

**O'ZBEKISTAN RESPUBLIKASI' JOQARI' HA'M WORTA ARNAWLI'
BILIMLENDIRIW MINISTRILIGI**

**BERDAQ ATI'NDAG'I' QARAQALPAQ MA'MLEKETLIK
UNIVERSITETI**

Magistratura bo'limi

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**“SHEKLI' O'LSHEMLI' LI ALGEBRALARI 2-LOKAL
DIFFERENCI'ALLAWLARI”**

5A 130101 Matematika (matematikaliq analiz) qa'nigelig

Magistr akademiya'li'q da'rejesin ali'w ushi'n jazi'lg'an

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No'kis 2016

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(matematikali'q analiz)

MAGISTRLIK DISSERTACIYASI' ANNOTACI'YASI'

Temani'n' tiykarlamasi' ha'm aktualli'g'i': Shekli' o'lshemli' Li algebralari 2-lokal differenci'allawlari'n xarakterlew usi' waqi'tqa deyin ashi'q turi'pti'. Bul jumi'sta bizin' maqsetimiz shekli' o'lshemli' Li algebralari 2-lokal differenci'allawlari'n izertlewden ibarat.

Jumi'sti'n' maqseti' ha'm wazi'ypalari': Shekli' o'lshemli' Li algebralari 2-lokal differenci'allawlari'n izertlewden ibarat.

Izertlew obekti ha'm predmeti: Li algebralari', nilpotent Li algebralari', differenciallawlar, lokal differenciallawar. 2-lokal differenciallawar.

Izertlew usi'llari'. Magistrlik dissertaciya jumi'si'nda tiykari'nan funkcionalli'q analiz ha'm Li algebralari' teoriyasi' usi'llari'nan paydalani'ladi'.

Izertlew na'tiyjelerinin' ilimiy ta'repten jan'ali'q da'rejesi. Jumi'sti'n' tiykarg'i' na'tiyjesi jan'a yesaplanadi'.

Izertlewdi'n' tiykarg'i' ma'seleleri ha'm boljawlari': U'sh o'lshemli nilpotent Li algebralari' 2-lokal differenciallawarin xarakterlew.

Izertlew na'tiyjelerinin' a'meliy a'hmiyeti ha'm qollani'li'wi': Jumi's teoriyali'q xarakterge iye. Dissertaciyada keltirilgen na'tiyjeler ha'm usi'llar funkcionalli'q analiz ha'm Li algebralari' teoriyasi'n izertlewde qollani'li'wi' mu'mkin.

Jumi'sti'n' ko'lemi ha'm du'zilisi: Magistrlik dissertaciya jumi'si' kirisiw, u'sh bap, alti' paragraf, juwmaqlaw ha'm paydalani'lg'an a'debiyatlar diziminen ibarat.

Wori'nleng'an jumi'sti'n' tiykarg'i' na'tiyjeleri: Jumi'sta u'sh o'lshemli' nilpotent Li algebralari 2-lokal differenci'allawlari' ni'n' uluima korinisi tabilg'an.

Juwmaq ha'm usi'ni'slardi'n' qi'sqasha uli'wmalasqan si'patlamasi': Jumi'sti'n' na'tiyjeleri Li ha'm operator algebralari'nda differenciallow tipindegi operatorlardi' xarakterlew ma'selelerinde qollani'li'wi' mu'mkin.

Ilimiy basshi':

f.-m.i.d. K.K.Kudaybergenov

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**MINISTRY OF HIGH AND SECONDARY SPECIAL EDUCATION OF
THE REPUBLIC OF UZBEKISTAN**

KARAKALPAK STATE UNIVERSITY

Faculty: Department of Masters Master student: Matyakupova Shaxnoza
Department: Algebra, functional Supervisor: K. K. Kudaybergenov
analysis and geometry Speciality: 5A130101- Mathematics
Academic year: 2014-2015-2016 (mathematical analysis)

THE ANNOTATION OF THE MASTER DISSERTATION

Relevance of the inquiry: Characterization of 2-local derivations on finite dimensional Lie algebras has been open so far. The present paper is devoted to study of characterization of 2-local derivations on finite dimensional Lie algebras.

Purpose and objectives of the inquiry: Description of 2-local derivations on finite dimensional Lie algebras.

Object and subject of the inquiry: Lie algebras, nilpotent Lie algebras, derivation, local derivation, 2-local derivation.

Methodology and methods of inquiry: In the work methods of functional analysis and of theory Lie algebras are used.

Degree of novelty of the work: The main result of the work is new.

Main problems and predictions of the research: Description of general form of 2-local derivations on three dimensional nilpotent Lie algebras.

The economic affectivity and sphere of usage of the work: The work has theoretical character. Results and methods introduced in the work can be used in special courses on functional analysis and theory of Lie algebras.

Structure and composition of the work: The thesis consists introduction, three volumes divided to six parts, conclusion and the referencee.

The main result of the work: In the work we give a general form of 2-local derivations on finite dimensional Lie algebras.

Brief description of the findings and conclusions: The results given in the work can be used in description of the maps of type derivation on Lie and operator algebras.

Supervisor:

K. K. Kudaybergenov

Master student:

Sh. Matyakupova

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KIRISIW

Magistrlik dissertaciya temasinin' aktualig'i: Operatorlar algebralari'nda lokal differenciallawlardi' uyreniw 1990 ji'li'birinshi ret R. Keydison [22], D. Larson ha'm A. Sururlardin' [25] jumi'si'nan baslang'an. R. Keydisonnin'' jumi'si'nda spetsifik qa'siyetlerge iye bo'lg'an lokal differensiallawlar qurilg'an. D. Larson ha'm R. Surur jumisinde bolsa X banax kenisliginde ani'qlang'an barliq shegaralang'an sizikli operatorlar algebrasinda aniqlang'an ha'r bir lokal differensiallawdin' differensiallaw bo'liwi daliyllengen. R. Kadisonnin' jumisinde fon Neymann algebralari va polinomlar algebralarinin local differensiallashlari uyrenilgen.

2-lokal differensiallaw ha'm avtomorfizmlar 1997 jili P. Shemrl [27] jumislarinda uyrenilp baslang'an. Ol [27] jumista sheksiz o'lsheimli separabel Gilbert kenisliginde aniqlang'an barliq shegaralang'an sizikli operatorlar algebrasinda ha'r bir 2-lokal differensiallawdin' differensiallaw bo'liwin ko'rsetken. Matritsalar algebrasinda 2-lokal differensiallawlar 2004 jili S. Kim ha'm J. Kim [23] tarepinen uyrenilgen.

Lokal va 2-lokal differensiallawlar va olarg'a jaqin sawlelendiriwler ko'plegen avtorlar tarepinen izertlengen (qaran' [10- 27]).

2014 jili Sh.A.Ayupov, K.K.Kudaybergenov ha'm I.S. Raximovlar [14] shekli o'lsheimli Li algebralari differensiallawlarin izertlegen. Olar [14] jumista shekli o'lsheimli yarim apiwayi Li algeblarinda ha'r bir 2-lokal differensiallawdin' differensiallaw ekenin dalillegen. O'lsheimi u'shten kishi bo'lmag'an nilpotent Li algebralari'nda ha'mde 2-lokal differensiallawlarinin' ba'zi bir qasiyetlerin izertlengen.

Obekti ha'm predmeti: Li algebralari', differenciallawlar, 2-lokal differenciallawar, nilpotent Li algebralari'.

Maqset ha'm wazi'ypalari': Shekli' o'lsheimli' Li algebralari 2-lokal differenci'allawlari'n izertlewden ibarat.

Ilimiy jan'ali'g'i: Shekli o'lishemli Li algebralari 2-lokal differenci'allawlari' xarakterlengen.

Izertlewdin' tiykarg'i' ma'seleleri ha'm boljawlari': Shekli o'lishemli Li algebralari 2-lokal differenci'allawlari'n uli'wma ko'rinishini ani'qlaw.

Izertlew temasi' boyi'nsha a'debiyatlar tu'sindirmesi (analizi):

Magistrlik dissertaciya jumi'si'nda uli'wma 29 turli adebayattan paydalani'ldi'. Normativ hu'jjetler boyi'nsha [1,2] a'debiyatlardan paydalani'ldi'. O'zbekistan Respublikasi' Prexidenti miynetlerinen [3,4] a'debiyatlardan paydalani'ldi'. Tiykarg'i' a'debiyatlar [5-9] a'debiyatlardan turadi'. Bul a'debiyatlardan magistrlik dissertaciya jumi'si' ushi'n za'ru'r bolg'an tiykarg'i' tu'sinikler, ani'qlamalar ha'mde ayri'm faktler ali'ndi'. Magistrlik dissertaciya jumi'si'nda paydalani'lg'an ilimiy maqalalar [10-27] a'debiyatlarlari'dan turadi'. Bul ilimiy maqalalardan Li algebralari' differenciallawlar', lokal ha'm 2-lokal differenciallawlari' haqqi'ndag'I' za'ru'rli mag'li'wmatlardan paydalani'ldi'.

Izertlewde qollani'lg'an metodika: Dissertaciya jumi'si'nda tiykari'nan Li algebralari' teoriyasi' ha'm funkcionalli'q analiz usullari'nan paydalani'ldi'.

Izertlew na'tiyjelerinin' teoriyalig' ha'm a'meliy a'h'miyeti: Jumi's teoriyalig' xarakterge iye. Dissertaciya keltirilgen na'tiyjeler ha'm usi'llar funkcionanal analiz ha'm Li algebralari' boyi'nsha izleniwlerde qollani'li'wi' mu'mkin.

Jumi'sti'n' du'zilisi: Magistrlik dissertaciya jumi'si' kirisiw, u'sh bap, alti' paragraf, juwmaq ha'm paydalani'lg'an a'debiyatlar diziminen ibarat.

Birinshi bap yeki paragraftan ibarat boli'p, bunda Li algebralari' ani'qlamalari', mi'sallar ha'm ayri'm qa'siyetleri qaralg'an. Birinshi paragrafta Li algebralari' haqqi'nda tiykarg'i' tu'sinikler keltirilgen. Yekinshi paragrafta bolsa ules algebralari' ha'm ideallar di'n` ani'qlamasi', bir neshe mi'sallar ha'm ayri'm belgili na'tiyjeler berilgen.

Yekinshi bapta kishi o'lishemli Li algebralari'ni'n` xarakteristikasi' qaralg'an. Birinshi paragrafta 1 ha'm 2 o'lishemli Li algebralari' dizimi, al 3

o'lishemli Li algebralari'nan' bazi'bir klaslari' qaralg'an. Yekinshi paragrafta nilpotent Li algebralari xarakteristikasi' u'yrenilgen.

U'shinshi bap yeki paragraftan ibarat boli'p, bunda Li algebralari'nda differenciallow, 2-lokal differenciallow ani'qlamalari', qa'siyetleri ha'm mi'sallar keltiriledi.

Meyli F maydan u'stinde L Li algebrası berilgen bolsın. Yeger $D: L \rightarrow L$ si'zi'qli' operatori' qa'legen $x, y \in L$ elementlaeri ushi'n

$$D[x, y] = [D(x), y] + [x, D(y)] \quad (1)$$

ten'likti qanaatlandiirsa, onda bul operator differenciallow delinedi.

3.1.1-Teorema. sl_2 algebrasının' ha'r bir differentsiallowı to'mendegishe ko'riniste boladı:

$$\begin{pmatrix} 0 & d_{12} & d_{13} \\ -2d_{13} & d_{22} & 0 \\ -2d_{12} & 0 & -d_{22} \end{pmatrix} \quad (2)$$

Meyli bizge $\Delta: L \rightarrow L$ sızıqlı sa'wlelendiririwi berilgen bolsın. Eger $\forall x \in L$ vektorı ushin sonday $D_x \in DerL$ tabılıp, $\Delta(x) = D_x(x)$ ten'ligi orınlansa, onda Δ operatori lokal differentsiallow dep ataladı.

3.1.2-Teorema. U'sh o'lishemli' Geizenberg algebrasında ha'r bir differentsiallowı to'mendegishe aniqlanadı:

$$\begin{pmatrix} d_{11} & d_{12} & 0 \\ d_{21} & d_{22} & 0 \\ d_{31} & d_{32} & d_{11} + d_{22} \end{pmatrix}. \quad (3)$$

Aniqlamadan L Li algebrasında qalegen differentsiallow lokal differentsiallow bolatuginligi keli'p shi'gadi'.

Tomendegi teoremani qarastirayiq.

3.1.3-Teorema. sl_2 algebrasının' ha'r bir lokal differentsiallanıwı differentsiallanıw boladı.

3.2.2-Teorema. U'sh o'lshepli'Heyzenberg algebrasında ha'r bir 2-lokal differentsiallaw tomendegi ko'riniste bo'ladi':

$$\Delta(x) = dx_3e_3 + f_1(x_1, x_2)e_1 + f_2(x_1, x_2)e_2 + f_3(x_1, x_2)e_3, \quad (4)$$

bul jerde d turaqli san, $f_1(x_1, x_2)$, $f_2(x_1, x_2)$, $f_3(x_1, x_2)$ bir tekli funkciyalar.

I BAP

LI ALGEBRALARI

Birinshi bap yeki paragraftan ibarat boli'p, bunda Li algebralasi' ani'qlamalari', mi'sallar ha'm ayri'm qa'siyetleri qaralg'an. Birinshi paragrafta Li algebralar haqqi'nda tiykarg'i' tu'sinikler keltirilgen. Yekinshi paragrafta bolsa Li algebralari'ni'n` u'les algebralar ha'm ideallar, bir neshe mi'sallar ha'm ayri'm belgili na'tiyjeler keltiriledi.

§ 1.1 Li algebralari

F sanli' maydani' ha'm L bul F maydani' u'stinde qanday da bir si'ziqli' ken'islik bolsi'n. Yeger

$$[\cdot, \cdot]: L \times L \rightarrow L$$

sa'wlelendiriwi to'mendegi qa'siyetlerge iye bolsa,

$$[\alpha_1 x_1 + \alpha_2 x_2, y] = \alpha_1 [x_1, y] + \alpha_2 [x_2, y]$$

$$[x, \beta_1 y_1 + \beta_2 y_2] = \beta_1 [x, y_1] + \beta_2 [x, y_2]$$

onda bul sa'wlelendiriw bisi'ziqli' forma delinedi.

1.1.1-Ani'qlama, L ken'isliginde $[\cdot, \cdot]$ ani'qlang'an bisi'ziqli' forma to'mendegi sha'rtlerdi qanaatlantirsa, onda L ken'isligine F maydani' u'stinde Li algebrasi' delinedi.

a) $[x, x] = 0, \quad \forall x \in L;$

$$b) [x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0, \forall x, y, z \in L.$$

$[x, y] \in L$ vektori' x, y elementlerinin' ko'beymesi yamasa kommutatori' delinedi.

b) sha'rti Yakobi birdeyligi delinedi.

Li algebrasi'nda to'mendegi birdeylik ori'nlanadi'.

$$[x, y] = -[y, x]. \quad (1.1)$$

Xaqiyqati'nda da

$$\begin{aligned} 0 &= [x + y, x + y] = [x, x + y] + [y, x + y] = \\ &= [x = 0, x] + [x, y] + [y, x] + [y = 0, y] = \\ &= [x, y] + [y, x]. \end{aligned}$$

Li algebrasi'ndag'i' a) sha'rtinin' ornina (1.1) birdeyliklide alsaq boladi'. (1.1) birdeyliktin' a)-ni'n' kelip shi'g'i'wi'n ko'remiz.

$$[x, y] = -[y, x].$$

(1.1) birdeyliktegi y tin' x qa qoyami'z. Onda

$$\begin{aligned} [x, x] + [x, x] &= 0, \\ 2[x, x] &= 0, \\ [x, x] &= 0. \end{aligned}$$

(1.1) birdeylik antikomutativ birdeylik delinedi.

1.1.2-Mi'sal. F maydani' u'stindegi assosativ algebradan paydalani'p, Li algebrasi'n quri'w mu'mkin. Buni'n' ushi'n Li ko'beymesi to'mendegishe ani'qlanadi'.

$$[x, y] = x \cdot y - y \cdot x, \quad x, y \in L. \quad (1.2)$$

Esletip otetug'in bolsaq, associativ algebra bul tomendegi ten'lik orinlanatug'in algebra:

$$(xy)z = x(yz) . \quad (1.3)$$

Li algebra'si' aksiomalarin teksereyik.

Da'slep bizi'zi'qli' ekenin tekseremiz. Onda

$$\begin{aligned} [\alpha_1 x_1 + \alpha_2 x_2, y] &= -[y, \alpha_1 x_1 + \alpha_2 x_2] = \\ &= (\alpha_1 x_1 + \alpha_2 x_2) y - y(\alpha_1 x_1 + \alpha_2 x_2) = \\ &= (\alpha_1 x_1) y + (\alpha_2 x_2) y - y(\alpha_1 x_1) - y(\alpha_2 x_2) = \\ &= \alpha_1 (x_1 y) + \alpha_2 (x_2 y) - \alpha_1 (y x_1) - \alpha_2 (y x_2) = \\ &= \alpha_1 (x_1 y - y x_1) + \alpha_2 (x_2 y - y x_2) = \alpha_1 [x_1, y] + \alpha_2 [x_2, y] \end{aligned}$$

ha'mde

$$\begin{aligned} [x, \beta_1 y_1 + \beta_2 y_2] &= x(\beta_1 y_1 + \beta_2 y_2) - (\beta_1 y_1 + \beta_2 y_2)x = \\ &= \beta_1 x y_1 + \beta_2 x y_2 - \beta_1 y_1 x - \\ &\quad - \beta_2 y_2 x = \beta_1 (x y_1 - y_1 x) + \beta_2 (x y_2 - y_2 x) = \\ &= \beta_1 [x, y_1] + \beta_2 [x, y_2]. \end{aligned}$$

Endi Yakobi birdeyligin tekseremiz.

$$1^\circ) [x, [y, z]] = x(yz - zy) - (yz - zy) \cdot x = xyz - xzy - yzx + zyx,$$

$$2^\circ) [y, [z, x]] = y(zx - xz) - (zx - xz) \cdot y = yzx - yxz - zxy + xzy,$$

$$3^\circ) [z, [x, y]] = z(xy - yx) - (xy - yx) \cdot z = z \cdot xy - zyx - xyz + yxz$$

ten'liklerin qossaq talap etilgen birdeylik kelip shi'd'adi'.

Komutativ assosativ algebra do'retken. Li algebra'si' trivial boladi', yag'ni'y

$$\forall x, y \in L \text{ elementleri ushi'n } [x, y] = -[y, x] = 0.$$

1.1.3-mi'sal. R^3 ken'isligin qarasti'rayi'q. Bunda $x = (\mu_1, \mu_2, \mu_3)$ $y = (v_1, v_2, v_3)$ vektorlari ko'beymesi

$$[x, y] = \left(\left| \begin{array}{cc} \mu_2 & \mu_3 \\ v_2 & v_3 \end{array} \right|; \left| \begin{array}{cc} \mu_3 & \mu_1 \\ v_3 & v_1 \end{array} \right|, \left| \begin{array}{cc} \mu_1 & \mu_2 \\ v_1 & v_2 \end{array} \right| \right). \quad (1.4)$$

(1.4) formula menen ani'qlang'an Li ko'beymesine qarata R^3 ken'isliginde Li algebrasi' ekenligin ko'rsetemiz. Oni'n' ushi'n da'slep to'mendegi birdeylikti da'lilleymiz.

$$[x, [y, z]] = (x, z)y - (x, y)z. \quad (1.5)$$

Yakobi birdeyligin da'lillew ushi'n da'slep (1.5) birdeylikti tekseremiz.

$$\begin{aligned} [x_1 [y, z]] &= \left(\left| \begin{array}{cc} \mu_2 & \mu_3 \\ v_2 & v_3 \end{array} \right|; \left| \begin{array}{cc} \mu_3 & \mu_1 \\ v_3 & v_1 \end{array} \right|, \left| \begin{array}{cc} \mu_1 & \mu_2 \\ v_1 & v_2 \end{array} \right| \right) = \\ &= \left| \begin{array}{cc} \mu_2 & \mu_3 \\ v_3 & v_1 \end{array} \right| \left| \begin{array}{cc} \mu_3 & \mu_1 \\ v_1 & v_2 \end{array} \right| - \left| \begin{array}{cc} \mu_3 & \mu_1 \\ v_1 & v_2 \end{array} \right| \left| \begin{array}{cc} \mu_2 & \mu_3 \\ v_2 & v_3 \end{array} \right| - \left| \begin{array}{cc} \mu_1 & \mu_2 \\ v_2 & v_3 \end{array} \right| \left| \begin{array}{cc} \mu_3 & \mu_1 \\ v_3 & v_1 \end{array} \right| + \left| \begin{array}{cc} \mu_1 & \mu_2 \\ v_3 & v_1 \end{array} \right| \left| \begin{array}{cc} \mu_2 & \mu_3 \\ v_1 & v_2 \end{array} \right|. \end{aligned}$$

1-koordinatasi'

$$\begin{aligned} &\left| \begin{array}{cc} \mu_2 & \mu_3 \\ v_3\gamma_1 - v_1\gamma_3 & v_1\gamma_2 - v_2\gamma_1 \end{array} \right| = \\ &= \mu_2 v_1 \gamma_2 - \mu_2 v_2 \gamma_1 - \mu_3 (v_3 \gamma_1 - v_1 \gamma_3) = \\ &= \mu_2 v_1 \gamma_2 - \mu_2 v_2 \gamma_1 - \mu_3 v_3 \gamma_1 + \mu_3 v_1 \gamma_3, \end{aligned}$$

2-koordinatasi'

$$\begin{aligned}
& \begin{vmatrix} \mu_3 & \mu_1 \\ v_1\gamma_2 - v_2\gamma_1 & v_2\gamma_3 - v_3\gamma_2 \end{vmatrix} = \\
& = \mu_3(v_2\gamma_3 - v_3\gamma_2) - \mu_1(v_1\gamma_2 - v_2\gamma_1) = \\
& = \mu_3v_2\gamma_3 - \mu_3v_3\gamma_2 - \mu_1v_1\gamma_2 + \mu_1v_2\gamma_1,
\end{aligned}$$

3-koordinatasi'

$$\begin{aligned}
& \begin{vmatrix} \mu_1 & \mu_2 \\ v_2\gamma_3 - v_3\gamma_2 & v_1\gamma_2 - v_2\gamma_1 \end{vmatrix} = \\
& = \mu_1(v_1\gamma_2 - v_2\gamma_1) - \mu_2(v_2\gamma_3 - v_3\gamma_2) = \\
& = \mu_1v_1\gamma_2 - \mu_1v_2\gamma_1 - \mu_2v_2\gamma_3 + \mu_2v_3\gamma_2.
\end{aligned}$$

Tomendegilerdi paydalanamiz:

$$\begin{aligned}
[y, [z, x]] &= (y, x)z - (y, z)x, \\
[z, [x, y]] &= (z, y)z - (z, x)y, \\
[x, [y, z]] &= (x, z)y - (x, y)z.
\end{aligned}$$

Onda

$$\begin{aligned}
[x(y, z)] + [y, [z, x]] + [z, [x, y]] &= \\
&= (x, z)y - (x, y)z + (y, x)z - (y, z)x + \\
&+ (z, y)x - (z, x)y = 0.
\end{aligned}$$

$x = (\mu_1, \mu_2, \mu_3)$, $y = (v_1, v_2, v_3)$, $z = (\gamma_1, \gamma_2, \gamma_3)$ ushi'n $(x, z)y - (x, y)z$ vector koordinatalri':

1-koordinatasi'

$$\begin{aligned}
& (\mu_1\gamma_1 + \mu_2\gamma_2 + \mu_3\gamma_3)v_1 - (\mu_1v_1 + \mu_2v_2 + \mu_3v_3)\gamma_1 = \\
& = \mu_1\gamma_1v_1 + \mu_2\gamma_2v_1 + \mu_3\gamma_3v_1 - \mu_1\gamma_1v_1 - \mu_2\gamma_2v_1 - \mu_3\gamma_3v_1 = \\
& = \mu_2(\gamma_2v_1 - v_2\gamma_1) + \mu_3(\gamma_3v_1 - v_3\gamma_1),
\end{aligned}$$

2-koordinatasi'

$$\begin{aligned}
& (\mu_1\gamma_1 + \mu_2\gamma_2 + \mu_3\gamma_3)v_2 - (\mu_1v_1 + \mu_2v_2 + \mu_3v_3)\gamma_2 = \\
& = \mu_1\gamma_1v_2 + \mu_2\gamma_2v_2 + \mu_3\gamma_3v_2 - \mu_1\gamma_1v_2 - \mu_2\gamma_2v_2 - \mu_3\gamma_3v_2 = \\
& = \mu_1(\gamma_1v_2 - v_1\gamma_2) - \mu_3(v_3\gamma_2 - \gamma_3v_2),
\end{aligned}$$

3-koordinatasi'

$$\begin{aligned}
& (\mu_1\gamma_1 + \mu_2\gamma_2 + \mu_3\gamma_3)v_3 - (\mu_1v_1 + \mu_2v_2 + \mu_3v_3)\gamma_3 = \\
& = \mu_1\gamma_1v_3 + \mu_2\gamma_2v_3 - \mu_1v_1\gamma_3 - \mu_2v_2\gamma_3.
\end{aligned}$$

Saykes koordinatalr ren'liginen Yakobi birdeyligi kelip shi'g'adi'.

Meyli V bazi' bir ken'islik bolsi'n. $End(V)$ belgisi menen V ken'isligin o'zin-o'zine sa'wlelendiriwshi barli'q si'zi'qli' operatorlar ko'pligin belgileymiz. Bul ko'plikte algebrali'q a'mellerdi to'mendegishe ani'qlayi'q: $x, y \in End(V)$ ushi'n

$$(x + y)(\xi) = x(\xi) + y(\xi), \quad \xi \in V,$$

$$(\lambda x)(\xi) = \lambda \cdot x(\xi), \quad \xi \in V,$$

$$(xy)(\xi) = y(x(\xi)), \quad \xi \in V$$

bul algebrali'q a'mellerge qarata $End(V)$ ko'pligi assosativ algebra boladi'.

Nollik vektori: $\theta(\xi) = 0, \quad \xi \in V.$

Algebrani'n' assosativlik sha'rtin teksereyik:

$$x(yz) = (xy)z.$$

Onda

$$\begin{aligned} (x(yz))(\xi) &= (yz)(x(\xi)) = z(y(x(\xi))), \\ ((xy)z)(\xi) &= z((xy)(\xi)) = z(y(x(\xi))), \\ (x(yz))(\xi) &= ((xy)z)(\xi), \quad \forall \xi \in V. \end{aligned}$$

Bunnan $x(yz) = (xy)z$.

Joqari'dag'i' 1.1.2-mi'saldan $End(V)$ ken'isligi $[x, y] = xy - yx$ ko'beymege qarata Li algebrasi' boladi'. Bul algebra $gl(V)$ arqali' belgilenedi.

1.1.4-mi'sal. Yeger $\dim V = n$ bolsa, onda

$$gl(V) \equiv M_n(F)$$

izomorfizm orinli' boladi'.

Da'lillew. V kenislikte e_1, e_2, \dots, e_n bazisin belgileyik. Onda

$\xi \in V$, $\xi = \sum_{i=1}^n \lambda_i e_i$ vektorin alsaq,

$$x(\xi) = x\left(\sum_{i=1}^n \lambda_i e_i\right) = \sum_{i=1}^n \lambda_i x(e_i) = \lambda_1 x(e_1) + \lambda_2 x(e_2) + \dots + \lambda_n x(e_n).$$

Bunnan

$$\begin{aligned} x(e_1) &= a_{11}e_1 + a_{21}e_2 + \dots + a_{n1}e_n, \\ x(e_2) &= a_{12}e_1 + a_{22}e_2 + \dots + a_{n2}e_n, \\ &\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\ x(e_n) &= a_{1n}e_1 + a_{2n}e_2 + \dots + a_{nn}e_n. \end{aligned}$$

Onda

$$x \mapsto \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$

ha'm

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n a_{1i} \lambda_i \\ \sum_{i=1}^n a_{2i} \lambda_i \\ \dots \dots \dots \\ \sum_{i=1}^n a_{ni} \lambda_i \end{pmatrix}.$$

Bunnan

$$y \mapsto \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & \dots & \dots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{pmatrix}.$$

Demek

$$x + y \mapsto \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \dots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \dots & a_{2n} + b_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} + b_{n1} & a_{n2} + b_{n2} & \dots & a_{nn} + b_{nn} \end{pmatrix}.$$

Onda

$$x \mapsto \begin{pmatrix} \lambda a_{11} & \lambda a_{12} & \dots & \lambda a_{1n} \\ \lambda a_{21} & \lambda a_{22} & \dots & \lambda a_{2n} \\ \dots & \dots & \dots & \dots \\ \lambda a_{n1} & \lambda a_{n2} & \dots & \lambda a_{nn} \end{pmatrix}$$

ha'm

$$y \cdot x \mapsto \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & \dots & \dots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{pmatrix}.$$

Bunnan

$$\begin{aligned} (xy)(e_1) &= y(x(e_1)) = y\left(\sum_{i=1}^n a_{i1}e_i\right) = \\ &= \sum_{i=1}^n a_{i1}y(e_i) = \\ &= a_{11}y(e_1) + a_{21}y(e_2) + \dots + a_{n1}y(e_n) = \\ &= a_{11}(b_{11}e_1 + b_{21}e_2 + \dots + b_{n1}e_n) + \\ &+ a_{21}(b_{12}e_1 + b_{22}e_2 + \dots + b_{n2}e_n) + \dots \\ &+ a_{n1}(b_{1n}e_1 + b_{2n}e_2 + \dots + b_{nn}e_n). \end{aligned}$$

§ 2.1. U'les algebralar ha'm ideallar

Meyli L bazı bir Li algebrası bolsın. E bul L algebrasının' qanday da bir u'les ken'isligi E u'les ken'islikten aling'an qa'legen eki elementtin' ko'beymesi de E de jatsa, onda E u'les ken'isligine u'les algebra delinedi.

Meyli I bul L algebrasının' u'les ken'isligi bolsın. Yeger I u'les ken'isliginen aling'an vektor menen L algebrasinan aling'an vektorlardın' ko'beymesi I u'les ken'isligine tiyisli bolsa, onda I ideal delinedi, yag'niy

$$\forall x \in I, \quad \forall y \in L, \quad [x, y] \in I.$$

Ko'beymenin' antikomutativlik qa'siyetinen to'mendegige iye bolamız.

$$[y, x] = -[x, y] \in I$$

Anıqlamadan to'mendegi tikkeley kelip shıg'adı. Qa'legen ideal u'les algebra boladı. Endi matritsalar algebrasının' u'les algebrasın h'a'm idealın qarastıramız.

$gl(n, F)$ bul F maydanı u'stindegi n -ta'rtpili kvadrat matritsalar Li algebrası:

$$[x, y] = xy - yx.$$

Endi matritsa izi anıqlamasın bereyik. Kvadrat matritsannın' barlıq diagonallıq elementleri qosındısına onın' izi delinedi.

$$\text{Matritsannın' izi} - tr(x) = \sum_{i=1}^n x_{ii} = x_{11} + x_{22} + x_{33} + \dots + x_{nn}.$$

Matritsannın' izi to'mendegi qa'siyetlerge iye boladı.

$$1^\circ. tr(x + y) = tr(x) + tr(y);$$

$$2^\circ. tr(\alpha x) = \alpha tr(x);$$

$$3^\circ. tr(xy) = tr(yx).$$

Da'lillew.

$$1^\circ. tr(x + y) = \sum_{i=1}^n (x_{ii} + y_{ii}) = \sum_{i=1}^n x_{ii} + \sum_{i=1}^n y_{ii} = tr(x) + tr(y).$$

$$2^\circ. tr(\alpha x) = \sum_{i=1}^n \alpha x_{ii} = \alpha \sum_{i=1}^n x_{ii} = \alpha tr(x).$$

$$3^\circ. tr(xy) = tr(yx). \text{ Diagonalli'q elementleri:}$$

$$x_{11}y_{11} + x_{12}y_{21} + \dots + x_{1n}y_{n1}$$

$$x_{21}y_{12} + x_{22}y_{22} + \dots + x_{2n}y_{n2}$$

... ..

$$x_{n1}y_{1n} + x_{n2}y_{2n} + \dots + x_{nn}y_{nn}$$

qatar ha'm bag'ana boyınsha $\sum_{i=1}^n \sum_{j=1}^n x_{ij}y_{ji} = \sum_{j=1}^n \sum_{i=1}^n y_{ji}x_{ij}$ qosindilardi' salıstırsaq,

talap etilgan ten'likti alamız.

Meyli $sl(n, F)$ izi nolge ten' bolg'an n -ta'rtpili kvadrat matritsalar bolsi'n.

$$sl(n, F) = \{x \in gl(n, F) : tr(x) = 0\}.$$

1.2.1-Lemma. $sl(n, F)$ ko'pligi $gl(n, F)$ algebrasinin' ideali boladi.

Da'lillew: Da'slep $sl(n, F)$ ko'pliginin' u'les ken'islik ekenligin matritsalar izinin' additivlik h'a'm birdeylik qa'siyetlerinen kelip shig'adi. Ha'r bir $x, y \in sl(n, F)$, $\alpha \in F$ ushi'n

$$tr(x + y) = tr(x) + tr(y) = 0 + 0 = 0.$$

Bunnan $x + y \in sl(n, F)$.

Endi $x, y \in sl(n, F)$, $\alpha \in F$ ushi'n $tr(\alpha x) = \alpha tr(x) = 0$, ten'liginen $\alpha x \in sl(n, F)$ keli'p shi'gadi'.

Ko'beyme ushi'n

$$tr([x, y]) = tr(xy - yx) = tr(xy) - tr(yx) = 0.$$

Lemma dalillendi.

Meyli $b(n, F)$ bul n -ta'rtpi u'stki u'shmu'yeshlik matritsalar ko'pligi:

$$b(n, F) = \{x \in gl(n, F); x_{ij} = 0, i > j\}.$$

1.2.2-Lemma. $b(n, F)$ ko'pligi $gl(n, F)$ algebrasinin' u'les algebrası boladi.

Da'lillew: U'shmu'yeshlik matritsasının' qosındısı da, sang'a ko'beymeside usı ko'riniske iye bolg'anlıqtan $b(n, F)$ te u'les ken'islik boladi.

$$x, y \in b(n, F) \quad [x, y] \in b(n, F)$$

ekenligin ko'rseteyik:

$$(xy)_{ij} = \sum_{s=1}^n x_{is} y_{sj} = \begin{cases} 1 \leq s \leq i \\ x_{is} = 0 \\ s > j \\ y_{sj} = 0 \end{cases} = \sum_{s=1}^i 0 x_{is} + \sum_{s=i+1}^n 0 y_{sj} = 0.$$

Bunnan

$$[x, y]_{ij} = 0, \quad i > j$$

ha'm

$$[x, y] \in b(n, F).$$

Lemma dalillendi.

$b(n, F)$ u'les algebrası $gl(n, F)$ tin' ideali emes. Buni $n = 2$ de tekseriyik.

$\begin{pmatrix} x_{11} & x_{12} \\ 0 & x_{22} \end{pmatrix} \in gl(n, F)$, $\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \in b(n, F)$ matritsaları'n alayıq. Onda

$$\begin{aligned} [x, y] &= xy - yx = \begin{pmatrix} x_{11} & x_{12} \\ 0 & x_{22} \end{pmatrix} \cdot \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} - \\ &- \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \cdot \begin{pmatrix} x_{11} & x_{12} \\ 0 & x_{22} \end{pmatrix} = \begin{pmatrix} x_{11}y_{11} + x_{12}y_{21} & x_{11}y_{12} + x_{12}y_{22} \\ x_{22}y_{21} & x_{22}y_{22} \end{pmatrix} - \\ &- \begin{pmatrix} y_{11}x_{11} & y_{21}x_{11} \\ y_{11}x_{12} + y_{12}x_{22} & y_{21}x_{12} + y_{22}x_{22} \end{pmatrix} = \begin{pmatrix} x_{12}y_{21} & x_{12}x_{22} \\ -y_{11}x_{12} & -y_{21}x_{12} \end{pmatrix} \notin b(n, F). \end{aligned}$$

$n(n, F)$ qatan' u'stki u'shmu'yeshlik matritsalar ko'pligi:

$$n(n, F) = \{x \in gl(n, F) : x_{ij} = 0, \quad i \geq j\}.$$

$n=3$ te

$$n(3, F) = \left\{ \begin{pmatrix} 0 & x_{12} & x_{13} \\ 0 & 0 & x_{23} \\ 0 & 0 & 0 \end{pmatrix} \mid x_{12}, x_{13}, x_{23} \in F \right\}.$$

1.2.3-Lemma. $n(n, F)$ ko'pligi $b(n, F)$ algebrasinin' ideali boladı.

Da'lillew: $n(n, F)$ tin' u'les ko'plik ekenligi ayqın. Ko'beymeni teksersek,

onda

$$x \in n(n, F), \quad y \in b(n, F), \quad [x, y] \in n(n, F)$$

ushi'n

$$(xy)_{ii} = \sum_{s=1}^n x_{is} y_{si} = \sum_{s=1}^i x_{is} y_{si} + \sum_{s=i+1}^n x_{is} y_{si} = 0.$$

$[x, y] \in n(n, F)$ diagonalli'q elementleri nol. Demek, $n(n, F) \subset sl(n, F)$. Lemma dalillendi.

Endi $gl(n, F)$, $sl(n, F)$, $b(n, F)$, $n(n, F)$ algebralar o'lishemlerin keltireyik.

$$n(3; F): \begin{pmatrix} 0 & x_{12} & x_{13} \\ 0 & 0 & x_{23} \\ 0 & 0 & 0 \end{pmatrix} \quad \dim 3$$

$$b(3; F): \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ 0 & x_{22} & x_{23} \\ 0 & 0 & x_{33} \end{pmatrix} \quad \dim 6$$

$$sl(3; F): \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & -x_{11} - x_{22} \end{pmatrix} \quad \dim 8.$$

Uluima jag'daya

$$gl(n, F) = \dim n^2,$$

$$sl(n, F) = \dim(n^2 - 1),$$

$$b(n, F) = \dim \left(\frac{n(n+1)}{2} \right),$$

$$n(n, F) = \dim \left(\frac{n(n-1)}{2} \right),$$

Meyli L li algebrası I J onın' u'les ken'islikleri bolsın. To'mendegi u'les ko'pliklerdi qarastıramız:

$$I \cap J = \{x \in L: x \in I, x \in J\}$$

$$I + J = \{x + y: x \in I, y \in J\}$$

$$[I, J] = \text{spam} \{[x, y]: x \in I, y \in J\} =$$

$$= \left\{ \sum_{i=1}^n \alpha_i [x_i \cdot y_i]; x_i \in I, y_i \in J, \alpha_i \in F, i \in \overline{1, n}, n \in N \right\}$$

1.2.4-Teorema. Meyli L Li algebrası bolsın. Eger I, J bul algebranın' idealları bolsa, onda $I \cap J, I + J, [\pm, J]$ ko'pliklerde bul algebranın' idealları boladı.

Da'lillew: $I \cap J$ ideal ekenin teksereyik. $x, y \in I \cap J, \alpha \in F$ qarastirayiq. I ha'm J leer ideallar bolg'anlıqtan to'mendegige iye bolamız.

$$\begin{aligned} x + y \in I, x + y \in J & \Rightarrow x + y \in I \cap J \\ \alpha x \in I, \alpha x \in J & \Rightarrow \alpha x \in I \cap J \\ [x, y] \in I, [x, y] \in J & \Rightarrow [x, y] \in I \cap J, y \in L \end{aligned}$$

bul qatnaslar tikkeley anıqlamadan kelip shi'g'ai'.

Endi $I + J$ dın' ideal ekenin teksereyik,

$$\forall x, y \in I + J, \alpha \in L$$

anıqlamadan to'mendegige iye bolamız.

$$x = a + b, \quad a \in I, \quad b \in J,$$

$$y = u + \mathcal{G}, \quad u \in I, \quad \mathcal{G} \in J$$

bul jazıwdan paydalansaq

$$x + y = (a + u) + (b + \mathcal{G}) \in I + J$$

ha'm

$$\alpha x = \alpha a + \alpha b \in I + J.$$

Jene

$$[x, y] = [a + b, u + \mathcal{G}] = [a, u] + [a, \mathcal{G}] + [b, u] + [b, \mathcal{G}] \in I + J,$$

$$[x, y] = [a + b, y] = [a, y] + [b, y] \in I + J.$$

$[I, J]$ kommutatorı u'les ken'islik ekenligi ayqın. Sebebi, bul ko'pliktin' ha'r bir elementi qanday da bir elementtin' sıızıqlı qabıg'ınan turadı.

$\forall x \in [I, J], y \in L \Rightarrow [x, y] \in [I, J]$ ekenin tekseremiz. Meyli $x = [u, \mathcal{G}], y \in L$ bo'lsin. Ona

$$[[u, \mathcal{G}], y] = -[y, [u, \mathcal{G}]] = [y, [u, \mathcal{G}]] = [u, [y, \mathcal{G}]] + [[y, u], \mathcal{G}] \in [I, J].$$

Endi $x = \sum_{i=1}^n \alpha_i [u_i, \mathcal{G}_i]$ korinistegi elementti alsaq, onda

$$[x, y] = \sum_{i=1}^n \alpha_i [[u_i, \mathcal{G}_i], y] \in [I, J].$$

Teorema alillendi.

1.2.5-Saldar. $[L, L]$ algebrasının' idealı boladı. Bul ideal L algebrasının' kvadratı delinedi. To'mendegi ten'likler orınlanadı.

$$1. [gl(n, F); gl(n, F)] = sl(n, F),$$

$$2. [sl(n, F); sl(n, F)] = sl(n, F),$$

3. $[b(n, F); b(n, F)] = n(n, F),$

4. $[n(n, F), n(n, F)] \neq n(n, F).$

Birinshi bap boyi'nsha juwmaq

Jumi'sti'n' birinshi babi' yeki paragraftan ibarat boli'p, bunda shekli o'lshemli Li algebralari' ani'qlamasi', mi'sallar ha'm ayri'm qa'siyetleri qaraldi'. Birinshi paragrafta Li algebralalar haqqi'nda tiykarg'i' tu'sinikler keltirilgen. Yekinshi paragrafta bolsa shekli o'lshemli Li algebralari' ani'qlamasi', mi'sallar ha'm ayri'm belgili natiyjeler berildi.

II BOB

LI ALGEBRALARI' KLASLARI'

Bul bapta kishi o'lishemli Li algebralari' ha'm nilpotent Li Li algebralari' qaralg'an. Birinshi paragrafta kishi o'lishemli Li algebralari' qaralg'an. Yekinshi paragrafta nilpotent Li Li algebralari' u'yrenilgen.

§ 2.1. Kishi o'lishemli Li algebralari'

Biz o'lishemi u'shten u'lken bolmag'an Li algebralari'ni u'zilis qarashtiramiz.

1. Bir o'lishemli Li algebrası

Meyli L bir o'lishemli Li algebrası bolsın. Onda $e \in L$, $e \neq 0$ vektorın alsaq, $L = \{\alpha e : \alpha \in F\}$ ten'lik orinlanadı. Onda $\forall x, y \in L$ $x = \alpha_1 e$, $y = \alpha_2 e$ ushi'n

$$[x, y] = [\alpha_1 e, \alpha_2 e] = \alpha_1 \alpha_2 [e, e] = 0.$$

Demek, $[x, y] = 0$.

Qa'legen eki elementtin' ko'beymesı nol bolg'an Li algebrasına abel algebrası delinedi.

2.1.1-Teorema. Bir o'lishemli Li algebrası abel algebrası boladı.

Da'lillew joqarıda keltirildi.

Meyli L n -o'lishemli Li algebrası bolsın. e_1, e_2, \dots, e_n bazis vektorların alayıq

$$[e_i, e_j] = \sum_{k=1}^n c_{ij}^k e_k \quad (2.1)$$

bul jerde $c_{ij}^k \in F$ $i, j, k \in \{1, 2, \dots, n\}$.

$$[e_j, e_i] = \sum_{k=1}^n c_{ji}^k e_{jk}. \quad (2.2)$$

(2.1) ha'm (2.2) salıstırsaq, onda $c_{ij}^k = -c_{ji}^k$. (2.1) c_{ij}^k sanları L algebrasının' strukturalıq konstantaları delinedi. L algebrasının' strukturası bul konstantalar menen birden-bir anıqlanadı. Eki dana

$$x = \sum_{i=1}^n x_i e_i, \quad y = \sum_{j=1}^n y_j e_j$$

elementleri ushi'n

$$\begin{aligned} [x, y] &= \left[\sum_{i=1}^n x_i e_i, \sum_{j=1}^n y_j e_j \right] = \\ &= \sum_{i,j=1}^n x_i y_j [e_i, e_j] = \sum_{i,j,x=1}^n x_i y_j c_{ij}^k e_k \end{aligned}$$

Eki o'lshefli Li algebralari.

Abel bolmag'an eki o'lshefli Li algebrasın qarastrayıq $\{x_1, x_2\}$ bul Li algebrasının' bazisi bolsın. Bul algebranın' kvadratın qarastramız. $x = \alpha_1 x_1 + \alpha_2 x_2, \quad y = \beta_1 x_1 + \beta_2 x_2$ elementlerin alsaq, onda

$$\begin{aligned} [x, y] &= [\alpha_1 x_1 + \alpha_2 x_2, \beta_1 x_1 + \beta_2 x_2] = [\alpha_1 x_1 \cdot \beta_2 x_2] + \alpha_1 \beta_1 \cdot 0 + \\ &+ [\alpha_2 \beta_1 x_2 x_1] + 0 = [\alpha_1 \beta_2 x_1 x_2] - [\alpha_1 \beta_1 x_1 x_2] = (\alpha_1 \beta_2 - \alpha_1 \beta_1) [x_1, x_2] \end{aligned}$$

Bunnan, $\dim [L, L] \leq 1$. Demek

- 1) $[x_1, x_2] = 0$ bolsa, onda $\dim [L, L] = 0$.
- 2) $[x_1, x_2] \neq 0$ bolsa, onda $\dim [L, L] = 1$.

x_1, x_2 baziste x_2 vektorın algebranın' kvadratın alayıq. $x_2 \in [L, L]$ onda $[x_1, x_2] \in [L, L]$ bunnan $J\alpha \neq 0, [x_1, x_2] = \alpha x_2$. Endi to'mendegi bazisti qarastramız.

$$e_1 = \frac{1}{\alpha} x_1, \quad e_2 = x_2,$$

$$[e_1, e_2] = \left[\frac{1}{2}x_1, x_2 \right] = \frac{1}{2}[x_1, x_2] = \frac{1}{\alpha} \alpha \cdot x_2 = x_2 = e,$$

$$[e_1, e_2] = e_2,$$

$$c_{ii}^1 = c_{ii}^2 = 0 \quad i = 1, 2, \dots,$$

$$c_{12}^1 = 0, \quad c_{12}^2 = 1, \quad c_{21}^1 = 0, \quad c_{21}^2 = -1.$$

Demek tomendegi teorema orinli boladi'.

2.1.2-Teorema. Meyli L eki o'lshefli Li algebrası bolsın. Onda L algebrası to'mendegilerdin' birewine izomorf boladı.

1. L abel Li algebrası;
2. $L: [e_1, e_2] = e_2$.

U'sh o'lshefli Li algebrasın u'yrenbesten aldın to'mendegilerdi qarastırayıq. L Li algebrasının' orayı dep

$$Z(L) = \{x \in L, [x, y] = 0, \forall y \in L\}.$$

2.1.3-Teorema.

1. Li algebrasının' orayı abel Li algebrası boladı.
2. $Z(L)$ orayı L algebrasının' idealı boladı.

Da'lillew. $\forall x_1, x_2 \in Z(L)$

$$[x_1 + x_2, y] = [x_1, y] + [x_2, y] = 0$$

$$x_1 + x_2 \in Z(L)$$

$$\alpha(x) \in Z(L)$$

$$[[x_1, x_2], y] = [0, y] = 0 \quad [x_1, x_2] \in Z(L)$$

$[x_1, x_2] = 0$ ten' orayda jatadı h'a'm abel algebrasına tiyisli.

$[x, y] = 0 \in Z(L)$ ideal boladı.

3. U'sh o'lshefli Li algebrası

Meyli $\dim L = 3$ u'sh o'lshefli algebra bolsın.

1. Yeger $\dim[L, L] = 0$ bolsa, onda L abel algebrası boladı.

2. $\dim[L, L] = 1$ jag'daydi qarastirayiq.

1. $[L, L] \subseteq Z(L)$ bul jag'dayda x_1, x_2, x_3 bazis vektorların alayıq. Bunda x_3 vektorın algebranın' kvadratınan alamız:

$$0 \neq x_3 \in [L, L] \subseteq Z(L) \text{ ha'm } [x_1, x_2] \in [L, L].$$

Onda sonday $\alpha \neq 0$ sani' tabilip, $[x_1, x_2] = \alpha x_3$. x_3 vektorı algebranın' orayında jatqanı ushın

$$[x_1, x_3] = 0, [x_2, x_3] = 0.$$

Endi jan'a baziske o'teyik.

$$\alpha_1 = \frac{1}{\alpha} x_1,$$

$$\alpha_2 = x_2,$$

$$\alpha_3 = x_3.$$

Bul Li algebrası to'mendegi Li algebrasına izomorf boladı.

$$n(3, F) = \left\{ \begin{pmatrix} 0 & \alpha_{12} & \alpha_{13} \\ 0 & 0 & \alpha_{23} \\ 0 & 0 & 0 \end{pmatrix} : \alpha_{ij} \in F, 1 \leq i < j \leq 3 \right\}$$

bul jag'dayda izomorfizmdi bazisler vektorlardı to'mendegishe an'latıwg'a boladı:

$$e_1 \mapsto e_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$e_2 \mapsto e_{23} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix},$$

$$e_3 \mapsto e_{13} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Buni'n ushi'n $[e_{12}, e_{23}] = e_{13}$ qatnasta tekseremiz:

$$[e_{12}, e_{23}] = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = e_{13}.$$

2. $[L, L] \not\subset Z(L)$ bolsi'n. Onda x_1, x_2, x_3 bazis vektorlarin alayiq. Bunda x_3 vektorin algebranın' orayinan alamiz: $x_3 \in Z(L)$.

Endi $[x_1, x_2]$ ko'beymeni qarastirayiq:

$$[x_1, x_2] = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3.$$

$\alpha_1 = \alpha_2 = 0$ dep boljayiq. Onda

$$[x_1, x_2] = \alpha_3 x_3 \in Z(L).$$

Son'g'ı qatnastan algebranın' kvadrati onin' orayında jatadı. Bul joqarıda berilgen sha'rtke qarama-qarsı. Payda bolg'an qarama-qarsılıqtan boljawımızdın' naduris ekenligi kelip shıg'adı. Demek, $\alpha_1 \neq 0$ yamasa $\alpha_2 \neq 0$.

Endi anılıq ushin $\alpha_1 \neq 0$. Onda to'mendegi baziske o'temiz,

$$e_2 = [x_1, x_2],$$

$$e_1 = -\frac{1}{\alpha_1} x_2,$$

$$e_3 = x_3.$$

Onda

$$[e_1, e_3] = [e_2, e_3] = 0,$$

$$[e_1, e_2] = \left[[x_1, x_2]; \frac{1}{\alpha_1} x_2 \right] = \left[\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3; \frac{1}{\alpha_1} x_2 \right] =$$

$$= \frac{\alpha_1}{\alpha_1} [x_1, x_2] + \frac{\alpha_2}{\alpha_1} [x_2, x_3] + \frac{\alpha_3}{\alpha_1} [x_3, x_2] = [x_1, x_2].$$

Demek $[e_1, e_2] = e_2$. Bul u'sh o'lishemli algebra eki o'lishemli Li bolmag'an algebra menen bir o'lishemli Li algebrasinin' tuvri qosindısına ten' boladı:

$$L = \text{span}\{e_1, e_2\} \oplus Z(L).$$

Biz tomendegi teoremag'a iye boldiq.

2.1.4-Teorema. Meyli L u'sh o'lishemli Li algebrası bolıp, onun' kvadratının' o'lishemi birden u'lken bolmasın. Onda bul algebra to'mendegi algebralardıń birewine izomorf boladı.

1. L abel Li algebrası;
2. $L: [e_1, e_2] = e_3$ (kvadratı orayda);
3. $L: [e_1, e_2] = e_2$ (kvadratı orayda jatpag'an).

Tomendegi matrirsaliq algebalardı qarastiramız.

- 1) $gl(n, F)$ bazisi $\{e_{ij}\}_{i,j=1}^n$;
- 2) $sl(n, F)$ bazisi $\{e_{ij}; i \neq j\} \cup \{h_i; i \in \overline{1, n}\}$, bunda $h_i = e_{ii} - e_{i+i+1}$, $i \in \overline{1, n-1}$;
- 3) $b(n, F)$ bazisi $\{e_{ij}; 1 \leq i \leq j \leq n\}$;

4) $n(n, F)$ bazisi $\{e_{ij} : 1 \leq i < j \leq n\}$.

Meyli $x \in sl(n, F)$ bolsin. Onda

$$x = \sum_{i \neq j} a_{ij} e_{ij} + \sum_{s=1}^n \alpha_s e_{ss}, \quad \sum_{s=1}^n \alpha_s = 0.$$

Ekinshi ta'repten $x = \sum_{i \neq j} \beta_{ij} e_{ij} + \sum_{s=1}^{n-1} \beta_s h_s$, $\beta_{ij} = \alpha_{ij}$, $i \neq j$. Bunnan

$$\sum_{s=1}^{n-1} \beta_s (e_{ss} - e_{s+1}) = \beta_1 e_{11} + \sum_{s=2}^{n-2} (\beta_{s+1} - \beta_s) e_{ss} - \beta_{n-1} e_{nn}.$$

Salistirsaq

$$\begin{array}{ll} \beta_1 = \alpha_1 & \beta_1 = \alpha_1 \\ \beta_2 - \beta_1 = \alpha_2 & \beta_2 = \alpha_1 + \alpha_2 \\ \beta_3 - \beta_2 = \alpha_3 & \beta_3 = \alpha_1 + \alpha_2 + \alpha_3 \\ \dots & \dots \\ \beta_{n-2} - \beta_{n-3} = \alpha_{n-2} & \beta_{n-2} = \alpha_1 + \dots + \alpha_{n-2} \\ -\beta_{n-1} = \alpha_n & \beta_{n-1} = -\alpha_n = \alpha_1 + \alpha_2 + \dots + \alpha_{n-1}. \end{array}$$

2.1.5-Lemma. $[g, l_n, gl_n] \subseteq sl_n$, $sl_n \subseteq [sl_n, sl_n]$.

Dalillew. Meyli $x \in [gl_n, gl_n]$ bolsin. Onda

$$x = \sum_{i=1}^n [a_i, b_i].$$

Bunnan

$$\begin{aligned} tr(x) &= tr\left(\sum_{i=1}^n [a_i, b_i]\right) = \\ &= \sum_{i=1}^n tr[a_i, b_i] = \sum_{i=1}^n tr(a_i b_i - b_i a_i) = 0. \end{aligned}$$

Demek $x \in sl_n$.

Ekinshi qatnasti' tekseriw ushın tek g'ana bazisli vektorlar ushın orınlı ekenligin ko'rsetiw jetkilikli.

$e_{ij} = [e_{ii}, e_{ij}]$ bul to'mendegi ten'liklerden kelip shi'g'ai'

$$[e_{ii}, e_{ij}] = e_{ii}e_{ij} - e_{ij}e_{ii} = e_{ij}.$$

$h_i = [e_{ij}, e_{ji}]$ $j = i + 1$ ushi'n

$$h_i = e_{ij} \cdot e_{ji} - e_{ji} \cdot e_{ij} = e_{ii+1} \cdot e_{i+1i} - e_{i+1i} \cdot e_{ii+1} = e_{ii} - e_{i(i+1)}.$$

Lemma dalillendi.

2.1.5-Lemmadan to'mendegiler kelip shıg'adı.

$$sl_n \subseteq [sl_n, sl_n] \subseteq [gl_n, gl_n] \subseteq sl_n.$$

Bunnan

$$[sl_n, sl_n] = [gl_n, gl_n] = sl_n.$$

$[b(n, F)]$, $b(n, F) = n(n, F)$ da'slep to'mendegi qatnasti da'lilleyik.

$n(n, F) \subseteq [b(n, F), b(n, F)]$ bul qatnas to'mendegi ten'likten kelip shıg'adı:

$$e_{ij} = [e_{ii}, e_{ij}], \quad i < j.$$

Endi joqarıdag'ı qatnastın' keriside orınlı ekenligin ko'rseteyik. Meyli $x, y \in b(n, F)$ bolsin. Ona

$$\begin{aligned} [x, y]_{ii} &= \sum_{s=1}^n x_{is} y_{si} - \sum_{s=1}^n y_{is} x_{si} = \sum_{s=1}^{i-1} (x_{is} y_{si} - y_{is} x_{si}) + \\ &+ (x_{ii} y_{ii} - y_{ii} x_{ii}) + \sum_{s=i+1}^n (x_{is} y_{si} - y_{is} x_{si}) = 0. \end{aligned}$$

U'sh o'lsheimli jag'aydi koreyik: $[n(3, F), n(3, F)]$

$$x = \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix}, \quad y = \begin{pmatrix} 0 & \alpha & \beta \\ 0 & 0 & \gamma \\ 0 & 0 & 0 \end{pmatrix}.$$

Ona

$$\begin{aligned} [x, y] &= xy - yx = \\ &= \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & \alpha & \beta \\ 0 & 0 & \gamma \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & \alpha & \beta \\ 0 & 0 & \gamma \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} = \\ &= \begin{pmatrix} 0 & 0 & a\gamma \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & \alpha c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & a\gamma - \alpha c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \end{aligned}$$

§ 2.1. Nilpotent li algebraları

Meyli L Li algebrası bolsın. To'mendegi izbe-izlikti qarastramız.

$$L^1 = [L, L], \quad L^k = [L^{k-1}, L], \quad k \geq 2$$

anıqlamadan to'mendegi kelip ıshg'adı.

$$L^1 \supseteq L^2 \supseteq \dots \supseteq L^n \supseteq \dots \quad (2.3)$$

2.2.1-Lemma. Ha'r bir natural k sanı ushın L^k ko'pligi L algebrasının' idealı boladı.

Da'lillew: Matematikalıq induktsiya boyınsha da'lilleyiz.

$k=1$ $L^1 = [L, L]$ idealdın' ko'beymesi ideal bolg'anlıqtan L^1 de ideal boladı.

$k-1$, $L^k = [L^{k-1}, L]$ ideal ekenligi joqarıdag'ıdan kelip shıg'adı. lemma da'lillendi.

Yeger sonay $n \in \mathbb{N}$ tabılıp, $L^n = \{0\}$ bolsa, onda L nilpotent algebra delinedi.

Abel Li algebrası Nilpotent boladı, sebebi $L^1 = [L, L] = \{0\}$.

Nilpotent bolmag'an algebrag'a mısıl to'mendegi eki o'lshemli Li algebrasın qarastırayıq.

$$L: [e_1, e_2] = e_2.$$

Bul algebra nilpotent emes, sebebi

$$L^1 = [L, L] = \{\alpha e_1 : \alpha \in F\},$$

$$L^2 = [L^1, L] = L^1,$$

$$L^k = L^1, \quad k \geq 2, \quad L^k \neq \{0\}.$$

2.2.2-Lemma. $n(n, F)$ algebrası nilpotent boladı.

Da'lillew. Bul algebra bazisin alayıq: $\{e_{ij}, i < j\}$. Ona

$$[x, y] = \left[\sum_{i < j} \alpha_{ij} e_{ij}, \sum_{k < s} \beta_{ks} e_{ks} \right] = \sum_{\substack{i < j \\ k < s}} \alpha_{ij} \beta_{ks} [e_{ij}, e_{ks}].$$

Bul jerde tomendegilerdi esapqa alamız:

$$[e_{ij}, e_{ks}] = \begin{cases} e_{is}, & j = k \\ -e_{kj}, & s = i \end{cases},$$

$$i < j = k < s, \quad i + 1 < s,$$

$$k < s = i < j, \quad k + 1 < j.$$

Demek L^1 bazisi: $\{e_{ij} : i + 1 < j\}$. Sondayaq, L^k bazisi: $\{e_{ij} : i + k < j\}$. Dawam ettirsek, onda $L^{n-1} = \{0\}$.

$b(n, F)$ algebrası nilpotent emes to'mendegi ten'likten paydalanayıq.

$$[b(n, F), b(n, F)] = n(n, F).$$

To'mendegi ten'lik orinli boladı:

$$[b(n, F), n(n, F)] = n(n, F)$$

ten'lik $e_{ij} = [e_{ij}, e_{ij}]$ ten'likten kelip shıg'adı. Bunnan u'shmu'yeshli matritsalar algebraları ushın to'mendegi qatnaslar kelip shıg'adı:

$$L = b(n, F), \quad L^1 = n(n, F), \quad L^k = n(n, F).$$

2.2.3-Teorema. n -o'lishemli nilpotent Li algebralarında to'mendegi qa'siyetlerge iye e_1, e_2, \dots, e_n bazis bar boladı:

$$[e_i, e_j] = \sum_{k=1}^n c_{ij}^k e_k,$$

bunda $c_{ij}^k = 0, \quad k \geq \min\{i, j\}$.

n o'lishemli algebrada ko'beytiw kestesi to'mendegishe bolsın.

$$[e_1, e_k] = e_{k+1} \quad 2 \leq k \leq n-1.$$

a) L Li algebrası boladı

b) L nilpotent boladı.

Da'lillew: a) Yakobi birdeyliginin' orinli ekenligin tekseresik.

$$[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$$

bul birdeyliktin' bazis vektorları ushın orinli ekenligin tekseriw jetkilikli.

$$[e_i, [e_j, e_k]] + [e_j, [e_k, e_i]] + [e_k, [e_i, e_j]] = 0,$$

$$i, j, k \in \overline{1, n}, \quad i \leq j \leq k.$$

1-jag'day. $i > 1$ bul jag'dayda joqarıdag'ı ten'ликтin' u'sh qosılıwshısında nolge ten'.

2. $i = 1$.

2₁) $j = 1$ bolsin. Onda

$$[e_1, [e_1, e_k]] + [e_1, [e_k, e_1]] + [e_k, [e_1, e_1]] = e_{k+2} - e_{k+2} = 0,$$

bul jerde $2 \leq k \leq n - 2$.

2₂) $j > 1$ bolsin. Onda

$$[e_1, [e_j, e_k]] + [e_j, [e_k, e_1]] + [e_k, [e_1, e_j]] = [e_1, 0] + [e_j - e_{k+1}] + [e_k - e_{j+1}] = 0.$$

Endi nilpotentlikke tekseremiz. Onda

$$\begin{aligned} [x \cdot y] &= \left[\sum_{i=1}^n \alpha_i e_i \sum_{j=1}^n \beta_j e_j \right] = \sum_{i,j=1}^n \alpha_i \beta_j [e_i, e_j] = \\ &= \sum_{j=2}^{n-1} \alpha_i \beta_j e_{j+1} - \sum_{i=2}^{n-1} \alpha_i \beta_1 e_{i+1} = \sum_{i=2}^{n-1} (\alpha_i \beta_i - \alpha_i \beta_i) e_{i+1}. \end{aligned}$$

Bunnan L^1 bazisi $\{e_3, \dots, e_n\}$. Dawam ettirsek, ona $x = \sum_{i=3}^n \alpha_i e_i$, $y = \sum_{j=1}^{n-1} \beta_j e_j$ ushi'n

$$[x, y] = \left[\sum_{i=3}^n \alpha_i e_i, \sum_{j=1}^n \beta_j e_j \right] = \sum_{i=3}^n \alpha_i \beta_1 [e_i, e_1] = - \sum_{i=3}^{n-1} \alpha_i \beta_i e_{i+1}.$$

Bunnan L^2 bazisi $\{e_4, \dots, e_n\}$. Uluima jag'ayda L^k bazisi $\{e_{k+2}, \dots, e_n\}$. Demek $L^{n-1} = \{0\}$. Bunnan algebra nilpotent algebra boladi'.

2.2.4-Teorema. Meyli L n o'lshemli nilpotent Li algebrası bolsın. Onda

$$\dim L^1 \leq n - 2 \quad (1)$$

Da'lillew. Kerisin boljap da'lillemiz. $\dim L^1 > n - 2$ dep boljayıq. Onda $\dim L^1 = n$ bolsa, $\dim L^1 = n$, $L^1 \leq L$ bolg'anlıqtan, $L = L^1$. Bunnan

$$L^k = [L^{k-1}, L] = L^1 = L,$$

$$L^2 = [L^1, L] = L,$$

$$L^k = [L^{k-1}, L] = [L, L] = L.$$

$L^k = L \neq \{0\}$ qatnas L din' nilpotent ekenligine qarama qarsı.

Meyli $\dim L^1 = n - 1$ dep alayiq. Onda L^1 ken'isliginin' qandayda bir bazisin qarastıramız. $L^1 : e_2, \dots, e_n$ bul bazisti L din' bazisine shekem toltıramız.

$$L : e_1, e_2, \dots, e_n$$

$$L^2 : x = \sum_{i=1}^n \alpha_i e_i \in L, \quad y = \sum_{j=2}^n \beta_j e_j \in L^1.$$

Bunnan

$$[x, y] = \sum_{i=1}^n \sum_{j=2}^n \alpha_i \beta_j [e_i, e_j] = \sum_{s=2}^n *e_s.$$

Demek $L^2 : e_2, \dots, e_n$ ha'm $L^2 = L^3 = \dots = L^k = \dots$ Bunnan $L^k = L^1 \neq \{0\}$ $k \geq 2$.

Son'g'ı qatnastan nilpotentlikke qarama-qarsı. Teorema da'lillendi.

II bap boyi'nsha juwmaq

Ekinshi bapta kishi o'lsheqli Li algebralari' qaraldi. Bir ha'm eki o'lsheqli Li algebralari' toliq dizimi keltirildi. Nilpotent Li algebralari'da qaraldi'. Birinshi paragrafta kishi o'lsheqli Li algebralari' qaralg'an. Yekinshi paragrafta nilpotent Li Li algebralari' u'yrenildi.

III BAP

LI ALGEBRALARI' 2-LOKAL DIFFERENCIALLAWLARI'

U'shinshi bap yeki paragraftan ibarat boli'p, bunda Li algebralari'nda differenciallawlar, 2-lokal differenciallawlar ani'qlamalari', ishki differenciallaw ha'm matrica izi qa'siyetleri, mi'sallar keltiriledi. Nilpotent Li algebralardi'n' 2-lokal differenciallawlari' xarakterlengen.

§ 3.1 Li algebra'lari' differenciallawlari'

Meyli F maydan u'stinde L Li algebrası berilgen bolsın. Yeger $D: L \rightarrow L$ si'zi'qli' operatori' qa'legen $x, y \in L$ elementlaeri ushi'n

$$D[x, y] = [D(x), y] + [x, D(y)] \quad (3.1)$$

ten'likti qanaatlandırsa, onda bul operator differenciallaw delinedi.

sl_2 algebrasin qarastirayik:

$$sl_2 = \left\{ \begin{pmatrix} a & b \\ c & -a \end{pmatrix} : a, b, c \in \mathbb{C} \right\}.$$

Bul algebrada tomendegi elementler basis boladi:

$$h = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad e = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

Ko'beytiw kelesi tomendegishe boladi:

$$\begin{aligned}
[h, e] &= h \cdot e - e \cdot h = 2e, & [h, e] &= 2e, \\
[h, f] &= hf - f \cdot h = -2f, & [h, f] &= -2f, \\
[e, f] &= e \cdot f - f \cdot e = h, & [e, f] &= h.
\end{aligned}$$

Endi bul algebrada diiferenciallawdin uluima korinisin tabamiz.

3.1.1-Teorema. sl_2 algebrasının' ha'r bir differentsiallawı to'mendegishe ko'riniste boladı:

$$\begin{pmatrix} 0 & d_{12} & d_{13} \\ -2d_{13} & d_{22} & 0 \\ -2d_{12} & 0 & -d_{22} \end{pmatrix} \quad (3.2)$$

Da'lillew. Da'slep izlenip atırğ'an matritsanı to'mendegishe jazıp alami'z:

$$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix}.$$

Endi bul ten'liklerdin' ha'r qaysı birinen differentsial alamız.

$$D[h, e] = D(2e)$$

Differentsiallawdın' anıqlamasına ko're

$$D[h, e] = [D(h), e] + [h, D(e)] = 2D(e).$$

Ori'nlari'na qoysaq, onda

$$[d_{11}h + d_{21}e + d_{31}f, e] + [h, d_{12}h + d_{22}e + d_{32}f] = 2(d_{12}h + d_{22}e + d_{32}f).$$

Bunnan

$$2d_{11}e - d_{31}h + 2d_{22}e - 2d_{32}f = 2d_{12}h + 2d_{22}e + 2d_{32}f.$$

Bul ten'liktegi bazis aldındag'ıların ten'lestirsek, onda

$$\begin{cases} 2d_{11} + 2d_{22} = 2d_{22}, \\ -d_{31} = 2d_{12}, \\ -2d_{32} = 2d_{32}. \end{cases}$$

Bul sistemadan tomendegige iye bolamız:

$$\begin{cases} d_{11} = 0, \\ d_{31} = -2d_{12}, \\ d_{32} = 0. \end{cases}$$

Bul'l sistemasındag'ı ten'liklerden paydalana otırıp, izlenip atırğ'an matritsa to'mendegishe ko'riniske keledi:

$$\begin{pmatrix} 0 & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ -2d_{12} & 0 & d_{33} \end{pmatrix}.$$

Endi tomendegi qatnastan faydalanamız:

$$D[h, f] = D(-2f).$$

Bunnan

$$[D(h), f] + [h, D(f)] = -2D(f).$$

Bazistagi jayilmasin esapqa alsaq, onda

$$[d_{11}h + d_{21}e + d_{31}f, f] + [h, d_{13}h + d_{23}e + d_{33}f] = -2(d_{13}h + d_{23}e + d_{33}f).$$

Bunnan

$$-2d_{11}f + d_{21}h + 2d_{23}e - 2d_{33}f = -2d_{13}h - 2d_{23}e - 2d_{33}f.$$

Bu'l ten'liktegi tabılg'an matritsadan paydalansaq

$$d_{21}h + 2d_{23}e - 2d_{33}f = -2d_{13}h - 2d_{23}e - 2d_{33}f.$$

Bunnan

$$\begin{cases} d_{21} = -d_{13}, \\ 2d_{23} = -2d_{23}, \\ -2d_{33} = -2d_{33}. \end{cases}$$

Bul sistemadan

$$d_{21} = -d_{13},$$

$$d_{23} = 0.$$

Bunnan izlenip atırg'an matritsa to'mendegishe ko'rinske keledi:

$$\begin{pmatrix} 0 & d_{12} & d_{13} \\ -2d_{13} & d_{22} & 0 \\ -2d_{12} & 0 & d_{33} \end{pmatrix}.$$

Endi $D[e, f] = D(h)$ ten'liginen paydalanamiz. Onda

$$[D(e), f] + [e, D(f)] = D(h).$$

Bunnan

$$[d_{12}h + d_{22}e + d_{32}f, f] + [e, d_{13}h + d_{23}e + d_{33}f] = d_{11}h + d_{21}e + d_{31}f.$$

Endi bazistagikelip shıqqan matritsadan paydalansaq, onda

$$-2d_{12}f + d_{22}h - 2d_{13}e + d_{33}h = -2d_{13}e - 2d_{12}f.$$

Bunnan

$$\begin{cases} d_{22} + d_{33} = 0, \\ -2d_{12} = -2d_{12}, \\ -2d_{13} = -2d_{13}. \end{cases}$$

Demek

$$d_{22} = -d_{33}.$$

Bunnan izlenip atırg'an matritsa, yag'nıy sl_2 algebrasının' ha'r bir differentsiallanıwı to'mendegi ko'riniske keledi:

$$\begin{pmatrix} 0 & d_{12} & d_{13} \\ -2d_{13} & d_{22} & 0 \\ -2d_{12} & 0 & -d_{22} \end{pmatrix}.$$

Teorema da'liyilendi.

3.1.2-Teorema. U'sh o'lishemli' Geizenberg algebrasında ha'r bir differentsiallanıwı to'mendegishe anıqlanadı:

$$\begin{pmatrix} d_{11} & d_{12} & 0 \\ d_{21} & d_{22} & 0 \\ d_{31} & d_{32} & d_{11} + d_{22} \end{pmatrix}. \quad (3.3)$$

Da'lillew. Da'slep izlenip atırg'an matritsanı to'mendegishe jazıp alsaq

$$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix}.$$

L_2 de bazislerin e_1, e_2, e_3 dep alsaq, onda olar to'mendegi ten'likti qanaatlandıradı.

$$[e_1, e_2] = e_3 \quad (3.4)$$

Al L_2 de qalg'an $[e_1, e_3], [e_2, e_3]$ ko'beymeleri nolge ten' dep alamız. Endi (3.4) ten'liginin' eki ta'repin differentsial alsaq, onda

$$D[e_1, e_2] = D(e_3).$$

Bunnan

$$[D(e_1), e_2] + [D; D(e_2)] = D(e_3).$$

Bazis vektorlardagi jayilmasin esapqa alsaq, onda

$$[d_{11}e_1 + d_{21}e_2 + d_{31}e_3, e_2] + [e_1, de_1 + d_{22}e_2 + d_{32}e_3] = d_{13}e_1 + d_{23}e_2 + d_{33}e_3.$$

Demek

$$d_{11}e_3 + d_{22}e_2 = d_{13}e_1 + d_{23}e_2 + d_{33}e_3.$$

Sa'ykes bazisler aldındag' ıların ten'lestirsek,

$$\begin{cases} d_{11} + d_{22} = d_{33}, \\ d_{13} = 0, \\ d_{23} = 0. \end{cases}$$

Demek differensiallaw matritsasi tomendegishe boladi':

$$\begin{pmatrix} d_{11} & d_{12} & 0 \\ d_{21} & d_{22} & 0 \\ d_{31} & d_{32} & d_{11+d_{22}} \end{pmatrix}.$$

Endi (3.4) matritsanin' differenciallaw ekenin ko'rseteyik. Meyli

$$x = x_1 e_1 + x_2 e_2 + x_3 e_3, \quad y = y_1 e_1 + y_2 e_2 + y_3 e_3 \in L$$

bolsin. Onda

$$[x, y] = (x_1 y_2 - x_2 y_1) e_3.$$

Buunan

$$D[x, y] = (d_{11} + d_{22})(x_1 y_2 - x_2 y_1) e_3.$$

Ekinshi ta'repten

$$[D(x), y] = [(d_{11} x_1 + d_{12} x_2) e_1 + (d_{21} x_1 + d_{22} x_2) e_2, y_1 e_1 + y_2 e_2] =$$

$$= (d_{11} x_1 y_2 + d_{12} x_2 y_2 - d_{21} x_1 y_1 - d_{22} x_2 y_1) e_3$$

ha'mde

$$\begin{aligned}
[x, D(y)] &= [x_1 e_1 + x_2 e_2, (d_{11} y_1 + d_{12} y_2) e_1 + (d_{21} y_1 + d_{22} y_2) e_2,] = \\
&= (d_{21} x_1 y_1 + d_{22} x_1 y_2 - d_{11} x_2 y_1 - d_{12} x_2 y_2) e_3.
\end{aligned}$$

Bunnan

$$D[x, y] = [D(x), y] + [x, D(y)].$$

Demek, (3.3) matritsanin' differenciallow ekenin ko'rsettik. Teorema da'lillendi.

Meyli bizge $\Delta: L \rightarrow L$ sızıqlı sa'wlelendiriwı berilgen bolsın. Eger $\forall x \in L$ vektorı ushın sonday $D_x \in DerL$ tabılıp, $\Delta(x) = D_x(x)$ ten'ligi orınlansa, onda Δ operatori lokal differentsiallanıw dep ataladı.

Anıqlamadan L Li algebrasında qalegen differentsiallanıw lokal differentsiallanıw bolatug'ınlığı keli'p shi'gádi'.

Tomendegi teoremani qarastirayıq.

3.1.3-Teorema. sl_2 algebrasının' ha'r bir lokal differentsiallanıwı differentsiallanıw boladı.

Da'lillew. 3.1.1-Teoremadan bizge ma'lim sl_2 algebrasının' ha'r bir differentsiallanıwı to'mendegishe ko'riniste boladı:

$$\begin{pmatrix} 0 & a & b \\ -2b & c & 0 \\ -2a & 0 & -c \end{pmatrix},$$

bul jerde $a, b, c \in C$. Endi bul matritsa menen aniqlangan operatorдын' lokal differentsiallanıw bolıwında ko'rseteyik. Bunın' ushın bazis vektorlar bolg'an

$$h = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad e = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad f = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

lerden paydalanamız. Lokal differentsiallanıw esabında

$$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix}$$

matritsanı alsaq lokal differentsiallanıwdın' anıqlamasına ko're $\forall x \in L$ vektor ushin sonday $D_x \in DerL$ tabılıp

$$\Delta(x) = D_x(x).$$

1) $x = h = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ dep alayıq. Onda

$$\Delta(h) = \begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} d_{11} \\ d_{21} \\ d_{31} \end{pmatrix}$$

ha'mde

$$D_h(h) = \begin{pmatrix} 0 & a_h & b_h \\ -2b_h & c_h & 0 \\ -2a_h & 0 & -c_h \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -2b_h \\ -2a_h \end{pmatrix}.$$

$\Delta(h) = D_h(h)$ ten'liginen

$$\begin{pmatrix} d_{11} \\ d_{21} \\ d_{31} \end{pmatrix} = \begin{pmatrix} 0 \\ -2b_h \\ -2a_h \end{pmatrix}.$$

Bunnan $d_{11} = 0$.

2) $x = e = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ dep alayıq. Onda

$$\Delta(e) = \begin{pmatrix} 0 & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} d_{12} \\ d_{22} \\ d_{32} \end{pmatrix}$$

ha'mde

$$D_e(e) = \begin{pmatrix} 0 & a_e & b_e \\ -2b_e & c_e & 0 \\ -2a_e & 0 & -c_e \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a_e \\ c_e \\ 0 \end{pmatrix}.$$

Bunnan $\Delta(e) = D_e(e)$ tengliginen, $d_{32} = 0$.

3) $x = f = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ vektorin alayiq. Onda

$$\Delta(f) = \begin{pmatrix} 0 & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} d_{13} \\ d_{23} \\ d_{33} \end{pmatrix},$$

$$D_f(f) = \begin{pmatrix} 0 & a_f & b_f \\ -2b_f & c_f & 0 \\ -2a_f & 0 & -c_f \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} b_f \\ 0 \\ -c_f \end{pmatrix}.$$

Bunnan $\Delta(f) = D_f(f)$ bolganliqtan $d_{23} = 0$.

4) $x = h + e = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ vektorin alamiz. Onda

$$\Delta(x) = \begin{pmatrix} 0 & d_{12} & d_{13} \\ d_{21} & d_{22} & 0 \\ d_{31} & 0 & d_{33} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} d_{12} \\ d_{21} + d_{22} \\ d_{31} \end{pmatrix},$$

$$D_x(x) = \begin{pmatrix} 0 & a_x & b_x \\ -2b_x & c_x & 0 \\ -2a_x & 0 & -c_x \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a_x \\ -2b_x + c_x \\ -2a_x \end{pmatrix}.$$

Bunan $\Delta(x) = D_x(x)$ tenligin esapqa alsaq, onda

$$d_{12} = a_x, d_{31} = -2a_x \Rightarrow d_{31} = -2d_{12}.$$

5) $x = e + f = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ desek, onda

$$\Delta(x) = \begin{pmatrix} 0 & d_{12} & d_{13} \\ d_{21} & d_{22} & 0 \\ -2d_{12} & 0 & d_{33} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} d_{12} + d_{13} \\ d_{22} \\ d_{33} \end{pmatrix},$$

$$D_x(x) = \begin{pmatrix} 0 & a_x & b_x \\ -2b_x & c_x & 0 \\ -2a_x & 0 & -c_x \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} a_x + b_x \\ c_x \\ -c_x \end{pmatrix}.$$

Bul vektolardi tenlestirsek, onda

$$d_{22} = c_x, d_{33} = -c_x.$$

Bunan $d_{33} = -d_{22}$.

6) $x = h + f = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ vektorin qaratirayiq. Onda

$$\Delta(x) = \begin{pmatrix} 0 & d_{12} & d_{13} \\ d_{21} & d_{22} & 0 \\ -2d_{12} & 0 & -d_{22} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} d_{13} \\ d_{21} \\ -2d_{12} - d_{22} \end{pmatrix},$$

$$D_x(x) = \begin{pmatrix} 0 & a_x & b_x \\ -2b_x & c_x & 0 \\ -2a_x & 0 & -c_x \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} b_x \\ -2b_x \\ -2a_x - c_x \end{pmatrix}.$$

Salistirsaq

$$\begin{cases} d_{13} = b_x, \\ d_{21} = -2b_x. \end{cases}$$

Bunnan $d_{21} = -2d_{13}$.

Demek izlenip aturg'an matritsa

$$\begin{pmatrix} 0 & d_{12} & d_{13} \\ -2d_{13} & d_{22} & 0 \\ -2d_{12} & 0 & -d_{22} \end{pmatrix}.$$

Teorema da'lillendi.

§ 3.2 Shekli o'lshemli Li algebra'lari' differenciallawlari'

3.2.1-Teorema. sl_2 algebrasin'n' ha'r bir 2-lokal differenciallawi differenciallaw bo'ladi'.

Dalillew. Meyli Δ bul sl_2 algebrasindagi' 2-lokal differenciallaw bo'lsi'n. Sonday $D_{h,e}$ differenciallawi tabilip,

$$\Delta(h) = D_{h,e}(h) \text{ ha'm } \Delta(e) = D_{h,e}(e)$$

ten'likleri ori'nlanadi'. Zarurlik bo'lsa, Δ ornina $\Delta - D_{h,e}$ 2-lokal differenciallawin qarastirip, biz $\Delta(h) = \Delta(e) = 0$ dep aliwimiz mumkin.

Qalegen $x = x_1h + x_2e + x_3f \in sl_2$ elementin alayiq. Onda ani'qlamadan sonday D differenciallawi tabilip,

$$\Delta(x) = D(x) \text{ ha'm } \Delta(h) = D(h)$$

ten'likleri ori'nlanadi'. Differenciallaw matritsasi' tomendegi ko'riniste boladi':

$$\begin{pmatrix} 0 & a_1 & b_1 \\ -2b_1 & c_1 & 0 \\ -2a_1 & 0 & -c_1 \end{pmatrix}.$$

Onda

$$0 = \Delta(h) = D(h) = \begin{pmatrix} 0 & a_1 & b_1 \\ -2b_1 & c_1 & 0 \\ -2a_1 & 0 & -c_1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -2b_1 \\ -2a_1 \end{pmatrix}.$$

Bunnan $b_1 = a_1 = 0$ ten'lilerine iye bolamiz. Onda

$$\Delta(x) = D(x) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & c_1 & 0 \\ 0 & 0 & -c_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ c_1 x_2 \\ -c_1 x_3 \end{pmatrix}. \quad (3.5)$$

Endi ani'qlamadan sonday basqa bir D differenciallawi tabilip,

$$\Delta(x) = D(x) \text{ ha'm } \Delta(e) = D(e)$$

ten'likleri ori'nlanadi'. Differenciallaw matritsasi' tomendegi ko'riniste boladi':

$$\begin{pmatrix} 0 & a_2 & b_2 \\ -2b_2 & c_2 & 0 \\ -2a_2 & 0 & -c_2 \end{pmatrix}.$$

Onda

$$0 = \Delta(e) = D(e) = \begin{pmatrix} 0 & a_2 & b_2 \\ -2b_2 & c_2 & 0 \\ -2a_2 & 0 & -c_2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a_2 \\ c_2 \\ 0 \end{pmatrix}.$$

Bunnan $a_2 = c_2 = 0$ ten'lilerine iye bolamiz. Onda

$$\Delta(x) = D(x) = \begin{pmatrix} 0 & 0 & b_2 \\ -2b_2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_2 x_3 \\ -2b_2 x_1 \\ 0 \end{pmatrix}. \quad (3.6)$$

Endi (3.5) ha'm (3.6) ten'liklerdi salistirip, $\Delta(x) = 0$ ten'ligine iye bolamiz.

Teorema dalillendi.

3.2.2-Teorema. U'sh o'lsheqli'Heyzenberg algebrasında ha'r bir 2-lokal differentsiallaw tomendegi ko'riniste bo'ladi':

$$\Delta(x) = dx_3 e_3 + f_1(x_1, x_2) e_1 + f_2(x_1, x_2) e_2 + f_3(x_1, x_2) e_3, \quad (3.7)$$

bul jerde d turaqli san, $f_1(x_1, x_2)$, $f_2(x_1, x_2)$, $f_3(x_1, x_2)$ bir tekli funkciyalar.

Dalillew. 3.1.2-Teoremadan u'sh o'lsheqli Geizenberg algebrasinin' differenciallawlari tomendegi koriniske iye.

$$\begin{pmatrix} d_{11} & d_{12} & 0 \\ d_{21} & d_{22} & 0 \\ d_{31} & d_{32} & d_{11} + d_{22} \end{pmatrix}.$$

Sonday D_{e_3} differenciallawi tabilip, $\Delta(e_3) = D_{e_3}(e_3)$ ten'ligi ori'nlanadi'. Zarurlik bo'lsa, Δ ornina $\Delta - D_{e_3}$ 2-lokal differenciallawin qarastirip, biz $\Delta(e_3) = 0$ dep aliwimiz mumkin.

Qalegen $x = x_1 e_1 + x_2 e_2 + x_3 e_3$ elementin alayiq. Onda ani'qlamadan sonday D differenciallawi tabilip,

$$\Delta(x) = D(x) \text{ ha'm } \Delta(e_3) = D(e_3)$$

ten'likleri ori'nlanadi'. Differenciallaw matritsasi' tomendegi ko'riniste boladi':

$$\begin{pmatrix} d_{11} & d_{12} & 0 \\ d_{21} & d_{22} & 0 \\ d_{31} & d_{32} & 0 \end{pmatrix}.$$

Meyli $\Delta(x)_1, \Delta(x)_2, \Delta(x)_3$ bul $\Delta(x)$ vektorinin' saykes turde 1, 2 ha'm 3 koordinatalari' bolsi'n. Onda

$$\Delta(x)_i = d_{i1}x_1 + d_{i2}x_2,$$

bul jerde $i \in \{1, 2, 3\}$. 2-lokal differenciallaw bir tekli bolg'anliqtan, bul funkciyalar da bir tekli boladi'. $f_1(x_1, x_2) = \Delta(x)_1$, $f_2(x_1, x_2) = \Delta(x)_2$, $f_3(x_1, x_2) = \Delta(x)_3$ dep aliw jetkilikli bo'ladi'.

Endi (3.7) ten'lik penen an'qlang'an operator'di'n' 2-lokal differenciallaw ekenin kotseteyik. Buni'n' ushi'n (3.3) korinisteg'e $\Delta(x) = D(x)$ ha'm $\Delta(y) = D(y)$ tenliklerin qanaatlandiriwshi differenciallawdi qidiramiz. Onda tomendegi sistemag'a iye bolamiz.

$$\begin{cases} d_{11}x_1 + d_{12}x_2 = f_1(x_1, x_2), \\ d_{11}y_1 + d_{12}y_2 = f_1(y_1, y_2), \\ d_{21}x_1 + d_{22}x_2 = f_2(x_1, x_2), \\ d_{21}y_1 + d_{22}y_2 = f_2(y_1, y_2), \\ d_{31}x_1 + d_{32}x_2 + (d_{11} + d_{22})x_3 = dx_3 + f_3(x_1, x_2), \\ d_{31}y_1 + d_{32}y_2 + (d_{11} + d_{22})y_3 = dy_3 + f_3(y_1, y_2). \end{cases}$$

Yeger $\begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} = 0$ bolsa, bul sistema sheksiz sheshimlarga iye boladi'.

Meyli $\begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} \neq 0$ bolsi'n. Onda joqaridag'i sistemag'i' birinshi ha'm

ekinshi ten'lemelerden d_{11}, d_{12} sanlari', u'shins ha'm tortinshi ten'lemelerden d_{21}, d_{22} sanlari', al besinshi ha'm alti'nshi ten'lemelerden d_{31}, d_{32}, d_{33} sanlari' tabiladi'. Teorema dalillendi.

III bap bo`yi'nsha juwmaq

U'shinshi bap yeki paragraftan ibarat boli'p, bunda Li algebralari'nda differenciallawlar, 2-lokal differenciallawlar ani'qlamalari', ishki differenciallaw ha'm matrica izi qa'siyetleri, mi'sallar keltiriledi. Nilpotent Li algebralardi'n' 2-lokal differenciallawlari' xarakterlengen.

JUWMAQLAW

Jumi'sti'n' birinshi babi' yeki paragraftan ibarat boli'p, bunda shekli o'lsheqli Li algebralari' ani'qlamasi', mi'sallar ha'm ayri'm qa'siyetleri qaraldi'. Birinshi paragrafta Li algebralari' haqqi'nda tiykarg'i' tu'sinikler keltirilgen. Yekinshi paragrafta bolsa shekli o'lsheqli Li algebralari' ani'qlamasi', mi'sallar ha'm ayri'm belgili natiyjeler berildi.

Ekinshi bapta kishi o'lsheqli Li algebralari' qaraldi. Bir ha'm eki o'lsheqli Li algebralari' toliq dizimi keltirildi. Nilpotent Li algebralari'da qaraldi'. Birinshi paragrafta kishi o'lsheqli Li algebralari' qaralg'an. Yekinshi paragrafta nilpotent Li Li algebralari' u'yrenildi.

U'shinshi bap yeki paragraftan ibarat boli'p, bunda Li algebralari'nda differenciallowlar, 2-lokal differenciallowlar ani'qlamalari', ishki differenciallow ha'm matrica izi qa'siyetleri, mi'sallar keltiriledi. Nilpotent Li algebralardi'n' 2-lokal differenciallowlari' xarakterlengen.

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