaki mahsulotlar miqdori mos ravishda 40, 30 va 20 birlikni tashkil qilsin. Po'lat xipchinlarni kesish yo'llari va ularga mos keluvchi xomaki mahsulotlar va chiqindilar miqdori quyidagi jadvalda keltirilgan.

Xomakimahsulotlar Uzunligi	**Kesish usullari**						**Xomaki mahsulotlar i/ch rejasi**
	1	2	3	4	5	6	
45 sm.	2	1	1	-	-	-	40
35 sm.	-	1	-	3	1	-	30
50 sm.	-	-	1	-	1	2	20
Chiqindilar	20	30	15	5	25	10	

Har bir kesish usuli bo'yicha qancha po'lat xipchinlar kesilganda tayyorlangan xomaki mahsulotlar miqdori rejadagiga teng bo'ladi va chiqindilarning umumiy miqdori minimal bo'ladi?

Yechish: $j-$usul bilan kesiladigan po'lat xipchinlar sonini x_j bilan belgilaymiz. U holda uzunligi 45sm bo'lgan xomaki mahsulotlardan ja'mi $2x_j + x_2 + x_3$ miqdorda tayyorlanadi. Rejaga ko'ra bunday mahsulotlar soni 40 taga teng bo'lishi kerak, ya'ni $2x_j + x_2 + x_3 = 40$.

Xuddi shuningdek, uzunliklari 35sm va 50sm bo'lgan xomaki mahsulotlarni ishlab chiqarish rejasini to'la bajarilishidan iborat shartlar mos ravishda $x_2 + 3x_4 + x_5 = 30$ va $x_3 + x_5 + 2x_6 = 20$ tenglamalar orqali ifodalanadi.

Iqtisodiy ma'nosiga ko'ra belgilangan noma'lumlar manfiy bo'la olmaydi, demak $x_1 \geq 0, x_2 \geq 0, \ ... \ , x_6 \geq 0.$

Rejadagi xomaki mahsulotlarni ishlab chiqarishda hosil bo'lgan chiqindilar-ning umumiy miqdorini quyidagi chiziqli funksiya ko'rinishida ifodalaymiz: $Y = 20x_1 + 30x_2 + 15x_3 + 5x_4 + 25x_5 + 10x_{6.}.$

Masalaning shartiga ko'ra bu funksiya minimum qiymatni qabul qilishi kerak, ya'ni $Y = 20x_1 + 30x_2 + 15x_3 + 5x_4 + 25x_5 + 10x_6 \to min.$

Shunday qilib, quyidagi chiziqli programmalashtirish masalasiga ega bo'lamiz:

$$\begin{cases} 2x_1 + x_2 + x_3 = 40, \\ x_2 + 3x_4 + x_5 = 30, \\ x_3 + x_5 + 2x_6 = 20 \end{cases}$$

$$x_1 \geq 0, x_2 \geq 0, \ ... \ , x_6 \geq 0,$$

$$Y = 20x_1 + 30x_2 + 15x_3 + 5x_4 + 25x_5 + 10x_6 \to min.$$

2-misol. Konditer fabrikasi uch turdagi A, B, C karamellarni ishlab chiqarish uchun uch xil xom-ashyo: shakar, qiyom va quruq mevalar ishlatadi. 1 tonna karamel turlarini ishlab chiqarish uchun sarf qilinadigan xom-ashyolar miqdori (me'yori), xom-ashyolarning zahirasi hamda 1 tonna karamelni sotishdan olinadigan daromad quyidagi jadvalda keltirilgan.

Xom-ashyo turlari	**1 tonna mahsulotga xom-ashyo sarfi (t. hisobida)**			**Xom-ashyo zahirasi (t)**
	A	B	C	
Shakar	0,8	0,5	0,6	800
Qiyom	0,4	0,4	0,3	600
Quruq mevalar	-	0,1	0,1	120
1 t karamel sotishdan olinadigan daromad (sh.b.)	108	112	126	

Fabrikaga maksimal foyda keltiruvchi karamel ishlab chiqarish rejasini toping.

Yechish: Konditer fabrikasida A turdagi karameldan x_1 miqdorda, B turdagi karameldan x_2 miqdorda va C turdagi karameldan x_3 miqdorda ishlab chiqarilsin deb belgilaymiz. U holda fabrikada ishlab chiqariladigan barcha karamellar uchun $0,8x_1 + 0,5x_2 + 0,6x_3$ miqdorda shakar sarf qilinadi. Bu miqdor shakarning zahirasidan, ya'ni 800 tonnadan oshmasligi kerak. Demak, $0,8x_1 + 0,5x_2 + 0,6x_3 \leq 800$ tengsizlik o'rinli bo'lishi kerak. Xuddi shunday yo'l bilan mos ravishda qiyom va quruq mevalar sarfini ifodalovchi quyidagi tengsizliklarni hosil qilish mumkin: $0,4x_1 + 0,4x_2 + 0,3x_3 \leq 600$, $0,1x_2 + 0,1x_3 \leq 120$. Fabrika ishlab chiqargan A karameldan $108x_1$, B karameldan $102x_2$, C karameldan $126x_3$ birlik va ja'mi $108x_1 + 112x_2 + 126x_3$ birlik daromad oladi. Bu yig'indini Y bilan belgilab uni maksimumga intilishini talab qilamiz. natijada quyidagi funksiyaga ega bo'lamiz: $Y = 108x_1 + 112x_2 + 126x_3 \to max$. Shunday qilib, berilgan masalaning matematik modelini quyidagi ko'rinishda yoziladi:

$$\begin{cases} 0,8x_1 + 0,5x_2 + 0,6x_3 \le 800, \\ 0,4x_1 + 0,4x_2 + 0,3x_3 \le 600, \\ \qquad\quad 0,1x_2 + 0,1x_3 \le 120. \end{cases}$$

$$x_1 \ge 0, x_2 \ge 0,\ x_3 \ge 0,$$

$$Y = 108x_1 + 112x_2 + 126x_3 \to max\,.$$

CHIZIQLI PROGRAMMALASHTIRISH MASALASINING GEOMETRIK TALQINI

***Tayanch so'z va iboralar**: Gipertekislik, gipertekisliklar oilasi, yechimlar ko'pburchagi, sath to'g'ri chizig'i, aktiv va passiv shartlar, kamyob xom-ashyo.*

REJA:

1. Chiziqli programmalashtirish masalasining geometirik talqini.

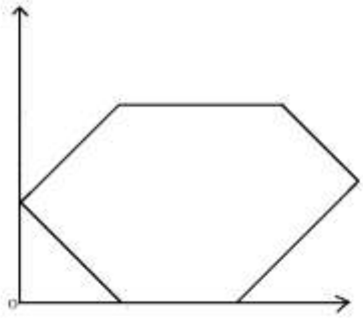
2. Chiziqli programmalashtirish masalasini grafik usulda yechish.

3. Iqtisodiy masalani grafik usulda yechish va tahlil qilish.

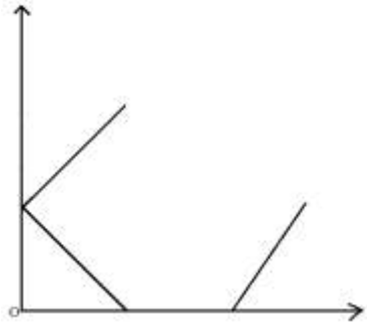
ChPMsini geometrik nuqtai nazardan tahlil qilish uchun quyidagi statandart masalani ko'ramiz:

$$\begin{aligned} & Ax \le B \\ & x_j \ge 0 \quad j = \overline{1,n} \\ & Y = CX \to \max \end{aligned} \qquad (1)$$

Ma'lumki, (1) masalaning har qanday rejasini n-o'lchovli fazoning nuqtasi deb qarash mumkin. Bizga yana shu ham ma'lumki, chiziqli tengsizliklar bilan aniqlangan bunday nuqtalar to'plami qavariq to'plamdan iborat bo'ladi. Bu holda qavariq to'plam (qavariq ko'pburchak yoki ko'pyoq) chegaralangan yoki chegaralanmagan bo’lishi, bitta nuqtadan iborat bo’lishi yoki bo’sh to’plam bo’lishi ham mumkin. Masalan, qavariq to’plamlar



a) chegaralangan



b) chegaralanmagan

(1) masalani geometrik nuqtai nazardan tahlil qilamiz. Buning uchun quyidagi

$$a_1x_1 + a_2x_2 + ... + a_nx_n \leq b, \tag{2}$$

tengsizlikni qanoatlantiruvchi nuqtalarning geometrik o’rni bilan tanishib chiqamiz.

Ma’lumki, koordinatalari

$$a_1x_1 + a_2x_2 + ... + a_nx_n = b, \tag{3}$$

tenglamani qanoatlantiruvchi $x_1, x_2, ... x_n$ nuqtalar to’plami gipertekislik deb ataladi. Demak, (1) masalada (3) kabi tengliklar qatnashsa ular gipertekisliklarni ifodalaydi. Har qanday gipertekislik fazoni ikki yarim fazoga ajratadi. Bu yarim fazolardan faqat bittasigina (2) tengsizlikni qanoatlantiradi. (2) tengsizlikni qanoatlantiradigan yarim fazoni aniqlash uchun $O(0,0,...,0)$ koordinata boshidan foydalanamiz, ya’ni:

agar $O(0,0,...,0)$ nuqta (2) tengsizlikni to’g’ri tengsizlikka aylantirsa, u holda $O(0,0,...,0)$ nuqtani o’z ichiga oluvchi yarim fazo (2) tengsizlikni qanoatlantiruvchi nuqtalarning geometrik o’rni bo’ladi;

agar $O(0,0,...,0)$ nuqta (2) tengsizlikni noto’g’ri tengsizlikka aylantirsa, u holda $O(0,0,...,0)$ nuiqtani o’z ichiga olmaydigan yarim fazo (2) tengsizlikni qanoatlantiruvchi nuqtalarning geometrik o’rni bo’ladi.

Bundan ko'rinadiki, (1) masalada nechta tensizlik qatnashsa ular shuncha yarim fazoni ifodalaydi. Bu yarim fazolarning kesishmasi esa (1) masalaning barcha joiz yechimlarini o'z ichiga oluvchi qavariq to'plamni tasvirlaydi. Bu qavariq to'plam masalaning joiz yechimlar sohasi deb ataladi.

(1) masalaning optimal yechimini topish uchun $Y = c_1x_1 + c_2x_2 + ... + c_nx_n$ maqsad funksiyasidan foydalanamiz. Buning uchun

$$c_1x_1 + c_2x_2 + ... + c_nx_n = const \tag{4}$$

tenglik bilan aniqlanuvchi gipertekisliklar oilasini qaraymiz.

Ma'lumki, bu yerda $const$ ning har bir qiymatiga bitta gipertekislik mos keladi. ChPMsining qavariq to'lami bilan ikkita umumiy nuqtaga ega bo'lgan gipertekisliklar "sath gipertekisliklar" deyiladi. ChPMsining qavariq to'plami bilan bitta umumiy nuqtaga ega bo'lgan gipertekislik, ya'ni urinma gipertekislik tayanch gipertekislik deyiladi. Tayanch gipertekislikni hosil qilish uchun (4) tenglikdagi $const$ ga turli qiymatlar berib uni gipertekislikning normal vektori bo'ylab parallel ko'chiramiz va urinma gipertekislikni hosil qilamiz.

Shuni ta'kidlaymizki, Y funksiyaning maksimal qiymatini topish uchun normal vektorning yo'nalashi bo'ylab, Y funksiyaning minimal qiymatini topish uchun normal vektorning yo'nalashiga qarama-qarshi harakatlanish kerak.

$n = 2$ o'lchovli fazoda, ya'ni tekislikda ChPMsini geometrik nuqtai nazardan ko'rib chiqamiz.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 \le b_1, \\ a_{21}x_1 + a_{22}x_2 \le b_2, \\ \dots\dots\dots\dots\dots\dots\dots, \\ a_{m1}x_1 + a_{m2}x_2 \le b._m \end{cases} \tag{5}$$

$$x_1, x_2 \ge 0 \tag{6}$$

$$Y = c_1x_1 + c_2x_2 \to \max \tag{7}$$

Faraz qilaylik, (5) sistema (6) shartni qanoatlantiruvchi yechimlarga ega va ulardan tashkil topgan to'plam chegaralangan bo'lsin.

Ma'lumki, (5) va (6) tengsizliklarning har biri

$$a_{i1}x_1 + a_{i2}x_2 = b_i, \ (i = \overline{1,m}),$$
$$x_1 = 0, \ \ x_2 = 0$$

to'g'ri chiziqlar bilan chegaralangan yarim tekisliklarni ifodalaydi. Bu tekisliklarni ko'rib chiqamiz.

$$a_{i1}x_1 + a_{i2}x_2 \le b_i, \ \ (i = \overline{1,m}) \tag{8}$$

tengsizlikni qanoatlantiruvchi nuqtalar to'plamini aniqlash uchun $O(0,0)$ nuqtadan foydalanamiz. Agar $O(0,0)$ nuqta (8) tengsizlikni qanoatlantirsa, u holda qidirilayotgan teksilik $O(0,0)$ nuqtani o'z ichiga oladi, aks holda qidirilayotgan tekislik $O(0,0)$ nuqtani o'z ichiga olmyadi. Yuqoridagi mulohazalar asosida (5) sistemaning yechimlaridan iborat qavariq ko'pburchakni topib olganimizdan so'ng (6) tengsizliklarni e'tiborga olamiz. (6) tengsizliklar (5) yordamida topilgan qavariq ko'pburchakning I chorakdagi qismini ajratib olishga yordam beradi. (5) va (6) cheklamalarni qanoatlantiruvchi qavariq ko'pburchakni reja ko'pburchagi deb ataymiz.

(5)-(7) masalaning optimal yechimini topish uchun (7) ifodada qatnashayotgan chiziqli funksiyadan hosil qilinadigan

$$c_1x_1 + c_2x_2 = const \tag{9}$$

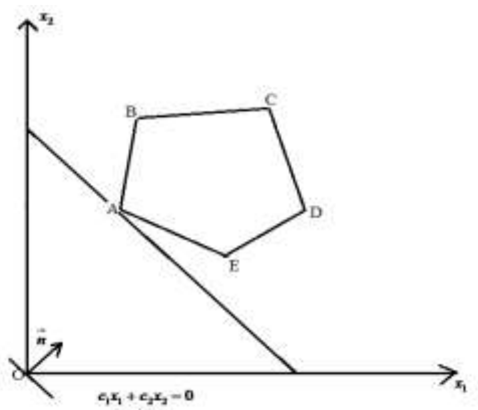
to'g'ri chiziqlar oilasidan foydalanamiz. Ma'lumki, (9) ifodadagi har bir ma'lum o'zgarmas $C_0 = const$ qiymatida bitta

$$c_1x_1 + c_2x_2 = C_0$$

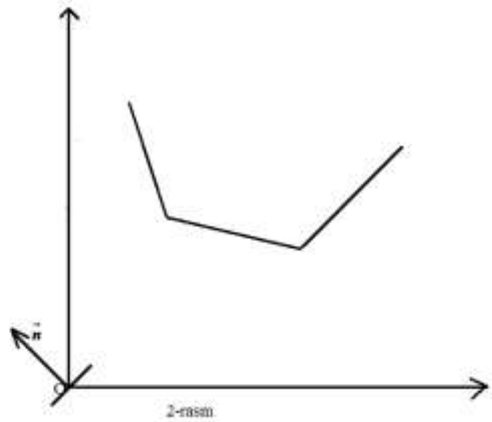
sath to'g'ri chizig'i to'g'ri keladi.

So'ngra, bu sath to'g'ri chiziqlardan birini, masalan, $c_1x_1 + c_2x_2 = C_0$ to'g'ri chiziqni chizib olamiz. $c_1x_1 + c_2x_2 = C_0$ chiziqni $\vec{n}(c_1, c_2)$ normal vektor bo'ylab parallel ko'chirib, reja ko'pburchagiga $c_1x_1 + c_2x_2 = C^0$ tayanch (urinma) to'g'ri chiziqni topib olamiz. Bu yerda C^0 (5)-(7) masalaning optimal yechimi yoki qiymati; $X^0(x_1^0, x_2^0)$ urinish nuqtasi esa (5)-(7) masalaning optimal rejasi deb ataladi.

Ba'zi xususiy hollarni ko'rib chiqamiz. Faraz qilaylik, reja ko'pburchakgi $ABCDE$ beshburchakdan iborat bo'lsin (1-rasm).



1-Rasm



2-rasm

1-rasmdan ko'rinib turibdiki, chiziqli funksiya o'zining minimal qiymatiga $ABCDE$ qavariq ko'pburchakning A nuqtasida erishadi. C nuqtada esa, u o'zining maksimal (eng katta) qiymatiga erishadi.

Yechimlardan tashkil topgan qavariq ko'pburchak chegaralanmagan bo'lsin. Bunday ko'pburchaklardan ba'zilarini ko'rib chiqamiz.

1-hol. 2-rasmdagi holatda $c_1x_1 + c_2x_2 = C_0$ to'g'ri chiziq $\vec{n}$ vektor bo'ylab siljib borib har vaqt qavariq ko'pburchakni kesib o'tadi.

Bu holda $c_1x_1 + c_2x_2 = C_0$ funksiya minimal qiymatga ham, maksimal qiymatga ham erishmaydi. Bu holda chiziqli funksiya (5) va (6) cheklamalar bilan aniqlangan sohada quyidan ham, yuqoridan ham chegaralanmagan bo'ladi.

1-misol. Masalani grafik usulda yeching.

$$\begin{cases} -2x_1 + x_2 \le 2 \\ -x_1 + x_2 \le 3 \\ x_1 \le 3 \end{cases}$$

$$Z = -x_1 - 2x_2 \to \min$$

$$x_1, x_2 \ge 0.$$

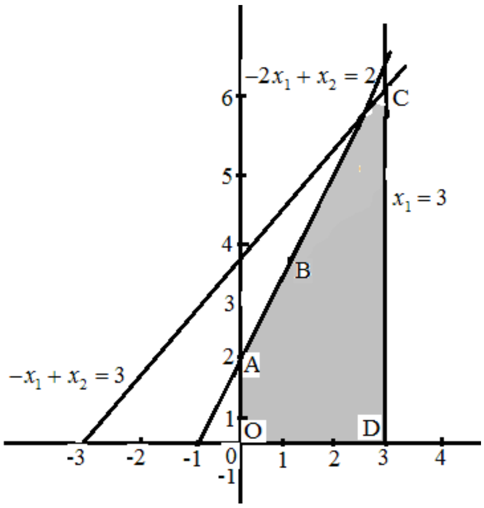
Yechish: Yechimlardan tashkil topgan qavariq ko'pburchak yasash uchun koordinatalar sistemasida

$$\begin{cases} -2x_1 + x_2 = 2 \\ -x_1 + x_2 = 3 \\ x_1 = 3 \end{cases}$$

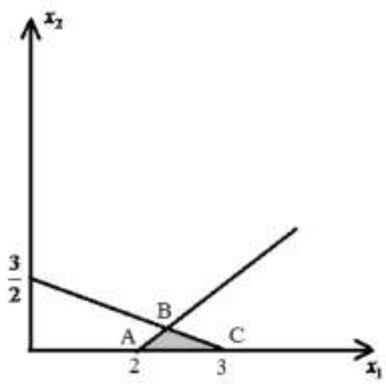
chiziqlarbilan chegaralangan

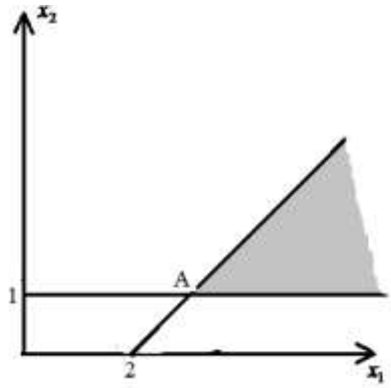
$$-2x_1 + x_2 \le 2, -x_1 + x_2 \le 3, x_1 \le 3$$

yarim tekisliklarni koordinatalar sistemasining I choragida yasaymiz, chunki $x_1, x_2 \ge 0$



Berilgan tengsizliklarni qanoatlantiruvchi yechimlar to'plami bo'yalgan *OABCD*-beshburchakni tashkil qiladi. Natijada $Z = -x_1 - 2x_2$ chiziqli funksiyaga minimal qiymat beruvchi $C(3;6)$ nuqtani topamiz. Bu nuqtaning koordinatalari masalaning optimal rejasi, $Z(C) = -15 \to \min$ esa masalaning optimal yechimi bo'ladi.





1-rasm **2-rasm**

2-misol. Berilgan ChPMsini grafik usulda yeching.

$$\begin{cases} x_1 + 2x_2 \le 3 \\ x_1 - x_2 \le 2 \end{cases}$$
$$Y = 2x_1 + 2x_2 \to \max$$
$$x_1, x_2 \ge 0.$$

Yechish: Bu yerda ham yuqoridagidek yechimlar ko'pburchagini hosil qilamiz.

Yuqoridagi 1-rasmdan ko'rinadiki, yechimlar ko'pburchagi yuqoridan chegaralanmagan. Koordinata boshidan $\vec{n}(2;2)$ vektorni yasaymiz va unga perpendikulyar bo'lgan $2x_1 + 2x_2 = 0$ to'g'ri chiziq o'tkazamiz. Bu to'g'ri chiziq $2x_1 + 2x_2 = const$ to'g'ri chiziqlar oilasidan biri bo'ladi. Shakldan ko'rinadiki, masalada maqsad funksiyaning qiymati yuqoridan chegaralanmagan.

3-misol. Masalani grafik usulda yeching.

$$\begin{cases} 2x_1 - x_2 \le 4, \\ x_1 \ge 0, \quad x_2 \ge 1, \end{cases}$$

$$Y = x_1 - 2x_2 \to \max$$

Yechish: Masalani yuqoridagi usul bilan yechib quyidagi shaklga ega bo'lamiz.

Yuqoridagi 2-rasmdan ko'rinadiki, yechimlar to'plami chegaralanmagan, lekin optimal yechim mavjud va u $X^0(2,5;1)$ nuqta koordinatalaridan iborat.

Shuni alohida ta'kidlash kerakki, agar ChPMda noma'lumlar soni $n = 2$ bo'lganda uning optimal yechimini gtafik usulida topish maqsadga muvofiq.

Agar ChPM kanonik ko'rinishda berilgan bo'lib, tenglamalar sistemasida noma'lumlar soni tenglamalar sonidan 2 taga ko'p bo'lsa, ya'ni $n - m = 2$ bo'lsa, bunday ChPMlarining optimal yechimlarini ham gtafik usulida topish maqsadga muvofiq.

Iqtisodiy masalalarning optimal yechimlarining tahlili. Endi ChPMsining optimal yechimini geometrik nuqtai nazardan tahlil qilib chiqamiz. Buning uchun quyidagi iqtisodiy masalaning optimal yechimini quramiz va tahlil qilamiz.

Faraz qilaylik, korxonada ikki xil bo'yoq ishlab chiqarilsin. Bu bo'yoqlarni ishlab chiqarish uchun 2 xil xom-ashyodan foydalanilsin. Xom-ashyolarning zahirasi 6 va 8 birlikni tashkil qilsin. Ikkinchi bo'yoqqa bo'lgan talab 2 birlikdan oshmasin va u birinchi bo'yoqqa bo'lgan talabdan 1 birlikka katta bo'lsin.

Har bir bo'yoqning bir birligini ishlab chiqarish uchun kerak bo'lgan xom-ashyolar miqdori, hamda korxonaning har bir birlik bo'yoqni sotishdan oladigan daromadi quyidagi jadvalda keltirilgan.

Xom-ashyolar / **Bo'yoqlar**	**1**	**2**	**Foyda**
I	1	2	3
II	2	1	2
Zahira	6	8	

Har bir bo'yoqdan qanchadan ishlab chiqarilganda ularga sarf qilingan xom-ashyolar miqdori ularning zahiralaridan oshmaydi, daromad eng yuqori bo'ladi, hamda talab bo'yicha shartlar bajariladi? Masalaning optimal rejasini toping.

Masaladagi noma'lumlarni belgilaymiz:

x_1 – ishlab chiqarish rejalashtirilgan I mahsulotning miqdori;

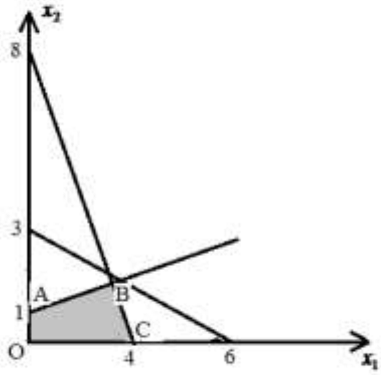
x_2 – ishlab chiqarish rejalashtirilgan II mahsulot miqdori.

U holda masalaning matematik modeli quyidagi ko'rinishda bo'ladi

$$\begin{cases} x_1 + 2x_2 \le 6 \\ 2x_1 + x_2 \le 8 \\ x_2 - x_1 \le 1 \\ x_1 \ge 0 \\ 0 \le x_2 \le 2 \end{cases}$$

$$Y = 3x_1 + 2x_2 \to \max.$$

Masalani grafik usulda yechib, $D(3\frac{1}{3};1\frac{1}{3})$ optimal nuqta ekanligini aniqlaymiz.



Optimalyechimquyidagicha bo'ladi: $x_1 = 3\frac{1}{3}$; $x_2 = 1\frac{1}{3}$; $Y_{\max} = 12\frac{2}{3}$. Demak, korxona birinchibo'yoqdan $3\frac{1}{3}$ birlik, ikkinchisidan $1\frac{1}{3}$ birlikishlabchiqarishikerak. Buholda uning oladigandaromadi $12\frac{2}{3}$ birlikka tengbo'ladi: $X^0\left(3\frac{1}{3},1\frac{1}{3}\right), Y = 12\frac{2}{3}$.

Endi masalaning optimal yechimini tahlil qilamiz. Buning uchun $D-$ optimal nuqtani qaraymiz. Bunuqta $2x_1 + x_2 = 8$ va $x_1 + 2x_2 = 6$ to'g'ri chiziqlarning kesishgan nuqtasi. Buesa, buyoq ishlab chiqarish uchun sarf qilinadigan ikkala xom-ashyoning ham kamyob ekanligini ko'rsatadi. Optimal nuqta bilan bog'liq bo'lgan bu shartlar aktiv shartlar , optimal nuqtaga bog'liq bo'lmagan shartlar esa passiv shartlar debataladi. Biz ko'rayotgan masalada mahsulotlarga bo'lgan talabga qo'yilgan $-x_1 + x_2 \le 1$ va $x_2 \le 2$ shartlar optimal nuqtaga bog'liq emas va shu sababli bu shartlar passiv shartlar.

Passiv shartlarga mos keluvchi resurslar kamyob bo'lmaydi va ularning ma'lum darajada o'zgarishi optimal yechimga ta'sir qilmaydi.

Aksincha, aktiv shartlarga mos keluvchi resurslarni bir birlikka oshirilishi optimal yechimning o'zgarishiga olib keladi.

Masalan, 1-xom-ashyo zahirasini bir birlikka oshirilishi optimal yechimga qanday ta'sir ko'rsatishini ko'rish uchun uning zahirasini 7 ga teng deb olamiz. U holda *CD*t o'g'ri chiziq o'ziga parallel ravishda yuqoriga ko'tariladi va *DCK* uchburchak reja ko'pburchagiga qo'shiladi. Natijada *K* nuqta optimal nuqtaga aylanadi.

Bu nuqtada $x_1 = 2;\ x_1 + 2x_2 = 7;\ 2x_1 + x_2 = 8$ to'g'ri chiziqlar kesishadi. Shuning uchun endi masalaning $0 \le x_2 \le 2;\ x_1 + 2x_2 \le 7;\ 2x_1 + x_2 \le 8$ shartlar aktiv shartlarga aylanadi. Demak, yangi optimal yechim: $X^0(2,3), Y_{\max} = 13.$

Xuddi shunday yo'l bilan 2-xom ashyoni bir birlikka oshirish optimal yechimni qanday o'zgartirishini ko'rsatish mumkin.

Bundan tashqari kamyob bo'lmagan xom-ashyolar miqdorini, optimal yechimga ta'sir qilmagan holda, qanchalik kamaytirish mumkinligini ham ko'rsatish mumkin.

Yuqoridagi 8-shaklda *BC* kesma $x_1 = 2$ chiziqni, ya'ni masalaning 4 shartini ifodalaydi. Ma'lumki, bu – passiv shart. Maqsad funksiya qiymatini o'zgartirmagan holda passiv shartni qanchalik o'zgartirish mumkin ekanligini aniqlash uchun *BC* kesmani o'ziga parallel ravishda pastga, *D* nuqta bilan kesishguncha siljitamiz. Bunuqtada $x_2 = \frac{4}{3}$ bo'ladi.

Demak, ikkinchi bo'yoqqa bo'lgan talabni optimal yechimga ta'sir qilmasdan $\frac{4}{3}$ gacha kamaytirish mumkin ekan.

Shunday yo'l bilan masalaning optimal yechimiga ta'sir etmasdan uning boshqa passiv shartning o'ng tomonini qanchaga kamaytirish mumkin ekanligini ko'rsatishmumkin.

TRANSPORT MASALASI

Tayanch so'z va iboralar: *Yopiq modelli transport masalasi, band katakchalar, ochiq modelli transport masalasi, "shimoliy-g'arb burchak" usuli, "minimal harajat" usuli.*

REJA:

1. Transport masalasining qo'yilishi va uning matematik modeli.
2. Transport masalasiyechimining xossalariga doir teoremalar.
3. Transport masalasining boshlang'ich joiz rejasini topish usullari.

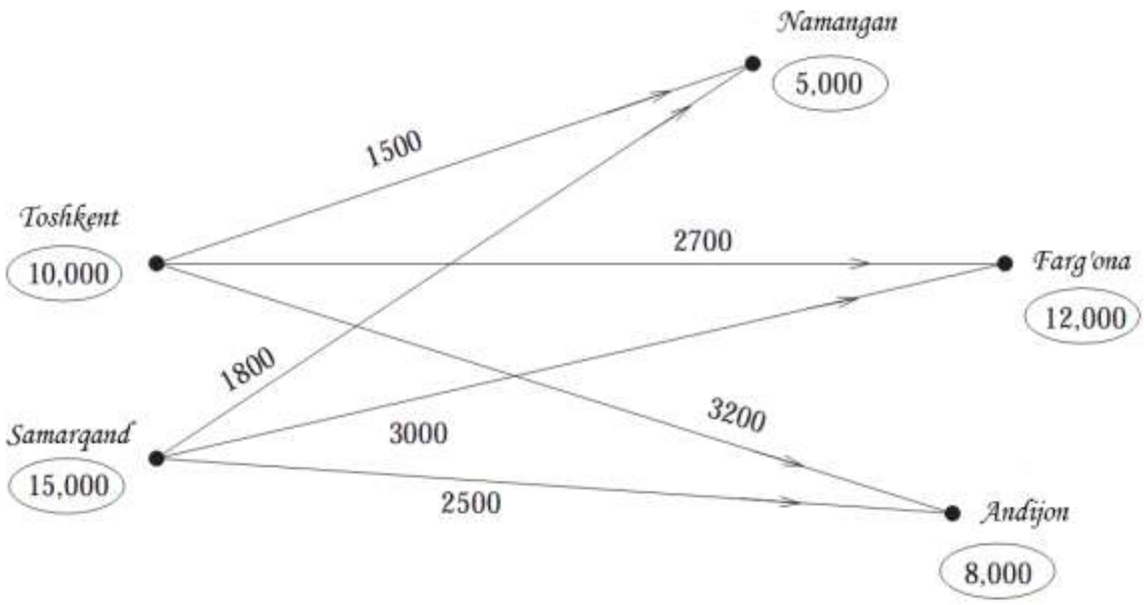
Transport masalasi – chiziqli programmalashtirishning alohidaxususiyatli masalasi bo'lib, bir jinsli yuk tashishning eng tejamli rejasini tuzish masalasidir. Bu masalaning qo'llanish sohasi juda kengdir.

Zahirasida b_i birlik mahsuloti bo'lgan i -ta'minotchidan mavjud bo'lgan istemolchilarga zahirasidagi mahsulotni to'la realizatsiya qilish shatri

$$\sum_j x_{i,j} = b_i$$

bu yerda $x_{i,j}$ – i -ta'minotchidan j -is'temolchiga tashilgan mahsulot hajmi.

1-misol. Faraz qilaylik, Toshkent va Samarqandga keltirilgan Xitoyda ishlab chiqariluvchi o'yinchoqlar Namangan, Farg'ona va Andijonga transport orqali tarqatilmoqda. Bunda Toshkentga 10000 ta va Samarqandga 15000 ta o'yinchoq keltirilgan bo'lib, Namanganga 5000 ta, Farg'onaga 12000 ta va Andijonga esa 8000 ta jo'natish rejalashtirilgan. Bitta o'yinchoqni yetkazib berishdagi transport harajatlari ta'minotchi va is'temolchilar orasidagi masofalarga to'g'ri proporsional bo'lib, masalaning tarmoq grafik ko'rinishi quyida keltirilgan.



Masalaning qo'yilishi va uning matematik modeli. m ta A_i ta'minotchilarda a_i miqdordagi bir xil mahsulotni n ta B_j iste'molchilarga mos ravishda b_j miqdordan yetkazib berish talab qilinsin. Har bir i -ta'minotchidan har bir j -iste'molchiga bir birlik mahsulotni tashishga sarf qilinadigan yo'l harajati c_{ij} pul birligini tashkil qilsin.

Mahsulot tashishning shunday rejasini tuzish kerakki, ta'minotchilardagi barcha mahsulotlar olib chiqib ketilsin, iste'molchilarning barcha talablari qondirilsin va shu bilan birga yo'l harajatlarining umumiy qiymati eng kichik bo'lsin.

Masalaning matematik modelini tuzish uchun i -ta'minotchidan j -iste'molchiga etkazib berish uchun rejalashtirilgan mahsulot miqdorini x_{ij} orqali belgilaymiz. U holda masalaning shartlarini quyidagi jadval ko'rinishda yozish mumkin:

Ta’minotchilar	Iste’molchilar				Zahiralar miqdori
	B_1	B_2	$\dots$	B_n	
A_1	c_{11} x_{11}	c_{12} x_{12}	$\dots$	c_{1n} x_{1n}	a_1
A_2	c_{21} x_{21}	c_{22} x_{22}	$\dots$	c_{2n} x_{2n}	a_2
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
A_m	c_{m1} x_{m1}	c_{m2} x_{m2}	$\dots$	c_{mn} x_{mn}	a_m
Talablar miqdori	b_1	b_2	$\dots$	b_n	$\sum a_i = \sum b_j$

Bunda harajatlarning umumiy qiymati

$$Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

ifoda bilan aniqlanadi.

Masalaning matematik modeli quyidagi ko’rinishni oladi:

$$\begin{cases} \sum\limits_{j=1}^{n} x_{ij} = a_i, & i = \overline{1,m} \\ \sum\limits_{i=1}^{m} x_{ij} = b_j, & j = \overline{1,n} \end{cases} \qquad (1)$$

chiziqli tenglamalar sistemasining

$$x_{ij} \geq 0, \quad (i = \overline{1,m}; \quad j = \overline{1,n}) \tag{2}$$

shartlarni qanoatlantiruvchi shunday yechimini topish kerakki, bu yechim

$$Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \tag{3}$$

chiziqli funksiyaga eng kichik qiymat bersin.

Jadvaldan va masalaning modelidan $0 \leq x_{ij} \leq \min\left(a_i, b_j\right)$ tengsizlikning bajariliahi ko'rinib turibdi.

Transport masalalarii ikki turga ajratib o'rganiladi:

1. Agar mahsulotga bo'lgan talab taklifga teng, ya'ni

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{4}$$

tenglik o'rinli bo'lsa, u holda bunday masala**yopiq modelli transport masalasi** deyiladi.

2. Agar mahsulotga bo'lgan talab taklifga teng bo'lmasa, ya'ni

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j \tag{5}$$

munosabat o'rinli bo'lsa, u holda bunday masalalar **ochiq modelli transport masalasi** deyiladi.

(1)-(3) masala uchun quyidagi teorema o'rinli.

1-teorema. Talablar hajmi takliflar hajmiga teng bo'lgan istalgan transport masalasining optimal yechimi mavjud bo'ladi.

Transport masalasi matematik modeli tenglamalar sistemasidagi bazis vektorlar sistemasining o'lchovini aniqlaymiz. Buning uchun sistema asosiy matrisasining rangini aniqlash kerak.

Agar x_{ij} o'zgaruvchilarni

$$x_{11}, x_{12}, ..., x_{1n}, x_{21}, x_{22}, ..., x_{2n}, ..., x_{m1}, x_{m2}, ..., x_{mn}$$

ko’rinishda joylashtirsak, u holda transport masalasining cheklamalar matrisasi quyidagi ko’rinishga ega bo’ladi.

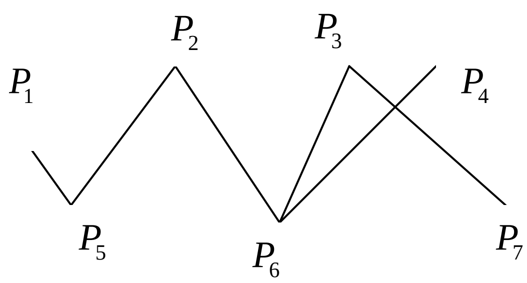
$$A=\begin{pmatrix} m\begin{cases} \overbrace{1\ 1\ \dots\ 1}^{n}\ \overbrace{0\ 0\dots 0}^{n}\ \dots\ \overbrace{0\ 0\dots 0}^{n} \\ 0\ 0\dots 0\ 1\ 1\ \dots\ 1\ \dots\ 0\ 0\dots 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ 0\ 0\dots 0\ 0\ 0\dots\ 0\ \dots 1\ 1\ \dots\ 1 \end{cases} \\ n\begin{cases} 1\ 0\dots 0\ 1\ 0\ \dots 0\ \dots 1\ 0\ \dots 0 \\ 0\ 1\dots 0\ 0\ 1\ \dots\ 0\ \dots 0\ 1\dots 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ 0\ 0\dots 1\ 0\ 0\dots\ 1\ \dots 0\ 0\ \dots\ 1 \end{cases} \end{pmatrix}$$

Bu matrisaning rangi: $rang(A)=m+n-1$ ekanligini ko’rish qiyin emas. Haqiqatdan ham, matrisada $m+n$ ta satr bo’lib ular chiziqli bog’liq. Chunki birinchi m ta satrni qo’shib undan oxirgi n ta satr yig’indisini ayirsak nol vektorni hosil qilamiz. Bu matrisaning ixtiyoriy $m+n-1$ satrini olsak chiziqli erkli vektorlar sistemasi hosil bo’ladi.

Demak, masalaning optimal yechimida musbat x_{ij} lar soni ko’pi bilan $m+n-1$ ta bo’ladi.

Transport masalasi rejalari ko’p takrorlanish xususiyatiga ega.

1-ta’rif. P_i nuqtalarning (punktlarning) chekli $P=\{P_1, P_2, ..., P_l\}$ to’plami va har bir elementi yoy deb ataluvchi tartiblanmagan (P_i, P_j) juftliklarning Ω to’plami berilgan bo’lsin. (P_i, P_j) yoy P_i va P_j nuqtalarni tutashtiradi, bu nuqtalar esa (P_i, P_j) yoyning oxiri deb ataladi. (P, Ω) juftlik esa **transport tarmog’i** deb ataladi. Masalan,



rasmda elementlari 7 ta nuqtadan iborat $P=\{P_1, P_2, ..., P_7\}$ to'plam va oltita:

$$(P_1, P_5), (P_2, P_5), (P_2, P_6), (P_3, P_6), (P_3, P_7), (P_4, P_6)$$

yoylarni o'zichiga oluvchi Ω to'plam tasvirlangan.

2-ta'rif. $P_{i_0}P_{i_1}...P_{i_k}$ ($P_{i_l} \in P,\ l=0,1,...,k$) ixtiyoriy chekli ketma-ketlik berilgan bo'lsin. Agar har qanday $(P_{i_r}, P_{i_{r+1}})$, $r=0,1,...,k-1$, juftlik yoy bo'lib ($(P_{i_r}, P_{i_{r+1}}) \in \Omega$), bu juftlik $P_{i_0}P_{i_1}...P_{i_k}$ ketma-ketlikda ko'pi bilan bir marta uchrasa, u holda $P_{i_0}P_{i_1}...P_{i_k}$ ketma-ketlik **marshrut** (yo'nalish) deb ataladi.

Yuqoridagi rasmda ikkita marshrut bor: $P_1P_5P_2P_6P_4$, $P_1P_5P_2P_6P_3P_7$.

3-ta'rif. $P_{i_0}P_{i_1}...P_{i_k}P_{i_0}$ ko'rinishdagi **marshrutsikli** deb ataladi.

Demak, marshrutda boshlang'ich holatga qaytilsa u **sikl** deb ataladi

Rasmdagi marshrutda sikl yo'q, ammo unga (P_4, P_7) yoy qo'shilsa, u holda bu marshrutda $P_3P_7P_4P_6P_3$ ko'rinishdagi sikl hosil bo'ladi.

Ma'lumki, ixtiyoriy chiziqli programmalashtirish masalasining optimal yechimini topish jarayoni boshlang'ich tayanch rejani topishdan boshlanadi.

Yopiq transport masalasining boshlang'ich tayanch rejasini topishning turli usullari mavjud bo'lib, ulardan ikkitasi bilan tanishib chiqamiz.

Boshlang'ich joiz rejani topish usullari. ***Masalaning aynimagan joiz rejasi*** $m+n-1$ ***ta musbat komponentalarni o'z ichigaoladi.***

Shunday qilib, transport masalasining aynimagan joiz rejasi biror usul bilan topilgan bo'lsa, matrisaning $m+n-1$ ta komponentalari musbat bo'lib, qolganlari nolga teng bo'ladi.

Agar transport masalasining shartlari va uning joiz rejasi yuqoridagi jadval ko'rinishda berilgan bo'lsa, noldan farqli x_{ij} lar joylashgan kataklar "**band kataklar**", qolganlari "**bo'sh kataklar**" deyiladi.

Yechim aynimagan bazis yechim bo'lishi uchun band kataklar soni $m+n-1$ ta bo'lib, u yerda sikllanish ro'y bermasligi kerak.

Shimoliy-g'arbiy burchak usuli. Quyidagi transport masalasi berilgan bo'lsin.

b_j a_i	b_1	b_2	$\ldots$	b_n
a_1	c_{11}	c_{12}	$\ldots$	c_{1n}
a_2	c_{21}	c_{22}	$\ldots$	c_{2n}
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
a_m	c_{m1}	c_{m2}	$\ldots$	c_{mn}

Ma'lumki, har bir bo'sh katakka x_{ij} noma'lumlardan biri to'g'ri keladi. Bu usulda bo'sh kataklrni x_{ij}^0 qiymatlar bilan to'ldiriladi deb faraz qilamiz.

Jadvalning shimoliy-g'arbiy burchagiga x_{11} o'zgaruvchi to'g'ri keladi. $x_{11}^0=\min\{a_1, b_1\}$ bo'lsin. Agar $x_{11}^0=a_1\ (a_1\le b_1)$ bo'lsa, u holda birinchi ta'minotchining barcha mahsuloti birinchi iste'molchiga jo'natilgan bo'ladi. Demak, $x_{1j}^0=0,\ j=\overline{1,n}$ bo'ladi.

II qadamda $x_{21}^0=\min\{b_1-a_1, a_2\}$ shart asosida x_{21} ning qiymatini aniqlaymiz. Bunda, agar $x_{21}^0=b_1-a_1\ (b_1-a_1\le a_2)$ bo'lsa, u holda $x_{s1}^0=0,\ s=\overline{3,m}$ bo'ladi. Bu jarayonni davom ettirib band kataklardagi x_{ij} larning qiymatlarini aniqlab olamiz.

Agar $x_{11}^0=b_1\ (b_1\le a_1)$ bo'lsa, u holda birinchi ta'minotchida a_1-b_1 miqdorda mahsulot qolgadi. Demak, $x_{i1}^0=0,\ i=\overline{2,m}$ bo'ladi. II qadamda $x_{12}^0=\min\{a_1-b_1, b_2\}$ shart asosida x_{21} ning qiymatini aniqlaymiz va hakozo.

2-misol. Shimoliy-g'arbiy burchak usulidan foydalanib, transport masalasining boshlang'ich yechimini toping.

Ta'minotchilar	Iste'molchilar					Zahira hajmi
	B_1	B_2	B_3	B_4	B_5	
A_1	10 100	7	4	1	4	100
A_2	2 100	7 150	10	6	11	250
A_3	8	5 50	3 100	2 50	2	200
A_4	11	8	12	16 50	13 250	300
Talab hajmi	200	200	100	100	250	

Minimal xarajatlar usuli. Buusulda boshlang'ich yechim qurish uchun x_{ij}^0 qiymat avvalambor yo'l harajati eng kichik bo'lgan katakka, ya'ni $\min\limits_{1\le i\le m, 1\le j\le n}\{c_{ij}\}$ shart o'rinli bo'ladigan katakka yoziladi. Masalan, $\min\limits_{1\le i\le m, 1\le j\le n}\{c_{ij}\} = c_{pq}$ bo'lsin. U holda $x_{pq}^0 = \min\{a_p, b_q\}$ qiymat aniqlanadi. $x_{pq}^0 = \min\{a_p, b_q\}$ $(a_p \le b_q)$ bo'lsin. Demak, $x_{pj} = a_p$, $j = 1, 2, ..., q-1, q+1, ..., n$ bo'ladi. Bundan keying qadamlarda ham $\min\limits_{1\le i\le m, 1\le j\le n}\{c_{ij}\} = c_{pq}$, $i \ne p$, $j \ne q$ shart asosida x_{ij}^0 qiymatlar aniqlanib boriladi. Bu usulda tuzilgan boshlang'ich yechimni sikllanishga tekshirish shart.

3-misol. Minimal harajatlar usuli bilan boshlang'ich yechimini toping.

Ta'minotchilar	**Iste'molchilar**					**Zahira hajmi**
	B_1	B_2	B_3	B_4	B_5	
A_1	10	7	4	1 100	4	100
A_2	2 200	7 50	10	6	11	250
A_3	8	5	3	2	2 200	200
A_4	11	8 150	12 100	16	13 50	300
Talab hajmi	200	200	100	100	250	

Transport masalasiga keltiriladigan taqsimot masalalari

Chiziqli programmalashtirishning maxsus masalalaridan bo'lgan transport masalasini yechish usuli bilan taqsimot masalasi deb ataluvchi masalalarni ham yechish mumkin. Bunday masalalar jumlasiga, mutaxassislarni ish o'rinlariga optimal taqsimlash, uskunalarni ish faoliyatiga optimal taqsimlash, shtatlar jadvalini optimal tuzish, samolyotlarni yo'nalishlarga optimal taqsimlash, haydovchilarni avtomashinalarga optimal taqsimlash kabi masalalar kiradi.

Yuqorida keltirilgan masalalardan mutaxassislarni ish o'rinlariga optimal taqsimlash masalasiga kengroq to'xtalamiz.

Masalaning iqtisodiy mohiyati quyidagicha: korxonadagi n ta $(A_1, A_2, ..., A_n)$ vakant o'ringa n ta kishi $(B_1, B_2, ..., B_n)$ da'vogar bo'lib, har bir bo'sh ish o'rni bo'yicha har bir da'vogar ish faoliyatini olib borishi mumkin, lekin ularning mehnat unumdorligi turlicha bo'ladi.

Shundan kelib chiqqan holda, har bir bo'sh ish joyiga bittadan da'vogarni shunday taqsimlash kerakki, natijada korxonaning umumiy mehnat unumdorligi maksimal bo'lsin.

Masalaning matematik modelini tuzish uchun quyidagi belgilashlarni kiritamiz:

a_{ij} - i – da'vogarning j– bo'sh ish joyi bo'yicha mehnat unumdorligi;

x_{ij} - i – da'vogarning j – bo'sh ish joyiga tayinlanishi deb belgilaymiz. Bunda $x_{ij} \geq 0\left(i = \overline{1,n};\ j = \overline{1,n}\right)$ bo'lib, ular 0 yoki 1 qiymatlarni qabul qiladi.

$$x_{ij} = \begin{cases} 1, & агар\ A_j\ вакант\ жойга\ B_i\ даъвогар\ тайинланса; \\ 0, & агар\ A_j\ вакант\ жойга\ B_i\ даъвогар\ тайинланмаса; \end{cases}$$

Yuqoridagi ma'lumotlarni jadval ko'rinishida quyidagicha ifodalash mumkin.

Ishga da'vogarlar	Vakant o'rinlar						Ishchi kuchi taklifi
	A_1	A_2	...	A_j	...	A_n	
B_1	x_{11} a_{11}	x_{12} a_{12}	...	x_{1j} a_{1j}	...	x_{1n} a_{1n}	1
...			...		...	...	...
B_i	x_{i1} a_{i1}	x_{i2} a_{i2}	...	x_{ij} a_{ij}	...	x_{in} a_{in}	1
...			...		...	...	...

B_n	x_{n1} a_{n1}		...	x_{nj} a_{nj}	...	x_{nn} a_{nn}	1
Bo'sh o'rinlar bo'yicha ishchilar talabi	1	1		1		1	

Shunday qilib, masalaning matematik modelini quyidagi ko'rinishda yozish mumkin:

$$\begin{cases} \sum_{i=1}^{n} x_{ij} = 1, \ \ (j = \overline{1,n}); \\ \sum_{j=1}^{n} x_{ij} = 1, \ \ (i = \overline{1,n}); \\ x_{ij} \geq 0. \end{cases}$$

$$Z = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_{ij} \rightarrow \max .$$

Mutaxassislarni bo'sh ish o'rinlariga optimal taqsimlash masalasini kombinatorik, potentsiallar usuli hamda vengercha usul bilan yechish mumkin bo'lib, quyida bulardan ayrimlarini misol orqali keltiramiz.

Korxonadagi uchta vakant joyga uch kishi da'vogar bo'lsin. Har bir da'vogarning ish turlari bo'yicha mehnat unumdorligi quyidagi jadvalda keltirilgan.

Ishga da'vogarlar	Ish turlari		
	A_1	A_2	A_3
B_1	5	10	15
B_2	10	15	5
B_3	25	10	10

Har bir bo'sh ish joyiga bittadan da'vogarni shunday taqsimlash kerakki, natijada korxonaning umumiy mehnat unumdorligi maksimal bo'lsin.

Masalani kombinatorik usul bilan echamiz. Uch da’vogarni uch vakant joyga taqsimlash mumkin bo’lgan barcha holatlarni aniqlaymiz.

$$(x_{11}, x_{22}, x_{33});\ (x_{11}, x_{23}, x_{32});(x_{12}, x_{21}, x_{33});$$
$$(x_{12}, x_{23}, x_{31});(x_{13}, x_{22}, x_{31});(x_{13}, x_{21}, x_{32}).$$

Har bir holat bo’yicha umumiy mehnat unumdorligini topamiz:

$$y_1 = 1\cdot 5 + 1\cdot 15 + 1\cdot 10 = 30; \qquad y_2 = 1\cdot 5 + 1\cdot 5 + 1\cdot 10 = 20;$$
$$y_3 = 1\cdot 10 + 1\cdot 10 + 1\cdot 10 = 30; \qquad y_4 = 1\cdot 10 + 1\cdot 5 + 1\cdot 25 = 40;$$
$$y_5 = 1\cdot 15 + 1\cdot 15 + 1\cdot 25 = 55; \qquad y_6 = 1\cdot 15 + 1\cdot 10 + 1\cdot 10 = 35;$$
$$\max(y_1, y_2, y_3, y_4, y_5, y_6) = \max(30;\ 20;\ 30;\ 40;\ 55;\ 35) = 55.$$

Demak, birinchi da’vogar uchinchi vakant joyga, ikkinchi da’vogar ikkinchi vakant joyga, uchinchi da’vogar birinchi vakant joyga taqsimlanganda, korxonaning umumiy mehnat unumdorligi 55 birlikni tashkil qilar ekan.

Endi bu masalani potentsiallar usuli bilan ko’rib chiqamiz. Masalani echish uchun uning berilganlarini quyidagi ko’rinishda yozib, boshlang’ich bazis echimni "shimoliy-g’arb burchak" usuli bilan topamiz. So’ngra potentsiallarni hisoblab, optimallik mezonini tekshiramiz.

b_j / a_i	1	1	1	u_i
1	5 1	10 0-θ	15 +θ -15	u_1=0
1	10 0	15 1+θ	5 0-θ	u_2=5
1	25 -10	10 10	10 1	u_3=10

v_j	v_1=5	v_2=10	v_3 =0	θ=0

Maqsad funktsiyasi *F(x)→max* ko’rinishda bo’lgani uchun optimallik mezoni $u_i+v_j \geq c_{ij}$ tengsizlikdan iborat bo’ladi. Demak, optimal yechimda band katakchalarga mos kelgan potentsiallar yig’indisi shu katakdagi mehnat unumdorligiga teng bo'lishi kerak, ya'ni

$$u_i+v_j=c_{ij}\ (x_{ij}\geq 0),$$

bo'sh katakchalarga mos kelgan potentsiallar esa

$$u_i+v_j\geq c_{ij}$$

tengsizlikni qanoatlantirishi kerak. Bo'sh kataklar bo'yicha optimal baho

$$\Delta_{ij}=u_i+v_j-c_{ij}>0$$

formula yordamida topiladi. Yuqoridagi masalaning (1;3) va (3;1) kataklarida bu shart bajarilmaydi. Shuning uchun bazisga $\min\limits_{\Delta_{ij}<0}(-15;-10)=\Delta_{31}=-15$ shartiga mos keluvchi x_{13} ni kiritamiz. (1;3) katakka θ son kiritib, potentsiallar usulini qo'llab, x_{12} ni bazisdan chiqaramiz. Natijada yangi bazis yechimga ega bo'lamiz:

a_i \ b_j	1	1	1	u_i
1	5 $1-\theta$	10 15	15 $0+\theta$	$u_1=0$
1	10 5	15 1	5 0	$u_2=-10$
1	25 $+\theta$ -25	10 10	10 $1-\theta$	$u_3=-5$
v_j	$v_1=5$	$v_2=25$	$v_3=15$	$\theta=1$

Bu bosqichda ham (3;1) katakda optimallik shart bajarilmaydi. Shuning uchun bazisga x_{31} ni kiritib, x_{12} ni bazisdan chiqaramiz. Natijada yangi bazis yechimga ega bo'lamiz.

Uchinchi bosqichda topilgan bazis yechim uchun optimallik sharti bajarilayapti.

a_i \ b_j	1	1	1	u_i
1	5 25	10 15	15 1	$u_1=0$
1	10 10	15 1	5 0	$u_2=-10$
1	25 1	10 10	10 0	$u_3=-5$
v_j	$v_1=30$	$v_2=25$	$v_3=15$	

Shunday qilib, birinchi da’vogar uchinchi vakant joyga, ikkinchi da’vogar ikkinchi vakant joyga, uchinchi da’vogar birinchi vakant joyga taqsimlansa, korxonaning umumiy mehnat unumdorligi

$$F_{max}=1\cdot 25+1\cdot 15+1\cdot 15=55$$

birlikni tashkil qilar ekan.

1-oraliq nazorati uchun mustaqil ish variantlari

1-misolda berilgan matritsalarning chiziqli kombinatsiyasini toping. 2-misolda matritsalar ko’paytmasi AB va BA ni toping (agar ular mavjud bo’lsa). 3- misolda berilgan matritsa rangini toping; 4-misolda berilgan matritsaga teskari matritsani toping; 5-misolda uchinchi tartibli determinantlarni qulay usulda hisoblang.

1-variant

1. $2A^T + 3B,\ A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 3 & -1 \end{pmatrix},\ B = \begin{pmatrix} -2 & 3 & 0 \\ 2 & 1 & 1 \end{pmatrix}.$ 2. $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \end{pmatrix},\ B = \begin{pmatrix} 3 & 4 & 5 \\ 6 & 0 & -2 \\ 7 & 1 & 8 \end{pmatrix}.$

3. $A = \begin{pmatrix} 1 & -3 & -2 & -2 \\ -3 & 10 & 2 & 1 \\ 7 & -24 & -2 & 1 \end{pmatrix}.$ 4. $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}.$ 5. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}.$

6. $A = \begin{pmatrix} 2 & 4 & 1 \\ 3 & 0 & 2 \\ -3 & 4 & -2 \end{pmatrix},\ B = \begin{pmatrix} 1 & 3 & -1 \\ 3 & -1 & 4 \\ -4 & 4 & -2 \end{pmatrix}\ A \cdot B = ?.$

$rangA = ?,\quad A^{-1} = ?$

. $\begin{cases} 2x_1 - x_2 + x_3 = 2, \\ x_1 + 2x_2 + x_3 = 4, \\ 5x_1 - x_2 - 3x_3 = 1. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} 2x_1 - 2x_2 + x_3 = 2, \\ x_1 + 2x_2 + 3x_3 = 12, \\ 5x_1 - 3x_2 - 3x_3 = -2. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1 - 5x_2 + x_3 = 2, \\ 2x_1 + 2x_2 + x_3 = -5, \\ 5x_1 - 2x_2 - 3x_3 = 0. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1 - x_2 + 4x_3 = 8, \\ 3x_1 + 2x_2 + 3x_3 = 4, \\ 5x_1 - 3x_2 - 3x_3 = 5. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

2-variant

1. $A^T - 3E,\ A = \begin{pmatrix} 2 & 5 & -1 \\ -1 & -3 & 0 \\ 2 & 3 & -2 \end{pmatrix}.$ 2. $A = \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix},\ B = \begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix}.$

3. $A = \begin{pmatrix} 1 & 2 & -1 & 2 \\ 9 & -12 & 15 & 0 \\ -2 & 6 & -6 & 2 \end{pmatrix}.$ 4. $\begin{pmatrix} 2 & 1 & 1 \\ 5 & 1 & 3 \\ 2 & 1 & 2 \end{pmatrix}.$ 5. $\begin{vmatrix} a & 1 & a \\ -1 & a & 1 \\ a & -1 & a \end{vmatrix}.$

6. $A = \begin{pmatrix} 2 & -3 & 1 \\ 1 & 3 & 25 \\ -3 & 4 & -2 \end{pmatrix},\ B = \begin{pmatrix} 1 & 2 & -1 \\ 3 & -21 & 4 \\ -4 & 4 & 5 \end{pmatrix}\ A \cdot B = ?.$

$rangA = ?,\quad A^{-1} = ?$

1. $\begin{cases} 2x_1 - x_2 + 2x_3 = -1, \\ x_1 + 2x_2 + 3x_3 = 4, \\ 5x_1 - x_2 - 3x_3 = -9. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} 2x_1 - 2x_2 + x_3 = 1, \\ x_1 + 2x_2 + 3x_3 = 6, \\ 5x_1 - x_2 - 3x_3 = 1. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1 - 5x_2 + x_3 = 8, \\ 2x_1 + 3x_2 + 2x_3 = 1, \\ 5x_1 - 2x_2 - 3x_3 = 4. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1 - x_2 + 4x_3 = -6, \\ x_1 + 2x_2 + x_3 = -4, \\ 5x_1 - 3x_2 - x_3 = -1. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

3-variant

1. $4A - 5B^T$, $A = \begin{pmatrix} 2 & -1 & 0 \\ 3 & 4 & -2 \\ -3 & 1 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -2 & 0 \\ 1 & 1 & 2 \\ 2 & 3 & -4 \end{pmatrix}$.

2. $A = \begin{pmatrix} 4 & 0 & -2 & 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 3 \\ 1 \\ -1 \\ 5 \\ 2 \end{pmatrix}$.

3. $A = \begin{pmatrix} 1 & 4 & -3 & 61 \\ 4 & 10 & 2 & -46 \\ 34 & -20 & 40 & 0 \end{pmatrix}$.

4. $\begin{pmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 7 & 9 & 1 \end{pmatrix}$.

5. $\begin{vmatrix} 1 & b & 1 \\ 0 & b & 0 \\ b & 0 & -b \end{vmatrix}$.

6. $A = \begin{pmatrix} 2 & -3 & 1 \\ 1 & 3 & 25 \\ -3 & 4 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & -1 \\ 3 & -21 & 4 \\ -4 & 4 & 5 \end{pmatrix}$ $A \cdot B = ?$.

$rangA = ?$, $\quad A^{-1} = ?$

1. $\begin{cases} 2x_1 - x_2 + 2x_3 = -1, \\ x_1 + 2x_2 + 3x_3 = 4, \\ 5x_1 - x_2 - 3x_3 = -9. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} 2x_1 - 2x_2 + x_3 = 1, \\ x_1 + 2x_2 + 3x_3 = 6, \\ 5x_1 - x_2 - 3x_3 = 1. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1 - 5x_2 + x_3 = 8, \\ 2x_1 + 3x_2 + 2x_3 = 1, \\ 5x_1 - 2x_2 - 3x_3 = 4. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1 - x_2 + 4x_3 = -6, \\ x_1 + 2x_2 + x_3 = -4, \\ 5x_1 - 3x_2 - x_3 = -1. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

4-variant

1. $3A^T + 4B,\ A = \begin{pmatrix} 7 & 0 & -5 \\ -2 & 2 & 3 \\ 3 & 1 & 2 \\ -4 & -1 & 0 \end{pmatrix},\ B = \begin{pmatrix} 2 & -1 & -3 & 1 \\ 7 & -1 & 0 & 4 \\ 8 & -2 & 1 & 5 \end{pmatrix}.$

2. $A = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix},\ B = \begin{pmatrix} 2 & 6 \\ -1 & -3 \end{pmatrix}.$

3. $A = \begin{pmatrix} 2 & 1 & -2 & 3 \\ -3 & 0 & 1 & 1 \\ 5 & 1 & -3 & 2 \end{pmatrix}.$

4. $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}.$

5. $\begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix}.$

4. $A = \begin{pmatrix} 2 & 7 & 1 \\ 3 & 3 & -6 \\ -3 & 4 & 5 \end{pmatrix},\ B = \begin{pmatrix} 12 & 3 & -11 \\ 3 & -2 & -2 \\ -4 & 4 & 5 \end{pmatrix}\ A \cdot B = ?.$

5. $A=\begin{pmatrix} 1 & -1 & 5 & -1 \\ 2 & -3 & 2 & 4 \\ 3 & 3 & -2 & 1 \\ 7 & 9 & 7 & 0 \end{pmatrix}$ $rangA=?,\quad A^{-1}=?$

1. $\begin{cases} x_1-2x_2+2x_3=2, \\ 3x_1+2x_2+3x_3=16, \\ 5x_1+2x_2-3x_3=8. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} 2x_1-2x_2+x_3=1, \\ x_1+2x_2+x_3=-2, \\ 5x_1+x_2-3x_3=-9. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1-5x_2+x_3=-4, \\ 2x_1+x_2+2x_3=1, \\ 5x_1+2x_2-3x_3=10. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1-x_2+4x_3=13, \\ x_1+2x_2+x_3=8, \\ 5x_1-3x_2+x_3=-2. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

5-variant

1. $3A-2B^T$, $A=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B=\begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}$.

2. $A=\begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$, $B=\begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$.

3. $A=\begin{pmatrix} 3 & 2 & 1 & 3 \\ 6 & 4 & 3 & 5 \\ 9 & 6 & 5 & 7 \end{pmatrix}.$ 4. $\begin{pmatrix} 1 & 3 & 4 \\ 2 & 0 & 3 \\ -2 & 1 & -3 \end{pmatrix}.$ 5. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}.$

6. $A=\begin{pmatrix} 2 & 14 & 1 \\ 3 & 13 & 12 \\ -3 & 4 & 15 \end{pmatrix}, B=\begin{pmatrix} 13 & 3 & -11 \\ 3 & -12 & 4 \\ -4 & 4 & 14 \end{pmatrix} A\cdot B=?.$

7. $A=\begin{pmatrix} 1 & -1 & 5 & 7 \\ -11 & -3 & 12 & 4 \\ 3 & 15 & 1 & 2 \\ 7 & 9 & 7 & 12 \end{pmatrix} rangA=?, \quad A^{-1}=?$

1. $\begin{cases} x_1-2x_2+2x_3=3, \\ 3x_1+2x_2+3x_3=-2, \\ x_1+2x_2+3x_3=0. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} 2x_1-2x_2+x_3=1, \\ x_1+2x_2+x_3=8, \\ 5x_1+x_2-3x_3=-2. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1-5x_2+x_3=-2, \\ 2x_1+x_2+2x_3=5, \\ 5x_1+2x_2-3x_3=4. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1-x_2+4x_3=12, \\ x_1+7x_2+x_3=18, \\ 5x_1+3x_2+x_3=18. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

6-variant

1. $2B-5A^T$, $A=\begin{pmatrix}0 & -6\\ 2 & 4\\ 4 & 0\end{pmatrix}$, $B=\begin{pmatrix}0 & 5 & 10\\ -15 & 10 & 0\end{pmatrix}$.

2. $A=\begin{pmatrix}2 & 4\\ -5 & 7\end{pmatrix}$, $B=\begin{pmatrix}2 & -2\\ -2 & 3\end{pmatrix}$.

3. $A=\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 3 & 4 & 5\\ 3 & 4 & 5 & 6\\ 4 & 5 & 6 & 7\end{pmatrix}$.

4. $\begin{pmatrix}1 & 2 & 3\\ 2 & 0 & -1\\ 3 & -1 & 4\end{pmatrix}$.

5. $\begin{vmatrix}\sin^2\alpha & \cos^2\alpha & \cos 2\alpha\\ \sin^2\beta & \cos^2\beta & \cos 2\beta\\ \sin^2\gamma & \cos^2\gamma & \cos 2\gamma\end{vmatrix}$.

6. $A=\begin{pmatrix}2 & 14 & 1\\ 3 & 13 & 12\\ -3 & 4 & 15\end{pmatrix}, B=\begin{pmatrix}13 & 3 & -11\\ 3 & -12 & 4\\ -4 & 4 & 14\end{pmatrix}$ $A\cdot B=?$.

7. $A=\begin{pmatrix}1 & -1 & 5 & 7\\ -11 & -3 & 12 & 4\\ 3 & 15 & 1 & 2\\ 7 & 9 & 7 & 12\end{pmatrix}$ $rangA=?,\quad A^{-1}=?$

1. $\begin{cases}x_1-2x_2+2x_3=3,\\ 3x_1+2x_2+3x_3=-2,\\ x_1+2x_2+3x_3=0.\end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases}2x_1-2x_2+x_3=1,\\ x_1+2x_2+x_3=8,\\ 5x_1+x_2-3x_3=-2.\end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases}2x_1-5x_2+x_3=-2,\\ 2x_1+x_2+2x_3=5,\\ 5x_1+2x_2-3x_3=4.\end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1 - x_2 + 4x_3 = 12, \\ x_1 + 7x_2 + x_3 = 18, \\ 5x_1 + 3x_2 + x_3 = 18. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

7-variant

1. $A^T - 2E,\ A = \begin{pmatrix} 2 & 3 \\ 3 & -2 \end{pmatrix}.$ 2. $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix},\ B = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}.$

3. $A = \begin{pmatrix} 1 & 8 & -1 & 2 \\ 2 & -1 & 8 & 5 \\ 1 & 10 & -6 & 8 \end{pmatrix}.$ 4. $\begin{pmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{pmatrix}.$ 5. $\begin{vmatrix} \sin^2\alpha & \cos^2\alpha & 1 \\ \sin^2\beta & \cos^2\beta & 1 \\ \sin^2\gamma & \cos^2\gamma & 1 \end{vmatrix}.$

6. $A = \begin{pmatrix} 2 & 14 & -2 \\ 13 & 13 & -12 \\ -3 & 4 & 15 \end{pmatrix},\ B = \begin{pmatrix} 3 & 3 & -11 \\ 13 & -12 & 4 \\ -4 & 12 & 4 \end{pmatrix}\ A \cdot B = ?.$

7. $A = \begin{pmatrix} 1 & -1 & 5 & 7 \\ 0 & -3 & 13 & 4 \\ 3 & 0 & 11 & 2 \\ -7 & -5 & 7 & 12 \end{pmatrix}\ rangA = ?,\quad A^{-1} = ?$

8-variant

1. $4A^T - 7B,\ A = \begin{pmatrix} 1 & 2 & 5 \\ -2 & 0 & -1 \\ 5 & -3 & 0 \\ 3 & 1 & 4 \end{pmatrix},\ B = \begin{pmatrix} 0 & 2 & 7 & -5 \\ -8 & 1 & 3 & 0 \\ 4 & 2 & -2 & 5 \end{pmatrix}.$

2. $A = (1 \quad -2 \quad 3 \quad 0),\ B = \begin{pmatrix} 5 \\ -3 \\ -4 \\ 1 \end{pmatrix}.$ 5. $\begin{vmatrix} a & a^2+1 & (a+1)^2 \\ b & b^2+1 & (b+1)^2 \\ c & c^2+1 & (c+1)^2 \end{vmatrix}.$

6. $A = \begin{pmatrix} 2 & 14 & -2 \\ 13 & 13 & -12 \\ -3 & 4 & 15 \end{pmatrix},\ B = \begin{pmatrix} 3 & 3 & -11 \\ 13 & -12 & 4 \\ -4 & 12 & 4 \end{pmatrix}\ A \cdot B = ?.$

7. $A=\begin{pmatrix} 1 & -1 & 5 & 7 \\ 0 & -3 & 13 & 4 \\ 3 & 0 & 11 & 2 \\ -7 & -5 & 7 & 12 \end{pmatrix}$ $rangA=?,\quad A^{-1}=?$

1. $\begin{cases} x_1-2x_2+2x_3=3, \\ 3x_1+5x_2+3x_3=22, \\ 2x_1+2x_2+3x_3=15 \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} 2x_1-2x_2+x_3=-1, \\ x_1+2x_2+2x_3=-5, \\ 5x_1+x_2-3x_3=-2. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1-5x_2+x_3=-8, \\ 2x_1+5x_2+2x_3=1, \\ 5x_1+2x_2+3x_3=-6. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1-x_2+4x_3=14, \\ x_1+7x_2+4x_3=20, \\ 3x_1+3x_2+2x_3=12. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

9-variant

1. ***a)*** $\begin{vmatrix} -27 & -23 \\ -17 & 15 \end{vmatrix}$**;** ***b)*** $\begin{vmatrix} -24 & -12 \\ 32 & -33 \end{vmatrix}$**;*v)*** $\begin{vmatrix} -27 & -31 \\ -52 & -24 \end{vmatrix}$**;**

2. a) $\begin{vmatrix} -8 & 10 & 3 \\ 5 & -10 & 12 \\ -10 & -4 & 13 \end{vmatrix}$; $\begin{vmatrix} 13 & -14 & 15 \\ 18 & -10 & 2 \\ -21 & 12 & 30 \end{vmatrix}$; $\begin{vmatrix} 4 & 13 & 5 \\ 13 & -5 & 7 \\ 5 & -13 & -24 \end{vmatrix}$; $\begin{vmatrix} 3 & -4 & 2 & 4 \\ -2 & 6 & 4 & 1 \\ 3 & -6 & -12 & -6 \\ -4 & 8 & -3 & 2 \end{vmatrix}$.

3. $A=\begin{pmatrix} 12 & 29 & 14 & -16 \\ 36 & -33 & -7 & 41 \end{pmatrix}, B=\begin{pmatrix} 12 & -31 & -23 & -21 \\ 13 & -12 & -24 & 15 \end{pmatrix}$ $A-B=?$.

4. $A=\begin{pmatrix} 12 & 18 & 12 \\ 5 & -4 & -12 \\ -3 & 4 & 8 \end{pmatrix}, B=\begin{pmatrix} -3 & 13 & -31 \\ -13 & -12 & 14 \\ -4 & 7 & -4 \end{pmatrix}$ $A\cdot B=?$.

5. $A=\begin{pmatrix} -1 & -1 & 5 & 7 \\ 1 & -3 & -12 & 4 \\ 3 & 10 & -1 & 8 \\ 2 & 10 & 2 & 4 \end{pmatrix}$ $rangA=?,\quad A^{-1}=?$

1. $\begin{cases} 8x_1 - x_2 - x_3 = 6, \\ 4x_1 + 2x_2 - x_3 = 5, \\ 2x_1 - x_2 + 3x_3 = 4. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} 2x_1 - 7x_2 + x_3 = -15, \\ 4x_1 - x_2 - x_3 = -7, \\ 6x_1 - x_2 - 3x_3 = -11. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1 - 5x_2 + 2x_3 = -2, \\ 3x_1 + x_2 - x_3 = 2, \\ 5x_1 + 2x_2 - 7x_3 = -12. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1 - x_2 + 4x_3 = -3, \\ x_1 + 6x_2 + x_3 = 5, \\ 2x_1 - 4x_2 - 7x_3 = -15. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

10-variant

1. ***a)*** $\begin{vmatrix} -3 & 23 \\ -7 & -25 \end{vmatrix}$***; b)*** $\begin{vmatrix} -32 & -32 \\ 39 & 31 \end{vmatrix}$***;v)*** $\begin{vmatrix} -27 & 25 \\ -52 & -24 \end{vmatrix}$***;***

2. a) $\begin{vmatrix} 12 & 10 & 3 \\ 25 & 1 & -36 \\ -10 & -4 & 3 \end{vmatrix}$***;*** $\begin{vmatrix} 3 & -14 & 15 \\ 28 & 3 & 22 \\ 21 & 12 & -30 \end{vmatrix}$***;)*** $A = \begin{pmatrix} 12 & -39 & 14 & -26 \\ 46 & -33 & 87 & -11 \end{pmatrix}$***;.***

$\begin{vmatrix} 3 & 3 & 2 & 4 \\ -12 & -6 & 6 & 1 \\ 2 & 6 & 1 & -6 \\ -4 & 8 & -3 & 2 \end{vmatrix}.$

3. $A = \begin{pmatrix} 12 & -59 & 14 & -26 \\ 96 & -33 & 87 & 41 \end{pmatrix}, B = \begin{pmatrix} 12 & 31 & -23 & -21 \\ 13 & -52 & -24 & 45 \end{pmatrix} \; A + B = ?.$

4. $A = \begin{pmatrix} 12 & 8 & 12 \\ 5 & 4 & 4 \\ -3 & 4 & 8 \end{pmatrix}, B = \begin{pmatrix} 13 & 13 & -31 \\ 23 & -12 & 24 \\ -4 & 17 & 14 \end{pmatrix} \; A \cdot B = ?.$

5. $A=\begin{pmatrix} -1 & -11 & 5 & 7 \\ 1 & -3 & 12 & 4 \\ 3 & 4 & -1 & 8 \\ 5 & 6 & 2 & 4 \end{pmatrix}$ $rangA=?,\quad A^{-1}=?$

1, $\begin{cases} x_1+3x_2-x_3=8, \\ 4x_1+2x_2-x_3=15, \\ 2x_1-x_2+3x_3=7. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} 2x_1-7x_2+x_3=-3, \\ 4x_1+x_2-x_3=3, \\ 6x_1-x_2-3x_3=-1. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1+5x_2+2x_3=-5, \\ 3x_1+x_2+x_3=-3, \\ 5x_1+2x_2-7x_3=-14. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1-x_2+4x_3=0, \\ x_1+6x_2+x_3=13, \\ 2x_1-4x_2+7x_3=-11. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

11-variant

1. ***a)*** $\begin{vmatrix} -2 & -3 \\ -77 & 25 \end{vmatrix}$; ***b)*** $\begin{vmatrix} -12 & -32 \\ 49 & -61 \end{vmatrix}$; ***v)*** $\begin{vmatrix} -27 & -5 \\ -52 & 4 \end{vmatrix}$;

2. a) $\begin{vmatrix} -12 & -10 & 13 \\ 5 & 13 & -36 \\ -14 & -4 & 3 \end{vmatrix}$**;** $\begin{vmatrix} 13 & -14 & 15 \\ -8 & 13 & 12 \\ 11 & -2 & -10 \end{vmatrix}$**;** $\begin{vmatrix} 4 & -12 & 5 \\ 14 & -12 & 7 \\ 15 & -13 & -14 \end{vmatrix}$**;** $\begin{vmatrix} -3 & 3 & 2 & 4 \\ 1 & 6 & 4 & 1 \\ 3 & 6 & 1 & -6 \\ -4 & 7 & -3 & 21 \end{vmatrix}$.

3. $A = \begin{pmatrix} 22 & -59 & 24 & -26 \\ 44 & -34 & 87 & 23 \end{pmatrix}, B = \begin{pmatrix} 23 & 31 & -23 & -21 \\ 13 & -32 & -24 & -35 \end{pmatrix}$ $A - B = ?$.

4. $A = \begin{pmatrix} 5 & 8 & 12 \\ 6 & -14 & 4 \\ -3 & 14 & 2 \end{pmatrix}, B = \begin{pmatrix} 3 & -13 & 9 \\ 13 & 2 & 14 \\ -4 & 7 & 8 \end{pmatrix}$ $A \cdot B = ?$.

5. $A = \begin{pmatrix} 5 & -1 & 5 & 7 \\ 1 & -3 & 12 & 4 \\ 3 & 5 & -11 & 8 \\ 7 & 6 & 12 & -4 \end{pmatrix}$ $rangA = ?, \quad A^{-1} = ?$

1. $\begin{cases} x_1 + 3x_2 - x_3 = 10, \\ x_1 + 3x_2 - x_3 = 13, \\ 2x_1 - x_2 + 3x_3 = 1. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} 2x_1 - 7x_2 + x_3 = -8, \\ 4x_1 + x_2 - x_3 = -4, \\ 6x_1 - x_2 - 3x_3 = -10. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1 + 4x_2 + 2x_3 = 10, \\ 3x_1 + x_2 + x_3 = 8, \\ 5x_1 + 2x_2 - 7x_3 = 5. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1 - x_2 + 4x_3 = 12, \\ x_1 + 6x_2 + x_3 = 10, \\ 2x_1 - 4x_2 + 7x_3 = 9. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

12-variant

1. ***a)*** $\begin{vmatrix} -3 & -13 \\ -25 & 12 \end{vmatrix}$; ***b)*** $\begin{vmatrix} -32 & -32 \\ 34 & 44 \end{vmatrix}$;***v)*** $\begin{vmatrix} -27 & -15 \\ -52 & 14 \end{vmatrix}$;

2. *a)* $\begin{vmatrix} 12 & 10 & 13 \\ -8 & -15 & 19 \\ -14 & -4 & 3 \end{vmatrix}$; $\begin{vmatrix} 13 & -14 & 4 \\ 1 & 13 & 12 \\ 11 & 1 & -10 \end{vmatrix}$; $\begin{vmatrix} 4 & -13 & 5 \\ 14 & 15 & 17 \\ 5 & -13 & 14 \end{vmatrix}$; $\begin{vmatrix} 5 & 3 & 2 & 4 \\ 4 & -6 & 0 & 1 \\ -3 & 6 & 0 & -6 \\ -4 & 8 & -3 & 11 \end{vmatrix}$.

1. $A = \begin{pmatrix} 12 & -59 & 24 & -26 \\ 36 & -44 & 37 & 23 \end{pmatrix}, B = \begin{pmatrix} 52 & 31 & -21 & -21 \\ 13 & -32 & -24 & 15 \end{pmatrix}$ $A + B = ?$.

2. $A = \begin{pmatrix} 7 & 8 & 4 \\ 5 & 1 & 4 \\ -3 & 5 & 8 \end{pmatrix}, B = \begin{pmatrix} 5 & 13 & -31 \\ 13 & 2 & -4 \\ -4 & 7 & 14 \end{pmatrix}$ $A \cdot B = ?$.

3. $A = \begin{pmatrix} -4 & -1 & 5 & 7 \\ 1 & -3 & 12 & 4 \\ 3 & 5 & -2 & 7 \\ 5 & 6 & 4 & 4 \end{pmatrix}$1. $\begin{cases} x_1 + 3x_2 - x_3 = 7, \\ 2x_1 + 3x_2 - x_3 = 9, \\ 2x_1 - x_2 + 2x_3 = 4. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} 2x_1 - 7x_2 + x_3 = 10, \\ 4x_1 + x_2 - 2x_3 = 1, \\ 6x_1 - 3x_2 - 3x_3 = 6. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1 + x_2 + 2x_3 = -1, \\ 3x_1 + 2x_2 + x_3 = 3, \\ 5x_1 + 2x_2 - 7x_3 = 21. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1 - 2x_2 + 4x_3 = 5, \\ x_1 + 6x_2 + 2x_3 = 9, \\ 2x_1 - x_2 + 7x_3 = 8. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

13-variant

1. ***a)*** $\begin{vmatrix} 3 & 4 \\ -25 & 41 \end{vmatrix}$; ***b)*** $\begin{vmatrix} 21 & 2 \\ 2 & -52 \end{vmatrix}$; ***v)*** $\begin{vmatrix} 2 & 23 \\ -31 & 4 \end{vmatrix}$;

2. *a)* $\begin{vmatrix} 2 & 15 & 3 \\ 5 & 3 & 2 \\ -13 & 4 & -12 \end{vmatrix}$; $\begin{vmatrix} 3 & 4 & -5 \\ 8 & 7 & -12 \\ 2 & -21 & 8 \end{vmatrix}$; $\begin{vmatrix} 5 & -3 & 5 \\ 3 & -12 & 9 \\ 13 & 17 & -5 \end{vmatrix}$; $\begin{vmatrix} 1 & 4 & 2 & 0 \\ -2 & 3 & -3 & 1 \\ 3 & 0 & 12 & -6 \\ 5 & -4 & 0 & 2 \end{vmatrix}$.

3. $\boldsymbol{A}=\begin{pmatrix} 1 & -3 & 4 & 0 \\ 2 & -1 & 3 & 5 \end{pmatrix}, B=\begin{pmatrix} 23 & 42 & -3 & 41 \\ 34 & 2 & -4 & -5 \end{pmatrix}\ A-B=?.$

4. $A=\begin{pmatrix} 4 & 4 & -13 \\ 3 & -20 & 2 \\ -3 & 14 & 5 \end{pmatrix}, B=\begin{pmatrix} 1 & 3 & -1 \\ 3 & -1 & 4 \\ -4 & 6 & 5 \end{pmatrix}\ A\cdot B=?.$

5. $A=\begin{pmatrix} 1 & -1 & 5 & 7 \\ 3 & -3 & 2 & 4 \\ 3 & 5 & 4 & -1 \\ 6 & 5 & 7 & 1 \end{pmatrix}\ rangA=?,\quad A^{-1}=?$

1. $\begin{cases} 2x_1 - x_2 + x_3 = -2, \\ 2x_1 - x_2 + x_3 = -2, \\ 5x_1 - x_2 - 3x_3 = -13. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} 2x_1 - 2x_2 + 3x_3 = 2, \\ 2x_1 + 2x_2 + 3x_3 = 6, \\ 5x_1 - 3x_2 - 3x_3 = -14. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1 - 5x_2 + x_3 = 16, \\ 2x_1 + 3x_2 + x_3 = 0, \\ 5x_1 - 2x_2 - 3x_3 = 8. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1 - x_2 + x_3 = -5, \\ 3x_1 + 2x_2 + 3x_3 = -4, \\ 5x_1 - 3x_2 - 3x_3 = -5. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

14-variant

1. ***a)*** $\begin{vmatrix} 3 & 6 \\ -25 & 23 \end{vmatrix}$; ***b)*** $\begin{vmatrix} 21 & 2 \\ 2 & -23 \end{vmatrix}$; ***v)*** $\begin{vmatrix} 2 & 41 \\ -31 & 4 \end{vmatrix}$;

2. *a)* $\begin{vmatrix} 2 & 11 & -3 \\ 5 & 3 & 2 \\ 13 & 4 & -12 \end{vmatrix}$; $\begin{vmatrix} 3 & 4 & -5 \\ 8 & 7 & -12 \\ 2 & -21 & 8 \end{vmatrix}$; $\begin{vmatrix} 4 & -3 & 5 \\ 3 & -12 & 7 \\ 3 & -17 & -5 \end{vmatrix}$; $\begin{vmatrix} 1 & 3 & 2 & 0 \\ -2 & 6 & 3 & 1 \\ 3 & 0 & 4 & -6 \\ 4 & -4 & 0 & 2 \end{vmatrix}$.

3. $A = \begin{pmatrix} 1 & -3 & 14 & 0 \\ 2 & -1 & 3 & -12 \end{pmatrix}, B = \begin{pmatrix} 23 & 33 & -3 & 41 \\ 24 & 2 & -4 & -5 \end{pmatrix}$ $A + B = ?$.

4. $A = \begin{pmatrix} 2 & 4 & 13 \\ 3 & -20 & 2 \\ -3 & 14 & 5 \end{pmatrix}, B = \begin{pmatrix} 1 & 3 & -1 \\ 3 & -1 & 4 \\ -4 & 4 & 2 \end{pmatrix}$ $A \cdot B = ?$.

5. $A=\begin{pmatrix} 1 & -1 & 6 & 7 \\ 3 & -3 & 6 & 4 \\ 3 & 5 & 4 & -1 \\ 7 & -3 & 7 & 1 \end{pmatrix}$ $rangA=?,\quad A^{-1}=?$

1. $\begin{cases} 2x_1 - x_2 + x_3 = 3, \\ 2x_1 - x_2 + x_3 = 3, \\ 5x_1 - x_2 - 3x_3 = -6. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} 2x_1 - 2x_2 + 3x_3 = 9, \\ 2x_1 + 2x_2 + 3x_3 = 1, \\ 5x_1 - 3x_2 - 3x_3 = 8. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1 - 5x_2 + x_3 = 8, \\ 2x_1 + 3x_2 + x_3 = 0, \\ 5x_1 - 2x_2 - 3x_3 = 15. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1 - x_2 + x_3 = 3, \\ 3x_1 + 2x_2 + 3x_3 = -5, \\ 5x_1 - 3x_2 - 3x_3 = 1. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

15-variant

1. ***a)*** $\begin{vmatrix} 12 & 4 \\ -23 & -11 \end{vmatrix}$**;** ***b)*** $\begin{vmatrix} 21 & 23 \\ 42 & -23 \end{vmatrix}$***;v)*** $\begin{vmatrix} 52 & -31 \\ -31 & 26 \end{vmatrix}$**;**

2. a) $\begin{vmatrix} -2 & 8 & 3 \\ 5 & 9 & 14 \\ 7 & 4 & -23 \end{vmatrix}$; $\begin{vmatrix} 3 & 4 & -5 \\ -8 & 7 & 6 \\ 12 & -4 & -3 \end{vmatrix}$; $\begin{vmatrix} 4 & -3 & 35 \\ 3 & 8 & 6 \\ 13 & 4 & -5 \end{vmatrix}$; $\begin{vmatrix} 3 & -2 & 2 & 0 \\ -2 & 3 & 3 & 1 \\ 3 & 7 & 4 & -6 \\ 4 & -4 & 0 & 2 \end{vmatrix}$.

3. $A=\begin{pmatrix} 61 & 23 & 4 & 40 \\ 42 & -81 & 3 & -5 \end{pmatrix}, B=\begin{pmatrix} -33 & 42 & -3 & 41 \\ 34 & 2 & -4 & -5 \end{pmatrix} \; A+B=?.$

4. $A=\begin{pmatrix} 2 & 4 & 13 \\ 3 & -12 & -2 \\ -3 & 14 & -5 \end{pmatrix}, B=\begin{pmatrix} 8 & 3 & -1 \\ 3 & -11 & 6 \\ -4 & 4 & 4 \end{pmatrix} \; A\cdot B=?.$

5. $A=\begin{pmatrix} 1 & -1 & 6 & 0 \\ 3 & -3 & 2 & 4 \\ 0 & 5 & 5 & -1 \\ 8 & 0 & 7 & 3 \end{pmatrix} \; rangA=?, \quad A^{-1}=?$

1. $\begin{cases} x_1 - x_2 + x_3 = 1, \\ 2x_1 - x_2 + x_3 = -1, \\ 5x_1 - x_2 - 3x_3 = -11. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} 2x_1 - 2x_2 + 3x_3 = 3, \\ 2x_1 - 3x_2 + 3x_3 = 2, \\ 5x_1 - 3x_2 - 3x_3 = -1. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1 - 5x_2 + x_3 = -17, \\ 2x_1 + 3x_2 + x_3 = -1, \\ 5x_1 - 2x_2 + 3x_3 = -22. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1 - x_2 - x_3 = 4, \\ 3x_1 + 2x_2 - 3x_3 = -9, \\ 5x_1 + 3x_2 - 3x_3 = -10. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

16-variant

. ***a)*** $\begin{vmatrix} 23 & 14 \\ -23 & 15 \end{vmatrix}$**;** ***b)*** $\begin{vmatrix} 31 & -23 \\ 42 & 22 \end{vmatrix}$***;v)*** $\begin{vmatrix} 52 & 41 \\ -61 & 34 \end{vmatrix}$**;**

2. a) $\begin{vmatrix} 2 & 8 & 23 \\ 5 & 9 & 2 \\ 7 & 4 & -3 \end{vmatrix}$**;** $\begin{vmatrix} 3 & 4 & -5 \\ 8 & -7 & 6 \\ 12 & -4 & 6 \end{vmatrix}$**;** $\begin{vmatrix} 4 & 13 & 15 \\ 3 & 8 & 7 \\ 13 & 14 & -5 \end{vmatrix}$**;** $\begin{vmatrix} 3 & 3 & 2 & 0 \\ 6 & 3 & 3 & 1 \\ 3 & 7 & 0 & -6 \\ 4 & -4 & 0 & 2 \end{vmatrix}$.

3. $A = \begin{pmatrix} 51 & 23 & 4 & 50 \\ 42 & -81 & 34 & 65 \end{pmatrix}, B = \begin{pmatrix} 23 & 42 & -3 & 41 \\ 34 & 23 & -24 & -75 \end{pmatrix} A - B = ?.$

4. $A = \begin{pmatrix} 12 & 14 & 13 \\ 3 & -10 & -2 \\ -3 & 14 & -5 \end{pmatrix}, B = \begin{pmatrix} 12 & 3 & -1 \\ 3 & -11 & 11 \\ -4 & 21 & 4 \end{pmatrix} A \cdot B = ?.$

5. $A = \begin{pmatrix} 1 & -1 & 6 & 0 \\ 3 & -3 & 2 & 4 \\ 5 & 5 & 3 & -1 \\ 7 & 2 & 7 & 1 \end{pmatrix} rangA = ?, \quad A^{-1} = ?$

1. $\begin{cases} x_1 - 3x_2 + 2x_3 = 0, \\ 2x_1 - 2x_2 + x_3 = 2, \\ 5x_1 - 2x_2 - 3x_3 = 0. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} 2x_1 - 2x_2 + 3x_3 = 6, \\ 2x_1 - 3x_2 + 3x_3 = 7, \\ 5x_1 - x_2 - 3x_3 = -10. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1 - 5x_2 + x_3 = -4, \\ 2x_1 + 3x_2 + x_3 = 4, \\ 5x_1 - x_2 + 3x_3 = 1. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1 - x_2 - x_3 = -7, \\ 3x_1 + x_2 - 3x_3 = -9, \\ 5x_1 + 3x_2 + 3x_3 = -7. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

17-variant

1. ***a)*** $\begin{vmatrix} 21 & -12 \\ 25 & 21 \end{vmatrix}$**;** ***b)*** $\begin{vmatrix} 14 & 23 \\ 12 & -33 \end{vmatrix}$***;v);*** $\begin{vmatrix} 1 & -22 \\ 32 & 53 \end{vmatrix}$

2. $\begin{vmatrix} 12 & 7 & 11 \\ 5 & 0 & 6 \\ 1 & 4 & -2 \end{vmatrix}$**;** $\begin{vmatrix} 4 & -3 & 0 \\ 1 & 11 & 5 \\ 1 & -4 & 7 \end{vmatrix}$**;** $\begin{vmatrix} 4 & -1 & 2 \\ 3 & -8 & 8 \\ 13 & 5 & -2 \end{vmatrix}$**;** $\begin{vmatrix} 8 & 2 & -1 & 0 \\ -1 & 0 & 3 & 1 \\ 3 & 5 & 0 & -2 \\ 1 & -4 & 1 & 9 \end{vmatrix}$.

3. $A=\begin{pmatrix} 17 & 42 & 38 & 55 \\ 34 & 32 & -27 & 44 \end{pmatrix}, B=\begin{pmatrix} 54 & 53 & -31 & 43 \\ 14 & 22 & -27 & 62 \end{pmatrix}$ $A+B=?$.

4. $A=\begin{pmatrix} 12 & -22 & 11 \\ 3 & 10 & -12 \\ -3 & -1 & 5 \end{pmatrix}, B=\begin{pmatrix} 10 & 31 & -1 \\ 3 & 12 & 14 \\ 4 & 0 & 3 \end{pmatrix}$ $A\cdot B=?$.

5. $A=\begin{pmatrix} 1 & -2 & 4 & 0 \\ 3 & 3 & 0 & 1 \\ 2 & 6 & 1 & -1 \\ 7 & 0 & -2 & 1 \end{pmatrix}$ $rangA=?,\quad A^{-1}=?$

1. $\begin{cases} 2x_1-3x_2+x_3=-7, \\ x_1-2x_2+3x_3=-4, \\ 4x_1-x_2-3x_3=-9. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} 2x_1-5x_2+4x_3=5, \\ 2x_1-3x_2+x_3=4, \\ 5x_1-x_2+3x_3=17. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1-5x_2+4x_3=17, \\ x_1+3x_2-x_3=-3, \\ 5x_1-x_2+x_3=13. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1-2x_2+x_3=6, \\ 3x_1+x_2-2x_3=0, \\ 5x_1-3x_2+3x_3=11. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

18-variant

1. ***a)*** $\begin{vmatrix} 21 & -27 \\ 25 & -21 \end{vmatrix}$***; b)*** $\begin{vmatrix} 11 & -13 \\ 12 & -30 \end{vmatrix}$***;v);*** $\begin{vmatrix} 15 & 22 \\ 32 & -45 \end{vmatrix}$

2. *a)* $\begin{vmatrix} 12 & -7 & 1 \\ 5 & 2 & -6 \\ 1 & 4 & 2 \end{vmatrix}$**;** $\begin{vmatrix} 4 & 3 & 1 \\ 1 & 27 & 5 \\ 1 & -4 & -7 \end{vmatrix}$ **;** $\begin{vmatrix} 4 & -1 & 12 \\ 3 & -2 & 1 \\ 11 & 3 & -2 \end{vmatrix}$ **;** $\begin{vmatrix} 2 & 0 & -1 & 3 \\ -1 & 2 & 3 & 1 \\ 3 & 5 & 1 & -2 \\ 1 & -4 & 3 & 5 \end{vmatrix}$.

3. $A = \begin{pmatrix} 21 & 33 & -16 & 34 \\ 31 & -32 & 27 & -43 \end{pmatrix}, B = \begin{pmatrix} 25 & 57 & -37 & 37 \\ -14 & 27 & -27 & 72 \end{pmatrix}$ $A - B = ?$.

4. $A = \begin{pmatrix} 2 & 11 & -12 \\ 3 & 1 & -2 \\ -3 & 0 & 5 \end{pmatrix}, B = \begin{pmatrix} 2 & 3 & -1 \\ 3 & 7 & 1 \\ 4 & -8 & 3 \end{pmatrix}$ $A \cdot B = ?$.

5. $A = \begin{pmatrix} 1 & 2 & -3 & 0 \\ 6 & 1 & 4 & 1 \\ 2 & -3 & 1 & -1 \\ 7 & 0 & 2 & 1 \end{pmatrix}$ 1. $\begin{cases} x_1 - 3x_2 + x_3 = -3, \\ x_1 - 4x_2 + 3x_3 = -2, \\ 4x_1 + x_2 - 3x_3 = -6. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} x_1 - 3x_2 + 4x_3 = -1, \\ 2x_1 + x_2 + x_3 = -2, \\ 5x_1 - x_2 + 3x_3 = -8. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1 - 5x_2 + 4x_3 = 9, \\ 4x_1 - 3x_2 + 2x_3 = 7, \\ 5x_1 + 3x_2 + x_3 = 11. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1 - 2x_2 + x_3 = 0, \\ x_1 + x_2 + 2x_3 = 0, \\ 5x_1 - 3x_2 + 3x_3 = 1. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

19-variant

1. ***a)*** $\begin{vmatrix} 11 & -13 \\ 31 & 43 \end{vmatrix}$**;** ***b)*** $\begin{vmatrix} 19 & -21 \\ 12 & 21 \end{vmatrix}$***;v);*** $\begin{vmatrix} 15 & -22 \\ 32 & 8 \end{vmatrix}$

2. ***a)*** $\begin{vmatrix} 11 & -7 & 15 \\ 5 & -4 & -6 \\ 1 & 6 & 2 \end{vmatrix}$**;** $\begin{vmatrix} 4 & 12 & -1 \\ 1 & 1 & 5 \\ 1 & -4 & -7 \end{vmatrix}$**;** $\begin{vmatrix} 4 & 1 & 12 \\ 3 & -2 & 1 \\ -12 & 0 & -2 \end{vmatrix}$**;** $\begin{vmatrix} 4 & 1 & 0 & -3 \\ -1 & 2 & 2 & 1 \\ 3 & 5 & 1 & 0 \\ 1 & 4 & -3 & 2 \end{vmatrix}$.

3. $A = \begin{pmatrix} 76 & 45 & -16 & 62 \\ 56 & -32 & 84 & -43 \end{pmatrix}, B = \begin{pmatrix} 57 & 53 & 37 & -41 \\ -14 & -27 & 41 & 53 \end{pmatrix} \; A + B = ?.$

4. $A = \begin{pmatrix} -3 & 11 & 0 \\ 3 & -1 & -2 \\ -3 & 0 & 5 \end{pmatrix}, B = \begin{pmatrix} 5 & 3 & -1 \\ 3 & 1 & 1 \\ 4 & 7 & -5 \end{pmatrix} \; A \cdot B = ?.$

5. $A = \begin{pmatrix} 1 & 0 & 3 & 2 \\ 3 & -1 & 2 & 0 \\ 2 & 3 & 1 & -1 \\ 6 & 0 & -2 & 1 \end{pmatrix} rangA = ?, \quad A^{-1} = ?$

1. $\begin{cases} 5x_1 - 3x_2 + 2x_3 = 5, \\ x_1 - x_2 + 3x_3 = 8, \\ 4x_1 + x_2 - 3x_3 = -3. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} x_1 + 3x_2 + 4x_3 = 5, \\ 2x_1 + x_2 - x_3 = -4, \\ 5x_1 - x_2 + 3x_3 = -8. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1 + 5x_2 + 4x_3 = -3, \\ 4x_1 - 3x_2 - 2x_3 = -3, \\ 5x_1 + 3x_2 - x_3 = -9. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1 - 2x_2 + x_3 = 6, \\ x_1 - x_2 + 2x_3 = 5, \\ 5x_1 - 3x_2 - 4x_3 = -1. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

20-variant

1. ***a)*** $\begin{vmatrix} 33 & -21 \\ 12 & 11 \end{vmatrix}$***;*** ***b)*** $\begin{vmatrix} 61 & -21 \\ 12 & 23 \end{vmatrix}$***;v);*** $\begin{vmatrix} 15 & -22 \\ 24 & 22 \end{vmatrix}$

2. *a)* $\begin{vmatrix} 14 & 7 & 5 \\ 5 & 4 & -6 \\ 1 & 3 & 2 \end{vmatrix}$**;** $\begin{vmatrix} 4 & 12 & -1 \\ -1 & 6 & 3 \\ 1 & -4 & -7 \end{vmatrix}$**;** $\begin{vmatrix} 4 & -1 & 8 \\ 3 & -2 & 1 \\ 11 & 0 & -2 \end{vmatrix}$**;** $\begin{vmatrix} 7 & 1 & 0 & -3 \\ -1 & 2 & 5 & 1 \\ 3 & 0 & 1 & 4 \\ 1 & 3 & -3 & 2 \end{vmatrix}$.

3. $A=\begin{pmatrix} 85 & 40 & 16 & 63 \\ 56 & -32 & 78 & -43 \end{pmatrix}, B=\begin{pmatrix} 57 & 67 & 38 & -41 \\ -14 & 27 & 61 & 59 \end{pmatrix}$ $A-B=?$.

4. $A=\begin{pmatrix} -2 & 6 & 0 \\ 3 & -1 & -7 \\ -3 & 8 & 5 \end{pmatrix}, B=\begin{pmatrix} 5 & 3 & -1 \\ 9 & 0 & 1 \\ 4 & -2 & 3 \end{pmatrix}$ $A\cdot B=?$.

5. $A=\begin{pmatrix} 5 & 0 & 3 & 2 \\ 3 & -1 & 4 & 0 \\ -2 & 3 & 0 & -1 \\ 7 & 0 & -2 & 1 \end{pmatrix}$ $rangA=?, \quad A^{-1}=?$

1. $\begin{cases} 2x_1-3x_2+2x_3=11, \\ x_1-x_2+5x_3=13, \\ -4x_1+x_2-3x_3=-15. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} x_1-x_2+4x_3=-6, \\ 2x_1+5x_2-x_3=4, \\ 5x_1-2x_2+3x_3=-10. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1+5x_2-2x_3=3, \\ -x_1-3x_2+2x_3=4, \\ 5x_1+3x_2-x_3=6. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1-2x_2-4x_3=-17, \\ x_1+5x_2+2x_3=10, \\ 5x_1-3x_2-4x_3=-20. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

21-variant

1. ***a)*** $\begin{vmatrix} 18 & -11 \\ 17 & 32 \end{vmatrix}$***; b)*** $\begin{vmatrix} 4 & -21 \\ -18 & 23 \end{vmatrix}$***;v);*** $\begin{vmatrix} 19 & -22 \\ -24 & -42 \end{vmatrix}$

2. a) $\begin{vmatrix} 14 & 7 & 15 \\ 5 & 0 & -6 \\ 1 & 2 & 2 \end{vmatrix}$**;** $\begin{vmatrix} -4 & 12 & -1 \\ -11 & 12 & 3 \\ 1 & 0 & -7 \end{vmatrix}$**;** $\begin{vmatrix} 4 & 1 & 8 \\ 3 & -2 & 12 \\ 1 & 0 & 2 \end{vmatrix}$**;** $\begin{vmatrix} 4 & 1 & 7 & -3 \\ -1 & 2 & -1 & 1 \\ 3 & 7 & 1 & 6 \\ 1 & 4 & -3 & 2 \end{vmatrix}$.

3. $A = \begin{pmatrix} -85 & 40 & 16 & 63 \\ 56 & -32 & -78 & -43 \end{pmatrix}, B = \begin{pmatrix} 57 & 67 & 38 & -41 \\ -14 & 27 & 61 & 59 \end{pmatrix} A + B = ?.$

4. $A = \begin{pmatrix} -2 & 11 & 0 \\ 3 & -1 & -7 \\ -3 & 5 & -5 \end{pmatrix}, B = \begin{pmatrix} 8 & 3 & -1 \\ 9 & 0 & 1 \\ 4 & -2 & -5 \end{pmatrix} A \cdot B = ?.$

5. $A = \begin{pmatrix} 2 & 0 & 3 & 2 \\ 3 & -1 & 5 & 0 \\ -2 & 3 & 0 & -1 \\ 7 & 0 & -2 & 1 \end{pmatrix} rangA = ?, \quad A^{-1} = ?$

1. $\begin{cases} 2x_1 - 3x_2 + 2x_3 = -4, \\ x_1 - x_2 + 5x_3 = -5, \\ -4x_1 + x_2 - 3x_3 = -3. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} x_1 - x_2 + 4x_3 = 1, \\ 2x_1 + 5x_2 - x_3 = 7, \\ 5x_1 - 2x_2 + 3x_3 = -6. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1+5x_2-2x_3=-3, \\ -x_1-3x_2+2x_3=3, \\ 5x_1+3x_2-x_3=6. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1-2x_2-4x_3=1, \\ x_1+5x_2+2x_3=14, \\ 5x_1-3x_2-4x_3=0. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

22-variant

1. ***a)*** $\begin{vmatrix} 21 & -33 \\ 31 & 13 \end{vmatrix}$***; b)*** $\begin{vmatrix} 19 & -21 \\ 12 & -13 \end{vmatrix}$***;v);*** $\begin{vmatrix} 15 & -22 \\ 32 & 38 \end{vmatrix}$

2. *a)* $\begin{vmatrix} 11 & -7 & 15 \\ 5 & 4 & -6 \\ 1 & 3 & 2 \end{vmatrix}$**;** $\begin{vmatrix} 4 & 12 & -1 \\ 1 & 14 & 5 \\ 1 & -4 & -7 \end{vmatrix}$**;** $\begin{vmatrix} 4 & 1 & 12 \\ 3 & -2 & 1 \\ -11 & 0 & -2 \end{vmatrix}$**;** $\begin{vmatrix} 2 & 1 & 0 & -3 \\ -1 & 2 & 3 & 1 \\ 3 & -4 & 1 & 0 \\ 1 & 4 & -3 & 2 \end{vmatrix}$.

3. $A=\begin{pmatrix} 76 & 45 & -16 & 62 \\ 56 & -32 & -84 & 43 \end{pmatrix}, B=\begin{pmatrix} 44 & 73 & 37 & -41 \\ -14 & -27 & 61 & -72 \end{pmatrix}\ A+B=?.$

4. $A=\begin{pmatrix} -3 & 11 & 0 \\ 3 & -1 & -2 \\ -3 & 0 & 5 \end{pmatrix}, B=\begin{pmatrix} 5 & 3 & -1 \\ 3 & 1 & 1 \\ 4 & 8 & -5 \end{pmatrix}\ A\cdot B=?.$

5. $A=\begin{pmatrix} 1 & 0 & 3 & -3 \\ 6 & -1 & 5 & 0 \\ 2 & 3 & 1 & -1 \\ 7 & 0 & -2 & 1 \end{pmatrix} rangA=?, \quad A^{-1}=?$

1. $\begin{cases} 8x_1 - x_2 - x_3 = 12, \\ 4x_1 + 2x_2 - x_3 = 13, \\ 2x_1 - x_2 + 3x_3 = 4. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} 2x_1 - 7x_2 + x_3 = 0, \\ 4x_1 - x_2 - x_3 = 10, \\ 6x_1 - x_2 - 3x_3 = 14. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1 - 5x_2 + 2x_3 = -10, \\ 3x_1 + x_2 - x_3 = 11, \\ 5x_1 + 2x_2 - 7x_3 = 9. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1 - x_2 + 4x_3 = 2, \\ x_1 + 6x_2 + x_3 = -6, \\ 2x_1 - 4x_2 - 7x_3 = -5. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

23-variant

1. ***a)*** $\begin{vmatrix} 13 & 4 \\ -25 & 31 \end{vmatrix}$**;** ***b)*** $\begin{vmatrix} 21 & -12 \\ 2 & 42 \end{vmatrix}$***;v)*** $\begin{vmatrix} 2 & 41 \\ -31 & 4 \end{vmatrix}$**;**

2. a) $\begin{vmatrix} 2 & -21 & 3 \\ 5 & 3 & 2 \\ 13 & 4 & 11 \end{vmatrix}$**;** $\begin{vmatrix} 3 & 4 & -5 \\ 8 & 7 & -12 \\ 2 & -21 & 8 \end{vmatrix}$**;** $\begin{vmatrix} 4 & -3 & 5 \\ 3 & -12 & 6 \\ 13 & -17 & -5 \end{vmatrix}$**;** $\begin{vmatrix} 1 & 3 & 2 & 0 \\ -2 & 3 & 3 & 1 \\ 3 & 0 & 8 & -6 \\ 4 & -4 & 0 & 2 \end{vmatrix}$.

3. $\boldsymbol{A}=\begin{pmatrix} 1 & -3 & 4 & 0 \\ 2 & -1 & 3 & 5 \end{pmatrix}, B=\begin{pmatrix} 23 & 42 & -3 & 41 \\ 34 & 2 & -4 & -5 \end{pmatrix}\ A-B=?.$

4. $A=\begin{pmatrix} 2 & 4 & 13 \\ 7 & 9 & 2 \\ -3 & 14 & 5 \end{pmatrix}, \boldsymbol{B}=\begin{pmatrix} 1 & 3 & -1 \\ 3 & -1 & 4 \\ -4 & 4 & 4 \end{pmatrix}\ A\cdot B=?.$

5. $A=\begin{pmatrix} 1 & -1 & 6 & 7 \\ 3 & -3 & 2 & 4 \\ 3 & 5 & 2 & -1 \\ 7 & 11 & 7 & 1 \end{pmatrix}\ rangA=?, \quad A^{-1}=?$

1. $\begin{cases} 2x_1-x_2+x_3=5, \\ 2x_1-x_2+2x_3=7, \\ 5x_1-x_2-3x_3=3. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} 2x_1-2x_2+3x_3=-1, \\ 2x_1+2x_2+3x_3=3, \\ 5x_1-3x_2-3x_3=-11. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1-5x_2+x_3=-5, \\ 2x_1+3x_2+x_3=11, \\ 5x_1-2x_2-3x_3=-8. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1-x_2+x_3=5, \\ 3x_1+2x_2+3x_3=13, \\ 5x_1-3x_2-3x_3=1. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

24-variant

1. ***a)*** $\begin{vmatrix} 21 & -33 \\ 31 & 12 \end{vmatrix}$***; b)*** $\begin{vmatrix} 19 & -21 \\ 12 & 23 \end{vmatrix}$***;v);*** $\begin{vmatrix} 15 & -22 \\ 32 & 38 \end{vmatrix}$

2. a) $\begin{vmatrix} 14 & -7 & 12 \\ 5 & 4 & -6 \\ 1 & 6 & 2 \end{vmatrix}$**;** $\begin{vmatrix} 4 & 12 & -1 \\ 1 & 14 & 5 \\ 1 & -4 & -7 \end{vmatrix}$**;** $\begin{vmatrix} 4 & 1 & 21 \\ 3 & -2 & 1 \\ -11 & 0 & -2 \end{vmatrix}$**;** $\begin{vmatrix} 7 & 1 & 0 & -3 \\ -1 & -3 & 3 & 1 \\ 3 & -5 & 1 & 0 \\ 1 & 4 & -3 & 2 \end{vmatrix}$.

3. $A=\begin{pmatrix} 76 & 45 & -16 & 62 \\ 56 & -32 & 84 & -43 \end{pmatrix}, B=\begin{pmatrix} 57 & -54 & 37 & -41 \\ -14 & -27 & 61 & 56 \end{pmatrix}\ A+B=?.$

4. $A=\begin{pmatrix} -3 & 21 & 0 \\ 3 & -1 & -2 \\ -3 & 0 & 4 \end{pmatrix}, B=\begin{pmatrix} 8 & 3 & -1 \\ 3 & 1 & 1 \\ 4 & 8 & -5 \end{pmatrix}\ A\cdot B=?.$

5. $A=\begin{pmatrix} -1 & 0 & 3 & 2 \\ 3 & -1 & 3 & 0 \\ 2 & 3 & 1 & -1 \\ 7 & 0 & -2 & 1 \end{pmatrix} rangA=?,\quad A^{-1}=?$

1. $\begin{cases} 2x_1-3x_2+2x_3=-7, \\ x_1-x_2+5x_3=1, \\ -4x_1+x_2-3x_3=10. \end{cases}$ tenglamalar sistemasini Gauss usulida yeching.

2. $\begin{cases} x_1-x_2+4x_3=15, \\ 2x_1+5x_2-x_3=-4, \\ 5x_1-2x_2+3x_3=21. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yeching.

3. $\begin{cases} 2x_1 + 5x_2 - 2x_3 = 9, \\ -x_1 - 3x_2 + 2x_3 = -6, \\ 5x_1 + 3x_2 - x_3 = 9. \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

4. $\begin{cases} 3x_1 - 2x_2 - 4x_3 = -5, \\ x_1 + 5x_2 + 2x_3 = -4, \\ 5x_1 - 3x_2 - 4x_3 = -6. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yeching.

2-mustaqil ish variantlari

1-variant

1.Toshko`mirni 3 ta shaxtadan 4 ta iste`molchiga tashish rejasini shunday tuzingki, unda jami transport xarajatlari minimallashtirilsin. Shaxtalarning bir sutkalik ishlab chiqarish hajmi, iste`molchi punktlarning ko`mirga bo`lgan talabi va 1 tonna ko`mirni tashish xarajatlari jadvalda keltirilgan. Masalaning matematik modelini tuzing.

Shaxtalar	1t ko`mirni iste`molchi-ga tashish xarajatlari				Shaxtalarning ishlab chiqarish hajmi, (ming t)
	B_1	B_2	B_3	B_4	
A_1	6	7	3	5	100
A_2	1	2	5	6	150
A_3	3	10	20	4	50
Buyurtmachilar talabi, (ming t)	75	80	60	85	300

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} x_1 - x_2 \geq 1 \\ x_1 - 2x_2 \geq 2 \end{cases}$$
$$x_1, x_2 \geq 0,$$
$$Z = -x_1 - x_2 \to \min.$$

3.Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

Ta'minotchilar	*Iste'molchilar*				*Zahra hajmi.*
	B_1	B_2	B_3	B_4	
A_1	8	1	9	7	110
A_2	4	6	2	12	190
A_3	3	5	8	9	90
Talab hajmi	80	60	170	80	

2-variant

1.Tikuv fabrikasida 4 turdagi mahsulot ishlab chiqarish uchun 3 artikuldagi gazlamalar ishlatiladi. Turli mahsulotning bittasini tikish uchun sarflanadigan turli artikuldagi gazlamalar normasi jadvalda keltirilgan. Fabrika ixtiyoridagi har bir artikuldagi gazlamalarning umumiy miqdori va mahsulotlar bahosi ham ushbu jadvalda berilgan. Fabrika har bir turdagi mahsulotdan qancha miqdorda ishlab chiqarsa, ishlab chiqarilgan mahsulotlar bahosi maksimal bo`ladi? Masalaning matematik modeli tuzilsin.

Gazlama artikuli	1 ta mahsulotga sarflanadigan gazlama normasi (m)				Gazlamalarning umumiy miqdori, (m)
	I	II	III	IV	
1	1	-	2	1	180
2	-	1	3	2	210
3	4	2	-	4	800
Mahsulotlarba-hosi,(sh.p.b.)	9	6	4	7	

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} x_1 - 2x_2 \geq 4 \\ x_1 + x_2 \leq 8 \end{cases}$$

$$x_1, x_2 \geq 0,$$

$$Z = x_1 - 2x_2 \to \min.$$

3. Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

Ta'minotchilar	*Iste'molchilar*				*Zahra hajmi.*
	B_1	B_2	B_3	B_4	
A_1	8	1	9	7	110
A_2	4	6	2	12	190
A_3	3	5	8	9	90
Talab hajmi	80	60	170	80	

3-variant

1.Korxona 4 xildagi mahsulot ishlab chiqarishda: tokarlik, frezerlik va silliqlash jihozlaridan foydalanadi. Har bir turdagi jihozning mahsulot birligini ishlab chiqarishga sarflaydigan vaqt normasi jadvalda keltirilgan. Har bir turdagi jihozning umumiy ish vaqti fondi, hamda turli mahsulot birliklarini sotishdan olinadigan daromad ham ushbu jadvalda berilgan. Eng ko`p daromad keltiradigan ishlab chiqarish rejasini topish masalasining matematik modeli tuzilsin.

Jihoz turi	Harbir turdagi bir birlik mahsulot ishlab chiqarishga sarflaydiganvaqtnormasi				Umumiy ish vaqti fondi (stanok-soat)
	I	II	III	IV	
Tokarlik	2	1	1	3	300
Frezerlik	1	-	2	1	70
Silliqlash	1	2	1	-	340
Bir birlik mahsulot sotishdan keladigan daromad (sh.p.b.)	8	3	2	4	

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} -3x_1 + 2x_2 \le 30 \\ -2x_1 + x_2 \le 12 \end{cases}$$
$x_1, x_2 \ge 0,$
$Z = -5x_1 - 7x_2 \to \min.$

3. Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

Ta'minotchilar	*Iste'molchilar*				*Zahira hajmi*
	B_1	B_2	B_3	B_4	
A_1	1	2	4	1	50
A_2	2	3	1	5	30
A_3	3	2	4	4	10
Talab hajmi	30	30	10	20	

4-variant

1.Uch xil A, B va C mahsulotlarni ishlab chiqarish uchun uch xil xom ashyolardan foydalanadi. Har bir xom ashyodan mos ravishda 180, 210 va 236 kg hajmdan ko'p bo'lmagan miqdorda ishlatish mumkin. Bir birlik mahsulotni ishlab chiqarish uchun har bir tur xom ashyolar sarfi, hamda bir birlik mahsulotdan olinadigan daromad quyidagi jadvalda berilgan.

Xom ashyo turi	Bir birlik mahsulotga sarflanadigan xom ashyolar normasi (kg)		
	A	B	C
I	4	2	1
II	3	1	3
III	1	2	5
Bir birlik buyumdan keladigan daromad	10	14	12

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} x_1 + x_2 \le 5 \\ x_1 + 2x_2 \le 6 \end{cases}$$
$Z = 2x_1 + 13x_2 \to \min$
$x_1, x_2 \ge 0.$

3. Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

Ta'minotchilar	*Iste'molchilar*				*Zahira hajmi*
	B_1	B_2	B_3	B_4	
A_1	1	7	9	5	120
A_2	4	2	6	8	230
A_3	3	8	1	2	160
Talab hajmi	130	220	90	70	

5-variant

1.Korxona ikki xil M_1 va M_2 mahsulotlarni ishlab chiqarish uchun A va B xom ashyolardan foydalanadi. Bir birlik M_1 va M_2 xil mahsulotga sarf qilinadigan turli xom ashyolar normasi, mahsulotning bir birligidan olinadigan daromad, hamda xom ashyolar zahirasi quyidagi jadvalda keltirilgan:

Xom ashyo turi	Bir birlik mahsulotga sarflanadigan xom ashyo normasi		Xom ashyolar zahirasi
	M_1	M_2	
A	2	3	9
B	3	2	13
Daromad	3	4	

Ish tajribasi shuni ko'rsatadiki, sutkasiga M_1 mahsulotga bo'lgan talab M_2 mahsulotga bo'lgan talabdan bir birlikdan ko'p bo'lmaydi. Bundan tashqari sutkasiga M_2 mahsulotga bo'lgan talab 2 birlikdan ko'p bo'lmaydi.

mahsulotishlabchiqarishningengkatta daromadberadiganrejasituzilsin

3.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} 2x_1 + x_2 \ge 12 \\ x_1 + x_2 \ge 5 \\ -x_1 + 3x_2 \le 3 \\ 6x_1 - x_2 \ge 12 \end{cases}$$

$$x_1, x_2 \ge 0, \qquad Z = x_1 + 2x_2 \to \max.$$

3.Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

Ta'minotchilar	*Iste'molchilar*				*Zahira hajmi*
	B_1	B_2	B_3	B_4	
A_1	1	7	9	5	120
A_2	4	2	6	8	230
A_3	3	8	1	2	160
Talab hajmi	130	220	90	70	

6-variant

1.Mexanika zavodi 2 turdagi detalni ishlab chiqarish uchun tokarlik, frezerlik va payvandlash jihozlarini ishlatadi. Shu borada har bir detalni 2 xil texnologik usul bilan ishlab chiqarish mumkin. Har bir jihozning samarali vaqt fondi berilgan. Har bir texnologik usul bilan turli moslamada detallar birligini ishlab chiqarish uchun sarflanadigan vaqt normasi va detallarni sotishdan olinadigan foydalar quyidagi jadvalda keltirilgan. Korxonaga maksimal foydani ta`minlovchi jihozlar yuklanishining optimal rejasini topish masalasining matematik modelini tuzing.

Jihoz turi	Detallar				Samarali vaqtfondi (stanok-soat)
	1		2		
	Texnologik usullar				
	1	2	1	2	
Tokarlik	3	2	3	0	20
Frezerlik	2	2	1	2	37
Payvandlovchi	0	1	1	4	30
Foyda, sh.p.b.)	11	6	9	6	

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} 2x_1 + x_2 \ge 12 \\ x_1 + x_2 \ge 5 \\ -x_1 + 3x_2 \le 3 \\ 6x_1 - x_2 \ge 12 \end{cases}$$

$$x_1, x_2 \ge 0, \qquad Z = x_1 + 2x_2 \to \max.$$

3.Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

Ta'minotchilar	*Iste'molchilar*				*Zahira hajmi*
	B_1	B_2	B_3	B_4	
A_1	4	2	3	1	70
A_2	6	3	5	6	140
A_3	3	2	6	3	80
Talab hajmi	80	50	50	110	

7-variant

1.Korxona omborida uzunligi 8,1 metr bo`lgan temir quymalar mavjud. Bu quymalardan ancha kichik bo`lgan 100 ta quyma mahsulotlar komplektini tayyorlash zarur bo`lsin. Har bir komplekt tarkibiga 2 ta 3 metrli, 1 ta 2 metrli va 1 ta 1,5 metrli quyma mahsulotlar kiradi. Berilgan materiallardan shunday foydalanish kerakki, quyma mahsulotlar komplekti talab qilingan miqdorda minimal chiqim bilan tayyorlansin. Turli xil usullardagi kesish natijasida bir quymadan olinadigan xomaki mahsulotlar soni, hamda chiqindilar miqdori jadvalda keltirilgan.

Mahsulotlar o`lchami, (m)	Kesish usullari								
	1	2	3	4	5	6	7	8	9
3 m	2	2	1	1	-	-	-	-	-
2 m	1	-	2	1	4	3	2	1	-
1,5 m	-	1	-	2	-	1	2	4	5
Chiqindilar, (m)	0,1	0,6	1,1	0,1	0,1	0,6	1,1	0,1	0,6

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} x_1 + 2x_2 \le 14, \\ -5x_1 + 3x_2 \le 15, \\ 2x_1 + 3x_2 \ge 12, \quad x_j \ge 0,\ j = 1,2,. \end{cases} \qquad F(x) = x_1 + x_2 \to \max .$$

3.Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

Ta'minotchilar	*Iste'molchilar*	*Zahira*

	B_1	B_2	B_3	B_4	*hajmi*
A_1	4	2	3	1	70
A_2	6	3	5	6	140
A_3	3	2	6	3	80
Talab hajmi	80	50	50	110	

8-variant

1.Xo`jalik karam, kartoshka va ko`p yillik o`tlar ekishga moslashgan. Buning uchun xo`jalik ixtiyorida 850 ga haydaladigan yer maydoni, 1500 tonna organik o`g`itlar, 50000 kishi-kun mehnat resurslari mavjud. 1 ga yerga ekiladigan mahsulotlar uchun sarf qilinadigan organik o`g`it va mehnat resurslari xarajati quyidagi jadvalda keltirilgan. Xo`jalik karam, kartoshka va ko`p yillik o`tlardan qancha hajmda eksa, pul ifodasidagi jami mahsulot miqdori maksimal bo`ladi? Masalaning matematik modelini tuzing.

Ko`rsatkichlar	Ekin turi		
	Karam	Kartoshka	Ko`p yillik o`t
Mehnat sarfi, (kishi-kun)	50	30	10
Organik o`g`itlar sarfi,(t)	20	15	10
1 ga yerdan olinadigan mahsulot bahosi,(sh.p.b.)	1000	800	200

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} 3x_1 - 2x_2 \le 12, \\ 2x_1 + 3x_2 \ge 6, \\ -x_1 + 2x_2 \le 8, \quad x_j \ge 0,\ j = 1,2,. \end{cases} \qquad F(x) = -2x_1 + x_2 \to \min .$$

3.Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

Ta'minotchilar	*Iste'molchilar*				*Zahira hajmi*
	B_1	B_2	B_3	B_4	
A_1	8	3	5	2	180
A_2	4	1	6	7	140

A_3	1	9	4	3	200
Talab hajmi	100	60	280	80	

9-variant

1.Quyidagi jadvalda berilgan ma`lumotlarga ko`ra hayvonlar ovqatlanishining optimal sutkalik ratsionini topish masalasining matematik modelini tuzing.

Ovqatbop mahsulotlar	Bir birlik yemish turidagi ovqatbop mahsulotlar miqdori		Iste`molning minimal sutkalik normasi,(sh.b.)
Xashak	1	0,5	5
Hazm qilinadigan protein	80	200	560
Kaltsiy	1	8	20
Bir birlik yemish narxi, (sh.p.b.)	3	5	

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} 3x_1 + 5x_2 \ge 15, \\ 3x_1 + x_2 \le 15, \\ -2x_1 + x_2 \le 3, \quad x_j \ge 0,\ j = 1,2,. \end{cases} \qquad F(x) = 2x_1 - 2x_2 \to \max(\min).$$

3.Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

Ta'minotchilar	*Iste'molchilar*				*Zahira hajmi*
	B_1	*B_2*	*B_3*	*B_4*	
A_1	8	3	5	2	180
A_2	4	1	6	7	140
A_3	1	9	4	3	200
Talab hajmi	100	60	280	80	

10-variant

1.Ratsion P_1 va P_2 mahsulotlardan tayyorlanadi. Ularning har biriga A, B va C vitaminlar kiradi. Bir sutkalik minimal iste`mol A vitamin uchun 100 birlik, B dan 80 birlik, C dan esa 160 birlikni tashkil qiladi. Bir birlik P_1 mahsulotning bahosi 2 shartli pul birligiga, P_2 niki esa 3 shartli pul birligiga teng. Quyidagi jadvalda har bir turdagi mahsulot tarkibidagi vitaminlar miqdori keltirilgan. Eng arzon tushadigan ratsion variantini topish masalasining matematik modelini tuzing.

Vitaminlar	Bir birlik mahsulot tarkibidagi vitaminlar miqdori	
	P_1	P_2
A	0,1	0,5
B	0,25	0,1
C	0,2	0,4

2.Quyidagi masalani grafikusuldan foyd alanib yeching

$$\begin{cases} 3x_1 - 2x_2 \geq 7, \\ x_1 - x_2 \geq 2, \\ 5x_1 - x_2 \geq 10, \end{cases} \quad x_1 \geq 3, \quad x_2 \geq 0, \quad F(x) = 4x_1 + 3x_2 \to \min .$$

3.Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

Ta'minotchilar	*Iste'molchilar*					*Zahira hajmi*
	B_1	B_2	B_3	B_4	B_5	
A_1	7	12	4	8	5	180
A_2	1	8	6	5	3	350
A_3	6	13	8	7	4	20
Talab hajmi	110	90	120	80	150	

11-variant

1.Savdo tashkiloti 3 turdagi tovarlarni sotish uchun quyidagi resurslardan foydalanmoqda: vaqt va sotuv muassasalarining maydoni. Har bir turdagi mahsulotning bir partiyasini sotish uchun resurslar xarajati jadvalda berilgan. 1- mahsulot turining 1- partiyasini ayirboshlashdan tushadigan daromad 50000 so`m, 2- sidan 80000 so`m va 3- sidan 60000 so`m. Savdo tashkilotiga maksimal foydani ta`minlovchi optimal tovar ayirboshlash rejasini topishning matematik modelini tuzing.

Resurslar	Tovarlar turlari			Resurslar hajmi
	1	2	3	

Vaqt,(soat)	0,5	0,7	0,6	970
Maydon,(m^2)	0,1	0,3	0,2	90

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} 2x_1 - x_2 \le 6, \\ -x_1 + 3x_2 \le 6, \\ x_1 + 2x_2 \ge 8, \end{cases} \qquad x_j \ge 0, \qquad j = 1, 2, \qquad F(x) = x_1 + 2x_2 \to \max .$$

3.Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

Ta'minotchilar	*Iste'molchilar*					*Zahira hajmi*
	B_1	B_2	B_3	B_4	B_5	
A_1	7	12	4	8	5	180
A_2	1	8	6	5	3	350
A_3	6	13	8	7	4	20
Talab hajmi	110	90	120	80	150	

12-variant

1.Aholining talabini hisobga olgan holda poyafzal do`koni reja davrida charm poyafzallardan kamida 140000 (sh.p.b.) va boshqa poyafzallardan kamida 40000 (sh.p.b.) sotishi kerak. Ayirboshlashdan tushadigan daromad va xarajatlarni bilgan holda, do`kon tovaroboroti kamida 200000 (sh.p.b.) va uning daromadi 2500 dan kam bo`lmaslik shartida xarajatlarni minimallashtiruvchi sotuv rejasini topish masalasining matematik modelini tuzing.

Ko`rsatkichlar	Poyafzal	
	Charm poyafzal	Boshqa poyafzallar
Daromad,(%)	1	2
Xarajatlar,(%)	6	5

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} x_1 + 4x_2 \geq 4, \\ 1.5x_1 + x_2 \geq 3, \\ x_j \geq 0, \qquad j = 1, 2, \end{cases} \qquad F(x) = 2x_1 + 3x_2 \to \min(\max).$$

3.Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

Ta'minotchilar	*Iste'molchilar*				*Zahira hajmi*
	B_1	B_2	B_3	B_4	
A_1	5	4	3	4	160
A_2	3	2	5	5	140
A_3	1	6	3	2	60
Talab hajmi	80	100	80	100	

13-variant

1.Kichik korhonada uch xil P_1, P_2 va P_3 mahsulotlarni ishlab chiqarish uchun to`rt xil S_1, S_2 ,S_3 va S_4 xom ashyolar ishlatiladi. Xom ashyolar zahirasi, har bir mahsulotga sarf qilinadigan xom ashyolarning texnologik normalari va bitta mahsulotning bahosi jadvalda keltirilgan. Ishlab chiqarilgan mahsulotning pul ifodasini maksimallashtiruvchi ishlab chiqarish rejasini aniqlash masalasining matematik modelini tuzing.

Xom ashyo turlari	Xom ashyo zahirasi (kg)	Bir mahsulotga ketgan xom ashyolar normasi (kg)		
		P_1	P_2	P_3
S_1	1500	4	2	1
S_2	1700	6	0	2
S_3	1000	0	2	4
S_4	2000	8	7	0
Bitta mahsulotning bahosi		100	150	200

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} 2x_1 - x_2 \ge 0, \\ x_1 - 5x_2 \ge -5, \\ x_j \ge 0, \qquad j = 1, 2, \end{cases} \qquad F(x) = 3x_1 - 10x_2 \to \min(\max).$$

3.Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

Ta'minotchilar	*Iste'molchilar*				*Zahira hajmi*
	B_1	B_2	B_3	B_4	
A_1	5	4	3	4	160
A_2	3	2	5	5	140
A_3	1	6	3	2	60
Talab hajmi	80	100	80	100	

14-variant

1.Yuzasi mos ravishda 0,8 va 0,6 mln. ga bo`lgan ikkita tuproq zonasi bor. Zonalar bo`yicha ekinlarning hosildorligi va 1 s donning bahosi jadvalda keltirilgan. Kuzgi ekinlarni 20 mln.s. dan kam bo`lmagan va bahorgi ekinlarni 6 mln.s. dan kam bo`lmagan miqdorda ishlab chiqarish talab qilinadi. Kuzgi va bahorgi donli ekinlar maydoni qanday bo`lganda pul ifodasidagi ishlab chiqarilgan jami mahsulotlar miqdori maksimal bo`ladi? Masalaning matematik modelini tuzing.

Mahsulotlar turi	Hosildorlik, s/ga		1 s mahsulot bahosi
	1-zona	2-zona	
Kuzgi ekinlar	20	25	8
Bahorgi ekinlar	25	20	7

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} x_1 + x_2 \le 18, \\ x_1 + 2x_2 \le 24, \\ 0 \le x_1 \le 12, \qquad 0 \le x_2 \le 8, \end{cases} \qquad F(x) = 4x_1 + 5x_2 \to \min(\max).$$

3.Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

Ta'minotchilar	*Iste'molchilar*				*Zahira hajmi*
	B_1	B_2	B_3	B_4	
A_1	6	7	3	2	180

A_2 A_3	5 3	1 2	4 6	3 2	90 170
Talab hajmi	95	85	100	160	

15-variant

1.Jadvalda berilgan ma`lumotlarga asoslanib mebel ishlab chiqarish rejasini shunday tuzingki, bunda mehnat rezervlaridan to`liq foydalangan holda ishlab chiqarilgan jami mahsulotning pul qiymati maksimallashtirilsin. Masalaningmatematikmodelinituzing.

Ishlab chiqarish faktorlari	Faner, (m^3)	Taxta, (m^3)	Mehnat, (kishi/smena)	Narxi (ming so`m)
1 ta servantga	0,2	0,1	2	150
1 ta shifonerga	0,1	0,2	1	120
I/ch faktorlari zahirasi	60	40	500	

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} -2x_1 + x_2 \le 2 \\ -x_1 + x_2 \le 3 \\ x_1 \le 3 \end{cases}$$

$Z = -x_1 - 2x_2 \to \min \qquad x_1, x_2 \ge 0.$

3.Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

Ta'minotchilar	*Iste'molchilar*				*Zahira hajmi*
	B_1	B_2	B_3	B_4	
A_1 A_2 A_3	6 5 3	7 1 2	3 4 6	2 3 2	180 90 170
Talab hajmi	95	85	100	160	

16-variant

1.Fermer xo`jalikka chorvachilikni rivojlantirish uchun 324 mln. so`m ajratilgan. Shundan 180 mln. so`mi ish haqiga, 144 mln. so`mi moddiy xarajatlar (texnik xizmat ko`rsatish) ga taqsimlangan. Jadvalda 1 s. sut va go`sht uchun sarflanadigan resurslar va sotuv narxlari ko`rsatilgan. Xo`jalik kamida 6000 s. sut va 1000 s. go`sht yetishtirishi kerak. Chorvachilik mahsulotlarini ishlab chiqarish rejasini shunday tuzingki, unda xo`jalikning chorvachilikdan oladigan daromadi maksimal bo`lsin. Masalaning matematik modelini tuzing.

Mahsulotlar	Mehnat sarfi, (ming so`m)	Moddiy xarajatlar	Sotuv narxi, (ming so`m)
Sut	12	8	25
Go`sht	90	80	200

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} x_1 + x_2 \geq 2, \\ x_1 - x_2 \leq 2, \\ -x_1 + x_2 \leq 2, \quad x_1 \geq 1, \quad 0 \leq x_2 \leq 4, \end{cases} \qquad F(x) = 4 + 6x_1 + 2x_2 \to \min(\max).$$

3. Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

a_j \ b_i	35	25	20
20	5	2	3
30	3	5	2
20	2	5	3

17-variant

1.Korxona har xil A,B,C buyumlarni ishlab chiqarishi uchun 3 tur xom ashyolardan foydalanadi. Bir birlik mahsulotni ishlab chiqarishda sarf qilingan xom ashyolar normasi, xom ashyolar zahiralari, hamda bir birlik mahsulotdan keladigan daromad quyidagi jadvalda keltirilgan.

Xom ashyo turi	Bir birlik mahsulotga sarflanadigan xom ashyolar normasi (kg)			Xom ashyolar zahiralari (kg)
	A	B	C	

I	18	15	12	360
II	6	4	8	192
III	5	3	3	180
Bir birlik buyumdan olinadigan daromad	9	10	16	

a) ishlab chiqarishning eng katta daromad beradigan rejasi tuzilsin.

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} x_1 - x_2 \geq 1 \\ x_1 - 2x_2 \geq 2 \end{cases}$$
$$x_1, x_2 \geq 0,$$
$$Z = -x_1 - x_2 \to \min.$$

3. Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

b_i / a_j	45	75	90	90
80	1	5	3	2
120	6	3	2	1
120	2	6	5	3

18-variant

1.Maishiy xizmat uyidagi duradgorlik ustaxonasida savdo tarmoqlari uchun stol va tumbochkalar ishlab chiqarish yo`lga qo`yilgan. Ularni tayyorlash uchun ikki turdagi 72 m^3 va 56 m^3 yog`och bor. Jadvalda bir dona mahsulot uchun ketadigan yog`ochlar miqdori ko`rsatilgan. Bitta stolni ishlab chiqarishdan ustaxona 44 birlik sof foyda oladi, bitta tumbochkadan 28 birlik foyda oladi. Ustaxona o`zida bor materialdan qancha stol va tumbochka ishlab chiqarsa, ko`proq foyda olish masalasining matematik modelini tuzing.

Mahsulot	Xom ashyolar	
	1- turdagi yog`och	2- turdagi yog`och
Stol	0,18	0,08
Tumbochka	0,09	0,28

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} x_1 - 2x_2 \geq 4 \\ x_1 + x_2 \leq 8 \end{cases}$$

$$x_1, x_2 \geq 0,$$

$$Z = x_1 - 2x_2 \rightarrow \min.$$

3. Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

b_i / a_j	45	75	90	90
80	1	5	3	2
120	6	3	2	1
120	2	6	5	3

19-variant

1.Chorva mollarini yaxshiroq boqish uchun kundalik ratsionda A vitamindan 6 birlik, B vitamindan 12 birlik, C vitamindan 4 birlik bo`lishi kerak. Mollarni boqish uchun ikki turdagi yemdan foydalaniladi. Jadvalda yem tarkibidagi foydali oziqa moddalari ulushi, oziqa moddalariga bo`lgan kundalik ehtiyoj va yemlar birligining narxi berilgan. Chorvani boqish uchun eng arzon bo`lgan kundalik ratsionni aniqlash masalasining matematik modelini tuzing.

Oziqa moddalari	Bir birlik yemdagi oziqa moddalari miqdori		Mollarning oziqa moddalariga bo`lgan kundalik ehtiyoji
	I	II	
A	2	1	6
B	2	4	12
C	0	4	4
Bir birlik yemning narxi,(so`m)	5000	6000	

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} -3x_1 + 2x_2 \le 30 \\ -2x_1 + x_2 \le 12 \end{cases}$$
$$x_1, x_2 \ge 0,$$
$$Z = -5x_1 - 7x_2 \to \min.$$

3.Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

a_j \ b_i	150	170	80	70
117	5	6	3	1
123	1	4	7	8
160	6	9	5	4

20-variant

1.Donli ekinlar uchun 900 ga yer maydoni ajratilgan. Mavjud zahiralar 1400 s. mineral o`g`it va 52000 kishi-kun mehnatdan iborat. Masalaga oid ma`lumotlar quyidagi jadvalda berilgan.

Ko`rsatkichlar	Ekinlar		
	Kuzgi bug`doy	Tariq	Arpa
Hosildorlik,(s/ga)	24	19	13
Mehnat sarfi,(kishi-kun)	3	4	5
O`g`itlar sarfi,(s/ga)	1,7	1,3	2
Tannarx, (sh.p.b.)	2	3	14
Sotish narhi, (sh.p.b.)	4,5	6	20

Engko`pfoydakeltiradiganekinekishrejasinianiqlashmasalaningmatematikm odelinituzing.

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} x_1 + x_2 \le 5 \\ x_1 + 2x_2 \le 6 \end{cases}$$
$$Z = 2x_1 + 13x_2 \to \min$$
$$x_1, x_2 \ge 0.$$

3. Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

a_j \ b_i	150	170	80	70
117	5	6	3	1
123	1	4	7	8
160	6	9	5	4

21-variant

1.Savdo tashkiloti 4 ta guruhdagi tovarlarni sotishda maksimal foydani ta`minlovchi tovarayirboshlash rejasi va hajmini aniqlashi kerak. Tovarlarni sotishda 3 xil resurslar: ish vaqti fondi – ko`pi bilan 1100 kishi-soat, savdo zallari hajmi - 900 kv.m., muomala xarajatlari – ko`pi bilan 1450 sh.p.b.da ishlatilishi mumkin. Guruhdagi bir birlik tovarlarni sotish uchun resurslarning sarflanish normasi quyidagicha:

Resurslar	Guruhdagi bir-birlik tovarlarni sotish uchun sarf-xarajatlar			
	1	2	3	4
Ish vaqti fondi,(kishi-soat)	3	5	4	3
Savdo zallari, (kv.m.)	4	3	2	4
Muomala xarajatlari, (sh.p.b.)	2	2	5	2
Foyda , (sh.p.b.)	7	6	5	8

Masalaning matematik modelini tuzing.

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} 2x_1 + x_2 \geq 12 \\ x_1 + x_2 \geq 5 \\ -x_1 + 3x_2 \leq 3 \\ 6x_1 - x_2 \geq 12 \end{cases}$$

$$x_1, x_2 \geq 0, \qquad Z = x_1 + 2x_2 \to \max.$$

3.Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

b_i	150	170	80	70

a_j				
117	5	6	3	1
123	1	4	7	8
160	6	9	5	4

22-variant

1.Korxona 4 xil mahsulotni ishlab chiqarishi uchun 3 tur resurslardan foydalanadi. Bir birlik mahsulotni ishlab chiqarishda sarf qilinadigan resurslar normasi, resurslarning zahiralari, hamda bir birlik mahsulot narxi quyidagi jadvalda berilgan.

Resurs turi	Bir birlik mahsulotga sarflanadigan resurslar normasi				Resurslar zahirasi
	A	B	C	D	
I	1	0	2	1	180
II	0	1	3	2	210
III	4	2	0	4	800
Bir birlik mahsulotning narxi	9	6	4	7	

Ishlab chiqarishning eng katta daromad beradigan rejasi tuzilsin.

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} x_1 + 2x_2 \le 14, \\ -5x_1 + 3x_2 \le 15, \\ 2x_1 + 3x_2 \ge 12, \quad x_j \ge 0,\ j = 1,2,. \end{cases} \qquad F(x) = x_1 + x_2 \to \max .$$

3. Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

a_j \ b_i	150	170	80	70
117	5	6	3	1
123	1	4	7	8
160	6	9	5	4

23-variant

1.Qandolatchilik fabrikasi 3 turdagi A, B va C karamellarni ishlab chiqarish uchun 3 turdagi xom ashyodan, ya`ni shakar, meva qiyomi va shinnidan foydalanadi. Har bir turdagi karameldan 1 t ishlab chiqarish uchun sarflanadigan turli xom ashyolar miqdori (normalari) quyidagi jadvalda keltirilgan. Shuningdek, jadvalda fabrika ishlatishi mumkin bo`lgan har bir turdagi xom ashyolarning umumiy miqdori va har bir turdagi karamelning 1 tonnasini sotishdan olinadigan daromad ham keltirilgan.

Xom ashyo turi	1t. karamel uchun sarflanadigan xom ashyo normasi, (t)			Xom ashyo-ning umumiy miqdori, (t)
	A	B	C	
Shakar	0,8	0,5	0,6	800
Shinni	0,4	0,4	0,3	600
Meva qiyomi	-	0,1	0,1	120
1t. mahsulotni sotishdan keladigan daromad, (sh. p.b.)*	108	112	126	

(*) - shartli pul birligi. Fabrikaning oladigan daromadini maksimallashtiruvchi karamel ishlab chiqarish rejasini topish masalasining matematik modeli tuzilsin.

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} x_1 + x_2 \le 18, \\ x_1 + 2x_2 \le 24, \\ 0 \le x_1 \le 12, \qquad 0 \le x_2 \le 8, \end{cases} \qquad F(x) = 4x_1 + 5x_2 \to \min(\max).$$

3. Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

a_j \ b_i	150	170	80	70
117	5	6	3	1
123	1	4	7	8
160	6	9	5	4

24-variant

1.Xususiy korxona ikki turdagi mahsulot ishlab chiqaradi. Bir kunlik reja bo`yicha birinchi turdagi mahsulotdan (№ 1) kamida 60 dona, ikkinchi turdagi mahsulotdan (№ 2) kamida 80 ta ishlab chiqarilishi kerak. Bir kunlik resurslar esa quyidagicha: ishlab chiqarish uskunalari 600 stanok-soat, xom ashyo 300 kg, 420 kishi-soat mehnat resursi va 450 kvt/soat elektroenergiya. Bir dona mahsulotga sarf qilinadigan resurslar miqdori quyidagi jadvalda berilgan. Birinchi mahsulotning narxi 5000 so`m, ikkinchi turdagi mahsulotning narxi 6000 so`m. Ishlab chiqariladigan mahsulotlardan maksimal foyda ko`rish uchun har bir mahsulotdan qanchadan ishlab chiqarish kerak? Masalaning matematik modelini tuzing.

Mahsulotlar	I/ch uskunasi, (st.-soat)	Xom ashyo, (kg)	Mehnat, (kishi-soat)	Elektroenergiya,(kvt)
№ 1	4	2	2	3
№ 2	3	1	3	2

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} 3x_1 - 2x_2 \le 12, \\ 2x_1 + 3x_2 \ge 6, \\ -x_1 + 2x_2 \le 8, \quad x_j \ge 0,\ j = 1,2,. \end{cases} \qquad F(x) = -2x_1 + x_2 \to \min .$$

3. Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

a_j \ b_i	150	170	80	70
117	5	6	3	1
123	1	4	7	8
160	6	9	5	4

25-variant

1.Aviakompaniya ikki xildagi samolyotda ma`lum bir yo`nalishda yo`lovchilarni tashishni amalga oshiradi. Birinchi xil samolyot ekipaji 3 kishidan iborat bo`lib, bir reysda 45 ta yo`lovchini tashiydi, ikkinchi xil samolyot ekipaji 6 kishidan iborat bo`lib, bir reysda 80 ta yo`lovchini tashiydi. Birinchi xil samolyotni ekspluatasiya qilishga 600 (sh.p.b.), ikkinchi xil samolyotni ekspluatasiya qilishga 900 (sh.p.b.)

sarflanadi. Rejadagi davr ichida ushbu yo`nalishda kamida 5000 ta yo`lovchini tashish kerakligi ma`lum. Agar samolyot ekipajini shakllantirishda 360 kishi-reysdan ortiq foydalanishi mumkin bo`lmasa, u holda ikkala xil samolyotlardagi reyslar miqdori qancha bo`lganda samolyotlarni ekspluatatsiya qilish xarajatlari minimal miqdorda bo`ladi? Masalaning matematik modelini tuzing.

2.Quyidagi masalani grafik usuldan foydalanib yeching

$$\begin{cases} 3x_1 + 5x_2 \ge 15, \\ 3x_1 + x_2 \le 15, \\ -2x_1 + x_2 \le 3, \quad x_j \ge 0,\ j = 1,2,. \end{cases} \qquad F(x) = 2x_1 - 2x_2 \to \max(\min).$$

3. Transport masalasining boshlang'ich yechimlaridan birini shimoliy-g'arbiy burchak usuli va minimal xarajatlar usuli bilan topinng

a_j \ b_i	150	170	80	70
117	5	6	3	1
123	1	4	7	8
160	6	9	5	4

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