O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA MAXSUS
TA'LIM VAZIRLIGI

Farg'na politexnika instituti
"Oliy Matematika" kafedrasi

Bilim sohasi 300000 "Ishlab chiqarish texnik soha".
Ta'lim sohasi: 320000 "Ishlab chiqarishlar texnalogiyasi" va 310000 "Muhandislik ishi"
talabalari uchun
"OLIY MATEMATIKA" fanidan
"Operatsion hisob" moduli bo'yicha

USLUBIY QO'LLANMA

Farg'ona 2018

Oliy matematika fanidan **Bilim sohasi 300000 "Ishlab chiqarish texnik soha".**

Ta'lim sohasi: 320000 "Ishlab chiqarishlar texnalogiyasi" va 310000 "Muhandislik ishi" talabalari uchun amaliy mashg'ulotlarga mo'jallab yozilgan ushbu uslubiy qo'llanmadan bakalavryatning boshqa ta'lim yo'nalishlari talabalari va matematikani mustaqil o'rganuvchilar ham foydalanishlari mumkin.

Uslubiy qo'llanma "Oliy matematika"
kafedrasi ilmiy uslubiy semiyarida
muhokama qilingan
Bayon №___ ____________2018y.

Uslubiy qo'llanma Ishlab chiqarishda
Boshqaruv fakulteti uslubiy kengashi
tomonidan tasdiqlangan
Bayon №___ ____________2018y

Tuzuvchi: Z. Z.Xo'jaxonov

Taqrizchi:
Fizika-matematika fanlari nomzodi, dotsent: J.D.Mamajonov

Kirish

Hozirgi kunda mamlakatimizda ta'lim sohasida olib borilayotgan tadbirlarning asosiy maqsadi mutaxassislar tayyorlash sifatini tubdan yaxshilashdan iborat. Bakalavriyatning texnika yo'nalishlari talabalari uchun matematikaning amaliy mashg'ulotlarini yaxshi o'zlashtirishi muhim ahamiyatga egadir. Oliy matematika fani o'quv rejalarida hisob grafik ishlarini bajarish fanni chuqurroq mustaqil o'zlashtirishga muhim vazifa sanaladi.

Tavsiya etilayotgan uslubiy qo'llanmada matematika fanining "Operatsion hisob" moduli bo'yicha talabalari uchun amaliy mashg'ulotlar uchun namunaviy misollar va mustaqil bajarish uchun misollar berilgan.

Laplas almashtirishi, uning xossalari.

Bizga quyidagi almashtirish berilgan bo'lsin:

$$L(p)=L\{f(t)\}=\int_0^\infty e^{-pt}f(t)dt$$

Bu almashtirish Laplas almashtirishi deyiladi. Bu yerda $f(t)$ boshlangich funksiya, $L(p)$ esa uning tasviri deyiladi

I. Ixtiyoriy o'zgaruvchi t ning masshtabi o'zgarganda $f(t)$ – funksiyani tasviri.

$a>0$ bo'lganda $f(at)$ – funksiyani tasvirini topamiz. Ta'rifga ko'ra

$$L\{f(at)\}=\int_0^\infty e^{-pt}f(at)dt$$

itegralda $z=at$ almashtirishni bajarsak $t=\frac{z}{a}$, $dt=\frac{1}{a}dz$ bo'ladi, demak

$$L\{f(at)\}=\frac{1}{a}\int_0^\infty e^{-\frac{p}{a}z}f(z)dz=\frac{1}{a}L\left(\frac{p}{a}\right)$$

ya'ni agar $L(p)\to f(t)$ bo'lsa $\frac{1}{a}L\left(\frac{p}{a}\right)\to f(at)$ bo'ladi

Misol. 1) $\sin at\leftarrow\frac{1}{a}\frac{1}{\left(\frac{p}{a}\right)^2+1}$ yoki $\sin at\leftarrow\frac{a}{p^2+a^2}$

2) $\cos at\leftarrow\frac{1}{a}\frac{\frac{p}{a}}{\left(\frac{p}{a}\right)^2+1}$ yoki $\cos at\leftarrow\frac{p}{p^2+a^2}$.

II. Tasvirning chiziqlili xossasi

Teorema. O'zgarmas ko'paytuvchiga ko'paytirilgan bir nechta funksiyalar yig'indisini tasviri shu funksiyalar tasvirlarini mos ko'paytuvchilarga ko'paytmalarining yig'indisiga tengdir, ya'ni

$$f(t)=\sum_{i=1}^{n} c_i f_i(t) \qquad (1.2.1)$$

(bunda c_i lar o'zgarmas sonlar), $L(p) \to f(t)$ va $L_i(p) \to f_i(t)$ bo'lsa, u holda

$$L(t)=\sum_{i=1}^{n} c_i L_i(p) \qquad (1.2.2)$$

Isbot. (1.2.1) tenglikni hadlab e^{-pt} ga ko'paytrib, 0 dan ∞ gacha oraliqda integrallasak (1.2.2) kelib chiqadi.

Misol. 1) $f(t) = 3\sin 4t - 2\cos 5t$

$$L\left\{\frac{1}{t}\right\} = 3\frac{4}{p^2+16} - 2\frac{p}{p^2+25} = \frac{12}{p^2+16} - \frac{2p}{p^2+25};$$

2) $L(p) = \dfrac{5}{p^2+4} + \dfrac{20p}{p^2+9}$ berilgan tasvirga ko'ra original funksiya topilsin.

$$L(p) = \frac{5}{2}\frac{2}{p^2+4} + 20\frac{p}{p^2+9}$$

$$f(t) = \frac{5}{2}\sin 2t + 20\cos 3t$$

III. Siljish teoremasi

Teorema. Agar $f(t)$ funksiyaning tasviri $L(p)$ bo'lsa, u holda $e^{-pt} f(t)$ ning tasviri $L(p+\alpha) \to e^{-\alpha t} f(t)$ boladi. Bunda $\mathrm{Re}(p+\alpha) > S_0$ deb faraz qilinadi.

$$L\left\{e^{-\alpha t} f(t)\right\} = \int_0^{\infty} e^{-pt} e^{-\alpha t} f(t)dt = \int_0^{\infty} e^{-(p+\alpha)t} f(t)dt$$

$$L\left\{e^{-\alpha t} f(t)\right\} = L(p+\alpha)$$

Bu teorema tasvirlar sinfini ancha kengaytiradi va ularning orginali oson topiladi.

Misol. $e^{-\alpha t}$, $sh\alpha t$, $ch\alpha t$, $\sin at, e^{-\alpha t}\cos at$ funksiyalarning tasvirini topaylik. bo'lgani uchun $\frac{1}{p+\alpha}\to e^{-\alpha t}$ bo'ladi, shunga o'xshash $\frac{1}{p-\alpha}\to e^{\alpha t}$. Bulardan ikkinchisidan birinchisini ayirib 2-ga bo'lamiz: $\frac{1}{2}(\frac{1}{p-\alpha}-\frac{1}{p+\alpha})\to\frac{1}{2}(e^{\alpha t}-e^{-\alpha t})$ yoki $\frac{\alpha}{p^2-\alpha^2}\to sh\alpha t$ agar ularni qo'shsak $\frac{p}{p^2-\alpha^2}\to ch\alpha t$ kelib chiqadi. $\frac{a}{p^2+a^2}\to\sin at$ va $\frac{p}{p^2+a^2}\to\cos at$ bo'lgani uchun siljish teoremasiga asosan:

$$\frac{a}{(p+\alpha)^2+a^2}\to e^{-\alpha t}\sin at \quad \text{va}$$

$$\frac{p+\alpha}{(p+\alpha)^2+a^2}\to e^{-\alpha t}\cos at$$

kelib chiqadi.

Misol: 1) Tasviri $L(p)=\frac{7}{p^2+10p+41}$ bo'lgan $f(t)$ funksiya topilsin.

$$L(p)=\frac{7}{p^2+10p+41}=\frac{7\cdot 4}{4[(p+5)^2+16]};\quad f(t)=\frac{7}{4}e^{-5t}\sin 4t$$

$$2)\quad L(p)=\frac{p+3}{p^2+2p+10}=\frac{p+3}{(p+1)^2+9}=\frac{p+1}{(p+1)^2+9}+\frac{2}{3}\frac{3}{(p+1)^2+9};$$

$$f(t)=e^{-t}\cos 3t+\frac{2}{3}e^{-t}\sin 3t$$

IV. Tasvirlarni differensiallash va integrallash

Teorema 1. Agar $L(p)\to f(t)$ bo'lsa, u holda

$$(-1)^n\frac{d^n}{dp^n}L(p)\to t^n f(t)\quad(2.1)$$

bo'ladi.

Isbot. $|f(t)| < Me^{S_0 t}$ bo'lganda quyidagi

$$\int_0^\infty e^{-pt}(-t^n)f(t)dt$$

integral mavjud ekanini isbot qilamiz. Shartga ko'ra

$$|f(t)| < Me^{S_0 t}, \quad p = a + ib, \quad a > S_0, \quad a > 0, \quad S_0 > 0$$

Bunga asosan shunday $\varepsilon > 0$ topiladiki, bu uchun $a > S_0 + \varepsilon$ tengsizlik bajariladi. Shuning uchun quyidagi integral yaqinlashadi:

$$\int_0^\infty e^{-(a-\varepsilon)t} f(t)dt < M\int_0^\infty e^{-(a-\varepsilon)t} e^{S_0 t}dt = M\int_0^\infty edt = \left.\frac{Me^{-(a-\varepsilon-S_0)}}{a-\varepsilon-S_0}\right|_0^\infty = \frac{M}{a-\varepsilon-S_0}$$

endi

$$\int_0^\infty \left|e^{-pt}t^n f(t)\right|dt = \int_0^\infty \left|e^{-(p-\varepsilon)t}e^{-\varepsilon t}t^n f(t)\right|dt$$

integralni ko'ramiz; bundagi $e^{-\varepsilon t}t^n$ funksiya chegaralangan va biror N sondan kichik, shuning uchun

$$\int_0^\infty \left|e^{-pt}t^n f(t)\right|dt < N\int_0^\infty \left|e^{-(p-\varepsilon)t} f(t)\right|dt = N\int_0^\infty e^{-(a-\varepsilon)t}|f(t)|dt < \infty .$$

Demak $\int_0^\infty e^{-pt}(-t)^n f(t)dt$ yaqinlashadi. Bu integralni quyidagi

$$L(p) = \int_0^\infty e^{-pt} f(t)dt$$

integralning p-parametr bo'yicha n - tartibli hosilasi deb qarash mumkin, ya'ni

$$\int_0^\infty e^{-pt}(-t)^n f(t)dt = \frac{d^n}{dp^n}\int_0^\infty e^{-pt} f(t)dt$$

oxirgi 2 tenglikdan :

$$(-1)^n \frac{d^n}{dp^n} L(p) = \int_0^\infty e^{-pt}t^n f(t)dt \text{ yoki}$$

$$(-1)^n \frac{d^n}{dp^n} L(p) \to t^n f(t) \quad \text{formula kelib chiqadi.}$$

Bu formuladan $f(t)=t^n$ - darajali funksiyaning tasvirini topamiz: $\frac{1}{p}\rightarrow 1$ formulaga asosan:

$$(-1)\frac{d}{dp}\left(\frac{1}{p}\right)\rightarrow t \quad \text{yoki} \quad \frac{1}{p^2}\rightarrow t$$

shunga o'xshash

$$(-1)\frac{d}{dp}\left(\frac{1}{p^2}\right)\rightarrow t^2 \quad \text{yoki} \quad \frac{1}{p3}\rightarrow t$$

$$(-1)\frac{d}{dp}\left(\frac{1}{p^3}\right)\rightarrow t^3 \quad yoki \quad \frac{3}{p^4}\rightarrow t$$

$$-------------------$$

$$-----------------\ \frac{n}{p^{n+1}}\rightarrow t^n$$

Misollar. 1) $\frac{a}{p^2+a^2}\rightarrow \sin at$ bo'lgani uchun formulaga asosan

$$(-1)\frac{d}{dp}\left(\frac{a}{p^2+a^2}\right)=\frac{2pa}{(p^2+a^2)}\rightarrow t\sin at$$

2) $\frac{p}{p^2+\alpha^2}\rightarrow \cos at$ dan $-\frac{a^2+p^2}{(p^2+a^2)^2}\rightarrow t\cos at;$

3) $\frac{1}{p+\alpha}\rightarrow e^{-\alpha t}$ dan $\frac{1}{(p+\alpha)^2}\rightarrow te^{-\alpha t}$

Teorema 2. Agar $L(p)\rightarrow f(t)$ bo'lsa, u holda:

$$\int_0^\infty L(p)dp\rightarrow \frac{f(t)}{t}$$

Haqiqatdan ham $\frac{f(t)}{t}\leftarrow \hat{O}(p)$ ya'ni $-F(p)=L(p)$. Bu tenglikni hadlab integrallash bilan $F(p)=\int_0^\infty L(p)dp$ ni olamiz yoki $\int_0^\infty L(p)dp\rightarrow \frac{f(t)}{t}$

V. Originalni differensiallash va integrallash

Teorema1. Agar $L(p) \to f(t)$ bo'lsa, u holda $pL(p) - f(0) \to f'(t)$ bo'ladi.

Isbot. Tasvirni ta'rifiga ko'ra:

$$L\{f'(t)\} = \int_0^\infty e^{-pt} f'(t)dt\,.$$

Bu integralni bo'laklab integrallaymiz:

$$I\{f'(t)\} = \int_0^\infty e^{-pt} f'(t)dt = e^{-pt} f(t)\Big|_0^\infty + p\int_0^\infty e^{-pt} f(t)dt = 0 - f(0) + pL(p);$$

(bunda $\lim\limits_{t\to\infty} e^{-pt} f(t) = 0$, $\int_0^\infty e^{-pt} f(t)dt = L(p)$). Demak $pL(p) - f(0) \to f'(t)$

Shu yo'l bilan:

$$p[pL(p) - f(0)] - f'(0) \to f''(t) \quad yoki$$

$$p^2 L(p) - pf(0) - f'(0) \to f''(t)$$

$$-----------------------$$

$$p^n L(p) - \left[p^{n-1} f(0) + p^{n-1} f'(0) + \ldots\ldots + pf^{(n-2)}(0) + f^{(n-1)}(0)\right] \to f^{(n)}(t)$$

Hususiy holda, agar

$$f(0) = f'(0) = \ldots\ldots\ldots = f^{(n-1)}(0) = 0$$

bo'lsa, u holda:

$$L(p) \to f(t)$$

$$pL(p) \to f'(t)$$

$$-----------$$

$$p^n L(p) \to f^{(n)}(t)$$

Teorema 2. Agar $f(t) \leftarrow F(p)$ bo'lsa, u holda $\int_0^t f(u)du \leftarrow \frac{L(p)}{p}$ bo'ladi.

Isbot. $\varphi(t) = \int_0^t f(u)du \leftarrow F(p)$ deb belgilasak, 1-teoremaga asosan:

$\varphi'(t) \leftarrow pF(p) - \varphi(0) = pF(p)$ bo'ladi $(\varphi(0) = 0)$.

$[\varphi(t)]' = \left[\int_0^t f(u)du\right]' = f(t)$ bo'lgani uchun $\int_0^t f(u)du \leftarrow \frac{L(p)}{p}$ kelib chiqadi.

Misol.

$$\cos t \leftarrow \frac{p}{p^2+1}; \quad \int \cos t dt = \sin t \leftarrow \frac{1}{p^2+1}.$$

Odatda funksiyaning tasviri uchun jadval tuziladi va amalda foydalaniladi.

Bazi bir originallar va ularni tasvirlari.

№	Original $f(t)$	tasvir $L(p)$
1.	$\delta_0(t)$	$\frac{1}{p}$
2.	$e^{\alpha t}$	$\frac{1}{p-\alpha}$
3.	$e^{-\alpha t}$	$\frac{1}{p-\alpha}$
4.	$\sin at$	$\frac{a}{p^2+a^2}$
5.	$\cos at$	$\frac{p}{p^2+a^2}$
6.	$shat$	$\frac{a}{p^2-a^2}$
7.	$chat$	$\frac{p}{p^2-a^2}$
8.	$e^{-\alpha t}\sin at$	$\frac{a}{(p+\alpha)^2+a^2}$
9.	$e^{-\alpha t}\cos at$	$\frac{p+\alpha}{(p+\alpha)^2+a^2}$
10.	t^n	$\frac{n!}{p^{n+1}}$
11.	$e^{-\alpha t}t^n$	$\frac{n!}{(p+\alpha)^{n+1}}$
12.	$e^{-\alpha t}t^n$	$\frac{n!}{(p-\alpha)^{n+1}}$

13.	$t\sin at$	$\dfrac{2pa}{(p^2+a^2)^2}$
14.	$t\cos at$	$\dfrac{p^2-a^2}{(p^2-a^2)^2}$
15.	$f(at)$	$\dfrac{1}{a}L(\dfrac{p}{a})$
16.	$e^{-\alpha t}f(t)$	$L(p+\alpha)$
17.	$f'(t)$	$pL(p)-f(0)$
18.	$\int\limits_0^t f(u)du$	$\dfrac{L(p)}{p}$
19.	$t^n f(t)$	$(-1)^n\dfrac{d^n}{dp^n}L(p)$
20.	$\dfrac{f(t)}{t}$	$\int\limits_0^\infty L(p)dp$
21.	$\int\limits_0^t f_1(\tau)f_2(t-\tau)d\tau$	$L_1(p)L_2(p)$

VI. Tasvirlari ratsional kasr bo'lgan funksiyani topish

Noma'lum $f(t)$- funksiyani tasviri $L(p)=\dfrac{\psi(p)}{\varphi(p)}$ to'g'ri ratsional kasr bo'lsin.

Ma'lumki, har qanday to'g'ri ratsional kasrni quyidagi ko'rinishdagi elementar kasrlarni yig'indisi shaklida ifodalash mumkin:

$$1.\ \frac{A}{p-a}\quad 2.\ \frac{A}{(p-a)^k}\quad 3.\frac{Ap+B}{p^2+ap+b}\quad 4.\ \frac{Ap+B}{(p^2+ap+b)^k}$$

bunda $\dfrac{a^2}{4}-b_2<0$ va $k\geq 2$

Bu kasrlarni har biriga nisbatan original funksiyasini topamiz.

1. $\dfrac{A}{p-a}\to Ae^{at}$

2. $\dfrac{A}{(p-a)^k} \to A\dfrac{1}{(k-1)!}t^{k-1}e^{at}$

3. $\dfrac{Ap+B}{p^2+ap+b} = \dfrac{Ap+B}{\left(p+\frac{a}{2}\right)^2+\left(\sqrt{b-\frac{a^2}{4}}\right)^2} = \dfrac{A\left(p+\frac{a}{2}\right)+\left(B-\frac{Aa}{2}\right)}{\left(p+\frac{a}{2}\right)^2+\left(\sqrt{b-\frac{a^2}{4}}\right)^2} =$

$$= A\frac{p+\frac{a}{2}}{\left(p+\frac{a}{2}\right)^2+\left(\sqrt{b-\frac{a^2}{4}}\right)^2} + \left(B-\frac{Aa}{2}\right)\frac{1}{\left(p+\frac{a}{2}\right)^2+\left(\sqrt{b-\frac{a^2}{4}}\right)^2}$$

Agar birinchi qo'shiluvchini M, ikkinchisini N bilan ifodalasak , u holda:

$$M \to Ae^{-\frac{a}{2}t}\cos t\sqrt{b-\frac{a^2}{4}}, \quad N \to \left(B-\frac{Aa}{2}\right)\frac{1}{\sqrt{b-\frac{a^2}{2}}}e^{-\frac{a}{2}t}\sin t\sqrt{b-\frac{a^2}{t}}$$

Shunday qilib:

$$\frac{Ap+B}{p^2+ap+b} \to e^{-\frac{a}{2}t}\left[A\cos t\sqrt{b-\frac{a^2}{4}} + \frac{b-\frac{Aa}{2}}{\sqrt{b-\frac{a^2}{4}}}\sin t\sqrt{b-\frac{a^2}{4}}\right]$$

Misollar. 1) $y''+4y=\sin 3x$ tenglamaning $y(0)=0,\ y'(0)=0$ boshlang'ich shartni qanoatlantruvchi yechimi topilsin.

Yechish. Yordamchi tenglamani tuzamiz:

$$\bar{y}(p)(p^2+4)=\frac{3}{p^2+9};\ \bar{y}(p)=\frac{3}{(p^2+9)(p^2+4)} \text{ yoki}$$

$$\bar{y}(p)=\frac{-\frac{3}{5}}{p^2+9}+\frac{\frac{3}{5}}{p^2+4}=-\frac{1}{5}\frac{3}{p^2+9}+\frac{3}{10}\frac{2}{p^2+4}$$

bundan tenglamaning yechimini topamiz:

$$y(t)=\frac{3}{10}\sin 2t-\frac{1}{5}\sin 3t$$

2) $y''+y=0$ tenglamaning $y(0)=1,\ y'(0)=3,\ y''(0)=6$ boshlang'ich shartlarni qanoatlantruvchi yechimi topilsin.

Yechish. Yordamchi tenglama:

$$\bar{y}(p)(p^3+1)=p^2\cdot 1+p\cdot 3+8; \quad \text{yoki } \bar{y}(p)=\frac{p^2+3p+8}{p^3+1}=\frac{p^2+3p+8}{(p+1)(p^2-p+1)}$$

$$\bar{y}(p)=\frac{2}{p+1}+\frac{-p+6}{p^2-p+1}=2\frac{1}{p+1}-\frac{p-\frac{1}{2}}{\left(p-\frac{1}{2}\right)^2+\left(\sqrt{\frac{3}{4}}\right)^2}+\frac{11}{\sqrt{3}}\frac{\frac{\sqrt{3}}{2}}{\left(p-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}.$$

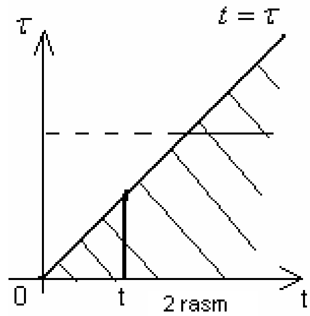
VII. Ko'paytirish teoremasi

Teorema. Agar $f_1(t)\ va\ f_2(t)$ funksiyalarni tasvirlari $L_1(p)\ va\ L_2(p)$ bo'lsa, ya'ni $L_1(p)\to f_1(t)$ va $L_2(p)\to f_2(t)$ u holda $\int_0^t f_1(\tau)f_2(t-\tau)d\tau$ funksiyani tasviri ko'paytmadan iborat bo'ladi; ya'ni

$$L_1(p)L_2(p)\to\int_0^t f_1(\tau)f_1(t-\tau)d\tau \qquad (1.2.3)$$

Isbot. $\int_0^t f_1(\tau)f_1(t-\tau)d\tau$ funksiyani tasvirini topamiz, tasvirni ta'rifiga asosan:

$$L\left\{\int_0^t f_1(\tau)f_1(t-\tau)d\tau\right\}=\int_0^\infty e^{-pt}\left[\int_0^t f_1(\tau)f_2(t-\tau)d\tau\right]dt$$



O'ng tomondagi ikki karrali integral $\tau=0,\ \tau=t$ chiziqlar bilan chegaralangan soha bo'yicha olinadi (2 rasm)

Bu integralda integrallash tartibini o'zgartiramiz:

$$L\left\{\int_0^t f_1(\tau)\, f_2(t-\tau)d\tau\right\} = \int_0^\infty \left[f_1(\tau)\int_\tau^\infty e^{-pt} f_2(t-\tau)dt \right] d\tau \qquad (1.2.4)$$

O’zgaruvchini almashtramiz: $t-\tau = z, \quad dt = dz$ ichki integralni hisoblaymiz:

$$\int_0^\infty e^{-pt} f_2(t-\tau)dt = \int_0^\infty e^{-p(z+\tau)} f_2(z)dz = e^{-p\tau}\int_0^\infty e^{-pz} f_2(z)dz = e^{-p\tau}F_2(p)$$

Ichki integralni qiymatini (1.2.4) ga qo’yamiz:

$$L\left\{\int_0^t f_1(\tau)\, f_2(t-\tau)d\tau\right\} = \int_0^\infty f_1(\tau)e^{-p\tau}L_2(p)d\tau = L_2(p)\int_0^\infty e^{-p\tau} f_1(\tau)d\tau = L_2(p)L_1(p)$$

Demak $\int_0^t f_1(\tau)\, f_2(t-\tau)d\tau$ ifoda berilgan ikkita $f_1(t)$ va $f_2(t)$ funksiyani o’ramasi deyiladi.

Bunda $\int_0^t f_1(\tau)\, f_2(t-\tau)d\tau = \int_0^t f_1(t-\tau)\, f_2(\tau)d\tau$ tenglik kuchga egadir.

Misol. $y'' + y = f(t)$ tenglamaning $y(0) = y'(0) = 0$ boshlang’ich shartni qanoatlantruvchi yechimi topilsin.

Yechish. Yordamchi tenglama tuzamiz:

$$\bar{y}(p)(p^2+1) = L(p)$$

bundan

$$\bar{y}(p) = \frac{L(p)}{p^2+1}$$

$\frac{1}{p^2+1} \to \sin t$ va $L(p) \to f(t)$ bo’lgani uchun $L(p) = L_1(p)$ orqali belgilab ko’paytrish teoremasini tadbiq etsak,

$$y(t) = \int_0^t f(\tau)\sin(t-\tau)d\tau$$

kelib chiqadi.

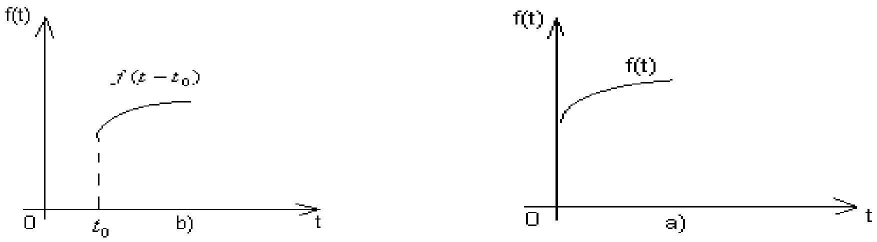
Izoh. 2) Agar $f_1(t) = f(t)$ va $f_2(t) = 1$ desak, u holda $L_1(p) = L(p)$ va $L_2(p) = \frac{1}{p}$ bo’ladi. Ko’paytirish teoremasiga asosan:

$$\frac{1}{p}L(p)=\int_0^t f(\tau)d\tau \qquad \text{yoki} \qquad \int_0^t f(\tau)d\tau=\frac{L(p)}{p} \quad \text{va} \quad f_2(t)=1$$

originalni integrallash haqidagi teorema kelib chiqadi.

VIII. Kechikish teoremasi

Agar $f(t)$ funksiya t<0 bo’lganda aynan nolga teng bo’lsa, u holda $f(t-t_0)$ funksiya $t<t_0$ bo’lganda aynan nolga teng bo’ladi (rasm 3 a) va b))



3 rasm

Quydagi kechikish teoremasini isbot qilamiz.

Teorema. Agar $f(t)$ funksiyani tasviri L(p) bo’lsa, u holda $f(t-t_0)$ funksiya tasviri $e^{-pt_0}F(p)$ bo’ladi, ya’ni: $f(t)\leftarrow L(p)$ bo’lsa $f(t-t_0)\leftarrow e^{-pt_0}L(p)$

Isbot. Tasvirni ta’rifiga asosan.

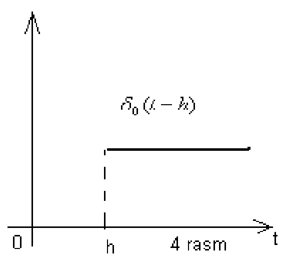
$$L\{f(t-t_0)\}=\int_0^{\infty} e^{-pt}f(t-t_0)dt=\int_0^{t_0} e^{-pt}f(t-t_0)dt+\int_{t_0}^{\infty} e^{-pt}f(t-t_0)dt$$

Tenglikni o’ng tomonidagi birinchi integral nolga teng, chunki $t<t_0$ bo’lganda $f(t-t_0)=0$. Keyingi integralda o’zgaruvchini almashtramiz: $t-t_0=z$, $dt=dz$;
U holda

$$L\{f(t-t_0)\}=\int_0^{\infty} e^{-p(z+t_0)}f(z)dz=e^{-pt_0}\int_0^{\infty} e^{-pz}f(z)dz=e^{-pt_0}L(p).$$

demak $f(t-t_0)\leftarrow e^{-pt_0}L(p)$.

Misollar. I. § 1.1 da Xevisaydning birlik funksiyasi uchun $\delta_0(t)\leftarrow\frac{1}{p}$ topilgan edi. Kechikish teoremasiga asosan $\delta_0(t-h)$ funksiya (rasm 4) uchun

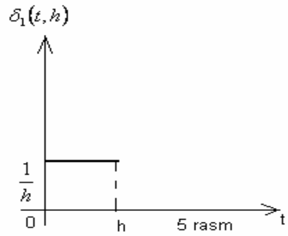


$\delta_0(t-h) \leftarrow \frac{1}{p}e^{-ph}$ boladi.

II. Quyidagi funksiyani qaraymiz:

$$\delta_1(t,h) = \frac{1}{h}\left[\delta_0(t) - \delta_0(t-h)\right] = \begin{cases} 0, \ t<0 \ \ bo'lg\,anda \\ \frac{1}{h}, \ \ 0 \le t < h\, bo'lg\,anda \\ 0, \quad h \le t \quad bo'lg\,anda \end{cases}$$

Agar bu funksiya 0 dan h gacha bo’lgan vaqt oralig’ida ta’sir etuvchi kuch deb qaralsa (boshqa vaqtlarda nolga teng), u holda bu kuch impulsi birga teng bo’ladi. (rasm 5)



Bu funksiya tasviri

$$\delta_1(t,h) \leftarrow \frac{1}{h}\left(\frac{1}{p} - \frac{1}{p}e^{-ph}\right) = \frac{1}{p}\left(\frac{1-e^{-ph}}{h}\right)$$

Mehanikada bazi qisqa vaqt oralig’ida ta’sir etuvchi oniy ta’sir etuvchi va chekli ipulsga ega kuch deb qarash qulay bo’ladi. Shunga ko’ra $h \to 0$ bo’lganda $\delta_1(t,h)$ funksiyaning limitiga teng bo’lgan $\delta(t)$ funksiya qaraladi:

$$\delta(t) = \lim_{h\to 0}\delta_1(t,h)$$

Bu funksiya impulsli funksiya yoki delta funksiya deyiladi. Ba’zan fizikada uni Dirak funksiyasi ham deyiladi. $\delta(t)$ funksiyani tasvirini $\delta_1(t,h)$ funksiya tasvirining $h \to 0$dagi limiti deb topamiz :

$$\delta_1(t,h) \leftarrow \int_0^\infty \delta_1(t,h)e^{-pt}dt = \frac{1}{ph}e^{-pt}\Big|_0^\infty = \frac{1}{ph} = \frac{1}{ph}(1-e^{-ph})$$

$t>0$ bo’lganda $\delta_1(t,h)=0$. Shuning uchun $\delta(t) \leftarrow \lim\limits_{h\to 0}\frac{1}{ph}(1-e^{-ph})$

Lopital qoidasiga asosan:

$$\lim_{h\to 0}\frac{1-e^{-pt}}{ph} = \lim_{h\to 0}\frac{pe^{-ph}}{p} = \lim_{h\to 0}e^{-ph} = 1.$$

Demak $\delta(t) \leftarrow 1$. Delta funksiya mehanikada, matematikani ko'pgina bo'limlarida, hususan matematik – fizikaning ko'p masalalarida uchraydi. Agar $\delta(t)$ funksiya massasi 1 ga teng bo'lgan moddiy nuqta $t=0$ momentda 1 ga teng tezlik beradigan kuch deb qarash mumkin.

Shunga o'xshash $\delta(t-t_0)$ funksiyani $t=t_0$ momentida birlik massaga 1 ga teng tezlik beruvchi kuch deb qarash mumkin. Kechikish teoremasiga asosan:

$$\delta(t-t_0) \leftarrow e^{-pt}$$

Yuqoridagi kabi $\int_t^{t_0} \delta(t-t_0)dt = 1$. Endi $\frac{d^2x}{dt^2} = f(t) + \delta(t)$ differensial tenglamaning $t=0$ bo'lganda $x_0 = 0,\ x_0' = 0$ boshlang'ich shartlarni qanoatlantruvchi yechimni ko'ramiz.

Yordamchi tenglama $p^2\overline{x}(p) = L(p) + 1$. Bundan $\overline{x}(p) = \frac{L(p)}{p^2} + \frac{1}{p^2}$ va

$$x(t) = \int_0^t f(\tau)(t-\tau)d\tau\ .$$

Delta funksiyani quyidagi xossalarini ko'rib o'tamiz:
$-\infty < t < 0$ bo'lganda 0 ga $0 \le t < \infty$ bo'lganda 1 ga teng bo'lgan quyidagi integral

$$\int_{-\infty}^{t} \delta(\tau)d\tau = \begin{cases} 0, & -\infty < t < 0 \\ 1, & 0 \le t < \infty \end{cases}$$

Xevisaydning birlik funksiyasini $\delta_0(t)$ ga tengdir:

$$\delta_0(t) = \int_{-\infty}^{t} \delta(\tau)d\tau$$

Bu tenglikni ikki tomonini differensiallab, quyidagi shartli tenglikni olamiz:

$$\delta_0'(t) = \delta(t) \tag{1.2.5}$$

Bu tenglikni ma'nosini tushuntirish uchun quyidagi funksiyani olamiz:

$$\overline{\delta}_0(t,h) = \begin{cases} 0 \ \ (\text{agar } t<0 \text{ bo'lsa}) \\ h \ \ (\text{agar } 0<t<h \text{ bo'lsa}) \\ 1 \ \ (\text{agar } t>h \text{ bo'lsa}) \end{cases}$$

Bu holda:

$$\bar{\delta}_0'(t,h) = \delta_1(t,h) \tag{1.2.6}$$

bunda

$$\lim_{h\to 0}\bar{\delta}_0(t,h) = \delta_0(t) \text{ va } \lim_{h\to 0}\bar{\delta}_0'(t,h) = \delta_0'(t) \tag{1.2.7}$$

(1.2.4) va (1.1.5) tenglika asosan (1.2.5) shartli tenglik kelib chiqadi:

$$\delta_0'(t) = \delta(t).$$

1-topshiriq

1. Quyidagi funksiyalarni Laplas operatoridan foydalangan holda tasvirini toping:

1. a) $f(t) = t^2 - \frac{sint}{5} + e^t$; b) $f(t) = tsin2t - \frac{4}{5t} + t^4$,

2. a) $f(t) = 2t^2 - \frac{sin2t}{5} + e^{5t}$; b) $f(t) = 3tsin2t - \frac{2}{5t} + 5t^4$,

3. a) $f(t) = 5t^2 - \frac{sin4t}{5} + e^{-t}$; b) $f(t) = 5tsin3t + \frac{3}{5t} + t^{-2}$,

4. a) $f(t) = 6t^2 - \frac{sint}{3} + 3e^{-2t}$; b) $f(t) = tsin2t - \frac{4}{5t} + t^3$,

5. a) $f(t) = at^2 - \frac{sint}{5} + 4e^t$; b) $f(t) = 5tsin2t - \frac{3}{5t} + t^2$,

6. a) $f(t) = 3t^2 - \frac{sint}{5} + e^{-2t}$; b) $f(t) = 3 + tsin2t - \frac{7}{5t} + t^2$,

7. a) $f(t) = t^2 + \frac{sint}{5} + 3e^t$; b) $f(t) = tsin2t - \frac{1}{5t} + t^4$,

8. a) $f(t) = 9t^2 + \frac{sint}{3} + e^{-t}$; b) $f(t) = tsin4t - \frac{3}{5t} + 5t^4$,

9. a) $f(t) = t^2 - \frac{sint}{5} + e^{-2t}$; b) $f(t) = 8tsin2t - \frac{2}{5t} + t^2$,

10. a) $f(t) = t^2 - \frac{sin2t}{5} + e^{-t}$; b) $f(t) = -tsin2t - \frac{4}{5t} + 2t^2$,

11. a) $f(t) = 3t^2 - \frac{cost}{5} + 4e^t$; b) $f(t) = tsin3t - \frac{9}{5t} + 5t^7$,

12. a) $f(t) = 5t^2 - \frac{cos3t}{5} + e^{-5t}$; b) $f(t) = tsin2t - \frac{8}{5t} + 5t^7$,

13. a) $f(t) = 6t^2 - \frac{cos4t}{5} + 3e^{-7t}$; b) $f(t) = tsint - \frac{7}{5t} + t^8$,

14. a) $f(t) = 4t^2 - \frac{cos2t}{5} + e^{-3t}$; b) $f(t) = 7tsin3t + \frac{3}{5t} + 4t^5$,

15. a) $f(t) = 2t^2 + \frac{cos3t}{7} + 4e^{-t}$; b) $f(t) = 3tsin2t - \frac{4}{5t} - 8t^5$,

16. a) $f(t) = t^2 - \frac{sint}{5} + e^t cost$; b) $f(t) = t^3 - \frac{4}{5t} + t^4$,

17. a) $f(t) = 2t^2 - \frac{sin2t}{5} + e^{5t} cost$; b) $f(t) = 3t^3 - \frac{2}{5t} + 5t^4$,

18. a) $f(t) = 5t^2 - \frac{sin4t}{5} + e^{-t} cost$; b) $f(t) = t^3 + \frac{3}{5t} + t^{-2}$,

19. a) $f(t) = 6t^2 - \frac{sint}{3} + 3e^{-2t} cost$; b) $f(t) = 6t^3 - \frac{4}{5t} + t^3$,

20. a) $f(t) = at^2 - \frac{sint}{5} + 4e^t cost$; b) $f(t) = 5t^3 - \frac{3}{5t} + t^2$,

21. a) $f(t) = 3t^2 - \frac{sint}{5} + e^{-2t} cost$; b) $f(t) = 3 + t^3 - \frac{7}{5t} + t^2$,

22. a) $f(t) = t^2 + \frac{sint}{5} + 3e^t cost$; b) $f(t) = t^3 - \frac{1}{5t} + t^4$,

23. a) $f(t) = 9t^2 + \frac{sint}{3} + e^{-t} cost$; b) $f(t) = 5t^3 - \frac{3}{5t} + 5t^4$,

24. a) $f(t) = t^2 - \frac{sint}{5} + e^{-2t} cost$; b) $f(t) = 5t^3 - \frac{2}{5t} + t^2$,

25. a) $f(t) = t^2 - \frac{sin2t}{5} + e^{-t} cost$; b) $f(t) = t^3 - \frac{4}{5t} + 2t^2$,

26. a) $f(t) = 3t^2 - \frac{cost}{5} + 4e^t cost$; b) $f(t) = t^3 - \frac{9}{5t} + 5t^7$,

27. a) $f(t) = 5t^2 - \frac{cos3t}{5} + e^{-5t} cost$; b) $f(t) = t^3 - \frac{8}{5t} + 5t^7$,

28. a) $f(t) = 6t^2 - \frac{cos4t}{5} + 3e^{-7t} cost$; b) $f(t) = t^5 - \frac{7}{5t} + t^8$,

29. a) $f(t) = 4t^2 - \frac{cos2t}{5} + e^{-3t} cost$; b) $f(t) = 7t^3 + \frac{3}{5t} + 4t^5$,

30. a) $f(t) = 2t^2 + \frac{cos3t}{7} + 4e^{-t} cost$; b) $f(t) = 3t^3 - \frac{4}{5t} - 8t^5$,

2. Quyidagi tasvirlarni boshlang'ich funksiyasini toping:

1. a) $F(p) = \frac{5p+4}{p^3-2p^2-3p}$; b) $F(p) = \frac{p^2+4p+3}{p^3-3p^2-4p} + \frac{p^2+4p+3}{p^2-1}$.

2. a) $F(p) = \frac{p+2}{p^3-2p^2-3p}$; b) $F(p) = \frac{p^2+p+3}{p^3-p^2-p} + \frac{p^2+7p+3}{p^2-4}$.

3. a) $F(p) = \frac{7p+2}{p^3-2p^2-3p}$; b) $F(p) = \frac{p+3}{p^3-4p^2-5p} + \frac{5p^2+3}{p^2-4}$.

4. a)$F(p) = \frac{3p+2}{p^3-3p^2-4p}$; b) $F(p) = \frac{p^2+p+3}{p^3+4p^2+13p} + \frac{p^2+p+3}{p^2-9}$.

5. a)$F(p) = \frac{p+2}{p^3-p^2-2p}$; b) $F(p) = \frac{7p+3}{p^3-5p^2-6p} + \frac{p^2+5p+3}{p^2-4}$.

6. a)$F(p) = \frac{7p}{p^3-2p^2-3p}$; b) $F(p) = \frac{ap^2+p+3}{p^3+6p^2+13p} + \frac{p^2+p+3}{p^2-25}$.

7. a)$F(p) = \frac{p+2}{p^3-4p^2-5p}$; b) $F(p) = \frac{p^2+p+3}{p^3+4p^2+8p} + \frac{p^2+p+3}{4p^2-1}$.

8. a)$F(p) = \frac{p+2}{p^3+4p^2+29p}$; b) $F(p) = \frac{p^2+p+3}{p^3-4p^2-12p} + \frac{p^2+6p+3}{p^2-4}$.

9. a)$F(p) = \frac{p+2}{p^3-p^2-2p}$; b) $F(p) = \frac{5p^2+p+3}{p^3+8p^2+32p} + \frac{p^2+3}{p^2-4}$.

10. a)$F(p) = \frac{p+2}{p^3+6p^2+18p}$; b) $F(p) = \frac{ap^2+p+3}{p^3-4p^2-5p} + \frac{p^2+p+3}{36p^2-1}$.

11. a)$F(p) = \frac{p+2}{p^3+2p^2 2p}$; b) $F(p) = \frac{p^2+p+3}{p^3-6p^2-7p} + \frac{ap^2+p+3}{p^2-4}$.

12. a)$F(p) = \frac{6p+2}{p^3+2p^2+5p}$; b) $F(p) = \frac{p^2+p+3}{p^3-2p^2-8p} + \frac{p^2+7p+3}{p^2-16}$.

13. a)$F(p) = \frac{-p+2}{p^3-2p^2+2p}$; b) $F(p) = \frac{p^2+p+3}{p^3-6p^2-27p} + \frac{p^2-6p+3}{p^2-1}$.

14. a)$F(p) = \frac{2p+2}{p^3-2p^2+5p}$; b) $F(p) = \frac{p^2+p+3}{p^3-8p^2-9p} + \frac{8p^2+p+3}{p^2-4}$.

15. a)$F(p) = \frac{p+2}{p^3-2p^2+10p}$; b) $F(p) = \frac{p^2+p+3}{p^3-7p^2-8p} + \frac{7p^2+6p+3}{p^2-4}$.

16. a)$F(p) = \frac{p+4}{p^3-2p^2-3p}$; b) $F(p) = \frac{p^2+p+3}{p^3-3p^2-4p} + \frac{p^2+4p+3}{p^2-1}$.

17. a)$F(p) = \frac{p-2}{p^3-2p^2-3p}$; b) $F(p) = \frac{p^2+4p+3}{p^3-p^2-p} + \frac{p^2+7p+3}{p^2-4}$.

18. a)$F(p) = \frac{p+2}{p^3-2p^2-3p}$; b) $F(p) = \frac{2p+3}{p^3-4p^2-5p} + \frac{5p^2+3}{p^2-4}$.

19. a)$F(p) = \frac{5p+1}{p^3-3p^2-4p}$; b) $F(p) = \frac{p^2+5p+3}{p^3+4p^2+13p} + \frac{p^2+5p+3}{p^2-9}$.

20. a)$F(p) = \frac{p+3}{p^3-p^2-2p}$; b) $F(p) = \frac{p+3}{p^3-5p^2-6p} + \frac{p^2+6p+3}{p^2-4}$.

21. a)$F(p) = \frac{6p}{p^3-2p^2-3p}$; b) $F(p) = \frac{ap^2+p+3}{p^3+6p^2+13p} + \frac{p^2+p+3}{p^2-25}$.

22. a)$F(p) = \frac{p-8}{p^3-4p^2-5p}$; b) $F(p) = \frac{p^2-p+3}{p^3+4p^2+8p} + \frac{p^2+7p+3}{4p^2-1}$.

23. a)$F(p) = \frac{7p-1}{p^3+4p^2+29p}$; b) $F(p) = \frac{2p^2-2p+3}{p^3-4p^2-12p} + \frac{5p^2+6p+3}{p^2-4}$.

24. a) $F(p)=\frac{4p+2}{p^3-p^2-2p}$; b) $F(p)=\frac{5p^2+p+3}{p^3+8p^2+32p}+\frac{p^2+3}{p^2-4}$.

25. a) $F(p)=\frac{8p+2}{p^3+6p^2+18p}$; b) $F(p)=\frac{ap^2+p+3}{p^3-4p^2-5p}+\frac{p^2+p+3}{36p^2-1}$.

26. a) $F(p)=\frac{9p+2}{p^3+2p^2 2p}$; b) $F(p)=\frac{p^2+9p+3}{p^3-6p^2-7p}+\frac{p^2+p+3}{p^2-4}$.

27. a) $F(p)=\frac{6p+9}{p^3+2p^2+5p}$; b) $F(p)=\frac{p^2+p+3}{p^3-2p^2-8p}+\frac{p^2+7p+3}{p^2-16}$.

28. a) $F(p)=\frac{-p+6}{p^3-2p^2+2p}$; b) $F(p)=\frac{p^2+6p+3}{p^3-6p^2-27p}+\frac{p^2-6p+3}{p^2-1}$.

29. a) $F(p)=\frac{7p-2}{p^3-2p^2+5p}$; b) $F(p)=\frac{p^2+10p+3}{p^3-8p^2-9p}+\frac{8p^2+5p+3}{p^2-4}$.

30. a) $F(p)=\frac{9p+2}{p^3-2p^2+10p}$; b) $F(p)=\frac{p^2+8p+3}{p^3-7p^2-8p}+\frac{7p^2+6p+3}{p^2-4}$.

Laplas almashtirishini tadbiqlari

2.1 § O’zgarmas koeffisiyentli chiziqli differensial tenglamalarni yechish

O’zgarmas koeffisiyentli chiziqli differensial tenglamalarning berilgan boshlang’ich shartni qanoatlantiruvchi hususiy yechimini Laplas almashtirishini qo’llash yo’li bilan topamiz:

1. Avval ikkinchi tartibli chiziqli differensial tenglamani yechamiz:

$$y''(t)+ay'(t)+a_2y(t)=f(t) \tag{2.1.1}$$

tenglama berilgan bo’lsin; a_1 *va* a_2 o’zgarmas sonlar. Bu tenglamaning $y(0)=y_0$ $y'(0)=y_0'$ boshlang’ich shartlarni qanoatlantruvchi *y(t)*-xususiy yechimini topish kerak. Tenglamani yechimi $y(t)$, uning hosilalari $y'(t), y''(t)$ va o’ng tomoni $f(t)$- originallar bo’lsin. Agar $y(t)\leftarrow\tilde{y}(p)$ va $f(t)\leftarrow F(p)$ desak, u holda originalni differensiallab

$$y'(t)\leftarrow p\bar{y}(p)-y_0,\quad y''(t)\leftarrow p^2\bar{y}(p)-py_0-y'$$

larni topamiz.

Tasvirni chiziqlik xossasiga va (2.1.1) tenglamaga asosan:

$$p^2\bar{y}(p)-py_0-y_0'+a(p\bar{y}(p)-y_0)+a_2\bar{y}(p)=F(p)$$

$$(p^2+a_1p+a_2)\bar{y}(p)=F(p)+py_0+y_0'+a_1y_0 \tag{2.1.2}$$

(2.1.2)- tenglama (2.1.1) differensial tenglamaning yordamchi tenglamasi yoki tasvirlovchi tenglamasi deyiladi.

Natijada original $y(t)$- uchun (2.1.1) differensial tenglama o'rniga uning tasviri $\bar{y}(p)$ - uchun (2.1.2) algebraik tenglamani hosil qildik.

(2.1.2) tenglikdan

$$\bar{y}(p)=\frac{L(p)+py_0+y_0'+a_1y_0}{p^2+a_1p+a_2} \tag{2.1.3}$$

(2.1.3) formula (2.1.2) tenglamaning operator yechimidir. $\bar{y}(p)$- tasvirga asosan $y(t)$ -originalni ya'ni (2.1.1) tenglamani yechimini topamiz.

Misol. 1) $y''-3y'+2y=2e^{3t}$ differensial tenglamaning $y(0)=1,\quad y'(0)=3$ boshlang'ich shartlarni qanoatlantiradigan hususiy yechimi topilsin.

Yechish.

$$L(p)=\frac{2}{p-3}\to 2e^{3t},\quad a_1=-3,\ a_2=2,\ y_0=1,\ y_0'=3$$

Bo'lgani uchun (2.1.3) formulaga asosan:

$$\bar{y}(p)=\frac{\frac{2}{p-3}+p\cdot 1+3+(-3)\cdot 1}{p^2-3p+2}=\frac{2+p^2-3p}{(p-3)(p^2-3p+2)}=\frac{1}{p-3}$$

jadvaldan $y(t)=e^{3t}$ - izlangan yechim bo'ladi.

2) $y''+4y=\sin t$ tenglamaning $y(0)=1,\quad y'(0)=1$ shartlarni qanoatlantiruvchi yechimi topilsin.

Yechish. $\sin t\leftarrow\frac{p}{p^2+1},\quad y_0=1,\ y_0'(0)=1$

$$\bar{y}(p)=\frac{\frac{1}{p^2+1}+p+1}{p^2+4}=\frac{1+(p+1)(p^2+1)}{(p^2+4)(p^2+1)}=\frac{p^3+p^2+p+2}{(p^2+4)(p^2+1)}$$

oxirgi kasrni elementar kasrlarga ajratamiz:

$$\bar{y}(p)=\frac{p}{p^2+4}+\frac{2}{3(p^2+4)}+\frac{1}{3(p^2+1)};$$

bundan jadvalga ko'ra

$$y(t) = \cos 2t + \frac{1}{3}\sin 2t + \frac{1}{3}\sin t \, .$$

2. Endi n- tartibli chiziqli differensial tenglama olamiz:

$$a_0 y^{(n)}(t) + a_1 y^{(n-1)}(t) + a_2 y^{n-2}(t) + ...+ a_{n-1} y'(t) + a_n y(t) = f(t) \qquad (2.1.5)$$

bunda $a_0, a_1, a_2, ..., a_n$ o'zgarmas koeffitsentlar. Bu tenglamaning $y(0) = y_0, y'(0) = y_0', ..., y^{(n-1)}(0) = y_0^{(n-1)}$ boshlang'ich shartlarni qanoatlantiruvchi xususiy yechimini topamiz.

(1) tenglamani hadlab e^{-pt} ga ko'paytramiz, (bunda $p = a + bi$) va 0 dan ∞ gacha oraliqda t bo'yicha integrallaymiz:

$$a_0 \int_0^\infty e^{-pt} y^{(n)}(t)dt + a_1 \int_0^\infty e^{-pt} y^{(n-1)} dt + ... + a_{n-1} \int_0^\infty e^{-pt} y'(t)dt + a_n \int_0^\infty e^{-pt} y(t)dt =$$

$$= \int_0^\infty e^{-pt} f(t)dt$$

Bu tenglamani chap tomonida $y(t)$ funksiya va uning hosilalarining tasvirlari, o'ng tomonida $f(t)$ funksiyaning tasviri turibdi:

$$a_0 L\left\{y^{(n)}(t)\right\} + a_1 L\left\{y^{(n-1)}(t)\right\} + ... + a_{n-1} L\left\{y'(t)\right\} + a_n L\left\{y(t)\right\} = L\left\{f(t)\right\}$$

yoki

$$a_0 \left\{ p^n \bar{y}(t) - (p^{n-1} y_0 + p^{n-2} y_0' + ... + y_0^{(n-1)}) \right\} +$$
$$+ a_1 \left\{ p^{n-1} \bar{y}(p) - (p^{n-2} y_0 + p^{n-3} y_0' + ... + y_0^{n-2}) \right\} + ... +$$
$$+ a_{n-1} \left\{ p\bar{y}(p) - y_0 \right\} + a_n \bar{y}(p) = L(p) \qquad (2.1.6)$$

(2.1.6)- tenglamani quyidagicha yozish mumkin:

$$\bar{y}(p)(a_0 p^n + a_1 p + ... + a_{n-1} p + a_n) = L(p) +$$
$$+ a_0 (p^{n-1} y_0 + p^{n-2} y_0' + ... + y_0^{(n-1)}) +$$
$$+ a_1 (p^{n-2} y_0 + p^{n-3} y_0' + ... + y_0^{(n-2)}) + a_{n-2}(py_0 + y_0') + a_{n-1} y_0 \qquad (2.1.7)$$

(2.1.6) va (2.1.7) tenglama (2.1.5) differensial tenglamaning yordamchi tenglamasi yoki tasvirlovchi tenglamasi deyiladi.

Agar $a_0 p^n + a_1 p^{n-1} + ... + a_{n-1} p + a_n = \varphi(p)$ va

$$a_0 (p^{n-1} y_0 + p^{n-2} y_0{}' + ... + y_0{}^{(n-1)}) + a_1 (p^{n-2} y_0 + p^{n-3} y_0{}' + ... + y_0{}^{(n-2)}) + ...$$

$$+ a_{n-2} (p y_0 + y_0{}') + a_{n-1} y_0 = \psi_{n-1}(p)$$

deb belgilasak, u holda

$$\bar{y}(p) \varphi_n(p) = L(p) + \psi_{n-1}(p)$$

bo'ladi

$$\bar{y}(p) = \frac{L(p)}{\varphi_n(p)} + \frac{\psi_{n-1}(p)}{\psi_n(p)} \qquad (2.1.8)$$

Bu esa $y(t)$ – funksiyani tasviridir, ya'ni $\bar{y}(p) \to y(t)$. Agar $y(0) = y'(0) = ... = y^{(n-1)}(0) = 0$ bo'lsa, u holda (2.1.8) dan

$$\bar{y}(p) = \frac{L(p)}{\varphi_n(p)} \qquad (2.1.9)$$

kelib chiqadi.

Misollar: 1) $y'(t) + y(t) = 1$ tenglamaning y(0)=0 boshlang'ich shartni qanoatlantiruvchi yechimi topilsin. $\bar{y}(p) = \dfrac{\frac{1}{p}}{p+1} = \dfrac{1}{p} - \dfrac{1}{p+1}$ jadvaldan

$y(t) = 1 - e^{-t}$ tenglamani yechimi ekanini topamiz.

2) $y''(t) + 9y(t) = 1$ boshlang'ich shart $y(0) = y'(0) = 0$

$$\bar{y}(p)(p^2 + 9) = \frac{1}{p}; \quad \bar{y}(p) = \frac{1}{p(p^2+9)} = -\frac{1}{9}\frac{1}{p^2+9} + \frac{1}{9}\frac{1}{p}$$

$$y(t) = -\frac{1}{9}\cos 3t + \frac{1}{9} \text{ tenglamani yechimi bo'ladi}.$$

3) $y''(t)+3y'(t)+2y(t)=t$ boshlang'ich shartli $y(0)=y'(0)=0$

$$\bar{y}(p)(p^2+3p+2)=\frac{1}{p^2};$$

$$\bar{y}(p)=\frac{1}{p^2(p+1)(p+2)}=\frac{1}{2}\frac{1}{p^2}-\frac{3}{4}\frac{1}{p}+\frac{1}{p+1}-\frac{1}{4(p+2)}$$

$$y(t)=\frac{1}{2}t-\frac{3}{4}+e^{-t}-\frac{1}{4}e^{-2t}.$$

4) $y''+2y'+5y=\sin t$ boshlang'ich shart $y(0)=1,\ y'(0)=2$

$$\bar{y}(p)(p^2+2p+5)=p*1+2+2*1+\frac{1}{p^2+1} \quad \text{yoki}$$

$$\bar{y}(p)=\frac{p+4+\dfrac{1}{p^2+1}}{p^2+2p+5}=\frac{p^3+4p^2+p+5}{(p^2+2p+5)(p^2+1)}=\frac{\dfrac{11}{10}p+4}{p^2+2p+5}+\frac{-\dfrac{1}{10}p+\dfrac{1}{5}}{p^2+1} \quad \text{yoki}$$

$$\bar{y}(p)=\frac{11}{10}\frac{p+1}{(p+1)^2+4}+\frac{29}{20}\frac{2}{(p+1)^2+4}-\frac{1}{10}\frac{p}{p^2+1}+\frac{1}{5}\frac{1}{p^2+1}$$

jadvaldan $y(t)=\frac{11}{10}e^{-t}\cos 2t+\frac{29}{20}e^{-t}\sin 2t-\frac{1}{10}\cos t+\frac{1}{5}\sin t$.

2.2 § Chiziqli differensial tenglamalar sistemasini operatsion metod bilan yechish.

O'zgarmas koeffitsientli chiziqli birinchi tartibli ikkita differensial tenglamalar sistemasini Laplas almashtirishi yordamida yechilishini ko'ramiz: $x(t)$ va $y(t)$ noma'lum funksiyalar

$$\left.\begin{aligned} \frac{dx}{dt} + a_{11}x(t) + a_{12}y(t) = f_1(t) \\ \frac{dy}{dt} + a_{21}x(t) + a_{22}y(t) = f_2(t) \end{aligned}\right\} \tag{2.2.1}$$

$x(0) = x_{0,}\ \ y(0) = y_0$ boshlang'ich shartlarni qanoatlantiruvchi hususiy yechimni topish kerak.

Izlanayotgan funksiyalar x(t) va $y(t)$, tenglamalarni o'ng tomonidagi $f_1(t)$ *va* $f_2(t)$ funksiyalar originallar bo'lsin. Ularning tasvirlarni mos ravishda quyidagicha belgilaylik:

$$\begin{aligned} &x(t) \leftarrow \overline{x}(p), &&y(t) \leftarrow \overline{y}(p) \\ &f(t) \leftarrow L_1(p) &&f_2(t) \leftarrow L_2(p) \end{aligned}$$

u holda:

$x'(t) \leftarrow p\overline{x}(p) - x_0$; $\qquad y'(t) \leftarrow p\overline{y}(p) - y_0$

Tasvirni chiziqli xossasiga asosan:

$$\left.\begin{aligned} p\overline{x}(p) - x_0 + a_{11}\overline{x}(p) + a_{12}\overline{y}(p) = L_1(p) \\ p\overline{y}(p) - y_0 + a_{21}\overline{x}(p) + a_{22}\overline{y}(p) = L_2(p) \end{aligned}\right\} \tag{2.2.2}$$

(2.2.2) sistema (2.2.1) sistemaga nisbatan yordamchi sistema deyiladi, uni yechib $\overline{x}(p)$ va $\overline{y}(p)$ tasvirlarni topamiz; bu tasvirning originallari esa berilgan sistemaning yechimlari $x(t)$ va $y(t)$ bo'ladi.

Misol. 1)

$$\left.\begin{aligned} \frac{dx}{dt} - x - y = t \\ \frac{dy}{dt} + 4x + 3y = 2t \end{aligned}\right\}$$

sistemaning $x(0)=x_0=0,\ y(0)=y_0=0$ boshlang'ich shartlarni qanoatlantiruvchi yechimi topilsin.

Yechish. $t \leftarrow \frac{1}{p^2},\ 2t \leftarrow \frac{2}{p^2}$ bo'lgani uchun yordamchi sistema quyidagicha bo'ladi:

$$\left.\begin{aligned} p\bar{x}(p)+\bar{x}(p)+\bar{y}(p)=\frac{1}{p^2} \\ p\bar{y}(p)+4\bar{x}(p)+3\bar{y}(p)=\frac{2}{p^2} \end{aligned}\right\} \text{ yoki } \left.\begin{aligned} \bar{x}(p)(p-1)-\bar{y}(p)=\frac{1}{p^2} \\ 4\bar{x}(p)+\bar{y}(p)(p+3)=\frac{2}{p^2} \end{aligned}\right\}$$

Bu sistemani yechamiz $\bar{x}(p)=\frac{p+5}{p^2(p+1)^2},\ \bar{y}(p)=\frac{2(p-3)}{p^2(p+1)^2}$

yoki

$$\bar{x}(p)=-\frac{9}{p}+\frac{5}{p}+\frac{9}{p+1}+\frac{4}{(p+1)^2}\to -9+5t+9e^{-t}+4te^{-t}=x(t)$$

$$\bar{y}(p)=\frac{14}{p}-\frac{6}{p^2}-\frac{14}{p+1}-\frac{8}{(p+1)^2}\to 14-6t-14e^{-t}-8te^{-t}=y(t)$$

Demak berilgan sistemani yechimi:

$$x=-9+5t+9e^{-t}+4te^{-t}+4te^{-t},\qquad y=14-6t-14e^{-t}-8te^{-t}$$

2)

$$\left.\begin{aligned} 3\frac{dx}{dt}+2x+\frac{dy}{dt}=1 \\ \frac{dx}{dt}+4\frac{dy}{dt}+3y=0 \end{aligned}\right\}$$

Sistemani $x(0)=0,\ \ y(0)=0$ boshlang'ich shartlarni qanoatlantruvchi yechimini topamiz.

Yechish. Yordamchi sistema

$$\left.\begin{aligned} (3p+2)\bar{x}(p)+p\bar{y}(p)=\frac{1}{p} \\ p\bar{x}(p)+(4p+3)\bar{y}(p)=0 \end{aligned}\right\}$$

Bu sistemani yechamiz $\bar{x}(p)$ va $\bar{y}(p)$ larni topamiz.

$$\bar{x}(p)=\frac{4p+3}{p(p+1)(11p+6)}=\frac{1}{2p}+\frac{1}{5(p+1)}+\frac{33}{10(11p+6)}$$

$$\bar{y}(p)=\frac{1}{(11p+6)(p+1)}-\frac{1}{5}(\frac{1}{p+1}-\frac{11}{11p+6})$$

topilgan tasvirga asosan noma'lum funksiyalar $x(t)$ va $y(t)$ ni topamiz:

$$x(t)=\frac{1}{2}-\frac{1}{5}e^{-t}-\frac{3}{10}e^{-\frac{6}{11}t}$$

$$y(t)=\frac{1}{5}(e^{-t}-e^{-\frac{6}{11}t})$$

Yuqori tartibli chiziqli tenglamalar sistemasi ham shunga o'xshash yechiladi.

2-topshiriq

1. Berilgan tenglamaning y(0)=0 boshlang'ich shartni qanoatlantiruvchi yechimi topilsin.

1) $y'(t)+2y(t)=-1;$

2) $y'(t)-y(t)=-2;$

3) $y'(t)-3y(t)=2;$

4) $y'(t)-2y(t)=1;$

5) $y'(t)-5y(t)=4;$

6) $y'(t)-2y(t)=5;$

7) $y'(t)+12y(t)=3;$

8) $y'(t)-3y(t)=-4;$

9) $y'(t)-4y(t)=12;$

10) $y'(t)+2y(t)=3;$

11) $y'(t)+7y(t)=4;$

12) $y'(t)+2y(t)=7;$

13) $y'(t)+2y(t)=1;$

14) $y'(t)-y(t)=-2;$

15) $y'(t)-3y(t)=2;$

16)$y'(t) + 3y(t) = 4$;

17) $y'(t) - 7y(t) = 4$;

18) $y'(t) - 5y(t) = 3$;

19) $y'(t) - 7y(t) = 1$;

20) $y'(t) - 5y(t) = 4$;

21) $y'(t) - 6y(t) = 5$;

22)$y'(t) + 12y(t) = -5$;

23) $y'(t) - 8y(t) = 4$;

24) $y'(t) - 9y(t) = 12$;

25) $y'(t) + 6y(t) = -3$;

26) $y'(t) + 9y(t) = 4$;

27) $y'(t) + 5y(t) = 7$;

28)$y'(t) + 11y(t) = 1$;

29) $y'(t) - 8y(t) = 2$;

30) $y'(t) - 7y(t) = -2$.

2. Berilgan tenglamaning $y(0) = y'(0) = 0$ boshlang'ich shartni qanoatlantiruvchi yechimi topilsin.

1) $y''(t) + 4y(t) = sin2t$;

2) $y''(t) + 5y(t) = 3sin2t$;

3) $y''(t) + 7y(t) = sin2t$;

4) $y''(t) - 4y(t) = -2sin2t$;

5) $y''(t) + 4y(t) = 4sin2t$;

6) $y''(t) + 7y(t) = sin2t$;

7) $y''(t) + 7y(t) = sin2t$;

8) $y''(t) + y(t) = sin2t$;

9) $y''(t) - y(t) = -cos2t$;

10) $y''(t) + 8y(t) = 4cos2t$;

11) $y''(t) + 5y(t) = cos2t$;

12) $y''(t) + 3y(t) = 3cos2t$;

13) $y''(t) + 8y(t) = cos2t;$

14) $y''(t) - 4y(t) = -cos2t;$

15) $y''(t) + 5y(t) = 4t;$

16) $y''(t) + 4y(t) = t;$

17) $y''(t) + 5y(t) = 3t;$

18) $y''(t) + 7y(t) = t;$

19) $y''(t) - 4y(t) = -2t;$

20) $y''(t) + 4y(t) = 4t;$

21) $y''(t) + 7y(t) = t;$

22) $y''(t) + 7y(t) = 3t;$

23) $y''(t) + y(t) = t;$

24) $y''(t) - y(t) = -2t;$

25) $y''(t) + 8y(t) = 4t;$

26) $y''(t) + 5y(t) = t;$

27) $y''(t) + 3y(t) = t;$

28) $y''(t) + 8y(t) = t;$

29) $y''(t) - 4y(t) = -t;$

30) $y''(t) + 5y(t) = t.$

3. Berilgan tenglamaning $y(0) = y'(0) = 0$ boshlang'ich shartni qanoatlantiruvchi yechimi topilsin.

1) $y''(t) + 3y'(t) + 2y(t) = t;$

2) $y''(t) + 2y'(t) + y(t) = 3t;$

3) $y''(t) + 4y'(t) + 3y(t) = t;$

4) $y''(t) + 7y'(t) + 6y(t) = 2t;$

5) $y''(t) + 6y'(t) + 5y(t) = t;$

6) $y''(t) + 5(t) + 4y(t) = t;$

7) $y''(t) + 3y'(t) + 2y(t) = 3t;$

8) $y''(t) + 4y'(t) + 3y(t) = t;$

9) $y''(t) + 6(t) + 5y(t) = 4t$;

10) $y''(t) + 8y'(t) + 7y(t) = t$;

11) $y''(t) + 4y'(t) + 3y(t) = 6t$;

12) $y''(t) + 8y'(t) + 7y(t) = -t$;

13) $y''(t) + 7(t) + 6y(t) = t$;

14) $y''(t) + 8y'(t) + 7y(t) = -2t$;

15) $y''(t) + 5y'(t) + 4y(t) = -5t$;

16) $y''(t) + 3y'(t) + 2y(t) = t$;

17) $y''(t) + 2y'(t) + y(t) = 3t$;

18) $y''(t) + 4y'(t) + 3y(t) = t$;

19) $y''(t) + 7y'(t) + 6y(t) = 2t$;

20) $y''(t) + 6y'(t) + 5y(t) = t$;

21) $y''(t) + 5(t) + 4y(t) = t$;

22) $y''(t) + 3y'(t) + 2y(t) = 3t$;

23) $y''(t) + 4y'(t) + 3y(t) = t$;

24) $y''(t) + 6(t) + 5y(t) = 4t$;

25) $y''(t) + 8y'(t) + 7y(t) = t$;

26) $y''(t) + 4y'(t) + 3y(t) = 6t$;

27) $y''(t) + 8y'(t) + 7y(t) = -t$;

28) $y''(t) + 7(t) + 6y(t) = t$;

29) $y''(t) + 8y'(t) + 7y(t) = -2t$;

30) $y''(t) + 5y'(t) + 4y(t) = -5t$.

4. Berilgan tenglamaning $y(0)=1,\ y'(0)=2$ boshlang'ich shartni qanoatlantiruvchi yechimi topilsin.

1) $y'' + 2y' + 5y = tsint$;

2) $y'' + 2y' + 37y = sin2t$;

3) $y'' + 2y' + 17y = sint$;

4) $y'' + 2y' + 26y = sint$;

5) $y'' + 2y' + 2y = sin3t$;

6) $y'' + 2y' + 5y = tsint$;

7) $y'' + 2y' + 2y = tsin8t$;

8) $y'' + 2y' + 5y = tsint$;

9) $y'' + 2y' + 5y = sin9t$;

10) $y'' + 2y' + 5y = sint$;

11) $y'' + 2y' + 5y = sin2t$;

12) $y'' + 2y' + 5y = 3tsint$;

13) $y'' + 2y' + 2y = tsin2t$;

14) $y'' + 2y' + 26y = tsint$;

15) $y'' + 2y' + 17y = tsint$;

16) $y'' + 2y' + 2y = 3tcos2t$;

17) $y'' + 2y' + 2y = tcos2t$;

18) $y'' + 2y' + 5y = tcos2t$;

19) $y'' + 2y' + 2y = 3cos2t$;

20) $y'' + 2y' + 5y = tcos2t$;

21) $y'' + 2y' + 10y = cos2t$;

22) $y'' + 2y' + 5y = tcos2t$;

23) $y'' + 2y' + 17y = cos2t$;

24) $y'' + 2y' + 5y = tcos2t$;

25) $y'' + 2y' + 10y = tcos2t$;

26) $y'' + 2y' + 2y = tcos4t$;

27) $y'' + 2y' + 5y = cos3t$;

28) $y'' + 2y' + 2y = tcos2t$;

29) $y'' + 2y' + 26y = cos2t$;

30) $y'' + 2y' + 50y = cos2t$.

5. Berilgan differensial tenglamalar sistemani $x(0)=0$, $y(0)=0$ boshlang'ich shartlarni qanoatlantruvchi yechimini toping:

1. $\begin{cases} \frac{dx}{dt}+3x+\frac{dy}{dt}=2 \\ \frac{dx}{dt}+\frac{dy}{dt}+3y=0 \end{cases}$;

2. $\begin{cases} \frac{dx}{dt}+x+\frac{dy}{dt}=2 \\ \frac{dx}{dt}+5\frac{dy}{dt}+3y=0 \end{cases}$;

3. $\begin{cases} \frac{dx}{dt}+3x+\frac{dy}{dt}=3 \\ \frac{dx}{dt}+5\frac{dy}{dt}+y=0 \end{cases}$;

4. $\begin{cases} \frac{dx}{dt}+x+\frac{dy}{dt}=2 \\ \frac{dx}{dt}+5\frac{dy}{dt}+3y=1 \end{cases}$;

5. $\begin{cases} \frac{dx}{dt}+2x+\frac{dy}{dt}=2 \\ \frac{dx}{dt}+5\frac{dy}{dt}+3y=3 \end{cases}$;

6. $\begin{cases} \frac{dx}{dt}+3x+2\frac{dy}{dt}=2 \\ \frac{dx}{dt}+5\frac{dy}{dt}+3y=0 \end{cases}$;

7. $\begin{cases} \frac{dx}{dt}+3x+3\frac{dy}{dt}=2 \\ \frac{dx}{dt}+\frac{dy}{dt}+3y=1 \end{cases}$;

8. $\begin{cases} \frac{dx}{dt}+3x+4\frac{dy}{dt}=2 \\ \frac{dx}{dt}+\frac{dy}{dt}+3y=0 \end{cases}$;

9. $\begin{cases} \frac{dx}{dt}+x+\frac{dy}{dt}=2 \\ \frac{dx}{dt}+5\frac{dy}{dt}+3y=2 \end{cases}$;

10. $\begin{cases} \frac{dx}{dt}+x+\frac{dy}{dt}=3 \\ \frac{dx}{dt}+5\frac{dy}{dt}+3y=1 \end{cases}$;

11. $\begin{cases} \frac{dx}{dt} + 3x + \frac{dy}{dt} = 2 \\ \frac{dx}{dt} + \frac{dy}{dt} + 3y = 4 \end{cases}$;

12. $\begin{cases} \frac{dx}{dt} + x + \frac{dy}{dt} = 2 \\ \frac{dx}{dt} + \frac{dy}{dt} + 3y = 0 \end{cases}$;

13. $\begin{cases} \frac{dx}{dt} + 3x + \frac{dy}{dt} = 2 \\ \frac{dx}{dt} + \frac{dy}{dt} + y = 3 \end{cases}$;

14. $\begin{cases} \frac{dx}{dt} + 3x + \frac{dy}{dt} = 2 \\ \frac{dx}{dt} + 2\frac{dy}{dt} + 3y = 1 \end{cases}$;

15. $\begin{cases} \frac{dx}{dt} + 3x + 2\frac{dy}{dt} = 2 \\ \frac{dx}{dt} + 6\frac{dy}{dt} + 3y = 0 \end{cases}$;

16. $\begin{cases} 3\frac{dx}{dt} + 3x + \frac{dy}{dt} = 2 \\ \frac{dx}{dt} + 5\frac{dy}{dt} + 3y = 0 \end{cases}$;

17. $\begin{cases} 2\frac{dx}{dt} + 3x + \frac{dy}{dt} = 2 \\ \frac{dx}{dt} + 5\frac{dy}{dt} + 3y = 2 \end{cases}$;

18. $\begin{cases} 2\frac{dx}{dt} + 3x + \frac{dy}{dt} = 2 \\ \frac{dx}{dt} + \frac{dy}{dt} + 3y = 0 \end{cases}$;

19. $\begin{cases} 3\frac{dx}{dt} + 3x + \frac{dy}{dt} = 2 \\ \frac{dx}{dt} + 5\frac{dy}{dt} + y = 1 \end{cases}$;

20. $\begin{cases} 3\frac{dx}{dt} + 3x + \frac{dy}{dt} = 2 \\ \frac{dx}{dt} + \frac{dy}{dt} + 3y = 3 \end{cases}$;

21. $\begin{cases} \frac{dx}{dt} + 3x + \frac{dy}{dt} = 2 \\ \frac{dx}{dt} + 5\frac{dy}{dt} + 3y = 3 \end{cases}$;

22. $\begin{cases} 3\frac{dx}{dt} + 3x + \frac{dy}{dt} = 2 \\ \frac{dx}{dt} + 5\frac{dy}{dt} + 3y = 0 \end{cases}$;

23. $\begin{cases} \frac{dx}{dt} + 3x + \frac{dy}{dt} = 2 \\ \frac{dx}{dt} + 5\frac{dy}{dt} + 3y = 1 \end{cases}$;

24. $\begin{cases} \frac{dx}{dt} + 3x + \frac{dy}{dt} = 2 \\ \frac{dx}{dt} + 5\frac{dy}{dt} + 3y = 4 \end{cases}$;

25. $\begin{cases} \frac{dx}{dt} + 3x + \frac{dy}{dt} = 1 \\ \frac{dx}{dt} + \frac{dy}{dt} + 3y = 0 \end{cases}$;

26. $\begin{cases} \frac{dx}{dt} + 3x + \frac{dy}{dt} = 2 \\ \frac{dx}{dt} + 5\frac{dy}{dt} + 3y = 3 \end{cases}$;

27. $\begin{cases} \frac{dx}{dt} + 3x + 5\frac{dy}{dt} = 2 \\ \frac{dx}{dt} + \frac{dy}{dt} + 3y = 0 \end{cases}$;

28. $\begin{cases} \frac{dx}{dt} + 3x + \frac{dy}{dt} = 4 \\ \frac{dx}{dt} + \frac{dy}{dt} + 3y = 0 \end{cases}$;

29. $\begin{cases} \frac{dx}{dt} + x + 3\frac{dy}{dt} = 5 \\ \frac{dx}{dt} + 5\frac{dy}{dt} + 3y = 0 \end{cases}$;

30. $\begin{cases} \frac{dx}{dt} + x + \frac{dy}{dt} = 1 \\ \frac{dx}{dt} + \frac{dy}{dt} + y = 0 \end{cases}$.

n-TARTIBLI CHIZIQLI TENGLAMALAR.

O’zgarmas koeffitsientli bir jinsli tenglamalar.

Ushbu

$$y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y = 0 \qquad (3.1)$$

tenglamani yechish uchun $y = e^{\lambda x}$ almashtirish yordamida

$$\lambda^n + a_2\lambda^{n-1} + ... + a_{n-2}\lambda + a_n = 0 \qquad (3.2)$$

xarakteristik tenglama hosil qilinadi va uni yechib $\lambda_1, \lambda_2, ..., \lambda_n$ ildizlari to'iladi.

Tenglamaning umumiy yechimini yozish uchun quyidagi xollarni qaraymiz:

1) $\lambda_1, \lambda_2, ..., \lambda_n$-haqiqiy va har xil. U holda (3.1) tenglamani umumiy yechimi

$$y = C_1e^{\lambda_1 x} + C_2e^{\lambda_2 x} + ... + C_ne^{\lambda_n x} \qquad (3.3)$$

ko'rinishda yoziladi.

2) $\lambda_1, \lambda_2, ..., \lambda_n$ ildizlar haqiqiy va ichida karralisi bor bo'lsin. Faraz qilaylik λ_j ildiz k karrali bo'lib, qolgan $n-k$ ta ildiz har xil bo'lsin. Bu xolda (3.1) tenglamaning umumiy yechimi

$$y = C_1e^{\lambda_j x} + C_2xe^{\lambda_j x} + ... + C_kx^{k-1}e^{\lambda_j x} + C_{k+1}e^{\lambda_{k+1} x} + ... + C_ne^{\lambda_n x} \qquad (3.4)$$

ko'rinishda ifodalanadi.

3) $\lambda_1, \lambda_2, ..., \lambda_n$ ildizlar ichida kom'leks yechimlar ham bor bo'lsin. Aniqlik uchun $\lambda_1 = a + ib,\ \lambda_2 = a - ib$ bo'lib qolganlari haqiqiy sonlar deb olaylik. U holda umumiy yechim

$$y = C_1e^{ax}\cos bx + C_2e^{ax}\sin x + C_3e^{\lambda_3 x} + ... + C_ne^{\lambda_n x} \qquad (3.5)$$

ko'rinishda bo'ladi.

Bu yerda $e^{(a+ib)x} = e^{ax}(\cos bx + i\sin bx)$ ko'rinishdagi formuladan foydalaniladi.

4) $\lambda_1, \lambda_2, ..., \lambda_n$ kom'leks ildizlar ichida k karralisi ham bo'lsin. Masalan, $\lambda_1 = a + ib$ k karrali $\left(k \le \frac{n}{2}\right)$, u holda $\lambda_2 = a - ib$ ildiz ham k karrali bo'ladi, unga mos umumiy yechim

$$\begin{aligned} y &= e^{ax}\left(c_1 + c_3x + ... + c_{2k-1}x^{k-1}\right)\cos bx + e^{ax}\left(c_2 + c_4x + ... + c_{2k}x^{k-1}\right)\sin bx + \\ &+ c_{2k+1}e^{\lambda_{2k+1} x} + ... + c_ne^{\lambda_n x} \end{aligned} \qquad (3.6)$$

ko'rinishda ifodalanadi.

Misol. Tenglamaning xarakteristik tenglamasini tuzing.

$$y''-4y'+8y=0.$$

Yechish. Berilgan tenglamada

$$y=e^{\lambda x}$$

almashtirish bajaramiz. Buning uchun hosila olib, $y'=\lambda e^{\lambda x}, y''=\lambda^2 e^{\lambda x}$ tenglamaga qo'ysak,

$$\lambda^2 e^{\lambda x}-4\lambda e^{\lambda x}+8e^{\lambda x}=0 \text{ yoki } \left(\lambda^2-4\lambda+8\right)e^{\lambda x}=0,$$

bundan $\lambda^2-4\lambda+8=0$ xarakteristik tenglamani hosil qilamiz. Demak, λ ning darajasi tenglamadagi hosila tartibi bilan bir xil ekan.

Misol. Tenglamani umumiy yechimini to'ing.

$$y'''-2y''-3y'=0$$

Yechish. Tenglamada $y=e^{\lambda x}$ almashtirish bajaramiz. Undan tenglamada qatnashgan hosilalarni hisoblaymiz

$$y'=\lambda e^{\lambda x},\ y''=\lambda^2 e^{\lambda x},\ y'''=\lambda^3 e^{\lambda x}.$$

To'ilgan hosilalarni tenglamaga qo'yib, $e^{\lambda x}$ ga bo'lish natijasida (3.2) ko'rinishdagi xarakteristik tenglamani hosil qilamiz:

$$\lambda^3-2\lambda^2-3\lambda=0.$$

Bu tenglamani $\lambda\left(\lambda^2-2\lambda-3\right)=0$ ko'rinishda yozib, $\lambda_1=0,\ \lambda_2=-1,\ \lambda_3=3$ ildizlarga ega bo'lamiz. Barcha ildizlar haqiqiy va har xil. SHuning uchun (3.3) ko'rinishdan foydalanib, umumiy yechimini yozish mumkin, yaoni

$$y=c_1+c_2e^{-x}+c_3e^{3x}.$$

Misol . Tenglamani yeching

$$y''-2y'+y=0.$$

Yechish. (3.2) formulaga ko'ra xarakteristik tenglamasi

$$\lambda^2-2\lambda+1=0$$

ko'rinishga ega. Uni ildizlari $\lambda_1 = 1,\ \lambda_2 = 1$ bo'lib 2 karrali ildiz. Umumiy yechim (3.4) formulaga ko'ra $y = (c_1 + c_2 x)e^x$ ko'rinishga ega bo'ladi.

Misol . Tenglamani integrallang:

$$y''' + 4y'' + 13y' = 0.$$

Yechish. Xarakteristik tenglamasi $\lambda^3 + 4\lambda^2 + 13\lambda = 0$ ko'rinishda bo'lib, $\lambda(\lambda^2 + 4\lambda + 13) = 0$ tenglamaga ekvivalent, ildizlari esa $\lambda_1 = 0,\ \lambda_2 = -2 + 3i,\ \lambda_3 = -2 - 3i$ ga teng. λ_2 va λ_3 ildizlar o'zaro qo'shma kom'leks bo'lganligi uchun (3.5) ko'rinishdagi formuladan foydalanamiz. Demak, umumiy yechim

$$y = c_1 + e^{-2x}(c_2 \cos 3x + c_3 \sin 3x).$$

Misol . Quyidagi tenglamani yeching.

$$y^{IV} + 4y''' + 8y'' + 8y' + 4y = 0.$$

Yechish. Berilgan tenglamani xarakteristik tenglamasi

$$\lambda^4 + 4\lambda^3 + 8\lambda^2 + 4 = 0 \text{ yoki } (\lambda^2 + 2\lambda + 2)^2 = 0$$

ko'rinishga ega. Uni ildizlari $\lambda_1 = \lambda_2 = -1 + i,\ \ \lambda_3 = \lambda_4 = -1 - i$. Ular kom'leks va 2 karrali bo'lganligi uchun (3.6) ga ko'ra

$$y = e^{-x}(c_1 + c_3 x)\cos x + e^{-x}(c_2 + c_4 x)\sin x$$

umumiy yechim bo'ladi.

O'zgarmas koeffitsientli bir jinsli bo'lmagan chiziqli tenglamalar.

Ushbu tenglama berilgan bo'lsin:

$$y^{(n)} + a_1 y^{(n-1)} + \dots + a_{n-1} y' + a_n y = q(x) \tag{3.7}$$

Bu tenglamani nomaolum koeffitsientlar usulida yechishni ko'rib chiqamiz. Avvalo (3.7) tenglamaga mos bir jinsli

$$y^{(n)} + a_1 y^{(n-1)} + \dots + a_{n-1} y' + a_n y = 0$$

tenglamani I bandda ko'rsatilgan usulda yechib, umumiy yechimini to'ib olamiz. So'ngra hususiy yechimini qidiramiz. Agar tenglamaning o'ng tomonini $q(x)$ maxsus ko'rinishga ega bo'lsa, u holda (3.7) tenglamaning xususiy yechimini nomaolum koeffitsientlar usulida to'ish mumkin. Baozi hollarni ko'rib chiqamiz:

a) $q(x)=\left(A_0x^m+A_1x^{m-1}+...+A_m\right)e^{\gamma x}$ ko'rinishda bo'lsa va agar γ (3.7) tenglamani xarakteristik tenglamasini yechimi bo'lmasa $\left(\gamma\neq\lambda_i,\ i=\overline{1,n}\right)$,

u xolda xususiy yechim

$$\bar{y}=\left(B_0x^m+B_1x^{m-1}+...+B_m\right)e^{\gamma x} \tag{3.8}$$

ko'rinishda qidiriladi, agar γ xarakteristik tenglamaning k karrali ildizi bo'lsa $\left(\gamma=\lambda_i,\ i=\overline{1,k}\right)$

$$\bar{y}=x^k\left(B_0x^m+B_1x^{m-1}+...+B_m\right)e^{\gamma x} \tag{3.9}$$

ko'rinishda qidiriladi,. Bunda B_i- nomaolum o'zgarmas sonlar $\left(i=1,m\right)$. (3.8) yoki (3.9) ko''xadni nomaolum B_i koeffitsientlarini to'ish uchun ularning ifodalarini (3.7) tenglamaga qo'yib, tenglikni har ikkala tomonidagi o'xshash xadlarning koeffitsientlarini tenglash lozim. Bulardan hosil bo'lgan algebraik tenglamalar sistemasini yechib, nomaolum koeffitsientlar to'iladi. So'ngra mos holda (3.8) yoki (3.9) ifodalarga qo'yib xususiy yechimning ko'rinishi aniqlanadi.

(3.7) tenglamaning umumiy yechimini esa $y=y_{б/ж}+\bar{y}$ ko'rinishda ifodalanadi. Bu yerda $y_{б/ж}$- (3.7) tenglamaga mos bir jinsli tenglamaning umumiy yechimi.

b) $q(x)=e^{ax}\left(P_m(x)\cos bx+R_l(x)\sin bx\right)$ ko'rinishda bo'lsa, xususiy yechimni

$$\bar{y}=x^ke^{ax}\left(\overline{P}_s(x)\cos bx+\overline{R}_s(x)\sin bx\right) \tag{3.10}$$

ko'rinishda qidiriladi. Bunda $s=\max(m,l)$ bo'lib, $\overline{P}_s(x)$ va $\overline{R}_s(x)$- nomaolum koeffitsientli s-tartibli ko''xadlar. Bu ko''xadlarning koeffitsientlarini to'ish uchun (3.10) ni (3.7) ga qo'yib, yuqoridagi kabi mos xadlarning koeffitsientlarini tenglashtiriladi.

Bunda agar $a+ib$ xarakteristik tenglamani ildizi bo'lmasa $k=0$, agar xarakteristik tenglamani ildizi bo'lsa, u holda k - ildizning karralisi ekanligini bildiradi.

Eslatma. Agar (3.7) tenglamaning o'ng tomonidan $q(x)$ funktsiya turli ko'rinishdagi bir nechta funktsiyalarning yig'indisidan iborat bo'lsa, u holda tenglama o'ng tomondagi funktsiyalarga aloxida-aloxida tenglab olinadi va har biri yechiladi. Umumiy ko'rinishdagi xususiy yechim har bir tenglama hususiy yechimlari yig'indisi sifatida olinadi.

Misol . Tenglamani umumiy yechimini to'ing:

$$y'''-y''+y'-y=x^2+x \tag{3.11}$$

Yechish. Tenglamaning xarakternistik tenglamasi

$$\lambda^3-\lambda^2+\lambda-1=0$$

bo'lib, ildizlari $\lambda_1=2, \lambda_2=-i, \lambda_3=i$ sonlarga teng. Berilgan tenglamaga mos bir jinsli

$$y'''-y''+y'-y=0$$

tenglamaning umumiy yechimi $y_{б/ж}=c_1e^x+c_2\cos x+c_3\sin x$ ko'rinishda bo'ladi ((3.6) formulaga qarang). Bu tenglamada $q(x)=x^2+x$ bo'lib, $\gamma=0$ xarakteristik tenglamani ildizi emas. $(\gamma\neq\lambda_i)$. SHuning uchun xususiy yechimni

$$\bar{y}=B_1x^2+B_2x+B_3 \tag{3.12}$$

(ikkinchi tartibli ko''xad) ko'rinishida qidiramiz. B_1, B_2, B_3- sonlar hozircha nomaolum. (3.12) ifodani (3.11) tenglamaga qo'yib,

$$-B_1x^2+(2B_1-B_2)x+(B_2-2B_2-B_3)=x^2+x$$

tenglikni hosil qilamiz. Undan mos koeffitsientlarni tenglaymiz:

$$\left.\begin{array}{ll} x^2: & -B_1=1 \\ x: & 2B_1-B_2=1 \\ x^0: & B_2-2B_1-B_3=0 \end{array}\right\}$$

Bu sistemani yechib, $B_1=-1,\ B_2=-3,\ B_3=-1$ ekanligini to'amiz. Bu sonlarni (3.12) ga qo'yib,

$$\bar{y}=-x^2-3x-1$$

ekanligini hosil qilamiz. (3.11) tenglamaning umumiy yechimi

$$y=c_1e^x+c_2\cos x+c_3\sin x-x^2-3x-1$$

ko'rinishda bo'ladi.

Misol . Tenglamani yeching:

$$y'''-y''=12x^2+6x.$$

Yechish. Xarakteristik tenglamasi $\lambda^3-\lambda^2=0$ va ildizlari $\lambda_1=\lambda_2=0,\ \lambda_3=1$. Bir jinsli tenglamaning umumiy yechimi:

$$y_{б/ж}=c_1+c_2x+c_3e^x.$$

Bu tenglamadagi $q(x)=12x^2+6x$ bo'lib, $\gamma=0$ harakteristik tenglamaning 2 karrali ildizi $(\gamma=\lambda_1=\lambda_2)$. Demak, xususiy yechim (3.9) ga ko'ra

$$\bar{y}=x^2\left(B_1x^2+B_2x+B_3\right) \qquad (3.13)$$

ko'rinishda qidiriladi. (3.13) ni tenglamaga qo'yib, 3.6-misoldagi kabi ushbu sistemani hosil qilamiz:

$$\begin{cases}-12B_1=12\\ 24B_1-6B_2=6\\ 6B_2-2B_3=0\end{cases}$$

Sistemaning ildizlari $B_1=-1,\ B_2=-5,\ B_3=-15$ bo'ladi. Xususiy yechim

$$\bar{y}=-x^4-5x^3-15x^2$$

ko'rinishda bo'lib, berilgan tenglamaning umumiy yechimi:

$$y=c_1+c_2x+c_3e^x-x^4-5x^3-15x^2.$$

Misol . Tenglamani yeching.

$$y''+2y'+5y=e^{-x}\cos 2x \qquad (3.14)$$

Yechish. Xarakteristik tenglama $\lambda^2+2\lambda+5=0$ bo'lib, ildizlari $\lambda_1=-1+2i,\ \lambda_2=-1-2i$ bo'ladi, yechim esa

$$y_{б/ж}=e^{-x}(c_1\cos x+c_2\sin 2x)$$

ko'rinishda ifodalanadi.

(3.14) da $q(x)=e^{-x}\cos 2x$ bo'lganligi uchun,

$$e^{-x}\cos 2x=\mathrm{Re}\left(e^{(-1+2i)x}\right),\quad e^{-x}\sin 2x=\mathrm{Im}\left(e^{(-1+2i)x}\right)$$

tengliklarni inobatga olsak $\gamma=-1+2i$ ekanligini ko'ramiz. Demak, γ harakteristik tenglamaning bir karrali ildizi. Hususiy yechim (3.10) formulaga asosan (bunda $P_m(x)=1,\ R_l(x)=0$)

$$\bar{y}=x(A\cos 2x+B\sin 2x) \tag{3.15}$$

ko'rinishda qidiriladi. (3.15) ni (3.14) ga qo'yib, $\cos 2x$ va $\sin 2x$ xadlarning oldidagi koeffitsientlarni mos holda tenglab, $A=0$ va $B=\frac{1}{4}$ ekanligini to'amiz. Demak

$$y=(c_1\cos 2x+c_2\sin 2x)e^{-x}+\frac{1}{4}x\sin 2x$$

ifoda (3.14) ning umumiy yechimini ifodalaydi.

Misol . Tenglamani umumiy yechimini to'ing:

$$y''-y'-2y=4x-2e^x.$$

Yechish. Yuqorida berilgan eslatmaga ko'ra bu tenglama quyidagi 2 ta tenglamaga ekvivalent.

$$y''-y'-2y=4x$$
$$y''-y'-2y=-2e^x$$

Bu tenglamalarning xususiy yechimlari yuqorida ko'rsatilgan usulda to'ishni o'quvchiga qoldirib, umumiy yechimning ko'rinishini yozamiz:

$$y=c_1e^{-x}+c_2e^{2x}-2x+1+e^x.$$

Endi (3.7) tenglamaning yechimini o'zgarmasni variatsiyalash usulida yechish bilan tanishamiz. Buning uchun avvalo (3.7) tenglamaga mos

$$y^{(n)} + a_1 y^{(n-1)} + \ldots + a_{n-1} y' + a_n y = 0 \qquad (3.16)$$

bir jinsli tenglamani yechib, chiziqli erkli yechimlarni to'amiz. Faraz qilaylik $y_1(x), y_2(x), \ldots, y_n(x)$ funktsiyalar (3.16) ning chiziqli erkli yechimlari bo'lsin. U holda umumiy yechim

$$y = c_1 y_1 + c_2 y_2 + \ldots + c_n y_n$$

bo'lib, (3.7) tenglamaning xususiy yechimini

$$y = c_1(x) y_1 + c_2(x) y_2 + \ldots + c_n(x) y_n \qquad (3.17)$$

ko'rinishda qidiramiz. Bunda $c_1(x), c_2(x), \ldots, c_n(x)$-hozircha nomaolum funktsiyalar. Ularni to'ish uchun quyidagi sistemani tuzamiz:

$$\begin{cases} c_1' y_1 + c_2' y_2 + \ldots + c_n' y_n = 0 \\ c_1' y_1' + c_2' y_2' + \ldots + c_n' y_n' = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ c_1' y_1^{(n-2)} + c_2' y_2^{(n-2)} + \ldots + c_n' y_n^{(n-2)} = 0 \\ c_1' y_1^{(n-1)} + c_2' y_2^{(n-1)} + \ldots + c_n' y_n^{(n-1)} = q(x) \end{cases} \qquad (3.18)$$

Bu sistemani $c_i'(x)$ larga nisbatan yechib, $c_i'(x) = \varphi_i(x)$ ko'rinishdagi tenglamalarni hosil qilamiz. Ulardan $c_i(x) = \int \varphi_i(x) dx + \tilde{c}_i$ to'ib (3.17) ga qo'yib, umumiy yechimni to'amiz.

Misol . Tenglamani yeching:

$$y'' + y = \frac{1}{\sin x}.$$

Yechish. Berilgan tenglamaga mos bir jinsli $y'' + y = 0$ tenglamani yechamiz. Harakteristik tenglamasi $\lambda^2 + 1 = 0$ bo'lib, $\lambda_1 = i, \lambda_2 = -i$ ildizlarga ega. Umumiy yechim $y = c_1 \cos x + c_2 \sin x$ funktsiya bo'ladi. Berilgan tenglamaning yechimini $y=c_1(x)\cos x+c_2(x)\sin x$ ko'rinishda izlaymiz. (3.18) sistema ushbu ko'rinishda bo'ladi.

$$\begin{cases} c_1'(x)\cos x+c_2'(x)\sin x=0 \\ -c_1'(x)\sin x+c_2'(x)\cos x=\dfrac{1}{\sin x} \end{cases}$$

Sistemani yechib $c_1'(x)=-1$ va $c_2'(x)=ctgx$ tenglamalarni yoki ularni integrallab, $c_1(x)=-x+\bar{c}_1$, $c_2(x)=\ln\sin x+\bar{c}_2$ ifodalarni to'amiz. Bulardan foydalanib, umumiy yechimni quyidagicha yozamiz:

$$y=\bar{c}_1\cos x+\bar{c}_2\sin x-x\cos x+\sin x\ln|\sin x|.$$

Mustaqil yechish uchun to'shiriqlar.

1. Tenglamalarning umumiy yechimini to'ing.

1. $y''-y=0$ 2. $3y''-2y'-8y=0$

3. $y''+2y'+y=0$ 4. $y^{iv}+2y^{v}+y^{iv}=0$

5. $y'''-2y''+2y'=0$ 6. $y''+2y'+y=-2$

7. $y''+9y-9=0$ 8. $y''-4y'+4y=x^2$

9. $y''+y=4x\cos x$ 10. $y''+3y'=3xe^{-3x}$

11. $y''-5y'=3x^2+\sin 5x$ 12. $y''-4y'+8y=e^{2x}+\sin 2x$

2. Quyidagi tenglamalarning xususiy yechimlari ko'rinishini yozing.

13. $y''-6y'+13y=x^2e^{3x}-3\cos 2x$ 14. $y^{IV}+y''=7x-3\cos x$

15. $y''+3y'+2y=e^{-x}\cos^2 x$ 16. $y''-y=4shx$

3. Ushbu tenglamalarni o'zgarmasni variatsiyalash usulida yeching.

17. $y''-y'=\dfrac{1}{e^x+1}$ 18. $y''-2y'+y=\dfrac{e^x}{x^2+1}$

19. $y''+2y'+2y=\dfrac{1}{e^x\sin x}$ 20. $y'''+y''=\dfrac{x-1}{x^2}$

UY I SH I.

1. Tenglamani yeching.

1. $y'''-3y''+3y'-y=0$.
2. $y'''+6y''+11y'+6y=0$.
3. $y''-2y'-2y=0$.
4. $y^{\mathrm{IV}}+2y^{\mathrm{V}}+y^{\mathrm{IV}}=0$.
5. $y'''-8y'+5y=0$.
6. $y'''-8y=0$.
7. $y'''+2y''-y'-2y=0$.
8. $y'''-2y''+2y'=0$.
9. $y^{\mathrm{IV}}-y=0$.
10. $y'''-3y''-2y'=0$.
11. $2y'''-3y''+y'=0$.
12. $y^{\mathrm{V}}-10y'''+9y'=0$.
13. $y'''+8y=0$.
14. $y^{\mathrm{IV}}+y=0$.
15. $y^{\mathrm{IV}}+10y''+9y=0$.

2. Tenglamaning umumiy yechimini nomaolum koeffitsientlar usulida to'ing.

1. $y''-2y'=2ch2x$.
2. $y''+y'=2\sin x-6\cos x+2e^{x}$.
3. $y'''-y'=2e^{x}+\cos x$.
4. $y''-3y'=2ch3x$.
5. $y''-4y'=16ch4x$.
6. $y''+2y'=2sh2x$.
7. $y''+3y'=2sh3x$.
8. $y''+y'=2shx$.
9. $y''+4y=-8\sin 2x+32\cos 2x+4e^{2x}$.
10. $y'''-y'=10\sin x+6\cos x+4e^{x}$.
11. $y''+9y=-18\sin 3x-18e^{3x}$.
12. $y''-4y=24e^{2x}4\cos 2x+8\sin 2x$.
13. $y''+16y=16\cos 4x-16e^{4x}$.
14. $y'''-9y'=-9e^{3x}+18\sin 3x-9\cos 3x$.
15. $y''+25y'=20\cos 5x-10\sin 5x+50e^{5x}$.

3. Tenglamani o'zgarmaslarni variatsiyalash usuli bilan yeching.

1. $y''+4y=\dfrac{1}{\cos 2x}$.
2. $y''+y=tgx$.

3. $y''-y=\dfrac{1}{x}$.

4. $y'''+y'=\dfrac{\sin x}{\cos^2 x}$.

5. $y''-2y'+y=\dfrac{x^2+2x+2}{x^3}$.

6. $y''-y'=\dfrac{2-x}{x^3}e^x$.

7. $y''-y=4\sqrt{x}+\dfrac{1}{x\sqrt{x}}$.

8. $y''+y=\dfrac{1}{\sin 2x\sqrt{\sin 2x}}$.

9. $y''+y=\dfrac{1}{\sin x}$.

10. $y''-y=\dfrac{1}{e^x+1}$.

11. $y''+y=\dfrac{1}{\cos^3 x}$.

12. $y''+y=\dfrac{1}{\sqrt{\sin^5 x\cos x}}$.

13. $y''-2y'+y=\dfrac{e^x}{x^2+1}$.

14. $y''+2y'+2y=\dfrac{1}{e^x\sin x}$.

15. $y''+y=\dfrac{1}{\sin^3 x}$.

DIFFERENTSIAL TENGLAMALARNING NORMAL SISTEMASI.

I. Kirish.

Oddiy differentsial tenglamalar sistemasini quyidagi ko'rinishi

$$\begin{cases} \dfrac{dy_1}{dx}=f_1(x,y_1,y_2,...,y_n) \\ \dfrac{dy_2}{dx}=f_2(x,y_1,y_2,...,y_n) \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ \dfrac{dy_n}{dx}=f_n(x,y_1,y_2,...,y_n) \end{cases} \tag{5.1}$$

normal sistema deyiladi. Bunda $y_1,y_2,...,y_n$ nomaolum funktsiyalar, x erkli o'zgaruvchi, n- normal sistemaning tartibi deyiladi.

(5.1) sistemani (a,b) intervaldagi yechimi deb shunday n ta ixtiyoriy

$$y_1=\varphi_1(x),\ \ y_2=\varphi_2(x),...,y_n=\varphi_n(x)$$

funktsiyalar to'plamiga aytiladiki, bu funktsiyalar uchun quyidagi shartlar bajariladi;

a) (a,b) intervalda aniqlangan;

b) (a,b) intervalda uzluksiz differentsiallanuvchi;

v) ixtiyoriy $x \in (a,b)$ uchun (5.1) sistemani ayniyatga aylantiradi.

(5.1) sistemani yechimlari ichidan $y_1 = y_1^0, y_2 = y_2^0, ..., y_n = y_n^0$ shartni qanoatlantiruvchi yechimini topish masalasi *Koshi masalasi* deyiladi.

Agar ushbu n ta o'zgarmasga bog'liq differentsiallanuvchi funktsiyalar sistemasi

$$y_i = y_i(x, c_1, c_2, ..., c_n), \qquad i = \overline{1,n} \tag{5.2}$$

a) $c_1, c_2, ..., c_n$ larning ixtiyoriy qiymatlari uchun (5.1) sistemani qanoatlantirsa;

b) Koshi teoremasi shartlarini qanoatlantiruvchi soxada Koshi masalasining yechimi bo'lsa, u holda (5.2) funktsiyalar sistemasi (1) sistemani umumiy yechimi deyiladi.

Misol. Berilgan funktsiyalar berilgan sistemaning yechimi ekanligini ko'rsating.

$$\begin{cases} x = t, \\ y = 2e^t \end{cases} \quad (5.6) \qquad \begin{cases} \dfrac{dx}{dt} = e^{t-x} \\ \dfrac{dy}{dt} = 2e^y \end{cases} \quad (5.7).$$

Yechish. (5.6) dan hosila olamiz,

$$\frac{dx}{dt} = 1, \quad \frac{dy}{dt} = 2e^t \qquad (5.8)$$

Endi (5.6) va (5.8) larni (5.7) ga qo'yib,

$$\begin{cases} 1 = e^{t-t} \\ 2e^t = 2e^t \end{cases}$$

tenglikni hosil qilamiz. Demak, (5.6) funktsiyalar (5.7) sistemaning yechimi ekan.

II. Differentsial tenglamalar sistemasini yechishning nomaolumni yo'qotish usuli.

(1) sistemani almashtirish yordamida n - tartibli tenglamaga keltirish mumkin. Buni nomaolumni yo'qotish usulida bajaramiz. Soddalik uchun bu usulni ikkita nomaolumli ikkita tenglamalar sistemasi uchun qo'llaymiz.

Bizga

$$\begin{cases} \dfrac{dx}{dt} = ax + by + f(t) \\ \dfrac{dx}{dt} = cx + dy + g(t) \end{cases} \tag{5.3}$$

sistema berilgan bo'lsin, bunda a, b, c, d - o'zgarmas sonlar, $f_1(t)$, $f_2(t)$- berilgan funktsiyalar, $x(t)$ va $y(t)$- nomaolum funktsiyalar.

(5.3) sistemaning birinchi tenglamasini y ga nisbatan yechib,

$$y = \frac{1}{b}\left(\frac{dx}{dt} - ax - f(t)\right) \tag{5.4}$$

ifodani hosil qilamiz. (5.4) ni (5.3) sistemaning ikkinchi tenglamasiga qo'yamiz. Buning uchun (5.4) dan hosila olamiz:

$$y' = \frac{1}{b}\left(\frac{d^2x}{dt^2} - a\frac{dx}{dt} - f(t)\right).$$

Bu ifodalarni (5.3) ni ikkinchi tenglamasiga qo'yib soddalashtirsak $x(t)$ ga nisbatan ikkinchi tartibli tenglama hosil bo'ladi:

$$a_0\frac{d^2x}{dt^2} + a_1\frac{dx}{dt} + a_2x = f_1(t) \tag{5.5}$$

Bunda a_0, a_1, a_2- o'zgarmas sonlar bo'lib, a, b, c, d -o'zgarmaslar, $f_1(t)$ funktsiya esa $f(t)$ va $g(t)$ funktsiyalar orqali ifodalanadi. (5.5) tenglama o'zgarmas koeffitsientli 2-tartibli bir jinsli bo'lmagan ko'rinishdagi tenglama bo'lib, uni yechish usuli 3-§ da keltirilgan.

Misol . Sistemani yechimini nomaolumni yo'qotish usulida toping.

$$\begin{cases} \dfrac{dx}{dt} = y + 1 \\ \dfrac{dx}{dt} = x + 1 \end{cases} \qquad 5.9)$$

Yechish. (5.9) sistemani birinchi tenglamasidan $y = \dfrac{dx}{dt} - 1$ tenglikni yozamiz. Undan hosila olamiz

$$\frac{dy}{dt} = \frac{d^2x}{dt^2}.$$

Bu tengliklarni (5.9) ni ikkinchi tenglamasiga qo'yib soddalashtiramiz

$$\frac{d^2x}{dt^2} = x + 1 \quad \text{yoki} \quad \frac{d^2x}{dt^2} - x = 1 \qquad (5.10)$$

So'ngi tenglama o'zgarmas koeffitsientli ikkinchi tartibli bir jinsli bo'lmagan tenglama bo'lib, uning umumiy yechimi

$$x = c_1e^t + c_2e^{-t} - 1 \qquad (5.11)$$

ko'rinishda ifodalanadi (3-§ ga qarang).

(5.11) dan hosila olib

$$y = \frac{dx}{dt} - 1$$

tenglikka qo'ysak

$$y = c_1e^t - c_2e^{-t} - 1$$

ifodani hosil qilamiz. Demak, (5.9) sistemaning umumiy yechimi

$$\begin{cases} x = c_1e^t + c_2e^{-t} - 1 \\ y = c_1e^t - c_2e^{-t} - 1 \end{cases}$$

funktsiyalar sistemasidan iborat bo'ladi.

III. Differentsial tenglamalar sistemasining simmetrik formasi.

Baozi hollarda (5.1) sistemani yechish uchun uni ushbu ko'rinishda yozib olish qulay bo'ladi.

$$\frac{dy_1}{f_1(x, y_1, y_2, ..., y_n)} = \frac{dy_2}{f_2(x, y_1, y_2, ..., y_n)} = ... = \frac{dy_n}{f_n(x, y_1, y_2, ..., y_n)} = \frac{dx}{1} \quad (5.12)$$

Sistemani (5.12) ko'rinishiga sistemaning *simmetrik formasi* deyiladi.

(5.12) sistemani yechish uchun o'zgaruvchilari ajraladigan tenglamaga keltiriladigan qilib juftlanadi yoki ushbu tenglikdan foydalaniladi:

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = ... \frac{a_n}{b_n} = \frac{\lambda_1 a_1 + \lambda_2 a_2 + ... + \lambda_n a_n}{\lambda_1 b_1 + \lambda_2 b_2 + ... + \lambda_n b_n} \quad (5.13)$$

Bunda $\lambda_1, \lambda_2, ..., \lambda_n$-koeffitsientlar ixtiyoriy bo'lib, ularni shunday tanlash mumkinki, kasrni surati maxrajining to'la differentsiali bo'lsin yoki surati to'la differentsial bo'lib, maxraji nolga teng bo'lsin.

Misol 5.3. Ushbu sistemani yeching:

$$\frac{dt}{4y - 5x} = \frac{dx}{5t - 3y} = \frac{dy}{3x - 4t} \quad (5.14)$$

Echish. (5.13) dan foydalanib, berilgan sistemani har bir hadi surat va maxrajini mos holda 3, 4, 5 ga ko'paytiramiz $(\lambda_1 = 3,\ \lambda_2 = 4,\ \lambda_3 = 5)$ va qo'shamiz:

$$\frac{3dt}{12y - 15x} = \frac{4dx}{20t - 12y} = \frac{5dy}{15x - 20t} = \frac{3dt + 4dx + 5dy}{0}.$$

Bundan $3dt + 4dx + 5dy = 0$ yoki $d(3t + 4x + 5y) = 0$ bo'lganligidan $3t + 4x + 5y = c_1$ ifodani olamiz.

Endi sistemani boshqa birinchi integralini topish uchun (5.13) dan $\lambda_1 = 2t,\ \lambda_2 = 2x,\ \lambda_3 = 2y$ deb olib, (5.13) dan foydalansak

$$\frac{2tdt}{8yt - 10xt} = \frac{2xdx}{10tx - 6yx} = \frac{2ydy}{6xy - 8ty} = \frac{2tdt + 2xdx + 2ydy}{0}$$

tenglikka ega bo'lamiz. Bundan $2tdt + 2xdx + 2ydy = 0$ yoki $d(t^2 + x^2 + y^2) = 0$.

Integrallab, $t^2 + x^2 + y^2 = c_2$ ifodani olamiz. Bu ikki o'zaro chiziqli erkli birinchi integrallar (5.14) sistemaning umumiy integralini ifodalaydi.

IV. Chiziqli tenglamalar sistemasi. Dalamber usuli.

Quyidagi sistemaga

$$\begin{cases} \dfrac{dy_1}{dx} = a_{11}y_1 + a_{12}y_2 + \ldots + a_{1n}y_n + b_1 x \\ \dfrac{dy_2}{dx} = a_{21}y_1 + a_{22}y_2 + \ldots + a_{2n}y_n + b_2 x \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \dfrac{dy_n}{dx} = a_{n1}y_1 + a_{n2}y_2 + \ldots + a_{nn}y_n + b_n x \end{cases} \tag{5.15}$$

differentsial tenglamalarning *chiziqli sistemasi* deb aytiladi.

Bunda a_{ij} o'zgarmas sonlar $(i, j = \overline{1, n})$, $y_i(x)$- nomaolum funktsiya, $b_i(x)$ berilgan funktsiyalar.

Agar $b_i(x) = 0$ bo'lsa (5.15) bir jinsli sistema deyiladi, $b_i(x) \neq 0$ bo'lsa, bir jinsli bo'lmagan sistema deyiladi.

Bir jinsli sistemaning umumiy yechimi xususiy yechimlar yordamida vektor ko'rinishida

$$y(x) = c_1 y_1(x) + c_2 y_2(x) + \ldots + c_n y_n(x) \tag{5.16}$$

yoki

$$\begin{aligned} y_1(x) &= c_1 y_{11} + c_2 y_{12} + \ldots + c_n y_{1n} \\ y_2(x) &= c_1 y_{21} + c_2 y_{22} + \ldots + c_n y_{2n} \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ y_n(x) &= c_1 y_{n1} + c_2 y_{n2} + \ldots + c_n y_{nn} \end{aligned}$$

ko'rinishda ifodalanadi. Bunda y_{ij} bir jinsli tenglamaning xususiy yechimlari.

Endi chiziqli sistemani yechishning *Dalamber usuli* bilan tanishamiz. Bu usulni ikki nomaolumli ikkita tenglamalar sistemasi uchun qo'llaymiz.

Ushbu sistema berilgan bo'lsin.

$$\begin{cases} \dfrac{dx}{dt} = ax + by + f_1(t) \\ \dfrac{dy}{dt} = cx + dy + f_2(t) \end{cases} \tag{5.17}$$

Bu sistemani ikkinchi tenglamasini nomaolum λ soniga ko'paytirib, birinchi tenglamaga hadma-had qo'shamiz, yig'indini differentsiallash formulasidan foydalansak

$$\frac{d(x+\lambda y)}{dt} = (a + c\lambda)x + (b + \lambda d)y + f_1(t) + \lambda f_2(t)$$

ifoda hosil bo'ladi.

Tenglikni o'ng tomonidagi birinchi hadni qavsdan tashqariga chiqarsak,

$$\frac{d(x+\lambda y)}{dt} = (a + c\lambda)\left(x + \frac{b+\lambda d}{a+c\lambda} y \right) + f_1(t) + \lambda f_2(t) \tag{5.18}$$

λ sonni shunday tanlaymizki, u ushbu $\frac{b+\lambda d}{a+c\lambda} = \lambda$ tenglamaning yechimi bo'lsin. Faraz qilaylik λ_1 va λ_2 sonlar so'ngi tenglamani ildizlari bo'lib $\lambda_1 \neq \lambda_2$. U holda bu sonlarni (5.18) ga qo'yib,

$$\frac{d(x+\lambda_i y)}{dt} = (a + c\lambda_i)(x + \lambda_i y) + f_1(t) + \lambda_i f_2(t), \qquad i = 1,2 \tag{5.19}$$

ko'rinishdagi $x + \lambda_i y$ ifodaga nisbatan chiziqli ikkita tenglama hosil bo'ladi. (5.19) tenglamalarni integrallab,

$$\begin{cases} x + \lambda_1 y = e^{(a+\lambda_1 c)t}\left(c_1 + \int \left(f_1(t) + \lambda_1 f_2(t)\right) e^{-(a+\lambda_1 c)\,t} dt\right) \\ x + \lambda_2 y = e^{(a+\lambda_2 c)t}\left(c_2 + \int \left(f_1(t) + \lambda_2 f_2(t)\right) e^{-(a+\lambda_2 c)\,t} dt\right) \end{cases} \tag{5.20}$$

ko'rinishdagi oddiy tenglamalar sistemasiga ega bo'lamiz. So'ngi sistemani x va y larga nisbatan yechib, (5.17) differentsial tenglamalar sistemasining umumiy yechimini hosil qilamiz.

Agar $\lambda_1 = \lambda_2$ bo'lsa, (5.19) faqat bitta tenglamadan iborat bo'lib,

$$x + \lambda y = e^{(a+\lambda c)t}\left(c + \int \left(f_1(t) + \lambda\, f_2(t)\right) e^{-(a+\lambda c)t} dt\right) \tag{5.21}$$

echimni olamiz. Buni x yoki y ga nisbatan yechib, (5.17) sistemani ixtiyoriy bitta tenglamasiga qo'yamiz va hosil bo'lgan tenglamani yechamiz.

Misol 5.4. Sistemani Dalamber usulida yeching:

$$\begin{cases} \dfrac{dx}{dt} = -2x + 2y + e^t \\ \dfrac{dx}{dt} = -x - 5y + e^{2t} \end{cases}$$

Echish. Berilgan tenglamada

$$a = -2,\ b = 2,\ c = -1,\ d = -5,\ f_1(t) = e^t,\ \ f_2(t) = e^{2t}.$$

Sistemani ikkinchi tenglamasini λ ga ko'paytirib, hadma-had qo'shsak, (5.18) ko'rinishdagi tenglama hosil bo'ladi:

$$\frac{d(x+\lambda y)}{dt} = (-2-\lambda)\left(x + \frac{2-5\lambda}{-2-\lambda}\, y\right) + e^t + \lambda e^{2t} \qquad (5.22)$$

Bundan $\dfrac{2-5\lambda}{-2-\lambda} = \lambda$ tenglamani yechib, $\lambda_1 = 1,\ \lambda_2 = 2$ ildizlarni topamiz. Bu sonlarni (5.22) ga qo'yamiz:

$\lambda_1 = 1$ uchun

$$\frac{d(x+y)}{dt} = -3(x+y) + e^t + e^{2t}$$

tenglamani va

$\lambda_2 = 2$ uchun

$$\frac{d(x+2y)}{dt} = -4(x+2y) + e^t + \lambda e^{2t}$$

tenglamani olamiz, bu tenglamalarni integrallab, (5.20) formuladan foydalansak,

$$\begin{cases} x + y = e^{-3t}\left(c_1 + \int\left(e^t + e^{2t}\right)e^{3t}dt\right), & (\lambda_1 = 1) \\ x + 2y = e^{-4t}\left(c_2 + \int\left(e^t + 2e^{2t}\right)e^{4t}dt\right), & (\lambda_2 = 2) \end{cases}$$

ifodaga ega bo'lamiz. Integralni hisoblab

$$\begin{cases} x + y = c_1 e^{-3t} + \dfrac{1}{4}e^t + \dfrac{1}{5}e^{2t} \\ x + 2y = c_2 e^{-4t} + \dfrac{1}{5}e^t + \dfrac{1}{3}e^{2t} \end{cases}$$

algebraik sistemaga kelamiz. Uni yechib ushbu ko'rinishdagi

$$\begin{cases} x = 2c_1e^{-3t} - c_2e^{-4t} + \dfrac{1}{10}e^t + \dfrac{1}{15}e^{2t} \\ y = -c_1e^{-3t} + c_2e^{-4t} - \dfrac{5}{100}e^t + \dfrac{2}{15}e^{2t} \end{cases}$$

umumiy yechimni hosil qilamiz.

V. Differentsial tenglamalar sistemasini o'zgarmasni variatsiyalash usulida yechish.

Yuqori tartibli chiziqli tenglamalarni yechishni o'zgarmasni variatsiyalash usuli bilan yechishni 3-paragrafda ko'rib o'tgan edik. SHu usulda chiziqli sistemani yechish uchun, avvalo bir jinsli sistemani umumiy yechimini biror usulda topib olamiz, yaoni (5.15) sistemada $b_i(x)$ lar aynan nolga teng

$$\begin{cases} \dfrac{dy}{dx} = a_{11}y_1 + a_{12}y_2 + ... + a_{1n}y_n \\ \dfrac{dy}{dx} = a_{21}y_1 + a_{22}y_2 + ... + a_{2n}y_n \\ .... \\ \dfrac{dy}{dx} = a_{n1}y_1 + a_{n2}y_2 + ... + a_{nn}y_n \end{cases}$$

holida yechib,

$$y_k = \sum_{i=1}^{n} c_i y_{ik} \tag{5.23}$$

umumiy yechimni topamiz. Bu ifodada $c_i = c_i(x)$ deb olib, (5.15) ni yechimini

$$y_k = \sum_{i=1}^{n} c_i(x) y_{ik} \qquad \left(k = \overline{1, n}\right) \tag{5.24}$$

ko'rinishda qidiramiz. Bunda $c_i(x)$ lar nomaolum funktsiyalar. (5.24) ni (5.15) ga qo'yib $c_i(x)$ larga nisbatan tenglamalar sistemasini hosil qilamiz. Undan nomaolum $c_i(x)$ larni topib, (5.24) ga qo'yamiz.

Bu usulning mazmunini quyidagi misol yordamida yoritamiz.

Misol 5.5. Sistemani o'zgarmasni variatsiyalash usulida yeching:

$$\begin{cases} \dfrac{dx}{dy} = y \\ \dfrac{dx}{dy} = -x + \dfrac{1}{\cos t} \end{cases} \tag{5.25}$$

Echish. Berilgan sistemaga mos bir jinsli

$$\begin{cases} \dfrac{dx}{dt} = y \\ \dfrac{dy}{dt} = -x \end{cases} \tag{5.26}$$

sistemani oldingi bandlarda ko'rsatilgan usullardan birini qo'llab, umumiy yechimini topamiz. (uni o'quvchiga havola qilamiz).

Echim ushbu

$$\begin{aligned} x &= c_1 \cos t + c_2 \sin t \\ y &= -c_1 \sin t + c_2 \cos t \end{aligned} \tag{5.27}$$

ko'rinishda bo'lib, c_1, c_2-o'zgarmaslarni $c_1 = c_1(t)$, $c_2 = c_2(t)$ deb almashtiramiz. U holda

$$\begin{aligned} x &= c_1(t)\cos t + c_2(t)\sin t \\ y &= -c_1(t)\sin t + c_2(t)\cos t \end{aligned} \tag{5.28}$$

ifodaga ega bo'lamiz. (5.28) ni (5.25) ga qo'yib, soddalashtirsak, $c_1'(t)$ va $c_2'(t)$ nomaolumlarga nisbatan quyidagi sistemani olamiz:

$$\begin{cases} c_1'(t)\cos t + c_2'(t)\sin t = 0 \\ -c_1'(t)\sin t + c_2'(t)\cos t = \dfrac{1}{\cos t} \end{cases}$$

Sistemani yechib, $c_1'(t) = -\dfrac{\sin t}{\cos t}$, $c_2'(t) = 1$ ekanligini ko'ramiz. Integrallash natijasida

$$c_1(t) = \ln|\cos t| + \tilde{c}_1, \quad c_2(t) = t + \tilde{c}_2$$

bo'lib, uni (5.28) ga qo'yamiz. Natijada

$$\begin{aligned} x &= \tilde{c}_1 \cos t + \tilde{c}_2 \sin t + \cos t \cdot \ln|\cos t| + t \cdot \sin t \\ y &= -\tilde{c}_1 \sin t + \tilde{c}_2 \cos t - \sin t \cdot \ln|\cos t| + t \cdot \cos t \end{aligned}$$

ko'rinishdagi umumiy yechim hosil bo'ladi.

VI. CHiziqli bir jinsli tenglamalr sistemasini Eyler usulida yechish.

Endi chiziqli bir jinsli sistemani yechishni *Eyler usuli* bilan tanishamiz. Bunda quyidagi

$$\begin{cases} \dfrac{dy_1}{dx} = a_{11}y_1 + a_{12}y_2 + ... + a_{1n}y_n \\ \dfrac{dy_2}{dx} = a_{21}y_1 + a_{22}y_2 + ... + a_{2n}y_n \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \dfrac{dy_n}{dx} = a_{n1}y_1 + a_{n2}y_2 + ... + a_{nn}y_n \end{cases} \qquad (5.29)$$

bir jinsli sistema berilgan bo'lsin. Uning yechimini

$$y = \alpha e^{\lambda x}, \quad \alpha = (\alpha_1, \alpha_2, ..., \alpha_n) \qquad (5.30)$$

ko'rinishda qidiramiz. Tenglamaga qo'yib, $e^{\lambda x}$ ga qisqartirib, α_i ga nisbatan sistema hosil qilamiz. Hosil bo'lgan sistema noldan farqli yechimga ega bo'lganligi uchun asosiy determinantini nolga teng, yaoni

$$\begin{vmatrix} a_{11} - \lambda & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} - \lambda & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{n1} & a_{n2} & ... & a_{nn} - \lambda \end{vmatrix} = 0 \qquad (5.31)$$

Bu determinantning yoyilmasi λ ga nisbatan n-tartibli tenglama bo'lib, *xarakteristik tenglama* deyiladi. Faraz qilaylik, $\lambda_1, \lambda_2, ..., \lambda_n$ sonlar xarakteristik tenglamasining ildizlari bo'lsin.

a) $\lambda_1, \lambda_2, ..., \lambda_n$ ildizlari har xil va xaqiqiy. $\lambda = \lambda_1$ deb

$$\begin{pmatrix} a_{11} - \lambda_1 & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} - \lambda_1 & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{n1} & a_{n2} & ... & a_{nn} - \lambda_1 \end{pmatrix} \begin{pmatrix} \alpha_{11} \\ \alpha_{21} \\ ... \\ \alpha_{n1} \end{pmatrix} = 0 \qquad (5.32)$$

sistemani yechib, $\alpha_{i\,1}$- larni topamiz $(i = \overline{1, n})$ va (5.30) ga qo'yamiz:

$$y_1=\alpha_{11}e^{\lambda_1 x},\ y_2=\alpha_{21}e^{\lambda_1 x},...,\ y_n=\alpha_{n1}e^{\lambda_1 x}$$

Xuddi shunday $\lambda=\lambda_i$ deb $\left(i=\overline{2,n}\right)$ (5.32) dan har bir λ uchun mos α_i larni topamiz. Natijada

$$y_{11}=\alpha_{11}e^{\lambda_1 x},\ y_{12}=\alpha_{12}e^{\lambda_1 x},...,\ y_{1n}=\alpha_{1n}e^{\lambda_1 x}$$
$$...$$
$$y_{n1}=\alpha_{n1}e^{\lambda_n x},\ y_{n2}=\alpha_{n2}e^{\lambda_n x},...,\ y_{nn}=\alpha_{nn}e^{\lambda_n x}$$

xususiy yechimlarni hosil qilamiz. Ular yordamida ushbu ko'rinishdagi

$$y_1=c_1\alpha_{11}e^{\lambda_1 x}+c_2\alpha_{12}e^{\lambda_2 x}+...+c_n\alpha_{n1}e^{\lambda_n x}$$
$$y_2=c_1\alpha_{12}e^{\lambda_1 x}+c_2\alpha_{22}e^{\lambda_2 x}+...+c_n\alpha_{n2}e^{\lambda_n x}$$
$$...$$
$$y_n=c_1\alpha_{1n}e^{\lambda_1 x}+c_2\alpha_{2n}e^{\lambda_2 x}+...+c_n\alpha_{nn}e^{\lambda_n x}$$

umumiy yechimni yozish mumkin.

b) $\lambda_1,\lambda_2,...,\lambda_n$-ildizlar xaqiqiy va ichida karralisi bor bo'lsin.

Faraz qilaylik λ_i-ildiz k_i karrali ildizi bo'lsin (bunda $k_1+k_2+...+k_m=n$).

Bunday xolda yechimni

$$y_j=\sum_{j=1}^{m}\left(\alpha_{1j}+\alpha_{2j}x+...+\alpha_{k_i j}x^{k_i-1}\right)e^{\lambda_i x}\qquad j=\overline{1,n}\qquad(5.33)$$

ko'rinishda qidiramiz. (5.33) ni (5.29) ga qo'yib, x larni tartibiga qarab mos koeffitsientlarni tenglab, α_{ij}-larga nisbatan sistema hosil qilamiz. Barcha α_{ij} larni topib, yuqoridagi kabi umumiy yechimini yozish mumkin.

Eslatma: $\lambda_1,\lambda_2,...,\lambda_n$ sonlar kompleks va kompleks karrali bo'lsa, u holda a) va b) hollar kabi yechiladi va $\lambda=a+bi$ ildizga mos kelgan yechimni haqiqiy va mavxum qismlari chiziqli erkli yechim ekanligidan hamda Eyler formulasidan (3-§ ga qarang) foydalaniladi.

ko'rinishda yoziladi. **Misol 5.6.** Sistemani Eyler usulida yeching.

$$\begin{cases} x' = 2x + y \\ y' = 3x + 4x \end{cases}$$

Echish. Berilgan tenglamaning xarakteristik tenglamasini tuzamiz:

$$\begin{vmatrix} 2-\lambda & 1 \\ 3 & 4-\lambda \end{vmatrix} = (2-\lambda)(4-\lambda) - 3 = 0$$

Bundan $\lambda^2 - 6\lambda + 5 = 0$ tenglamadan $\lambda_1 = 5,\ \lambda_2 = 1$ ildizlarga egamiz. Xususiy yechimni $x = \alpha e^{\lambda t},\ y = \beta e^{\lambda t}$ ko'rinishda qidiramiz ($\lambda_1,\ \lambda_2$ haqiqiy va har xil bo'lganligi uchun).

$\lambda_1 = 5$ ildizga mos koeffitsentni topish uchun

$$\begin{pmatrix} 2-\lambda_1 & 1 \\ 3 & 4-\lambda_1 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 2-5 & 1 \\ 3 & 4-5 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} -3 & 1 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = 0$$

sistemani yechamiz. Demak,

$$\begin{cases} -3\alpha + \beta = 0 \\ 3\alpha - \beta = 0 \end{cases}$$

sistemadan $3\alpha - \beta = 0$ tenglamaga egamiz. Bu cheksiz ko'p yechimga ega. Xususiy holda $\alpha = 1$ deb olsak $\beta = 3$ bo'ladi. U holda xususiy yechim

$$x_1 = e^{5t}, \quad y_1 = 3e^{5t}$$

ko'rinishda bo'ladi.

$\lambda_2 = 1$ ildizga mos koeffitsientlarni

$$\begin{pmatrix} 1 & 1 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = 0$$

sistemadan topamiz. Bundan $\alpha + \beta = 0$ tenglama cheksiz ko'p yechimga ega bo'lib, xususiy holda $\alpha = 1$ bo'lsa $\beta = -1$ bo'ladi. Xususiy yechim

$$x_2 = e^x, \quad y_2 = -e^x$$

bo'lib, ular yordamida umumiy yechimni

$$\begin{cases} x = c_1 x_1 + c_2 x_2 = c_1 e^{5t} + c_2 e^t \\ y = c_1 y_1 + c_2 y_2 = 3c_1 e^{5t} - c_2 e^t \end{cases}$$

Misol 5.7. Sistemani Eyler usulida yeching:

$$\begin{cases} \dfrac{dx}{dt} = 2x + y \\ \dfrac{dy}{dt} = 4y - x \end{cases} \tag{5.34}$$

Echish. Sistemaning xarakteristik tenglamsi

$$\begin{vmatrix} 2-\lambda & 1 \\ -1 & 4-\lambda \end{vmatrix} = \lambda^2 - 6\lambda + 9 = 0$$

ko'rinishda bo'lib, ildizi $\lambda_1 = \lambda_2 = 3$ ikki karrali ildiz bo'ladi.

Sistemaning yechimini (5.33) ga asosan

$$x = (a_1 + b_1 t)e^{3t}, \quad y = (a_2 + b_2 t)e^{3t} \tag{5.35}$$

ko'rinishda qidiramiz. Buni (5.34) ni birinchi tenglamasiga qo'yib, e^{3t} ga qisqartiramiz:

$$3(a_1 + b_1 t) + b_1 = 2(a_1 + b_1 t) + (a_2 + b_2 t).$$

Endi t ning mos darajalari oldidagi koeffitsientlarni tenglaymiz

$$\left.\begin{aligned} t^0 &: \quad 3a_1 + b_1 = 2a_1 + a_2 \\ t &: \quad 3b_1 = 2b_1 + b_2 \end{aligned}\right\} \tag{5.36}$$

Bu sistemadan $b_2 = b_1$ va $a_2 = a_1 + b_1$ ekanligini topamiz. (5.36) sistema to'rt nomaolumli ikkita tenglamalar sistemasi bo'lganligi uchun (algebra fanining sistemalar haqidagi tushunchalariga ko'ra) cheksiz ko'p yechimga ega. SHuning uchun a_1 va b_1 koeffitsientlar ixtiyoriy bo'lib, ularni mos holda c_1 va c_2 orqali belgilasak, (5.35) ga asosan yechim

$$x = (c_1 + c_2 t)e^{3t}, \quad y = (c_1 + c_2 + c_2 t)e^{3t}$$

ko'rinishda yoziladi.

SHuni aytish lozimki, (5.35) ni (5.34) ni ikkinchi tenglamasiga qo'yib hisoblasak ham xuddi shunday natijaga ega bo'lamiz. (Tekshirishni o'quvchiga havola qilamiz).

Misol 5.8. Sistemani yeching:

$$\begin{cases} \dfrac{dy}{dx} = 2y - z \\ \dfrac{dz}{dx} = y + 2z \end{cases} \tag{5.37}$$

Yechish. Sistemaning xarakteristik tenglamasi

$$\begin{vmatrix} 2-\lambda & -1 \\ 1 & 2-\lambda \end{vmatrix} = \lambda^2 - 4\lambda + 5 = 0$$

ko'rinishda bo'lib, uning ildizlari $\lambda_1 = 2 + i,\ \lambda_2 = 2 - i$ o'zaro qo'shma kompleksdir. $\lambda_1 = 2 + i$ ildizga mos kelgan yechimni

$$y = \alpha e^{(2+i)x}, \quad z = \beta e^{(2+i)x}$$

ko'rinishda qidiramiz. α va β koeffitsientlarni

$$\begin{pmatrix} 2-(2+i) & -1 \\ 1 & 2-(2+i) \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} -i & -1 \\ 1 & -i \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = 0 \quad \text{yoki} \quad \left.\begin{matrix} -i\alpha - \beta = 0 \\ \alpha - i\beta = 0 \end{matrix}\right\}$$

sistemadan topamiz. Sistemani birinchi tenglamasini i ga ko'paytirsak

$$\left.\begin{matrix} \alpha - i\beta = 0 \\ \alpha - i\beta = 0 \end{matrix}\right\}$$

sistema hosil bo'ladi. Bunda $\alpha = 1$ deb olsak $\beta = -i$ bo'lib, yechim

$$y = e^{(2+i)x}, \quad z = -ie^{(2+i)x} \tag{5.38}$$

ko'rinishni oladi. Endi

$$e^{(a+ib)x} = e^{ax}(\cos bx + i \sin bx)$$

formuladan foydalanib, (5.38) ni haqiqiy va mavxum qismlarini ajratamiz.

$$y = e^{2x}(\cos x + i \sin x), \quad z = -i\, e^{2x}(\cos x + i \sin x).$$

Bundan

$$\operatorname{Re} y = y_1 = e^{2x}\cos x, \quad \operatorname{Re} z = z_1 = e^{2x}\sin x,$$
$$\operatorname{Im} y = y_2 = e^{2x}\sin x, \quad \operatorname{Im} z = z_2 = -e^{2x}\cos x.$$

Bu yechimlar fundamental yechimlar sistemasini tashkil etib, umumiy yechim

$$y = e^{2x}\left(c_1 \cos x + c_2 \sin x\right)$$
$$z = e^{2x}\left(c_1 \sin x - c_2 \cos x\right)$$

ko'rinishda ifodalanadi.

Chiziqli bir jinsli bo'lmagan o'zgarmas koeffitsientli tenglamalar sistemasini nomaolum koeffitsientlar usulida yechish.

Agar sistemaning o'ng tomoni maxsus ko'rinishga ega bo'lsa, uni *nomaolum koeffitsientlar usulida* yechish mumkin.

Bu usul o'zgarmas koeffitsientli $n-$tartibli tenglamalarni yechish kabi bo'ladi.

Faraz qilaylik,

$$y_i' = a_{i1}y_1 + a_{i2}y_2 + ... + a_{in}y_n + b_i(x) \quad \left(i = \overline{1,n}\right) \qquad (5.39)$$

sistema berilgan bo'lsin. Bunda $b_i(x)$ funktsiyalar

$$A_0 x^m + A_1 x^{m-1} + ... + A_0$$

ko'rinishdagi ko'phad, $e^{\gamma x}$ yoki $\sin bx$, $\cos bx$ ko'rinishdagi funktsiyalardan iborat bo'lsin, yaoni $b_i(x) = R_{m_i}(x)e^{\gamma x}$ ko'rinishda bo'lsin. U holda xususiy yechim

$$y_i = p^i_{m+s}(x)e^{\gamma x}, \quad i = 1,2,...,n$$

ko'rinishda qidiriladi. Bu yerda P^i_{m+s} $-$ $m+s$ tartibli nomaolum koeffitsientli ko'pxad, m $-$ m_i tartibli ko'pxadlarning eng yuqori darajasi, agar γ - xarakteristik tenglamani yechimi bo'lmasa $s = 0$, agar -γ xarakteristik tenglamani k karrali yechimi bo'lsa, $s = k$ deb olinadi.

Yozib olingan xususiy yechimni tenglamalar sistemasiga qo'yib, x ni mos darajalari oldidagi koeffitsientlarni tenglab (3.6- misolga qarang), nomaolum koeffitsientlarni topish uchun algebraik sistema hosil qilinadi.

Umumiy yechim esa bir jinsli tenglamaning umumiy yechimi bilan bir jinsli bo'lmagan tenglamaning xususiy yechimi yig'indisi ko'rinishida ifodalanadi.

Misol 5.9. Sistemani yeching:

$$\begin{cases} \dfrac{dx}{dt} = y + \sin t \\ \dfrac{dy}{dt} = -x \end{cases} \tag{5.40}$$

Echish. Berilgan sistemaga mos bir jinsli sistemani

$$\begin{cases} \dfrac{dx}{dt} = y \\ \dfrac{dy}{dt} = -x \end{cases}$$

echish uchun xarakteristik tenglamasini tuzamiz

$$\begin{vmatrix} -\lambda & 1 \\ -1 & -\lambda \end{vmatrix} = 0$$

Uning ildizlari $\lambda_1 = i,\ \lambda_2 = -i$. Bir jinsli sistemaning umumiy yechimi

$$\begin{cases} x = c_1 \cos t + c_2 \sin t \\ y = -c_1 \sin t + c_2 \cos t \end{cases} \tag{5.41}$$

ko'rinishda yoziladi.

Misoldan ko'rinadiki, $\sin t = e^{0t} \sin t,\ \ \gamma = 0 + i \cdot 1 = i$ yoki $a = 0,\ b = 1$ bo'lib, $\gamma = \lambda_1$, yaoni γ xarakteristik tenglamaning bir karrali ildizi. SHuning uchun, yechimni

$$\begin{aligned} \bar{x} &= (a_1 + a_2 t)\sin t + (a_3 + a_4 t)\cos t \\ \bar{y} &= (b_1 + b_2 t)\sin t + (b_3 + b_4 t)\cos t \end{aligned} \tag{5.42}$$

ko'rinishda qidiriladi. (5.42) ni (5.40) ga qo'yamiz hamda $\sin t, \cos t$ va $t\sin t,\ t\cos t$ oldidagi koeffitsientlarni tenglab, sistema hosil qilamiz. Undan $b_1 = -\dfrac{1}{2},\ a_2 = b_4 = \dfrac{1}{2},\ a_1 = a_3 = a_4 = b_2 = b_3 = 0$ ekanligini topamiz. Demak, (5.42)ga asosan $\bar{x} = \dfrac{t}{2}\sin t,\ \ \bar{y} = -\dfrac{1}{2}\sin t + \dfrac{t}{2}\cos t$, umumiy yechim esa

$$\begin{aligned} x &= c_1 \cos t + c_2 \sin t + \frac{t}{2}\sin t \\ y &= -c_1 \sin t + c_2 \cos t - \frac{1}{2}\sin t + \frac{t}{2}\cos t \end{aligned}$$

ko'rinishda ifodalanadi.

Mustaqil yechish uchun topshiriqlar.

1. Quyidagi sistemalarni nomaolumni yo'qotish usulida yeching.

1. $$\begin{cases} \dfrac{dx}{dt} = y + t \\ \dfrac{dy}{dt} = x - t \end{cases}$$

2. $$\begin{cases} \dfrac{dx}{dt} - \dfrac{1}{4}\dfrac{dy}{dt} = -\dfrac{3}{4}x + \sin t \\ \dfrac{dx}{dt} = -y + \cos t \end{cases}$$

3. $$\begin{cases} \dfrac{dx}{dt} = y + z \\ \dfrac{dy}{dt} = x + z \\ \dfrac{dz}{dt} = x + y \end{cases}$$

4. $$\begin{cases} \dfrac{d^2x}{dt^2} + \dfrac{dy}{de} = -x \\ \dfrac{dx}{dt} + \dfrac{d^2y}{dt^2} = 0 \end{cases}$$

2. Sistemani Dalamber usulida yeching.

5. $$\begin{cases} \dfrac{dx}{dt} = 6x + y \\ \dfrac{dy}{dt} = 4x + 3y \end{cases}$$

6. $$\begin{cases} \dfrac{dx}{dt} = 3x + y + e^t \\ \dfrac{dy}{dt} = x + 3y - e^t \end{cases}$$

3. Simmetrik ko'rinishda berilgan sistemalarni yeching.

7. $\dfrac{dx}{x}=\dfrac{dy}{y}$

8. $\dfrac{dt}{xy}=\dfrac{dx}{yt}=\dfrac{dy}{xt}$

9. $\dfrac{dx}{2xy}=\dfrac{dz}{2yz}=\dfrac{dy}{y^2-x^2-z^2}$

10. $\dfrac{dx}{y+x}=\dfrac{dy}{x+z}=\dfrac{dz}{x+y}$

4. Eyler usulidan foydalanib, quyidagi sistemalarni yeching.

11. $\begin{cases} \dfrac{dx}{dt}=8y-x \\ \dfrac{dy}{dt}=x+y \end{cases}$

12. $\begin{cases} \dfrac{dx}{dt}=4y-2z-3x \\ \dfrac{dy}{dt}=z+x \\ \dfrac{dz}{dt}=6x-6y+5z \end{cases}$ $\quad (\lambda_1=1,\ \lambda_2=2,\ \lambda_3=-1)$

13. $\begin{cases} \dfrac{dx}{dt}=3x+2y \\ \dfrac{dy}{dt}=-x-y \end{cases}$

14. $\begin{cases} \dfrac{dx}{dt}=y+z \\ \dfrac{dy}{dt}=x+z \\ \dfrac{dz}{dt}=x+y \end{cases}$

5. Bir jinsli bo’lmagan tenglamalar sistemasini o’zgarmasni variatsiyalash usulida yeching.

15. $\begin{cases} \dfrac{dx}{dt} = y - 2x - e^{2t} \\ \dfrac{dy}{dt} = 2y - 3x + 6e^{2t} \end{cases}$

16. $\begin{cases} \dfrac{dx}{dt} = y + tg^2 t - 1 \\ \dfrac{dy}{dt} = -x + tgt \end{cases}$

17. $\begin{cases} \dfrac{dx}{dt} = 5x + 4y + e^{t} \\ \dfrac{dy}{dt} = 4x + 5y + 1 \end{cases}$

18. $\begin{cases} \dfrac{dx}{dt} = 4x - y - 5t + 1 \\ \dfrac{dy}{dt} = x + 2y + t - 1 \end{cases}$

6. Sistemalarni nomaolum koeffitsientlar usulidan foydalanib yeching.

19. $\begin{cases} \dfrac{dx}{dt} = 3 - 2y \\ \dfrac{dy}{dt} = 2x - 2t \end{cases}$

20. $\begin{cases} \dfrac{dx}{dt} = -y + \sin t \\ \dfrac{dy}{dt} = x + \cos t \end{cases}$

21. $\begin{cases} \dfrac{dx}{dt} = y - x + e^{t} \\ \dfrac{dy}{dt} = x - y + e^{t} \end{cases}$

22. $\begin{cases} \dfrac{dx}{dt} = -y + t^2 \\ \dfrac{dy}{dt} = x + t \end{cases}$

Differensial tenglamalarni injinerlik masalalarga tadbiqlari

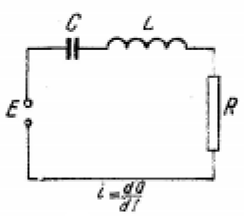
Mexanik tebranishlarning differensial tenglamalari. Elektr zanjirlari nazariyasining differensial tenglamalari

Mexanikadan ma'lumki, massasi ***m*** bo'lgan moddiy nuqtaning tebranishi ushbu differensial tenglama bilan ifodalanadi:

$$\frac{d^2x}{dt^2}+\frac{\lambda}{m}\frac{dx}{dt}+\frac{k}{m}x=\frac{1}{m}f_1(t) \qquad (1)$$

Bu yerda $x-$nuqtaning biron xolatdan chetlanishi, $k-$elastik sistemaning (masalan, prujinaning qattiqligi, harakatga qarshilik kuchi tezligining birinchi darajasiga proporsional ($\lambda-$proporsionallik koeffisienti), $f_1(t)$- tashqi kuch, yoki g'alayonlantiruvchi kuch. Bir erkinlik darajaga ega bo'lgan bashqa mexanik sistemalarning kichik tebranishlari ham (1) tipdagi tenglamaning echimi orqali ifodalanadi. Masalan, elastik o'qdagi maxovixning aylanma tebranishi ham shu tenglamaga olib keladi; bunda $x-$maxovikning aylanma burchagi, m-maxovikning inersiya momenti, $k-$o'qning burilma qattiqligi, $mf_1(t)-$tashqi kuchlarning aylanish o'qiga nisbatan momenti. (1) tipdagi tenglamalar faqat mexanik tebranishlarnigina emas, balki elektr zanjiridagi hodisalarni ham ifodalaydi.

L induktivlik, R qarshilik, C sig'imdan iborat bo'lgan elektr zanjiri berilgan bo'lib, unda E elektr yurituvchikuch (E.Yu.K) qo'yilgan bo'lsin (1-rasm).



1-rasm

Zanjirdagi tokni i bilan, kondensator zaryadini Q bilan belgilaymiz; elektrotexnikadan ma'lumki, i va Q ushbu tenglamalarni qanoatlantiradi:

$$L\frac{di}{dt}+Ri+\frac{Q}{C}=E \qquad (2)$$

$$\frac{dQ}{dt}=i \qquad (3)$$

(3)-tenglamadan:

$$\frac{d^2Q}{dt^2}=\frac{di}{dt}. \qquad (3')$$

(3) va (3') ni (2) tenglamaga qo'yib, Q uchun (1) tipdagi tenglamani hosil qilamiz:

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{1}{C}Q=E. \qquad (4)$$

(2) tenglamaning ikkala qismini differensiallab va (3) tenglamadan foydalanib i ni aniqlash uchun ushbu tenglamani hosil qilamiz:

$$L\frac{d^2i}{dt^2}+R\frac{di}{dt}+\frac{1}{C}i=\frac{dE}{dt} \qquad (5)$$

(4) va (5) tenglamalar (1) tipdagi tenglamalardir.

Tebranishlar differensial tenglamasini yechish

Tebranishlar tenglamasini ushbu ko'rinishda yozamiz:

$$\frac{d^2x}{dt^2}+a_1\frac{dx}{dt}+a_2x=f(t), \qquad (6)$$

Bu yerda noma'lum funksiya x ning a_1 va a_2 koeffisientlarning va $f(t)$ funksiyaning mexanik va fizik ma'nolarini (1), (4) va (5) tenglamalarni solishtirish yo'li bilan aniqlash oson. (6) tenglamaning $t=0$ va $x=x_0$, $x'=x'_0$ boshlang'ich topamiz.

(6) tenglama uchun yordamchi tenglama tuzamiz:

$$\bar{x}(p)(p^2+a_1p+a_2)=x_0p+x'_0+a_1x_0+L(p), \qquad (7)$$

bu yerda $F(p)$ funksiya $f(t)$ funksiyaning tasviri. (7) tenglikdan shuni topamiz:

$$\bar{x}(p)=\frac{x_0p+x'_0+a_1x_0+L(p)}{(p^2+a_1p+a_2)}. \qquad (8)$$

Chunonchi (4) tenglamaning $t=0$ va $Q=Q_0$, $Q'=Q'_0$ boshlang'ich shartlarni qanoatlantiruvchi $Q(t)$ yechimi uchun tasviri ushbu ko'rinishda bo'ladi:

$$\bar{Q}(p)=\frac{L(Q_0p+Q'_0)+RQ_0+\bar{E}(p)}{Lp^2+Rp+\dfrac{1}{C}}.$$

$\bar{Q}(p)$ tasvir uchun avval o'rganganlarimizdan uning boshlang'ich funksiyasini topish kifoya.

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