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BITIRUV

MALAKAVIY ISHI

**Mavzu: Ko'p hadli interpolatsion formulalarni dasturiy
amalga oshirish**

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Kirish

Interpolyatsiyalash masalalarini yechish amali berilgan barcha nuqtalar orqali, ya'ni interpolyatsiyalash tugunlari orqali o'tishi va jadval turida berilgan ba'zi bir $f(x)$ funksiyasini, taqribiy turda almashtiruvchi $L_n(x)$ interpolyatsiyali funksiyasini yasash bilan taminlanadi. Bu funksiya yordamida hohlagan nuqtada dastlabki funksiyaning izlanuvchi qiymatini hisoblab olishga bo'ladi[2,7].

Interpolyatsiyalash masalasiga bog'liq b'lgan uch asosiy muommolari qaraladi:

- 1) $L_n(x)$ interpolyatsiyalash funksiyasini tanlab olish;
- 2) Interpolyatsiyalash amalining $R(x)$ xotaligini baholash;
- 3) Funksiyani tiklashda eng yuqori hisoblash aniqligini taminlash uchun interpolyatsiyalash tugunlarini joylashtirish[4].

Interpolyatsiyalash amalining maxsus usullari funksiyaning izlanuvchi qiymatini bevosita interpolyatsiyon k'op 'ozgaruvchisining tuzmay aniqlashga yordam beradi. Interpolyatsion funkstiyasi sifatida ko'phadlardan paydalanadigan barcha interpolyatsion usillari birdek natijani beradi. Shuning uchun interpolyatsion funkstiyasi sifatida ko'phadini saylap olish masalasi birinchi navbatda yechiladi. Interpolyatsion funksiyasining turini saylap olish masalasi eng ahamiyatli masalasi boladi[5].

Bitiruv malakaviy ishi kirish qismi, besh paragrafdan, xulosa qismidan, foydalanilgan adabiyotlar va qo'shimchalardan iborat.

Bitiruv malakaviy ishining birinshi paragrafida Lagranjning interpolyatsion ko'phadi qaraladi. Bunda $[a,b]$ kesmasida $f(x)$ funksiyasi jadval turida berilgan bo'ladi, va funksiya uchun $\varphi(x)$ interpolyatsiyalovchi funksiyasi bo'lib hisoblanadi, agarda berilgan x_0, x_1, \dots, x_n nuqtalarda uning $\varphi(x_0), \varphi(x_1), \dots, \varphi(x_n)$ qiymatlari berilgan $f(x)$ funksiyaning y_0, y_1, \dots, y_n qiymatlari bilan teng bo'lsa. Bunda interpolyatsion ko'phadning geometrialik ko'rinishi keltirildi. Bunda n -

darajali $L_n(x)$ ko'phadi n -darajali $l_i(x)$ ko'phalardining chiziqli kombinatsiyasi turini yasaydikesmasida.

Bitiruv malakaviy ishining ikkinchi paragrafida interpolatsion formulaning oddiy turga keltirish maqsadida chekli ayirmalar teoriyasining elementlarini qaraldi. Bunda chekli ayirmalarda Nyutonning 1-chi va 2-chi interpolatsion ko'phadlari qaraladi. Shuning bilan birga teng emas oraliqlar uchun Nyutonning interpolatsion formulalari qaraldi. Bunda chekli ayirmalar o'rniga bo'lingan ayirmalardan foydalanildi. Shundan bu paragrafdafunksiyalarning qiymatlari orqali 1-chi tartibli bo'lingan ayirmalar kiritildi, bu ayirmalar asosida 2-chi tartibli bo'lingan ayirmalar kiritildi va k -tartibli ayirmali formulasi kiritildi.

Bitiruv malakaviy ishining u'chinchi paragrafida Ermitning interpolatsion ko'phadi qaraladi. Ayrim hollarda $f(x)$ funktsiyaning berilish sohasining ma'lum bir nuqtalarida bu funktsiyaning o'zining va ushbu funktsiyaning bir nechsha tartibli hosilalarining ko'rsatilgan qiymatlari boyicha bu funktsiyani interpolatsiyalash ko'phadi yaqinlashishini tuzish talab qilinadi. Bu ko'phad Ermitning interpolatsion ko'phadi hisoblanadi.

Bitiruv malakaviy ishining to'rtinchi paragrafida interpolatsion ko'phadlarning yaqinlashuvchilik masalasi qaraladi. Bul paragrafta interpolatsion jarayonlarning yaqinlashuv shartlari haqida ta'riflar ko'rsatigan. Bu yerda ko'phadli interpolatsiyon jarayonlari funktsiyaning xususiyatlariga, shu bilan birga saylab olingan tu'rning turlariga ham bog'liq va ko'phadli interpolatsiyon jarayonlari tu'rda yaqinlashuv bolmaydiganligi haqida Faberning teoremasi keltirildi.

Bitiruv malakaviy ishining besinshi paragrafida sonli misollar qaraladi, bunda jadval turida berilgan $f(x)$ funktsiyasining qiymatleri Lagranjning, Nyutonning va Ermitting inerpolatsion ko'hadlari yordamida tiklandi.

1-§. Funktsiyalarning approksimatsiyalash masalasining qo'yilishi.

Lagranjning interpolyatsion ko'phadi

Matematik analizning ko'pchilik sonli usullarning asosiga bir $f(x)$ funksiyaning unga yaqin bo'lgan boshqa bir $\varphi(x)$ funksiyasi bilan almashtirish usuli olingan. Bu $\varphi(x)$ funksiyasi «yaxshi» xususiyatga ega bo'ladi va bu funksiyasi o'z ustida har turli analitik yoki hisoblash amallarini bajarishga mumkinchilik beradi. Bunday almashtirishlardi *approksimatsiyalash* deb ataymiz.

Funksiyani approksimatsiyalash amali $n \in N_0$ darajali ko'phadi bilan ishga bajariladi deb faraz qilamiz. U holda saylab olingan muvofik kriteriyasiga bog'lanishli va xususiy hollarda $f(x)$ funksiyasi bilan $\varphi(x)$ funksiyasining nuqtalar sonining o'zaro mos kelishiga bog'liq, ularni tugun deb ataymiz, ya'ni $f(x)$ funksiyasi haqida ma'lum axborot bor nuqtalari va balki uning hosilalari haqida ma'lumotlar bor bo'lib va approksimatsiyalashning har turli amallarini ko'rib chiqishga bo'ladi.

1.1. Lagranjning interpolyatsion ko'phadi. Ko'phadli interpolyatsion masalasining ko'yilishi. Mayli x_0, x_1, \dots, x_n nuqtalarda $y = f(x)$ funksiyaning qiymatlari ma'lum bo'lsin, bunda $a \leq x_0 < x_1 < \dots < x_n \leq b$ bo'ladi, ya'ni, $[a, b]$ kesmada jadval shakldagi funksiya berilgan bo'ladi

$$f(x) : \frac{x | x_0 | x_1 | \dots | x_n}{y | y_0 | y_1 | \dots | y_n} . \quad (1.1)$$

$f(x)$ funksiyasi uchun $\varphi(x)$ funksiyasi interpolyatsion funksiya deb ataladi, agarda berilgan interpolyatsion deb atalishi x_0, x_1, \dots, x_n tugunlarda $\varphi(x_0), \varphi(x_1), \dots, \varphi(x_n)$ funksiyaning qiymatlari berilgan $f(x)$ funksiyaning qiymatlari bilan teng bo'lsa, ya'ni y_0, y_1, \dots, y_n qiymatlari mos bo'lsa. Unda interpolyatsion masalasi, aniqrog'i, ko'phadli, algebralik yoki parabo'lik interpolyatsiyalash quyidagicha bo'ladi: (1.1) jadvali ko'rinishida berilgan $f(x)$ funksiyasi uchun quyida berilgan interpolyatsion shartlari o'rinlanadigan

$$l_i(x_j) = \delta_{ij} = \begin{cases} 0, & i \neq j, \\ 1, & i = j, \quad i, j \in \overline{\{0, n\}}. \end{cases} \quad (1.4)$$

Bu hollarda
$$L_n(x) = \sum_{i=0}^n y_i l_i(x),$$

ko'p hadi uchun har bir $x_j, j \in \{0, 1, \dots, n\}$ tug'un uchun (1.4) sababli quyidagi o'zaro nisbati to'g'ri bo'ladi

$$\begin{aligned} L_n(x_j) &= l_0(x_j)y_0 + \dots + l_{j-1}(x_j)y_{j-1} + l_j(x_j)y_j + l_{j+1}(x_j)y_{j+1} + \dots \\ &\quad + l_n(x_j)y_n = 0 + \dots + 0 + y_j + 0 + \dots + 0 = y_j, \end{aligned}$$

ya'ni (2)-chi interpolatsion shartlari o'rinlanadi.

$l_i(x)$ bazis ko'phadlarning shaklini konkretlashtirish uchun, ular (1.4) shartini ta'minlash kerakligini hisobga olamiz. i -chi nuqtasidan boshqa borlik nuqtalarida i -chi ko'phadning nolga teng bo'ladiganligi, $l_i(x)$ ko'phadning quyidagicha turda yozishga mumkin ekanligini ko'rsatadi:

$$l_i(x) = A_i(x - x_0)\dots(x - x_{i-1})(x - x_{i+1})\dots(x - x_n),$$

bu ko'rinishdagi A_i koeffitsienti (1.4) dan $l_i(x_i) = 1$ talabnomadan oson kelib chiqadi. $l_i(x)$ ifodaga $x = x_i$ qiymatini olib qo'yib va natijani birga teng deb

olsak, unda
$$A_i = \frac{1}{(x_i - x_0)\dots(x_i - x_{i-1})(x_i - x_{i+1})\dots(x_i - x_n)}$$

ega bo'lamiz.

Demak, Lagranjning bazis ko'phadlari quyidagicha turga ega bo'ladi

$$l_i(x) = \frac{(x - x_0)(x - x_1)\dots(x - x_{i-1})(x - x_{i+1})(x - x_n)}{(x_i - x_0)(x_i - x_1)\dots(x_i - x_{i-1})(x_i - x_{i+1})(x_i - x_n)},$$

va izlanuvchi Lagranjning interpolatsion ko'phadi bo'lib quyidagi hisoblanadi.

$$L_n(x) = \sum_{i=0}^n \frac{(x - x_0)(x - x_1)\dots(x - x_{i-1})(x - x_{i+1})\dots(x - x_n)}{(x_i - x_0)(x_i - x_1)\dots(x_i - x_{i-1})(x_i - x_{i+1})\dots(x_i - x_n)} y_i \quad (1.5)$$

Misol uchun birinchi va ikkinchi darajali Lagranjning interpolatsion ko'phadlarini keltiramiz.

$n=1$ da interpolatsion $y = f(x)$ funksiyasi haqidagi axborot (x_0, y_0) va (x_1, y_1) ikki nuqtasida to'plangan bo'ladi. Lagranj ko'phadni bu holda yasash uchun birinchi darajali ikki bazis ko'p hadlari ($l_0(x)$ va $l_1(x)$) yordamida tuziladi va u quyidagicha bo'ladi:

$$L_1(x) = \frac{x - x_1}{x_0 - x_1} y_0 + \frac{x - x_0}{x_1 - x_0} y_1 \quad . \quad (1.6)$$

$n=2$ bo'lganda uch nuqtali jadvali bo'yicha

$$f(x) : \frac{x | x_0 | x_1 | x_2}{y | y_0 | y_1 | y_2},$$

$l_0(x), l_1(x)$ va $l_2(x)$ uch bazis ko'phadlarini paydo qilish mumkin va mos keluvchi ikkinchi darajali Lagranjning interpolatsion ko'phadi quyidagicha bo'ladi:

$$L_n(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1 + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2 \quad (1.7)$$

Taqribiy turdagi tengliklari

$$f(x) \approx l_1(x) \text{ va } f(x) \approx l_2(x)$$

mos ravishda chiziqli va kvadratlik interpolatsion formulalari deb ataladi.

Mayli berilgan $f(x)$ funksiyasi uchun $L_n(x)$ ko'phadi tuzilgan bo'lsin, unda $[a, b] \supseteq [x_0, x_n]$ kesmasida $f(x)$ funksiyasini taxmin turda ko'rsatish uchun quyida berilgan interpolatsion formulasidan foydalaniladi

$$f(x) \approx L_n(x) \quad (1.8)$$

Bu yerda taqribiy tenglikning xatoligi qanday bo'ladi degan savol paydo bo'ladi? Boshqacha aytganda, $f(x)$ interpolatsion funksiyaning qiymatlari va tugun nuqtalari bilan bir-biriga mos bo'lmagan $[a, b]$ kesmaning nuqtalarida $L_n(x)$ interpolatsion ko'p hadning mos qiymatlari bilan farqi qanchali bo'ladi.

1.3. Interpolatsion xatoliklarini baholash: $f(x)$ funksiyasi $(n + 1)$ -marta differensiallanuvchi deb faraz qilaylik, agar qoldik qismi haqida bilimga ega

bo'lsak, unda uning $L_n(x)$ interpolyatsion ko'phadi orqali $f(x)$ funksiyaning aniq ko'rinishini yozishga imkoniyat beradi:

$$f(x) = L_n(x) + \frac{f^{(n+1)}(\xi)}{(n+1)!} \prod_{n+1}(x), \quad (1.9)$$

bunda $\xi \in (a, b)$ interpolyatsion oraliqning ba'zi bir nuqtasi va

$\prod_{n+1}(x) = \prod_{i=0}^n (x - x_i) = (x - x_0)(x - x_1)\dots(x - x_n)$ - bu x_0, x_1, \dots, x_n nuqtalar

orqali aniqlangan $(n+1)$ -darajali ko'phadi.

Agarda

$$M_{n+1} = \max_{x \in [a, b]} |f^{(n+1)}(\xi)| \quad (1.10)$$

qiymati aniq bo'lsa, unda (1.8) interpolyatsion formulasining absolyut xatoligini har qanday $\tilde{x} \in [a, b]$ nuqtada quyidagi tengsizlikdan foydalanib baholash mumkin bo'ladi:

$$|R_n(\tilde{x})| = |f(\tilde{x}) - L_n(\tilde{x})| \leq \frac{M_{n+1}}{(n+1)!} |\prod_{n+1}(\tilde{x})| \quad (1.11)$$

$[a, b]$ kesmasida maksimal interpolyatsion xatoligi quyidagi qiymati bilan baholanadi:

$$\max_{x \in [a, b]} |R_n(x)| \leq \frac{M_{n+1}}{(n+1)!} |\prod_{n+1}(x)| \quad (1.12)$$

1-misol: $[0, \pi/2]$ kesmada $y = \sin(x)$ funksiyasining kvadratlik interpolyatsion masalani $\sin(0) = 0$, $\sin(\pi/4) = \sqrt{2}/2$, $\sin(\pi/2) = 1$ nuqtalari bo'yicha karaymiz.

Lagranjning ko'phadini yasash uchun uch bazis ko'phadlari kerak bo'ladi:

$$l_0 = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)},$$

$$l_1 = \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)},$$

$$l_2 = \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}$$

(1.7) formulasiga muvofiq Lagranjning ko'phadi quyidagicha bo'ladi:

$$L_2(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} f(x_0) + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} f(x_1) + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} f(x_2).$$

Demak, bizning hol uchun

$$L_2(x) = \frac{(x - \pi/4)(x - \pi/2)}{(0 - \pi/4)(0 - \pi/2)} \cdot 0 + \frac{(x - 0)(x - \pi/2)}{(\pi/4 - 0)(\pi/4 - \pi/2)} \cdot \sqrt{2}/2 + \frac{(x - 0)(x - \pi/4)}{(\pi/2 - 0)(\pi/2 - \pi/4)} \cdot 1,$$

yoki qayta o'zgartirilgandan so'ng

$$L_2(x) = \frac{8}{\pi^2} \cdot x \left[(1 - \sqrt{2})x + (\sqrt{2}/2 - 1/4)\pi \right]. \quad (1.13)$$

Bu hol uchun qoldik hadni (11) formulasi bo'yicha olamiz, bunda $n = 2$ da $(\sin(x))^m = -\cos(x)$ va $\prod_3(x) = x * (x - \pi/4)(x - \pi/2)$ bo'ladiganligini hisobga olamiz.

Natijada ega bo'lamiz:

$$R_2(x) = \sin(x) - L_2(x) = \frac{-\cos \xi}{3!} x(x - \pi/4)(x - \pi/2).$$

$\xi \in (0, \pi/2)$ nuqtasi nomalum bo'gani sababli, unda $M_3 = \max_{x \in [0, \pi/2]} |-\cos(x)| = 1$

deb hisoblab, faqatgina $|R_2(x)|$ baholashga bo'ladi. Berilgan kesmaning ikki nuqtasida $x_{1,2} = \pi/12(3 + \sqrt{3})$ amalga oshiruvchi va 0,568 dan oshmaydigan $|\prod_3(x)|$ ning maksimal qiymatini topib, interpalyatsion oraliqda berilgan sinus funksiyaning (13) parabo'laning mumkin bo'gan qiymati yuqoridan baholaymiz

$$\max_{x \in [0, \pi/2]} |\sin(x) - L_2(x)| \leq \frac{1}{3!} \max_{x \in [0, \pi/2]} |L_2(x)| < \frac{0,568}{6} \approx 0,095.$$

Unda kelip chiqqan (13) interpolatsion ko'phadiga $\bar{x} = \pi/6$ nuqtani olib qo'yamiz. Natijada, $L_2(\pi/6) = \frac{4\sqrt{2}-1}{9} \approx 0.517$ taxminan qiymatiga ega bo'lamiz va u $\sin(\pi/6) = 0.5$ ning qiymatidan 0,17 qiymatga faqr qiladi, bu (12) formulasi bo'yicha bo'lishi mumkun bo'lsa. Bunday baho (10) formulasi bo'yicha ruxsat etiladi:

$$\left| \sin\left(\frac{\pi}{6}\right) - L_2\left(\frac{\pi}{6}\right) \right| \leq \frac{M_3}{3!} \left| \prod_3\left(\frac{\pi}{6}\right) \right| = \frac{1}{6} \left(\frac{\pi}{6}\right)^3 \approx 0,075.$$

2-§. Nyutonning interpolatsion ko'phadi

2.1. Chekli ayirmalar. Interpolatsion formulasini Teylor formulasi turga o'xshash ancha oddiy ko'rinishiga keltiramiz. Agarda, (1.5) Lagranjning interpolatsion ko'phadida barcha qo'shiluvchilar bir turda bo'lsa va natijani keltirib chiqarishga bir xil vazifa bajaradigan bo'lsa, unda interpolatsion ko'phadning shunday bir ko'rinishga ega bo'lish kerak bo'ladi, bunda Teylor ko'phadiga o'xshash barcha qo'shiluvchilar ularning qiymati kamayish tartibida joylashgan bo'lsin. Bunda uning boshidan uzoqda joylashgan a'zosini olib tashlab yoki qo'shib ancha oddiy turda, uning darajalarini o'zgartirish mumkin.

Dastlab Interpolatsion masalasining qo'yilishining xususiy holini karaymiz. Mayli, tengday oraliqda joylashgan x_0, x_1, \dots, x_n nuqtalar sistemasida interpolatsion $y = f(x)$ funksiyasining y_0, y_1, \dots, y_n qiymatlari berilgan bo'lsin, ya'ni bu turning hohlagan x_i tugunini quyidagicha ko'rinishda yozib ko'rsatish mumkin:

$$x_i = x_0 + ih, i = \overline{0, n},$$

bunda $h > 0$ - turning qadami deb ataladigan bazi – bir o'zgarmas qiymat.

Interpolatsion formulalar ko'rmasdan oldin chekli ayirmalar teoryasining elementlarini qarab chiqamiz.

y_0, y_1, \dots, y_n - $n + 1$ chekli sonlar ketma-ketligini har bir keyingi hadidan oldingi hadni ayirib olib n sonli birinchi tartibli chekli ayirmalarini paydo etamiz

$$\Delta y_0 = y_1 - y_0, \Delta y_1 = y_2 - y_1, \dots, \Delta y_{n-1} = y_n - y_{n-1},$$

yoki berilgan jadval funksiyaning birinchi n sonli ayirmalariga ega bo'lamiz. Ulardan, o'z navbatda xuddi shunday yo'l bilan soni $n - 1$ bo'lgan ikkinchi tartibli chekli ayirmalarini yozishga bo'ladi:

$$\Delta^2 y_0 = \Delta y_1 - \Delta y_0, \quad \Delta^2 y_1 = \Delta y_2 - \Delta y_1, \quad \dots, \quad \Delta^2 y_{n-2} = \Delta y_{n-1} - \Delta y_{n-2}$$

Chekli ayirmalarni tuzishning bu jarayonni davom ettiramiz va u natijada bir rekurent formulasi bilan yoziladi. Bu formulasi k tartibli $\Delta^k y_i$ chekli ayirmalarni ($k - 1$) tartibli ayirmalar orqali ifodalaydi:

$$\Delta^k y_i = \Delta^{k-1} y_{i+1} - \Delta^{k-1} y_i, \quad k = 1, 2, \dots, n$$

bunda $\Delta^0 y_i = y_i$ bo'ladi.

Chekli ayirmalari va hosilalar o'rtasida to'g'ri bog'liqlik bor bo'lib hisoblanadi. Aytilgan vaxtimizda, agarda quyidagi munosabatlarni hisobga olsak:

$$\lim_{h \rightarrow 0} \frac{y_{i+1} - y_i}{h} = \lim_{h \rightarrow 0} \frac{f(x_i + h) - f(x_i)}{h} = f'(x_i),$$

unda kichik h da quyidagi taqribiy tenglik o'rinli bo'ladi deb aytish mumkin:

$$\Delta y_i \approx f'(x_i)h,$$

ya'ni birinchi tartibli ayirmalar $f(x)$ funksiya-sining birinchi hosilani sifatlarini, sababi ular bu funksiya'ning qiymatlari bo'yicha tuzilgan. Shu aytilganlardan foydalanib, ikkinchi tartibli chekli ayirmalari uchun:

$$\frac{\Delta^2 y_i}{h^2} = \frac{\Delta y_{i+1} - \Delta y_i}{h^2} = \frac{\frac{y_{i+2} - y_{i+1}}{h} - \frac{y_{i+1} - y_i}{h}}{h} \approx \frac{f'(x_{i+1}) - f'(x_i)}{h} \approx f''(x_i)$$

ya'ni $\Delta^2 y_i \approx f''(x_i)h^2$ va umuman:

$$\Delta^k y_i \approx f^k(x_i)h^k. \quad (2.1)$$

Demak, chekli ayirmalar munosabatlarini hosilalarning bazi – bir o'xshashligi deb qarash mumkin.

x_0, x_1, \dots, x_n nuqtalarida y_0, y_1, \dots, y_n qiymatlari jadval turida berilgan $y = f(x)$ funksiyasi uchun, har turli tartibdagi chekli ayirmalarni, nuqtalari bilan undagi funksiyaning qiymatlarini bir umumiy jadvalga joylashtirish qulay bo'ladi, bunda $x_i = x_0 + ih$. Bunday umumiy jadvalini **chekli ayirmalar jadvalsi** deb ataydi.

1 – Jadval

| | | | | | |
|-------|-------|--------------|----------------|----------------|----------------|
| x_0 | y_0 | | | | |
| | | Δy_0 | | | |
| x_1 | y_1 | | $\Delta^2 y_0$ | | |
| | | Δy_1 | | $\Delta^3 y_0$ | |
| x_2 | y_2 | | $\Delta^2 y_1$ | | $\Delta^4 y_0$ |
| | | Δy_2 | | $\Delta^3 y_1$ | |
| x_3 | y_3 | | $\Delta^2 y_2$ | | |
| | | Δy_3 | | | |
| x_4 | y_4 | | | | |

2.2. Chekli ayirmali interpolatsion formulalari: Mayli bir biriga tengday masofada joylashgan $x_i = x_0 + ih$, $i = \overline{0, n}$ to'rda $y = f(x)$ funksiyasi berilgan bo'lsin va bu funksiyasi uchun 1-jadvalga o'xshash chekli ayirmalar munosabatlari tuzilgan bo'lsin.

Lagranj interpolatsion formulasini belgilangan modifikatsiyasiga mos $P_n(x)$ interpolatsion ko'phadini quyidagicha formada yashaladi:

$$P_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \dots + a_n(x - x_0)(x - x_1)\dots(x - x_{n-1}) \quad (2.2)$$

Uning $n + 1$ koeffitsentlarin a_0, a_1, \dots, a_n ni ketma – ket quyidagi interpoliyatsion ko'phadidan topamiz:

$$P_n(x) = y_i, i = 0, 1, 2, \dots, n.$$

Jumladan, $i = 0$ deb hisoblab, ya'ni $x = x_0$ deb, (2.2) formulanida $P_n(x_0) = a_0$ ega bo'lamiz, va interpoliyatsion shartlari bo'yicha $P_n(x_0) = y_0$. Shuning uchun $a_0 = y_0$ bo'ladi.

Bundan so'ng, $i = 1$ da shunga o'xshash bo'gan quyidagi tengligiga ega bo'lamiz:

$$a_0 + a_1(x_1 - x_0) = y_1,$$

bu tenglamasiga hisoblanganda $a_0 = y_0$ olib qo'yamiz.

Bu tengligini a_1 qarata hal etamiz va chekli ayirmalar belgilashlarni foydalanib, quyidagiga ega bo'lamiz:

$$a_1 = \frac{y_1 - y_0}{x_1 - x_0} = \frac{\Delta y_0}{h}.$$

$i = 2$ keyingi qadami quyidagi tenglamani beradi:

$$a_0 + a_1(x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1) = y_2,$$

$$y_0 + \frac{\Delta y_0}{h} * 2h + a_2 * 2h^2 = y_2, \Leftrightarrow$$

$$a_2 = \frac{y_2 - 2y_1 + y_0}{2!h^2} = \frac{\Delta^2 y_0}{2!h^2}.$$

To'liq induksiyasi bilan quyidagi ifodaning to'g'riligini ko'rsatish mumkin:

$$a_k = \frac{\Delta^k y_0}{k!h^k}, \quad \forall k \in \{0, 1, 2, \dots, n\}. \quad (2.3)$$

Hisoblanib topilgan a_0, a_1, \dots, a_n koeffitsentlarning qiymatlarini (2.2) qo'yib, quyidagi ko'phadiga ega bo'lamiz:

$$P_n(x) = y_0 + \frac{\Delta y_0}{h}(x - x_0) + \frac{\Delta^2 y_0}{2!h^2}(x - x_0)(x - x_1) + \dots + \frac{\Delta^n y_0}{n!h^n}(x - x_0)(x - x_1)\dots(x - x_{n-1}) \quad (2.4)$$

(2.4) ko'phadini Nyutonning birinchi interpolyatsion ko'phadi deb ataydi.

2.3. Teng emas oraliqlar uchun Nyutonning interpolyatsion ko'phadi.

Umumiy hol uchun, ya'ni $[a, b]$ oralig'ida bir birga teng emas, biroq tartiblashgan x_0, x_1, \dots, x_n nuqtalarida o'xshash interpolyatsion formulalarini tuzish uchun chekli ayirmalarning o'rniga bo'lingan ayirmalardan yoki boshqacha aytganda, ayirma munosabatlaridan foydalaniladi.

Dastlab funksiya'ning $f(x_0), f(x_1), \dots, f(x_n)$ qiymatlari orqali *birinchi tartibli bo'lingan ayirmalarin* aniqlaydi:

$$f(x_0; x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$f(x_1; x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

.....

$$f(x_{n-1}; x_n) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}.$$

Bu ayirmalarga *ikkinchi tartibli bo'lingan ayirmalar* asoslangan

$$f(x_0; x_1; \dots; x_2) = \frac{f(x_1; x_2) - f(x_0; x_1)}{x_2 - x_0}$$

.....

$$f(x_{n-2}; x_{n-1}; \dots; x_n) = \frac{f(x_{n-1}; x_n) - f(x_{n-2}; x_{n-1})}{x_n - x_{n-2}},$$

va shunday etib davom etamiz. Demak, agarda k - tartibli $f(x_{i-1}; x_i; \dots; x_{i+k})$ bo'lingan munosabatlari aniqlangan bo'lsa, unda keyingi $(k+1)$ chi tartibli bo'lingan munosabatlari quyidagi tengligi bilan aniqlanadi:

$$f(x_{i-1}; x_i; \dots; x_{i+k}) = \frac{f(x_i; x_{i+1}; \dots; x_{i+k}) - f(x_{i-1}; x_i; \dots; x_{i+k-1})}{x_{i+k} - x_{i-1}}. \quad (2.5)$$

Chekli ayirmalar holga o'xshash, bo'lingan ayirmalarni olish operatsiyasi additive va bir turli, ya'ni chiziqli bo'ladiganligi oson teksherishga bo'ladi. Bundan boshqa, bo'lingan ayirmalar $-o'z$ argumentlaridan simmetrik funksiyasi bo'ladi, shuning uchun bo'lingan ayirmalarning argumentlaring o'rnini almashtirish mumkin. Bundan, *teng emas tugunlari uchun Nyutonning interpolyatsion formulasiga ega bo'lamiz:*

$$f(x) \approx P_n(x) = y_0 + f(x_0; x_1)(x - x_0) + \dots + f(x_0; x_1; x_2)(x - x_0)(x - x_1) + \\ + f(x_0; x_1; x_2; \dots; x_n)(x - x_0)(x - x_1) \dots (x - x_{n-1}), \quad (2.6)$$

Ixtiyoriy bir, belgilangan x nuqta uchun aniq tengligini olishimiz mumkin:

$$f(x) = P_n(x) + f(x, x_0; \dots; x_n)(x - x_0) \dots (x - x_{n-1})(x - x_n),$$

bu tenglikdagi ikkinchi yig'indi qoldik had sifatida qarash mumkin, ya'ni:

$$R_n(x) = f(x) - P_n(x) = f(x; x_0; \dots; x_n) \prod_{n+1}(x), \quad (2.7)$$

bunda $\prod_{n+1}(x)$ kiritilgan ko'phadi.

3-§. Ermitning interpolyatsion ko'phadi.

1.Karrali tugunlariga ega interpolyatsiya. Bazi-bir holda $f(x)$ funksiyasining berilgan sohasining bazi-bir nuqtalarida bu funksiyaning o'zining va uning bir necha tartibli hosilalarning berilgan qiymatlari bo'yicha shu funksiyani interpolyatsiyalavchi ko'phadli yaqinlashuvini tuzish talab etiladi.

Mayli bir-biri mos kelmaydigan $x_i \in [a, b]$, $i = \overline{0, m}$, tugunlarida funksiyaning $f(x_i)$ qiymatlari va uning $(k_i - 1)$ - tartibgacha hosilalarning $f^s(x_i)$, $s = 1, 2, \dots, k_i - 1$ qiymatlari berilgan bo'lsin. Demak, har bir $x_i \in [a, b]$, $i = \overline{0, m}$ nuqtasida $f(x_i), f'(x_i), \dots, f^{k_i-1}(x_i)$ qiymatlari berilgan. Unda, $f(x)$ funksiyasi haqidagi berilgan ma'lumotlarning soni $k_0 + k_1 + \dots + k_m$ yig'indisiga teng bo'ladi. Shunda darajasi $n = k_0 + k_1 + \dots + k_m - 1$ teng bo'lgan va

$H_n(x) \equiv 0$ bo'lgandagina mumkin bo'ladi. Demak, bu ko'phadning koeffitsientlari nolga teng bo'ladi va tenglamalarning (3.3) bir jinsli sistemasi faqatgina $a_0 = a_1 = \dots = a_n = 0$ yechimiga ega bo'ladi. Shuning uchun, (3.2) bir jinsli emas sistemasi, uning yon qismi hohlagan sonlar bo'lganda faqat bir yechimga ega bo'ladi.

$f^{(s)}(x_i)$, $i=0,1,\dots,m$, $s=0,1,\dots,k_i-1$ qiymatlari (3.2) sistemaning faqat uning yon qismida joylashgandan $H_n(x)$ ko'phadining a_j , $j=\overline{0,n}$ koeffitsientlari $f^{(s)}(x_i)$ qiymatlari bilan chiziqli ifodalaydi. Shuning uchun bu ko'phadi

$$H_n(x) = \sum_{i=0}^m \sum_{s=0}^{k_i-1} C_{is} f^{(s)}(x_i)$$

chiziqli birikmasi turida ko'rsatishga bo'ladi, bunda C_{is} - n darajali ko'phadlilar. Bu ko'phadlilarni keltirib chiqarish murakkab bolg'ani uchun, Ermitning interpolyatsion ko'phadining umumiy formulasini keltiramiz:

$$H_n(x) = \sum_{i=0}^m \sum_{s=0}^{k_i-1} \sum_{k=0}^{k_i-s-1} f^{(s)}(x_i) \frac{1}{k!} \cdot \frac{1}{s!} (x-x_i)^{s+k} \times \prod_{n \neq i} (x-x_i)^{k_i} \left\{ \frac{d^k}{dx^k} \prod_{r \neq i} (x-x_i)^{-k_i} \Big|_{x=x_i} \right\} \quad (3.4)$$

Xususiyl holda $n=3$ bo'lganda bu ko'phadli quyidagicha bo'ladi:

$$H_3(x) = y_0 + (x-x_0)y_0 + (x-x_0)^2 \left[\frac{y_1 - y_0}{(x_1 - x_0)^2} - \frac{y_0}{x_1 - x_0} \right] + \\ + (x-x_0)^2 (x-x_1) \left[\frac{y_1 + y_0}{(x_1 - x_0)^2} - \frac{2(y_1 - y_0)}{(x_1 - x_0)^3} \right] \quad (3.5)$$

Bo'lingan ayirmalar yordamida, karrali tugunlariga ega torda tor funksiyasi bo'yicha Nyutonning interpolyatsion ko'phadini formal turda tuzishga harakat etamiz. Oldinda berilgan bo'lingan ayirmalarning to'gridan-to'gri berilgan aniqlamasi, argumentning takrorlovchi qiymatlari holi uchun qo'llash mumkin

emas, sababi bunda zarur turda $\frac{0}{0}$ munosabatlari uchrashadi. Shuning uchun bu yerda qo'shimcha anilamalar zarur bo'ladi. Mayli, $x_i^{(j)} \rightarrow x_i$ bo'lganda va barcha $x_i^{(j)}$ lar har xil bo'lganda quyidagicha bo'ladi deb faraz qilamiz:

$$\begin{aligned} f(\underbrace{x_0; x_0; \dots; x_0}_{k_0 \text{ pa3}}; \underbrace{x_1; x_1; \dots; x_1}_{k_1 \text{ pa3}}; \dots; \underbrace{x_n; x_n; \dots; x_n}_{k_n \text{ pa3}}) = \\ = \lim f(x_0; x_0^1; \dots; x_0^{(k_0-1)}; x_1; x_1^1; \dots; x_1^{(k_1-1)}; \\ x_n; x_n^1; \dots; x_n^{(k_n-1)}). \end{aligned} \quad (3.6)$$

Bunday limitning bo'lishi uchun $f(x)$ funksiyasiga qo'shimcha shartlari kerak bo'ladi. Masalan, biz berilgan tarifga muvofiq:

$$f(x_0; x_1) = \lim_{x_1 \rightarrow x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0} = f'(x_0),$$

va bundan $f'(x_0)$ hosilasining bor boladiganligini taxmin qilamiz. Avvalgi vaqtda $f(x)$ funksiyasining barcha bizga uchrashadigan hosilalari bor va ular uzluksiz bo'ladi deb faraz qilamiz.

3.2. Bo'lingan ayirmalariga ega Nyutonning umumiylashgan formulasi.

Karrali tugunlarlar holida, teng emas oraliqlar uchun Nyutonning interpolyatsion formulasini umumiylashtirishga o'tamiz. Mayli (3.1) interpolyatsion talablari qo'yilgan bo'lsin, bunda y_i qiymatlari $[a, b]$ kesmada aniqlangan va uzluksiz, bazi bir $f(x)$ funksiyaning shu kesmada joylashgan x_i tugunlarlaridagi qiymatlari bo'lib, $y_i^{(j)}$ - bu x_i tugunlardagi $f(x)$ funksiyasidan olingan j -chi hosilasining qiymati. Barcha kerakli hosilalar uzluksiz bo'ladi deb faraz qilamiz. $[a, b]$ kesmada x_0, x_1, \dots, x_m tugunlarlari bilan birgalikda,

$x_0^1, \dots, x_0^{(a_0-1)}, \dots, x_1^1, \dots, x_1^{(a_1-1)}, \dots, x_n^1, \dots, x_n^{(a_n-1)}$ tugunlarlarini ham karaymiz. Bu tugunlarlarning ichida bir birga teng tugunlarlar yo'q etib saylab olingan.

Unda

$$\begin{aligned}
f(x) &= f(x_0) + (x - x_0)f(x_0; x_0^1) + (x - x_0)(x - x_0^1)f(x_0; x_0^1; x_0^2) + \dots \\
&+ (x - x_0)(x - x_0^1)\dots(x - x_0^{\alpha_0-1})f(x_0; x_0^1; \dots; x_0^{\alpha_0-1}; x_1) + \\
&+ (x - x_0)\dots(x - x_0^{\alpha_0-1})(x - x_1)f(x_0; x_0^1; \dots; x_0^{\alpha_0-1}; x_1; x_1^1) + \\
&\dots + (x - x_0)(x - x_0^1)\dots(x - x_n^{\alpha_n-2})f(x_0; x_0^1; \dots; x_n^{\alpha_n-2}) + \\
&+ (x - x_0)(x - x_0^1)\dots(x - x_n^{\alpha_n-1})f(x; x_0; x_0^1; \dots; x_n^{\alpha_n-1}).
\end{aligned}
\tag{3.7}$$

x_1, \dots, x_n nuqtalarida misol sifatida interpolyatsion ko'phadini karaymiz, bunda bir bo'lagini, ya'ni p nuqtalari x_j nuqtaning atrofida quyidagi formula bo'yicha yasaladi:

$$x_j^k = x_j + (k - 1)\varepsilon, \quad k = 1, 2, \dots, p \quad \varepsilon \leq \varepsilon_0,$$

bunda $k = 1, 2, \dots, p$ da $x_j^k \xrightarrow{\varepsilon \rightarrow 0} x_j$ bo'ladi.

Bo'lingan ayirmalar jadvalini tuzamiz, bunda bir guruhiga tegishli bo'lgan nuqtalar, $\varepsilon \rightarrow 0$ intilganda bir biriga teng boladigan nuqtalarni qarab joylashtirib va $x_j^k \neq x_l, l \neq j$. Misoli, uch nuqta bir guruxga tegishli bo'lgan besh nuqtasi uchun quyidagi 2-jadvaliga ega bo'lamiz.

2-jadvali.

| | | | | |
|-------|------------|------------------------|-------------------------------|------------------------------------|
| x_1 | $f(x_1)$ | | | |
| | | $f(x_1; x_2^1)$ | | |
| x_1 | $f(x_2^1)$ | $f(x_1; x_2^1; x_2^2)$ | | |
| | | | $f(x_1; x_2^1; x_2^2; x_2^3)$ | |
| x_2 | $f(x_2^2)$ | | | $f(x_1; x_2^1; x_2^2; x_2^3; x_3)$ |
| | | | $f(x_2^1; x_2^2; x_2^3)$ | |
| x_3 | $f(x_2^3)$ | $f(x_2^2; x_2^3; x_3)$ | | |
| | | $f(x_2^3; x_3)$ | | |
| x_4 | $f(x_3)$ | | | |

Endi (3.5) tengliginida $x_i^j \rightarrow x_i$ intilganda limitga o'tamiz. Natijada ega bo'lamiz

$$f(x) = f(x_0) + (x - x_0)f(x_0; x_0) + (x - x_0)^2 f(x_0; x_0; x_0) + \dots + (x - x_0)^{(a_0-1)} \underbrace{f(x_0; x_0; \dots; x_0)}_{a_0pa3} +$$

$$\begin{aligned} & (x - x_0)^{(a_0)} (x - x_1)^{a_1} \dots (x - x_n)^{(a_n-1)} \underbrace{f(x_0; x_0; \dots; x_0)}_{a_0pa3} \underbrace{x_1; x_1; \dots; x_1}_{a_1pa3} \dots \underbrace{x_n; x_n; \dots; x_n}_{a_npa3} \\ & (x - x_0)^{(a_0)} \underbrace{f(x_0; x_0; \dots; x_0; x_1)}_{a_0pa3} + (x - x_0)^{(a_0)} (x - x_1) \underbrace{f(x_0; x_0; \dots; x_0; x_1; x_1)}_{a_0pa3} + \dots + \\ & (x - x_0)^{(a_0)} (x - x_1)^{a_1} \dots (x - x_n)^{(a_n-1)} \underbrace{f(x_0; x_0; \dots; x_0)}_{a_0pa3} \underbrace{x_1; x_1; \dots; x_1}_{a_1pa3} \dots \underbrace{x_n; x_n; \dots; x_n}_{a_npa3} + \\ & (x - x_0)^{(a_0)} (x - x_1)^{a_1} \dots (x - x_n)^{(a_n)} \underbrace{f(x; x_0; x_0; \dots; x_0)}_{a_0pa3} \underbrace{x_1; x_1; \dots; x_1}_{a_1pa3} \dots \underbrace{x_n; x_n; \dots; x_n}_{a_npa3}. \end{aligned}$$

Birinchi $m + 1$ hadlari interpolatsion ko'phadi uchun ifodasin bizga beradi, eng keyingi hadi qoldiq hadi bo'ladi. Aniqlangan interpolatsion ko'phadi qo'yilgan (3.1) shartlarini qanoatlantiradiganligini ko'rsatishga bo'ladi. Shunga bogliq bu ko'phadini $H_m(x)$ deb belgilaymiz.

4-§. Interpolatsion jarayonining yaqinlashuvchiligi

1-ta'rif. $\Omega_n = \{x_i \in [a, b], i = \overline{0, n}\}$ turlar sistemasida interpolatsion jarayoni $f(x)$ funksiyasi uchun $x \in [a, b]$ nuqtaga yaqinlashuvchi bo'ladi, agarda quyida berilgan sharti bajarilsa:

$$\lim_{n \rightarrow \infty} |P_n(x) - f(x)| = 0.$$

2-ta'rif. $\Omega_n = \{x_i \in [a, b], i = \overline{0, n}\}$ turlar sistemasida interpolatsion jarayoni $f(x)$ funksiyasi uchun $[a, b]$ intervalida yaqinlashuvchi bo'ladi, agarda bu oraliqning har bir nuqtasida yaqinlashuvchi bo'lsa.

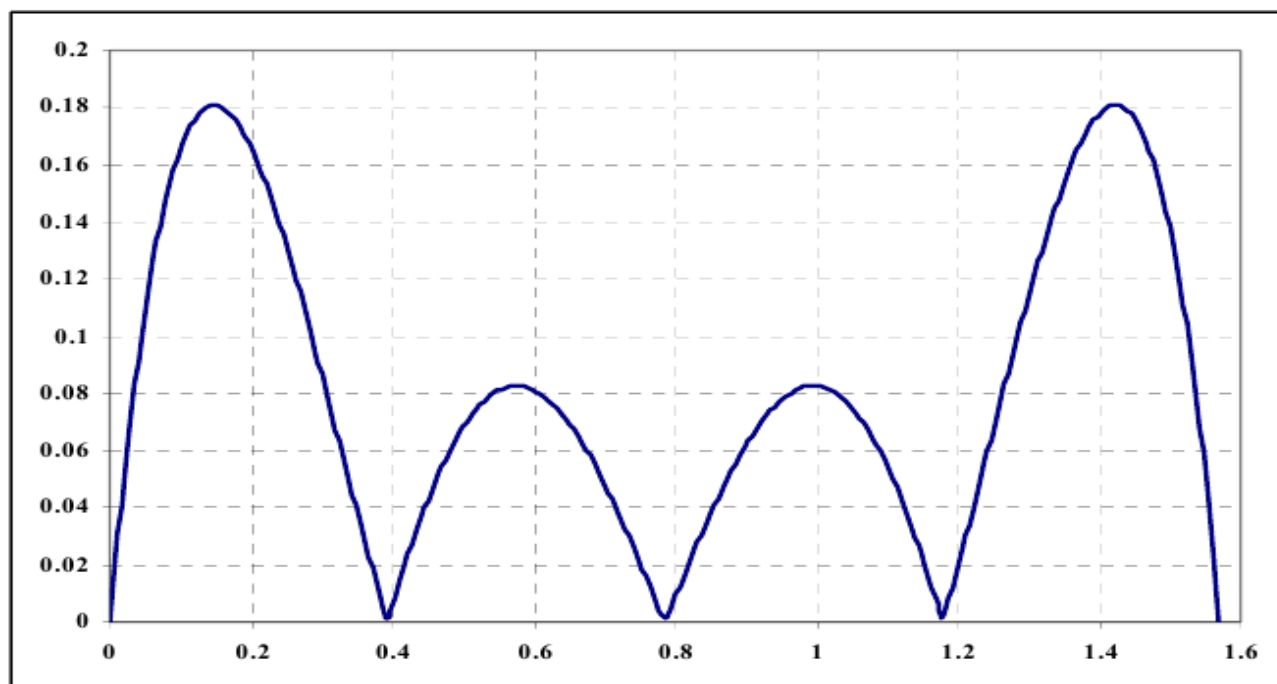
3-ta'rif. Interpolatsion jarayonsi $\Omega_n = \{x_i \in [a, b], i = \overline{0, n}\}$ turlar sistemasida $f(x)$ funksiyasi uchun oralig'ida yaqinlashuvchiligi bo'ladi, agarda quyida berilgan sharti bajarilsa:.

$$\lim_{n \rightarrow \infty} \left(\max_{x \in [a,b]} |P_n(x) - f(x)| \right) = 0.$$

Interpolyatsion formulaning qoldiq hadni baholash uchun (1.11) ifodani tahlil qilganda, interpolyatsiyalavchi funksiyasining $(n+1)$ hosilasi cheklangan shartida, interpolyatsiyalavchi ko'phadlining yetarli katta tartibida yaqinlashuv xatoligi mumkin bo'lganicha kichik bo'ladi degan xulosa chiqarildi. Ammo, yuqori darajali hosilalarning bunday harakati faqatgina butun funksiyalar uchun harakterli bo'lib va yuqori darajali hosilalari $n!$ darajada bo'lib ko'payishi ancha ko'p bo'ladi. Demak, hojlagan funksiya uchun ko'phadlili interpolyatsiyalash jarayonning yaqinlashuvchi boladiganligiga kafolat berishga bo'lmaydi.

Birinchi martaba ko'phadli interpolyatsiyalash jarayonining yaqinlashuvchili bo'lmaydiganligini Runqa (1901) ko'rsatib beradi. Runqa teng o'lchamli turlar sistemasidan foydalanib $[-1,1]$ intervalida $f(x) = \frac{1}{\sqrt{1+25x^2}}$ funksiya'ni interpolyatsiyalashda va $n > 9$ uchun yaqinlashuvning teng o'lchamli xatoligi cheklanmagan turda o'sishini ko'rsatib beradi.

Yuqorida qaralgan 1 va 2- misollar uchun 1- 3- rasmlarida $P_n(x) = \sin 4x$ funksiya'ning mos turlari $n = 5$, $n = 20$ va $n = 60$ ko'rsatilgan.



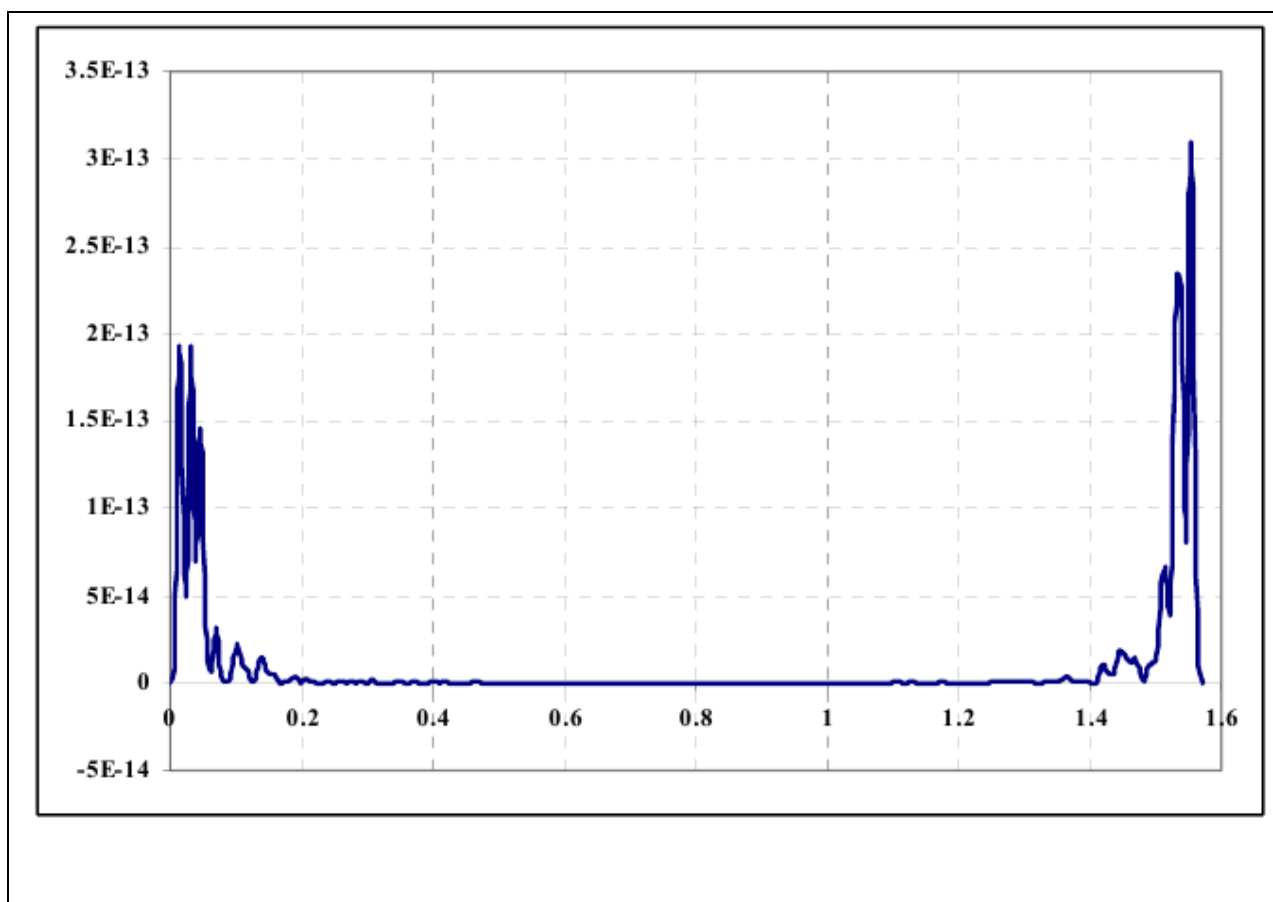
1-rasm. $y = \sin 4x$ funksiya'ning interpolatsion xatoligi, $n = 5$

Interpolatsion jarayonlarning keyingi izlanishlari soni ko'rsatdi, bunda funksiya'ning xossalari bilan birgalikda turlar turida ahamiyatli bo'lib hisoblanadi. Faber tomonidan quyidagi teoremasi isbotlandi.

Faber teoremasi. Ω_n turlar ketma-ketligi qanday bo'lishiga qaramasdan, barcha vaqtda uzluksiz funksiyasi topilib, bu funksiya uchun ko'phadli interpolatsiyalash jarayonni bu turlar yaqinlashuvchi bo'lmaydi.

O'z navbatda Marksenivich tomonidan quyidagi berilgan tasdiralar isbotlandi.

Teorema. Hoxlagan uzluksiz funksiyasi uchun barcha vaqtda Ω_n guruxlar sistemasi bor bo'lib va bu guruxlarda ko'phadli interpolatsiyalash jarayonsi teng o'lchamli yaqinlashuvchili bo'ladi.



2-rasm. $y = \sin 4x$ funksiya'ning interpolatsion xatoligi, $n = 20$

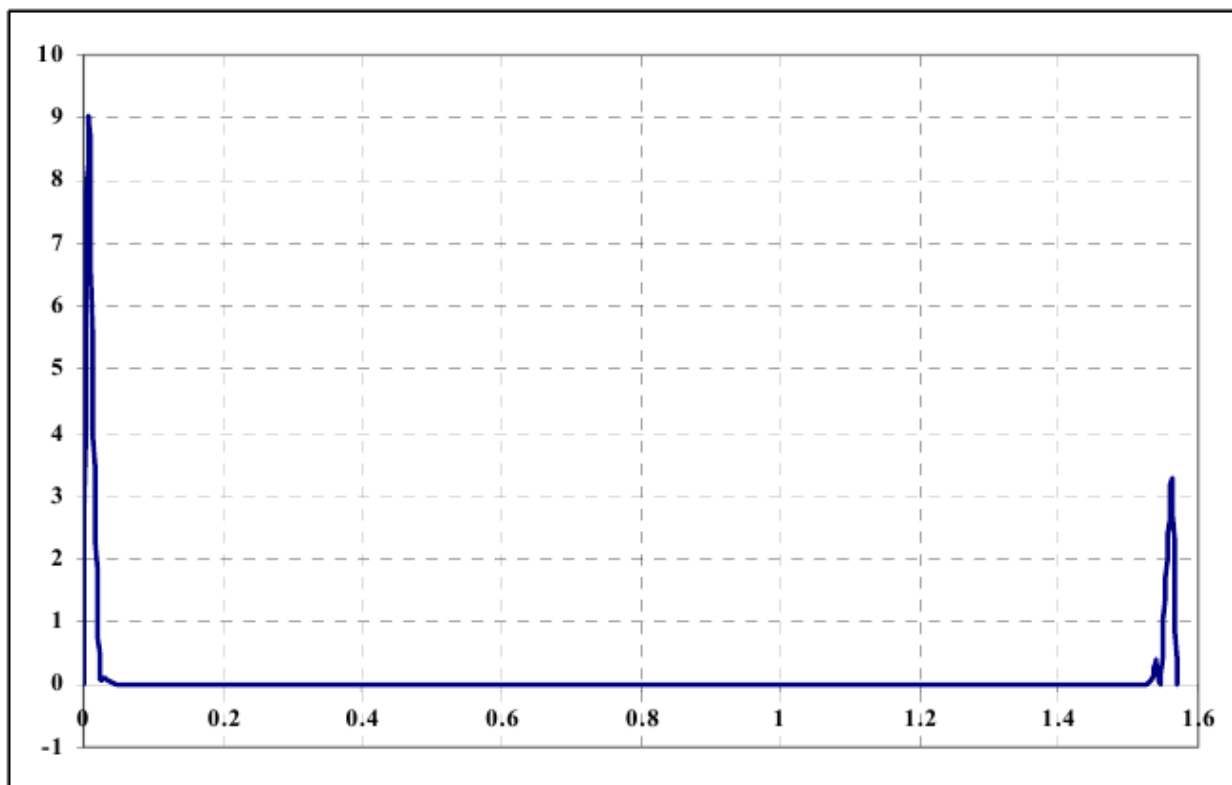
Interpolatsion jarayoni barcha vaqtda teng o'lchamli yaqinlashuvchi boladiganni «universal» gurux bor bo'lishi uchun, uzluksiz funksiyalar guruxi ancha keng boladigani aniqlandi. Shuning, global ko'phadli interpolatsiyalash jarayonni (funksiyani kesmada bir ko'phadli yordamida interpolatsiyalash) hisoblash amaliyotida kam qo'llanishning bir sababi bo'lib hisoblanadi. Shuning bilan birga differensiallanuvchi funksiyalar guruxiga bunday gurux bo'lib Chebishev guruxi hisoblanadi.

Teorema. Agarda $[a, b]$ kesmada interpolatsion tugunlarlari sifatida Chebishev ko'phadining ildizlarini saylab olsak, ya'ni

$$x_k = 0.5(a + b) + 0.5(b - a) \cos(2k + 1) / (2n + 2) \pi$$

bunda $k = 0, 1, \dots, n$, unda $[a, b]$ kesmadagi hohlagan uzluksiz differensiallanuvchi $f(x)$ funksiyasi uchun n ning cheksiz o'sishida shu yaqinlashuvchi bo'ladi, ya'ni

$$\max_{x \in [a, b]} |f(x) - P_n(x)| \rightarrow 0.$$



3-rasm. $y = \sin 4x$ funksiya'ning interpolatsion xatoligi, $n = 69$.

Ammo «optimal» guruxda saylab olganda holda ham kirish malumotlarning oz muqdorda o'zgarishiga va yuqori darajali ko'phadlilarning qiymatlarini hisoblash murakkabligi, interpolatsion ko'phadlining sezilarliligi kuchli bo'lib qaladi. Bu bo'lsa bunday ko'phadlilari bilan interpolatsiyalashni ishonchsiz va natijali emas etib tasdiqladi.

5-§. Sonli misol

1- misol: $y = \lg x$ berilgan jadvaldan foydalanib funksiyaning qiymatini toping

$$\lg(2,25) - ? \quad x = 2,25 \quad n = 4$$

| | | | | | |
|---|---------|---------|---------|---------|---------|
| x | 2 | 2,1 | 2,2 | 2,3 | 2,4 |
| y | 0,30103 | 0,32222 | 0,34242 | 0,36173 | 0,38021 |

Yechish:

Buning uchun bir-biridan tengday masofada joylashgan interpolatsion nuqtalari uchun Langranjning interpolatsion formulasidan foydalanamiz.

$$y = \lg x \quad x = 2,25$$

h-nuqtalar orasidagi masofa

$$h = x_{i+1} - x_i = 0,1$$

$$t = \frac{x - x_0}{h} = \frac{2,25 - 2}{0,1} = 2,5$$

$$f(x) \approx f(x_0 + th) \approx L_4(x_0 + th) =$$

$$= \frac{\prod_5^*(t)}{4!} \sum_{k=0}^4 (-1)^{4-k} \left[\frac{y_0}{t} C_4^0 - \frac{y_1}{t-1} C_4^1 + \frac{y_2}{t-2} C_4^2 - \frac{y_3}{t-3} C_4^3 + \frac{y_4}{t-4} C_4^4 \right]$$

$$C_n^k = \frac{n!}{k!(n-k)!}$$

$$C_4^0 = \frac{4!}{0!4!} = 1$$

$$C_4^1 = \frac{4!}{1!(4-1)!} = 1$$

$$C_4^2 = \frac{4!}{2!2!} = 6$$

$$C_4^3 = \frac{4!}{3! * 1!} = 4$$

$$C_4^4 = \frac{4!}{4!0!} = 1$$

$$\prod_5^*(t) = t(t-1)(t-2)(t-3)(t-4) = 2,5 * 1,5 * 0,5 * (-0,5) * (-1,5) = 1,40625$$

$$\prod_5^*(2,5) = 1,40625$$

$$\begin{aligned} f(2,25) &= \frac{1,40625}{24} \left[\frac{0,30103}{2,5} * 1 - \frac{0,32222}{1,1} * 4 + \frac{0,34242}{0,5} * 6 + \frac{0,36173}{0,5} * 4 \right. \\ &\quad \left. + \frac{0,38021}{(-1,5)} * 1 \right] = \\ &= \frac{1,40625}{24} [0,120412 - 0,859253 + 4,10904 + 2,89384 - 0,25347] = \\ &= 0,35218 \end{aligned}$$

№1 tengligini baholaymiz:

$$|R_4(x)| \leq \frac{M_5}{5!} \left| \prod_5^*(t) \right|$$

$$x = 2,25 \quad t = 2,5 \quad \prod_5^*(2,5) = 1,40625$$

$$M_5 = \max_{[2;2,4]} |f^{(5)}(x)|$$

$$f(x) = \lg x \Rightarrow f(x) = \frac{\ln x}{\ln 10} \Rightarrow f(x) = \frac{1}{\ln 10} \ln x$$

$$f'(x) = \frac{1}{\ln 10} * \frac{1}{x}$$

$$f''(x) = -\frac{1}{\ln 10} * \frac{1}{x^2}$$

$$f'''(x) = \frac{1}{\ln 10} * \frac{2}{x^3}$$

$$f^{IV}(x) = -\frac{1}{\ln 10} * \frac{6}{x^4}$$

$$f^V(x) = -\frac{1}{\ln 10} * \frac{24}{x^5}$$

$$M_5 = \max_{[2;2,4]} \frac{24}{2^5} * \frac{1}{\ln 10} = \frac{24}{32 \ln 10} = 0,32572$$

$$|R_4(x)| \leq \frac{0,32572}{5!} * 1,40625$$

$$|R_4(x)| \leq 0,003817$$

Javobi: $L(2,25) = 0,35218$

2-Misol: Argumentning ko'rsatilgan qiymatlariga mos berilgan funksiyaning qiymatini aniqlang $y = \sin x$ funksiya jadval usulida berilgan

| | | | | |
|-------|---------|----------|---------|---------|
| x_i | 1,1 | 1,25 | 1,6 | 1,8 |
| y_i | 0,89129 | 0,994899 | 0,99957 | 0,97385 |

$f(1,7)$ -qiymat toping va xatoligini baxolang

Yechish: Buning uchun Langranjning bir-biridan teng emas oraliqlari uchun interpolyatsion formulalaridan foydalanin $x=1,7$ $n=3$

$$\begin{aligned} \mathcal{E}_3(x) &= \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \\ &+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3 = \\ &= \frac{(1,7-1,25)(1,7-1,6)(1,7-1,8)}{(1,1-1,25)(1,1-1,6)(1,1-1,8)} * 0,89129 + \\ &+ \frac{(1,7-1,1)(1,7-1,6)(1,7-1,8)}{(1,25-1,1)(1,25-1,6)(1,25-1,8)} * 0,94899 + \\ &+ \frac{(1,7-1,1)(1,7-1,25)(1,7-1,8)}{(1,6-1,1)(1,6-1,25)(1,6-1,8)} * 0,99957 + \\ &+ \frac{(1,7-1,1)(1,7-1,25)(1,7-1,6)}{(1,8-1,1)(1,8-1,25)(1,8-1,6)} * 0,97385 = \frac{0,45 * 0,1 * (-0,1)}{-0,15 * (-0,5) * (-0,7)} * \end{aligned}$$

$$* 0,89129 + \frac{0,6 * 0,1 * (-0,1)}{0,15 * (-0,35) * (-0,55)} * 0,94899 + \frac{0,6 * 0,45 * (-0,1)}{0,5 * 0,35 * (-0,2)} * 0,97385 = 0,07639 - 0,19719 + 0,77109 + 0,34147 = 0,99176$$

Javobi: $f_3(1.7)=0,99176$

Xatoligini baxolang

n=3

$$|R_n(x)| \leq \frac{M_{n+1}}{(n+1)!} \left| \prod_{n+1} (x) \right|$$

$$|R_3(x)| \leq \frac{M_4}{4!} \left| \prod_4 (x) \right|$$

$$\prod_4 (x) = (x - x_0)(x - x_1)(x - x_2)(x - x_3) =$$

$$= (1,7 - 1,1)(1,7 - 1,25)(1,7 - 1,6)(1,7 - 1,8) = 0,6 * 0,45 * 0,1 * (-0,1) = -0,0027$$

$$M_4 = \max_{[1,1;1,8]} |f^{(IV)}(x)|$$

$$f(x) = \sin x$$

$$f'(x) = \cos x$$

$$f''(x) = -\sin x$$

$$f'''(x) = -\cos x$$

$$f^{IV}(x) = \sin x$$

$$M_4 = \max_{[1,1;1,8]} |\sin x| = 0,97385$$

$$|R_3(x)| \leq \frac{0,97385}{4!} |-0,0027|$$

$$|R_3(x)| \leq 0,0001$$

3-Misol: $y=f(x)$ funksiyasi jadval usulida berilgan.

| | | | | | |
|---|---------|---------|----------|----------|----------|
| X | 0,5 | 0,7 | 0,8 | 1,1 | 1,5 |
| Y | 0,87758 | 0,76484 | 0,696707 | 0,453596 | 0,070737 |

a) $x=0,6$

b) $x=1,3$

Yechish: a) Funksiyaning $x=0,6$ nuqtasidagi qiymatini aniqlash uchun Nyutonning oldinga qarab interpolyatsion formulasidan foydalanamiz.

| i | x_i | y_i | $f(x_i, x_{i+1})$ | $f(x_i, x_{i+1}, x_{i+2})$ | $f(x_i, x_{i+1}, x_{i+2}, x_{i+3})$ | $f(x_i, x_{i+1}, x_{i+2}, x_{i+3}, x_{i+4})$ |
|---|-------|----------|-------------------|----------------------------|-------------------------------------|--|
| 0 | 0,5 | 0,877583 | -0,563705 | -0,39215 | 0,116 | 0,0251 |
| 1 | 0,7 | 0,764842 | -0,68135 | -0,32255 | 0,1411 | |
| 2 | 0,8 | 0,696707 | -0,81037 | -0,2097 | | |
| 3 | 1,1 | 0,453596 | -0,95714 | | | |
| 4 | 1,5 | 0,070737 | | | | |

$$f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0} = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

$$\begin{aligned} f(x_0, x_1, x_2, x_3) &= \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0} = \\ &= \frac{\frac{f(x_1, x_3) - f(x_1, x_2)}{x_3 - x_1} - \frac{f(x_1, x_2) - f(x_0, x_1)}{x_1 - x_0}}{x_3 - x_0} \end{aligned}$$

$$f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{0,764842 - 0,877583}{0,2} = -0,563705$$

$$f(x_1, x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{0,696707 - 0,764842}{0,1} = -0,68135$$

$$f(x_2, x_3) = \frac{f(x_3) - f(x_2)}{x_3 - x_2} = \frac{0,453596 - 0,69707}{0,3} = -0,81037$$

$$f(x_3, x_4) = \frac{f(x_4) - f(x_3)}{x_4 - x_3} = \frac{-0,382859}{0,4} = -0,95714$$

$$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0} = \frac{-0,68135 + 0,563705}{0,3} = -0,39215$$

$$f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1} = \frac{0,12902}{0,4} = -0,32255$$

$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2} = -\frac{0,14677}{0,7} = -0,20967$$

$$f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0} = \frac{0,0696}{0,6} = 0,116$$

$$f(x_1, x_2, x_3, x_4) = \frac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1} = \frac{0,11288}{0,8} = 0,1411$$

$$f(x_0, x_1, x_2, x_3, x_4) = \frac{f(x_1, x_2, x_3, x_4) - f(x_0, x_1, x_2, x_3)}{x_4 - x_0} = 0,02251$$

$$f(x) = P_n(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1) * \\ * f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2)f(x_0, x_1, x_2, x_3) + (x - x_0) * \\ * (x - x_1)(x - x_2)(x - x_3)f(x_0, x_1, x_2, x_3, x_4)$$

$$f(0,6) = P_4(0,6) = 0,877583 + (0,6 - 0,5)(-0,563705) + (0,6 - 0,5) * \\ * (0,6 - 0,7)(-0,39215) + (0,6 - 0,5)(0,6 - 0,7)(0,6 - 0,8) * 0,116 + \\ + (0,6 - 0,5)(0,6 - 0,7)(0,6 - 0,8)(0,6 - 1,1) * 0,0251 = 0,877583 - \\ - 0,056370 + 0,003921 + 0,000232 - 0,0000251 = 0,82534$$

$$\text{Javobi: } P_4(0,6)=0,82534$$

b) $x=1,3$ Bunda Nyutonnig ortga qaray interpolyatsion formulasidan foydalanamiz.

$$f(x) = P_n(x) = f(x_n) + (x - x_n)f(x_n, x_{n-1}) + (x - x_n)(x - x_{n-1}) * \\ * f(x_n, x_{n-1}, x_{n-2}) + \dots + (x - x_n)(x - x_{n-1}) \dots (x - x_1) * \\ * f(x_n, x_{n-1}, x_{n-2}, \dots, x_0)$$

Bizning misol uchun:

$$f(1,3) = P_n(1,3) = f(x_4) + (x - x_4)f(x_4, x_3) + (x - x_4)(x - x_3) * \\ * f(x_4, x_3, x_2) + (x - x_4)(x - x_3)(x - x_2)f(x_4, x_3, x_2, x_1) + (x - x_4)(x - x_3) * \\ * (x - x_2)(x - x_1)f(x_4, x_3, x_2, x_1, x_0) = 0,070737 + (1,3 - 1,5)(-0,95714) + \\ + (1,3 - 1,5)(1,3 - 1,1)(-0,2097) + (1,3 - 1,5)(1,3 - 1,1)(1,3 - 0,8) * \\ * (0,1411) + (1,3 - 1,5)(1,3 - 1,1)(1,3 - 0,8)(1,3 - 0,7) * \\ * 0,0251 = 0,267431$$

4-Misol:

Teng oraliqlar

Nyutonning birinchi va ikkinchi interpolyatsion formulasidan foydalanib argumetning berilgan funksiyaning qiymatini aniqlaymiz.

Chekli ayirmalar jadvali yordamida hisoblab boramiz.

| x_i | y_i | Δy_i | $\Delta^2 y_i$ | $\Delta^3 y_i$ |
|----------|----------|--------------|----------------|----------------|
| 1,215 | 0,106044 | 0,007232 | -0,000837 | 0,000025 |
| 1,220 | 0,113276 | 0,006395 | -0,000742 | 0,000093 |
| 1,225 | 0,119671 | 0,005693 | -0,000649 | 0,000093 |
| 1,230 | 0,12532 | 0,005004 | -0,000556 | 0,000091 |
| 1,235 | 0,130328 | 0,004448 | -0,000465 | 0,000090 |
| 1,240 | 0,134776 | 0,003983 | -0,000375 | 0,000088 |
| 1,245 | 0,138759 | 0,003608 | -0,000287 | 0,000087 |
| 1,250 | 0,142367 | 0,003321 | -0,0002 | |
| 1,255 | 0,145688 | 0,003121 | | |
| 1,260 | 0,148809 | | | |
| Σ | | 0,042765 | -0,004111 | 0,000637 |
| Π | 0,042765 | -0,00411 | 0,000637 | |

$$x=1,2273$$

a) $x=1,253$

$$\Delta y_0 = y_1 - y_0$$

$$\Delta y_1 = y_2 - y_1$$

$$\Delta y_2 = y_3 - y_2$$

... ..

$$y_i = y_{i+1} - y_i$$

$$\Delta y_i = y_{i+1} - y_i$$

$$\Delta^2 y_i = \Delta y_{i+1} - \Delta y_i$$

$$\Delta^3 y_i = \Delta^2 y_{i+1} - \Delta^2 y_i$$

... ..

$$\Delta^n y_i = \Delta^{n-1} y_{i+1} - \Delta^{n-1} y_i$$

a) $x=1,2273$ nuqtasidagi funksiyaning qiymatini aniqlash uchun Nyutonning olg'a qaray interpolyatsion usulidan foydalanamiz.

$$f(x) = f(x_0 + th) \approx P_n(x) = y_0 + t\Delta y_0 + \frac{t(t-1)}{2!} \Delta^2 y + \frac{t(t-1)(t-2)}{3!} * \Delta^3 y_0$$

$$x = 1,2273$$

$$x_0 = 1,225$$

$$h = x_{i+1} - x_i = 0,005$$

$$t = \frac{x - x_0}{h} = \frac{1,2273 - 1,225}{0,005} = 0,46$$

$$f(1,2273) = 0,119671 + 0,46 * 0,005653 + \frac{0,46 * (0,46 - 1)}{2!} (-0,000649) + \frac{0,46(0,46 - 1)(0,46 - 2)}{6} * 0,000093 \approx 0,122358$$

Misolning xatoligini baxolang

$$|R_n(x)| \leq \frac{M_{n+1}}{(n+1)!} * h^{n+1} \left| \prod_{n+1} (t) \right|$$

$$M_{n+1} = \max_{x \in [a,b]} |f^{(n+1)}(x)|$$

$$f^{(n+1)}(x) = \lim_{h \rightarrow 0} \frac{\Delta^{n+1} y}{h^{n+1}}$$

$$f^{(n+1)}(\varepsilon) = \frac{\Delta^{n+1} y}{h^{(n+1)}}$$

$$|R_n(x)| \leq \frac{\Delta^{n+1} y}{(n+1)!} \left| \prod_{n+1} (t) \right|$$

$$\Delta^{n+1} y = \max_{0 \leq k \leq n} |\Delta^{n+1} y|$$

n=3

$$\Delta^4 y = 0,000002$$

$$|R_n(x)| \leq \frac{\Delta^{n+1} y}{(n+1)!} \left| \prod_{n+1} (t) \right|$$

$$|R_3(x)| \leq \frac{0,000002}{4!} |t(t-1)(t-2)(t-3)|$$

$$|R_3(x_0 + th)| \leq \frac{0,000002}{24} * 0,97164114$$

$$|R_3(x_0 + th)| \leq 0,00000008$$

$$\text{Javobi: } f(1,2273) = 0,122358$$

b) $x=1,253$ nuqtasidagi funksiyaning qiymatini aniqlash uchun Nyutonning ortga qaray interpolatsion usulidan foydalanamiz.

$$f(x) - f(x_n + th) = P_n(x) = y_n + t\Delta y_{n-1} + \frac{t(t+1)}{2!} \Delta^2 y_{n-2} + \frac{t(t+1)(t+2)}{3!} \Delta^3 y_{n-3}$$

$$x_n = 1,255$$

$$t = \frac{x - x_n}{h} = \frac{1,253 - 1,255}{0,005} = -0,4$$

$$f(1,253) = 0,145688 + (-0,4)0,003321 + \frac{(-0,4)(-0,4+1)}{2}(-0,000287) + \frac{(-0,4)(-0,4+1)(-0,4+2)}{6}0,00088 \approx 0,144388$$

Misolning xatoligini baholaymiz:

$$|R_3(x_0 + th)| \leq \frac{\Delta^4 y}{4!} \left| \prod_4(t) \right|$$

$$\prod_4(-0,4) = t(t+1)(t+2)(t+3) = -0,4(-0,4+1)(-0,4+2)(-0,4+3)$$

$$= -0,4 * 0,6 * 1,6 * 2,6 = -0,9984$$

$$|R_3(x_0 + th)| \leq \frac{0,000002}{4!} * 0,9984$$

$$|R_3(x_0 + th)| \leq 0,00000083$$

$$\text{Javobi: } f(1,253) = 0,144388$$

5–Misol. Mayli $x=0,1,2$ nuqtalarda $f(x)$ funksiyasi va uning tugunlarining qiymatlari jadvalda berilgan bo'lsin.

| x | y | y' | y'' |
|-----|-----|------|-------|
| 0 | 1 | 0 | 0 |
| 1 | 2 | 7 | 448 |

| | | | |
|---|-----|--|------|
| 2 | 129 | | 1344 |
|---|-----|--|------|

Ermittling H_m interpolatsion ko'phadini toping.

Yechish. (3.1) belgilashlariga mos bu yerda $m=2$, $k_0-1=2$, $k_1-1=2$, $k_2-1=1$. Bundan $n+1=k_0+k_1+k_2=3+3+2=8$ bo'ladi, $n=8-1=7$. Demak 3, $y=f(x)$ funkstiyasi haqida ma'lumatlar uch tugunda $x_0=0$, $x_1=1$, $x_2=2$ toplangan, karrali mos 3, 2, 3 teng bo'ladi. Bundan $H_7(x)$ Ermitning interpolatsion ko'phadini tuzish kerak boladi.

Bu $H_7(x)$ ko'phadi bilan birga ikkinchi darajali $L_2(x)$ Lagranjning interpolatsion ko'phadini ko'rib chiqamiz va ol $x_0=0, x_1=1, x_2=2$ nuqtalarida mos $y_0=1, y_1=2, y_2=129$ manislerini qabul qiladi. $H_7(x)-L_2(x)$ ayirmasi $x_0=0, x_1=1, x_2=2$ nuqtalarida nolga teng bolishini va darajasi $m=7$ dan yuqori bolmagan ko'phad boladi. Demak

$$H_7(x) - L_2(x) = \varpi_2(x)H_4(x),$$

bunda
$$\varpi_2(x) = (x-x_0)(x-x_1)(x-x_2) = x \cdot (x-1)(x-2).$$

Ixtiyoriy $H_4(x)$ ko'phadida

$$H_7(x) = L_2(x) + \varpi_2(x)H_4(x)$$

funkstiyasi interpolatsion tugunlarida $y_i, i=0, \overline{2}$ qiymatlarini qabul qiladi.

Bu holda Lagranjning interpolatsion ko'phadi teng bo'ladi:

$$L_2(x) = 1 \cdot \frac{(x-1)(x-2)}{2} + 2 \cdot \frac{x(x-2)}{-1} + 129 \cdot \frac{x(x-1)}{2} = 63x^2 - 62x + 1.$$

Bundan

$$H_7(x) = 63x^2 - 62x + 1 + x(x-1)(x-2)H_4(x).$$

Bu ifodani differensiallaymiz va natijada ega bo'lamiz:

$$H_7'(x) = 126x - 62 + (3x^2 - 6x + 2)H_4(x) + (x^3 - 3x^2 + 2x)H_4'(x).$$

Bunga $x=0,1,2$ qiymatlarni qo'yib, quyidagiga ega bo'lamiz:

$$H_4(0) = 31, H_4(1) = 57, H_4(2) = 129.$$

$H_7(x)$ ko'phadining ikkinchi nuqtasida quyidagi turda bo'ladi:

$H_7''(x) = 126 + (6x + -6)H_4(x) + 2(3x^2 - 6x + 2)H_4'(x) + (x^3 - 3x^2 + 2x)H_4''(x)$
 Bundan $H_4'(x)$ qiymatin $x=0$ va $x=2$ nuqtalarida topamiz:

$$H_7'(0) = 15, H_7'(2) = 111.$$

Quyidagi shartlarini qanoatlantiradigan $H_4(x)$ ko'phadini topish kerak bo'ladi

$$H_4(0) = 31; \quad H_4(1) = 57; \quad H_4(2) = 129;$$

$$H_7'(0) = 15; \quad H_7'(2) = 111.$$

$H_4(x)$ ko'phadini quyidagicha yozamiz:

$$H_4(x) = 31 \frac{(x-1)(x-2)}{2} + 57 \frac{x(x-2)}{-1} + 129 \frac{x(x-1)}{2} + x(x-1)(x-2)H_1(x) =$$

$$= 23x^2 + 3x + 31 + (x^3 - 3x^2 + 2x)H_1(x)$$

$H_4(x)$ ko'phadini differensiallaymiz va quyidagi natijaga ega bo'lamiz:

$$H_4'(x) = 46x + 3 + (3x^2 - 6x + 2)H_1(x) + (x^3 - 3x^2 + 2x)H_1'(x).$$

Bunga $x=0$ va $x=2$ qiymatlarini olib borib qo'yib, quyidagi ega bo'lamiz:

$$H_1(0) = 6, H_1(2) = 8.$$

Bundan

$$H_1(x) = x + 6$$

$$H_7(x) = 63x^2 - 62x + 1 + x(x-1)(x-2)[23x^2 + 3x + 31 + x(x-1)(x-2)(x+6)] = 1 + x^7$$

6-misol. Bolingan ayirmalarina ega Nyutonning umumiy lashgan formulasidan foydalanib Ermit ko'phadini tuzing

| x | y | y' | y'' |
|-----|-----|------|-------|
| 0 | 1 | 0 | 0 |
| 1 | 2 | 7 | 448 |
| 2 | 129 | | 1344 |

Bo'lingan ayirmalarni hisoblaymiz:

| x | y | Δy | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ | $\Delta^5 y$ | $\Delta^6 y$ | $\Delta^7 y$ |
|-----|-----|------------|--------------|--------------|--------------|--------------|--------------|--------------|
| 0 | 1 | | | | | | | |

| | | | | | | | | |
|---|-----|-----|-----|-----|-----|----|---|---|
| 0 | 1 | 0 | | | | | | |
| 0 | 1 | 0 | 0 | | | | | |
| 1 | 2 | 1 | 1 | 1 | | | | |
| 1 | 2 | 7 | 6 | 5 | 4 | | | |
| 2 | 129 | 127 | 120 | 57 | 26 | 11 | | |
| 2 | 129 | 448 | 321 | 201 | 72 | 23 | 6 | |
| 2 | 129 | 448 | 672 | 351 | 150 | 39 | 8 | 1 |

Bunda bo'lingan ayirmali Nyutonning interpolyatsion ko'phadi quyidagicha boladi:

$$y(x) = y(x_0) + (x - x_0)y(x_0; x_0) + (x - x_0)^2 y(x_0; x_0; x_0) + (x - x_0)^3 y(x_0; x_0; x_1) + \\ + (x - x_0)^3 (x - x_1)y(x_0; x_0; x_0; x_1; x_1) + (x - x_0)^3 (x - x_1)^2 y(x_0; x_0; x_0; x_1; x_1; x_2) + \\ + (x - x_0)^3 (x - x_1)^2 (x - x_2) y(x_0; x_0; x_0; x_1; x_1; x_2; x_2) + \\ + (x - x_0)^3 (x - x_1)^2 (x - x_2)^2 y(x_0; x_0; x_0; x_1; x_1; x_2; x_2; x_2).$$

$$y(x) = y(x_0) + (x - x_0)\Delta y + (x - x_0)^2 \Delta^2 y + (x - x_0)^3 \Delta^3 y + (x - x_0)^3 (x - x_1)\Delta^4 + \\ + (x - x_0)^3 (x - x_1)^2 \Delta^5 y + (x - x_0)^3 (x - x_1)^2 (x - x_2)\Delta^6 y + (x - x_0)^3 (x - x_1)^2 (x - x_2)^2 \Delta^7 y.$$

Jadvalda berilgan ma'lumotlardan va bo'lingan ayirmalardan foydalanib Ermit ko'phadini keltirip chiqaramiz:

$$H(x) = 1 + x \cdot 0 + x^2 \cdot 0 + x^3 \cdot 1 + x^3(x-1) \cdot 4 + \\ + x^3(x-1)^2(x-2) \cdot 6 + x^3(x-1)^2(x-2)^2 \cdot 1 = x^7 + 1.$$

Xulosa

Ushbu bitiruv malakaviy ishida funkstiyalarni approksimastiyalash masalasi qaraldi. Approksimatsiyalanuvchi $\varphi(x)$ funksiyasi faqatgina ko'phadlar yoki ko'phadlarni tuzgan funkstiyalardan foydalaniladi. Shuning uchun bu yerda ko'phadlarni approksimatsiyalanuvi haqida gap ketdi. Bunda Lagranjning, Nyutonning va Ermitning interpolyatsion ko'phadlari qaraladi.

Bitiruv malakaviy ishining asosiy natijalari:

- Lagranjning interpolyatsion ko'phadi, bazis polinomlari va interpolyastiyalash xatoliklarini baholashlar keltirildi va o'rganilib chiqildi;
- Teng oraliqlar holi uchun chekli ayirmalar tushunchalari kiritilib, ularning oddiy xossalari keltirib o'tildi. Chekli ayirmalar interpolyatsion ko'phadlari tuzildi. Bunda chekli ayirmalar nisbatlari chekli ayirmalarga asoslangan Nyutonning birinchi va ikkinch interpolyatsion ko'phadlari bilan interpolyatsiyalash qaralgan, interpolyatsion fo'rmulalari haqida nazariy ma'lumotlar berildi va ularni o'rganib chiqildi.
- Nyutonning interpolyatsion ko'phadi, chekli ayirmalar nisbatlari, chekli ayirmali interpolyatsion formulalari haqida teoriyalik ma'lumotlar berildi va ular o'rganilib chiqildi;
- Karrali tugunlar bilan interpolyastiyalash masalasining qo'yilishi o'rganildi;
- Ermitning interpolyatsion ko'phadi bor bo'ladigan va u yagona ko'phadi bo'lishi isbotlab, o'rganildi;
- Ermitning interpolyatsion ko'phadining umumiy formasi keltirilib berildi va bul formula asosida interpolyatsion ko'phadi tuzildi;
- Interpolyatsion jarayonning yaqinlashuvchi bo'lishi holatlari qaraldi va o'rganildi.

Asosiy bo'limda qaralgan barcha interpolyatsion formulalardan foydalanib misollar yechildi va hisoblash eksperimentlari o'tkazildi.

Foydalangan adabiyotlar

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Internet manbalari

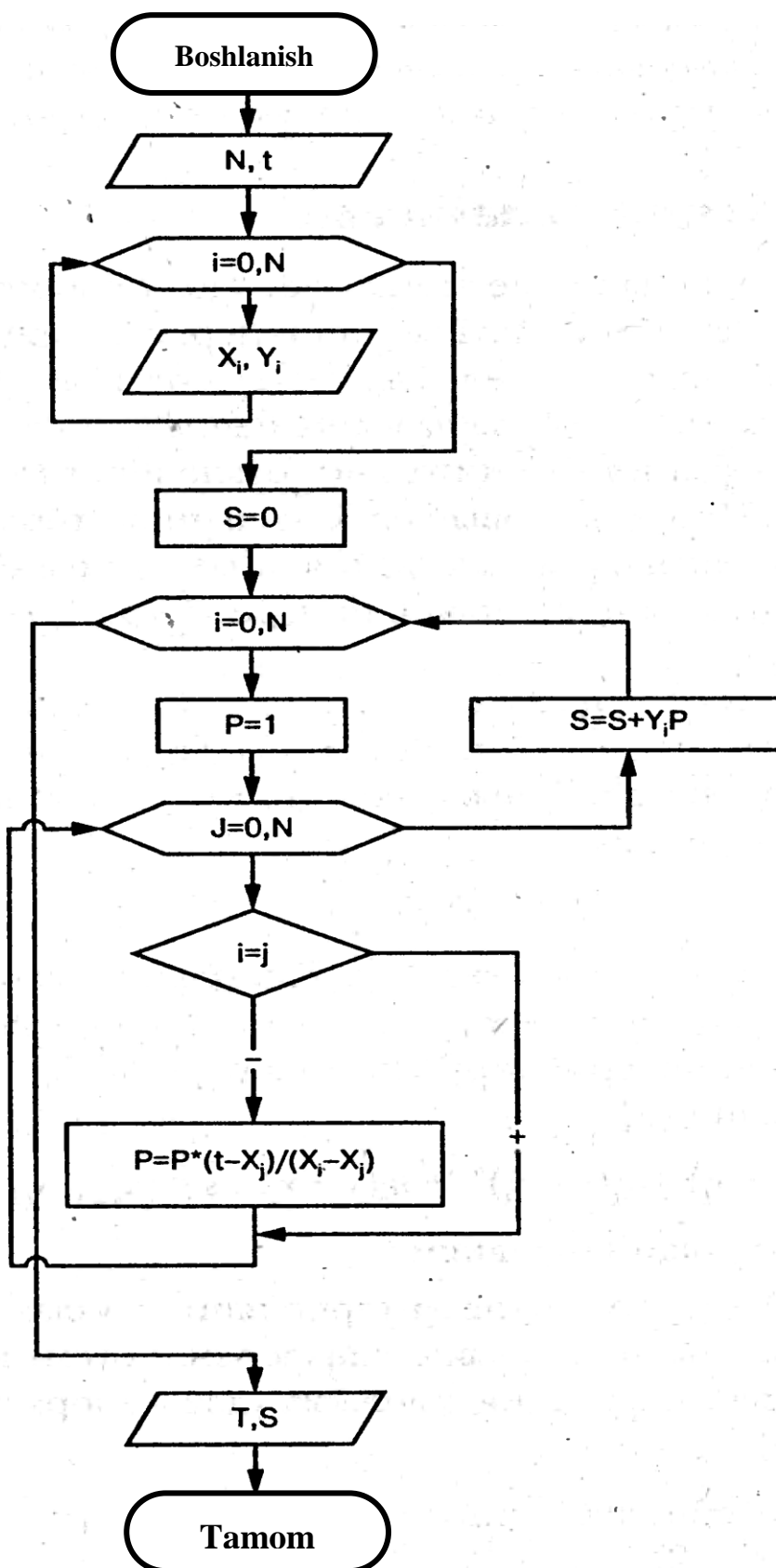
1. https://ru.wikipedia.org/wiki/Интерполяционный_многочлен_Лагранжа
2. <http://yukhym.com/ru/matematika/mnogochlen-lagranzha-postroenie-v-maple.html>
3. ru.wikipedia.org/wiki/Эрмитова_интерполяция
4. <http://window.edu.ru/resource/248/63248/files/interpolation.pdf>

Qo`shimchalar

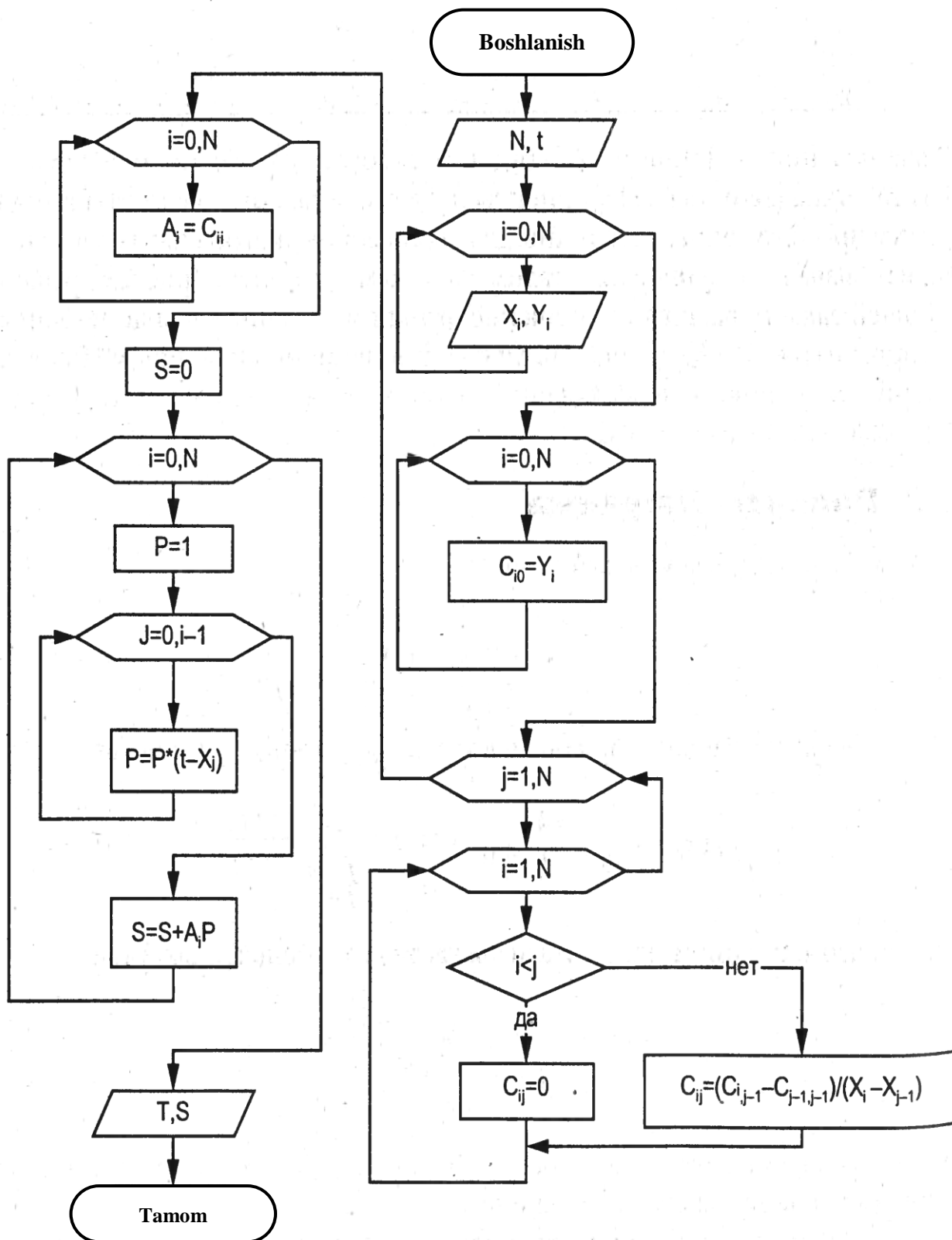
(EHM uchun tuzilgan pragrammalarning va ularning bajarishdan olingan sonli natijalarning bosib chiqarilgan nusxalari hamda blok sxemalari)

Berilgan masalalar uchun blok- sxema:

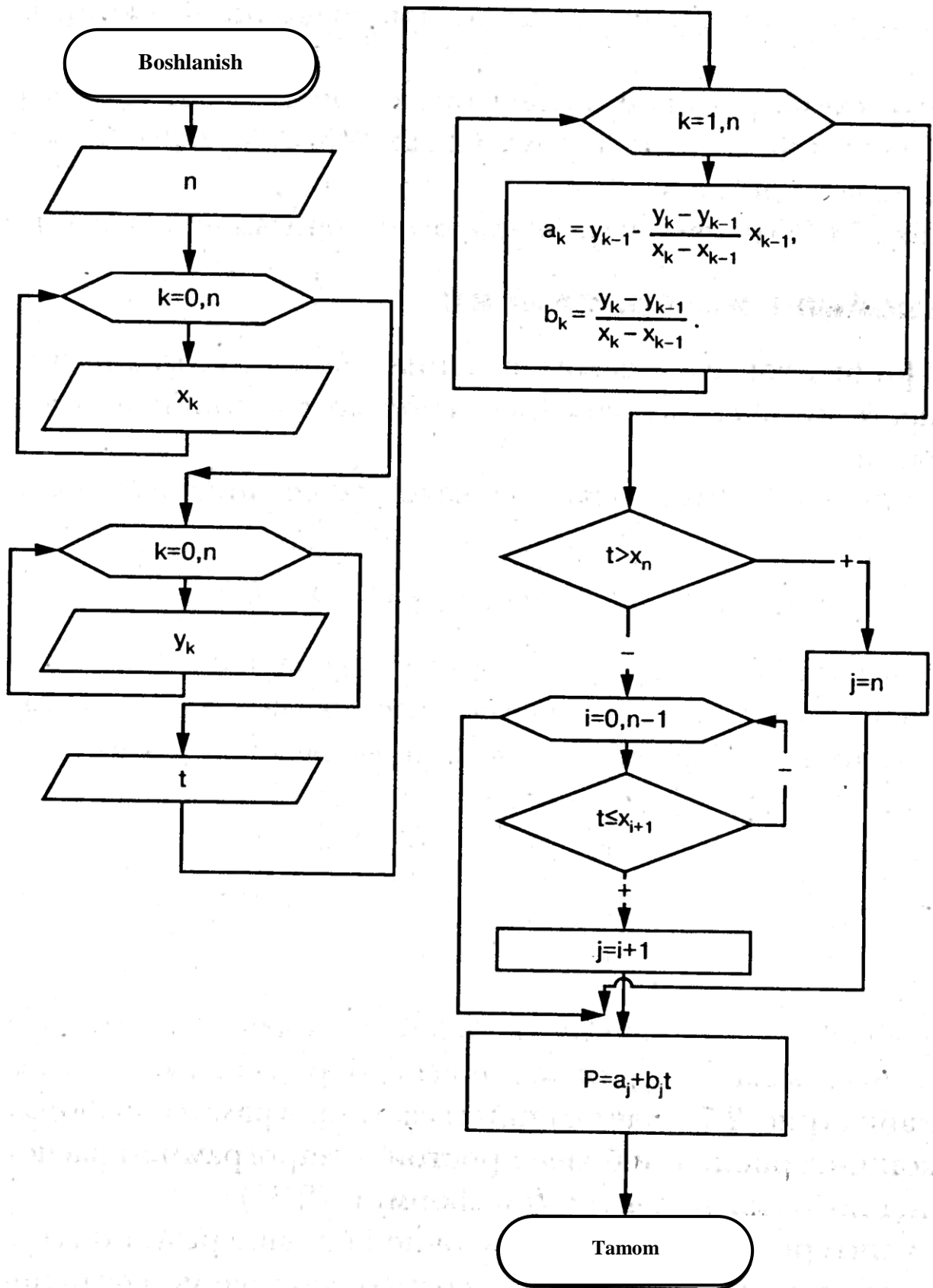
a) Lagranj usuli:



b) Nyuton usuli



c) Ermit usuli




```

cout<<"\n";}
cout<<"KHKHHHHHHHKHHHHHHHHHKHHHHHHHHHHKHHHHHHHHHHH
HKHHHHHHHHHHHK"<<"\n";
s=0;
if(k>=n/2){s=y[k+1];for(j=1;j<=n-2;j++){p=p*(x1-x[k-j+1]);p1=p1*j;s=s+p*a[k-
j+1][j-1]/p1;
cout<<x[k-j+1]<<" "<<p<<" "<<p1<<" "<<a[k-j+1][j-1]<<"\n";}}
if(k<=n/2){s=y[k];for(j=1;j<=3;j++){p=p*(x1-x[k+j-1]);p1=p1*j;s=s+p*a[k][j-
1]/p1;
cout<<x[k+j-1]<<" "<<p<<" "<<p1<<" "<<a[k][j-1]<<"\n";}}
cout.precision(8);cout<<"f("<<x1<<")="<<s;
getch();}

```

*****NYUTON USULI*****
*****TENG EMAS ORALIQLAR UCHUN*****

| i | x _i | Y _i | ¹ F | ² F | ³ F | ⁴ F |
|---|----------------|----------------|----------------|----------------|----------------|----------------|
| 0 | 0.5 | 0.87758 | -0.56370 | -0.392100 | 0.160202 | 0.02514 |
| 1 | 0.7 | 0.78484 | -0.68133 | -0.322550 | 0.141125 | 0 |
| 2 | 0.8 | 0.69670 | -0.81046 | -0.209457 | 0 | 0 |
| 3 | 1.1 | 0.45356 | -0.95708 | 0 | 0 | 0 |
| 4 | 1.5 | 0.07073 | 0 | 0 | 0 | 0 |

f(0.6)=0.8252929

3-b-misol

```

#include<iostream>
#include<conio.h>
#include<math.h>
using namespace std;
main()
{int i,j,z,n,m,l=0,k=0,p1=1;;
double a[100][100],x1,h,t,d[100],c[100],s,p=1;
n=5 ;x1=1.3;

```



```
getch();}
```

*****NYUTON USULI*****
*****TENG EMAS ORALIQLAR UCHUN*****

| i | x _i | Y _i | ¹ F | ² F | ³ F | ⁴ F |
|---|----------------|----------------|----------------|----------------|----------------|----------------|
| 0 | 0.5 | 0.87758 | -0.56370 | -0.392100 | 0.160202 | 0.02514 |
| 1 | 0.7 | 0.78484 | -0.68133 | -0.322550 | 0.141125 | 0 |
| 2 | 0.8 | 0.69670 | -0.81046 | -0.209457 | 0 | 0 |
| 3 | 1.1 | 0.45356 | -0.95708 | 0 | 0 | 0 |
| 4 | 1.5 | 0.07073 | 0 | 0 | 0 | 0 |

f(1.3)=0.267429

4-a-misol

```
#include <iostream>
#include<string.h>
#include<stdio.h>
#include<math.h>
#include<iomanip>
#include<conio.h>
using namespace std;
int main(){ char d=187,ss=188;
float x[10]={ 1.215,1.220,1.225,1.230,1.235,1.24,1.245,1.250,1.255,1.260};
float y[10]={0.106044,0.113276,0.119671,0.12532,0.130328,0.134776, 0.138759,
0.142367,0.145688,0.148809};
int i,j,n=9,m=1;
float p4,x1=1.2273,t,dy1[100],dy2[100],dy3[100],dy4[100],dy5[100],dy6[100],
dy7[100],dy8[100],dy9[100],dy10[100]; t=(x1-x[0])/(x[1]-x[0]);
cout<<"\n\n TENG ORALIQLAR UCHUN "<<endl;
cout<<" NYUTONNING OLG'A QARAB
INTERPalyatsiyasi"<<endl<<endl;
cout<<"ЙННЛННННННЛННННННННЛННННННННЛННННННННЛНННННННН
НН"<<d<<"\n";
```

```

cout<<"e ie xi e yi e Dyi e D^2yi e D^3yi e\n";
cout<<"MHHOHHHHHHOHHHHHHHHOHHHHHHHHHHOHHHHHHHHHHOHHHHHH
HHHN\n";
for(i=0;i<n;i++) dy1[i]=y[i+1]-y[i];for(i=0;i<n-1;i++) dy2[i]=dy1[i+1]-dy1[i];
for(i=0;i<n-2;i++) dy3[i]=dy2[i+1]-dy2[i];
for(i=0;i<=n;i++){
cout<<"e";if(i<10)cout<<" ";
cout<<i<<"e"<<setprecision(2)<<setiosflags(ios::fixed|ios::showpoint)<<x[i]<<"e";
cout<<setprecision(5)<<y[i]<<setprecision(5)<<"e";
if(i<=10)cout<<dy1[i]; else cout<<" ";cout<<"e";if(i<=9)cout<<dy2[i];
else cout<<" ";cout<<" e";if(i<=8)cout<<setprecision(5)<<dy3[i]; else cout<<"
";cout<<"e"; cout<<"\n";}
p4=y[0]+(t*dy1[0])+(t*(t-1)*dy2[0])/2+(t*(t-1)*(t-2)*dy3[0])/6;
cout<<"ИHHKHHHHHHKHKKHHHHHHHHKHKKHHHHHHHHHHKHKKHHHHHHHH
HH"<<ss<<"\n\n";
cout<<" F("<<setprecision(4)<<x1<<")="<<setprecision(9)<<p4; getch(); }

```

**TENG ORALIQLAR UCHUN
NYUTONNING OLG'A QARAB INTERPALYATSIYASI**

| i | xi | yi | Dyi | D^2yi | D^3yi |
|---|------|---------|---------|----------|---------|
| 0 | 1.22 | 0.10604 | 0.00723 | -0.00084 | 0.00009 |
| 1 | 1.22 | 0.11328 | 0.00640 | -0.00075 | 0.00011 |
| 2 | 1.23 | 0.11967 | 0.00565 | -0.00064 | 0.00008 |
| 3 | 1.23 | 0.12532 | 0.00501 | -0.00056 | 0.00010 |
| 4 | 1.24 | 0.13033 | 0.00445 | -0.00046 | 0.00009 |
| 5 | 1.24 | 0.13478 | 0.00398 | -0.00038 | 0.00009 |
| 6 | 1.25 | 0.13876 | 0.00361 | -0.00029 | 0.00009 |
| 7 | 1.25 | 0.14237 | 0.00332 | -0.00020 | 0.00000 |
| 8 | 1.25 | 0.14569 | 0.00312 | 0.00000 | 0.00000 |
| 9 | 1.26 | 0.14881 | 0.00000 | 0.00000 | |

$$F(1.2273)=0.122356728$$

4-b-misol

```

#include <iostream>
#include<string.h>
#include<stdio.h>
#include<math.h>
#include<iomanip>
#include<conio.h>
using namespace std;
float sss(float zz,float jj,float xx);
int main(){
char d=187,ss=188;
float n[10]={ 1.215,1.220,1.225,1.230,1.235,1.24,1.245,1.250,1.255,1.260};
float m[10]={0.106044,0.113276,0.119671,0.12532,0.130328,0.134776,0.138759,
0.142367,0.145688,0.148809};
double qq[12]={0,0,0,0,0,0,0,0,0,0,0,0};
double ww[11]={0,0,0,0,0,0,0,0,0,0,0};
double ee[10]={0,0,0,0,0,0,0,0,0,0};
double rr[9]={0,0,0,0,0,0,0,0,0};
int i,j,ii=1,kk=1;
double x=1.253,cc,t,p,pl,ll,oo=0,nn=1;t=(x-n[12])/(n[1]-n[0]);
cout<<"\n\n    ***    TENG ORALIQLAR UCHUN    ***"<<endl;
cout<<"    ***    NYUTONNING ORTGA QARAB INTERPALATSIYASI
***"<<endl<<endl;
cout<<"ЙННЛНННННННННЛНННННННННЛНННННННННЛНННННННННЛНННННН
ННН"<<d<<"\n";
cout<<"ei e xi e yi e Dyi e D^2yi e D^3yi e \n";
cout<<"МННОНННННННННОНННННННННОНННННННННОНННННННННОННННН
ННННН№\n"; for(i=0;i<=9;i++) qq[i]=m[i+1]-m[i];

```

```

for(i=0;i<=8;i++) ww[i]=qq[i+1]-qq[i];
for(i=0;i<=7;i++) ee[i]=ww[i+1]-ww[i];
for(i=0;i<=6;i++) rr[i]=ee[i+1]-ee[i];
for(j=0;j<=9;j++) { cout<<"e";
if(j<8) cout<<" "; cout<<j<<"e";
cout<<" "<<setprecision(3)<<setiosflags(ios::fixed|ios::showpoint)<<n[j]<<"
"<<"e";cout<<setprecision(5)<<setiosflags(ios::fixed|ios::showpoint)<<m[j]<<"e";
sss(j,11,qq[j]); sss(j,10,ww[j]); sss(j,9,ee[j]); cout<<endl; }
cout<<"ИHKHHHHHHHKHHHHHHHKHHHHHHHKHHHHHHHKHHHHHH
HHH"<<ss<<"\n\n";
pl=m[11]+qq[10]*t+(t*(t+1))/2*ww[9]+(t*(t+1)*(t+2))/6*ee[8];
cout<<"      F("<<setprecision(4)<<x<<")="<<setprecision(9)<<pl; getch(); }
float sss(float zz,float jj,float xx) { char e=186;
if(jj==8&&zz<jj) {
cout<<setprecision(5)<<setiosflags(ios::fixed|ios::showpoint)<<xx<<"e";
return 0; } if(zz<jj) {
cout<<setprecision(5)<<setiosflags(ios::fixed|ios::showpoint)<<xx<<"e";}
else {cout<<"      "<<e; }return 0;}

```

*** TENG ORALIQLAR UCHUN ***
 *** NYUTONNING ORTGA QARAB INTERPALATSIYASI ***

| i | x _i | y _i | Dy _i | D ² y _i | D ³ y _i |
|---|----------------|----------------|-----------------|-------------------------------|-------------------------------|
| 0 | 1.215 | 0.10604 | 0.00723 | -0.00084 | 0.00009 |
| 1 | 1.220 | 0.11328 | 0.00640 | -0.00075 | 0.00011 |
| 2 | 1.225 | 0.11967 | 0.00565 | -0.00064 | 0.00008 |
| 3 | 1.230 | 0.12532 | 0.00501 | -0.00056 | 0.00010 |
| 4 | 1.235 | 0.13033 | 0.00445 | -0.00046 | 0.00009 |
| 5 | 1.240 | 0.13478 | 0.00398 | -0.00038 | 0.00009 |
| 6 | 1.245 | 0.13876 | 0.00361 | -0.00029 | 0.00009 |
| 7 | 1.250 | 0.14237 | 0.00332 | -0.00020 | -0.15173 |
| 8 | 1.255 | 0.14569 | 0.00312 | -0.15193 | 0.00000 |

|| 9 || 1.260 || 0.14881 || -0.14881 || 0.00000 ||

F(1.2530)=0.144400