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GLUON PROPAGATOR IN THE INSTANTON VACUUM AT NON-ZERO TEMPERATURE

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Annotation

We consider the modifications of gluon properties in the instanton liquid model (ILM) for the QCD vacuum at non-zero temperature T. Main parameters of ILM are average instanton size $\bar{\rho}(T)$ and instanton density $n(T) = NT/V_3$.Using the typical phenomenological values of the average instanton size $\bar{\rho}(0) = 1/3\ fm$ and average instanton density $n(0) = 1\, fm^{-4}$ we get $M_{el}(0,0) = 362\, MeV$. At small $T \leq T_c \approx 150\, MeV$ we have rather weak T-dependence as $M_{el}(q,T) = M_{el}(0,0)(1 - 1/6\, \pi^2 \bar{\rho}^2 T^2) F(q, T = 0)\ MeV$, where form-factor $F(0,0) = 1,\ F(q,T) \leq F(q,0) = q\bar{\rho}K_1(q\bar{\rho})$. Dynamical gluon mass in QCD instanton vacuum at non-zero temperature is calculated by using approximation methods. In this work my personal contribution was averaging $\Delta_{2,\mu\nu}(x,y)$ and we work with N. Rakhimov to calculate $\Delta_{1,\mu\nu}(x,y)$ [18]. $\Delta_{0,\mu\nu}(x,y)$ was calculated with my supervisor. Finally, we conclude that only singular part in $\Delta(x,y)$ has significant contribution to the "Electrical" mass

Annotatsiya

Biz nol bo'lmagan haroratli KXD vakuumi uchun instanton suyuqlik modelida (ILM) gluon xossalarining o'zgarishlarini ko'rib chiqdik. Instanton suyuqlik modelining asosiy parametrlari instanton o'lchami $\bar{\rho}(T)$ va instanton zichligi $n(T) = NT/V_3$ hisoblanadi. Instantonning o'rtacha o'lchami $\bar{\rho}(0) = 1/3\ fm$ va o'rtacha zichligining $n(0) = 1\,fm^{-4}$ fenomenologik qiymatlaridan foydalanib biz $M_{el}(0,0) = 362\,MeV$ qiymatni oldik. Kichik $T \leq T_c \approx 150\,MeV$ temperaturalarda $M_{el}(q,T) = M_{el}(0,0)(1 - 1/6\,\pi^2\bar{\rho}^2T^2)F(q,T = 0)\ MeV$ ko'rinishida temperaturaga kuchsiz bog'lanish aniqlandi,bu yerda $F(0,0) = 1$, $F(q,T) \leq F(q,0) = q\bar{\rho}K_1(q\bar{\rho})$-form-factor. Nol bo'lmagan temperaturali KXD instanton vakuumida dinamik gluon massasi taqribiy metodlar yordamida hisoblandi. Bu ishda mening shaxsiy hissam $\Delta_{2,\mu\nu}(x,y)$ o'rtachalashdandan iborat bo'lib, $\Delta_{1,\mu\nu}(x,y)$ N. Rahimov bilan birgalikda hisoblandi. $\Delta_{0,\mu\nu}(x,y)$ ilmiy rahbarim bilan birgalikda hisoblab chiqildi. Biz $\Delta(x,y)$ da "Elektrik" massaga faqat singular qism assosiy hissa qo'shishini aniqladik

Contents

Topicality of the theme and tasks of dissertation.

Topicality of the theme: The question, whether creation of a mass for gauge bosons is possible without gauge symmetry breaking, is very old. The explicit introduction of a gluon mass into the Lagrangian causes several difficulties. To preserve gauge invariance the gauge fields how to be coupled to massless scalar particles, which decouple from physical matrix elements [2, 3]. The mass cannot be a constant, but must vanish for large momentum in order to ensure renormalization of the theory. Both phenomenons occur in every model which dynamically creates a gluon mass. One of the main ingredients in QCD is heavy-quark potential which is Coulomb potential generated by gluon exchange and confinement potential. Although Coulomb potential is dominant at the distances of around charmonium and bottomonium sizes, confinement potential is dominant one at distances around 1 fm. So, any modification of Coulomb potential is an essential problem.

The objects of the research: Heavy quarks at non-zero temperature in quantum chromodynamics vacuum. Till now static potential between heavy quarks is thought to be Coulomb potential which do not include modifications of gluon fluctuations. In J.M. Cornwall's [4] work a gauge invariant self energy has been defined and computed. Solving the Schwinger Dyson equations leads to a mass of $500 \pm 200\ MeV$. Due to asymptotic freedom the mass vanishes logarithmically for large momenta $M_{gluon}(p) \sim (\ln p^2)^{-12/11}$. non-perturbative arguments lead to a decay $(\ln p^2)^{12/11}/p^2$ [5, 6].

The subjects of the research: Gluon fluctuations in the instanton vacuum model at non-zero temperature. Momentum and temperature dependence of the dynamical gluon mass.

The methods of the research:. As we said above, we have to average gluon propagator in order to find its modifications. For this we use some approximation methods for gluon propagator equation. In section III we average gluon propagator over orientation and get the expression including three terms. Then we average it over position. The first term can be calculated by hand but last two terms are a bit complicated one. To deal with this situation we have to use approximation which helps to estimate last terms contribution at large and small momentum. We also use approximation method to derive Pobylitsa's equation. When we illustrate scalar gluon propagator at non-zero temperature in graphs it takes more simple form. Then further calculations include some approximation methods(we neglect non-planar graphs to get analytic form of the equations).

The aim of the research: We consider the modifications of gluon properties in the instanton liquid model (ILM) for the QCD vacuum at non-zero temperature T.

The chapters of the dissertation:: Dissertation consists of three chapters. First chapter gives simple introduction to QCD instanton vacuum. Second one is about thermal QCD properties. The calculations simplifies significantly if we use gluon-like scalar propagator instead of real gluon propagator. So, the third chapter includes two interconnecting sections: firstly, we will work with gluon-like propagator to warm up and find dependence of gluon-like dynamical mass to the temperature. This includes extending Pobylitsa's equation for gluons and neglecting non-planar graphs. Then we will work with real gluon propagator and find dynamical gluon mass which leads to modifications of the Electrical part of quark-antiquark potential.

The novelty of the dissertation:

In my dissertation work we discussed modifications of gluon fluctuation and we found temperature dependence of dynamical mass in momentum space.

Introduction

We consider the modifications of gluon properties in the instanton liquid model (ILM) for the QCD vacuum at non-zero temperature T. Main parameters of ILM are average instanton size $\rho(\bar{T})$ and instanton density $n(T) = NT/V_3$. There are two possible scenarios: 1) At small $T \leq T_c$ (T_c is a critical temperature) $\rho(\bar{T})$ and $n(T)$ are almost constant and at $T \geq T_c$ they are falling functions as it was demonstrated by lattice measurements [7]; 2) Another one, when $\rho(\bar{T})$ and $n(T)$ are gradually decreasing functions of temperature T [8]. Re-scattering of gluons on instantons generates the dynamical 3-momentum q, temperature T-dependent "electric" gluon mass $M_{el}(q,T)$. Using the typical phenomenological values of the average instanton size $\rho(\bar{0}) = 1/3\ fm$ and average instanton density $n(0) = 1\,fm^{-4}$ we get $M_{el}(0,0) = 362\,MeV$. At first scenario at small $T \leq T_c \approx 150\,MeV$ we have rather weak T-dependence as $M_{el}(q,T) = M_{el}(0,0)(1 - 1/6\,\pi^2\bar{\rho}^2\bar{T^2})F(q,T=0)\ MeV$, where form-factor $F(0,0) = 1,\ F(q,T) \leq F(q,0) = q\bar{\rho}K_1(q\bar{\rho})$. At $T \geq T_c$ $M_{el}(0,T)$ is a fast falling function. In second scenario $M_{el}(0,T)$ is a falling function in whole region of T. One-loop thermal gluon corrections give a rise with temperature contribution $M_{pert,el}(0,T) \sim T$ and its account provides a possibility to reproduce lattice measurements of the dynamical gluon mass [1]. The gluodynamics at non-zero temperature $T(\equiv 1/\beta)$ is described by the partition function

$$Z = \int DA_\mu \exp\{-\frac{1}{2g^2}\int_0^\beta dx_4 \int d^3x \,\mathrm{tr}\, F_{\mu\nu}F_{\mu\nu}\},$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$ and the gauge field A_μ are obeying strict periodic condition $A_\mu(\vec{x}, x_4+\beta) = A_\mu(\vec{x}, x_4)$. The extension of the zero-temperature instanton solution [9] caloron was found in [10] and has the form

$$\Pi^{-1}\partial^2\Pi = 0, A^I_\mu = \Pi\bar{\eta}^a_{\mu\nu}(\tau_a/2i)\partial_\nu\Pi^{-1}, F_{\mu\nu} = \frac{1}{2}\Pi(\tau\partial)\bar{\eta}^a_{\mu\nu}(\tau_a/2i)(\tau^+\partial)\Pi^{-1},$$
$$\Pi(r,t) = 1 + \frac{\pi\rho^2}{\beta r}\sinh\frac{2\pi r}{\beta}/(\cosh\frac{2\pi r}{\beta} - \cos\frac{2\pi t}{\beta}) =$$

$$= 1 + \sum_{n=-\infty}^{\infty}\frac{\rho^2}{r^2 + (t - n\beta)^2}$$

where $r = |\vec{x}|$, $t = x_4$ and $\tau_\mu = (\vec{\tau}, i)$. For distances $r, t << \beta$

$$\Pi(x) \approx \Pi_0(x) = \left(1 + \frac{\lambda^2}{3}\right) + \rho^2/x^2,$$

where $\lambda = \pi\rho/\beta$. At these distances

$$A^{I,a}_\mu = \frac{2\rho^{'2}}{x^2}\frac{\bar{\eta}^a_{\mu\nu}x_\nu}{(x^2 + \rho^{'2})}, \quad \rho^{'2} = \rho^2/(1 + \frac{\lambda^2}{3}).$$

In fact, the accuracy of this representation is about 1% up-to $r, t \sim \beta$.

By the straightforward extension of the instanton liquid model (ILM) (see reviews [11, 12]) for the vacuum at non-zero temperature it was obtained the temperature dependencies of main parameters of the model – average instanton size $\bar{\rho}(T)$ and average instanton density $n(T) = NT/V_3 = 1/R^4(T)$, where N is the total number of instantons [8]. The general conclusion on $\bar{\rho}(T)$ and $n(T)$ in ILM – they are gradually decreasing functions of T.

But it is too simplified conclusion did not take into account critical temperature $T_c \sim \Lambda_{QCD}$. When at $T < T_c$ all color objects are bounded as a colorless hadrons and the heat bath consists of weakly interacting pions the T dependence of instanton density $n(T)$ should be rather weak, most likely inside the range $n(T) = n_0(1 + O(T^2/(6f_\pi^2)))$. The conclusion is the instanton density is expected to be essentially constant below the phase transition $T < T_c$, but exponentially suppressed at large temperature [12].

The extension of ILM is so-called dyon-instanton liquid model (DLM) [13]. DLM is able to describe confinement at $T \leq T_c$ and deconfinement at $T \geq T_c$. On the other hand these authors concluded later that at very low temperature the semi-classical description of the Yang-Mills state at very low temperature appears to reconcile the instanton liquid model without confinement, with the t'Hooft-Mandelstam proposal with confinement. In the former, the low temperature thermal state is composed of a liquid of instanton and anti-instantons, while in the latter it is a superfluid of monopoles and anti-monopoles [14].

In the series of papers on lattice QCD [1] it was found "electric" gluon mass $M_{el}(T)$ consistent with Debye screening at $T \geq T_c$, and has a minimum at $T \sim T_c$. It means that at $T \leq T_c$ $M_{el}(T)$ is decreasing function of temperature T.

On the other hand we may immediately guess that the dynamical gluon "electric" mass M_{el} which naturally has dependencies like

$$M_{el} \sim (packing\ \ parameter(T))^{1/2}\bar{\rho}^{-1}(T) = \bar{\rho}(T)n^{1/2}(T)$$

$(packing\ \ parameter = \bar{\rho}^4(T)n(T))$ will also be decreasing function of temperature at $T \leq T_c$. Also, one-loop thermal gluon contribution to the gluon propagator gives a rise with temperature contribution $M_{pert,el}(0,T) \sim T$ and its account provides a possibility to reproduce lattice measurements of the dynamical gluon mass [1].

The calculations of gluon propagator in ILM are related with two essential problems: the account of zero-modes (fluctuations along of instanton collective coordinates) and the averaging over the collective coordinates of all instantons.

The first problem we solve by using the approach from ref. [15], while for the treating of the second one we extend Pobylitsa Eq. [16] also applied for the gluons at $T = 0$ [17] and finally we find ILM averaged gluon propagator at $T \neq 0$ [18]. My own contribution in [18] this work is calculation of corrections of $\Delta_1(x, y)$ and $\Delta_2(x, y)$ to dynamical gluon mass. I made it clear that there is no (or negligible) contribution of these terms

Chapter 1

An introduction to QCD instanton vacuum

1.1 Instanton Solutions in Yang-Mills theory

In perturbation theory one deals with zero-point quantum-mechanical fluctuations of the YM fields which are near to one of the minima, say, at $N_{CS} = 0$ [19]. The non-linearity of the YM theory is taken into account as a perturbation, and it results in series in g^2 where g is the gauge coupling. In that approach one is apparently missing a possibility for the system to tunnel to another minimum, say, at $N_{CS} = 1$. The tunnelling is a typical non-perturbative effect in the coupling constant [20].

Instanton is a large fluctuation of the gluon field in imaginary (or Euclidean) time corresponding to quantum tunnelling from one minimum of the potential energy to the neighbor one. Mathematically, it was discovered by Belavin, Polyakov, Schwartz and Tyupkin (1975) [9]; the tunnelling interpretation was given by V. Gribov (1976). The name "instanton" has been introduced by 't Hooft (1976) [21] who studied many of the key properties of those fluctuations. Anti-instantons are similar fluctuations but they are tunnelling in the opposite direction. Physically, one can think of instantons in two ways: on the one hand it is a tunnelling process occurring in time, on the other hand it is a localized pseudoparticle in the Euclidean space. Following the WKB approximation, the tunnelling amplitude can be estimated as $exp(-S)$, where S is the action along the classical trajectory in imaginary time, leading from the minimum at $N_{CS} = 0$ at $t = -\infty$ to that at $N_{CS} = 1$ at $t = +\infty$.

The 4-dimensional topological charge of such trajectory is $Q_T = N_{CS}(+\infty) - N_{CS}(-\infty) = 1$. To find the best tunnelling trajectory having the largest amplitude one has thus to minimize the YM action provided with the topological charge is fixed to be unity. This can be done using the following trick.

$$0 \leq \int d^4x(F^a_{\mu\nu} - \tilde{F}^a_{\mu\nu})^2 \tag{1.1}$$

$$= \int d^4(2F^2 - 2F\tilde{F}) = 8g^2S - 62\pi^2Q_T, \tag{1.2}$$

hence, the action is restricted from:

$$S \geq \frac{8\pi^2}{g^2}Q_T = \frac{8\pi^2}{g^2} \tag{1.3}$$

Therefore, the minimal action for a trajectory with a unity topological charge is equal to $8\pi^2/g^2$, which is achieved if the trajectory satisfies the self-duality equation:

$$F^a_{\mu\nu} = \tilde{F}^a_{\mu\nu} \tag{1.4}$$

Notice that any solution of eq. (1.4) is simultaneously a solution of the general YM equation of motion $D^{ab}_\mu F^b_{\mu\nu} = 0$: that is because the "second pair" of the Maxwell equations, $D^{ab}_\mu \tilde{F}^b_{\mu\nu}$, is satisfied identically. Thus, the tunnelling amplitude can be estimated as

$$A \sim exp(-S) = exp(-\frac{1}{4g^2}\int d^4x F^2_{\mu\nu}) = exp(-\frac{8\pi^2}{g^2}) = exp(-\frac{2\pi}{\alpha_s}). \quad (1.5)$$

It is non-analytic in the gauge coupling constant and hence instantons are missed in all orders of the perturbation theory. However, it is not a reason to ignore tunnelling. For example, tunnelling of electrons from one atom to another in a metal is also a non perturbative effect, but we would nowhere get in understanding metals had we ignored it.

To solve equation (1.4) let us recall a few facts about the Lorentz group $SO(3,1)$ [22] . Since we are talking about the tunnelling fields which can only develop in imaginary time, it means that we have to consider the fields in Euclidean space-time, so that the Lorentz group is just $SO(4)$ isomorphic to $SU(2)$ x $SU(2)$. The gauge potentials A_μ belong to the $(\frac{1}{2}, \frac{1}{2})$ representation of the $SU(2)$ x $SU(2)$ group, while the field strength $F_{\mu\nu}$, belongs to the reducible $(1,0)+(0,1)$ representation. In other words it means that one linear combination of $F_{\mu\nu}$ transforms as a vector of the left $SU(2)$, and another combination transforms as a vector of the right $SU(2)$. These combinations are

$$F^A_L = \eta^A_{\mu\nu}(F_{\mu\nu} + \tilde{F}_{\mu\nu}), \qquad F^A_R = \bar{\eta}^A_{\mu\nu}(F_{\mu\nu} - \tilde{F}_{\mu\nu}) \quad (1.6)$$

Where η, $\tilde{\eta}$ are the so-called 't Hooft symbols. We see therefore that a self-dual field strength is a vector of the left $SU(2)$ while its right part is zero. Keeping that experience in mind we look for the solution of the self-dual equation in the form

$$A^a_\mu = \bar{\eta}^a_{\mu\nu} x_\nu \frac{1+\Phi(x^2)}{x^2} \quad (1.7)$$

$$\eta^a_{ij} = \epsilon_{aij}, \qquad \eta^a_{4j} = \eta^a_{j4} = -\delta_{aj},$$

$$\bar{\eta}^a_{ij} = \epsilon_{aij}, \qquad \bar{\eta}^a_{4j} = \bar{\eta}^a_{j4} = +\delta_{aj}$$

Using the formulae for the 't Hoofts η symbols one can easily check that the YM action can be rewritten as

$$S = \frac{8\pi^2}{g^2}\frac{3}{2}\int d\tau[\frac{1}{2}(\frac{d\Phi}{d\tau})^2 + \frac{1}{8}(\Phi^2-1)^2], \qquad \tau = \ln(\frac{x^2}{\rho^2}) \tag{1.8}$$

This can be recognized as the action of the double-well potential whose minima lie at $\Phi = \pm 1$, and τ plays the role of "time"; ρ is an arbitrary scale. The trajectory which tunnels from 0 at $\tau = -\infty$ to 1 at $\tau = +\infty$ is

$$\Phi = -\mathsf{tanh}(\frac{\tau}{2}) \tag{1.9}$$

And its action (1.8) is $S = 8\pi^2/g^2$, as needed. Substituting the solution (1.9) into (1.7) we get

$$A^a_\mu(x) = \bar{\eta}^a_{\mu\nu}x_\nu \frac{1+\mathsf{tanh}(\frac{1}{2}\ln\frac{x^2}{\rho^2})}{x^2} = \frac{2\bar{\eta}^\mu_{\nu a}\rho^2}{x^2(x^2+\rho^2)} \tag{1.10}$$

The correspondent field strength is

$$F^a_{\mu\nu} = -\frac{4\rho^2}{(x^2+\rho^2)^2}(\bar{\eta}^\mu_{\nu a} - 2\bar{\eta}^\mu_{\alpha a}\frac{x_\alpha x_\nu}{x^2} - 2\bar{\eta}^\beta_{\nu a}\frac{x_\mu x_\beta}{x^2}), F^a_{\mu\nu}F^a_{\mu\nu} = \frac{192\rho^4}{(x^2+\rho^2)^4} \tag{1.11}$$

and satisfies the self-duality condition (1.4).

The anti-instanton corresponding to tunnelling in the opposite direction, from $N_{CS} = 1$ to $N_{CS} = 0$, satisfies the anfi-self-dual equation, with $\tilde{F} \to -\tilde{F}$; its concrete form is given by eqs. (1.10)- (1.11) with the replacement $\bar{\eta} \to \eta$.

1.2 Gluon condensate

The QCD perturbation theory implies that the fields $A_i^a(x)$ are performing quantum zero-point oscillations; in the lowest order these are just plane waves with arbitrary frequencies. The aggregate energy of these zero-point oscillations, $(\mathbf{B}^2 + \mathbf{E}^2)/2$, is divergent as the fourth power of the cutoff frequency, however for any state one has $< F_{\mu\nu}^2 >= 2(\mathbf{B}^2 - \mathbf{E}^2) = 0$, which is just a manifestation of the virial theorem for harmonic oscillators: the average potential energy is equal to the kinetic one (we are temporarily in the Minkowski space) [23]. One can prove that this is also true in any order of the perturbation theory in the coupling constant, providing that one does not violate the Lorentz symmetry and the renormalization properties of the theory. Meanwhile, we know from the QCD sum rules phenomenology that the QCD vacuum possesses what is called gluon condensate:

$$\frac{1}{32\pi^2} < F_{\mu\nu}^a F_{\mu\nu}^a >= \frac{1}{16\pi^2} < \mathbf{B}^2 - \mathbf{E}^2 > \approx (200\ MeV)^4 > 0 \qquad (1.12)$$

Instantons suggest an immediate explanation of this basic property of QCD. Indeed, instanton is a tunnelling process, it occurs in imaginary time; therefore in Minkowski space one has $E_i^a = \pm i B_i^a$(this is actually the duality equation (1.4)). Therefore, during the tunnelling $\mathbf{B} - \mathbf{E}$ is positive, and one gets a chance to explain the gluon condensate. In Euclidean space the electric field is real as well as the magnetic one, and the gluon condensate is just the average action density. Let us make a quick estimate of its value. Let the total number of instantons and anti-instantons in the 4-dimensional volume V be N. Assuming that the average separations of instantons are larger than their average sizes, we can estimate the total action of the ensemble as the sum of individual actions:

$$< F_{\mu\nu}^2 > V = \int d^4x F_{\mu\nu}^2 \approx N \cdot 32\pi^2 \tag{1.13}$$

hence the gluon condensate is directly related to the instanton density in the 4-dimensional Euclidean space-time:

$$\frac{1}{32\pi^2} < F_{\mu\nu}^a F_{\mu\nu}^a > \approx \frac{N}{V} = \frac{1}{R^4} \tag{1.14}$$

In order to get the phenomenological value of the condensate one thus needs to have the average separation between pseudoparticles

$$R \approx \frac{1}{200\ MeV} = 1\ fm \tag{1.15}$$

There is another point of view on the gluon condensate. In principle, all information about field theory is contained in the partition function being the functional integral over the fields. In the Euclidean formulation it is

$$Z = \int DA_\mu exp(-\frac{1}{4g^2} \int d^4x F_{\mu\nu}^2), \qquad \lim_{T\to\infty} = exp(\varepsilon T) \tag{1.16}$$

where we have used that at large (Euclidean) time T the partition function picks up the ground state or vacuum energy ε. If the state is homogeneous, the energy can be written as $\varepsilon = \theta_{44} V^{(3)}$ where θ is the stress-energy tensor and V_3 is the 3-volume of the system. Hence, at large 4-volumes $V = V^{(3)} T$ the partition function is $Z = exp(-\theta_{44} V)$. This θ_{44} includes zero-point oscillations and diverges badly. The more reasonable quantity is the partition function, normalized to the partition function which is understood as a perturbative expansion about the zero-field vacuum,

$$\frac{Z}{Z_{P.T.}} = exp[-(\theta_{44} - \theta_{44}^{P.T.})V]. \tag{1.17}$$

We expect that the non-perturbative vacuum energy density $\theta_{44} - \theta_{44}^{P.T.}$ is a negative quantity since we have allowed for tunnelling: as usual in quantum mechanics, it lowers the ground state energy. If the vacuum is isotropic, one has $\theta_{44} = \theta_{\mu\mu}/4$. Using the trace anomaly,

$$\theta_{\mu\mu} = \frac{\beta(\alpha_s)}{16\pi^2\alpha_s^2}(F^a_{\mu\nu}) \approx -b\frac{F^{a\ 2}_{\mu\nu}}{32\pi^2} \tag{1.18}$$

where β_{α_s} is the Gell-Mann-Low function,

$$\beta_{\alpha_s} = \frac{d\alpha_s(\mu)}{d\ln\mu} = -b_1\frac{\alpha_s^2(\mu)}{2\pi} - \frac{b}{2}\frac{\alpha_s^3(\mu)}{(2\pi)^2} - ... \tag{1.19}$$

with $b_{1,2}$ given below, one gets:

$$\frac{Z}{Z_{P.T.}} = exp(\frac{b_1}{4}V < F^2_{\mu\nu}/32\pi^2 >_{NP}) \tag{1.20}$$

where $< F^2_{\mu\nu} >_{NP}$ is the gluon field vacuum expectation value which is due to non-perturbative fluctuations, *i.e.* the gluon condensate. The aim of any QCD-vacuum builder is to minimize the vacuum energy or, equivalently, to maximize the gluon condensate. It is important that it is a renormalization-invariant quantity, meaning that its dependence on the ultraviolet cutoff μ and the bare charge $\alpha_s(\mu)$ given at this cutoff is such that it is actually cutoff-independent. Such combination is called Λ. The gluon condensate has to be proportional to Λ^4 by dimensions.

The fact that the vacuum energy or, equivalently, the gluon condensate is a renormalization-invariant quantity leads to an infinite number of low-energy theorems. Translated into the instanton-vacuum language, the renormalizability of the QCD implies that the probability that there are N instantons and anti-instantons in the vacuum is

$$P(N) \sim exp[-\frac{b}{4}(\ln\frac{N}{< N >} - 1)], \tag{1.21}$$

where $< N > \approx V < F^a_{\mu\nu} F^a_{\mu\nu} >$ is the average number of instantons and anti-instantons.

1.3 The Instanton Liquid Model

The sum of well-separated instantons is also an approximate solution of Yang-Mills equations: [23]

$$A = \sum_{I=1}^{N} A_I, \;\; S[A] \approx N S_0 \tag{1.22}$$

The partition function of this instanton gas

$$Z = \sum_{N=0}^{\infty} Z_N, \;\; Z_N \approx \frac{1}{N!} (V_4 \bar{D})^N \tag{1.23}$$

The sum is dominated by instanton configuration with density $N/V = \bar{D}$. Unfortunately $\bar{D}$ is infinite and the assumption of a diluted gas turns out to be wrong. The probability of small size instantons is low because $D(\rho)$ vanishes rapidly for small distances [24]. On the other hand for large distances $D(\rho)$ blows up and soon gets large. This is the origin of the infrared problem which made a lot of people no longer believing in instanton physics. Those who were not deterred by that have thought of the following outcome [25]. For large distances, the vacuum gets more and more filled with instantons of increasing size. At some scale the instanton gas approximation breaks down and one has to consider the interaction between instantons which might be repulsive to stabilize the medium. The stabilization might occur at distances which a semiclassical treatment is still possible and at densities which the various instantons are still well separated objects say not much deformed through their interaction. So there is a narrow region of allowed values for the instanton radius. This picture of the vacuum is called the instanton liquid model. The simplest suggestion is to introduce a cutoff ρ c and to ignore large instantons

$$\bar{D}_{\rho_c} = \int_0^{\rho_c} d\rho D(\rho) \tag{1.24}$$

The cutoff has to be chosen sufficiently small such that the space-time fraction f filled with instantons is smaller than 1 in order to justify the model of a diluted gas

$$f = \frac{2}{N_c} \int_0^{\rho_c} d\rho \frac{1}{2} \pi^2 \rho^4 D(\rho) \qquad < \qquad 1 \tag{1.25}$$

This simple cutoff procedure can be improved by introducing a scale invariant (hardcore) repulsion between instantons, which effectively suppresses large instantons. This procedure has the advantage of respecting the scaling Ward identities which are otherwise violated by the simple cutoff ansatz. Unfortunately this repulsion is an artifact of the sum-ansatz as it has been shown by. Therefore, the infrared problem is still unsolved. Nevertheless, it is possible to make successful prediction by simply assuming a certain instanton density and some average radius. It seems that the vacuum can be described by effectively independent instantons of size $\rho = 600MeV^{-1}1$ and mean distance $L_0 = 200MeV$. The integral instanton density is fixed by the experimentally known gluon condensate

$$n = N/V_4 = 1/L_0^4 = \frac{1}{32\pi^4} < G^a_{\mu\nu} G^{\mu\nu}_a >= (200MeV)^4_{exp} \tag{1.26}$$

The Instanton Liquid Model is therefore defined by the following assumption on $D(\rho_I)$:

$$D(\rho_I) = n\delta(\rho_I - \rho)\,, \; n = (200MeV)^4\,, \; \rho = 600MeV^{-1} \tag{1.27}$$

The model describes very successfully the physics of light hadrons [26]. In high energy processes with momentum transfer p of $1-10GeV$, $D(\rho)$ is often multiplied with function, which is sharply peaked around $\rho \sim \rho^{-1}$. The integral over ρ is then dominated by small instantons and it is infrared convergent. The results are, hence, independent of the infrared cutoff

Chapter 2

QCD vacuum at finite temperature

2.1 The finite temperature QCD

The finite-temperature behaviour of any theory is specified by the partition function

$Z = \mathrm{Tr}\exp(-\beta H)$

and the thermal expectations of physical observables,

$< O >= \mathrm{Tr}(\exp(-\beta H)O)/Z$

Here $\beta = 1/T$, T is temperature ($k_B = 1$).

This exhibits Z as a functional integral over fields which are periodic up to a "twist".

$$Z = \int_{A_\mu(\beta,\vec{x})=A_\mu(0,\vec{x})} DA_\mu(t,\vec{x}) \exp(-\frac{1}{4g^2}\int_0^\beta dt \int d^3x F_{\mu\nu}^2) \qquad (2.1)$$

One is not interested in the minimal energy of the quark pair at finite temperature, but rather in the aver- age over the thermal ensemble of the energy of the quark and antiquark. In other words, one wishes to compute the trace over all states containing an external source and sink of color electric flux, separated by the distance R. This is given by the expectation of "Wilson strings, "

$$\Omega(\vec{x}) = P\exp(\int_0^\beta dt A_0(t,\vec{x})) \tag{2.2}$$

owing to the periodic boundary conditions, $\Omega(\vec{x})$ may be considered as a closed, timelike Wilson loop. One easily finds that

$$(\mathrm{tr}[\Omega(\vec{x})]\mathrm{tr}[\Omega^+(0)]) = \exp[-\beta V(\vec{x},\beta)] \tag{2.3}$$

where V(R, p) is the finite-temperature static quark potential.

2.2 Variational estimation

Feynman variational principle was successfully applied to the QCD vacuum and lead to ILM [27]. Further it was generalized to non-zero temperatures [8]. Main ingredient of this approach is one-instanton size distribution function $\mu(\rho,T,n)$, where $n(T) = NT/V_3$ – is density of instantons. The single instanton one-loop distribution function is $d(\rho,T) = C\rho^{b-5}\exp(-A_{N_c}T^2)$, where the exponential suppression factor is the one-loop contribution of thermal gluon fluctuations [28]. Accordingly, [8, 12, 29] $\mu(\rho,T,n)$ in pure gluodynamics with account of $d(\rho)$ is

$$\mu(\rho,T,n) = d(\rho)\exp[-\bar{\beta}\gamma^2 n\bar{\rho}^2)\rho^2] = C\rho^{b-5}\exp[-\Phi(n,T)\rho^2], \tag{2.4}$$

$$\Phi(n,T) = \frac{1}{2}A_{N_c}T^2 + [\frac{1}{4}A_{N_c}^2T^4 + \nu\bar{\beta}\gamma^2 n]^{1/2}. \tag{2.5}$$

Here $A_{N_c} = 1/3\,[11/6\,N_c - 1]\pi^2$, $b = 11/3\,N_c$, $\nu = \frac{b-4}{2}$, $\bar{\beta} = -b\log(\Lambda\bar{\rho})$, $\gamma^2 = \frac{27\pi^2 N_c}{4(N_c^2-1)}$. Also,

$$\bar{\rho}^2(T,n) = \frac{1}{\mu_0(T,n)}\int_0^\infty d\rho\mu(\rho,T,n)\rho^2, \;\; \mu_0(T,n) = \int_0^\infty d\rho\mu(\rho,T,n). \tag{2.6}$$

The content of variational ILM partition function [8] is given by

$$Z \geq Z_1 \exp(- < E - E_1 >), \tag{2.7}$$

$$Z_1 = \frac{1}{(N/2\,!)^2}(\frac{2\mu_0(T,n)V}{N})^N, \;\; < E - E_1 >= -\frac{N}{2}\bar{\beta}\gamma^2 n\bar{\rho^2}^2 \tag{2.8}$$

Right side here is a trial partition function, which must be maximized by variations of $\mu(\rho,T,n)$ and the density n. In fact, the $\mu(\rho,T,n)$ in the Eq. (2.4) is a result of the variation of Eq. (2.8) over μ. The next step is a minimization of free energy $F = -T/V_3 \log Z$ by variation over n.

$$\Phi = \frac{1}{2}A_{N_c}T^2 + (A_{N_c}^2 T^4 + \bar{\beta}\gamma^2 n\nu)^{1/2} \tag{2.9}$$

$$\mu_0(T,n)\int_0^\infty d\rho\rho^{b-5}\exp[\Phi(T,n)\rho^2] = C\Phi^{-\nu}, \nu = \frac{b-4}{2} \tag{2.10}$$

$$\bar{\rho^2} = \frac{1}{\mu_0}\int_0^\infty d\rho\rho^{b-5}\exp[\Phi(T,n)\rho^2]\rho^2 = \nu\Phi^{-1} \tag{2.11}$$

$$Z \geq (\frac{\mu_0 V}{N})^N \exp(N)\exp(\frac{N}{2}\bar{\beta}\gamma^2 n\nu^2\Phi^{-2}) \tag{2.12}$$

$$-F = \frac{1}{V}\log Z \geq n(\log\frac{2C\Phi^{-\nu}}{n} + 1 + \bar{\beta}\gamma^2 n\nu^2\Phi^{-2}) \tag{2.13}$$

$$\frac{dF}{dn} = \log\frac{2C\Phi^{-\nu}}{n} + \bar{\beta}\gamma^2 n\nu^2\Phi^{-2} - \nu n\frac{1}{\Phi}\frac{d\Phi}{dn} - \bar{\beta}\gamma^2 n^2\nu^2\Phi^{-3}\frac{d\Phi}{dn} \tag{2.14}$$

$$\bar{\beta}\gamma^2 n\nu^2\frac{1}{\Phi^{-2}} = \frac{\bar{\beta}\gamma^2 n\nu^2}{[\frac{1}{2}A_{N_c}T^2 + (A_{N_c}^2T^4 + \bar{\beta}\gamma^2 n\nu^2)^{1/2}]^2} \tag{2.15}$$

$$\nu n\frac{1}{\Phi}\frac{d\Phi}{dn} = \nu/2\frac{\bar{\beta}\gamma^2 n\nu}{[\frac{1}{2}A_{N_c}T^2 + (A_{N_c}^2T^4 + \bar{\beta}\gamma^2 n\nu^2)^{1/2}](A_{N_c}^2T^4 + \bar{\beta}\gamma^2 n\nu)}$$

$$\frac{dF}{dn} = \log\frac{2C\Phi^{-\nu}}{n} + \frac{\bar{\beta}\gamma^2 n\nu^2}{\Phi^2}[1 - \frac{\Phi^2 + \bar{\beta}\gamma^2 n\nu}{2\Phi(\Phi - \frac{1}{2}A_{N_c}T^2)}] = \log\frac{2C\Phi^{-\nu}}{n} \tag{2.16}$$

$$\frac{dF}{dn} = \log\frac{2C\Phi^{-\nu}}{n} = 0 \ \ or \ \ n = 2C\Phi^{-\nu} \tag{2.17}$$

This step leads to the Eq. for the density

$$n(T) = 2\mu_0(n, T) \tag{2.18}$$

The variational estimates demonstrated that $\bar{\rho}(T)$ and $n(T)$ are decreasing functions of temperature T(see fig.2.1). On the other hand, lattice data show that instanton density n is not modified by temperature up-to critical temperature T_c [7]. In numerical simulations of ILM [12] it was chosen an interpolation between no suppression ($A_{N_c} = 0$ in Eq.(2.5)) below T_c and full suppression ($A_{N_c} \neq 0$ in Eq.(2.5)) above $T_c \sim 150MeV$, with a width $\Delta T = 0.3T_c$ to be in the correspondence with lattice result [7]. We are following this suggestion and are repeating the calculations with the modification in the Eq.(2.5) as $A_{N_c} \to A_{N_c}\Theta_{\Delta x}(x - x_c)$, where $x_c = 0.25 \sim T_c = 150\,MeV$, $\Delta x = 0.075 \sim \Delta T = 0.3T_c$ and step-like function

$$\Theta_{\Delta x}(x - x_c) = 1/2[1 + \tanh((x - x_c)/\Delta x)]. \tag{2.19}$$

Then, we have $\bar{\rho}^2(x)/\bar{\rho}^2(0)$, $n(x)/n(0)$ given by Fig. 2.1.

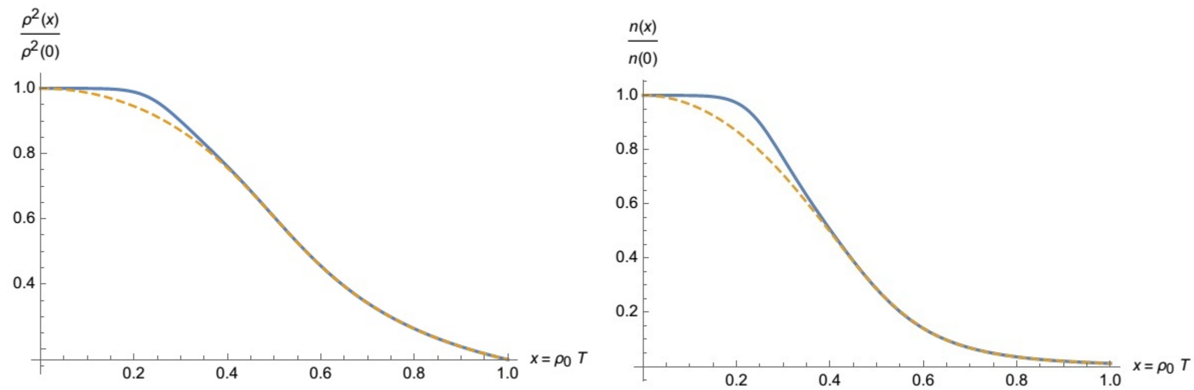


Figure 2.1: The figure on the left represents ratio of instanton sizes $\bar{\rho}^2(x)/\bar{\rho}^2(0)$ while right one ratio of instanton densities $n(x)/n(0)$ as functions of $x = \bar{\rho}_0 T$ corresponding to the variational estimates from Refs. [8, 12, 29] at the phenomenological values of $\bar{\rho}(0) = 1/3\, fm$ and $n(0) = 1\, fm^{-4}$. A Full line correspond the modification $A_{N_c} \to A_{N_c}\Theta_{\Delta x}(x - x_c)$ (see Eq.(2.5) and $\Theta_{\Delta x}(x - x_c)$ is the step-like function Eq.(2.19)) to interpolate between no suppression below T_c and full suppression above $T_c = 150 MeV$, with a width $\Delta T = 0.3 T_c$ [12]. Dashed lines correspond to the full suppression at the whole region of T.

2.3 HS-caloron

Now we can consider solution with zero magnetic charge. Such solutions can be described by periodic instantons. We show how such solutions can be constructed from the multiple-instanton solutions. The general $SU(N)$ multi-instanton solution with Pontryagin index K contains $4NK$parameters [30, 31]. This solution describes K instantons with independent positions, sizes, and group orientations. Periodic instantons(calorons) may be constructed from multi- instanton solutions which describes an infinite string of instantons located at $x = 0$ and $x_0 = n\beta$, $n = Z$. Periodic multiple-instanton solutions may be had similarity when they constructed from zero-temperature solutions with several strings of instantons. These solutions can describe any number of periodic instantons, each with an independent size, position and orientation.

If we examine the behaviour of the fields of the periodic instantons. Unlikely, explicit parametrization of the zero-temperature multi-instanton solutions are available only when all instantons have identical gauge orientations. In this case, we may use convenient 't Hooft solution [32] to describe aligned instantons

$$\Pi^{-1}\partial^2\Pi = 0$$

$$A_\mu = \Pi\bar{\eta}^a_{\mu\nu}(\tau^a/2i)\partial_\nu\Pi^{-1},$$

$$F_{\mu\nu} = \frac{1}{2}\Pi\tau\partial\bar{\eta}^a_{\mu\nu}(\tau^a/2i)\tau^\dagger\partial\Pi^{-1} \tag{2.20}$$

Explicitly,

$$\Pi(x) = 1 + \sum_{n=1}^{K}\rho_n^2/(x-z_n)^2$$

describes K instantons with positions z_n and sizes ρ_n. Taking $\rho_n = \rho$ and $z_n = n\beta\hat{t_0}$ one finds the periodic single instanton

$$\Pi(t,x) = 1 + \frac{\pi\rho^2}{\beta r}\frac{\sinh\frac{2\pi}{\beta}r}{\cosh\frac{2\pi}{\beta}r - \cos\frac{2\pi}{\beta}t} \tag{2.21}$$

This time periodic instanton solution, also calls HS caloron, was discovered by Harrington and Shepard [10]. The HS caloron can be written in the following form

$$\psi(x) = \cosh(x) - \cos(2\pi t) + \frac{\pi\rho^2}{r}\sinh(2\pi r)$$

$$\hat{\psi}(x) = \cosh(2\pi r) - cos(2\pi t)$$

$$\chi(x) = 1 - \frac{1}{\phi} = \frac{\pi\rho^2\sinh(2\pi r)}{\psi}$$

$$\phi(x) = \frac{\psi(x)}{\hat{\psi}(x)} = 1 + \frac{\pi\rho^2}{r}\frac{\sinh(2\pi r)}{\cosh(2\pi r) - \cos(2\pi t)} \tag{2.22}$$

$$A_\mu = \frac{\tau_3}{2}\bar{\eta}^3_{\mu\nu}\partial_\nu \ln\phi(x) + \frac{\phi(x)}{2}Re((\bar{\eta}^1_{\mu\nu} - \bar{\eta}^2_{\mu\nu})(\tau_1 + i\tau_2)\partial_\nu\chi(x)) \tag{2.23}$$

For distances $|x| << \beta$

$$\Pi(x) = 1 + \frac{\pi\rho^2}{\beta r}\left(\frac{2\pi r}{\beta} + 1/6(\frac{2\pi r}{\beta})^3\right)\left[1/2(\frac{2\pi r}{\beta})^2 + \right. \tag{2.24}$$

$$\left. + 1/2(\frac{2\pi t}{\beta})^2 + 1/24(\frac{2\pi r}{\beta})^4 - 1/24(\frac{2\pi t}{\beta})^4\right]^{-1} = 1 + \frac{\rho^2}{r^2+t^2}\left(1 + 1/6(\frac{2\pi r}{\beta})^2\right)$$

$$\left(1 - \frac{\pi^2(r^2-t^2)}{3\beta^2}\right) = \left(1 + \frac{\lambda^2}{3}\right) + \rho^2/x^2 + \lambda^2 O(x^2/\beta^2), \tag{2.25}$$

where $\lambda = \pi\rho/\beta$.

Another way of the estimate for $r \sim t << \beta$:

$$\Pi(x) = 1 + \sum_{n=-\infty}^{\infty}\frac{\rho^2}{r^2 + (t - n\beta)^2} = 1 + \frac{\rho^2}{r^2+t^2} + 2\sum_{n=1}^{\infty}\frac{\rho^2}{\beta^2 n^2} =$$

$$= 1 + \frac{\rho^2}{r^2+t^2} + \frac{\rho^2\pi^2}{3\beta^2} \tag{2.26}$$

If we let $\rho^{'2} = \rho^2/(1 + \frac{\lambda^2}{3})$, then

$$A^a_\mu = \frac{2\rho^{'2}}{x^2}\frac{\bar{\eta}^a_{\mu\nu}x_\nu}{(x^2 + \rho^{'2})} + (1 + O(x^2/\beta^2)^2), F^a_{\mu\nu} =$$

$$-4\rho^{'2}\frac{\bar{\eta}^a_{\alpha\beta}}{(x^2+\rho^{'2})^2}I_{\alpha\mu}I_{\beta\nu} + O(x^2/\beta^4), \tag{2.27}$$

$I_{\alpha\mu} = \delta_{\alpha\mu} - 2x_\alpha x_\mu/x^2$.

Thus, viewed on scales much less than β, the finite-temperature instanton is identical to a zero-temperature instanton with a renormalized size

$\rho'^2 = \rho^2/(1+\frac{\lambda^2}{3})$. Note that this exhibits the periodic instanton in a "singular" gauge where A_μ has a pure gauge singularity at $\vec{x} = t = 0$. This gauge singularity may be removed by a periodic gauge transformation. (For example, transforming to axial gauge results in an every- where regular, periodic solution.) Note that (in any periodic gauge) $Q(x) = P\exp(\int_0^\beta A_0)$ equals 1 at $\vec{x} = 0$ and approaches +1 as $r \to \infty$.

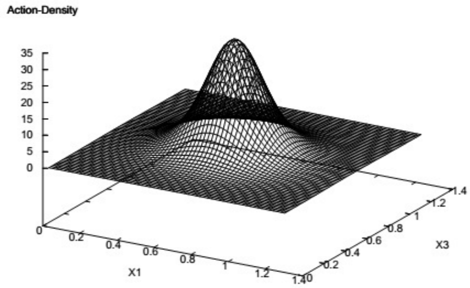
2.4 KvB caloron

A generalization of the time periodic caloron was discovered in 1998, almost simultaneously, by Kraan and van Baal and by Lee and Lu [33, 34]. The new solution, also called KvB caloron, provides an additional degree of freedom, namely the asymptotic holonomy $P^\infty = e^{2\pi i\bar{\omega}\tau}$which can be set to an arbitrary element of $SU(2)$ through the parameter $\bar{\omega}$. The construction of the KvB caloron is very similar to the construction of the HS caloron except for the fact that every instanton in the infinite chain is gauge rotated by the holonomy P^∞ relative to the preceding one. Again, the caloron action lump is $O(4)$-symmetric in space-time only as long as $\rho << \beta$. The corresponding vector potential is analytically given by (2.23) with $\vec{r} = \vec{x} + \vec{z_1}$, $\vec{s} = \vec{x} - \vec{z_2}$, $r = |\vec{r}|$, $s = |\vec{s}|$ and newly defined function $\phi(x)$, $\chi(x)$ according to

$$\psi(x) = \cosh(4\pi\bar{\omega})\cosh(4\pi s\omega) + \frac{r^2+s^2+\pi^2\rho^4}{2rs}\sinh(4\pi\bar{\omega})\sinh(4\pi s\omega) - \cos(2\pi t)$$
$$+\pi\rho^2[s^{-1}\sinh(4\pi\omega)\cosh(4\pi s\bar{\omega}) + r^{-1}\sinh(4\pi\bar{\omega})\cosh(4\pi s\omega)]$$

$$\hat{\psi}(x) = \cosh(4\pi\bar{\omega})\cosh(4\pi s\omega)\frac{r^2+s^2-\pi^2\rho^4}{2rs}\sinh(4\pi\bar{\omega})\sinh(4\pi s\omega) - \cos(2\pi t)$$

$$\chi(x) = e^{4\pi it\omega}\frac{\pi\rho^2}{\psi}[e^{-2\pi it}s^{-1}\sinh(4\pi s\omega) + r^{-1}\sinh(4\pi r\bar{\omega})]$$



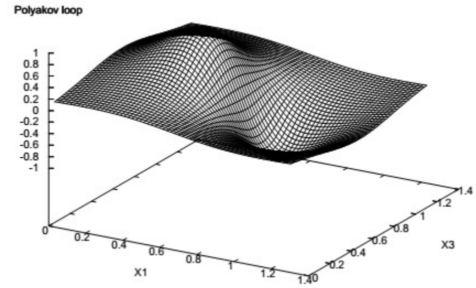


Figure 2.2: Action density (left) and Polyakov lop distribution(right)of a KvB caloron with maximally non-trivial holonomy $\omega = 0.25$, $\rho = 0.33\ fm$ and $\beta = 1fm$, where the action density is given in units of S_I. For $\rho << \beta$ the action lump is $O(4)$-symmetric and rho describes its radius. [35]

$$\phi(x) = \frac{\psi(x)}{\hat{\psi}(x)} \tag{2.28}$$

where the asymptotic holonomy was set to $\vec{\omega}\vec{\tau} = \omega\tau_3$ for convenience. So far, the given gauge field is in the so called algebraic gauge, where $A_{mu}(x\vec{x}, t+\beta) = P_\infty A_\mu(\vec{x}, t) P_\infty^\dagger$ is not periodic in time. Periodicity is achieved by the gauge transformation

$$A_\mu^{per} = \Omega A_\mu \Omega^\dagger - i\Omega\partial_\nu\Omega^\dagger, \qquad \Omega(x) = e^{-2\pi i\tau_3\omega t} \tag{2.29}$$

bringing the given potential into the so called periodic gauge.

Chapter 3

Dynamical gluon mass at non-zero temperature

3.1 Scalar gluon dynamical mass at non-zero T

In this section we will calculate the scalar gluon mass at finite temperature. Relation between the gluon mass $M(p)$ and gluon propagator is

$$\bar{S}^{-1}_{\mu\nu} = (p^2 + M^2(p))\delta_{\mu\nu} \tag{3.1}$$

Our purpose is finding $M(p, T)$ so, we need to calculate $\bar{S}_{\mu\nu}$. Gluon field is a bit complicated, therefore we will use the propagator of gluon-like field (pseudoparticle field) Δ^{ab} to simplify our calculations. Then we will extend our calculations to non-zero case, and we go further to the gluon field, after learning some techniques when we work with gluon-like field.

This gluon-like propagator Δ in the background field A is given by

$$\Delta = (p + A)^{-2} = (p^2 + \sum_i (\{p, A_i\} + A_i^2) + \sum_{i \neq j} A_i A_j)^{-1},$$

$$\tilde{\Delta} = (p^2 + \sum_i (\{p, A_i\} + A_i^2))^{-1}, \qquad \Delta_i = (p^2 + \{p, A_i\} + A_i^2)^{-1},$$

$$\Delta_0 = p^{-2} \tag{3.2}$$

Now we have to find $< \Delta >$.From (3.1) we may write

$$\Delta^{-1} - \Delta_0^{-1} = \sum_{i \neq j} A_i A_j, \qquad \tilde{\Delta} - \Delta = \tilde{\Delta} \sum A_i A_j \Delta$$

$$\Delta = \tilde{\Delta} - \tilde{\Delta} \sum A_i A_j \Delta = \tilde{\Delta} - \tilde{\Delta} \sum A_i A_j \tilde{\Delta} + \tilde{\Delta} \sum_{i \neq j} A_i A_j \tilde{\Delta} \sum_{k \neq l} A_k A_l \tilde{\Delta} - ...$$

$$< \Delta >=< \tilde{\Delta} > - < \tilde{\Delta} \sum A_i A_j \tilde{\Delta} > + < \tilde{\Delta} \sum_{i \neq j} A_i A_j \tilde{\Delta} \sum_{k \neq l} A_k A_l \tilde{\Delta} > - ... \tag{3.3}$$

In order to get $< \Delta >$ we must find $< \tilde{\Delta} >$, $< \tilde{\Delta} \sum A_i A_j \tilde{\Delta} >$, ...

a. Firstly we find $< \tilde{\Delta} >$.

$$\tilde{\Delta} = (p^2 + \sum_i (\{p, A_i\} + A_i^2))^{-1} = (p^2 + \sum_i B_i)^{-1}, \qquad B_i = \{p, A_i\} + A_i^2$$

$$\tilde{\Delta} = \frac{1}{\Delta_0^{-1} + \sum B_i} = \Delta_0 \frac{1}{1 + \sum B_i \Delta_0} = \{\tilde{\Delta} \rightarrow -\tilde{\Delta}, \Delta_0 \rightarrow -\Delta_0\} =$$

$$= \Delta_0 + \Delta_0 \sum_i B_i \Delta_0 + \Delta_0 \sum B_i \Delta_0 \sum B_j \Delta_0 + ...$$

$$\tilde{\Delta} - \Delta_0 = \Delta_0 \sum_i B_i \Delta_0 + \Delta_0 \sum_i B_i \Delta_0 \sum_j B_j \Delta_0 + ... =$$

$$= 1 \; term + 2 \; term + ... \tag{3.4}$$

After summation the first term gives us $\sum(\Delta_i - \Delta_0)$, e.g.:

$$\sum_i (\Delta_i - \Delta_0) = \sum_i \Delta_0 B_i \Delta_0 + \sum_i \Delta_0 B_i \Delta_0 B_i \Delta_0 + ... \tag{3.5}$$

Second term is

$$\sum_{i \neq j} (\Delta_i - \Delta_0)\Delta_0^{-1}(\Delta_j - \Delta_0) = \sum_{i \neq j} \Delta_0 B_i \Delta_0 B_j \Delta_0 + \sum_{i \neq j} (\Delta_0 B_i \Delta_0 B_i \Delta_0 B_j \Delta_0$$

$$+ \Delta_0 B_i \Delta_0 B_j \Delta_0 B_j \Delta_0) + ... \tag{3.6}$$

The inner lines can be written next

$$a. \quad \sum_{i \neq j} (\Delta_i - \Delta_0)\Delta_0^{-1}(\Delta_j - \Delta_0)\Delta_0^{-1}(\Delta_i - \Delta_0) = \sum_{i \neq j} \Delta_0 B_i \Delta_0 B_j \Delta_0 B_i \Delta_0 + ...$$

$$b. \quad \sum_{i \neq j \neq k} (\Delta_i - \Delta_0)\Delta_0^{-1}(\Delta_j - \Delta_0)\Delta_0^{-1}(\Delta_k - \Delta_0) = \sum_{i \neq j \neq k} \Delta_0 B_i \Delta_0 B_j \Delta_0 B_k \Delta_0 + ...$$

We may write them together

$$\sum_{i \neq j,\ j \neq k} (\Delta_i - \Delta_0)\Delta_0^{-1}(\Delta_j - \Delta_0)\Delta_0^{-1}(\Delta_k - \Delta_0) = \sum_{i \neq j \neq k} \Delta_0 B_i \Delta_0 B_j \Delta_0 B_k \Delta_0 +$$

$$+ \sum_{i \neq j} \Delta_0 B_i \Delta_0 B_j \Delta_0 B_i \Delta_0 + ... \tag{3.7}$$

Using them we may write the equation (3.3) in the different form

$$\tilde{\Delta} - \Delta_0 = \sum_i (\Delta_i - \Delta_0) + \sum_{i \neq j} (\Delta_i - \Delta_0)\Delta_0^{-1}(\Delta_j - \Delta_0) +$$

$$+ \sum_{i \neq j,\ j \neq k} (\Delta_i - \Delta_0)\Delta_0^{-1}(\Delta_j - \Delta_0)\Delta_0^{-1}(\Delta_k - \Delta_0) + ... \tag{3.8}$$

Now we have to average this over collective coordinates of the instanton medium. This problem can be simplified drastically if we neglect the correlation of different pseudoparticles. It can be justified by the diluteness of the instanton medium. We get from last equation

$$< \tilde{\Delta} > -\Delta_0 = \sum_i < (\Delta_i - \Delta_0) > +$$

$$\sum_{i \neq j} < (\Delta_i - \Delta_0) > \Delta_0^{-1} < (\Delta_j - \Delta_0) > +... \quad (3.9)$$

A diagram technique can be developed for the expansion (3.8) as shown in (3.2). A circle with i inside denotes $\Delta_i - \Delta_0$ and a solid line represents Δ_0^{-1}.

Figure 3.1: Notations for diagram technique

The circles of the same pseudoparticles are connected with a pink line (or with a bunch of pink lines if the pseudoparticle appears more than twice). Then we get the following graph describing the last equation

We will now study the diagram expansion of figure (3.2) at large number of colours N_c. The diagrams surviving at large N_c make possible a simple description. A graph for the gluon propagator will be called planar if the pink lines corresponding to different pseudoparticles do not intersect and at large N_c planar graphs dominate. We may neglect non-planar graphs.

Thus, our task is to sum up all the planar graphs for the gluon propagator. Although this cannot be done directly, it is possible to write down, when a closed equation for the quark propagator with all planar graphs are taken

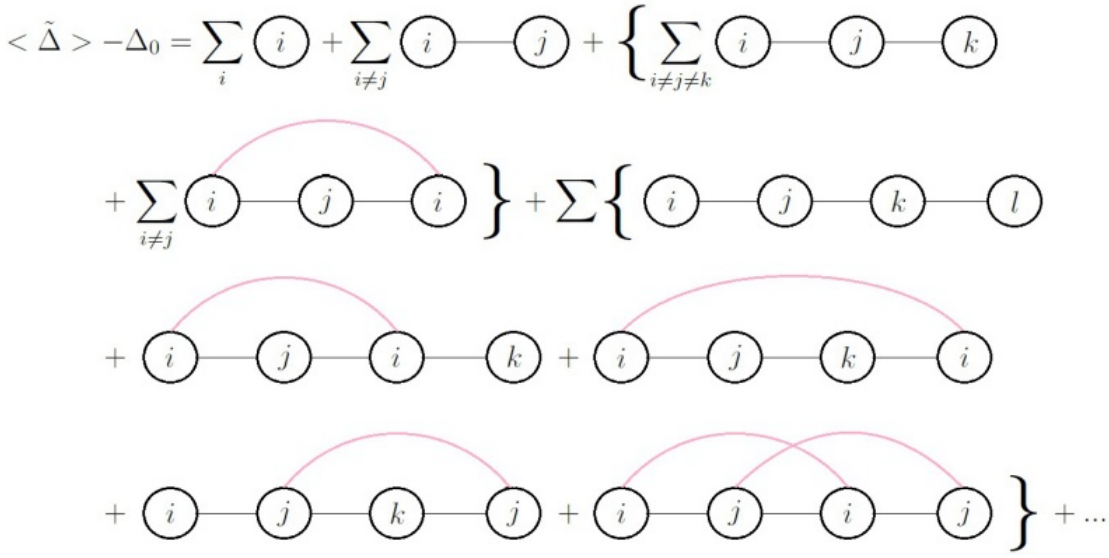


Figure 3.2: Graph for the gluon propagator

into account. First of all, let us construct a graph expressing only planar graphs. To this end we fix the first left pseudoparticle in each planar graph. In general, this pseudoparticle may appear in the same graph more than once. If one sums over all other pseudoparticles of the graph one will obtain a graph with bold lines between the circles of the fixed pseudoparticle (figure (3.3)).

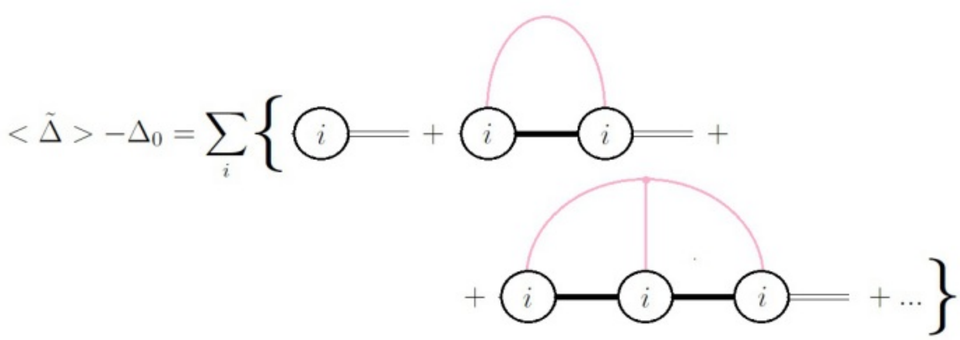


Figure 3.3: Gluon propagator at large N_c

A diagram expansion can be written for the bold lines (3.3). If one compares it with the diagram expansion for the gluon propagator $\bar{\tilde{\Delta}}$ ((3.2)) one can see that the bold line is equal to $\Delta_0^{-1}\bar{\bar{\tilde{\Delta}}}\Delta_0^{-1}$. The same situation for the parallel lines (3.5).

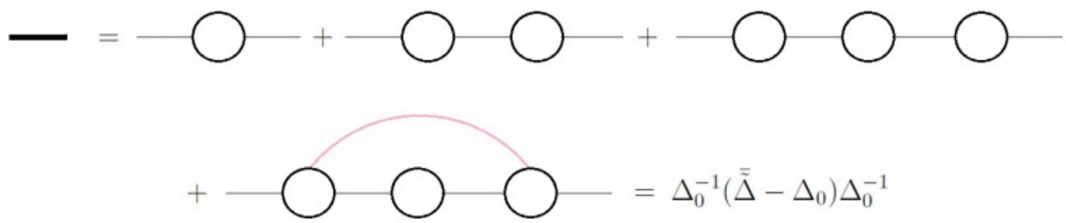


Figure 3.4: A diagram expansion for the bold line

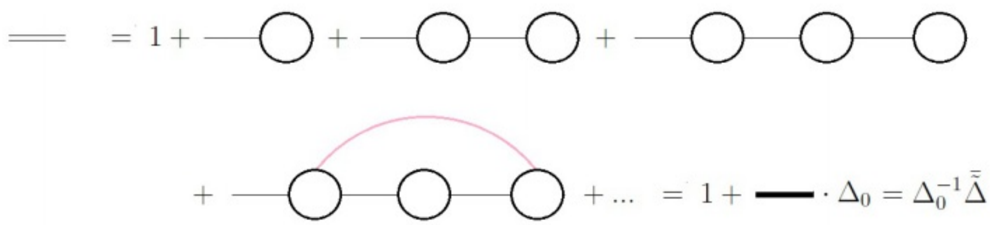


Figure 3.5: A diagram expansion for the parallel lines

Thus

$$\bar{\bar{\tilde{\Delta}}} - \Delta_0 = \sum < (\Delta_i - \Delta_0) \cdot \Delta_0^{-1}\bar{\bar{\tilde{\Delta}}} +$$

$$+ (\Delta_i - \Delta_0) \cdot \Delta_0^{-1}(\bar{\bar{\tilde{\Delta}}} - \Delta_0)\Delta_0 \cdot (\Delta_i - \Delta_0) \cdot \Delta_0^{-1}\bar{\bar{\tilde{\Delta}}} + ... >$$

$$= \sum < [(\Delta_i - \Delta_0) + (\Delta_i - \Delta_0)(\Delta_0^{-1}\bar{\bar{\tilde{\Delta}}}\Delta_0^{-1} - \Delta_0^{-1})(\Delta_i - \Delta_0) + ...]\Delta_0^{-1}\bar{\bar{\tilde{\Delta}}} >$$

$$= \sum < [\{a_i + a_i b a_i + a_i b a_i b a_i + ...\}\Delta_0^{-1}\bar{\bar{\tilde{\Delta}}}] >, \qquad \begin{cases} a_i = \Delta_i - \Delta_0 \\ b = \Delta_0^{-1}\bar{\bar{\tilde{\Delta}}}\Delta_0^{-1} - \Delta_0^{-1} \end{cases}$$

Here we have

$$a_i + a_i b a_i + a_i b a_i b a_i + ... = a_i \frac{1}{1 - b a_i} = \frac{1}{a_i^{-1} - b}$$

$$\{a_i + a_i b a_i + a_i b a_i b a_i + ...\}\Delta_0^{-1}\bar{\bar{\tilde{\Delta}}} = \frac{1}{a_i^{-1} - b}\Delta_0^{-1}\bar{\bar{\tilde{\Delta}}} = \frac{1}{\bar{\bar{\tilde{\Delta}}}^{-1}\Delta_0 (a_i^{-1} - b)}$$

$$= [\bar{\bar{\tilde{\Delta}}}^{-1}\Delta_0 \cdot (\Delta_i - \Delta_0)^{-1} - \bar{\bar{\tilde{\Delta}}}\Delta_0 \cdot (\Delta_0^{-1}\bar{\bar{\tilde{\Delta}}}\Delta_0^{-1} - \Delta_0^{-1})]^{-1}$$

$$= [\bar{\bar{\tilde{\Delta}}}^{-1}(\Delta_0^{-1} - \Delta_i^{-1})^{-1}\Delta_i^{-1} - (\Delta_0^{-1} - \bar{\bar{\tilde{\Delta}}}^{-1})]^{-1} \tag{3.10}$$

So we have

$$\bar{\bar{\tilde{\Delta}}} - \Delta_0 = \sum < [\bar{\bar{\tilde{\Delta}}}^{-1}(\Delta_0^{-1} - \Delta_i^{-1})^{-1}\Delta_i^{-1} - (\Delta_0^{-1} - \bar{\bar{\tilde{\Delta}}}^{-1})]^{-1} > \tag{3.11}$$

Multiplying Δ_0^{-1} from left and $\bar{\bar{\tilde{\Delta}}}^{-1}$ from right, we get

$$\Delta_0^{-1} - \bar{\bar{\tilde{\Delta}}}^{-1} = \sum < \{\bar{\bar{\tilde{\Delta}}}^{-1}[\bar{\bar{\tilde{\Delta}}}^{-1}(\Delta_0^{-1} - \Delta_i^{-1})^{-1}\Delta_i^{-1} - (\Delta_0^{-1} - \bar{\bar{\tilde{\Delta}}}^{-1})]\Delta_0\}^{-1} >$$

$$= \sum < \{(\Delta_0^{-1} - \Delta_i^{-1})^{-1}\Delta_i^{-1}\Delta_0 - (\bar{\bar{\tilde{\Delta}}}^{-1} - \Delta_0)\}^{-1} >$$

$$= \sum < \{(1 + \frac{1}{\Delta_i\Delta_0^{-1} - 1})\Delta_0 - \bar{\bar{\tilde{\Delta}}}\}^{-1} >= \sum < \{\frac{1}{\Delta_0^{-1} - \Delta_i^{-1}} - \bar{\bar{\tilde{\Delta}}}\}^{-1} >$$

$$= -\sum < \{\bar{\bar{\tilde{\Delta}}} - (\Delta_0^{-1} - \Delta_i^{-1})^{-1}\}^{-1} > \tag{3.12}$$

It is enough for us the firs order on density term, so we use $\bar{\bar{\tilde{\Delta}}} = \Delta_0$ for the right side of the last equation.

$$\bar{\bar{\tilde{\Delta}}}^{-1} - \Delta_0^{-1} = \sum_i < (\Delta_0 - (\Delta_i - \Delta_0)^{-1})^{-1} >$$

$$= -\sum_i < \{(\Delta_0(\Delta_0^{-1} - \Delta_i^{-1}) - 1)(\Delta_0^{-1} - \Delta_i^{-1})^{-1}\}^{-1} >$$

$$= -\sum_i < (\Delta_0^{-1} - \Delta_i^{-1})\Delta_i\Delta_0^{-1} >= -\sum_i < \Delta_0^{-1}(\Delta_i - \Delta_0)\Delta_0^{-1} > \quad (3.13)$$

In order to get this equation we used that $\tilde{\Delta} = -\tilde{\Delta},\ \Delta_0 = -\Delta_0,\ \Delta_i = -\Delta_i$. So, if we note that, we may write the following equation.

$$\bar{\tilde{\Delta}}^{-1} - \Delta_0^{-1} = -\sum < \Delta_0^{-1}(\Delta_i - \Delta_0)\Delta_0^{-1} > \quad (3.14)$$

b. Now we must find $< \tilde{\Delta} \sum A_i A_j \tilde{\Delta} >$. We have the followings:

$$< \sum_i A_i >= N < A_i >= N\bar{A}_i = O(N),\ \ (first\, order\, on\, density\, term\, n = \frac{N}{V})$$

$$< \sum_{i \neq j} A_i A_j >=< \sum A_i >< \sum_j A_j >= N^2 < A_i >< A_j >= 0 \quad (3.15)$$

According to last expression, $< \tilde{\Delta} \sum_{i \neq j} A_i A_j \tilde{\Delta} >$ is at least second order on density term and thus negligible.

Finally, we found that in the first-order expansion with respect to the density is [17]

$$\bar{\tilde{\Delta}}^{-1} - \Delta_0^{-1} =< \sum_i \{\Delta_0 + (\Delta_i^{-1} - \Delta_0^{-1})^{-1}\}^{-1} >$$

$$= N\Delta_0^{-1}(\bar{\Delta}_I - \Delta_0)\Delta_0^{-1} + O(n^2), \quad (3.16)$$

where $\bar{\Delta}_I = \int d\gamma_I\, \Delta_I$. It is obvious that in this order of expansion $\bar{\Delta}^{-1} = \bar{\tilde{\Delta}}^{-1} = p^2 + M_s^2$, where we introduced squared dynamical scalar "gluon" mass operator M_s^2. Fortunately, the correct propagators may be easily constructed by noting that the solution to $-D^2\Delta(x,y) = \delta^4(x-y)$ with periodic or antiperiodic boundary conditions in time is equivalent to the solution of [28]

$$D^2\Delta(x,y) = \sum_{n=-\infty}^{n=+\infty} (\pm)^n \delta^4(x-y+n\beta\hat{t}) \tag{3.17}$$

with boundary conditions of regularity at infinity. Consequently, the correct finite temperature propagators are given by

$$\Delta^{\pm}(x,y) = \sum_{n=-\infty}^{n=+\infty} (\pm)^n \bar{\Delta}(x, y+n\beta\hat{t}) \tag{3.18}$$

where $\bar{\Delta}(x,y)$ is the aperiodic scalar propagator in [15]. Note that only the $n=0$ term of this sum involves $(x-y)^{-2}$ singularity of the scalar propagator and all other terms are finite. Our remarks is given by these formulas:

Euclid time operator $\hat{t}$ is a hermitian one, eigenstates are $\hat{t}|t>= t|t>$ and total system is $\sum_t |t><t| = 1$,

$$<t'|t>= \delta(t'-t), <t'|\theta|t>= \theta(t'-t),$$

$$\frac{d}{dt'}\theta(t'-t) = \delta(t'-t) \;\; means \;\; \theta \equiv \frac{d}{dt'} \tag{3.19}$$

Periodical Propagator

$$\Delta^{-1}\Delta = 1 \qquad <t'|\Delta^{-1}\Delta|t_\beta>= \delta(t'-t_\beta) \equiv \sum_{n=-\infty}^{\infty} \delta(t'-(t-n\beta)) \tag{3.20}$$

Since Pobylitsa Eqs are written in operator form can be easily extended from temperature $T=0$ to $T \neq 0$ case. Just by calculating of matrix elements of propagator Δ using periodical state $|t_\beta>$ on the right side. One can consider inverse free propagator

$$\Delta_0^{-1} = p^2 \qquad <t'|p^2|t_\beta>= \delta(t'-t_\beta)(\vec{p}^2 - \frac{\partial^2}{\partial t^2}) = \delta(t'-t_\beta)p^2. \tag{3.21}$$

Inverse propagator takes next form

$$\Delta^{-1} = P^2$$

$$< t'|P^2|t_\beta >= \delta(t'-t_\beta)(\vec{P}^2 + (i\frac{\partial}{\partial t} + A_4)^2) = \delta(t'-t_\beta)P^2. \tag{3.22}$$

The equation for the propagator now is

$$P^2 < t'|\Delta|t_\beta >= \delta(t'-t_\beta), \tag{3.23}$$

Accordingly[28] periodic scalar "gluon" propagator in periodical instanton field is

$$\Delta_I^{ab}(x,y) = \Delta_0^{ab}(x,y) + \Delta_1^{ab}(x,y) + \Delta_2^{ab}(x,y) \tag{3.24}$$

$$\Delta_0^{ab}(x,y) = 1/2\,\mathrm{tr}\frac{\tau_a F(x,y)\tau_b F(y,x)}{\Pi(x)4\pi^2(x-y)^2\Pi(y)} \tag{3.25}$$

$$F(x,y) = 1 + \sum_m \frac{\rho^2(\tau x_m)(\tau^+ y_m)}{x_m^2 y_m^2}, \quad (x_m \equiv x - m\beta\hat{t},\ y_m \equiv y - m\beta\hat{t})$$

$$\Delta_1^{ab}(x,y) = 1/2\,\mathrm{tr}{\sum_m}' \frac{\tau_a F(x,y_m)\tau_b F(y_m,x)}{\Pi(x)4\pi^2(x-y_m)^2\Pi(y)} \tag{3.26}$$

$$\Delta_2^{ab}(x,y) = \sum_m \frac{C^{ab}(x,y_m)}{\Pi(x)4\pi^2\Pi(y)}, \tag{3.27}$$

$$C^{ab}(x,y) = \sum_{r\neq s}\frac{2\Phi_{rs}^a(x)\Phi_{rs}^b(y)}{\beta^2(r-s)^2} - \sum_{r\neq s}\sum_{t\neq u}\frac{\rho^2\Phi_{rs}^a(x)}{\beta^2(r-s)^2}\frac{\Phi_{tu}^b(y)}{\beta^2(t-u)^2}h_{rs,tu},$$

$$\sum_m C^{ab}(x,y_m) = \sum_m\sum_{r\neq s}\frac{2\Phi_{rs}^a(x)\Phi_{rs}^b(y_m)}{\beta^2(r-s)^2} = \sum_{r\neq s}\frac{\rho^2 x^a}{x_r^2 x_s^2}\sum_m \frac{\rho^2 y^b}{y_{r+m}^2 y_{s+m}^2}$$

here $\Phi_{rs}^a(x) = \frac{\rho^2\beta(r-s)x^a}{x_r^2 x_s^2}$ Then

$$\Delta_2^{ab}(x,y) = \sum_{r\neq s}\frac{\rho^2 x^a}{x_r^2 x_s^2}\sum_m \frac{\rho^2 y^b}{y_{r+m}^2 y_{s+m}^2}\frac{1}{\Pi(x)4\pi^2\Pi(y)} \tag{3.28}$$

Let's consider the region $r \sim t \leq \beta$. Then caloron field become instanton-like (2.26) with the modification $\rho \to \rho^{'}$, $\rho^{'2} = \rho^2/(1+1/3\,\lambda^2)$, $\lambda = \pi\rho/\beta$. In this region

$$\Delta^{ab}_{I,0} = \frac{1}{2}\mathrm{tr}\frac{\tau_a F_0(x,y)\tau_b F_0(y,x)}{4\pi^2(x-y)^2\Pi_0(x)\Pi_0(y)}, \quad \Pi_0(x) = \frac{x^2+\rho^{'2}}{x^2}, \tag{3.29}$$

$$\tau_\mu = (\vec{\tau}, i), \;\; \tau^+_\mu = (\vec{\tau}, -i), \;\; \tau_\mu\tau^+_\nu = \delta_{\mu\nu} + i\bar{\eta}_{a\mu\nu}\tau_a, \tag{3.30}$$

$$F_0(x,y) = 1 + \rho^{'2}\frac{(\tau x)(\tau^+ y)}{x^2y^2} = 1 + \rho^{'2}\frac{(xy)}{x^2y^2} + \rho^{'2}\frac{i\bar{\eta}_{a\mu\nu}\tau_a x_\mu y_\nu}{x^2y^2}, \tag{3.31}$$

than we can find

$$\Delta^{ab}_{I,0} - \Delta^{ab}_0 = -\frac{\delta_{ab}\rho^{'2}}{4\pi^2(x^2+\rho^{'2})(y^2+\rho^{'2})} +$$

$$+\frac{2\rho^{'4}}{4\pi^2(x-y)^2}\frac{\delta_{ab}(-x^2y^2+(xy)^2) + \bar{\eta}_{a\mu\nu}x_\mu y_\nu \bar{\eta}_{b\rho\sigma}x_\rho y_\sigma}{x^2y^2(x^2+\rho^{'2})(y^2+\rho^{'2})} \tag{3.32}$$

where $\bar{\eta}_{a\mu\nu} = -\bar{\eta}_{a\nu\mu}$ is the 'tHooft symbol. the definitions of $\Delta^{ab}_{I,1}$ and $\Delta^{ab}_{I,2}$ see above. In Eq.(3.29) it is assumed the position of the instanton $z = 0$ and the orientation $U = 1$. In order to average over the position z, we have to change $x \to x - z$, $y \to y - z$ and perform integration $\int_0^\beta dz_4 \int_{V_3} d^3z$. To make a color orientation averaging we introduce the orientation factor $O^{ab} = \mathrm{tr}(U^+ t^a U \tau^b)$, where t_a are $SU(N_c)$- matrices, change Δ^{ab}_I to $O^{ab}O^{a'b'}\Delta^{bb'}_I$, and we carry our integration $\int dO$. Here $\int dO O^{ab}O^{ab'} = \delta_{bb'}$, $\int dO O^{ab}O^{a'b'} = (N_c^2 - 1)^{-1}\delta_{aa'}\delta_{bb'}$. Also, $\int dO O^{ab}\bar{\eta}_{b\mu\nu}O^{a'b'}\bar{\eta}_{b'\mu'\nu'} = (N_c^2-1)^{-1}\delta_{aa'}(\delta_{\mu\mu'}\delta_{\nu\nu'} - \delta_{\mu\nu'}\delta_{\nu\mu'})$. The contribution of the $\Delta^{ab}_{I,0}$ to the scalar "gluon" dynamical mass operator is given by

$$M^2_{s,0}\delta_{ab} = N(p^2\bar{\Delta}^{ab}_{I,0}p^2 - \delta_{ab}p^2). \tag{3.33}$$

Here we need to take into account (3.19)- (3.23)

$$< t'|M^2_s|t_\beta > \delta_{ab} = N\, p^2(< t'|\bar{\Delta}^{ab}_I|t_\beta > - < t'|\Delta^{ab}_0|t_\beta >)p^2. \tag{3.34}$$

In coordinate space, we find

$$\bar{\Delta}^{aa'}_{I,0}(x,y) - \Delta^{aa'}_0(x,y) = \tag{3.35}$$
$$= \int d^4z dO O^{ac} O^{a'c'} (\Delta^{cc'}_{I,0}(x',y') - \Delta^{cc'}_0(x',y')) \;\; (x' \equiv x - z, \;\; y' \equiv y - z),$$
$$= \int \frac{\delta_{aa'}}{N_c^2 - 1}[-\frac{3\rho'^2}{4\pi^2(x'^2+\rho'^2)(y'^2+\rho'^2)} +$$

$$+\frac{2\rho'^4[(x'y')^2 - x'^2y'^2]}{4\pi^2(x'-y')^2x'^2y'^2(x'^2+\rho'^2)(y'^2+\rho'^2)}] =$$

$$= \delta_{aa'} \int d^4z [\frac{3\rho'^2}{4\pi^2(N_c^2-1)} f_1(x')f_1(y') + \frac{2\rho'^4}{N_c^2-1} f_2(x')g(x'-y')f_2(y')],$$
$$f_1(x) = \frac{1}{(x^2+\rho'^2)}, \;\; f_2(x) = \frac{(x_\mu x_\nu, ix^2)}{x^2(x^2+\rho'^2)}, \;\; g(x-y) = \frac{1}{4\pi^2(x-y)^2}.$$

We have to calculate "electric" dynamical gluon mass corresponding to the 3-momentum space $\vec{q}$ Fourier components and $n=0$ Matsubara modes ($\omega_n = 2\pi n T$) $M^2_{s,0}(\vec{q}, n=0)$. The contribution of the first term in Eq. (3.36) is given by Eqs.

$$I_1 = \int_{-\beta/2}^{\beta/2} dz_4 \int d^3z f_1(x-z) f_1(y-z) = \tag{3.36}$$
$$= \sum_{m,n=-\infty}^{\infty} \int_{-\beta/2}^{\beta/2} dz_4 \int d^3z \frac{d^3p}{(2\pi)^3}\frac{d^3p_1}{(2\pi)^3} =$$
$$= \sum_{n=-\infty}^{\infty} \frac{d^3p}{(2\pi)^3} \exp[i\vec{p}(\vec{x}-\vec{y}) + i2\pi n(x-y)_4/\beta] f_1(\vec{p},n) f_1(-\vec{p},-n)$$

here

$$f_1(x-z) = \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} \exp[i\vec{p}(\vec{x}-\vec{z})] \exp(2\pi n(x-z)_4/\beta) f_1(\vec{p},n),$$

So, the contribution of the first term to the "scalar" gluon dynamical mass $M^2_{s,0,1}(q,T)$ is given by $f_1(\vec{q}, n=0) f_1(-\vec{q}, -n=0)$. We may use $\vec{q} = (q,0,0)$

to simplify our calculations, and we redefine variables

$$q^2 f(\vec{q}, n=0) = q^2 f(q, n=0) = \tag{3.37}$$

$$= \int_{-\beta/2}^{\beta/2} dx_4 \int d^3x \frac{e^{iqx_4}}{\sum_{i=2}^{4} x_i^2 + x_1^2 + \rho'^2} =$$

$$= q^2\rho'^2 \int_{-\beta/2\rho'}^{\beta/2\rho'} dx_4 \int_{-\infty}^{\infty} dx_2 dx_3 \pi \exp(-q\rho'(\sum_{i=2}^{4} x_i{}^2 + 1)^{1/2})) \frac{1}{(\sum_{i=2}^{4} x_i{}^2 + 1)^{1/2}}$$

$$\leq q^2\rho'^2 \int_{-\infty}^{\infty} dx_4 dx_2 dx_3 \pi \frac{\exp(-q\rho'(\sum_{i=2}^{4} x_i{}^2 + 1)^{1/2}))}{(\sum_{i=2}^{4} x_i{}^2 + 1)^{1/2}} = 4\pi^2 q\rho' K_1(q\rho').$$

Here $K_1(x)$ modified Bessel function of the second kind, $\lim_{x\to 0} xK_1(x) = 1$. We can see $\lim_{q\to 0} q^2 f(q,0) = 4\pi^2$ It means that temperature slightly affect dynamical mass form-factor, and we may neglect by this modification at small temperature.

Our main purpose is to analyse contributions of $\Delta_{I,1}$ and $\Delta_{I,2}$ to dynamical mass. First we need to calculate the integral corresponding to $\Delta_0(x - y_m) = \frac{1}{(x-y_m)^2}$

$$J_m(q) = \int d^3x \int_{-\beta/2}^{\beta/2} dx_4 \exp(i\vec{q}\vec{x}) \frac{1}{x_m^2}, \quad (x_m^2 = \vec{x}^2 + (x_4 + m\beta)^2) \tag{3.38}$$

$$= \int 2\pi r^2 dr d(-\cos\Theta) \exp(iqr\cos\Theta) \int_{-\beta/2}^{\beta/2} dx_4 \frac{1}{r^2 + (x_4 + m\beta)^2}, \quad (q = |\vec{q}|)$$

$$= 2\pi \int_{-\infty}^{\infty} r^2 dr (\frac{\exp(iqr)}{iqr} + k.c.) \frac{1}{r^2 + (x_4 + m\beta)^2}, \quad (r_\pm = \pm i(x_4 + m\beta)),$$

$$= \frac{4\pi^2}{q^2} \exp(-q|m|\beta) \int_0^{\beta/2} q dx_4 \exp(-qx_4) =$$

$$= \frac{4\pi^2}{q^2}\exp(-q|m|\beta)(1-\exp(-q\beta/2)). \quad (3.39)$$

Now we take $f(x) = x^2/(x^2+\rho^2)$. As previous we have to make an estimation assuming $\beta \to \infty$, then

$$f(q) = \int dx_1 d^3x\, e^{iqx_1}\frac{x_1^2+\vec{x}^2}{x_1^2+\vec{x}^2+\rho^2}$$

$$= \rho^4 4\pi \int_{-\infty}^{\infty} dy x^2 dx e^{iq\rho y}\frac{y^2+x^2}{y^2+x^2+1}$$

$$= -\rho^4 4\pi^2 \int x^2 dx \frac{e^{-q\rho(x^2+1)^{1/2}}}{(1+x^2)^{1/2}} = -4\pi^2\rho^2(q\rho\cdot K_1(q\rho))/q^2 \quad (3.40)$$

We can find

$$F(x,y_m) = F(x,y) = 1 + \sum_m \frac{\rho^2(\tau x_m)(\tau^+ y_m)}{x_m^2 y_m^2},$$

$$(x_m \equiv x - m\beta\hat{t},\ \ y_m \equiv y - m\beta\hat{t})$$

$$F(x,y_m) = 1 + \sum_m \frac{\rho^2(\tau x_m)(\tau^+ y_m)}{x_m^2 y_m^2} \approx 1 + O(1/\beta) \quad (3.41)$$

$$\Delta_{I,1}^{ab} = 1/2\sum_m \frac{\delta_{ab}}{\Pi(x)4\pi^2(x-y_m)^2\Pi(y)} \quad (3.42)$$

Then as previous

$$\Pi(x) = 1 + \frac{\rho'^2}{x^2} \quad (3.43)$$

$$\bar{\Delta}_{I,1}^{aa'}(x,y) = \int d^4z dO O^{ac} O^{a'c'}(\Delta_{I,1}^{cc'}(x',y')) =$$

$$\frac{3\delta_{aa'}}{4\pi^2(N_c^2-1)}\int d^4z \frac{x'^2}{(x'^2+\rho'^2)}\frac{y'^2}{(y'^2+\rho'^2)}\sum_m{}' \frac{1}{(x-y_m)^2}$$

$$(x - z \equiv x', \;\; y' \equiv y - z, y_m = (\vec{y}, y_4 + m\beta)), \tag{3.44}$$

$$F(x-y) = \int d^4z \frac{x'^2}{(x'^2+\rho^2)} \frac{y'^2}{(y'^2+\rho^2)} = \int \frac{d^4Q}{(2\pi)^4} \exp(iQ(x-y)) f(Q) f(-Q),$$

$$\bar{\Delta}^{aa'}_{I,1}(x,y) = \frac{3\delta_{aa'}}{4\pi^2(N_c^2-1)} F(x-y) \int \frac{d^4q}{(2\pi)^4} \exp(iq(x-y)) {\sum_m}' J_m(q)$$

$$\bar{\Delta}^{aa'}_{I,1}(Q) = \frac{3\delta_{aa'}}{4\pi^2(N_c^2-1)} \int \frac{d^4q}{(2\pi)^4} {\sum_m}' J_m(Q-q) f(q) f(-q)$$

$$= \frac{3\delta_{aa'}}{4\pi^2(N_c^2-1)} \int \frac{d^4q}{(2\pi)^4} \frac{4\pi^2}{(Q-q)^2} \Bigg[(1 - \exp(-\frac{|Q-q|\beta}{2})) \times$$

$$\times (4\pi^2\rho^2 \frac{q\rho K_1(q\rho)}{q^2})^2 {\sum_m}' \exp(-|Q-q||m|\beta) \Bigg] \tag{3.45}$$

Here q_μ is 4-dim vector. We can take $Q_\mu = (Q, 0, 0, 0)$. Then $(Q-q)^2 = (Q-q_1)^2 + \vec{q}^2$, $d^4q = dq_1 d^3\vec{q}$.

$$\bar{\Delta}^{aa'}_{I,1}(Q) = \frac{3\delta_{aa'}}{4\pi^2(N_c^2-1)} \int \frac{d^4q}{(2\pi)^4} \frac{4\pi^2}{(Q-q)^2} (1 - \exp(-\frac{|Q-q|\beta}{2}))$$

$$\frac{\exp(-|Q-q|\beta)}{1-\exp(-|Q-q|\beta)} (4\pi^2\rho^2 \frac{q\rho K_1(q\rho)}{q^2})^2 \tag{3.46}$$

Our estimation is based on the following $(1-\exp(-x/2))\exp(-x)/(1-\exp(-x)) \leq 0.5$, $(xK_1(x))^2 \leq 1$, where $x \geq 0$.

Then,

$$\bar{\Delta}^{aa'}_{I,1}(Q) \leq 0.5 \frac{3\delta_{aa'}}{(N_c^2-1)(4\pi^2\rho^2)^2} \int \frac{d^4q}{(2\pi)^4} \frac{1}{(Q-q)^2} (\frac{1}{q^2})^2$$

$$= 0.5 \frac{3\delta_{aa'}}{(N_c^2-1)(4\pi^2\rho^2)^2} T(Q)$$

The calculation of above-given integral with $Q_\mu = (Q, 0, 0, 0)$

$$T(Q) = \int_0^\infty 4\pi q^2 dq \int_{-\infty}^\infty dq_1 \frac{1}{(q_1^2+q^2)^2((Q-q_1)^2+q^2)} \qquad (3.47)$$

There are poles are given by $q_{1\pm} = \pm iq$ (second order poles) and $q'_{1\pm} = Q \pm iq$. We take a anticlockwise contour from $-\infty$ to ∞ and may neglect by the semicircle of the radius R contribution since at $R \to \infty$ its contribution behave as $1/R^5$. In this case we have to take the contributions from the poles q_{1+} and q'_{1+}.

$(q_1^2+q^2)^2((Q-q_1)^2+q^2) = (q_1-q_{1+})^2(q_1-q_{1-})^2(q_1-q'_{1+})(q_1-q'_{1-})$.

The residue at the pole q_{1+} is

$$Res = \frac{d}{dq_1}\frac{1}{(q_1-q_{1-})^2(q_1-q'_{1+})(q_1-q'_{1-})}|_{q_1=q_{1+}} =$$

$$= -2\frac{1}{(q_{1+}-q_{1-})^3(q_{1+}-q'_{1+})(q_{1+}-q'_{1-})} - \frac{1}{(q_{1+}-q_{1-})^2(q_{1+}-q'_{1+})^2(q_{1+}-q'_{1-})}$$

$$-\frac{1}{(q_{1+}-q_{1-})^2(q_{1+}-q'_{1+})(q_{1+}-q'_{1-})^2} = -2\frac{1}{(2iq)^3(iq-(Q+iq))(iq-(Q-iq))}$$

$$-\frac{1}{(2iq)^2(iq-(Q+iq))^2(iq-(Q-iq))} - \frac{1}{(2iq)^2(iq-(Q+iq))(iq-(Q-iq))^2}$$

$$= 2\frac{1}{(2iq)^3Q(2iq-Q)} - \frac{1}{(2iq)^2Q^2(2iq-Q)} + \frac{1}{(2iq)^2Q(2iq-Q)^2}$$

$$= 2\frac{1}{(2iq)^3Q(2iq-Q)} - \frac{1}{(2iq)^2Q^2(2iq-Q)} + \frac{1}{(2iq-Q)}$$

$$= (\frac{1}{iq} - \frac{1}{Q} + \frac{1}{2iq-Q})\frac{1}{(2iq)^2Q(2iq-Q)} \qquad (3.48)$$

while the residue at the pole $q^{'}_{1+}$ is

$$Res' = \frac{1}{(q_1 - q_{1+})^2 (q_1 - q_{1-})^2 (q_1 - q^{'}_{1-})}|_{q_1 = q^{'}_{1+}} = \tag{3.49}$$

$$= \frac{1}{(q^{'}_{1+} - q_{1+})^2 (q^{'}_{1+} - q_{1-})^2 (q^{'}_{1+} - q^{'}_{1-})} =$$

$$= \frac{1}{((Q+iq) - iq)^2 ((Q+iq) + iq)^2 ((Q+iq) - (Q-iq))} = \frac{1}{Q^2 (Q+2iq)^2 (2iq)}$$

Now

$$Q^4 T(Q) = 4\pi 2\pi i Q^4 \int_0^\infty q^2 dq (Res + Res') = \tag{3.50}$$

$$= 8\pi^2 i Q^4 \int_0^\infty dq [(\frac{1}{iq} - \frac{1}{Q} + \frac{1}{2iq - Q}) \frac{1}{(2i)^2 Q (2iq - Q)} + \frac{q}{Q^2 (Q+2iq)^2 (2i)}]$$

We have to remember that convergence of the integral on the upper limit is provided in fact by $\exp(-|Q-q||m|\beta)$ in our actual integral. So, we may safely consider limit $\lim_{Q\to 0} Q^4 T(Q)$ before integration over q. So, we have

$$\lim_{Q\to 0} Q^4 T(Q) = 4\pi^2 \lim_{Q\to 0} Q^2 \int_0^\infty dq \frac{q}{(Q+2iq)^2} = 0 \tag{3.51}$$

It means $M_{s,1} = 0$ Now we average $\Delta_2^{ab}(x,y)$. In order to average over the position z, we have to change $x \to x - z$, $y \to y - z$ and perform integration $\int_{-\beta/2}^{\beta/2} dz_4 \int_{whole 3-dim} d^3 z$. Also, we average over the color orientation U. $\int dO O^{ab} O^{a'b'} = (N_c^2 - 1)^{-1} \delta_{aa'} \delta_{bb'}$. Then we have

$$\bar{\Delta}_2^{aa'}(x,y) =$$

$$= \frac{\delta_{aa'}}{N_c^2 - 1)} \int_{-\beta/2}^{\beta/2} dz_4 \int d^3 z \sum_{r\neq s} \frac{\rho^2 x'^b}{x'^2_r x'^2_s} \sum_m \frac{\rho^2 y'^b}{y'^2_{r+m} y'^2_{s+m}} \frac{1}{\Pi(x') 4\pi^2 \Pi(y')} \tag{3.52}$$

we can use following notations here

$$f^b_{rs}(x') = \frac{x'^b}{x'^2_r x'^2_s}\frac{1}{\Pi(x')}, \quad f^b_{rsm}(y') = \frac{y'^b}{y'^2_{r+m} y'^2_{s+m}}\frac{1}{\Pi(y')} \tag{3.53}$$

$$f_{rs}(x') = \frac{1}{x'^2_r x'^2_s}\frac{1}{\Pi(x')}, \quad f_{rsm}(y') = \frac{1}{y'^2_{r+m} y'^2_{s+m}}\frac{1}{\Pi(y')} \tag{3.54}$$

Then $\Delta^{ab}_{I,2}$ will

$$\bar{\Delta}^{aa'}_2(\vec{q}) = \int d^4z dO O^{ac} O^{a'c'} (\Delta^{cc'}_{I,2}(x', y')) =$$

$$= \int \frac{d^4q}{2\pi^4} \frac{\delta_{aa'}\rho^4}{4\pi^2(N_c^2 - 1)} \sum_{r \neq s, m} f^b_{rs}(\vec{q}, n = 0) f^b_{rsm}(-\vec{q}, -n = 0) \tag{3.55}$$

$$f^b_{rs}(\vec{q}, n = 0) = \frac{1}{\beta}\int_{-\beta/2}^{\beta/2} dx_4 \int d^3x \exp(i\vec{q}\vec{x}) f^b_{rs}(x) \tag{3.56}$$

$$f^b_{rsm}(\vec{q}, n = 0) = \frac{1}{\beta}\int_{-\beta/2}^{\beta/2} dx_4 \int d^3x \exp(i\vec{q}\vec{x}) f^b_{rsm}(x) \tag{3.57}$$

It is easy to find that

$$f^b_{rs}(\vec{q}, 0) = -i\frac{\partial}{\partial q} f_{rs}(\vec{q}, 0),$$

$$f_{rs}(\vec{q}, 0) = \frac{1}{\beta}\int_{-\beta/2}^{\beta/2} dx_4 \int d^3x \exp(i\vec{q}\vec{x}) f_{rs}(x) \tag{3.58}$$

$$f^b_{rsm}(\vec{q}, 0) = -i\frac{\partial}{\partial q} f_{rsm}(\vec{q}, 0)$$

$$f_{rsm}(\vec{q}, 0) = \frac{1}{\beta}\int_{-\beta/2}^{\beta/2} dx_4 \int d^3x \exp(i\vec{q}\vec{x}) f_{rsm}(x) \tag{3.59}$$

$$f_{rs}(x) = \frac{1}{x_r^2 x_s^2}\frac{1}{\Pi(x)} \approx \frac{1}{x_r^2 x_s^2}\frac{1}{\Pi_0(x)} =$$

$$= \frac{1}{(\vec{x}^2 + (x_4 + r\beta)^2)(\vec{x}^2 + (x_4 + s\beta)^2)} \frac{x^2}{x^2 + \rho'^2} \tag{3.60}$$

$$f_{rs}(q) = \int dx_4 d^3x \exp(i\vec{q}\vec{x}) f_{rs}(x) =$$

$$= 2\pi \int_0^\infty r^2 dr dx_4 \left[\frac{\exp(iqr) - \exp(-iqr)}{iqr} \times \right.$$
$$\left. \times \frac{1}{(r^2 + (x_4 + r\beta)^2)(r^2 + (x_4 + s\beta)^2)} \frac{x^2}{x^2 + \rho'^2} \right] =$$
$$= 2\pi \int_0^\infty dx_4 \int_{-\infty}^\infty r dr \left[\frac{\exp(iqr)}{iq} \times \right.$$
$$\left. \times \frac{1}{(r^2 + (x_4 + r\beta)^2)(r^2 + (x_4 + s\beta)^2)} \frac{r^2 + x_4^2}{r^2 + x_4^2 + \rho'^2} \right] \tag{3.61}$$

The poles there are $r_{r\pm} = \pm i((x_4 + r\beta)^2))^{1/2}$, $r_{s\pm} = \pm i((x_4 + s\beta)^2))^{1/2}$, $r_\pm = \pm i(x_4^2 + \rho'^2)^{1/2}$. Their contributions are

$$\int d^3x \exp(i\vec{q}\vec{x}) f_{rs}(x) = \frac{2\pi^2}{iq} \left[\exp(-q((x_4 + r\beta)^2))^{1/2}) \times \right. \tag{3.62}$$
$$\times \frac{1}{i((x_4 + r\beta)^2))^{1/2}((-(x_4 + r\beta)^2)) + (x_4 + s\beta)^2)} \times$$
$$\times \frac{-((x_4 + r\beta)^2)) + x_4^2}{-((x_4 + r\beta)^2)) + x_4^2 + \rho'^2} + (s \leftrightarrow r) + \exp(-q(x_4^2 + \rho'^2))^{1/2}) \times$$
$$\left. \times \frac{1}{(-(x_4^2 + \rho'^2) + (x_4 + r\beta)^2)(-(x_4^2 + \rho'^2) + (x_4 + s\beta)^2)} \frac{-\rho'^2}{i(x_4^2 + \rho'^2))^{1/2}} \right]$$
$$= -\frac{2\pi^2}{q} [\exp(-q|x_4 + r\beta|) \frac{1}{|x_4 + r\beta|((-(x_4 + r\beta)^2)) + (x_4 + s\beta)^2)} \times$$
$$\frac{-((x_4 + r\beta)^2)) + x_4^2}{-((x_4 + r\beta)^2)) + x_4^2 + \rho'^2} + (s \leftrightarrow r) + \exp(-q(x_4^2 + \rho'^2))^{1/2}) \times$$
$$\times \frac{1}{(-(x_4^2 + \rho'^2) + (x_4 + r\beta)^2)(-(x_4^2 + \rho'^2) + (x_4 + s\beta)^2)} \frac{-\rho'^2}{(x_4^2 + \rho'^2))^{1/2}}$$

We have plotted graph of $F(q) = q^2 f_{rs}(q)$ and from graph we can sea probably it leads to the convergent integral over x_4 and at $q- > 0$ limit we have $F(q) \to 0$ and $f_{rs}(\vec{q}, 0) \sim 1/q$, $f_{rsm}(\vec{q}, 0) \sim 1/q$ and $f_{rs}^b(\vec{q}, 0) \sim q_b/q^2$, $f_{rsm}^b(\vec{q}, 0) \sim q_b/q^2$

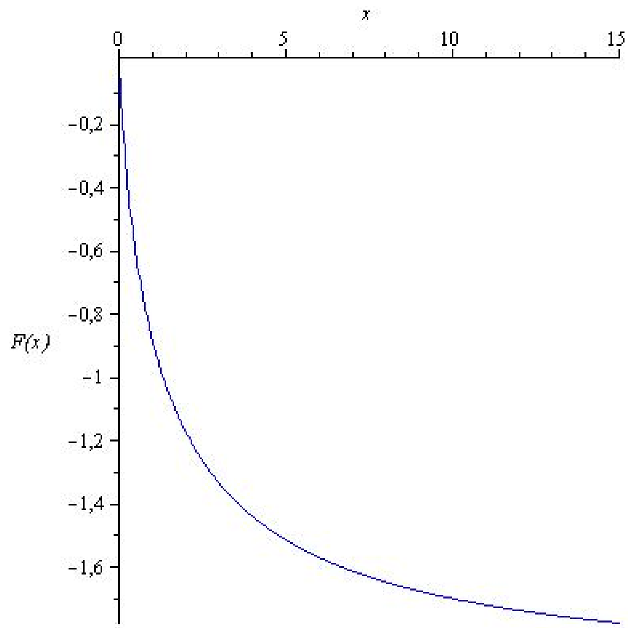


Figure 3.6: F(x) is function of $x = q\rho$

It means in this limit $q^4 \bar{\Delta}_2^{aa'}(\vec{q}) -> 0$ and no contribution to the dynamical mass. Our careful analysis show that second term in Eq. (3.36) and all of other terms including Δ_1^{ab} and Δ_2^{ab} give zero or negligible contribution and we finally have a

$$M_s(q,T) \approx M_{s,0,1}(q,T) = [\frac{3\bar{\rho}'^2(T)n(T)}{(N_c^2-1)} 4\pi^2]^{1/2} F(q,T), \tag{3.63}$$
$$F(0,0) = 1, \;\; F(q,T) \leq F(q,0) = q\bar{\rho}K_1(q\bar{\rho}).$$

3.2 Real Gluon propagator at non-zero temperature

3.2.1 Zero Modes

Let's assume that the gauge field is represented in the form

$$A_\mu = A_\mu^{inst}(\alpha) + a_\mu \tag{3.64}$$

and α denotes generically the set of all relevant collective coordinates.

The action of this field is as following if we take into account that A_μ^{inst} is the solution of the equation of motion. Hence,

$$S = S[A_\mu^{inst}(\alpha)] + \int d^4x a_\mu(x) L^{\mu\nu}(\alpha) a_\nu(x) \tag{3.65}$$

where $L^{\mu\nu}$ is some linear differential operator of the second order.

The fact that the operator L must and does have zero modes is rather obvious [37]. Each collective coordinate reflecting the existence of a non trivially realized symmetry produces its own zero mode. Thus, for SU(2) instanton we have 8 zero modes [38].

Indeed, the symmetry of the action implies that there is a variation of the classical field generated by a change in the corresponding collective coordinate which gives no variation of the action. For instance, $A_\mu^{inst}(\rho)$ and have the same action, $8\pi^2/g^2$ hence,

$$a_\mu^{(0)} \sim A_\mu^{inst}(\rho + \delta\rho) - A_\mu^{inst}(\rho) \tag{3.66}$$

has to be the zero mode of $L^{\mu\nu}$.

In general the non-Abelian gauge field is governed by the

$$g^2 S = -\int d^4x [\frac{1}{4}(F_{\mu\nu}^a) + \frac{1}{2\xi}(D_{\mu ab}^{cl} A_{\mu b})^2] \tag{3.67}$$

With our normalization of the gauge field the coupling constant g appears as an overall factor. Here we have chosen a "background gauge" specified by the parameter ξ. The operator $D_{\mu ab}^{cl}$ is the gauge-covariant derivative

$$D_{\mu ab}^{cl} = \partial_\mu \delta_{ab} + f_{abc} A_{\mu c}^{cl}, \tag{3.68}$$

where the vector potential $A_{\mu a}^{cl}$ describes a classical solution of the non-Abelian field equations and is fixed in all variations of the action. We shall study the small fluctuations $\phi_{\mu a}$ of the gauge field about the classical solution,

$$A_{\mu a} = A^{cl}_{\mu a} + \phi_{\mu a}. \tag{3.69}$$

Inserting this decomposition of the vector potential into the field-strength tensor and extracting pieces quadratic in ϕ_μ from the resulting action (3.67) yields the small-fluctuation, vector-field action

$$g^2 S_2 = -\frac{1}{2}\int d^4x \phi_\mu[-D^2\delta_{\mu\nu} - 2F_{\mu\nu} + (1-1/\xi)D_\mu D_\nu]\phi_\nu. \tag{3.70}$$

As we know that the dual tensor $\tilde{F}^a_{\mu\nu}$ obeys

$$D_{\mu ab}\tilde{F}^b_{\mu\nu} = 0 \tag{3.71}$$

Hence any gauge field with a self-dual (or anti self-dual) field-strength tensor $\tilde{F}^a_{\mu\nu} = \pm F^a_{\mu\nu}$ provides automatically a solution of the field equations

$$D_{\mu ab}F^b_{\mu\nu} = 0 \tag{3.72}$$

Moreover, by a suitable gauge transformation, we may impose the background gauge condition

$$D_{\mu ab}A_{\mu b} = \partial_\mu A_{\mu a} = 0 \tag{3.73}$$

In general, a continuously connected family of self-dual (or anti-self-dual) fields exists, labeled by some set of continuously varying parameters. [For example, if $A_\mu(x)$ yields a self-dual field-strength tensor, then so does the translated field $A_\mu(x-z)$, where z_μ are four constant parameters]. Thus, given a self-dual (or anti-self-dual) field, we can take its derivative with respect to one of its parameters to get a small-fluctuation field which is also self-dual (or anti-self-dual). Moreover, an infinitesimal gauge transformation can be added to this small-fluctuation field to bring it into the background gauge [39]. We find that any self-dual (or anti-self-dual) field will support some number of zero-mode fluctuation fields $\phi^{(s)}_{\mu,a}$, $s = 1,\ 2, ...,\ N$, satisfying

$$D_\mu \phi^{(s)}_\mu = 0 \tag{3.74}$$

and the field equation which follows from the action (3.70),

$$(D^2 \delta_{\mu\nu} + 2F_{\mu\nu})\phi^{(s)}_\nu = 0. \tag{3.75}$$

The small fluctuations of the gauge field are described by a massless spin-1 propagator $\mathbf{G}_{(\mathbf{0})\mu\nu}$. It may be convenient, however, to introduce an artificial regulating mass and define a massive spin-1 propagator $\mathbf{G}_{m,\mu\nu}$ by

$$(-D^2\delta_{\mu\nu} - 2F_{\mu\nu} + (1 - 2/\xi)D_\mu D_\nu + m^2\delta_{\mu\nu})\mathbf{G}_{m,\mu\nu} = \delta_{\mu\nu} \tag{3.76}$$

Here $F_{\mu\nu}$ is the gauge field-strength tensor (a matrix in the group space indices) and ξ parametrizes a particular background gauge condition. To solve the spin-1 propagator equation (3.76) for an arbitrary self-dual or anti-self-dual field-strength tensor $F_{\mu\nu}$, we introduce the notation [40]

$$\{X\}^{(\pm)}_{\mu\nu} = q^{(\pm)}_{\mu\nu\lambda k} D_\lambda X D_k, \tag{3.77}$$

for an arbitrary operator X. The numerical tensor $q^{(\pm)}_{\mu\nu\lambda k}$ is given by

$$q^{(\pm)}_{\mu\nu\lambda k} = \delta_{\mu\nu}\delta_{\lambda k} + \delta_{\mu\lambda}\delta_{\nu k} - \delta_{\mu k}\delta_{\nu\lambda} \pm \epsilon_{\mu\nu\lambda k}. \tag{3.78}$$

In terms of the anti symmetrical symbol $\eta^{(\pm)}_{\mu\nu a} = -\eta^{(\pm)}_{\nu\mu a}$ defined by $\eta^{(\pm)}_{kla} = \epsilon_{kla},\ \eta^{(\pm)}_{k4a} = \pm\delta_{ka}$, we also have

$$q^{(\pm)}_{\mu\nu\lambda k} = \delta_{\mu\lambda}\delta_{\nu k} + \eta^{(\pm)}_{\mu\lambda a}\eta^{(\pm)}_{\nu k a} \tag{3.79}$$

The bracket notation has several useful properties if the field-strength tensor $F_{\mu\nu}$ is self-dual (+) or anti-self-dual (−). Here they are:

$$\{\mathrm{b}f1\}^{(\pm)}_{\mu\nu} = D^2\delta_{\mu\nu} + 2F_{\mu\nu} \tag{3.80}$$

$$D_\mu \{X\}^{(\pm)}_{\mu\nu} = D^2 X D_\nu \tag{3.81}$$

$$\{X\}^{(\pm)}_{\mu\nu} D_\nu = D_\mu X D^2 \tag{3.82}$$

and

$$\{X\}^{(\pm)}_{\mu\sigma}\{Y\}^{(\pm)}_{\sigma\nu} = \{X D^2 Y\}^{(\pm)}_{\mu\nu} \tag{3.83}$$

To achieve a simpler notation we shall henceforth consider only self-dual fields and delete the superscript $(\pm)$. (The treatment of the anti-self-dual case is an obvious parallel.)

We express the first two pieces of the field operator

$$-D^2\delta_{\mu\nu} - 2F_{\mu\nu} + (1 - 2/\xi)D_\mu D_\nu + m^2\delta_{\mu\nu} \tag{3.84}$$

in terms of the bracket notation $\{\mathbf{1}\}^{(\pm)}_{\mu\nu}$ and the bracket divergence conditions [equations (3.81)- (3.82)] to conclude that

$$\bar{\mathbf{G}}_{m,\mu\nu} = \{\frac{1}{D^2}\frac{1}{-D^2+m^2}\}_{\mu\nu} - (\xi - 1)D_\mu \frac{1}{-D^2+\xi m^2}\frac{1}{-D^2+m^2}D_\nu \tag{3.85}$$

obeys

$$(-D^2\delta_{\mu\lambda} - 2F_{\mu\lambda} + (1 - 2/\xi)D_\mu D_\lambda + m^2\delta_{\mu\lambda})\mathrm{b}\bar{f}G_{m,\lambda\nu} = \{\frac{1}{D^2}\}_{\mu\nu} \tag{3.86}$$

The operator

$$Q_{\mu\nu} = \{\frac{1}{D^2}\}_{\mu\nu} \tag{3.87}$$

which appears in equation (3.86) is a projection operator into the subspace of all the nonzero modes of the spin-1 fluctuations. Using the properties of the bracket notation it is a simple matter to prove that $Q_{\mu\nu}$ is indeed a projection operator,

$$Q_{\mu\lambda}Q_{\lambda\nu} = Q_{\mu\nu} \tag{3.88}$$

A zero-mode potential $\phi_\mu^{(0)}$, as we said above, obeys the background gauge condition

$$D_\mu \phi_\mu^{(0)} = 0 \tag{3.89}$$

and produces a self-dual field-strength fluctuation

$$f_{\mu\nu} = D_\mu \phi_\nu^{(0)} - D_\nu \phi_\mu^{(0)} = \frac{1}{2}\epsilon_{\mu\nu\lambda k} f_{\lambda k} \tag{3.90}$$

Since the symbols $\eta_{\mu\nu a}^{(-)}$ with $a = 1,\ 2,\ 3$ form three independent anti-self-dual tensors, this constraint can be written as

$$\eta^{(-)} D_\mu \phi_\nu^{(0)} = 0 \tag{3.91}$$

Therefore, the expression (3.100) quoted above for the numerical tensor $q_{\mu\nu\lambda k}$ shows that the bracket notation of any operator is orthogonal to any zero mode,

$$\phi_\mu^{(0)} \{X\}_{\mu\nu} = 0 = \{X\}_{\mu\nu} \phi_\nu^{(0)} \tag{3.92}$$

In particular, the projection operator $Q_{\mu\nu}$ is orthogonal to a zero mode,

$$\phi_\mu^{(0)} Q_{\mu\nu} = 0 = Q_{\mu\nu} \phi_{nu}^{(0)} \tag{3.93}$$

It follows from equation (3.89) and (3.90) that a zero mode satisfies the field equation

$$(D^2 \delta_{\mu\nu} + 2F_{\mu\nu}) \phi_\nu^{(0)} = 0 \tag{3.94}$$

On the other hand, the properties of the bracket notation give

$$(D^2\delta_{\mu\lambda} + 2F_{\mu\lambda})Q_{\lambda\nu} = \{\mathbf{1}\}_{\mu\lambda}\{\frac{1}{D^2}\}_{\lambda\nu} = \{\mathbf{1}\}_{\mu\nu} = D^2\delta_{\mu\nu} + 2F_{\mu\nu} \qquad (3.95)$$

which implies that $Q_{\mu\nu}$ contains all the non-zero modes. The zero modes must be removed from the spin-1 propagator if a well-defined massless limit is to be achieved. Therefore, equation (3.86) is the proper Greens function equation in the massless case, and the massless propagator is given by

$$\bar{\mathbf{G}}_{(0)\mu\nu} = \lim_{m\to 0} \bar{\mathbf{G}}_{m,\mu\nu} = -\{(\frac{1}{D^2})^2\}_{\mu\nu} - (\xi - 1)D_\mu(\frac{1}{D^2})^2 D_\nu \qquad (3.96)$$

The propagator displayed in equation (3.96) suffers from a logarithmic infrared divergence. The infrared divergence arises from the large spatial regions in the convolution integral that defines the product of the two massless propagators $(1/D^2)^2$. This trouble motivates the introduction of a mass in the vector propagator. We could use the massive propagator $\bar{\mathbf{G}}_{m,\mu\nu}$ which is orthogonal to the zero modes, but it is also a simple matter to construct the propagator $\mathbf{G}_{m,\mu\nu}$ defined by equation (3.76) which has the full identity $\delta_{\mu\nu}$ on its right-hand side, not the projection $Q_{\mu\nu}$ into the non-zero modes. To effect this construction, we note that

$$P_{\mu\nu} = \delta_{\mu\nu} - Q_{\mu\nu} = \delta_{\mu\nu} - \{\frac{1}{D^2}\}_{\mu\nu} \qquad (3.97)$$

projects into the zero-mode subspace, with

$$[-D^2\delta_{\mu\lambda} - 2F_{\mu\lambda} + (1 - \frac{1}{\xi})D_\mu D_\lambda]P_{\lambda\nu} = 0 \qquad (3.98)$$

Hence, equation (3.86) implies that

$$[-D^2\delta_{\mu\nu} - 2F_{\mu\nu} + (1 - 2/\xi)D_\mu D_\nu + m^2\delta_{\mu\nu}](\bar{\mathbf{G}}_{m,\lambda\nu} + \frac{1}{m^2}P_{\lambda\nu}) = \delta_{\mu\nu}, \qquad (3.99)$$

and we conclude that the propagator $\mathbf{G}_{m,\mu\nu}$ defined by equation (3.76) is given by

$$\mathbf{G}_{m,\mu\nu} = \bar{\mathbf{G}}_{m,\mu\nu} + \frac{1}{m^2} P_{\mu\nu} \tag{3.100}$$

The product of spin-0 propagators of different masses that appear in the definition (3.85) of the operator $\bar{\mathbf{G}}_{m,\mu\nu}$ corresponds to a convolution integral of Greens functions. This convolution is trivially performed, however, by simple operator identities such as

$$\frac{1}{D^2}\frac{1}{-D^2+m^2} = \frac{1}{m^2}(\frac{1}{D^2} + \frac{1}{-D^2+m^2}) \tag{3.101}$$

Hence, using also equations (3.86) and (3.97), we may rewrite the result (3.100) as

$$\mathbf{G}_{m,\mu\nu} = \frac{1}{m^2}\{\frac{1}{-D^2+m^2}\}_{\mu\nu} + \frac{1}{m^2}\delta_{\mu\nu} +$$

$$+\frac{1}{m^2} D_\mu (\frac{1}{-D^2+\xi m^2} - \frac{1}{-D^2+m^2}) D_\nu \tag{3.102}$$

This expresses the propagator $\mathbf{G}_{m,\mu\nu}$ directly in terms of covariant derivative operators acting upon massive spin-0 propagators.

The precise definition of the spin-1 propagator depends upon the particular form of the functional integral used to determine the quantum transition amplitudes. Recently it has been suggested that the zero modes be treated in the functional integral in the same manner as are the gauge degrees of freedom which are, in fact, special types of zero modes. The effect of this uniform treatment is to alter the spin-1 field operator:

$$-D^2\delta_{\mu\nu} - 2F_{\mu\nu} + (1 - 2/\xi) D_\mu D_\nu + m^2 \delta_{\mu\nu} \rightarrow \tag{3.103}$$

$$-D^2\delta_{\mu\nu} - 2F_{\mu\nu} + (1 - 2/\xi) D_\mu D_\nu + m^2 \delta_{\mu\nu} + \lambda \phi^{(0)}_\mu \phi^{(0)}_\nu \tag{3.104}$$

Here, for the sake of simplicity, we have assumed that only a single zero mode $\phi_\mu^{(0)}$ is present; in general a sum of such zero-mode terms will appear. With only a single (normalized) zero mode present in the theory,

$$\phi_\mu^{(0)}\phi_\nu^{(0)} = P_{\mu\nu}, \tag{3.105}$$

and the new propagator $\mathbf{G}^{(\phi)}_{m,\mu\nu}$ is defined by

$$[-D^2\delta_{\mu\sigma} - 2F_{\mu\sigma} + (1-2/\xi)D_\mu D_\sigma + m^2\delta_{\mu\sigma} + \lambda P_{\mu\sigma}]\mathbf{G}^{(\phi)}_{m,\sigma\nu} = \delta_{\mu\nu} \tag{3.106}$$

Since $\bar{\mathbf{G}}_{m,\mu\nu}$ is orthogonal to the zero mode, equation (3.100) implies that

$$P_{\mu\sigma}\mathbf{G}_{m,\sigma\nu} = \frac{1}{m^2}P_{\mu\nu} \tag{3.107}$$

Therefore, in view of equation (3.98), we find that the new propagator $\mathbf{G}^{(\phi)}_{m,\mu\nu}$ is simply related to the old propagators.

3.2.2 Real gluon dynamical mass in thermal QCD

Now we will find gluon field propagator. For this we have to expand $L_{QCD}(\bar{A}+B)$ over the fluctuations B_μ^a around background $\bar{A}_\mu{}^a$.

$$L_{QCD}(\bar{A}+B) = \frac{1}{4g^2}G^a_{\mu\nu}(\bar{A}+B)G_a^{\mu\nu}(\bar{A}+B) \tag{3.108}$$

Here

$$G^a_{\mu\nu}(\bar{A}+B) = \partial_\mu(\bar{A}+B)^a_\nu - \partial_\nu(\bar{A}+B)^a_\mu + gf^{abc}(\bar{A}+B)^b_\mu(\bar{A}+B)^c_\nu \tag{3.109}$$

$$G_a^{\mu\nu}(\bar{A}+B) = \partial^\mu(\bar{A}+B)^\nu_a - \partial^\nu(\bar{A}+B)^\mu_a + gf_{ade}(\bar{A}+B)^\mu_d(\bar{A}+B)^\nu_e \tag{3.110}$$

By multiplying and simplifying them, we get

$$4g^2L_{QCD} = G^a_{\mu\nu}G_a^{\mu\nu} = O(B^0) + O(B^1) + O(B^2) + ... \tag{3.111}$$

Firstly we will calculate the first term $O(B^0)$ which doesn't include field fluctuations B^a_μ.

$$O(B^0) = \partial_\mu A^a_\nu \partial^\mu A^\nu_a - \partial_\mu A^a_\nu \partial^\nu A^\mu_a + gf^{ade}\partial_\mu A^a_\nu \cdot A^\mu_d A^\nu_e - \partial_\nu A^a_\mu \partial^\mu A^\nu_a +$$

$$+\partial_\nu A^a_\mu \partial^\nu A^\mu_a - gf^{ade}\partial_\nu A^a_\mu \cdot A^\mu_d A^\nu_e + gf^{abc}A^b_\mu A^c_\nu \partial^\mu A^\nu_a - gf^{abc}A^b_\mu A^c_\nu \partial^\nu A^\mu_a +$$

$$+gf^{abc} \cdot gf^{ade} \cdot A^b_\mu A^c_\nu \cdot A^\mu_d A^\nu_e =$$

$$\partial_\mu A^a_\nu \{\partial^\mu A^\nu_a - \partial^\nu A^\mu_a + gf^{ade}A^\mu_d A^\nu_e\} - \partial_\nu A^a_\mu \{\partial^\mu A^\nu_a - \partial^\nu A^\mu_a + gf^{ade}A^\mu_d A^\nu_e\} +$$

$$+gf^{abc}A^b_\mu A^c_\nu \{\partial^\mu A^\nu_a - \partial^\nu A^\mu_a + gf^{ade}A^\mu_d A^\nu_e\} =$$

$$= \{\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu\}\{\partial^\mu A^\nu_a - \partial^\nu A^\mu_a + gf^{ade}A^\mu_d A^\nu_e\} =$$

$$= G^a_{\mu\nu}(A) G^{\mu\nu}_a(A) \tag{3.112}$$

In this way we may find $O(B^1)$, $O(B^2)$, ...the final result for them is as following.

$$O(B^1) = 4B^a_\mu D^{ab}_\nu G^{\mu\nu}_a \tag{3.113}$$

$$O(B^2) = \frac{1}{2}B^a_\mu(-D^{ac}_\rho \delta_{\mu\nu} - 2f_{acb}G^c_{\mu\nu} + D^{ac}_\mu D^{cb}_\nu)B^b_\nu \tag{3.114}$$

So the Lagrangian(3.108) takes the expended form over B

$$g^2 L_{QCD}(A+B) = \frac{1}{4}G^a_{\mu\nu}(A)G^{\mu\nu}_a(A) + B^a_\mu D^{ab}_\nu G^{\mu\nu}_a + \tag{3.115}$$

$$+\frac{1}{2}B^a_\mu(-D^{ac}_\rho \delta_{\mu\nu} - 2f_{acb}G^c_{\mu\nu} + D^{ac}_\mu D^{cb}_\nu)B^b_\nu + ... \tag{3.116}$$

here $D_\mu^{ab} = \partial_\mu \delta_{ab} + f_{acb} A_\mu^c$.If we choose background gauge $D_\nu^{ac} B_\nu^c = 0$ the last term of the $O(B^2)$-contribution vanishes. Now we can read the universe gluon propagator from the terms quadratic in B.

$$(S^{-1})^{ab}_{\mu\nu} = \frac{1}{g^2}(-D^{ac}_\rho \delta_{\mu\nu} - 2 f_{acb} G^c_{\mu\nu}) \tag{3.117}$$

We will also omit the $1/g^2$ in front of the propagator which is a result of the rescaling of fields anyway. Taking into account that $(\hat{P}_\mu)^{ab} = 2iG_{\mu\nu}, \quad \hat{P}_\mu = \hat{p}_\mu + A_\mu$, we can write

$$S^{-1}_{\mu\nu} = \hat{P}^2 \delta_{\mu\nu} + 2iG_{\mu\nu} + (1 - \frac{1}{\xi}) P_\mu P_\nu \tag{3.118}$$

In the instanton background A and in the case $G_{\mu\nu} = \tilde{G}_{\mu\nu}$, we may find the following form of the gluon propagator

$$S_{\mu\nu} = q_{\mu\nu\rho\sigma} P_\rho \tilde{\Delta}^2 P_\sigma - (1 - \xi) P_\mu \Delta^2 P_\nu \tag{3.119}$$

where

$$q_{\mu\nu\rho\sigma} = \delta_{\mu\nu}\delta_{\rho\sigma} + \delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho} + \epsilon_{\mu\nu\rho\sigma} \tag{3.120}$$

We will work at $\xi = 1$. We have also following notations:

$$A = \sum_i A_i, \qquad \Delta^{-1} = P^2 = \tilde{\Delta}^{-1} + \sum A_i A_j \approx \tilde{\Delta}^{-1} \tag{3.121}$$

$$G_{\mu\nu} = -i[(p_\mu + \sum A^i_\mu), (p_\nu + \sum A^j_\nu)] = \sum_i G^i_{\mu\nu} - i \sum ([A^i_\mu, p_\nu] +$$

$$+ [p_\mu, A^j_\nu] + [A^i_\mu, A^j_\nu]) \tag{3.122}$$

Using them we may write that

$$S^{-1}_{\mu\nu} = \hat{P}^2 \delta_{\mu\nu} + 2iG_{\mu\nu} = \tilde{\Delta}^{-1}\delta_{\mu\nu} + 2i \sum_i G^i_{\mu\nu} + \sum_{i \neq j} A_i A_j \delta_{\mu\nu} - \tag{3.123}$$

$$-i\sum([A^i_\mu, p_\nu] + [p_\mu, A^j_\nu] + [A^i_\mu, A^j_\nu]) \tag{3.124}$$

We define

$$\tilde{S}^{-1}_{\mu\nu} = \tilde{\Delta}^{-1}\delta_{\mu\nu} + 2i\sum_i G^i_{\mu\nu} \tag{3.125}$$

When we calculated gluon-like field propagator, we have seen how to expend this kind of propagator. We use that result in here too.

$$\tilde{S}_{\mu\nu} = S^0_{\mu\nu} + \sum_i (S^i_{\mu\nu} - S^0_{\mu\nu}) + \sum (S^i_{\mu\rho} - S^0_{\mu\rho}) {S^0_{\rho\sigma}}^{-1} (S^j_{\sigma\nu} - S^0_{\sigma\nu}) + ... \tag{3.126}$$

Here $S^0_{\mu\nu}$ is a free gluon propagator and $S^i_{\mu\nu}$ is a single instanton propagator.

$$S^0_{\mu\nu} = \Delta_0\delta_{\mu\nu}, \qquad S^i_{\mu\nu} = q_{\mu\nu\rho\sigma} P^i_\sigma \Delta^2_i P^i_\sigma \tag{3.127}$$

At first order on density we will get:

$$\bar{\tilde{S}}_{\mu\nu}^{\ -1} - {S^0_{\mu\nu}}^{-1} =< \sum_i \{S^0_{\mu\nu} + ({S^i}^{-1}_{\mu\nu} - {S^0_{\mu\nu}}^{-1})^{-1}\}^{-1} >= \tag{3.128}$$

$$= - < \sum_i {S^0_{\mu\rho}}^{-1} (S^i_{\rho\sigma} - S^0_{\rho\sigma}) {S^0_{\sigma\nu}}^{-1} >= - < \sum \Delta_0^{-1}\delta_{\mu\rho}(S^i_{\rho\sigma} - S^0_{\rho\sigma})\Delta_0^{-1}\delta_{\sigma\nu} >$$

$$= - < \sum {\Delta_0}^{-1}(S^i_{\mu\nu}\Delta_0\delta_{\mu\nu}){\Delta_0}^{-1} >= - < \sum_i p^2(q_{\mu\nu\rho\sigma}P^i_\rho\Delta^2_i P^i_\sigma - p^{-2}\delta_{\mu\nu})p^2 >$$

There is also other way to get this equation. For it we use eq. (3.118) and write the following

$$S_{\mu\nu} = (\hat{P}^2\delta_{\mu\nu} + 2iG_{\mu\nu})^{-1} = (S^{-1}_{0\ \mu\nu} + V_{\mu\nu})^{-1} = \tag{3.129}$$

$$= S_{0\ \mu\rho}(1 + V_{\mu\nu}S_{\rho\mu})^{-1} = S_{0\ \mu\rho}(1+T)^{-1}_{\rho\nu} = S_{0\ \mu\rho}(1 - T + T^2 - T^3 + ...)_{\rho\nu}$$

here

$$V_{\mu\nu} = (P^2 - p^2)\delta_{\mu\nu} + 2iG_{\mu\nu} = (A^2 + pA + Ap)\delta_{\mu\nu} + 2iG_{\mu\nu} \tag{3.130}$$

$$T_{\mu\nu} = V_{\mu\rho} S_{0\rho\nu} \tag{3.131}$$

Now we may average gluon propagator (3.130), taking into account only first order on density terms O(N) and neglect others.

$$< S_{\mu\nu} > = < S_{0\ \mu\rho}(1 - T + T^2 - T^3 + ...)_{\rho\nu} > =$$

$$= S_{0\ \mu\rho}(1 - < T > + < T^2 > - < T^3 > + ...)_{\rho\nu} \tag{3.132}$$

Using $< T^n > = N < T_i^n >$, we get

$$< S_{\mu\nu} > = S_{0\ \mu\rho}(1 - N < T_i > + N < T_i^2 > - N < T_i^3 > + ...)_{\rho\nu} =$$

$$= S_{0\ \mu\rho}(1 - N(\bar{T} - \bar{T}^2 + \bar{T}^3 - ...)_{\rho\nu} = S_{0\ \mu\rho}(1 - N < \frac{T_i}{1 + T_i} >)_{\rho\nu} =$$

$$= S_{0\ \mu\rho}(1 - N < T_{eff} >)_{\rho\nu} \tag{3.133}$$

Here we define $T_{eff} = T_i - T_i^2 + T_i^3 - ... = \frac{T_i}{1+T_i}$. Similarly, we define $T_{eff} = V_{eff} S_0$, and for V_{eff} we get the following equation

$$V_{eff} = V_i - V_i S_0 V_{eff} \tag{3.134}$$

$$V_{eff} = (1 + V_I S_0)^{-1} V_I = [1 + (S_I^{-1} - S_0^{-1}) S_0]^{-1} (S_I^{-1} - S_0^{-1}) =$$

$$= S_0^{-1}(S_0 - S_I) S_0^{-1} \tag{3.135}$$

with $S_I^{-1} = S_0^{-1} + V_I$ is a propagator in one instanton background Now we can write Pobylsta's equation for gluon field in this form

$$< S_{\mu\nu} >^{-1} = (S_{0\ \mu\rho}\{1 - N < T_{eff} >\}_{\rho\nu})^{-1} = (1 - N < T_{eff} >)^{-1}_{\mu\rho} S^{-1}_{0\ \rho\nu} \approx$$

$$\approx (1 + N < T_{eff} >)_{\mu\rho} S^{-1}_{0\ \rho\nu} = S^{-1}_{0\ \mu\nu} + N < T_{eff} >_{\mu\rho} \cdot S^{-1}_{0\ \rho\nu} =$$

$$= S_0{}^{-1}_{\mu\nu} + N < V_{eff} >_{\mu\nu} = S_0{}^{-1}_{\mu\nu} + N(S_0^{-1}(S_0 - S_i)S_0^{-1})_{\mu\nu} \tag{3.136}$$

The relation between gluon mass and gluon propagator is

$$\bar{S}^{-1}_{\mu\nu} = (p^2 + M^2(p))\delta_{\mu\nu} \tag{3.137}$$

Firstly we will consider two cases.

a. At large p ($p \gg \rho^{-1} = 600\ MeV$)

$$P = p + A \approx p \tag{3.138}$$

$$\Delta_i^{-1} = p^2 + (\{p, A_i\} + A_i{}^2) \approx p^2 = \Delta_0^{-1} \tag{3.139}$$

Then the gluon propagator is

$$\bar{S}^{-1}_{\mu\nu} - S^{0}_{\mu\nu}{}^{-1} = - < \sum_i p^2(q_{\mu\nu\rho\sigma} P^i_\rho \Delta^2_i P^i_\sigma - p^{-2}\delta_{\mu\nu})p^2 >= \tag{3.140}$$

$$= -N < p^2(q_{\mu\nu\rho\sigma} p_\rho \Delta_0{}^2 p_\sigma - p^{-2}\delta_{\mu\nu})p^2 >= N < q_{\mu\nu\rho\sigma} p_\rho p_\sigma - p^{-2}\delta_{\mu\nu} >= 0$$

It means $M(p \to \infty) \to 0$.

b. At small p ($p \ll \rho^{-1}$).

Now we must expend $\bar{S}(p)$ over small momentum.

$$\bar{S}^{-1}_{\mu\nu} - S^{0}_{\mu\nu}{}^{-1} = - < \sum_i p^2(q_{\mu\nu\rho\sigma} P^i_\rho \Delta^2_i P^i_\sigma - p^{-2}\delta_{\mu\nu})p^2 > \tag{3.141}$$

Small p corresponds to large x and for $x \gg \rho'$ ρ' is negligible. Then we have

$$\Delta^I_{ab} = \frac{\delta_{ab}}{4\pi^2(x-y)^2} - \rho^{'2}\left(\frac{\delta_{ab}}{4\pi^2 x^2 y^2} + \frac{2\epsilon_{abc}\bar{\eta}_{c\mu\nu}x_\mu y_\nu}{4\pi^2(x-y)^2 x^2 y^2}\right) +$$

$$+ \frac{2\rho^{'4}}{4\pi^2(x-y)^2} \frac{\delta_{ab}(-x^2y^2 + (xy)^2) + \bar{\eta}_{a\mu\nu}x_\mu y_\nu \bar{\eta}_{b\rho\sigma}x_\rho y_\sigma}{x^4 y^4} = \Delta^0_{ab} - \rho^{'2} W_{ab} + O(\rho^{'4})$$

here

$$W_{ab}(x,y) = \frac{\delta_{ab}}{4\pi^2 x^2 y^2} + \frac{2\epsilon_{abc}\bar{\eta}_{c\mu\nu}x_\mu y_\nu}{4\pi^2 (x-y)^2 x^2 y^2} \tag{3.142}$$

And also taking into account that $P_\mu = p_\mu + \rho^{'2}\tilde{A}_\mu + O(\rho^{'4})$, we may find $P^i_\mu \Delta^2_i P^i_\nu$

$$P^i_\mu \Delta^2_i P^i_\nu = (p_\mu + \rho^{'2}A_\mu + O(\rho^{'4}))(\Delta^0_{ab} - \rho^{'2}W_{ab} + O(\rho^{'4}))^2(p_\mu + \rho^{'2}A_\nu + O(\rho^{'4})) \approx$$

$$\approx (p_\mu + \rho^{'2}A_\mu)(\Delta_0^2 - \rho^{'2}(\Delta_0 W + W\Delta_0))(p_\mu + \rho^{'2}A_\nu) =$$

$$= p_\mu \Delta_0^2 p_\nu - \rho^2 p_\mu(\Delta_0 W + W\Delta_0)p_\nu + p_\mu \Delta_0 A^i_\nu + A^i_\mu \Delta_0 p_\nu \approx \tag{3.143}$$

$$\approx p_\mu \Delta_0^2 p_\nu - p_\mu(\Delta_0(\Delta^0 - \Delta_I) + (\Delta^0 - \Delta_I)\Delta_0)p_\nu + p_\mu \Delta_0 A^i_\nu + A^i_\mu \Delta_0 p_\nu$$

If we integrate A^i in color space, we get zero. For this we have to change $\bar{\eta}_{a\mu\nu} \to \bar{\eta}'_{a\mu\nu} O^{ab} \bar{\eta}_{b\mu\nu}$.

$$\int dO O^{ab} = 0$$

$$\bar{A}^{ai}_\mu = \int dO A^{ai}_\mu = \int dO O^{ab} \bar{\eta}_{b\mu\nu} \frac{x_\nu}{x^2} \frac{2\rho^{'2}}{x^2 + \rho^{'2}} = 0 \tag{3.144}$$

Our purpose to calculate averaged full gluon propagator $\bar{G}^{ab}_{\mu\nu}$ [17], at non-zero temperature. Most essential point here is the luck of relativistic covariance, since Euclidean time is bounded as $0 \le x_4 \le \beta$ and all the bosonic fields – background A_μ, the fluctuations a_μ and zero-modes ϕ_μ must be periodical like $A_\mu(\vec{x}, x_4 + \beta) = A_\mu(\vec{x}, x_4)$. The Eq. for the gluon propagator can be written in the operator form [17] (where the zero-modes problem was solved in operator form accordingly [15]). It is simplified our problem very much, since at the end we have just to calculate the matrix element of the operators between time state $|t' >$ and periodical state $|t_\beta >$ defined at Eq (3.20).

Then, we expect in the 3-momentum, Matsubara frequencies representation $\bar{G}^{ab}_{\mu\nu}(\omega_n, \vec{k}) = [(\omega_n^2+k^2)\delta_{ab}\delta_{\mu\nu}+\Pi^{ab}_{\mu\nu}(\omega_n, \vec{k})]^{-1}$ (we take here and in the following Eqs. The gauge fixing parameter $\xi = 1$). On the other hand, at first order on density n we can write the solution of the Pobylitsa Eq. for the polarization operator in the form

$$\Pi_{\rho\nu} = N {S^0}^{-1}_{\rho\sigma}(\bar{S}^I_{\sigma\mu} - {S^0}_{\sigma\mu}){S^0}^{-1}_{\mu\nu}. \quad (3.145)$$

where ${S^0}_{\mu\nu} = \delta_{\mu\nu}/p^2$ is the free gluon propagator and the single instanton gluon propagator [40] is given as

$$S^I_{\mu\nu} = q_{\mu\nu\rho\sigma} P^I_\rho \Delta^2_I P^I_\sigma, \quad (3.146)$$

where $q^I_{\mu\nu\rho\sigma} = \delta_{\mu\nu}\delta_{\rho\sigma} + \delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho} + \epsilon_{\mu\nu\rho\sigma}$.

"Electric" gluon mass is defined by Eq.

$$M^2_{el}(\vec{k}, T)\delta_{ab} = \Pi^{ab}_{44}(\vec{k}, n = 0).$$

We above expect that the most slowly decreasing part part of the matrix elements of $S^I_{\nu\mu} - {S^0}_{\nu\mu}$ will only contribute to M_{el}. In coordinate space comparing the effects from $i\partial_\mu$ with A^I_μ, we can conclude from Eq. (3.146) that the the most slowly decreasing part part of the $S^I_{\nu\mu} - {S^0}_{\nu\mu}$ in Eq. (3.145) comes from

$$p_\rho(\textit{the most slowly decreasing part of } (\Delta_I - \Delta_0)\Delta_0 + \Delta_0(\Delta_I - \Delta_0))p_\sigma$$

and only this term will contribute to M_{el}. Comparing it with Eq. (3.33), we conclude that $M^2_{el}(\vec{k}, T) = 2M^2_s(\vec{k}, T)$, where T and q dependencies are represented by Fig.3.7. Using the phenomenological values of $\bar{\rho}$ and n at $T = 0$, we obtain $M_{el}(0, 0) = 362\, MeV$.

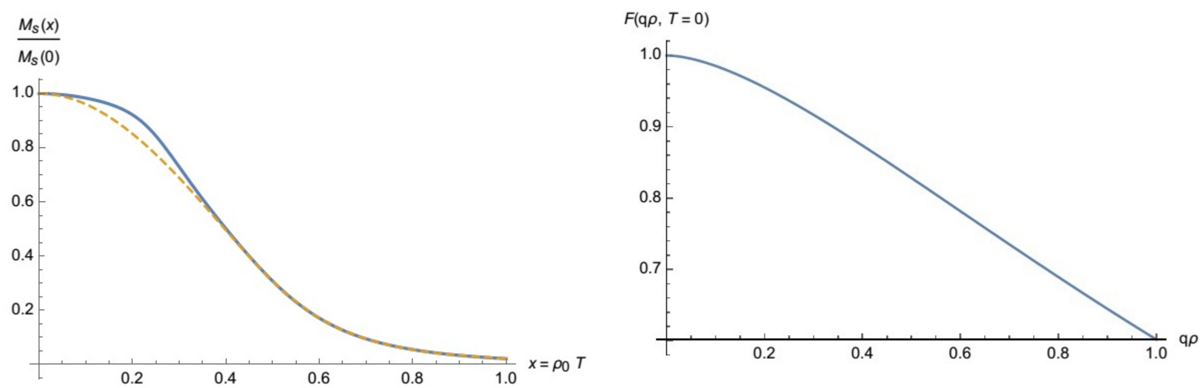


Figure 3.7: Full line on the left side – temperature dependencies of "electric" and "scalar" gluon dynamical masses $M_{el}(0,T)/M_{el}(0,0) = M_s(0,T)/M_s(0,0)$ (since $M_{el}(q,T) = 2^{1/2}M_s(q,T)$) with modification $A_{N_c} \to A_{N_c}\Theta_{\Delta x}(x - x_c)$ (see Eqs.(2.5, 2.19)) to interpolate between no suppression below and full suppression above $T_c = 150\ MeV$, with a width $\Delta T = 0.3\,T_c$ [12]. At small $T \leq T_c$ full line correspond to the $M_{el}(0,T)/M_{el}(0,0) = \bar{\rho}^{'}(T)/\bar{\rho}(T) = (1 - 1/6\,\pi^2\bar{\rho}_0^2T^2)$. Dashed line here correspond to the full suppression at the whole region of T (A_{N_c} is not modified). Here $M_{el}(0,0) = 2^{1/2}M_s(0,0) = 362\,MeV$ at the phenomenological values of $\bar{\rho}(0) = 1/3\,fm$ and $n(0) = 1\,fm^{-4}$. Right side – form-factor of dynamical mass $F(q,0)$, Eq. (3.63).

Conclusion

We extended the calculations of dynamical gluon mass in ILM [17] to non-zero temperature. In this case we are interesting by so-called "electric" gluon mass $M_{el}(q,T)$, which is correspond Π_{44} component of polarization operator. First, we have to find main parameters of ILM – averaged instanton size $\bar{\rho}(T)$ and instanton density $n(T)$ and its dependencies on temperature T. In ILM they are falling functions of temperature due to influence of thermal gluon fluctuations [8]. On the other hand, lattice investigations demonstrated that $\bar{\rho}(T)$, $n(T)$ are almost constant till critical temperature T_c and fast falling ones at $T \geq T_c$ [7]. We took into account this scenario by neglecting of thermal gluon fluctuations contribution at low temperature $T \leq T_c$ [12]. The comparison both of these scenarios is presented at the Fig.2.1. In order to find gluon propagator in the ILM background field at $T \neq 0$ we have to solve gluon zero-modes problem and to average full gluon propagator over collective coordinates of all instantons. It were done by extension of our previous work [17] to non-zero temperature case. First, we found 3-momentum and temperature dependent scalar "gluon" dynamical mass $M_s(q,T)$. The solution of zero-modes problem leads to the relation $M_{el}^2(q,T) = 2M_s^2(q,T)$. Finally, we have "electric" gluon dynamical mass $M_{el}(q,T)$ presented at the Fig.3.7. The result is in agreement with the values calculated by Cornwall

$(500 \pm 200 MeV)$ [4] or extracted from pp scattering $(370 MeV)$ [41]

It is interesting to compare our result with the result of lattice calculations of dynamical "electric" gluon mass, where it was observed that $M_{el}(0,T)$ is a decreasing function of T for $T \leq T_c$, and is an increasing function of T above the confinement-deconfinement phase transition [1]. At $T \geq T_c$ the $M_{el}(0,T)$ follows the expected perturbative functional dependence.

So, since thermal gluon corrections gives a rising with temperature contribution $M_{pert,el}(0,T) \sim T$, there is easy possibility to reproduce within ILM lattice measurements of the dynamical "electric" gluon mass. In this work my personal contribution was averaging $\Delta_{2,\mu\nu}(x,y)$ and we work with Nurmukhammad to calculate $\Delta_{1,\mu\nu}(x,y)$ [18]. $\Delta_{0,\mu\nu}(x,y)$ was calculated with my supervisor. Finally, we conclude that only singular part in $\Delta(x,y)$ has significant contribution to the "Electrical" mass We assume to apply our result to the calculations of temperature dependencies of the heavy quarkonium properties.

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