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INTERCEPT PROBLEM IN DYNAMIC FLOW FIELD

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Abstract. In this article is considered the Intercept Problem, when objects move in dynamic flow field. The proposed method substantiates the parallel approach strategy, i.e., the Π-strategy in the nonlinear differential games. The new sufficient solvability conditions are obtained for problem of the pursuit.

Keywords: Differential game, Caratheodory's conditions, Lipschitz's condition, players, geometrical constraints, Pursuit, Evasion, strategy, Gronwall's inequalities.

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1 Introduction

The concept of "Differential Game" first appeared in a series works of American mathematician R.Isaacs, made in the early 50's of the 20th-century. R.Isaacs studies were published in 1965 in the form of monographs [19], in which numerous examples were examined and theoretical questions were only touched upon. The foundation of the modern theory of the Differential Game was settled mathematicians Isaacs [19], Bercovitz [5], Fleming [10], Friedman [11], Ho, Bryson, Baron [16], Pontryagin [24], Krasovskiy [20], Petrosyan [23], Pshenichnyi [25].

According to the fundamental approaches in the theory of differential games developed by Pontryagin [24] and Krasovskiy [20], a differential game is considered as a control problem from the point of view of either the pursuer or the evader. According to this view, the game reduces to either pursuit (convergence) problem or to evasion (escape) problem. Pursuit-Evasion differential games have been studied extensively in the literature (see e.g. [3], [7], [12], [17], [18], [26], [27] and in others).

The book of Isaacs [19] contains specific game problems that were discussed in details and proposed for further study. One of these problems is the so-called "Life-line" problem that was initially formulated and studied for certain special cases in [19] (of the Problem 9.5.1). For the case when controls of both players are subject to geometric constraints, this game has been rather comprehensively studied in the works of Petrosyan [23] based on approximating measurable controls with most efficient piecewise constant controls that realize the parallel convergence strategy. Later this approach to control in differential pursuit games was termed the Π-strategy. The strategy proposed in [2],[25],[23] for a simple pursuit game with geometric constraints became the starting point for the development of the pursuit method in games with multiple pursuers (see e.g. [3], [6], [7], [12], [18], [26], [27]).

Some optimal control problem formulations have taken into account the effect of an external flow field. For example, in [21], the authors addressed the problem of optimal guidance to a specified position of a Dubins vehicle [8] under the influence of an external flow. The minimum-time guidance problem for an isotropic rocket in the presence of wind has been studied in [4]. The problem of minimizing the expected time to steer a Dubins vehicle to a target set in a stochastic wind field has also been discussed in [1]. However, the same level of attention in the literature has not been devoted to pursuit-evasion games with two (or more) competing agents under the influence of external disturbances (e.g., winds or currents).

In paper [29]–[30], is considered a multi pursuer and one-evader for pursuit-evasion game in an external dynamic flow field. Due to the generality of the external flow, Isaacs's approach is not readily applicable [19]. Instead, in [30] is use a different approach and is find the optimal trajectories of the players through a reachable set method.

In this paper, we consider the Intercept Problem, when objects move in an external dynamic flow field. We will be based on Pontryagin's formalization [24] and use Chikrii's method of resolving functions [7] and the proposed method substantiates the parallel approach strategy, i.e., the Π-strategy. The new sufficient solvability conditions are obtained for this problem of the pursuit.

2 Formulation of the problems

Consider the differential game when Pursuer X and Evader Y having radius vectors x and y correspondingly move in the $\mathbb{R}^n$ and their dynamics will be described by the equations:

$$\dot{x} = u + f(t, x), \quad x(0) = x_0, \tag{2.1}$$

$$\dot{y} = v + f(t, y), \quad y(0) = y_0, \tag{2.2}$$

where $x, y \in \mathbb{R}^n, n \geq 2$; x_0, y_0 are the initial positions of the objects **X** and **Y**.

In (2.1) u is a control function of the Pursuer X and the temporal variation of u must be a measurable function $u(\cdot) : R_+ \to \mathbb{R}^n$ such that

$$|u(t)| \leq \alpha, \quad \text{for almost every } t \geq 0, \tag{2.3}$$

where α is a nonnegative parametric number. Similarly, in (2.2) v is a control function of the Evader Y and the temporal variation of v must be a measurable function $v(\cdot) : R_+ \to \mathbb{R}^n$ such that

$$|v(t)| \leq \beta, \quad \text{for almost every } t \geq 0, \tag{2.4}$$

where β is a nonnegative parametric number.

In the theory of Differential Games an inequalities of the forms (2.3) and (2.4) are usually called *an geometrical constraints for control functions* (briefly G-constraints) and denoted the class of admissible controls pursuer, i.e., of all measurable functions satisfying an G-constraint (2.3) by U and denoted the class all of the admissible controls evader satisfying G-constraint (2.4) by V.

Assumption 2.1. (Caratheodory's conditions [15].) Let the function $f(t, x)$ is defined on a domain $D := R_+ \times \mathbb{R}^n$, $R_+ := [0, \infty)$ and satisfies the following three conditions: 1) $f(t, x)$ is continuous in x for each fixed t; 2) $f(t, x)$ is measurable in t for each fixed x; 3) for each compact set Q of D, there is an integrable function $m(\cdot) : R_+ \to R_+$ such that $|f(t, x)| \leq m(t)$, $(t, x) \in Q$.

In equations (2.1) and (2.2), $f(t, x)$ and $f(t, y)$ represents an exogenous dynamic flows, but it could also represent an endogenous drift owing to the nonlinear dynamics of the players ([30]). It is reasonable to assume that the magnitude of this flows (e.g., winds or currents) is bounded from above by some function $m(t)$ in $t \in R_+$ such that holds Assumption 1.1.3.

Assumption 2.2. (Lipschitz's condition.) There is a Lebesgue-integrable function $k : R_+ \to R_+$ such that

$$|f(t, x) - f(t, y)| \leq k(t)|x - y| \tag{2.5}$$

for all $x, y \in \mathbb{R}^n$.

In the differential game (2.1)-(2.2) the objective of the Pursuer X is to catch the Evader Y, i.e., reach the equality

$$x(t) = y(t) \tag{2.6}$$

where $x(t)$ and $y(t)$ are trajectories generated during the game. The notion of a "trajectories generated during the game" requires clarification. The Evader Y tries to avoid the meeting, and if it is impossible, postpone the moment of meeting as far as possible. Naturally, this is a preliminary problem setting.

Definition 2.3. If $u(\cdot) \in U$ and $v(\cdot) \in V$ then the Caratheodory's differential equations ([15])

$$\dot{x}(t) = u(t) + f(t, x(t)), \quad x(0) = x_0, \tag{2.7}$$

$$\dot{y}(t) = v(t) + f(t, y(t)), \quad y(0) = y_0, \tag{2.8}$$

generate a unique trajectories: $x(t) = x(t; x_0, u(\cdot))$ and $y(t) = y(t; y_0, v(\cdot))$ respectively. In this case, $x(t)$ is called the pursuer's motion trajectory and $y(t)$ is called the evader's motion trajectory.

Definition 2.4. Let the function $\mathbf{u}(t, x, y, v) : R_+ \times \mathbb{R}^n \times \mathbb{R}^n \times S_\beta \to S_\alpha$ satisfies the following conditions: 1) $\mathbf{u}(t, x, y, v)$ is continuous in (x, y, v) for each fixed t; 2) $\mathbf{u}(t, x, y, v)$ is measurable in t for each fixed (x, y, v), where S_ϱ is a ball of a radius ϱ in $\mathbb{R}^n$. Then the function $\mathbf{u}(t, x, y, v)$ is called the strategy for X, where S_ϱ is the ball of a radius ϱ in $\mathbb{R}^n$.

Definition 2.5. A strategy $\mathbf{u}(t, x(t), y(t), v(\cdot))$ is called intercepting for $\mathbf{X}$ on the interval $[0, T]$ in the game (2.1)-(2.5), if for every $v(\cdot) \in V$ there exists a moment $t^* \in [0, T]$ that is to reach the equality $x(t^*) = y(t^*)$.

3 The main result

In this section, we construct a strategies for the pursuer and give a solve the intercept problem.

3.1 Construction of the Π-strategy

To construct a strategy for the pursuer, first we assume that pursuer knows $t, x(t), y(t), v(t)$ at the current time t. Let $x(t) \neq y(t)$, $\xi = \xi(t) = z(t)/|z(t)|$,

$z(t) = x(t) - y(t)$. Based on the classical method for deriving a Π-strategy (see, for example, [2],[23],[25],[27]) we assume that, for the constant vector $v \in \mathbb{R}^n$, the velocity $u \in \mathbb{R}^n$ is chosen so that the following relation hold

$$u = w - \lambda\xi, \tag{3.1}$$

where

$$w = w(t, x, y, v) = v + f(t, y) - f(t, x) \tag{3.2}$$

and λ is a non-negative parameter. From (3.1) we obtain a quadratic equation for λ:

$$\lambda^2 - 2\langle \xi, w\rangle\lambda - \alpha^2 + |w|^2 = 0,$$

where $\langle v, \xi\rangle$ denotes the scalar product of the vectors v and ξ in $\mathbb{R}^n$. To construct the strategy, we use the following root

$$\lambda(w, z) = \langle w, \xi\rangle + \sqrt{\langle w, \xi\rangle^2 + \alpha^2 - |w|^2}. \tag{3.3}$$

Note that $\lambda(w, z)$ may not be positive for all w and z. We call the root (3.3) resolving function (see [7]) and present some of its important properties.

Property 3.1. *If* $\alpha \geq |w|$ *then the function* $\lambda(w, z)$ *is continuous, non-negative in* $\mathbb{R}^n \times \mathbb{R}^n \setminus \{0\}$ *and holds*

$$\alpha - |w| \leq \lambda(w, z) \leq \alpha + |w|. \tag{3.4}$$

Now, substituting the resolving function (3.3) into (3.1), we obtain

$$\mathbf{u}(w, z) = w - \lambda(w, z)\xi. \tag{3.5}$$

Property 3.2. *If* $\alpha \geq |w|$ *then the function* $\boldsymbol{u}(w,z)$ *is continuous and holds* $|\boldsymbol{u}(w, z)| = \alpha$ *in* $\mathbb{R}^n \times \mathbb{R}^n \setminus \{0\}$.

Remark 3.3. Let $l = l(t) : R_+ \to \mathbb{R}^n$ is some a measurable function so that $\alpha \geq |l(t)|$. Then we consider the differential equation

$$\dot{z} = -\lambda(l(t), z)\frac{z}{|z|}, \quad z(0) = z_0, \tag{3.6}$$

for $z \neq 0$, where $\lambda(l(t), z) = \langle l(t), \xi\rangle + \sqrt{\langle l(t), \xi\rangle^2 + \alpha^2 - |l(t)|^2}$, $\xi = z/|z|$. For the initial values problem (3.6), the hypotheses of the existence theorem of Caratheodory are satisfied and therefore it has a unique absolutely continuous solution $z(t)$, which starts out from the point $z_0 \neq 0$. The equation can be transformed to the form

$$z(t) = z_0 \Lambda(t, z(\cdot)), \tag{3.7}$$

where

$$\Lambda(t, z + (\cdot)) = \exp\left\{ -\int_0^t \frac{1}{|z(s)|} \lambda(s, l(s), z(s)) ds \right\}.$$

Hence we have the equivalent equation in form

$$\dot{z} = -\lambda(l(t), z_0) \frac{z_0}{|z_0|}, \quad z(0) = z_0,$$

to (3.6).

Lemma 3.4. *If $\alpha \geq |w(t)|$ and $z \neq 0$, then for every z_0, $z_0 \neq 0$, and $v(\cdot) \in V$, the following equation*

$$\boldsymbol{u}(w(t), z(t)) = \boldsymbol{u}(w(t), z_0) \tag{3.8}$$

holds on some time interval $[0, t^*]$, where $w(t) = v(t) + f(t, y(t)) - f(t, x(t))$.

Proof. Since $\mathbf{u}(w, z)$ is homogeneous in z, therefore by (3.7) and Remark 2.3 we obtain (3.8).

By (3.8) the pursuer constructs its strategy based on the information about the current time t, the value $w(t)$, and the initial data z_0, α, β.

Definition 3.5. If $\alpha \geq |w|$, then the function

$$\mathbf{u}(t, w) = w - \lambda(t, w)\xi_0, \quad \lambda(t, w) = \langle w, \xi_0 \rangle + \sqrt{\langle w, \xi_0 \rangle^2 + \alpha^2 - |w|^2}, \tag{3.9}$$

is called Π-strategy of pursuer in the game (2.1)-(2.5), where $\xi_0 = z_0/|z_0|$; and here $|\mathbf{u}(t, w)| = \alpha$.

Let $v(\cdot) \in V$ be an arbitrary control of evader, and let the pursuer use Π-strategy (3.9). Then from Assumptions 2.1-2.2 and (3.9) we have the Caratheodory's differential equation

$$\dot{x}(t) = v(t) + f(t, y(t)) - \lambda(t, w(t, x(t), y(t), v(t)))\xi_0, \quad x(0) = x_0, \tag{3.10}$$

generate a unique trajectory: $x(t) = x(t; x_0, \mathbf{u}(\cdot))$ for of the pursuer.

3.2 Intercept problem

We use the following statement.

Lemma 3.6. *(Generalized Gronwall inequality [13], [15]). If $\varphi(t)$ is real valued and continuous function, $k(t) \geq 0$ is integrable on $t \geq 0$ function and*

$$|z(t)| \leq \varphi(t) + \int_0^t k(s)|z(s)|ds,$$

then

$$|z(t)| \leq \varphi(t) + \int_0^t k(s)\varphi(s) \left(\exp \int_s^t k(\tau)d\tau \right) ds.$$

Assumption 3.7. Let

a) there exists positive root of the equation

$$\Psi(t) = 0 \tag{3.11}$$

with respect to t, where $\Psi(t) = |z_0| - (\alpha - \beta) \int_0^t \exp \left(- \int_0^s k(\tau)d\tau \right) ds$;

b) satisfies the following condition

$$\alpha > \beta + k(t) \max_{t \in [0,T]} \Phi(t), \tag{3.12}$$

where $T = \min\{t : \Psi(t) = 0\}$ and $\Phi(t) = \exp \int_0^t k(s)ds \ \Psi(t)$.

Theorem 3.8. *If Assumptions 2.1-2.2 and Assumption 3.7 are satisfied. Then the* Π*-strategy (3.9) for the player* X *is intercepting on the interval* $[0, T]$ *in the game (2.1)-(2.5), where* T *is the smallest positive root of the equation (3.11).*

Proof. Let $v(\cdot) \in V$ be an arbitrary control of evader, and let the pursuer use Π-strategy (3.9). Use equations (2.8) and (3.10) to get the following initial value problem

$$\dot{z} = \mathbf{u}(t, w(t)) - w(t) = -\lambda(t, w(t))\xi_0, \quad z(0) = z_0,$$

where $w(t) = w(t, x(t), y(t), v(t))$ (see (3.2)). From this we see that

$$z(t) = \Lambda(t, w(\cdot))z_0, \tag{3.13}$$

where $\Lambda(t, w(\cdot)) = 1 - \frac{1}{|z_0|}\int\limits_0^t \lambda(s, w(s))ds$. Using the definition of function $\lambda(t, w)$ in (3.9) we have

$$|z(t)| = |z_0| - \int\limits_0^t [\sqrt{\langle w(s), \xi_0\rangle^2 + \alpha^2 - |w(s)|^2} + \langle w(s), \xi_0\rangle]ds.$$

From Property 3.1 and (3.2) we get: $|z(t)| \leq |z_0| - \int\limits_0^t (\alpha - |w(s)|)ds = |z_0| - \alpha t + \int\limits_0^t |v(s) + f(s, y(s)) - f(s, x(s))|ds$. Hence by (2.4) and (2.5) we obtain

$$|z(t)| \leq |z_0| - (\alpha - \beta)t + \int\limits_0^t |f(s, y(s)) - f(s, x(s))|ds \leq$$

$$\leq |z_0| - (\alpha - \beta)t + \int\limits_0^t k(s)|x(s)) - y(s)|ds$$

or Gronwall's inequality in the form

$$|z(t)| \leq |z_0| - (\alpha - \beta)t + \int\limits_0^t k(s)|z(s)|ds.$$

By Lemma 3.6 we have

$$|z(t)| \leq |z_0| - (\alpha - \beta)t + \int\limits_0^t [|z_0| - (\alpha - \beta)s]\, k(s) \left(\exp\int\limits_s^t k(\tau)d\tau\right) ds.$$

Calculating right side this inequality we obtain

$$|z_0| - (\alpha - \beta)t + \int\limits_0^t [|z_0| - (\alpha - \beta)s]\, k(s) \left(\exp\int\limits_s^t k(\tau)d\tau\right) ds = \Phi(t)$$

and

$$|z(t)| \leq \Phi(t). \tag{3.14}$$

From the Assumption 3.7. a) follows that the function $\Psi(t)$ is monotone deceasing for every $t \in [0,T] \geq 0$ when holds (3.12). Then from the Assumption 3.7 we have that $\Phi(T) = 0$ and $\Lambda(t, w(\cdot))|z_0| \leq \Phi(t)$ in $[0,T]$. Consequently, there exists time $t^* \in [0,T]$ such that $\Lambda(t^*, w(\cdot)) = 0$ or $z(t^*) = 0$ and this implies $x(t^*) = y(t^*)$.

Now, prove the admissibility of the realization (3.9) for $t \in [0,T]$. Here we show that $\alpha \geq |w(t)|$ when $t \in [0,T]$. Let the Assumption 3.7 satisfying. Then from conditions (2.4), (2.5) and inequalities (3.12), (3.14) for $t \in [0,T]$ we obtain $\alpha > \beta + k(t)\Phi(t) \geq |v(t)| + k(t)|z(t)| \geq |v(t)| + |f(t, y(t)) - f(t, x(t))| \geq |v(t) + f(t, y(t)) - f(t, x(t))|$ which completes the proof of the theorem 3.8.

Remark 3.9. If $k(t)$ is increasing function on $t \geq 0$ then we get inequality $\alpha > \beta + k(t)|z_0|$ instead (3.12). This following from of the function $\Phi(t)$.

Example 3.10. Consider the differential game:

$$\dot{x} = u + Ax,\ x(0) = x_0, \quad \dot{y} = v + Ay,\ y(0) = y_0, \tag{3.15}$$

where $x, y \in \mathbb{R}^n,\ n \geq 2;\ |u| \leq \alpha,\ |v| \leq \beta;\ A$ is an $n \times n$ real constant matrix.

Here represent the Assumption 3.7 in the form: a) $|z_0| - (\alpha - \beta)[1 - \exp(-|A|t)] = 0$ if $|A| \neq 0$ or $|z_0| - (\alpha - \beta)t = 0$ if $|A| = 0$; b) $\alpha > \beta + |A||z_0|$. Then we have the following statement.

Theorem 3.11. *If $\alpha > \beta + |A||z_0|$ then the Π-strategy (3.9) for the player X is intercepting on the interval $[0, T_A]$ in the game (3.15), where $T_A = \frac{1}{|A|} \ln \frac{\alpha - \beta}{\alpha - \beta - |A||z_0|}$ if $|A| \neq 0$ or $T_A = \frac{|z_0|}{\alpha - \beta}$ if $|A| = 0$.*

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