O’ZBEKISTON RESPUBLIKASI OLIY BA O’RTA MAXSUS TA’LIM VAZIRLIGI

SAMARQAND DAVLAT UNIVERSITETI

XUDAYOROV ULUG’BEK OBILMALIK O’G’LINING

POTENSIALLAR NAZARIYASINING TESKARI MASALASI

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DISSERTATSIYASI

Ish ko’rib chiqildi va himoyaga ruxsat berildi

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MUNDARIJA

Kirish.

T soha $B_R = \{|x| \le R\}, \quad R<1$ doira ichida saqlansin. $u(x,T)$ orqali $\frac{1}{2\pi}\int_T \ln|x-y|dy$

T sohaning zichligi birga teng bo'lgan soha logarifmik potensialini belgilaymiz.
γ birlik doiraning biror yoyi qismi bo'lsin.

Logarifmik potensialning teskari masalasi deb $u(x,T)$ potensialning γ yoydagi Koshi shartlariga ko'ra T sohani topishni tushunamiz.

Bu masala eoydali qazilmalarni topishdagi gravimetriyaning asosiy masalalaridan hisoblanib, matematik fizikaning shartli korrakt masalalaridan bo'lib hisoblanadi. Bu masalani yechishning effiktiv usullari 1960-yillarda A.N.Tixonov, B.K.Ivanov va M.M.Lavrintevlar tomonidan yaratilgan..
Masala shartli korrektligini isbotlashning asosini yechishning biror M to'plamda yagonaligi va turg'unligini isbotlashdan iboratdir. Bunda M to'plam korrektlik sinfi deb ataladi.

A.N.Tixonovning [19] ishida korrektlik sinfi kompakt bo'lganda yechimning yagonaligidan uning turg'unligi kelib chiqadi. Bu masala yechimining mavjudligi masalaning fizik xususiyatlaridan kelib chiqadi.
Qaralayodgan masalaning yechimi yagonaligi yulduzli sohalar uchun P.S.Novikov tomonidan isbotlangan [11]. L.N.Sretenskiy o'rta tekisligi mavjud bo'lgan sohalar uchun yagonalik teoremasini isbotlagan. Masala yechimining biror sohaga yaqin ekanligini bolgan holda mavjudligini V.K.Ibanov isbotlagan. Soha zichligi o'zgaruvchi bo'lganda yechimning yagonaligini Yu.A.Shankin[26] isbotlagan. Masalaning ko'p o'lchovli holi yulduzli sohalar va kontaktli sohalar uchun o'rganib yagonalik teoremalari A.I.Prilenko tomonidan isbotlangan[13,14]. Masala yechimi mavjud emasligini ko'rsatuvchi misollar [6] ishda

keltirilgan.M.M.Lavrentev tomonidan masala yechimining turg'unligi teoremasi yulduzli sohalar uchun isbotlangan. Potensiallar nazariyasi teskari masalasi V.M.Isanov tomonidan Gelmgols teoremasi uchun o'rganilgan.

Ushbu ishda logarifmik potensialning teskari masalasi o'rganilgan bo'lib, masala yechimining yagonaligi birlik zichlikga ega bo'lgan va biror egri chiziqli koordinatalarga nisbatan yulduzli bo'lgan sohalar uchun isbotlangan bo'lib, uch bo'limdan iborat.Birinchi bo'limning 1.1-1.5 parograflarida soha potensiali, oddiy qatlam , qo'sh qatlam potensiallarining xossalari o'rganilgan bo'lib, ular yordamida Dirixli va Neyman masalalarini Fredgolm integral tenglamariga keltirish masalalari hamda potensiallarni hisoblash masalalari qaralgan.

Ikkinchi bo'limning 2.1-2.2 parograflarida logarifmik potensiallarning massa tarqatmasining xossalari o'rganilgan. Massa zichligi μ_0 bo'lgan tekis D sohaning logarifmik potensial uchun massa tarqatmasida D dagi μ_0 gi $\Gamma \in \partial T$ da shunday μ zichlik qo'yiladiki, μ massa $\Gamma \in \partial T,\ T \supset D$.

§2.1 da , agar $\mu_0 > 0$ bo'lsa, u holda μ ham musbat funksiyadan iborat bo'lishi isbotlanadi

§2.2 da ixtiyoriy sohalar uchun tarqatma masalasi o'rganiladi va komform akslash natijasida tarqatma masalasi birlik doirada keltiriladi. §2.3 da μ tarqatmaning $C^{(0,\lambda)}$ dagi normasi bahosi keltiriladi.
Uchinchi bobda muhim yordamchi teorema isbotlanadi ba dissertatsiyaning asosiy natijasi isbotlanadi. §3.1 ga bazi belgilashlar kiritiladi va asosiy teorema isbotlanadi. §3.2 da potensialning oddiy qatlam potensiali bilan yaqinlashtirishning baholashlari keltiriladi. §3.3 da bo'lakli silliq sohalar uchun konform akslantirish o'rganiladi va conform akslash modulini soha chegarasining $C^{(1,\lambda)}$ normasi bilan bog'lovchi tengsizliklar keltiriladi.

§2.4 da yordamchi tengsizliklar o'rnatiladi va ular yordamida §3.5 ning natijasi isbotlanadi. §3.5 da M.M. Lavrentivning usuli bilan tarqatmaning muhim usulini qo'llab, tarqatmaning yordamchi teoremasi isbotlanadi.

Potensiallar nazariyasi

1§. Potensiallarning xossalari

$D \in E_n$ chegaralangan sohada zichligi $\mu(y)$ elektr zaryadlari taqsimlangan bo'lsin.

Bo'nda $y \bar{\in} \bar{D}$ bo'lganda $\mu(y)=0$. $\mathrm{E}_n(x,y)$ – Laplas tenglamasining fundamental yechimi bo'lsa,

$$u(x)=\int_D \mu(y)\mathrm{E}_n(x,y)dx \quad x=(x_1,x_2,x_3) \qquad (1.1)$$

funksiyaga hajm potensiali yoki uch o'lchovli potensial deb aytamiz, agar $D \subset E_3$
Agar $D \subset E_2$ bo'lsa, (1) ga sohaning logarifmik potensiali deymiz.

Agar $\mu(y)$zaryadlar bo'lakli silliq σ-sirtda taqsimlangan bo'lsa, u holda

$$u(x)=\int_\sigma \mu(y)\mathrm{E}_n(x,y)d\sigma_y \qquad (1.2)$$

integralga $n=3$ bo'lganda oddiy qatlam potensiali, $n=2$ bo'lganda oddiy qatlamning logarifmik potensiali deymiz.
$\sigma \subset E_n$ sirtda musbat yunalishga ega bo'lgan ν normal yunalishni tanlaymiz.

$y \in \sigma$ nuqtadagi ν normalga $\overset{1}{y}$ va $\overset{2}{y}$ nuqtalarni olamiz. $\overset{1}{y}$ dan $\overset{2}{y}$ gacha yunalish

ν yunalish bilan bir xil bo'lsin. $\overset{1}{y}$, $\overset{2}{y}$ nuqtalarda $-\mu_0<0,\ \mu_0>0$ zaryadlarni joylashtiramiz x nuqtada bu zaryadlarning potensiali

$u_0(x)=\mu_0\mathrm{E}_n(x,\overset{2}{y})-\mu_0\mathrm{E}_n(x,\overset{1}{y})$ ko'rinishda bo'ladi. Agar $\overset{1}{y}$ va $\overset{2}{y}$ lar y

ga intilsa va $\mu_0\left|\overset{2}{y}-\overset{1}{y}\right|=\mu(y)$ o'zgarmas bo'lib qolsa, u holda zaryadlarning limitik taqsimlanishiga dipol deb aytamiz, ν yunalishga dipol o'qi deymiz.

x nuqtada dipol potensiali limit orqali aniqlanadi

$$u_0(x)=\lim_{\left|\overset{2}{y}-\overset{1}{y}\right|\to 0}(\mu_0 \mathrm{E}_n(x,\overset{2}{y})-\mu_0 \mathrm{E}_n(x,\overset{1}{y}))=\lim_{\left|\overset{2}{y}-\overset{1}{y}\right|\to 0}\mu(y)\frac{\mathrm{E}_n(x,\overset{2}{y})-\mathrm{E}_n(x,\overset{1}{y})}{\left|\overset{2}{y}-\overset{1}{y}\right|}=\mu(y)\frac{\partial_y \mathrm{E}_n(x,y)}{\partial \nu}$$

Bunda $x=\left(x_{1,}x_{2,}x_3\right)-$paramitrik nuqtadan iborat

$$u(x)=\int_\sigma \mu(y)\frac{\partial_y \mathrm{E}_n(x,y)}{\partial \nu}d\sigma_y \qquad (1.3)$$

(3) ko'rinishidagi potensialga $n=3$ bo'lgan qo'sh qatlam potensiali, $n=2$ bo'lganda qo'sh qatlam logarifmik potensiali deb aytamiz.

$$\frac{\partial_y \mathrm{E}_3(x,y)}{\partial \nu}=\frac{\partial}{\partial \nu}\frac{1}{|x-y|}=-\frac{1}{|x-y|^2}\cos(x-y,\nu)$$
$$\frac{\partial_y \mathrm{E}_2(x,y)}{\partial \nu}=\frac{\partial}{\partial \nu}\ln\frac{1}{|x-y|}=-\frac{1}{|x-y|}\cos(x-y,\nu)$$

bo'lgani uchun (3) ni $\cos(x-y,\nu)=\varphi_{xy}$ desak

$$u(x)=\int_\sigma \mu(y)\frac{\cos\varphi_{xy}}{|x-y|}d\sigma_y \qquad (4)$$

ko'rinishida yozamiz.

E_3 da σ sirt (E_2 da σ chiziq) ni Lyapunov sirti (chizig'i) de ataymiz, agar

1^0 har bir $y\in\sigma$ nuqtada ν_y normal mavjud,

2^0 shunday $\delta>0$ son mavjudki, har bir x nuqta uchun $|x-y|<\delta$ sfera σ sirtni ikki qismga ajratadi: $\sigma_\delta^{'}$ $|y-x|\le\delta$ da yotadi, bu σ_δ sharning tashqarisida yotadi hamda ν_δ ga parallel to'g'ri chiziq $\sigma_\delta^{'}$ ni farqli boshqa nuqtada kesadi.

3^0 $C_1>0,\quad 0<\alpha<1$ o'zgarmaslar mavjudki

$$\left|\nu_{\overset{1}{y}} - \nu_{\overset{2}{y}}\right| < C_1 \left|\overset{1}{y} - \overset{2}{y}\right|^{\alpha} \qquad \forall \overset{1}{y}, \overset{2}{y} \in \sigma$$ bo'ladi.

2§. Soha potensialining xossalari

D^+ sohada taqsimlangan zaryadlarning $n=3$ bo'lgandagi potensialini hajm potensiali , $n=2$ dagi potensialga sohaning logarifmik potensiali deb ataymiz.

Bu potensiallar qo'yidagicha bo'ladi:

$$u(x) = \int_{D^+} \mu(y) \mathrm{E}_n(x,y) dy \qquad (n = 2,3) \tag{2.1}$$

$$\mathrm{E}_n(x,y) = \begin{cases} \ln \dfrac{1}{|y-x|}, & n = 2 \\ \dfrac{1}{|y-x|}, & n = 3 \end{cases}$$

Teorema 2.1. Agar $\mu(y)$ D^+ sohada chegaralangan va integrallanuvchi bo'lsa, u holda $u(x)$ funksiya E_n da uzluksiz bo'ladi.

Teorema 2.2. Agar $\mu(y)$ funksiya D^+ sohada chegaralangan uzluksiz bo'lsa, yani $|\mu(y)| < A$, u holda (2.1) potensial E_n da uzluksiz birinchi tartibli hosilaga ega va bu hosilani integral ostida bajarish mumkin.

Teorema 2.3. Agar $\mu(y)$ funksiya D^+ sohada uzluksiz va chegaralangan bo'lsa, u holda (2.1) potensial E_3 da cheksiz nuqta atrofida regulyar bo'ladi.

Teorema 2.4. Agar $\mu(y)$ funksiya D^+ sohada uzluksiz va chegaralangan bo'lsa, u holda n-o'lchovli potensial D^- da garmonik bo'ladi.

Teorema 2.5. Agar $\mu(y)$ funksiya D^+ sohada uzluksiz va chegaralangan hosilaga ega bo'lsa, u holda $n -$ o'lchovli potensial D^+ sohada uzluksiz ikkinchi tartibli hosilaga ega bo'ladi.

Nateja. Agar $\mu(y) \in C^1(D^+)$ bo'lib, chegaralangan bo'lsa, u holda (2.1) potensial qo'yidagi Puasson tenglamasini qanoatlantiradi:

$$\Delta u(x) = -\omega_n \mu(x) \quad (2.2)$$

Bu tenglikdan

$$u_0(x) = -\frac{1}{\omega_n} \int_{D^+} f(y) \mathrm{E}_n(x,y) d\sigma_y \quad (2.3)$$

funksiya Puasson tenglamasining xususiy yechimi bo'ladi. Agar biz (2.2) tenglamada

$u(x) = u_0(x) + v(x)$ almashtirish olsak, $v(x)$ funksiya $\Delta v(x) = 0$ Laplas tenglamasining yechimi bo'ladi.

3§. Qo'sh qatlam potensialining hossalari

Qosh qatlam potensiali E_n da

$$u(x) = \int_\sigma \mu(y) \frac{\partial_y \mathrm{E}_n(x,y)}{\partial \nu} d\sigma_y = \int_\sigma \mu(y) \frac{\cos\varphi_{xy}}{|x-y|^{n-1}} d\sigma_y \quad (3.1)$$

Teorema 3.1 Agar σ Lyapunov sirti bo'lsa va $\mu(y)$ integrallanuvchi bo'lsa hamda chegaralangan bo'lsa, u holda (3.1) qo'sh qatlam porensiali fazoda aniqlangan bo'ladi.

Teorema 3.2. Agar qo'sh qatlam potensiali $\mu = 1$ bo'lsa hamda σ Lyapunov sirti bo'lsa, u holda qo'sh qatlam potensiali bo'lakli o'zgarmas bo'ladi. Yani

$$u(x) = \int_\sigma \frac{\cos\varphi_{xy}}{|x-y|^{n-1}} d\sigma_y = \begin{cases} -\omega_n, & x \in D^+ \\ -\frac{1}{2}\omega_n, & x \in \sigma \\ 0, & x \in D^- \end{cases}$$

$\omega_3 = 4\pi \quad n = 3, \qquad \omega_2 = 2\pi \quad n = 2$

Agar $x^0 \in \sigma$ da $u(x^0) = \int_\sigma \mu(y) \frac{\partial \mathrm{E}_n(x^0, y)}{\partial \nu} d\sigma_y$ –qosh qatlam potensialining to'g'ri qiymati deymiz.

$$u^-(x^0) = \lim_{\substack{x \to x^0 \\ x \in D^-}} \int_\sigma \mu(y) \frac{\partial \mathrm{E}_n(x^0, y)}{\partial \nu} d\sigma_y$$

$$u^{+}(x^{0})=\lim_{\substack{x\to x^{0}\\ x\in D^{+}}}\int_{\sigma}\mu(y)\frac{\partial \mathrm{E}_{n}(x^{0},y)}{\partial \nu}d\sigma_{y}$$

deb belgilaymiz.

Teorema 3.3. Agar σ Lyapunov sirti bo'lib, $\mu(y)$ σ da uzluksiz bo'lsa, u holda qo'sh qatlam potensiali $u^{-}(x^{0})$ $u^{+}(x^{0})$ limitlarga ega bo'ladi va bu limitlar qo'yidagicha aniqlanadi

$$u^{-}(x^{0})=u(x^{0})+\frac{1}{2}\omega_{n}\mu(x^{0}), \quad u^{+}(x^{0})=u(x^{0})-\frac{1}{2}\mu(x^{0})$$

Bo'larda ko'ramizki qo'sh qatlam potensiali D^{+} dan σ orqali D^{-} ga o'tishda sakrashga ega bo'lib, bu sakrash $u^{+}(x^{0})-u^{-}(x^{0})=-\omega_{n}\mu(x^{0})$, $\omega_{3}=4\pi$, $\omega_{2}=2\pi$ orqali aniqlanadi.

4§. Oddiy qatlam potensialining hossalari

Agar σ yopiq sirt, $\mu(y)$ zichlik bo'lsa,

$$u(x)=\int_{\sigma}\mu(y)\mathrm{E}_{n}(x,y)dy \qquad (4.1)$$

funksiyaga oddiy qatlam potensiali degan edik.

$\mathrm{E}_{n}(x,y)$ – Laplas tenglamasining fum\ndamintal yechimidan iborat.

Teorema 4.1 Agar σ Lyapunov sirti bo'lsa va $\mu(y)$ σ da integrallanuvchi chegaralangan bo'lsa, u holda oddiy qatlam potensiali E_{n} da aniqlangan va uzluksiz funksiyadan iborat bo'ladi.

$n=3$ da cheksiz nuqta atrofida regulyar, $n=2$ da cheksiz nuqta atrofida logarifmik maxsuslikga ega

Teorema 4.2. Agar σ Lyapunov sirti bo'lsa va $\mu(y)$ σ da integrallanuvchi chegaralangan bo'lsa, u holda (4.1) funksiya $D^{+}\cup D^{-}$ da garmonik funksiya bo'ladi.

Teorema 4.3. Agar σ Lyapunov sirti bo'lsa va $\mu(y)$ σ da uzluksiz bo'lsa, u holda oddiy qatlam potensiali to'g'ri normal hosila ega bo'lib,

$\frac{\partial u^+(x)}{\partial \nu_{x^0}}$, $\frac{\partial u^-(x)}{\partial \nu_{x^0}}$ limitik qiymatlar σ da uzluksiz bo'ladi.

Teorema 4.4. Agar σ Lyapunov sirti bo'lsa va $\mu(y)$ σ da uzluksiz bo'lsa, u holda limitik normal bilan , normal to'g'ri hosilasi orasida

$$\frac{\partial u^-(x^0)}{\partial \nu_{x^0}} = \frac{\partial u(x^0)}{\partial \nu_{x^0}} - \frac{\omega_n}{2}\mu(x^0)$$

$$\frac{\partial u^+(x^0)}{\partial \nu_{x^0}} = \frac{\partial u(x^0)}{\partial \nu_{x^0}} + \frac{\omega_n}{2}\mu(x^0)$$

munosabat o'rinli bo'ladi.

Potensiallar keltirilgan xossalariga qarab, D^+, D^- Dirixli masalalari hamda N^+, N^- Neyman masalalari yechimini potensiallar ko'rinishida izlaymizki, bu potensiallarda zichlik $\mu(y)$ funksiyalar nomalum bo'ladi. Bunda $\mu(y)$ zichliklarga nisbatan Fridgolimning integral tenglamalariga kelamiz.

Dirixlining D^+ ichki masalasi yechimini qo'sh qatlam potensiali ko'rinishida izlaymiz.

$$u(x) = \int_\sigma \mu(y)\frac{\cos\varphi_{xy}}{|x-y|}d\sigma_y,\ \ \varphi_{xy} = (x-y.\nu_y),\ \ x \in D^+$$

Dirixlening D^- tashqi masalasi yechimini qo'sh qatlam potensiali ba o'zgarmas α yig'indisi ko'rinishida izlaymiz:

$$u(x) = \int_\sigma \mu(y)\frac{\cos\varphi_{xy}}{|x-y|}d\sigma_y + \alpha,\quad x \in D^-$$

Neyman N^+ ichki masalasi yechimini oddiy qatlam potensiali kurinishida izlaymiz:

$$u(x) = \int_\sigma \mu(y)\ln\frac{1}{|x-y|}d\sigma_y,\quad x \in N^+$$

Neyman N^- ichki masalasi yechimini oddiy qatlam potensiali kurinishida izlaymiz:

$$u(x)=\int_{\sigma}\mu(y)\ln\frac{1}{|x-y|}d\sigma_y,\quad x\in N^{-}$$

II-BOB. TARQATMANING BA'ZI XOSSALARI

§ 2.1 Tekis sohalarda massa tarqatmasining masalasi haqida.

T tekislikdagi bo'lakli silliq Γ chegaraga ega bo'lgan soha bo'lib, D, T sohaga tegishli ochiq soha bo'lsin. T ning har bir chegaraviy nuqtasiga ega bo'sh bo'lmagan konus bilan o'ranish mumkin bo'lsin.

Agar ν yurituvchisi kompakt bo'lgan o'lchov bo'lsa,

$$u(x,\nu)=\frac{1}{2\pi}\int_{\Gamma}\ln|x-y|\,d\nu(y)$$

funksiyaga ν o'lchovining logarifmik potensial deb aytamiz.

Γ da $\mu(y)$ zichlikga ega bo'lgan oddiy qatlamning potensiali;

$$V(x,\Gamma,\mu)=\frac{1}{2\pi}\int_{\Gamma}\mu(y)\ln|x-y|\,d\Gamma(y).$$

$\mu_0(y)$ zichlikga ega bo'lgan D sohaning logarifmik potensiali

$$u(x,D,\mu_0)=\frac{1}{2\pi}\int_{D}\mu_0(y)\ln|x-y|\,dy$$

bo'lib, $\mu_0(y)$ $\bar{D}$ sohada uzluksiz bo'lgan funksiya.

Massa tarqatma masalasida Γ da shunday $\mu(y)$ funksiyani topish talab qilinadiki,

$$u(x,D,\mu_0)=V(x,\Gamma,\mu),\qquad x\in\square\setminus\overline{T}\qquad(2.1)$$

biz qo'yidagi $T=\{|y|<1\}$ bo'lganda tarqatmaning ba'zi xossalarini keltiramiz. Bunda oldin keyinchalik qo'llaniladigan qo'yidagi ortaganallik munosabatini isbotlaymiz.

<u>Limma 2.1</u> Agar T chekli sohaning tutashmasi $\bar{T}$ bog'lamli bo'lsa va uning chegarasi Γ bo'lakli silliq bo'lsa, hamda yurituvchisi Γ bo'lgan ν o'lchov bo'lib, T ning ochiq to'plam ostisi bo'lsa, u holda

$$u(x,\nu)=u(x,D,\mu_0),\quad x\in \mathbb{R}\setminus T \tag{2.2}$$

tenglik

$$\int_{\Gamma} h(y)d\nu(y)=\int_{D} h(y)\mu_0(y)dy \tag{2.3}$$

tenglikka teng kuchli bo'ladi.

bunda $h(y)$ ixtiyoriy T da garmonik va $\bar{T}$ da uzluksiz bo'lgan funksiya.

Isbot: (2.2) dan (2.3) kelib chiqishini isbotlaymiz. $\tilde{h}$ $\bar{T}$ ning ochiq atrofi bo'lgan W atrofida garmonik bo'lsin. Cheksiz differensiallanuvchi f funksiyani shunday tuzamizki $\bar{T}$ ning ochiq W_1 atrofida $f=1$: $\bar{T}\subset W_1\subset \overline{W_1}\subset W$ va $\bar{T}$ ning ochiq W_2 atrofida $f=0$: $\overline{W_1}\subset W_2\subset \overline{W_2}\subset W$

$$\begin{aligned}\int_W u(x,\nu)\Delta(f(x)h(x))dx&=\int_{W_1} u(x,\nu)\Delta(f(x)h(x))dx+\int_{W\setminus W_1} u(x,\nu)\Delta(f(x)h(x))dx=\\&=\int_{W\setminus W_1} u(x,\nu)\Delta(f(x)h(x))dx=\int_{W\setminus W_1} u(x;D,\mu_0)\Delta(f(x)h(x))dx,\end{aligned}$$

Demak

$$\int_W dx\int_{\Gamma}\ln|x-y|\Delta(f(x)h(x))d\nu(y)=\int_W dx\int_D \mu_0(y)\ln|x-y|\Delta(f(x)h(x))dy$$

Oxirgi integrallarda x *va* y larning tartibini o'zgartirsak (Fubini teoremasi)

$$\int_W d\nu(y)\int_{\Gamma}\ln|x-y|\Delta(f(x)h(x))d\nu(y)=\int_D \mu_0(y)f(y)h(y)dy\int_W \ln|x-y|\Delta(f(x)h(x))dx \tag{2.4}$$

Endi fundamental yechimning ta'rifiga ko'ra $\left(f, \tilde{h} \in C_0^\infty\right)$ W bo'yicha integral

$f(y)\tilde{h}(y)$ ga teng, shuning uchun (2.4) dan

$$\int_\Gamma f(y)h(y)dv(y) = \int_D \mu_0(y) f(y)h(y)dy \qquad (2.5)$$

(2.5) da $T \supset (D \cup \Gamma)$ da $f = 1$ bo'lgani uchun (2.5) dan

$$\int_\Gamma h(y)dv(y) = \int_D \mu_0(y)h(y)dy \qquad (2.6)$$

tenglikni hosil qilamiz.

Agar $h \in (\overline{T})$ T da garmonik bo'lsa, [8] ga ko'ra shunday garmonik $\tilde{h}_n$ ketma-ketlik mavjudki, ular T ning atrofida garmonik bo'lib, $C\left(\overline{T}\right)$da $\tilde{h}(y)$ ga intiladi. (2.6) da $(\tilde{h} = \tilde{h}_n)$

deb limitga o'tib, (2.3) ni hosil qilamiz. Endi (2.3) dan (2.2) kelib chiqadi. Limma 2.1 isbotlandi.

Natija: (2.1) shartda $\mu - \Gamma$da summalanuvchi funksiya bo'lsin. U holda (2.1) qo'yidagi ortogonallik munosabatiga ekvivalent bo'ladi.

$$\int_\Gamma h(y)\mu(y)d\Gamma(y) = \int_D h(y)\mu_0(y)dy \qquad (2.3^*)$$

barcha T da garmonik va $\overline{T}$ da uzluksiz funksiyalar uchun.
Natija isbotini ν o'lchov va $\mu(y)$ zichlik uchun isbotlash yetarli.

T birlik doiradan iborat bo'lsin, yani $T = \{|y| < 1\}$

<u>Teorema 2.1</u> Ixtiyoriy $D, \overline{D} \in T$ uchun $\mu \in C^{(0,\lambda)}(\Gamma)$ zichlik mavjudki, bu zichlik $u(x, D, \mu_0)$ ning Γ dagi tarqatmasidan iborat bo'ladi.

Isbot: (2.1) tenglikni $\mu(x)$ ga nisbatan quyidagi integral tenglamaga teng kuchli ekanligini ko'rsatamiz.

$$-\pi\mu(x)+\int_{\Gamma}\mu(y)\frac{\partial}{\partial|x|}\ln|x-y|d\Gamma(y)=f(x),\ x\in\Gamma \tag{2.7}$$

bunda $f(x)=\lim\frac{\partial}{\partial|x|}u\left(x;D,\mu_0\right),\ x\to x$ (bunda integral T dan tashqaridan)

$x\in n(x)-\Gamma$ ning normali (x nuqtadagi)

(2.1) dan (2.7) kelib chiqadi. Xaqiqatdan ham (2.1) dan $\mu\in C^{(0,\lambda)}(\Gamma)$ shartdan differensiallab qo'yidagini hosil qilamiz

$$\frac{\partial}{\partial|x|}u\left(x,D,\mu_0\right)=\frac{\partial}{\partial|x|}V\left(x,\Gamma,\mu\right)\quad x\in R^2\setminus T \tag{2.8}$$

(2.8) da tashqaridan $x\to x$ limitga o'tsak $f(x)$ ni hosil qilamiz. Bunda limitga o'tish mumkin, chunki $u\left(x,D,\mu_0\right)$ Γ ning atrofida analitik funksiya. O'ng tomonida potensialning normal hosilasini sakrashdan foydalanib, (2.7) ning chap tomoninihosil qilamiz.

Endi (2.7) dan (2.1) ni kelib chiqishini ko'rsatamiz. Haqiqatdan ham V va u funksiyalarning to'g'ri normal hosilalarini

$\left(\frac{\partial}{\partial n}V\right)_+$ *va* $\left(\frac{\partial}{\partial n}u\right)_+$ orqali belgilasak (u *va* V larning to'g'ri normal hosilari)

(2.7) dan qo'yidagini topamiz

$$\left(\frac{\partial}{\partial n}u(D,\mu_0)\right)_+(x)=\left(\frac{\partial}{\partial n}V(\Gamma,\mu)\right)_+(x)\quad x\in\Gamma \tag{2.9}$$

$|x|>1$, $y\in T\cup\Gamma$ dan $\ln|x-y|=\ln|x|+\ln\left|1-\frac{y}{x}\right|$ hosil bo'ladi, shuning uchun

$$u(x,D,\mu_0)=M_1\ln|x|+O_1\left(\frac{1}{|x|}\right) \tag{2.10}$$

bunda

$$M_1 = \int_D \mu_0(y)dy\,, \quad O_1(\frac{1}{|x|}) = \int_D \mu_0(y)\ln(1-\frac{y}{x})dy$$

$O_1(\frac{1}{|x|}) \le C(\mu_0)\frac{1}{|x|},$ agar $|x|>1$ da

Shunga o’xshash

$$V(x,\Gamma,\mu) = M_2 \ln|x| + O_2(\frac{1}{|x|}) \qquad (2.11)$$

(2.10) va (2.11) dan

$$\vartheta(x) = u(x,D,\mu_0) - V(x,\Gamma,\mu) = M\ln|x| + O(\frac{1}{|x|}) \qquad (2.12)$$

bunda $M = M_1 - M_2, \quad O = O_1 - O_2$

Endi $M = 0$ ekanligini ko’rsatamiz.

$M > 0$ bo’lsin. U holda (2.12) ga asosan $\lim\limits_{|x|\to\infty} \vartheta = +\infty$.

Shuning uchun $\vartheta(x)$ funksiya o’zining maksimumini $R^2\backslash T$ dagi biror $x_0 \in R^2\backslash T$ da erishadi. Garmonik funksiyalarning maksimum prinsipiga ko’ra $x_0 \in \Gamma$.
Zarima-Jaro Prinsipiga ko’ra [2.31 bet]

$$\left(\frac{\partial}{\partial n}\vartheta\right)_+ (x_0) \ne 0, \quad x_0 \in \Gamma$$

Bu (2.9) ga qarama-qarshidir. Shuning uchun (2.1) va (2.7) lar tengkuchli bo’ladi.

(2.7) integral tenglama $C(\Gamma) = C^{(0,0)}(\Gamma)$ da yechimga ega. Shuning uchun (2.7) ni qanoatlantiruvchi $\mu \in C(\Gamma)$ funksiya mavjud bo’ladi [3.461 bet].
Zichligi chegaralangan va o’lchovli bo’lgan potensial tipidagi integrallarning baholanishiga ko’ra [7.330 bet] f funksiya Gyolder sinfiga tegishli bo’ladi.

$\Gamma = \{|y| = 1\}$ bo’lgani uchun (2.7) tenglama sodda ko’rinishga ega [3.461 bet].

$$\mu(x) + \frac{1}{2\pi}\int_\Gamma \mu(y)d\tilde{A}(y) = -\frac{f(x)}{\pi}$$

bundan ko’rinadiki $\mu \in \square^{(0,\lambda)}(\Gamma)$

Endi μ ning manfiy emasligi ko’rsatamiz. Faraz qilaylik $\mu(y) < 0$ y_0

nuqtaning atrofida bo’lsin. U holda uzluksizlik prinsipidan $l \in \Gamma$ da $\mu(y) > 0$ bo’ladi. h orqali qo’yidagi Dirixle masalasi yechimini belgilaylik:

$$\Delta h = 0, \quad \{|y| < 1\}$$
$$h = h_0, \quad \{|y| = 1\}$$

Bundah_0 biror uzluksiz funksiya bo’lib , $h_0(y_0) > 0$, Γ da $h_0 \geq 0$ va $\Gamma \setminus l$ da $h_0 = 0$.

Limma 2.1 ga ko’ra qo’yidagi ortogonallik prinsipi o’rinli

$$\int_{\tilde{A}} \mu(y) h(y) d\tilde{A}(y) = \int_{D} \mu_0(y) h(y) dy \tag{2.13}$$

Chegaraviy shart h_0 ning tanlanishiga ko’ra (2.13) dagi chap integral noldan kichik. Ikkinchi tomondan $h \geq 0$ va $\mu_0 > 0$ bo’ladi. Shuning uchu garmonik funksiyalarning maksimum prinsipiga ko’ra (2.13) ning o’ng tomoni musbat bo’ladi.

Bu topilgan qarama-qarshilik μ ning manfiy emasligini ko’rsatadi.

§ 2.2 Ixtiyoriy tarqatmani doira holiga keltirish

T bir bog’lamli soha, $D \in T$ bo’lganda $u(D, \mu_0)$ potensialning $\Gamma \in C^{(1,\lambda)}$ ga tarqatmasi masalasini qaraylik.

$W(z)$ orqali T sohani birlik doiraga akslaylik. μ^* $u(D^*, \mu_0^*)$ ni

$\left(D^* = W(D), \mu_0^*(W) = \mu_0^*(W) \cdot |\frac{dz}{dW}|^2 (W) \right)$ birlik doira chegarasidagi tarqatmasi, μ esa

$u(D, \mu_0)$ ni Γ ga massa tarqatmasi.

Limma 2.2 Qo’yidagi tenglik o’rinli

$$\mu(z) = \mu^*(W(z)) \cdot \frac{1}{\left|\frac{dz}{dW}\right|(W(z))} \tag{2.14}$$

bunda $\frac{dz}{dW}$ hosilani $|W| < 1$ doira ichidagi hosilani tushunamiz.

Isbot. $u(D, \mu)$ potensialning T ning chegarasi μ zichlik bilan Γ ga tarqatmasiga qo’yidagi potensiallarning tengliklari o’rinli bo’ladi.

$$u(x;D,\mu_0)=V(x;\Gamma,\mu) \quad x\in R^2\setminus\overline{T} \tag{2.15}$$

Limma2.1 ga ko'ra (2.15)tenglik qo'yidagi ortogonallik munosabati o'rinli

$$\int_\Gamma \mu(y)h(y)d\tilde{A}(y)=\int_D \mu_0(y)h(y)dy \tag{2.16}$$

Bunda $h(y)$ ixtiyoriy garmonik funksiya agar $y\in T$ va $\overline{T}$ da uzluksiz funksiya. (2.16) tenglikda konform akslanish $(y=z\to w)$ ni bajaramiz, integral qo'yidagi ko'rinishni oladi.

$$\int_{|W|=1} \mu(z(w))\cdot h(z(w))\left|\frac{dz}{dw}\right|d\Gamma(w)=\int_{D^*}\mu_0(z(w))h(z(w))\left|\frac{dz}{dw}\right|dw$$

μ_0^* ning belgilanishidan

$$\int_{|W|=1} \mu(z(w))\cdot h^*(z(w))\left|\frac{dz}{dw}\right|d\Gamma(w)=\int_{D^*}\mu_0^*(w)h^*(w)dw \tag{2.17}$$

ixtiyoriy $h^*(w)=h(z(w))$ funksiyalar uchun.

Bunday funksiyalar to'plami $\{|w|\le 1\}$ da uzluksiz va $\{|w|<1\}$ da garmonik funksiyalardan iborat. (2.17) dagi $h^*(w)$ ixtiyoriyligidan $\mu(z(w))\left|\frac{dz}{dw}\right|$ $u(D^*,\mu^*)$ ning $\{|w|=1\}$ tarqatmasidan iborat bo'ladi. $\mu^*(w)$ ning ta'rifida $w\to z$ da Limma 2.2 ni isbotini hosil qilamiz. Limma 2.2 dan trqatma masalasi doiraga keladi.

Endi tarqatmaning $C^{(0,\lambda)}$ normasida baholaymiz.

$(x,y)\to(\tilde{x},\tilde{y})$ akslash E_2 ni E_2 ga akslantiradigan bir qiymatli ikki marta uzluksiz differensiallanuvchi xosmas bo'lsin.$(\xi,\eta)-$ qutb koordenatalaridan iborat. Komform akslashning soha chegarasining tenglamasi orqali baholanishlaridan [18.127-129 bet] qo'yidagi baholash o'rinli

$$|w(z)|^{(1,\lambda)}(\overline{T})+|z(w)|^{(1,\lambda)}(|w|\le 1)\le C(|\varphi|^{(1,\lambda)}[0,2\pi]) \tag{2.18}$$

bunda $\eta=\varphi(z)$ ∂T ning tenglamasi.

Limma2.3 Qo'yidagi baholar o'rinli

$$\left|\mu^*\right|^{(0,\lambda)} \le C\left(mesD^*\right)^{1/q}, \quad 0<\lambda<1$$

bunda $\quad C=C\left(\lambda,|\varphi|^{(1,\lambda)}\right), \quad q=q(\lambda)>1$

<u>Isbot</u> Teorema 2.1 da μ^* ni aniqlash qo'yidagi integral tenglamaga teng kuchli edi

$$-\pi\mu^*(x)+\int_\Gamma \mu^*(y)\frac{\partial}{\partial n_x}\ln|x-y|\,d\Gamma(y)=f(x)$$

birlik doira uchun bu integral tenglama

$$\mu^*(x)+\frac{1}{2\pi}\int_{|y|=1}\mu^*(y)\,d\Gamma(y)=f^*(x) \qquad (2.19)$$

ko'rinishni oladi. Bunda

$$f^*(x)=-\frac{1}{\pi}\lim\frac{\partial}{\partial n}u(x;D^*,\mu_0^*), \ \ x\to x\in\Gamma=\{|\,y\,|=1\}$$

(2.18) va potensial tagidagi operatorlarning xossalariga ko'ra

$$\left|u(D^*,\mu_0{}^*)\right|^{(1,\lambda)}(\overline{B})\le C\left(q,\lambda,|\varphi|^{(1,\lambda)}[0,\pi]\right)(mesD^*)^{1/q} \qquad (2.20)$$

bunda $B=\{|y|<1\}$

(2.20) dan quyidagilarni hosil qilamiz

$$\left|f^*\right|^{(0,\lambda)}=\left|\frac{\partial}{\partial n}u(D^*,\mu_0^*\right|^{(0,\lambda)}\le C\left(mesD^*\right)^{1/q} \qquad (2.21)$$

(2.19) tenglama $C^{(0,0)}(\Gamma)$ da yechimga ega

Shuning uchun (2.19) operator chegaralangan teskari operatorga ega $C^{(0,0)}(\Gamma)$ da

Shuning uchun $\left|\mu^*\right|^{(0,0)}(\tilde{A}^*)\le C\left|f^*\right|^{(0,0)}(\tilde{A}^*)\le C\left|f^*\right|^{(0\lambda)}(\tilde{A}^*)$ (2.19) tenglamadan $|...|^{(0,\lambda)}$ norma $C\left|f^*\right|^{(0,\lambda)}(\tilde{A}^*)$ dan oshmaydi.

shuning uchun

$$\left|\mu^*\right|^{(0,\lambda)}(\tilde{A}^*)\le C\left|f^*\right|^{(0,\lambda)}(\tilde{A}^*) \qquad (2.22)$$

(2.21) va (2.22) dan Limma 2.3 kelib chiqadi.

Limma 2.4 $u(D,\mu_0)$ potensialning μ tarqatmasi quyidagi tengsizlikni qanoatlantiradi

$$|\mu|^{(0,\lambda)}(\tilde{A}) \le C(mesD)^{1/q},$$

bunda $C = C(q, |\varphi|^{(1,\lambda)}[0,2\pi], \quad \eta = \varphi(\xi) \quad \Gamma = \partial T$ ning tenglamasi.

<u>Isbot</u> $mesD^* = mesw(D)$ ni (2.18) orqali baholaymiz

$$mes = \int_D \left|\frac{dw}{dz}\right|^2 (y)dy \le C(|\varphi|^{(1,\lambda)}) \int_D dy = C(|\varphi|^{(1,\lambda)}) mesD.$$

Shuning uchun Limma 2.3 ga asosan

$$|\mu^*|^{(0,\lambda)}(\tilde{A}) \le C|\varphi|^{(0,\lambda)}(mesD)^{1/q} \qquad (2.23)$$

(2.23) , (2.13) va (2.18) yordamida $\mu^*(w(z))$ superpozitsiyaning limma 2.3 dan qo'yidagini topamiz.

$$|\mu|^{(0,\lambda)}(\tilde{A}) \le C|\varphi|^{(0,\lambda)}(mesD)^{1/q}$$

Bundan 2.4 limmaning tastig'i kelib chiqadi.

<u>Limma 2.5</u> Limma 2.4ning shartlaridan tashqari D va Γ to'plamlar orasidagi masofa ε dan kichik bo'lmasa, u holda

$$|\mu|^{(0,\lambda)} \le C(mesD)$$

bunda $C = C(\varepsilon, |\varphi|^{(1,\lambda)})$

Isbot xuddi Limma 2.3, Limma 2.4 limmalardek isbotlanadi, faqat bunda (2.20) qo'shimcha shartlarda

$u(D^*, \mu_0^*)(\overline{B}) \le CmesD^*$ bo'lib,

bunda $C = C(\varepsilon, |\varphi|^{(1,\lambda)})$

III-BOB TESKARI MASALA YECHIMINING EGRI CHIZIQLI YULDUZLI SOHALARDA YAGONALIGI HAQIDA

§ 3.1 Teskari masala yechimining egri chiziqli yulduzli sohalarda yagonaligi haqida

Asosiy tushunchalar va belgilashlar m sohalar sinfini qo'yidagicha kiritamiz.

$$T_\varphi = \{(x,y): \eta < \varphi(\xi)\},\ \varphi \in C^{(1,\lambda)}[0,2\pi],\ \varphi > 0$$

<u>Teorema</u>(asosiy) Agar $T_\varphi\ T_\psi \in m$ va T_φ, T_ψ larning massa zichligi 1 ga teng bo'lib, logarifmik potensiali teng bo'lsa, u holda

$$T_\varphi = T_\psi$$

bo'ladi.

Avval bir qancha belgilashlar kiritamiz . O'rtachalash yadrosini qo'yidagicha kiritamiz

$$g_n(x) = \begin{cases} C_n e^{-\frac{h^2}{h^2 - x^2}}, & |x| \le h \\ 0\ , & |x| > h \end{cases} \tag{3.1}$$

Bunda C_n, $\int_{-h}^{h} g_n(x)dx = 1$ shartdan tanlanadi. Davri 2π bo'lgan φ funksiyaning

φ_n o'rtachasini $C^{(0,\lambda)}[0,2\pi]$ $(0 < \lambda < 1)$

$$\varphi_n(\xi) = \int_{-\infty}^{\infty} g_n(\xi - \zeta)\varphi(\zeta)\,d\zeta \tag{3.2}$$

$\varphi_n \in C^\infty[0,2\pi]$, $C[0,2\pi]$ fazo normasida va $\varphi_n \to \varphi$, agar $h \to 0$ bo'lsa.

$\overline{\Phi}(\xi;\varphi,\psi)$ orqali $\overline{\Phi}=\max(\varphi,\psi)$ ni belgilaymiz.

$J_0=\{\xi:\varphi(\xi)=\psi(\xi)\}$ deb olamiz.

$$J^+=\{\xi:\psi(\xi)>\varphi(\xi)\},\qquad J^-=\{\xi:\varphi(\xi)>\psi(\xi)\}$$

Agar $\varepsilon>0$ bo'la $J_{4\varepsilon}$ orqali uzunligi $4\varepsilon>0$ bo'lgan J^+ *va* J^- larning birlashmalarini belgilaymiz. $J_0^{4\varepsilon}=J_0\cup J_{4\varepsilon}$ *va* $\overline{J}_0^{4\varepsilon}$ orqali $J_0^{4\varepsilon}$ ning $[0.2\pi]$ ga <u>qo'shimchasini</u> begilaymiz. J^{ε} orqali $\{\xi_1,\xi_2,...,\xi_n\}$ to'plamning ε atrofi, bunda $\xi_j\in J_0^{4\varepsilon}$ $\left(j=\overline{1,n}\right)$ bo'lib, $\overline{J}_0^{4\varepsilon}$ intervallarning chetidan iborat.

$\overline{\Phi}_{\varepsilon}$ orqali

$$\overline{\Phi}'(\xi)=\overline{\Phi}(\xi)\left[1-f(\xi)\right]+cf(\xi)$$ funksiyani belgilaymiz, bunda

$$f(\xi)=\begin{cases}\left[1-\dfrac{R(\xi)}{\xi}\right]^2, & agar\ \ R(\xi)\le\varepsilon\\ 0, & agar\ \ R(\xi)>\varepsilon\end{cases}$$

$C=\max\overline{\Phi}(\xi)$ $\quad\overline{\Phi}_{\xi}\ge\overline{\Phi}$ $\quad$ *va* $\quad\overline{\Phi}_{\varepsilon}\in C^{(1,\lambda)}$ $\quad\xi\in[0,2\pi]$

§ 3.2 Oddiy qatlam potensialini hajm potensiali bilan yaqinlashtirish

Bu paragrafda tarqatmaning yordamchi teoremasi isbotlanadi

<u>Limma 3.1</u> Agar $0\le\xi_1<\xi_2<...<\xi_{2n}<2\pi$ va $J=[\xi_1,\xi_2]\cup...\cup[\xi_{2n-1},\xi_{2n}]$ hamda $\mu(\xi)$ J da uzluksiz, $\max|\mu|\le\varepsilon$, $0<\delta<\varphi<1$ va $\varphi\epsilon C^{(1,\lambda)}(J)$bo'lsa, u holda

$$\int_j d\xi\int_{\varphi(\xi)}^{\varphi(\xi)+\mu(\xi)}\ln\left|\sqrt{\left[x(\xi,\eta)-x_0\right]^2+\left[y(\xi,\eta)-y_0\right]^2}\right|J(\xi,\eta)\,d\eta-$$
$$-\int_J\mu(\xi)\ln\left|\sqrt{\left[x(\xi,\varphi(\xi))-x_0\right]^2+\left[y(\xi,\varphi(\xi))-y_0\right]^2}\right|J(\xi,\varphi(\xi))\,d\xi=o(\varepsilon)\cdot\varepsilon$$

$J(\xi,\eta)$ - yakabian.

Bunda $x_0^{\,2}+y_0^2=R^2$, $\;R=\max\limits_{x^2+y^2\le1}\sqrt{x^2(x,y)+y^2(x,y)}$ (3.3)

hamda $0(\varepsilon)\to0$ agar $\varepsilon\to0$.

Isbot Baholanuvchi ifoda

$$|\int_j d\xi\left[\int_{\varphi(\xi)}^{\varphi(\xi)+\mu(\xi)}\ln\sqrt{\left[x(\xi,\eta)-x_0\right]^2+\left[y(\xi,\eta)-y_0\right]^2}J(\xi,\eta)d\eta-\mu(\xi)\ln\sqrt{\left[x(\xi,\varphi(\xi))-x_0\right]^2+\left[y(\xi,\varphi(\xi))-y_0\right]^2}J(\xi,\varphi(\xi))\right]|=$$

Lagranjning o'rta qiymat formulasidan

$$=\left|\int_J[\ln\sqrt{[x(\xi,\varphi(\xi)+\theta\mu(\xi)-x_0]^2+[y(\xi,\varphi(\xi)+\theta\mu(\xi)-y_0]^2}\right|J+$$

$$+\theta\mu(\xi)-\ln\sqrt{\left[x(\xi,\varphi(\xi))-x_0\right]^2+\left[y(\xi,\varphi(\xi))-y_0\right]^2}J(\xi,\varphi(\xi))]\mu(\xi)d\xi\le$$

$$\le[\ln\sqrt{[x(\xi,\varphi(\xi)+\theta\mu(\xi)-x_0]^2+[y(\xi,\varphi(\xi)+\theta\mu(\xi)-y_0]^2}J$$

Chegaralangan birinchi hosilaga ega (3.3) ga asosan, $|x-x_0|>\delta, |y-y_0|>\delta$ bo'lgani uchun

$$\le\int_J\left[C\theta\mu(\xi)\right]\mu(\xi)d\xi\le 2\pi C\varepsilon^2$$

Limma 3.1 isbotlandi.

Limma3.2 Agar $D_\varepsilon=\{\eta<f_\varepsilon(\xi),\ \delta<f_\varepsilon,\ \varepsilon\in(0,1)\}$, $\gamma=\{\eta=f_\varepsilon(\xi),\ 0\le\xi\le\xi_1\}$ yoy ε ga bog'liq emas. $f_\varepsilon\in C^{(1,\lambda)}[0,\xi_1]$, $|f_\varepsilon|^{(0,\lambda)}[0,2\pi]<C$, $\gamma_\delta=\{\eta=f_\varepsilon(\xi),\ \delta\le\xi_1\le\xi_2-\delta\}$,

$z_\varepsilon=z_\varepsilon(w)-$ birlik doirani D_ε ga konform akslantiruvchi funksiya

$z_\varepsilon(\delta)=\gamma$, $\delta=\{e^{i\theta},\ 0<\theta<\pi\}$, $z(i)=\{\frac{\xi_1}{2}, f_\varepsilon(\frac{\xi_1}{2})\}$ bo'lsa, u holda

$$|z_\varepsilon|^{(1,\lambda)}(K_\delta)<C,\quad |z_\varepsilon^{-1}|^{(1,\lambda)}(M_\delta)<C$$

bunda K_δ $z_\varepsilon^{-1}(\gamma_\delta)$ ning birlik aylana yoyining δ atrofidan iborat, M_δ D_ε dagi γ_δ yoyning δ atrofi.

Limmaning va uning natijalari isboti [24] da keltirilgan

Natija 1. Agar $|f|^{(1,0)}<C_0$ *va* $0\le\overline{\Phi}_\varepsilon(\xi)-f(\xi)\le\delta_0$ u holda yetarlicha kichik δ_0 uchun tarqatma f uchun

$$\int_{J_j} f(\xi)d\xi \geq \delta_1 mesD(f, J_j), \quad \delta_1 = \delta_1(\delta_0)$$

Nateja 2. Ixtiyoriy $\delta_2 > 0$ uchun shunday δ_0 mavjudki nateja 1 shartlari bajarilganda qo'yidagi baholash o'rinli bo'ladi

$$\int_{[0,2\pi]\setminus J_j} f(\xi)d\xi \leq \delta_2 mesD(f, J_j)$$

§ 3.3 Bo'rchakli sohalarni komform akslantirishning hosilalari haqida

Chekli D sohaning chegarasi bo'lgan $\tilde{A}$ chiziq bo'lakli silliq bo'lsin va $a \in \tilde{A}$ o'rchakli bo'rchakli nuqta. Bu nuqtada o'ng va chap o'rinmalarni $\tau^+(a) va\ \tau^-(a)$ deymiz.

$$\tau^{\pm}(a) = lim \frac{z^{\pm} - a}{|z^{\pm} - a|}, \quad z^{\pm} \in \tilde{A}^{\pm}$$

Bunda Γ^+ *va* Γ^- lar Γ ning qismlari bo'lib , ular qo'yidagicha aniqlanadi:

$z(\theta)$ Γ ning shunday paramertlashidan iboratki θ ning o'sishi bo'yicha harakterlanganda soha chap tomonda qolsin. Tanlangan $\theta_0 \in [0, 2\pi]$ bo'lganda

$$\tilde{A}^+ = \{z(\theta) : \theta_0 < \theta < 2\pi\} \quad \tilde{A}^- = \{z(\theta) : 0 < \theta < \theta_0\}$$

O'rinma $\tau^+ va\ \tau^-$ vektorlar orasidagi bo'rchakni soat mili bo'yicha hisoblaymiz.

Limma 3.2

$$0 \leq \xi_1 < ... < \xi_k < 2\pi, \quad |\xi_j - \xi_i| \geq \delta_0 > 0, \ \varphi \in C[0, 2\pi],$$
$$\varphi(0) = \varphi(2\pi)\ va\ |\varphi|^{(1,\lambda)}[\xi_j, \xi_{j+1}] \leq c_0, 0 < \delta_0 < \varphi < 1$$

Hamda o'rinmalar orasidagi $(\widehat{\tau^+, \tau^-})$ burchaklar barcha burchakli nuqtalarda π dan oshmaydi.

Bunda birlik doiraga akslaydigan $w(z)$ funksiya uchun

$$|w'(z)| < C, \ C = C(C_0, \delta_0, k)$$

Tengsizlik o'rinli.

Limma 3.2 ning isboti qisimlarga bo'lib isbotlaymiz.

1) Darajali funksiyalardan birining tarmog'ini qo'yidagicha olamiz

$$w_1(z) = w_{12}(w_{11}(z))$$

Bo'lsinki, bunda $w_{11}(z)$ qutb nuqtasi $\{\eta < \varphi(\xi)\}$ dan tashqarida bo'lgan kasr-chiziqli akslashdan iborat bo'lib, radiusi δ bo'lgan B_1 doira uchun

$\bar{B}_1 \cap \{\eta < \varphi(\xi)\}$ chap yarim tekislikga akslasin va haqiqiy yarim o'q 0 nuqtada

$\{\eta < \varphi(\xi)\}$ sohaning bissektrisasi bo'lsin, hamda $\{\eta < \varphi(\xi)\}$ ning aksi

$-\frac{1}{2}\alpha_1 < \arg w_{11} < \frac{1}{2}\alpha_1$ bo'rchakda joylashadi. Bunda $\alpha_1 < \alpha_1 < \min(\pi, 2\alpha_1)$

$w_{12}(w_{11})$ ning $-\frac{1}{2}\alpha_1 < \arg w_{11} < \frac{1}{2}\alpha_1$ bo'rchakda yotuvchi $1^{\frac{\pi}{\alpha_1}} = 1$ shartdagi tarmog'ini

$w_{11}^{\frac{\pi}{\alpha_1}}$ deymiz.

Shunday qilib $w_1(z) = w_{11}^{\frac{\pi}{\alpha_1}}$, $w_2, ..., w_n$ larni induksiya bo'yicha tuzamiz.

$w_1(z), ..., w_{l-1}(z)$ tuzilgan bo'lsin va bunda $a_l = w_{l-1}(...w_1(\xi_l, \varphi(\xi_l)))$ bo'lsin.

Xuddi yuqoridagidek $w_l(w_{l-1})$ ni $w_l(w_{l1}) = w_{i1}^{\frac{\pi}{\alpha_l}}$ $1^{\frac{\pi}{\alpha_l}} = 1$ bo'lgan tarmog'ini olamiz.

Bunda $\alpha_l = (\tau_l^+ \hat{,} \tau_l^-)$, $\tau_l^{\pm}$ o'ng va chap o'rinmalar orasidagi bo'rchak, $w_{l+1}(z)$ $\{\eta < \varphi(\xi)\}$ sohani $w_{11}(...w_1(z(\xi, \eta)))$ akslashdan iborat bo'lib $B_l -$ radiusi δ bo'lgan doira bo'lib, $\overline{B_l} \cap w_{l-1}\{\eta < \varphi(\xi)\}$ chap yarimi tekislikga akslanadi. $(0, \infty)$

yarim o'q $\{\eta<\varphi(\xi)\}$ sohaning $w_{l-1}(...w_1(z))$ akslashda 0 nuqtadagi bissektrisa bo'ladi.

Bu akslashda $\tilde{A}$ ning obrazi $\tilde{A}_0$ silliq chiziqdan iborat bo'ladi. $w_k(...w_1(z))$ akslashda D soha D_0 sohaga akslanadi. Bunda har bir w_l akslash a_l nuqtadagi chiziqni silliqlashtiradi.

Silliqlashda soha tenglamasini baholaymiz. $B(0,\delta)$ doira ichida silliqlangan kontur qismini baholash yetarli. D_ε ning silliqlangan $B(0,\delta)$ doiraning s_δ sektori ichida yotadi. Bu sektor

$$|z|\le\delta$$

$$-\frac{3\pi}{4}\le\arg z\le\frac{3\pi}{4}$$

dan iborat bo'ladi, x o'qi silliqlangan bo'rchakning bissektrisasida yotadi.

δ istalgancha kichik olib, $\tilde{A}_l^{\pm}$ grafikning tenglamasini $x=f_l^{\pm}(y)$ deb yozish mumkin.

Bu funksiya uchun

$$\left|f^{\pm}\right|_{C^{(1,\lambda)}[-1,1]}\le C,\quad C=C(C_0,\delta_0) \tag{3.1}$$

bo'ladi.

Bunda silliqlanadigan akslash $w_l(z)z^{\gamma}$ dan iborat bo'lib, $\gamma=\dfrac{\pi}{\alpha_l}\in(1,c)$. Bunda

tarmoqni ajratishda $(-\infty,0)$ o'q bo'yicha kesimni bajarish kerak.

2) Aniqlik uchun $\tilde{A}_l^{+}$ egri chizig'ining aksi uchun $w_l(z)=x_1(y)+iy_1(y)=(f(y)+iy)^{\gamma}$ bunda $f(y)=f_l^{+}(y)$ bo'lganda

$$\frac{\partial x_1}{\partial y_1}=\frac{\frac{\partial x_1}{\partial y}}{\frac{\partial y_1}{\partial y}}=\frac{\operatorname{Re}(\gamma(f(y)+iy)^{\gamma-1}(f'(y)+i)}{J_m(\gamma(f(y)+iy)^{\gamma-1}(f'(y)+i)}$$

Agar $g(y)=\int_0^1 f'(ty)dt$ demak $yg(y)=f(y)$ bo'lgani uchun

$$\frac{\partial x_1}{\partial y_1}=\frac{\operatorname{Re}(yg(y)+iy)^{\gamma-1}(f'(y)+i)}{J_m(yg(y)+iy)^{\gamma-1}(f'(y)+i)}=\frac{\operatorname{Re}(g(y)+i)^{\gamma-1}(f'(y)+i)}{J_m(g(y)+i)^{\gamma-1}(f'(y)+i)} \tag{3.2}$$

(3.2) ning maxrajining modulini quyidan baholaymiz. Bu maxrajni

$$\operatorname{Im} AB \tag{3.3}$$

desak bunda $A=(g(y)+i)^{\gamma}$, $B=\frac{f'(y)+i}{g(y)+i}$

δ ni shunday tanlaymizki

$$|A-zi|<\frac{1}{8},\quad |B-1|<\frac{1}{8C},\quad |B|<2 \tag{3.4}$$

bunda $|y|<\delta$ va $\gamma(g(0)+i)^{\gamma}=zi,\ 1\le z\le c$

Bularga asosan $|AB-zi|\le|AB-izB|+|zB-z|\le|A-zi|-|B|+|z|\cdot|B-1|\le\frac{1}{8}\cdot 2+\frac{2}{8c}<\frac{1}{2}$

Demak,

$$|ImAB|>\frac{1}{2} \tag{3.5}$$

Endi (3.4) ning mumkinligini ko'rsatamiz

$$\gamma\in(1,c)\text{dan}\left|z_1^{\gamma}-z_2^{\gamma}\right|\le|z_1-z_2| \text{ agar } z\in s_{\delta}\text{bo'lsa} \tag{3.6}$$

$z_1=g(0)+i,\ z_2=g(y)+i$ demak, (3.6) dan va $g(y)$ ning belgilanishidan

$|A-iz|\le C|z_1-z_2|=C|g(0)-g(y)|=C\left|g'(\bar{y})\right||y|\le C\cdot\delta$ agar $|y|<\delta$ bunda $0<\bar{y}<y$.

Agar δ ni $\delta < \frac{1}{8c}$ shartdan tanlasak, $|A - iz| < \frac{1}{8}$ bo'ladi $\left|\frac{i+t_2}{i+s_2} - \frac{i+t_1}{i+s_1}\right| \le C(|t_1 - t_2| + |s_1 - s_2|)$

tengsizlikdan $\frac{i+t}{i+s}$ kasr

funksiyasining t va S ga nisbatan differensiallanganligidan

$(|t_j| + |s_j| < C \;\; va \;\; t_j + s_j \in R)$ bo'lganda hamda $t_2 = t_1 = s_1 = f^{'}(y) \;\; va \;\; s_2 = g(y)$ lar uchun

$|B - i| \le C|g(y) - f^{'}(y)| = C\left|f^{'}(\overline{y_1}) - f^{'}(y)\right| \le C\left|\overline{y} - y\right|^{\lambda} < C|y|^{\lambda} < C\delta$ bunda $C = C(C_0), \; |y| < \delta$.

Endi δ ni tanlash natijasida (3.4)ni hosil qilishimiz mumkin.

$\frac{dx_1}{dy}(y_1)$ ning normasini $C^{(0,\lambda)}[-1,1]$da baholaymiz. (3.5)ning maxrajining modulini

$\frac{1}{2}$ dan kattaligini isbotladik.

Lemma shartiga ko'ra $g(y)$ va $f^{'}(y)$ funksiyalar $C^{(0,\lambda)}[-1,1]$ da normaga ega va bu norma $C(C_0)$ dan kata emas (3.2). Kasrning gyolder o'zgarmasi $C(C_0)$ dan oshmaydi. Shu bilan birgalikda radiusi δ bo'lgan doirada $x_1(y_1)$ ning grafigi $w_l(f(y) + iy)$ egri chiziqdan iborat ekanligi kelib chiqadi.

3. Oldingi qismda $\tilde{A}$ egri chiziq $w(z) = w_k(...w(z))$ akslashda $\tilde{A}_0$ silliq chiziqqa o'tadi. $\tilde{A}_0$ ning silliqligi baholashlari ham shu 2) qismda keltirildi. Bu baholashlardan $w(w)$ akslash uchun quyidagi bog'lanish kelib chiqadi. (18.127-129 betlar)

$$\left|w_{\overline{w}}(w)\right| \le C(C_0) \qquad (3.7)$$

$w(w)$ funksiya, D sohani birlik doiraga akslaydi. _ $w = w_k(...w_1(z))$ bo'lib $w_k, ..., w_1$ lar w_{l1} $(l = 1, 2, ..., k)$ funksiyaning tarmoqlari. w_{l1} funksiyani yuqorida keltirgan edik. $(\gamma > 1)$.

Shuning uchun $\left|(w_k)_{w_{k-1}}\right| ... \left|(w_1(z))_z\right| \leq C(C_0)$ bo'ladi. Shu sababli $w_k(....w_1(z))$ superpozitsiya ham chegaralangan hosilaga ega:_

$$\left|\overline{w_z}\right| \leq C(C_0) \qquad (3.8)$$

(3.7) va (3.8) bog'lanishlardan lemma 3.2 ning isboti kelib chiqadi.

§ 3.4 Ba'zi yordamchi baholashlar

Lemma 2.3 Agar $D_\varepsilon = \{\eta < f_\varepsilon(\xi)\},\ \delta < f_\varepsilon,\ (\varepsilon \in (0,1)),\ \ \gamma = \{\eta = f_\varepsilon(\xi)\},\ (0 < \xi < \xi_1)$ yoy ε ga bog'liq bo'lmasa, $f_\varepsilon \in C^{(1,\lambda)}[0,\xi_1],\ |f_\varepsilon|^{(0,\lambda)}[0,2\pi] < C\ \ \gamma_\delta = \{\eta = f_\varepsilon(\xi),\ \delta \leq \xi \leq \xi_1 - \delta\}$ va $z_\varepsilon = z_\varepsilon(w) -$ birlik doirani D_ε ga konform akslovchi funksiya bo'lsa hamda bu akslash $z_\varepsilon(\sigma) = \gamma,\ \ \sigma = \{e^{i\theta}, 0 < \theta < \pi\}$ $\quad z(i) = \left(\frac{\xi_1}{2}, f_\varepsilon(\frac{\xi_1}{2})\right)$ shartlarga bo'ysunsa, u holda $|z_\varepsilon|^{(1,\lambda)}(K_\delta) < C,\ \ |z_\varepsilon^{-1}|(M_\delta) < C$ bo'ladi, bunda $K_\delta - z_\varepsilon^{-1}(\gamma_\delta)$ ning $\_\delta$ atrofi, $M_\delta - \lambda_\delta$ ning D_ε dagi atrofi.

Isbot. Teorema shartiga ko'ra z_ε *va* z_ε^{-1} funksiyalar tekis Gyolder shartlariga bo'ysinadi (λ ko'rsatkich bo'yicha) . Shuning uchun σ ning chetki nuqtalaridan $\sigma_\delta = z_\varepsilon^{-1}(\gamma_\delta)$ chetki qiymatlarigacha bo'lgan masofalar δ dan kichik emas.

f_ε funksiyani $[\delta, \xi_1 - \delta]$ dan $[0, 2\pi]$ ga shunday davom ettiramizki, davom ettirilgan f_ε funksiya $f > f_\varepsilon$ bo'lib, $\left|f\right|^{(1,\lambda)}[0,2\pi] < C$ bo'lsin. $D = \{\eta < f(\xi)\}$ yuqori yarim tekislikda chegarasi $\partial D_1 \in C^{(1,\lambda)}$ bo'lgan D_1 sohasi belgilaymizki, $[0,1] \subset \partial D_1$ bo'lsin.

D ni D_1 ga konform akslantiruvchi funksiya $\tilde{z}$ bo'lsin va $\tilde{z}(\gamma) = [0,1]$ $\tilde{z}(\frac{\xi_1}{2}, f(\frac{\xi_1}{2})) = \frac{1}{2} + 0 \cdot i$

Reman – Shvarstning simmetriya prinsipuga ko’ra $\tilde{z}(z_\varepsilon)$ funksiya $C\setminus\sigma_0$ ga analitik davom ettiriladi $(\sigma_0=\{e^{i\varphi},-\pi\le\theta\le\pi\}$ va $C\setminus\sigma_0$ da moduli bo’yicha chegaralangan bo’ladi.

Isbotdagi birinchi abzasga ko’ra $z_\varepsilon^{-1}(\gamma_\varepsilon)$ dan σ_0 gacha masofa δ dan kichik bo’lmagani uchun $\left|\tilde{z}(z_\varepsilon)\right|^{(1,\lambda)}<C$ _tengsizlik $z_\varepsilon^{-1}(\gamma_\varepsilon)$ _ ning biror K_δ atrofida urinma bo’ladi. Bu lemmani isbotlaydi. Lemmaning ikkinchi baholashi shunga o’xshash isbotlanadi.

Natija 2.1. Agar w_n funksiya D_ε $\{\eta=f_\varepsilon(\xi),\ \varepsilon_0-h<\xi<\varepsilon_0+h\},\ 0<\varepsilon_0<\varepsilon_1$ yoyining garmonik o’lchovi bo’lsa, u holda $\{|z-z_0|>\delta\}\cap D_\varepsilon,\ \ z_0=(\xi_0,f(\xi_0))$ da $|w_n|<ch$ tengsizlik o’rinli bo’ladi.

Isbot. $[-h,h]$ oraliqqa mos garmonik o’lchovning ko’rinishi Puasson formulasiga natija isboti kelib chiqadi. Natijadagi holda konform akslash natijasida yuqori yarim tekislikga keltiriladi. Bu konform akslash z_ε^{-1} lemma 2.3 baholangan. Shuning uchun garmonik funksiyalar uchun maksimum qiymati prinsipidan natija isbotlanadi.

Natija 2.2 Agar $|f|^{(1,0)}<C_0$ va $0\le\overline{\Phi}_\varepsilon(\xi)-f(\xi)\le\delta_0$ ($\overline{\Phi}_\varepsilon(\xi)$ 18 betda aniqlangan) bo’lsa, u holda f tarqatma uchun quyidagi baholash o’rinli

$$\int_{J_j} f(\xi)\,d\xi\ge\delta_1 mesD(f_j;J_j),\ \ \delta_1=\delta_1(\delta_0)$$

Isbot. f funksiyaning Lipshist shartini qanoatlantirganligi uchun va Lemma 2.3 dan w_j $D(f,J_j)$ sohaning $\{y:\xi\in J_j,\ \eta=\overline{\Phi}_\varepsilon(\xi)\}$ yoyining garmonik o’lchovi bo’lsa, u holda bu funksiya δ va C_0 ga bog’liq bo’ladi. Ortogonallik prinsipiga ko’ra

$$\int_{\{y:\xi\in J_j,\ \eta=\overline{\Phi}_\varepsilon(\xi)\}} H\,f\,d\tilde{A}(y)=\int_D H dy \qquad (*)$$

$\{\eta < \overline{\Phi}_\varepsilon(\xi)\}$ da garmonik va $\{\eta = \overline{\Phi}_\varepsilon(\xi)\}$ da uzluksiz bo'lgan $H(y)$ funksiya uchun o'rinli. (*) da $H = w$ desak

$$\int\limits_{\{y:\xi\in J_j,\ \eta=\overline{\Phi}_\varepsilon(\xi)\}} d\Gamma(y) = \int\limits_{\partial D_{\overline{\Phi}_\varepsilon(\xi)}} f d\Gamma(y) = \int\limits_{D(f_j, J_j)} w_j dy \ge \delta mesD(f_j, J_j).$$

Natija 2 isboti bo'ldi.

Natija 3. Ixtiyoriy $\delta_2 > 0$ uchun shunday δ_0 muvjudki natija 2 dan quyidagi tengsizlik kelib chiqadi.

$$\int\limits_{[0,2\pi]\setminus J_j} f(\xi) d\xi \le \delta_2 mesD(f, J_j)$$

Isbot. Agar w, $\{y : \xi \in J_j,\ \eta = \overline{\Phi}_\varepsilon(\xi)\}$ yoyining garmonik o'lchovi bo'lsin. w_1 orqali $z^{-1}(\gamma(\delta))$ yoynming garmonik o'lchovini belgilaymiz, bunda z funksiya birlik doirani $\{\eta < \overline{\Phi}_\varepsilon(\xi)\}$ sohaga konfort akslovchi funksiya. Ixtiyoriy $\delta_3 > 0$ uchun shunday $\delta_4 > 0$ mavjudki $\{y : \xi \in J_j,\ \eta = \overline{\Phi}_\varepsilon(\xi)\}$ ning δ_4 atrofida $w_1 > 1 - \delta_3$ bo'ladi $w = w_1(z^{-1})$ dan lemma 2.3 dan $w > 1 - \delta_3$ tengsizlik δ_5 atrofida bajariladi $\delta_5 = \delta_5(\delta_0)$

Natija 2 dek $\int\limits_{\gamma(\delta)} f d\Gamma \ge (1 - \delta_3) mesD(f, J_j)$

Tarqatma natijasida massa saqlanganligidan δ_0 bo'yicha aniqlanadigan δ_3 yetarli kichik bo'lganda natijaning isboti kelib chiqadi.

§ 3.5 Asosiy teotemani isbotlash uchun muhim natejalari

Bu parografda asosiy teotemani isbotlash uchun muhim natejalarni hosil qilamiz

Teorema Agar $\varphi,\psi \in C^{(1,\lambda)}[0,2\pi],\ \ \varphi \neq \psi$ bo'lsa, u holda shunday chekli bir bog'lamli D soha mavjudki, $\partial D \in C^{(1,\lambda)}$ bu D soha $T_\varphi \cup T_\psi$ o'zida saqlaydi va ∂Dda mavjud nolmas μ o'lchov uchun $U(T_\psi)-U(T_\varphi)=V(M)$ tenglik $\tilde{N}\setminus\overline{D}$ da o'rinli bo'ladi.

Isbot $\varphi,\psi \in C^{(1,\lambda)}[0,2\pi]$ va $U(x,T_\psi)-U(x,T_\varphi)=f(x)$

Qo'ydagi funksiyalarni tuzamiz

$$\varphi_{-1}(\xi)=\begin{cases}\varphi(\xi)\ ,agar\ \ (J^+\setminus J^{2\varepsilon})\cap\overline{J}_0^{4\varepsilon}\\ \psi(\xi)+[\varphi(\xi)-\psi(\xi)]\dfrac{1}{\varepsilon^2}\rho^2(\xi),\ agar\ J^+\cap(J^{2\varepsilon}\setminus J^{\varepsilon})\\ \overline{\Phi}_\varepsilon(\xi)\ ,\quad agar\ \ ([0,2\pi]\setminus(J^+\cap\overline{J}_0^{4\varepsilon}))\cup J^\varepsilon\end{cases}$$

$$\psi_{-1}(\xi)=\begin{cases}\psi(\xi)\ ,agar\ \ (J^-\setminus J^{2\varepsilon})\cap\overline{J}_0^{4\varepsilon}\\ \varphi(\xi)+[\psi(\xi)-\varphi(\xi)]\dfrac{1}{\varepsilon^2}\rho^2(\xi),\ agar\ J^-\cap(J^{2\varepsilon}\setminus J^{\varepsilon})\\ \overline{\Phi}_\varepsilon(\xi)\ ,\quad agar\ \ ([0,2\pi]\setminus(J^-\cap\overline{J}_0^{4\varepsilon}))\cup J^\varepsilon\end{cases}$$

bunda $\rho(\xi)$ ξ va $\xi_j \pm \varepsilon$ nuqtalar orasidagi masofalarning minimum. Bunda $\varphi_{-1},\psi_{-1} \in C^{(1,\lambda)}[0,2\pi]$ bo'ladi.

Teorema isbotini qutblarga ajratamiz.

I. Bunda biz tarqatmaning tuzilishini keltiramiz

Endi $\varphi_{nk},\psi_{nk}\ va\ \mu_{nk}$ $(k=0,1,2,...)$ funksiyalarni qo'yidagicha kiritamiz:

$$\varphi_{n0} = \varphi_{-1},$$
$$\varphi_{nk+1} = \varphi_{nk} + \delta_{nk}^{\varphi};$$
$$\psi_{nk+1} = \psi_{nk} + \delta_{nk}^{\psi},$$
$$\mu_{n0} = 0,$$
$$\mu_{nk+1} = \mu_{nk} + \delta_{nk},$$

bunda $\delta_{nk}^{\varphi}, \delta_{nk}^{\psi}$ *va* δ_{nk} funksiyalar ushbu ko'rinishda tuziladi:

$$t_0^{\varphi} = \begin{cases} \dfrac{1}{n} \ , & J^+ \\ 0 \ , & J^- \end{cases}$$

$$\delta_{k0}^{\varphi} = \min(t_0^{\varphi}, \psi_{n0} - \varphi_{n0})$$

$$\mu_{n0}^{+} = B\psi_{n0}(U(T_0^{\varphi})) \qquad T_0^{\varphi} = \{\varphi_{n0} < \varphi < \varphi_{n0} + \delta_{n0}^{\varphi}\}$$

bo'lakning $\{\eta = \psi_{n0}(\xi)\}$ egri chiziqqa tarqatmasi.

$\overset{+}{M}_{n0}$ orqali μ_{n0}^{+} ning izini Uitni bo'yicha [16] davomini

$\left\{\overline{\Phi}_{\varepsilon}(\xi) \geq \psi_{n0}(\xi) + \mu_{n0}^{+}(\xi)\right\}$ to'plamga davomini belgilaymiz.

Tarqatmaning xossasidan [24] limma 3.2 ga asosan μ_{n0}^{+} funksiya uchun

$$\mu_{n0}^{+} = \mu_{n0}^{1}\left|\xi - \xi_0\right|^{\delta} \ agar \ \left|\xi - \xi_0\right| < \delta_1$$
$$bunda \quad \xi_0 = \left\{\xi : \varphi_{n0}(\xi) = \psi_{n0}(\xi)\right\} \cap \left\{\xi : \overline{\Phi}_{\varepsilon}(\xi) \geq \psi_{n0}(\xi)\right\}$$

$\delta = \dfrac{\pi}{\alpha_{\psi_{n0}(\xi_0)}} - 1 \geq 0, \quad bunda \ \ \alpha_{\psi_{n0}(\xi_0)}$ $\{\eta = \psi_{n0}(\xi)\}$ chiziqning $(\xi_0, \psi_{n0}(\xi))$ nuqtada

chap va o'ng urinmalar orasidagi burchak.

$\psi_{n0} \in C^{(1,\lambda)}[0.2\pi]$ bo'lgani uchun $\delta = 0$ bo'ladi. Shuning uchun (3.1) dan $\mu_{n0}^{+} = \mu_{n0}^{+1}$

Endi t_0^{ψ} ni qo'yidagicha tanlaymiz.

$$t_0^{\psi} = \begin{cases} \overset{+h}{\mu}_{n0} + \dfrac{3}{4\pi}, & agar \ J^- \\ 0 \ , & agar \ \ J^+ \end{cases}$$

bunda $\overset{+h}{\mu}_{n0}$ $\overset{+}{\mu}_{n0}$ ning Steklov ma'nosidagi o'rtachasi:

Bundagi paramert h ning $Ch^{\lambda} < \frac{1}{4}, \quad C = C(\varepsilon, |\psi|^{(1,\lambda)}$ (3.2)

shartdan tanlaymiz. Bu tanlash va limma 2.5 ga ko'ra qo'yidagi baholash o'rinli bo'ladi

$$\left|\overset{+}{\mu}_{n0} - \overset{+h}{\mu}_{n0}\right| \le \int_{-\infty}^{\infty} \left|\overset{+}{\mu}_{n0}(\zeta) - \mu_{n0}^{+h}(\zeta)\right| g_n(\xi-\zeta)d\zeta \le \int_{-\infty}^{\infty} h^{\lambda} \left|\overset{+}{\mu}_{n0}\right|^{(0,\lambda)} [0,2\pi] g_n(\xi-\zeta)d\zeta \le$$

$$\frac{Ch^{\lambda}}{n} \int_{-\infty}^{\infty} g(\zeta)d\zeta = C\frac{h^{\lambda}}{n} \qquad C = C(\varepsilon, |\psi|^{(1,\lambda)})$$

$$\delta_{n0}^{\psi} = \begin{cases} \min[t_0^{\psi}, \varphi_{n0} - \left(\psi_{n0} + t_0^{\psi}\right), & J^{-1} \ da \\ 0, & J^{+1} \ da \end{cases}$$

$\mu_{n0}^{-} = -B\varphi_{n1}\left(U(T_0^{\psi})\right)$ belgilash $T_0^{\psi} = \left\{\psi_{n0} + \mu_{n0}^{+} < \eta < \psi_{n0} + \delta_{n0}\right\}$ yulakning $\{\eta = \psi_{n1}(\xi)\}$ chiziqqa tarqatmasidan iborat.

$$\delta_{n0} = \begin{cases} \mu_{n0}^{+}, & \{\xi : \psi_{n0}(\xi) > \varphi_{n0}(\xi)\} \ bo'lsa \\ \mu_{n0}^{-}, & \{\xi : \varphi_{n0}(\xi) > \psi_{n0}(\xi)\} \ bo'lsa \\ \mu_{n0}^{+} + \mu_{n0}^{-}, & \{\xi : \varphi_{n0}(\xi) = \psi_{n0}(\xi)\} \ bo'lsa \end{cases}$$

Bu keltiringan tarqatma jarayoniga jarayonning birinchi qadami deymiz.

Endi tarqatma jarayonining $k+1$ –qadamini keltiramiz.

$$t_{nk}^{\varphi} = \begin{cases} -\mu_{nk+1}^{-h} + \frac{3}{4}\cdot\frac{1}{n}, & J^{+} \ da \\ 0, \quad J^{-} \ da \end{cases}$$

Bunda $\overset{-h}{\mu}_{nk-1}$ funksiya μ_{nk-1}^{-h} izining $[0,2\pi]$ oraliqqa Uitni manosida $\left\{\xi : \overline{\Phi}_{\varepsilon}(\xi) \ge \varphi_{nk-1} - \mu_{nk-1}^{-}\right\}$ ga davomining o'rtachasi

$$\delta_{nk}^{\varphi} = \begin{cases} \min[t_k^{\varphi}, \psi_{nk} - \left(\varphi_{nk} + t_k^{\varphi}\right), & J^{+} \ da \\ 0, & J^{-} \ da \end{cases}$$

Bunda $\overset{+h}{\mu}_{nk}$ funksiya μ_{nk}^{+} izining $[0,2\pi]$ oraliqqa Uitni manosi

bo'yicha davomining o'rtachasi. Bunda o'rtachalashish parametri h ni $Ch^{\lambda} < \frac{1}{4}$ shartdan tanlaymiz. Bunda $\overset{+}{\mu}_{nk}$ va $\overset{+h}{\mu}_{nk}$ orasidagi farqni baholaymiz.

$$\left|\overset{+}{\mu}_{nk}-\overset{+h}{\mu}_{nk}\right|\le\int_{-\infty}^{\infty}\left|\overset{+}{\mu}_{nk}(\xi)-\overset{+}{\mu}_{nk}(\zeta)\right|g_n(\xi-\zeta)\,d\zeta\le$$

$$\le\int_{-\infty}^{\infty}h^{\lambda}\left|\overset{+}{\mu}_{nk}\right|^{(0,\lambda)}[0,2\pi]g(\xi-\zeta)d\zeta\le\frac{Ch^{\lambda}}{n}<\frac{1}{4n}.$$

$$C=C(\varepsilon,h,|\psi_{nk}|^{(1,\lambda)})$$

Keyingi yozuvlarda Cning $\varepsilon\ va\ |\psi_{nk}|^{(1,\lambda)},\ |\varphi_{nk}|^{(1,\lambda)}$ larga bog'liqligini tashlab yozamiz

$$\delta_{nk}^{\psi}=\begin{cases}\min[t_k^{\psi},\psi_{nk}-\left(\varphi_{nk}+t_k^{\psi}\right), & J^{+}\ da\\ 0, & J^{-}\ da\end{cases}$$

$\overset{-}{\mu}_{nk}=-B\varphi_{nk+1}\left(U(T_k^{\psi})\right)$ orqali $T_0^{\varphi}=\left\{\psi_{nk}+\overset{+}{\mu}_{nk+1}<\eta<\psi_{nk}+\delta_{nk}^{\psi}\right\}$ yulakning $\{\eta=\varphi_{nk+1}(\xi)\}$ chiziqqa tarqatmasini belgilaymiz.

$$\delta_{nk}=\begin{cases}\overset{+}{\mu}_{nk}, & \{\xi:\psi_{nk}(\xi)>\varphi_{nk}(\xi)\}\ bo'lsa\\ \overset{-}{\mu}_{nk}, & \{\xi:\varphi_{nk}(\xi)>\psi_{nk}(\xi)\}\ bo'lsa\\ \overset{+}{\mu}_{nk}+\overset{-}{\mu}_{nk}, & \{\xi:\varphi_{nk}(\xi)=\psi_{nk}(\xi)\}\ bo'lsa\end{cases}$$

$\overset{+}{\mu}_{nk}$ va $\overset{-}{\mu}_{nk}$ funksiyalar uchun qo'yidagi formulalarni yozamiz

$$\overset{+}{\mu}_{nk}=\mu_{nk}^{+1}\left|\xi-\xi_0\right|^{\delta},\quad \overset{-}{\mu}_{nk}=\mu_{nk}^{-1}\left|\xi-\xi_0\right|^{\delta},\quad \left|\xi-\xi_0^{\pm}\right|<\delta_1 \tag{3.4}$$

Bunda $\mu_{nk}^{+1},\mu_{nk}^{-1}\in C^{(0,\lambda)}[0,2\pi],\quad \xi_0^{+},\xi_0^{-}\ \{\eta=\psi_{nk}\}\ va\{\eta=\varphi_{nk+1}\}$ larning burchakli nuqtalari bo'lib, $\delta_0\ va\ \delta_1$ yetarlicha kichik somlar. (3.4) tengsizliklardan $\overset{+}{\mu}_{nk}\in C^{(0,\lambda)}[0,2\pi]$ kelib chiqadi.

Agar $\max\limits_{\xi}|\varphi-\psi|=\left|\varphi(\xi_{\max})-\psi(\xi_{\max})\right|=M$, $J(\delta)$ $\xi_{\max}$ ning δ atrofi bo'lsin.

$\xi_{\max}\in J^{+}\ va\ z\in\{\eta=\psi_{nk}(\xi),\ \xi\in J(\delta)\}$ bo'sa, [25] ga ko'ra

$$\left|\overset{-}{\mu}_{nk}(z)\right|^{(0,\lambda)}\le\frac{C(h)}{n}$$

bu tengsizlikka ko'ra tarqatma massa $\frac{C}{n}$ dan oshmaydi va z nuqtadan δ masofada toqlashadi. Shuning uchun $\left(\varphi_{nk+1}-\varphi_{nk}\right)(\xi_{\max})=\mu_{nk}^{+h}(\xi_{\max})+\frac{3}{4}\cdot\frac{1}{n}<\frac{C(h)}{n}$ agar $h<\delta$ bo'lsa.

Tarqatma jarayoning tugashi uchun $\frac{M}{C(h)} \cdot n$ qadamda jarayon davom etishi kerak.

$\delta_0 = \frac{C(h)}{M}$ bo'lsin, ε ni shunday tanlaymizki J^{ε} *va* $J(\delta)$ lar orasidagi masofa δ dan oshmasin. $\varepsilon > 0$ ni shunday tanlaymizki $4C_0\varepsilon^{1+\lambda} < \delta_1\delta^2$ hamda J_j *va* J^{ε} to'plamlar orasidagi masofa δ dan katta bo'lsin.

Tarqatma jarayoning eng oxirgidan k oldingi qadamida φ_{nk} *va* ψ_{nk} larni shunday tanlaymizki $\max[\left|\overline{\Phi}_{\varepsilon} - \psi_{nk}\right|^{(0,0)}, \left|\overline{\Phi}_{\varepsilon} - \varphi_{nk}\right|^{(0,0)}] < \delta$ jarayonning oxirga qadamida funksiyalarni qo'yidagicha tanlaymiz:

$f^{+} = B_{\overline{\Phi}_{\varepsilon}}(U(D_{+})) \quad D_{+} = \{\varphi_{nk}(\xi) < \eta < \overline{\Phi}_{\varepsilon}(\xi)\}$ bo'lsa

$f^{-} = B_{\overline{\Phi}_{\varepsilon}}(-U(D_{-})) \quad D_{-} = \{\psi_{nk}(\xi) < \eta < \overline{\Phi}_{\varepsilon}(\xi)\}$ bo'lsa

$f = B_{\overline{\Phi}_{\varepsilon}}(U(D_{\varepsilon}^{+}) - U(D_{\varepsilon}^{-}))$

Bunda $D_{\varepsilon}^{-} = \left\{\psi(\xi) < \eta < \varphi(\xi), \quad \xi \in J^{\varepsilon}\right\}$

$D_{\varepsilon}^{+} = \left\{\varphi(\xi) < \eta < \psi(\xi), \quad \xi \in J^{\varepsilon}\right\}$

II. φ_{nk} *va* ψ_{nk} **larni** $C^{(1,\lambda)}$ **normasida baholash.**

$C_k = \left|\varphi_{nk}\right|^{(1,\lambda)}, \ \overline{C}_k = \left|\psi_{nk}\right|^{(1,\lambda)}$ deb olaylik. φ_{nk} larning tuzilishidan

$$\varphi_{nk} = \varphi_{n0} + \sum_{j=1}^{k-1} \delta_{nj}^{\varphi}$$

$$C_1 = \left|\varphi_{n0} + \delta_{n0}^{\varphi}\right|^{(1,\lambda)} \leq \left|\varphi_{n0}\right|^{(1,\lambda)} + \left|\delta_{n0}^{\varphi}\right|^{(1,\lambda)}$$

Limma 2.5 ga asosan tarqatmaning ko'rinishidan

$\left|\delta_{n0}^{\varphi}\right|^{(0,\lambda)} \leq \frac{C\left|\varphi_{n0}^{\varphi}\right|^{(1,\lambda)}}{n}$ shuning uchun

$$C_1 \leq \left|\varphi_{n0}^{\varphi}\right|^{(1,\lambda)} + \frac{C\left|\varphi_{n0}\right|^{(1,\lambda)}}{nh} = (1 + \frac{C}{nh}) \cdot \left|\varphi_{n0}\right|^{(1,\lambda)} \qquad (3.6)$$

Ixtiyoriy l uchun (3.6) ning to'g'ri ekanini ko'rsatamiz. $l = 1$ da (3.6) isbotlandi. Endi $l + 1$ da ham (3.6) ni isbotlaymiz.

$$C_{l+1} = \left|\varphi_{nl} + \delta_{nl}^{\varphi}\right|^{(1,\lambda)} \leq \left|\varphi_{nl}\right|^{(1,\lambda)} + \left|\delta_{nl}^{\varphi}\right|^{(1,\lambda)} \leq \left|\varphi_{nl}\right|^{(1,\lambda)} + \frac{C}{nh}\left|\varphi_{nl}\right|^{(1,\lambda)} = (1 + \frac{C}{nh})\left|\varphi_{nl}\right|^{(1,\lambda)} = (1 + \frac{C}{nh})^{l+1}\left|\varphi_{n0}\right|^{(1,\lambda)}$$

Oxirgi (3.6) ni $l+1$ uchun o'rinliligi isbotlandi.

$\left\{(1+\frac{C}{nh})^n\right\}$ ning manoton o'suvchi ekanligi uchun qo'yidagi baholashlarni hosil qilamiz.

$$C_k \le |\varphi_{n0}|^{(1,\lambda)} (1+\frac{C}{nh})^k \le |\varphi_{n0}|^{(1,\lambda)} e^{\frac{C}{h}}$$

Huddi shunga o'xshash

$$\overline{C}_k \le |\psi_{n0}|^{(1,\lambda)} e^{\frac{C}{h}}$$

(3.3) ga ko'ra $e^{\frac{C}{h}} < \infty$ ekanligini ko'ramiz.

III. $\{\mu_n^{\pm}\} = \left\{\sum_k \mu_{nk}^{\pm}\right\}$ larning yaqinlashishini isbotlaymiz.

$\mu_n^{\pm} = \sum_k \mu_{nk}^{\pm}$ bo'lsin.

Limma 2.1 ning natejasidan hamma $H(y)$ garmonik funksiyalar uchun chegarasi Γ bo'lgan sohada chegaragacha uzluksiz bo'lgan funksiyalar uchun

$$\int_D \mu_0(y) H(y) dy = \int_{\tilde{A}} \mu(y) H(y) d\tilde{A}(y)$$

tenglik o'rinli bo'ladi. Bunda $H(y) = 1$ bo'lganda

$$\int_D \mu_0(y) dy = \int_{\tilde{A}} \mu(y) d\tilde{A}(y)$$

bo'ladi. Bundan tarqatma vaqtida massa o'zgarmas bo'lar ekan . Shuning uchun

$$\int_{\{x:\eta<\overline{\Phi}_\varepsilon(\xi)\}} \left|\overline{\mu}^{\pm}\right| d\tilde{A}(x) < C, \quad \mu_n^{+} \ge 0, \quad \overline{\mu}_n^{-} \le 0 \qquad (3.7)$$

$\overline{\mu}^{\pm}$ o'lchovni qo'yidagicha aniqlaymiz

$$\overline{\mu}_n^{\pm}(F) = \int_{\{y:\eta<\overline{\Phi}_\varepsilon(\xi)\}} F(y) \overline{\mu}_n^{\pm}(\xi(y)) d\tilde{A}(y)$$

(3.4) ga ko'ra $\overline{\mu}_n^{\pm}$ ning normal o'lchovlar fazosida (3.7) ga asosan chegaralangan, shuning uchun $\left\{\overline{\mu}_n^{\pm}\right\}$ ketma-ketliklar bo'sh kompakt bo'ladi (o'lchovlar fazosida) Shuning uchun yengil yaqinlashuvchi qismiy ketma-ketlik $\overline{\mu}_{nq}^{\pm}$

ajratish mumkinki

$$\overline{\mu}_{nq}^{\pm} \xrightarrow{yengil} \mu^{\pm}$$

IV. Tarqatma jarayonida oddiy qatlam potensialini hajm potensiali bilan yaqinlashtirishning yig'indisini baholash.

zichligi μ_{nk} $(1 \le k \le m)$ bo'lgan oddiy qatlam potensialini massasi zichligi birga teng bo'lgan hajm potensiali bilan yaqinlashini summalar baholashini keltiramiz.

Limma 1.5 ga ko'ra $|\mu_{nk}|^{(0,\lambda)} \le \frac{C}{n}$

Limma 3.1 ga ko'ra

$$|\int\limits_J d\xi \int\limits_{\varphi_{nk}}^{\varphi_{nk}+\delta_{nk}^{\varphi}(\xi)} \ln\sqrt{\left[x(\xi,\eta)-x_0\right]^2+\left[y(\xi,\eta)-y_0\right]^2}\, J(\xi,\eta)d\eta -$$

$$-\mu_{nk}(\xi)\ln\sqrt{\left[x(\xi,\varphi_{nk}(\xi))-x_0\right]^2+\left[y(\xi,\varphi_{nk}(\xi))-y_0\right]^2}\, Jd\xi| \le$$

$$\le o(\frac{1}{n})\left|\mu_{nk}(\xi)\right|^{(0,\lambda)} \le o(\frac{1}{n})\frac{C}{n}$$

bo'ladi limma 2.1 ga asosan. Bu yerda $J=\{\xi:\varphi_{nk}(\xi)<\overline{\Phi}_{\varepsilon}(\xi)\}$ $\qquad J(\xi,\eta)=\frac{D(x,y)}{D(\xi,\eta)}$

Endi $k=1,2,\ldots,m$ uchun yig'sak

$$\sum_k o(\frac{1}{n})\left|\mu_{nk}(\xi)\right|^{(0,\lambda)} \le o\left(\frac{1}{n}\right) \to 0 \quad \text{agar } m \to \infty$$

V. μ larni tuzish masalasi.

μ larni qo'yidagicha tuzamiz $\quad \mu = f + f_{+} + f_{-} + \lim\limits_{n\to\infty}\overline{\mu}_n^{+} + \lim\limits_{n\to\infty}\overline{\mu}_n^{-}$

$\int\limits_{\partial\overline{\Phi}_{\varepsilon}} f_{+} > \int\limits_{\partial\overline{\Phi}_{\varepsilon}} f_{-}$ o'rinli deb hisoblash mumkin.

Limma 2.2 ning ikkinchi natejasida $f=\varphi_{nk}$ desak, u holda $l=\left\{\xi:\overline{\Phi}>\varphi_{nk}\right\}$ $\qquad \mu_{nk}^{-}=0$

Limma 2.2 dan $\int\limits_{l} f_{+} \ge \delta_0 \int\limits_{\partial\overline{\Phi}_{\varepsilon}} f_{+} \ge \delta_0 \int\limits_{\partial\overline{\Phi}_{\varepsilon}} f_{-}$

Limma 2.2 ning ikkinchi natejasidan $-\int\limits_{l} f_{-} < -\frac{\delta_0}{2}\int\limits_{\partial\overline{\Phi}_{\varepsilon}} f_{-}$, $\quad$ shuning uchun

$$\int_l (f_+ + f_-) \geq \frac{\delta_0}{2}\int_l f_+ \geq \delta_1\delta^2$$

tengsizlik bajariladi φ_{nk} larning Lipshis shartini qanoatlantirgani uchun yani nateja 2 dan

ε ning tanlanishiga asosan $\mu(l) \geq \int_l (f + f_+ - f_-) > 0$

Bundan teorema isboti kelib chiqadi.

Asosiy teoremani isbotlaymiz.

Isbotni teskarisidan faraz qilamiz. Teskari teoremadan yurituvchisi ∂D da bo'lgan $\mu \neq 0$ o'lcham mavjudki

$V(D:\mu) = 0,\ x \in C \setminus \overline{D}$

Limma 1.1 ga asosan

$$\int_{\partial D} h(x)d\mu(x) = 0 \tag{3.8}$$

Ixtiyoriy $h(x)$ garmonik funksiya uchun o'rinli bo'ladi. f ∂D da uzluksiz funksiya bo'lsin, h_f $\Delta h_f,\ D\ da\ h_f = f\ \ \partial D\ da$ Dirixli masalasining yechimi bo'lsin. Garmonik polinomlar $C(\overline{D})$ metrikasida zich joylashgani uchun

$$\int_{\partial D} f(x)d\mu(x) = 0$$

Bundan $\mu = 0$ hosil bo'ladi. Qarama-qarshilik asosiy teoremani isbotlaydi.

XULOSA

Xulosa qilib shuni aytish mumkinki   qo’yilgan masala yer osti qazilmalarini izlashda katta ahamiyatga ega bo’lib , qazilma boyligini ma’lum bir joyda borligini bilgan holda bu qazilma qancha joyga tarqalganligini bilish mumkin bo’lar ekan. Bu geologiya sohasida katta ahamiyatga ega bo’lgan masalalardan hisoblanadi.

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