

THE PROBLEM OF MATHEMATICAL MODELING OF A VIBRATION PROTECTED ROD UNDER KINEMATIC EXCITATIONS

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Abstract. In this paper, we consider the problem of mathematical modeling of nonlinear vibrations of a rod with distributed parameters and elastic-dissipative characteristics of the hysteresis type, and also having a liquid section dynamic absorber under kinematic excitations. The method of obtaining the differential equations of motion of the system protected from vibrations under consideration in the work using the structural method - the method of bond graph. An expression is obtained for the transfer function of a vibration-protective system for analyzing the dynamics and stability of the system and evaluating the effectiveness of a dynamic absorber.

Keywords: Liquid section dynamic absorber, rod, hysteresis, bond graph, junctions, vibrations, mathematical modeling

1 Introduction

Particular attention is paid to the effective use of parts of machines and mechanisms, devices widely used in all areas of industry and technology by reducing harmful vibrations caused by the movement of their parts, identifying factors that time-proof working and taking measures to reduce them. In this regard, one of the urgent tasks is to scientifically substantiate the existing problems associated with the motion of mechanical systems, to study the dynamics, to choose the optimal parameters, to make the necessary recommendations. In solving such problems, the effect of using liquid section dynamic absorbers in reducing harmful vibrations of low-frequency systems is high.

In the problem of reducing the harmful vibrations of different types of mechanical systems, the correct approach to the distribution of internal energy, taking into account the properties of elastic dissipation, plays an important role in finding the correct solution to the problem. In many scientific works, the problems of complex expression of nonlinear functions expressing the properties of elastic dissipative properties in systems by the method of harmonic linearization have been studied under the influence of various external excitations.

In the article [1] studied the dynamic properties of the system hysteresis loop.

In the article [2] mathematically modeled hysteresis-type elastic dissipative characteristics of systems. The geometrical position of each parameter involved in the formula representing the hysteresis curve is shown graphically on this curve, and the change of the curve with the change of the parameters is analyzed.

The dissertation [3] presents analytical expressions of different types of elastic dissipative characteristics of the hysteresis type of systems, which are compared with each other. For the mathematical model of the elastic dissipative characteristic of the hysteresis type, the concept of the hysteresis operator is introduced, its definition, properties and classes are studied.

The work [4] presents general equations of motion, nonlinear conservative, nonlinear dissipative forces and mathematical expression of elastic dissipative characteristic of hysteresis type and application of averaging method for systems with elastic dissipative characteristic of hysteresis type.

In the article [5] the elastic dissipative characteristic of the system of hysteresis type is considered in parametric form. In this case, the parametric changes of the characteristic were analyzed numerically, that is, it was shown that each parameter change leads to changes in a certain part of the hysteresis loop.

In the article [6] the forced vibrations of viscoelastic systems are learned and discussed problem of mathematical modeling.

The article [7] presents nonaxisymmetric vibrations of systems, which have associated masses and hollows. Systems have been chosen as axisymmetric structures.

In the work [8] the study of the use and design of a console-type dynamic absorber in the reducing of vibrations of micro-mechanisms. On this basis, systems with six degrees of freedom are mathematically modeled and analyzed by solving solutions of the differential equation of motion when the external force is harmonic.

The article [9] shows the method of obtaining equations of normal form by the method of averaging and the expression of the determinant form of the characteristic equation in the form of the Jacobi determinant.

The article [10] studied the effect of a dynamic absorber with elastic dissipative characteristics of the hysteresis type which leads to unstable motion. It solves the problem of radically reducing the amplitudes that cause instability.

In the article [11] solved problem that investigation and choosing the optimal parameters of dynamic absorbers. Expression for optimal parameters is analyzed for a dissipative mechanical system.

Liquid section dynamic absorbers have been studied in the works [12, 13], and it is emphasized that unlike traditional dynamic absorbers, very low frequency mechanical systems give high efficiency in reducing the vibrational motions.

Systems consisting of liquids and solids as well as distributed parametric systems were mathematically modeled separately, their dynamics were studied and the results were analyzed [13].

The method of bond graph allows mathematical modeling of the mechanical systems under consideration, regardless of the complexity of the processes, through a single structural approach [14–21].

2 Formulation of the problem

We consider the transverse vibrations of a rod with an elastic dissipative characteristic of the hysteresis type in conjunction with a liquid section dynamic absorber under the influence of kinematic excitations (Figure 1). The elastic dissipative characteristic of the hysteresis type of the rod material is obtained according to the Pisarenko-Koginich hypothesis [22].

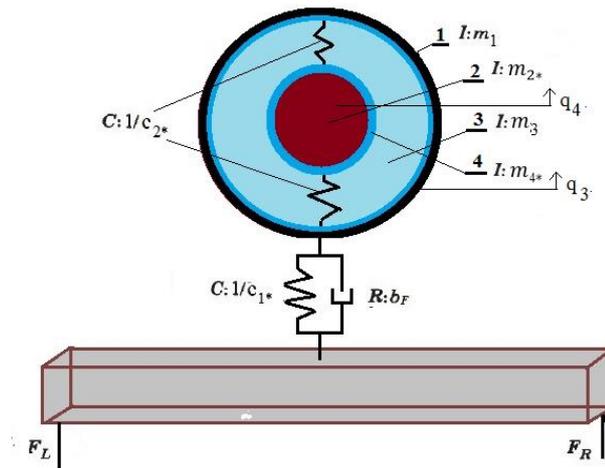


Figure 1. Schematic of a physical model of an elastic rod and liquid section dynamic absorber situated on it

We use the method of bond graph to mathematical model a system that is protected from the vibrations under consideration. To do this, we express the system parameters by the method of bond graph. In this case $I: (m_{1*})$ is the inertial dimension (mass) of the outer body of the dynamic absorber, which surrounds the liquid; $I: (m_{2*})$ is the inertial dimension (mass) of solid of a dynamic absorber; $I: (m_{3*})$ is the inertial dimension (mass) of liquid; $I: (m_{4*})$ is the inertial dimension of liquid attached to the body-2 (mass); $R: (b_F)$ is coefficient of resistance of damper (viscosity coefficient); $C: (c_{1*}^{-1})$ and $C: (c_{2*}^{-1})$ are compliances; $F_L(t)$ and $F_R(t)$ are external forces; assume that the effect of hydrodynamic force of a liquid on a solid-2 is F_S .

3 Method of solving

In the mathematical modeling of the system under consideration, the structural method - the method of bond graph and the method of harmonic linearization in the expression of nonlinear elastic dissipative characteristics is used.

Given that the method of bond graph is based on the exchange of power between the elements of mechanical systems, we determine the elements of the system with total velocities ($f_i = f, i = 1 \dots n$) and forces ($e_i = e, i = 1 \dots n$) and connect them to 1 and 0 junctions, respectively.

The shell liquid surrounding and the liquid have the same absolute velocity, so they are attached 1 junction has following property:

$$I: m_{3*} \leftarrow \mathbf{1} \rightarrow I: m_{1*} \Rightarrow I: m_{1*} + m_{3*}, (m_{1*} + m_{3*} = m_{13*}).$$

According to Archimedes' principle, a force equals to the weight $m_v g$ exerted by a solid surrounded by a liquid acts on it, and the body is balanced by gravity, elastic forces of the springs and inertial forces in the liquid. If we denote by F_1 and F_2 the inertial forces generated by Archimedes' principle in the shell surrounding the liquid and the solid surrounding the liquid, then these forces are considered as an external source attached to the 1 junction attached to the inertia of shell surrounding the liquid and the solid surrounding the liquid. As a result, the bond graph is as follows:

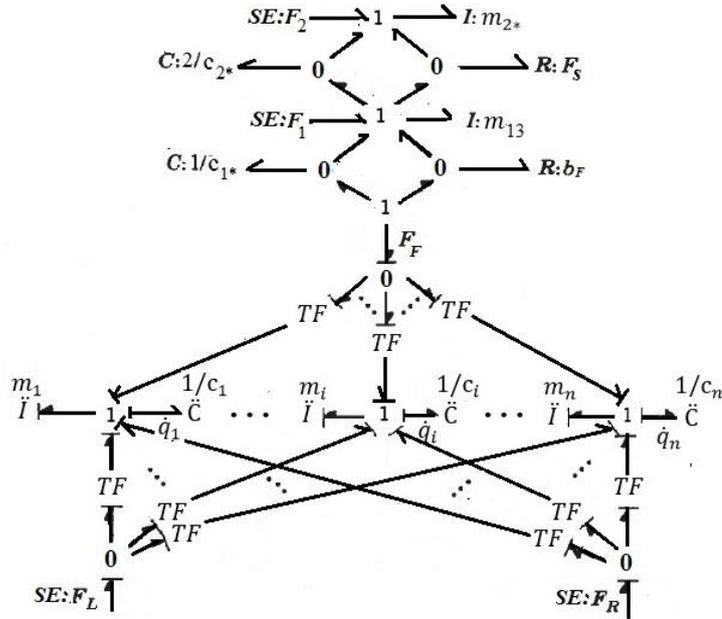


Figure 2. 0 and 1 junctions for a system consisting of a liquid section dynamic absorber with a rod under the influence of forces $F_L(t)$ and $F_R(t)$ at the left and right ends, respectively

where F_F is the force representing the interaction of the dynamic absorbers with the rod; \dot{q}_i are velocities ($i = 1 \dots n$); m_i and c_i are the modal mass and stiffness, expressed as follows ($i = 1 \dots n$):

$$m_i = \int_0^L \rho A u_i^2 dx; \quad (1)$$

$$c_i = \left[\int_0^L \rho A (1 + C_0 (-\eta_1 + j\eta_2)) u_i^2 dx + \frac{3EI}{\omega_{*i}^2} (-\eta_1 + j\eta_2) \times \sum_{i^*=1}^n C_{i^*} q_{ia}^{i^*} \frac{h^{i^*}}{2^{i^*} (i^* + 3)} \int_0^L u_i \frac{\partial^2}{\partial x^2} \left(\frac{\partial^2 u_i}{\partial x^2} \left| \frac{\partial^2 u_i}{\partial x^2} \right|^{i^*} \right) dx \right] \omega_{*i}^2, \quad (2)$$

A and ρ are the cross-sectional area and density of the rod; C_0, C_1, \dots, C_n are experimentally determined coefficients of the hysteresis loop, depending on the damping properties of the rod material [23]; E is Yong's module; I is moment of inertia; $q_{1a}, q_{2a}, \dots, q_{na}$ are amplitude values of rod vibration forms; h and ω_{*i} are the thickness and natural frequency of the rod; u_i is natural vibration forms;

$\eta_1, \eta_2 = \text{sign}(\omega) \eta_{22}$ are constant coefficients depending on the dissipative properties of the rod material, determined from the hysteresis loop, $\text{sign}(\omega)$ is the sign of ω , η_{22} is constant coefficient [22, 23]; L is length of the rod; $j^2 = -1$.

We define causality of junctions 0 and 1 for a system consisting of liquid section dynamic absorber and rod under the influence of forces $F_L(t)$ and $F_R(t)$, at the left and right ends, respectively, described in Figure 2. (Figure 3.). Given the determination of the energy variables p_m and q_m from the inertia and compliance elements and defining them as integral causality, then the compliance gives forces to the system and the other graphs in the 0 bonds to which they are attached give speed to the system. The elements of inertia give speed to the system, and

From this, it is possible to form a differential equation of motion for arbitrary i -sets. For convenience, we highlight the bond graph connected to the i -sets and number them accordingly as shown (Figure 5.).

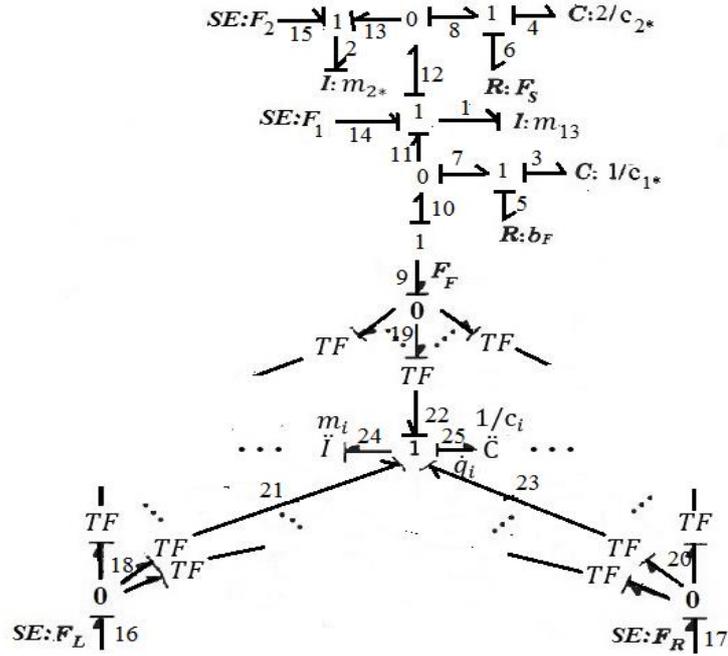


Figure 5. Bond graph for i -sets of a system consisting of a liquid section dynamic absorber with a rod

From the junctions 0 and 1 we obtain the following relations for the velocities and forces determined by the graphs numbered, respectively:

$$\begin{aligned}
 f_{21} &= f_{22} = f_{23} = f_{24} = f_{25}; & f_{23} &= u_i^{-1}(L)f_{20}; \\
 e_{24} &= e_{21} + e_{22} + e_{23} - e_{25}; & e_7 &= e_{10} = e_{11}; \\
 e_{18} &= e_{16}; & f_{10} &= f_7 + f_{11}; \\
 e_{19} &= e_9; & f_3 &= f_5 = f_7; \\
 e_{20} &= e_{17}; & e_7 &= e_3 + e_5; \\
 e_{10} &= e_9; & f_1 &= f_{11} = f_{12} = f_{14}; \\
 f_{10} &= f_9; & e_{11} &= e_1 + e_{12} - e_{14}; \\
 e_{21} &= u_i(0)e_{18}; & e_{12} &= e_{13} = e_8; \\
 f_{21} &= u_i^{-1}(0)f_{18}; & f_{12} &= f_8 + f_{13}; \\
 e_{22} &= u_i(x_1)e_{19}; & f_4 &= f_6 = f_8; \\
 f_{22} &= u_i^{-1}(x_1)f_{19}; & e_8 &= e_4 + e_6; \\
 e_{23} &= u_i(L)e_{20}; & f_{13} &= f_{15} = f_2; \\
 & & e_2 &= e_{13} + e_{15},
 \end{aligned} \tag{3}$$

where $u_i(0)$, $u_i(L)$ and $u_i(x_1)$ are the values of natural vibration forms of rod at the points $x = 0$, $x = L$ and $x = x_1$, they are the transformer modulus; x_1 is the point at which the liquid section dynamic absorber is installed.

We create expressions of the effect of elements to the system.

$$\begin{aligned}
f_{24} &= m_i^{-1} p_{24}; & e_9 &= F_F; \\
f_1 &= m_{13*}^{-1} p_1; & f_9 &= -\frac{\partial w_i(x_1, t)}{\partial t}; \\
f_2 &= m_{2*}^{-1} p_2; & e_3 &= c_{1*} q_3; \\
e_{25} &= c_i q_{25}; & e_4 &= 2c_{2*} q_4; \\
e_{14} &= F_1; & e_5 &= b_F \dot{q}_5 = b_F \dot{q}_7 = b_F (\dot{q}_{10} - \\
e_{15} &= F_2; & & \dot{q}_{11}) = b_F (\dot{q}_9 - \dot{q}_1) = \\
e_{16} &= F_L; & & b_F \left(-\frac{\partial w_i(x_1, t)}{\partial t} - m_{13*}^{-1} p_1 \right); \\
e_{17} &= F_R; & e_6 &= -F_S,
\end{aligned} \tag{4}$$

where $\frac{\partial w_i(x_1, t)}{\partial t}$ are the first-order derivatives of the rod vibration forms at points $x = x_1$, which represent the velocity of the rod point on which the liquid section dynamic absorber is situated.

We determine the expressions of the effect of the system to the elements of inertia and compliance.

$$\begin{aligned}
\dot{p}_{24} &= e_{24} = e_{21} + e_{22} + e_{23} - e_{25} = u_i(0)e_{18} + u_i(x_1)e_{19} + u_i(L)e_{20} - e_{25} = \\
&= u_i(0)e_{16} + u_i(x_1)e_9 + u_i(L)e_{17} - e_{25} = u_i(x_1)F_F + u_i(0)F_L + u_i(L)F_R - c_i q_{25}. \tag{5}
\end{aligned}$$

$$\begin{aligned}
\dot{p}_1 &= e_1 = e_{11} - e_{12} - e_{14} = e_7 - e_8 - e_{14} = e_3 + e_5 - e_4 - e_6 - e_{14} = \\
&= c_{1*} q_3 + b_F \left(-\frac{\partial w_i(x_1, t)}{\partial t} - m_{13*}^{-1} p_1 \right) - 2c_{2*} q_4 + F_S - F_1. \tag{6}
\end{aligned}$$

$$\dot{p}_2 = e_2 = e_{13} + e_{15} = e_8 + e_{15} = e_4 + e_6 + e_{15} = 2c_{2*} q_4 - F_S + F_2. \tag{7}$$

$$\dot{q}_{25} = f_{25} = f_{24} = m_i^{-1} p_{24}. \tag{8}$$

$$\dot{q}_3 = f_3 = f_7 = f_{10} - f_{11} = f_9 - f_1 = -\frac{\partial w_i(x_1, t)}{\partial t} - m_{13*}^{-1} p_1. \tag{9}$$

$$\dot{q}_4 = f_4 = f_8 = f_{12} - f_{13} = f_1 - f_2 = m_{13*}^{-1} p_1 - m_{2*}^{-1} p_2. \tag{10}$$

in the differential equation (5)

$$F_F = e_9 = e_{10} = e_7 = e_3 + e_5 = c_{1*} q_3 + b_F \left(-\frac{\partial w_i(x_1, t)}{\partial t} - m_{13*}^{-1} p_1 \right), \tag{11}$$

given that, we form a system of differential equations of state variables

$$\begin{aligned}
\dot{p}_{24} &= u_i(x_1)(c_{1*} q_3 + b_F \left(-\frac{\partial w_i(x_1, t)}{\partial t} - m_{13*}^{-1} p_1 \right)) + u_i(0)F_L + u_i(L)F_R - c_i q_{25}; \\
\dot{p}_1 &= c_{1*} q_3 + b_F \left(-\frac{\partial w_i(x_1, t)}{\partial t} - m_{13*}^{-1} p_1 \right) - 2c_{2*} q_4 + F_S - F_1; \\
\dot{p}_2 &= 2c_{2*} q_4 - F_S + F_2; \\
\dot{q}_{25} &= m_i^{-1} p_{24}; \\
\dot{q}_3 &= -\frac{\partial w_i(x_1, t)}{\partial t} - m_{13*}^{-1} p_1; \\
\dot{q}_4 &= m_{13*}^{-1} p_1 - m_{2*}^{-1} p_2.
\end{aligned} \tag{12}$$

This obtained system of first-order differential equations is a mathematical model of nonlinear vibrations of a rod with an elastic dissipative characteristic of the hysteresis type in conjunction with a liquid section dynamic absorber.

From the system of differential equations (12), it is possible to derive mathematical models of several problems as special cases. Including, if $m_{13*} \rightarrow \infty$, $m_{2*} \rightarrow \infty$, $c_{1*} \rightarrow 0$, $c_{2*} \rightarrow 0$, $b_F \rightarrow 0$ the limit is reached when $F_S = F_1 = F_2 = 0$ can be obtained from the differential equations of motion of a rod without a dynamic absorber under the influence of forces $F_L(t)$ and $F_R(t)$. D.C. Karnopp obtained this result for a rod with a linear elastic characteristic [15], and for a rod with an elastic dissipative characteristic of the hysteresis type was obtained by the method of bond graph [13].

$$\begin{aligned}
\dot{p}_{24} &= u_i(0)F_L + u_i(L)F_R - c_i q_{25}; \\
\dot{q}_{25} &= m_i^{-1} p_{24}.
\end{aligned} \tag{13}$$

If in the system of differential equations (12) $m_{3*} \rightarrow 0, m_{2*} \rightarrow \infty, c_{2*} \rightarrow 0$ the limit is reached, when $F_S = F_1 = F_2 = 0$, it is possible to obtain a system of differential equations of motion of rod with a dynamic absorber consists only of solid [2]. In this case, the last differential equation becomes that expressing the velocity of the load of mass m_{1*} , namely $\dot{q}_4 = \dot{q}_1 \equiv m_{1*}^{-1} p_1$. So,

$$\begin{aligned} \dot{p}_{24} &= u_i(x_1)(c_{1*}q_3 + b_F \left(-\frac{\partial w_i(x_1, t)}{\partial t} - m_{1*}^{-1} p_1 \right)) + u_i(0)F_L + u_i(L)F_R - c_i q_{25}; \\ \dot{p}_1 &= c_{1*}q_3 + b_F \left(-\frac{\partial w_i(x_1, t)}{\partial t} - m_{1*}^{-1} p_1 \right); \\ \dot{q}_{25} &= m_i^{-1} p_{24}; \\ \dot{q}_3 &= -\frac{\partial w_i(x_1, t)}{\partial t} - m_{1*}^{-1} p_1. \end{aligned} \quad (14)$$

Hydrodynamic force in the system of differential equations (12) as the inertial force of the fluid attached to a body of mass m_{2*} and the viscosity of the fluid with the coefficient b_S are given by [12]:

$$F_S = -(m_{4*}\ddot{q}_4 + b_S\dot{q}_4). \quad (15)$$

We express the inertial forces $F_1 = m_v\ddot{q}_2$ and $F_2 = m_v\ddot{q}_1$ as follows:

$$F_1 = m_v\ddot{q}_2 = \left\{ \begin{aligned} \ddot{q}_2 = \ddot{q}_{13} = \ddot{q}_{12} - \ddot{q}_8 = \ddot{q}_{11} - \ddot{q}_4 = \\ = \ddot{q}_{10} - \ddot{q}_7 - \ddot{q}_4 = \ddot{q}_9 - \ddot{q}_3 - \ddot{q}_4 \end{aligned} \right\} = m_v(\ddot{q}_9 - \ddot{q}_4 - \ddot{q}_3); \quad (16)$$

$$F_2 = m_v\ddot{q}_1 = \{ \ddot{q}_1 = \ddot{q}_{11} = \ddot{q}_{10} - \ddot{q}_7 = \ddot{q}_9 - \ddot{q}_3 \} = m_v(\ddot{q}_9 - \ddot{q}_3).$$

Putting expressions (15) and (16) into the system of differential equations (12), after making some simplifications, we obtain

$$A\ddot{Q} + B\dot{Q} + CQ = F, \quad (17)$$

where

$$\begin{aligned} \ddot{Q} &= \begin{bmatrix} \ddot{q}_i \\ \ddot{q}_3 \\ \ddot{q}_4 \end{bmatrix}; \quad \dot{Q} = \begin{bmatrix} \dot{q}_i \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix}; \quad Q = \begin{bmatrix} q_i \\ q_3 \\ q_4 \end{bmatrix}; \quad F = \begin{bmatrix} u_i(0)F_L + u_i(L)F_R \\ 0 \\ 0 \end{bmatrix}; \\ A &= \begin{bmatrix} m_i & 0 & 0 \\ (m_{13*} + m_{2*})u_i(x_1) & (m_{13*} + m_{2*}) & m_{2*} + m_v \\ (m_{2*} - m_v)u_i(x_1) & m_{2*} - m_v & m_{2*} + m_{4*} \end{bmatrix}; \\ B &= \begin{bmatrix} 0 & -u_i(x_1)b_F & 0 \\ 0 & b_F & 0 \\ 0 & 0 & b_S \end{bmatrix}; \quad C = \begin{bmatrix} c_i & -u_i(x_1)c_{1*} & 0 \\ 0 & c_{1*} & 0 \\ 0 & 0 & 2c_{2*} \end{bmatrix}. \end{aligned}$$

This system of differential equations (17) determined using the method of bond graphic is a mathematical model of motion of rod with an elastic dissipative characteristic of the hysteresis type and liquid section dynamic absorber.

We analyze the dynamics of the system under consideration using the system of differential equations defined (17). First, we will get the transfer functions of the system. To do this, we bring the system of differential equations (17) to the system of algebraic equations using the differential operator $S = \frac{d}{dt}$.

$$\begin{aligned} (m_i S^2 + c_i)q_i - u_i(x_1)(b_F S + c_{1*})q_3 &= u_i(0)F_L + u_i(L)F_R; \\ M_1 u_i(x_1) S^2 q_i + (M_1 S^2 + b_F S + c_{1*})q_3 + M_2 S^2 q_4 &= 0; \\ M_3 u_i(x_1) S^2 q_i + M_3 S^2 q_3 + (M_4 S^2 + b_S S + 2c_{2*})q_4 &= 0, \end{aligned} \quad (18)$$

where $M_1 = m_{13*} + m_{2*}$; $M_2 = m_{2*} + m_v$; $M_3 = m_{2*} - m_v$; $M_4 = m_{2*} + m_{4*}$.

We solve this system of equations on the basis of Cramer's rule with respect to the variables q_i, q_3, q_4 .

$$\begin{aligned} q_i(S) &= \frac{a_3(b_2 d_3 - b_3 d_2)}{a_1(b_2 d_3 - b_3 d_2) + a_2(b_3 d_1 - b_1 d_3)}; \\ q_3(S) &= \frac{a_3(b_3 d_1 - b_1 d_3)}{a_1(b_2 d_3 - b_3 d_2) + a_2(b_3 d_1 - b_1 d_3)}; \end{aligned} \quad (19)$$

$$q_4(S) = \frac{a_3(b_1d_2 - b_2d_1)}{a_1(b_2d_3 - b_3d_2) + a_2(b_3d_1 - b_1d_3)};$$

where $a_1 = m_i S^2 + c_i$; $a_2 = -u_i(x_1)(b_F S + c_{1*})$; $a_3 = u_i(0)F_L + u_i(L)F_R$;

$$b_1 = M_1 u_i(x_1) S^2; b_2 = M_1 S^2 + b_F S + c_{1*};$$

$$b_3 = M_2 S^2; d_1 = M_3 u_i(x_1) S^2; d_2 = M_3 S^2;$$

$$d_3 = M_4 S^2 + b_S S + 2c_{2*}.$$

To find the expression for the transfer function, we determine the absolute acceleration of the system under consideration.

Let the external forces F_L and F_R acting on the left and right ends of the rod through the lying base give the system W_0 acceleration.

In that case,

$$F_L = F_R = -m_i W_0. \quad (20)$$

Absolute acceleration of the rod protected from vibrations

$$\ddot{W}_a = \ddot{w}_i + W_0. \quad (21)$$

We put the expression of forces (20) in the system of equations (19) and, as a result, using them and (21) the expression of absolute acceleration, we obtain the ratio of the expression of acceleration to the expression of basic acceleration as follows:

$$W_i(S, x) = 1 + \frac{u_i(x) S^2 q_i(S)}{W_0}. \quad (22)$$

The result obtained (22) is called the transfer function of the system under consideration.

We put the first equation of the system of equations (19) into the transfer function (22).

$$W_i(S, q_{ma}, x) = \frac{\sum_{l=0}^6 \mu_l S^l}{\sum_{l=0}^6 \alpha_l S^l}, \quad (23)$$

where $\mu_0 = 2c_{1*}c_{2*}c_i$; $\mu_1 = (2b_F c_{2*} + b_S c_{1*})c_i$;

$$\mu_2 = (c_{1*}M_4 + b_F b_S + 2c_{2*}M_1)c_i + 2c_{2*}c_{1*}(m_i + u_i^2(x_1)M_1 - u_i(x)m_i(u_i(0) + u_i(L)));$$

$$\mu_3 = (b_F M_4 + b_S M_1)c_i + (m_i + u_i^2(x_1)M_1 - u_i(x)m_i(u_i(0) + u_i(L)))(c_{1*}b_S + 2c_{2*}b_F);$$

$$\mu_4 = \Delta c_i + M_1 u_i^2(x_1) b_F b_S + u_i^2(x_1) \Delta c_{1*} + (1 - u_i(x)(u_i(0) + u_i(L)))m_i(M_4 c_{1*} + b_F b_S + 2c_{2*}M_1);$$

$$\mu_5 = u_i^2(x_1) b_F \Delta + (1 - u_i(x)(u_i(0) + u_i(L)))m_i(M_4 b_F + b_S M_1);$$

$$\mu_6 = \Delta m_i (1 - u_i(x)(u_i(0) + u_i(L)));$$

$$\alpha_0 = \mu_0 = 2c_{1*}c_{2*}c_i; \alpha_1 = \mu_1 = (2b_F c_{2*} + b_S c_{1*})c_i;$$

$$\alpha_2 = (c_{1*}M_4 + b_F b_S + 2c_{2*}M_1)c_i + 2c_{2*}c_{1*}(m_i + u_i^2(x_1)M_1);$$

$$\alpha_3 = (b_F M_4 + b_S M_1)c_i + (m_i + u_i^2(x_1)M_1)(b_S c_{1*} + 2b_F c_{2*});$$

$$\alpha_4 = \Delta c_i + m_i(M_4 c_{1*} + b_F b_S + 2c_{2*}M_1) + u_i^2(x_1)c_{1*}\Delta + u_i^2(x_1)b_F b_S M_1;$$

$$\alpha_5 = m_i(b_F M_4 + b_S M_1) + u_i^2(x_1)b_F \Delta;$$

$$\alpha_6 = m_i \Delta; \Delta = M_1 M_4 - M_2 M_3.$$

In (23) we put $j\omega$ instead of S

$$W_i(j\omega, q_{ma}, x) = \frac{\sum_{l=0}^6 \mu_l (j\omega)^l}{\sum_{l=0}^6 \alpha_l (j\omega)^l}. \quad (24)$$

The expressions of stiffness c_i are complex expression in the coefficients $\mu_0, \mu_1, \mu_2, \mu_3, \mu_4, \alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$. It can be written from the expression of stiffness (2) as follows:

$$c_i = c_{1i} + jc_{2i}, \quad (25)$$

where

$$c_{1i} = \left[\int_0^L \rho A (1 - C_0 \eta_1) u_m^2 dx - \frac{3EI}{\omega_{*m}^2} \eta_1 \times \right. \\ \left. \times \sum_{i^*=1}^n C_{i^*} q_{ma}^{i^*} \frac{h^{i^*}}{2i^*(i^*+3)} \int_0^L u_m \frac{\partial^2}{\partial x^2} \left(\frac{\partial^2 u_m}{\partial x^2} \left| \frac{\partial^2 u_m}{\partial x^2} \right|^{i^*} \right) dx \right] \omega_{*m}^2; \quad (26)$$

$$c_{2i} = \left[\int_0^L \rho A C_0 \eta_2 u_m^2 dx + \frac{3EI}{\omega_{*m}^2} \eta_2 \times \right. \\ \left. \times \sum_{i^*=1}^n C_{i^*} q_{ma}^{i^*} \frac{h^{i^*}}{2i^*(i^*+3)} \int_0^L u_m \frac{\partial^2}{\partial x^2} \left(\frac{\partial^2 u_m}{\partial x^2} \left| \frac{\partial^2 u_m}{\partial x^2} \right|^{i^*} \right) dx \right] \omega_{*m}^2. \quad (27)$$

According to stiffness expression (25), the coefficients $\mu_0, \mu_1, \mu_2, \mu_3, \mu_4, \alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ will be as follows:

$$\mu_0 = \mu_{10} + j\mu_{20}; \quad \mu_1 = \mu_{11} + j\mu_{21}; \quad \mu_2 = \mu_{12} + j\mu_{22}; \quad \mu_3 = \mu_{13} + j\mu_{23};$$

$$\mu_4 = \mu_{14} + j\mu_{24}; \quad \alpha_0 = \alpha_{10} + j\alpha_{20} = \mu_0 = \mu_{10} + j\mu_{20}; \quad \alpha_1 = \alpha_{11} + j\alpha_{21} =$$

$$= \mu_1 = \mu_{11} + j\mu_{21}; \quad \alpha_2 = \alpha_{12} + j\alpha_{22}; \quad \alpha_3 = \alpha_{13} + j\alpha_{23}; \quad \alpha_4 = \alpha_{14} + j\alpha_{24};$$

$$\mu_{10} = \alpha_{10} = 2c_{1*}c_{2*}c_{1i}; \quad \mu_{20} = \alpha_{20} = 2c_{1*}c_{2*}c_{2i};$$

$$\mu_{11} = \alpha_{11} = (2b_F c_{2*} + b_S c_{1*})c_{1i}; \quad \mu_{21} = \alpha_{21} = (2b_F c_{2*} + b_S c_{1*})c_{2i};$$

$$\mu_{12} = (c_{1*}M_4 + b_F b_S + 2c_{2*}M_1)c_{1i} + 2c_{1*}c_{2*}(m_i + u_i^2(x_1)M_1) - u_i(x)m_i(u_i(0) + u_i(L));$$

$$\mu_{22} = \alpha_{22} = (c_{1*}M_4 + b_F b_S + 2c_{2*}M_1)c_{2i};$$

$$\mu_{13} = (b_F M_4 + b_S M_1)c_{1i} + (m_i + u_i^2(x_1)M_1 - u_i(x)m_i(u_i(0) + u_i(L)))(c_{1*}b_S + 2c_{2*}b_F);$$

$$\mu_{23} = \alpha_{23} = (b_F M_4 + b_S M_1)c_{2i};$$

$$\mu_{14} = \Delta c_{1i} + M_1 u_i^2(x_1) b_F b_S + u_i^2(x_1) \Delta c_{1*} + (1 - u_i(x)(u_i(0) + u_i(L)))m_i(M_4 c_{1*} + b_F b_S + 2c_{2*}M_1);$$

$$\mu_{24} = \alpha_{24} = \Delta c_{2i};$$

$$\alpha_{12} = (c_{1*}M_4 + b_F b_S + 2c_{2*}M_1)c_{1i} + 2c_{2*}c_{1*}(m_i + u_i^2(x_1)M_1);$$

$$\alpha_{13} = (b_F M_4 + b_S M_1)c_{1i} + (2b_F c_{2*} + b_S c_{1*})(m_i + u_i^2(x_1)M_1);$$

$$\alpha_{14} = \Delta c_{1i} + m_i(M_4 c_{1*} + b_F b_S + 2c_{2*}M_1) + u_i^2(x_1)c_{1*}\Delta + u_i^2(x_1)b_F b_S M_1.$$

We put these coefficients to the transfer function (24)

$$W_i(j\omega, q_{ma}, x) = \frac{E_0 + jE_1}{N_0 + jN_1}, \quad (28)$$

where

$$E_0 = \mu_{10} - \mu_{21}\omega - \mu_{12}\omega^2 + \mu_{23}\omega^3 + \mu_{14}\omega^4 - \mu_6\omega^6; \quad (29)$$

$$E_1 = \mu_{20} + \mu_{11}\omega - \mu_{22}\omega^2 - \mu_{13}\omega^3 + \mu_{24}\omega^4 + \mu_5\omega^5; \quad (30)$$

$$N_0 = \mu_{10} - \mu_{21}\omega - \alpha_{12}\omega^2 + \mu_{23}\omega^3 + \alpha_{14}\omega^4 - \alpha_6\omega^6; \quad (31)$$

$$N_1 = \mu_{20} + \mu_{11}\omega - \mu_{22}\omega^2 - \alpha_{13}\omega^3 + \mu_{24}\omega^4 + \alpha_5\omega^5. \quad (32)$$

Since it is of practical importance that the absolute accelerations of the rod points, which are determined from the expression of the transfer function in dynamic reducing of vibrations (28), reach a minimum value, we test this function to a minimum.

(28) The absolute value of the transfer function depends on the variables ω and q_{ma} . In that case from expression (28)

$$|W_i(j\omega, q_{ma}, x)| = \Phi_1(\omega, q_{ma}, x) = \sqrt{\frac{E_0^2 + E_1^2}{N_0^2 + N_1^2}}. \quad (33)$$

From this it is possible to form the following equations which allow to define stationary points:

$$\frac{\partial \Phi_1}{\partial q_{ma}} = \frac{E_0(E_0)'_{q_{ma}} + E_1(E_1)'_{q_{ma}}}{E_0^2 + E_1^2} - \frac{N_0(N_0)'_{q_{ma}} + N_1(N_1)'_{q_{ma}}}{N_0^2 + N_1^2}. \quad (34)$$

$$\frac{\partial \Phi_1}{\partial \omega} = \frac{E_0(E_0)'_{\omega} + E_1(E_1)'_{\omega}}{E_0^2 + E_1^2} - \frac{N_0(N_0)'_{\omega} + N_1(N_1)'_{\omega}}{N_0^2 + N_1^2}. \quad (35)$$

We define second-order partial differentials from first-order partial differentials (34) and (35).

$$\begin{aligned} \frac{\partial^2 \Phi_1}{\partial q_{ma} \partial \omega} &= \frac{\partial^2 \Phi_1}{\partial \omega \partial q_{ma}} = \frac{((E_0)'_{\omega}(E_0)'_{q_{ma}} + E_0(E_0)''_{q_{ma}\omega} + (E_1)'_{\omega}(E_1)'_{q_{ma}} + \\ &+ E_1(E_1)''_{q_{ma}\omega})(E_0^2 + E_1^2) - 2(E_0(E_0)'_{q_{ma}} + E_1(E_1)'_{q_{ma}})(E_0(E_0)'_{\omega} + E_1(E_1)'_{\omega})}{(E_0^2 + E_1^2)^2} \\ &- \frac{((N_0)'_{\omega}(N_0)'_{q_{ma}} + N_0(N_0)''_{q_{ma}\omega} + (N_1)'_{\omega}(N_1)'_{q_{ma}} + N_1(N_1)''_{q_{ma}\omega})(N_0^2 + N_1^2) - \\ &- 2(N_0(N_0)'_{q_{ma}} + N_1(N_1)'_{q_{ma}})(N_0(N_0)'_{\omega} + N_1(N_1)'_{\omega})}{(N_0^2 + N_1^2)^2}; \end{aligned} \quad (36)$$

$$\begin{aligned} \frac{\partial^2 \Phi_1}{\partial q_{ma}^2} &= \frac{(((E_0)'_{q_{ma}})^2 + E_0(E_0)''_{q_{ma}q_{ma}} + ((E_1)'_{q_{ma}})^2 + E_1(E_1)''_{q_{ma}q_{ma}}) \times \\ &\times (E_0^2 + E_1^2) - 2(E_0(E_0)'_{q_{ma}} + E_1(E_1)'_{q_{ma}})^2}{(E_0^2 + E_1^2)^2} - \frac{(((N_0)'_{q_{ma}})^2 + N_0(N_0)''_{q_{ma}q_{ma}} \\ &+ ((N_1)'_{q_{ma}})^2 + N_1(N_1)''_{q_{ma}q_{ma}})(N_0^2 + N_1^2) - 2(N_0(N_0)'_{q_{ma}} + N_1(N_1)'_{q_{ma}})^2}{(N_0^2 + N_1^2)^2}; \end{aligned} \quad (37)$$

$$\begin{aligned} \frac{\partial^2 \Phi_1}{\partial \omega^2} &= \frac{(((E_0)'_{\omega})^2 + E_0(E_0)''_{\omega\omega} + ((E_1)'_{\omega})^2 + E_1(E_1)''_{\omega\omega})(E_0^2 + E_1^2) - \\ &- 2(E_0(E_0)'_{\omega} + E_1(E_1)'_{\omega})^2}{(E_0^2 + E_1^2)^2} - \frac{(((N_0)'_{\omega})^2 + N_0(N_0)''_{\omega\omega} + ((N_1)'_{\omega})^2 + \\ &+ N_1(N_1)''_{\omega\omega})(N_0^2 + N_1^2) - 2(N_0(N_0)'_{\omega} + N_1(N_1)'_{\omega})^2}{(N_0^2 + N_1^2)^2}. \end{aligned} \quad (38)$$

The stationary values of the variables q_{ma} and ω are determined from the following system of equations:

$$\frac{\partial \Phi_1}{\partial q_{ma}} = 0; \quad \frac{\partial \Phi_1}{\partial \omega} = 0. \quad (39)$$

Based on the above results and the theorem that a function of two known variables has a minimum can be defined for the absolute value of the transfer function $\Phi_1(\omega, q_{ma}, x)$ as follows:

If the variables q_{ma} and ω satisfy system of equations (39) and

$$\frac{\partial^2 \Phi_1}{\partial q_{ma}^2} > 0, \quad (40)$$

$$\frac{\partial^2 \Phi_1}{\partial \omega^2} > 0, \quad (41)$$

along with satisfying inequalities

$$\frac{\partial^2 \Phi_1}{\partial q_{ma}^2} \frac{\partial^2 \Phi_1}{\partial \omega^2} - \left(\frac{\partial^2 \Phi_1}{\partial q_{ma} \partial \omega} \right)^2 > 0, \quad (42)$$

satisfy the inequalities, then the absolute value of the transfer function $|W_i(j\omega, q_{ma}, x)| = \Phi_1(\omega, q_{ma}, x)$ reaches a minimum at these values of the variable.

4 Conclusion

1. The system of differential equations that is governed above determined by the method of bond graph represents a mathematical model of the motion of a rod with elastic dissipative characteristics of the hysteresis type in conjunction with a liquid section dynamic absorber.
2. The transfer function of the system, which is protected from vibrations under the influence of kinematic excitations, is obtained. The expression of this transfer function allows a complete analysis of the dynamics and stability of motion of the system protected from vibrations.
3. Given that the problem of stability of motion in nonlinear cases is solved using the equations of the normal form of systems, the traditional methods of converting the differential equations of motion to the equations of the normal form require certain deviations and complex operations. To this end, it has been shown that the method of bond graph in solving problems allows to bypass this problem, that is, to create a system of equations of normal form, called the Cauchy form, directly relative to state variables.
4. Based on the given theorem, it is possible to determine the variables q_{ma} and ω , where the absolute value of the transmission function $\Phi_1(\omega, q_{ma}, x)$ reaches a minimum, and select the optimal values of the system parameters protected from vibrations connecting these variables.
5. The structural approach in mathematical modeling allows to fully express the elastic dissipative nonlinear characteristics of system elements and to evaluate the efficiency of dynamic reducing at a wide range of frequencies, taking into account the inert properties of the fluid.

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