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THE SOLUTION OF NONLINEAR PROGRAMMING PROBLEMS IN A PROJECTION-DIFFERENTIAL METHOD

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Summary. *In this article differential descent method made for solving non-linear equation systems is generalised for solving non-linear programming problems (N-lP). For this N-lP problems are exchanged with the sequence of the unconstrained extremum problems by using the crowd and fine functions. To solve the last problem the projection-differential descent method is used.*

Key words. *Equation, function, iteration, projection, differential, extremum, local minimum, unconstrained minimum.*

Introduction. From the second half of 50-years of the 20th century the rapid development of mathematical programming informs that the optimization problems are of great importance in various fields of human services.

On solving the practical problems especially the linear programming problems are often met. So, for these problems many highly effective exact (right) and integration methods are made [1,4,5].

Nowadays, the main attention according to the practice requirements, is being paid to the development of methods of solving the general problems of non-linear programming [1,4,7]. The algorithms of solving these problems are being formed in the establishing directions two different methods of organizing the searching the constrained extremum of non-linear functions.

In the first of these directions, the subject directly controls the constraints of the subject and performs the sequence of possible or nearly impossible points to the optimal point of the target function. In this case, the purpose of the target function is to produce a sequence of monotony drops or monotonous increments, i.e. a minimized sequence of target functions, at each step of the reproduction process, is brought to solve a very simple problem with linear programming problem solution. This kind of solving algorithms is called descent method. They include gradient projection, meeting methods and many variants of possible direction methods.

In the second direction the constrained extremum will be brought to the sequence of non-constrained extremum of support (miscellaneous) functions made in special method [4,5,7]. These support functions are called fine functions and the used algorithms are methods of fine functions [4,7]. This group include barrier functions and fine function methods, centers and methods of Lagrangian generalized functions and etc [4,5,7].

Last methods differ from descent method with their comfortability of performing and character of high convergence. Therefore, in producing these methods and developing overall nowadays much force is being spent.

At the same time, optimization in the process of solving various practical issues has been solved by the methods of solving the problem of non-constrained extremum problems, individually, main defects of fine function methods were opened: these methods were determined to be useless for solving the non-linear programming problems with high clearness.[4,7]. Because using the big or small meaning of fine coefficient brings to find the local minimums of support functions, which its degree lines outstretched, i.e. is linear. This condition complicates the completing the calculation. So, If it is necessary to determine the non-linear programming problem solving with high accuracy, then it is best to receive the found solution in according of primary approximating and use other quick-convergence methods.

These methods include iteration methods based on projection-differential descent methods.[2,5,7]. Accordingly, the using problems the iteration methods made in the basis of projection-differential descent methods in solving the non-linear programming problems put in general will be discussed and researched. In its turn, in the basis of this generalizing the mentioned method comes turning methods in the sequence of non-constrained optimization problems by using support fine functions from constrained extremum problems. Last method, given constrained extremum problems are used to bring to solve the sequence of systems of non-linear equation, which is equally strong. To solve the sequence of systems of non-linear equation projection-differential descent methods are used. [5,7].

1-task. Solve the following non-linear programming problem.

$$\begin{aligned} &f(x)=0.5(x_1+x_2)^2+50(x_2-x_1)^2+x_3^2 \to \min, \\ &\varphi_1(x)=-\left[(x_1-1)^2+(x_2-1)^2+(x_3-1)^2-1.5\right]\geq 0, \\ &\varphi_2(x)=\sin(x_1+x_2)-x_3=0 \end{aligned} \qquad (1)$$

In the given task because of having different limitation in the kind of equality and inequality we bring the constrained minimum of following support function to the sequence-finding task:

$$P(x,r)=f(x)+\frac{1}{2r}\left\{\min\left[0,\varphi_1(x)\right]^2\right\}+\frac{1}{r}\left[\varphi_2(x)\right]^2 \qquad (2)$$

This function will have continuous derivate in necessary order. So, to determine the local minimum points we use its first and second ordinal derivate. By using the requirement of local minimum of (2) function, we decide to the following non-linear equation system:

$$f_1(x)=101x_1-99x_2+\frac{2}{r}\varphi_1(x)(x_1-1)+\frac{2}{r}\varphi_2(x)\cos(x_1+x_2)=0,$$
$$f_2(x)=101x_2-99x_1+\frac{2}{r}\varphi_1(x)(x_2-1)+\frac{2}{r}\varphi_2(x)\cos(x_1+x_2)=0, \quad (3)$$
$$f_3(x)=2x_3+\frac{2}{r}\varphi_1(x)(x_3-1)-\frac{2}{r}\varphi_2(x)=0,$$

The meaning of qualifier r parameter in the equation system (3) was determined by the formula $r_k=r_{k-1}\cdot 10^{k-1},\ k=1,2,...(r_0=1)$ and solved by using the following (4) algorithm of projection-differential descent methods:

$$x_i^{(k+1)}=x_i^{(k)}-\frac{a_i^{i-1}(x_i^{(k)})h_i}{(f_i'(x_i^{(k)}),a_i^{i-1}(x_i^{(k)}))}, \quad i=1,2,..,n;k=0,1,2,.. \quad (4)$$

Here :

$h_i=f_i(x_i^{(k)})$

$a_i^j(x)=a_i^{j-1}(x)-a_j^{j-1}\frac{(f_j',a_i^{j-1})}{(f_j',a_j^{j-1})},\ j<i,\ i,j$ - Integers.

$a_i^0=f_i'\in E^n,\ i=1,2,...$

To the solution, $x_0=(-1,4,5)$ point was taken in according to primary approximating.

After completing the 15 iteration following approximate solution was taken:

$\bar{x}^{(15)}=(0.2290,\ 0.2290,\ 0.4422);\ f_1(\bar{x}^{(15)})=3.2656\cdot 10^{-14};$

$f_2(\bar{x}^{(15)})=3.2655\cdot 10^{-14};\ f_3(\bar{x}^{(15)})=-0.3447\cdot 10^{-13};$

$\varphi_1(\bar{x}^{(15)})=4.6233\cdot 10^{-9};\ \varphi_2(\bar{x}^{(15)})=3.6432\cdot 10^{-9};\ \min f(\bar{x}^{(15)})=0.3004;$

(results are rounded after comma till 4 tens symbols)

This solved task is for ensnare many definite algorithms, i.e. this is specially thought to bring the non-optimal stationary point [7]. Actually, it was like this. For example, with simple descent methods and Newton's methods this task wasn't solved. If it was $x_0=(-3,-1,-3)$, these methods were convergence to the point $\bar{x}=(-1.07,1.16,-0.86)$, the meaning $f(\bar{x})=7.89$ was taken.[7]

2-task. Solve this non-linear programming problem:

$$f(x)=e^{x_1x_2x_3x_4x_5}-0.5(x_1^3+x_2^3+1)^2\to\min.$$
$$\varphi_1(x)=x_1^2+x_2^2+x_3^2+x_4^2+x_5^2-10=0,$$
$$\varphi_2(\bar{x})=x_2x_3-5x_4x_5=0, \quad (5)$$
$$\varphi_3(\bar{x})=x_1^3+x_2^3+1=0$$

To bring this given problem to the sequence of non-constrained minimum problems we use following support function:

$$P(x,r)=f(x)+\frac{1}{2r}\left\{[\varphi_1(x)]^2+[\varphi_2(x)]^2+[\varphi_3(x)]^2\right\} \qquad (6)$$

We have the following system of non-linear equations from the first ordinal requirements of the local minimum of this function.

$$\begin{aligned}
&f_1(x)=x_2x_3x_4x_5e^{x_1x_2x_3x_4x_5}-3x_1^2(x_1^3+x_2^3+1)+\frac{1}{r}\left[2\varphi_1(x)\cdot x_1+3\varphi_3(x)\cdot x_1^2\right]=0,\\
&f_2(x)=x_1x_3x_4x_5e^{x_1x_2x_3x_4x_5}-3x_2^2(x_1^3+x_2^3+1)+\\
&+\frac{1}{r}\left[2\varphi_1(x)\cdot x_2+\varphi_2(x)\cdot x_2+3\varphi_3(x)\cdot x_2^2\right]=0,\\
&f_3(x)=x_1x_2x_4x_5e^{x_1x_2x_3x_4x_5}+\frac{1}{r}\left[2\varphi_1(x)\cdot x_3+\varphi_3(x)\cdot x_2\right]=0,\\
&f_4(x)=x_1x_2x_3x_5e^{x_1x_2x_3x_4x_5}+\frac{1}{r}\left[2\varphi_1(x)\cdot x_4-5\varphi_2(x)\cdot x_5\right]=0,\\
&f_5(x)=x_1x_2x_3x_4e^{x_1x_2x_3x_4x_5}+\frac{1}{r}\left[2\varphi_1(x)\cdot x_5-5\varphi_2(x)\cdot x_4\right]=0
\end{aligned} \qquad (7)$$

(7) Non-linear equation system $x_0=(-2;2;2;-1;-1)$ is solved by primary approximating; the meaning of r parameter is determined by the formula $r_k=10^{k-1},\ k=1,2,...(r_0=1)$, by using the algorithm of (4) the projection-differential descent method. The results taken from 10-iteration of iteration process are rounded after comma until 4 tens symbols:

$\bar{x}^{(10)}=(1,7171;\ 1,5957;\ 1,8273;\ -0,7636;\ -0,7636);$

$f_1(\bar{x}^{(10)})=-4.4597\cdot 10^{-12};\ f_2(\bar{x}^{(10)})=2.3799\cdot 10^{-12};\ f_3(\bar{x}^{(10)})=3.7485\cdot 10^{-12};$

$f_4(\bar{x}^{(10)})=-1.1439\cdot 10^{-13};\ f_5(\bar{x}^{(10)})=-1.1298\cdot 10^{-13};$

$\varphi_1(\bar{x}^{(10)})=5\cdot 10^{-12};\ \varphi_2(\bar{x}^{(10)})=-3.947\cdot 10^{-7};\varphi_3(\bar{x}^{(10)})=5,7\cdot 10^{-8};f(\bar{x}^{(10)})=0.05395$

Having solved this problem with the scratch method in the [6] book, after 32 iterations these results are taken:

$x_0=(-2;2;2;-1;-1);\ \bar{x}=(-1.7171;1.596;1.827;-0.7636;-0.7636);\ f(\bar{x})=0.0595;$

Comparing the results taken by scratch method with the results taken by projection-differential descent method again claims the first method as a most effective.

Conclusions:

So, we tell some advantages and differences of this method from others.

1. In projection-differential descent methods the solution $x_i(t_i)$ of recurrent sequence of Cauchy problems is determined in the finite $[t_i,0]$ meet and first integrals of these problems are definite [5,7]. Therefore, the reliability of the descent method on system solutions (3),(7) which is compatible with solutions of problems (1),(5), will be monitored at each stage of the iteration process.

2. If many iteration methods (e.g. simple iteration method, Newton method, Aitken – Stevenson method and etc. [2,3]) are used in solving the (3),(7) systems of non-linear equation, determining the primary approximating which is closer to the solution will be demanded. Because the convergence of iteration method is depended on convenient choosing the primary approximating. This problem was solved in high degree by the called projection-differential descent method, i.e. convergence area of iteration methods based on this method is comparatively wide.

3. When using the usual counting algorithms of descent methods, finding the minimum point of an indefinite function is demanded in each step. Solving this problem is demanded the completing much counting works in condition of non-linear equation. On we mentioned method this kind of extra counting works are not demanded and its algorithm is convenient in establishing on computers.

4. This method is not convergent, when minimizing some of the real $\varphi(x) \geq 0$ functionality, while reducing iterative processes to its local minimum. However, on above mentioned projection-differential descent method the connection with variation problem of equation systems (3) is not used. Therefore, this disadvantage of descent method is not met in we mentioned method.

5. The theory of the solution of complex problems is confirmed by the results of its application to practical problems in the workplace. From this point of view, the above-mentioned, rather complex, non-linear programming problem solving results prove once again that the method of projection-differential descent method is a highly effective method.

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Rezyume. *Mazkur maqolada chiziqli emas tenglamalar sistemalarini echish uchun ishlab chiqilgan differentsial pasga tushish usuli chiziqli emas dasturlash (ChED) masalalarini echish uchun umumiylashtiriladi. Buning uchun ChED masalalari, irkinish va shtraf funksiyalaridan foydalanib, shartsiz ekstremum masalalarining ketma-ketligi bilan almashtiriladi. Songi masalani echish uchun proektsion-differentsial pasga tushish usuli qo'llaniladi.*

Резюме. *В данной статье обобщен метод дифференциального спуска, предназначенный для решения систем нелинейных уравнений, для решения задач нелинейного программирования (НеЛП). Для этого задачи НеЛП обмениваются* *с последовательностью неограниченных экстремальных задач с использованием функций толпы и точных функций. Для решения последней задачи используется проекционно-дифференциальный метод спуска.*

Kalit so'zlar. *Tenglama, funksiya, iteratsiya, proektsion, differentsial, ekstremum, lokal minimum, shartsiz minimum.*

Ключевые слова. *Уравнение, функция, итерация, проекция, дифференциал, экстремум, локальный минимум, безусловный минимум.*