O’ZBEKISTON RESPUBLIKASI OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

BUXORO DAVLAT UNIVERSITETI

Fizika –matematika fakulteti

5130100- “Matematika” kafedrasi

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“4×4 O`LCHAMLI OPERATORLI MATRITSALARNING SPEKTRAL XOSSALARI” mavzusida

5130100-“Matematika” ta’lim yo’nalishi bo’yicha bakalavr darajasini olish uchun

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“Ish ko’rildi va himoya qilishga ruxsat berildi”

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Buxoro-2019

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KIRISH

“Yoshlarimizni mustaqil fikrlaydigan, yuksak
intellektual va ma`naviy salohiyatga ega bo`lib,
dunyo miqyosida o`z tengdoshlariga hech qaysi sohada
bo`sh kelmaydigan insonlar bo`lib kamol topishi,
baxtli bo`lishi uchun davlatimiz va jamiyatimizning
bor kuch va imkoniyatlarini safarbar etamiz”.
Sh.M.Mirziyoyev

Biz mustaqil O`zbekiston yoshlari har tomonlama yetuk barkamol avlod bo`lib yetishimiz uchun keng imkoniyatlar yaratib, Prezidentimiz Sh.M.Mirziyoyev yosh avlod tarbiyasiga juda katta e`tibor berib kelmoqdalar. Jumladan, O`zbekiston Respublikasi Prezidenti Sh.M.Mirziyoyevning mamlakatimizning 2017 – yilga mo`ljallangan iqtisodiy dasturning eng muhim ustuvor yo`nalishlariga bag`ishlangan Vazirlar Mahkamasining kengaytirilgan majlisidagi ma`ruzasida:

“Oldimizda yoshlarga tarbiya berish, psixologiya va boshqa turli sohalarda kadrlarni tayyorlash va qayta tayyorlash bo`yicha murakkab vazifalar turibdi. Yana bir muammoni hal etish muhim hisoblanadi: bu – pedagoglar va professor – o`qituvchilar tarkibining professional darajasi, ularning maxsus bilimlaridir. Bu borada ta`lim olish, ma`naviy – ma`rifiy kamolot masalalari va haqiqiy qadriyatlarni shakllantirish jarayonlariga faol ko`mak beradigan muhitni yaratish zarur” ekanligi ta`kidlab o`tildi.

Mustaqil va erkin fikrlayotgan, ongli yashaydigan, o’z haq – huquqlarini yaxshi taniydigan, o’z kuchi va aqliga ishonadigan, ma’naviy – axloqiy yetuk barkamol bo’lgan avlodni, mustaqil fikrlashga qodir, jasoratli, fidoiy va tashabbuskor kishilarni tarbiyalab yetkazadigan xalq va millat kelajakka ochiq ko’z, katta ishonch, umid va ixlos bilan qaray oladi. Fuqarolarni ana shunday noyob xislat va fazilat sohiblari qilib shakllantirilgan davlatning istiqboli porloq bo’ladi.

Mamlakatimiz kuch-qudrati aholining soni bilan emas, balki odamlarning intellektual salohiyati, zamonaviy bilim asoslarini texnika va texnologiyalarni, ko’plab xorijiy tillarni egallaganligi, g’oyaviy – siyosiy yetukligi, huquqiy bilimlilik va madaniylik darajasi, irodasini mustahkamligi, inson e’tiqodining butunligi, o’z shaxsiy manfaatlarini shaxs, vatan manfaatlari bilan uyg’un holda ko’ra olishi, elim deb, yurtim deb yashashi, faoliyat ko’rsatishi bilan kuchlidir. O’zbekiston davlati hozirgi paytda o’z oldiga ana shunday oliy maqsadni qo’yib kelajak sari intilmoqda.

O`zbekiston Respublikasi Prezidenti Sh.M.Mirziyoyevning mamlakatimizni 2016 – yilda ijtimoiy – iqtisodiy rivojlantirishning asosiy yakunlari va 2017 – yilga mo`ljallangan iqtisodiy dasturning eng muhim ustuvor yo`nalishlariga bag`ishlangan Vazirlar Mahkamasining kengaytirilgan majlisidagi “Tanqidiy tahlil, qat`iy tartib – intizom va shaxsiy javobgarlik – har bir rahbar faoliyatining kundalik qoidasi bo`lishi kerak” deb nomlangan ma`ruzasida “…ta`limning yangi zamonaviy usullarini, jumladan, axborot – kommunikatsiya texnologiyalarini joriy etish…, bu boradagi dolzarb vazifalarni amalga oshirish yoshlarimiz, jamiyatimiz va mamlakatimizning kelajagi uchun strategik ahamiyatga ega ekani…” alohida ta`kidlab o`tildi.

2017 – yil 20 – aprelda Oliy ta`lim tizimini tubdan takomillashtirish, mamlakatimizni ijtimoiy – iqtisodiy rivojlantirishning ustuvor vazifalaridan kelib chiqqan holda, kadrlar tayyorlash mazmunini tubdan qayta tayyorlash, xalqaro standartlar darajasiga mos oliy ma`lumotli mutaxassislar tayyorlash uchun zarur sharoitlar yaratilishini ta`minlash maqsadida, O`zbekiston Respublikasi Prezidenti Sh.M.Mirziyoyevning “Oliy ta`lim tizimini yanada rivojlantirish chora – tadbirlari to`g`risida”gi PQ – 2909 sonli qarorlari chiqdi. Qaror 16 ta band va 75 ta ilovadan iborat. Qarorning asosiy maqsadi Oliy ta`lim tizimini tubdan takomillashtirish, mamlakatni ijtimoiy – qitisodiy rivojlantirishning ustuvor vazifalaridan kelib chiqqan holda, kadrlar tayyorlash mazmunini tubdan qayta ko`rish, xalqaro standartlar darajasiga mos oliy ma`lumotli mutaxassislar tayyorlash uchun zarur sharoitlar yaratilishini ta`minlashdan iborat.

Oliy ta`lim tizimida o`z yo`nalishlari bo`yicha dunyoning yetakchi ilmiy – ta`lim muassasalari bilan yaqin hamkorlik aloqalari o`rnatish, o`quv jarayoniga ilg`or xorijiy tajribalarni joriy etish, ayniqsa, istiqbolli pedagog va ilmiy kadrlarni xorijning yetakchi ilmiy ta`lim muassasalarida stajirovkadan o`tkazish va malakasini oshirish borasidagi ishlar talab darajasida emasligini, oliy ta`limda ishlovchi pedagog zamon bilan hamnafas bo`lishi kerakligi ta`kidlab o`tildi.

Biz iqtisodiy o`nglanish, iqtisodiy tiklanish, iqtisodiy rivojlanishni ma`naviy o`nglash, ma`naviy poklanish, ma`naviy yuksalish harakakatlari bilan tamomila uyg`un bo`lishni istayotgan bir davrda yashayapmiz. Bu harakatlar mamlakatimizning barcha jabhalarida mehnat qilayotgan kishilardan yuksak professional tayyorgarlikka, g`oyaviy – siyosiy barkamol, tashkilotchilikka va boshqaruvchilikka ega bo`lish lozimligini taqozo qilmoqda. Chunonchi, ta`lim sohasidagi tub islohotlar, yangilanayotgan, ta`lim – tarbiyaning mazmuni, shakli, usullari, vositalari milliylashib, o`zbekona urf – odatlar o`quv – tarbiya jarayoniga faol kirib borayotgan bir sharoitda yuz bermoqda. Bu o`zgarishlar har bir o`qituvchini yangicha fikrlashga, sharqona ish yuritishga, tadbirkorlikka, ishbilarmonlikka, ma`naviy – ma`rifiy ishlarning faol ishtirokchisi bo`lishga undaydi.

Shuni alohida ta`kidlash lozimki, ta`lim – tarbiya sohasida islohotlar o`tkazish va ularning asosiy yo`nalishlari, talablari va maqsadlarini aniqlash, hamda tegishli xulosalarni chiqarishda, bugungi muhokama qilinadigan hujjatlarda keng jamoatchiligimizning fikr – mulohazalari, tarbiya va izohlari ifoda topgan desak hech qanday mubolag`a bo`lmas. Shuning uchun ham amaldagi ta`lim – tarbiya tizimining zaif tomonlarini, zamon talablari, jamiyatimiz kelajagi va maqsadlariga javob bermaydigan jihatlarini chuqur tasavvur qilish, erkin, badavlat yashayotgan mamlakatlar tajribasini o`rganish, o`z o`lkamizga yuksak malakali, har jihatdan yetuk kadrlar tayyorlash dasturining asosiy sharti bo`lmog`i lozim. Shuning uchun ta`lim – tarbiya sohasida ham belgilanayotgan islohotlarni hayotga tatbiq qilishda islohotlarni bosqichma – bosqich o`tkazish prinsipi qo`yilgan.

Jumladan, O`zbekiston Respublikasining Prezidenti Sh.M.Mirziyoyevning 2018 – yil 28 – dekabrdagi “2019 – yil uchun mo`ljallangan eng muhim ustuvor vazifalar haqidagi Oliy Majlisga Murojaatnomasi”da davlatimiz rahbari “Rivojlangan mamlakatlarda ta`limning to`liq sikliga investitsiya kiritishga, ya`ni, bola 3 yoshdan 22 yoshgacha bo`lgan davrda uning tarbiyasiga sarmoya sarflashga katta e`tibor beriladi. Chunki ana shu sarmoya jamiyatga 15 – 17 barobar miqdorda foyda keltiradi. Bizda esa bu ko`rsatkich atigi 4 barobarni tashkil etadi” deb ta`kidladilar. Bu esa oliy ta`lim tizimida o‘qitish sistemasini isloh qilishni taqozo etadi. Shuning uchun zamon talabiga mos o’quv qo‘llanmalar yaratish milliy dasturni amalga oshirish vazifalaridan biridir. Matematika, informatika, axborot texnologiyalari o‘sib borayotgan yosh avlodni kamol toptirishda o‘quv fanlari sifatida keng imkoniyatlarga ega. Ular tafakkurni rivojlantirib, aqlni charxlaydi, fikrlashni tartibga soladi.

Matematik mutaxassislar, jumladan, matematika o’qituvchilari o’z fani va mutaxassisligini atroflicha egallagan va chuqur kasbiy ko’nikmalar hosil qilgan bo’lishi zarur. Shu nuqtai nazardan qaraganda, bakalavriatning «Matematika» yo’nalishini bitiruvchilariga Davlat standartida ko’zda tutilgan bilimlar hajmidan yuqoriroq bilim berish va ularni shu yo’l bilan ma’lum ma’noda ijodiy fikrlashni, ilmiy izlanishini va iqtidorlilarni rivojlantirish maqsadga muvofiq. Buni va yuqoridagilarni e’tiborga olsak, mazkur bitiruv malakaviy ishining mavzusi dolzarb deb hisoblash mumkin.

<u>Bitiruv malakaviy ishi mavzusining dolzarbligi.</u> Zamonaviy matematik fizikaning, xususan chiziqli operatorlar spektral nazariyasining ko`plab masalalari panjaradagi soni saqlanmaydigan chekli sondagi zarrachalar sistemasini tavsiflovchi operatorli matritsalar (Gamiltonian) ning spektral xossalarni o`rganish masalasiga keltiriladi. Jumladan uzluksiz fazodagi va panjaradagi qirqilgan spin –

bozon modelini o`rganishda operatorli matritsalar muhim ahamiyat kasb etadi. Shu nuqtai nazardan bitiruv malakaviy ishi mavzusi dolzarb hisoblanadi.

Bitiruv malakaviy ishining maqsadi. Bitiruv malakaviy ishining asosiy maqsadi ikkinchi, uchinchi va to`rtinchi tartibli operatorli matritsalarni mos ravishda Fok fazosining ikki, uch va to`rt zarrachali qism fazolaridagi chiziqli, chegaralangan va o`z – o`ziga qo`shma operator sifatida aniqlash, ularning muhim va diskret spektrlarini tadqiq qilish hamda rezolventa operatorlarini topishdan iboratdir.

Bitiruv malakaviy ishining vazifalari. Ishning asosiy vazifalari quyidagilardan iborat:

1) Chiziqli operatorlar spektral nazariyasining asosiy elementlarini o`rganish;
2) 2 – tartibli operatorli matritsaning muhim spektrini, Fredgolm determinantini, diskret spektrini va rezolventa operatorini aniqlash;
3) 2 – tartibli operatorli matritsa uchun olingan natijalar yordamida 3 – tartibli operatorli matritsaning muhim spektrini tavsiflash, diskret spektrini aniqlash, rezolventa operatorini qurish;
4) 2 – va 3 – tartibli operatorli matritsalarning spektrlari yordamida 4 – tartibli operatorli matritsaning muhim spektrini o`rganish, diskret spektri uchun formula hosil qilish, rezolventa operatorini topish.

Ta`kidlash joizki, 3 – va 4 – tartibli operatorli matritsalarning muhim spektrini aniqlash jarayonida Veyl kriteriyasi qo`llaniladi va qaralayotgan operatorlarning xos funksiyalari uchun Faddeyev tenglamalari qurilib, uning xarakteristik xossalaridan foydalaniladi.

**Bitiruv malakaviy ishining o`rganilganlik darajasi.** Bitiruv malakaviy ishi to`liq o`rganilgan. Unda olingan barcha natijalar qat`iy matematik isbotlangan.

Bitiruv malakaviy ishining predmeti. Matematik analiz, chiziqli algebra, funksional analiz va kompleks analiz usullaridan foydalanilgan.

Bitiruv malakaviy ishining obyekti. Fok fazosi, operatorli matritsalar, muhim spektr, diskret spektr, Fredgolm determinanti, rezolventa operatori.

Bitiruv malakaviy ishining ilmiy farazi. Ish ilmiy xarakterga ega bo`lib, n – tartibli (n=3,4) operatorli matritsalarning spektral xossalarini o`rganishda o`zidan pastki tartibli operatorli matritsalarning xossalaridan o`rganish g`oyasi ilgari surilgan.

Bitiruv malakaviy ishining amaliy ahamiyati. Bitiruv malakaviy ishida keltirilgan ma`lumotlardan bakalavriatning 3 – , 4 – bosqich talabalarini funksional analiz, funksional analizning tanlangan boblari fanlarini, magistraturaning matematika mutaxassisligi bo`yicha tahsil olayotgan magistrlarni “O`z – o`ziga qo`shma operatorlar”, “Operatorli matritsalarning spektral nazariyasi” fanlarini o`qitishda o`quv – uslubiy qo`llanma sifatida foydalanish mumkin. Bundan tashqari, undan operatorlarning spektral nazariyasi bo`yicha ilmiy – tadqiqot ishlari olib borayotgan iqtidorli talabalar, magistrlar, yosh olimlar foydalanishlari mumkin.

Bitiruv malakaviy ishining metodologik asosi. Chiziqli operatorlar spektral nazariyasi elementlarini operatorli matritsalarning muhim va diskret spektrlarini hamda rezolventa operatorini o`rganishga qo`llash.

Bitiruv malakaviy ishining metodlari. Bitiruv malakaviy ishida olingan natijalarni isbotlashda funksional va kompleks analiz usullaridan foydalanildi.

Bitiruv malakaviy ishining tarkibi va hajmi. Bitiruv malakaviy ishi mundarija, kirish, 2 ta bob, 5 ta paragraf, har bir bob uchun xulosa, xotima va foydalanilgan adabiyotlar ro`yxatidan iborat bo`lib, 66 sahifadan iborat.

I BOB. 2 – VA 3 – TARTIBLI OPERATORLI MATRITSALARNING SPEKTRI VA REZOLVENTA OPERATORI.

1.1. 2 – tartibli operatorli matritsalarning spektri va rezolventa operatori

$H_0 := \mathbb{C}$ - bir o`lchamli kompleks fazo, $H_1 := L_2(\mathbb{T}^d)$ - $\mathbb{T}^d$ da aniqlangan kvadrati bilan integrallanuvchi, (umuman olganda, kompleks qiymatli) funksiyalar Gilbert fazosi bo`lsin. H orqali H_0 va H_1 fazolarning to`g`ri yig`indisini belgilaymiz, ya`ni $H := H_0 \oplus H_1$.

Odatda, H Gilbert fazosiga Fok fazosining qirqilgan ikki zarrachali qism fazosi deyiladi. H Gilbert fazosining ixtiyoriy f elementi $f = (f_0, f_1)$ kabi tasvirlanadi, bunda $f_0 \in H_0, f_1 \in H_1$.

$f = (f_0, f_1) \in H$ elementning normasi

$$\|f\| = \sqrt{\|f_0\|_0^2 + \|f_1\|_1^2}$$

formula yordamida topiladi:

$$\|f_0\|_0 = |f_0|; \;\; \|f_1\|_1 = \sqrt{\int_{\mathbb{T}^d} |f_1(t)|^2 dt}.$$

Ikkita $f = (f_0, f_1), g = (g_0, g_1) \in H$ elementlarning skalyar ko`paytmasi

$$(f, g) = (f_0, g_0)_0 + (f_1, g_1)_1$$

kabi topiladi:

$$(f_0, g_0)_0 = f_0 \cdot \overline{g_0}\,, \;\; (f_1, g_1)_1 = \int_{\mathbb{T}^d} f_1(t)\overline{g_1(t)}dt.$$

H Gilbert fazosida quyidagi 2 – tartibli

$$A_2 := \begin{pmatrix} A_{00} & A_{01} \\ A_{01}^* & A_{22} \end{pmatrix} \qquad (1.1.1)$$

operatorli matritsani qaraymiz, bu yerda

$$A_{ij}: H_j \to H_i, i \leq j, i,j = 0,1$$

operatorlar quyidagicha aniqlangan:

$A_{00}f_0 = \varepsilon f_0, \ f_0 \in H_0;$

$A_{01}f_1 = \alpha \int_{\mathbb{T}^d} v(t) f_1(t) dt, \ f_1 \in H_1;$

$(A_{11}f_1)(x) = (\varepsilon + \omega(x)) f_1(x), \ f_1 \in H_1.$

Bunda, ε – haqiqiy son, $v(\cdot)$ va $\omega(\cdot)$ lar haqiqiy qiymatli uzluksiz funksiyalar, $\alpha > 0$ – esa "ta`sirlashuv parametri".

1.1.1 – Lemma. H Gilbert fazosida (1.1.1) formula orqali ta`sir qiluvchi 2 – tartibli $A$ operatorli matritsa chiziqli, chegaralangan va o`z – o`ziga qo`shma operator bo`ladi.

Isbot. Ixtiyoriy $a, b \in \mathbb{C}$ kompleks sonlar va ixtiyoriy $f = (f_0, f_1), g = (g_0, g_1) \in H$ elementlar uchun

$$A(af + bg) = aAf + bAg$$

tenglikni tekshirish orqali A_2 operatorli matritsaning chiziqli operator ekanini isbotlaymiz.

Dastlab,

$$Af = \begin{pmatrix} A_{00} & A_{01} \\ A_{01}^* & A_{11} \end{pmatrix} \begin{pmatrix} f_0 \\ f_1 \end{pmatrix} = \begin{pmatrix} A_{00}f_0 + A_{01}f_1 \\ A_{01}^* f_0 + A_{11}f_1 \end{pmatrix} = \begin{pmatrix} \varepsilon f_0 + \alpha \int_{\mathbb{T}^d} v(t) f_1(t) dt \\ \alpha v(x) f_0 + \big(\varepsilon + \omega(x)\big) f_1(x) \end{pmatrix}$$

ekanligini ta`kidlab o`tamiz.

$$A(af + bg) = \begin{pmatrix} \varepsilon(af_0 + bg_0) + \alpha \int_{\mathbb{T}^d} v(t)(af_1(t) + bg_1(t)) dt \\ \alpha v(x)(af_0 + bg_0) + \big(\varepsilon + \omega(x)\big)(af_1(x) + bg_1(x)) \end{pmatrix} =$$

$$\begin{pmatrix} \varepsilon a f_0 + \alpha \int_{\mathbb{T}^d} v(t) a f_1(t)dt + \varepsilon b f_0 + \alpha \int_{\mathbb{T}^d} v(t) b f_1(t)dt \\ \alpha v(x) a f_0 + \big(\varepsilon + \omega(x)\big) a f_1(x) + \alpha v(x) b f_0 + \big(\varepsilon + \omega(x)\big) b f_1(x) \end{pmatrix} =$$

$$= a \begin{pmatrix} \varepsilon f_0 + \alpha \int_{\mathbb{T}^d} v(t) f_1(t)dt \\ \alpha v(x) f_0 + \big(\varepsilon + \omega(x)\big) f_1(x) \end{pmatrix} + b \begin{pmatrix} \varepsilon g_0 + \alpha \int_{\mathbb{T}^d} v(t) g_1(t)dt \\ \alpha v(x) g_0 + \big(\varepsilon + \omega(x)\big) g_1(x) \end{pmatrix} =$$

$$= aAf + bAg.$$

Demak, A_2 chiziqli operator ekan.

Endi A_2 operatorni chegaralanganlikka tekshiramiz. Buning uchun shunday $C > 0$ soni topilib, ixtiyoriy $f \in H$ uchun

$$\|Af\| \le C\|f\|$$

tengsizlik bajarilishini ko`rsatamiz.

Buning uchun quyidagi munosabatlardan foydalanamiz:

$$\|f_0\|_0 = |f_0|,\ \|f_1\|_1^2 = \int_{\mathbb{T}^d} |f_1(t)|^2 dt,\ \|f\|^2 = \|f_0\|_0^2 + \|f_1\|_1^2;$$

$$\|Af\|^2 = \|(Af)_0\|_0^2 + \|(Af)_1\|_1^2;$$

$$\|(Af)_0\|_0^2 = \left|\varepsilon f_0 + \alpha \int_{\mathbb{T}^d} v(t) f_1(t)dt\right|^2,$$

$$\|(Af)_1\|_1^2 = \int_{\mathbb{T}^d} \left|\alpha v(x) f_0 + \big(\varepsilon + \omega(x)\big) f_1(x)\right|^2 dx,$$

$$\|Af\|^2 = \left|\varepsilon f_0 + \alpha \int_{\mathbb{T}^d} v(t) f_1(t)dt\right|^2 + \int_{\mathbb{T}^d} \left|\alpha v(x) f_0 + \big(\varepsilon + \omega(x)\big) f_1(x)\right|^2 dx.$$

So`ngra ikki haqiqiy qiymatli a va b soblar uchun

$$|a + b|^2 \le 2|a|^2 + 2|b|^2$$

tengsizlikdan foydalanamiz.

$$\|Af\|^2 = \left|\varepsilon f_0 + \alpha\int_{\mathbb{T}^d} v(t)f_1(t)dt\right|^2 + \int_{\mathbb{T}^d}\left|\alpha v(x)f_0 + \big(\varepsilon+\omega(x)\big)f_1(x)\right|^2 dx \le$$

$$2|\varepsilon f_0|^2 + 2\left|\alpha\int_{\mathbb{T}^d} v(t)f_1(t)dt\right|^2 + \int_{\mathbb{T}^d}\left(2|\alpha v(x)f_0|^2 + 2\left|\big(\varepsilon+\omega(x)\big)f_1(x)\right|^2\right)dx$$

$$= 2|\varepsilon|^2|f_0|^2 + 2|\alpha|^2\left|\int_{\mathbb{T}^d} v(t)f_1(t)dt\right|^2 +$$

$$+\int_{\mathbb{T}^d}(2|\alpha|^2|v(x)|^2|f_0|^2 + 2|\varepsilon+\omega(x)|^2|f_1(x)|^2)dx.$$

Endi yuqoridagi munosabatlar uchun quyidagi Koshi – Bunyakovskiy tengsizligini qo`llaymiz:

$$|\int f(x)g(x)dx|^2 \le \int|f(x)|^2dx \cdot \int|g(x)|^2dx.$$

$$\|Af\|^2 \le 2|\varepsilon|^2|f_0|^2 + 2|\alpha|^2\left|\int_{\mathbb{T}^d} v(t)f_1(t)dt\right|^2 + 2|\alpha|^2|f_0|^2\int_{\mathbb{T}^d}|v(x)|^2dx +$$

$$+2\int_{\mathbb{T}^d}|\varepsilon+\omega(x)|^2|f_1(x)|^2dx \le 2|\varepsilon|^2|f_0|^2 + 2|\alpha|^2\int_{\mathbb{T}^d}|v(t)|^2dt\cdot\int_{\mathbb{T}^d}|f_1(t)|^2dt$$

$$+2|\alpha|^2|f_0|^2\int_{\mathbb{T}^d}|v(x)|^2dx + 2\int_{\mathbb{T}^d}|\varepsilon+\omega(x)|^2|f_1(x)|^2dx.$$

$|\omega(x)| \le M$ munosabatdan foydalangan holda, quyidagiga ega bo`lamiz:

$$\|Af\|^2 \le 2|\varepsilon|^2\|f_0\|_0^2 + 2|\alpha|^2\|v\|^2\|f_1\|_1^2 + 2|\alpha|^2\|f_0\|_0^2\|v\|^2 + 2(\varepsilon+M)^2\|f_1\|_1^2 =$$

$$=\|f_0\|_0^2(2|\varepsilon|^2 + 2|\alpha|^2\|v\|^2) + \|f_1\|_1^2(2|\alpha|^2\|v\|^2 + 2(\varepsilon+M)^2);$$

Quyidagicha belgilash kiritamiz:

$$C^2 = max\{(2|\varepsilon|^2 + 2|\alpha|^2\|v\|^2);\ (2|\alpha|^2\|v\|^2 + 2(\varepsilon+M)^2)\}.$$

U holda

$$\|Af\|^2 \le C^2(\|f_0\|_0^2 + \|f_1\|_1^2) \Rightarrow \|Af\|^2 \le C^2\|f\|^2 \Rightarrow$$
$$\|Af\| \le C\|f\|.$$

Demak, A_2 chegaralangan operator ekan.

Endi A_2 operatorning o`z – o`ziga qo`shma ekanligini ko`rsatamiz, ya`ni

$f=(f_0,f_1), g=(g_0,g_1)\in H$ elementlar uchun $(Af,g)=(f,Ag)$ tenglik o`rinli bo`lishi kerak.

Ma`lumki,

$$(f_0,g_0)=f_0\cdot\overline{g_0}\,;\ (f_1,g_1)=\int_{\mathbb{T}^d} f_1(x)\overline{g_1(x)}dx$$

hamda

$$(Af,g)=((Af)_0,g_0)_0+((Af)_1,g_1)_1.$$

$$(Af,g)=\left(\varepsilon f_0+\alpha\int_{\mathbb{T}^d} v(t)f_1(t)dt\right)\overline{g_0}+\int_{\mathbb{T}^d}(\alpha v(x)f_0+\left(\varepsilon+\omega(x)\right)\cdot$$
$$f_1(x))\overline{g_1(x)}dx=\varepsilon f_0\overline{g_0}+\alpha\int_{\mathbb{T}^d} v(x)f_0\overline{g_1(x)}dx+\alpha\int_{\mathbb{T}^d} v(t)f_1(t)dt\cdot\overline{g_0}+$$
$$\int_{\mathbb{T}^d}\left(\varepsilon+\omega(x)\right)f_1(x)\overline{g_1(x)}dx=f_0\left(\overline{\varepsilon g_0+\alpha\int_{\mathbb{T}^d} v(t)g_1(t)dt}\right)+$$

$$+\int_{\mathbb{T}^d} f_1(x)\left(\overline{\alpha\int_{\mathbb{T}^d} v(t)g_0dt+\left(\varepsilon+\omega(x)\right)g_1(x)}\right)dx=(f_0,(Ag)_0)+(f_1,(Ag)_1)$$

$$=(f,Ag).$$

Demak, A_2 o`z – o`ziga qo`shma operator ekan.

1.1.1 – lemma to`liq isbotlandi.

1.1.1 – Teorema. A_2 operatorning muhim spektri uchun quyidagi tenglik o`rinlidir:

$$\sigma_{ess}(A_2)=[\varepsilon+m,\varepsilon+M],$$

bu yerda

$$m=\min_{x\in\mathbb{T}^d}\omega(x),\ M=\max_{x\in\mathbb{T}^d}\omega(x).$$

Isbot. 1.1.1 – teoremani isbotlash maqsadida $H_0\oplus H_1$ Gilbert fazosida

$$A_{2,0} := \begin{pmatrix} 0 & 0 \\ 0 & A_{11} \end{pmatrix}$$

kabi ta`sir qiluvchi operatorni qaraymiz.

U holda

$$A_2 - A_{2,0} = \begin{pmatrix} A_{00} & A_{01} \\ A_{01}^* & A_{22} \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & A_{11} \end{pmatrix} = \begin{pmatrix} A_{00} & A_{01} \\ A_{01}^* & 0 \end{pmatrix}$$

qo`zg`alish operatori qiymatlar sohasi

$$Im(A_2 - A_{2,0}) = \{f = (f_0, f_1) \in H_0 \oplus H_1 : f_0 = a, f_1(x) = \alpha v(x) a, a \in \mathbb{C}\}$$

kabi bo`ladi. Ushbu qism fazodan

$$f^{(1)} = (1,0)$$

$$f^{(2)} = (0, v(x))$$

elementlarni tanlasak, ular chiziqli bog`lanmagan hamda ixtiyoriy

$$f = (f_0, f_1) \in Im(A_2 - A_{2,0})$$

elementlar $f^{(1)}$ va $f^{(2)}$ elementlar orqali

$$f = af^{(1)} + \alpha a f^{(2)}$$

kabi tasvirlanadi. Shu sababli

$$dimIm(A_2 - A_{2,0}) = 2.$$

Chekli o`lchamli qo`zg`alishlarda muhim spektrning o`zgarmasligi haqidagi mashhur G.Veyl teoremasiga ko`ra, A_2 operatorning muhim spektri bilan ustma – ust tushadi.

Bizga yaxshi ma`lumki,

$$\sigma_{ess}\left(A_{2,0}\right) = [\varepsilon + m, \varepsilon + M].$$

Yuqoridagi fikr va mulohazalardan

$$\sigma_{ess}(A_2) = [\varepsilon + m, \varepsilon + M]$$

tenglik kelib chiqadi.

1.1.1 – teorema to`liq isbot bo`ldi.

$\mathbb{C} \backslash [\varepsilon + m, \varepsilon + M]$ sohada regulyar bo`lgan

$$\Omega_2(z) = \varepsilon - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(s)ds}{\varepsilon + \omega(s) - z}$$

funksiyani qaraymiz.

Quyidagi lemma A_2 operator xos qiymatlari va $\Omega_2(\cdot)$ funksiya nollari orasidagi bog`lanishni ifodalaydi.

1.1.2 – Lemma. $z \in \mathbb{C} \backslash \sigma_{ess}(A_2)$ soni A_2 operatorning xos qiymati bo`lishi uchun $\Omega_2(z) = 0$ bo`lishi zarur va yetarlidir.

Isbot. Faraz qilaylik, $z \in \mathbb{C} \backslash \sigma_{ess}(A_2)$ soni A_2 operator uchun xos qiymat, $f = (f_0, f_1) \in H_0 \oplus H_1$ esa unga mos xos vektor bo`lsin. U holda $A_2 f = zf$ tenglama yoki

$$\begin{cases} (\varepsilon - z) f_0 + \alpha \int_{\mathbb{T}^d} v(s) f_1(s) ds = 0 \\ \alpha v(x) f_0 + (\varepsilon + \omega(x) - z) f_1(x) = 0 \end{cases} \tag{1.1.2}$$

tenglamalar sistemasi nolmas $f = (f_0, f_1) \in H_0 \oplus H_1$ yechimga ega bo`ladi.

$z \notin [\varepsilon + m, \varepsilon + M]$ bo`lgani uchun barcha $x \in \mathbb{T}^d$ larda $\varepsilon + \omega(x) - z \neq 0$ munosabat o`rinli. Shuning uchun (1.1.2) tenglamalar sistemasining ikkinchi tenglamasidan $f_1(x)$ uchun

$$f_1(x) = -\frac{\alpha v(x) f_0}{\varepsilon + \omega(x) - z} \qquad (1.1.3)$$

ifodani topamiz. $f_1(x)$ uchun topilgan (1.1.3) ifodani (1.1.2) tenglamalar sistemasining birinchi tenglamasiga qo`yib,

$$\left(\varepsilon - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(s)\,ds}{\varepsilon + \omega(s) - z}\right) f_0 = 0,$$

ya`ni

$$\Omega_2(z) f_0 = 0 \qquad (1.1.4)$$

tenglikni hosil qilamiz. Agar (1.1.4) tenglikda $f_0 = 0$ bo`lsa, u holda (1.1.3) ga ko`ra, $f_1(x) = 0$, ya`ni $f = (f_0, f_1) = \theta$. Bu esa f ning xos vektor ekanligiga zid. Demak, (1.1.4) tenglikdan $\Omega_2(z) = 0$ kelib chiqar ekan.

1.1.2 – lemma to`liq isbotlandi.

1.1.2 – lemmadan quyidagi natija kelib chiqadi:

1.1.1 – natija. A_2 operatorning diskret spektri uchn quyidagi tenglik o`rinli:

$$\sigma_{disc}(A_2) = \{z \in \mathbb{C} \backslash [\varepsilon + m, \varepsilon + M] : \Omega_2 = 0\}.$$

Quyidagi lemma A_2 operator xos qiymatlari soni va joylashuv o`rnini tavsiflaydi.

1.1.3 – Lemma. A_2 operatorning $(-\infty;\ \varepsilon + m)$ oraliqda yotuvchi ko`pi bilan bitta oddiy xos qiymati mavjud.

Isbot. 1.1.2 – lemmaga ko`ra, A_2 operatorning xos qiymatlari $\Omega_2(\cdot)$ funksiya nollari orqali aniqlanadi. Shu sababli dastlab $\Omega_2(\cdot)$ funksiyani $(-\infty;\ \varepsilon + m)$ va $(\varepsilon + M;\ +\infty)$ oraliqlarda monotonlikka tekshiramiz. Ushbu

$$\frac{d\Omega(z)}{dz} = -1 - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(s)ds}{(\varepsilon + \omega(s) - m)^2} < 0$$

munosabat barcha $z \in (-\infty;\ \varepsilon + m) \cup (\varepsilon + M;\ +\infty)$ lar uchun o`rinlidir. Shu sababli $\Omega_2(\cdot)$ funksiya $(-\infty;\ \varepsilon + m)$ oraliqda ko`pi bilan bitta oddiy nolga ga, ya`ni 1.1.2 – lemmaga ko`ra A_2 operator $(-\infty;\ \varepsilon + m)$ oraliqda ko`pi bilan bitta oddiy xos qiymatga ega.

1.1.3 – lemma to`liq isbot bo`ldi.

Quyidagi lemma 1.1.3 – lemma kabi isbotlanadi.

1.1.4 – Lemma. A_2 operatorning $(\varepsilon + M;\ +\infty)$ oraliqda yotuvchi ko`pi bilan bitta oddiy xos qiymati mavjud.

1.1.3 – va 1.1.4 – lemmalarga asosan, $\sigma_{disc}(A_2)$ to`plam ko`pi bilan ikkita sondan iborat bo`lishi mumkin.

Demak,

$$\sigma(A_2) = [\varepsilon + m, \varepsilon + M] \cup \{z \in \mathbb{C}\backslash[\varepsilon + m, \varepsilon + M]: \Omega_2 = 0\}, \rho(A_2) = \mathbb{C}\backslash\sigma(A_2).$$

Endi A_2 operatorga mos rezolventa operatorini o`rganamiz.

1.1.5 – Lemma. Har bir fiksirlangan $z \in \rho(A_2)$ soni uchun A_2 operatorga mos rezolventa operatori $H_0 \oplus H_1$ Gilbert fazosida 2 – tartibli

$$R_\lambda(A_2) = \begin{pmatrix} R_{00}(\lambda;\ A_2) & R_{01}(\lambda;\ A_2) \\ R_{10}(\lambda;\ A_2) & R_{11}(\lambda;\ A_2) \end{pmatrix}$$

operatorli matritsa ko`rinishida ta`sir qiladi, bunda $R_{ij}(\lambda;\ A_2): H_j \to H_i; i, j = 0,1$ operatorlar quyidagi formulalar yordamida aniqlanadi:

$$R_{00}(\lambda;\ A_2)g_0 = \frac{g_0}{\Omega_2(z)};$$

$$R_{01}(\lambda;\ A_2)g_1 = -\frac{\alpha}{\Omega_2(z)}\int_{\mathbb{T}^d}\frac{v(s)g_1(s)ds}{\varepsilon+\omega(s)-z};$$

$$(R_{10}(\lambda;\ A_2)g_0)(x) = -\frac{\alpha v(x)g_0}{\Omega_2(z)(\varepsilon+\omega(x)-z)};$$

$$(R_{11}(\lambda;\ A_2)g_1)(x) = \frac{g_1(x)}{\varepsilon+\omega(x)-z} - \frac{\alpha^2 v(x)}{\Omega_2(z)(\varepsilon+\omega(x)-z)}\int_{\mathbb{T}^d}\frac{v(s)g_1(s)ds}{\varepsilon+\omega(s)-z}.$$

Isbot. A_2 operatorga mos rezolventa operatorini qurish maqsadida ixtiyoriy $f = (f_0, f_1), g = (g_0, g_1) \in H_0 \oplus H_1$ elementlar uchun

$$A_2 f - zf = g$$

tenglamani yoki

$$\begin{cases} (\varepsilon - z)f_0 + \alpha\int_{\mathbb{T}^d} v(s)f_1(s)ds = g_0 \\ \alpha v(x)f_0 + (\varepsilon + \omega(x) - z)f_1(x) = g_1(x) \end{cases} \qquad (1.1.5)$$

tenglamalar sistemasini qaraymiz. $z \notin [\varepsilon + m, \varepsilon + M]$ bolgani uchun ixtiyoriy $x \in \mathbb{T}^d$ uchun $\varepsilon + \omega(x) - z \neq 0$ munosabat o`rinli. Shu sababli (1.1.5) tenglamalar sistemasining ikkinchi tenglamasidan $f_1(x)$ uchun

$$f_1(x) = \frac{g_1(x)}{\varepsilon+\omega(x)-z} - \frac{\alpha v(x)f_0}{\varepsilon+\omega(x)-z} \qquad (1.1.6)$$

ifodani topamiz. $f_1(x)$ uchun topilgan (1.1.6) ifodani (1.1.5) sistemaning birinchi tenglamasiga qo`yamiz:

$$\left(\varepsilon - z - \alpha^2\int_{\mathbb{T}^d}\frac{v^2(s)\,ds}{\varepsilon+\omega(s)-z}\right)f_0 + \alpha\int_{\mathbb{T}^d}\frac{v(s)g_1(s)ds}{\varepsilon+\omega(s)-z} = g_0,$$

ya`ni

$$\Omega_2(z)f_0 + \alpha\int_{\mathbb{T}^d}\frac{v(s)g_1(s)ds}{\varepsilon+\omega(s)-z} = g_0 \qquad (1.1.7)$$

$z \notin \sigma_{disc}(A_2)$ bo`lgani uchun $\Omega_2(z) \neq 0$ bo`ladi. Shuning uchun (1.1.7) tenglikdan

$$f_0 = \frac{g_0}{\Omega_2(z)} - \frac{\alpha}{\Omega_2(z)} \int_{\mathbb{T}^d} \frac{v(s) g_1(s) ds}{\varepsilon + \omega(s) - z} \tag{1.1.8}$$

ni topamiz. f_0 uchun topilgan (1.1.8) ifodani (1.1.6) tenglikka qo`yamiz:

$$f_1(x) = \frac{g_1(x)}{\varepsilon + \omega(x) - z} - \frac{\alpha v(x) f_0}{\Omega_2(z)(\varepsilon + \omega(x) - z)} + \frac{\alpha^2 v(x)}{\Omega_2(z)(\varepsilon + \omega(x) - z)} \int_{\mathbb{T}^d} \frac{v(s) g_1(s) ds}{\varepsilon + \omega(s) - z}.$$

Shunday qilib,

$$\begin{cases} f_0 = R_{00}(\lambda;\, A_2) g_0 + R_{01}(\lambda;\, A_2) g_1 \\ f_1 = R_{10}(\lambda;\, A_2) g_0 + R_{11}(\lambda;\, A_2) g_1 \end{cases},$$

ya`ni $f = R_\lambda(A_2) g$ ekan.

1.1.5 – lemma to`liq isbotlandi.

$$R_{00}(\lambda;\, A_2) g_0 = \frac{g_0}{\Omega_2(z)};$$

$$R_{01}(\lambda;\, A_2) g_1 = -\frac{\alpha}{\Omega_2(z)} \int_{\mathbb{T}^d} \frac{v(s) g_1(s) ds}{\varepsilon + \omega(s) - z};$$

$$(R_{10}(\lambda;\, A_2) g_0)(x) = -\frac{\alpha v(x) g_0}{\Omega_2(z)(\varepsilon + \omega(x) - z)};$$

$$(R_{11}(\lambda;\, A_2) g_1)(x) = \frac{g_1(x)}{\varepsilon + \omega(x) - z} - \frac{\alpha^2 v(x)}{\Omega_2(z)(\varepsilon + \omega(x) - z)} \int_{\mathbb{T}^d} \frac{v(s) g_1(s) ds}{\varepsilon + \omega(s) - z}.$$

1.2. 3– tartibli operatorli matritsalarning spektri va rezolventa operatori

Faraz qilaylik, $H_0 := \mathbb{C}$, $H_1 := L_2(\mathbb{T}^d)$ va $H_2 := (L_2(\mathbb{T}^d)^2)$ hamda $H^{(3)} := H_0 \oplus H_1 \oplus H_2$ bo`lsin. Odatda, $H^{(3)}$ Gilbert fazosiga Fok fazosining qirqilgan uch zarrachali qism fazosi deyiladi.

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$$A_3 := \begin{pmatrix} A_{00} & A_{01} & 0 \\ A_{01}^* & A_{11} & A_{12} \\ 0 & A_{12}^* & A_{22} \end{pmatrix} \qquad (1.2.1)$$

formula orqali ta`sir qiluvchi A_3 3 – tartibli operatorli matritsani qaraymiz, bu yerda

$$A_{ij}: H_j \to H_i, i \leq j, i, j = 0,1,2, |i-j| \leq 1,$$

operatorlar quyidagi formulalar yordamida ta`sir qiladi:

$A_{00} f_0 = \varepsilon f_0, \ f_0 \in H_0;$

$A_{01} f_1 = \alpha \int_{\mathbb{T}^d} v(s) f_1(s) ds, \ f_1 \in H_1;$

$(A_{11} f_1)(x) = (\varepsilon + \omega(x)) f_1(x), \ f_1 \in H_1;$

$(A_{12} f_2)(x) = \alpha \int_{\mathbb{T}^d} v(s) f_2(x,s) ds, f_2 \in H_2;$

$(A_{22} f_2)(x,y) = (\varepsilon + \omega(x) + \omega(y)) f_2(x,y), \ f_2 \in H_2.$

Bunda, A_{01}^* va A_{12}^* operatorlar mos ravishda A_{01} va A_{12} operatorlarga qo`shma operatorlar bo`lib, ular quyidagicha aniqlanadi:

$(A_{01}^* f_0)(x) = \alpha v(x) f_0, f_0 \in H_0;$

$(A_{12}^* f_1)(x,y) = \alpha v(y) f_1(x), \ f_1 \in H_1.$

1.2.1 – Lemma. $H^{(3)}$ Gilbert fazosida (1.2.1) formula orqali ta`sir qiluvchi $A_3$ operator chiziqli, chegaralangan va o`z – o`ziga qo`shma operator bo`ladi.

Isbot. Dastlab, A_3 operatorli matritsaning chiziqli ekanligini isbotlaymiz. Buning uchun ixtiyoriy $a, b \in \mathbb{C}$ kompleks sonlar va ixtiyoriy $f = (f_0, f_1, f_2), g = (g_0, g_1, g_2) \in H^{(3)}$ elementlar uchun

$$A(af + bg) = aAf + bAg$$

tenglikni tekshiramiz.

$$Af = \begin{pmatrix} A_{00} & A_{01} & 0 \\ A_{01}^* & A_{11} & A_{12} \\ 0 & A_{12}^* & A_{22} \end{pmatrix} \begin{pmatrix} f_0 \\ f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} A_{00}f_0 + A_{01}f_1 \\ A_{01}^* f_0 + A_{11}f_1 + A_{12}f_2 \\ A_{12}^* f_1 + A_{22}f_2 \end{pmatrix} =$$

$$= \begin{pmatrix} \varepsilon f_0 + \alpha \int_{\mathbb{T}^d} v(s) f_1(s) ds \\ \alpha v(x) f_0 + \big(\varepsilon + \omega(x)\big) f_1(x) + \alpha \int_{\mathbb{T}^d} v(s) f_2(x, s) ds \\ \alpha v(y) f_1(x) + (\varepsilon + \omega(x) + \omega(y)) f_2(x, y) \end{pmatrix}.$$

$$A(af + bg) = \begin{pmatrix} \varepsilon a f_0 + \alpha \int_{\mathbb{T}^d} v(s) a f_1(s) ds \\ \alpha v(x) a f_0 + \big(\varepsilon + \omega(x)\big) a f_1(x) + \alpha \int_{\mathbb{T}^d} v(s) a f_2(x, s) ds \\ \alpha v(y) a f_1(x) + \big(\varepsilon + \omega(x) + \omega(y)\big) a f_2(x, y) \end{pmatrix} +$$

$$+ \begin{pmatrix} \varepsilon b f_0 + \alpha \int_{\mathbb{T}^d} v(s) b f_1(s) ds \\ \alpha v(x) b f_0 + \big(\varepsilon + \omega(x)\big) b f_1(x) + \alpha \int_{\mathbb{T}^d} v(s) b f_2(x, s) ds \\ \alpha v(y) b f_1(x) + \big(\varepsilon + \omega(x) + \omega(y)\big) b f_2(x, y) \end{pmatrix} =$$

$$= a \begin{pmatrix} \varepsilon f_0 + \alpha \int_{\mathbb{T}^d} v(s) f_1(s) ds \\ \alpha v(x) f_0 + \big(\varepsilon + \omega(x)\big) f_1(x) + \alpha \int_{\mathbb{T}^d} v(s) f_2(x, s) ds \\ \alpha v(y) f_1(x) + (\varepsilon + \omega(x) + \omega(y)) f_2(x, y) \end{pmatrix} +$$

$$+ b \begin{pmatrix} \varepsilon f_0 + \alpha \int_{\mathbb{T}^d} v(s) f_1(s) ds \\ \alpha v(x) f_0 + \big(\varepsilon + \omega(x)\big) f_1(x) + \alpha \int_{\mathbb{T}^d} v(s) f_2(x, s) ds \\ \alpha v(y) f_1(x) + (\varepsilon + \omega(x) + \omega(y)) f_2(x, y) \end{pmatrix} = aAf + bAg.$$

Demak, A_3 chiziqli operator ekan.

Endi A_3 operatorni chegaralanganlikka tekshiramiz. Buning uchun shunday $C > 0$ soni topilib, ixtiyoriy $f \in H^{(3)}$ uchun

$$\|Af\| \leq C\|f\|$$

tengsizlik bajarilishini ko`rsatamiz.

Buning uchun quyidagi munosabatlardan foydalanamiz:

$$\|f_0\|_0 = |f_0|, \ \|f_1\|_1^2 = \int_{\mathbb{T}^d} |f_1(t)|^2 dt, \ \|f_1\|_1^2 = \int_{\mathbb{T}^d} \int_{\mathbb{T}^d} |f_2(x,y)|^2 dxdy.$$

$$\|Af\|^2 = \|(Af)_0\|_0^2 + \|(Af)_1\|_1^2 + \|(Af)_2\|_2^2;$$

$$\|(Af)_0\|_0^2 = \left|\varepsilon f_0 + \alpha \int_{\mathbb{T}^d} v(s) f_1(s) ds\right|^2,$$

$$\|(Af)_1\|_1^2 = \int_{\mathbb{T}^d} \left|\alpha v(x) f_0 + \big(\varepsilon + \omega(x)\big) f_1(x) + \alpha \int_{\mathbb{T}^d} v(s) f_2(x,s) ds\right|^2 dx,$$

$$\|(Af)_2\|_2^2 = \int_{\mathbb{T}^d} \int_{\mathbb{T}^d} |\alpha v(y) f_1(x) + (\varepsilon + \omega(x) + \omega(y)) f_2(x,y)|^2 dxdy.$$

So`ngra, a, b, c haqiqiy sonlar uchun $|a+b|^2 \leq 2|a|^2 + 2|b|^2$

hamda
$|a+b+c|^2 \leq 3|a|^2 + 3|b|^2 + 3|c|^2$

tengsizliklardan foydalanamiz.

$$\|(Af)_0\|_0^2 = \left|\varepsilon f_0 + \alpha \int_{\mathbb{T}^d} v(s) f_1(s) ds\right|^2 \leq 2|\varepsilon f_0|^2 + 2\left|\alpha \int_{\mathbb{T}^d} v(s) f_1(s) ds\right|^2 =$$

$$= 2|\varepsilon|^2 |f_0|^2 + 2|\alpha|^2 \left|\int_{\mathbb{T}^d} v(s) f_1(s) ds\right|^2;$$

$$\|(Af)_1\|_1^2 = \int_{\mathbb{T}^d} \left|\alpha v(x) f_0 + \big(\varepsilon + \omega(x)\big) f_1(x) + \alpha \int_{\mathbb{T}^d} v(s) f_2(x,s) ds\right|^2 dx \leq$$

$$\leq \int_{\mathbb{T}^d}(3|\alpha v(x) f_0|^2 + 3\left|\left(\varepsilon + \omega(x)\right) f_1(x)\right|^2 + 3\left|\alpha\int_{\mathbb{T}^d} v(s) f_2(x,s)ds\right|^2)dx \leq$$

$$\leq 3|\alpha|^2|f_0|^2\int_{\mathbb{T}^d}|v(x)|^2dx + 3|\varepsilon + M|^2\int_{\mathbb{T}^d}|f_1(x)|^2dx +$$

$$+3|\alpha|^2\left|\int_{\mathbb{T}^d} v(s) f_2(x,s)ds\right|^2 dx;$$

$$\|(Af)_2\|_2^2 = \int_{\mathbb{T}^d}\int_{\mathbb{T}^d}|\alpha v(y) f_1(x) + (\varepsilon + \omega(x) + \omega(y)) f_2(x,y)|^2 dxdy \leq$$

$$\leq 2\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}|\alpha v(y) f_1(x)|^2 dxdy + 2\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}|(\varepsilon + \omega(x) + \omega(y)) f_2(x,y)|^2 dxdy \leq$$

$$\leq 2|\alpha|^2\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}|v(y) f_1(x)|^2 dxdy + 2|\varepsilon + 2M|^2\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}|f_2(x,y)|^2 dxdy.$$

Endi Koshi – Bunyakovskiy tengsizligi deb ataluvchi

$$|\int f(x)g(x)dx|^2 \leq \int|f(x)|^2dx \cdot \int|g(x)|^2dx$$

tengsizlikdan foydalansak, quyidagilarga ega bo`lamiz:

$$\|(Af)_0\|_0^2 \leq 2|\varepsilon|^2|f_0|^2 + 2|\alpha|^2\int_{\mathbb{T}^d}|v(s)|^2ds\int_{\mathbb{T}^d}|f_1(s)|^2ds =$$

$$= 2|\varepsilon|^2\|f_0\|^2 + 2|\alpha|^2\|v\|^2\|f_1\|^2.$$

$$\|(Af)_1\|_1^2 \leq 3|\alpha|^2|f_0|^2\int_{\mathbb{T}^d}|v(x)|^2dx + 3|\varepsilon + M|^2\int_{\mathbb{T}^d}|f_1(x)|^2dx +$$

$$+3|\alpha|^2\int_{\mathbb{T}^d}|v(s)|^2ds\int_{\mathbb{T}^d}|f_2(x,s)|^2dx.$$

$$\|(Af)_2\|_2^2 \leq 2|\alpha|^2\int_{\mathbb{T}^d}|v(y)|^2dy\int_{\mathbb{T}^d}|f_1(x)|^2dx +$$

$$+2|\varepsilon + 2M|^2\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}|f_2(x,y)|^2dxdy.$$

Natijada,

$$\|(Af)_0\|_0^2 \leq 2|\varepsilon|^2\|f_0\|^2 + 2|\alpha|^2\|v\|^2\|f_1\|^2,$$

$$\|(Af)_1\|_1^2 \leq 3|\alpha|^2\|f_0\|^2\|v\|^2 + 3|\varepsilon + M|^2\|f_1\|^2 + 3|\alpha|^2\|v\|^2\|f_2\|^2,$$

$$\|(Af)_2\|_2^2 \le 2|\alpha|^2\|v\|^2\|f_1\|^2 + 2|\varepsilon + 2M|^2\|f_2\|^2,$$

so`ngra,

$$\|Af\|^2 = \|(Af)_0\|_0^2 + \|(Af)_1\|_1^2 + \|(Af)_2\|_2^2 \le$$

$$\le \|f_0\|^2(2|\varepsilon|^2 + 3|\alpha|^2\|v\|^2) + \|f_1\|^2(2|\alpha|^2\|v\|^2 + 3|\varepsilon + M|^2 + 2|\alpha|^2\|v\|^2) +$$

$$+\|f_2\|^2(3|\alpha|^2\|v\|^2 + 2|\varepsilon + 2M|^2);$$

hosil bo`ladi.

$$C^2 = max\{(2|\varepsilon|^2 + 3|\alpha|^2\|v\|^2), (2|\alpha|^2\|v\|^2 + 3|\varepsilon + M|^2 + 2|\alpha|^2\|v\|^2), (3|\alpha|^2\|v\|^2 + 2|\varepsilon + 2M|^2)\}$$

deb belgilash kiritamiz. U holda,

$$\|Af\|^2 \le C^2(\|f_0\|^2 + \|f_1\|^2 + \|f_2\|^2) = C^2\|f\|^2 \Rightarrow \|Af\| \le C\|f\|.$$

Demak, A_3 chegaralangan operator ekan.

Endi A_3 operatorning o`z – o`ziga qo`shma operator ekanligini ko`rsatamiz, ya`ni $f = (f_0, f_1, f_2), g = (g_0, g_1, g_2) \in H^{(3)}$ elementlar uchun $(Af, g) = (f, Ag)$ tenglik o`rinli bo`lishi kerak.

Bizga ma`lumki,

$$(f_0, g_0) = f_0 \cdot \overline{g_0};\ (f_1, g_1) = \int_{\mathbb{T}^d} f_1(x)\overline{g_1(x)}dx;$$

$$(f_2, g_2) = \int_{\mathbb{T}^d}\int_{\mathbb{T}^d} f_2(x, y)\overline{g_2(x, y)}dxdy$$

hamda

$$(Af, g) = ((Af)_0, g_0)_0 + ((Af)_1, g_1)_1 + ((Af)_2, g_2)_2.$$

$$(Af, g) = \left(\varepsilon f_0 + \alpha\int_{\mathbb{T}^d} v(s)f_1(s)ds\right)\overline{g_0} + \int_{\mathbb{T}^d}(\alpha v(x)f_0 + \left(\varepsilon + \omega(x)\right)\cdot f_1(x) +$$

$$+\alpha\int_{\mathbb{T}^d} v(s) f_2(x,s)ds\overline{g_1(x)}dx + \int_{\mathbb{T}^d}\int_{\mathbb{T}^d}(\alpha v(y) f_1(x) + (\varepsilon + \omega(x) + \omega(y))\cdot$$

$$\cdot f_2(x,y))\overline{g_2(x,y)}dxdy = f_0\,(\overline{\varepsilon g_0 + \alpha\int_{\mathbb{T}^d} v(s) g_1(s)ds}) +$$

$$+\int_{\mathbb{T}^d} f_1(x)\left(\overline{\alpha\int_{\mathbb{T}^d} v(s) g_0 ds + \big(\varepsilon + \omega(x)\big)g_1(x) + \int_{\mathbb{T}^d}\alpha v(y)dy}\right)dx +$$

$$+\int_{\mathbb{T}^d}\int_{\mathbb{T}^d} f_2(x,y)\left(\overline{\alpha v(s) g_1(x) + \big(\varepsilon + \omega(x) + \omega(y)\big)g_2(x,y)}\right)dxdy =$$

$$(f_0, (Ag)_0) + (f_1, (Ag)_1) + (f_2, (Ag)_2) = (f, Ag).$$

Demak, A_3 o`z – o`ziga qo`shma operator ekan.

1.2.1 – lemma to`liq isbotlandi.

1.2.1 – Teorema. A_3 operatorning muhim spektri uchun

$$\sigma_{ess}(A_2) = [-\varepsilon + 2m, -\varepsilon + 2M] \cup \bigcup_{x\in\mathbb{T}^d}\{\omega(x) + \sigma_{disc}(A_2)\}$$

tenglik o`rinlidir.

Odatda,

$[-\varepsilon + 2m, -\varepsilon + 2M]$ va $\bigcup_{x\in\mathbb{T}^d}\{\omega(x) + \sigma_{disc}(A_2)\}$ to`plamlarga mos ravishda A_3 operator muhim spektrining mos ravishda uch va ikki zarrachali qism fazolari deyiladi.

Isbot. Avvalo, $[-\varepsilon + 2m, -\varepsilon + 2M] \subset \sigma_{ess}(A_3)$ munosabatni isbotlaymiz.

Faraz qilaylik, $z_0 \in [-\varepsilon + 2m, -\varepsilon + 2M]$ ixtiyoriy nuqta bo`lsin. $z_0 \in \sigma_{ess}(A_3)$ ekanligini isbotlaymiz. Bunda biz Veyl kriteriyasidan foydalanamiz:

H Gilbert fazosi va $A\colon H \to H$ chiziqli chegaralangan va o`z – o`ziga qo`shma operator bo`lsin. z_0 soni $\sigma_{ess}(A)$ ga tegishli bo`lishi uchun shunday $\{f^{(n)}\} \subset H$ ortonormal vektor funksiyalar ketma – ketligi topilib,

$$\|(A - z_0)f^n\| \to 0, n \to \infty$$

bo`lishi zarur va yetarlidir.

$\omega(\cdot) - \mathbb{T}^d$ dagi uzluksiz funksiya bo`lgani uchun shunday $p_0, q_0 \in \mathbb{T}^d$ nuqtalar topilib, $z_0 = -\varepsilon + \omega(p_0) + \omega(q_0)$ tenglik o`rinli bo`ladi. $(p_0, q_0) \in (\mathbb{T}^d)^2$ nuqtaning quyidagi atrofini qaraymiz:

$$W_n(p_0, q_0) := V_n(p_0) \times V_n(q_0),$$

bu yerda

$$V_n(p_0) := \left\{x \in \mathbb{T}^d : \frac{1}{n+1} < |x - p_0| < \frac{1}{n}\right\}, n \in N, p_0 \in \mathbb{T}^d$$

nuqtaning o`yilgan atrofi $\mu(W_n(p_0, q_0))$ orqali $W_n(p_0, q_0)$ to`plamning Lebeg o`lchovini belgilaymiz.

$f^{(n)}$ ortonormal vektor – funksiyalar ketma – ketligini ushbu ko`rinishda tanlaymiz:

$$f^{(n)} := \begin{pmatrix} 0 \\ 0 \\ f_2^{(n)}(x, y) \end{pmatrix},$$

$$f_2^{(n)}(x, y) := \begin{cases} \dfrac{\chi_{W_n(p_0, q_0)}(p, q)}{\sqrt{\mu(W_n(p_0, q_0))}}, (x, y) \in W_n(p_0, q_0) \\ 0, (x, y) \notin W_n(p_0, q_0) \end{cases}.$$

$n \neq m$ lar uchun $\left(f_2^{(n)}, f_2^{(m)}\right)$ ni hisoblaymiz:

$$\left(f_2^{(n)}, f_2^{(m)}\right) = \int_{\mathbb{T}^d} \int_{\mathbb{T}^d} f_2^{(n)}(x, y) \overline{f_2^{(m)}(x, y)} dxdy =$$

$$= \int_{W_n(p_0, q_0)} f_2^{(n)}(x, y) \overline{f_2^{(m)}(x, y)} dxdy +$$

$$+\int_{W_m(p_0,q_0)} f_2^{(n)}(x,y) f_2^{(m)}(x,y)dxdy+$$

$$+\int_{\mathbb{T}^3\setminus(W_n(p_0,q_0)\cup W_m(p_0,q_0))} f_2^{(n)}(x,y) f_2^{(m)}(x,y)dxdy=0$$

tenglik o`rinli, chunki $n\neq m$ uchun $W_n(p_0,q_0)\cap W_m(p_0,q_0)=\emptyset$. Demak, $\left\{f_2^{(n)}\right\}$ – ortogonal sistema ekan.

Aniqlanishiga ko`ra,

$$\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\left|f_2^{(n)}(x,y)\right|^2 dxdy=\int_{V_n(p_0)}\int_{V_n(q_0)}\frac{dxdy}{\mu(V_n(p_0))\mu(V_n(q_0))}=1.$$

Shunday qilib, $\left\{f_2^{(n)}\right\}$ ortonormal sistema ekan. Bundan esa, $\left\{f^{(n)}\right\}$ ortonormal sistema bo`lishi kelib chiqadi.

Endi $\left\|(A_3-z_0)f^{(n)}\right\|$ ni qaraymiz va uni baholaymiz:

$$\left\|(A_3-z_0)f^{(n)}\right\|^2=\alpha^2\int_{\mathbb{T}^d}\left|\int_{\mathbb{T}^d} v(t) f_2^{(n)}(x,t)dt\right|^2 dx+$$

$$+\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}|-\varepsilon+\omega(x)+\omega(y)-z_0|^2\cdot\left|f_2^{(n)}(x,y)\right|^2 dxdy\leq$$

$$\leq\alpha^2\left[\max_{x\in V_n(p_0)} v(x)\right]^2\mu\big(V_n(p_0)\big)+\sup_{(x,y)\in W_n(p_0,q_0)}|-\varepsilon+\omega(x)+\omega(y)-z_0|^2.$$

Qurilishiga ko`ra, $\mu(V_n)\to 0, n\to\infty$ $\omega(\cdot)$ funksiyaning uzluksizligiga ko`ra,

$$\sup_{(x,y)\in W_n(p_0,q_0)}|-\varepsilon+\omega(x)+\omega(y)-z_0|^2\to 0, n\to\infty.$$

Shunday qilib,

$$\left\|(A_3-z_0)f^{(n)}\right\|^2\to 0, n\to\infty$$

ekan. Veyl kriteriyasiga ko`ra, $z_0 \in \sigma_{ess}(A_2)$. Endi $z_0$ nuqtaning ixtiyoriyligiga ko`ra,

$$[-\varepsilon + 2m, -\varepsilon + 2M] \subset \sigma_{ess}(A_3)$$

munosabatni hosil qilamiz.

Endi

$$\cup_{x \in \mathbb{T}^d} \{\omega(x) + \sigma_{disc}(A_2)\} \subset \sigma_{ess}(A_3)$$

ekanligini isbotlaymiz. Faraz qilaylik, $z_0 \in \cup_{x \in \mathbb{T}^d} \{\omega(x) + \sigma_{disc}(A_2)\}$ – ixtiyoriy nuqta bo`lsin. U holda shunday $x_0 \in \mathbb{T}^d$ nuqta topilib,

$$z_0 = \omega(x_0) + \lambda_0, \lambda_0 \in \sigma_{disc}(A_2)$$

tenglik o`rinli bo`ladi. Ta`rifga ko`ra, shunday $\psi = (\psi_0, \psi_1) \in H_0 \oplus H_1$ funksiya topilib,

$$(A_2 - \lambda_0)\psi = 0$$

bo`ladi. $f^{(n)} = (0, f_1^{(n)}, f_2^{(n)}) \in H_0 \oplus H_1 \oplus H_2$ vektor – funksiyalar ketma – ketligini quyidagicha tanlaymiz:

$$f_1^{(n)}(x) := \frac{\chi_{V_n(p_0)}(p)}{\sqrt{\mu(V_n(p_0))}} \cdot \frac{\psi_0}{\|\psi\|},$$

$$f_2^{(n)}(x, y) := \frac{\chi_{V_n(p_0)}(p)}{\sqrt{\mu(V_n(p_0))}} \cdot \frac{\psi_1(q)}{\|\psi\|}.$$

Bunda $\chi_{V_n(p_0)}(\cdot) - V_n(p_0)$ to`plamning xarakteristik funksiyasi. $V_n(p_0)$ ning qurilishiga va $f^{(n)}$ ning tanlanishiga ko`ra $\{f^{(n)}\}$ ortonormal sistema bo`ladi. U holda

$$\left\|(A_3 - z_0)f^{(n)}\right\|^2 \le \alpha^2 \left[\max_{x \in V_n(p_0)} v(x)\right]^2 \mu\big(V_n(p_0)\big) + \frac{1}{\mu\big(V_n(p_0)\big)} \cdot$$

$$\cdot \int_{V_n(p_0)} |\omega(x) - \omega(p_0) + (A_2 - z_0)\psi|^2 dx \to 0, n \to \infty.$$

Demak, $z_0 \in \sigma_{ess}(A_3)$ ekan. z_0 nuqtaning ixtiyoriyligiga ko`ra,

$$\cup_{x \in \mathbb{T}^d}\{\omega(x) + \sigma_{disc}(A_2)\} \subset \sigma_{ess}(A_3)$$

ekan.

Endi teskari munosabatni isbotlaymiz.

Teskari munosabatni isbotlash uchun avvalo, Faddeyev tenglamasini quramiz. Shu maqsadda A_3 operator uchun $A_3 f = zf$ tenglamani qaraymiz. Bu tenglamani

$$\begin{cases} (\varepsilon - z)f_0 + \alpha \int_{\mathbb{T}^d} v(s) f_1(s) ds = 0 \\ \alpha v(x) f_0 + (\varepsilon + \omega(x) - z) f_1(x) + \alpha \int_{\mathbb{T}^d} v(s) f_2(x,s) ds = 0 \\ \alpha v(y) f_1(x) + (\varepsilon + \omega(x) + \omega(y) - z) f_2(x,y) = 0 \end{cases} \qquad (1.2.2)$$

kabi yozish mumkin.

(1.2.2) tenglamalar sistemasinng uchinchi tenglamasidan $f_2(x,y)$ ni topamiz:

$$f_2(x,y) = -\frac{\alpha v(y) f_1(x)}{\varepsilon + \omega(x) + \omega(y) - z} \qquad (1.2.3)$$

Topilgan (1.2.3) ifodani (1.2.2) tenglamalar sistemasining ikkinchi tenglamasiga qo`yamiz:

$$\begin{cases} (\varepsilon - z)f_0 + \alpha \int_{\mathbb{T}^d} v(s) f_1(s) ds = 0 \\ \alpha v(x) f_0 + (\varepsilon + \omega(x) - z) f_1(x) - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(s) f_1(x) ds}{\varepsilon + \omega(x) + \omega(s) - z} = 0 \end{cases} \qquad (1.2.4)$$

(1.2.4) tenglamalar sistemasini quyidagi ko`rinishda yozib olamiz:

$$\begin{cases} (\varepsilon - z + 1)f_0 + \alpha\int_{\mathbb{T}^d} v(s)f_1(s)ds = f_0 \\ f_1(x) = -\frac{\alpha v(x) f_0}{\Delta_1(x,z)} \end{cases},$$

bu yerda, $\Delta_1(x,z) = \varepsilon + \omega(x) - z - \alpha^2\int_{\mathbb{T}^d} \frac{v^2(s)ds}{\varepsilon+\omega(x)+\omega(s)-z}$.

Endi $\mathbb{C}\bigoplus L_2(\mathbb{T}^d)$ da ta`sir qiluvchi

$$T(z) = \begin{pmatrix} T_{00}(z) & T_{01}(z) \\ T_{10}(z) & 0 \end{pmatrix}$$

operatorni qaraymiz, bunda

$$T_{00}(z)f_0 = (\varepsilon - z + 1)f_0,$$

$$T_{01}(z)f_1 = \alpha\int_{\mathbb{T}^d} v(s)f_1(s)ds,$$

$$(T_{10}(z)f_0)(x) = -\frac{\alpha v(x) f_0}{\Delta_1(x,z)}.$$

Shunday qilib, quyidagi tasdiq isbotlandi:

1.2.1 – Tasdiq. z soni A_3 operatorning xos qiymati bo`lishi uchun $1$ soni $T(z)$ operatorning xos qiymati bo`lishi zarur va yetarlidir.

$T_{00}(z)$, $T_{01}(z)$ va $T_{10}(z)$ operatorlar bir o`lchamli operatorlar bo`lganligi uchun $T(z)$ kompakt qiymatli analitik funksiya bo`ladi. $A_3$ operatorning o`z – o`ziga qo`shmaligiga ko`ra,

$$\left(I - T(z)\right)^{-1}$$

operator qiymatli funksiya barcha $Imz \neq 0$ larda mavjud bo`ladi.

Fredgolmning analitik teoremasiga ko`ra,

$$\left(I - T(z)\right)^{-1}$$

operator qiymatli funksiya

$$\mathbb{C}\backslash\{[-\varepsilon+2m,-\varepsilon+2M]\cup\bigcup_{x\in\mathbb{T}^d}\{\omega(x)+\sigma_{disc}(A_2)\}\}$$

to`plamning chekli rangli qoldiqlariga ega bo`ladigan S diskret to`plamdan boshqa hamma yerda mavjud bo`ladi. Shu sababli

$$\sigma(A_3)\backslash\{[-\varepsilon+2m,-\varepsilon+2M]\cup\bigcup_{x\in\mathbb{T}^d}\{\omega(x)+\sigma_{disc}(A_2)\}\}$$

to`plam faqat yakkalangan nuqtalardan iborat bo`lib, ularning limitik nuqtalari faqat

$$[-\varepsilon+2m,-\varepsilon+2M]\cup\bigcup_{x\in\mathbb{T}^d}\{\omega(x)+\sigma_{disc}(A_2)\}$$

to`plamning chegaralari bo`lishi mumkin.

Demak,

$$\sigma(A_3)\backslash\{[-\varepsilon+2m,-\varepsilon+2M]\cup\bigcup_{x\in\mathbb{T}^d}\{\omega(x)+\sigma_{disc}(A_2)\}\}\subset\sigma(A_3)\backslash\sigma_{ess}(A_3)$$

munosabat o`rinli bo`ladi, ya`ni

$$\sigma_{ess}(A_3)\subset\{[-\varepsilon+2m,-\varepsilon+2M]\cup\bigcup_{x\in\mathbb{T}^d}\{\omega(x)+\sigma_{disc}(A_2)\}\}.$$

Xulosa qilib aytganda,

$$\sigma_{ess}(A_3)=[-\varepsilon+2m,-\varepsilon+2M]\cup\bigcup_{x\in\mathbb{T}^d}\{\omega(x)+\sigma_{disc}(A_2)\}.$$

1.2.1 – teorema to`liq isbot bo`ldi.

1.2.2 – Lemma. $z\in\mathbb{C}\backslash\sigma_{ess}(A_3)$ soni A_3 operatorning xos qiymati bo`lishi uchun $\Omega_3(z)=0$ bo`lishi zarur va yetarlidir.

Isbot. Faraz qilaylik, $z\in\mathbb{C}\backslash\sigma_{ess}(A_3)$ soni A_3 operator uchun xos qiymat bo`lsin. $(f_0,f_1,f_2)\in H_0\oplus H_1\oplus H_2$ vektor esa bu xos qiymatga mos xos vektor bo`lsin. U holda $A_3 f = zf$ tenglama yagona nolmas $(f_0, f_1, f_2) \in H_0 \oplus H_1 \oplus H_2$ yechimga ega bo`ladi.

$$\begin{cases} (\varepsilon - z) f_0 + \alpha \int_{\mathbb{T}^d} v(s) f_1(s) ds = 0 \\ \alpha v(x) f_0 + (\varepsilon + \omega(x) - z) f_1(x) + \alpha \int_{\mathbb{T}^d} v(s) f_2(x,s) ds = 0 \\ \alpha v(y) f_1(x) + (\varepsilon + \omega(x) + \omega(y) - z) f_2(x,y) = 0 \end{cases} \qquad (1.2.2)$$

tenglamalar sistemasiga ega bo`lamiz. Bu sistemaning uchinchi tenglamasidan $f_2(x,y)$ funksiyani topamiz va ikkinchi tenglamasiga qo`yamiz:

$$f_2(x,y) = -\frac{\alpha v(y) f_1(x)}{\varepsilon + \omega(x) + \omega(y) - z}.$$

$$\alpha v(x) f_0 + \left(\varepsilon + \omega(x) - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(s) ds}{\varepsilon + \omega(x) + \omega(s) - z}\right) \cdot f_1(x) = 0.$$

Oxirgi tenglikdan esa $f_1(x)$ funksiyani topamiz:

$$f_1(x) = -\frac{\alpha v(x) f_0}{\varepsilon + \omega(x) - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(s) ds}{\varepsilon + \omega(x) + \omega(s) - z}}.$$

Endi $\Delta_1(x,z) = \varepsilon + \omega(x) - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(s) ds}{\varepsilon + \omega(x) + \omega(s) - z}$ deb belgilash kiritamiz. Bunda, $\Delta_1(x,z) \neq 0$. U holda $f_1(x) = -\frac{\alpha v(x) f_0}{\Delta_1(x,z)}$.

Topilgan $f_1(x)$ va $f_2(x,y)$ funksiyalarni (1.2.2) sistemaning birinchi tenglamasiga qo`yib, $f_0$ ning ham ko`rinishini aniqlaymiz.

$$\left(\varepsilon - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(s) ds}{\Delta_1(s,z)}\right) f_0 = 0.$$

$$\Omega_3(z) = \varepsilon - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(s) ds}{\Delta_1(s,z)}$$

deb belgilaymiz, ya`ni $\Omega_3(z) f_0 = 0$ tenglik hosil bo`ladi. Agar bu tenglikda $f_0 = 0$ bo`lsa, u holda $f_1(x) = 0$ bo`ladi. Bundan esa $f_2(x,y) = 0$ kelib chiqadi.

Ya`ni $f=(f_0,f_1,f_2)=\theta$. Bu esa f ning xos vektor ekanligiga ziddir. Demak, $\Omega_3(z)=0$ ekanligi kelib chiqar ekan.

Shunday qilib, $z\in\mathbb{C}\backslash\sigma_{ess}(A_3)$ soni A_3 operatorning xos qiymati bo`lishi uchun $\Omega_3(z)=0$ bo`lishi zarur va yetarlidir.

1.2.2 – lemma to`liq isbotlandi.

Endi A_3 operatorga mos rezolventa operatorini qaraymiz:

1.2.3 – Lemma. Har bir fiksirlangan $z\in\rho(A_3)$ soni uchun A_3 operatorga mos rezolventa operatori $H_0\oplus H_1\oplus H_2$ Gilbert fazosida 3 – tartibli

$$R_\lambda(A_3)=\begin{pmatrix} R_{00}(\lambda,A_3) & R_{01}(\lambda,A_3) & R_{02}(\lambda,A_3)\\ R_{10}(\lambda,A_3) & R_{11}(\lambda,A_3) & R_{12}(\lambda,A_3)\\ R_{20}(\lambda,A_3) & R_{21}(\lambda,A_3) & R_{22}(\lambda,A_3)\end{pmatrix}$$

operatorli matritsa ko`rinishida ta`sir qiladi, bunda $R_{ij}(\lambda;\ A_3): H_j\to H_i; i,j=0,1,2.$

Isbot. A_3 operatorga mos rezolventa operatorini qurish uchun ixtiyoriy $f=(f_0,f_1,f_2)$ va $g=(g_0,g_1,g_2)\in H_0\oplus H_1\oplus H_2$ elementlar uchun

$$A_3f-zf=g$$

tenglamani yechish kerak bo`ladi. Ya`ni quyidagi tenglamalar sistemasini qaraymiz:

$$\begin{cases} (\varepsilon-z)f_0+\alpha\int_{\mathbb{T}^d}v(s)f_1(s)ds=g_0\\ \alpha v(x)f_0+(\varepsilon+\omega(x)-z)f_1(x)+\alpha\int_{\mathbb{T}^d}v(s)f_2(x,s)ds=g_1(x)\\ \alpha v(y)f_1(x)+(\varepsilon+\omega(x)+\omega(y)-z)f_2(x,y)=g_2(x,y)\end{cases}\qquad(1.2.5)$$

$z\notin\sigma_{ess}(A_3)$ bo`lganligidan ixtiyoriy $x,y\in\mathbb{T}^d$ uchun

$$\varepsilon+\omega(x)-z\neq 0 \text{ va } \varepsilon+\omega(x)+\omega(y)-z\neq 0$$

munosabatlar o`rinli bo`ladi.

Endi (1.2.5) tenglamalar sistemasining uchinchi tenglamasidan $f_2(x,y)$ ni topamiz:

$$f_2(x,y) = \frac{g_2(x,y)}{\varepsilon+\omega(x)+\omega(y)-z} - \frac{\alpha v(y) f_1(x)}{\varepsilon+\omega(x)+\omega(y)-z} \qquad (1.2.6)$$

Ushbu ifodani (1.2.5) tenglamalar sistemasining ikkinchi tenglamasiga qo`yamiz:

$$\alpha v(x) f_0 + \left(\varepsilon + \omega(x) - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(s)\,ds}{\varepsilon+\omega(x)+\omega(s)-z}\right) f_1(x) +$$

$$+\alpha \int_{\mathbb{T}^d} \frac{v(s) g_2(x,s)\,ds}{\varepsilon+\omega(x)+\omega(s)-z} = g_1(x),$$

ya`ni

$$\alpha v(x) f_0 + \Delta_1(x,z) f_1(x) + \alpha \int_{\mathbb{T}^d} \frac{v(s) g_2(x,s)\,ds}{\varepsilon+\omega(x)+\omega(s)-z} = g_1(x)$$

tenglikni hosil qilamiz. Bu tenglikdan $f_1(x)$ ni topamiz:

$$f_1(x) = \frac{1}{\Delta_1(x,z)}\left(g_1(x) - \alpha v(x) f_0 - \alpha \int_{\mathbb{T}^d} \frac{v(s) g_2(x,s) ds}{\varepsilon+\omega(x)+\omega(s)-z}\right) \qquad (1.2.7)$$

Topilgan (1.2.7) ifodani (1.2.5) sistemaning birinchi tenglamasiga qo`yamiz va $f_0$ ning ko`rinishini aniqlaymiz:

$$\left(\varepsilon - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(s) ds}{\Delta_1(s,z)}\right) f_0 + \alpha \int_{\mathbb{T}^d} \frac{g_1(s) v(s)\,ds}{\Delta_1(s,z)} - \alpha^2 \int_{\mathbb{T}^d}\int_{\mathbb{T}^d} \frac{v^2(s) g_2(x,s)\,ds^2}{\varepsilon+\omega(x)+\omega(s)-z} = g_0,$$

ya`ni

$$\Omega_3(z) f_0 + \alpha \int_{\mathbb{T}^d} \frac{g_1(s) v(s)\,ds}{\Delta_1(s,z)} - \alpha^2 \int_{\mathbb{T}^d}\int_{\mathbb{T}^d} \frac{v^2(s) g_2(x,s)\,ds^2}{\varepsilon+\omega(x)+\omega(s)-z} = g_0.$$

$z \notin \sigma_{disc}(A_3)$ bo`lganligi uchun $\Omega_3(z) \neq 0$ bo`ladi. Shuning uchun oxirgi tenglikdan quyidagini hosil qilamiz:

$$f_0=\frac{1}{\Omega_3(z)}\left(g_0-\alpha\int_{\mathbb{T}^d}\frac{g_1(s)v(s)ds}{\Delta_1(s,z)}+\alpha^2\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\frac{v^2(s)g_2(x,s)ds^2}{\varepsilon+\omega(x)+\omega(s)-z}\right) \qquad (1.2.8)$$

f_0 uchun topilgan ifodani (1.2.7) tenglikka qo`yamiz. Natijada,

$$f_1(x)=\frac{1}{\Delta_1(x,z)}(g_1(x)-\alpha v(x)\frac{g_0}{\Omega_3(z)}+\frac{\alpha^2 v(x)}{\Omega_3(z)}\int_{\mathbb{T}^d}\frac{g_1(s)v(s)ds}{\Delta_1(s,z)}-$$

$$-\frac{\alpha^3 v(x)}{\Omega_3(z)}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\frac{v^2(s)g_2(x,s)ds^2}{\varepsilon+\omega(x)+\omega(s)-z}-\alpha\int_{\mathbb{T}^d}\frac{v(s)g_2(x,s)ds}{\varepsilon+\omega(x)+\omega(s)-z})$$

yoki

$$f_1(x)=\frac{1}{\Delta_1(x,z)}(g_1(x)-\alpha v(x)\frac{g_0}{\Omega_3(z)}+\frac{\alpha^2 v(x)}{\Omega_3(z)}\int_{\mathbb{T}^d}\frac{g_1(s)v(s)ds}{\Delta_1(s,z)}-$$

$$-\alpha\int_{\mathbb{T}^d}\frac{v(s)g_2(x,s)ds}{\varepsilon+\omega(x)+\omega(s)-z}(\frac{\alpha^2 v(x)}{\Omega_3(z)}\int_{\mathbb{T}^d}v(s)g_2(x,s)ds+1)) \qquad (1.2.9)$$

ni hosil qilamiz.

(1.2.9) formuladan foydalanib, $f_2(x,y)$ funksiyaning ko`rinishini topamiz, ya`ni (1.2.9) formulani (1.2.6) tenglikka qo`yamiz:

$$f_2(x,y)=\frac{g_2(x,y)}{\varepsilon+\omega(x)+\omega(y)-z}-\frac{\alpha v(y)}{(\varepsilon+\omega(x)+\omega(y)-z)\Delta_1(x,z)}[g_1(x)-\alpha v(x)\frac{g_0}{\Omega_3(z)}+$$

$$+\frac{\alpha^2 v(x)}{\Omega_3(z)}\int_{\mathbb{T}^d}\frac{g_1(s)v(s)ds}{\Delta_1(s,z)}-\alpha\int_{\mathbb{T}^d}\frac{v(s)g_2(x,s)ds}{\varepsilon+\omega(x)+\omega(s)-z}(\frac{\alpha^2 v(x)}{\Omega_3(z)}\int_{\mathbb{T}^d}v(s)g_2(x,s)ds+1)].$$

Boshqacha,

$$f_2(x,y)=\frac{g_2(x,y)}{\varepsilon+\omega(x)+\omega(y)-z}-\frac{\alpha^2 v(y)}{(\varepsilon+\omega(x)+\omega(y)-z)\Delta_1(x,z)}\int_{\mathbb{T}^d}\frac{v(s)g_2(x,s)ds}{\varepsilon+\omega(x)+\omega(s)-z}\cdot$$

$$\cdot\left(\frac{\alpha^2 v(x)}{\Omega_3(z)}\int_{\mathbb{T}^d}v(s)g_2(x,s)ds+1\right)-\frac{\alpha v(y)g_1(x)}{(\varepsilon+\omega(x)+\omega(y)-z)\Delta_1(x,z)}-$$

$$-\frac{\alpha^3 v(x)v(y)}{(\varepsilon+\omega(x)+\omega(y)-z)\Delta_1(x,z)\Omega_3(z)}\int_{\mathbb{T}^d}\frac{g_1(s)v(s)ds}{\Delta_1(s,z)}+\frac{\alpha^2 v(x)v(y)g_0}{(\varepsilon+\omega(x)+\omega(y)-z)\Delta_1(x,z)\Omega_3(z)}.$$

Shunday qilib,

$$\begin{cases} R_{00}g_0 + R_{01}g_1 + R_{02}g_2 = f_0 \\ R_{10}g_0 + R_{11}g_1 + R_{12}g_2 = f_1 \\ R_{20}g_0 + R_{21}g_1 + R_{22}g_2 = f_2 \end{cases}$$

bo`ladi va quyidagi formulalar o`rinlidir:

$$R_{00}(\lambda, A_3)g_0 = \frac{g_0}{\Omega_3(z)},$$

$$R_{01}(\lambda, A_3)g_1 = -\frac{\alpha}{\Omega_3(z)}\int_{\mathbb{T}^d}\frac{g_1(s)v(s)ds}{\Delta_1(s,z)},$$

$$R_{02}(\lambda, A_3)g_2 = \frac{\alpha^2}{\Omega_3(z)}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\frac{v^2(s)g_2(x,s)ds^2}{\varepsilon+\omega(x)+\omega(s)-z},$$

$$(R_{10}(\lambda, A_3)g_o)(x) = -\frac{\alpha v(x)g_0}{\Delta_1(x,z)\Omega_3(z)},$$

$$(R_{11}(\lambda, A_3)g_1)(x) = \frac{g_1(x)}{\Delta_1(x,z)} + \frac{\alpha^2 v(x)}{\Delta_1(x,z)\cdot\Omega_3(z)}\int_{\mathbb{T}^d}\frac{g_1(s)v(s)ds}{\Delta_1(s,z)},$$

$$(R_{12}(\lambda, A_3)g_2)(x) = -\frac{\alpha}{\Delta_1(x,z)}\int_{\mathbb{T}^d}\frac{v(s)g_2(x,s)ds}{\varepsilon+\omega(x)+\omega(s)-z}\cdot\left(\frac{\alpha^2 v(x)}{\Omega_3(z)}\int_{\mathbb{T}^d}v(s)g_2(x,s)ds + 1\right)$$

$$(R_{20}(\lambda, A_3)g_0)(x,y) = \frac{\alpha^2 v(x)v(y)g_0}{(\varepsilon+\omega(x)+\omega(y)-z)\Delta_1(x,z)\Omega_3(z)},$$

$$(R_{21}(\lambda, A_3)g_1)(x,y) = -\frac{\alpha v(y)g_1(x)}{(\varepsilon+\omega(x)+\omega(y)-z)\Delta_1(x,z)} -$$

$$-\frac{\alpha^3 v(x)v(y)}{(\varepsilon+\omega(x)+\omega(y)-z)\Delta_1(x,z)\Omega_3(z)}\int_{\mathbb{T}^d}\frac{g_1(s)v(s)ds}{\Delta_1(s,z)},$$

$$(R_{22}(\lambda, A_3)g_2)(x,y) = \frac{g_2(x,y)}{\varepsilon+\omega(x)+\omega(y)-z} - \frac{\alpha^2 v(y)}{(\varepsilon+\omega(x)+\omega(y)-z)\Delta_1(x,z)}\cdot$$

$$\cdot\int_{\mathbb{T}^d}\frac{v(s)g_2(x,s)ds}{\varepsilon+\omega(x)+\omega(s)-z}\cdot\left(\frac{\alpha^2 v(x)}{\Omega_3(z)}\int_{\mathbb{T}^d}v(s)g_2(x,s)ds + 1\right).$$

1.2.3 – lemma to`liq isbotlandi.

I BOB XULOSASI

Bitiruv malakaviy ishining birinchi bobi “2 – va 3 – tartibli operatorli matritsalarning spektri va rezolventa operatori” deb nomlangan bo`lib, u ikkita bo`limdan tashkil topgan. 1.1 – bo`limda 2 – tartibli operatorli matritsa Fok fazosining qirqilgan ikki zarrachali qism fazosida chiziqli, chegaralangan va o`z-o`ziga qo`shma operator sifatida qaraladi. Uning muhim spektri kesmadan iborat bo`lishi isbotlangan. $A_2$ operatorga mos Fredgolm determinanti qurilgan hamda uning analitik xossalari o`rganilgan. Topilgan Fredgolm determinantining nollari to’plami o’rganilayotgan operatorning diskret spektri bilan ustma-ust tushishi isbotlangan. Mos rezolventa operatori uchun aniq formula topilgan.

2.2 – bo`limda esa 3 – tartibli $A_3$ operatorli matritsa Fok fazosining qirqilgan uch zarrachali qism fazosida chiziqli, chegaralangan va o`z – o`ziga qo`shma operator sifatida o`rganilgan. $A_3$ operator muhim spektrining joylashuv o`rni topilgan. Bunda Veyl mezoni va mos Faddeyev tenglamasining xossalaridan foydalanilgan. Uning diskret spektrini aniqlash imkonini beruvchi analitik funksiyaning ko`rinishi aniq topilgan. $A_3$ operatorning rezolventa operatorining ta`sir formulasi aniq topilgan.

I bobdagi barcha natijalarni isbotlashda matematik analiz, kompleks analiz va funksional analiz usullaridan foydalanilgan.

II BOB. 4 – TARTIBLI OPERATORLI MATRITSALARNING SPEKTRAL XOSSALARI.

2.1. 4 – tartibli operatorli matritsalarning muhim spektri

Biz quyidagi Fok fazosining qirqilgan to`rt zarrachali qism fazosi $H^{(4)} := H_0 \oplus H_1 \oplus H_2 \oplus H_3$ Gilbert fazosini qaraymiz. Bu yerda $H_0 := \mathbb{C}$, $H_1 := L_2(\mathbb{T}^d)$, $H_2 := (L_2(\mathbb{T}^d)^2)$ va $H_3 := (L_2(\mathbb{T}^d)^3)$.

$H^{(4)}$ Gilbert fazosida

$$A_4 := \begin{pmatrix} A_{00} & A_{01} & 0 & 0 \\ A_{01}^* & A_{11} & A_{12} & 0 \\ 0 & A_{12}^* & A_{22} & A_{23} \\ 0 & 0 & A_{23}^* & A_{33} \end{pmatrix}$$

formula orqali ta`sir qiluvchi A_4 4 – tartibli operatorli matritsani qaraymiz, bunda $A_{ij}: H_j \to H_i, i,j = 0,1,2,3$ operatorlar quyidagi formulalar yordamida aniqlangan:

$$A_4 f := \begin{pmatrix} A_{00}f_0 + A_{01}f_1 \\ A_{01}^* f_0 + A_{11}f_1 + A_{12}f_2 \\ A_{12}^* f_1 + A_{22}f_2 + A_{23}f_3 \\ A_{23}^* f_2 + A_{33}f_3 \end{pmatrix}, f_i \in H_i, i = 0,1,2,3. \qquad (2.1.1)$$

$$A_{00}f_0 = \varepsilon f_0, \; A_{01}f_1 = \alpha \int_{\mathbb{T}^d} v(t) f_1(t) dt,$$

$$A_{01}^* f_0 = \alpha v(x) f_0, \; A_{11}f_1 = \big(\varepsilon + \omega(x)\big) f_1(x),$$

$$A_{12}f_2 = \alpha \int_{\mathbb{T}^d} v(t) f_2(x,t) dt, \; A_{12}^* f_1 = \alpha v(y) f_1(x),$$

$$A_{22}f_2 = (\varepsilon + \omega(x) + \omega(y)) f_2(x,y), \; A_{23}f_3 = \alpha \int_{\mathbb{T}^d} v(t) f_3(x,y,t) dt,$$

$$A_{23}^* f_2 = \alpha v(t) f_2(x,y), \; A_{33}f_3 = (\varepsilon + \omega(x) + \omega(y) + \omega(t)) f_3(x,y,t).$$

2.1.1 –Lemma. $H^{(4)}$ Gilbert fazosida (2.1.1) formula orqali ta`sir qiluvchi $A_4$ operator chiziqli, chegaralangan va o`z – o`ziga qo`shma operator bo`ladi.

Isbot. A_4 operatorni chiziqlilikka tekshiramiz. Buning uchun ixtiyoriy $f = (f_0, f_1, f_2, f_3), g = (g_0, g_1, g_2, g_3) \in H^{(4)}$ elementlar uchun

$$A(af + bg) = aAf + bAg$$

tenglik o`rinli ekanligini ko`rsatamiz:

$$Af := \begin{pmatrix} \varepsilon f_0 + \alpha\int_{\mathbb{T}^d} v(t) f_1(t)dt \\ \alpha v(x) f_0 + \big(\varepsilon + \omega(x)\big) f_1(x) + \alpha\int_{\mathbb{T}^d} v(t) f_2(x,t)dt \\ \alpha v(y) f_1(x) + \big(\varepsilon + \omega(x) + \omega(y)\big) f_2(x,y) + \alpha\int_{\mathbb{T}^d} v(t) f_3(x,y,t)dt \\ \alpha v(t) f_2(x,y) + (\varepsilon + \omega(x) + \omega(y) + \omega(t)) f_3(x,y,t) \end{pmatrix}.$$

$$A(af + bg) =$$

$$\begin{pmatrix} \varepsilon a f_0 + \alpha\int_{\mathbb{T}^d} v(t) a f_1(t)dt \\ \alpha v(x) a f_0 + \big(\varepsilon + \omega(x)\big) a f_1(x) + \alpha\int_{\mathbb{T}^d} v(t) a f_2(x,t)dt \\ \alpha v(y) a f_1(x) + \big(\varepsilon + \omega(x) + \omega(y)\big) a f_2(x,y) + \alpha\int_{\mathbb{T}^d} v(t) a f_3(x,y,t)dt \\ \alpha v(t) a f_2(x,y) + (\varepsilon + \omega(x) + \omega(y) + \omega(t)) a f_3(x,y,t) \end{pmatrix} +$$

$$+\begin{pmatrix} \varepsilon b f_0 + \alpha\int_{\mathbb{T}^d} v(t) b f_1(t)dt \\ \alpha v(x) b f_0 + \big(\varepsilon + \omega(x)\big) b f_1(x) + \alpha\int_{\mathbb{T}^d} v(t) b f_2(x,t)dt \\ \alpha v(y) b f_1(x) + \big(\varepsilon + \omega(x) + \omega(y)\big) b f_2(x,y) + \alpha\int_{\mathbb{T}^d} v(t) b f_3(x,y,t)dt \\ \alpha v(t) b f_2(x,y) + (\varepsilon + \omega(x) + \omega(y) + \omega(t)) b f_3(x,y,t) \end{pmatrix} =$$

$$= a\begin{pmatrix} \varepsilon f_0 + \alpha\int_{\mathbb{T}^d} v(t) f_1(t)dt \\ \alpha v(x) f_0 + \big(\varepsilon + \omega(x)\big) f_1(x) + \alpha\int_{\mathbb{T}^d} v(t) f_2(x,t)dt \\ \alpha v(y) f_1(x) + \big(\varepsilon + \omega(x) + \omega(y)\big) f_2(x,y) + \alpha\int_{\mathbb{T}^d} v(t) f_3(x,y,t)dt \\ \alpha v(t) f_2(x,y) + (\varepsilon + \omega(x) + \omega(y) + \omega(t)) f_3(x,y,t) \end{pmatrix} +$$

$$
+b\begin{pmatrix} \varepsilon f_0+\alpha\int_{\mathbb{T}^d} v(t)f_1(t)dt \\ \alpha v(x)f_0+\big(\varepsilon+\omega(x)\big)f_1(x)+\alpha\int_{\mathbb{T}^d} v(t)f_2(x,t)dt \\ \alpha v(y)f_1(x)+\big(\varepsilon+\omega(x)+\omega(y)\big)f_2(x,y)+\alpha\int_{\mathbb{T}^d} v(t)f_3(x,y,t)dt \\ \alpha v(t)f_2(x,y)+(\varepsilon+\omega(x)+\omega(y)+\omega(t))f_3(x,y,t) \end{pmatrix}.
$$

$\Rightarrow A(af+bg)=aAf+bAg$. Demak, A_4 chiziqli operator ekan.

A_4 operatorni chegaralanganlikka tekshirish uchun $\|Af\|\le C\|f\|$ tengsizlikni ko`rsatamiz. Buning uchun quyidagi munosabatlardan foydalanish kerak:

$$
|a+b|^2\le 2|a|^2+2|b|^2,\ \ |a+b+c|^2\le 3|a|^2+3|b|^2+3|c|^2.
$$

$$
\|(Af)_0\|_0^2=\left|\varepsilon f_0+\alpha\int_{\mathbb{T}^d} v(t)f_1(t)dt\right|^2\le 2|\varepsilon|^2|f_0|^2+
$$

$$
+2|\alpha|^2\int_{\mathbb{T}^d}|v(t)|^2dt\int_{\mathbb{T}^d}|f_1(t)|^2dt=2|\varepsilon|^2\|f_0\|^2+2|\alpha|^2\|v\|^2\|f_1\|^2;
$$

$$
\|(Af)_1\|_1^2=\int_{\mathbb{T}^d}\left|\alpha v(x)f_0+\big(\varepsilon+\omega(x)\big)f_1(x)+\alpha\int_{\mathbb{T}^d} v(t)f_2(x,t)dt\right|^2dx\le
$$

$$
\le 3|\alpha|^2|f_0|^2\int_{\mathbb{T}^d}|v(x)|^2dx+3|\varepsilon+M|^2\int_{\mathbb{T}^d}|f_1(x)|^2dx+
$$

$$
+3|\alpha|^2\left|\int_{\mathbb{T}^d} v(t)f_2(x,t)dt\right|^2dx\le
$$

$$
\le 3|\alpha|^2\|v\|^2\|f_0\|^2+3|\varepsilon+M|^2\|f_1\|^2+3|\alpha|^2\|v\|^2\|f_2\|^2;
$$

$$
\|(Af)_2\|_2^2=\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\left|\alpha v(y)f_1(x)+\big(\varepsilon+\omega(x)+\omega(y)\big)f_2(x,y)+\right.
$$
$$
\left.\alpha\int_{\mathbb{T}^d} v(t)f_3(x,y,t)dt\right|^2dxdy\le 3|\alpha|^2\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}|v(y)f_1(x)|^2dxdy+
$$

$$
+3|\varepsilon+2M|^2\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}|f_2(x,y)|^2dxdy+3|\alpha|^2\int_{\mathbb{T}^d}|v(t)f_3(x,y,t)dt|^2dxdy\le
$$

$$
\le 3|\alpha|^2\int_{\mathbb{T}^d}|v(y)|^2dy\int_{\mathbb{T}^d}|f_1(x)|^2dx+3|\varepsilon+2M|^2\|f_2\|^2+
$$

$$
+3|\alpha|^2\int_{\mathbb{T}^d}|v(t)|^2dt\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}|f_3(x,y,t)|^2dxdydt\le 3|\alpha|^2\|v\|^2\|f_1\|^2+
$$

$$+3|\varepsilon+2M|^2\|f_2\|^2+3|\alpha|^2\|v\|^2\|f_3\|^2;$$

$$\|(Af)_3\|_3^2=\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}|\alpha v(t)f_2(x,y)+(\varepsilon+\omega(x)+\omega(y)+\omega(t))f_3(x,y,t)|^2$$

$$dxdydt\le 2|\alpha|^2\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}|v(t)f_2(x,y)|^2dxdydt+2|\varepsilon+3M|^2\cdot$$

$$\cdot\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}|f_3(x,y,t)|^2dxdydt\le 2|\alpha|^2\int_{\mathbb{T}^d}|v(t)|^2dt\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}|f_2(x,y)|^2dxdy+$$

$$+2|\varepsilon+3M|^2\|f_3\|^2\le 2|\alpha|^2\|v\|^2\|f_2\|^2+2|\varepsilon+3M|^2\|f_3\|^2.$$

Endi $\|Af\|^2$ ni hisoblaymiz:

$$\|Af\|^2=\|(Af)_0\|_0^2+\|(Af)_1\|_1^2+\|(Af)_2\|_2^2+\|(Af)_3\|_3^2\le$$

$$\le\|f_0\|^2(2|\varepsilon|^2+3|\alpha|^2\|v\|^2)+\|f_1\|^2(2|\alpha|^2\|v\|^2+3|\varepsilon+M|^2+3|\alpha|^2\|v\|^2)+$$
$$+\|f_2\|^2(5|\alpha|^2\|v\|^2+3|\varepsilon+2M|^2)+\|f_3\|^2(3|\alpha|^2\|v\|^2+2|\varepsilon+3M|^2).$$

Quyidagi belgilashlarni kiritamiz:

$$a=2|\varepsilon|^2+3|\alpha|^2\|v\|^2,$$

$$b=2|\alpha|^2\|v\|^2+3|\varepsilon+M|^2+3|\alpha|^2\|v\|^2,$$

$$c=5|\alpha|^2\|v\|^2+3|\varepsilon+2M|^2,$$

$$d=3|\alpha|^2\|v\|^2+2|\varepsilon+3M|^2$$

va $C^2=max\{a,b,c,d\}$.

U holda $\|Af\|^2\le C^2(\|f_0\|^2+\|f_1\|^2+\|f_2\|^2+\|f_3\|^2)\Rightarrow$

$$\|Af\|^2\le C^2\|f\|^2\ \Rightarrow\ \|Af\|\le C\|f\|.$$

Demak, A_4 – chegaralangan operator.

A_4 operatorni o`z – o`ziga qo`shma operator ekanligiga tekshirish uchun

$$(Af,g)=(f,Ag)$$

tenglikni isbotlaymiz. Bu yerda $f=(f_0,f_1,f_2,f_3), g=(g_0,g_1,g_2,g_3)\in H^{(4)}$.

Ma`lumki,

$$(f_0,g_0)=f_0\cdot\overline{g_0};\ (f_1,g_1)=\int_{\mathbb{T}^d}f_1(x)\overline{g_1(x)}dx;$$

$$(f_2,g_2)=\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}f_1(x,y)\overline{g_2(x,y)}dxdy;$$

$$(f_3,g_3)=\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}f_3(x,y,t)\overline{g_3(x,y,t)}dxdydt.$$

$$(Af,g)=\sum_{i=0}^{3}((Af)_i,g_i)_i.$$

$$(Af,g)=\left(\varepsilon f_0+\alpha\int_{\mathbb{T}^d}v(t)f_1(t)dt\right)\overline{g_0}+\int_{\mathbb{T}^d}(\alpha v(x)f_0+\left(\varepsilon+\omega(x)\right)\cdot f_1(x)+$$

$$+\alpha\int_{\mathbb{T}^d}v(t)f_2(x,t)dt)\overline{g_1(x)}dx+\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}(\alpha v(y)f_1(x)+(\varepsilon+\omega(x)+\omega(y))\cdot$$

$$\cdot f_2(x,y)+\alpha\int_{\mathbb{T}^d}v(t)f_3(x,y,t)dt)\overline{g_2(x,y)}dxdy+\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}(\alpha v(t)f_2(x,y)+$$

$$+(\varepsilon+\omega(x)+\omega(y)+\omega(t))f_3(x,y,t))\overline{g_3(x,y,t)}dxdydt.$$

So`ngra yuqoridagi ifodani quyidagi ko`rinishda o`zgartiramiz:

$$(Af,g)=f_0\,\overline{(\varepsilon g_0+\alpha\int_{\mathbb{T}^d}v(t)g_1(t)dt)}++\int_{\mathbb{T}^d}f_1(x)\cdot$$

$$\cdot\left(\overline{\alpha\int_{\mathbb{T}^d}v(t)g_0dt+\left(\varepsilon+\omega(x)\right)g_1(x)+\alpha\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}v(y)g_2(x,y)dxdy}\right)dx+$$

$$+\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}f_2(x,y)\left(\overline{\alpha v(t)g_1(x)+\left(\varepsilon+\omega(x)+\omega(y)\right)g_2(x,y)+\alpha\int_{\mathbb{T}^d}v(t)dt}\right)\cdot$$

$$\cdot\,dxdy+\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}f_3(x,y,t)(\overline{\alpha\int_{\mathbb{T}^d}v(t)g_2(x,y)dt+}$$

$$\overline{+(\varepsilon+\omega(x)+\omega(y)+\omega(t))g_3(x,y,t)})dxdydt.$$

Demak,

$$(f_0,(Ag)_0)+(f_1,(Ag)_1)+(f_2,(Ag)_2)+(f_3,(Ag)_3)=(f,Ag).$$

A_4 o`z – o`ziga qo`shma operator ekan.

2.1.1 – Teorema. A_4 matritsaviy operatorning muhim spektri uchun $\sigma_{ess}(A_4)=\Sigma$ tenglik o`rinlidir. Bunda Σ to`plam

$$\Sigma := \bigcup_{x\in\mathbb{T}^d}\{\sigma_{disc}(A_3)+\omega(x)\}\cup\bigcup_{x,y\in\mathbb{T}^d}\{\sigma_{disc}(A_2)+\omega(x)+\omega(y)\}\cup$$

$$\cup\,[\varepsilon+3m,\varepsilon+3M]$$

kabi aniqlanadi.

Bundan tashqari, $\sigma_{ess}(A_4)$ to`plam ko`pi bilan 7 ta kesmalar birlashmasidan iborat bo`ladi.

Isbot. Avvalo, $[\varepsilon+3m,\varepsilon+3M]\subset\sigma_{ess}(A_4)$ munosabatni isbotlaymiz.

Faraz qilaylik, $z_0\in[\varepsilon+3m,\varepsilon+3M]$ ixtiyoriy nuqta bo`lsin. $z_0\in\sigma_{ess}(A_4)$ ekanligini isbotlaymiz. Bunda biz 1.2.1 – teorema isbotida bayon qilingan Veyl kriteriyasidan foydalanamiz.

$\omega(\cdot)-\mathbb{T}^d$ dagi uzluksiz funksiya bo`lgani uchun shunday $p_0,q_0,r_0\in\mathbb{T}^d$ nuqtalar topilib,

$$z_0=\varepsilon+\omega(p_0)+\omega(q_0)+\omega(r_0)$$

temglik o`rinli bo`ladi. $(p_0,q_0,r_0)\in(\mathbb{T}^d)^3$ nuqtaning quyidagi atrofini qaraymiz.

$$W_n(p_0,q_0,r_0):=V_n(p_0)\times V_n(q_0)\times V_n(r_0),$$

bu yerda

$$V_n(p_0):=\left\{x\in\mathbb{T}^d:\frac{1}{n+1}<|x-p_0|<\frac{1}{n}\right\},n\in N,p_0\in\mathbb{T}^d$$

nuqtaning o`yilgan atrofi $\mu(W_n(p_0, q_0, r_0))$ orqali $W_n(p_0, q_0, r_0)$ to`plamning Lebeg o`lchovini belgilaymiz.

$f^{(n)}$ ortonormal vektor – funksiyalar ketma – ketligini ushbu ko`rinishda tanlaymiz:

$$f^{(n)} := \begin{pmatrix} 0 \\ 0 \\ 0 \\ f_3^{(n)}(x, y, t) \end{pmatrix},$$

$$f_3^{(n)}(x, y, t) := \begin{cases} \frac{\chi_{W_n(p_0, q_0, r_0)}(p, q, r)}{\sqrt{\mu(W_n(p_0, q_0, r_0))}}, (x, y, t) \in W_n(p_0, q_0, r_0) \\ 0, (x, y, t) \notin W_n(p_0, q_0, r_0) \end{cases}.$$

Endi $n \neq m$ lar uchun $\left(f_3^{(n)}, f_3^{(m)}\right)$ ni hisoblaymiz:

$$\left(f_3^{(n)}, f_3^{(m)}\right) = \int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d} f_3^{(n)}(x, y)\overline{f_3^{(m)}(x, y, t)}dxdy =$$

$$= \int_{W_n(p_0, q_0, r_0)} f_3^{(n)}(x, y, t)\overline{f_3^{(m)}(x, y, t)}dxdydt +$$

$$+ \int_{W_m(p_0, q_0, r_0)} f_3^{(n)}(x, y, t) f_3^{(m)}(x, y, t)dxdydt +$$

$$+ \int_{\mathbb{T}^4 \setminus (W_n(p_0, q_0, r_0) \cup W_m(p_0, q_0, r_0))} f_3^{(n)}(x, y, t) f_3^{(m)}(x, y, t)dxdydt = 0$$

tenglik o`rinli, chunki $n \neq m$ uchun $W_n(p_0, q_0, r_0) \cap W_m(p_0, q_0, r_0) = \emptyset$. Demak, $\left\{f_3^{(n)}\right\}$ – ortogonal sistema ekan.

Aniqlanishiga ko`ra,

$$\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d} \left|f_3^{(n)}(x, y, t)\right|^2 dxdydt =$$

$$= \int_{V_n(p_0)}\int_{V_n(q_0)}\int_{V_n(r_0)} \frac{dxdydt}{\mu(V_n(p_0))\mu(V_n(q_0))\mu(V_n(q_0))} =$$

$$= \frac{1}{\mu(V_n(p_0))\mu(V_n(q_0))\mu(V_n(q_0))} \cdot \int_{V_n(p_0)} dx \int_{V_n(q_0)} dy \int_{V_n(r_0)} dt =$$

$$= \frac{1}{\mu(V_n(p_0))\mu(V_n(q_0))\mu(V_n(q_0))} \cdot \mu\big(V_n(p_0)\big) \cdot \mu\big(V_n(q_0)\big) \cdot \mu\big(V_n(q_0)\big) = 1.$$

Shunday qilib, $\left\{f_3^{(n)}\right\}$ ortonormal sistema ekan. Bundan esa, $\left\{f^{(n)}\right\}$ ortonormal sistema bo`lishi kelib chiqadi.

Endi $\left\|(A_4 - z_0)f^{(n)}\right\|$ ni baholaymiz:

$$\left\|(A_4 - z_0)f^{(n)}\right\|^2 = \alpha^2 \int_{\mathbb{T}^d} \int_{\mathbb{T}^d} \left| \int_{\mathbb{T}^d} v(t) f_3^{(n)}(x,y,t) \right|^2 dxdydt +$$

$$+ \int_{\mathbb{T}^d} \int_{\mathbb{T}^d} \int_{\mathbb{T}^d} |\varepsilon + \omega(x) + \omega(y) - z_0|^2 \cdot \left| f_3^{(n)}(x,y,t) \right|^2 dxdydt \leq$$

$$\leq \alpha^2 \max_{x \in V_n(p_0)} |v(x)|^2 \int_{\mathbb{T}^d} \int_{\mathbb{T}^d} \int_{\mathbb{T}^d} \left| f_3^{(n)}(x,y,t) \right|^2 dxdydt \cdot \int_{V_n(r_0)} dt +$$

$$+ \sup_{(x,y,t) \in W_n(p_0,q_0,r_0)} |\varepsilon + \omega(x) + \omega(y) - z_0|^2 \cdot \left| f_3^{(n)}(x,y,t) \right|^2 dxdydt \leq$$

$$\leq \alpha^2 \max_{x \in V_n(p_0)} |v(x)|^2 \mu\big(V_n(r_0)\big) +$$

$$+ \sup_{(x,y,t) \in W_n(p_0,q_0,r_0)} |\omega(x) + \omega(y) + \omega(t) - \omega_0(x) - \omega_0(y) - \omega_0(t)|^2.$$

Qurilishiga ko`ra, $\mu(V_n) \to 0, n \to \infty$ $\omega(\cdot)$ funksiyaning uzluksizligiga ko`ra,

$$\sup_{(x,y,t) \in W_n(p_0,q_0,r_0)} |\omega(x) + \omega(y) + \omega(t) - \omega_0(x) - \omega_0(y) - \omega_0(t)|^2 \to 0.$$

Shunday qilib,

$$\left\|(A_4 - z_0)f^{(n)}\right\| \to 0, n \to \infty$$

ekan. Endi z_0 nuqtaning ixtiyoriyligiga ko`ra, $[\varepsilon+3m,\varepsilon+3M]\subset\sigma_{ess}(A_4)$ munosabatni hosil qilamiz.

Endi

$$\bigcup_{x,y\in\mathbb{T}^d}\{\sigma_{disc}(A_2)+\omega(x)+\omega(y)\}\subset\sigma_{ess}(A_4)$$

ekanligini isbotlaymiz. Faraz qilaylik, $z_0\in\bigcup_{x,y\in\mathbb{T}^d}\{\sigma_{disc}(A_2)+\omega(x)+\omega(y)\}$ – ixtiyoriy nuqta bo`lsin. $z_0\in\sigma_{ess}(A_4)$ ekanligini isbotlaymiz.

Aniqlanishiga ko`ra, shunday $x_0,y_0\in\mathbb{T}^d$ nuqtalar topilib, z_0 nuqtani

$$z_0=\omega(x_0)+\omega(y_0)+\lambda_0,\ \lambda_0\in\sigma_{disc}(A_2)$$

kabi tasvirlash mumkin. Xos qiymat va xos vektorlarning ta`rifiga ko`ra shunday $\psi=(\psi_0,\psi_1)\in H_0\oplus H_1$ vektor – funksiya topilib,

$$(A_2-\lambda_0)\psi=0$$

bo`ladi. $f^{(n)}=(0,0,f_2^{(n)},f_3^{(n)})\in H_0\oplus H_1\oplus H_2\oplus H_3$ vektor – funksiyalar kerakligini qaraymiz. Bunda

$$f_2^{(n)}(x,y):=\frac{\chi_{V_n(x_0)}(x)\cdot\chi_{V_n(y_0)}(y)}{\sqrt{\mu(V_n(x_0))}\cdot\sqrt{\mu(V_n(y_0))}}\cdot\frac{\psi_0}{\|\psi\|},$$

$$f_3^{(n)}(x,y,t):=\frac{\chi_{V_n(x_0)}(x)\cdot\chi_{V_n(y_0)}(y)}{\sqrt{\mu(V_n(x_0))}\cdot\sqrt{\mu(V_n(y_0))}}\cdot\frac{\psi_1(t)}{\|\psi\|}.$$

$V_n(x_0)$ – to`plamning qurilishiga va $f^{(n)}$ – vektor – funksiyalar kerakligining aniqlanishiga ko`ra, $f^{(n)}$ ortonormal sistema bo`ladi.

Haqiqatdan ham, $n\neq m$ uchun $V_n(x_0)\cap V_m(x_0)=\emptyset$, $V_n(y_0)\cap V_m(y_0)=\emptyset$ va shu sababli

$$(f^{(n)}, f^{(m)}) = \left(f_2^{(n)}, f_2^{(m)}\right) + \left(f_3^{(n)}, f_3^{(m)}\right) = 0 + 0 = 0.$$

Demak, $\{f^{(n)}\}$ – ortogonal sistema ekan. Endi $\|f^{(n)}\|$ ni hisoblaymiz.

$$\|f^{(n)}\|^2 = \left\|f_2^{(n)}\right\|^2 + \left\|f_3^{(n)}\right\|^2 = \int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\left|f_2^{(n)}(x,y)\right|^2 dxdy +$$

$$+\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\left|f_3^{(n)}(x,y,t)\right|^2 dxdydt = \frac{1}{\mu(V_n(x_0))}\cdot\frac{1}{\mu(V_n(y_0))}\cdot$$

$$\cdot\int_{V_n(x_0)}dx\int_{V_n(y_0)}dy\cdot\frac{\|\psi_0\|^2}{\|\psi\|^2} + \frac{1}{\mu(V_n(x_0))}\cdot\frac{1}{\mu(V_n(y_0))}\cdot\int_{V_n(x_0)}dx\int_{V_n(y_0)}dy\cdot$$

$$\cdot\frac{1}{\|\psi\|^2}\cdot\int_{\mathbb{T}^d}|\psi_1(t)|^2dt = \frac{\|\psi_0\|^2+\|\psi_1\|^2}{\|\psi\|^2} = \frac{\|\psi\|^2}{\|\psi\|^2} = 1.$$

Demak, $\{f^{(n)}\}$ – ortonormal sistema ekan.

Endi $\|(A_4 - z_0)f^{(n)}\|$ ni baholaymiz:

$$(A_4 - z_0)f^{(n)} =$$

$$= \begin{pmatrix} 0 \\ \alpha\int_{\mathbb{T}^d}v(t)f_2^{(n)}(x,t)dt \\ (\varepsilon + \omega(x) + \omega(y) - z_0)f_2^{(n)}(x,y) + \alpha\int_{\mathbb{T}^d}v(s)f_3^{(n)}(x,y,s)dt \\ \alpha v(t)f_2^{(n)}(x,y) + (\varepsilon + \omega(x) + \omega(y) + \omega(t))f_3^{(n)}(x,y,t) \end{pmatrix}.$$

U holda,

$$\|(A_4 - z_0)f^{(n)}\|^2 \le \alpha^2\left[\max_{x\in V_n(y_0)} v(x)\right]^2 \mu(V_n(x_0)) + \frac{1}{\mu(V_n(x_0))}\cdot\frac{1}{\mu(V_n(y_0))}\cdot$$

$$\cdot\int_{V_n(x_0)}\int_{V_n(y_0)}|\omega(x) + \omega(y) - \omega(x_0) - \omega(y_0)|^2dxdy \le$$

$$\le \alpha^2\left[\max_{x\in V_n(y_0)} v(x)\right]^2 \mu(V_n(x_0)) +$$

$$+\sup_{(x,y)\in V_n(x_0)\times V_n(y_0)}|\omega(x)+\omega(y)-\omega(x_0)-\omega(y_0)|^2.$$

Oxirgi tengsizlikning birinchi qo`shiluvchisi $n\to\infty$ da 0 ga intiladi, chunki

$$\max|v(x)|^2<+\infty,\ \lim_{n\to\infty}\mu\big(V_n(x_0)\big)=0.$$

Uning ikkinchi qo`shiluvchisining $n\to\infty$ da 0 ga intilishi

$$\lim_{n\to\infty}\sup_{(x,y)\in V_n(x_0)\times V_n(y_0)}|\omega(x)+\omega(y)-\omega(x_0)-\omega(y_0)|^2=0$$

munosabatdan kelib chiqadi. Demak, $\lim\limits_{n\to\infty}\|(A_4-z_0)f^{(n)}\|=0.$

Shu sababli, $z_0\in\sigma_{ess}(A_4)$. Bu nuqtaning ixtiyoriy ekanligidan

$$\cup_{x,y\in\mathbb{T}^d}\{\sigma_{disc}(A_2)+\omega(x)+\omega(y)\}\subset\sigma_{ess}(A_4)$$

munosabat kelib chiqadi.

Endi

$$\cup_{x\in\mathbb{T}^d}\{\sigma_{disc}(A_3)+\omega(x)\}\subset\sigma_{ess}(A_4)$$

ekanligini isbotlaymiz.

Faraz qilaylik, $z_1\in\cup_{x\in\mathbb{T}^d}\{\sigma_{disc}(A_3)+\omega(x)\}$ – ixtiyoriy nuqta bo`lsin. $z_1\in\sigma_{ess}(A_4)$ ekanligini ko`rsatamiz. $z_1\in\cup_{x\in\mathbb{T}^d}\{\sigma_{disc}(A_3)+\omega(x)\}$ bo`lganligi uchun shunday $x_1\in\mathbb{T}^d$ nuqta topilib,

$$z_1=\omega(x_1)+\lambda_1,\lambda_1\in\sigma_{disc}(A_3)$$

tenglik bajariladi. Xos qiymat va xos vektor – funksiya ta`rifiga ko`ra shunday $\psi=(\psi_0,\psi_1,\psi_2)\in H_0\oplus H_1\oplus H_2$ vektor – funksiya topilib,

$$(A_3-\lambda_1)\psi=0$$

bo`ladi. $f^{(n)}=\left(0,f_1^{(n)},f_2^{(n)}(x,y),f_3^{(n)}(x,y,t)\right)$ vektor – funksiyalar ketma – ketligini qaraymiz, bunda

$$f_1^{(n)}(x):=\frac{\chi_{V_n(x_0)}(x)}{\sqrt{\mu(V_n(x_0))}}\cdot\frac{\psi_0}{\|\psi\|},$$

$$f_2^{(n)}(x,y):=\frac{\chi_{V_n(x_0)}(x)}{\sqrt{\mu(V_n(x_0))}}\cdot\frac{\psi_1(y)}{\|\psi\|};$$

$$f_3^{(n)}(x,y,t):=\frac{\chi_{V_n(x_0)}(x)}{\sqrt{\mu(V_n(x_0))}}\cdot\frac{\psi_2(y,t)}{\|\psi\|}.$$

Ko`rinib turibdiki, $V_n(x_0)$ to`plamning qurilishiga va $\{f^{(n)}\}$ – vektor – funksiyalar ketma – ketligining tanlanishiga ko`ra u ortogonal sistema bo`ladi. Haqiqatdan ham, ixtiyoriy $n\neq m$ uchun

$$(f^{(n)},f^{(m)})=\left(f_1^{(n)},f_1^{(m)}\right)+\left(f_2^{(n)},f_2^{(m)}\right)+\left(f_3^{(n)},f_3^{(m)}\right)=0.$$

Bundan tashqari,

$$\|f^{(n)}\|^2=\left\|f_1^{(n)}\right\|^2+\left\|f_2^{(n)}\right\|^2+\left\|f_3^{(n)}\right\|^2=$$

$$=\frac{1}{\mu(V_n(x_0))}\cdot\mu\big(V_n(x_0)\big)\cdot\frac{\|\psi_0\|^2}{\|\psi\|^2}+\frac{1}{\mu(V_n(x_0))}\cdot\mu\big(V_n(x_0)\big)\cdot\frac{1}{\|\psi\|^2}\cdot\int_{\mathbb{T}^d}|\psi_1(t)|^2dt+$$

$$+\frac{1}{\mu(V_n(x_0))}\cdot\mu\big(V_n(x_0)\big)\cdot\frac{1}{\|\psi\|^2}\cdot\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\left|\psi_2(t,t`)\right|^2dtdt`=$$

$$=\frac{\|\psi_0\|^2+\|\psi_1\|^2+\|\psi_2\|^2}{\|\psi\|^2}=\frac{\|\psi\|^2}{\|\psi\|^2}=1.$$

Shunday qilib, $\{f^{(n)}\}$ – ortonormal sistema ekan.

Endi $\left\|(A_4-\lambda_1)f^{(n)}\right\|^2$ ni qaraymiz va uni baholaymiz:

$$\left\|(A_4-\lambda_1)f^{(n)}\right\|^2=\alpha^2\left[\max_{t\in V_n(x_0)}v(t)\right]^2\mu\big(V_n(x_0)\big)+\frac{1}{\mu\big(V_n(p_0)\big)}\cdot$$

$$\cdot\int_{V_n(p_0)}|\omega(x)-\omega(x_0)|^2dx\leq\alpha^2\left[\max_{t\in V_n(x_0)}v(t)\right]^2\mu\big(V_n(x_0)\big)+$$

$$+\sup_{x\in V_n(x_0)}|\omega(x)-\omega(x_0)|^2.$$

$V_n(x_0)$ ning ta`rifidan $\mu\big(V_n(x_0)\big)\to 0, n\to\infty$ ekanligini, $\omega(\cdot)$ funksiyaning $x_0\in\mathbb{T}^d$ nuqtada uzluksiz ekanligidan $\sup\limits_{x\in V_n(x_0)}|\omega(x)-\omega(x_0)|^2\to 0, n\to\infty$ munosabatni hosil qilamiz.

Shunday qilib, $\left\|(A_4-\lambda_1)f^{(n)}\right\|\to 0, n\to\infty$ munosabat o`rinli ekan. Veyl kriteriyasiga ko`ra, $\lambda_1\in\sigma_{ess}(A_4)$ o`rinli.

Teskari munosabatni isbotlash uchun Faddeyev tenglamasini quramiz. Buning uchun A_4 operatorning

$$A_4f=zf$$

xos qiymatga nisbatan tenglamani qaraymiz.

Bu tenglamani

$$\begin{cases}(\varepsilon-z)f_0+\alpha\int_{\mathbb{T}^d}v(t)f_1(t)dt=0\\ \alpha v(x)f_0+(\varepsilon+\omega(x)-z)f_1(x)+\alpha\int_{\mathbb{T}^d}v(t)f_2(x,t)dt=0\\ \alpha v(y)f_1(x)+(\varepsilon+\omega(x)+\omega(y)-z)f_2(x,y)+\alpha\int_{\mathbb{T}^d}v(t)f_3(x,y,t)dt=0\\ \alpha v(t)f_2(x,y)+(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)f_3(x,y,t)=0\end{cases}$$

(2.1.2)

kabi yozib olamiz.

(2.1.2) tenglamalar sistemasining 4 – tenglamasidan $f_3(x,y,t)$ ni topamiz:

$$f_3(x,y,t) = -\frac{\alpha v(t) f_2(x,y)}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z} \qquad (2.1.3)$$

Hosil bo`lgan (2.1.3) ifodani (2.1.2) tenglamalar sistemasining 3 – tenglamasiga qo`yamiz:

$$\alpha v(y) f_1(x) + (\varepsilon + \omega(x) + \omega(y) - z) f_2(x,y) -$$

$$-\alpha \int_{\mathbb{T}^d} v(t) \frac{\alpha v(t) f_2(x,y)}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z} dt = 0 \Rightarrow$$

$$\alpha v(y) f_1(x) + \left(\varepsilon + \omega(x) + \omega(y) - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(t)dt}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z}\right) f_2(x,y) = 0.$$

Ya`ni

$$\alpha v(y) f_1(x) + \Delta_2(x,y,z) f_2(x,y) = 0 \qquad (2.1.4)$$

bu yerda, $\Delta_2(x,y,z) = \varepsilon + \omega(x) + \omega(y) - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(t)dt}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z}$.

Hosil bo`lgan (2.1.4) ifodadan $f_2(x,y)$ ni topib, (2.1.2) tenglamalar sistemasining 2 – tenglamasiga qo`yamiz:

$$f_2(x,y) = -\frac{\alpha v(y) f_1(x)}{\Delta_2(x,y,z)}$$

$$\alpha v(x) f_0 + \left(\varepsilon + \omega(x) - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(t)dt}{\Delta_2(x,y,z)}\right) f_1(x) = 0 \Rightarrow$$

$$\alpha v(x) f_0 + \Delta_3(x,z) f_1(x) = 0. \qquad (2.1.5)$$

Bu yerda $\Delta_3(x,z) = \varepsilon + \omega(x) - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(t)dt}{\Delta_2(x,y,z)}$.

(2.1.5) tenglamadan $f_1(x)$ ni topamiz:

$$f_1(x) = -\frac{\alpha v(x) f_0}{\Delta_3(x,z)}.$$

Yuqoridagi hisoblashlarga ko`ra, (2.1.2) tenglamalar sistemasini quyidagicha yozib olamiz:

$$\begin{cases} f_0 = (\varepsilon - z + 1)f_0 + \alpha \int_{\mathbb{T}^d} v(t) f_1(t) dt \\ f_1(x) = -\frac{\alpha v(x) f_0}{\Delta_3(x,z)} \end{cases}.$$

Endi $\mathbb{C} \oplus L_2(\mathbb{T}^d)$ da ta`sir qiluvchi

$$T(z) = \begin{pmatrix} T_{00}(z) & T_{01}(z) \\ T_{10}(z) & 0 \end{pmatrix}$$

operatorni qaraymiz, bunda

$$T_{00}(z) f_0 = (\varepsilon - z + 1) f_0,$$

$$T_{01}(z) f_1 = \alpha \int_{\mathbb{T}^d} v(t) f_1(t) dt,$$

$$(T_{10}(z) f_0)(x) = -\frac{\alpha v(x) f_0}{\Delta_3(x,z)}.$$

Shunday qilib, quyidagi tasdiqni isbotladik:

2.1.1 – Tasdiq. z soni A_4 operatorning xos qiymati bo`lishi uchun $1$ soni $T(z)$ operatorning xos qiymati bo`lishi zarur va yetarlidir.

$T_{00}(z)$, $T_{01}(z)$ va $T_{10}(z)$ operatorlar bir o`lchamli operatorlar bo`lganligi uchun $T(z)$ kompakt qiymatli analitik funksiya bo`ladi. $A_4$ operatorning o`z – o`ziga qo`shmaligiga ko`ra,

$$\big(I - T(z)\big)^{-1}$$

operator qiymatli funksiya barcha $Imz \neq 0$ larda mavjud bo`ladi.

Fredgolmning analitik teoremasiga ko`ra,

$$\left(I - T(z)\right)^{-1}$$

operator qiymatli funksiya

$$\mathbb{C}\backslash\Sigma$$

to`plamning chekli rangli qoldiqlariga ega bo`ladigan S diskret to`plamdan boshqa hamma yerda mavjud bo`ladi. Shu sababli $\sigma(A_4)\backslash\Sigma$ to`plam faqat yakkalangan nuqtalardan iborat bo`lib, ularning limitik nuqtalari faqat

$$\bigcup_{x\in\mathbb{T}^d}\{\sigma_{disc}(A_3) + \omega(x)\} \cup \bigcup_{x,y\in\mathbb{T}^d}\{\sigma_{disc}(A_2) + \omega(x) + \omega(y)\} \cup$$

$$\cup\ [\varepsilon + 3m, \varepsilon + 3M]$$

to`plamning chegaralari bo`lishi mumkin.

Demak,

$$\sigma(A_4)\backslash\Sigma \subset \sigma(A_4)\backslash\sigma_{ess}(A_4)$$

munosabat o`rinli bo`ladi, ya`ni $\sigma_{ess}(A_4) \subset \Sigma$.

Xulosa qilib aytganda, $\sigma_{ess}(A_4) = \Sigma$.

2.1.1 – teorema isbot bo`ldi.

2.2. 4 – tartibli operatorli matritsalarning diskret spektri

2.2.1 – Lemma. $z \in \mathbb{C} \setminus \sigma_{ess}(A_4)$ soni A_4 operatorning xos qiymati bo`lishi uchun $\Omega_4(z) = 0$ bo`lishi zarur va yetarlidir.

Isbot. Faraz qilaylik, $z \in \mathbb{C} \setminus \sigma_{ess}(A_4)$ soni A_4 operatorning xos qiymati bo`lsin. $(f_0, f_1, f_2, f_3) \in H^{(4)}$ vektor esa bu xos qiymatga mos xos vektor bo`lsin. U holda $A_4 f = zf$ tenglamani yechib, noldan farqli $(f_0, f_1, f_2, f_3) \in H^{(4)}$ xos vektorni aniqlaymiz.

$$\begin{pmatrix} A_{00} & A_{01} & 0 & 0 \\ A_{01}^* & A_{11} & A_{12} & 0 \\ 0 & A_{12}^* & A_{22} & A_{23} \\ 0 & 0 & A_{23}^* & A_{33} \end{pmatrix} \cdot \begin{pmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{pmatrix} zf_0 \\ zf_1 \\ zf_2 \\ zf_3 \end{pmatrix}$$

$$A_{00} f_0 + A_{01} f_1 = zf_0$$

$$A_{01}^* f_0 + A_{11} f_1 + A_{12} f_2 = zf_1$$

$$A_{12}^* f_1 + A_{22} f_2 + A_{23} f_3 = zf_2$$

$$A_{23}^* f_2 + A_{33} f_3 = zf_3,$$

yoki

$$\begin{cases} \varepsilon f_0 + \alpha \int_{\mathbb{T}^d} v(t) f_1(t) dt = zf_0 \\ \alpha v(x) f_0 + \big(\varepsilon + \omega(x)\big) f_1(x) + \alpha \int_{\mathbb{T}^d} v(t) f_2(x,t) dt = zf_1 \\ \alpha v(y) f_1(x) + \big(\varepsilon + \omega(x) + \omega(y)\big) f_2(x,y) + \alpha \int_{\mathbb{T}^d} v(t) f_3(x,y,t) dt = zf_2 \\ \alpha v(t) f_2(x,y) + \big(\varepsilon + \omega(x) + \omega(y) + \omega(t)\big) f_3(x,y,t) = zf_3 \end{cases} \quad (2.2.1)$$

tenglamalar sistemasiga ega bo`lamiz.

(2.2.1) sistemaning 4 – tenglamasidan $f_3(x, y, t)$ ni topamiz:

$$f_3(x,y,t) = -\frac{\alpha v(t) f_2(x,y)}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z}.$$

Topilgan ifodani (2.2.1) tenglamalar sistemasining 3 – tenglamasiga qo`yamiz va $f_2(x,y)$ ning ko`rinishini aniqlaymiz:

$$\alpha v(y) f_1(x) + (\varepsilon+\omega(x)+\omega(y)-z) f_2(x,y) - \alpha^2 \int_{\mathbb{T}^d} \frac{f_2(x,y)\cdot v^2(t)\,dt}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z} = 0,$$

$$\left(\varepsilon+\omega(x)+\omega(y)-z-\alpha^2 \int_{\mathbb{T}^d} \frac{v^2(t)\,dt}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z}\right) f_2(x,y) = -\alpha v(y) f_1(x).$$

Endi

$$\Delta_2(x,y,z) = \varepsilon+\omega(x)+\omega(y)-z-\alpha^2 \int_{\mathbb{T}^d} \frac{v^2(t)\,dt}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z}$$

deb belgilash kiritamiz. U holda

$$f_2(x,y) = -\frac{\alpha v(y) f_1(x)}{\Delta_2(x,y,z)}.$$

Bunda $\Delta_2(x,y,z) \neq 0$.

Topilgan $f_2(x,y)$ ning ifodasini (2.2.1) tenglamalar sistemasining 2 – tenglamasiga qo`yamiz hamda $f_1(x)$ ni aniqlaymiz:

$$\alpha v(x) f_0 + \left(\varepsilon+\omega(x)-z-\alpha^2 \int_{\mathbb{T}^d} \frac{v^2(t)\,dt}{\Delta_2(x,t,z)}\right) f_1(x) = 0,$$

$$f_1(x) = -\frac{\alpha v(x) f_0}{\varepsilon+\omega(x)-z-\alpha^2 \int_{\mathbb{T}^d} \frac{v^2(t)\,dt}{\Delta_2(x,t,z)}}.$$

So`ngra,

$$\Delta_3(x,z) = \varepsilon+\omega(x)-z-\alpha^2 \int_{\mathbb{T}^d} \frac{v^2(t)\,dt}{\Delta_2(x,t,z)}$$

deb belgilash kiritamiz, $\Delta_3(x,z) \neq 0$. U holda

$$f_1(x) = -\frac{\alpha v(x) f_0}{\Delta_3(x,z)}.$$

Endi $f_1(x)$ ning qiymatini (2.2.1) tenglamalar sistemasining 1 – tenglamasiga qo`yamiz:

$$\varepsilon f_0 + \alpha \int_{\mathbb{T}^d} v(t) \left(-\frac{\alpha v(x) f_0}{\Delta_3(x,z)}\right) dt = z f_0,$$

$$\left(\varepsilon - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(t)\, dt}{\Delta_3(t,z)}\right) f_0 = 0.$$

$$\Omega_4(z) = \varepsilon - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(t)\, dt}{\Delta_3(t,z)}$$

deb belgilasak, $\Omega_4(z) f_0 = 0$ hosil bo`ladi. Agar bu tenglikda $f_0 = 0$ bo`lsa, u holda $f_1(x) = 0, f_2(x,y) = 0, f_3(x,y,t) = 0$ ekanligi kelib chiqadi, ya`ni $f = (f_0, f_1, f_2, f_3) = \theta$ hosil bo`ladi. Bu esa f vektorning xos vektor bo`lishiga ziddir. Demak, $\Omega_4(z) = 0$ ekanligi kelib chiqadi.

2.2.1 – lemma to`liq isbotlandi.

2.3. 4 – tartibli operatorli matritsalarning rezolventa operatori

A_4 operatorga mos rezolventa operatorining ko`rinishini izlash masalasini qaraymiz.

2.3.1 – Lemma. Har bir fiksirlangan $z \in \rho(A_4)$ soni uchun A_4 operatorga mos rezolventa operatori $H_0 \oplus H_1 \oplus H_2 \oplus H_3$ Gilbert fazosida to`rtinchi tartibli

$$R_\lambda(A_4) = \begin{pmatrix} R_{00}(\lambda, A_4) & R_{01}(\lambda, A_4) & R_{02}(\lambda, A_4) & R_{03}(\lambda, A_4) \\ R_{10}(\lambda, A_4) & R_{11}(\lambda, A_4) & R_{12}(\lambda, A_4) & R_{13}(\lambda, A_4) \\ R_{20}(\lambda, A_4) & R_{21}(\lambda, A_4) & R_{22}(\lambda, A_4) & R_{23}(\lambda, A_4) \\ R_{30}(\lambda, A_4) & R_{31}(\lambda, A_4) & R_{32}(\lambda, A_4) & R_{33}(\lambda, A_4) \end{pmatrix}$$

operatorli matritsa ko`rinishida ta`sir qiladi.

Isbot. Rezolventa operatorni qurish uchun

$$A_4 f - zf = g$$

tenglamani yechamiz, bunda (f_0, f_1, f_2, f_3) va $g = (g_0, g_1, g_2, g_3) \in H_0 \oplus H_1 \oplus H_2 \oplus H_3$. Ya`ni quyidagi tenglamalar sistemasini qaraymiz:

$$\begin{cases} (\varepsilon - z)f_0 + \alpha \int_{\mathbb{T}^d} v(t) f_1(t) dt = g_0 \\ \alpha v(x) f_0 + (\varepsilon + \omega(x) - z) f_1(x) + \alpha \int_{\mathbb{T}^d} v(t) f_2(x,t) dt = g_1(x) \\ \alpha v(y) f_1(x) + (\varepsilon + \omega(x) + \omega(y) - z) f_2(x,y) + \alpha \int_{\mathbb{T}^d} v(t) f_3(x,y,t) dt = g_2(x,y) \\ \alpha v(t) f_2(x,y) + (\varepsilon + \omega(x) + \omega(y) + \omega(t) - z) f_3(x,y,t) = g_3(x,y,t) \end{cases} \tag{2.3.1}$$

$z \notin \sigma_{ess}(A_4)$ bo`lganligidan ixtiyoriy $x, y, t \in \mathbb{T}^d$ uchun

$$\varepsilon + \omega(x) - z \neq 0, \ \ \varepsilon + \omega(x) + \omega(y) - z \neq 0 \text{ va}$$

$$\varepsilon + \omega(x) + \omega(y) + \omega(t) - z \neq 0$$

munosabatlar o`rinli bo`ladi.

Endi (2.3.1) tenglamalar sistemasining 4 – tenglamasidan $f_3(x,y,t)$ ni topamiz va 3 – tenglamasiga qo`yamiz:

$$f_3(x,y,t)=\frac{g_3(x,y,t)}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z}-\frac{\alpha v(t)f_2(x,y)}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z} \tag{2.3.2}$$

$$\alpha v(y)f_1(x)+(\varepsilon+\omega(x)+\omega(y)-z)f_2(x,y)+$$

$$+\alpha\int_{\mathbb{T}^d}v(t)\left(\frac{g_3(x,y,t)}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z}-\frac{\alpha v(t)f_2(x,y)}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z}\right)dt=g_2(x,y);$$

$$\alpha v(y)f_1(x)+\left(\varepsilon+\omega(x)+\omega(y)-z-\alpha^2\int_{\mathbb{T}^d}\frac{v^2(t)\,dt}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z}\right)f_2(x,y)+$$

$$+\alpha\int_{\mathbb{T}^d}\frac{v(t)\cdot g_3(x,y,t)\,dt}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z}=g_2(x,y);$$

So`ngra oldingi rejadagi quyidagi

$$\Delta_2(x,y,z)=\varepsilon+\omega(x)+\omega(y)-z-\alpha^2\int_{\mathbb{T}^d}\frac{v^2(t)dt}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z}$$

belgilashdan foydalanamiz. U holda

$$\alpha v(y)f_1(x)+\Delta_2(x,y,z)f_2(x,y)=g_2(x,y)-\alpha\int_{\mathbb{T}^d}\frac{v(t)\cdot g_3(x,y,t)dt}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z}$$

kelib chiqadi. Oxirgi tenglamadan $f_2(x,y)$ ni topamiz:

$$f_2(x,y)=\frac{g_2(x,y)}{\Delta_2(x,y,z)}-\frac{\alpha}{\Delta_2(x,y,z)}\int_{\mathbb{T}^d}\frac{v(t)\cdot g_3(x,y,t)\,dt}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z}-\frac{\alpha v(y)f_1(x)}{\Delta_2(x,y,z)} \tag{2.3.3}$$

(2.3.3) ifodani (2.3.1) tenglamalar sistemasining 2 – tenglamasiga qo`yamiz:

$$\alpha v(x)f_0+\left(\varepsilon+\omega(x)-z-\alpha^2\int_{\mathbb{T}^d}\frac{v^2(t)dt}{\Delta_2(x,t,z)}\right)f_1(x)+\alpha\int_{\mathbb{T}^d}\frac{g_2(x,t)v(t)dt}{\Delta_2(x,t,z)}-$$

$$-\alpha^2\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\frac{v^2(t)g_3(x,y,t)\,dt^2}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\Delta_2(x,t,z)}=g_1(x).$$

Oldingi rejadagi

$$\Delta_3(x,z) = \varepsilon + \omega(x) - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(t)dt}{\Delta_2(x,t,z)}$$

belgilashdan foydalanamiz va oxirgi ifodadan $f_1(x)$ ni topamiz:

$$f_1(x) = \frac{g_1(x)}{\Delta_3(x,z)} - \frac{\alpha v(x) f_0}{\Delta_3(x,z)} - \frac{\alpha}{\Delta_3(x,z)} \int_{\mathbb{T}^d} \frac{g_2(x,t)v(t)dt}{\Delta_2(x,t,z)} +$$

$$+ \frac{\alpha^2}{\Delta_3(x,z)} \int_{\mathbb{T}^d} \int_{\mathbb{T}^d} \frac{v^2(t)\, g_3(x,y,t)dt^2}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\Delta_2(x,t,z)} \qquad (2.3.4)$$

Va nihoyat, topilgan $f_1(x)$ ning qiymatini (2.3.1) tenglamalar sistemasining 1 – tenglamasiga qo`yamiz:

$$\left(\varepsilon - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(t)dt}{\Delta_3(t,z)}\right) f_0 + \alpha \int_{\mathbb{T}^d} \frac{v(t) g_1(t)dt}{\Delta_3(t,z)} - \alpha^2 \int_{\mathbb{T}^d} \int_{\mathbb{T}^d} \frac{v^2(t) g_2(x,t)dt^2}{\Delta_2(x,t,z)\Delta_3(t,z)} +$$

$$+\alpha^3 \int_{\mathbb{T}^d} \int_{\mathbb{T}^d} \int_{\mathbb{T}^d} \frac{v^3(t)\, g_3(x,y,t)\, dt^3}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\Delta_2(x,t,z)\Delta_3(t,z)} = g_0.$$

Oldingi rejadagi

$$\Omega_4(z) = \varepsilon - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(t)\, dt}{\Delta_3(t,z)}$$

belgilashdan foydalangan holda hosil bo`lgan tenglikdan $f_0$ ning ko`rinishini aniqlaymiz:

$$f_0 = \frac{g_0}{\Omega_4(z)} - \frac{\alpha}{\Omega_4(z)} \int_{\mathbb{T}^d} \frac{v(t) g_1(t)dt}{\Delta_3(t,z)} + \frac{\alpha^2}{\Omega_4(z)} \int_{\mathbb{T}^d} \int_{\mathbb{T}^d} \frac{v^2(t)\, g_2(x,t)dt^2}{\Delta_2(x,t,z)\Delta_3(t,z)} -$$

$$- \frac{\alpha^3}{\Omega_4(z)} \int_{\mathbb{T}^d} \int_{\mathbb{T}^d} \int_{\mathbb{T}^d} \frac{v^3(t) g_3(x,y,t)dt^3}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\Delta_2(x,t,z)\Delta_3(t,z)}.$$

Oxirgi topilgan f_0 ning ifodasini (2.3.4) tenglikka qo`yamiz:

$$f_1(x) = \frac{g_1(x)}{\Delta_3(x,z)} - \frac{\alpha v(x)\, g_0}{\Omega_4(z)\cdot\Delta_3(x,z)} + \frac{\alpha^2 v(x)}{\Omega_4(z)\cdot\Delta_3(x,z)} \int_{\mathbb{T}^d} \frac{v(t) g_1(t)}{\Delta_3(t,z)} dt -$$

$$- \frac{\alpha^3 v(x)}{\Omega_4(z)\cdot\Delta_3(x,z)} \int_{\mathbb{T}^d} \int_{\mathbb{T}^d} \frac{v^2(t) g_2(x,t)dt^2}{\Delta_2(x,t,z)\Delta_3(t,z)} + \frac{\alpha^4 v(x)}{\Omega_4(z)\cdot\Delta_3(x,z)} \cdot$$

$$\cdot\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\frac{v^3(t)g_3(x,y,t)dt^3}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\Delta_2(x,t,z)\Delta_3(t,z)}.$$

Oxirgi $f_1(x)$ ning ifodasini (2.3.3) tenglikka qo`yamiz:

$$f_2(x,y)=\frac{g_2(x,y)}{\Delta_2(x,y,z)}-\frac{\alpha}{\Delta_2(x,y,z)}\int_{\mathbb{T}^d}\frac{v(t)\cdot g_3(x,y,t)dt}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z}-\frac{\alpha v(y)g_1(x)}{\Delta_2(x,y,z)\Delta_3(x,z)}+$$

$$+\frac{\alpha^2 v(x)v(y)g_0}{\Omega_4(z)\cdot\Delta_3(x,z)\cdot\Delta_2(x,y,z)}-\frac{\alpha^3 v(x)v(y)}{\Omega_4(z)\cdot\Delta_3(x,z)\cdot\Delta_2(x,y,z)}\int_{\mathbb{T}^d}\frac{v(t)g_1(t)}{\Delta_3(t,z)}dt+$$

$$+\frac{\alpha^4 v(x)v(y)}{\Omega_4(z)\cdot\Delta_3(x,z)\cdot\Delta_2(x,y,z)}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\frac{v^2(t)g_2(x,t)dt^2}{\Delta_2(x,t,z)\Delta_3(t,z)}-\frac{\alpha^5 v(x)v(y)}{\Omega_4(z)\cdot\Delta_3(x,z)\cdot\Delta_2(x,y,z)}\cdot$$

$$\cdot\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\frac{v^3(t)g_3(x,y,t)dt^3}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\Delta_2(x,t,z)\Delta_3(t,z)}.$$

Endi $f_2(x,y)$ ni (2.3.2) ga qo`yamiz:

$$f_3(x,y,t)=\frac{g_3(x,y,t)}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z}-\frac{\alpha v(t)g_2(x,y)}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\cdot\Delta_2(x,y,z)}+$$

$$+\frac{\alpha^2 v(t)}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\cdot\Delta_2(x,y,z)}\int_{\mathbb{T}^d}\frac{v(t)\cdot g_3(x,y,t)dt}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z}+$$

$$+\frac{\alpha^2 v(t)v(y)g_1(x)}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\cdot\Delta_2(x,y,z)\cdot\Delta_3(x,z)}-\frac{\alpha^3 v(x)v(y)v(t)g_0}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\cdot\Delta_2(x,y,z)\cdot\Delta_3(x,z)\cdot\Omega_4(z)}+$$

$$+\frac{\alpha^4 v(x)v(y)v(t)}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\cdot\Delta_2(x,y,z)\cdot\Delta_3(x,z)\cdot\Omega_4(z)}\int_{\mathbb{T}^d}\frac{v(t)g_1(t)}{\Delta_3(t,z)}dt-$$

$$-\frac{\alpha^5 v(x)v(y)v(t)}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\cdot\Delta_2(x,y,z)\cdot\Delta_3(x,z)\cdot\Omega_4(z)}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\frac{v^2(t)g_2(x,t)dt^2}{\Delta_2(x,t,z)\Delta_3(t,z)}+$$

$$+\frac{\alpha^6 v(x)v(y)v(t)}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\cdot\Delta_2(x,y,z)\cdot\Delta_3(x,z)\cdot\Omega_4(z)}\cdot$$

$$\cdot\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\frac{v^3(t)g_3(x,y,t)dt^3}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\Delta_2(x,t,z)\Delta_3(t,z)}$$

tengliklarga ega bo`ldik.

$$\begin{cases} R_{00}g_0 + R_{01}g_1 + R_{02}g_2 + R_{03}g_3 = f_0 \\ R_{10}g_0 + R_{11}g_1 + R_{12}g_2 + R_{13}g_3 = f_1 \\ R_{20}g_0 + R_{21}g_1 + R_{22}g_2 + R_{23}g_3 = f_2 \\ R_{30}g_0 + R_{31}g_1 + R_{32}g_2 + R_{33}g_3 = f_3 \end{cases}$$

sistemadan foydalanib, quyidagi rezolventa operatorlariga ega bo`lamiz:

$$R_{00}(\lambda, A_4)g_0 = \frac{g_0}{\Omega_4(z)},$$

$$R_{01}(\lambda, A_4)g_1 = -\frac{\alpha}{\Omega_4(z)}\int_{\mathbb{T}^d}\frac{v(t)g_1(t)dt}{\Delta_3(t,z)},$$

$$R_{02}(\lambda, A_4)g_2 = \frac{\alpha^2}{\Omega_4(z)}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\frac{v^2(t)g_2(x,t)dt^2}{\Delta_2(x,t,z)\Delta_3(t,z)},$$

$$R_{03}(\lambda, A_4)g_3 = -\frac{\alpha^3}{\Omega_4(z)}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\frac{v^3(t)g_3(x,y,t)dt^3}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\Delta_2(x,t,z)\Delta_3(t,z)},$$

$$(R_{10}(\lambda, A_4)g_0)(x) = -\frac{\alpha v(x)g_0}{\Omega_4(z)\cdot\Delta_3(x,z)},$$

$$(R_{11}(\lambda, A_4)g_1)(x) = \frac{g_1(x)}{\Delta_3(x,z)} + \frac{\alpha^2 v(x)}{\Omega_4(z)\cdot\Delta_3(x,z)}\int_{\mathbb{T}^d}\frac{v(t)g_1(t)}{\Delta_3(t,z)}dt,$$

$$(R_{12}(\lambda, A_4)g_2)(x) = -\frac{\alpha^3 v(x)}{\Omega_4(z)\cdot\Delta_3(x,z)}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\frac{v^2(t)g_2(x,t)dt^2}{\Delta_2(x,t,z)\Delta_3(t,z)},$$

$$(R_{13}(\lambda, A_4)g_3)(x) = \frac{\alpha^4 v(x)}{\Omega_4(z)\cdot\Delta_3(x,z)}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\frac{v^3(t)g_3(x,y,t)dt^3}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\Delta_2(x,t,z)\Delta_3(t,z)},$$

$$(R_{20}(\lambda, A_4)g_0)(x,y) = \frac{\alpha^2 v(x)v(y)g_0}{\Omega_4(z)\cdot\Delta_3(x,z)\cdot\Delta_2(x,y,z)},$$

$$(R_{21}(\lambda, A_4)g_1)(x,y) = -\frac{\alpha v(y)g_1(x)}{\Delta_2(x,y,z)\Delta_3(x,z)} - \frac{\alpha^3 v(x)v(y)}{\Omega_4(z)\cdot\Delta_3(x,z)\cdot\Delta_2(x,y,z)}\int_{\mathbb{T}^d}\frac{v(t)g_1(t)}{\Delta_3(t,z)}dt,$$

$$(R_{22}(\lambda, A_4)g_2)(x,y) = \frac{g_2(x,y)}{\Delta_2(x,y,z)} + \frac{\alpha^4 v(x)v(y)}{\Omega_4(z)\cdot\Delta_3(x,z)\cdot\Delta_2(x,y,z)}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\frac{v^2(t)g_2(x,t)dt^2}{\Delta_2(x,t,z)\Delta_3(t,z)},$$

$$(R_{23}(\lambda, A_4)g_3)(x,y) = -\frac{\alpha}{\Delta_2(x,y,z)}\int_{\mathbb{T}^d}\frac{v(t)\cdot g_3(x,y,t)dt}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z} - \frac{\alpha^5 v(x)v(y)}{\Omega_4(z)\cdot\Delta_3(x,z)\cdot\Delta_2(x,y,z)}\cdot$$

$$\cdot \int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d} \frac{v^3(t) g_3(x,y,t)\, dt^3}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\Delta_2(x,t,z)\Delta_3(t,z)},$$

$$(R_{30}(\lambda,A_4)g_0)(x,y,t) = -\frac{\alpha^3 v(x)v(y)v(t)g_0}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\cdot\Delta_2(x,y,z)\cdot\Delta_3(x,z)\cdot\Omega_4(z)},$$

$$(R_{31}(\lambda,A_4)g_1)(x,y,t) = \frac{\alpha^2 v(t)v(y)g_1(x)}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\cdot\Delta_2(x,y,z)\cdot\Delta_3(x,z)} +$$

$$+\frac{\alpha^4 v(x)v(y)v(t)}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\cdot\Delta_2(x,y,z)\cdot\Delta_3(x,z)\cdot\Omega_4(z)}\int_{\mathbb{T}^d}\frac{v(t)g_1(t)}{\Delta_3(t,z)}dt,$$

$$(R_{32}(\lambda,A_4)g_2)(x,y,t) = -\frac{\alpha v(t)g_2(x,y)}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\cdot\Delta_2(x,y,z)} -$$

$$-\frac{\alpha^5 v(x)v(y)v(t)}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\cdot\Delta_2(x,y,z)\cdot\Delta_3(x,z)\cdot\Omega_4(z)}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\frac{v^2(t)g_2(x,t)dt^2}{\Delta_2(x,t,z)\Delta_3(t,z)},$$

$$(R_{33}(\lambda,A_4)g_3)(x,y,t) = \frac{g_3(x,y,t)}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z} + \frac{\alpha^2 v(t)}{(\varepsilon+\omega(x)+\omega(y)+\omega(t)-z)\cdot\Delta_2(x,y,z)}\cdot$$

$$\cdot\int_{\mathbb{T}^d}\frac{v(t)\cdot g_3(x,y,t)dt}{\varepsilon+\omega(x)+\omega(y)+\omega(t)-z}\left(1+\frac{\alpha^4 v(x)v(y)}{\Delta_3(x,z)\cdot\Omega_4(z)}\int_{\mathbb{T}^d}\int_{\mathbb{T}^d}\frac{v^2(t)dt^2}{\Delta_2(x,t,z)\Delta_3(t,z)}\right.$$

2.3.1 – lemma to`liq isbotlandi.

II BOB XULOSASI

Bitiruv malakaviy ishining ikkinchi bobi “4 – tartibli operatorli matritsalarning spektral xossalari” deb nomlangan bo`lib, u uchta bo`limdan iborat.

2.1 – bo`limda 4 – tartibli $A_4$ operatorli matritsa Fok fazosining to`rt zarrachali qirqilgan qism fazosida, ya’ni nol zarrachali, bir zarrachali, ikki zarrachali va uch zarrachali qism fazolarning to’gri yig’indisida chiziqli, chegaralangan va o`z – o`ziga qo`shma operator sifatida o`rganilgan. A_4 operatorli matritsa muhim spektrining joylashuv o`rni tavsiflangan.

Muhim spektrni tashkil qilgan to`plamlarni aniqlashda Veyl kriteriyasi va A_4 operator xos funksiyalariga mos keluvchi Faddeyev tenglamasi xossalaridan foydalanilgan. Muhim spektrga tegishli kesmalarning maksimal sonini aniqlashda xos qiymatlar soni, ularning mavjudlik shartlari hamda yoyiluvchi operatorlarning spektri haqidagi teoremalardan foydalanilgan.

2.2 – bo`limda diskret spektrni topish va o`rganish imkonini beruvchi regulyar funksiya topilgan. Bunda chiziqli tenglamalar sistemasida noma’lumlarni ketma-ket topish usulidan foydalanilgan. Bu funksiyaning monotonlik xossalari o’rganilgan.

2.3 – bo`limda esa 4 – tartibli operatorli matritsaning rezolventa operatorining ko`rinishi aniq topilgan. Qaralayotgan operator 4-tartibli operatorli matritsa bo’lganligi uchun hosil bo’lgan rezolventa operatori ham 4-tartibli operatorli matritsa bo’lishi ko’rsatilgan

II bobda olingan natijalarni isbotlashda funksional analiz va kompleks analiz elementlaridan foydalanilgan.

XOTIMA

Bitiruv malakaviy ishi mavzusi “4×4 – o`lchamli operatorli matritsalarning spektral xossalari” bo`lib, u ikkita bobga bo`lib o`rganilgan. Shuni alohida ta’kidlab o’tish lozimki, bunday operatorli matrisalar zamonaviy matematik fizikada panjaradagi soni saqlanmaydigan ko’pi bilan to’rtta zarrachalar sistemasiga mos Gamil’tonianni tavsiflaydi. Odatda uning diagonal elementlari qo’zg’almas operatorlar deyiladi, diagonal bo’lmagan elementlar esa paydo qilish va yo’qotish operatorlari deyiladi. Yo’qotish operatorining qo’shmasi paydo qilish operatori bo’ladi va aksincha.

I bobda asosan 2 – va 3 – tartibli operatorli matritsalar (elementlari Gil’bert fazosida ta’sir qiluvchi chiziqli chegaralangan operatorlar bo’lgan matrisalar) chiziqlilikka, chegaralanganlikka va o`z – o`ziga qo`shmalikka tekshirilgan. 2 – tartibli operatorli matritsaning muhim spektri Veyl teoremasi yordamida aniqlangan. Unga mos Fredgolm determinanti qurilib, u berilgan operatorning diskret spektrini tahlil qilishda qo`llanilgan.

3 – tartibli operatorli matritsa muhim spektri 2 – tartibli operatorli matritsa spektri yordamida tavsiflangan. Uni o’rganishda Veyl mezoni, Faddeyev tenglamasi va Fredgolm nazariyasidan foydalanilgan. Diskret spektr va rezolventa operatorlari uchun aniq formulalar topilgan.

II bob 4 – tartibli operatorli matritsalarning spektral xossalarini o`rganishga bag`ishlangan bo`lib, avvalo qaralayotgan operator chiziqlilikka, chegaralanganlikka va o`z – o`ziga qo`shmalikka tekshirilgan. Uning muhim spektri 2 – va 3 – tartibli operatorli matritsalarning muhim spektrlari yordamida aniqlangan. Diskret spektrni aniqlashda asos bo`lib xizmat qiladigan regulyar funksiya qurilgan. Funksional analiz elementlari yordamida berilgan operatorga mos rezolventa operatori topilgan.

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