O`zbekiston Respublikasi Oliy va O`rta maxsus ta'lim vazirligi

Jizzax politexnika instituti

«Oliy matematika» kafedrasi.

Texnika yo`nalishi bo`yicha ta'lim olayotgan I-kurs talabalari uchun mustaqil ta'lim darsi bo`yicha uslubiy qo`llanma.

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UDK 517 (2-3) oliy matemaika

Ushbu uslubiy qo`llanma “Oily matematika” kafedrasining 2005 yil “__”_____ dagi № ___ -sonli yig`ilishida tasdiqlandi.

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Ushbu uslubiy qo`llanma Institut ilmiy kengashining 2005 yil “___” dekabrdagi № ___ -sonli yig`ilishida tasdiqlandi.

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Ushbu uslubiy qo`llanma texnika `nalishi bo`yicha ta'lim olayotgan 1-kurs talabalari uchun oliy matematikaning Chiziqli algebra qismi bo`yicha yozilgan bo`lib, talabaning mustaqil ishi darsi uchun mo`ljallangan. Oliy matematikaning bu qismi uchun 12 soatlik auditoriya darsi o`tiladi va kurs nihoyasida talabalar reyting nazorati tizimi bo`yicha reyting ballari bilan baholanadi.

Ushbu qo`llanma «Chiziqli algebra»ning asosiy tuchunchalari bo`lgan matritsa, determinant va ularning Chiziqli tenglamalar sistemalarini yechishga qo'llash mavzulari kiritilgan bo`lib, har bir qism bo`yicha namunaviy misollar yetarli darajada sodda, tuchunarli uslubda yechib ko`rsatilgan. Shu bilan birgalikda har bir talabaning mustaqil bajarishi uchun nazariy savollar va amaliy mashqlar keltirilgan. Talaba oliy matematikaning bu bo'limi o`quv soatlari yakunida qismning nazariy qismi bo`yicha referat, amaliy qismi bo`yicha hisob ishlarini topshiradi va to`plagan baliga ko`ra ushbu bo'limini uning «o`zlashtirganligi» yoki «o`zlashtirmaganligi» talaba mustaqil ishini tashkil etish, nazorat qilish va baholash tartibi to`g`risidagi nizomga ko`ra aniqlanadi.

1-Mustaqil ish darsi.

Mavzu : Matritsa tuchunchasi. Matritsalar ustida bajariladigan amallar**.**

Matritsa tuchunchasi, Chiziqli algebraning asosiy tuchunchalaridan biri bo`lib, uning talaba tomonidan chuqur o`zlashtirilishini muhim ahamiyatga ega. Chunki, bu tuchunchaning tatbiqlari zamonaviy ishlab chiqarishdagi muhim iqtisodiy, texnikaviy masalalarni yechishda keng qo`llaniladi.

Ta'rif; Quyidagi m ta satr va n ta ustundan iborat bo`lgan va to`g`ri to`rtburchak shaklidagi sonlar jadvali:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{33} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{pmatrix} \quad \text{yoki} \quad \begin{Vmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{33} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{Vmatrix} \qquad (1)$$

m satrli va n ustunli matritsa deyiladi.

(1) ni tashkil etuvchi a_{ij} lar uning elementlari deyiladi. i- satr nomeri j-ustun nomerini bildiradi.

Masalan: a_{34}- bu 3-satr va 4-ustun elementlari kesishuvida joylashgan elementdir.

Ta'rif 2. Agar matritsa n ta satr va n ta ustundan iborat bo`lsa, u holda u n-tartibli ***kvadrat matritsa*** deyiladi.

n-tartibli kvadrat matritsaning umumiy ko`rinishi quyidagichadir:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{33} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{pmatrix} \qquad (2)$$

Matritsalar umumiy holda lotin alifbosining bosh harflari: A, B, C va hokazolar bilan belgilanadi.

Masalan:

$$A = \begin{pmatrix} 3 & 8 & 4 \\ -2 & 1 & 5 \end{pmatrix}, \qquad B = \begin{pmatrix} -1 & 2 & 4 \\ 5 & 6 & 7 \\ 3 & 0 & 1 \end{pmatrix}$$

Ta'rif3 . $a_{11}, a_{22} \ldots . a_{nn}$ elementlar (2) ko`rinishdagi kvadrat matritsaning asosiy dioganal elementlari deyiladi (dioganal yuqoridan pastga qarab yo`naladi).

Ta'rif 4. Agar (2) kvadrat matritsa elementlari orasida $a_{ij} = b_{ji}$ munosabat o`rinli bo`lsa, u ***simmetrik matritsa*** deyiladi.

Masalan:

$$A=\begin{pmatrix} 2 & 3 & -1 \\ 3 & 5 & 8 \\ -1 & 8 & 4 \end{pmatrix}$$

Ta'rif 5. Agar (2) kvadrat matritsada asosiy diagonal elementlari 1 ga teng, qolgan barcha elementlari 0 dan iborat bo`lsa, u holda bunday matritsa ***birlik matritsa*** deb ataladi.

$$E=\begin{pmatrix} 1 & 0 & 0\ldots\ldots 0 \\ 0 & 1 & 0\ldots\ldots 0 \\ \multicolumn{3}{c}{----------} \\ 0 & 0 & 0\ldots\ldots 1 \end{pmatrix}$$

Ta'rif 6. Barcha elementlari nol lardan iborat bo`lgan matritsa ***nol matritsa*** deb ataladi.

Agar ikkita A va B matritsalarda barcha mos elementlar o`zaro teng bo`lsa, ya'ni $a_{ij}=b_{ij}$; u holda ular ***o'zaro teng*** deyiladi.

$$A=B$$

2- Mustaqil ish darsi.

Mavzu: Matritsalarni qo`shish va songa ko`paytirish.

m ta satrdan va n ta ustundan iborat bo`lgan

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{33} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{pmatrix}, \quad B=\begin{pmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1n} \\ b_{21} & b_{22} & b_{33} & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ b_{m1} & b_{m2} & b_{m3} & \cdots & a_{mn} \end{pmatrix}$$

matritsalar berilgan bo`lsin.

A va B matritsalarning yig'indisi shunday C matritsadan iboratki, uning elementlari $s_{ij} = a_{ij} + b_{ij}$ orqali topiladi.

Masalan:

$$A=\begin{pmatrix} 1 & 3 & -4 \\ -2 & 5 & 3 \end{pmatrix} \text{ va } B=\begin{pmatrix} -2 & 4 & 1 \\ 3 & 5 & 2 \end{pmatrix}$$

$$A+B=\begin{pmatrix} -1 & 7 & -3 \\ 1 & 10 & 5 \end{pmatrix}$$

Matritsalarni ayirish amali ham xuddi qo`shish kabi bajariladi.

Matritsani songa ko`paytirish uchun uning har bir elementini shu songa ko`paytirish kifoyadir.

Masalan:

$$A=\begin{pmatrix} 3 & 4 & 1 \\ -2 & 0 & 5 \end{pmatrix} \qquad \alpha = 2$$

$$2A=\begin{pmatrix} 6 & 8 & 2 \\ -4 & 0 & 10 \end{pmatrix}$$

Bizga

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1k} \\ a_{21} & a_{22} & a_{33} & \cdots & a_{2k} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mk} \end{pmatrix}, \qquad B=\begin{pmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1n} \\ b_{21} & b_{22} & b_{33} & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ b_{к1} & b_{к2} & b_{к3} & \cdots & a_{кn} \end{pmatrix}$$

matritsalar berilgan bo`lsin. A matritsaning ustunlari soni, B matritsaning satrlari soniga teng bo`lsin. A va B matritsaning ko`paytmasi deb, elementlari

$S_{ij} = a_{i1}\ b_{1j} + a_{i2}\ b_{2j} + \ldots + a_{in}\ b_{nj} = \sum_{S=1}^{m} a_{is}\, b_{sj}$ formula bilan aniqlanadigan S matritsaga aytiladi.

Masalan:

$$1)\quad A=\begin{pmatrix} 1 & 2 & 3 \\ -4 & 5 & 1 \end{pmatrix};\ B=\begin{pmatrix} 1 & 2 & -1 \\ 3 & 4 & 1 \\ 5 & 1 & 2 \end{pmatrix}$$

$$C=A*B=\begin{pmatrix} 1+6+15 & 2+8+3 & -1+2+6 \\ -4+15+5 & -8+20+1 & 4+5+2 \end{pmatrix}=\begin{pmatrix} 15 & 13 & 7 \\ 16 & 2 & 11 \end{pmatrix}$$

$$2)\quad A=\begin{pmatrix} 2 & 1 & -1 \\ 0 & 3 & 5 \end{pmatrix};\ B=\begin{pmatrix} 1 & 2 \\ 1 & -2 \\ 1 & 3 \end{pmatrix}$$

$$C=A*B=\begin{pmatrix} 2+1-1 & 4-2-3 \\ 0+3+5 & 0-6+15 \end{pmatrix}=\begin{pmatrix} 2 & -1 \\ 8 & 9 \end{pmatrix}$$

$$3)\ A=\begin{pmatrix} 1 & 3 & 2 & -1 \\ 5 & -2 & 6 & 2 \\ -3 & 1 & 0 & 4 \end{pmatrix};\ B=\begin{pmatrix} 5 & 3 \\ 1 & 0 \\ 2 & -1 \\ 3 & 4 \end{pmatrix}$$

$$A*B=\begin{pmatrix} 5+3+4-3 & 3-2-4 \\ 25-1+12+6 & 15-6+8 \\ -15+1+0+12 & -9+16 \end{pmatrix}=\begin{pmatrix} 9 & -3 \\ 42 & 17 \\ -2 & 7 \end{pmatrix}$$

A va B matritsalar berilgan. a) A* B; b) B* A ko`paytmalar topilsin.

1. $A=\begin{pmatrix}1&2&3\\4&1&-7\\5&4&-2\end{pmatrix}$ va $B=\begin{pmatrix}-1&-3&2\\3&1&-3\\-2&-2&-3\end{pmatrix}$

2. $A=\begin{pmatrix}1&3&5\\-1&2&3\\-3&-1&1\end{pmatrix}$ va $B=\begin{pmatrix}2&-3&3\\2&-3&1\\3&-2&3\end{pmatrix}$

3. $A=\begin{pmatrix}3&-2&-1\\1&-2&1\\2&1&2\end{pmatrix}$ va $B=\begin{pmatrix}1&1&2\\2&-2&-1\\1&2&2\end{pmatrix}$

3. $A=\begin{pmatrix}3&2&3\\1&-1&0\\-1&2&1\end{pmatrix}$ va $B=\begin{pmatrix}4&-3&2\\2&1&-2\\-2&1&1\end{pmatrix}$

4. $A=\begin{pmatrix}1&2&3\\-2&3&-1\\-2&-4&1\end{pmatrix}$ va $B=\begin{pmatrix}1&-1&-1\\2&1&-1\\-1&1&-4\end{pmatrix}$

5. $A=\begin{pmatrix}-3&5&0\\-2&1&2\\4&-2&-1\end{pmatrix}$ va $B=\begin{pmatrix}3&0&2\\1&-1&3\\4&-1&1\end{pmatrix}$

6. $A=\begin{pmatrix}3&1&2\\0&3&-1\\4&4&1\end{pmatrix}$ va $B=\begin{pmatrix}-1&0&-1\\-1&1&-1\\2&-1&-12\end{pmatrix}$

7. $A=\begin{pmatrix}2&3&-1\\1&2&2\\5&0&-1\end{pmatrix}$ va $B=\begin{pmatrix}-4&1&-1\\2&-4&0\\3&1&-1\end{pmatrix}$

8. $A=\begin{pmatrix}0&-1&2\\-2&1&5\\5&1&-1\end{pmatrix}$ va $B=\begin{pmatrix}1&2&1\\-1&3&1\\-1&2&-3\end{pmatrix}$

9. $A=\begin{pmatrix}2&0&-3\\-4&2&0\\1&3&2\end{pmatrix}$ va $B=\begin{pmatrix}-3&-2&0\\3&-2&1\\1&-1&2\end{pmatrix}$

10. $A=\begin{pmatrix}1&-3&0\\2&2&3\\-11&2&-1\end{pmatrix}$ va $B=\begin{pmatrix}4&3&2\\-2&-6&-3\\-3&2&1\end{pmatrix}$

11. $A=\begin{pmatrix}4&1&2\\-3&2&1\\2&2&1\end{pmatrix}$ va $B=\begin{pmatrix}2&2&2\\1&-1&2\\-3&5&3\end{pmatrix}$

12. $A=\begin{pmatrix}-2&2&2\\3&4&-1\\3&-2&2\end{pmatrix}$ va $B=\begin{pmatrix}-4&0&2\\2&-2&-3\\3&-2&3\end{pmatrix}$

13. $A=\begin{pmatrix}-5&1&3\\2&2&-2\\3&3&2\end{pmatrix}$ va $B=\begin{pmatrix}2&-2&3\\2&2&1\\-3&1&2\end{pmatrix}$

14. $A=\begin{pmatrix}3&-2&2\\2&3&-3\\1&-1&0\end{pmatrix}$ va $B=\begin{pmatrix}2&2&-2\\-3&1&3\\2&-1&3\end{pmatrix}$

15. $A=\begin{pmatrix}1&2&3\\4&1&-7\\5&4&-2\end{pmatrix}$ va $B=\begin{pmatrix}-1&-3&2\\3&1&-3\\-2&-2&-3\end{pmatrix}$

16. $A=\begin{pmatrix}1&3&5\\-1&2&3\\-3&-1&1\end{pmatrix}$ va $B=\begin{pmatrix}2&-3&3\\2&-3&1\\3&-2&3\end{pmatrix}$

17. $A=\begin{pmatrix}3&-2&-1\\1&-2&1\\2&1&2\end{pmatrix}$ va $B=\begin{pmatrix}1&1&2\\2&-2&-1\\1&2&2\end{pmatrix}$

18. $A=\begin{pmatrix}3 & 2 & 3\\ 1 & -1 & 0\\ -1 & 2 & 1\end{pmatrix}$ va $B=\begin{pmatrix}4 & -3 & 2\\ 2 & 1 & -2\\ -2 & 1 & 1\end{pmatrix}$

19. $A=\begin{pmatrix}1 & 2 & 3\\ -2 & 3 & -1\\ -2 & -4 & 1\end{pmatrix}$ va $B=\begin{pmatrix}1 & -1 & -1\\ 2 & 1 & -1\\ -1 & 1 & -4\end{pmatrix}$

20. $A=\begin{pmatrix}-3 & 5 & 0\\ -2 & 1 & 2\\ 4 & -2 & -1\end{pmatrix}$ va $B=\begin{pmatrix}3 & 0 & 2\\ 1 & -1 & 3\\ 4 & -1 & 1\end{pmatrix}$

21. $A=\begin{pmatrix}3 & 1 & 2\\ 0 & 3 & -1\\ 4 & 4 & 1\end{pmatrix}$ va $B=\begin{pmatrix}-1 & 0 & -1\\ -1 & 1 & -1\\ 2 & -1 & -12\end{pmatrix}$

22. $A=\begin{pmatrix}2 & 3 & -1\\ 1 & 2 & 2\\ 5 & 0 & -1\end{pmatrix}$ va $B=\begin{pmatrix}-4 & 1 & -1\\ 2 & -4 & 0\\ 3 & 1 & -1\end{pmatrix}$

23. $A=\begin{pmatrix}0 & -1 & 2\\ -2 & 1 & 5\\ 5 & 1 & -1\end{pmatrix}$ va $B=\begin{pmatrix}1 & 2 & 1\\ -1 & 3 & 1\\ -1 & 2 & -3\end{pmatrix}$

24. $A=\begin{pmatrix}2 & 0 & -3\\ -4 & 2 & 0\\ 1 & 3 & 2\end{pmatrix}$ va $B=\begin{pmatrix}-3 & -2 & 0\\ 3 & -2 & 1\\ 1 & -1 & 2\end{pmatrix}$

25. $A=\begin{pmatrix}1 & -3 & 0\\ 2 & 2 & 3\\ -11 & 2 & -1\end{pmatrix}$ va $B=\begin{pmatrix}4 & 3 & 2\\ -2 & -6 & -3\\ -3 & 2 & 1\end{pmatrix}$

26. $A=\begin{pmatrix}4 & 1 & 2\\ -3 & 2 & 1\\ 2 & 2 & 1\end{pmatrix}$ va $B=\begin{pmatrix}2 & 2 & 2\\ 1 & -1 & 2\\ -3 & 5 & 3\end{pmatrix}$

27. $A=\begin{pmatrix}-2 & 2 & 2\\ 3 & 4 & -1\\ 3 & -2 & 2\end{pmatrix}$ va $B=\begin{pmatrix}-4 & 0 & 2\\ 2 & -2 & -3\\ 3 & -2 & 3\end{pmatrix}$

28. $A=\begin{pmatrix}-5 & 1 & 3\\ 2 & 2 & -2\\ 3 & 3 & 2\end{pmatrix}$ va $B=\begin{pmatrix}2 & -2 & 3\\ 2 & 2 & 1\\ -3 & 1 & 2\end{pmatrix}$

29. $A=\begin{pmatrix}3 & -2 & 2\\ 2 & 3 & -3\\ 1 & -1 & 0\end{pmatrix}$ va $B=\begin{pmatrix}2 & 2 & -2\\ -3 & 1 & 3\\ 2 & -1 & 3\end{pmatrix}$

30. $A=\begin{pmatrix}4 & 1 & 2\\ -3 & 2 & 1\\ 2 & 2 & 1\end{pmatrix}$ va $B=\begin{pmatrix}2 & 2 & 2\\ 1 & -1 & 2\\ -3 & 5 & 3\end{pmatrix}$

3-Mustaqil ish darsi.

Mavzu: 2- va 3-tartibli determinantlar.

Determinantlar tuchunchasi matritsaning xususiy holi bo'lib, ular hisoblanadi. Biz faqat 2- va 3-tartibli determinantlar bilan tanishamiz.

Ta'rif. 2-tartibli determinant deb

$$\det = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

ko`rinidagi ifodaga aytiladi, bu yerda $a_{11}\ a_{12}$- asosiy, $a_{21}\ a_{22}$-, -yordamchi diogonal elementlari deyiladi. 2-tartibli determinant qiymati $a_{11}\ a_{12}$-$a_{21}\ a_{22}$- ifodaga teng.

Masalan:

$$\det = \begin{vmatrix} 2 & -3 \\ 4 & 5 \end{vmatrix} = 10-(-3)*4=10+12=22.$$

Ta'rif. 3-tartibli determinant deb, 3 ta satr va 3 ta ustundan iborat bo`lgan kvadrat matritsaga aytiladi, ya'ni

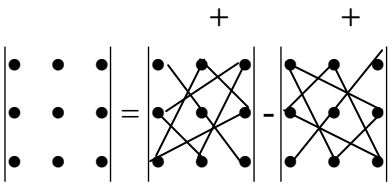
$$\det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

3-tartibli determinantlarni hisoblashning bir necha usullari mavjud bo`lib, biz ularni alohida-alohida qaraymiz.

1-usul. Ushburchak usuli.

$$\det = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{12}a_{23}a_{31} - a_{13}a_{22}a_{31} - a_{23}a_{32}a_{11} - a_{12}a_{12}a_{33}$$

Bu amalning sxematik bajarilishini ko'rsatamiz:



Masalan**:**

$$\det\Delta=\begin{vmatrix}1 & -2 & 3\\ 4 & 1 & 0\\ 5 & 7 & 2\end{vmatrix}=2+4*7*3+(-2)*0*5-3*1*5-0*7*1-4*(-2)*2=2+84-15+16=87$$

2-usul. Satr yoki ustun elementlarini parallel ko`chirish.

Bizga 3-tartibli determinant berilgan bo`lsin, ya'ni

$$\det\Delta=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}$$

1- va 2-satr elementlarini parallel holda determinant quyi qismiga ko`chiramiz va asosiy va yordamchi diagonal , ularga parallel turgan elementlarni o`zaro ko`paytirib, dastlabki 3 ta ko`paytma yig`indisidan, keyingi 3 ta ko`paytma yig`indisini ayiramiz, ya'ni

$$\det\Delta=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\\ a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\end{vmatrix}=a_{11}a_{22}a_{33}+a_{21}a_{32}a_{13}+$$

$$+\ a_{31}a_{12}a_{23}-a_{13}a_{22}a_{31}-a_{23}a_{32}a_{11}-a_{33}a_{12}a_{21}$$

Masalan:

$$\det\Delta=\begin{vmatrix}-1 & 2 & 3\\ 4 & 2 & 4\\ 3 & 2 & 1\\ -1 & 2 & 3\\ 4 & -5 & 4\end{vmatrix}=\begin{array}{l}(-1)*(-5)*1+4*7*3+3*2*4-3*(-5)*3-4*7(-1)-\\ -1*2*4=5+84+24+45+28-8=178\end{array}$$

Endi matritsa 1- va 2-ustun elementlarini parallel holda ko`chirib, determinant yoniga qo`yamiz, ya'ni

$$\det\Delta=\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{vmatrix} = a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32}-a_{13}a_{22}a_{31}-\ a_{11}a_{23}a_{32}-a_{12}a_{21}a_{33}$$

Masalan:

$$\det\Delta=\begin{vmatrix} 2 & 4 & -1 & 2 & 4 \\ 3 & 1 & 3 & 3 & 1 \\ 5 & 0 & 7 & 5 & 0 \end{vmatrix} \quad =14+60+5-84= \quad -5$$

Ta'rif 3. Matritsa a_{ij} elementining ***minori*** deb, shu element joylashgan satr va ustun elementlarini o'chirishdan hosil bo`lgan matritsaga aytiladi va M_{ij} deb belgilanadi.

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

a_{32} elementining minori $M_{32}=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

Masalan:

$$A=\begin{pmatrix} -2 & 4 & 5 \\ 6 & 7 & 1 \\ 3 & 2 & 4 \end{pmatrix} \quad , \quad M_{23}=\begin{pmatrix} -2 & 4 \\ 3 & 2 \end{pmatrix}, \quad M_{11}=\begin{pmatrix} 7 & 1 \\ 2 & 4 \end{pmatrix}, \quad M_{12}=\begin{pmatrix} 6 & 1 \\ 3 & 4 \end{pmatrix}$$

Ta'rif 4 . Matritsa a_{ij} elementining algebraik to`ldiruvchisi deb $A_{ij}=(-1)^{i+j}M_{ij}$ ifodaga aytiladi.

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$A_{23}=(-1)^{2+3}\begin{pmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{pmatrix} \qquad A_{13}=(-1)^{1+3}\begin{pmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$$

Masalan :

$$A=\begin{pmatrix} 4 & -1 & 2 \\ 3 & 5 & 1 \\ 1 & 4 & -2 \end{pmatrix}$$

$$A_{32}=(-1)^{3+2}\begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix} \qquad A_{11}=(-1)^{1+1}\begin{pmatrix} 5 & 1 \\ 4 & -2 \end{pmatrix}$$

va hokazolar.

Ta'rif 5 . A matritsaga teskari matritsa deb,

$$\mathbf{A^{-1}} = \begin{pmatrix} \frac{A_{11}}{\Delta} & \frac{A_{21}}{\Delta} & \frac{A_{31}}{\Delta} \\ \frac{A_{12}}{\Delta} & \frac{A_{22}}{\Delta} & \frac{A_{23}}{\Delta} \\ \frac{A_{13}}{\Delta} & \frac{A_{23}}{\Delta} & \frac{A_{33}}{\Delta} \end{pmatrix}$$

matritsaga aytiladi, bu erda Δ - shu matritsaning determinanti

Masalan: $A = \begin{pmatrix} 1 & -2 & 3 \\ 4 & 5 & 2 \\ 6 & 3 & -2 \end{pmatrix}$ $A^{-1} = ?$

$$\Delta = \begin{vmatrix} 1 & -2 & 3 \\ 4 & 5 & 2 \\ 6 & 2 & -2 \end{vmatrix} = -10 + 24 - 24 - 90 - 4 - 16 = -120 \neq 0$$

$\mathbf{A_{11}} = \begin{vmatrix} 5 & 2 \\ 2 & -2 \end{vmatrix}$ **= -10 –4 =-14**

$\mathbf{A_{12}} = \mathbf{(-1)^{1+2}} \begin{vmatrix} 4 & 2 \\ 6 & -2 \end{vmatrix}$ **= - (-8 -12) =20**

$\mathbf{A_{13}} = \begin{vmatrix} 4 & 5 \\ 5 & 3 \end{vmatrix}$ **= 12-30 = -18**

$\mathbf{A_{21}} = \begin{vmatrix} -2 & 3 \\ 2 & -2 \end{vmatrix}$ **= - (4-6) = 2**

$\mathbf{A_{22}} = \begin{vmatrix} 1 & 3 \\ 6 & -2 \end{vmatrix}$ **= -2 –18 = -20**

$\mathbf{A_{23}} = - \begin{vmatrix} 1 & -2 \\ 6 & 2 \end{vmatrix}$ **= (2+12) = -14**

$$\mathbf{A_{31}} = \begin{vmatrix} -2 & 3 \\ 5 & 2 \end{vmatrix} = \mathbf{-4 - 15 = -19}$$

$$\mathbf{A_{32}} = - \begin{vmatrix} 1 & 3 \\ 4 & 2 \end{vmatrix} = \mathbf{-(2 - 12) = 10}$$

$$\mathbf{A_{33}} = \begin{vmatrix} 1 & -2 \\ 4 & 5 \end{vmatrix} = \mathbf{5 + 8 = 13}$$

$$A^{-1} = \begin{pmatrix} \frac{-14}{-120} & \frac{2}{-120} & \frac{-19}{-120} \\ \frac{20}{-120} & \frac{-20}{-120} & \frac{10}{-120} \\ \frac{-18}{-120} & \frac{-14}{-120} & \frac{13}{-120} \end{pmatrix} = \begin{pmatrix} \frac{14}{120} & -\frac{2}{120} & \frac{19}{120} \\ -\frac{20}{120} & \frac{20}{120} & -\frac{10}{120} \\ \frac{18}{120} & \frac{14}{120} & -\frac{13}{120} \end{pmatrix}$$

3 - tartibli determinantlarni algebraik to`ldiruvchilar yordamida quyidagicha hisoblash mumkin.

$$\Delta = \mathbf{a_{11}\, A_{11} + a_{12}\, A_{12} + a_{13}\, A_{13}}$$
$$\Delta = \mathbf{a_{21}\, A_{21} + a_{22}\, A_{22} + a_{23}\, A_{23}}$$
$$\Delta = \mathbf{a_{31}\, A_{31} + a_{32}\, A_{32} + a_{33}\, A_{33}}$$

$$\Delta = \mathbf{a_{11}\, A_{11} + a_{21}\, A_{21} + a_{31}\, A_{31}}$$
$$\Delta = \mathbf{a_{12}\, A_{12} + a_{22}\, A_{22} + a_{32}\, A_{32}}$$
$$\Delta = \mathbf{a_{13}\, A_{13} + a_{23}\, A_{23} + a_{33}\, A_{33}}$$

Sinov mashqlari.

1) Quyidagi determinantlarni hisoblang.

1. Determinantlarni hisoblang:

$$\Delta = \begin{vmatrix} a_{11} a_{12} \\ a_{21} a_{22} \end{vmatrix}, \qquad \Delta = \begin{vmatrix} a_{11} a_{12} a_{13} \\ a_{21} a_{22} a_{23} \\ a_{31} a_{32} a_{33} \end{vmatrix}, \qquad \Delta = \begin{vmatrix} a_{11} a_{12} a_{13} a_{14} \\ a_{21} a_{22} a_{23} a_{24} \\ a_{31} a_{32} a_{33} a_{34} \\ a_{41} a_{42} a_{43} a_{44} \end{vmatrix}$$

Variant	a_{11}	a_{12}	a_{13}	a_{14}	a_{21}	a_{22}	a_{23}	a_{24}	a_{31}	a_{32}	a_{33}	a_{34}	a_{41}	a_{42}	a_{43}	a_{44}
1.	2	3	-1	0	-4	-1	-3	3	3	4	2	-2	1	2	0	4
2.	0	3	2	-4	-2	4	-3	-1	1	3	-2	0	3	-1	4	2
3.	0	4	2	1	1	0	3	2	3	-3	-1	4	-2	2	4	3
4.	1	3	-2	0	-3	1	-4	-2	0	6	4	-8	-2	4	-3	-1
5.	1	-4	0	3	-4	3	2	-3	-2	3	-1	4	3	2	5	0

6.	-4	3	2	3	1	-2	3	-4	3	4	0	-3	0	-1	-2	5
7.	1	-4	0	3	-4	3	2	-3	-2	-1	3	4	3	2	5	0
8.	-4	2	3	0	-1	-3	4	-2	2	4	-1	3	0	-5	2	1
9.	2	-1	0	5	-1	-3	2	-4	4	2	-1	3	3	0	-1	-2
10.	4	2	0	-3	-1	3	2	4	-2	4	3	1	0	5	-1	2
11.	0	-1	1	-4	2	5	0	6	1	-1	2	-1	4	2	1	0
12.	1	2	-1	4	-2	1	3	-5	3	7	5	1	5	3	-1	2
13.	3	0	1	-3	1	7	1	3	2	-1	0	2	-2	3	7	1
14.	5	-8	-4	7	0	-5	4	1	2	-1	-3	-2	1	5	-5	-1
15.	2	-2	0	5	4	1	1	-1	2	-3	4	-3	1	2	3	-5
16.	1	-2	4	-3	-4	1	-1	2	0	5	-3	-4	-3	2	2	-1
17.	1	7	-1	0	2	6	2	-1	1	-3	4	0	4	5	1	3
18.	8	5	-1	1	5	3	1	1	0	4	-7	-6	3	2	-1	0
19.	1	4	-4	2	3	1	2	1	-4	-3	4	2	1	-5	3	0
20.	2	-1	-4	4	-9	3	2	-7	-1	0	4	5	6	4	7	-4
21.	3	-1	0	3	5	1	4	-7	5	-1	0	2	1	-8	5	3
22.	-1	3	-1	2	-2	1	1	3	1	6	5	4	4	3	0	-3
23.	3	-7	2	1	-8	3	4	-2	3	0	5	-3	2	4	1	5
24.	8	4	-1	-1	3	1	5	2	1	3	7	1	5	-1	6	0
25.	8	-1	-1	5	-5	1	10	3	2	3	-2	-5	3	2	1	0
26.	2	1	8	7	1	3	1	0	6	0	5	-3	4	-1	-4	3
27.	3	1	0	4	3	2	2	-2	-7	2	7	0	-5	3	1	-6
28.	6	9	4	1	3	4	-3	-1	-1	-1	10	7	5	2	0	3
29.	5	5	2	1	2	4	-4	3	0	-2	5	3	-5	3	-7	2
30.	4	-3	2	-4	0	5	3	2	-3	2	6	1	13	3	-2	0

1.Berilgan matritsaga A^{-1} -teskari matritsani toping.

1. $A=\begin{pmatrix} 1 & 2 & 3 \\ 4 & 1 & -7 \\ 5 & 4 & -2 \end{pmatrix}$

2. $A=\begin{pmatrix} -1 & -3 & 2 \\ 3 & 1 & -3 \\ -2 & -2 & -3 \end{pmatrix}$

3. $A=\begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 3 \\ -3 & -1 & 1 \end{pmatrix}$

4. $A=\begin{pmatrix} 2 & -3 & 3 \\ 2 & -3 & 1 \\ 3 & -2 & 3 \end{pmatrix}$

5. $A=\begin{pmatrix} 3 & -2 & -1 \\ 1 & -2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$

6. $A=\begin{pmatrix} 1 & 1 & 2 \\ 2 & -2 & -1 \\ 1 & 2 & 2 \end{pmatrix}$

7. $A=\begin{pmatrix} 3 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{pmatrix}$

8. $A=\begin{pmatrix} 4 & -3 & 2 \\ 2 & 1 & -2 \\ -2 & 1 & 1 \end{pmatrix}$

9. $A=\begin{pmatrix} 1 & 2 & 3 \\ -2 & 3 & -1 \\ -2 & -4 & 1 \end{pmatrix}$

10. $A=\begin{pmatrix} 1 & -1 & -1 \\ 2 & 1 & -1 \\ -1 & 1 & -4 \end{pmatrix}$

11. $A=\begin{pmatrix}-3 & 5 & 0\\ -2 & 1 & 2\\ 4 & -2 & -1\end{pmatrix}$

12. $A=\begin{pmatrix}3 & 0 & 2\\ 1 & -1 & 3\\ 4 & -1 & 1\end{pmatrix}$

13. $A=\begin{pmatrix}3 & 1 & 2\\ 0 & 3 & -1\\ 4 & 4 & 1\end{pmatrix}$

14. $A=\begin{pmatrix}-1 & 0 & -1\\ -1 & 1 & -1\\ 2 & -1 & -12\end{pmatrix}$

15. $A=\begin{pmatrix}2 & 3 & -1\\ 1 & 2 & 2\\ 5 & 0 & -1\end{pmatrix}$

16. $A=\begin{pmatrix}-4 & 1 & -1\\ 2 & -4 & 0\\ 3 & 1 & -1\end{pmatrix}$

17. $A=\begin{pmatrix}0 & -1 & 2\\ -2 & 1 & 5\\ 5 & 1 & -1\end{pmatrix}$

18. $A=\begin{pmatrix}1 & 2 & 1\\ -1 & 3 & 1\\ -1 & 2 & -3\end{pmatrix}$

19. $A=\begin{pmatrix}2 & 0 & -3\\ -4 & 2 & 0\\ 1 & 3 & 2\end{pmatrix}$

20. $A=\begin{pmatrix}-3 & -2 & 0\\ 3 & -2 & 1\\ 1 & -1 & 2\end{pmatrix}$

21. $A=\begin{pmatrix}1 & -3 & 0\\ 2 & 2 & 3\\ -11 & 2 & -1\end{pmatrix}$

22. $A=\begin{pmatrix}4 & 3 & 2\\ -2 & -6 & -3\\ -3 & 2 & 1\end{pmatrix}$

23. $A=\begin{pmatrix}4 & 1 & 2\\ -3 & 2 & 1\\ 2 & 2 & 1\end{pmatrix}$

24. $A=\begin{pmatrix}2 & 2 & 2\\ 1 & -1 & 2\\ -3 & 5 & 3\end{pmatrix}$

25. $A=\begin{pmatrix}-2 & 2 & 2\\ 3 & 4 & -1\\ 3 & -2 & 2\end{pmatrix}$

26. $A=\begin{pmatrix}-4 & 0 & 2\\ 2 & -2 & -3\\ 3 & -2 & 3\end{pmatrix}$

27. $A=\begin{pmatrix}-5 & 1 & 3\\ 2 & 2 & -2\\ 3 & 3 & 2\end{pmatrix}$

28. $A=\begin{pmatrix}2 & -2 & 3\\ 2 & 2 & 1\\ -3 & 1 & 2\end{pmatrix}$

29. $A=\begin{pmatrix}3 & -2 & 2\\ 2 & 3 & -3\\ 1 & -1 & 0\end{pmatrix}$

30. $A=\begin{pmatrix}2 & 2 & -2\\ -3 & 1 & 3\\ 2 & -1 & 3\end{pmatrix}$

4. Mustaqil ish darsi

Mavzu: Chiziqli tenglamalar sistemasini yechish.

Bizga uchta noma'lumli uchta chiziqli tenglamalar sistemasi berilgan bo`lsin.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$$

Bu sistemaning yechimini topish uchun bir necha usullar mavjud, bu usulllarni alohida - alohida qaraymiz.

1-usul (Kramer qoidasi).

Aytaylik, berilgan sistemaning asosiy determinanti noldan farqli bo`lsin, ya'ni

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$$

Bu holda uning qolgan uchta yordamchi determinantlarini topamiz. Buning uchun asosiy determinantdagi ustun elementlarini navbatma - navbat ozod hadlar bilan almashtirib uchta sistemaning yordamchi determinatlarini tuzish mumkin.

$$\Delta_{x_1} = \begin{vmatrix} e_1 & a_{12} & a_{13} \\ e_2 & a_{22} & a_{23} \\ e_3 & a_{32} & a_{33} \end{vmatrix}, \qquad \Delta_{x_2} = \begin{vmatrix} a_{11} & e_1 & a_{13} \\ a_{21} & e_2 & a_{23} \\ a_{31} & e_3 & a_{33} \end{vmatrix}, \Delta_{x_3} = \begin{vmatrix} a_{11} & a_{12} & e_1 \\ a_{21} & a_{22} & e_2 \\ a_{31} & a_{32} & e_3 \end{vmatrix}$$

Endi berilgan sistemaning yechimini ya'ni ildizlarini:

$X_1 = \frac{\Delta_{x_1}}{\Delta}$, $X_2 = \frac{\Delta_{x_2}}{\Delta}$, $X_3 = \frac{\Delta_{x_3}}{\Delta}$.

tengliklar orqali topish mumkin.

Masalan:

$$\begin{cases} 2x - 4y = z = 3 \\ x - 5y + 3z = -1 \\ x - y + z = 1 \end{cases}, \quad \Delta = \begin{vmatrix} 2 & -4 & 1 \\ 1 & -5 & 3 \\ 1 & -1 & 1 \end{vmatrix} = -10 - 1 - 12 + 5 + 6 + 4 = -8 \neq 0.$$

$$\Delta_{x_1} = \begin{vmatrix} 3 & -4 & 1 \\ -1 & -5 & 3 \\ 1 & -1 & 1 \end{vmatrix} = -15 + 1 - 12 + 5 + 9 - 4 = -16 \neq 0, \quad \Delta_{x_2} = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = -2 + 1 + 9 + 1 - 6 - 3 = 0,$$

$$\Delta_{x_3} = \begin{vmatrix} 2 & -4 & 3 \\ 1 & -5 & -1 \\ 1 & -1 & 1 \end{vmatrix} = -10 - 3 + 4 + 15 - 2 + 4 = 8, \qquad \text{demak, } X_1 = \frac{\Delta_{x_1}}{\Delta} = \frac{-16}{-8} = 2,$$

$X_2 = \frac{\Delta_{x_2}}{\Delta} = \frac{0}{-8} = 0, \quad X_3 = \frac{\Delta_{x_3}}{\Delta} = \frac{8}{-8} = -1,$ Javob: {2,0,-1}.

2-usul (Matritsalar yordamida)

Yuqoridagi tenglamalar sistemasini matritsalar yordamida quyidagicha yozish mumkin:

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad B=\begin{pmatrix} в_1 \\ в_2 \\ в_3 \end{pmatrix}$$

Demak AX=B yoki

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} в_1 \\ в_2 \\ в_3 \end{pmatrix}$$

Agar bundan X ni topsak, $X=\frac{B}{A}=A^{-1}B.$

Sistemani matritsalar yordamida yechish uchun sistema matritsasiga teskari bo`lgan matritsani topish talab etiladi.

A matritsaga teskari bo`lgan matritsa

$$A^{-1}=\begin{pmatrix} \frac{A_{11}}{\Delta} & \frac{A_{21}}{\Delta} & \frac{A_{31}}{\Delta} \\ \frac{A_{12}}{\Delta} & \frac{A_{22}}{\Delta} & \frac{A_{32}}{\Delta} \\ \frac{A_{13}}{\Delta} & \frac{A_{23}}{\Delta} & \frac{A_{33}}{\Delta} \end{pmatrix}$$

ko'rinishda topiladi.

Bu erda A_{ij} lar a_{ij} elementlarning algebraik to`ldiruvchilari, berilgan matritsa determinantidir. Endi matritsani matritsaga ko`paytirish amalidan foydalanib, berilgan sistemasini yechimini topish mumkin.

Masalan:

$$\begin{cases} 2x-4y+z=3 \\ x-5y+3z=-1 \\ x-y+z=1 \end{cases}$$

$$\det A=\begin{vmatrix} 2 & -4 & 1 \\ 1 & -5 & 3 \\ 1 & -1 & 1 \end{vmatrix}=-10-12-1+5+4+6=-8$$

$$A_{11}=(-1)^{1+1}M_{11}=(-1)^2\begin{vmatrix} -5 & 3 \\ -1 & 1 \end{vmatrix}=-5+3=-2$$

$$A_{21}=(-1)^{2+1}M_{21}=(-1)^3\begin{vmatrix} -4 & 1 \\ -1 & 1 \end{vmatrix}=-(-4+1)=3$$

$$A_{31}=(-1)^{3+1}M_{31}=(-1)^4\begin{vmatrix} -4 & 1 \\ -5 & 3 \end{vmatrix}=-12+5=-7$$

$$A_{12}=(-1)^{1+2}M_{12}=(-1)^3\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix}=-(1-3)=2$$

$$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 2 - 1 = 1$$

$$A_{32} = (-1)^{3+2} M_{32} = (-1)^5 \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = -(6-1) = -5$$

$$A_{13} = (-1)^{1+3} M_{13} = (-1)^4 \begin{vmatrix} 1 & -5 \\ 1 & -1 \end{vmatrix} = -1+5 = 4$$

$$A_{23} = (-1)^{2+3} M_{23} = (-1)^5 \begin{vmatrix} 2 & -4 \\ 1 & -1 \end{vmatrix} = -(-2+4) = -2$$

Chiziqli tenglamalar sistemasini Kramer va matritsa usulida yeching.

1. $\begin{cases} 2x_1 + 2x_2 = 2 \\ -x_1 + 2x_2 - 2x_3 = 3 \\ 2x_1 - 2x_2 + 3x_3 = 0 \end{cases}$

2. $\begin{cases} 5x_1 - 2x_2 - x_3 = 0 \\ 2x_1 - x_2 - x_3 = -1 \\ 2x_1 - 4x_2 - 8x_3 = 1 \end{cases}$

3. $\begin{cases} 2x_1 + 2x_3 = -1 \\ 5x_1 - 2x_2 + 5x_3 = 2 \\ 2x_1 + 2x_2 + 5x_3 = 0 \end{cases}$

4. $\begin{cases} 5x_1 - 2x_2 + x_3 = 2 \\ 2x_1 - x_2 + x_3 = -1 \\ 18x_1 - 8x_2 + 4x_3 = 0 \end{cases}$

5. $\begin{cases} 5x_1 + 2x_2 - 2x_3 = 0 \\ 3x_1 - 3x_2 - x_3 = 1 \\ 2x_1 + 4x_2 - x_3 = -1 \end{cases}$

6. $\begin{cases} 2x_1 - 2x_2 + 5x_3 = 2 \\ -2x_1 + 3x_2 + 6x_3 = -1 \\ -10x_1 + 12x_2 - 4x_3 = 2 \end{cases}$

7. $\begin{cases} 3x_1 + 12x_2 - 5x_3 = 1 \\ 3x_1 + 5x_2 - 2x_3 = 1 \\ -x_1 + 2x_2 - x_3 = 1 \end{cases}$

8. $\begin{cases} 5x_1 + 3x_2 - 2x_3 = -1 \\ 2x_1 + x_2 - x_3 = 0 \\ 3x_1 - 2x_2 - 3x_3 = 2 \end{cases}$

9. $\begin{cases} -10x_1 - x_2 + 3x_3 = -1 \\ 8x_1 + 7x_2 + 2x_3 = 0 \\ 6x_1 + 6x_2 + 2x_3 = 2 \end{cases}$

10. $\begin{cases} 7x_1 - 3x_2 + 2x_3 = 1 \\ x_1 - 8x_2 + x_3 = 1 \\ 6x_1 - 6x_2 + 2x_3 = -1 \end{cases}$

11. $\begin{cases} 6x_1 + 12x_2 + 7x_3 = 3 \\ -3x_1 - 5x_2 + x_3 = -2 \\ 2x_1 + 4x_2 + 2x_3 = -1 \end{cases}$

12. $\begin{cases} -x_1 + 7x_2 + 5x_3 = -1 \\ 2x_1 + 4x_2 + 3x_3 = 1 \\ 5x_1 - 3x_2 - 2x_3 = 0 \end{cases}$

13. $\begin{cases} 3x_1 + 4x_2 + 7x_3 = 1 \\ 5x_1 + 7x_2 + 12x_3 = 0 \\ 2x_1 - 3x_2 + x_3 = 2 \end{cases}$

14. $\begin{cases} 6x_1 + 4x_2 - 2x_3 = 1 \\ 4x_1 + 3x_2 + 2x_3 = 0 \\ -2x_1 - x_2 + 5x_3 = -1 \end{cases}$

15. $\begin{cases} -4x_1 + 6x_2 + 2x_3 = 5 \\ 3x_1 - 4x_2 - x_3 = -2 \\ 2x_1 - 3x_2 - 2x_3 = -2 \end{cases}$

16. $\begin{cases} 6x_1 - x_2 + 4x_3 = 1 \\ 8x_1 - x_3 = 1 \\ -4x_1 + 3x_2 - 3x_3 = 1 \end{cases}$

17. $\begin{cases} -3x_1 + 2x_2 - 4x_3 = 1 \\ -7x_1 + 6x_2 - 10x_3 = 1 \\ -4x_1 + 5x_2 - 6x_3 = 0 \end{cases}$

18. $\begin{cases} 2x_1 + 3x_3 = 0 \\ -x_1 + 5x_2 + 2x_3 = -2 \\ 3x_1 + x_2 + 5x_3 = 1 \end{cases}$

19. $\begin{cases} 5x_1 + x_2 - 3x_3 = 1 \\ 2x_1 + 5x_2 - x_3 = 0 \\ -3x_1 + 2x_2 + 2x_3 = -1 \end{cases}$

20. $\begin{cases} -3x_1 + 2x_2 + 5x_3 = 0 \\ -2x_1 + 10x_2 + 4x_3 = 2 \\ 2x_1 + 2x_2 - 3x_3 = 1 \end{cases}$

21. $\begin{cases} 2x_1 - 3x_2 + 3x_3 = 0 \\ -3x_1 + 5x_2 + 2x_3 = 1 \\ x_1 - x_2 + 6x_3 = -1 \end{cases}$

22. $\begin{cases} -x_1 + 2x_2 + 3x_3 = 1 \\ 2x_1 + 3x_2 + 4x_3 = 0 \\ x_1 - 3x_2 - 5x_3 = -1 \end{cases}$

23. $\begin{cases} x_1 - 3x_2 + 3x_3 = 1 \\ 5x_1 + 2x_2 + 12x_3 = 0 \\ 3x_1 + 20x_2 + 4x_3 = 2 \end{cases}$

24. $\begin{cases} -2x_1 + x_2 + 3x_3 = 2 \\ 6x_1 + 9x_2 + x_3 = 1 \\ -3x_1 + 2x_2 + 5x_3 = 1 \end{cases}$

25. $\begin{cases} 4x_1 - 3x_2 + 7x_3 = -1 \\ 13x_1 - 2x_3 = -1 \\ 2x_1 + 2x_2 + 3x_3 = 0 \end{cases}$

26. $\begin{cases} 5x_1 - 2x_2 + 4x_3 = -3 \\ 2x_1 - 3x_2 + 7x_3 = 1 \\ 8x_1 - 5x_2 + 11x_3 = 1 \end{cases}$

27. $\begin{cases} 7x_1 + 4x_2 + 3x_3 = 1 \\ 3x_1 + 2x_2 + x_3 = -2 \\ 5x_1 + 2x_2 + 2x_3 = 0 \end{cases}$

28. $\begin{cases} -3x_1 + 7x_2 + 5x_3 = -1 \\ 4x_1 + 3x_2 - x_3 = 1 \\ 2x_1 + 9x_2 + 3x_3 = 0 \end{cases}$

9. $\begin{cases} -3x_1 + 2x_2 + 5x_3 = 2 \\ -5x_1 + 3x_2 + 8x_3 = -1 \\ -2x_1 - x_2 + 3x_3 = -1 \end{cases}$

30. $\begin{cases} 3x_1 + x_2 + 5x_3 = 0 \\ 5x_1 + 4x_2 + 9x_3 = 0 \\ 2x_1 + 2x_2 + 4x_3 = -1 \end{cases}$

«Oliy matematika» fanidan mustaqil ta'limning amaliy mashg'ulotiga ajratilgan masalalarni yechish namunalari

Mazkur mavzuda talaba bajarishi lozim bo'lgan variantlaridan namunalar keltiramiz:

I- SEMESTR

Hisoblang.

1) $\begin{vmatrix} 5 & -7 \\ 4 & 3 \end{vmatrix}$; 2) ; $\begin{vmatrix} 1 & 9 & 0 \\ 4 & 5 & 3 \\ -1 & 2 & -3 \end{vmatrix}$; 3) $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & -3 & 1 \\ 1 & 0 & 2 & 3 \\ 3 & 4 & 0 & 1 \end{vmatrix}$

Yechish: $\begin{vmatrix} 5 & -7 \\ 4 & 3 \end{vmatrix} = 5 \cdot 3 - 4 \cdot (-7) = 15 + 28 = 43$

2) $\begin{vmatrix} 1 & 9 & 0 \\ 4 & 5 & 3 \\ -1 & 2 & -3 \end{vmatrix}$ determinantni ([9]) ushburchak usuli, Sirius usuli va algebraik to`ldiruvchilarga yoyib hisoblaymiz:

a) $\begin{vmatrix} 1 & 9 & 0 \\ 4 & 5 & 3 \\ -1 & 2 & -3 \end{vmatrix} = 1 \cdot 5 \cdot (-3) + 4 \cdot 2 \cdot 0 + 9 \cdot 3 \cdot (-1) - (-1) \cdot 5 \cdot 0 - 2 \cdot 3 \cdot 1 - 4 \cdot 9 \cdot (-3) = -15 - 27 - 6 + 108 = 60$

b) $\begin{vmatrix} 1 & 9 & 0 \\ 4 & 5 & 3 \\ -1 & 2 & -3 \end{vmatrix} = 1\cdot 5\cdot(-3)+4\cdot 2\cdot 0+9\cdot 3\cdot(-1)-(-1)\cdot 5\cdot 0-2\cdot 3\cdot 1-4\cdot 9\cdot(-3)=60$

$\begin{matrix} 1 & 9 & 0 \\ 4 & 5 & 3 \end{matrix}$

v) Determinantni hisoblash uchun birinshi satr elementlar buyisha algebraik to'ldiruvchilarga yoyamiz:

$$\begin{vmatrix} 1 & 9 & 0 \\ 4 & 5 & 3 \\ -1 & 3 & -3 \end{vmatrix} = 1\cdot A_{11}+9A_{12}+0\cdot A_{13}=1\cdot M_{11}-9M_{12}=\begin{vmatrix} 5 & 3 \\ 2 & -3 \end{vmatrix}-9\begin{vmatrix} 4 & 3 \\ -1 & -2 \end{vmatrix}=(-15-6)-9(-8+3)=$$

$$=-21-9(-5)=-21+45=24$$

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & -3 & 1 \\ 1 & 0 & 2 & 3 \\ 3 & 4 & 0 & 1 \end{vmatrix} = 3A_{13}+2A_{33}=3M_{11}+2M_{33}=3\begin{vmatrix} 3 & 5 & 5 \\ 1 & 0 & 3 \\ 3 & 4 & 1 \end{vmatrix}+2\begin{vmatrix} 1 & 2 & 4 \\ 3 & 5 & 5 \\ 3 & 4 & 1 \end{vmatrix}=3(20+45-36-5)+2)5+48+30-60-20-6)=$$

$$=3\cdot 24+2\cdot(-13)=72-26=46$$

2. Quyidagi $A=\begin{pmatrix} 4 & 5 \\ -3 & 2 \end{pmatrix}$, $B=\begin{pmatrix} -1 & 0 \\ 3 & 2 \end{pmatrix}$, $C=\begin{pmatrix} 3 & 5 \\ 1 & -2 \end{pmatrix}$ matritsalar berilgan. A+B, 2A-C, 3C-A, $A\cdot B, B\cdot C, A\cdot C$ matritsalar topilsin.

Yechish. A, B, C matritsalar bir xil o`lchamli bo'lganidan ular ustidan chisiqli amallar bajarish va songa ko`paytirish mumkin. [9]

$$A+B=\begin{pmatrix} 4 & 5 \\ -3 & 2 \end{pmatrix}+\begin{pmatrix} -1 & 0 \\ 3 & 2 \end{pmatrix}=\begin{pmatrix} 4-1 & 5+0 \\ -3+3 & 2+2 \end{pmatrix}=\begin{pmatrix} 3 & 5 \\ 0 & 4 \end{pmatrix}$$

$$2A-C=2\begin{pmatrix} 4 & 5 \\ -3 & 2 \end{pmatrix}-\begin{pmatrix} 3 & 5 \\ 1 & -2 \end{pmatrix}=\begin{pmatrix} 8 & 10 \\ -6 & 4 \end{pmatrix}-\begin{pmatrix} 3 & 5 \\ 1 & -2 \end{pmatrix}=\begin{pmatrix} 8-3 & 10-5 \\ -6-1 & 4-(2) \end{pmatrix}=\begin{pmatrix} 5 & 5 \\ -7 & 6 \end{pmatrix}$$

$$3C-A=3\begin{pmatrix} 3 & 5 \\ 1 & -2 \end{pmatrix}-\begin{pmatrix} 4 & 5 \\ -3 & 2 \end{pmatrix}=\begin{pmatrix} 9 & 15 \\ 3 & -6 \end{pmatrix}-\begin{pmatrix} 4 & 5 \\ -3 & 2 \end{pmatrix}=\begin{pmatrix} 9-4 & 15-5 \\ 3-(-3) & -6-2 \end{pmatrix}=\begin{pmatrix} 5 & 10 \\ 6 & -8 \end{pmatrix}$$

A Matritsani B matritsaga ko'paytirish uchun A matritsani mos satr elementlarini B matritsani mos ustun elementlariga ko`paytirib qo`shish kerak:

$$A\cdot B=\begin{pmatrix} 4 & 5 \\ -3 & 2 \end{pmatrix}\cdot\begin{pmatrix} -1 & 0 \\ 3 & 2 \end{pmatrix}=\begin{pmatrix} 4\cdot(-1)+5\cdot 3 & 4\cdot 0+5\cdot 2 \\ (-3)(-1)+2\cdot 3 & -3\cdot 0+2\cdot 2 \end{pmatrix}=\begin{pmatrix} 11 & 10 \\ 9 & 4 \end{pmatrix}$$

$$B\cdot C=\begin{pmatrix} -1 & 0 \\ 3 & 2 \end{pmatrix}\cdot\begin{pmatrix} 3 & 5 \\ 1 & -2 \end{pmatrix}=\begin{pmatrix} -3 & -5 \\ 11 & 6 \end{pmatrix};$$

$$A\cdot C=\begin{pmatrix}4 & 5\\ -3 & 2\end{pmatrix}\cdot\begin{pmatrix}3 & 5\\ 1 & -2\end{pmatrix}=\begin{pmatrix}17 & -19\\ -7 & -19\end{pmatrix}$$

3. $A=\begin{pmatrix}4 & 1\\ 0 & 5\end{pmatrix}$ va $B=\begin{pmatrix}4 & 3 & 2\\ 0 & 1 & -1\\ -2 & 1 & 0\end{pmatrix}$ matritsalarga teskari matritsa topilsin.

Yechish: $A=\begin{pmatrix}4 & 1\\ 0 & 5\end{pmatrix}$; $\det A=20-0=20\neq 0,$ demak A xos matritsa ekan, shu sababli A matritsaga teskari matritsa mavjud. A^{-1} teskari matritsani $A^{-1}=\frac{1}{\det A}\begin{pmatrix}A_{11} & A_{21}\\ A_{12} & A_{22}\end{pmatrix}$ formula bilan hisoblaymiz:

$$A_{11}=5 \quad A_{12}=0 \qquad A^{-1}=\frac{1}{20}\begin{pmatrix}5 & -1\\ 0 & 4\end{pmatrix}$$
$$A_{21}=-1 \quad A_{22}=4$$

A^{-1} matritsani to'g'ri topilganligini tekshiramiz:

Buning uchun $A\cdot A^{-1}=E$ tenglik bajarilishini tekshiramiz:

$$A\cdot A^{-1}=\frac{1}{20}\begin{pmatrix}4 & 1\\ 0 & 5\end{pmatrix}\cdot\begin{pmatrix}5 & -1\\ 0 & 4\end{pmatrix}=\frac{1}{20}\begin{pmatrix}20 & 0\\ 0 & 20\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=E$$

Demak, A^{-1} to`g`ri topilgan.

Endi B matritsaga teskari B^{-1} matritsani topamiz. Buning uchun det B hisoblaymiz:

$\det B=\begin{vmatrix}4 & 3 & 2\\ 0 & 1 & -1\\ -2 & 1 & 0\end{vmatrix}=0+0+6+4+4=14\neq 0$ demak B xos matritsa ekan, B^{-1} ni quyidagi

$B^{-1}=\frac{1}{\det B}\begin{pmatrix}A_{11} & A_{21} & A_{13}\\ A_{12} & A_{22} & A_{23}\\ A_{13} & A_{23} & A_{33}\end{pmatrix}$ formula bilan topamiz. Buning uchun V matritsani barcha elementlarini algebraik to`ldiruvchilarini hisoblaymiz [9] ;

$$A_{11}=M_{11}\begin{vmatrix}1 & -1\\ 1 & 0\end{vmatrix}=1\,, \qquad A_{21}=-M_{21}=-\begin{vmatrix}3 & 2\\ 1 & 0\end{vmatrix}=2\,, \qquad A_{31}=M_{31}=\begin{vmatrix}3 & 2\\ 1 & -1\end{vmatrix}=-5$$

$$A_{12} = -M_{12} = -\begin{vmatrix} 0 & -1 \\ -2 & 0 \end{vmatrix} = 2\ ; \qquad A_{22} = M_{22}\begin{vmatrix} 4 & 2 \\ -2 & 0 \end{vmatrix} = 4; \qquad A_{32} = -M_{32} = -\begin{vmatrix} 4 & 2 \\ 0 & -1 \end{vmatrix} = 4$$

$$A_{13} = -M_{13} = \begin{vmatrix} 0 & 1 \\ -2 & 0 \end{vmatrix} = 2; \qquad A_{23} = -M_{23}\begin{vmatrix} 4 & 3 \\ -2 & 1 \end{vmatrix} = -10; \quad A_{33} = M_{32} = \begin{vmatrix} 4 & 3 \\ 0 & 1 \end{vmatrix} = 4$$

A_{ij} (i=1,2,3) lardan foydalanib, B ni yozamiz.

$$B^{-1} = \frac{1}{14}\begin{pmatrix} 2 & 4 & -5 \\ 2 & 4 & 4 \\ 2 & -10 & 4 \end{pmatrix}$$

B^{-1} ni to'g'ri topilganligini tekshirish uchun $B \cdot B^{-1} = E$ ekanini isbotlaymiz:

$$B \cdot B^{1} = \frac{1}{14}\begin{pmatrix} 4 & 3 & 2 \\ 0 & 1 & -1 \\ -2 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 4 & -5 \\ 2 & 4 & 4 \\ 2 & -10 & 4 \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 14 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 14 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = E$$

4. Quyidagi tenglamalar sistemasi Kramer formulasi va teskari matritsa usuli bilan yechilsin:

1) $\begin{cases} 3x_1 + 2x_2 = 0 \\ 2x_1 - x_2 = -7 \end{cases}$ 2) $\begin{cases} x_1 + x_2 + x_3 = 9 \\ 2x_1 - x_2 + x_3 = 5 \\ x_1 + 2x_2 - x_3 = 4 \end{cases}$

Yechish: 1. $\begin{cases} 3x_1 + 2x_2 = 0 \\ 2x_1 - x_2 = -7 \end{cases}$

a) berilgan tenglamalar sistemasini Kramer formulasi ([9]) yordamida echamiz; sistemani asosiy determinantini hisoblaymiz:

$\Delta = \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} = -3 - 4 = -7 \neq$; endi Δx_1 va Δx_2 larni xisoblaymiz

$\Delta x_1 = \begin{vmatrix} 0 & 2 \\ -7 & -1 \end{vmatrix} = 14$; $\Delta x_2 = \begin{vmatrix} 3 & 0 \\ 2 & 7 \end{vmatrix} = -21$; $x_1 = \frac{\Delta x_1}{\Delta} = \frac{14}{-7} - 2$; $x_2 = \frac{\Delta x2}{\Delta} = \frac{-21}{-7} 3$

Demak berilgan sistemani yechimi (-2;3) ekan.

Endi berilgan sistemani teskari matritsa usuli bilan yechamiz:

$A=\begin{pmatrix}3 & 2\\ 2 & -1\end{pmatrix}$, $\det A=\Delta=-7\neq 0$ bo'lganidan A^{-1} mavjud.

A^{-1} ni topamiz ; $\begin{matrix}A_{11}=-1 & A_{21}=-2\\ A_{12}=-2 & A_{22}=3\end{matrix}$; $A^{-1}=\frac{1}{-7}\begin{pmatrix}-1 & -2\\ -2 & 3\end{pmatrix}$

$X--=\begin{pmatrix}x_1\\ x_2\end{pmatrix}=A^1\cdot B=-\frac{1}{7}\begin{pmatrix}-1 & -2\\ -2 & 3\end{pmatrix}\cdot\begin{pmatrix}0\\ -7\end{pmatrix}=-\frac{1}{7}\begin{pmatrix}14\\ -21\end{pmatrix}=\begin{pmatrix}-2\\ 3\end{pmatrix}=\begin{pmatrix}x_1\\ x_2\end{pmatrix}$

$x_1=-2,\quad x_2=3$ yoki (-2;3)

2) a) berilgan tenglamalarni sistemasini Kramer formulasi bilan yechamiz:

$\Delta=\begin{vmatrix}1 & 1 & 1\\ 2 & -1 & 1\\ 1 & 2 & -1\end{vmatrix}=1+4+1+1-2+2=7\neq 0$ endi $\Delta x_1,\Delta x_2,\Delta x_3$ larni hisoblaymiz.

$\Delta x_1=\begin{vmatrix}9 & 1 & 1\\ 5 & -1 & 1\\ 4 & 2 & -1\end{vmatrix}=9+10+4+4-18+5=32-18=14;$ $X_1=\frac{\Delta x_1}{\Delta}=\frac{14}{7}=2;$

$\Delta x_2=\begin{vmatrix}1 & 9 & 1\\ 2 & 5 & -1\\ 1 & 4 & -1\end{vmatrix}=-5+8+9-5-4+18=35-14=21$ $X_2=\frac{\Delta x_2}{\Delta}=\frac{21}{7}=3;$

$\Delta x_3=\begin{vmatrix}1 & 1 & 2\\ 2 & -1 & 5\\ 1 & 2 & 4\end{vmatrix}=-4+36+5+9-10-8=50-22=28$ $X_3=\frac{\Delta x_3}{\Delta}=\frac{28}{7}=4;$

Demak, berilgan sistema yechimi (2;3;4) ekan. Endi ushbu sistemani teskari matritsa usuli bilan yechamiz:

$A=\begin{pmatrix}1 & 1 & 1\\ 2 & -1 & 1\\ 1 & 2 & -1\end{pmatrix}$, $\det A=\Delta=7\neq 0$ xos matritsadir. A^{-1} matritsani topamiz:

$$A_{11}=M_{11}=\begin{vmatrix}-1 & 1\\ 2 & -1\end{vmatrix}=-1; \qquad A_{21}=M_{21}=\begin{vmatrix}1 & 1\\ 2 & -1\end{vmatrix}=3; \qquad A_{31}=M_{31}=\begin{vmatrix}1 & 1\\ -1 & 1\end{vmatrix}=2;$$

$$A_{12}=M_{12}=\begin{vmatrix}2 & 1\\ 1 & -1\end{vmatrix}=3; \qquad A_{22}=M_{22}=\begin{vmatrix}1 & 1\\ 1 & -1\end{vmatrix}=-2;$$

$$A_{32}=-M_{23}=-\begin{vmatrix}1 & 1\\ 2 & 1\end{vmatrix}=1;$$

$$A_{13}=M_{13}=\begin{vmatrix}2 & -1\\ 1 & 2\end{vmatrix}=5; \qquad A_{23}=M_{23}=-\begin{vmatrix}1 & 1\\ 1 & 2\end{vmatrix}=-1;$$

$$A_{33}=M_{33}=\begin{vmatrix}1 & 1\\ 2 & -1\end{vmatrix}=-3;$$

$$A^{-1}=\frac{1}{7}\begin{pmatrix}-1 & 3 & 2\\ 3 & -2 & 1\\ 5 & -1 & -3\end{pmatrix},\ A\cdot A^{-1}=\frac{1}{7}\begin{pmatrix}1 & 1 & 1\\ 2 & -1 & 1\\ 1 & 2 & 1\end{pmatrix}\begin{pmatrix}-1 & 3 & 2\\ 3 & -2 & 1\\ 5 & -1 & 3\end{pmatrix}=$$

$$=\frac{1}{7}\begin{pmatrix}7 & 0 & 0\\ 0 & 7 & 0\\ 0 & 0 & 7\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$X--=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=A^{-1}B=\frac{1}{7}\begin{pmatrix}-1 & 3 & 2\\ 3 & -2 & 1\\ 5 & -1 & 3\end{pmatrix}\begin{pmatrix}9\\ 5\\ 4\end{pmatrix}=\frac{1}{7}\begin{pmatrix}14\\ 21\\ 28\end{pmatrix}=\begin{pmatrix}2\\ 3\\ 4\end{pmatrix}; \qquad \begin{matrix}x_1=2\\ x_2=3\\ x_3=4\end{matrix}$$

Tenglamalar sistemasini har ikki usul bilan yechsak ham, sistema yechimlari bir xil bo'lar ekan.

5. $M_1(4;3)$, $M_2(-3;4)$, $M_3(-3;-3)$ nuqtalarni yasang va $|M_1M_2|, |M_1M_3|, |M_2M_3|$ larni toping.

Yechish: Maktab kursidan ma'lumki, $M_1(4;3)$ nuqtani yasash uchun OX o'qidan 4 kesma OY o'qidan 3 kesma ajratib, topilgan kesmalarni oxiriga koordinata o`qlariga parallel qilib, uzuq to'g'ri chiziqlar o'tkaziladi, ularni kesishgan nuqtasi Dekart koordinata sistemasida $M_1(4;3)$ nuqtalarni aniqlaydi.

Endi M_1M_2, M_1M_3, M_2M_3 kesmalarni uzunliklarini topamiz. $(x_{1;}y_1)$va V$(x_2;y_2)$nuqtalari orasidagi masofa $d=|AB|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ formula bilan ([1])

topilar edi, M_1 va M_2 nuqtalar orasidagi masofani topsak, $x_1 = 4,\ y_1 = 3,\ x_2 = -3,$ $y_2 = 4$ bo'lganidan

$$|M_1M_2| = \sqrt{(-3-4)^2 + (4-3)^2} = \sqrt{(-7)^2 + 12} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$$

Xuddi shuningdek

$$|M_1M_3| = \sqrt{(-3-4)^2 + (-3-4)^2} = \sqrt{49+36} = \sqrt{85}$$

$$+(-3-4)^2 = \sqrt{0+72} = 7$$

$$|M_2M_3| = \sqrt{(-3-(-3))^2 + (-3-4)^2} = \sqrt{0+7^2} = 7$$

6. A (3;4;5) , V(-3;-2;3) , S(-5;3;-3) va D(2;-3;-4) nuqtalarni yasang va qaysi oktantda joylashganini aniqlang.

Yechish: Fazoda A nuqtani aniqlash uchun OX va OY o`qlaridan A nuqtani abstsissa va ordinatalariga teng kesmani topib olamiz va kesmaning oxirgi nuqtalaridan OX va OY uklariga parallel qilib, uzuq chiziqlar o'tkazib, ularni kesishishi nuqtasini topamiz. Sungra shu topilgan nuqtadan OZ o'qiga parallel kilib , uzuk chiziq o'tkazamiz va A nuqtani aplikatasini ishorasiga qarab shu topilgan nuqtadan boshlab uchunchi koordinatasiga teng kesma topamiz, shu topilgan kesmani oxiri A nuqtani bildiradi. Qolgan V,S va D nuqtalari ham xuddi shunday aniqlanadi.

A nuqta I – oktantda joylashgan, chunki hamma koordinatalari musbat. V nuqta II oktantda joylashgan, chunki $X = 0, Y\langle 0$ va S nuqta VI- oktantda joylashgan, chunki $x\langle 0, y\rangle 0, z\langle 0,$ D nuqta esa VII – oktantda joylashgan, chunki $x\rangle 0, y\langle 0, z\langle 0$.

7. $\overline{n_1} = 4\overline{i} - 5\overline{j} + 6\overline{k},$ $\overline{n_2} = 3\overline{i} + 5\overline{j} + 4\overline{k}$ $\overline{n_3} = -3\overline{i} + 4\overline{j} - 5\overline{k},$ vektorlarni yasang va $\overline{n_1} + \overline{n_2}$, $2\overline{n_1} + \overline{n_2}$, $3\overline{n_1} + 2\overline{n_2} - \overline{n_3}$ vektorlarni toping.

Yechish: $\overline{n_1} = x_1\overline{i} + y_1\overline{j} + z_1\overline{k},$ vektorni yasash uchun shu vektorni oxirgi nuqtasi M($x_1; y_1; z$) nuqtani yasab, so`ngra koordinata boshi bilan M nuqtani birlashtirib, $\overline{OM}$ vektor yasaladi. Masalan, $\overline{n_1} = 4\overline{i} - 5\overline{j} + 6\overline{k}$ vektorni yasash uchun $M_1(4;-5;6)$ nuqtani topib, so`ngra O nuqta bilan birlashtirib, $\overline{OM_1}$ vektorni hosil qilamiz. Endi $\overline{n_1} + \overline{n_2}$, $2\overline{n_1} - \overline{n_2}$ va $3\overline{n_1} + 2\overline{n_2} - \overline{n_3}$ vektorlarni yasash uchun

komponentalari bilan berilgan vektorlarning ustida chiziqli amallarni topamiz:

$$\begin{array}{ll} \bar{n}_1 = 4\bar{i}+5\bar{j}+5\bar{k} & 2\bar{n}_1 = 8\bar{i}-10\bar{j}+12\bar{k} \\ \bar{n}_2 = 3\bar{i}+5j+4k & \bar{n}_2 = 3\bar{i}+5\bar{j}+4k \\ \hline \bar{n}_1+\bar{n}_2 = 7\bar{i}+10\bar{k}; & -\overline{2n_1-n_2 = 5\bar{i}-15\bar{j}+16\bar{k}}; \end{array}$$

$$\begin{array}{r} 3\bar{n}_1 = 12\bar{i}-15\bar{j}+18\bar{k} \\ 2\bar{n}_2 = 6\bar{i}+10\bar{j}+8\bar{k} \\ -\bar{n}_3 = 3\bar{i}-4\bar{j}+5k \\ \hline 3\bar{n}_1+2\bar{n}_2-\bar{n}_3 = 21\bar{i}-9\bar{j}+31\bar{k} \end{array}$$

$\bar{n}_1+\bar{n}_2 = 7\bar{i}+10\bar{k}$, $2\bar{n}_1-\bar{n}_2 = 5\bar{i}-15\bar{j}+16\bar{k}$, $3\bar{n}_1+2\bar{n}_2-\bar{n}_3 = 21\bar{i}-9\bar{j}+31\bar{k}$ vektorlar yukoridagi kabi yasaladi.

8. $\bar{n}_1 = 3\bar{i}+4\bar{j}-2\bar{k}$, $\bar{n}_2 = 3\bar{j}-4\bar{k}$, $\bar{n}_3 = 5\bar{i}+3\bar{j}$ vektorlar berilgan. $(\bar{n}_1,\bar{n}_2)$, $(2\bar{n}_1,3\bar{n}_3)$, $[\bar{n}_2,\bar{n}_3]$, $([\bar{n}_3,\bar{n}_2]\bar{n}_1)$ lar topilsin.

Yechish: $(\bar{n}_1,\bar{n}_2)$ ni hisoblaymiz. Komponentalari bilan berilgan ikki vektorni skalyar -ko'paytmasi mos komponentalari ko'paytmasining yig'indisiga teng edi:
$(\bar{a},\bar{в}) = x_1x_2+y_1y_2+z_1z_2$
$(\bar{n}_1,\bar{n}_2) = 3\cdot 0+4\cdot 3+(-2)\cdot(-4) = 12+8 = 20$;

$(2\bar{n}_1,3\bar{n}_3)$ ni hisoblaymiz. Kompanentalari bilan berilgan ikki vektorlarni ko'paytmasi

$[\bar{a},\bar{в}] = \begin{vmatrix} \bar{i} & \bar{y} & \bar{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$ formula bilan hisoblanar edi.

$$[\bar{n}_1,\bar{n}_2] = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & 4 & -2 \\ 0 & 3 & -4 \end{vmatrix} = (-4)\cdot(4)\bar{i}+3\cdot 3\cdot\bar{k}-(-2)\cdot 3\bar{i}-(-4)3\bar{j} = -16\bar{j}+9\bar{k}-6\bar{i}+12\bar{j} = -22\bar{i}+12\bar{j}+9\bar{k}$$

Endi uch vektorni aralash ko'paytmasini, ya'ni $([\bar{n}_3,\bar{n}_2]\bar{n}_1)$ xisoblaymiz;

$\bar{a}$, $\bar{в}$, $\bar{c}$ uch vektorni aralash ko'paytmasi

$([\bar{a},\bar{в},]\bar{c}) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$ formula bilan hisoblanar edi;

$$\left(\left[\overline{n_3},\overline{n_1}\right]\overline{n_1}\right)=\begin{vmatrix}5 & 3 & 0\\ 0 & 3 & -4\\ 3 & 4 & -2\end{vmatrix}=5\cdot3\cdot(-2)+3\cdot3\cdot(-4)-5\cdot4\cdot(-4)=-30-36+80=-66+80=14$$

9. A(7;0;0), V(0;-4;0), S(0;0;3), D(4;3;-3) nuqtalar berilgan. Uchlari A,V,S,D nuqtalarda bo'lgan piramidani yasang va quyidagilarni toping:

1) ΔABC yuzasini va perimetirini va $\angle CBA$ ni toping.
2) Piramida hajmini toping;
3) D uchidan AVS asosga tushirilgan balandligini toping.

Yechish : A,V, S va D nuqtalarni topib ularni birlashtirsak, AVSD piramida hosil bo'ladi. 1) $S\Delta ABC$ ni topish uchun $\overline{AB}$ va $\overline{AC}$ vektorlarni komponentlarini topamiz.
$\overline{AB}=(0-Z)\bar{i}+(-4-0)\bar{j}+(0-0)\bar{k}=-7\bar{i}-4\bar{j}$; $\overline{AC}=-7\bar{i}+3$ endi $\overline{AB}$ va $\overline{AC}$ vektorlarni ko'paytiramiz.

$$\left[\overline{AB},\overline{AC}\right]=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ -7 & -4 & 0\\ -7 & 0 & 3\end{vmatrix}=-12\bar{i}-28\bar{k}+21\bar{j}=-12\bar{i}+21\bar{j}-28\bar{k}$$

$$\left[\overline{AB},\overline{AC}\right]=\sqrt{(-12)^2+(21)^2+(-28)^2}=\sqrt{144+441+784}=\sqrt{1369}$$

$$S\Delta ABC\ \frac{1}{2}\left\|\left[\overline{AB},\overline{AC}\right]\right\|=\frac{\sqrt{1369}}{2}=\frac{37}{2}\ кв.б$$

Endi ΔABC ni perimetirini topamiz, $\left|\overline{AB}\right|=\sqrt{49+16}=\sqrt{65}$, $\left|\overline{ac}\right|=\sqrt{49+9}=\sqrt{65}$, $\left|BC\right|=\sqrt{0^2+4^2+3^2}=5$; p=$\left|\overline{AO}\right|+\left|\overline{AC}\right|+\left|\overline{BC}\right|=2\sqrt{65}+5$;

$$COS\angle A=\frac{\left(\overline{AB},\overline{AC}\right)}{\left|\overline{AB}\right|\left|AC\right|}=\frac{49-12}{\sqrt{65\cdot5}}=\frac{37}{5\sqrt{65}},\quad \angle A=ar\cos\left(\frac{37}{5\sqrt{65}}\right)$$

2) Endi uchlari A,V,S,D nuqtalarda bo'lgan piramida hajmini topamiz; Ma'lumki

$\overline{AB},\overline{AC},\overline{AD},$ vektorlarga ko`rilgan piramida hajmi shu uch vektor aralash ko'paytmasi absolyut miqdorini $\frac{1}{6}$ bo'lagiga teng; $\overline{AD}=-3\bar{i}+3\bar{j}-3\bar{k}$ bo'lganidan

$$Vnn=\pm\frac{1}{6}\begin{vmatrix}-7 & -4 & 0\\ -7 & 0 & 3\\ -3 & 3 & -3\end{vmatrix}=\pm\frac{3}{6}\begin{vmatrix}-7 & -4 & 0\\ -7 & 0 & 3\\ -1 & 1 & -1\end{vmatrix}=\pm\frac{1}{2}(12+21+28)=\frac{61}{2}\text{ k.b.}$$

3) Piramidaning D uchidan AVS asosga tushirilgan balandlikni h deb belgilasak,

$V = \frac{1}{3} Saaco \cdot h = \frac{1}{3}\frac{37}{2} \cdot h = \frac{61}{2}$ yoki $\frac{37}{3}h = 61;$ $h = \frac{61 \cdot 3}{37} = \frac{183}{37}$ *бирлик*

10.Quyidagi to'g'ri chiziqlarni yasang: 5x+4y-20=0; 3x-2y+12=0, 5x-6y = 0, 3x-9 = 0, 3y+6 = 0.

Yechish. To'g'ri chiziq umumiy tenglamasi bilan berilganda, uning tenglamasini kesmalar shaklldagi tenglamaga keltirib yoki to'g'ri chiziq ustida ikkita nuqta topib yasash mumkin; $5x + 4y = 20;$ $\frac{5x}{20} + \frac{4y}{20} = 1,$ $\frac{x}{4} + \frac{y}{5} = 1;$ yoki x=0 desak y=0 va y=0 desak x=4; $3x - 2y = -12;$ $\frac{3x}{-12} - \frac{2y}{-12} = 1;$ $\frac{x}{-4} + \frac{y}{6} = 1.$

Endi $5x - 6y = 0$ to'g'ri chiziqni yasaymiz; bu to'g'ri chiziq koordinata boshidan o`tadi, uning ustida yotgan bita nuqta topamiz; y=5 desak x=6 bo'ladi. 3x-4=0 va 3x+6=0 to'g'ri chiziqlar x=3 va u=-2 bo'lib mos ravishda OX o'qidan 3 birlik ajratib OY o'qiga parallel, OY o'qidan -2 birlik kesma ajratib, OX o'qiga parallel to'g'ri chiziqdir.

11. $M_o(3;-5)$ nuqtadan o'tib $\bar{n} = 2\bar{i} + 5\bar{j}$ vektorga parallel va perpendikulyar bo'lgan to'g'ri chiziqlar tenglamasini tuzing va yasang

Yechish. M_o nuqtadan o'tib to'g'ri chiziq $\bar{n}$ vektorga parallel bo'lsa, $\bar{n}$ vektor to'g'ri chiziqni yo'naltiruvchi vektori bo'ladi, ya'ni $\frac{x - x0}{m} = \frac{y - y0}{n};$ x_0=3, y_0=-5, m=2; n=5 $\frac{x-3}{2} = \frac{y+5}{5}$

Endi M_o nuqtadan o'tib $\bar{n}$ vektorga perpendikulyar bo'lgan to'g'ri chiziq tenglamasini yozamiz; $A(x - x_0) + B(y - y_0) = 0$ $x_0 = 3,\ y_0 = -5,$ A=2, B=5

$M_o(3;5),\ 2(x-3) + 5(y+5) = 0,$ $2x + 5y + 19 = 0$

12. OY o'qidan v=4 birlik kesma ajratib OX o'qining musbat yunalishi bilan α=60^0 burchak ajratgan to'g'ri chiziq tenglamasini tuzing va yasang.

Yechish: Ma'lumki OX o'qining musbat yo`nalishi bilan α burchak tashkil qilib, OY o'qidan v kesma ajratgan to'g'ri chiziq tenglamasi $y = kx + b,$ $k = t\ g$ edi. $k = tg60^0 = \sqrt{3}$

bo'lganidan biz izlayotgan to'g'ri chiziq tenglamasi. $y=\sqrt{3}x+4$

13. $3x-4y-7=0$ va $x+3y+8=0$ to'g'ri chiziqlar orasida burchakni toping.

Yechish: Umumiy tenglamalari $A_1x+B,Y+C_1$=0 va $A_2x+B_2y+C_2=0$ bilan berilgan ikki to'g'ri chiziq orasidagi burchakni tangensi

$tg\alpha=\dfrac{A_2B_1-A_1B_2}{A_1A_2+B_1B_2}$ formula bilan hisoblanar edi.

$tg\alpha=\dfrac{1\cdot(-4)-3\cdot3}{3\cdot1+(-4)(3)}=\dfrac{-4-9}{3-12}=\dfrac{-13}{-9}=\dfrac{13}{9}\qquad \alpha=arctg\left(\dfrac{13}{9}\right)$

$3x-4y-7=0;\quad x=o,\quad y=-\dfrac{7}{4};\quad y=0,\quad x=\dfrac{7}{3.}$

$x+3y+8=0;\quad x=o,\quad y=-\dfrac{8}{3};\quad y=0,\quad x=-8.$

14. Uchlari A(5;0), V(0;-3) va S(-2;-5) nuqtalarda bo'lgan uchburchakning ichki burchaklarini toping.

Yechish: Uchlari A,B.C nuqtalari bo'lgan uchburchakning ichki burchaklarini topish uchun A va B; A va C; C va B nuqtalardan o`tuvchi to'g'ri chiziq tenglamalarini tuzamiz va ketma-ket ular orasidagi burchaklarni topamiz;

$AB: y=o;\quad AC:\dfrac{x-5}{-2-7}=\dfrac{y}{-5};\quad \dfrac{x-5}{-7}=\dfrac{y}{-5};\quad 5(-5)=7y;$

$5x-7y-25=0\quad CB:\dfrac{x-0}{-2-0}=\dfrac{y+3}{-5};\quad \dfrac{x}{-2}=\dfrac{y+3}{-5}$

$5x-2y-6=0$

Endi BA va CA orasidagi $\angle A$ burchakni, so`ngra BC va CA orasidagi $\angle C$ ni va nixoyat AB va BC orasidagi $\angle B$ ni topamiz.

1) $\begin{cases} y=0 \\ 5x-7y-25=0 \end{cases}$ 2) $\begin{cases} 5x-2y-6=0 \\ 5x-7y-25=0 \end{cases}$ 3) $\begin{cases} y=0 \\ 5x-2y-6=0 \end{cases}$

$tg\angle A=\dfrac{5\cdot1-0}{0+(-7)\cdot1}=-\dfrac{5}{7};$ $tg\angle B=\dfrac{5\cdot1-0}{0+(-2)\cdot1}=-\dfrac{5}{7};$ $tg\angle C=\dfrac{-5\cdot2-(-7)\cdot5}{25+14}=\dfrac{25}{39};$

$\angle A = arctg\left(\frac{5}{7}\right);\quad \angle C = arctg\left(\frac{25}{39}\right);\quad \angle B = arctg\left(\frac{5}{2}\right);$

$\angle A = 35^0 30^1 \qquad \angle C = 88^0 30^1 \qquad \angle B = 56^0$

$\angle A + \angle C + \angle B = 35^0 30^1 + 88^0 30^1 + 56^0 = 180^0$

15. $7x - 8y + 56 = 0$ va $3x + 7y - 21 = 0$ to'g'ri chiziqlarni kesishish nuqtasi topilsin.

Yechish: Ikkita to'g'ri chiziqning kesishishi nuqtasi har ikki to'g'ri chiziqqa ham tegishli bo'lganligidan, ularning tenglamalarini sistema qilib yechamiz;

$\begin{cases} 7x - 8y = -56 \\ 3x + 7y = 21 \end{cases}$ bu tenglamalar sistemasini Kramer usuli bilan yechamiz: 1) $\Delta = \begin{vmatrix} 7 & -8 \\ 3 & 7 \end{vmatrix} = 49 + 24 = 73 \neq 0$

2) $\Delta_x = \begin{vmatrix} -56 & -8 \\ 21 & 7 \end{vmatrix} = -392 + 168 = -224$

$\Delta_y = \begin{vmatrix} 7 & -56 \\ 3 & 21 \end{vmatrix} = 147 + 168 = 315$

$x = \frac{\Delta x}{\Delta} = -\frac{224}{73} = -3{,}07; \qquad y = \frac{315}{73} = 4{,}3 \quad (-3{,}07;4{,}3)$

Endi to'g'ri chiziqlarni yasaymiz:

$\frac{x}{-8} + \frac{y}{7} = 1$ va $\frac{x}{7} + \frac{y}{3} = -1$

16. $M_1(-4;5)$ nuqtadan $3x + 4y - 8 = 0,\quad 4x - 3y + 5 = 0$ to'g'ri chiziqlargacha bo'lgan masofalarni toping.

Yechish. Avvalo M_1 nuqta va berilgan to'g'ri chiziqlarni yasaymiz.

$\frac{x}{\frac{8}{3}} + \frac{y}{2} = 1, \qquad \frac{x}{\frac{-5}{4}} + \frac{y}{\frac{5}{3}} = 1$

Endi berilgan to'g'ri chiziqlardan M_1 nuqtagacha bo'lgan masofadan chiziq tenglamalarini normal ko'rinishga keltiramiz:

$3x+4y-8=0, \qquad 4x-3y+5=0$

$\mu_1=\frac{1}{\sqrt{9+16}}=\frac{1}{5}; \qquad \mu_2=\frac{1}{\sqrt{9+16}}=\frac{1}{5};$

$\frac{3x+4y-8}{5}=0; \qquad \frac{4x-3y+5}{-5}=0;$

$dm_1=\left|\frac{3(-4)+4\cdot 5-8}{5}\right|=0, \qquad dm_2=\left|\frac{4(-4)-3\cdot 5+5}{-5}\right|=\left|\frac{-31+5}{-5}\right|=\frac{29}{5};$

$dm_1=0$ bo'lgani uchun $M_1(-4;5)$ nuqta $3x+4y-8=0$ to'g'ri chiziq ustida yotar ekan.

17. Koordinata o`qlaridan mos ravishda $a=4$ va $b=5$ kesmalar ajratgan to'g'ri chiziq tenglamasi tuzilsin va yasalsin.

Yechish: Koordinata o`qlaridan a va v kesmalar ajratgan to'g'ri chiziq tenglamasi ([1],137*b*.) $\frac{x}{a}+\frac{y}{b}=1$ ko'rinishda edi, ya'ni $\frac{x}{4}+\frac{7}{-5}=1$ yoki $-5x+4y=20, \quad 5x-4y-20=0;$

18. Quyidagi $x+3y-7=0$ va $2x+6y+11=0$ va parallel to'g'ri chiziqlar orasidagi masofani toping.

Yeshish: Ikki parallel to'g'ri chisiqlar orasidagi masofani topishg uchun bittasi ustida yotuvchi nuqta topib, shu nuqtadan ikkinshi to'g'ri chiziqqacha masofani hisoblash kifoya.

$x+3y-7=0$ to'g'ri chiziq ustida yotuvchi nuqtani topamiz; x=1 bo`lsa, u=2 ya'ni $M_1(1;2)$

Endi $M_1(1;2)$ nuqtadan $2x+6y+11=0$ to'g'ri chiziqqacha bo'lgan masofani topamiz. Buning uchun $2x+6y+11=0$ tenglamani normal ko'rinishga keltiramiz: $\mu=-\frac{1}{\sqrt{4+36}}=-\frac{1}{2\sqrt{10}}$

Demak, $-\frac{2x+6y+11}{2\sqrt{10}}=0$ yoki $-\frac{x}{\sqrt{10}}-\frac{3y}{\sqrt{10}}-\frac{11}{\sqrt{10}}=0$

$dm_1=\left|-\frac{1}{\sqrt{10}}-\frac{6}{\sqrt{10}}-\frac{11}{\sqrt{10}}\right|=\left|-\frac{18}{\sqrt{10}}\right|=\frac{18}{\sqrt{10}}$

19. $x^2+y^2-4x+6y-12=0$ tenglama bilan ifodalangan chiziqni yasang.

Yechish: Bu tenglamani x va y ga nisbatan to`liq kvadratini ajratamiz;
$x^2-2\cdot 2x+4-4+y^2+2\cdot 3y+9-9-12=0.\ (a\pm b)^2=a^2\pm 2ab+b^2$
formulaga asosan $(x-2)^2+(y+3)^2=25$ tenglama hosil bo'ladi. Bu tenglama markazi $M_0(2;-3)$ nuqtada va radusi R=5 , bo'lgan aylananing kononik tenglamasidir.

20. $9x^2+4y^2=36$ tenglama bilan ifodalangan chiziqni yasang.

Yechish : $9x^2+4y^2=36$ tenglamani , har ikki tomonlama 36 ga bo'lib kanonik ko'rinishga keltiramiz:

$\frac{9x^2}{36}+\frac{4y^2}{36}=1$ yoki $\frac{x^2}{4}+\frac{y^2}{36}=1.$ $\frac{x^2}{4}+\frac{y^2}{9}=1$ tenglik yarim o`qlari a=2, b=3 bo'lgan ellipsning kanonik tenglamasidir. $a\langle b$ bo'lgani uchun bu ellipsni uchlari fokuslari OY o'qida yotadi. Bu ellipsni uchlari $A_1(+2;0), A_2(-2;0),$ $B_1(0;2)$ va $B_2(0;-2)$ nuqtalarda bo'ladi. Endi $x=\pm 2$ va $y=\pm 3$ to'g'ri chiziqlari yasab parallelogram hosil qilamiz.

Bu ellips uchlari A_1, A_2, B_1, B_2 nuqtalarda bo'lib yasalgan parallelogram ichida yotadi.

21. $2x^2+3y^2=20$ va $x-y=0$ chiziqlarni kesishishi nuqtasini toping va chiziqlarni yasang.

Yechish: Ikki chiziqning kesishish nuqtasi ulrani har ikkalasiga ham tegishli bo'lganidan bu tenglamalarni sistema qilib yechamiz.

$\begin{cases}2x^2+3y=20\\ x-y=0\end{cases}$, $y=x$, $2x^2+3x^2=20$, $x^2=4$, $x_{12}=\pm 2.$

$y=\pm 2$, Ya'ni bu chiziqlar $M_1(-2;-2), M_2(2;2)$ nuqtalarda kesishar ekan. Endi bu chiziqlarni yasaymiz. $2x^2+3y^2=20$ yoki $\frac{2x2}{20}+\frac{3y^2}{20}=1$ yoki $\frac{x^2}{10}+\frac{y^2}{\frac{20}{3}}=11$

Demak, $2x^2+3y^2=20$ tenglama yarim o`qlari $a=\sqrt{10}$, $b=\sqrt{\frac{20}{3}}$ bo'lgan ellipiss ekan. $2x^2+3y^2=20$ ellpis va $x-y=0$ to'g'ri chiziqni yasaymiz.

22. 1) $2x+3y+4z-12=0$, 2) $2x+3y-8=0$, 3) $2x+zy-12=0$,
4) $2x-7=0$, 5) $3y+7=0$, 6) $4z-9=0$ tekisliklarni yasang.

Yechish : 1) $2x+3y+4z-12=0$ tenglamani kesmalar shakliga keltirsak, $\frac{x}{6}+\frac{4}{4}+\frac{z}{3}=1$ tenglama hosil bo'ladi.

2) $2x+3y-8=0$, $3x+4y-12=0$ tenglamalarga mos ravishda z va y qatnashmaganidan ular oz va oy o'qiga parallel tekisliklarini ifodalaydi;

$\frac{x}{4}+\frac{y}{\frac{8}{3}}=1;\quad \frac{x}{4}+\frac{z}{3}=1$

3) $2x-7=0$, 5) $3y+7=0$, $4z-y=0$ tengalamalar mos ravishda *yoz*, *xoz* va *xoy* tekisliklarga parallel tekisliklar tenglamalaridir, haqiqatdan

$x=\frac{7}{z};\quad y=\frac{7}{3};\quad z=\frac{9}{4};$

23. $M_o(4;-3;5)$ nuqtadan o'tib $\bar{n}=3\bar{i}-4\bar{j}+5\bar{k}$ vektorga perpendikulyar bo'lgan tekislik tenglamasini tuzing va yasang.

Yechish: $M_o(x_0;y_0;z_0)$ nuqtadan o'tib $\bar{n}=\bar{i}+B\bar{j}+C\bar{k}$ vektorga perpendikulyar bo'lgan tekislik tenglamasi $A(x-xo)+B(y-yo)+C(7\cdot 70)=0$ edi.

Demak, $x_0=4,\ yo=-3,\quad 70=5;\ A=3,\ B=-4,\ C=5$ bo'lgani uchun $3(x-4)+(-4)(y-(-3))+5(z-5)=0$ yoki $3x-12-4y-12+5z-25=0,\ 3x-4y+5z-49=0$ bo'ladi.

Bu tekislikni umumiy tenglamasidir. Uni kesmalar shaklidagi tenglamaga keltirib, so`ngra yasaymiz.

$\frac{3x}{49}-\frac{4y}{49}+\frac{5z}{49}=1$ yoki $\frac{x}{\frac{49}{3}}+\frac{y}{-\frac{49}{4}}+\frac{z}{\frac{89}{5}}=1$

24. $M_1(3;4;-2)$, $M_2(-3;-4;1)$, $M_3(5;-3;4)$ nuqtalardan o`tuvchi tekislik tenglamasi tuzilsin va yasalsin.

Yechish: Berilgan $M_1(x_1;y_1;z_1), M_2(x_2;y_2;z_2), M_3(x_3;y_3;z_3)$ nuqtalardan o`tuvchi tekislik tenglamasi ([1]156*b*.)

$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$ ko'rinishga ega edi,

$\begin{vmatrix} x-3 & y-4 & z-z_1 \\ -3-3 & -4-4 & 1+2 \\ 5-3 & -3-4 & 4+2 \end{vmatrix}=0;$ $\begin{vmatrix} x-3 & y-4 & z+2 \\ -6 & -8 & 3 \\ 2 & -7 & 6 \end{vmatrix}=0$ determinantni hisoblaymiz.

$-48(x-3)+42(z+2)+6(y-4)+16(z+2)+21(x-3)+36(y-4)=0$
$-27(x-3)+58(z+2)+42(y-4)=0;-27x+58z++42y+81+116-168=0$
$-27x+42y+58z+29=0$

Tenglamani kesmalar shakliga keltirib tekislikni yasaymiz. $\frac{x}{\frac{29}{27}}+\frac{y}{\frac{29}{42}}+\frac{z}{\frac{29}{58}}=1$

25. $M_0(5;-4;3)$ nuqtadan o'tib $\overline{n_1}=2\bar{i}+\bar{j}-3\bar{k}$, $\overline{n_2}=3\bar{i}+3\bar{k}$ vektorlarga perpendikulyar bo'lgan tekislik tenglamasi tuzilsin va yasalsin.

Yechish : Bu masalalarni yechish uchun M_0 nuqtadan o'tib normal vektori $\overline{n_1}=A\bar{i}+B\bar{j}+C\bar{k}$ bo'lgan tekislik tenglamasini yozamiz: $A(x-5)+B(y+4)+C(z-3)=0$

Masala sharti bo`yicha bu tekislik $\overline{n_1}$ va $\overline{n_2}$ vektorlarga perpendikulyar bulishi kerak , ya'ni $\begin{cases} A(x-5)+B \\ A\cdot 2+B\cdot 1+C(-3)=0 \\ B\cdot 3+C\cdot 4=0 \end{cases}$

Bu A,B va C ga nisbatan chiziqli bir jinsli tenglamalar sistemasi notrvial echimga ega bo`lishi uchun , bu sistemani asosiy determinanti nolga teng bo`lishi kerak, ya'ni,

$\begin{vmatrix} x-5 & y+1 & 7-3 \\ 2 & 1 & -3 \\ 0 & 3 & 4 \end{vmatrix}=0,$ $4(x-5)+6(z-3)+9(x-5)-8(y+1)=0$

$13(x+5)-8(y+1)+6(z-3)=0$ yoki

$13x-8y+6z=-39,$ $\frac{x}{-3}+\frac{y}{\frac{32}{8}}+\frac{7}{\frac{-39}{6}}1,$ Endi tekislikni yasaymiz.

26. $x+2y-2z+6=0$; $2x-y-2z-9=0$ tekisliklar orasidagi burchak topilsin.

Yechish:Berilgan $A_1x+B_1y+C_1z+D_1=0$, $A_2x+B_2y+C_1z+D_2=0$ tekisliklar orasida burchak kosinusi ([1]160б)

$$\cos\alpha=\frac{A_1A_2+B_1b_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}$$ formula bilan h isoblanar edi. $A_1=1, B_2=-1, C2=-2$ va $A_2=2, B_2=-1, C_2=-2$ bo'lganidan $Cos\alpha=\frac{1\cdot2+2\cdot(-1)+(-2)(-2)}{\sqrt{1+4+4}\sqrt{4+1+4}}=\frac{4}{9}$

$\alpha=arcas\left(\frac{4}{9}\right)\approx63^0 40^1$ Berilgan tekisliklarni umumiy tenglamalarini kesmalar shakldagi tenglamaga keltirib yasaymiz.

$x+2y-2z=-6;$ $\quad 2x-y-2z=9;$

$\frac{x}{-6}+\frac{y}{-3}+\frac{z}{3}=1$ $\quad \frac{x}{\frac{2}{2}}+\frac{y}{-9}+\frac{z}{\frac{9}{2}}=1$

27. $M(7;-3;4)$ nuqtadan o'tib $2x+3y-5z+6=0$ tekislikka parallel tekislik tenglamasini tuzing va yasang.

Yechish: $M_0(x_0;y_0;z_0)$ nuqtadan o'tib $Ax+By+Cz+D=0$ tekislikka parallel bo'lgan tekislik tenglamasi $A(x-x0)+B(y-y_0)+C(z+z_0)=0$ bo'lar edi. ([1,]153б.)

Demak, $A=2$, $B=3$, $C=-5$, $X_0=7$, $Y_0=7$, $Y_0=-3$, $Z_0=4$ bo'lganidan $2(x-7)+3(z+3)+(-5)(z-4)=0$ yoki $2x+3z-5z-14+9+20=0$ yoki $2x+3z-5z+15=0$

bu tekislikning umumiy tenglamasini kesmalar shakldagi tenglamaga keltirsak, $2x+3z-5z=0$ yoki $\frac{x}{-\frac{15}{2}}+\frac{y}{-5}+\frac{z}{3}=1$ bo'ladi. Koordinata o`qlaridan mos ravishda $a=-\frac{15}{2}$, $b=-5$ va $C=3$ kesmalar ajratgan tekislikni yasaymiz;

28. $M_0(4;-3;5)$ nuqtadan $x+3y-2z+9=0$ tekislikkacha bo'lgan masofa topilsin.

Yechish: Ma'lumki, ([1,]163б) $M_0(x_0;y_0;z_0)$ nuqtadan $Ax+By+Cz+D=0$ tekislikkacha bo'lgan masofa $d\mu_0=\left|\frac{Ax_0+By+Cz_0+D}{\pm\sqrt{A^2+B^2+C^2}}\right|$ formula bilan topilar edi.

$A=1,\ B=2,\ C=-2$ bo'lganidan

$$d\mu_0=\left|\frac{4+2(-3)-2\cdot5+9}{\pm\sqrt{1+4+4}}\right|=\left|\frac{4-6-10+9}{\pm3}\right|=\left|\frac{-3}{\pm3}\right|=1$$

Endi M_0 nuqtani topamiz va $x+2y-27+9=0$ tekislikni umumiy tenglamasini kesmalar shaklidagi tenglamaga keltirib yasaymiz: $x+2y-2z=-9,\ \frac{x}{-9}+\frac{y}{-\frac{9}{2}}+\frac{7}{\frac{3}{2}}=1$

29. $M_0(5;-4;3)$ nuqtadan o'tib $\bar{S}=4\bar{i}-3\bar{j}+5\bar{k}$ vektorga parallel bo'lgan to'g'ri chiziq tenglamasi tuzilsin va yasalsin.

Yechish : $M_0(x_0;y_0;z_0)$ nuqtadag o'tib $\bar{S}=m\bar{i}-n\bar{j}+p\bar{k}$ vektorga parallel bo'lgan to'g'ri chiziqni kanonik tenglamasi ([1]165*б*.)

$\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$ ko'rinishga ega edi.

$x_0=5,\ y=-4,\ z=3$ va $m=4,\ m=-3,\ p=5$ bo'lganidan

$$\frac{x-5}{4}=\frac{y+4}{-3}=\frac{z-3}{5}$$

Bu tenglama $M_0(5;-4;3)$ nuqtadan o`tuvchi va yo`naltiruvchi vektori $\bar{S}=4\bar{i}-3\bar{j}+5\bar{k}$ bo'lgan to'g'ri chiziqning kanonik tenglamasidir. Shu to'g'ri chiziqni yasaymiz.

30. $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z}{-2}$ to'g'ri chiziq va $x+2y+2z-9=0$ tekislik orasidagi burchak va ularni kesishi nuqtasi topilsin.

Yechish: $\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$ to'g'ri chiziq va $Am+Bn+Cp+D=0$ tekislik orasidagi burchak sinusi quyidagi ([1],172*б*) formula bilan topilar edi.

$$Sin\gamma=\frac{Am+Bn+CP}{\sqrt{m^2+n^2+n^2}\sqrt{A^2+B^2+C^2}}$$

$m=2,\ n=-2,\ p=-2,$ va $A=1,\ B=2, C=2$ bo'lganda

$$Sin\gamma=\frac{1\cdot2+2\cdot(-1)+2(-2)}{\sqrt{2^2+(-1)^2+(-2)^2}\sqrt{1^2+2^2+2^2}}=\frac{2-2-4}{9}\qquad Sin\gamma=-\frac{4}{9},$$

$$y=arcSin\left(-\frac{4}{9}\right)$$

Endi to'g'ri chiziq bilan tekislikning kesishish nuqtasini topamiz; buning uchun to'g'ri chiziqning kanonik tenglamasini parametrik ko'rinishga keltiramiz va x,y,z larga nisbatan yechib tekislik tenglamasidagi x,y,z urniga qo`yamiz, ya'ni

$\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z}{-2}=t,\quad x=2t+3,\quad y=-t-2,\quad z=-2t;\qquad 2t+3+2(-t-2)+2(-2t)-9=0,$

$2t+3-2t-4-4t-9=0,\ -4t-10=0,\quad t=-\frac{5}{2}.$

Topilgan t ning qiymatlarini $x=2t+3,\quad y=-t-2,\quad t=-2t$ tengliklarga qo`ysak

$$x=2\left(-\frac{5}{2}\right)+3=-5+3=-2$$

$$y=-\left(-\frac{5}{2}\right)-2=\frac{5}{2}-2=\frac{1}{2}$$

$$z=-2\left(-\frac{5}{2}\right)=5$$

Demak, $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z}{-2}$ to'g'ri chiziq va $x+2y+2z-9=0$ tekislik $A=\left(-2;\frac{1}{2};5\right)$ nuqtada kesishar ekan. Endi to'g'ri chiziq va tekislikni yasaymiz. $x+2y+2z-9=0$ yoki $\frac{x}{9}+\frac{y}{\frac{9}{2}}+\frac{z}{\frac{9}{2}}=1$

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