Tasodifiy jarayonlar nazariyasi.

Ma'ruzalar matni

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Ushbu o'quv qo'llanma "Tasodifiy jarayonlar nazariyasi " kursi bo'yicha ma'ruzalar matni saqlangan. Mazmuni tasodifiy jarayonlar kursi bo'yicha ikki semestrli ishchi dasturning ma'ruza qismiga mos keladi. Tasodifiy jarayonlarning asosiy matematik modelini va ularning modellashtirish usullari va analizi qaralgan. Hususiy holda tasodifiy jarayonlarning asosiy tushunchalari va termenalogiyasi, ehtimoli taqsimot va moment funksiyalari bog’liqsiz orttirmali jarayonlar, keng ma'noda statsionar jarayonlar va ularning korrelatsion va spektral harakteristikalari, diskret va uzluksiz vaqtli Markov zanjiri, Kolmogorov diffensial tenglamalari, vaqtli qatorlar va avtoregressini modeli- o'rtacha qaralgan.
Ma'ruzalar matni "matematika" yo'nalishida 7,8 semester "Tasodifiy jarayonlar nazariyasi " predmetini o'rganuvchi bakalavrlar tayyorlash uchun mo'ljallangan.
"Ehtimollar nazariya va matematik statistika matematika o'qitish metodikasi" kafedrasida ishlab chiqilgan.

Samarqand -2019

Tasodifiy jarayonlar nazariyasi

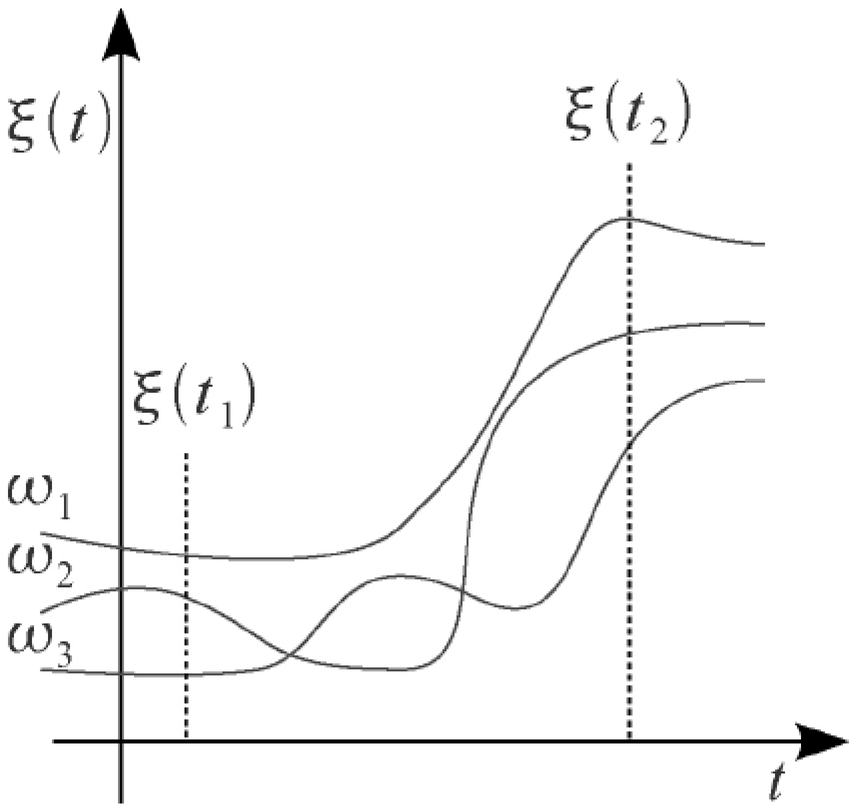
Ma’ruzalar matni

Mundarija

1.Tasodifiy funksiyalar va tasodifiy jarayon tushunchasi

1.1-ta'rif

t parametrdan bog'liq $\varepsilon:\Omega\to R^n$ elementar hodisalar fazosi Ω ni R^nga akslantiruvchi $\xi(t)$tasodifiy funksiya deyiladi



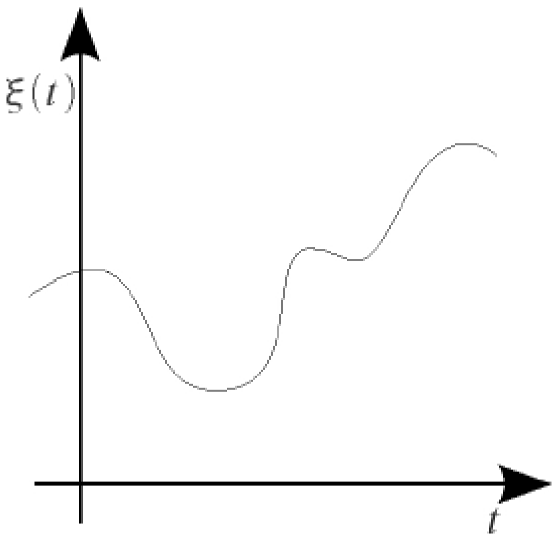
Eslatma.

$X_{t_1}=\xi(t_1)$ va $X_{t_2}=\xi(t_2)$ tasodifiy miqdorlarga tasodifiy jarayonlarning kesishmasi deyiladi. $\omega_1,\omega_2,\omega_3$ tasodifiy miqdorlar to'plamiga tasodifiy jarayonlarning taqsimoti deyiladi

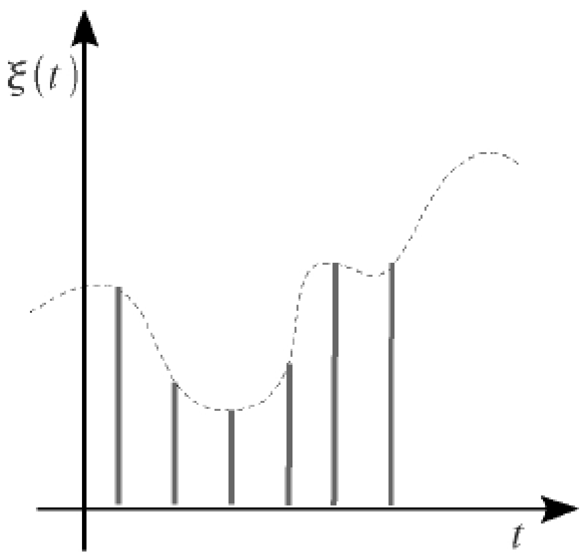
1.2. Ta'rif

Tasodifiy miqdor-bu qandaydir $\xi(t_1)=X(t)$ tasodifiy funksiya. Ixtiyoriy tasodifiy jarayon 4ta sinfga bo'linishi mumkin:

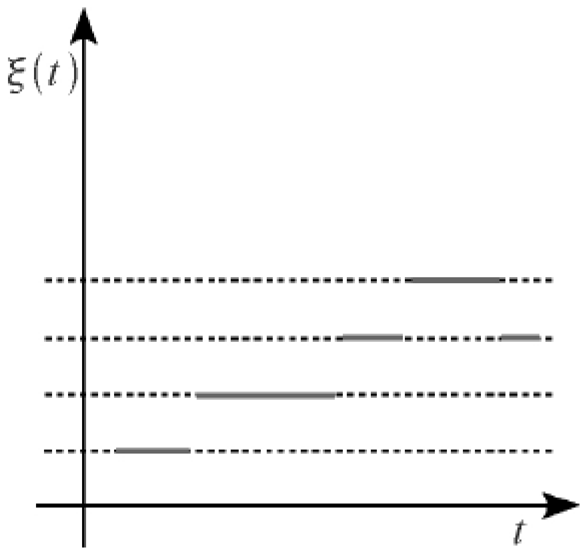
1) X -uzluksiz tasodifiy miqdor, t-uzluksiz parametr. Bunday jarayonlarga misol vaqt bo'yicha temperatura, vaqt bo'yicha avtomobil tezligi



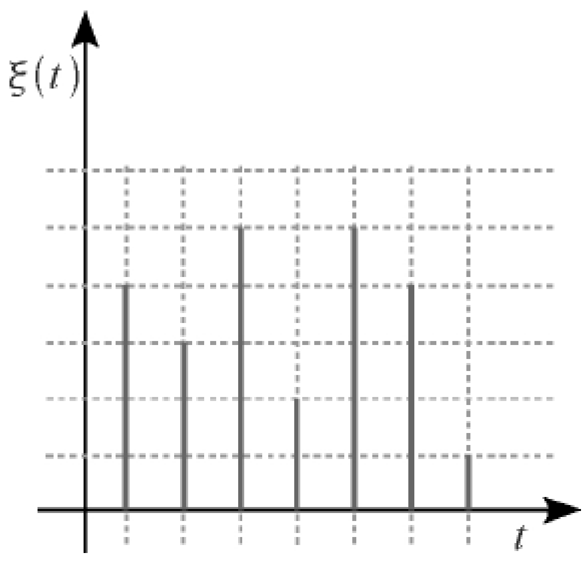
2) X -uzluksiz tasodifiy miqdor, t-diskret vaqt. Bu ham tasodifiy ketma-ketligi deyiladi. $\xi_n = \xi(t)\big|_{t=n+\Delta t}$ kesim bilan berilgan uzluksiz jarayonni diskretlashtirilganini misol bo'ladi.



3) X - diskret tasodifiy miqdor, t-uzluksiz parametr. Misollar- navbatdagi kishilar soni, radiaktiv so'nish, jarayoni.



4) X-diskret tasodifiy miqdor, t-diskret parametr. Misol - raqamlashtirilgan tovush.



Bir o'lchovli tasodifiy miqdorlar holatida

$F_X(x) = \mathsf{P}\{X < x\}$ taqsimot funksiyasi bu tasodifiy jarayonni harakteristikasi bo'ladi (ya'ni bunday harakteristika orqali bu tasodifiy miqdorning barcha parametrlari topilishi mumkin).

Tasodifiy vektor uchun tugatuvchi harakteristika

$$F_X(x_1, x_2, \dots, x_n) = \mathsf{P}\{X_1 < x_1, X_2 < x_2, \dots, X_n < x_n\}.$$

taqsimot funksiyasi bo'ladi. Shunga o'xshash tasodidiy jarayon uchun quyidagi tasdiqni ifodalash mumkin.

1.1-Tasdiq.

Tasodifiy jarayon quyidagi taqsimot funksiyasi:

$$F_\xi(t_1, \dots, t_n, x_1, \dots, x_n) = \mathsf{P}\{\xi(t_1) < x_1, \dots, \xi(t_n) < x_n\}.$$

bilan tugallanuvchi harakterlangan bo'lishi mumkin.
Berilgan taqsimot funksiyasi orqali

$$F_\xi(t_1, \dots, t_n, x_1, \dots, x_n) = \mathsf{P}\{\xi(t_1) < x_1, \dots, \xi(t_n) < x_n\},$$

Quyidagi taqsimot funksiyasini topamiz:

$$F_\xi(t_1, \dots, t_m, x_1, \dots, x_m) = \mathsf{P}\{\xi(t_1) < x_1, \dots, \xi(t_m) < x_m\}, \ m < n.$$

$\triangleleft$ zichlik funksiyasi ehtimoli uchun moslik kriteriyasiga ko'ra:

$$\iint\limits_{\mathbb{R}^n} \dots \int f_{t_1,\dots,t_m,t_{m+1},\dots,t_n}(x_1, \dots, x_n)\, dx_{m+1} \dots dx_n = f_{t_1,\dots,t_m}(x_1, \dots, x_m),$$

U holda taqsimot funksiyaga o'tib:

$$F_\xi(t_1, \dots, t_m, x_1, \dots, x_m) = F_\xi(t_1, \dots, t_n, x_1, \dots, x_m, \infty, \dots, \infty),$$

Berilgan funksiya orqali izlanuvchi taqsimot funksiyani to'la aniqlaydi.

2. Tasodifiy jarayonlarning moment funksiyasi

2.1-Ta'rif.
X -tasodifiy miqdorning boshlang'ich momenti deb,

$$m_k = \mathsf{M}X^k,$$

ga aytiladi.

2.2-Ta'rif.
X -tasodifiy miqdorning markaziy momenti deb,

$$\mu_k = \mathsf{M}(X - \mathsf{M}X)^k.$$

ga aytiladi.

2.3-Ta'rif.

Tasodifiy jarayonning boshlang'ich momenti deb,

$$m_k(t) = \mathsf{M}\xi^k(t).$$

ga aytiladi.

2.4-Ta'rif.

Tasodifiy jarayonning markaziy momenti deb,

$$\mu_k(t) = \mathsf{M}(\xi(t) - \mathsf{M}\xi(t))^k.$$

ga aytiladi.

Tasodifiy jarayonlarning muhim moment funksiyalari

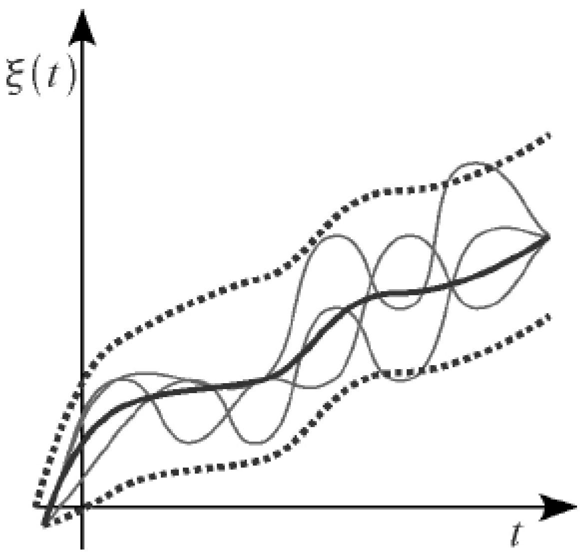
$$k = 1\text{:}\ \ m_1(t) = \mathsf{M}_\xi(t) = m(t)\ -$$

Tasodifiy jarayonni matematik kutilmasi;

$$k = 2\text{:}\ \ \mu_2(t) = \mathsf{D}_\xi(t) = m_2(t) - m_1^2(t) = \mathsf{M}\xi^2(t) - (\mathsf{M}_\xi(t))^2\ -$$

Tasodifiy jarayonning dispersiyasi.

Geometrik jihatdan matematik kutilmasi tasodifiy jarayonning barcha taqsimotlari uchun o'rtacha qiymatini, dispersiya esa bu qiymatlarni tarqalishini harakterlaydi.



Taqsimot funksiyasi harakteristika orqali tugallangadi, shuning uchun jarayonning ixtiyoriy parametri shu orqali topilishi mumkin. Bu yerdan boshlang'ich va markaziy moment uchun

$$m_1 = M_\xi(t) = \int_{-\infty}^{\infty} x dF_\xi(t,x) \quad \text{o'rinli.} \quad m_k = M\xi^k(t) = \int_{-\infty}^{\infty} x^k dF_\xi(t,x) \qquad (2.1)$$

$$\mu_k = M(\xi(t) - M_\xi(t))^k = \int_{-\infty}^{\infty} (x - M_\xi(t))^k dF_\xi(t,x) \qquad (2.2)$$

Agar tasodifiy jarayon bir nechta parametrga ega bo'lsa, bu holda har bir bo'yicha integrallash amalga oshiriladi:

$$m_{k_1,..k_m}(t_1,...,t_m) = M\left|\xi_1(t_1)...\xi_m(t_m)\right| = \iint_{R^m} ...\int x_1^{k_1}...x_m^{k_m} dF_\xi(t_1,...,t_m,x_1,...,x_m)$$

Eslatma:

Fiksirlangan $t_1, t_2 \in T$ uchun tasodifiy jarayondan ikkinchi tartibli markaziy moment korrelyasion funksiya (kovariasion funksiya) deyiladi.

$$R_\xi(t_1,t_2) \in M\xi(t_1)\xi(t_2) - M_\xi(t_1)M_\xi(t_2) \qquad (2.3)$$

Eslatma: ξ **tasodifiy miqdorning normallashgan korrelyasion funksiyasi deb,**

$$r_\xi(t_1,t_2) = \frac{R_\xi(t_1,t_2)}{\sqrt{D_\xi(t_1)|\cdot|, D_\xi(t_2)}}$$

aytiladi.

2.1-mashq

Korrelyasion funksiya quyidagi xossalarga ega.

1) $R_\xi(t_1,t_2) = R_\xi(t_2,t_1)$

2) $D_\xi(t) = R_\xi(t,t)$

3) $\left|R_\xi(t_1,t_2)\right| \le \sqrt{D_\xi(t_1)|\cdot|, D_\xi(t_2)}$

4) $\sum_{k=1}^{N}\sum_{i=1}^{N} x_i x_k R_\xi(t_i,t_k) \ge 0 \qquad \forall N, t_1,...,t_N, x_1,...,x_N$

3.Statsionar tasodifiy jarayonlar

Statsionar jarayon (qat’iy bo’lmagan) – shunday jarayon, vaqt o’tish ehtimoli harakteristikasi o’zgarmaydi. Tasodifiy jarayon tor ma’noda statsionar jarayon deyiladi, agar bu jarayonning kesishmalari to’plami sinxron siljishda o’zgarmasa. Tahminan:

3.1-ta'rif

$\xi(t)$ tasodifiy jarayon tor ma'noda statsionar jarayon deyiladi, agar

$F_\xi(t_1,...,t_n,x_1,...,x_n)=F_\xi(t_1+\tau,...,t_n+\tau,x_1,...,x_n)\ \forall n,\tau,t_1,...,t_n,x_1,...,x_n$

tor ma'noda statsionar bo'lgan tasodifiy jarayon uchun:

1) $F_\xi(t,x)\Big|_{\substack{n=1\\ \tau=-t}}=F_\xi(x)$, ya’ni $F_\xi(t,x)$ funksiya t parametrdan bog’liq emas;

2) $F_\xi(t_1,t_2,x_1,x_2)\Big|_{\substack{n=2\\ \tau=-t_1}}=F_\xi(0,t_2-t_1,x_1,x_2)=F_\xi(,t_2-t_1,x_1,x_2)$ ya’ni $F_\xi(\tau,x_1,x_2)$

Uchta parametrdan bog’liq, bu yerda τ -kesish vaqti momentlarining farqi.

3.2- ta'rif

$\xi(t)$ tasodifiy jarayon keng ma'noda statsionar jarayon deyiladi, agar quyidagi xossalar bajarilsa, (bunda tor ma'noda statsionarligidan Keng ma'noda statsionarligi kelib chiqadi):

1) $\mathsf{M}_\xi(t)=M_\xi$;

2) $\mathsf{D}_\xi(t)=D_\xi$;

3) $R_\xi(t_1,t_2)=R_\xi(\tau)$ bu yerda $\tau=t_2-t_1$

bunda (2) shart "Ortiqcha"dir va (3) shartdan kelib chiqish mumkin.

Eslatma

Statsionar jarayonning $R_\xi(\tau)$ korrelatsion funksiyasi quyidagi xossalarga ega (bu korrelatsion funksiyaning xossasi 2.1 tasdiqdan kelib chiqadi):

1) $\mathsf{R}_\xi(\tau) = \mathsf{R}_\xi(-\tau)$;

2) $\mathsf{D}_\xi = \mathsf{R}_\xi(0)$;

3) $|\mathsf{R}_\xi(\tau)| \leqslant \mathsf{D}_\xi$;

4) $\sum_{k=1}^N \sum_{i=1}^N x_i x_k \mathsf{R}_\xi(t_i - t_k) \geqslant 0, \ \forall \ N, t_1, \ldots, t_N, x_1, \ldots, x_N.$

3.1-tasdiq

Tasodifiy jarayon gaussli deyiladi, agar uning ko'p o'lchovli taqsimotining ixtiyoriy ehtimoli kesishmasi gaussli tasodifiy miqdor bo'lsa:

$$\forall n, t_1, \ldots, t_n : \{\xi(t_1), \ldots, \xi(t_n)\} \sim \mathcal{N}(\overline{a}, \mathsf{R}),$$

gaussli tasodifiy jarayon,uchun keng ma'noda statsionarligi kelib chiqadi, buni isbotlash uchun ko'p o'lchovli gaussli taqsimot ehtimolligi zichlik funksiyasini yozamiz:

$$f_\xi(t_1, \ldots, t_n, x_1, \ldots, x_n) = \frac{1}{(2\pi)^{\frac{n}{2}} |\mathsf{R}_\xi|^{\frac{1}{2}}} \exp\left\{-\frac{1}{2}(\overline{x} - \overline{a})^\mathsf{T} \cdot R_\xi^{-1}(\overline{x} - \overline{a})\right\},$$

$$\overline{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}, \qquad \overline{a} = \begin{pmatrix} \mathsf{M}_\xi(t_1) \\ \vdots \\ \mathsf{M}_\xi(t_n) \end{pmatrix} \qquad \mathsf{R}_\xi = \{\mathsf{R}_\xi(t_i - t_k)\}_{i,k=1}^n,$$

qandaydir τ miqdorga siljishini amalga oshiramiz, u holda

$$f_\xi(t_1 + \tau, \ldots, t_n + \tau, x_1, \ldots, x_n) = \frac{1}{(2\pi)^{\frac{n}{2}} |\mathsf{R}_\xi|^{\frac{1}{2}}} \exp\left\{-\frac{1}{2}(\overline{x} - \overline{a})^\mathsf{T} \cdot R_\xi^{-1}(\overline{x} - \overline{a})\right\},$$

ya'ni

$$f_\xi(t_1 + \tau, \ldots, t_n + \tau, x_1, \ldots, x_n) = f_\xi(t_1, \ldots, t_n, x_1, \ldots, x_n),$$

bundan kelib chiqadiki

$$F_\xi(t_1 + \tau, \ldots, t_n + \tau, x_1, \ldots, x_n) = F_\xi(t_1, \ldots, t_n, x_1, \ldots, x_n).$$

Misol

$$\xi(t) = X \cos \omega t + Y \sin \omega t,$$

tasodifiy miqdor uchun moment bu yerda

$$(X, Y) \sim \mathcal{N}(0, \sigma^2, 0, \sigma^2, 0)$$

tasodifiy vektor funksiyasini topamiz, jarayonni keng ma'noda, tor ma'noda statsionar bo'ladimi aniqlaymiz.

◁ Moment funksiyani topamiz:

$$m_1 = \mathsf{M}_\xi(t) = \mathsf{M}\left[X\cos\omega t + Y\sin\omega t\right] = \mathsf{M}\left[X\cos\omega t\right] + \mathsf{M}\left[Y\sin\omega t\right] = \\ = \underbrace{\mathsf{M}X}_{0}\cdot\cos\omega t + \underbrace{\mathsf{M}Y}_{0}\cdot\sin\omega t = 0,$$

$$\mu_2 = M\xi^2(t) - \underbrace{(M\xi(t))^2}_{0} = M\xi^2(t) = M\left[X^2\cos^2 wt + Y^2\sin^2 wt + 2XY\cos wt\sin wt\right] =$$

$$= MX^2\cos^2 wt + MY^2\sin^2 wt + 2MXY\cos wt\sin wt = \cos^2 wt\underbrace{MX^2}_{\sigma^2} + \sin^2 wt\underbrace{MY^2}_{\sigma^2} + 2\cos wt\sin wt$$

$$\underbrace{MXY}_{\text{korrelyar bolmagan}, MX\cdot MY} = \sigma^2$$

jarayonni statsionar bo'lishligini aniqlash uchun korrelatsion funksiyani topamiz:

$$R_\xi(t_1,t_2) = M\xi(t_1)\xi(t_2) - \underbrace{M\xi(t_1)M\xi(t_2)}_{0} =$$

$$M\left[X^2\cos wt_1\cos wt_2 + Y^2\sin wt_1\sin wt_2 + XY\cos wt_1\sin wt_2 + XY\cos wt_2\sin wt_1\right] =$$

(karraligi uchun almashtirishni tushurib, $\tau = t_1 - t_2$ dan funksiyani hosil qilamiz)? $= \cos(wt_2 - wt_1)R_\xi(\tau)$ U holda seziladiki,

1) $M_\xi(t) = M_\xi = 0$

2) $D_\xi(t) = D_\xi = \sigma^2$

3) $R_\xi(t_1,t_2) = R_\xi(\tau)$

Ya'ni jarayon keng ma'noda statsionardir, bunda $\xi(t)$ jarayon normal tasodifiy jarayonlarning chiziqli kombinatsiyasidan iborat ekanligidan buning o'zi normal tasodifiy jarayon bo'ladi. U holda 3.1-tasdiqga asoslanib $\xi(t)$ jarayonni tor ma'noda statsionar deb hisoblash mumkin.

4. Kompleks qiymatli va vektorli tasodifiy jarayonlar eslatma

Agar tasodifiy jarayon C kompleks sonlar fazosida qiymat qabul qilsa, uni kompleks qiymatli deyiladi. Ixtiyoriy kompleks qiymatli $\xi(t)$ jarayonni $\zeta(t) = \xi(t) + i\eta(t)$ ko'rinishda tasvirlash mumkin, bu yerda $\xi(t), \eta(t)$ haqiqiy tasodifiy jarayonlar.

4.1- tasdiq

Kompleks qiymatli tasodifiy jarayonlarning moment funksiyalari bunda, kompleks qiymatli jarayonning korrelatsion funksiyasi quyidagi xossalarga ega:

$$M_\zeta(t) = M_\xi(t) + iM_\eta(t)$$
$$D_\zeta(t) = M(\zeta(t) - M_\zeta(t))\overline{(\zeta(t) - M_\zeta(t))} = M\zeta(t)\overline{\zeta(t)} - M_\zeta(t)\overline{M_\zeta(t)}$$
$$R_\zeta(t_1 - t_2) = M\left[(\zeta(t_1) - M_\zeta(t_1))\overline{(\zeta(t_2) - M_\zeta(t_2))}\right]$$
$$1) R_\zeta(t_1, t_2) = \overline{R_\zeta(t_2, t_1)}$$
$$2) D_\zeta(t) = R_\zeta(t, t) \geq 0$$
$$3) \left|R_\zeta(t_1, t_2)\right| \leq \sqrt{D_\xi(t_1) D_\xi(t_2)}$$

$$4) \sum_{k=1}^{N}\sum_{i=1}^{N} x_i \overline{x_k} R_\xi(t_i - t_k) \geq 0 \qquad \forall N, t_1, ..., t_N, x_1, ..., x_N$$

Kompleks qiymatli jarayonlar keng ma'noda statsionar bo'ladi, xuddi haqiqiy qiymatli (3.2-qarang) ga o'xshash shartlarda

$$1)\ \ \mathsf{R}_\zeta(t_i, t_k) = \overline{\mathsf{R}_\zeta(t_k, t_i)};$$

$$2)\ \ |R_\zeta(\tau)| \leqslant \mathsf{D}_\zeta = \mathsf{R}_\zeta(0);$$

$$3)\ \ \sum_{k=1}^{N}\sum_{i=1}^{N} x_i\overline{x_k}\mathsf{R}_\xi(t_i - t_k) \geqslant 0,\ \forall\ N, t_1, \ldots, t_N, x_1, \ldots, x_N.$$

Eslatma

$$\zeta(t) = \big(\xi(t), \eta(t)\big).$$

jarayonni ikki o'lchovli vektorli tasodifiy jarayon deyiladi. Bunday jarayonning matematik kutilmasi

$$\mathsf{M}_\zeta(t) = \big(\mathsf{M}_\xi(t), \mathsf{M}_\eta(t)\big),$$

korrelatsion funksiya matritsali ko'rinishda

$$\mathsf{R}_\zeta(t_1, t_2) = \begin{pmatrix} \mathsf{R}_\xi(t_1, t_2) & \mathsf{R}_{\xi\eta}(t_1, t_2) \\ \mathsf{R}_{\eta\xi}(t_1, t_2) & \mathsf{R}_\eta(t_1, t_2) \end{pmatrix}$$

4.1- ta'rif

Ikkita ξ,ζ tasodifiy jarayonlarning o'zaro korrelatsion funksiyasi deb

$$\mathsf{R}_{\xi\eta} = \mathsf{M}\Big[\big(\xi(t_1) - \mathsf{M}_\zeta(t_1)\big)\big(\eta(t_2) - \mathsf{M}_\eta(t_2)\big)\Big],$$

quyidagi xossalarga ega bo'lgan:

1) $\mathsf{R}_{\xi\eta}(t_1, t_2) = \mathsf{R}_{\eta\xi}(t_2, t_1);$

2) $\big|\mathsf{R}_{\xi\eta}(t_1, t_2)\big| \leqslant \sqrt{\mathsf{R}_\xi(t_1, t_2)\mathsf{R}_\eta(t_1, t_2)}.$

aytiladi,

5. Bog'liq bo'lmagan jarayonlar

5.1-ta'rif

$\xi(t)$ tasodifiy jarayon bog'liq bo'lmagan qiymatli jarayon bo'ladi, agar uning vaqtni ixtiyoriy momentidagi kesishmasi to'plamga bog'liq bo'lmasa. Bunday jarayonning taqsimot funksiyasi quyidagi tarzda faktorizatsiyalanadi:

$$F_\xi(t_1, \dots, t_n, x_1, \dots, x_n) = \mathsf{P}\{\xi(t_1) \leqslant x_1, \dots, \xi(t_n) \leqslant x_n\} = \\ = \mathsf{P}\{\xi(t_1) \leqslant x_1\} \dots \mathsf{P}\{\xi(t_n) \leqslant x_n\} = F_\xi(t_1, x_1) \dots F_\xi(t_n, x_n). \quad \textbf{(5.1)}$$

Eslatma

$\xi(t)$ qiymatlari bog'liq bo'lmagan jarayon ko'p hollarda oq shovqin ham deyiladi. Qiymatlari bog'liq bo'lmagan statsionar jarayon qaysiki

$$F_\xi(t, x) = F_\xi(x),$$

statsionar oq shovqin deyiladi

6. Orttirmalari bog'liqsiz jarayonlar

6.1-ta'rif

$\xi(t)$ tasodifiy jarayon orttirmalari bog'liqsiz jarayon bo'ladi, agar

$$\xi(t_0), \underbrace{\xi(t_1) - \xi(t_0)}_{\Delta\xi_1}, \dots, \underbrace{\xi(t_n) - \xi(t_{n-1})}_{\Delta\xi_n}$$

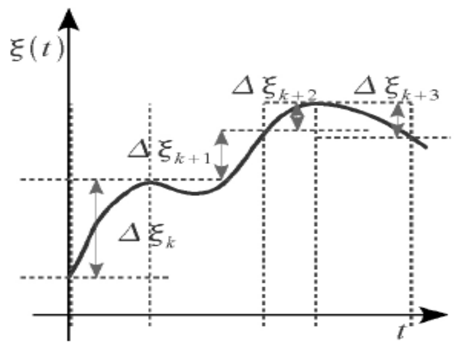
tasodifiy qiymatlar ixtiyoriy

$$n, t_0, \dots, t_n,$$

parametrda, qaysiki

$$t_0 < t_1 < \cdots < t_n$$

to'plamda bog’liqsiz bo'lsa.



Eslatma

X tasodifiy miqdorning harakteristik funksiya deb

$$\varphi(\lambda) = \mathsf{M}e^{i\lambda X},$$

aytiladi,

$$(X_1, \ldots, X_n)$$

tasodifiy vektor uchun funksiya

$$\varphi(\lambda_1, \ldots, \lambda_n) = \mathsf{M}e^{i(\lambda_1 X_1 + \cdots + \lambda_n X_n)}.$$

ko'rinishda bo'ladi.

Harakteristik funksiya muhim xossaga ega: agar X vaY bog’liq bo'lmagan tasodifiy miqdorlar bo'lsa, u holda

$$\varphi_{X+Y}(\lambda) = \varphi_X(\lambda)\varphi_Y(\lambda).$$

6.1-tasdiq

Bog’liq bo’lmagan orttirmali tugatuvchi jarayon o'zining ikki o'lchovli taqsimot funksiyasi bilan ifodalanadi.

Isbot:

$$X = (X_1, \ldots, X_n) = (\xi(t_0), \xi(t_1) - \xi(t_0), \ldots, \xi(t_n) - \xi(t_{n-1}))$$

tasodifiy vektor va uning harakteristik funksiyasi

$$\varphi_X(t_0, \ldots, t_n, \lambda_0, \ldots, \lambda_n) = \prod_i \mathsf{M}e^{i\lambda_i X_i} = \mathsf{M}e^{i\lambda_0 X_0}\mathsf{M}e^{i\lambda_1(\xi(t_1)-\xi(t_0))} \ldots \mathsf{M}e^{i\lambda_n(\xi(t_n)-\xi(t_{n-1}))}$$

qaraymiz. Umumiy harakteristik funksiya

$\varphi_X(t_0,\dots,t_n,\lambda_0,\dots,\lambda_n)$, $\varphi_\xi(t_1,t_2,\lambda_1,\lambda_2)=\mathsf{M}e^{i(\lambda_1\xi(t_1)+\lambda_2\xi(t_2))}$.

orqali ifodalanishini ko'rsatamiz. X,ξ matritsali ko'rinishda o'zaro quyidagi tarzda ifodalanadi:

$$\underbrace{\begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{pmatrix}}_{X} \cdot \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ -1 & 1 & 0 & 0 & \dots & 0 \\ 0 & -1 & 1 & 0 & \dots & 0 \\ 0 & 0 & -1 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 \end{pmatrix}} = \underbrace{\begin{pmatrix} \xi_1(t) \\ \xi_2(t) \\ \vdots \\ \xi_n(t) \end{pmatrix}}_{\xi},$$

Yoki

$$\underbrace{\begin{pmatrix} \xi_1(t) \\ \xi_2(t) \\ \vdots \\ \xi_n(t) \end{pmatrix}}_{\xi} \cdot \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & 0 & \dots & 0 \\ 1 & 1 & 1 & 0 & \dots & 0 \\ 1 & 1 & 1 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & 1 & \dots & 1 \end{pmatrix}}_{A^{-1}} = \underbrace{\begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{pmatrix}}_{X}$$

$$\varphi x(t,\lambda) = Me^{i\lambda^T A^{-1} X} = \left[\mu = \lambda^T A^{-1}\right] = Me^{i(\mu_0 X_0+\dots+\mu_n X_n)} = Me^{i\mu_0 X_0}\dots Me^{i\mu_n X_n} =$$

$$\underbrace{Me^{i\mu_0\xi(t_0)}}_{\varphi_\xi(t_0,\mu_0)}\underbrace{Me^{i\mu_1(\xi(t_1)-\xi(t_0))}}_{\varphi_\xi(t_0,t_1-\mu_1,\mu_1)}\dots\underbrace{Me^{i\mu_n(\xi(t_n)-\xi(t_{n-1}))}}_{\varphi_\xi(t_{n-1},t_n-\mu_n,\mu_n)} =$$

$$= \varphi_\xi(t_0,\mu_0)\prod_k \varphi_\xi(t_{k-1},t_k-\mu_k,\mu_k) \qquad \mu = \lambda^T A^{-1}$$

7. Mustaqil argumentli jarayonlarga misollar

7.1-ta'rif

Mustaqil argumentli $\xi(t)$ tasodifiy jarayon vinerli deyiladi, agar shartlar bajarilsa:

$$1)\ \ w(0)=0;$$

$$2)\ \ w(s)-w(t)\sim\mathcal{N}\Big(0,\sigma^2(s-t)\Big),\ \ s\geqslant t\geqslant 0.$$

7.1-teorema
Viner jarayoni gaussli deyiladi
7.1-tasdiq
Viner jarayoni moment funksiyasi

$$1)\ \ \mathsf{M}w(t)=\mathsf{M}\underbrace{\Big(w(t)-w(0)\Big)}_{\mathcal{N}\left(0,\sigma^2(t-0)\right)}=0;$$

$$2)\ \ \mathsf{R}_w(s,t)=\mathsf{M}w(t)w(s)-\underbrace{\mathsf{M}w(t)}_{0}\underbrace{\mathsf{M}w(s)}_{0}=\mathsf{M}w(t)w(s)=\mathsf{M}\left[w(t)(w(s)-w(t))\right]+\mathsf{M}\omega^2(t)=$$

$$\underbrace{\mathsf{M}w(t)}_{0}\mathsf{M}(w(s)-w(t))+\mathsf{M}w^2(t)=\mathsf{M}\Big[\underbrace{w(t)-w(0)}_{\mathcal{N}\left(0,\sigma^2(t-0)\right)}\Big]^2=\sigma^2 t,\quad \text{если } s\geqslant t.$$

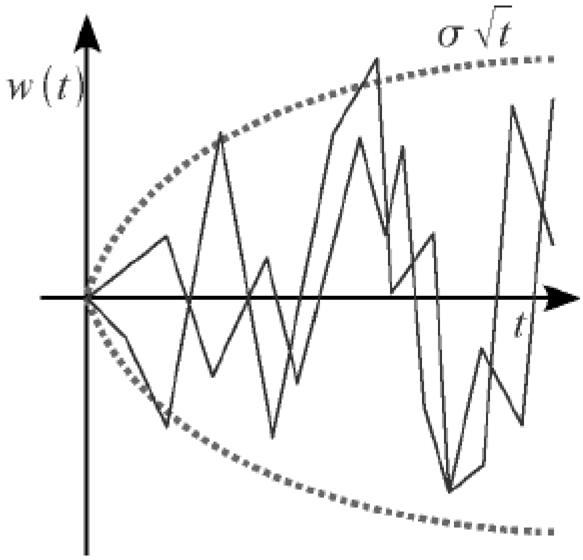
Agar bo'lsa bu holda

$$s\leqslant t,\ \text{то}\ \mathsf{R}_w(s,t)=\sigma^2 s.$$

u holda viner jarayonini korrelatsion funksiyasi uchun formula:
1. Vinerli jarayon fizikada broun harakatini harakterlaydi.

$$\mathsf{R}_w(s,t)=\sigma^2\min(s,t)$$

$$3)\ \ \mathsf{D}_w(t)=\mathsf{R}_w(t,t)=\sigma^2 t.$$



Misol.

$$\xi(t) = e^{iw(t)} \ npu \ \sigma^2 = 1$$

tasodifiy jarayon uchun $\sigma^2 = 1$ bo'lganda moment funksiyasini topamiz va statsionarlik jarayoniga tekshiramiz

◃Jarayonning matematik kutilmasi:

$$\mathsf{M}e^{iw(t)} = \int\limits_{-\infty}^{+\infty} e^{ix} f_w(x)\,dx = \frac{1}{\sqrt{2\pi t}} \int\limits_{-\infty}^{+\infty} e^{ix} e^{-\frac{x^2}{2t}}\,dx = e^{-\frac{t}{2}} \underbrace{\frac{1}{\sqrt{2\pi t}} \int\limits_{-\infty}^{+\infty} e^{-\frac{(x-it)^2}{2t}}}_{1} = e^{-\frac{t}{2}}.$$

Korrelatsion funksiyasi

$$\mathsf{R}_\xi(t_1, t_2) = \underbrace{\mathsf{M}\left[e^{iw(t_1)}\right]\overline{\left[e^{iw(t_2)}\right]}}_{\mathsf{M}e^{i(w(t_1)-w(t_2))}} - \underbrace{\mathsf{M}e^{iw(t_1)}}_{e^{-\frac{t_1}{2}}}\underbrace{\mathsf{M}e^{iw(t_2)}}_{e^{-\frac{t_2}{2}}} = e^{\frac{t_1-t_2}{2}} - e^{-\frac{t_1+t_2}{2}}.$$

Jarayon statsionar bo'lmaydi, chunki korrelatsion funksiyasi orqali $\tau = t_2 - t_1$ ifodalanishi mumkin emas.

7.2. Puasson jarayon

Eslatma

X tasodifiy miqdor puassonli deyiladi

$$X \sim \Pi(a), \quad \mathsf{P}\{X = k\} = \frac{a^k}{k!} e^{-a}$$

agar bunda bunday tasodifiy miqdor quyidagi momentga ega bo'ladi:

$$1)\ \mathsf{M}X = a;$$

$$2)\ \mathsf{D}X = a;$$

$$3)\ \mathsf{M}X^2 = a + a^2.$$

7.2-ta'rif

Mustaqil orttirmali $\tau(t)$ tasodifiy jarayon puassonli deyiladi, agar:

$$1)\ \pi(0) = 0;$$

$$2)\ \pi(s) - \pi(t) \sim \Pi\big(\lambda(s-t)\big), \quad s \geqslant t \geqslant 0.$$

7.2-tasdiq

Puasson jarayonini momentli funksiyasi:

$$1)\ \mathsf{M}\pi(t) = \mathsf{M}\underbrace{\big[\pi(t) - \pi(s)\big]}_{\Pi(\lambda t)} = \lambda t;$$

$$2)\ \mathsf{R}_\pi(s,t) = \mathsf{M}\pi(s)\pi(t) - \mathsf{M}\pi(s)\mathsf{M}\pi(t) = \mathsf{M}\pi(t)\left[\pi(s) - \pi(t) + \pi(t)\right] - \lambda^2 st = \mathsf{M}\pi(t)\left[\mathsf{M}\pi(s) - \mathsf{M}\pi(t)\right] + \mathsf{M}\underbrace{\big[\pi(t) - \pi(0)\big]}_{\Pi(\lambda t)}^2 = \lambda t,$$

agar $s \geq t$ bo'lsa, korrelatsion funksiya

$$\mathsf{R}_\pi(s,t) = \lambda s.$$

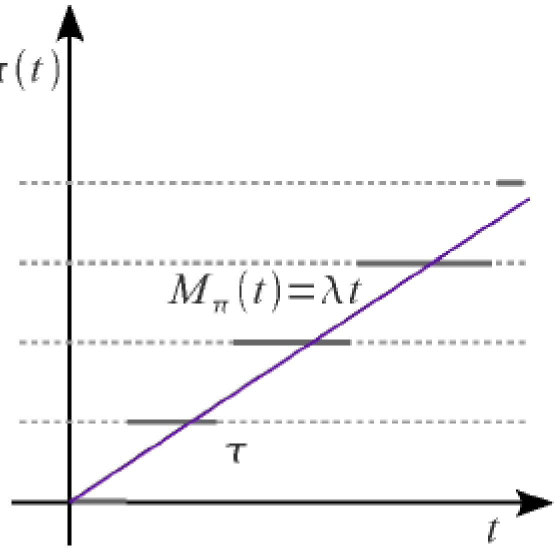
u holda:

$$\mathsf{R}_\pi(s,t) = \lambda \min(s,t)$$

$$3)\ \mathsf{D}_\pi(t) = \lambda t.$$

Eslatma.

λ parametr matematik kutilma to'g'ri chizig'ining buriliah burchagi tangasiga teng va puasson jarayonini intensivligi deyiladi. $T = \frac{1}{2}$ miqdor sakrashlar orasidagi o'rta vaqt



8. Statsionar jarayonlarning spektral xossasi

Eslatma

Muhim bo'lmagan chegaralanishlar bilan Ixtiyoriy davriy funksiya fur'e qatoriga yoyish mumkin.

8.1-ta'rif

$x(t)$ funksiyani T davr bilan fur'e qatoriga yoyilmasi quyidagi ko'rinishga ega:

$$x(t)=\frac{a_0}{2}+\sum_{k=1}^{\infty}\left(a_k\cos\frac{2\pi}{T}kt+b_k\sin\frac{2\pi}{T}kt\right),$$

bu yerda,

$$a_k=\frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}x(t)\cos\frac{2\pi}{T}ktdt, \qquad k=0,1,\ldots$$

$$b_k=\frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}x(t)\sin\frac{2\pi}{T}ktdt, \qquad k=0,1,\ldots$$

yoyilma kompleks ko'rinishda quyidagi ko'rinishni oladi:

$$x(t)=\sum_{k=\infty}^{+\infty}x(t)c_k e^{i\frac{2\pi}{T}kt}$$

Bu yerda,

$$c_k=\frac{1}{T}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}e^{-i\frac{2\pi}{T}kt}dt, \qquad k=0,1,\ldots$$

Eslatma

Qismiy yig'indi

$$\hat{x}(t) = \sum_{k=-N}^{N} c_k e^{i\frac{2\pi}{T}kt} \approx x(t).$$

$$\mathsf{M}\xi_k = 0, \quad \mathsf{D}\xi_k = \sigma_k^2.$$

bo'lgan T davrli $\xi(t)$ tasodifiy jarayonni qaraymiz. Bu jarayonni Fur'e qatoriga yoyilmasi quyidagi ko'rinishda bo'ladi:

$$\xi(t) = \sum_k \xi_k e^{i\frac{2\pi}{T}kt}. \qquad \mathbf{(8.1)}$$

1) Fur'e qatoriga yoyilgan matematik kutilmasi

$$\mathsf{M}_\xi(t) = \mathsf{M}\sum_k \xi_k e^{i\frac{2\pi}{T}kt} = \sum_k e^{i\frac{2\pi}{T}kt}\mathsf{M}\xi_k = 0. \qquad \mathbf{(8.2)}$$

2) jarayonning korrelatsion funksiyasi:

$$\mathsf{R}_\xi(t_1, t_2) = \mathsf{M}\xi(t_1)\overline{\xi(t_2)} = \mathsf{M}\left(\sum_k \xi_k e^{i\frac{2\pi}{T}kt_1}\sum_m \overline{\xi_m} e^{-i\frac{2\pi}{T}mt_2}\right) =$$

$$= \sum_k\sum_m e^{i\frac{2\pi}{T}(kt_1 - mt_2)}\underbrace{\mathsf{M}\xi_k\overline{\xi_m}}_{\delta_{km}\sigma_k^2} = \sum_k \sigma_k^2 e^{-i\frac{2\pi k}{T}(t_2 - t_1)},$$

$\tau = t_2 - t_1$ orqali ifodalanib:

$$\mathsf{R}_\xi(\tau) = \sum_k \sigma_k^2 e^{-i\frac{2\pi k}{T}\tau}. \qquad \mathbf{(8.3)}$$

3) jarayonni dispersiyasi:

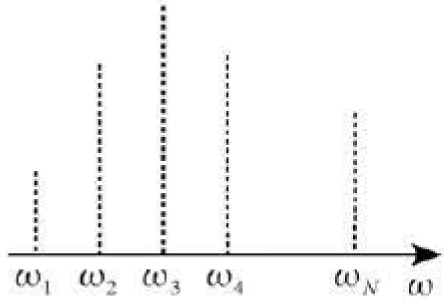
$$\mathsf{D}_\xi = \mathsf{R}_\xi(0) = \sum_k \sigma_k^2. \qquad \mathbf{(8.4)}$$

$$\xi_k(t) = \xi_k e^{i\frac{2\pi}{T}kt} = \xi_k e^{i\omega_k t},$$

Agar

$$\omega_k = \frac{2\pi}{T}k$$

ko'rinishda Fur'e qatoriga yoyilgan tasodifiy jarayon uchun w_k garmonik tashkil etuvchili D_ξ taqsimot spektrini tashkil etadi.



ξ_κ garmoniklar sonini ko'paytirib, ya'ni $\sum_k \xi_\kappa(t)$, yig'indida k ni orttirib:

$$\lim_{T\to\infty} \mathsf{R}_\xi(\tau) = \lim_{T\to\infty} \sum_{k=-\infty}^{+\infty} e^{-i\omega_k\tau}\sigma_k^2 = \lim_{\Delta\omega\to 0} \sum_{k=-\infty}^{+\infty} e^{-i\omega_k\tau}\Delta\omega \underbrace{\frac{\sigma_k^2}{\Delta\omega}}_{\substack{\text{спектральная}\\ \text{плотность}\\ \text{мощности } S(\omega)}} = \int_{-\infty}^{+\infty} S(\omega)e^{-i\omega t}d\omega.$$

8.1-tasdiq.

$S_\xi(w)$ va $R_\xi(\tau)$ orasidagi quyidagi almashtirishlar o'rinlidir:

$$S_\xi(\omega) = \frac{1}{2\pi}\int^{+\infty} \mathsf{R}_\xi(\tau)e^{i\omega\tau}d\tau \quad -$$

teskari fur'e almashtirishni,

$$\mathsf{R}_\xi(\tau) = \int_{-\infty}^{+\infty} S_\xi(\omega)e^{-i\omega\tau}d\omega \quad -$$

to'g'ri fur'e almashtirishni

Eslatma

$\frac{1}{2\pi}$ ko'paytma teskari fur'e almashtirishda mavjud va to'g'risida bo'lmaydi deb hisoblaymiz.

8.2- tasdiq

$\xi(t)$ jarayonni $S_\xi(w)$ quvvatni spektral zichligi quyidagi xossalarga ega:

1) $S_\xi(\omega) \geqslant 0$;

2) $S_\xi(\omega)$ – haqiqiy funksiya

haqiqiy tasodifiy jarayonlarning keng ma'nodagi statsionar bo'lsin. U holda quvvatning spektral zichligi va korrelatsion funksiyasi

Quydagi tarzda:

$$S_\xi(\omega) = \frac{1}{2\pi}\left(\int_{-\infty}^{+\infty} \mathsf{R}_\xi(\tau)\cos\omega\tau\, d\tau + i\underbrace{\int_{-\infty}^{+\infty} \mathsf{R}_\xi(\tau)\sin\omega\tau\, d\tau}_{0}\right) = \frac{1}{\pi}\int_{0}^{+\infty} \mathsf{R}_\xi(\tau)\cos\omega\tau\, d\tau \qquad \mathbf{(8.7)}$$

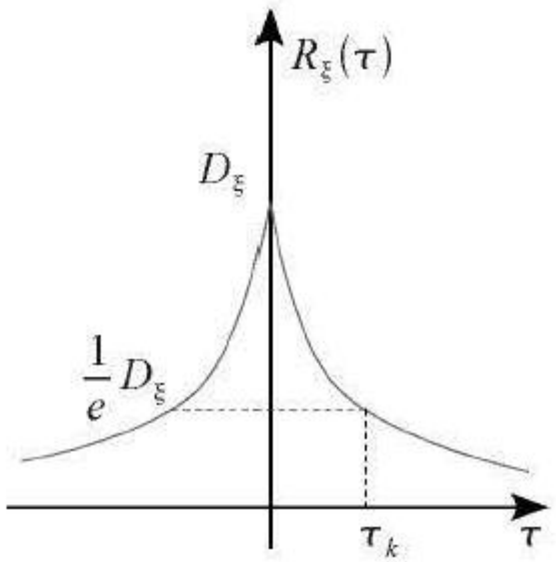
$$\mathsf{R}_\xi(\tau) = 2\int_{0}^{+\infty} S_\xi(\omega)\cos\omega\tau\, d\omega. \qquad \mathbf{(8.8)}$$

9. Keng ma’noda statsionar jarayonlarga misollar

9.1 Eksponentsial korrelyotsion funksiyali tasodifiy jarayon.

9.1-ta’rif, Eksponentsial korrelyotsion funksiyali tasodifiy jarayon quydagi tarzda harakterlanadi:

$$R_\xi(\tau) = D_\xi e^{-\alpha[\tau]}, \qquad \alpha > 0$$



Eslatma. Karrelyatsiya radiusi $\tau_k = \frac{1}{\sigma}$ tasodifiy jarayon kesishi orasidagi bog’lanish qanchaga “ chizishini ” ifodalaydi. τ_k qancha katta bo’lsa, tasodifiy jarayon shuncha korrelirovan.

9.1 – tasdiq.

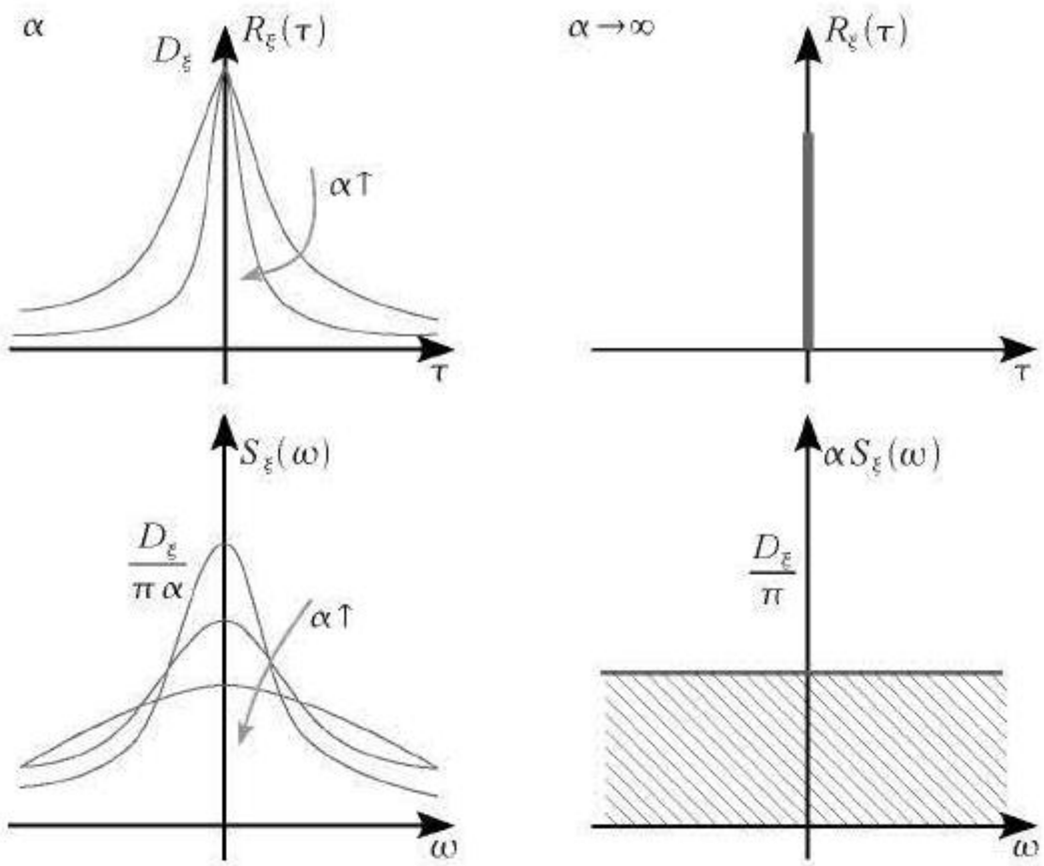
Korreliyatsiyaning eksponentsial funksiyali jarayonning quvvatini spectral zichligi quydagi tarzda ifodalanadi:

$$S_\xi(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathsf{R}_\xi(\tau)e^{i\omega\tau}d\tau = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathsf{D}_\xi e^{-\alpha|\tau|}e^{i\omega\tau}d\tau =$$

$$= \frac{\mathsf{D}_\xi}{2\pi}\left(\int_{-\infty}^{0} e^{(\alpha+i\omega)\tau}d\tau + \int_{0}^{+\infty} e^{(-\alpha+i\omega)\tau}d\tau\right) = \frac{\mathsf{D}_\xi}{2\pi}\left(\frac{e^{(\alpha+i\omega)\tau}}{\alpha+i\omega}\Bigg|_{\tau=-\infty}^{0} + \frac{e^{(-\alpha+i\omega)\tau}}{\alpha+i\omega}\Bigg|_{\tau=0}^{+\infty}\right) =$$

$$= \frac{\mathsf{D}_\xi}{2\pi}\left(\frac{1}{\alpha+i\omega} - \frac{e^{(\alpha+i\omega)\tau}\big|_{\tau=-\infty}}{\alpha+i\omega} + \frac{e^{(-\alpha+i\omega)\tau}\big|_{\tau=+\infty}}{-\alpha+i\omega} + \frac{1}{\alpha-i\omega}\right) = \frac{\alpha\mathsf{D}_\xi}{\pi(\alpha^2+\omega^2)}.$$

9.2 Oq shovqin



Qandaydir eksponentsial korrelyatsion funksiyali

$$R_\xi(\tau) = D_\xi e^{-\alpha|\tau|}$$

tasodifiy jarayonni qaraymiz, Faraz qilamizki, $\alpha \to \infty$, u holda:

$$\begin{cases} \mathrm{R}_\xi = \begin{cases} \mathrm{D}_\xi, & \tau = 0 \\ 0, & \tau \neq 0 \end{cases} \\ \alpha S_\xi(\omega) = \dfrac{\mathrm{D}_\xi}{\pi} \end{cases}$$

9.2-tasdiq.

Yuqorida ko’rsatilgan ξ tasodifiy jarayon oq shavqin deyiladi.

10. Keng ma'noda statsionar tasodif jarayonlar uchun katta sonlar qonuni

Eslatma.

$\{\xi_n\}$ tasodifiy miqdorlar ketma-ketligi ehtimollik bo'yicha ξ tasodifiy miqdorga yaqinlashadi deyiladi, agar ixtiyoriy $\varepsilon > 0$ uchun

$$\lim_{n\to\infty} \mathsf{P}\{|\xi_n - \xi| \leqslant \varepsilon\} = 1.$$

10.1-tasdiq.

$\xi(t)$ tasodifiy jarayon katta sonlar qonuniga bo'ysinadi deyiladi, agar vaqt bo'yicha o'rtacha ehtimollik bo'yicha matematik kutilmaga yaqinlashsa:

$$\frac{1}{T}\int_{t_0}^{t_0+T} \xi(t)\,dt \xrightarrow[T\to\infty]{\mathsf{P}} \mathsf{M}_\xi.$$

10.1-ta'rif.

Katta sonlar qonuniga bo'ysinuvchi tasodifiy jarayonlar Irgogik deyiladi.

10.1-teorema.

Tasodifiy jarayonni Irgogik bo'lishi uchun

$$\lim_{\tau\to\infty} \mathsf{R}_\xi(\tau) = 0. \qquad \textbf{(10.1)}$$

Tenglikni bajarilishi yetarli.

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Isbot, Soddalik uchun $t_0 = 0$ olamiz. U holda

$$\frac{1}{T}\int_0^T \xi(t)dt \xrightarrow[T\to\infty]{P} M_\xi$$

Yanada kuchliroq o'rta kvadratik yaqinlashishdan kelib chiqadiki:

$$\frac{1}{T}\int_0^T \xi(t)dt \xrightarrow[T\to\infty]{o'r,kv} M_\xi$$

O'rta kvadratik yaqinlashishi ta'rifdan

$$\lim_{T\to+\infty} M\left(\frac{1}{T}\left[\int_0^T \xi(t)dt - M_\xi\right]\right)^2 = 0$$

Isbotlash uchun limit ostidagi ifodani almashtiramiz:

U holda

$$\lim_{T\to\infty}\frac{2}{T}\left|\int_0^T R_\xi(\tau)\left(1-\frac{\tau}{T}\right)d\tau\right| \le \lim_{T\to\infty}\frac{2}{T}\int_0^T |R_\xi(\tau)|d\tau = 2\lim_{T\to\infty}\frac{\int_0^T |R_\xi(\tau)|d\tau}{(T)'_T} = 2\lim_{T\to\infty} R_\xi(\tau)$$

Yaqinlashish

$$\lim_{\tau\to\infty} R_\xi(\tau) = 0$$

Yetarli shartda o'rinli bo'ladi, bu shartda ehtimollik bo'yicha yanada " kuchsizroq " yaqinlashishi bo'ladi.

11. Tasodifiy jarayonlarning uzluksizligi

Eslatma, x(t) funksiya t_0 nuqtada uzluksiz deyiladi, agar

$$\lim_{\Delta t\to+0}[x(t_0+\Delta t)-x(t_0)] = 0$$

11.1-tasdiq. Tasodifiy jarayon t_0 nuqtada uzluksiz deyiladi,agar

$$\lim_{\Delta t\to+0} M(\xi(t_0+\Delta t)-\xi[(t]_0))^2 = 0$$

11.2-tasdiq. Keng ma'noda statsionar ξ tasodifiy jarayonni o'rta kvadratik uzluksizning zaruriy va yetarli sharti uning korrelyatsion funksiyasi R_ξ ni nolda uzluksiz bo'lishidir;

$$\lim_{\tau\to 0} R_\xi(\tau) = R_\xi(0)$$

zarur bo'laolmaydi.

11.3-tasdiq

Agar keng ma'noda statsionar ξ tasodifiy jarayonni R_ξ korrelatsion funksiya nolda uzluksiz bo'lsa, u holda u ixtiyoriy nuqtada uzluksizdir.

12. Karunena-Loeva orthogonal yoyilmasi

$$\xi(t), t\in[0,\tau].$$

tasodifiy jarayonni qaraymiz, soddalik uchun

$$\mathsf{M}\xi(t) = 0.$$

deb hisoblaymiz.

12.1-ta'rif

Orthogonal yoyilma deb uni qator yig'indisi ko'rinishidagi ifodasiga aytiladi:

$$\xi(t) = \sum_k \xi_k\varphi_k(t),\ t\in[0,T], \qquad \textbf{(12.1)}$$

Bu yerda ξ_κ korrelatsiya bo'lmaydigan tasodifiy miqdor,

$$\mathsf{M}_{\xi_k} = 0, \mathsf{D}\xi_k = \sigma_k^2,$$

ya'ni

$$\mathsf{M}\xi_k\xi_i = \sigma_k^2\delta_{ki},$$

$\varphi_\kappa(t)$ ortonormallangan bazis funksiya. Ikkilangan orthogonalli bir vaqtda orthogonal deyiladi

$$\xi_k \perp \xi_i,\ \varphi_k(t) \perp \varphi_i(t),\ i \neq k.$$

Mulohaza
Jarayonlarning modellashtirish uchun, ma'lumotlarni qisqartirish raqam ko'rinishi uchun orthogonal yoyilma qo'llanadi.
12.1-tasdiq
Orthogonal yoyilgan tasodifiy jarayonni korrelatsion funksiyasi

$$\mathsf{R}_\xi(t,s) = \mathsf{M}\xi(t)\xi(s) = \mathsf{M}\left[\sum_k \xi_k\varphi_k(t)\sum_i \xi_i\varphi_i(s)\right] = \sum_k\sum_i \mathsf{M}\underbrace{\xi_k\xi_i}_{\delta_{ki}}\varphi_k(t)\varphi_i(s) = \sum_i \varphi_i(t)\varphi_i(s)\sigma_i^2,$$

S bo'yicha integrallab,

$$\varphi(t) = \lambda\int_0^T K(t,s)\varphi(s)\,ds$$

$\varphi_\xi(t)$ bazis funksiyasi va tasodifiy miqdorning dispersiyasi chiziqli bir jinsli Fregrol'm II tur integral tenglamasi yechimi sifatida aniqlanadi:

$$\varphi(t) = \lambda\int_0^T K(t,s)\varphi(s)\,ds,$$

Bu yerda K(t,s)yadro tasodifiy jarayonning korrelatsion funksiyasidan iborat. Tenglamaning yechimi $\varphi_\xi(t)$ xos funksiyadan iborat, qaysiki orthogonal yoyilmaning funksiyasidan va $\frac{1}{\sigma_k}$ xos son qaysiki yoyilmasining tasodifiy koeffisiyenti dispersiyasiga teskari miqdordan iborat.
Mulohoza. Agar

$$\mathsf{M}\xi(t) \neq 0,$$

u holda markazlashgan tasodifiy miqdorni qarash mumkin.

$$\dot{\xi}(t) = \xi(t) - \mathsf{M}_\xi(t),$$

U holda

$$\mathsf{M}\dot{\xi}(t) = 0$$

va

$$\mathsf{R}_{\dot{\xi}}(t,s) = \mathsf{R}_\xi(t,s)$$

va orthogonal yoyilma

$$\xi(t) = \mathsf{M}_\xi(t) + \sum_{k=1}^{\infty} \xi_k \varphi_k(t).$$

12.1. Karunena-Loeva yoyilmas optimalligi

12.2-ta'rif

Orthogonal yoyilmani to'lasini o'rniga qaraladigan qismiy qatorida paydo bo'luvchi xatolik

$$\xi_N(t) = \sum_{k=1}^{N} \xi_k \varphi_k(t).$$

ko'rinishda ifodalanadi, bunday atroksimasiya integral o'rta kvadratik xatosi

$$\overline{\varepsilon_N^2} = \int_0^T \mathsf{M}\varepsilon_N^2(t)\,dt = \int_0^T \mathsf{M}\left(\sum_{k=N+1}^{+\infty} \xi_k \varphi_k(t)\right)^2 = \left[\begin{smallmatrix} \mathsf{M}\xi_i\xi_j = \sigma^2\delta_{ij} \\ \forall i \neq j \varphi_i(t)\varphi_j(t) = 0 \end{smallmatrix}\right] = \sum_{k=N+1}^{+\infty} \sigma_k^2$$

Agar dispersiya komponent

$$D\xi_k = \sigma_k^2$$

k ga nisbatan kamayish tartibida joylashgan bo'lsa, xatolik minumumi bajarilishini ko'rish qiyin emas.

12.2. Viner jarayoni uchun Karunena-Loeva orthogonal yoyilmasiga misol

Misol

W(t) Viner jarayonini $t \in [0,1]$dispersiya $\sigma^2 = 1$ korrelatsion funksiyasi

$$\mathsf{R}_w(t, s) = \min(t, s).$$

uchun orthogonal yoyilmani topamiz.

$\triangleleft$(12.1) formula bo'yicha ma 'lum bo'lgan R_w qo'yib, bazis funksiyani ifodalaymiz:

$$\varphi(t) = \lambda \int_0^1 \min(t, s)\varphi(s)\,ds = \lambda \int_0^t s\varphi(s)\,ds + \lambda \int_t^1 t\varphi(s)\,ds,$$

differensiallab;

$$\varphi'(t) = \lambda t\varphi(t) + \left(\lambda t \int_0^1 \varphi(s)\,ds\right)',$$

$$\frac{\varphi'(t)}{\lambda} = t\varphi(t) + \int\limits_t^1 \varphi(s)\,ds + t\left(\int\limits_t^1 \varphi(s)\,ds\right)' = t\varphi(t) - t\varphi(t) + \int\limits_t^1 \varphi(s)\,ds,$$

yana bir marta differensiallab

$$\frac{\varphi''(t)}{\lambda} = -\varphi(s),$$

ya'ni

$$\varphi''(t) = -\lambda\varphi(t).$$

$$\varphi''(t) = -\lambda\varphi(t)$$

tenglamaning yechimi

$$\varphi(t) = A\sin(\omega t + \alpha),\ \ \lambda = \omega^2.$$

Iborat. U holda

$$A\sin(\omega t + \alpha) = \omega^2 \int\limits_0^1 \min(t,s)\varphi(s)\,ds,$$

$$\frac{A\sin(\omega t + \alpha)}{\omega^2} = A\int\limits_0^t s\sin(\omega s + \alpha)\,ds + A\int\limits_t^1 t\sin(\omega t + \alpha)\,ds, \sin(\omega t + \alpha),$$

$$\sin(\omega t + \alpha) = \Big[\sin(\omega s + \alpha) - \omega s\cos(\omega s + \alpha)\Big]_{s=0}^{s=t} + \omega t\Big[\cos(\omega s + \alpha)\Big]_{s=t}^{s=1},$$

Bundan

$$\sin\alpha + t\omega\cos(\omega t + \alpha) = 0, \forall t.$$

$$\sin\alpha + t\omega\cos(\omega t + \alpha) = 0$$

tenglamani yechimi sistemadan topiladi:

$$\begin{cases} \sin\alpha & = 0 \\ \cos(\omega t + \alpha) & = 0 \end{cases} \Rightarrow \begin{cases} \alpha & = 0 \\ \omega & = \dfrac{\pi}{2} + \pi m,\ \ m \in \mathbb{Z} \end{cases}$$

bazis funksiya faqat ishorasiga farq qilishi mumkin emas, shuning uchun

$$\varphi(t) = A\sin(\omega t)$$

yoki

$$\varphi(t) = -A\sin(\omega t),$$

bu yerda,

$$\omega = \frac{\pi}{2} + \pi m,\ m \in \mathbb{Z}.$$

orthogonal bazisning elementlari

$$\varphi_k(t) = A \sin \frac{\pi}{2}(2k-1)t,\ k = 1, 2, 3, \dots,$$

dispersiya

$$\sigma_k^2 = \frac{1}{\lambda_k} = \frac{4}{\pi^2(2k-1)^2},$$

A koeffisiyent

$$\int_0^1 \varphi_k^2(t)\, dt = 1$$

tenglamadan topiladi va $\sqrt{2}$ tenglamadan, natijada jarayon quyidagi orthogonal yoyilmaga ega bo'ladi:

$$\xi(t) = \sum_{k=1}^{\infty} \xi_k \sqrt{2} \sin\left[\frac{\pi}{2}(2k-1)t\right],$$

$$\mathsf{D}\xi_k = \frac{4}{\pi^2(2k-1)^2}, \quad \mathsf{M}\xi_k = 0.$$

13. Markov jarayonlari

13.1-ta'rif.

$\xi(t) \in E$ tasodifiy jarayon bi'lsin, bu yerda E-holat to'plami (diskret yoki uzluksiz). $\xi(t)$ jarayon markovli deyiladi, agar

$$\forall t_1 < \cdots < t_n: \quad \mathsf{P}\{\xi(t_n) \in A | \xi(t_1) = x_1, \dots, \xi(t_{n-1}) = x_{n-1}\} = \mathsf{P}\{\xi(t_n) \in A | \xi(t_{n-1}) = x_{n-1}\},$$

bu yerda

$$A \subset E, \forall x_1, \dots, x_n \in E.$$

boshqacha aytganda, Markov jarayoni -"xotirasiz" jarayon, faqat oxirgi holatdan bog'liq bo'lgan.

Misol

Markov jarayonlari:

◃Braun harakati markov jarayoni, xokkeydagi manbaning holati- qisman markovli (qat'iy aytganda markovli emas) jarayon.

13.1-tasdiq

Ixtiyoriy markov jarayonini tugatuvchi harakteristikasi o'tish funksiyasi bo'ladi.

$$p(s,x,t,y) = \mathsf{P}\left\{\xi(t) = y | \xi(s) = x\right\}, t \geqslant s, \quad x, y \in E.$$

$$\mathsf{P}\left\{\underbrace{\xi(t_1) = x_1}_{A}, \underbrace{\xi(t_2) = x_2}_{B}\right\} = \mathsf{P}(A)\mathsf{P}(B|A) = \mathsf{P}\left\{\xi(t_1)\right\} p(t_1, x_1, t_2, x_2)$$

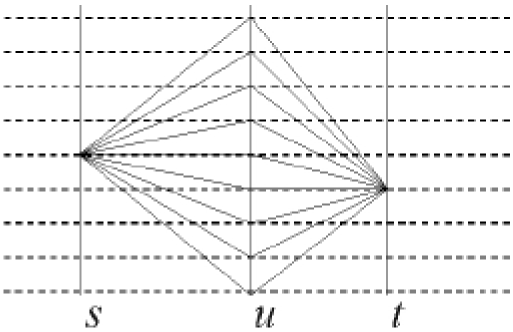
13.2-tasdiq

O'tish funksiyasi quyidagi xossalarga egadir:

$$1)\ \ p(s,x,s,y) = \begin{cases} 1, & x = y \\ 0, & x \neq y \end{cases}$$

2) Kolmogorov - Chepmen tenglamasi

$$p(s,x,t,y) = \sum_{z \in E} p(s,x,u,z) p(u,z,t,y), \quad \forall s \leqslant u \leqslant t, \forall x, y$$



13.2-ta'rif

Markov jarayoni bir jinsli deyiladi, agar o'tish ehtimoli absolyut vaqtdan bog’liq bo'lmasdan, bu holatdagi vaqtlar orasidagi farqdan bog’liq bo'lsa:

$$p(s,x,t,y) = p(\underbrace{t-s}, x, y).$$

14. Markov zanjiri

14.1. Bir jinsli Markov zanjiri

14.1-ta'rif

Diskret t vaqt va chekli yoki sanoqli E holat to'plami markov jarayoni markov zanjiri deyiladi

$$p(m,i,n,j) = \mathsf{P}\left\{\xi_n = E_j | \xi_m = E_i\right\},\ m,i,n,j \in \mathbb{N}_0,\ n > m.$$

Bir jinsli markov zanjiri quyidagi o'tish funksiyasiga egadir:

$$p(m,i,n,j) = p(n-m,i,j).$$

Mulohaza. Quyidagi belgilardan foydalanamiz:

- $p_{ij}(m,n) = p(m,i,n,j)$

ehtimollik, qaysiki n vaqt holayida markov jarayoni j holatda, bo'lgan, m vaqt holatidagi shartga i holatda bo'lgan;

- $p_{ij}(n-m) = p_{ij}(k)$

Shartli ehtimollik, qaysiki k vaqtga jarayon i holatdan j holatga o'tadi.

14.1-tasdiq

Bir jinsli Markov zanjiri uchun Kolmogorov tenglamasi:

$$p_{ij}(m+n) = \sum_k p_{ik}(m)p_{kj}(n)\ , m \geqslant 0, n \geqslant 0.$$

14.2-tasdiq

Markov zanjiri quyidagi Xossalarga ega:

1) $p_{ij}(0) = \delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$

2) $p_{ij}(t-s) = \sum_k p_{ik}(u-s)p_{kj}(t-u)\ , m \geqslant 0, n \geqslant 0;$

3) O'tish funksiyasi k qadamda bir qadamga o'tish funksiyasi orqali bir qiymatli aniqlanadi:

$$p_{ij}(1) = p_{ij};$$

4) $p_{ij}(2) = \sum_k p_{ik}p_{kj};$

14.3-tasdiq

Agar $|E| = N$ u holda o'tish ehtimolligi matritsasi ma'no egadir:

$$\mathbb{P} = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1N} \\ p_{21} & p_{22} & \cdots & p_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ p_{N1} & p_{N2} & \cdots & p_{NN} \end{pmatrix} \qquad \mathbf{(14.1)}$$

O'tish ehtimolligi matritsaning m-darajasi m qadamdagi o'tish ehtimolini saqlaydi:

$$\mathbb{P}(m) = \mathbb{P}^m = \{p_{ij}(m)\}$$

o'tkazish ehtimolligi matritsasi quyidagi xossalarga ega:

- $0 \leqslant p_{ij} \leqslant 1;$
- $\sum_{j=1}^{N} p_{ij} = 1;$
- $p(n+m) = p(n)\mathbb{P}^m$

Mulohaza

P Matritsani darajaga ko'tarishda orthogonal yoyilmadan foydalanish qulay

$$\mathbb{P} = Q\Lambda Q^{\top}, \qquad \textbf{(14.2)}$$

bu yerda

$$\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_n), \quad \lambda_i -$$

ning xos qiymati

$$|\mathbb{P} - \lambda_i E_N| = 0,$$

tenglamadan topiladigan,

$$Q = (X_1, \ldots, X_n), X_i -$$

ning xos vektori xos qiymatga mos keluvchi va

$$(\mathbb{P} - \lambda_i E_n)X_i = 0.$$

dan topiladigan.

Mulohaza.

Markov zanjiri yaqqollik uchun grafik ko'rinishda tasvirlash qulay, bu yerda uchlari holati, qirra vaznlik bilan mos keluvchi holatlar orasidagi o'tishi uning ehtimolligi bilan.

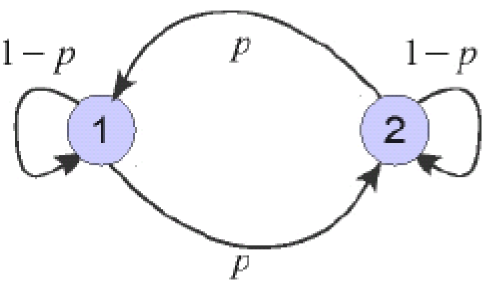
Misol

Ikkita holat bilan markov zanjirini qaraymiz, qaysiki

$$\begin{pmatrix} 1-p & p \\ p & 1-p \end{pmatrix}, \quad p(100) = p(0)\mathbb{P}^{100}.$$

topamiz

Tenglamadan matritsaning xos qiymatini topamiz:



$$|\mathbb{P} - \lambda E_2| = 0:$$

Tenglamadan matritsaning xos qiymatini topamiz:

$$\begin{vmatrix} 1-p-\lambda & p \\ p & 1-p-\lambda \end{vmatrix} = 0,$$

Bundan

$$\lambda_1 = 1, \lambda_2 = 1-2p$$

xos vektorlar

$$X_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, X_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Orthogonal yoyilma va darajaga ko'tarish

$$\mathbb{P}^n = \frac{1}{2}\overbrace{\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}}^{Q}\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & (1-2p)^n \end{pmatrix}}_{\Lambda^n}\overbrace{\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}}^{Q^{-1}} = \frac{1}{2}\begin{pmatrix} 1+(1-2p)^n & 1-(1-2p)^n \\ 1-(1-2p)^n & 1+(1-2p)^n \end{pmatrix}$$

O'tish ehtimoli:

$$1)\ \ p_1(n) = \tfrac{1}{2} + \tfrac{(1-2p)^n}{2};$$

$$2)\ \ p_2(n) = \tfrac{1}{2} - \tfrac{(1-2p)^n}{2}.$$

$n \to \infty$ *da ehtimollik* $p(\infty) = (\frac{1}{2}, \frac{1}{2})$

14.2. Markov zaniiri holati limitik ehtimoli

14.4-tasdiq

O'tish ehtimoli limitda qandaydir π vektorga yaqinlashadi

$$p(n) = p(0)\mathbb{P}^n \xrightarrow[n\to\infty]{} \pi,$$

bu yerda

$$\pi = \begin{pmatrix}\pi_1 & \dots & \pi_n\end{pmatrix},\ \pi_k = \lim_{n\to\infty} p_k(n).$$

14.2-ta'rif

Holatning ehtimoli

$$\pi = \lim_{n\to\infty} p(0)\mathbb{P}^n,$$

deb, aytiladi, agar bu limit mavjud bo'lib P(0) dan bog’liq bo'lmasa. π komponentalari

$$\pi_k = \sum_i \pi_i p_{ik},\ \ k = 1, \dots, N \qquad \textbf{(14.3)}$$

va tenglamalar sistemasi

$$\begin{cases}\pi_k & = \sum_i \pi_i p_{ik}, \quad k = 1, \dots, N \\ \sum_k \pi_k & = 1\end{cases}$$

formulalardan topiladi,

Misol

Qaytada ikki holatli markov zanjirini qaraymiz, qaysiki

$$P = \begin{pmatrix}1-p & p \\ p & 1-p\end{pmatrix}$$

va holatning ehtimoli yordamga P(100) topamiz.

◃Tenglamalar sistemasini tuzamiz

$$\begin{cases}\pi_1 & = \pi_1(1-p) + \pi_2 p \\ \pi_2 & = \pi_1 p + \pi_2(1-p) \\ \pi_1 + \pi_2 & = 1\end{cases}$$

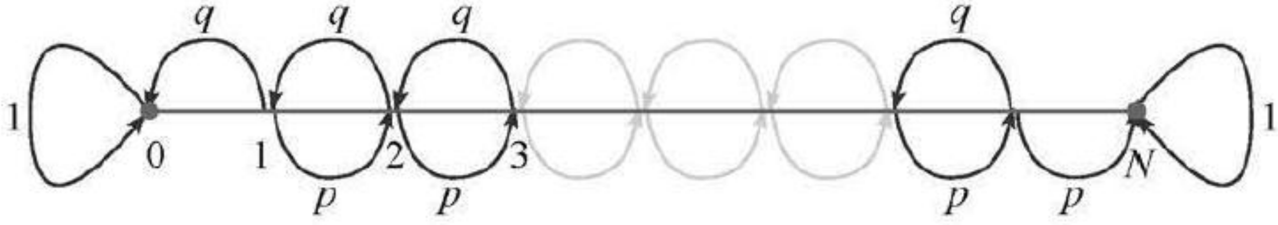
bundan hosil qilamizki

$$\pi_1 = \pi_2 = \tfrac{1}{2}.$$

14.3. Bir o’lchovli tasodifiy adashish

14.3.1. Yutilish uchli bir o’lchovli tasodifiy adashish

Quyidagi tasodifiy adashish modelida p,q o'tish ehtimoli va N holat soni berilgan bo'lsin:



0 va N holatda o'tish ehtimolligidan boshlanganda rekkurentli quyidagi tarzda ifodalanadi:

$$P(i) = p \cdot P(i+1) + q \cdot P(i-1).$$

bu rekkurent munosabatni yechimini topib:

$$\begin{cases} P(i) & = p \cdot P(i+1) + q \cdot P(i-1) \\ P(0) & = 0 \\ P(N) & = 1 \end{cases}$$

$$P(i) = \begin{cases} \dfrac{1-\left(\frac{q}{p}\right)^{i}}{1-\left(\frac{q}{p}\right)^{N}}, & p \neq q \\ \dfrac{i}{N}, & p = q = \frac{1}{2} \end{cases}$$

o'rnatamiz. Yutilish ehtimoli 0da i dan jo'natilganda shunga o'xshash quyidagi tarzda rekkurent ifodalanadi:

$$Q(i) = p \cdot Q(i+1) + q \cdot Q(i-1),$$

Aniq ko'rinishda (simmetriya munosabatidan)

$$Q(i) = \begin{cases} \dfrac{1-\left(\frac{p}{q}\right)^{N-i}}{1-\left(\frac{p}{q}\right)^{N}}, & p \neq q \\ 1-\dfrac{i}{N}, & p = q = \frac{1}{2} \end{cases}$$

bitta uchda yutilishga qadar qadamlar miqdori matematik kutilmasi (i holatdan jo'natilganda) rekkurent munosabat bilan ifodalanadi:

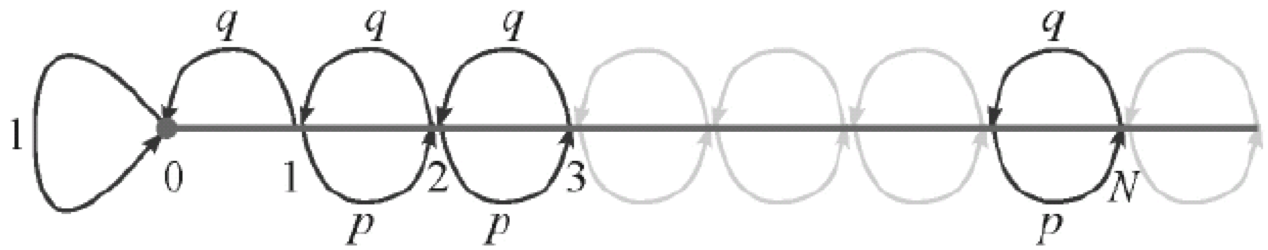
$$\begin{cases} \mathsf{M}(i) & = p \cdot \mathsf{M}(i+1) + q \cdot \mathsf{M}(i-1) + 1 \\ \mathsf{M}(0) & = 0 \\ \mathsf{M}(N) & = 0 \end{cases}$$

Qaysiki yechimga ega

$$\mathsf{M}(i) = \begin{cases} \dfrac{i}{q-p} - \dfrac{N}{q-p} \cdot \dfrac{1-\left(\frac{q}{p}\right)^i}{1-\left(\frac{q}{p}\right)^N}, & p \neq q \\ i(N-i), & p = q \end{cases}$$

14.3.2.Yarim cheksiz bir o'lchovli tasodifiy adashish

Yarim cheksiz bir o’lchovli tasodifiy adashish quyidagi adashish modeli tavsiflanadi:



Ya’ni $N=\infty$ yutilish bilan tasodifiy adashishga o’xshash. Yarim cheksiz bir o'lchovli tasodifiy adashishning barcha harakteristikalari limitga o'tish orqali topiladi:

$$\lim_{N\to\infty}.$$

u holda $P^{\infty}(i)$ cheksiz adashish ehtimolini quyidagi tarzda ifodalanadi:

$$\mathsf{P}^{\infty}(i) = \begin{cases} 0, & q \geqslant p \\ 1-\left(\frac{q}{p}\right)^i, & q < p \end{cases} \qquad p \neq q$$

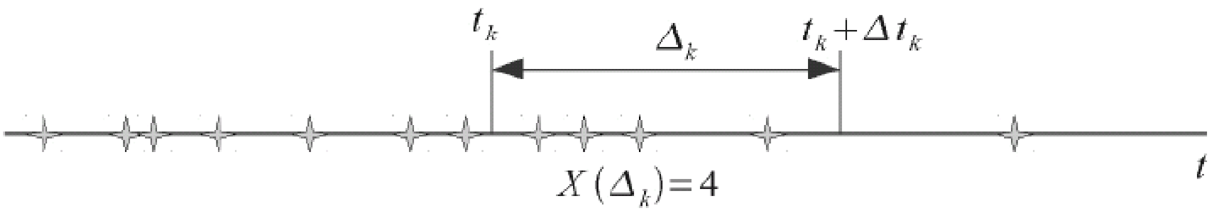
holatda qadamlar soni matematik kutilmasi:

$$\mathsf{M}^{\infty}_{(i)} = \lim_{N\to\infty}\left[\frac{i}{q-p} - \frac{N}{q-p}\cdot\frac{1-\left(\frac{q}{p}\right)^i}{1-\left(\frac{q}{p}\right)^N}\right] = \frac{i}{q-p} - \frac{1-\left(\frac{q}{p}\right)^i}{q-p}\cdot\lim_{N\to\infty}\frac{N}{1-\left(\frac{q}{p}\right)^N} = \begin{cases} \frac{i}{q-p}, & p<q \\ \infty, & p>q \end{cases}$$

P=q holatda qadamlar soni kutilmasi

$$\mathsf{M}^{\infty}_{(i)} = \lim_{N\to\infty} i(N-i) = \infty, \quad i \neq 0.$$

14.4. Hodisalar oddiy (puassonli) oqimi



14.3-ta'rif.
Hodisalar oqimi ma'lum bir vaqtda paydo bo'luvchi hodisalarni tasvirlaydi
14.4-ta'rif

$$\Delta_k = (t_k, t_k + \Delta t_k)$$

Vaqt oralig'ida sodir bo'luvchilar soni $X\Delta_k$ tasodifiy miqdordan iborat
14.5-tasdiq
Hodisalar oddiy oqimi aksiomalar orqali aniqlanadi:
1) erkinlik (bog'lanmaganlik) aksiomasi. Kesishmaydigan Δ_k intervallar uchun $X(\Delta_k)$ tasodifiy miqdorlar to’plamiga erkin (bog'lanmagan)

$$\Delta_1, \ldots, \Delta_n : \Delta_i \cap \Delta_j = \emptyset \Rightarrow X(\Delta_1), \ldots, X(\Delta_n) -$$

to'plamda erkin
2) bir jinslilik aksiomasi. $X(\Delta_k)$ taqsimot t_k interval holatidan bog’liq emas, faqat uning uzunligi Δt_k dan bog'liq
3) hodisalarni ajratish aksiomasi. Bir va ortiq hodisalarni paydo bo'lish ehtimoli quyidagi tenglikka bo'ysinadi.

$$\mathsf{P}\{X(\Delta_k) = 1\} = \lambda \Delta t_k + o(\Delta t_k), \qquad \mathsf{P}\{X(\Delta_k) > 1\} = o(\Delta t_k).$$

Navbatdagi hodisani kutish vaqti τ tasodifiy miqdordam iboratdir. Kutish vaqtning taqsimot funksiyasi

$$F_\tau(t) = \begin{cases} 1 - e^{-\lambda t}, & t \geqslant 0 \\ 0, & t < 0 \end{cases}$$

ehtimolning zichligi

$$f_\tau(t) = \begin{cases} \lambda e^{-\lambda t}, & t \geqslant 0 \\ 0, & t < 0 \end{cases}$$

hodisalar orasidagi masofa ξ tasodifiy miqdordan iboratdir. Uning harakteristikasi navbatdagi hodisa

$$F_\tau(t) = F_\xi(t).$$

kutish vaqtiga o'xshash.
t vaqtdagi hodisalar

$$p_m(t) = \mathsf{P}\left\{X(\Delta_k) = m\right\}, m = 0, 1, \ldots,$$

ehtimollik bilan aniqlanadi, puassonning taqsimot qonuniga ega

$$p_m(t) = \frac{(\lambda t)^m}{m!} e^{-\lambda t}$$

λ parametr hodisalar oqimi intensivligi deyiladi va

$$\lambda = \frac{\mathsf{M}X(t)}{t}$$

ko'rinishda aniqlanadi.

14.6-tasdiq

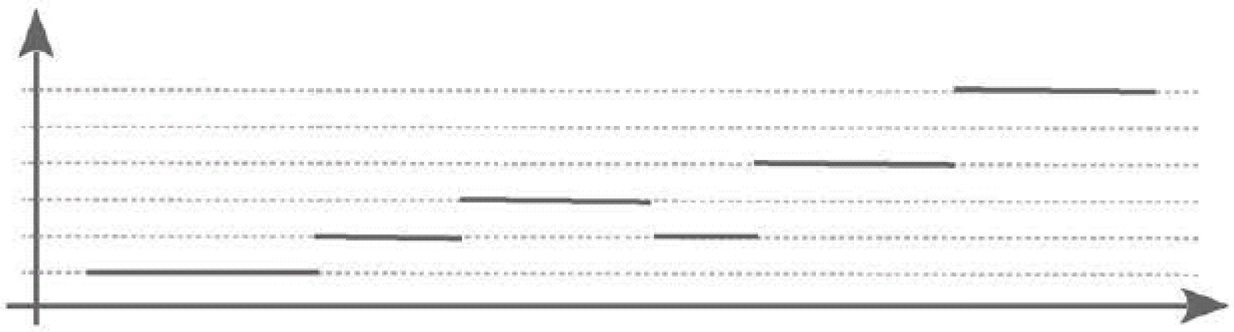
Aksiomalardan foydalangan holda ko'rsatish mumkinmi:

1)Kutish vaqti va hodisalar orasidagi vaqt intervali λ parametrli ko'rsatkichli taqsimotga egadir;

2)Vaqt intervalidagi hodisalar soni λt parametrli puasson taqsimotiga egadir.

3) λ-hodisalar oqimi intensivligi ya'ni birlik vaqt ichida sodir bo'luvchi hodisalarni o'rtu quyidagi xossalarga ega o'rtacha soni

14.5.Uzluksiz vaqtli bir jinsli Markov zanjiri



14.7-tasdiq

$P_{ij}(t)$ o'tish ehtimoli

$$p_{ij}(t) = \mathsf{P}\left\{\xi(t+s) = E_j | \xi(s) = E_i\right\}.$$

ko'rinishda aniqlanadi

U quyidagi xossalarga ega:

1) $p_{ij}(\Delta t) = \lambda_{ij}\Delta t + o(\Delta t),$

Bu yerda $\lambda_{ij} - i$ dan j gao'tish ehtimoli;

2) $p_{ii}(\Delta t) = 1 - \sum_{j\neq i} p_{ij}(\Delta) = 1 - \sum_{j\neq i}\lambda_{ij} + o(\Delta t) = \left[\lambda_{ii} = \sum_{j\neq i}\lambda_{ij}\right] = 1 + \lambda_{ii}\Delta t + o(\Delta t);$

Bir jinsli Markov zanjiri uchun Kolmogorov -chepmen tenglamasidan kelib chiqadiki,

$$p_{ij}(s+t) = \sum_k p_{ik}(s)p_{kj}(t), \forall t \geqslant 0, s \geqslant 0,$$

u holda

$$s = \Delta t:$$

$$p_{ij}(t+\Delta t) = \sum_k p_{ik}(\Delta t)p_{kj}(t) = \sum_{k\neq i} p_{ik}(\Delta t)p_{kj}(t) + p_{ii}(\Delta t)p_{ij}(t),$$

1)xossani qo'llab,

$$p_{ij}(t+\Delta t) = \sum_{k\neq i}(\lambda_{ik}\Delta t + o(\Delta t))p_{kj}(t) + (1 + \lambda_{ii}\Delta t + o(\Delta t))p_{ij}(t),$$

Hosil qilamiz;

Shunday qilib,

$$p_{ij}(t+\Delta t) - p_{ij}(t) = \sum_k (\lambda_{ik}\Delta t + o(\Delta t))p_{kj}(t),$$

taxminan Δ ga bo'lib, limitga o'tish bilan hosilani hosil qilamiz

Shunday qilib tahminan ga bo'lib, limitga o'tish yo'li bilan hosilani hoail qilamiz.

$$p'_{ij}(t) = \lim_{\Delta t\to 0}\frac{p_{ij}(t+\Delta t) - p_{ij}(t)}{\Delta t} = \sum_k \lim_{\Delta t\to 0}\frac{\lambda_{ik}\Delta t + o(\Delta t)}{\Delta t}p_{kj}(t) = \sum_k \lambda_{ik}p_{kj}(t)$$

Shunday qilib hosil qilingan tenglamalar sistemasi kolmogorov differensial tenglamalari sistemasi teskari

$$p'_{ij}(t) = \sum_k \lambda_{ik}p_{kj}(t)$$

Qaysiki boshlang'ich shart:

$$p_{ik}(0) = \delta_{ik}.$$

shunga o'xshash

$$p_{ij}(t+\Delta t) = \sum_k p_{ik}(t)p_{kj}(\Delta t),$$

tenglamadan limitga o'tish orqali kolmogorov diffensial tenglamalar to'g'ri sistemasi hosil qilingan:

$$p'_{ij} = \sum_k p_{ik}(t)\lambda_{kj},$$

boshlang'ich shart qaysiki teskari sitemaga o'xshash)

$$p_{ik}(0) = \delta_{ik}.$$

14.5-ta'rif

$\wedge$matritsa o'tish intensivligi matritsasi deyiladi

$$\Lambda = \begin{pmatrix} \lambda_{11} & \lambda_{12} & \dots & \lambda_{1N} \\ \lambda_{21} & \lambda_{22} & \dots & \lambda_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{N1} & \lambda_{N2} & \dots & \lambda_{NN} \end{pmatrix}$$

bunda

$$\textstyle\sum_j \lambda_{ij} = 0.$$

14.8-tasdiq.

Markov zanjirining

$$\pi_k = \lim_{t\to\infty} p_{ik}(t)$$

holatini limitik ehtimoli uzluksiz vaqt bilan sistemadan topiladi:

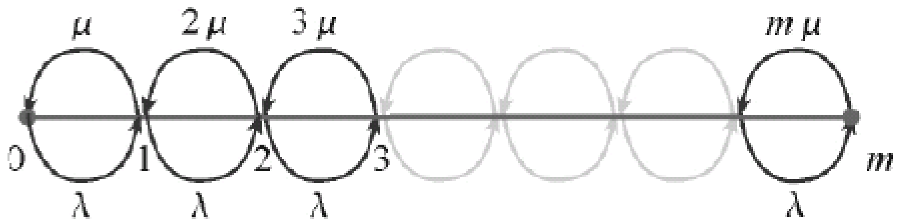
$$\begin{cases} \sum_i \pi_i \lambda_{ik} & = \pi_k \\ \sum_i \pi_i & = 1 \end{cases}$$

14.6.Ommaviy xizmat ko'rsatishning sodda sistemasi

Ommaviy Xizmat ko'rsatish sistemalariga avtomobillar narxi,.supermarketdagi kassalr misol bo'la oladi. Bunday holatda parkdagi joy va kaasalar xizmat ko'rsatish chizig'i, avtomobil va haridorlar- buyurtma

14.6-ta'rif.

Xizmat ko'rsatish sistemasi λ parametrga ega buyurtmalar oqimi intensivligi $\mu = \frac{1}{T}$ xizmat ko'rsatish intensivligi m+1 har xil holatlar bo'lish mumkin bu yerda i-holat ifodalaydiki, i-holat ifodalaydiki, xizmat ko'rsatish zichig'i band.



Xizmat ko'rsatish oddiy sistemasi o'tkazish intensivligi matritsasi:

$$\Lambda = \begin{pmatrix} -\lambda & \lambda & 0 & \dots & 0 \\ \mu & -(\mu+\lambda) & \lambda & \dots & 0 \\ \vdots & \vdots & \ddots & \dots & 0 \\ \vdots & \vdots & (m-1)\mu & -(m-1)\mu+\lambda & \lambda \\ 0 & \dots & \dots & 0 & -\mu m \end{pmatrix}$$

$$\sum_k \pi_k \lambda_{kj} = 0,$$

shart bo'yicha chiziqli tenglamalar sistemasini tashkil qiladiki, qaysiki yechimi:

$$\pi_m = \frac{1}{m!}\left(\frac{\lambda}{\mu}\right)^m \pi_0,$$

buyurtmani yo'qotish ehtimolidan iborat. π_m uchun ifodadan Erlang formulasi keltirib chiqariladi,

$$\pi_k = \frac{\frac{1}{k!}\left(\frac{\lambda}{\mu}\right)^k}{\sum_i \frac{1}{i!}\left(\frac{\lambda}{\mu}\right)^i} \qquad \textbf{(14.4)}$$

xizmat ko'rsatishning oddiy sistemasini ifodalovchi 5T xizmat ko'rsatishning o'rtacha vaqti

Foydalanilgan adabiyotlar

1 **Сборник задач по теории вероятностей, математической статистике и теории случайных функций** [Текст]: учеб. пособие / [Б. Г. Володин и др.]; под общ. ред. А.А. Свешникова. – Изд. 4-е, стер. – СПб. Лань, 2008. – 445 с. –ISBN 978-5-8114-0708-8.

2 **Прохоров, С. А.** Структурно-спектральный анализ случайных процессов [Текст] / С.А. Прохоров, В.В. Графкин. – Самара: СНЦ РАН, 2010. – 147 с. – ISBN 978-5-93424-469-0.

3 **Миллер, Б. М.** Теория случайных процессов в примерах и задачах [Текст] / Б. М. Миллер, А. Р. Панков; под ред. А. И. Кибзуна. – М.: Наука: Физматлит. – 2007. – 317с.

4 **Розанов, Ю. А.** Введение в теорию случайных процессов [Текст] / Ю. А. Розанов. – М.: Наука, 1982. – 128с.

5 **Вентцель, Е. С.** Теория вероятностей [Текст] / Е. С. Вентцель. – М.: Высшая школа, 2002. – 576с.

6 **Яглом, А. М.** Корреляционная теория стационарных случайных функций / А. М. Яглом. – Л.: Гидрометеоиздат, 1981. – 282с.