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GEOMETRICAL AND TOPOLOGICAL PROPERTIES OF SUBSPACES OF THE SPACE $P(X)$ OF PROBABILITY MEASURES THAT ARE MANIFOLDS

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Abstract. We study subspaces of the space $P(X)$ of probability measures that are finite-dimensional and infinite-dimensional topological manifolds. Studying various properties of the subspaces of the space $P(X)$ of probability measures, the following are proved: for any closed subset A of the compactum X other than X itself, there exists a strong deformation retraction $r: P(X) \to P(A)$, for any infinite X and any of its closed subset $A \subset X$ other than X itself, subspace $P(A)$ is barycentrically open, for any compact set X and for any $n \in N$ compact set $P_n(X)$ is FAR compact, for any infinite compact set X and any of its closed subset $A \subset X$ other than X, subspace $S_P(A) \setminus P(A)$ is homeomorphic to the Hilbert space ℓ_2, for any infinite compact set X and any of its open subset $A \subset X$ other than X, the subspace $S_P(X \setminus A) \setminus P(X \setminus A)$ is homeomorphic to the Hilbert space ℓ_2, for any compact set X and for any $n \in N$ the factor space $P(X)|_{P_n(X)}$ is ANR compact and the projection $P: P(X) \to P(X)|_{P_n(X)}$ is a homotopy equivalence, for any compact set X and for any $n \in N$ the following conditions are equivalent: a) $P_n(X)$ shape is dominated by some FAR compact; b) $P_n(X)$ has a point shape; c) Any map $P_n(X)$ into an arbitrary ANR-compactum is homotopic to zero; d) $P_n(X)$ is a fundamental retract of the Hilbert brick Q; e) $P_n(X)$ is a movable compact set that is approximately connected in all dimensions; f) $P_n(X)$ is approximately connected in the class of all polyhedra; g) $P_n(X)$ has finite fundamental dimension and is approximately connected in all dimensions; h) $P_n(X)$ is the limit of the inverse spectrum of FAR-compact sets; i) $P_n(X)$ is cell-like, i.e. $P_n(X)$ can be nested in cells into Q; j) There is a sequence of Hilbert bricks $Q_n \subseteq Q$ such that $Q_{n+1} \subseteq \operatorname{int} Q_n$ and $\bigcap_{n=1}^{\infty} Q_n = P_n(X)$.

In particular, it is proved that the subspace $P_{n,n-1}(X)$ consisting of all probability measures whose supports consisting of exactly n points of the space $P_n(X)$ in a certain zero-dimensional infinite compactum X are $(n-1)$-dimensional manifolds.

Keywords: *probability measures, manifolds, gomotopy dense, fundamentaly absolute retract, shape, movable compact, dominated.*

AMS subject classification: 54B15, 54B30, 54B35, 54C05, 54C15, 54C60, 54O30.

Introduction

The space $P(X)$ of all probability measures of a compactum X is the set of all regular Borel probability measures on X, equipped with the weakest of the topologies, for which each functional $f_u : C(X) \to R$ is continuous, taking measure μ to $\mu(U)$ ($U -$ is an open set into X). It is known that the space $P(X)$ of probability measures of any infinite metric compactum X is homeomorphic to the Hilbert cube $Q = I^{\chi_0}$. It is also known that for any $\chi_1 -$ degree of a non-one-point compactum the K space of probability measures $P(K^{\chi_1})$ is homeomorphic to the Tikhonov cube I^{χ_1}. $P(K^{\chi_1}) \cong I^{\chi_1}$, $I -$segment [0,1]. Note, in particular, that all these spaces are topologically homogeneous. And for spaces $P(K^{\chi})$ at $\tau > \chi_1$ the situation is different [1-4].

For an arbitrary compact set X and measure $\mu \in P(X)$, its support $\operatorname{supp}(\mu) -$is defined, this is the smallest of the closed sets $F \subset X$, for which $\mu(F) = \mu(X)$, i.e. $\operatorname{supp}(\mu) = \bigcap \{A : A \subset X, A = \bar{A}, \mu \in P(A)\}$.

$$P_n(X) = \{\mu \in P(X) : |\operatorname{supp} \mu| \le n\}$$

$P_\omega(X) = \bigcup_{n=1}^{\infty} P_n(X) -$ is the set of all probabilistic measures with finite supports.

We recall that space $P_f(X) \subset P(X)$ consists of all probability measures

$$\mu = m_1\delta(x_1) + m_2\delta(x_2) + \ldots + m_n\delta(x_n)$$

with finite supports, for each of which $m_i \ge \dfrac{n}{n+1}$ for some i.

$$P_{f,n}(X) = \{\mu \in P_f(X) : |\operatorname{supp} \mu| \le n\}.$$

Obviously, for a metric compactum X and any $n \in N$ the sets $P_n(X)$ are closed in $P(X)$, $P_f(X)$ and $P_{f,n}(X)$ are closed in $P(X)$ [6-8].

Therefore, the subspaces $P_\omega(X) \subset P(X)$ and $P_\omega(X)$ are everywhere dense in $P_\omega(X)$. Consequently, the compactum $P_f(X)$ is the union of the compacta $P_{f,n}(X)$ and $P_f(X)=\bigcup_{n=1}^{\infty}P_{f,n}(X)$. Obviously, $P_{f,n}(X)\subseteq \mathrm{P}_n(X)$ and $P_f(X)\subseteq P_{\omega}(X)$ [9-11].

Recall that a topological space Y is called an absolute / neighborhood / retract in the class K / is written $Y\in A(N)R(K)$ if $Y\in K$ and for any homeomorphism h mapping Y to a closed subset $h(Y)$ of a space X from the class K, the set $h(y)$ is a retract / neighborhood / of X [4] ...

Definition [4]. A topological space X is called a manifold modeled on space Y, or $Y-$a manifold if every point of space X has a neighborhood homeomorphic to an open subset of space Y.
It follows from the results of Keller-Cayley [2] that the space $P(X)$ of probability measures on an infinite compact set X is homeomorphic to a Hilbert brick Q'; where $Q'=\prod_{n=1}^{\infty}[0,\frac{1}{2^n}]$.

A Q-manifold is a separable metric space locally homeomorphic to a Hilbert cube Q,, where $Q=\prod_{i=1}^{\infty}[-1,1]_i$ Hilbert cube $W_i^{\pm}=\{(g_j)\in Q\,|\,g_i=\pm 1$ The $i-$rd face of the Hilbert cube Q, $BdQ=\bigcup_{i=1}^{\infty}W_i^{\pm}-$ is called the pseudo-boundary of the cube Q, and $S=Q\setminus BdQ-$ is the pseudo-interior of the cube Q [5].

In the theory of infinite-dimensional manifolds, three objects play an important role: the Hilbert cube Q, the separable Hilbert space ℓ_2, and $\sum-$ the linear hull of a standard brick Q' in the Hilbert space ℓ_2. By the Anderson-Kadets theorem, ℓ_2 is homeomorphic to S. It follows from the results of Bessagi-Pelchinski that $\sum$ is homeomorphic to $\mathrm{rint}Q$ [5]. Here, $\mathrm{rint}Q$ denotes the set $\{x=(x_n)\in Q\,|\,|x_n|<t<1$ for all $n\in N\}$. Further, $\mathrm{rint}\,Q\approx BdQ$ [5] means $BdQ\approx\sum$.. By ℓ_2^f we denote the linear subspace of the Hilbert space ℓ_2, consisting of all points, only a finite number of coordinates of which are different from zero.

It is known that spaces $Q,\sum$ and ℓ_2 are strongly infinite-dimensional, and space ℓ_2^f is weakly infinite-dimensional, and these spaces are homogeneous [5], where ℓ_2 is a Hilbert space; $Q=\prod_{i=1}^{\infty}[-1,1]_i$ $-$Hilbert cube, $\sum-$linear shell of standard Hilbert brick Q' to ℓ_2; $Q'=\prod_{i=1}^{\infty}[-1,\frac{1}{2^i}]$, , Q^f $-$subspace Q of the Hilbert cube, consisting of all points, only a finite number of coordinates which are different from zero.

The closed set A of the space X is called the Z-set in X if the identity mapping id_X of the space X can be approximated arbitrarily closely by the mappings $f:X\to X\setminus A$.

A countable union of Z-sets in X is called a $\sigma-Z-$set in X .

Following [4]. $\sigma-Z-$is a set, the Hilbert cube Q is called a boundary set in Q / is denoted by $B(Q)$ / if $Q\setminus B\approx\ell_2$. More generally, a boundary set in a Q-manifold is $\sigma-Z-$a set whose complement is ℓ_2- a manifold.

From the above it follows that the pseudo-boundary is BdQ. the Hilbert cube Q is the boundary set for the Hilbert cube Q.

Let X be a topological space. A set $A \subset X$ is called homotopy dense in X [5] if there exists a homotopy $h(x,t): X \times [0,1] \to X$ such that $h(x,0) = id_X$ and $h(X \times (0,1]) \subset A$.

A a set $A \subset X$ is homotopically negligible in X if $X \setminus A$ is homotopically dense in X.

Embedding $e: Y \to X$ is homotopically dense (respectively, homotopically negligible) if $e(Y)$ is a homotopically dense set (correspondingly homotopically negligible) in X.

The missing geometric and topological concepts and generally accepted notation related to the functor of $P: Comp \to Comp$ probability measures of its subfunctors can be found in [1,5-9], the general properties of acting in category $Comp$ in [1], dimensional properties of topological spaces and notations can be found in [2-5].

For each infinite compactum $X \in Comp$ and a normal (or seminormal) functor $F: Comp \to Comp$ of infinite degree, following the Zarichny M.M. [13] we take the following designations:

a) $F_{\nabla}(X) = F(X) \setminus \eta_F(X)$;

b) $F_{\nabla n}(X) = F(X) \setminus F_n(X)$, for $n = 1$ we identify $F_{\nabla_1}(X) \cong F_{\nabla}(X)$.

c) $F_{nk}(X) = F_n(X) \setminus F_k(X), n > k, n \geq 2$;

d) $F_{\omega}(X) = \bigcup_{n=1}^{\infty} F_n(X)$;

e) $F_{\nabla\omega}(X) = F(X) \setminus F_{\omega}(X)$;

f) $F_{\omega n}(X) = F_{\omega}(X) \setminus F_n(X)$.

Obviously, the subspaces $F_{\nabla}(X)$ and $F_{\nabla n}(X)$ are open at $F(X)$, $F_{n,k}(X)$ open at $F_{n,k}(X)$, $F_{\omega}(X)$ is a countable union of compact sets in $F(X)$, i.e. $F_{\omega}(X) - \sigma -$ is compact, the other side $F_{\omega}(X)$ is everywhere dense in $F(X)$; and $F_{\nabla\omega}(X)$ is $F_{\sigma} -$ subspace of space $F(X)$, subspace $F_{\omega n}(X)$ is open set in $F_{\omega}(X)$.

Let X be a zero-dimensional infinite compact, then it is known that $P(X) \cong Q$.

Proposition 1. For any closed subset A of compactum X other than X itself, there is a strong deformation retraction $r: P(X) \to P(A)$.

Proof. Let X be an infinite zero-dimensional compact set $A \subset X$, $A \neq X$, $A = \overline{A}$. It is known that $P(A) \subset P(X)$ and $P(A)$ are $Z-$ sets in $P(X)$ [7]. If A is a finite set, then it is known that $P(A) \cong \sigma^{|A|-1}$ is a simplex of dimension $|A| - 1$. those. $P(A)$ is a $(|A| - 1)$-dimensional manifold. If the set A is infinite, then $P(A) \cong Q$ and $P(A) \subset P(X)$. In any case $P(A)$ is AR compact set lying in the Hilbert cube $P(X)$. It is known that a closed subset $A \subset X$ of a zero-dimensional compactum X different from X itself is a retract of a compactum X [3], that is, there is a continuous retraction $r: X \to A$.

Consider a continuous map $P(r): P(X) \to P(A)$. It is known that the functor P preserves retraction, therefore $P(r)$ will also be a retraction, i.e. points of set $P(A)$ remain fixed. Now we construct a deformation retraction $h(\mu,t): P(X)\times[0,1]\to P(A)$ by setting

$$h(\mu,t)=(1-t)\mu+t\cdot r(\mu)$$

where $t\in[0,1]$, $\mu\in P(X)$, $r(\mu)=P(r)(\mu)$.

If $t=0$, then $h(\mu,0)=(1-0)\mu+0\cdot r(\mu)=\mu$. those. $h(\mu,0)=id_{P(X)}$.

If $t=1$, then $h(\mu,1)=(1-1)\mu+1\cdot r(\mu)=r(\mu)\in P(A)$. This means that $P(A)$ has a strong deformation retraction. Proposition 1 is proved.

It follows from Proposition 1 and from the definition of barycentrically open sets [2].

Proposition 2. For any infinite X and any of its closed subset $A\subset X$ other than X itself, the subspace $P(A)$ is barycentrically open.

A space X is called a fundamental absolute retract [4] (notation: $X\in FAR$) if for each space containing X as a closed subset, the set X is a fundamental retract of the space X.

The fundamental sequence $r=\{r_k : X, X\}_{M.M}$ is called the fundamental space retraction X' by X (M or M), if $r_k(x)=x$ for each $x\in X$ and each $k=1,2,\ldots$.

If there is a fundamental retraction of X' to X to M, then X is called a fundamental retraction of X' to M.

Let X and Y be two compact sets lying in the spaces M and N, respectively, where $M,N\in AR$ [4]. A sequence of mappings $f_k : M\to N$, where $k=1,2,\ldots$ is called a fundamental sequence from X to Y, if for each neighborhood V (by a neighborhood of the set Y we mean everywhere such a set whose open kernel contains Y, i.e., $A^0=X\setminus(\overline{X\setminus A})-$is the interior-open kernel) of compact Y (in N) there is a neighborhood U of the set in M for which the homotopy is $\varphi_k : U\times[0,1]\to V$, $\varphi_k(x,0)=f_k(x)$, and $\varphi_k(x,1)=f_{k+1}(x)$ for all $x\in U$ and for almost all k.

Theorem 1. For any compactum X and for any $n\in N$ compactum $P_n(X)$ is FAR compact.

Proof. Let X be an arbitrary compact. If X consists of a finite number of points, then there is a number $n\in N$ such that $P_n(X)$ is homeomorphic to a $P(X)=\sigma^{|X|-1}$-simplex of dimension $|X|-1$. It is known that simplices are FAR-compact sets and, moreover, are AR compacta. Now let X be an infinite compact. It is known that $P(X)\cong Q$. In [6] for any $n\in N$ it was shown that $P_n(X)$ is $Z-$set in $P(X)$. On the other hand, the compact $P_n(X)$ consists of a linear combination of $m_1\delta_{x_1}+m_2\delta_{x_2}+\ldots+m_n\delta_{x_n}$ Dirac measures δ_{x_1}, where δ_{x_i} is the Dirac measure of the compact X at points x_i, $0\le m_i\le 1$ and $\sum_{i=1}^{n} m_i=1$. those. $P_n(X)=\{\sum_{i=1}^{n} m_i\delta_{x_i} : m_i\ge 0, m_i\le 1, \sum_{i=1}^{n} m_i=1$, $\delta_{x_i}\in\delta(X)-$space of Dirac measures$\}$.

In this case, it follows from Theorem 1 [4, §8, p. 217] that $P_n(X)$ is FAR compact. Theorem 1 is proved.

Theorem 1 and Theorem 1 [4, §8, p. 217] imply

Corollary 1. For any compact set X and for any $n\in N$ the following conditions are equivalent:

1) $P_n(X)$ shape is dominated by some FAR compact;

2) $P_n(X)$ has a point shape;

3) Any map $P_n(X)$ into an arbitrary ANR -compactum is homotopic to zero;

4) $P_n(X)$ is a fundamental retract of the Hilbert cube Q;

5) $P_n(X)$ is a movable compact set that is approximately connected in all dimensions;

6) $P_n(X)$ is approximately connected in the class of all polyhedra;

7) $P_n(X)$ has a finite fundamental dimension and is approximately connected in all dimensions;

8) $P_n(X)$ is the limit of the inverse spectrum of FAR-compact sets;

9) $P_n(X)$ is cell-like, i.e. $P_n(X)$ can be nested in cells in Q;

10) There is a sequence of $Q_n \subseteq Q$ Hilbert bricks such that $Q_{n+1} \subseteq \operatorname{int} Q_n$ and $\bigcap_{n=1}^{\infty} Q_n = P_n(X)$.

From Theorem 8.1. [4] and Corollary 1 implies

Corollary 2. For any closed subsets A and B of the infinite compactum X, the following holds:

a) $P_{\nabla n}(A)$ and $P_{\nabla n}(B)$ are homeomorphic.

b) $P(X)|_{P_n(A)} \cong P(X)|_{P_n(B)}$.

Here $P(X)|_{P_n(A)}$ is the space factor of spaces $P(X)$ to $P_n(A)$.

Theorem 2. For any zero-dimensional compactum X, subspace $P_{n,n-1}(X)$ of space $P(X)$.

a) $(n-1)-$dimensional variety:

b) $P_{n,n-1}(X)$ is homotopically dense in $P_n(X)$.

Proof. Let X be a zero-dimensional compactum.

a) If X consists of a finite number of points, then $P_n(X) \cong P(X)$, where $n = |X| - 1$, $P(X) = \sigma^{n-1}$. Then $P_{n,n-1}(X)$ is the interior of the simplex σ^{n-1}, i.e. $P_{n,n-1}(X) = \operatorname{int}\sigma^{n-1}$. It is known that $\operatorname{int}\sigma^{n-1} \cong R^{n-1}$.

b) Let X be infinite. First, we prove that $P_{n,n-1}(X)$ is homotopy dense in $P_n(X)$.

Take an arbitrary point $\mu \in P_{n,n-1}(X)$, i.e.

$$\mu = m_1\delta_{x_1} + m_2\delta_{x_2} + \ldots + m_n\delta_{x_n}$$

where, $m_i > 0$, $m_i < 1$, $\sum_{i=1}^{n} m_i = 1$, $x_1, \ldots, x_n$ are mutually different, since the support $\operatorname{supp}\mu$ of the measure μ consists of n points, i.e. $|\operatorname{supp}\mu| = n$. Since the compact set X is zero-dimensional, the closed set $\operatorname{supp}\mu \subset X$ by the result of [3] is a retract of the compact set X, that is, there is a continuous retraction $r(\mu): X \to \operatorname{supp}\mu$.

We construct the required homotopy $h(\mu, t): P_n(X) \times [0,1] \to P_n(X)$ by setting

$$h(\mu, t) = (1-t)\mu + t \cdot r'(\mu)$$

where $\mu \in P_n(X)$, $t \in [0,1]$, $r'(\mu) = P_n(r_\mu)(\mu)$. From the definition of retraction for any $\mu \in P(\mu)$, the carrier $\operatorname{supp} r'(\mu)$ consists of points $\operatorname{supp}\mu$, i.e. $|\operatorname{supp}\mu| = |\operatorname{supp} r'(\mu)|$.

If $t=0$, then $h(\mu,0)=(1-0)\mu+0\cdot r'(\mu)=\mu$ i.e. $h(\mu,0)=id_{P_n(X)}$.

If $t>0$, then $h(\mu,t)=(1-t)\mu+tr'(\mu)\in P_n(X)$, since $|\operatorname{supp} h(\mu,t)|=|\operatorname{supp}\mu|=|\operatorname{supp} r'(\mu)|=\{x_1,x_2,...,x_n\}$. those. for $t>0$, measure $h(\mu,t)\in P_{n,n-1}(X)$. Hence, the subspace $P_{n,n-1}(X)$ is homotopy dense in $P_n(X)$.

c) Now we show that the subspace $P_{n,n-1}(X)$ of $P_n(X)$ is R^{n-1}-manifolds, that is, $(n-1)-$dimensional manifolds.

It is known that $P_{n,n-1}(X)$ is an open subspace of $P_n(X)$.

Let's take $\mu\in P_{n,n-1}(X)$. Consider $\operatorname{supp}\mu$ the carrier of measure μ. Obviously, $\operatorname{supp}\mu$ consists of n points $\{x_1,x_2,...,x_n\}$, i.e. A $\mu\in\operatorname{int}\sigma^{n-1}(x_1,x_2,...,x_n)-(n-1)$-dimensional simplex whose vertices consist of points $\{x_1,x_2,...,x_n\}$, $\operatorname{int}\sigma^{n-1}(x_1,x_2,...,x_n)$ The interior of a $(n-1)$-dimensional simplex σ^{n-1} is homeomorphic to the space R^{n-1}. Obviously, $\operatorname{int}\sigma^{n-1}-(n-1)-$dimensional manifolds. For $\mu\in P_{n,n-1}(X)$, put $O(\mu)=\operatorname{int}\sigma^{n-1}$. This means that space $P_{n,n-1}(X)$ is $(n-1)$-dimensional manifolds. Theorem 2 is proved.

Remark. From the result of [12] any compact set X lying in a Hilbert space ℓ_2 is negligible in ℓ_2, then this is $\ell_2\setminus Y\approx\ell_2$.

It was shown in [7] that for any infinite compactum X and any closed subset $A\subset X$ other than X, subspace $S_P(A)$ of space $P(X)$ is homeomorphic to Hilbert space ℓ_2. Therefore, from the remark and the above, we obtain

$S_P(A)$ denotes the subspace $\{\mu\in P(X):\operatorname{supp}\mu\bigcap A\neq\varnothing\}$ of $P(X)$.

Theorem 3. For any infinite compactum X and any of its closed subset $A\subset X$ other than X, subspace $S_P(A)\setminus P(A)$ is homeomorphic to the Hilbert space ℓ_2.

Theorem 4. For any infinite compactum X and any of its open subset $A\subset X$ other than X, subspace $S_P(X\setminus A)\setminus P(X\setminus A)$ is homeomorphic to Hilbert space ℓ_2.

From Theorem 1 from Theorems 1 [4: §8 p. 217] we obtain

Theorem 5. For any compact set X and for any $n\in N$ the factor space $P(X)|_{P_n(X)}$ is ANR compact and the projection $P:P(X)\to P(X)|_{P_n(X)}$ is a homotopy equivalence.

We note in [6-9] it was shown that for any infinite compactum X and a functor of $P:Comp\to Comp$ probability measures, the subspaces:

1) $P_\nabla(X)$ are Q–manifolds;

2) $P_{\nabla n}(X)$ are also Q -manifolds;

3) $P_{n,k}(X)$- Q - are manifolds if $X-Q-$ are manifolds;

4) $P_\omega(X)$ is a boundary set at $P(X)$;

5) $P_\omega(X)$ there are ℓ_2^f-manifolds if X is finite-dimensional;

6) $P_{\omega}(X)$ there are $\sum$ - manifolds, if X there are Q - manifolds;

7) $P_{\nabla\omega}(X)$ there are ℓ_2 manifolds;

8) $P_{\omega n}(X)$ is ℓ_2^f -manifolds if X is finite-dimensional;

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