Sinfdan va darsdan tashqari mashg`ulotlarda foydalanish uchun maslalar.
Chust tumanidagi 52-maktab o`qituvchisi Mansurali Tashov tomonidan tayyorlandi va maktabning 9-sinf o`quvchilari o`rtasida o`tkazilgan „O`tkir zehnlilar mushoirasida“ foydalanildi.

**№ 1**

It o`zidan 30 m masofada turgan tulkini quvib ketdi. It bir sakrashda 2 m, tulki esa 1 m masofani bosib o`tadi. Tulki 3 marta sakraganda it 2 marta sakraydi. Tulkiga yetib olish uchun it qancha masofani bosib o`tishi kerak?

***Yechilishi:*** Tulki 3 marta sakraganda it 2 marta sakraydi, bunda tulki 3 metr, it esa 4 metr masofani bosib o`tadi. Demek, it har ikki marta sakraganida oradgi masofani 1 metr qisqaradi. It tulkiga yetib olishi uchun (oradagi masofa 30 metrga qisqarishi uchun) it 60 marta sakarshi, ya'ni $2 \cdot 60 = 120$ metr masofani bosib o`tishi kerak

**№ 2**

Uchta o`rtoq bayram dasturxoniga qo`yish uchun 12 shirin kulcha sotib olmoqchi bo`lishdi. Ulardan biri 5 ta , ikkinchisi 7 ta kulcha sotib oldi. Uchinchisi esa 1200 so`m pul berdi. Bu pulni dastlabki ikki o`rtoq qanday bo`lib olishi kerak?

***Yechilishi:*** Har biri 12:3=4 tadan kulcha yegan. Uchinchisi yegan 4 ta kulcha uchun 1200 so`m to`lagan bo`lsa, demak 1 ta kulcha 1200:4=300 so`m turadi. 1-bola 5 kulcha olgan va 4 tasini o`zi yegan, 1 tasini 3-bolaga bergan, shuning uchun 1-bola $1 \cdot 300$ so`m=300 so`m olishi kerak. 2-bola 7 kulcha olgan va 4 tasini o`zi yegan, 3 tasini 3-bolaga bergan, shuning uchun 2-bola $3 \cdot 300$ so`m=900 so`m olishi kerak.

**№ 3**

Idishdagi suv Aliga 14 kunga yetadi. Bu suv Ali bilan Valiga 10 kunga yetadi. Suv Valining yol`giz o`ziga necha kunga yetadi?

***Yechilishi:*** Suv Valiga x kunda yetadi deb fraz qilamiz.

Ali 1 kunda idishdagi suvning $\frac{1}{14}$ qismini ichadi;

Vali 1 kunda idishdagi suvning $\frac{1}{x}$ qismini ichadi;

Ikkalasi birgalikda 1 kunda idishdagi suvning $\frac{1}{10}$ qismini ichadi;

Tenglama tuzamiz: $\frac{1}{14}+\frac{1}{x}=\frac{1}{10}$. Tenglamani yechib, x=35 ni topamiz. **Javob**: suv Valiga 35 kunga yetadi.

**№ 4**

Ona olma sotib oldi. Ikkita olmani o`zi yedi. Birinchi o`g`liga qolgan olmalarning yarmini va yana yarimta olma berdi. Ikkinchi o`g`liga qolgan olmalarning yarmini va yana yarimta olma berdi. Uchinchi o`g`liga qolgan olmalarning yarmini va qolgan yarimta olmani berdi. Agar bitta ham olma kesilmagan bo`lsa, ona nechta olma sotib olgan?

**Yechilishi:** 3-o`gliga 0,5+0,5=1 ta;

2-o`gliga $0{,}5 \cdot 2+1=2$ ta;

1-o`gliga 1,5·2+1=4 ta;
2 tasini o`zi egan;

Demak, ona 1+2+4+2=9 ta olma olgan.

**№ 5**

$\sqrt[3]{x+\sqrt[3]{x+\sqrt[3]{x+\ldots}}}=4$ tenglamani yeching.

***Yechilishi:*** Tenglamaning har ikki tomonini kubga ko`taramiz: $x+\sqrt[3]{x+\sqrt[3]{x+\sqrt[3]{x+\ldots}}}=64$. $\sqrt[3]{x+\sqrt[3]{x+\sqrt[3]{x+\ldots}}}=4$ bo`lgani uchun: x+4=64; bundan: x=60.

**№ 6**

3 litrli va 5 litrli ikkita idish yordamida qanday qilib hovuzdan 4 litr suv olish mumkin?

**Yechiloshi:** 3 litrli idishda ikki marta 5 litrli idishga suv quyamiz. Ikkinchisida 3 litrli idishda 1 litr suv ortib qoladi. 5 litrli idishdagi suvni hovuzga tokib, unga 3 litrli idishda qolgan 1 litr suvni quyamiz. So`ngra 3 litrli idishda suv olib uni ham 5 litrli idishga quyamiz: 1+3=4 litr bo`ladi.

**№ 7**

$x$ va $y$ musbat sonlar uchun $x^2+y^2+1\geq xy+x+y$ tengsizlik bajarilishini isbotlang.

***Yechilishi:*** $x^2+y^2+1=\frac{x^2}{2}+\frac{x^2}{2}+\frac{y^2}{2}+\frac{y^2}{2}+\frac{1}{2}+\frac{1}{2}=\left(\frac{x^2}{2}+\frac{y^2}{2}\right)+\left(\frac{x^2}{2}+\frac{1}{2}\right)+\left(\frac{y^2}{2}+\frac{1}{2}\right)\geq 2\sqrt{\frac{x^2}{2}\cdot\frac{y^2}{2}}+2\sqrt{\frac{x^2}{2}\cdot\frac{1}{2}}+2\sqrt{\frac{y^2}{2}\cdot\frac{1}{2}}=xy+x+y$

**№ 8**

Sinfdagi 30 o`quvchidan 18 tasi matematika to`garagiga, 8 tasi informatika to`garagiga, 3 tasi har ikkala to`garakka qatnashadi. Sinfda bu ikki to`garakning hech biriga qatnashmaydigan nechta o`quvchi bor?

***Yechilishi:*** 30-(18+8-3)=7 ta

**№ 9**

Kema A shahardan B ga 5 kunda, teskarisiga esa 7 kunda keladi. A shahardan B ga sol necha kunda keladi?

**Yechilishi.** Shaharlar orasidagi masofa AB=S bo`lsin.

Kemaning turg`un suvdagi tezligi $V_k$;

Oqimning (solning) tezligi $V_s$ bo`lsin.

$\begin{cases}\frac{S}{V_k+V_s}=5\\ \frac{S}{V_k-V_s}=7\end{cases}\Rightarrow\begin{cases}\frac{V_k+V_s}{S}=\frac{1}{5}\\ \frac{V_k-V_s}{S}=\frac{1}{7}\end{cases}$ birinchi tenglamadan ikkinchisini hadlab ayiramiz: $\frac{2V_s}{S}=\frac{2}{35}$; $\frac{V_s}{S}=\frac{1}{35}$ bundan $\frac{S}{V_s}=35$.

Demak, sol A shahardan B shaharga 35 kunda yetib keladi.

**№ 10**

Shunday ketma-ket keluchi beshta natural sonni toping-ki, ularning dastlabki uchtasi kvadratlarining yig`indisi qolgan ikkitasi kvadratlarining yig`indisiga teng bo`lsin.

**Yechilishi:** 1-son — x; 2-son — x+1; 3-son — x+2; 4-son x+3; 5-son — x+4 bo`lsin.

$x^2+(x+1)^2+(x+2)^2=(x+3)^2+(x+4)^2$; $x^2+x^2+2x+1+x^2+4x+4=x^2+6x+9+x^2+8x+16$; $x^2-8x-20=0$; $x_1=10$, $x2=-2$;

izlanayotgan sonlar natural sonlar bo`lgani uchun $x_1=10$ ni olamiz.

Demak izlanayotgan sonlar: 10, 11, 12, 13, 14

**№ 11**

$(a+b)(b+c)(a+c)\geq 8abc$ tengsizlikni isbotlang.

***Yechilishi:*** Koshi tengsizligiga asosan, $a+b\geq 2\sqrt{ab}$ (1), $b+c\geq 2\sqrt{bc}$ (2), $a+c\geq 2\sqrt{ac}$ (3).

(1), (2), (3) tengsizliklarni hadlab ko`paytirsak: $(a+b)(b+c)(a+c)\geq 8\sqrt{abbcac}$ bundan $(a+b)(b+c)(a+c)\geq 8abc$ kelib chiqadi.

**№ 12**

$x^2-y^2=31$ tenglamaning natural yechimlarini toping.

***Yechilishi:*** 31 tub son bo`lgani uchun, uni 1·31 yoki 31·1 kabi yozish mumkin. Tenglamaning faqat natural yechimlarini topish talab qilinayotgan uchun (x+y)(x-y)=31·1, (x+y)(x-y)=1·31 hollarni qarab chiqamiz.

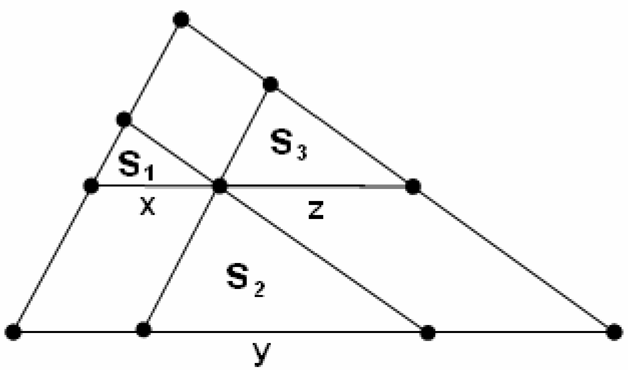
| | |
|---|---|
| 1) $\begin{cases} x+y=31 \\ x-y=1 \end{cases}$ | 2) $\begin{cases} x+y=1 \\ x-y=31 \end{cases}$ |
| Hadlab qo`shsak | Hadlab qo`shsak |
| 2x=32 | 2x=32 |
| x=16 | x=16 |
| y=15 | y=-15 |

Tenglamaning faqat natural yechimlari izlanayotgani uchun 1-holdagi yechimlarnigina olamiz.

Javob: x=16, y=15

**№ 13**

ABC uchburchak ichida ixtiyoriy nuqta olingan va bu nuqta orqali uchburchak tomonlariga parallel qilib to`g`ri chiziqlar o`tkazilgan. Bu to`g`ri chiziqlar ABC uchburchakni oltita bo`lakka bo`ladi, ulardan uchtasi uchburchaklardir. Bu uchburchaklarning yuzlari $S_1$, $S_2$, $S_3$ ga teng. ABC uchburchakning yuzi $S=\left(\sqrt{S_1}+\sqrt{S_2}+\sqrt{S_3}\right)^2$ ga teng bo`lishini isbotlang.




***Yechilishi:*** uchburchakning ikki tomonini kesib o`tuvchi va uchinchi topomoniga parallel to`g`ri chiziq berilgan uchburchakka o`xshash uchburchak ajratadi. Shuning uchun bu uchta uchburchak ham o`zaro, ham berilgan uchburchakka o`xshash bo`ladi. Uchburchaklarning o`xshashik koeffitsiyenti mos tomonlari nisbatiga teng. O`xshash uchburchaklar yuzlarining nisbati esa, o`xshashlik koeffitsiyentining kavadratiga teng. Uchburchaklarning mos tomonlari uzunliklarini x,y,z bilan belgilasak:

$$\frac{S_1}{S}=\left(\frac{x}{x+y+z}\right)^2 \Rightarrow \frac{\sqrt{S_1}}{\sqrt{S}}=\frac{x}{x+y+z} \quad (1)$$

$$\frac{S_2}{S}=\left(\frac{y}{x+y+z}\right)^2 \Rightarrow \frac{\sqrt{S_2}}{\sqrt{S}}=\frac{y}{x+y+z} \quad (2)$$

$$\frac{S_3}{S}=\left(\frac{z}{x+y+z}\right)^2 \Rightarrow \frac{\sqrt{S_3}}{\sqrt{S}}=\frac{z}{x+y+z} \quad (3)$$

Bu uchta tenglikni hadlab qo`shsak, $\frac{\sqrt{S_1}}{\sqrt{S}}+\frac{\sqrt{S_2}}{\sqrt{S}}+\frac{\sqrt{S_3}}{\sqrt{S}}=\frac{x}{x+y+z}+\frac{x}{x+y+z}+\frac{x}{x+y+z}$

$\frac{\sqrt{S_1}+\sqrt{S_2}+\sqrt{S_3}}{\sqrt{S}}=\frac{x+y+z}{x+y+z}=1$. Bundan $\sqrt{S_1}+\sqrt{S_2}+\sqrt{S_3}=\sqrt{S}$. Bu tenglikni har ikki tomonini kvadaratga ko`tarib, chap va ong qismlari o`rinlarini almashtirib yozsak: $S=\left(\sqrt{S_1}+\sqrt{S_2}+\sqrt{S_3}\right)^2$

**№ 14**

Tengsizlikni isbotlang: $\frac{a+b+c+d}{4}\geq\sqrt[4]{abcd}$. (a,b,c,d — musbat sonlar).

***Yechilishi:*** $\frac{a+b+c+d}{4}=\frac{\frac{a+b}{2}+\frac{c+d}{2}}{2}\geq\frac{\sqrt{ab}+\sqrt{cd}}{2}\geq\sqrt{\sqrt{ab}\sqrt{cd}}=\sqrt[4]{abcd}$

№ 15

$\underbrace{\sqrt{20+\sqrt{20+...+\sqrt{20+\sqrt{20}}}}}_{n\ ta}<5$ tengsizlikni isbotlang.

***Yechilishi:***

$$\sqrt{20+\sqrt{20+...+\sqrt{20+\sqrt{20}}}}<\sqrt{20+\sqrt{20+...+\sqrt{20+\sqrt{25}}}}=\sqrt{20+\sqrt{20+...+\sqrt{20+5}}}=...=\sqrt{20+\sqrt{20+5}}=\sqrt{20+\sqrt{25}}=\sqrt{20+5}=\sqrt{25}=5$$

**№ 16**

Ifodani soddalashtiring: $2(3^1+1)(3^2+1)(3^4+1)(3^8+1)\cdot\ldots\cdot(3^{64}+1)$ ifodani soddalashtiring.

***Yechilishi:*** $2(3^1+1)(3^2+1)(3^4+1)\cdot\ldots\cdot(3^{64}+1)=(3^1-1)(3^1+1)(3^2+1)(3^4+1)\cdot\ldots\cdot(3^{64}+1)=(3^2-1)(3^2+1)(3^4+1)\cdot\ldots\cdot(3^{64}+1)=$ $=(3^4-1)(3^4+1)\cdot\ldots\cdot(3^{64}+1)=\ldots=(3^{64}-1)\cdot(3^{64}+1)=3^{128}+1$

**№ 17**

Uchta musbat a,b,c sonlar uchun $\frac{b+c}{a}+\frac{a+c}{b}+\frac{a+b}{c}\geq 6$ tengsizlik o`rinli bo`lishini isbotlang.

***Yechilishi:*** $\frac{b+c}{a}+\frac{a+c}{b}+\frac{a+b}{c}=\left(\frac{b}{a}+\frac{c}{a}\right)+\left(\frac{a}{b}+\frac{c}{b}\right)+\left(\frac{a}{c}+\frac{b}{c}\right)=\left(\frac{b}{a}+\frac{a}{b}\right)+\left(\frac{b}{c}+\frac{c}{b}\right)+\left(\frac{a}{c}+\frac{c}{a}\right)\geq 2\sqrt{\frac{b}{a}\cdot\frac{a}{b}}+2\sqrt{\frac{b}{c}\cdot\frac{c}{b}}+2\sqrt{\frac{a}{c}\cdot\frac{c}{a}}=2+2+2=6$

№ 18

Ixtiyoriy uchburchak uchun $\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_a}=\frac{1}{r}$ tenglik o`rinli bo`lishini isbotlsng. Bunda $h_a$, $h_b$, $h_c$ — uchburchakning mos holda $a,b,c$ tomonlariga tushirilgan balandliklar, $r$ — uchbburchakka ichki chizilgan aylana radiusi.

***Yechilishi:***

1) Uchburchak yuzi $S_\Delta=\frac{1}{2}pr$ ga teng, bunda $p=a+b+c$. $r=\frac{2S}{p}$ ; $\frac{1}{r}=\frac{p}{2S}$ (1).

2) $S_\Delta=\frac{1}{2}ah_a$, $S_\Delta=\frac{1}{2}bh_b$, $S_\Delta=\frac{1}{2}ch_c$. Bu uch tenglikdan mos ravishda $\frac{1}{h_a}=\frac{a}{2S}$, $\frac{1}{h_b}=\frac{b}{2S}$, $\frac{1}{h_c}=\frac{c}{2S}$ tengliklar kelib chiqadi.

Bu tengliklarni hadlab qo`shamiz: $\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}=\frac{a}{2S}+\frac{b}{2S}+\frac{c}{2S}=\frac{a+b+c}{2S}=\frac{p}{2S}$ (2)

(1) va (2) tengliklardan $\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_a}=\frac{1}{r}$ tenglik kelib chiqadi.

**№ 19**

Shaxmat turnirida har bir ishtirokchi boshqalari bilan bir martadan kuch sinashdi. Hammasi bo`lib 66 partiya o`nalgan bo`lsa, turnirda nechta ishtirokchi qatnashgan?

***Yechilishi:*** ishtiropkchilar soni x ta bo`lsin. Har bir ishtirokchi x-1 partiya o`ynagan. Har bir partiya ikki kishiga tegishli bo`gani uchun umumiy partiyalar soni $\frac{(x-1)\cdot x}{2}$ bo`ladi. $\frac{(x-1)\cdot x}{2}=66$. $x^2$-x=66, $x^2$-x-66=0. Bu tenglamani yechib, $x_1$= 12,$x_2$=-11 ga ega bo`lamiz. Ishirokchilar soni musbat bo`lgani uchun, $x_1$=12 ni olamiz. Demak, hammasi bo`lib 12 ta ishtirokchi bo`lgan.

**№ 20**

$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}}$ ifodaning qiymatini toping.

***Yechilishi:*** $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}}=x$ deb olamiz va tenglikning ikkala qismini kvadratga ko`taramiz: $2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}=x^2$ ; $\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}=x$ ekanligidan : 2+x=$x^2$. $x^2$-x-2=0; bu tenglamani yechsak: $x_1$=2, $x_2$= -1. Arifmetik kvadart ildich nomanfiy son bo`lgani uchun $x_1$=2 ni olamiz. Demak, $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}}=2$

**№ 21**

1. Tengsizlikni isbotlang: $a^2+b^2+c^2 \geq ab+ac+bc$

***Yechilishi:*** $a^2+b^2+c^2=\frac{a^2}{2}+\frac{a^2}{2}+\frac{b^2}{2}+\frac{b^2}{2}+\frac{c^2}{2}+\frac{c^2}{2}=\left(\frac{a^2}{2}+\frac{b^2}{2}\right)+\left(\frac{a^2}{2}+\frac{c^2}{2}\right)+\left(\frac{b^2}{2}+\frac{c^2}{2}\right)$; (1)

$\frac{a^2}{2}+\frac{b^2}{2}\geq 2\sqrt{\frac{a^2}{2}\cdot\frac{b^2}{2}}=ab$ (2); $\frac{a^2}{2}+\frac{c^2}{2}\geq 2\sqrt{\frac{a^2}{2}\cdot\frac{c^2}{2}}=ac$ (3); $\frac{b^2}{2}+\frac{c^2}{2}\geq 2\sqrt{\frac{b^2}{2}\cdot\frac{c^2}{2}}=bc$ (4)

(2), (3), (4) tengsizliklarni hadlab qo`shsak, (1) tenglikka asosan $a^2+b^2+c^2 \geq ab+ac+bc$ *tengsizlik kelib chiqadi.*

**№ 22**

$\sqrt{2\sqrt{2\sqrt{2\sqrt{2\ldots}}}}$ ifodaning qiymatini toping.

***Yechilishi:*** $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\ldots}}}}=x$ deb olamiz va tenglikning ikkala qismini kvadratga ko`taramiz: $2\sqrt{2\sqrt{2\sqrt{2\ldots}}}=x^2$. $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\ldots}}}}=x$ ekanligini hisobga olsak, $2x=x^2$; $x^2-2x=0$; $x(x-2)=0$; $x1=0$, $x_2=2$; berilgan ifodaning qiymati 0 ga teng bo`lmaydi, shuning uchun x=2 ni olamiz. Demak, $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\ldots}}}}=2$

**№ 23**

$\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2-\ldots}}}}$ ifodaning qiymatini toping.

***Yechilishi:*** $\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2-\ldots}}}}=x$ deb olamiz va tenglikning ikkala qismini kvadratga ko`taramiz: $2-\sqrt{2-\sqrt{2-\sqrt{2-\ldots}}}=x^2$. $\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2-\ldots}}}}=x$ ekanligini hisobga olsak, $2-x=x^2$; $x^2+x-2=0$, $x_1=1$, $x_2=-2$. berilgan ifodaning qiymati nomanfiy bo`lgani uchun $x_1=1$ ni olamiz. Demak, $\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2-\ldots}}}}=1$

**№ 24**

Ixtiyoriy $a,b,c$ sonlar uchun $2a^2+b^2+c^2 \geq 2a(b+c)$ bo`lishini sibotlang.

***Yechilishi:*** $2a^2+b^2+c^2-2a(b+c)\geq 0$ ko`rinishda yozib olamiz va uni isbotlaymiz:

$2a^2+b^2+c^2-2a(b+c)=2a^2+b^2+c^2-2ab-2ac=(a^2-2ab+b^2)+(a^2-2ac+c^2)=(a-b)^2+(a-c)^2\geq 0$.

Demak, $2a^2+b^2+c^2-2a(b+c)\geq 0$ tengsizlik o`rinli ekan. Bundan $2a^2+b^2+c^2\geq 2a(b+c)$ tengsizlik ham o`rinli ekanligi kelib chiqadi.

Chust tumani 52-maktab. M. Tashov. E-mail: tmj1958@inbox.uz, mansurali@mail.ru