O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS

TA’LIM VAZIRLIGI

BUXORO DAVLAT UNIVERSITETI

Fizika–matematika fakulteti

“Matematika” kafedrasi

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“KUCHSIZ GORIZONTAL MUHITDA XOTIRALI TENGLAMA UCHUN TESKARI MASALA” mavzusida

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“Ish ko‘rildi va himoya qilishga	**Ilmiy rahbar________o‘qit.Z.R.Bozorov**
ruxsat berildi”	**“_______”________________2019y.**
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«______»__________________2019 y.	

“Himoya qilishga ruxsat berildi”

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“_______”______________2019 y.

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MUNDARIJA.

Kirish.

> Biz uchun dolzarbligi va ahamiyatini yo‘qotmaydigan masala bu – farzandlarimizni mustaqil fikrli, bilim va hayotiy pozitsiyaga ega chinakam vatanparvar insonlar etib tarbiyalash vazifasidir.
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> SH. M. MIRZIYOYEV

Prezidentimiz Shavkat Mirziyoyev 2019 yilga “Faol investitsiyalar va ijtimoiy rivojlanish yili” deb nom berdilar va joriy yilda amalga oshiriladigan ishlar rejasi belgilab berildi.

O‘zbekiston Respublikasi prezidenti “2017-2021 yillarda O‘zbekiston Respublikasini rivojlantirishning beshta ustuvor yo‘nalishi bo‘yicha Harakatlar strategiyasini “Faol investitsiyalar va ijtimoiy rivojlanish yili”da amalga oshirishga oid Davlat dasturi to‘g‘risida”gi farmonni imzoladi. Farmon bilan umumiy qiymati 16,9 trillion so‘m va 8,1 milliard AQSh dollariga teng loyihalarni amalga oshirishni nazarda tutuvchi hamda quyidagilarga qaratilgan 2017-2021 yillarda O‘zbekiston Respublikasini rivojlantirishning beshta ustuvor yo‘nalishi bo‘yicha Harakatlar strategiyasini “Faol investitsiyalar va ijtimoiy rivojlanish yili”da amalga oshirishga oid davlat dasturi tasdiqlandi:

davlat va jamiyat qurilishi tizimini takomillashtirish sohasida – parlamentning muhim qarorlar qabul qilish va qonunlar ijrosini nazorat etish faoliyatini kuchaytirish, ijro etuvchi hokimiyat tizimini optimallashtirish, ma'muriy islohotlarni davom ettirish, davlat boshqaruvida zamonaviy menejment uslublarini keng qo‘llash, davlat xizmatlarini rivojlantirish, davlat xizmatiga malakali mutaxassislarni jalb etishga qaratilgan yagona kadrlar siyosatini shakllantirish, mahalliy davlat hokimiyati organlarining vakolat va mas'uliyatini qayta ko‘rib chiqish, ularning mustaqilligini oshirish;

qonun ustuvorligini ta'minlash va sud-huquq tizimini yanada isloh qilish sohasida – sud hokimiyatining haqiqiy mustaqilligiga erishish, aholi tinchligi, xavfsizligini va qonuniylikni ta'minlash, jinoyatchilikning barvaqt oldini olish bo‘yicha zarur choralar ko‘rish, jinoyat qonunchiligini takomillashtirish va liberallashtirish;

iqtisodiyotni rivojlantirish va investitsiyalarni faol jalb etish sohasida – makroiqtisodiy barqarorlikni ta'minlash, sog‘lom raqobat uchun zarur sharoitlarni yaratish, ishbilarmonlik va investitsiya muhitini tubdan yaxshilash, iqtisodiyotda davlat ishtirokini jiddiy ravishda kamaytirish, yuqori iqtisodiy o‘sish sur'atlarini saqlab qolish, “xufiyona” iqtisodiyotga qarshi kurashish va uning ulushini keskin qisqartirish, valyuta siyosatini erkinlashtirishni davom ettirish;

ijtimoiy rivojlanish sohasida – aholi o‘rtasida ishsizlikni kamaytirish, odamlarning daromadlarini oshirish, fan va uzluksiz ta'limni rivojlantirish, tibbiy xizmatlar sifatini yaxshilash va ular bilan aholini qamrab olishni kengaytirish, xotin-qizlar va yoshlarni ijtimoiy qo‘llab-quvvatlashni kuchaytirish, odamlarning turmush sharoitlarini yaxshilash, ularni munosib turar joy bilan ta'minlash va farovonligini oshirish, jamiyatda sog‘lom turmush tarzini qaror toptirish, jismoniy tarbiya va sportni yanada ommalashtirish, turizmni rivojlantirish;

xavfsizlik, millatlararo totuvlik va diniy bag‘rikenglikni ta'minlash, shuningdek, tashqi siyosat sohasida – mamlakatimiz mudofaa qudratini oshirish, O‘zbekiston Respublikasi Qurolli Kuchlarining salohiyatini mustahkamlash, milliy mudofaa sanoati kompleksini shakllantirish, ekologik xavfsizlikni ta'minlash, suv va boshqa tabiiy resurslardan oqilona foydalanish, ochiqlik, o‘zaro teng va manfaatli hamkorlik tamoyillariga asoslangan tashqi siyosiy faoliyat samaradorligini yanada oshirish.

O‘zbekiston Respublikasi Prezidenti Shavkat Mirziyoyev raisligida 2019-yil 19-mart kuni yoshlarga e’tiborni kuchaytirish, ularni ma’daniyat, san’at, jismoniytarbiya va sportga keng jalb etish, ularga axborot texnologiyalaridan foydalanish ko‘nikmalarini singdirish, yoshlaro‘rtasida kitobxonlikni targ‘ib qilish, xotin-qizlar bandligini oshirish masalalariga bag‘ishlangan videoselektor yig‘ilishi o‘tkazildi.

Mamlakatimiz aholisining 30 foizini 14 yoshdan 30 yoshgacha bo‘lgan yigit-qizlar tashkil etadi.Ularning ta’lim olishi, kasb-hunar egallashi uchun keng sharoit yaratilgan. Shu bilan birga, yoshlarning bo‘sh vaqtlarini mazmunli o‘tkazishni tashkil etish dolzarb masala hisoblanadi. Yoshlar qanchalik ma’naviy barkamol bo‘lsa, turli yot illatlarga qarshi immuniteti ham shunchalik kuchli bo‘ladi.

Ma’lumki davlatimiz rahbari ijtimoiy, ma’naviy-ma’rifiy sohalardagi ishlarni yangi tizim asosida yo‘lga qo‘yish bo‘yicha 5 ta muhim tashabbusni ilgari surgan edi.

Birinchi tashabbus– yoshlarning musiqa, rassomlik, adabiyot, teatr va san’atning boshqa turlariga qiziqishlarini oshirishga, iste’dodini yuzaga chiqarishga xizmat qiladi.

Ikkinchi tashabbus – yoshlarni jismoniy chiniqtirish, sport sohasida qobiliyatini namoyon qilishlari uchun zarur sharoitlar yaratishga yo‘naltirilgan.

Uchinchi tashabbus – aholi va yoshlar o‘rtasida kompyuter texnologiyalari va internetdan samarali foydalanishni tashkil etishga qaratilgan.

To‘rtinchi tashabbus – yoshlar ma’naviyatini yuksaltirish, ular o‘rtasida kitobxonlikni keng targ‘ib qilish bo‘yicha tizimli ishlarni tashkil etishga yo‘naltirilgan.

Beshinchi tashabbus – xotin-qizlarni ish bilan ta’minlash masalalarini nazarda tutadi.

Ana shu ezgu g‘oya Prezidentimizning Sirdaryo viloyatiga tashrifi chog‘ida boshlanib, qisqa vaqtda ulkan ishlar amalga oshirildi. Sirdaryo viloyatidagi tuman va shaharlar kutubxonalariga 300 ming nusxada badiiy adabiyotlar yetkazib berildi. Musiqa va san’at maktablari cholg‘u asboblari, sport obyektlari jihozlar bilan ta’minlandi.

Bu ishlar Namangan viloyatida ham davom ettirilib, “Ma’rifat karvoni” tashkil etildi. Yoshlar uchun 25 ming dona kitob, 80 turdagi sport jihozlari va musiqa asboblari yetkazib berildi. Bir so‘z bilan aytganda, ushbu 5 ta tashabbus xalqimiz, ayniqsa, yoshlarimiz tomonidan kata qiziqish bilan kutib olindi.

Ma’daniyat vazirligi va Xalq ta’limi vazirligiga hokimliklar bilan birgalikda tuman (shahar) ma’daniyat markazlari va umumta’lim maktablarida yoshlarning qiziqishlaridan kelib chiqib, qo‘shimcha 1,5 mingta to‘garak tashkil etish vazifasi qo‘yildi. Musiqa va san’at sohasida oliy ma’lumotli kadrlarni ko‘paytirish masalasiga ham e’tabor qaratildi.

Mamlakatimizda 12 mingdan ziyod sport inshootlari borligi, lekin yoshlarni jismoniy tarbiya va ommaviy sportga qamrab olish darajasi yetarli emasligi qayd etildi. Axborot texnologiyalari va kommunikatsiyalarini rivojlantirish vazirligiga ilg‘or xalqaro tajribalar asosida barcha shahar va tuman markazlarida Raqamli texnologiyalar o‘quv markazlarini tashkil etish bo‘yicha topshiriq berildi. Bu markazlarda elektron tijorat va dasturlash bepul o‘rgatiladi, axborot texnologiyalari sohasida tadbirkorlik bilan shug‘ullanish bo‘yicha innovatsion ko‘nikmalar beriladi, “startap” loyihalariga yordam ko‘rsatiladi.

Yoshlarda bolalik chog‘idan kitobga mehr uyg‘otish, mustaqil fikr va keng dunyoqarashni shakllantirish ularning hayot yo‘llarida mustahkam zamin bo‘ladi.

Axborot va ommaviy kommunikatsiyalar agentligiga barcha shahar va tumanlarda ko‘chma kitob pavilyonlarini joylashtirish, Qoraqalpog‘iston Respublikasi va barcha viloyatlarda “Bibliobus”larni yo‘lga qo‘yish orqali qishloq va ovullar aholisiga kutubxona xizmati ko‘rsatish vazifasi qo‘yildi.

Yig‘ilishda xotin-qizlarni ish bilan ta’minlash masalalari ham muhokama qilindi.Shu maqsadda 2019-2020 – yillarda barcha tumanlarda sendvich panellardan 195 ta tikuv – tirikotaj korxonalarini qurish belgilangan.Ularni qurish uchun aholi zich yashaydigan, mehnat resurslari ko‘p joylar tanlab olingan. Bu korxonalarni tashkil etish natijasida xotin-qizlar uchun 24 mingdan ortiq doimiy ish o‘rni yaratish ko‘zda tutilgan.

Bitiruv malakaviy ishining dolzarbligi. Fan, texnika va ishlab chiqarishning barcha sohalarida mavjud muammolarni hal etishda matematik-fizika bugungi kunning eng dolzarb va zamonaviy fanlaridan biri hisoblanadi. Bu fan matematikaning bir qancha jumladan, matematik analiz, algebra, optimallashtirish, differensial tenglamalar, funksional analiz fanlari bilan uzviy bog’liq. Yuqorida aytib o‘tganimizdek sanoat usulida o‘zlashtirish uchun yaroqli mineral-xom ashyo zaxiralari hosil bo‘lishiga olib keladigan geologik jarayonlarning qonuniyatlarini o‘rganish, shuningdek, tektonika, geofizika, seysmologiya kabi sohalarda olib boriladigan izlanishlar asosan matematik fizika tenlamalari yordamida o‘z yechimini topadi. Qaralayotgan masala geofizikada foydali qazilmalarni qidirilishida har xil tabiiy ofatlarni oldindan aytib berish va amaliyotning boshqa sohalarida vujudga keladi. Giperbolik tenglamalarda xotirani tiklash masalasi teskari masalalar nazariyasida nisbatan yangi jadal rivojlanayotgan yo‘nalish bo‘lib, bugungi kunda dolzarb hisoblaniladi.

Bitiruv malakaviy ishining maqsadi. Bitiruv malakaviy ishida xotirali tenglama uchun teskari masalani yechish metodlarini qurish, shuningdek, bu teskari masalalar yechimlarining mavjudligi va yagonaligini tadqiq etish asosiy maqsad qilib belgilangan.

Bitiruv malakaviy ishining vazifalari. Ushbu bitiruv malakaviy ishining asosiy vazifalari sifatida quyidagilar belgilangan:

1) ikkinchi tartibli differensial tenglamalarning klassifikatsiyasi va kanonik ko‘rinishga keltirishni o‘rganish;

2) integral va integro-differensial tenglamalarni o‘rganish;

3) siqiluvchan akslantirishlar prinsipini o‘rganish;

4) kuchsiz gorizontal muhitda xotirali tenglama uchun teskari masalani yechishdan iborat.

Bitiruv malakaviy ishining tadqiqot usuli va uslubiyoti. Xususiy hosilali differensial tenglamalar va integral tenglamalar nazariyasi usullari, qo‘zg’almas nuqta haqidagi teorema.

Bitiruv malakaviy ishining obyekti va predmeti. Xususiy hosilali differensial tenglamalar, Vol’terra tipidagi integral tenglamalar, integro-differensial tenglamalar, del’ta funksiya.

Olingan asosiy natijalar. Ushbu bitiruv malakaviy ishida bo‘sh gorizontal muhitlarda integro-differensial tor tebranish tenglamasidan yadroni aniqlash bo‘yicha natija olingan. Bitiruv malakaviy ishining asosiy natijasi 2.2.1-teorema, va 2.2.1-lemmadan iborat bo‘lib, noma’lum funksiya $O(\varepsilon)$ aniqlikda qurilgan. Olingan natijalar qat’iy matematik isbotlangan.

Natijalarning ilmiy yangiligi va amaliy ahamiyati. Bitiruv malakaviy ishida ko‘rilgan tenglama uchun olingan natijalar yangi hisoblanadi. Ular ushbu sohadagi keyingi tadqiqotlarda ishlatilishi mumkin.

Tadbiq etish darajasi va iqtisodiy samaradorligi. Bitiruv malakaviy ishida yechim haqida qo‘shimcha ma’lumotga asoslanib giperbolik tenglamalar va sistemalarning koeffitsiyentlarini aniqlash masalasi katta ahamiyatga ega. Qidirilayotgan koeffitsiyentlar ma’lum bo‘lsa, o‘rganilayotgan sohalarning muhim xarakteristikalari bo‘lgan parametrlari elastiklik nazariyasida teskari masala hamda, muhitda to‘lqinlarning tarqalishi va akustikasining teskari masalalardan juda ko‘p foydalaniladi.

Bitiruv malakaviy ishining tuzilishi va hajmi. Mazkur bitiruv malakaviy ishi kirish, 2 ta bob, 4 ta paragraf, xotima va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan bo‘lib, ishning hajmi 69 sahifadan iborat. Bunda 2 ta teorema, 2 ta lemma, 2 ta chizma va 18 ta adabiyotlardan foydalanilgan.

I BOB. XUSUSIY HOSILALI DIFFERENSIAL TENGLAMALAR VA INTEGRAL TENGLAMALAR.

1.1. Ikkinchi tartibli differensial tenglamalarning klassifikatsiyasi va kanonik ko‘rinishi.

1.1.1-ta’rif. Oddiy differensial tenglama deb, o‘zgaruvchi x, noma’lum $y = f(x)$ funksiya va uning $y', y'', \dots, y^{(n)}$ hosilalari orasidagi bog‘lanishni ifodalaydigan tenglamaga aytiladi. Differensial tenglama umumiy holda quyidagicha yoziladi:

$$F\left(x, y, y', y'', \dots, y^{(n)}\right) = 0 \qquad (1.1.1)$$

Noma’lum funksiya ko‘p argumentli bo‘lgan holda differensial tenglama xususiy hosilali differensial tenglama deyiladi.

Xususiy hosilali differensial tenglamalar va ularning yechimi to‘g‘risida tushuncha.

D orqali dekart ortogonal koordinatalari $x_1, x_2, \dots x_n,\ n \geq 2$ bo‘lgan x nuqtaning $n-$o‘lchovli E^n Yevklid fazosidagi sohani, ya’ni ochiq bog‘langan (bo‘sh bo‘lmagan) to‘plamni belgilaymiz. Tartiblangan manfiy bo‘lmagan n ta butun sonning $|\alpha| = (\alpha_1, \alpha_2, \dots, \alpha_n)$ ketma-ketligi $n-$ tartibli multiindeks deyiladi, $|\alpha| = \alpha_1 + \ \alpha_2 + \dots + \ \alpha_n$ son bu multiindeksning uzunligi deb ataladi.

$u(x) = u(x_1, x_2, \dots x_n)$ funksiyaning $x \in D$ nuqtadagi $|\alpha| = \alpha_1 + \ \alpha_2 + \dots + \ \alpha_n$ tartibli hosilasini

$$D^\alpha u = \ D_1^{\alpha_1} D_2^{\alpha_2} \dots D_n^{\alpha_n} u \ = \frac{\partial^{|\alpha|} u}{\partial x_1^{\alpha_1} \partial x_2^{\alpha_2} \dots \partial x_n^{\alpha_n}}\,, \qquad D^0 u = u(x) \qquad (1.1.2)$$

ko‘rinishda yozib olamiz. Xususiy holda $\alpha = \alpha_i$ bo‘lganda

$$D^\alpha u = \ \frac{\partial^{\alpha_i} u}{\partial x_i^{\alpha_i}} = D_i^{\alpha_i} u\,, \qquad D_i u = \ \frac{\partial u}{\partial x_i} = \ u_{x_i}\,, \qquad D_i^2 u = \ \frac{\partial^2 u}{\partial x_i^2} = \ u_{x_i x_i} \qquad (1.1.3)$$

$\mathrm{F} = \mathrm{F}(\mathrm{x}, \dots, P_\alpha, \dots)$ funksiya D soha x nuqtalarining va $P_\alpha = P_{\alpha_1\alpha_2\dots\alpha_n} = D^\alpha u$, $\alpha_i = 0,1,\dots$ haqiqiy o‘zgaruvchilarning berilgan funksiyasi bo‘lib, kamida bitta $\frac{\partial F}{\partial P_\alpha}$, $|\alpha| = \mathrm{m} > 0$ hosila noldan farqli bo‘lsin.

1.1.2-ta’rif. Ushbu

$$\mathrm{F}(\mathrm{x}, \dots, D^\alpha u, \dots) = 0 \qquad (1.1.4)$$

tenglik noma’lum $\mathrm{u}(\mathrm{x}) = \mathrm{u}(x_1, x_2, \dots x_n)$ funksiyaga nisbatan $m-$ tartibli xususiy hosilali differensial tenglama deyiladi.

(1.1.4) tenglamaning chap tomoni esa xususiy hosilali differensial operator

deb ataladi.

Agar F barcha $P_\alpha(|\alpha| = 0,1,\dots,\mathrm{m})$ o‘zgaruvchilarga nisbatan chiziqli funksiya bo‘lsa, (1.1.4) tenglama chiziqli differensial tenglama deyiladi.

Agarda F, $|\alpha| = \mathrm{m}$ bo‘lganda barcha P_α o‘zgaruvchilarga nisbatan chiziqli funksiya bo‘lsa, (1.1.4) tenglama kvazichiziqli differensial tenglama deyiladi.

D sohada aniqlangan $\mathrm{u}(\mathrm{x})$ funksiya (1.1.4) tenglamada ishtirok etuvchi barcha hosilalari bilan uzluksiz bo‘lib uni ayniyatga aylantirsa, $\mathrm{u}(\mathrm{x})$ ni (1.1.4) tenglamaning regulyar (klassik) yechimi deyiladi.

Xususiy hosilali $m-$ tartibli chiziqli differensial tenglamani ushbu

$$Lu \equiv \sum_{|\alpha|\le m} a_\alpha(x) D^\alpha u = f(x) \qquad (1.1.5)$$

ko‘rinishda yozib olish mumkin.

Barcha $\mathrm{x} \in \mathrm{D}$ lar uchun (1.1.5) tenglamaning o‘ng tomoni f(x) nolga teng bo‘lsa, (1.1.5) tenglama bir jinsli, f(x) funksiya nolga teng bo‘lmasa, bir jinsli bo‘lmagan tenglama deyiladi.

Agar $\mathrm{u(x)}$ va $\upsilon(\mathrm{x})$ funksiyalar bir jinsli bo‘lmagan (1.1.5) tenglamaning yechimlari bo‘lsa, ravshanki $\mathrm{w(x)} = \mathrm{u(x)} - \upsilon(\mathrm{x})$ ayirma bir jinsli $(\mathrm{f} = 0)$ tenglamaning yechimi bo‘ladi.

Agarda $u_i(\mathrm{x})$, $\mathrm{i} = 1, \dots, \mathrm{k}$ funksiyalar bir jinsli $(\mathrm{f} = 0)$ tenglamaning yechimlari bo‘lsa,

$$u(x) = \sum_{i=1}^{k} c_i\, u_i(x) \qquad (1.1.6)$$

funksiya ham, bu yerda c_i −haqiqiy o‘zgarmaslar, shu tenglamaning yechimi bo‘ladi.

Xususiy hosilali ikkinchi tartibli chiziqli differensial tenglama

$$\sum_{i,j=1}^{n} A_{ij}(x)\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{n} B_i\,(x)\frac{\partial u}{\partial x_i} + C(x)u = f(x) \qquad (1.1.7)$$

ko‘rinishda yoziladi, bu yerda A_{ij}, B_i, C, f – D sohada berilgan haqiqiy funksiyalardir.

(1.1.7) tenglamaning barcha A_{ij}, $\mathrm{i, j} = 1, \dots, \mathrm{n}$ koeffitsientlari nolga teng bo‘lgan $\mathrm{x} \in \mathrm{D}$ nuqtalarda tenglama ikkinchi tartibli bo‘lmay qoladi, ya’ni bu nuqtalarda (1.1.7) tenglamaning tartibi buziladi. Bundan keyin biz (1.1.7) tenglama berilgan sohada uning tartibi ikkiga teng deb hisoblaymiz. (1.1.7) tenglamada $\mathrm{i} \neq \mathrm{j}$ bo‘lganda alohida-alohida $A_{ij}u_{x_i x_j}$, $A_{ji}u_{x_j x_i}$ qo‘shiluvchilar ishtirok etmay, balki ularning yig‘indisi $(A_{ij} + A_{ji})u_{x_i x_j}$ ishtirok etadi. Shu sababli ham umumiyatlikka ziyon yetkazmay hamma vaqt $A_{ij} = A_{ji}$ deb hisoblaymiz.

Eslatib o‘tamiz, Dsohada aniqlangan va k −tartibgacha xususiy hosilalari bilan uzluksiz bo‘lgan haqiqiy u(x) funksiyalarning to‘plami C^k(D) orqali belgilanadi.

Xarakteristik forma tushunchasi, ikkinchi tartibli differensial tenglamalarning klassifikatsiyasi va kanonik ko‘rinishi.

Faraz qilaylik (1.1.4) tenglamada ishtirok etayotgan $F(x, \dots, P_\alpha, \dots)$ funksiya $P_\alpha = P_{\alpha_1 \dots \alpha_n}$, $|\alpha| = m$ o‘zgaruvchilar bo‘yicha uzluksiz birinchi tartibli hosilalarga ega bo‘lsin. (1.1.4) tenglama nazariyasida $\lambda_1, \dots, \lambda_n$ haqiqiy o‘zgaruvchilarga nisbatan ushbu

$$K(\lambda_1, \dots, \lambda_n) = \sum_{\alpha=m} \frac{\partial F}{\partial P_\alpha} \lambda^\alpha \quad \lambda^\alpha = \lambda_1^{\alpha_1} \lambda_2^{\alpha_2} \dots \lambda_n^{\alpha_n} \qquad (1.1.8)$$

$m-$ tartibli forma $-$ m darajali bir jinsli ko‘phad muhim rol o‘ynaydi. Bu forma (1.1.4) tenglamaga mos bo‘lgan xarakteristik forma deyiladi.

Ikkinchi tartibli kvazichiziqli

$$\sum_{i,j=1}^{n} A_{i,j}(x) u_{x_i x_j} + \Phi\left(x, u, u_{x_1}, \dots, u_{x_n}\right) = 0 \qquad (1.1.9)$$

differensial tenglama uchun, bu yerda $A_{i,j}(x) \epsilon\, C(D)$, (1.1.8) forma

$$Q(\lambda_1, \dots, \lambda_n) = \sum_{i,j=1}^{n} A_{i,j}(x) \lambda_i \qquad (1.1.10)$$

kvadratik formadan iborat bo‘ladi. Xususiy hosilali differensial tenglamalar, shu jumladan (1.1.9) ko‘rinishdagi ikkinchi tartibli tenglama tekshirilayotganda, iloji boricha erkli o‘zgaruvchilarni almashtirib, tenglamalarni soddaroq ko‘rinishga keltirishga harakat qilinadi. Shu maqsadda, avvalo (1.1.9) tenglamada erkli o‘zgaruvchilarni almashtirganda uning $A_{i,j}(x)$ koeffitsientlari qanday qonun bilan o‘zgarishini tekshiramiz.

$x = (x_1, \dots, x_n)$ o‘zgaruvchilar o‘rniga $y = y(x)$, ya’ni $y_k = y_k(x_1, \dots, x_n)$,

$k = 1,2, \dots, n$ o‘zgaruvchilarni kiritamiz. x o‘zgaruvchilarning biror atrofida $y_k \epsilon C^2$ bo‘lsin va ushbu yakobian

$$\frac{D(y_1, \dots, y_n)}{D(x_1, \dots, x_n)} \neq 0 \qquad (1.1.11)$$

deb hisoblaymiz. Bu shartga ko‘ra x o‘zgaruvchilarni y lar orqali ifodalashimiz mumkin, ya’ni $\mathrm{x} = \mathrm{x(y)}$. (1.1.9) tenglamaga kirgan $\mathrm{u(x)}$ funksiyaning hosilalarini yangi y o‘zgaruvchilarga nisbatan hisoblaymiz:

$$\frac{\partial u}{\partial x_i} = \sum_{k=1}^{n} \frac{\partial u}{\partial y_k} \frac{\partial y_k}{\partial x_i},$$

$$\frac{\partial^2 u}{\partial x_i \partial x_j} = \sum_{k,l=1}^{n} \frac{\partial^2 u}{\partial y_k \partial y_l} \frac{\partial y_k}{\partial x_i} \frac{\partial y_l}{\partial x_j} + \sum_{k=1}^{n} \frac{\partial u}{\partial y_k} \frac{\partial^2 y_k}{\partial x_i \partial x_j} \qquad (1.1.12)$$

bu ifodalarni (1.1.9) tenglamaga qo‘yib, uni ushbu ko‘rinishda yozib olamiz:

$$\sum_{k,l=1}^{n} \tilde{\mathrm{A}}_{kl}\, u_{y_k y_l} + \tilde{\mathrm{V}}\left(y, u, u_{y_1}, \dots, u_{y_n}\right) = 0 \qquad (1.1.13)$$

bu yerda

$$\tilde{\mathrm{A}}_{kl} = \sum_{i,j=1}^{n} A_{ij} \frac{\partial y_k}{\partial x_i} \frac{\partial y_l}{\partial x_j} \qquad (1.1.14)$$

$\tilde{\mathrm{V}}$ esa, Φ dan va birinchi tartibli hosilalar ishtirok etgan hadlardan tashkil topgan ifoda.

(1.1.9) tenglama tekshirilayotgan D sohada aniq x_0 nuqtani olamiz va

$$y_0 = y(x_0), \beta_{ki} = \frac{\partial y_k(x_0)}{\partial x_i} \qquad (1.1.15)$$

belgilashlarni kiritamiz.

(1.1.14) formula x_0 nuqtada quyidagicha yoziladi:

$$\tilde{A}_{kl}(y_0) = \sum_{i,j=1}^{n} A_{i,j}(x_0)\beta_{ki}\beta_{lj} \qquad (1.1.16)$$

(1.1.9) kvadratik formani x_0 nuqtada yozib olamiz:

$$Q = \sum_{i,j=1}^{n} A_{i,j}\,(x_0)\lambda_i\lambda_j. \qquad (1.1.17)$$

Maxsus bo‘lmagan ushbu

$$\lambda_i = \sum_{k=1}^{n} \beta_{ki}\xi_k\ , \qquad \det(\beta_{ki}) \neq 0 \qquad (1.1.18)$$

affin almashtirish natijasida (1.1.17) kvadratik forma

$$Q = \sum_{k,l=1}^{n} \tilde{A}_{kl}(y_0)\xi_k\,\xi_l \qquad (1.1.19)$$

ko‘rinishga keladi. Bu kvadratik formaning koeffitsientlari ham (1.1.16) formula bilan aniqlanadi.

Shunday qilib, (1.1.9) tenglamani x_0 nuqtada x o‘zgaruvchilar o‘rniga yangi $y = y(x)$ o‘zgaruvchilar kiritib soddalashtirish uchun, shu nuqtada (1.1.17) kvadratik formani maxsus bo‘lmagan (1.1.18) chiziqli almashtirish yordami bilan soddalashtirish yetarlidir.

Algebra kursida isbot qilinadiki, hamma vaqt shunday maxsus bo‘lmagan (1.1.18) almashtirish mavjud bo‘lib, uning yordami bilan (1.1.17) kvadratik forma quyidagi ko‘rinishga olib kelinadi:

$$Q = \sum_{k=1}^{n} \mu_k\xi_k^2 \qquad (1.1.20)$$

bu yerda $\mu_k,\ k = 1,\dots,n$ koeffitsientlar $1,-1,0$ qiymatlarni qabul qiladi. Shu bilan birga musbat (manfiy) koeffitsientlar soni (inersiya indeksi) va nolga teng bo‘lgan koeffitsientlar soni (forma defekti) affin invariantdir, ya’ni bu sonlar faqat (1.1.17) forma bilan aniqlanib, (1.1.18) almashtirishning tanlab olinishiga bog‘liq bo‘lmaydi.

Bu narsa (1.1.9) differensial tenglama $A_{ij}(x)$ koeffitsientlarining x_0 nuqtada qabul qiladigan qiymatlariga qarab, klassifikatsiya qilish imkonini beradi.

Yuqorida aytilganlarga asosan (1.1.13) tenglama

$$\sum_{k=1}^{n} \mu_k u_{y_k y_k} + \tilde{V}(y, u, u_{y_1}, \dots, u_{y_n}) = 0 \qquad (1.1.21)$$

ko‘rinishda yoziladi.

Ikkinchi tartibli differensial tenglamaning aralash hosilalar qatnashmagan bunday ko‘rinishi, odatda uning kanonik ko‘rinishi deyiladi.

(1.1.9) tenglamani bitta nuqtada emas, hech bo‘lmaganda $x_0 \epsilon D$ nuqtaning biror kichik atrofida kanonik ko‘rinishga olib keluvchi o‘zgaruvchilarning almashtirishini (affin bo‘lishi shart emas) topish mumkinmi degan savol tug‘iladi.

Bu savolga ijobiy javob faqat $n = 2$ bo‘lgandagina ma’lum. Bu holni biz alohida ko‘ramiz.

Agar barcha $\mu_k = 1$ yoki barcha $\mu_k = -1$, $k = 1, \dots, n$ bo‘lsa, ya’ni Q forma mos ravishda musbat yoki manfiy aniqlangan (definit) bo‘lsa, (1.1.9) tenglama $x \epsilon D$ nuqtada elliptik tipdagi yoki elliptik tenglama deyiladi.

Agar μ_k koeffitsientlardan bittasi manfiy, qolganlari musbat (yoki aksincha) bo‘lsa, (1.1.9) tenglama $x \epsilon D$ nuqtada giperbolik tenglama deb ataladi.

μ_k koeffitsientlardan l tasi, $1 < \mathrm{l} < n - 1$, musbat, qolgan $n - l$ tasi manfiy bo‘lsa, (1.1.9) tenglama ultragiperbolik tipdagi tenglama deyiladi.

Agar μ_k koeffitsientlardan bittasi nolga teng, qolganlari noldan farqli va bir xil ishorali bo‘lsa, (1.1.9) tenglama $x \epsilon D$ nuqtada parabolik tenglama deyiladi.

Agar koeffitsientlardan kamida bittasi nolga teng bo‘lsa, (1.1.9) tenglama keng ma’noda $x \epsilon D$ nuqtada parabolik tenglama deb ataladi.

Agar (1.1.9) tenglama D sohaning har bir nuqtasida elliptik, giperbolik yoki parabolik bo‘lsa, u holda D sohada mos ravishda elliptik, giperbolik yoki parabolik tipdagi tenglama deb ataladi.

Agar noldan farqli bo‘lgan, bir xil ishorali k_0 , k_1 haqiqiy sonlar mavjud bo‘lib, barcha $x \epsilon D$ nuqtalar uchun ushbu

$$k_0 \sum_{i=1}^{n} \lambda_i^2 \leq Q(\lambda_1, \dots, \lambda_n) \leq k_1 \sum_{i=1}^{n} \lambda_i^2 \qquad (1.1.22)$$

tengsizlik bajarilsa, D sohada elliptik bo‘lgan (1.1.9) tenglama tekis elliptik tenglama deyiladi.

D sohaning turli qismida (1.1.9) tenglama har xil tipga tegishli bo‘lsa, uni aralash tipdagi tenglama deyiladi.

Yuqorida bayon qilingan (1.1.9) tenglamaning klassifikatsiyasini ekvivalent tarzda $A = ||A_{i,j}||$ matritsaning xarakteristik sonlariga asoslanib ham berish mumkin. Buning uchun algebradan ma’lum bo‘lgan (1.1.17) kvadratik formaning (1.1.20) kanonik ko‘rinishidagi μ_k, $k = 1, \dots, n$ sonlar A matritsaning xarakteristik sonlaridan iborat ekanligini eslash kifoyadir. Ma’lumki, simmetrik ($A_{i,j} = A_{j,i}$) matritsaning barcha xarakteristik sonlari haqiqiy sonlardan iboratdir.

Eslatib o‘tamiz, A matritsaning xarakteristik sonlari ushbu

$$det\,(A - \lambda\, E) = 0 \qquad (1.1.23)$$

algebraik tenglamaning ildizlaridan iborat, bu yerda E – birlik matritsa.

Demak, (1.1.9) tenglama berilgan D sohaning ixtiyoriy x nuqtasida A matritsa xarakteristik sonlarining ishorasini aniqlab, (1.1.9) tenglamaning qaysi tipga tegishli ekanini darhol bilib olish mumkin.

Ikkinchi tartibli ikki o'zgaruvchili differensial tenglamalarni kanonik ko'rinishga keltirish.

(1.1.9) tenglama $n = 2$ bo'lgan holda, ya'ni ikkita x va y haqiqiy o'zgaruvchili ikkinchi tartibli kvazichiziqli tenglama ushbu

$$A(x,y)\frac{\partial^2 u}{\partial x^2} + 2B(x,y)\frac{\partial^2 u}{\partial x \partial y} + C(x,y)\frac{\partial^2 u}{\partial y^2} + \Phi\left(x,y,u,\frac{\partial u}{\partial x},\frac{\partial u}{\partial y}\right) = 0 \quad (1.1.24)$$

ko'rinishda yoziladi.

(1.1.24) tenglama xarakteristikalarining tenglamasi chiziqli bo'lmagan

$$A\left(\frac{\partial \omega}{\partial x}\right)^2 + 2B\ \frac{\partial \omega}{\partial x}\frac{\partial \omega}{\partial y} + C\left(\frac{\partial \omega}{\partial y}\right)^2 = 0 \quad (1.1.25)$$

tenglamadan iboratdir.

Bu tenglamani soddagina usul bilan oddiy differensial tenglamaga keltirish mumkin. Haqiqatan ham, $\omega(x,y)$ funksiya (1.1.25) tenglamaning yechimi bo'lsa,

$\omega(x,y) = const$ egri chiziq (1.1.24) tenglamaning xarakteristikasidir. Bu xarakteristika atrofida

$$\frac{\partial \omega}{\partial x}dx + \frac{\partial \omega}{\partial y}dy = 0$$

yoki

$$\frac{\partial \omega}{\partial x} : \frac{\partial \omega}{\partial y} = dy : (-dx)$$

munosabat bajariladi. (1.1.25) tenglamada $\frac{\partial \omega}{\partial x} : \frac{\partial \omega}{\partial y}$ nisbatni $dy : (-dx)$ ga almashtirib,

$$Ady^2 - 2\,B\,dydx + C\,dx^2 = 0 \quad (1.1.26)$$

oddiy differensial tenglamaga ega bo'lamiz.

Aksincha, agar $\omega = const$ (1.1.26) tenglamaning integrali bo‘lsa, $\omega(x,y)$ funksiya (1.1.25) xarakteristikalar tenglamasini qanoatlantiradi. (1.1.26) tenglik xarakteristik egri chiziqlarning oddiy differensial tenglamasidan iboratdir.

(1.1.26) tenglik bilan aniqlangan (dx, dy)yo‘nalish odatda xarakteristik yo‘nalish deyiladi.

Yuqorida aytilganlarga asosan $Ady^2 - 2\,B\,dydx + C\,dx^2$ kvadratik formaning aniqlangan (musbat yoki manfiy), ishorasi o‘zgaruvchan yoki yarim aniqlangan (buzilgan) bo‘lishiga qarab, (1.1.24) tenglama elliptik, giperbolik yoki parabolik tipga tegishli bo‘ladi. Shunga muvofiq

$Ady^2 - 2\,B\,dydx + C\,dx^2$ kvadratik formaning diskriminanti $B^2 - AC$ noldan kichik, kata yoki nolga teng bo‘lishiga qarab, mos ravishda, (1.1.24) tenglama elliptik, giperbolik yoki parabolik bo‘ladi.

Shuning uchun ham (1.1.24) tenglama elliptik sohasida haqiqiy xarakteristik yo‘nalishlarga ega emas, har bir giperbolik nuqtasida esa ikita turli haqiqiy xarakteristik yo‘nalish, har bir parabolik nuqtada bitta haqiqiy xarakteristik yo‘nalish mavjud. Demak, A, B va C koeffitsientlar yetarli silliq funksiyalar bo‘lganda, (1.1.24) tenglamaning giperbolik sohasi xarakteristik egri chiziqlarning ikita oilasi to‘ri bilan, paraboliklik sohasi esa xarakteristik egri chiziqlarning bitta oilasi to‘ri bilan qoplanadi.

Agar (1.1.24) tenglamaning A, B va C koeffitsientlari tenglama berilgan D sohada yetarli silliq funksiyalar bo‘lsa, x va y o‘zgaruvchilarning shunday maxsus bo‘lmagan $\xi = \xi\,(x,y)$, $\eta = \eta\,(x,y)$ almashtirish mavjud bo‘ladiki, (1.1.24) tenglama D sohada bu almashtirish yordamida quyidagi kanonik ko‘rinishlardan biriga keladi.

Elliptik holda

$$u_{\xi\xi} + u_{\eta\eta} + \tilde{V}(\xi,\eta,u,u_\xi,u_\eta) = 0, \qquad (1.1.27)$$

giperbolik holda

$$u_{\xi\xi} - u_{\eta\eta} + \tilde{\mathrm{V}}(\xi, \eta, u, u_{\xi}, u_{\eta}) = 0, \qquad (1.1.28)$$

yoki

$$u_{\xi\eta} + \tilde{\mathrm{V}}(\xi, \eta, u, u_{\xi}, u_{\eta}) = 0 \qquad (1.1.28')$$

va parabolik holda

$$u_{\eta\eta} + \tilde{\mathrm{V}}(\xi, \eta, u, u_{\xi}, u_{\eta}) = 0. \qquad (1.1.29)$$

(1.1.24) tenglama berilgan barcha D sohada, ya’ni global uni (26), (27) yoki (28) kanonik ko‘rinishga keltirish katta qiyinchiliklar bilan bog‘liqdir. Shuning uchun ham, (1.1.24) tenglamani D soha (x, y) nuqtasining yetarli kichik atrofida, ya’ni lokal kanonik ko‘rinishga keltirish bilan chegaralanamiz. (1.1.24) tenglamaning A, B va C koeffitsientlari (x, y) nuqtaning biror atrofida C^2 sinfga tegishli bo‘lib,

$$Ady^2 - 2\,B\,dydx + C\,dx^2 \neq 0$$

bo‘lsin.

Umumiyatlikka ziyon yetkazmay $A(x, y) \neq 0$ deb hisoblashimiz mumkin. Haqiqatan ham, aks holda $C(x, y) \neq 0$ bo‘lishi mumkin. Bu holda x va y ning o‘rnini almashtirib, shunday tenglama hosil qilamizki, unda $A(x, y) \neq 0$ bo‘ladi.

Agarda A va C bir vaqtda biror nuqtada nolga teng bo‘lsa, bu nuqta atrofida $B(x, y) \neq 0$ bo‘ladi. Bu holda $x' = x + y,\ y' = x - y$ almashtirish natijasida hosil bo‘lgan tenglamada $A(x, y) \neq 0$ bo‘ladi. (1.1.24) tenglamada erkli o‘zgaruvchilarni ixtiyoriy qaytariluvchi almashtiramiz:

$$\xi = \xi\,(x, y), \quad \eta = \eta\,(x, y) \qquad (1.1.30)$$

Birinchi va ikkinchi tartibli hosilalar quyidagicha almashtiriladi:

$$u_x = u_{\xi}\xi_x + u_{\eta}\eta_x\,, \quad u_y = u_{\xi}\xi_y + u_{\eta}\eta_y$$

$$u_{xx} = \xi_x^2 u_{\xi\xi} + 2\xi_x\eta_x u_{\xi\eta} + \eta_x^2 u_{\eta\eta} + \xi_{xx}u_\xi + \eta_{xx}u_\eta ,$$

$$u_{xy} = \xi_x\xi_y u_{\xi\xi} + \left(\xi_x\eta_y + \xi_y\eta_x\right)u_{\xi\eta} + \eta_x\eta_y u_{\eta\eta} + \xi_{xy}u_\xi + \eta_{xy}u_\eta ,$$

$$u_{yy} = \xi_y^2 u_{\xi\xi} + 2\xi_x\eta_x u_{\xi\eta} + \eta_y^2 u_{\eta\eta} + \xi_{yy}u_\xi + \eta_{yy}u_\eta .$$

Bularga asosan (1.1.24) tenglama

$$A_1\frac{\partial^2 u}{\partial\xi^2} + 2B_1\frac{\partial^2 u}{\partial\xi\partial\eta} + C_1\frac{\partial^2 u}{\partial\eta^2} + ?\left(\xi,\eta,u,\frac{\partial u}{\partial\xi},\frac{\partial u}{\partial\eta}\right) = 0 \qquad (1.1.31)$$

ko‘rinishda yoziladi. Bu yerda

$$A_1(\xi,\eta) = A\xi_x^2 + 2B\xi_x\xi_y + C\xi_y^2 ,$$

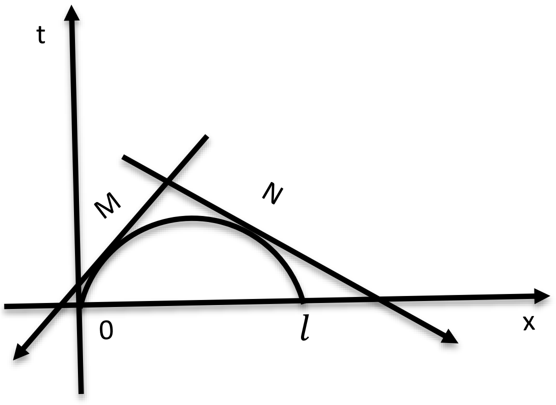
$$B_1(\xi,\eta) = A\xi_x\eta_x + B\left(\xi_x\eta_y + \xi_y\eta_x\right) + C\xi_y\eta_y ,$$

$$C_1(\xi,\eta) = A\eta_x^2 + 2B\eta_x\eta_y + C\eta_y^2 ,$$

$$u(\xi,\eta) = u[x(\xi,\eta), y(\xi,\eta)] ,$$

$x = x(\xi,\eta), y = y(\xi,\eta)$ almashtirish esa $\xi = \xi(x,y), \eta = \eta(x,y)$ ga teskari almashtirishdir.

Tor tebranish tenglamasi. Faraz qilaylik, uzunligi l ga teng bo‘lgan tor Ox o‘qning o va l nuqtalariga tortilib biriktirilgan bo‘lib, T taranglik kuchi ta’siri ostida muvozanat holatda turgan bo‘lsin. Agar tashqi kuch ta’sirida tor muvozanat holatdan chetlashtirilsa, umuman olganda uning uzunligi va taranglik kuchi o‘zgaradi. Lekin biz torning kichik tebranishlarini qaraymiz. Bu holda torning uzunligi va taranglik kuchi $\vec{T}$ o‘zgarmaydi.



1.1.1 − *chizma.*

Agar tor chizmada ko‘rsatilgandek, muvozanat holatdan (x, u) tekislik bo‘ylab chetlashtirilib, so‘ngra qo‘yib yuborilgan bo‘lsa, u ko‘ndalang tebranma harakat qiladi. Torning x absissali nuqtasiga mos kelgan siljishni u bilan belgilasak, $u(x,t)$ x koordinata va t vaqtning funksiyasi bo‘ladi, ya’ni $u = u(x,t)$.

Bu funksiyaning grafigi t ning har bir qiymati uchun torning formasini tasvirlaydi. Kichik tebranishlarni qarayotganimiz uchun u va $\frac{\partial u}{\partial x}$ lar juda kichik bo‘lib, $(\frac{\partial u}{\partial x})^2$ ni hisobga olmasligimiz mumkin. Bu holda muvozanat holatdan siljigan torning uzunligi l_1 desak,

$$l_1 = \int_0^l \sqrt{1 + (\frac{\partial U}{\partial x})^2}\, dx \approx \int_0^l dx = l \qquad (1.1.32)$$

bo‘ladi, ya’ni deformatsiya natijasida torning uzunligi o‘zgarmaydi deb qaraymiz. Bundan Guk qonuniga asosan vaqt o‘tishi bilan torning barcha nuqtalarida taranglik kuchi $\vec{T}$ o‘zgarmaydi degan xulosaga kelamiz.

Faraz qilaylik, torning boshlang‘ich holatdagi $dx = M_1N_1$ elementi deformatsiyadan keyin $ds = MN$ ga teng bo‘lsin. Tor juda ingichka deb qaralsa, uning har qanday elementining og‘irligini hisobga olmasak ham bo‘ladi. Demak, qaralayotgan holda tor elementining ikkala tomoniga ta’sir etuvchi $\vec{T}$ taranglik kuchi e’tiborga olinadi. Bu kuch torning elastikligiga ko‘ra M nuqtaga urinma bo‘ylab chap tomonga, N nuqtaga urinma bo‘ylab esa o‘ng tomonga yo‘nalgan bo‘ladi.

Agar M nuqtaga qo‘yilgan kuchni $\vec{T}_1$, N nuqtaga qo‘yilgan kuchni $\vec{T}_2$ deb belgilasak, bu kuchlarning moduli T ga teng, yo‘nalishlari esa turlicha bo‘lib, nuqtadan nuqtaga o‘tishda o‘zgarib turadi. Nyuton qonuniga asoslanib, torning muvozanat tenglamasini tuzamiz. Ko‘ndalang tebranishlarni qarayotganimiz uchun elementga ta’sir etayotgan kuchlarning faqat vertikal o‘qqa proyeksiyalarining yig‘indisini qarasak bo‘ladi (kuchlarning gorizontal o‘qqa proyeksiyalarining yig‘indisi nolga teng). Agar M nuqtaga qo‘yilgan taranglik kuchining Ox o‘q bilan hosil qilgan burchagini α_1 bilan, N nuqtaga qo‘yilgan taranglik kuchining Ox o‘q bilan hosil qilgan burchagini α_2 bilan belgilasak, bu kuchlarning vertikal o‘qqa proyeksiyalari mos ravishda $-T\ \sin\alpha_1$ va $T\ \sin\alpha_2$ lardan iborat bo‘ladi. Demak, qaralayotgan kuchlarning vertikal o‘qqa proyeksiyalarining yig‘indisi

$$\Phi = T(\sin\alpha_2 - \sin\alpha_1) \tag{1.1.33}$$

ga teng. Biz kichik tebranishlarni qarayotganimiz uchun α_1 va α_2 lar juda kichik bo‘lib,

$$\sin\alpha_1 \approx \tan\alpha_1 = \left(\frac{\partial u}{\partial x}\right)_x$$

$$\sin\alpha_2 \approx \tan\alpha_2 = \left(\frac{\partial u}{\partial x}\right)_{x+dx} \tag{1.1.34}$$

Endi $\left(\frac{\partial u}{\partial x}\right)_{x+dx}$ ni Teylor qatoriga yoyamiz.

$$\left(\frac{\partial u}{\partial x}\right)_{x+dx} = \left(\frac{\partial u}{\partial x}\right)_x + \left(\frac{\partial^2 u}{\partial x^2}\right)_x dx + \cdots$$

bundan

$$\left(\frac{\partial u}{\partial x}\right)_{x+dx} = \left(\frac{\partial u}{\partial x}\right)_x + \left(\frac{\partial^2 u}{\partial x^2}\right)_x dx$$

taqribiy tenglikni hosil qilamiz. Demak,

$$\Phi \approx T\frac{\partial^2 u}{\partial x^2}dx \tag{1.1.35}$$

Nyutonning ikkinchi qonuniga ko‘ra $dm = \rho(x)dx$ ($\rho(x)$ torning chiziqli zichligi.) massasining $\frac{\partial^2 u}{\partial x^2}$ tezlanishga ko‘paytmasi qaralayotgan elementga qo‘yilgan kuchlarning vertikal o‘qqa proyeksiyalarining yig‘indisiga teng, ya’ni

$$\rho(x)\frac{\partial^2 u}{\partial t^2} = \mathrm{T}\frac{\partial^2 u}{\partial x^2} \tag{1.1.36}$$

tenglamaga aytiladi. Agar torni bir jinsli deb faraz qilsak, u holda $\rho(x) = const$ bo‘ladi va

$$\frac{T}{\rho} = a^2 \tag{1.1.37}$$

deb belgilasak, (1.1.36) tenglama

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2} \tag{1.1.38}$$

ko‘rinishga keladi. Bu tenglama torning erkin tebranish tenglamasi deyiladi. (1.1.38) tenglama qaralayotgan fizik jarayonning matematik modelidir. Endi ∂x element gorizontal holatdan chiqarilgandan so‘ng, unga taranglik kuchlaridan tashqari $F(t,x)dx$ tashqi kuch ham ta’sir etyapti deb faraz qilaylik. Bu holda (1) tenglama quyidagi ko‘rinishda bo‘ladi.

$$\rho(x)dx\ \frac{\partial^2 u}{\partial t^2} = \Phi + F(t,x)dx \tag{1.1.39}$$

Bundan yuqoridagi kabi

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2} + f(x,t) \tag{1.1.40}$$

tenglamaga ega bo‘lamiz. Bu yerda $f(x,t) = \frac{F(x,t)}{\rho}$ tenglama bir jinsli torning majburiy tebranish tenglamasi deyiladi.

Yuqorida chiqarilgan (1.1.38) va (1.1.40) tenglamalar 2- tartibli xususiy hosilali differensial tenglamalar yoki bir o‘lchovli to‘lqin tenglamalari deyiladi. Bu tenglamalar matematik-fizikaning eng soda va muhim tenglamalaridan biri bo‘lib, bu tenglamalar yordamida ko‘plab fizik masalalarning matematik modelini tuzish mumkin.

Tor tebranish tenglamasi. Dalamber formulasi.

Ma'lumki torning erkin tebranishi

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} \qquad (1.1.41)$$

tenglama bilan tasvirlanadi. (1.1.41) tenglamaning xarakteristikalar tenglamasi

$$dx^2 - a^2 dt^2 = 0 \qquad (1.1.42)$$

dan iborat. Bundan darxol (1.1.41) tenglamaning xarakteristikalari $x - at = c_1$, $x + at = c_2$, c_1, c_2 – $const$, to'g'ri chiziqlardan iborat ekanligi kelib chiqadi.

Umumiy nazariyaga asosan

$$\xi = x - at, \quad \eta = x + at \qquad (1.1.43)$$

deb belgilab olamiz.

Bu yangi o'zgaruvchilarga nisbatan (1.1.41) tenglama

$$\frac{\partial^2 u}{\partial \xi \partial \eta} = 0 \qquad (1.1.44)$$

tenglamaga keladi. (1.1.44) tenglamani

$$\frac{\partial}{\partial \xi}\left(\frac{\partial u}{\partial \eta}\right) = 0$$

ko'rinishda yozib olamiz. Bundan

$$\frac{\partial u}{\partial \eta} = f(\eta),$$

bu yerda $f(\eta)$ – ixtiyoriy funksiya. Hosil bo'lgan tenglamani, ξ ni paramatr deb qarab, η bo'yicha integrallaymiz u holda

$$u(\xi,\eta) = \int f(\eta)d\eta + f_1(\xi).$$

Ushbu

$$\int f(\eta)d\eta = f_2(\eta)$$

belgilashni kiritib,

$$u(\xi,\eta) = f_1(\xi) + f_2(\eta) \tag{1.1.45}$$

formulani hosil qolamiz. Bunda $f_1(\xi)$, $f_2(\eta)$ – ixtiyoriy funksiyalar.

Eski x va t o‘zgaruvchilarga qaytsak, avvalgi formula

$$u(x,t) = f_1(x-at) + f_2(x+at) \tag{1.1.46}$$

ko‘rinishda yoziladi.

Agar f_1, f_2 ikki marta uzluksiz differensiallanuvchi funksiyalar bo‘lsa, (1.1.46) formula bilan aniqlangan $u(x,t)$ funksiya (1.1.41) tenglamaning yechimi ekanligiga ishonch hosil qilish qiyin emas. (1.1.41) tenglamaning (1.1.46) yechimi Dalamber yechimi deyiladi. Bu yechimdan foydalanib, chegaralanmagan tor uchun Koshi masalasini, ya’ni ushbu

$$u|_{t=0} = \varphi(x), \qquad \frac{\partial u}{\partial t}|_{t=0} = \psi(x), \ -\infty < x < \infty \tag{1.1.47}$$

boshlang‘ich shartlarni qanoatlantiruvchi (1.1.41) tenglamaning yechimini topish qiyin emas. Bu masalaning yechimi mavjud deb faraz qilamiz. U holda, bu yechim (1.1.46) formula bilan aniqlanadi (chunki u umumiy yechim). (1.1.46) formuladagi f_1 va f_2 funksiyalarni shunday topamizki, natijada (1.1.47) boshlang‘ich shartlar qanoatlantirilsin, ya’ni

$$u(x,0) = f_1(x) + f_2(x) = \varphi(x),$$

$$\frac{\partial u}{\partial t}|_{t=o} = -af_1'(x) + af_2'(x) = \psi(x) \qquad (1.1.48)$$

Ikkinchi tenglikni integrallab,

$$-f_1(x) + f_2(x) = \frac{1}{a}\int_0^x \psi(s)ds + C \qquad (1.1.49)$$

ni hosil qilamiz, C – biror o‘zgarmas. (1.1.48) tenglik birinchisidan va (1.1.49) dan quyidagilar kelib chiqadi.

$$f_1(x) = \frac{1}{2}\varphi(x) - \frac{1}{2a}\int_0^x \psi(s)ds - \frac{C}{2},$$

$$f_2(x) = \frac{1}{2}\varphi(x) + \frac{1}{2a}\int_0^x \psi(s)ds + \frac{C}{2}. \qquad (1.1.50)$$

f_1 va f_2 ning bu qiymatlarini (1.1.46) formulaga qo‘yib, ushbu

$$u(x,t) = \frac{\varphi(x-at) + \varphi(x+at)}{2} + \frac{1}{2a}\int_{x-at}^{x+at} \psi(s)ds \qquad (1.1.51)$$

formulaga ega bo‘lamiz. (1.1.51) ni Dalamber formulasi deyiladi. Koshi masalasining yechimi mavjud deb, (1.1.51) formulani keltirib chiqardik.

1.2. Integral va integro-differensial tenglamalar.

1.2.1-ta'rif. Agar tenglamada noma'lum funksiya shu funksiyaning argumenti bo'yicha olingan integral belgisi ostida albatta qatnashsa, bunday tenglama integral tenglama deb ataladi.

1.2.2-ta'rif. Agar funksional tenglamada noma'lum funksiya integral ishorasi ostida qatnashsa, u holda bu tenglama integral tenglama deb ataladi.

1.2.3-ta'rif. Agar integral tenglamada integral belgisi ostidagi funksiya va tenglamaning boshqa qismlari (hadlari) noma'lum funksiyaga nisbatan chiziq bo'lsa, u holda tegishli tenglama chiziqli integral tenglama deyiladi.

Mexanika, matematik-fizika va texnikaning juda ko'p masalalari ushbu

$$\varphi(x) - \lambda \int_a^b K(x,y)\varphi(y)dy = f(x) \qquad (1.2.1)$$

ko'rinishdagi integral tenglamalarni tekshirishga olib kelinadi, bu yerda $\varphi(x) -$ noma'lum funksiya, $K(x,y)$ va $f(x)$ funksiyalar mos ravishda $a \le x \le b,\ a \le y \le b$ va $a \le x \le b$ (a, b–o'zgarmas sonlar) yopiq sohalarda berilgan uzluksiz haqiqiy funksiyalardir. $f(x)$ funksiya (1.2.1) integral tenglamaning ozod hadi, $K(x,y)$ uning yadrosi, sonli λ ko'paytma tenglamaning parametri deyiladi. Bunday parametrni kiritish shart emas, agar $\lambda K(x,y)$ ko'paytmani $K_1(x,y)$ bilan belgilab, $K_1(x,y)$ yangi yadro deb qarasak, parameter 1 ga teng bo'lib qoladi. Biz hozir noma'lum funksiya chiziqli ishtirok etgan tenglamalarni, ya'ni chiziqli integral tenglamalarni tekshiramiz. (1.2.1) tenglama ikkinchi tur chiziqli integral tenglama yoki bunday tenglamalarni birinchi bo'lib o'rgangan matematik nomi bilan Fredgol'm integral tenglamasi deyiladi.

1.2.4-ta'rif. Fredgol'mning birinchi tur tenglamasi deb,

$$\int_a^b K(x,y)\varphi(y)dy = f(x) \tag{1.2.2}$$

ko‘rinishdagi integral tenglamaga aytiladi.

Agar (1.2.1) tenglamada $f(x) \equiv 0$ bo‘lsa, ya’ni

$$\varphi(x) - \lambda \int_a^b K(x,y)\varphi(y)dy = 0 \tag{1.2.3}$$

tenglama (1.2.1) ga mos bo‘lgan bir jinsli integral tenglama deyiladi. Bir jinsli

$$\psi(x) - \lambda \int_a^b K(x,y)\psi(y)dy = 0 \tag{1.2.4}$$

tenglama (1.2.3) bir jinsli tenglamaga qo‘shma integral tenglama deyiladi.

1.2.5-ta’rif. Agar (1.2.1) tenglamada integral chegaralaridan bittasi, masalan, yuqori chegara o‘zgaruvchi bo‘lsa, ya’ni tenglama

$$\varphi(x) - \lambda \int_a^x K(x,y)\varphi(y)dy = f(x), \qquad x > a \tag{1.2.5}$$

ko‘rinishda bo‘lsa, u Vol’terraning ikkinchi tur integral tenglamasi,

$$\int_a^x K(x,y)\varphi(y)dy = f(x) \tag{1.2.6}$$

tenglama esa Vol’terraning birinchi tur integral tenglamasi deyiladi.

1.2.6-ta’rif. Ushbu

$$\alpha(x)\varphi(x) - \lambda \int_a^b K(x,y)\varphi(y)dy = f(x) \tag{1.2.7}$$

tenglama uchinchi tur integral tenglama deb ataladi.

Agar $a \leq x \leq b$ kesmada $\alpha(x) \equiv 0$ bo‘lsa, (1.2.7) dan (1.2.2) tenglama, $\alpha(x) \equiv 1$ bo‘lsa, (1.2.1) tenglama kelib chiqadi. Lekin bu kesmaning ayrim nuqtalarida $\alpha(x)$ funksiya nolga teng bo‘lib, qolgan nuqtalarida esa noldan farqli bo‘lishi ham mumkin. Bu holda (1.2.7) tenglamani maxsus tekshirishga to‘g‘ri keladi. Amaliy masalalarda tez-tez shunday integral tenglamalarni yechishga to‘g‘ri keladiki, bularda noma’lum funksiya ko‘p argumentli bo‘ladi. E^n Yevklid fazosidagi chegaralangan sohani D orqali, bu sohaga tegishli nuqtalarni

$x = (x_1, \dots, x_n)$, $y = (y_1, \dots, y_n)$ orqali belgilab olamiz. $\varphi(x) = \varphi(x_1, \dots, x_n)$ noma’lum, $f(x)$ berilgan funksiya, $\lambda -$ berilgan sonli parametr bo‘lsin. Qisqalik uchun D soha bo‘yicha integrallashni bitta integral bilan belgilaymiz. Bu holda Fredgol’mning ko‘p o‘lchovli ikkinchi tur integral tenglamasi

$$\varphi(x) - \lambda \int_D K(x, y)\varphi(y)dy = f(x) \qquad (1.2.8)$$

ko‘rinishda yoziladi.

Tekshirib ko‘rish qiyin emaski, agar ikkinchi tur Fredgol’m integral tenglamasining umumiy yechimi $\Phi(x)$ mavjud bo‘lsa, u

$$\Phi(x) = \varphi^0(x) + \varphi(x) \qquad (1.2.9)$$

ko‘rinishga ega bo‘ladi, bunda $\varphi^0(x)$ (1.2.3) tenglamaning umumiy yechimi, $\varphi(x)$ esa (1.2.1) tenglamaning xususiy yechimidir.

Haqiqatan ham, agar $\Phi(x)$ va $\varphi(x)$ mos ravishda bir jinsli bo‘lmagan (1.2.1) tenglamaning umumiy va xususiy yechimlari bo‘lsa, bularning ayirmasi

$\varphi^0(x) = \Phi(x) - \varphi(x)$ (1.2.3) tenglamaning yechimidan iborat bo‘ladi. Bundan darhol (1.2.9) tenglik kelib chiqadi.

Agar tenglamadagi noma’lum funksiya bir vaqtda ham integral ishorasi ostida qatnashsa, ham uning hosilalari qatnashsa bunday tenglama integro-differensial tenglama deyiladi.

Xulosa.

Bitiruv malakaviy ishining birinchi bobi Xususiy hosilali differensial tenglamalar va integral tenglamalar deb nomlangan bo‘lib u ikki paragrafdan iborat.

Birinchi bo‘limda xususiy hosilali differensial tenglamalarning yechimi, klassifikatsiyasi va kanonik ko‘rinishi, tor tebranish tenglamasi uchun Dalamber formulasi haqida ma’lumotlar keltirilgan. Ikkinchi bo‘limda integral va integro-differensial tenglamalar haqida umumiy tushunchalar keltirilgan.

II bob. Kuchsiz gorizontal muhitda xotirali tenglama uchun teskari masala.

2.1. Umumiy tushunchalar va yadroni aniqlash bo‘yicha teskari masalaning qo‘yilishi

2.1.1. Dirakning del’ta funksiyasi. Bir o‘lchovli holni qaraylik. Soddalik uchun $R^1 = R$ deb olamiz. $C^\infty(R)$ bilan R da cheksiz marta differensiallanuvchi funksiyalar fazosini belgilaymiz.

2.1.1-ta’rif. Biror chegaralangan to‘plamdan tashqarida nolga teng bo‘lgan funksiya finit deyiladi. Bir o‘lchovli funksiya uchun boshqacha aytadigan bo‘lsak, agar $\varphi: R \to R$ funksiya uchun shunday $[a_\varphi, b_\varphi] \subset R$ kesma topilib, $x \overline{\in} [a_\varphi, b_\varphi]$ larda $\varphi(x) \equiv 0$ bo‘lsa, bu funksiya finit funksiya va $[a_\varphi, b_\varphi]$ kesma uning tashuvchisi deyiladi.

2.1.2-ta’rif. Cheksiz marta differensiallanuvchi finit funksiyalar to‘plamiga asosiy funksiyalar fazosi deyiladi va u $D(R)$ orqali belgilanadi. Shunday qilib,

$$D(R) = \{\varphi \in C^\infty(R) | [a_\varphi, b_\varphi] \text{ kesmadan tashqarida } \varphi(x) \equiv 0\}.$$

2.1.1-lemma. [a,b] kesmada uzluksiz va finit bo‘lgan ixtiyoriy $f(x)$ funksiya ushbu

$$\int_a^b f(x)\varphi(x)dx \ , \qquad \forall\varphi \in D(R) \qquad (2.1.1)$$

integral bilan bir qiymatli aniqlanadi.

Uzluksiz finit $f(x)$ funksiyani nuqtaviy berish va uning D(R) funksiyalar to‘plamidagi “proyeksiya” larini berish $f(x)$ funksiyani berishga teng kuchli. Ammo ikkinchi usulda funksiyani berish qaysidir ma’noda qulaydir. Ya’ni u funksiya tushunchasini kengaytirishga, fanga yangi ba’zi nuqtalarda ma’noga ega bo‘lmagan biror funksiyalar to‘plamida o‘zining qiymatlari bilan to‘liq aniqlanadigan funksiyalarni kiritishga imkon beradi. Bunday funksiyaga misol tariqasida

$$\int_{a}^{b} \delta(x)\varphi(x)dx = \varphi(0), \qquad \forall\varphi \in D(R) \tag{2.1.2}$$

tenglik bilan aniqlangan Dirakning $\delta(x)$ del'ta $-$ funksiyasini keltirish mumkin. $\delta(x)$ funksiya $\omega_h(x)$ ning h $\rightarrow$0 dagi kuchsiz limiti hisoblanadi. Haqiqatdan ham $\forall\varphi \in D(R)$ funksiyalar uchun

$$\int_{-\infty}^{\infty} \varphi(x)\omega_h(x)dx = \varphi(\theta h)F\int_{-\infty}^{\infty} \omega_h(x)dx = \varphi(\theta h), \qquad -1 < \theta < 1 \tag{2.1.3}$$

Bundan

$$\lim_{h\rightarrow 0}\int_{-\infty}^{\infty} \omega_h(x)\,\varphi(x)dx = \varphi(0) = \int_{-\infty}^{\infty} \delta(x)\varphi(x)dx\,, \quad \forall\varphi \in D(R^1) \tag{2.1.4}$$

2.1.2. $\boldsymbol{\delta(x)}$ −del'ta funksiyaning xossalari.

2.1.1-xossa. $\delta(x - x^0),\ x^0 \in R^n$ funksiyani aniqlaymiz. $\varphi \in D(R^n)$ funksiya uchun

$$\big(\delta(x - x^0), \varphi(x)\big) = \big(\delta(y), \varphi(y + x^0)\big) = \varphi(x^0) \tag{2.1.6}$$

tenglik o'rinli, ya'ni $\delta(x - x^0)$ quyidagi tenglik bilan aniqlanadi:

$$\big(\delta(x - x^0), \varphi(x)\big) = \varphi(x^0), \varphi \in D(R^n) \tag{2.1.7}$$

2.1.2-xossa. $\delta(x) = \delta(-x)$ ekanligini ko'rsatamiz.

Ixtiyoriy $\varphi \in D(R^n)$ uchun

$$\big(\delta(-x), \varphi(x)\big) = \big(\delta(x), \varphi(-x)\big) = \varphi(0) = \big(\delta(x), \varphi(x)\big) \tag{2.1.8}$$

ekanligidan yuqoridagi tenglik kelib chiqadi.

2.1. 3-xossa. Agar $a(x)\ \ x = x_0$ da uzluksiz funksiya, x$\in R^n,\ x^0 \in R^n$ bo'lsa,

$$a(x)\delta(x - x^0) = a(x^0)\delta(x - x^0) \tag{2.1.9}$$

bo‘ladi.

2.1.4-xossa. $x^n\delta^n(x) = (-1)^n n!\ \delta(x)\,,\ x \in R',\ n = 1,2,\dots;$ (2.1.10)

2.1.3. Umumlashgan funksiya tushunchasi. Regulyar va singulyar funksiyalar.

2.1.3-ta’rif. $f\colon D(R) \to C$ −akslantirish $D(R)$ da funksional deyiladi, bu yerda C − kompleks sonlar maydoni.

f funksionalning $\varphi \in D(R)$ funksiyadagi qiymati $(f\,,\varphi)$ bilan belgilanadi. Ta’rifga asosan $(f\,,\varphi)$ kompleks son.

2.1.4-ta’rif. Funksional $f\colon D(R) \to C$ chiziqli deyiladi, agarda ixtiyoriy $\varphi,\psi \in D(R)$ va $\alpha,\beta \in C$ lar uchun

$$(f,\alpha\varphi + \beta\,\psi) = \alpha(f,\varphi) + \beta(f,\psi) \qquad (2.1.5)$$

tenglik o‘rinli bo‘lsa.

2.1.5-Ta’rif. Chiziqli fuknksional $f\colon D(R) \to C$ uzluksiz deyiladi, agarda $D(R)$ da nolga intiluvchi ixtiyoriy funksiyalar ketma - ketligi $\{\varphi_k(x)\} \in D(R)$ uchun k →∞ da $(f,\varphi_k) \to 0$ bo‘lsa .

2.1.6-ta’rif. Chiziqli, uzluksiz funksional $f\colon D(R) \to C$ umumlashgan funksiya deyiladi.

$D' = D'(R)$ orqali barcha umumlasgan funksiyalardan tuzilgan to‘plamni belgilaymiz.

Lokal integrallovchi funksiyaning ta’rifini $n-$ o‘lchovli funksiya uchun keltiramiz.

2.1.7-ta’rif. $f\colon R^n \to R$ funksiya ixtiyoriy chegaralangan $Q \subset R^n$ to‘plam uchun u Q da absolyut integrallanuvchi bo‘lsa, u R da lokal integrallanuvchi deyiladi.

Bu ta'rif yanada tushunarli bo'lishi uchun uni bir o'lchovli hol uchun quyidagicha ta'riflaymiz. R da barcha lokal integrallanuvchi funksiyalar to'plamini $L_1^{loc}(R)$ ko'rinishida belgilaymiz, u holda

$$L_1^{loc}(R) = \left\{ f: R \to C: \int_a^b |f(x)| dx < \infty \right\},$$

bu yerda $a, b -$ chekli haqiqiy sonlar.

2.1.1-Misol. Faraz qilaylik, $f(x)\, R$ da lokal integrallanuvchi funksiya bo'lsin. U holda ixtiyoriy $\varphi(x) \in D(R)$ uchun

$$(f, \varphi) = \int_R f(x)\varphi(x)dx \qquad (*)$$

funksional chiziqli va uzluksizdir. Shuning uchun bu funksional umumlashgan funksiyadir.

2.1.8-ta'rif. Lokal integrallanuvchi funksiya yordamida qurilgan umumlashgan funksiya regulyar umumlashgan funksiya (yoki regulyar funksiya) deyiladi.

2.1.9-ta'rif. Regulyar bo'lmagan umumlashgan funksiyalar singulyar umumlashgan funksiya deyiladi.

Ixtiyoriy $\varphi(x) \in D(R^n)$ funksiyalarda $(\delta(x), \varphi(x)) = \varphi(0)$ tenglik bilan aniqlangan $\delta(x)$ funksionalni qaraymiz. Bu funksional chiziqli va uzluksiz. Demak, u umumlashgan funksiyani aniqlaydi. $\delta(x)$ umumlashgan funksiya singulyardir. Uni (*) ko'rinishda hech bir lokal integrallanuvchi f funksiya yordamida ifodalab bo'lmaydi.

2.1.4. Xevisayda funksiyasi. Quyidagi

$$\theta(x) = \begin{cases} 1, & x \geq 0, \\ 0, & x < 0 \end{cases} \qquad (2.1.11)$$

funksiya Xevisayda funksiyasi deyiladi. Bu funksiya uchun

$$\frac{d}{dx}\,\theta(x) = \delta(x) \tag{2.1.12}$$

tenglik o‘rinli. Yana bu yerdan

$$\frac{d^{k+1}}{dx^{k+1}}\theta(x) = \frac{d^k}{dx^k}\delta(x) = \delta^{(k)}(x), \qquad k = 0,1,2\,... \tag{2.1.13}$$

ekanligi kelib chiqadi.

2.1.5. Siqiluvchan akslantirishlar prinsipi.

Berilgan shartlarda tenglama yechimining mavjudligi va yagonaligi bilan bog‘liq masalalarni mos metrik fazolardagi biror akslantirishning qo‘zg‘almas nuqtasi mavjudligi va yagonaligi haqidagi masala ko‘rinishida ifodalash mumkin. Qo‘zg‘almas nuqta mavjudligi va yagonaligi belgilari ichida eng sodda va shu bilan birga juda muhim belgi - bu «siqiluvchan akslantirishlar prinsipi» deb nomlanuvchi belgidir.

2.1.10-ta’rif. X metrik fazo va uni o‘zini-o‘ziga akslantiruvchi A akslantirish berilgan bo‘lsin. Agar shunday $\alpha \in (0,1)$ son mavjud bo‘lib, barcha $x, y \in X$ nuqtalar uchun

$$\rho(Ax, Ay) = \alpha\rho(x, y) \tag{2.1.14}$$

tengsizlik bajarilsa, A siqiluvchan akslantirish deb ataladi.

Har bir siqiluvchan akslantirish uzluksizdir. Haqiqatan ham, agar $x_n \to x$ $(\rho(x_n, x) \to 0)$ bo‘lsa, u holda

$$\rho(Ax_n, Ax) \le \alpha\rho(x_n, x) \tag{2.1.15}$$

bo‘lgani uchun $Ax_n \to Ax$.

2.1.11-ta'rif. Agar $A: X \to X$akslantirish uchun shunday $x \in X$ nuqta mavjud bo'lib $Ax = x$ tenglik bajarilsa, x nuqta A akslantirishning qo'zg'almas nuqtasi deyiladi.

2.1.1-teorema. (Siqiluvchan akslantirishlar prinsipi). To'la metrik fazoda aniqlangan har qanday siqiluvchan akslantirish yagona qo'zg'almas nuqtaga ega.

2.1.1-teorema isbot. X metrik fazodan ixtiyoriy x_0 nuqtani olamiz. Keyin

$$x_1 = Ax_0, x_2 = Ax_1 = A^2x_0, x_3 = Ax_2 = A^3x_0, \dots, x_n = Ax_{n-1} = A^nx_0, \dots$$

nuqtalar ketma-ketligini qaraymiz va bu ketma-ketlikning fundamentalligini ko'rsatamiz.

Haqiqatan ham, (2.1.14) va uchburchak aksiomasiga muvofiq har qanday m, n $(m > n)$ natural sonlar uchun

$$\rho(x_n, x_m) = \rho(A^nx_0, A^mx_0) = \rho(A^nx_0, A^nx_{m-n}) \leq \alpha^n\rho(x_0, x_{m-n})$$

$$\leq \alpha^n(\rho(x_0, x_1) + \rho(x_1, x_2) + \cdots \rho(x_{m-n-1}, x_{m-n})) \leq$$

$$\leq \alpha^n\rho(x_0, x_1)(1 + \alpha + \alpha^2 + \cdots + \alpha^{m-n-1}) \leq\leq \alpha^n \rho(x_0, x_1)\frac{1}{1-\alpha} \quad (2.1.16)$$

n yetarli katta bo'lganda bu tengsizlikning o'ng tomonini istalgancha kichik qilinishi mumkin, chunki $\alpha < 1$. Demak, $\{x_n\}$ ketma-ketlik fundamentaldir. X fazo to'la bo'lganligi uchun $\{x_n\}$ ketma-ketlikning fundamentalligidan uning yaqinlashuvchiligi kelib chiqadi, ya'ni:

$$\lim_{n\to\infty} x_n = x$$

F aks ettirish uzluksiz bo'lganligi uchun

$$Fx = F\left(\lim_{n\to\infty} x_n\right) = \lim_{n\to\infty} Fx_n = \lim_{n\to\infty} x_{n+1} = x. \qquad (2.1.17)$$

Demak, x qo'zg'almas nuqta.

Endi qo'zg'almas nuqtaning yagonaligini isbotlaymiz. Agar:

$$Ax = x, \qquad Ay = y$$

desak, (2.1.15) tengsizlikka ko‘ra

$$\rho(x, y) = \rho(Ax, Ay) \le \alpha\rho(x, y). \tag{2.1.18}$$

Bundan $\alpha \in (0,1)$ bo‘lgani uchun:

$$\rho(x, y)(1 - \alpha) \le 0 \Rightarrow \rho(x, y) = 0 \tag{2.1.19}$$

ya’ni $x = y$ bo‘lishi kelib chiqadi. Qo‘zg‘almas nuqta yagona ekan.

Siqiluvchan akslantirishlar prinsipi $Ax = x$ tenglama yechimi mavjudligi va yagonaligini isbotlash uchungina qo‘llanib qolmay, bu tenglama yechimini topish usulini ham beradi.

2.1.6. Masalaning qo‘yilishi.

Integro $-$ differensial to‘lqin tenglamasi uchun boshlang‘ich-chegaraviy masalani qaraymiz.

$$U_{tt} - U_{xx} - U_{zz} = \int_0^t K(x, \tau) U_{xx}(t - \tau, x, z) d\tau, \quad t \in R, \ (x, z) \in R_+^2 \tag{2.1.20}$$

$$\mathrm{U}|_{t<0} \equiv 0 \tag{2.1.21}$$

$$U_z|_{z=0} = \delta'(x)\delta'(t) \tag{2.1.22}$$

Bu yerda $R_+^2 = \{(x, z) \in R^2 | \, z > 0\}$, $\delta'(x)$ $-$Dirakning del’ta funksiyasining hosilasi.

Teskari masalani quyidagicha qo‘yamiz:

Agar (2.1.20) − (2.1.22) masala yechimining z = 0 dagi qiymati, ya’ni quyidagi funksiya berilgan bo‘lsa:

$$U(t, x, 0) = g(t, x), \quad t > 0, \qquad x \in R \tag{2.1.23}$$

(2.1.20) dagi integral haddan $K(t,x),\ t > 0,\ x \in R$ yadroni topish talab qilinadi.

2.1.12-ta'rif. $C([0,\infty] \times R)$ uzluksiz funksiyalar sinfiga tegishli $K(t,x)$funksiya $(2.1.20) - (2.1.22)$ masalaning yechimi deyiladi, agar unga mos $(2.1.20) - (2.1.22)$ masalaning $D'(R \times R_+^2)$ umumlashgan funksiyalar sinfiga tegishli $U(t,x,z)$ yechimi $D'([0,\infty] \times R)$ umumlashgan funksiyalar sinfiga tegishli $g(t,x)$ funksiya uchun (2.1.23) ni qanoatlantirsa.

(2.1.20) tenglama xotirali muhitlarda sodir bo‘ladigan to‘lqin jarayonlarida hosil bo‘ladi. Bunda muhit holati ayni vaqtdagi harakat bilan aniqlanmaydi, uning "tarixiga" bog‘liq bo‘ladi. $(2.1.20) - (2.1.23)$ masala differensial tenglamalar uchun ko‘p o‘lchovli teskari masalalar qatoriga kiradi. Ushbu bitiruv malakaviy ishida Banax fazolari shkalalar metodiga asosan x o‘zgaruvchi bo‘yicha analitik va t o‘zgaruvchi bo‘yicha silliq funksiyalar sinfida qo‘yilgan masalalarning lokal bir qiymatli yechimga ega bo‘lishi ko‘rsatiladi.

Bunda $K(t,x)$ yadro gorizontal x o‘zgaruvchiga kuchsiz bog‘liq deb faraz qilamiz:

$$K(x,t) = K_0(t) + \varepsilon x K_1(t) + \cdots \qquad (2.1.24)$$

bu yerda ε kichik parametr.

Ushbu ishning asosiy natijasi $K_0(t)$ ni topish usulini qurish.

2.2. Kuchsiz gorizontal muhitda xotirali tenglama uchun teskari masalani yechish.

Biz (2.1.20) − (2.1.23) masalani ko‘p o‘lchovli teskari masaladan bir o‘lchovli teskari masalaga keltiramiz. (2.1.20) − (2.1.22) masalaning yechimini ε ning darajalari qatori ko‘rinishida, ya’ni quyidagi ko‘rinishda qidiramiz:

$$U(t,x,z) = U_0(t,x,z) + \varepsilon U_1(t,x,z) + \cdots \qquad (2.2.1)$$

(2.2.1) ni (2.1.20) ga qo‘yib ε oldidagi bir xil darajalarni tenglashtirib natijada to‘g‘ri masalalarning rekkurent sistemasini hosil qilamiz, undan U_0, U_1 va hokazolarni topamiz.

$$\varepsilon^0(U_{0tt} - U_{0xx} - U_{0zz}) + \varepsilon(U_{1tt} - U_{1xx} - U_{1zz}) + \varepsilon^2(U_{2tt} - U_{2xx} - U_{2zz}) +$$

$$+\cdots = \int_0^t (\varepsilon^0 K_0(\tau) + \varepsilon x K_1(\tau) + \varepsilon^2 x^2 K_2(\tau) + \cdots)(\varepsilon^0 U_{0xx}(t-\tau,x,z) +$$

$$+\varepsilon U_{1xx}(t-\tau,x,z) + \varepsilon^2 U_{2xx}(t-\tau,x,z) + \ \ldots)d\tau =$$

$$= \int_0^t (\varepsilon^0\, K_0(\tau) U_{0xx}(t-\tau,x,z) +$$

$$+\varepsilon\big(K_0(\tau)U_{1xx}(t-\tau,x,z) + xK_1(\tau)U_{0xx}(t-\tau,x,z)\big) +$$

$$+\varepsilon^2(K_0(\tau)U_{2xx}(t-\tau,x,z) + xK_1(\tau)U_{1xx}(t-\tau,x,z) +$$

$$+x^2K_2(\tau)U_{0xx}(t-\tau,x,z))+\ldots)d\tau$$

$$U_{0tt} - U_{0xx} - U_{0zz} = \int_0^t K_0(\tau)U_{0xx}(t-\tau,x,z)d\tau$$

$$U_{1tt} - U_{1xx} - U_{1zz} = \int_0^t (K_0(\tau)U_{1xx}(t-\tau,x,z) + xK_1(\tau)U_{0xx}(t-\tau,x,z))d\tau$$

$$U_{2tt} - U_{2xx} - U_{2zz} = \int_0^t (K_0(\tau)U_{2xx}(t-\tau,x,z) + xK_1(\tau)U_{1xx}(t-\tau,x,z) +$$

$$+x^2K_2(\tau)U_{0xx}(t-\tau,x,z))d\tau$$

va hokazo

$$U_{ntt}-U_{nxx}-U_{nzz}=$$

$$=\int_0^t (K_0(\tau)U_{nxx}(t-\tau,x,z)+xK_1(\tau)U_{(n-1)xx}(t-\tau,x,z)+\cdots+$$

$$+x^{n-1}K_{n-1}(\tau)U_{1xx}(t-\tau,x,z)+x^nK_n(\tau)U_{0xx}(t-\tau,x,z))d\tau=$$

$$=\int_0^t \sum_{j=0}^n x^j\, K_j(\tau)U_{(n-j)xx}(t-\tau,x,z)d\tau.$$

(2.1.21)−(2.1.22) formulalar quyidagi ko‘rinishni oladi.

$$\mathrm{U}|_{t<0}=(U_0(t,x,z)+\ \varepsilon U_1(t,x,z)+\cdots\ )|_{t<0}\equiv 0=(0+\varepsilon 0+\cdots)$$

Bundan

$$U_0|_{t<0}\equiv 0$$

$$U_1|_{t<0}\equiv 0$$

…………..

$$U_n|_{t<0}\equiv 0$$

$$U_z|_{z=0}=(U_{0z}(t,x,0)+\ \varepsilon U_{1z}(t,x,0)+\cdots\ )=\delta'(x)\delta'(t)+\varepsilon 0+\cdots$$

Bundan

$$U_{0z}|_{z=0}=\delta'(x)\delta'(t)$$

$$U_{1z}|_{z=0}=0$$

……………..

$$U_{nz}|_{z=0}=0$$

U holda (2.1.23) formulaga asosan $g(x,t)$ funksiya $U(t,x,z)$ funksiya kabi tuzilishga ega bo‘ladi.

$$g(t,x) = \varepsilon^0 g_0(t,x) + \varepsilon g_1(t,x) + \varepsilon^2 g_2(t,x) + \cdots \qquad (2.2.2)$$

Bundan kelib chiqadiki

$$U_0|_{z=0} = g_0(t,x)$$

$$U_1|_{z=0} = g_1(t,x)$$

…………………

$$U_n|_{z=0} = g_n(t,x)$$

$U(t,x,z)$ funksiyaning (2.2.1) formula bo‘yicha $K(x,t)$ funksiyaning (2.1.24) formula bo‘yicha qatorga yoyilishidan foydalanib, (2.1.20) − (2.1.23) teskari masala K_0, K_1 ... larni ketma-ket aniqlash uchun quyidagi masalalarga bo‘linadi.

U_0, K_0 uchun

$$U_{0tt} - U_{0xx} - U_{0zz} = \int_0^t K_0(\tau) U_{0xx}(t-\tau,x,z)d\tau, t \in R, (x,z) \in R_+^2 \qquad (2.2.3)$$

$$U_0|_{t<0} \equiv 0\,, \qquad U_{0z}|_{z=0} = \delta'(x)\delta'(t) \qquad (2.2.4)$$

$$U_0|_{z=0} = g_0(x,t), \qquad (x,t) \in R^2 \qquad (\,2.2.5)$$

U_n, K_n uchun

$$U_{ntt} - U_{nxx} - U_{nzz} = \int_0^t \sum_{j=0}^n x^j K_j\,(\tau) U_{(n-j)xx}(t-\tau,x,z)d\tau,$$

$$t \in R, (x,z) \in R_+^2 \qquad (2.2.6)$$

$$U_n|_{t<0} \equiv 0\,, \qquad U_{nz}|_{z=0} = 0 \qquad (2.2.7)$$

$$U_n|_{z=0} = g_n(x,t), \qquad (x,t) \in R^2 \qquad n = 1,2,\ldots \qquad (2.2.8)$$

$U_j(t,x,z)$ $j=1,2,...$ funksiyalardan bularning x o‘zgaruvchi bo‘yicha eksponensial Fur’ye obraziga o‘tamiz.

$$\tilde{U}_j(t,\lambda,z)=\int_R U_j(t,x,z)e^{-i\lambda x}dx, \quad \lambda\in R \qquad (2.2.9)$$

Har bir chekli t uchun $U_j(t,x,z)$ $j=1,2,...$ funksiya uchun Fur’ye almashtirishi mavjud, chunki har bir U_j regulyar funksiya va chekli tartibdagi qandaydir singulyar umumlashgan funksiyaning yig‘indisi bo‘lib, U_j funksiyalarning tashuvchilari chegaralangan.

$\tilde{U}_0(t,\lambda,z)$ ning mos hosilalarini olib (2.1.20) tenglamaga keltirib qo‘yamiz.

$$\tilde{U}_0(t,\lambda,z)=\int_R U_0(t,x,z)e^{-i\lambda x}dx$$

$$\tilde{U}_{0tt}(t,\lambda,z)=\int_R U_{0tt}(t,x,z)e^{-i\lambda x}dx$$

$$\tilde{U}_{0zz}(t,\lambda,z)=\int_R U_{0zz}(t,x,z)e^{-i\lambda x}dx$$

$$\tilde{U}_{0xx}(t,\lambda,z)=\int_R U_{0xx}(t,x,z)e^{-i\lambda x}dx=\left|\begin{matrix}U_{0xx}dx=dvU_{0x}=v\\ e^{-i\lambda x}=u \;\; -i\lambda e^{-i\lambda x}dx=du\end{matrix}\right|=$$

$$=U_{0x}e^{-i\lambda x}|_{-\infty}^{\infty}+\ i\lambda\int_R U_{0x}\,e^{-i\lambda x}dx=\left|\begin{matrix}U_{0x}dx=dv\;\; U_0=v\\ e^{-i\lambda x}=u \;\; -i\lambda\, e^{-i\lambda x}dx=du\end{matrix}\right|=$$

$$i\lambda\left(U_0(t,x,z)e^{-i\lambda x}|_{-\infty}^{\infty}+i\lambda\int_R U_0(t,x,z)\,e^{-i\lambda x}dx\right)=-\lambda^2\tilde{U}_0(t,\lambda,z).$$

Tenglamani va shartlarni qaytadan yozib olamiz:

$$\tilde{U}_{0tt}-\tilde{U}_{0zz}+\lambda^2\tilde{U}_0=\int_0^t K_0(\tau)\tilde{U}_{0xx}(t-\tau,\lambda,z)d\tau=$$

$$=-\lambda^2\int_0^t K_0(\tau)\tilde{U}_0(t-\tau,\lambda,z)d\tau$$

$$\tilde{U}_0|_{t<0} = (\int_R U_0\,(t,x,z)e^{-i\lambda x}dx)|_{t<0} \equiv 0$$

$$\tilde{U}_{0z}|_{z=0} = \int_R U_{0z}\,(t,x,0)e^{-i\lambda x}dx = \int_R \delta'(x)\delta'(t)\,e^{-i\lambda x}dx =$$

$$= \delta'(t)\int_R \delta'(x)\,e^{-i\lambda x}dx = \begin{vmatrix} \delta'(x)dx = dv\delta(x) = v \\ e^{-i\lambda x} = u \quad -i\lambda e^{-i\lambda x}dx = du \end{vmatrix} =$$

$$= \delta'(t)\left(\delta(x)e^{-i\lambda x}|_{-\infty}^{\infty} + \ i\lambda\int_R \delta(x)\,e^{-i\lambda x}dx\right) = i\lambda\delta'(t)$$

$$\tilde{U}_0|_{z=0} = \int_R U_0\,(t,x,0)e^{-i\lambda x}dx = \int_R g_0\,(t,x)e^{-i\lambda x}dx = \breve{g}_0(t,\lambda)$$

(2.2.3) − (2.2.5) va (2.2.6) − (2.2.8) teskari masalalar $\tilde{U}_j$ funksiyalar atamasida K_0, K_1 , ... larni topish uchun quyidagi masalalardan iborat.

$$\tilde{U}_{0tt} - \tilde{U}_{0zz} + \lambda^2\tilde{U}_0 = -\lambda^2\int_0^t K_0(\tau)\tilde{U}_0(t-\tau,\lambda,z)d\tau, t \in R, (\lambda,z) \in R_+^2 \quad (2.2.9)$$

$$\tilde{U}_0|_{t<0} \equiv 0, \qquad \tilde{U}_{0z}|_{z=0} = i\lambda\delta'(t) \qquad (2.2.10)$$

$$\tilde{U}_0|_{z=0} = \breve{g}_0(x,t), \qquad (\lambda,t) \in R^2 \qquad (2.2.11)$$

$$\tilde{U}_{ntt} - \tilde{U}_{nzz} + \lambda^2\tilde{U}_n = -\lambda^2\int_0^t \sum_{j=0}^n (i)^j K_j\,(\tau)\frac{\partial^j}{\partial\lambda^j}\tilde{U}_{n-j}(t-\tau,\lambda,z)d\tau,$$

$$t \in R, \qquad (\lambda,z) \in R_+^2 \qquad (2.2.12)$$

$$\tilde{U}_n|_{t<0} \equiv 0\,, \quad \tilde{U}_{nz}|_{z=0} = 0 \qquad (2.2.13)$$

$$\tilde{U}_n|_{z=0} = \breve{g}_n(t,\lambda), \qquad (t,\lambda) \in R^2, \qquad n = 1,2,\ldots \qquad (2.2.14)$$

bu yerda

$$\breve{g}_m(t,\lambda) = \int_R g_m(t,x)e^{-i\lambda x}\,dx, \quad m = 0,1,2,\ldots$$

K_0 va $\tilde{U}_0$ funksiyalarni aniqlash masalasi.

(2.2.9)-(2.2.11) teskari masala to‘la aniqlangan, chunki $K(t)$ bitta funksiyani aniqlash uchun ikki o‘zgaruvchining funksiyasi ((2.11) shart) beriladi. Teskari masalaning bir qiymatli yechilishi uchun $g_0(t,x)$ funksiyaning Fur’ye obrazini berish yetarli. Almashtirish tayin bitta parametr qiymati uchun (2.2.9)− (2.2.11) da λ fiksirlangan va $\lambda \neq 0$.

Giperbolik tenglamalar nazariyasidan, $0 < t < z,\ z > 0$ da $\tilde{U}_0(t,\lambda,z) = 0$. Shuning uchun (2.2.9), (2.2.10) masalaning yechimini quyidagi ko‘rinishda belgilash qulay:

$$\tilde{U}_0(t,\lambda,z) = -i\lambda\delta(t-z) + v(t,\lambda,z)\theta(t-z) \qquad (2.2.15)$$

bu yerda $v(t,\lambda,z)$ regulyar funksiya.

Quyidagi belgilash kiritamiz:

$$v(z,\lambda,z) = \beta(\lambda,z) \qquad (2.2.16)$$

$$v(t,\lambda,z)\delta(t-z) = \beta(\lambda,z)\delta(t-z)$$

bo‘ladi, $\delta(x)$ funksiyaning (2.1.3)-xossasiga asosan:

$$\frac{d}{dz}v(z,\lambda,z) = v_z(z,\lambda,z) + v_t(z,\lambda,z) = \frac{d}{dz}\beta(\lambda,z) = \beta_z(\lambda,z). \qquad (2.2.17)$$

$\tilde{U}_0(t,\lambda,z)$ ning hosilalarini hisoblaymiz.

$$\tilde{U}_{0t} = -i\lambda\delta'(t-z) + v_t(t,\lambda,z)\theta(t-z) + v(t,\lambda,z)\delta(t-z)$$

$$\tilde{U}_{0tt} = -i\lambda\delta''(t-z) + v_{tt}\theta(t-z) + v_t(t,\lambda,z)\delta(t-z) + \beta(\lambda,z)\delta'(t-z)$$

$$\tilde{U}_{0z} = i\lambda\delta'(t-z) + v_z(t,\lambda,z)\theta(t-z) - v(t,\lambda,z)\delta(t-z)$$

$$\tilde{U}_{0zz} = -i\lambda\delta''(t-z) + v_{zz}\theta(t-z) - v_z(t,\lambda,z)\delta(t-z) - \beta_z(\lambda,z)\delta(t-z) +$$

$$+\beta(\lambda,z)\delta'(t-z).$$

Hosil bo‘lgan tengliklarni (2.2.9) tenglamaga keltirib qo‘yamiz:

$$-i\,\lambda\delta''(t-z)+v_{tt}\theta(t-z)+v_t(t,\lambda,z)\delta(t-z)+\ \beta(\lambda,z)\delta'(t-z)+$$

$$+i\,\lambda\delta''(t-z)-v_{zz}\theta(t-z)+v_z(t,\lambda,z)\delta(t-z)+\beta_z(\lambda,z)\delta(t-z)-$$

$$-\,\beta(\lambda,z)\delta'(t-z)-i\lambda^3\delta(t-z)+\lambda^2 v(t,\lambda,z)\theta(t-z)=$$

$$=-\lambda^2\int_0^t K_0(\tau)\left(-i\lambda\delta(t-\tau-z)+v(t-\tau,\lambda,z)\theta(t-\tau-z)\right)d\tau.$$

Tenglama quyidagi ko‘rinishga keladi:

$$(v_{tt}-v_{zz})\theta(t-z)+(v_t(t,\lambda,z)+v_z(t,\lambda,z))\delta(t-z)+\beta_z(\lambda,z)\delta(t-z)-$$

$$-i\lambda^3\delta(t-z)+\lambda^2 v(t,\lambda,z)\theta(t-z)=$$

$$=i\lambda^3\int_0^t K_0(\tau)\,\delta(t-\tau-z)d\tau-\lambda^2\int_0^t K_0(\tau)\ v(t-\tau,\lambda,z)\theta(t-\tau-z)d\tau.$$

(2.2.17) belgilashdan foydalansak tenglama quyidagi ko‘rinishga keladi:

$$(v_{tt}-v_{zz})\theta(t-z)+2\beta_z(\lambda,z)\delta(t-z)-i\lambda^3\delta(t-z)+\lambda^2 v(t,\lambda,z)\theta(t-z)=$$

$$=i\lambda^3\int_0^t K_0(\tau)\,\delta(t-\tau-z)d\tau-\lambda^2\int_0^t K_0(\tau)\ v(t-\tau,\lambda,z)\theta(t-\tau-z)d\tau.$$

Integrallarni alohida hisoblaymiz. Birinchi integralni hisoblashda del’ta funksiyaning (2.1.2) va (2.1.1) - xossalaridan foydalanamiz:

$$\int_0^t K_0(\tau)\,\delta(t-\tau-z)d\tau=\int_0^t K_0(\tau)\,\delta\big((t-z)-\tau\big)d\tau=$$

$$=\int_0^t K_0(\tau)\,\delta\big(\tau-(t-z)\big)d\tau=K_0(t-z).$$

Ikkinchi integralni hisoblashda $\theta(x)$ funksiyaning xossasidan foydalanamiz:

$$\int_0^t K_0(\tau)\,v(t-\tau,\lambda,z)\theta(t-\tau-z)d\tau=\int_0^{t-z} K_0(\tau)\,v(t-\tau,\lambda,z)\theta(t-\tau-z)d\tau$$

$$+\int_{t-z}^{t} K_0(\tau)\ v(t-\tau,\lambda,z)\theta(t-\tau-z) = \theta(t-z)\int_0^{t-z} K_0(\tau)\ v(t-\tau,\lambda,z).$$

Hosil bo‘lganlarni tenglamaga keltirib qo‘yamiz:

$$(v_{tt}-v_{zz})\theta(t-z) + 2\beta_z(\lambda,z)\delta(t-z) - i\lambda^3\delta(t-z) + \lambda^2 v(t,\lambda,z)\theta(t-z) =$$

$$= i\lambda^3\theta(t-z)K_0(t-z) - \lambda^2\theta(t-z)\int_0^{t-z} K_0(\tau)\ v(t-\tau,\lambda,z)d\tau.$$

Tenglikning chap tomonida $\delta(t-z)$ qatnashgan had mavjud. O‘ng tarafida esa mavjud emas. Shu sababli biz $\delta(t-z)$ oldidagi koeffitsientni nolga tenglaymiz:

$$2\beta_z(\lambda,z)\delta(t-z) - i\lambda^3\delta(t-z) = 0,$$

$$2\beta_z(\lambda,z) - i\lambda^3 = 0$$

ekanligi kelib chiqadi.

$t-z > 0$ bo‘lganligi sababli $\theta(t-z) = 1$ bo‘ladi.

$$v_{tt} - v_{zz} + \lambda^2 v = \ i\lambda^3 K_0(t-z) - \lambda^2\int_0^{t-z} K_0(\tau)\, v(t-\tau,\lambda,z)d\tau$$

$$\tilde{U}_{0z}|_{z=0} = i\lambda\delta'(t) + v_z|_{z=0}\theta(t) - \beta(\lambda,0)\delta(t) = \ i\lambda\delta'(t).$$

(2.2.10) shartga asosan tenglik o‘rinli. Bu yerdan

$$v_z|_{z=0} - \beta(\lambda,0)\delta(t) = 0 + 0\ \delta(t)$$

kelib chiqadi. Ya’ni:

$$\begin{cases} v_z|_{z=0} = 0 \\ \beta(\lambda,0) = 0 \end{cases}.$$

Yuqoridagiga asosan:

$$\begin{cases} 2\beta_z(\lambda,z) - i\lambda^3 = 0 \\ \beta(\lambda,0) = 0 \end{cases}.$$

$$\beta_z(\lambda,z) = \frac{i\lambda^3}{2} \Rightarrow \frac{d\beta(\lambda,z)}{dz} = \frac{i\lambda^3}{2} \Rightarrow \int_0^z d\beta(\lambda,z) = \int_0^z \frac{i\lambda^3}{2} dz$$

$$\beta(\lambda,z)|_0^z = \frac{i\lambda^3}{2} z|_0^z \ \Rightarrow \ \beta(\lambda,z) - \beta(\lambda,0) = \frac{i\lambda^3}{2} z \ \Rightarrow \ \beta(\lambda,z) = \frac{i\lambda^3}{2} z.$$

$v(t,\lambda,z)$ funksiya tayin λ uchun $t > z > 0$ sohada quyidagi tenglamalarni qanoatlantiradi:

$$v_{tt} - v_{zz} + \lambda^2 v = i\lambda^3 K_0(t-z) - \lambda^2 \int_0^{t-z} K_0(\tau)\ v(t-\tau,\lambda,z)d\tau \qquad (2.2.18)$$

$$v|_{t=z+0} = \frac{i\lambda^3}{2} z \qquad (2.2.19)$$

$$v_z|_{z=0} = 0 \qquad (2.2.20)$$

(2.2.15) ifodaga asosan teskari masala yechimga ega bo‘lishi uchun $g_0(t,\lambda)$ funksiya quyidagicha ifodalanishi kerak:

$$\breve{g}_0(t,\lambda) = i\lambda\delta(t) + r(t,\lambda)\theta(t), \quad t\epsilon R, \qquad (2.2.21)$$

Bu yerda $r(t,\lambda)$ funksiya t argument bo‘yicha ba’zi bir silliqlik shartlarini qanoatlantirishi kerak. Bunga bog‘liq r funksiya uchun shart quyidagicha:

$$v|_{z=0} = r(t,\lambda), \qquad t > 0 \qquad (2.2.22)$$

2.2.1-teorema. $r(t,\lambda) \in C^2[0,T]$ va $r(0,\lambda) = 0,\ r'(0,\lambda) = \frac{i\lambda^3}{4}$ bo‘lsin. U holda ixtiyoriy tayin $T > 0$ uchun (2.2.18)-(2.2.20), (2.2.22) teskari masala $K_0(t) \in C[0,T]$ yagona yechimga ega.

2.2.1-lemma. 2.2.1-teoremaning shartlari bajarilganda (2.2.18)-(2.2.20), (2.2.22) teskari masala, $(z,t) \in D_T,\ D_T = \{(z,t)|0 \le z \le t \le T - z\}$ shar uchun quyidagi tenglamalar sistemasidan $v(t,\lambda,z),\ v_t(t,\lambda,z),\ K_0(t)$ vektor funksiyalarni topish masalasiga teng kuchli.

$$v(t,\lambda,z)=\frac{i\lambda^3}{2}z+\int_z^t v_t(\tau,\lambda,z)d\tau \qquad (2.2.23)$$

$$v_t=\frac{i\lambda^3}{4}+\frac{1}{2}\int_z^{\frac{z+t}{2}}\lambda^2 v(-\xi+t+z,\lambda,\xi)d\,\xi-\frac{i\lambda^3}{2}\int_z^{\frac{z+t}{2}}K_0(t+z-2\xi)d\xi+$$

$$+\frac{\lambda^2}{2}\int_z^{\frac{z+t}{2}}\int_0^{t+z-2\xi}K_0(\tau)\ v(-\xi+t+z-\tau,\lambda,\xi)d\tau d\,\xi+\frac{1}{2}r'(t-z,\lambda)-$$

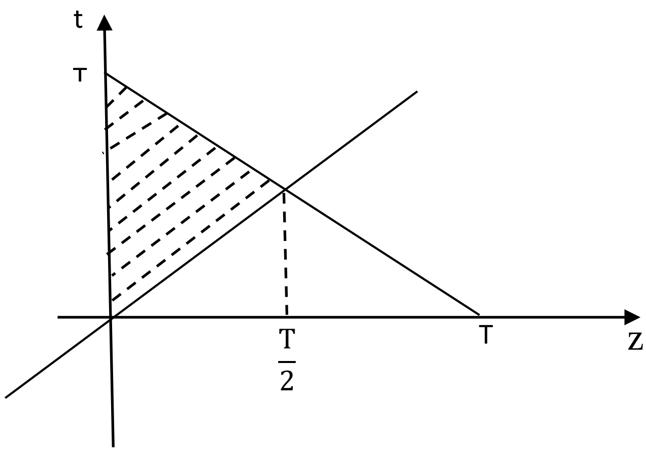
$$-\frac{\lambda^2}{2}\int_0^z v(\xi+t-z,\lambda,\xi)d\,\xi+\frac{i\lambda^3}{2}zK_0(t-z)-$$

$$-\frac{\lambda^2}{2}\int_0^z\int_0^{t-z}K_0(\tau)\ v(\xi+t-z-\tau,\lambda,\xi)d\tau d\xi \qquad (2.2.24)$$

$$K_0(t)=\frac{\lambda^2}{4}t+\frac{2i}{\lambda^3}r''(t,\lambda)-\frac{2i}{\lambda}\int_0^{\frac{t}{2}}v_t(-\xi+t,\lambda,\xi)d\,\xi+\lambda^2\int_0^{\frac{t}{2}}\xi K_0(t-2\xi)d\xi-$$

$$-\frac{2i}{\lambda}\int_0^{\frac{t}{2}}\int_0^{t-2\xi}K_0(\tau)\,v_t(-\xi+t-\tau,\lambda,\xi)d\tau\,d\xi \qquad (2.2.25)$$

2.2.1-lemma isboti. $D_T=\{(z,t)|0\le z\le t\le T-z\}$ sohadan (ξ,θ) tekislikda $D_T=\{(\xi,\theta)|0\le \xi\le \theta\le T-\xi\}$ sohaga o‘tamiz.



$2.2.1-chizma.$

$$v_{\theta\theta} - v_{\xi\xi} + \lambda^2 v = i\lambda^3 K_0(\theta - \xi) - \lambda^2 \int_0^{\theta-\xi} K_0(\tau)\ v(\theta - \tau, \lambda, \xi) d\tau.$$

Quyidagi tenglik o‘rinli:

$$v_{\theta\theta} - v_{\xi\xi} = \left(\frac{\partial}{\partial\theta} - \frac{\partial}{\partial\xi}\right)\left(\frac{\partial}{\partial\theta} + \frac{\partial}{\partial\xi}\right) v(\xi, \lambda, \theta) = \left(\frac{\partial}{\partial\theta} - \frac{\partial}{\partial\xi}\right)\left(v_\theta - v_\xi\right) =$$

$$= \left(\frac{\partial}{\partial\theta} + \frac{\partial}{\partial\xi}\right)(v_\theta - v_\xi)$$

$$\left(\frac{\partial}{\partial\theta} - \frac{\partial}{\partial\xi}\right);\ \frac{\partial\theta}{\partial\xi} = -1 \Rightarrow \begin{cases} \theta = -\xi + C_1 \\ t = -z + C_1 \end{cases} \Rightarrow C_1 = t + z \Rightarrow \theta = -\xi + t + z$$

$$\left(\frac{\partial}{\partial\theta} + \frac{\partial}{\partial\xi}\right);\ \frac{\partial\theta}{\partial\xi} = 1 \Rightarrow \begin{cases} \theta = \xi + C_2 \\ t = z + C_2 \end{cases} \Rightarrow C_2 = t - z \Rightarrow \theta = \xi + t - z.$$

Bular (z, t) nuqtadan o‘tuvchi xarakteristikalar. $\left(\frac{\partial}{\partial\theta} - \frac{\partial}{\partial\xi}\right)$ operatorni

$\theta = -\xi + t + z$ xarakteristika bo‘ylab (z, t) nuqtadan $\left(\frac{z+t}{2}, \frac{z+t}{2}\right)$ nuqtagacha integrallaymiz:

$$\frac{\partial}{\partial\xi}\left(v_\theta + v_\xi\right)(\theta, \lambda, \xi) = -\lambda^2 v + i\lambda^3 K_0(\theta - \xi) - \lambda^2 \int_0^{\theta-\xi} K_0(\tau)\, v(\theta - \tau, \lambda, \xi) d\tau.$$

$\theta = -\xi + t + z$ ifodani o‘rniga qo‘yib, tenglamaning ikkala tomonini $d\xi$ ga ko‘paytirib integrallaymiz.

$$\frac{\partial}{\partial\xi}\left(v_\theta + v_\xi\right)(-\xi + z + t, \lambda, \xi) = -\lambda^2 v(-\xi + t + z, \lambda, \xi) + i\lambda^3 K_0(t + z - 2\xi) -$$

$$-\lambda^2 \int_0^{t+z-2\xi} K_0(\tau)\ v(-\xi + t + z - \tau, \lambda, \xi) d\tau$$

$$\int_z^{\frac{z+t}{2}} d\left(v_\theta + v_\xi\right)(-\xi + t + z, \lambda, \xi) = \int_z^{\frac{z+t}{2}} (-\lambda^2 v(-\xi + t + z, \lambda, \xi) +$$

$$+i\lambda^3 K_0(t+z-2\xi)-\lambda^2\int_0^{t+z-2\xi} K_0(\tau)\ v(-\xi+t+z-\tau,\lambda,\xi)d\tau)d\xi$$

Chap tomon integralini hisoblaymiz:

$$\left(v_\theta+v_\xi\right)(-\xi+t+z,\lambda,\xi)|_z^{\frac{z+t}{2}}=(v_t+v_z)\left(\frac{z+t}{2},\lambda,\frac{z+t}{2}\right)-(v_t+v_z)(t,\lambda,z)$$

$$=\frac{i\lambda^3}{2}-(v_t+v_z)(t,\lambda,z)$$

$$(v_t+v_z)(t,\lambda,z)=\frac{i\lambda^3}{2}-[\int_z^{\frac{z+t}{2}}(-\lambda^2 v(-\xi+t+z,\lambda,\xi)+i\lambda^3 K_0(t+z-2\xi)-$$

$$-\lambda^2\int_0^{t+z-2\xi} K_0(\tau)\, v(-\xi+t+z-\tau,\lambda,\xi)d\tau)d\xi] \qquad (2.2.26)$$

$\left(\frac{\partial}{\partial\theta}+\frac{\partial}{\partial\xi}\right)$ operatorni

$\theta=\xi+t-z$ xarakteristika bo‘ylab $(0,t-z)$ nuqtadan (z,t) nuqtagacha integrallaymiz:

$$\frac{\partial}{\partial\xi}\left(v_\theta-v_\xi\right)(\theta,\lambda,\xi)=-\lambda^2 v+i\lambda^3 K_0(\theta-\xi)-\lambda^2\int_0^{\theta-\xi} K_0(\tau)\, v(\theta-\tau,\lambda,\xi)d\tau$$

$\theta=\xi+t-z$ ifodani o‘rniga qo‘yib, tenglamaning ikkala tomonini $d\xi$ ga ko‘paytirib integrallaymiz:

$$\frac{\partial}{\partial\xi}\left(v_\theta+v_\xi\right)(\xi+t-z,\lambda,\xi)=-\lambda^2 v(\xi+t-z,\lambda,\xi)+i\lambda^3 K_0(t-z)-$$

$$-\lambda^2\int_0^{t-z} K_0(\tau)\ v(\xi+t-z-\tau,\lambda,\xi)d\tau.$$

Endi tenglikni integrallaymiz:

$$\int_0^z d\left(v_\theta-v_\xi\right)(\xi+t-z,\lambda,\xi)=\int_0^z(-\lambda^2 v(\xi+t-z,\lambda,\xi)+i\lambda^3 K_0(t-z)-$$

$$-\lambda^2\int_0^{t-z}K_0(\tau)\ v(\xi+t-z-\tau,\lambda,\xi)d\tau)d\xi.$$

Chap tomon integralini hisoblaymiz:

$$\left(v_\theta-v_\xi\right)(\xi+t-z,\lambda,\xi)|_0^z=(v_t-v_z)(t,\lambda,z)-(v_t-v_z)(t-z,\lambda,0)=$$

$$=\left|\begin{matrix}v_z|_{z=0}=0\\ v|_{z=0}=r(t-z,\lambda)\\ v_t|_{z=0}=r'(t-z,\lambda)\end{matrix}\right|=(v_t-v_z)(t,\lambda,z)-r'(t-z,\lambda)$$

$$(v_t-v_z)(t,\lambda,z)=r'(t-z,\lambda)+[\int_0^z(-\lambda^2v(\xi+t-z,\lambda,\xi)+i\lambda^3K_0(t-z)-$$

$$-\lambda^2\int_0^{t-z}K_0(\tau)\,v(\xi+t-z-\tau,\lambda,\xi)d\tau)d\xi] \qquad (2.2.27)$$

(2.2.26) va (2.2.27) ifodalarni qo‘shib quyidagiga ega bo‘lamiz:

$$2v_t=\frac{i\lambda^3}{2}-\left[\int_z^{\frac{z+t}{2}}\left(-\lambda^2v(-\xi+t+z,\lambda,\xi)+i\lambda^3K_0(t+z-2\xi)\right.\right.$$

$$\left.\left.-\lambda^2\int_0^{t+z-2\xi}K_0(\tau)\ v(-\xi+t+z-\tau,\lambda,\xi)d\tau\right)d\xi\right]+r'(t-z,\lambda)+$$

$$+\left[\int_0^z(-\lambda^2v(\xi+t-z,\lambda,\xi)+i\lambda^3K_0(t-z)-\lambda^2\int_0^{t-z}K_0(\tau)\ v(\xi+t-z\right.$$

$$\left.-\tau,\lambda,\xi)d\tau)d\xi\right];$$

$$v_t=\frac{i\lambda^3}{4}+\frac{1}{2}\int_z^{\frac{z+t}{2}}\lambda^2v(-\xi+t+z,\lambda,\xi)d\,\xi-\frac{i\lambda^3}{2}\int_z^{\frac{z+t}{2}}K_0(t+z-2\xi)d\xi+$$

$$+\frac{\lambda^2}{2}\int_z^{\frac{z+t}{2}}\int_0^{t+z-2\xi}K_0(\tau)\ v(-\xi+t+z-\tau,\lambda,\xi)d\tau d\,\xi+\frac{1}{2}r'(t-z,\lambda)-$$

$$-\frac{\lambda^2}{2}\int_0^z v(\xi+t-z,\lambda,\xi)d\,\xi+\frac{i\lambda^3}{2}zK_0(t-z)-$$

$$-\frac{\lambda^2}{2}\int_0^z\int_0^{t-z}K_0(\tau)\ v(\xi+t-z-\tau,\lambda,\xi)d\tau d\xi.$$

Ushbu tenglikdan $v_t(t,\lambda,0)=r'(t,\lambda)$ ni hisoblaymiz:

$$r'(t,\lambda)=\frac{i\lambda^3}{4}+\frac{1}{2}\int_0^{\frac{t}{2}}\lambda^2 v(-\xi+t,\lambda,\xi)d\,\xi-\frac{i\lambda^3}{2}\int_0^{\frac{t}{2}}K_0(t-2\xi)d\xi+$$

$$+\frac{\lambda^2}{2}\int_0^{\frac{t}{2}}\int_0^{t-2\xi}K_0(\tau)\ v(-\xi+t-\tau,\lambda,\xi)d\tau d\,\xi+\frac{1}{2}r'(t,\lambda).$$

Tenglikni soddalashtiramiz:

$$r'(t,\lambda)=\frac{i\lambda^3}{2}+\int_0^{\frac{t}{2}}\lambda^2 v(-\xi+t,\lambda,\xi)d\,\xi-i\lambda^3\int_0^{\frac{t}{2}}K_0(t-2\xi)d\xi+$$

$$+\lambda^2\int_0^{\frac{t}{2}}\int_0^{t-2\xi}K_0(\tau)\ v(-\xi+t-\tau,\lambda,\xi)d\tau d\xi.$$

Tenglikning ikkala tomonini t bo‘yicha differensiallaymiz:

$$r''(t,\lambda)=\frac{\partial}{\partial t}(\int_0^{\frac{t}{2}}\lambda^2 v(-\xi+t,\lambda,\xi)d\,\xi)-\frac{\partial}{\partial t}(i\lambda^3\int_0^{\frac{t}{2}}K_0(t-2\xi)d\xi)+$$

$$+\frac{\partial}{\partial t}\left(\lambda^2\int_0^{\frac{t}{2}}\int_0^{t-2\xi}K_0(\tau)\ v(-\xi+t-\tau,\lambda,\xi)d\tau d\,\xi\right).$$

Har bir differensialni alohida $I^{(1)},I^{(2)},I^{(3)}$ belgilab olib hisoblaymiz:

$$I^{(1)}=\frac{\partial}{\partial t}\left(\int_0^{\frac{t}{2}}\lambda^2 v(-\xi+t,\lambda,\xi)d\,\xi\right)=\frac{1}{2}\lambda^2 v\left(\frac{t}{2},\lambda,\frac{t}{2}\right)+\int_0^{\frac{t}{2}}\lambda^2 v_t(-\xi+t,\lambda,\xi)d\,\xi$$

$$=\frac{1}{2}\lambda^2\cdot\frac{i}{2}\lambda^3\frac{t}{2}+\int_0^{\frac{t}{2}}\lambda^2 v_t(-\xi+t,\lambda,\xi)d\,\xi=\frac{i\lambda^5}{8}t+\int_0^{\frac{t}{2}}\lambda^2 v_t(-\xi+t,\lambda,\xi)d\,\xi;$$

$$I^{(2)}=\frac{\partial}{\partial t}(i\lambda^3\int_0^{\frac{t}{2}}K_0(t-2\xi)d\xi)=\frac{1}{2}i\lambda^3 K_0(0)+i\lambda^3\int_0^{\frac{t}{2}}K_{0t}(t-2\xi)d\xi)=$$

$$\frac{i\lambda^3}{2}K_0(0)-\frac{i\lambda^3}{2}K_0(t-2\xi)|_0^{\frac{t}{2}}=\frac{i\lambda^3}{2}K_0(0)-\frac{i\lambda^3}{2}K_0(0)+\frac{i\lambda^3}{2}K_0(t)=\frac{i\lambda^3}{2}K_0(t);$$

$$I^{(3)}=\frac{\partial}{\partial t}\left(\lambda^2\int_0^{\frac{t}{2}}\int_0^{t-2\xi}K_0(\tau)\,v(-\xi+t-\tau,\lambda,\xi)d\tau d\xi\right)=$$

$$=\lambda^2(\frac{1}{2}\int_0^{t-2\cdot\frac{t}{2}}K_0(\tau)v(\frac{t}{2}-\tau,\lambda,\frac{t}{2})d\tau+\int_0^{\frac{t}{2}}K_0(t-2\xi)v(\xi,\lambda,\xi)\,d\xi+$$

$$+\int_0^{\frac{t}{2}}\int_0^{t-2\xi}K_0(\tau)\,v_t(-\xi+t-\tau,\lambda,\xi)d\tau d\,\xi)=\int_0^{\frac{t}{2}}\frac{i\lambda^5}{2}\xi K_0(t-2\xi)d\xi+$$

$$+\lambda^2\int_0^{\frac{t}{2}}\int_0^{t-2\xi}K_0(\tau)\,v_t(-\xi+t-\tau,\lambda,\xi)d\tau\,d\xi.$$

Hisoblangan differensiallarni keltirib o‘rniga qo‘yib kerakli hisoblashlarni bajaramiz.

$$r''(t,\lambda)=\frac{i\lambda^5}{8}t+\int_0^{\frac{t}{2}}\lambda^2 v_t(-\xi+t,\lambda,\xi)d\,\xi-\frac{i\lambda^3}{2}K_0(t)+\int_0^{\frac{t}{2}}\frac{i\lambda^5}{2}\xi K_0(t-2\xi)d\xi$$
$$+\lambda^2\int_0^{\frac{t}{2}}\int_0^{t-2\xi}K_0(\tau)\,v_t(-\xi+t-\tau,\lambda,\xi)d\tau\,d\xi.$$

Hosil bo‘lgan haddan $K_0(t)$ ning ifodasini topib olamiz:

$$K_0(t)=\frac{\lambda^2}{4}t+\frac{2i}{\lambda^3}r''(t,\lambda)-\frac{2i}{\lambda}\int_0^{\frac{t}{2}}v_t(-\xi+t,\lambda,\xi)d\,\xi+\lambda^2\int_0^{\frac{t}{2}}\xi K_0(t-2\xi)d\xi-$$

$$-\frac{2i}{\lambda}\int_0^{\frac{t}{2}}\int_0^{t-2\xi}K_0(\tau)\,v_t(-\xi+t-\tau,\lambda,\xi)d\tau\,d\xi.$$

$$v(t,\lambda,z)=\frac{i\lambda^3}{2}z+\int_z^t v_t(\tau,\lambda,z)d\tau.$$

2.2.1-lemma isbotlandi.

2.2.1-teorema isboti: (2.2.23)-(2.2.25) tenglamalar sistemasini operator tenglama ko‘rinishida yozamiz.

$$\varphi = A\varphi \tag{2.2.28}$$

Bu yerda

$$\varphi = [\varphi_1(t,\lambda,z), \varphi_2(t,\lambda,z), \varphi_3(t)]^* =$$

$$= \left[v(t,\lambda,z), v_t(t,\lambda,z) - \frac{i\lambda^3}{2} z K_0(t-z), K_0(t)\right]^*$$

φ_i $(i = 1,2,3)$ komponentali vektor funksiya, $*-$ transponirlash belgisi. A operator $\varphi \epsilon C(D_T)$ funksiyalar to‘plamida aniqlangan va $A = (A_1, A_2, A_3)$ ko‘rinishga ega.

Bu yerda:

$$A_1\varphi = \frac{i\lambda^3}{2} z + \int_z^t \left(\varphi_2(\tau,\lambda,z) + \frac{i\lambda^3}{2} z\varphi_3(\tau - z)\right) d\tau\,;$$

$$A_2\varphi = \frac{i\lambda^3}{4} + \int_z^{\frac{z+t}{2}} \Big(\frac{\lambda^2}{2}\varphi_1(t+z-\xi,\lambda,\xi) - \frac{i\lambda^3}{2}\varphi_3(t+z-2\xi) +$$

$$+\frac{\lambda^2}{2}\int_0^{t+z-2\xi} \varphi_3(\tau)\,\varphi_1(t+z-\xi-\tau,\lambda,\xi)d\tau\Big)d\xi - \frac{\lambda^2}{2}\int_0^z \Big(\varphi_1(\xi+t-z,\lambda,\xi) +$$

$$+\int_0^{t+z} \varphi_3(\tau)\,\varphi_1(\xi+t-z-\tau,\lambda,\xi)d\tau\Big)\,d\xi + \frac{1}{2} r'(t-z,\lambda);$$

$$A_3\varphi = \frac{\lambda^2 t}{4} + \frac{2i}{\lambda^3} r''(t,\lambda) - \int_0^{\frac{t}{2}} \Big(\frac{2i}{\lambda}\varphi_2(t-\xi,\lambda,\xi) - 2\lambda^2\xi\varphi_3(t-\xi)\Big)d\xi -$$

$$-\frac{2i}{\lambda}\int_0^{\frac{t}{2}}\int_0^{t-2\xi} \varphi_3(\tau)\Big(\varphi_2(t-\xi-\tau,\lambda,\xi) + \frac{i\lambda^3}{2}\xi\varphi_3(t-\xi-\tau,\lambda,\xi)\Big)\,d\tau d\xi.$$

$C_\rho(D_T)$ $(\rho \geq 0)$ orqali uzluksiz funksiyalarning Banax fazosini (ya'ni ixtiyoriy funksional ketma-ketlik yaqinlashuvchi normallangan fazo) belgilaymiz. Normani quyidagicha kiritamiz:

$$\|\varphi\|_\rho = max\left\{\sup_{(z,t)\epsilon D_T}|\varphi_i(t,\lambda,z)e^{-\rho t}|,(i=1,2),\ \sup_{t\epsilon[0,T]}|\varphi_3(t)e^{-\rho t}|\right\} \qquad (2.2.29)$$

$\rho = 0$ da bu fazo uzluksiz funksiyalar fazosi bo'ladi, uning normasini $\|\varphi\|$ deb belgilaymiz. Har qanday tayin $T\epsilon(0,\infty)$ uchun quyidagi tengsizlik o'rinli.

$$e^{-\rho T}\|\varphi\| \leq \|\varphi\|_\rho \leq \|\varphi\| \qquad (2.2.30)$$

$S_\rho(\varphi_0,R) -$ radiusi R, markazi φ_0 nuqtadagi yopiq shar, bu yerda:

$$\varphi_0 = [\varphi_{01},\varphi_{02},\varphi_{03}]^* = \left[\frac{i\lambda^3}{2}z, \frac{i\lambda^3}{4} + \frac{1}{2}r'(t-z,\lambda), \frac{\lambda^2 t}{4} + \frac{2i}{\lambda^3}r''(t,\lambda)\right]^*$$

$\varphi\epsilon S_\rho(\varphi_0,R)$ uchun quyidagi tengsizlik o'rinli:

$$\|\varphi\|_\rho \leq \|\varphi_0\|_\rho + R \leq \|\varphi_0\| + R \coloneqq R_0 \qquad (2.2.31)$$

$\varphi\epsilon S_\rho$ (φ_0,R) bo'lsin. $\rho > 0$ ni to'g'ri tanlaganda, A operator sharni o'zi-o'ziga akslantirishini ko'rsatamiz, ya'ni $A\varphi\epsilon S_\rho$ (φ_0,R). (2.2.18) tenglik yordamida

$(z,t)\epsilon D_T$ uchun $(A_i\varphi - \varphi_{0i})(i = 1,2,3)$ ayirma normalarini tuzib hisoblaymiz.

$$\|A_1\varphi - \varphi_{01}\|_\rho = \sup_{(z,t)\epsilon D_T}|(A_1\varphi - \varphi_{01})e^{-\rho t}| =$$

$$= \sup_{(z,t)\epsilon D_T}\left|\int_z^t\left(\varphi_2(\tau,\lambda,z) + \frac{i\lambda^3}{2}z\varphi_3(\tau - z)\right)e^{-\rho t}d\tau\right| =$$

$$= \sup_{(z,t)\epsilon D_T}\left|\int_z^t\left(\varphi_2(\tau,\lambda,z)e^{-\rho\tau}e^{-\rho(t-\tau)} + \frac{i\lambda^3}{2}z\varphi_3(\tau-z)e^{-\rho(\tau-z)}e^{-\rho(t-\tau+z)}\right)d\tau\right|$$

$$\leq \|\varphi_2\|_\rho \sup_{(z,t)\epsilon D_T}\left|\int_z^t e^{-\rho(t-\tau)}\,d\tau\right| + \|\varphi_3\|_\rho \sup_{(z,t)\epsilon D_T}\left|\int_z^t \frac{i\lambda^3}{2}ze^{-\rho(t-\tau+z)}\,d\tau\right| =$$

$$= \|\varphi_2\|_\rho \sup_{(z,t)\epsilon D_T} \left|\frac{1}{\rho} e^{-\rho(t-\tau)}|_z^t\right| + \|\varphi_3\|_\rho \sup_{(z,t)\epsilon D_T} \left|\frac{i\lambda^3}{2} z \frac{1}{\rho} e^{-\rho(t-\tau+z)}|_z^t\right| =$$

$$= \|\varphi_2\|_\rho \sup_{(z,t)\epsilon D_T} \left|\frac{1}{\rho}\left(1 - e^{-\rho(t-z)}\right)\right| + \frac{|i\lambda^3|}{2} \|\varphi_3\|_\rho \sup_{(z,t)\epsilon D_T} \left|\frac{1}{\rho} z e^{-\rho z}\left(1 - e^{-\rho(t-z)}\right)\right|$$

$$\leq \|\varphi_2\|_\rho \frac{1}{\rho} + \frac{|\lambda^3|}{2} \|\varphi_3\|_\rho \frac{T}{2}\frac{1}{\rho} \leq R_0 \left(1 + \frac{|\lambda^3|}{4} T\right)\frac{1}{\rho} \coloneqq \alpha_1 \frac{1}{\rho} \qquad (2.2.32)$$

$$\|A_2\varphi - \varphi_{02}\|_\rho = \sup_{(z,t)\epsilon D_T} |(A_2\varphi - \varphi_{02})e^{-\rho t}| =$$

$$= \sup_{(z,t)\epsilon D_T} |(\int_z^{\frac{z+t}{2}} (\frac{\lambda^2}{2}\varphi_1(t+z-\xi,\lambda,\xi) - \frac{i\lambda^3}{2}\varphi_3(t+z-2\xi) +$$

$$+\frac{\lambda^2}{2}\int_0^{t+z-2\xi} \varphi_3(\tau)\,\varphi_1(t+z-\xi-\tau,\lambda,\xi)d\tau)d\xi - \frac{\lambda^2}{2}\int_0^z (\,\varphi_1(\xi+t-z,\lambda,\xi) +$$

$$+\int_0^{t+z} \varphi_3(\tau)\,\varphi_1(\xi+t-z-\tau,\lambda,\xi)d\tau)\,d\xi)\,e^{-\rho t}| =$$

$$\sup_{(z,t)\epsilon D_T} |\int_z^{\frac{z+t}{2}} (\frac{\lambda^2}{2}\varphi_1(t+z-\xi,\lambda,\xi)\,e^{-\rho(t+z-\xi)}e^{-\rho(\xi-z)} - \frac{i\lambda^3}{2}\varphi_3(t+z-2\xi)$$

$$e^{-\rho(t+z-2\xi)}e^{-\rho(2\xi-z)} + \frac{\lambda^2}{2}\int_0^{t+z-2\xi} \varphi_3(\tau)e^{-\rho\tau}\,\varphi_1(t+z-\xi-\tau,\lambda,\xi)$$

$$e^{-\rho(t+z-\xi-\tau)}e^{-\rho(\xi-z)}d\tau)d\xi - \frac{\lambda^2}{2}\int_0^z (\,\varphi_1(\xi+t-z,\lambda,\xi)e^{-\rho(\xi+t-z)}e^{-\rho(z-\xi)} +$$

$$+\int_0^{t+z} \varphi_3(\tau)\,e^{-\rho\tau}\varphi_1(\xi+t-z-\tau,\lambda,\xi)e^{-\rho(\xi+t-z-\tau)}e^{-\rho(z-\xi)}d\tau)\,d\xi| \leq$$

$$\frac{\lambda^2}{2}\|\varphi_1\|_\rho \sup_{(z,t)\epsilon D_T} \left|\int_z^{\frac{z+t}{2}} e^{-\rho(\xi-z)}\,d\xi\right| + \frac{|i\lambda^3|}{2}\|\varphi_3\|_\rho \sup_{(z,t)\epsilon D_T} \left|\int_z^{\frac{z+t}{2}} e^{-\rho(2\xi-z)}\,d\xi\right| +$$

$$\frac{\lambda^2}{2}\|\varphi_3\|_\rho\|\varphi_1\|_\rho \sup_{(z,t)\epsilon D_T}\left|\int_z^{\frac{z+t}{2}}\int_0^{t+z-2\xi} e^{-\rho(\xi-z)}d\tau d\xi\right| +$$

$$\frac{\lambda^2}{2}\|\varphi_1\|_\rho \sup_{(z,t)\epsilon D_T}\left|\int_0^z e^{-\rho(z-\xi)}\, d\xi\right| +$$

$$+\frac{\lambda^2}{2}\|\varphi_3\|_\rho\|\varphi_1\|_\rho \sup_{(z,t)\epsilon D_T}\left|\int_0^z\int_0^{t+z} e^{-\rho(z-\xi)}d\tau d\xi\right| =$$

$$=\frac{\lambda^2}{2}\|\varphi_1\|_\rho \sup_{(z,t)\epsilon D_T}\left|-\frac{1}{\rho}e^{-\rho(\xi-z)}|_z^{\frac{z+t}{2}}\right| + \frac{|\lambda^3|}{2}\|\varphi_3\|_\rho \sup_{(z,t)\epsilon D_T}\left|-\frac{1}{2\rho}e^{-\rho(2\xi-z)}|_z^{\frac{z+t}{2}}\right| +$$

$$+\frac{\lambda^2}{2}\|\varphi_3\|_\rho\|\varphi_1\|_\rho \sup_{(z,t)\epsilon D_T}\left|\int_z^{\frac{z+t}{2}} e^{-\rho(\xi-z)}\,(t+z-2\xi)d\xi\right| +$$

$$+\frac{\lambda^2}{2}\|\varphi_1\|_\rho \sup_{(z,t)\epsilon D_T}\left|\frac{1}{\rho}e^{-\rho(z-\xi)}|_0^z\right| +$$

$$+\frac{\lambda^2}{2}\|\varphi_3\|_\rho\|\varphi_1\|_\rho \sup_{(z,t)\epsilon D_T}|\int_0^z e^{-\rho(z-\xi)}(\,t+z)d\xi| \leq$$

$$\leq \frac{\lambda^2}{2}\|\varphi_1\|_\rho \sup_{(z,t)\epsilon D_T}\left|-\frac{1}{\rho}(e^{-\rho\left(\frac{t-z}{2}\right)}-1)\right| + \frac{|\lambda^3|}{2}\|\varphi_3\|_\rho \sup_{(z,t)\epsilon D_T}\left|-\frac{1}{2\rho}(e^{-\rho t}-e^{-\rho z})\right|$$

$$+\frac{\lambda^2}{2}\|\varphi_3\|_\rho\|\varphi_1\|_\rho \sup_{(z,t)\epsilon D_T}\left|-\frac{(t-z)}{\rho}e^{-\rho(\xi-z)}|_z^{\frac{z+t}{2}}\right| +$$

$$+\frac{\lambda^2}{2}\|\varphi_1\|_\rho \sup_{(z,t)\epsilon D_T}\left|\frac{1}{\rho}(1-e^{-\rho z})\right| + \frac{\lambda^2}{2}\|\varphi_3\|_\rho\|\varphi_1\|_\rho \sup_{(z,t)\epsilon D_T}\left|\frac{(t+z)}{\rho}e^{-\rho(z-\xi)}|_0^z\right| =$$

$$=\frac{\lambda^2}{2}\|\varphi_1\|_\rho \sup_{(z,t)\epsilon D_T}\left|\frac{1}{\rho}(1-e^{-\rho\frac{t-z}{2}})\right| + \frac{|\lambda^3|}{2}\|\varphi_3\|_\rho \sup_{(z,t)\epsilon D_T}\left|\frac{1}{2\rho}e^{-\rho z}(1-e^{-\rho(t-z)})\right|$$

$$+\frac{\lambda^2}{2}\|\varphi_3\|_\rho\|\varphi_1\|_\rho \sup_{(z,t)\epsilon D_T}\left|\frac{(t-z)}{\rho}(1-e^{-\rho\left(\frac{t-z}{2}\right)})\right| +$$

$$+\frac{\lambda^2}{2}\|\varphi_1\|_\rho \sup_{(z,t)\epsilon D_T}\left|\frac{1}{\rho}(1-e^{-\rho z})\right| + \frac{\lambda^2}{2}\|\varphi_3\|_\rho\|\varphi_1\|_\rho \sup_{(z,t)\epsilon D_T}\left|\frac{(t+z)}{\rho}(1-e^{-\rho z})\right|$$

$$\leq \frac{\lambda^2}{2\rho}\|\varphi_1\|_\rho + \frac{|\lambda^3|}{2}\cdot\frac{1}{2\rho}\|\varphi_3\|_\rho + \frac{\lambda^2}{2}\cdot\frac{T}{2\rho}\|\varphi_3\|_\rho\|\varphi_1\|_\rho + \frac{\lambda^2}{2\rho}\|\varphi_1\|_\rho +$$

$$+\frac{\lambda^2}{2}\cdot\frac{3T}{2\rho}\|\varphi_3\|_\rho\|\varphi_1\|_\rho \leq R_0\left(\frac{\lambda^2}{2}+\frac{|\lambda^3|}{4}+\frac{\lambda^2}{4}TR_0+\frac{\lambda^2}{2}+\frac{3\lambda^2}{4}TR_0\right)\frac{1}{\rho} =$$

$$= R_0\left(\frac{4\lambda^2+|\lambda^3|+4\lambda^2 TR_0}{4}\right)\frac{1}{\rho} \coloneqq \alpha_2\frac{1}{\rho}. \qquad (2.2.33)$$

$$\|A_3\varphi-\varphi_{03}\|_\rho = \sup_{(z,t)\epsilon D_T}|(A_3\varphi-\varphi_{03})e^{-\rho t}| =$$

$$= \sup_{(z,t)\epsilon D_T}|(\int_0^{\frac{t}{2}} 2\lambda^2\xi\varphi_3(t-\xi)d\xi - \int_0^{\frac{t}{2}}\frac{2i}{\lambda}\varphi_2(t-\xi,\lambda,\xi)d\xi -$$

$$-\frac{2i}{\lambda}\int_0^{\frac{t}{2}}\int_0^{t-2\xi}\varphi_3(\tau)(\,\varphi_2(t-\xi-\tau,\lambda,\xi)+\frac{i\lambda^3}{2}\xi\varphi_3(t-\xi-\tau,\lambda,\xi))\,d\tau d\xi)e^{-\rho t}|$$

$$= \sup_{(z,t)\epsilon D_T}|\int_0^{\frac{t}{2}} 2\lambda^2\xi\varphi_3(t-\xi)e^{-\rho(t-}\xi)e^{-\rho\xi}d\xi - \int_0^{\frac{t}{2}}\frac{2i}{\lambda}\varphi_2(t-\xi,\lambda,\xi)$$

$$e^{-\rho(t-\xi)}e^{-\rho\xi}d\xi - \frac{2i}{\lambda}\int_0^{\frac{t}{2}}\int_0^{t-2\xi}\varphi_3(\tau)e^{-\rho\tau}\,\varphi_2(t-\xi-\tau,\lambda,\xi)e^{-\rho(t-\xi-\tau)}$$

$$e^{-\rho\xi}d\tau d\xi - \frac{2i}{\lambda}\int_0^{\frac{t}{2}}\int_0^{t-2\xi}\frac{i\lambda^3}{2}\xi\varphi_3(\tau)e^{-\rho\tau}\,\varphi_3(t-\xi-\tau,\lambda,\xi)e^{-\rho(t-\xi-\tau)}$$

$$e^{-\rho\xi}d\tau d\xi| \leq \|\varphi_3\|_\rho\sup_{(z,t)\epsilon D_T}|\int_0^{\frac{t}{2}} 2\lambda^2\xi\, e^{-\rho\xi}d\xi| + \|\varphi_2\|_\rho\sup_{(z,t)\epsilon D_T}|\int_0^{\frac{t}{2}}\frac{2i}{\lambda}e^{-\rho\xi}d\xi| +$$

$$\|\varphi_3\|_\rho\|\varphi_2\|_\rho\sup_{(z,t)\epsilon D_T}\left|\frac{2i}{\lambda}\int_0^{\frac{t}{2}}\int_0^{t-2\xi}e^{-\rho\xi}d\tau d\xi\right| +$$

$$\|\varphi_3\|_\rho\|\varphi_3\|_\rho\sup_{(z,t)\epsilon D_T}\left|\frac{2i}{\lambda}\cdot\frac{i\lambda^3}{2}\int_0^{\frac{t}{2}}\int_0^{t-2\xi}\xi e^{-\rho\xi}d\tau d\xi\right| =$$

$$= \|\varphi_3\|_\rho \sup_{(z,t)\epsilon D_T} \left| -2\lambda^2 \cdot \frac{t}{2} \cdot \frac{1}{\rho} e^{-\rho\xi} |_0^{\frac{t}{2}} \right| + \|\varphi_2\|_\rho \sup_{(z,t)\epsilon D_T} \left| -\frac{2i}{\lambda} \cdot \frac{1}{\rho} e^{-\rho\xi} |_0^{\frac{t}{2}} \right| +$$

$$+\|\varphi_3\|_\rho \|\varphi_2\|_\rho \sup_{(z,t)\epsilon D_T} \left| \frac{2i}{\lambda} \int_0^{\frac{t}{2}} e^{-\rho\xi} (t - 2\xi) d\xi \right| +$$

$$+\|\varphi_3\|_\rho \|\varphi_3\|_\rho \sup_{(z,t)\epsilon D_T} \left| -\lambda^2 \int_0^{\frac{t}{2}} \xi e^{-\rho\xi} (t - 2\xi) d\xi \right| =$$

$$= \|\varphi_3\|_\rho \sup_{(z,t)\epsilon D_T} \left| -\lambda^2 \frac{t}{\rho} (e^{-\rho\frac{t}{2}} - 1) \right| + \|\varphi_2\|_\rho \sup_{(z,t)\epsilon D_T} \left| -\frac{2i}{\lambda\rho} (e^{-\rho\frac{t}{2}} - 1) \right| +$$

$$+\|\varphi_3\|_\rho \|\varphi_2\|_\rho \sup_{(z,t)\epsilon D_T} \left| -\frac{2i}{\lambda} \cdot \frac{t}{\rho} e^{-\rho\xi} |_0^{\frac{t}{2}} \right| +$$

$$+\|\varphi_3\|_\rho \|\varphi_3\|_\rho \sup_{(z,t)\epsilon D_T} \left| \lambda^2 \cdot \frac{t}{2} \cdot \frac{t}{\rho} e^{-\rho\xi} |_0^{\frac{t}{2}} \right| =$$

$$= \|\varphi_3\|_\rho \sup_{(z,t)\epsilon D_T} \left| \lambda^2 \frac{t}{\rho} (1 - e^{-\rho\frac{t}{2}}) \right| + \|\varphi_2\|_\rho \sup_{(z,t)\epsilon D_T} \left| \frac{2i}{\lambda\rho} (1 - e^{-\rho\frac{t}{2}}) \right| +$$

$$+\|\varphi_3\|_\rho \|\varphi_2\|_\rho \sup_{(z,t)\epsilon D_T} \left| \frac{2i}{\lambda} \cdot \frac{t}{\rho} (1 - e^{-\rho\frac{t}{2}}) \right| +$$

$$+\|\varphi_3\|_\rho \|\varphi_3\|_\rho \sup_{(z,t)\epsilon D_T} \left| -\lambda^2 \frac{t^2}{2\rho} (1 - e^{-\rho\frac{t}{2}}) \right| \leq$$

$$\leq \lambda^2 \|\varphi_3\|_\rho \frac{T}{\rho} + \frac{2}{|\lambda|} \|\varphi_2\|_\rho \frac{1}{\rho} + \frac{2}{|\lambda|} \|\varphi_3\|_\rho \|\varphi_2\|_\rho \frac{T}{\rho} + \lambda^2 \|\varphi_3\|_\rho \|\varphi_3\|_\rho \frac{T^2}{2\rho} \leq$$

$$\leq R_0 \left(\lambda^2 T + \frac{2}{|\lambda|} + \frac{2}{|\lambda|} R_0 T + \lambda^2 \frac{R_0 T^2}{2} \right) \frac{1}{\rho} := \alpha_3 \frac{1}{\rho} \qquad (2.234)$$

$\rho \geq \alpha_0 \coloneqq (1/R) max\{\alpha_1, \alpha_2, \alpha_3\}$, ni tenglab A $S_\rho(\varphi_0, R)$ sharni $S_\rho(\varphi_0, R)$ sharga o‘tkazishini olamiz.

$\varphi^1, \varphi^2 - S_\rho(\varphi_0, R)$ shardagi ikkita ixtiyoriy elementlarni olamiz.

$$\left|\varphi_i^1\varphi_j^1-\varphi_i^2\varphi_j^2\right|e^{-\rho t}\le\left|\varphi_i^1\right|\left|\varphi_j^1-\varphi_j^2\right|e^{-\rho t}+\left|\varphi_j^2\right|\left|\varphi_i^1-\varphi_i^2\right|e^{-\rho t}\le$$

$$\le 2R_0\|\varphi^1-\varphi^2\|_\rho,\qquad (z,t)\in D_T \tag{2.2.35}$$

yordamchi tengliklardan foydalanib, quyidagi hisoblashlarni bajaramiz:

$$\|A\varphi^1-A\varphi^2\|_\rho=$$

$$=\max\left\{\sup_{(z,t)\epsilon D_T}|(A\varphi^1-A\varphi^2)_i e^{-\rho t}|\ (i=1,2),\ \sup_{t\epsilon[0,T]}|(A\varphi^1-A\varphi^2)_3 e^{-\rho t}|\ \right\},$$

normalarni alohida hisoblaymiz:

$$\|(A\varphi^1-A\varphi^2)_1\|_\rho=\sup_{(z,t)\epsilon D_T}|(A\varphi^1-A\varphi^2)_1 e^{-\rho t}|=$$

$$\sup_{(z,t)\epsilon D_T}\left|\int_z^t\left[\,(\varphi_2^1-\varphi_2^2)(\tau,\lambda,z)e^{-\rho\tau}e^{-\rho(t-\tau)}+\frac{i\lambda^3}{2}z(\varphi_3^1-\varphi_3^2)(\tau-z)e^{-\rho(\tau-z)}\times\right.\right.$$

$$\left.\left.\times\, e^{-\rho(t-\tau+z)}\right]d\tau\right|\le\ \|\varphi^1-\varphi^2\|_\rho\sup_{(z,t)\epsilon D_T}\left|\int_z^t e^{-\rho(t-\tau)}\,d\tau\right|+\frac{|\lambda^3|}{2}\|\varphi^1-\varphi^2\|_\rho\times$$

$$\times\sup_{(z,t)\epsilon D_T}\left|\int_z^t ze^{-\rho(t-\tau+z)}\,d\tau\right|=\|\varphi^1-\varphi^2\|_\rho\sup_{(z,t)\epsilon D_T}\left|\frac{1}{\rho}e^{-\rho(t-\tau)}\Big|_z^t\right|+$$

$$\frac{|\lambda^3|}{2}\|\varphi^1-\varphi^2\|_\rho\sup_{(z,t)\epsilon D_T}\left|\frac{z}{\rho}e^{-\rho(t-\tau+z)}\Big|_z^t\right|=\|\varphi^1-\varphi^2\|_\rho\sup_{(z,t)\epsilon D_T}\left|\frac{1}{\rho}(1-e^{-\rho(t-z)})\right|$$

$$+\frac{|\lambda^3|}{2}\|\varphi^1-\varphi^2\|_\rho\sup_{(z,t)\epsilon D_T}\left|\frac{z}{\rho}e^{-\rho z}(1-e^{-\rho(t-z)})\right|\le\frac{\|\varphi^1-\varphi^2\|_\rho}{\rho}+$$

$$+\frac{|\lambda^3|}{2}\cdot\frac{T}{2\rho}\|\varphi^1-\varphi^2\|_\rho=\left(1+\frac{|\lambda^3|}{4}T\right)\|\varphi^1-\varphi^2\|_\rho\frac{1}{\rho}:=$$

$$=\beta_1\|\varphi^1-\varphi^2\|_\rho\frac{1}{\rho}, \tag{2.2.36}$$

$$\|(A\varphi^1 - A\varphi^2)_2\|_\rho = \sup_{(z,t)\epsilon D_T} |(A\varphi^1 - A\varphi^2)_2 e^{-\rho t}| =$$

$$\sup_{(z,t)\epsilon D_T} |\int_z^{\frac{z+t}{2}} [\frac{\lambda^2}{2}(\varphi_1^1 - \varphi_1^2)(t+z-\xi,\lambda,\xi)e^{-\rho(t+z-\xi)}e^{-\rho(\xi-z)} - \frac{i\lambda^3}{2}\times$$

$$\times(\varphi_3^1 - \varphi_3^2)(t+z-2\xi)\, e^{-\rho(t+z-2\xi)}e^{-\rho(2\xi-z)} +$$

$$+\frac{\lambda^2}{2}\int_0^{t+z-2\xi}(\varphi_3^1\,(\tau)e^{-\rho\tau}(\varphi_1^1 - \varphi_1^2)(t+z-\xi-\tau,\lambda,\xi)e^{-\rho(t+z-\xi-\tau)}e^{-\rho(\xi-z)} +$$

$$+\varphi_1^2(t+z-\xi-\tau)e^{-\rho(t+z-\xi-\tau)}(\varphi_3^1 - \varphi_3^2)(\tau)e^{-\rho\tau}e^{-\rho(\xi-z)})d\tau]d\xi -$$

$$-\frac{\lambda^2}{2}\int_0^z [(\varphi_1^1 - \varphi_1^2)(\xi+t-z,\lambda,\xi)\, e^{-\rho(\xi+t-z)}e^{-\rho(z-\xi)} +$$

$$+\int_0^{t+z}(\,\varphi_3^1(\tau)e^{-\rho\tau}(\varphi_1^1 - \varphi_1^2)(\xi+t-z-\tau,\lambda,\xi)e^{-\rho(\xi+t-z-\tau)}e^{-\rho(z-\xi)} +$$

$$+\varphi_1^2(\xi+t-z-\tau,\lambda,\xi)e^{-\rho(\xi+t-z-\tau)}(\varphi_3^1 - \varphi_3^2)(\tau)e^{-\rho\tau}e^{-\rho(z-\xi)})d\tau]d\xi| \le$$

$$\le \frac{\lambda^2}{2}\|\varphi^1 - \varphi^2\|_\rho \sup_{(z,t)\epsilon D_T}\left|\int_z^{\frac{z+t}{2}} e^{-\rho(\xi-z)}d\xi\right| + \frac{|\lambda^3|}{2}\|\varphi^1 - \varphi^2\|_\rho \times$$

$$\times \sup_{(z,t)\epsilon D_T}\left|\int_z^{\frac{z+t}{2}} e^{-\rho(2\xi-z)}d\xi\right| + \frac{\lambda^2}{2}2R_0\|\varphi^1 - \varphi^2\|_\rho \times$$

$$\times \sup_{(z,t)\epsilon D_T}\left|\int_z^{\frac{z+t}{2}}\int_0^{t+z-2\xi} e^{-\rho(\xi-z)}d\tau d\xi\right| + \frac{\lambda^2}{2}\|\varphi^1 - \varphi^2\|_\rho \times$$

$$\times \sup_{(z,t)\epsilon D_T}\left|\int_0^z e^{-\rho(z-\xi)}d\xi\right| + \frac{\lambda^2}{2}2R_0\|\varphi^1 - \varphi^2\|_\rho \sup_{(z,t)\epsilon D_T}\left|\int_0^z\int_0^{t+z} e^{-\rho(\xi-z)}d\tau d\xi\right| =$$

$$= \frac{\lambda^2}{2}\|\varphi^1 - \varphi^2\|_\rho \sup_{(z,t)\epsilon D_T}\left|-\frac{1}{\rho}(e^{-\rho\frac{t-z}{2}} - 1)\right| + \frac{|\lambda^3|}{2}\|\varphi^1 - \varphi^2\|_\rho \times$$

$$\times \sup_{(z,t)\epsilon D_T}\left|-\frac{1}{2\rho}(e^{-\rho t} - e^{-\rho z})\right| + \lambda^2 R_0\|\varphi^1 - \varphi^2\|_\rho \times$$

$$\sup_{(z,t)\epsilon D_T}\left|\int_z^{\frac{z+t}{2}} e^{-\rho(\xi-z)}\ (t+z-2\xi)d\xi\right|+\frac{\lambda^2}{2}\|\varphi^1-\varphi^2\|_\rho \sup_{(z,t)\epsilon D_T}\left|\frac{1}{\rho}(1-e^{-\rho\frac{t-z}{2}})\right|$$

$$+\lambda^2 R_0\|\varphi^1-\varphi^2\|_\rho \sup_{(z,t)\epsilon D_T}\left|\int_0^z e^{-\rho(z-\xi)}\,(t+z)d\xi\right|\le\frac{\lambda^2}{2}\|\varphi^1-\varphi^2\|_\rho\times$$

$$\times\sup_{(z,t)\epsilon D_T}\left|\frac{1}{\rho}(1-e^{-\rho\frac{t-z}{2}})\right|+\frac{|\lambda^3|}{2}\|\varphi^1-\varphi^2\|_\rho \sup_{(z,t)\epsilon D_T}\left|\frac{1}{2\rho}e^{-\rho z}(1-e^{-\rho(t-z)})\right|+$$

$$+\lambda^2 R_0\|\varphi^1-\varphi^2\|_\rho \sup_{(z,t)\epsilon D_T}\left|\frac{(t-z)}{\rho}(1-e^{-\rho\frac{t-z}{2}})\right|+\frac{\lambda^2}{2}\|\varphi^1-\varphi^2\|_\rho\times$$

$$\times\sup_{(z,t)\epsilon D_T}\left|\frac{1}{\rho}(1-e^{-\rho\frac{t-z}{2}})\right|+\lambda^2 R_0\|\varphi^1-\varphi^2\|_\rho \sup_{(z,t)\epsilon D_T}\left|\frac{(t+z)}{\rho}(1-e^{-\rho z})\right|\le$$

$$\le\frac{\lambda^2}{2}\|\varphi^1-\varphi^2\|_\rho\frac{1}{\rho}+\frac{|\lambda^3|}{2}\|\varphi^1-\varphi^2\|_\rho\frac{1}{2\rho}+\lambda^2 R_0\|\varphi^1-\varphi^2\|_\rho\frac{T}{2\rho}+$$

$$+\frac{\lambda^2}{2}\|\varphi^1-\varphi^2\|_\rho\frac{1}{\rho}+\lambda^2 R_0\|\varphi^1-\varphi^2\|_\rho\frac{3T}{2\rho}=(\frac{\lambda^2}{2}+\frac{|\lambda^3|}{4}+\frac{\lambda^2 R_0 T}{2}+\frac{\lambda^2}{2}+$$

$$+\frac{3\lambda^2 R_0 T}{2})\|\varphi^1-\varphi^2\|_\rho\frac{1}{\rho}=\left(\frac{4\lambda^2+|\lambda^3|+8\lambda^2 R_0 T}{4}\right)\|\varphi^1-\varphi^2\|_\rho\frac{1}{\rho}\coloneqq$$

$$=\beta_2\|\varphi^1-\varphi^2\|_\rho\frac{1}{\rho}, \qquad (2.2.37)$$

$$\|(A\varphi^1-A\varphi^2)_3\|_\rho=\sup_{(z,t)\epsilon D_T}|(A\varphi^1-A\varphi^2)_2 e^{-\rho t}|=$$

$$\sup_{(z,t)\epsilon D_T}|\int_0^{\frac{t}{2}}[2\lambda^2\xi\,(\varphi_3^1-\varphi_3^2)(t-\xi)e^{-\rho(t-\xi)}e^{-\rho\xi}-\frac{2i}{\lambda}(\varphi_2^1-\varphi_2^2)(t-\xi,\lambda,\xi)\times$$

$$\times e^{-\rho(t-\xi)}e^{-\rho\xi}-\frac{2i}{\lambda}\int_0^{t-2\xi}(\varphi_3^1\,(\tau)e^{-\rho\tau}(\varphi_2^1-\varphi_2^2)(t-\xi-\tau,\lambda,\xi)e^{-\rho(t-\xi-\tau)}e^{-\rho\xi}$$

$$+\varphi_2^2(t-\xi-\tau,\lambda,\xi)e^{-\rho(t-\xi-\tau)}(\varphi_3^1-\varphi_3^2)(\tau)e^{-\rho\tau}e^{-\rho\xi})d\tau]d\xi+$$

$$+\lambda^2 \int_0^{\frac{t}{2}} \int_0^{t-2\xi} \xi \, (\varphi_3^1(\tau) e^{-\rho\tau} (\varphi_3^1 - \varphi_3^2)(t-\xi-\tau,\lambda,\xi) e^{-\rho(t-\xi-\tau)} e^{-\rho\xi} +$$

$$+\varphi_3^2(t-\xi-\tau,\lambda,\xi) e^{-\rho(t-\xi-\tau)} (\varphi_3^1 - \varphi_3^2)(\tau) e^{-\rho\tau} e^{-\rho\xi})| \le$$

$$\le 2\lambda^2 \|\varphi^1 - \varphi^2\|_\rho \sup_{(z,t)\epsilon D_T} \left| \int_0^{\frac{t}{2}} \xi e^{-\rho\xi} \, d\xi \right| + \frac{2}{|\lambda|} \|\varphi^1 - \varphi^2\|_\rho \sup_{(z,t)\epsilon D_T} \left| \int_0^{\frac{t}{2}} e^{-\rho\xi} \, d\xi \right| +$$

$$+\frac{2}{|\lambda|} 2R_0 \|\varphi^1 - \varphi^2\|_\rho \sup_{(z,t)\epsilon D_T} \left| \int_0^{\frac{t}{2}} \int_0^{t-2\xi} e^{-\rho\xi} \, d\tau d\xi \right| +$$

$$+2\lambda^2 R_0 \|\varphi^1 - \varphi^2\|_\rho \sup_{(z,t)\epsilon D_T} \left| \int_0^{\frac{t}{2}} \int_0^{t-2\xi} \xi e^{-\rho\xi} \, d\tau d\xi \right| \le$$

$$\le 2\lambda^2 \|\varphi^1 - \varphi^2\|_\rho \sup_{(z,t)\epsilon D_T} \left| -\frac{t}{2\rho} (e^{-\rho\frac{t}{2}} - 1) \right| + \frac{2}{|\lambda|} \|\varphi^1 - \varphi^2\|_\rho \times$$

$$\times \sup_{(z,t)\epsilon D_T} \left| -\frac{1}{\rho} (e^{-\rho\frac{t}{2}} - 1) \right| + \frac{4R_0}{|\lambda|} \|\varphi^1 - \varphi^2\|_\rho \sup_{(z,t)\epsilon D_T} \left| \int_0^{\frac{t}{2}} e^{-\rho\xi} (t-2\xi) d\xi \right| +$$

$$+2\lambda^2 R_0 \|\varphi^1 - \varphi^2\|_\rho \sup_{(z,t)\epsilon D_T} \left| \int_0^{\frac{t}{2}} \xi e^{-\rho\xi} (t-2\xi) d\xi \right| \le$$

$$\le 2\lambda^2 \|\varphi^1 - \varphi^2\|_\rho \sup_{(z,t)\epsilon D_T} \left| \frac{t}{2\rho} (1 - e^{-\rho\frac{t}{2}}) \right| + \frac{2}{|\lambda|} \|\varphi^1 - \varphi^2\|_\rho \times$$

$$\times \sup_{(z,t)\epsilon D_T} \left| \frac{1}{\rho} (1 - e^{-\rho\frac{t}{2}}) \right| + \frac{4R_0}{|\lambda|} \|\varphi^1 - \varphi^2\|_\rho \sup_{(z,t)\epsilon D_T} \left| \frac{t}{\rho} (1 - e^{-\rho\frac{t}{2}}) \right| +$$

$$+2\lambda^2 R_0 \|\varphi^1 - \varphi^2\|_\rho \sup_{(z,t)\epsilon D_T} \left| \frac{t}{2} \cdot \frac{t}{\rho} (1 - e^{-\rho\frac{t}{2}}) \right| \le$$

$$\le 2\lambda^2 \|\varphi^1 - \varphi^2\|_\rho \frac{T}{2\rho} + \frac{2}{|\lambda|} \|\varphi^1 - \varphi^2\|_\rho \frac{1}{\rho} + \frac{4R_0}{|\lambda|} \|\varphi^1 - \varphi^2\|_\rho \frac{T}{\rho} +$$

$$+2\lambda^2 R_0 \|\varphi^1 - \varphi^2\|_\rho \frac{T^2}{2\rho} = \left(\lambda^2 T + \frac{2}{|\lambda|} + \frac{4R_0 T}{|\lambda|} + \lambda^2 R_0 T^2 \right) \|\varphi^1 - \varphi^2\|_\rho \frac{1}{\rho} \coloneqq$$

$$= \beta_3 \|\varphi^1 - \varphi^2\|_\rho \frac{1}{\rho}. \qquad (2.2.38)$$

$\beta_0 := max\{\beta_1, \beta_2, \beta_3\}$ bo‘lsin. Agar ρ son $\rho > max\{\alpha_0, \beta_0\}$, shartdan tanlangan bo‘lsa, u holda A operator $S_\rho(\varphi_0, R)$ shar uchun siquvchi operator bo‘ladi. U holda Banax prinsipiga asosan, (2.2.28) tenglama ixtiyoriy tayin $T > 0$ uchun $S_\rho(\varphi_0, R)$ sharda yagona yechimga ega. 2.2.1-teorema isbotlandi.

Xulosa.

Bitiruv malakaviy ishining ikkinchi bobi Kuchsiz gorizontal muhitda xotirali tenglama uchun teskari masala deb nomlangan bo‘lib, u ikki paragrafdan iborat.

Birinchi paragrafda Dirakning del’ta funksiyasi va uning xossalari, umumlashgan funksiyalar, siqiluvchan akslantirishlar, Xevisayda funksiyalari haqida umumiy tushunchalar keltirib o‘tilgan. Bundan tashqari birinchi bo‘limda teskari masalaning qo‘yilishi ham berib o‘tilgan.

Ikkinchi paragrafda yuqorida kiritilgan tushunchalardan foydalangan holda kuchsiz gorizontal muhitda xotirali tenglama uchun teskari masalaning $K_0(t)$ yechimini topish bilan shug‘ullanilgan. Bu bo‘limda bir teorema va bir lemma keltirilgan bo‘lib, ularni isbotlash jarayonida $K_0(t)$ yechi aniqlangan.

XOTIMA.

Bitiruv malakaviy ishi Kuchsiz gorizontal muhitda xotirali tenglama uchun teskari masala deb nomlangan bo‘lib ikki bobdan iborat.

Bitiruv malakaviy ishining birinchi bobi Xususiy hosilali differensial tenglamalar va integral tenglamalar deb nomlangan bo‘lib u ikki paragrafdan iborat.

Birinchi paragrafda xususiy hosilali differensial tenglamalarning yechimi, klassifikatsiyasi va kanonik ko‘rinishi, tor tebranish tenglamasi uchun Dalamber formulasi haqida ma’lumotlar keltirilgan. Ikkinchi bo‘limda integral va integro-differensial tenglamalar haqida umumiy tushunchalar keltirilgan.

Bitiruv malakaviy ishining ikkinchi bobi Kuchsiz gorizontal muhitda xotirali tenglama uchun teskari masala deb nomlangan bo‘lib, u ikki bo‘limdan iborat.

Birinchi bo‘limda Dirakning del’ta funksiyasi va uning xossalari, umumlashgan funksiyalar, siqiluvchan akslantirishlar, Xevisayda funksiyalari haqida umumiy tushunchalar keltirib o‘tilgan. Bundan tashqari birinchi bo‘limda teskari masalaning qo‘yilishi ham berib o‘tilgan.

Ikkinchi bo‘limda yuqorida kiritilgan tushunchalardan foydalangan holda Kuchsiz gorizontal muhitda xotirali tenglama uchun teskari masalaning $K_0(t)$ yechimini topish bilan shug‘ullanilgan. Bu bo‘limda bir teorema va bir lemma keltirilgan bo‘lib, ularni isbotlash jarayonida $K_0(t)$ yechim aniqlangan.

Ushbu bitiruv malakaviy ishida ko‘rilgan tenglama uchun olingan natijalar yangi hisoblanadi. Ular ushbu sohadagi keyingi tadqiqotlarda ishlatilishi mumkin.

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