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DIFFERENSIAL TENGLAMALAR KAFEDRASI

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USLUBIY KO’RSATMA

“HOSILAGA NISBATAN YECHILGAN BIRINCHI TARTIBLI DIFFERENSIAL TENGLAMALAR” mavzusi bo’yicha uy va nazorat ishlarini bajarishga doir

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"Ushbu differensial tenglamalar" fanining Hosilaga nisbatan yechilgan birinchi tartibli differensial tenglamalar bo'limi bo'yicha uy ishlarini bajarishga hamda auditoriyada mustaqil ishlarni bajarishga doir metodik ko'rsatma.

Bu ko'rsatma 2-kurs talabalariga uy ishlari hamda nazorat ishlarini bajarishga yaqindan yordam berishi kerak.

Metodik ko'rsatmada eng sodda tenglamalar chiziqli va unga keltiriladigan, Bernulli, Rikkati tenglamalrini to'liq differensial tenglamalarini integrallovchi ko'paytuvchi yordamida misol va masalalarni yechishda yaqindan yordam beradi, chunki bunda har bir tenglama turiga namunaviy misollar izohlar bilan to'liq yechib ko'rsatilgan.

Metodik ko'rsatma to'rt bo'limdan iborat. Mavzular bo'yicha testlar va javoblari keltirilgan.

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Kirish

Noma’lum funksiya, uning turli hosilalari va erkli o‘zgaruvchilar qatnashgan tenglama differensial tenglama deyiladi. Agar noma’lum funksiya ko‘p argumentli bo‘lsa va tenglamaga funksiyaning shu argumentlar bo‘yicha xususiy hosilalari qatnashsa, bunday tenglama xususiy hosilali differensial tenglama deyiladi. Masalan:

$$x\frac{\partial z}{\partial y}-y\frac{\partial z}{\partial x}=0$$

bunda noma’lum funksiya, ikkita erkli o‘zgaruvchilarga bog‘liq, ya’ni $z=z(x,y)$.

Agar noma’lum funksiya $y=y(x)$ bitta erkli o‘zgaruvchining funksiyasi bo‘lsa va tenglamaga odatdagi hosilalar qatnashsa, bunday tenglama oddiy differensial tenglama deyiladi, u quyidagicha belgilanadi:

$$\Phi(x,y,y',...,y^{(n)})=0 \quad .$$

Bu kitobda biz faqat oddiy differensial tenglamalarni qaraymiz. Differensial tenglama tartibi unda qatnashayotgan hosilaning yuqori tartibi bilan aniqlanadi. Yuqoridagi tenglama *n* - tartibli oddiy differensial tenglama.

Ta’rif. Differensial tenglamaning yechimi deb, biror *I* oraliqda aniklangan *n* marta differensiallanuvchi va barcha $x\in I$ uchun berilgan differensial tenglamani qanoatlantiradigan, ya’ni $y=\varphi(x)$ tenglamaga, $y^{(k)}=\varphi^{(k)}(x)$ ($k=1,2,...,n$) larni qo‘yganda, uni ayniyatga aylantiradigan $y=\varphi(x)$ funksiyaga aytiladi. $y=\varphi(x)$ silliq chiziq bunda $\varphi(x)$ differensial tenglamaning yechimi integral chiziq (integral egri chiziq) deyiladi.

Misol. Ushbu $x^2+y^2-2cx=0 \quad (-\infty,+\infty)$ integralda aniqlangan funksiya, quyidagi $x^2-y^2+2xyy'=0$ differensial tenglama yechimi ekanini ko‘rsatamiz.

Yechish: haqiqatdan dastlabki tenglikni differensiallab, hosil qilamiz:

$$2x-2yy'-2c=0\ ,\ y'=\frac{c-x}{y}.$$

Endi hosilaning topilgan ifodasini berilgan differensial tenglamaga qo‘ysak, unda quyidagicha bo‘ladi:

$$x^2-y^2+2cx-2x^2=-(x^2+y^2-2cx)\equiv 0\ .$$

Demak, $x^2+y^2-2cx=0$ funksiya berilgan differensial tenglama yechimi ekan.

1. Eng sodda differensial tenglamalar

1. Ushbu

$$\frac{dy}{dx}=f(x) \tag{1.1}$$

tenglamani qaraymiz.

Aytaylik $f(x)\in C(a,b)$ bo‘lsin. Ravshanki (1.1) tenglamaning umumiy yechimi

$$y=\int f(x)dx+C=F(x)+C$$

bo‘ladi, bunda *S* – ixtiyoriy o‘zgarmas son, *F(x)* esa *f(x)* funksiyaning biror boshlang‘ich funksiyasi: ya’ni $F'(x)=f(x)$.

Agar (1.1) differensial tenglamani

$$y(x_0) = y_0 \qquad (x_0 \in (a,b))$$

boshlang'ich shartda qaraydigan bo'lsak, unda

$$y_0 = F(x_0) + C,$$

ya'ni,

$$C = y_0 - F(x_0)$$

bo'lib,

$$y = F(x) + y_0 - F(x_0) = y_0 + [F(x) - F(x_0)] = y_0 + \int_{x_0}^{x} f(x)dx$$

bo'ladi.

Shunday qilib, (1.1) differensial tenglamaning boshlang'ich shartni qanoatlantiruvchi yechimi

$$y = y_0 + \int_{x_0}^{x} f(x)dx \tag{1.2}$$

bo'lar ekan. (1.2) ko'rinishdagi yechimni Koshi formasidagi umumiy yechim deyiladi.

2. Endi

$$y' = f(y) \qquad f(y) \in c(c,d) \tag{1.3}$$

differensial tenglamani qaraymiz.

Aytaylik $f(y) \in C(c,d)$ va $f(y) \neq 0 \quad (c,d)$ intervalda. Unda (1.3) tenglikdan

$$dx = \frac{dy}{f(y)}$$

kelib chiqadi. Keyingi tenglikning har ikki tomonini integrallab

$$x = \int \frac{dy}{f(y)} + C, \quad \text{ёки} \quad x - x_0 = \int_{y_0}^{y} \frac{dy}{f(y)}$$

bunda $y_0 \in (c,d)$ umumiy integralni (umumiy yechimni oshkormas ko'rinishda) hosil qilamiz.

3. Ushbu

$$M(x)dx + N(y)dy = 0 \tag{1.4}$$

bunda *dx* oldidagi funksiya faqat *x* ga, *dy* oldidagi funksiya esa faqat *y* ga bog'liq, ko'rinishdagi differensial tenglama o'zgaruvchilari ajralgan differensial tenglama deyiladi.

Agar *M (x)* va *N (y)* koeffisientlarni biror intervalda uzluksiz funksiyalar deb faraz etsak, unda (1.4) ni quyidagicha yozish mumkin

$$d\left(\int_{x_0}^{x} M(x)dx + \int_{y_0}^{y} N(y)dy\right) = 0$$

bunda x_0 va y_0 – tayinlangan sonlar, *M (x)* va *N(y)* funksiyalarning aniqlanish va uzluksiz intervallaridan olingan $M^2(x_0) + N^2(y_0) \neq 0$.

Demak

$$\int_{x_0}^{x} M(x)dx + \int_{y_0}^{y} N(y)dy = C \tag{1.5}$$

bunda C – ixtiyoriy o‘zgarmas. Keyingi tenglik (1.4) differensial tenglamaning umumiy integrali. Ravshanki (1.5) ni quyidagi

$$\int M(x)dx + \int N(y)dy = C$$

ko‘rinishda ham yozish mumkin.

Ushbu

$$m(x)n(y)dx + m_1(x)n_1(y)dy = 0 \qquad (1.6)$$

ko‘rinishdagi differensial tenglama o‘zgaruvchilari ajraladigan differensial tenglama deyiladi. Bunda $m(x)$, $n(y)$, $m_1(x)$ va $n_1(y)$ - biror (a,b) da aniqlangan va uzluksiz funksiyalar, undan tashqari shu intervalda $n(y)\cdot m_1(x) \neq 0$, $m_1(x)$ $n(y)$ ga bo‘lib olsak,

$$\frac{m(x)}{m_1(x)}dx + \frac{n_1(y)}{n(y)}dy = 0$$

(1.4) tipdagi differensial tenglamani hosil qilamiz.

Aytaylik, $n(y)=0$ yoki $m_1(x)=0$ bo‘lsin. Agar bu tenglamalar $y=y_k$ va $x=x_k$ $k=1,2,\dots$ yechimga ega bo‘lsa, bular (1.6) differensial tenglamaning ham yechimlari bo‘ladilar. Bu yechimlar – maxsus yechimlar bo‘lishi mumkin.

1 – Misol. Ushbu differensial tenglamaning

$$\frac{dy}{dx} = 4x^3$$

$y(0)=1$ boshlang‘ich shartni qanoatlantiruvchi yechimi topilsin.

Yechish. Bu tenglikning har ikki tomonini integrallab topamiz:

$$y = x^4 + C\,.$$

Bu berilgan tenglamaning umumiy yechimi, bunda S ixtiyoriy o‘zgarmas.

Boshlang‘ich shartga binoan $x=0$ da $y=1$. Shunga ko‘ra

$$1=0+S \quad S=1$$

shunday qilib, $y = x^4+1$ masala yechimi.

2 – Misol. Ushbu

$$y' = 5\sqrt{y}$$

differensial tenglamaning $y(0)=25$ boshlang‘ich shartni qanoatlantiruvchi yechimi topilsin.

Yechish. Berilgan tenglamani dx ga ko‘paytirib $\sqrt{y}$ ga bo‘lamiz:

$$\frac{dy}{5\sqrt{y}} = dx$$

hosil bo‘ladi. Bu tenglikning har ikki tomonnini integrallab

$$\frac{1}{5}\int y^{-\frac{1}{2}}dy = x + C \Rightarrow \quad \frac{2}{5}\sqrt{y} = (x+c)$$

$$y = \frac{25}{4}(x+c)^2$$

berilgan differensial tenglamaning umumiy yechimini topamiz, bunda S ixtiyoriy o‘zgarmas.

Boshlang‘ich shartga binoan $x=0$ da $u=25$. Shunga ko‘ra

$$25=\frac{25}{4}(0+C)^2 \Rightarrow C=2$$

bo‘ladi. Demak, berilgan differensial tenglamaning boshlang‘ich shartni qanoatlantiruvchi yechimi

$$y=\frac{25}{4}(x+2)^2$$

<u>3 – Misol.</u> Ushbu

$$\sqrt{1-y^2}\,dx+\sqrt{1-x^2}\,dy=0$$

tenglamani yeching.

Yechish. O‘zgaruvchilarni ajratamiz, bo‘ladi

$$\frac{dx}{\sqrt{1-x^2}}+\frac{dy}{\sqrt{1-y^2}}=0$$

integrallab differensial tenglamaning umumiy integrallarini topamiz:

$$arcsinx + arcsin\, y = S.$$

Endi ixchamlash uchun tenglamaning ikkala tomonidan sinusni olamiz, bo‘ladi.

$$\sin(\arcsin x+\arcsin y)=\sin c \Rightarrow \sin\arcsin x\cos\arcsin y+\sin\arcsin y\cos\arcsin x=$$

$$=x\cdot\sqrt{1-\sin^2\arcsin y}+y\cdot\sqrt{1-\sin^2\arcsin x}=\sin c$$

Shunday qilib umumiy integral bo‘ladi:

$x\cdot\sqrt{1-y^2}+y\cdot\sqrt{1-x^2}=c_1$, bunda $|\sin c|=|c_1|\le 1$, yechim $|x|\le 1,\ |y|\le 1$ kvadratda aniqlangan. Bundan tashqari berilgan tenglama $x=\pm 1,\ y=\pm 1$ yechimga ham ega. Bu yechimlarni umumiy integraldan hosil qilish iloji yo‘k.

<u>4 – Misol.</u> Ushbu tenglamani yeching.

$$\frac{dy}{dx}=\frac{1}{x}.$$

Yechish. $f(x)=\frac{1}{x}$ funksiya x=0 dan boshqa barcha nuqtalarda uzluksiz. Agar $x\neq 0$ desak, berilgan differensial tenglamaning yechimlari

$$y=\ln|x|+C$$

bo‘ladi.

Endi $\frac{dx}{dy}=x$ differensial tenglamani qarasak, $x=0$ bu differensial tenglamaning yechimi bo‘ladi.

Shunday qilib yuqoridagi differensial tenglamaning yechimlari $y=\ln|x|+C$ va x=0 bo‘ladi.

<u>5 – Misol.</u> Shunday egri chiziqlar topilsinki, ular quyidagi xossaga ega bo‘lsin: bu egri chiziqning ixtiyoriy *M(x, y)* nuqtasida normal osti bo‘lgan kesma *a* ga teng bo‘lsin.

Yechish. Chizmadan *NB* normal osti bo‘lgan kesma, $\angle NBM=90^0-\alpha$

$$y = NB \cdot tg(90^0 - \alpha) = NB\ ctg\,\alpha = \frac{NB}{tg\,\alpha} = \frac{NB}{y'} \Rightarrow NB = yy'$$

Boshqa tomondan masala shartiga ko‘ra $NB=a$. Bu ikki tenglamalardan

$$y\,y' = a$$

o‘zgaruvchilari ajralgan differensial tenglama hosil bo‘ladi.

Integrallab izlanayotgan egri chiziqlar oilasining tenglamasini topamiz,

$$y^2=2ax+c$$

bunda *S* ixtiyoriy o‘zgarmas.

<u>6 – Misol.</u> Berilgan (0; 1) nuqtadan o‘tuvchi shunday egri chiziq tenglamasini tuzingki, bu egri chiziq yoyi bilan chegaralangan egri chiziqli trapesiya yuzi shu yoy uzunligiga teng bo‘lsin.

Yechish. Masala shartiga ko‘ra

$$\int_0^x ydx = \int_0^x \sqrt{1+y'^2}\,dx\ ,$$

bunda chap tomonga egri chiziqli trapesiya yuzasi, o‘ngda esa yoy uzunligi ifodalangan. Tenglamaning ikkala tomonini differensiallab topamiz.

$$y = \sqrt{1+y'^2} \Rightarrow y' = \pm\sqrt{y^2-1}, \quad (y \ge 1)$$

Bu differensial tenglamada o‘zgaruvchilarni ajratib integrallaymiz.

$$\int \frac{dy}{\sqrt{y^2-1}} = \pm\int dx + c, \Rightarrow \ln\left(y+\sqrt{y^2-1}\right) = \pm x + c$$

egri chiziqlar oilasini topamiz. Masala shartiga muvofiq $x=0$ da $y=1$, demak $s=0$.

Shunday qilib $y+\sqrt{y^2-1} = e^{\pm x}$ oxirgi tenglikdan

$$\frac{1}{y+\sqrt{y^2-1}} = e^{\mp x}$$

yoki

$$y-\sqrt{y^2-1} = e^{\mp x}$$

$y+\sqrt{y^2-1} = e^{\pm x}$ va $y-\sqrt{y^2-1} = e^{\mp x}$ tengliklarni qo‘shib topamiz

$$2y = e^{\pm x} + e^{\mp x}\ .$$

Demak,

$$y = \frac{e^x + e^{-x}}{2}\ ,$$

zanjirli chiziq bu masala yechimi.

2. Bir jinsli va unga keltiriladigan differensial tenglamalar

Eng avval bir jinsli funksiyaga ta’rif beramiz.

Ta’rif. Agar *f(x,y)* funksiyada *x* va *y* o‘zgaruvchilarni mos ravishda *tx* va *ty* ga almashtirganda (bu yerda $0 \ne t \in R$)

$$f(tx, ty) \equiv t^n f(x,y) \quad (n\text{–o‘zgarmas son}) \qquad (2.1)$$

shart bajarilsa, *f(x,y)* funksiya *n* o‘lchovli bir jinsli funksiya deyiladi.

Masalan, $f(x,y)=x\sqrt{x^2+y^y}$ funksiya ikki o‘lchovli bir jinsli funksiya bo‘ladi, chunki

$$f(tx,ty)=tx\sqrt{t^2x^2+t^2y^2}=t^2x\sqrt{x^2+y^2}=t^2\cdot f(x,y),$$

$f(x,y)=\dfrac{3x^2-y^2}{x^2+5y^2}$ esa nol o‘lchovli bir jinsli funksiya, chunki

$$f(tx,ty)=\frac{t^2(3x^2-y^2)}{t^2(x^2+y^2)}=f(x,y)$$

ya’ni

$$f(tx,ty)\equiv f(x,y) \tag{2.2}$$

ayniyat o‘rinli bo‘ladi.

Nol o‘lchovli bir jinsli funksiyani $\varphi(\frac{y}{x})$ ko‘rinishda yozish mumkin. Haqiqatan ham t parametrni ixtiyoriy tanlab olish mumkin bo‘lgani uchun $t=1/x$ deb olamiz, u holda bo‘ladi:

$$f(x,y)=f(tx,ty)=f(1,y/x)\equiv\varphi(y/x).$$

Ta’rif. Agar birinchi tartibli $y'=g(x,y)$ diffrensial tenglamaning o‘ng tomoni x va y ga nisbatan nol o‘lchovli bir jinsli funksiya bo‘lsa, bunday tenglama bir jinsli tenglama deyiladi.

Shunday qilib, bir jinsli tenglamani quyidagi ko‘rinishda yozish mumkin:

$$y'=\varphi\left(\frac{y}{x}\right) \tag{2.3}$$

Bir jinsli (2.3) tenglamani $y/x=u(x)$ o‘rniga qo‘yish yordamida o‘zgaruvchilari ajraladigan tenglamaga keltirish mumkin, u holda $y=u(x)x$ bu yerda $u=u(x)$–yangi noma’lum funksiya. Keyingi tenglikni differensiallab, $y'=u'x+u$ ni hosil qilamiz, y va y' ning qiymatlarini (2.3) tenglamaga qo‘yib, quyidagi o‘zgaruvchilari ajraladigan tenglamani topamiz:

$$u'\cdot x=\varphi(u)-u \Rightarrow xdu=(\varphi(u)-u)dx.$$

O‘zgaruvchilarni ajaratamiz:

$$\frac{du}{\varphi(u)-u}=\frac{dx}{x} \qquad (\varphi(u)-u\neq 0).$$

Integrallab topamiz

$$\int\frac{du}{\varphi(u)-u}=\int\frac{dx}{x}+C.$$

Integrallashdan keyin u o‘rniga y/x nisbatni qo‘yib, (2.3) tenglamaning umumiy integralini hosil qilamiz.

Aytaylik $\varphi(u)-u=0$ bo‘lsin. Agar $u=u_0$ bu tenglamaning ildizi bo‘lsa, unda bir jinsli tenglamaning yechimi $y=u_0x$ bo‘ladi.

Izoh. Ushbu

$$M(x,y)\,dx+N(x,y)\,dy=0 \tag{2.4}$$

tenglamada $M(x,y)$, $N(x,y)$ lar bir xil o‘lchovli bir jinsli funksiyalar bo‘lganda bir jinsli differensial tenglama bo‘ladi.

1 – Misol. $(x^2+y^2)dy+2xy\ dx=0$ $f_1(x, y)=x^2+y^2$ va $f_2(x, y)=2xy$ differensial tenglama bir jinslidir, chunki x^2+y^2 va $2xy$ funksiyalar ikki o‘lchovli bir jinslidir:

Haqiqatan $f_1(tx, ty) = (tx)^2+(ty)^2=t^2(x^2+y^2)=t^2 f_1(x, y)$

$$f_2(tx,ty)= 2(tx)(ty)=t^2 2xy = t^2 f_2(x, y).$$

Endi differensial tenglamani yechamiz, ya’ni $u=u(x)$ funksiya kiritib $y=ux$, $dy=u\ dx+x\ du$. Unda

$$(x^2 + x^2 u^2)(u\ dx+x\ du) + 2x^2 u\ dx = 0$$

yoki ixchamlab,

$$(1+u^2)dx+2ux\ dx=0$$

o‘zgaruvchilarni ajratib,

$$\frac{dx}{x}+\frac{2u}{1+u^2}du = 0$$

hosil kilamiz.

Integrallab, $lnx+ln(l+u^2)=lnc$ yoki $x(l+u^2)=C$ ni topamiz. $u=y/x$ almashtirishni hisobga olsak, berilgan tenglamaning umumiy integralini hosil qilamiz:

$$x^2+y^2=Sx.$$

2 – Misol. Ushbu

$$y' = \frac{y+\sqrt{x^2-y^2}}{x} \text{ yoki } y' = \frac{y}{x}+\sqrt{1-\left(\frac{y}{x}\right)^2} \qquad (x \neq 0)$$

bir jinsli tenglamani yeching.

Yechish. O‘ng tomoni nol o‘lchovli bir jinsli funksiyadan iborat, $y/x = u$ almashtirish bajaramiz, u holda $y = ux,\ y' = u'x + u,$ y va y' ning ifodalarini differensial tenglamaga qo‘yamiz:

$$u'x + u = u + \sqrt{1-u^2}, \quad u'x = \sqrt{1-u^2}$$

o‘zgaruvchilari ajraladigan tenglama hosil bo‘ladi.

Oxirgi tenglikni dx ga ko‘paytirib $x\cdot\sqrt{1-u^2} \neq 0$ ga bo‘lamiz, o‘zgaruvchilar ajraladi.

Integrallab, topamiz: $\arcsin u = \ln x + \ln C$. Bu yerdan $y = x\sin(\ln Cx)$.

3 – Misol. Ushbu $xy' = y\cos\ln\frac{y}{x}$ differensial tenglamani yeching.

Yechish. Berilgan tenglamani x ga bo‘lamiz, bo‘ladi

$$y' = \frac{y}{x}\cos\ln\frac{y}{x} \equiv \varphi\left(\frac{y}{x}\right).$$

Demak, qaralayotgan tenglama bir jinsli differensial tenglama, quyidagi $y=z\ x$, $z=z\ (x)$ almashtirishni bajaramiz. Unda $y' = z + xz'$ bo‘lib, berilgan differensial tenglama, ushbu

$$xz' + z = z\cos\ln z$$

yoki

$$x\frac{dz}{dx} = z(\cos\ln z - 1)$$

ko‘rinishda bo‘ladi. Bu o‘zgaruvchilari ajraladigan tenglama. Unda o‘zgaruvchilarni ajratsak bo‘ladi

$$\frac{dx}{x}=\frac{dz}{z(\cos\ln z-1)},\qquad (\cos\ln z=1?)$$

Integrallaymiz

$$\ln|x|+\ln|c|=\int\frac{d\ln z}{\cos\ln z-1}$$

yoki

$$\ln cx=\int\frac{du}{\cos u-1}=-\int\frac{du}{2\sin^2\frac{u}{2}}\Rightarrow(u=\ln z)\Rightarrow\ln cx=ctg\frac{u}{2}$$

yoki

$$\ln cx=ctg\frac{\ln z}{2}\Rightarrow\ln cx=ctg\left(\frac{1}{2}\ln\frac{y}{x}\right)$$

berilgan differensial tenglamaning umumiy yechimi, bunda *c* ixtiyoriy o‘zgarmas. Endi *cos ln z*=1 tenglikni ko‘ramiz, bundan

$$z=e^{2k\pi},\quad k=0,\pm1,\pm2,...,\Rightarrow y=xe^{2\pi k},\quad k=0,\pm1,\pm2,...$$

yechim hosil bo‘ladi.

4–Misol: $\left(xe^{\frac{y}{x}}-y\right)dx+xdy=0$ simmetrik ko‘rinishdagi differensial tenglama integrallansin.

Yechish. Misolda $M(x,y)=xe^{\frac{y}{x}}-y$ va $N(x,y)=x$ funksiyalar birinchi tartibli bir jinsli funksiyalar. Haqiqatan,

$$M(tx,ty)=(tx)e^{\frac{ty}{tx}}-(ty)=t\left(xe^{\frac{y}{x}}-y\right)=tM(x,y)\;N(tx,ty)=tx=tN(x,y).$$

Demak, berilgan tenglama bir jinsli differensial tenglama va uni yechish uchun $\frac{y}{x}=z$ almashtirish kiritamiz. Unda *y=xz, dy=xdz+zdx* ni tenglamaga qo‘ysak, bo‘ladi

$$(x e^z-x z)\,dx+x\,(xdz+zdx)=0$$

yoki $x\neq0$ ga qisqartirib ixchamlasak, bo‘ladi

$$x\,dz+e^z\,dx=0.$$

O‘zgaruvchilari ajraladigan differensial tenglama hosil bo‘ladi, $x\,e^z\neq0$ deb, topamiz

$$-e^{-z}dz=\frac{dx}{x}.$$

Integrallaymiz, bo‘ladi

$$e^{-z}=\ln|x|+\ln|c|\text{ yoki }e^{-z}=\ln|cx|\Rightarrow z=-\ln\ln cx\text{ yoki }\frac{y}{x}=-\ln\ln cx,$$

bundan *y=–x ln lncx* – differensial tenglamaning umumiy yechimini topamiz, bunda *S* – ixtiyoriy o‘zgarmas.

Quyidagi tenglamani qaraymiz:

$$\frac{dy}{dx}=\frac{ax+by+c}{a_1x+b_1y+c_1}\tag{2.3}$$

Bunda a,b,c,a_1,b_1,c_1 lar haqiqiy sonlar.

Agar $s=s_1=0$ bo‘lsa (2.3) tenglama bir jinsli tenglama bo‘ladi. Aytaylik, $|c|+|c_1|\neq 0$, unda ikki hol bo‘lishi mumkin.

1) Determinant $\Delta=\begin{vmatrix} a & b \\ c & d \end{vmatrix}\neq 0$.

$x=x_1+\alpha,\quad y=y_1+\beta$ deb yangi x_1 va u_1 o‘zgaruvchilarni kiritamiz, α va β lar hozircha noma’lum o‘zgarmaslar. Unda quyidagini hosil qilamiz

$$\frac{dy_1}{dx_1}=\frac{ax_1+by_1+a\alpha+b\beta+c}{a_1x_1+b_1y_1+a_1\alpha+b_1\beta+c_1} \tag{2.4}$$

quyidagi tengliklar bajarilsa,

$$\begin{cases} a\alpha+b\beta+c=0 \\ a_1\alpha+b_1\beta+c_1=0 \end{cases} \tag{2.5}$$

ya’ni α,β lar (2.5) algebraik sistemaning yechimi bo‘lsa bir jinsli tenglamani hosil qilamiz:

$$\frac{dy_1}{dx_1}=\frac{ax_1+by_1}{a_1x_1+b_1y_1}.$$

2) Determinant $\begin{vmatrix} a & b \\ a_1 & b_1 \end{vmatrix}=0$ bo‘lsa, (2.5) algebraik sistema yechimga ega bo‘lmaydi, bunday holda $\frac{a_1}{a}=\frac{b_1}{b}=k$, ya’ni $a_1=ak$, $b_1=b\,k$ bo‘ladi, (2.3) differensial tenglamani $\frac{dy}{dx}=\frac{ax+by+c}{k\cdot(ax+by)+c_1}$ ko‘rinishda yozish mumkin. $z=ax+by$ almashtirsak, bu tenglamani o‘zgaruvchilari ajraladigan tenglamaga keltiriladi.

Izoh: (2.3) tenglamani integrallashda qo‘llanilgan usul

$$\frac{dy}{dx}=f\left(\frac{ax+by+c}{a_1x+b_1y+c_1}\right),$$

bu yerda f – ixtiyoriy uzluksiz funksiya tenglamani integrallashda ham qo‘llaniladi.

<u>5 – Misol.</u> Ushbu $y'=\frac{x+y-3}{x-y-1}$ tenglamaning umumiy integralini toping.

Yechish. Determinant: $\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}=-2\neq 0$

Quyidagi almashtirishni bajaramiz:

$$\begin{cases} x=x_1+\alpha \\ y=y_1+\beta \end{cases}$$

U holda quyidagiga ega bo‘lamiz:

$$\frac{dy_1}{dx_1}=\frac{x_1+y_1+\alpha+\beta-3}{x_1-y_1+\alpha-\beta-1}$$

Endi $\begin{cases}\alpha+\beta-3=0\\ \alpha-\beta-1=0\end{cases}$ sistemani yechib, α=2, β=1 ekanini topamiz. natijada bir jinsli $\frac{dy_1}{dx_1}=\frac{x_1+y_1}{x_1-y_1}$ tenglamaga ega bo‘lamiz, uni $\frac{y_1}{x_1}=u$ o‘rniga qo‘yish yordamida yechamiz: demak,

$$y_1=ux_1,\ y_1'=u'x_1+u,\ u'x_1+u=\frac{1+u}{1-u}.$$

Soddalashtirishlardan so‘ng o‘zgaruvchilari ajraladigan tenglamani hosil qilamiz:

$$x_1\frac{du}{dx_1}+\frac{1+u^2}{1-u}\ \text{yoki}\ \frac{1-u}{1+u^2}du=\frac{dx_1}{x_1}$$

Tenglamani integrallab, topamiz:

$$u-\frac{1}{2}\ln(1+u^2)=\ln|x_1|+\ln|C|\ \text{yoki}\ Cx_1\sqrt{1+u^2}=e^{arctgu}.$$

$u=\frac{y_1}{x_1}$ ni o‘rniga qo‘ysak, quyidagiga ega bo‘lamiz:

$$C\sqrt{x_1^2+y_1^2}=e^{arctg\frac{y_1}{x_1}}.$$

Nihoyat, $x_1=x-2$, $y_1=y-1$ almashtirishlarni bajarib, x va u o‘zgaruvchilarga o‘tamiz:

$$C=\sqrt{(x-2)^2+(y-1)^2}=e^{arctg\frac{y-1}{x-2}}$$

<u>6 – Misol.</u> Ushbu $y'=\frac{2x+y-1}{4x+2y+5}$ tenglamaning umumiy integralini toping.

Yechish. Determinant: $\begin{vmatrix}2 & 1\\ 4 & 2\end{vmatrix}=0$ demak, tenglamani $\begin{cases}x=x_1+\alpha\\ y=y_1+\beta\end{cases}$ o‘rniga qo‘yish yordamida yechish mumkin emas. Bu tenglamani $2x+y=z$ o‘rniga qo‘yish yordamida o‘zgaruvchilari ajraladigan tenglamaga keltiramiz, u holda $y'=z'-2$ desak, tenglama $z'-2=\frac{z-1}{2z+5}$ yoki $z'=\frac{5z+9}{2z+5}$ ko‘rinishga keladi. Uni yechib topamiz:

$$\frac{2}{5}z+\frac{7}{25}\ln|5z+9|=x+C$$

Endi $z=2x+y$ almashtirish bajarib x va y o‘zgaruvchilariga o‘tamiz:

$$10y-5c=c-7\ln|10x+5y+9|$$

Ba’zi tenglamalarni $y=z^m$ almashtirish orqali, bir jinsli differensial tenglamaga keltirish mumkin. Bunda m – sonini shunday tanlab olish kerakki, differensial tenglamaning barcha a’zolari bir xil o‘lchovli (tartibli) bo‘lsin, agar

x - 1 o‘lchovli, y - m o‘lchovli, $\frac{dy}{dx}$ - m – 1 o‘lchovli $c = const$ - 0 o‘lchovli deb hisoblasak, o‘lchovlar, darajalar kursatkichlardek ko‘paytirilganda qo‘shiladi, bo‘lishda esa ayiriladi.

Masalan, ushbu tenglamada $yy' + 7xy = 5x^3$ birinchi had o‘lchovi $m+(m–1)$, ikkinchi had ulchovi 1+m, uchinchi 3.
Bu sonlarni tenglashtirib, hosil qilamiz.

$$m + (m – 1) = 1 + m = 3.$$

Bundan m=2. Demak, $y=z^2$ almashtirish berilgan differensial tenglamani bir jinsliga aylantiradi. Hosil bo‘lgan bir jinsli tenglama $2z^3z' + 7xz^2 = 5x^3$ odatdagidek yechiladi.

$$yy' + 7xy = 5x^3 \,, \quad y = z^2, \qquad y' = 2zz'$$

$$2z^2zz' + 7xz^2 = 5x^3$$

7 –Misol. Ushbu $(4x^6 - y^4)dx - 2x^4ydy = 0$ tenglamani yeching.

Yechish. $y = z^m, \quad dy = mz^{m-1}dz$ almashtirish kiritamiz, bunda m hozircha noma’lum son. Endi y va dy ifodalarini berilgan tenglamaga qo‘ysak, bo‘ladi

$$(4x^6 - z^{4m})dx - 2x^4z^m \cdot mz^{m-1}dz = 0 \,.$$

Bu tenglama bir jinsli differensial tenglama bo‘lishi uchun unga qatnashgan funksiyalarning o‘lchovi teng bo‘lishi kerak, ya’ni

$$6 = 4m = 4 + m + m - 1 \Rightarrow 6 = 4m = 3 + 2m$$

Bundan $m = \frac{3}{2}$ kelib chiqadi. Aytaylik $y = z^{\frac{3}{2}}$, unda berilgan differensial tenglama ushbu ko‘rinishda bo‘ladi

$$(4x^6 - z^6)dx - 2x^4z^{\frac{3}{2}} \cdot \frac{3}{2} \cdot z^{\frac{1}{2}}dz = 0$$

yoki

$$(4x^6 - z^6)dx - 3x^4z^2dz = 0$$

Endi $z = u \cdot x, \quad dz = udx + xdu$ deymiz, unda x^6 - ga qisqartirib hosil qilamiz:

$$(4 - u^6)dx - 3u^2(udx + x\,du) = 0$$

yoki

$$(4 - 3u^3 - u^6)dx - 3u^2x\,du = 0 \,.$$

Bu tenglamada o‘zgaruvchilarni ajratamiz:

$$\frac{dx}{x} = \frac{3u^2du}{u^6 + 3u^3 - 4} \,.$$

Integralaymiz, o‘ng tomondagi integralda $u^3 = t, dt = 3u^2du$ deb, unda

$$\ln c + \ln|x| = -\int \frac{dt}{t^2 + 3t - 4} = -\int \frac{dt}{(t-1)(t+4)} = -\frac{1}{5}\int\left(\frac{1}{t-1} - \frac{1}{t+4}\right)dt = \frac{1}{5}\ln\left|\frac{t+5}{t-1}\right|$$

yoki

$$cx^5 = \frac{t+4}{t-1}, \qquad t = u^3$$ - ni xisobga olsak

$$cx^5 \cdot (u^3 - 1) = u^3 + 4$$

lekin $u=\frac{z}{x}$, $cx^5(z^3-x^3)=z^3+4x^3$, $z=y^{\frac{2}{3}}$ edi,

demak, $cx^5(y^2-x^3)=y^2+4x^3$ berilgan tenglamaning umumiy integrali.

Ushbu $y'=f(ax+by+c)$ ko‘rinishdagi tenglamalar $z=ax+by+c$ almashtirish orqali o‘zgaruvchilari ajraladigan differensial tenglamaga keltiriladi.

<u>8-Misol.</u> Ushbu $(x+2y)y'=1$ differensial tenglamaning $x=0$ da $y=-1$ boshlang‘ich shartni qanoatlantiruvchi yechimini toping.

Yechish. Yangi $z=z(x)\in c'$ funksiya kiritamiz, $z=x+2y$ deb, unda $y'=\frac{1}{2}(z'-1)$. Endi y va y' ifodalarini berilgan differensial tenglamaga qo‘yamiz, bo‘ladi:

$$\frac{1}{2}z(z'-1)=1\Rightarrow zz'-z=2\Rightarrow zz'=z+2.$$

O‘zgaruvchilari ajraladigan tenglama hosil bo‘ldi. dx ga ko‘paytirib $z+2\neq 0$ ga bo‘lsak, o‘zgaruvchilari ajralgan tenglamani topamiz:

$$\frac{z}{z+2}dz=dx.$$

Integrallaymiz

$$\int\frac{z+2-2}{z+2}dz+\ln|c|=x\Rightarrow z-2\ln|z+2|+\ln c-x$$

yoki oldingi funksiya y ga qaytsak:

$x+2y-2\ln|x+2y+2|+2\ln|c|=x\Rightarrow$ ixchamlasak, bo‘ladi $y=\ln\left|\frac{x+2y+2}{c}\right|$ yoki $x+2y+2=c\cdot e^y$ tenglamaning umumiy integrali x=0 da y=–1 boshlang‘ich shartdan $0=c\cdot e^{-1}$ ya’ni $c=0$ ni topamiz.

Shunday qilib boshlang‘ich shartni qanoatlantiruvchi yechim:

$$x+2y+2=0.$$

<u>9-Misol.</u> Urinma osti urinish nuqtasining abssissasi va ordinatasining yig‘indisiga teng bo‘lgan egri chiziqlarni toping.

Yechish. Masala shartidan ushbu differensial tenglamani tuzamiz:

$$\frac{y}{y'}=x+y\Rightarrow ydx=(x+y)dy.$$

Bu tenglamada $x=y\cdot z$ almashtirishni bajarish qulaydir. U holda $dx=ydz+zdy$ va tenglama $y(ydz+zdy)=y(z+1)dy\Rightarrow ydz=dy;\ dz=\frac{dy}{y}$ ko‘rinishga keladi. Integrallaymiz $z=\ln y+\ln c,\quad \Rightarrow y=ce^z$ egri chiziqlar oilasidan iborat yoki $y=c\cdot e^{\frac{x}{y}}$.

<u>10-Misol.</u> $y=f(x)$ egri chiziq (1,1) nuqtadan o‘tadi. Koordinatalar boshidan bu egri chiziqning ixtiyoriy nuqtasida o‘tkazilgan urinmagacha bo‘lgan masofa urinish nuqtasining abssissasiga teng. Ko‘rsatilgan egri chiziq tenglamasini toping.

Yechish. $M(x, y)$ egri chiziqning ixtiyoriy nuqtasi bo‘lsin, bu nuqtadan o‘tuvchi urinma tenglamasini yozamiz:

$$Y - y = y'(X - x),$$

bunda (X, Y) urinma nuqtalarining o‘zgaruvchi koordinatalari, yoki

$$X - \frac{1}{y'}Y + y - xy' = 0$$

urinmaning umumiy tenglamasi. Endi, nuqtadan to‘g‘ri chiziqgacha bo‘lgan masofani topish formulasidan foydalanib topamiz, (0; 0) nuqtadan urinmagacha bo‘lgan masofa, bo‘ladi

$$d = \frac{|y - xy'|}{\sqrt{1 + (y')^2}},$$

bu masofa masala shartiga ko‘ra x ga teng. Demak izlanayotgan egri chiziqning differensial tenglamasi bo‘ladi:

$$\frac{\left|y - x\frac{dy}{dx}\right|}{\sqrt{1 + (y')^2}} = x$$

yoki

$$y^2 - 2xy\frac{dy}{dx} + x^2\left(\frac{dy}{dx}\right)^2 = x^2 + x^2\left(\frac{dy}{dx}\right)^2,$$

ya’ni

$$2xy\frac{dy}{dx} = y^2 - x^2$$

bir jinsli differensial tenglama hosil bo‘ldi. Almashtirish kiritamiz $y = u \cdot x$, unda $\frac{dy}{dx} = u + x\frac{du}{dx}$, x^2 qisqartirib hosil qilamiz:

$$2xu\frac{du}{dx} + u^2 + 1 = 0.$$

O‘zgaruvchilarni ajratib topamiz:

$$\frac{2u}{u^2 + 1}du + \frac{dx}{x} = 0,$$

$$\ln(u^2 + 1) + \ln|x| = \ln c \Rightarrow (u^2 + 1)x = c.$$

Lekin $u = \frac{y}{x}$ edi, $y^2 + x^2 = cx$. Masala shartiga muvofiq egri chiziq (1; 1) nuqtadan utadi: 1+1=*s*, *s*=2. Shunday qilib, izlanayotgan egri chiziq tenglamasi

$$y^2 + x^2 = 2x$$

yoki

$$(x - 1)^2 + y^2 = 1,$$

bu markazi (1; 0) nuqtada radiusi 1 ga teng bo‘lgan aylana.

3. Birinchi tartibli chiziqli va unga keltiriladigan tenglamalar

Birinchi tartibli chiziqli tenglama deb noma'lum funksiya y va uning hosilasiga y' nisbatan chiziqli bo'lgan tenglamaga aytiladi.

Uning ko'rinishi

$$\frac{dy}{dx} + p(x)y = q(x), \tag{3.1}$$

bu yerda $p(x)$, q(x) lar *x* ning *(a,b)* dagi qiymatlari uchun uzluksiz funksiyalardir, xususiy holda bulardan bittasi o'zgarmas son bo'lishi mumkin.

Agar $q(x) \equiv 0$ bo'lsa, bu holda tenglama quyidagi

$$\frac{dy}{dx} + p(x)y = 0 \tag{3.2}$$

ko'rinishda bo'ladi va u (3.1) differensial tenglamaga mos chiziqli bir jinsli tenglama deyiladi. Bu o'zgaruvchilari ajraladigan tenglama, uning umumiy yechimi:

$$y = c \cdot e^{-\int p(x)dx} \tag{3.3}$$

Teorema. Birinchi tartibli chiziqli differensial tenglamaning umumiy yechimi:

$$y = e^{-\int p(x)dx}\left(c + \int q(x)e^{\int p(x)dx}dx\right) \tag{3.4}$$

formula orqali ifodalanadi, bunda *s* – ixtiyoriy o'zgarmas son.

Bu teoremaning isbot etish uchun bir nechta usullardan foydalanish mumkin, masalan:

a) o'zgarmasni variasiya usuli bilan, bunda (3.1) tenglamaning yechimini

$$y = c(x) \cdot e^{-\int P(x)dx} \tag{3.5}$$

ko'rinishda izlaydilar, bunda *c(x)* – yangi noma'lum differensiallanuvchi funksiya.

1 – Misol: Lagranj - o'zgarmasni variasiyalash usuli bilan ushbu

$$y' - y\sin x = \sin x\cos x$$

chiziqli differensial tenglamaning umumiy yechimini toping.

Yechish. Avvalo bu tenglamaga mos bir jinsli tenglamani yechamiz:

$$y' - y\sin x = 0$$

$$\frac{dy}{dy} = y\sin x \Rightarrow \frac{dy}{y} = \sin xdx$$

integrallaymiz:

$$\ln|y| = -\cos x + \ln|c| \Rightarrow y = C \cdot e^{-\cos x}$$

bu bir jinsli tenglamaning umumiy yechimi, bunda *S* – ixtiyoriy o'zgarmas.

Endi bu tenglikda *S=S(x)* deb berilgan differensial tenglamaning yechimini quyidagi ko'rinishda izlaymiz:

$$y = C(x) \cdot e^{-\cos x} \Rightarrow y' = C'(x)e^{-\cos x} + C(x) \cdot \sin x \cdot e^{-\cos x}$$

Endi berilgan tenglamaga *u* va y' ifodalarni qo'yamiz:

$$C'(x) \cdot e^{-\cos x} + C(x)\sin x \cdot e^{-\cos x} - C(x)\sin x \cdot e^{-\cos x} = \sin x\cos x$$

yoki

$$C'(x)\cdot e^{-\cos x} = \sin x\cos x \Rightarrow C'(x) = \sin x\cos xe^{\cos x}.$$

Keyingi tenglikdan topamiz:

$$C(x) = \int \sin x\cos x\cdot e^{\cos x}dx + C_1$$

O‘ng tomondagi integralda *t=cosx* deb oson topish

$$C(x) = -\cos xe^{\cos x} + e^{\cos x} + C.$$

Demak, berilgan chiziqli differensial tenglamaning umumiy yechimi

$$y = C(x)\cdot e^{-\cos x} = e^{-\cos x}(-\cos xe^{\cos x} + e^{\cos x} + C)$$

yoki

$$y = Ce^{-\cos x} - \cos x + 1$$

bo‘ladi, bunda *S* - ixtiyoriy o‘zgarmas.

<u>2-Misol.</u> Ushbu $y' - \frac{1}{x}y = x\cos x$ chiziqli tenglamani integrallang.

Yechish. Oldin mos bir jinsli tenglamaning umumiy yechimini topamiz:

$$\frac{dy}{dx} - \frac{y}{x} = 0 \text{ dan } \frac{dy}{y} = \frac{dx}{x},$$

integrallaymiz

$$\ln|y| = \ln|x| + \ln C, \quad C > 0 \Rightarrow y = C\cdot x,$$

bunda *C* – ixtiyoriy o‘zgarmas.

Endi $C = C(x)$ deb, berilgan tenglamaning yechimini $y = C(x)\cdot x$ ko‘rinishda izlaymiz, ya’ni variasiya usulini qo‘llaymiz:

$$y' = C'(x)\cdot x + C(x).$$

u va y' ifodalarni berilgan tenglamaga qo‘yib, berilgan differensial tenglamaning umumiy yechimini

$y = Cx + \sin x$

ko‘rinishda topamiz, bunda *C* – ixtiyoriy o‘zgarmas.

<u>3 – Misol.</u> Ushbu $\frac{dy}{dx} - \frac{y}{x} = x^2$ chiziqli tenglamaning boshlang‘ich shartni $y(1) = \frac{1}{2}$ qanoatlantiruvchi yechimini toping.

Yechish. Oldin mos bir jinsli $\frac{dy}{dx} - \frac{y}{x} = 0$ tenglamaning umumiy yechimini topamiz.

Ravshanki, $\frac{dy}{y} = \frac{dx}{x}$, bundan *u =Cx*, *C* – ixtiyoriy o‘zgarmas.

Endi berilgan tenglamaning yechimini

$$y = C(x)\cdot x \qquad (*)$$

ko‘rinishda izlaymiz, *C(x)* – noma’lum funksiya.

Endi *u* va y' ifodalarini berilgan tenglamaga qo‘yamiz:

$$C'(x)\cdot x + C(x) - C(x) = x^2$$

yoki

$$C'(x) = x$$

bundan integrallab topamiz

$$C(x) = \frac{x^2}{2} + C \ ,$$

buni (*) tenglamaga qo'yib, berilgan tenglamaning umumiy yechimini

$$y = Cx + \frac{1}{2}x^3 ,$$

bunda *S* – ixtiyoriy o'zgarmas, ko'rinishda topamiz.

2-chi o'rniga qo'yish usuli

b) (3.1) differensial tenglamaning yechimini quyidagi ko'rinishda izlaymiz

$$y = u \cdot v \tag{3.6}$$

bunda $u = u(x), \quad v = v(x)$ noma'lum funksiyalar bo'lib, birini aytaylik v ni ixtiyoriy tanlab olish mumkin.

Almashtirishni bajarsak, ya'ni $y = u \cdot v$, $y' = u'v + uv'$ larni (3.1) tenglamaga qo'ysak

$$u'v + uv' + p(x)u \cdot v = q(x)$$

tenglamani hosil qilamiz. Bundan

$$u'v + (v' + p(x)v)u = q(x) .$$

Yuqorida aytib o'tdik $v(x)$ funksiyani ixtiyoriy tanlab olish mumkin deb, uni $v' + p(x)v = 0$ shartdan topib olamiz

$$v = e^{-\int p(x)dx} \tag{3.7}$$

Ikkinchi noma'lum funksiya $u(x)$ ushbu $u' \cdot e^{-\int p(x)dx} = q(x)$ tenglamani integrallash orqali hosil bo'ladi:

$$u(x) = \int q(x)e^{\int p(x)dx}dx + C \ , \tag{3.8}$$

endi, $u(x)$ va funksiyalar uchun topilgan (3.7) va (3.8) ifodalarni ko'paytirsak (3.4) formulani hosil qilamiz.

<u>4-Misol.</u> $y' - 3y = 2e^x$ tenglamani yeching.

Yechish. Yechimni $y = u(x) \cdot v(x)$ ko'rinishda izlaymiz, bunda $u = u(x), \quad v = v(x)$ hozircha noma'lum funksiyalar, y ва $y' = u'v + uv'$ larni tenglamaga qo'yib, topamiz:

$$u'v + uv' + 3uv = 2e^x$$

$$u'v + u\left(\frac{dv}{dx} - 3v\right) = 2e^x$$

$v(x)$ noma'lum funksiyani shunday tanlab olamizki qavsdagi ifoda nolga teng bo'lsin:

$$\frac{dv}{dx} - 3v = 0 \Rightarrow \frac{dv}{v} = 3dx$$

integrallaymiz

$\ln|v| = 3x + \ln|c|, \quad v = c \cdot e^{3x}, \quad c = 1$ desak,

bo'ladi $v(x) = e^{3x}$.

$u(x)$ noma'lum funksiyani $e^{3x} \cdot \frac{du}{dx} = 2e^x$ tenglamadan topamiz, bo'ladi

$$du = 2e^{-2x} \cdot dx .$$

Integrallasak, $u(x) = -e^{-2x} + c$.

Endi, $u(x)$ va $v(x)$ funksiyalarning ifodalarini ko‘paytirib, berilgan differensial tenglamaning umumiy yechimini hosil qilamiz

$$y = c \cdot e^{3x} - e^{x} ,$$

bunda *c* – ixtiyoriy o‘zgarmas.

Ba’zi differensial tenglamalar chiziqli bo‘ladi, agar noma’lum funksiya *u* va argument *x* ning rollarini almashtirsa.

<u>5-Misol.</u> $\frac{dy}{dx} = \frac{y}{8x - y^3}$ tenglamani yeching.

Yechish. Bu tenglamada *u* – noma’lum funksiya bo‘lib, *x* argument. Ravshanki bu tenglama *u* ga nisbatan chiziqli emas.

Agar *x* va *u* larning rollarini almashtirsak, ushbu

$$\frac{dx}{dy} = \frac{8}{y}x - y^2, \quad x'(y) = a(y)x + b(y)$$

chiziqli tenglamaga kelamiz. Buning umumiy yechimini, qo‘yidagi

$$x(y) = c \cdot e^{\int a(y)dy} + e^{\int a(y)dy} \cdot \int b(y) \cdot e^{-\int a(y)dy} dy$$

formula orqali topish mumkin, tenglamamizda $a(y) = \frac{8}{y}, \; b(y) = -y^2$.

Natijada, bo‘ladi:

$$X(y) = c \cdot e^{\int \frac{8}{y}dy} - e^{\int \frac{8}{y}dy} \cdot \int y^2 \cdot e^{-\int \frac{8}{y}dy} dy = c \cdot e^{8\ln|y|} - e^{8\ln|y|} \cdot \int y^2 \cdot e^{8\ln|y|} dy = c \cdot y^8 - y^8 \cdot \int y^2 \cdot y^{-8} dy$$

Demak, tenglamaning umumiy yechimi

$X = cy^8 + \frac{1}{5}y^8 \cdot y^{-5}$ ёки $X = cy^8 + \frac{1}{5}y^3$, bunda *s* – ixtiyoriy o‘zgarmas.

Ushbu

$$\ddot{y} + a(x)y = b(x)y^m \tag{3.9}$$

differensial tenglama (bunda m – haqiqiy son, $m \neq 0, \; m \neq 1$) Bernulli tenglamasi deyiladi.

Bu tenglamani chiziqli differensial tenglamaga keltirish oson buning uchun (3.9) ning ikkala tomonini $y^m \neq 0$ bo‘lamiz, bo‘ladi

$$y^{-m}y' + a(x)y^{1-m} = b(x) ,$$

endi $z = z(x)$ yangi funksiya kiritamiz $y^{1-m} = z$ deb, unda $z' = (1-m)y^{-m} \cdot y'$ va yuqoridagi differensial tenglama quyidagi ko‘rinishga keltiriladi

$$z' + (1-m) \cdot a(x)z = (1-m)b(x)$$

ya’ni chiziqli differensial tenglamagani hosil qildik.

Izoh. Bernulli tenglamasining yechimini to‘g‘ridan-to‘g‘ri $y = u(x) \cdot v(x)$ almashtirish yordamida ham topish mumkin. Buni misolda ko‘rsatamiz.

<u>6- Misol.</u> Ushbu $xy' + y = y^2 \ln x$ Bernulli tenglamasini yeching.

Yechish. Bu tenglamani yechish uchun $y = u \cdot v,\ (u = u(x),\ v = v(x))$ almashtirishni bajaramiz, $y' = u'v + uv'$ bo‘ladi

$$xu'v + xv'u + uv = u^2v^2 \ln x,$$

yoki

$$xu'v + u(xv' + v) = u^2v^2 \ln x$$

$xv' + v = 0$ bo‘lishini talab qilamiz.

Unda $\frac{dv}{v} = -\frac{dx}{x} \Rightarrow \ln v = -\ln x + \ln c,\ c = 1$ deb $v = \frac{1}{x}$ ni topamiz.

Ikkinchi noma’lum $u = u(x)$ funksiyani $xu'v = u^2v^2 \ln x$ tenglamadan topamiz $v = \frac{1}{x}$ ni hisobga olsak, bo‘ladi

$$X\frac{du}{dx} = u^2\frac{\ln x}{x} \Rightarrow \frac{du}{u^2} = \frac{\ln x}{x^2}dx$$

integrallab hosil qilamiz

$$\frac{1}{u} = \frac{1}{x}(\ln x + 1 + cx) \Rightarrow u = \frac{x}{\ln x + 1 + cx}, \quad y = u \cdot v$$

ekanligini hisobga olsak, bo‘ladi

$$y = \frac{1}{\ln x + 1 + cx}$$

bu berilgan differensial tenglamaning umumiy yechimi bo‘ladi.

Ravshanki, bizning tenglamamiz bundan tashqari $y = 0$ yechimga ega, bu yechim $c = \infty$ da umumiy yechimdan hosil bo‘ladi.

7-Misol. Shunday egri chiziqlar topilsinki, uning ixtiyoriy nuqtasidan o‘tkazilgan urinma, shu nuqtadagi radius-vektor va absissa o‘qining kesmasi bilan hosil bo‘lgan uchburchak yuzi o‘zgarmas a^2 ga teng bo‘lsin.

Yechish. Egri chiziqda ixtiyoriy *M(x,y)* nuqta olamiz.

Uchburchak yuzi:

$$S = \frac{1}{2}OA \cdot MN$$

$$ON = X, \quad MN = Y$$

O‘rinma osti

$AN = \frac{y}{y'}$; chizmadan ko‘rinadiki

$$OA = ON - AN \quad \text{ёки} \quad OA = X - \frac{y}{y'}$$

Masala shartiga ko‘ra

$$\frac{1}{2}y\left(x - \frac{y}{y'}\right) = a^2 \Rightarrow y' = \frac{y^2}{xy - 2a^2}$$

yoki

$$\frac{dx}{dy} = \frac{1}{y}x - \frac{2a^2}{y^2} \quad (*)$$

x-ga nisbatan chiziqli differensial tenglamani hosil qildik.

Bu tenglamani o'zgarmasni variasiya metodi bilan yechamiz, shu uchun oldin unga mos bo'lgan chiziqli bir jinsli tenglamani yechamiz:

$$\frac{dx}{dy}=\frac{x}{y}\Rightarrow x=c\cdot y,$$ c – ixtiyoriy o'zgarmas.

Ushbu $x=c(y)\cdot y$, bunda *c(y)* hozircha noma'lum, (*) ko'rinishda differensial tenglama yechimini izlaymiz:

$$\frac{dx}{dy}=c(y)+y\cdot c'(y)$$

$$c(y)+y\cdot c'(y)=c(y)-\frac{2a^2}{y^2}\Rightarrow$$ soddalashtiramiz,

bo'ladi: $c'(y)=-\frac{2a^2}{y^3}$, integrallaymiz: $c(y)=\frac{a^2}{y^2}+c$.

Endi *c(y)* ifodasini $x=c(y)\cdot y$ ga qo'yib, berilgan tenglamaning umumiy integralini topamiz:

$$x=\left(\frac{a^2}{y^2}+c\right)\cdot y \quad \text{ёки} \quad xy=a^2+cy^2,$$

bunda *c* – ixtiyoriy o'zgarmas.

8-Misol. Shunday egri chiziqlar topilsinki, uning ixtiyoriy nuqtasidan o'tkazilgan urinmaning urinma osti uzunligi urinish nuqtasining o'rta arifmetigiga teng bo'lsin.

Yechish. Egri chiziq tenglamasi *y=f(x)* bo'lsin, uning grafigida ixtiyoriy nuqta *M(x,u)* bo'lsin.

Urinma osti: $AN=\frac{y}{y'}$. Masala shartiga ko'ra $|AN|=\frac{x+y}{2}$.

Shunday qilib oxirgi ikkita tenglamalardan, ushbu

$$\frac{y}{y'}=\frac{x+y}{2} \quad \text{ёки} \quad \frac{dy}{dx}=\frac{2y}{x+y}$$

bir jinsli differensial tenglamani topamiz, yoki

$$\frac{dx}{dy}=\frac{1}{2y}x+\frac{1}{2}$$

x – ga nisbatan chiziqli differensial tenglamani topamiz.

Differensial tenglamani yechib, egri chiziq tenglamasini hosil qilamiz (tenglamani yechish kitobxonga topshiriladi).

Rikkati tenglamasi

Ushbu

$$\frac{dy}{dx}=p(x)y^2+q(x)y+f(x) \quad (3.10)$$

ko'rinishdagi tenglama Rikkati tenglamasi deyiladi, bu yerda *p(x)*, *q(x)*, *f(x)* lar *xϵ(a,b)* dagi qiymatlarida uzluksiz funksiyalar bo'lib, $p(x)\cdot f(x)\neq 0$. Bu shartlarga ko'ra (3.10) tenglama yagona *u=u(x)* yechimga ega bo'lib, $x=x_0$, $u(x_0)=u_0$

qiymatlarni qabul qiladi, $x_0\epsilon(a,b)$. Rikkati tenglamasi maxsus yechimga ega emas. Umumiy holda elementar funksiyalar orqali tenglamani kvadraturada integrallab bo‘lmaydi. Shu sababga ko‘ra Rikkati tenglamasining xossalari, umumiy yechimining ko‘rinishi, integrallash masalalari xususiy yechimlar berilgandagina bajariladi.

Agar $u=u_1(x)$ Rikkati tenglamaning xususiy yechimi bo‘lsa, bu tenglamani chiziqli tenglamaga keltirish mumkin. Buning uchun

$u=z+y_1(x)$ (3.11)

almashtirishdan foydalanamiz, bu yerda $u_1(x)$ Rikkati tenglamasining xususiy yechimi.

Misollar yechishda ba’zi hollarda Rikkati tenglamasi uchun xususiy yechimni biror ko‘rinishda izlash va uni topish mumkin bo‘ladi.

<u>1-Misol.</u> Ushbu $x^2\frac{dy}{dx}+xy+x^2y^2=4 \Rightarrow y'+\frac{1}{x}y+y^2=\frac{4}{x^2}$ tenglama Rikkati tenglamasi bo‘lib, uning xususiy yechimini $y=\frac{a}{x}$ ko‘rinishda izlash maqsadga muvofiqdir. Bundan: $y'=-\frac{a}{x^2}$

unda

$$x^2\cdot\left(-\frac{a}{x^2}\right)+x\cdot\frac{a}{x}+x^2\cdot\frac{a^2}{x^2}=4 \qquad (**)$$

yoki

$$-a+a+a^2=4, \quad a^2=4 \Rightarrow a=\pm 2$$

Oson tekshirishki (**) ga o‘rniga ±2 qo‘yib

$$y_1=\frac{2}{x} \quad \text{ва} \quad y_2=-\frac{2}{x}$$

funksiya berilgan tenglamani qanoatlantiradi. Agar $y=\frac{2}{x}$ ni olsak va $y=z+\frac{2}{x}$ almashtirishni bajaramiz

$$x^2\left(z'-\frac{2}{x^2}\right)+x\cdot\frac{2}{x}+xz+x^2\left(\frac{4}{x^2}+4\cdot\frac{7}{x}+z^2\right)=4 .$$

Ixchamlab Bernulli tenglamasini topamiz:

$$z'+\frac{5}{x}z=-z^2 .$$

Bu tenglamani yechishni kitobxonga qoldiramiz. Javob: $z=\frac{4}{4c_1x^5-x}$.

Undan keyin $y=z+\frac{2}{x}$ dan $y=\frac{4}{cx^5-x}+\frac{2}{x}$ - Rikkati tenglamasining umumiy yechimini hosil qilamiz. Bunda *s* – ixtiyoriy o‘zgarmas.

<u>2-Misol.</u> Ushbu $(x^2-1)y'+y^2-2xy+1=0$ tenglmani yeching.

Yechish. Tenglamani

$$y'=-\frac{1}{x^2-1}y^2+\frac{2x}{x^2-1}y-\frac{1}{x^2-1}$$

shaklda yozamiz, demak bu Rikkati tenglamasi. Xususiy yechimi $u=ax+b$ ko‘rinishda izlaymiz, a,b lar noma’lum sonlar, $y'=a$ Endi u va y' ifodalarini berilgan tenglamaga qo‘yib hosil qilamiz:

$$\left(x^2-1\right)a+a^2x^2+2abx+b^2-2ax^2-2bx+1=0$$

yoki

$$\left(a^2-2a+1\right)x^2-2bx(a-1)+b^2-a+1=0 \quad \Rightarrow \quad a^2-2a+1=0, \Rightarrow \quad a=1, \mathrm{b}=0\ .$$

Demak $u_1=x$ Rikkati tenglamasining xususiy yechimi. Dastlabki differensial tenglamaga $u=z+x$ almashtirish kiritamiz $y'=z'+1$ ifodalarini tenglamaga qo‘yib topamiz.

$$\left(x^2-1\right)\left(z'+1\right)+z^2+2zx+x^2-2x^2-2zx+1=0 \quad \Rightarrow$$

$$\left(x^2-1\right)z'=-z^2 \quad \Rightarrow \quad -\frac{dz}{z^2}=\frac{dx}{x^2-1}$$

integrallaymiz:

$$\frac{1}{z}=\frac{1}{2}\ln\frac{x-1}{x+1}+\frac{1}{2}\ln|c| \quad \Rightarrow \quad (x+y)\ln c\frac{x-1}{x+1}=2\ .$$

<u>3-Misol.</u> $y'-2x+y^2=5-x^2$ Rikkati tenglamasining xususiy yechimini topib Bernulli tenglamasiga keltiring va yeching.

Yechish. Bu tenglamaning xususiy yechimini

$$u=ax+b,$$

bunda a, b hozircha noma’lum sonlar, ko‘rinishda izlash maqsadga muvofiqdir, bundan $y'=a$.

u, y' ifodalarini berilgan differensial tenglama qo‘yamiz:

$$a-2x(ax+b)+(ax+b)^2\equiv 5-x^2 \quad \Rightarrow \quad a-2ax^2-2bx+a^2x^2+2abx+b^2\equiv 5-x^2$$

$x^2; x; x^0$ –larning chap va o‘ng tomondagi koeffisiyentlarini tenglashtirib hosil qilamiz:

$$\left.\begin{array}{l} -2a+a^2=-1 \\ -2b+2ab=0 \\ a+b^2=5 \end{array}\right\} \Rightarrow \quad a=1; \quad b=\pm 2$$

$a=1$, $b=2$ ni olsak, xususiy yechim $u_1=x+2$ bo‘ladi.

Endi Rikkati tenglamasida

$$y=z+x+2$$

bunda $z \in c'$ yangi noma’lum funksiya, almashtirish bajaramiz. Bunda

$$y'=z'+1.$$

Unda

$$z'+1-2x(z+x+2)+(z+x+2)^2=5-x^2$$

$$z'+1-2xz-2x^2-4x+z^2+x^2+4+2zx+4z+4x=5-x^2$$

yoki ixchamlab, $z'+4z=z^2$ hosil qilamiz:

Bernulli tenglamasi hosil bo‘ldi.

Yechimini: $z=u\cdot v$, $u,v\in c'$ ko‘rinishda izlaymiz, bunda $z'=u'v+uv'$

$$u'v + uv' + 4uv = -u^2v^2 \Rightarrow u'v + u(v' + 4v) = -u^2v^2$$

$$v' + 4v = 0 \Rightarrow \frac{dv}{v} = -4dx \quad \ln v = -4 \cdot x + c_{4c}$$

$$V = Ce^{-4x}, \quad c = 1 \quad \text{десак} \quad V = e^{-4x}$$

$e^{-4x} \cdot u' = -e^{-8x} \cdot u^2$ dan *u* ni topamiz.

O‘zgaruvchilarni ajratib topamiz:

$$\frac{du}{u^2} = -e^{-4x}dx \Rightarrow -\frac{1}{u} = \frac{1}{4}e^{-4x} + \frac{C}{4}$$

$$u = \frac{-4}{e^{-4x} + c}; \quad u = \frac{4e^{4x}}{-c_1e^{4x} - 1} = \frac{4e^{4x}}{ce^{4x} - 1} \quad (-c_1 = c)$$

$$z = u \cdot v = \frac{4}{ce^{4x} - 1}$$

Javob: $y = x + 2 + \frac{4}{ce^{4x} - 1}, \quad y_1 = x + 2$.

4. To‘liq differensialli tenglama. Integrallovchi ko’paytuvchi

Agar

$M(x, y)dx + N(x, y)dy = 0$ (4.1)

tenglamaning chap tomoni birorta *u(x,u)* funksiyaning to‘liq differensialli, ya’ni

$$M(x, y)dx + N(x, y)dy = du(x, y)$$

bo‘lsa, (4.1) tenglama to‘liq differensialli tenglama deyiladi. Bu holda uni quydagicha yozish mumkin: $du(x, y) = 0$.

Shuning uchun uning umumiy integrali *u(x,y)=c* bo‘ladi, bu yerda *s* - ixtiyoriy o‘zgarmas.

<u>1-Misol.</u> Ushbu tenglama integrallansin.

$$(2x - y)dx + (4y - x)dy = 0$$

Yechish. Bu tenglamani quyidagicha yozamiz.

$$2xdy + 4ydy - (ydx + xdx) = 0$$

ravshanki tenglamaning chap tomoni $u(x, y) = x^2 + 2y^2 - xy$ funksiyaning to‘liq differensiali. Shuning uchun tenglamani

$$d(x^2 + 2y^2 - xy) = 0$$

ko‘rinishda yozish mumkin, bundan

$$x^2 + 2y^2 - xy = c$$

umumiy integralni topamiz, *s* - ixtiyoriy o‘zgarmas.

<u>2-Misol.</u> $y(1 + xy)dx - xdy = 0$ tenglamani yeching.

Yechish. Bu tenglamani quyidagicha yozib olamiz:

$$ydx - xdy + xy^2dx = 0$$

Tenglamaning ikkala tomonini $y^2 \neq 0$ bo‘lib olsak, chap tomoni to‘la differensial bo‘ladi

$$\frac{ydx - xdy}{y^2} + xdx = 0, \quad d\left(\frac{x}{y} + \frac{x^2}{2}\right) = 0 \Rightarrow \frac{x}{y} + \frac{x^2}{2} = c$$

Demak, tenglamaning umumiy integrali $2x + x^2 y = cy$ bo‘ladi.

<u>3-Misol.</u> $y(1 + xy)dx - xdy = 0$ tenglamaning *u(*1*)*=1 boshlang‘ich shartni qanoatlantiruvchi yechimi topilsin.

Yechish. Tenglamani quyidagicha yozib olamiz:

$$ydx - xdy + xy^2 dx = 0.$$

Endi buning ikkala tomonini $y^2 \neq 0$ bo‘lib olamiz

$$\frac{ydx - xdy}{y^2} + xdx = 0 \Rightarrow \quad d\left(\frac{x}{y} + \frac{x^2}{2}\right) = 0$$

to‘la differensial tenglama hosil bo‘ldi.

Bundan: $\frac{x}{y} + \frac{x^2}{2} = c, \quad \Rightarrow \quad 2x + x^2 y = cy$ umumiy integralni yozib olamiz, bunda *c* - ixtiyoriy o‘zgarmas.

Endi *x*=1 da *u*=1 deb olsak, *c*=3 bo‘ladi va izlanayotgan xususiy yechim hosil bo‘ladi

$$2x + x^2 y = 3y.$$

Teorema. Ushbu

$$M(x, y)dx + N(x, y)dy = 0$$

differensial tenglamaning (bu yerda *M(x,u)* va *N(x,u)* funksiyalar biror *D* sohada *x* va *u* bo‘yicha uzluksiz hosilalarga ega) to‘liq differensialli tenglama bo‘lishi uchun

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \tag{4.2}$$

shart bajarilishi zarur va yetarlidir.

(4.1) tenglikning umumiy integralini

$$\int_{x_0}^{x} M(x, y)dx + \int_{y_0}^{y} N(x_0, y)dy = c \tag{4.3}$$

yoki

$$\int_{x_0}^{x} M(x, y_0)dx + \int_{y_0}^{y} N(x, y)dy = c \tag{4.4}$$

ko‘rinishda bo‘ladi, bunda $(x_0, y_0) \in D$.

<u>4-Misol.</u> Ushbu $(2xy + 3y^2)dx + (x^2 + 6xy - 3y^2)dy = 0$ tenglama integrallansin.

Yechish. Bu yerda

$$M(x, y) = 2xy + 3y^2$$

$$N(x, y) = x^2 + 6xy - 3y^2$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 2x + 6y.$$

(4.2) shart bajariladi.

Berilgan tenglama to'liq differensialli tenglama ekan, ya'ni

$$\left(2xy+3y^2\right)dx+\left(x^2+6xy-3y^2\right)dy\equiv\frac{\partial M}{\partial y}dx+\frac{\partial N}{\partial x}dy.$$

Bundan,

$$\frac{\partial u}{\partial x}=2xy+3y^2,\quad \frac{\partial u}{\partial y}=x^2+6xy-3y^2$$

oxirgi ikki tengliklardan birinchi tenglikni *u* ni o'zgarmas deb, *x* bo'yicha integrallaymiz

$$u(x,y)=\int(2xy+3y^2)dx+\varphi(y)=x^2y+3xy^2+\varphi(y),$$

bunda $\varphi(y)\in c'$ funksiya *u* ning hozircha noma'lum funksiyasi. Bu munosabatni *y* bo'yicha differensiallab va

$$\frac{\partial u}{\partial y}=x^2+6xy-3y^2$$

ekanini e'tiborga olib,

$$x^2+6xy+\varphi'(y)=x^2+6xy-3y^2$$

bo'lishini topamiz. Demak,

$$\varphi'(y)=-3y^2,\quad \varphi(y)=-y^3+c_1,$$

bunda c_1- ixtiyoriy o'zgarmas.

Shunday qilib, dastlabki tenglamaning umumiy integrali

$$x^2y+3xy^2-y^3=C.$$

Ushbu

$$M(x,y)dx+N(x,y)dy=0 \qquad (4.1)$$

tenglamaning chap tomoni biror funksiyaning to'la differensialli bo'lmasin. Ba'zan holda shunday $\mu(x,y)\neq 0$ funksiya tanlab olish mumkin bo'ladiki, tenglamaning barcha hadlarini $\mu(x,y)$ ga ko'paytirishda tenglamani chap tomoni biror funksiyaning to'la differensialli bo'lib qoladi.

Shu usul bilan hosil qilingan tenglamaning umumiy integrali dastlabki tenglamaning umumiy yechimi bilan bir xil bo'ladi: $\mu(x,y)$ funksiya (4.1) tenglamaning integrallovchi ko'paytuvchisi deyiladi. Har qanday *U(x,y)=C* umumiy integralga ega bo'lgan (5.1) differensial tenglama uchun integrallovchi ko'paytuvchi mavjud, biroq bu uni topish oson degan so'z emas.

Teorema. Ushbu

$$\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N\frac{\partial\omega}{\partial x}-M\frac{\partial\omega}{\partial y}}\equiv\psi(\omega) \qquad (4.5)$$

shartning bajarilishi (4.1) differensial tenglamaning $\omega=\omega(x,y)$ funksiyaga bog'liq bo'lgan $\mu=\mu(\omega)$ integrallovchi ko'paytuvchisining mavjudligi uchun yetarli va zaruriy shart, shu bilan birga

$$\mu=e^{\int\psi(\omega)d\omega} \qquad (4.6)$$

bo'ladi.

Ba'zi xususiy hollarda, jumladan, integrallovchi ko'paytuvchini faqat *x* ga, *u* ga, *xu* ga, *x+u* ga, x^2+u ga, x^2+u^2 ga va h.k. bog'liq deb qarab (4.1) tenglamani integrallash mumkin.

Masalan, a) $\mu = \mu(x)$, ya'ni $\omega = x$ bo'lsin, unda $\omega'_x = 1,\ \omega'_y = 0$ va (4.5) ifoda soddalashib, quyidagi ko'rinishga keladi:

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} \equiv \psi \tag{4.7}$$

Demak,

$$\mu(x) = e^{\int \psi(x)dx} \tag{4.8}$$

shunday qilib, agar $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$ ifoda faqat *x* ning funksiyasi bo'lsa, u holda *x* ga bog'liq bo'lgan integrallovchi ko'paytuvchi $\mu(x)$ mavjud bo'ladi va u (4.8) formula bilan aniqlanadi.

<u>5- Misol.</u> Birinchi tartibli chiziqli tenglama uchun faqat *x* ning funksiyasidan iborat integrallovchi ko'paytuvchi mavjudligini isbotlaymiz.

Yechish. Haqiqatan,

$$\frac{dy}{dx} + p(x)y = q(x)$$

birinchi tartibli chiziqli differensial tenglamani simvolik ravishda quyidagicha yozamiz:

$$[P(x)y - q(x)]dx + dy = 0\ .$$

Oddiy hisoblashdan

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = p(x)$$

kelib chiqadi. Demak, faqat *x* ga bog'liq $\mu(x)$ mavjud va uning qiymati (4.7) dan topiladi:

$$\mu(x) = e^{\int p(x)dx}$$

b) $\mu = \mu(y)$, ya'ni $\omega = y$ bo'lsin, unda $\omega'_x = 0,\ \omega'_y = 0$ va (4.5) ifoda

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M} = \psi(y) \tag{4.9}$$

ko'rinishni oladi va (4.6) dan topamiz:

$$\mu(y) = e^{\int \psi(y)dy} \tag{4.10}$$

<u>6 - Misol.</u> $(2xy^2 - y)dx + (y^2 + x + y)dy = 0$ tenglama integrallansin.

Yechish. $M(x,y) = 2xy^2 - y,\ \ N(x,y) = y^2 + x + y$

$$\frac{\partial M}{\partial y}=4xy-1,\quad \frac{\partial N}{\partial x}=1,\quad \frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$$

(4.9) ni topamiz

$$\psi(y)=\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{-M}=\frac{4xy-1-1}{-y(2xy-1)}=\frac{2(2xy-1)}{-y(2xy-1)}=-\frac{2}{y}.$$

Demak, berilgan tenglama faqat u ga bog‘liq bo‘lgan integrallovchi ko‘paytuvchiga ega ekan. Bu misol uchun

$$\mu=e^{\int\psi(y)dy}=e^{-\int\frac{2}{y}dy}=e^{-2iny}=y^{-2}=\frac{1}{y^2}$$

$$\mu=\frac{1}{y^2}$$

Berilgan tenglamani $\mu=\frac{1}{y^2}$ ga ko‘paytirsak,

$$\left(2x-\frac{1}{y}\right)dx+\left(1+\frac{x}{y^2}+\frac{1}{y}\right)dy=0$$

to‘liq differensialli tenglama hosil bo‘ladi.

U holda $u(x,y)=\int\left(2x-\frac{1}{y}\right)dx+\varphi(y)$ yoki $u(x,y)=x^2-\frac{x}{y}+\varphi(y)$,

bu yerdan $\frac{\partial u}{\partial y}=\frac{x}{y^2}+\varphi'(y)$

Ikkinchi tomondan $\frac{\partial u}{\partial y}=N(x,y)$, ya’ni $\frac{\partial u}{\partial y}=1+\frac{x}{y^2}+\frac{1}{y}$.

U holda

$$\frac{x}{y^2}+\varphi'(y)=1+\frac{x}{y^2}+\frac{1}{y}\Rightarrow\varphi'(y)=1+\frac{1}{y}\ \text{yoki}\ \varphi(y)=y+\ln y+C_1.$$

Demak, $x^2-\frac{x}{y}+y+\ln y=C$ tenglamaning umumiy integrali ekan, bunda S - ixtiyoriy o‘zgarmas.

s) $\mu=\mu(x\cdot y)$, ya’ni $\omega=x\cdot y$ bo‘lsin, unda $\omega'_x=y,\ \omega'_y=x$ va (4.5) ifoda ushbu ko‘rinishni oladi:

$$\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{y\cdot N-x\cdot M}=\psi(x\cdot y) \tag{4.11}$$

Demak,

$$\mu(x\cdot y)=e^{\int\psi(x\cdot y)d(x\cdot y)} \tag{4.12}$$

<u>7-Misol.</u> $(x^2y^3+y)dx+(x^3y^2-x)dy=0$ tenglamani $\mu=\mu(x\cdot y)$ ko‘rinishdagi ko‘paytuvchiga ega ekanligini ko‘rsating.

Yechish. $M(x,y)=x^2-y^3+y,\quad N(x,y)=x^3y^2-x,$

$$\frac{\partial M}{\partial y}=3x^2y^2+1,\quad \frac{\partial N}{\partial x}=3x^2y^2-1,\quad \omega=x\cdot y,\quad \omega'_x=y,\ \omega'_y=x$$

$$\varphi(\omega)=\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N\frac{\partial\omega}{\partial x}-M\frac{\partial\omega}{\partial y}}=\frac{-1}{x\cdot y}=\frac{-1}{\omega}$$

Demak, berilgan tenglama uchun $\mu=\mu(\omega)$, $\omega=x\cdot y$ ko‘rinishdagi integrallovchi ko‘paytuvchi mavjud ekan u

$$\mu=e^{\int\psi(\omega)d\omega}=e^{-\int\frac{1}{\omega}d\omega}=e^{-\ln\omega}=\omega^{-1}=\frac{1}{xy}$$

Dastlabki tenglamani $\mu=\frac{1}{xy}$ ko‘paytiramiz, bo‘ladi:

$$\left(xy^2+\frac{1}{x}\right)dx+\left(x^2y-\frac{1}{y}\right)dy=0\,.$$

$$M(x,y)=xy^2+\frac{1}{x},\quad N(x,y)=x^2y-\frac{1}{y}$$

ravshanki $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}=2xy$, ya’ni to‘liq differensialli tenglamani hosil qildik. Bu tenglamani $x_0=1$, $u_0=1$ olib (4.3) formuladan foydalanib umumiy integralini topamiz

$$\int_1^x\left(xy^2+\frac{1}{x}\right)dx+\int_1^y\left(y-\frac{1}{y}\right)dy=c\Rightarrow x^2y^2+\ln\frac{x}{y}=c,\quad c=const\quad.$$

d) $\mu=\mu(x+y^2)$, ya’ni $\omega=x+y^2$ bo‘lsin, unda $\omega'_x=1,\ \omega'_y=2y$ va (4.5) ifoda quyidagi ko‘rinishda bo‘ladi:

$$\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N\cdot 1-2y\cdot M}=\psi(x+y^2)\tag{4.12}$$

$$\mu=e^{\int\psi(x+y^2)d(x+y^2)}\tag{4.13}$$

<u>8-Misol.</u> $(3y^2-x)dx+(2y^3-6xy)dy=0$ tenglamani $\mu=\mu(x+y^2)$ ko‘rinishdagi integrallovchi ko‘paytuvchiga ega ekanligini ko‘rsating.

Yechish. Berilgan

$$M(x,y)=3y^2-x,\quad N=2y^3-6xy\quad \omega=\mathrm{x}+\mathrm{y}^2\,,$$

u holda

$$\frac{\partial M}{\partial y}=6y,\ \frac{\partial N}{\partial x}=-6y,\ \frac{\partial\omega}{\partial x}=1,\ \frac{\partial\omega}{\partial \mathrm{y}}=2y$$

bo‘lib, (4.12) quyidagi ko‘rinishni oladi:

$$\psi(\omega)=\frac{6y+6y}{(2y^3-6xy)\cdot 1-(3y^2-x)\cdot 2y}=\frac{12y}{-4y^3-4xy}=\frac{-3}{x+y^2}=\frac{-3}{\omega}$$

Demak, berilgan tenglama $\mu=\mu(\omega),\ \omega=\frac{-3}{\mathrm{x}+\mathrm{y}^2}$ ko‘rinishdagi integrallovchi ko‘paytuvchiga ega ekan.

(4.6) formulaga asosan

$$\mu=e^{\int\psi(\omega)d\omega}=e^{-3\int\frac{d\omega}{\omega}}=e^{-3\ln\omega}=\omega^{-3}=(x+y^2)^{-3}=\frac{1}{(x+y^2)^3}$$

Dastlabki tenglamani $\mu = \frac{1}{(x+y^2)^3}$ ko‘paytirib, hosil qilamiz:

$$\frac{3y^2 - x}{(x+y^2)^3}dx + \frac{2y(y^2-3x)}{(x+y^2)^3}dy = 0. \qquad (*)$$

Bundan,

$$M(x,y) = \frac{3y^2 - x}{(x+y^2)^3}, \qquad N(x,y) = \frac{2y(y^2-3x)}{(x+y^2)^3}dy.$$

Oson ko‘rsatishki $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, ya’ni $\mu(x,y) = \frac{1}{(x+y^2)^3}$, tenglamaning integrallovchi ko‘paytuvchisi ekan. Demak, (*) tenglama to‘liq differen-sialli tenglama, uning umumiy integralini (4.3) yoki (4.4) formula orqali topish mumkin.

1. Izoh. $\mu M(x,y)dx + \mu N(x,y)dy = du(x,y)$ dan

$$M(x,y)dx + N(x,y)dy = \frac{1}{\mu}\cdot du = 0$$

kelib chiqadi. (4.1) tenglamaning yechimi $du(x,y) = 0$ va $\frac{1}{\mu(x,y)} = 0$,

tenglamalardan topiladi, ya’ni

$$u(x,y) = c \qquad \text{ва} \qquad \mu(x,y) = \infty$$

yechimni beradi. Ayrim hollarda bu yechimlar maxsus yechim bo‘lishi mumkin.

<u>9-Misol.</u> $2\sqrt{y}dx + dy = 0$ agar bu tenglama uchun $\mu = \mu(y)$ mavjud bo‘lsa, tenglamani yeching.

Yechish. Bunda $M = 2\sqrt{y}$, $N = -1$, $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ chunki $\frac{\partial M}{\partial y} = \frac{1}{\sqrt{y}}$, $\frac{\partial N}{\partial x} = 0$.

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M} = \frac{\frac{1}{\sqrt{y}}}{-2\sqrt{y}} = -\frac{1}{2y} = \psi(y)$$

Haqiqatan, $\mu = \mu(y)$ mavjud ekan, uni topamiz

$$\mu = e^{\int \psi(y)dy} = e^{-\int \frac{dy}{2y}} = \frac{1}{\sqrt{y}}.$$

Dastlabki tenglamani $\mu = \frac{1}{\sqrt{y}}$ ko‘paytiramiz, unda $2dx - \frac{dy}{\sqrt{y'}} = 0$ hosil qilamiz. $x - \sqrt{y} = c$ umumiy integralni $\mu = \frac{1}{\sqrt{y}} = \infty$ dan y=0 maxsus yechim kelib chiqadi.

2. Izoh. Integrallovchi ko‘paytuvchi usuli yordamida o‘zgaruvchilarga ajraladigan, bir jinsli, birinchi tartibli chiziqli differensial tenglamalarni integrallash mumkin.

<u>10-Misol.</u> $xydx + dy = 0$ tenglamani yeching.

Yechish. Bu o‘zgaruvchilari ajraladigan differensial tenglama.

$$M = xu,\ N = 1,\ \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}.$$

Ravshanki $\mu = \frac{1}{y}$ ga tenglamani ikkala tomonini ko‘paytirsa to‘liq differensialli tenglama hosil bo‘ladi:

$$xdx + \frac{1}{y}dy = 0, \quad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 0 .$$

(boshqa tomondan bu o‘zgaruvchilari ajralgan tenglama)

$$\frac{x^2}{2} + \ln|y| = \ln|c| \Rightarrow y = c \cdot e^{-\frac{x^2}{2}}$$

dastlabki tenglamaning umumiy yechimi, bunda c - ixtiyoriy o‘zgarmas.

$\mu = \frac{1}{y} = \infty$ dan y=0 xususiy yechim kelib chiqadi, chunki u $y = c \cdot e^{-\frac{x^2}{2}}$ dan $c=0$ da hosil bo‘ladi.

Mustaqil ish topshiriqlari 8-ta vazifadan iborat:

1) O’zgaruvchilari ajraladigan tenglamalar.
2) Bir jinsli va unga keltiriladigan differentsial tenglamalar.
3) Chiziqli differentsial tenglamalarni yeching.
4) Koshi shartlari berilgan differentsial teglamaning ko’rsatilgan boshlang’ich shartini qanoatlantiruvchi yechimini toping
5) Egri chiziqlar oilasining differensial tenglamasini tuzing va turini aniqlang.
6) Bernulli tenglamasini quyidagi usullar bilan yeching.
   a) Chiziqli tenglamaga keltirib;
   b) O’zgarmasni variasiya usuli bilan.
7) Quyidagi tenglamalar to’liq difrentsiali tenglamalar ekanligini tekshiring va integrallang.
8) Qulay almashtirish yoki integrallovchi ko’paytuvchi yordamida quyidagi differentsial tenglamalarni integrallang.

5. Namunaviy misollar va topshiriqlar

1) O’zgaruvchilari ajraladigan teglamalar.

Berilgan differentsial tenglamaning umumiy yechimini toping.

$(xy^2 + x)dx + (y - x^2 y)dy = 0$

<u>Yechish:</u> Tenglamani quyidagicha yozib olamiz.

$$y(1 - x^2)dy = -x(1 + y^2)dx$$

Bu tenglama o`zgaruvchilari ajraladigan tenglamadir.

O`zgaruvchilarini ajratamiz. Buning uchun tenglamani ikkala tomonini $(1 - x^2)(1 + y^2)$ ifodaga bo`lamiz.

$$\frac{y(1 - x^2)}{(1 - x^2)(1 + y^2)}dy = -\frac{x(1 + y^2)}{(1 - x^2)(1 + y^2)}dx \quad \text{yoki}$$

$$\frac{y}{1+y^2}dy = -\frac{x}{1-x^2}dx$$

Tenglamani ikkala tomonini integrallaymiz:

$$\frac{1}{2}\ln\left|1+y^2\right| = \frac{1}{2}\ln\left|1-x^2\right| + \frac{1}{2}\ln C \quad \text{ёки}$$

$$1+y^2 = C\left|x^2-1\right|$$

$$y^2 = C\left|x^2-1\right|-1$$

U holda berilgan tenglamani umumiy yechimi:

$$y = \pm\sqrt{C\left|x^2-1\right|-1}\,.$$

2) Bir jinsli va unga keltiriladigan differential tenglamalar.

Differentsial tenglamaning umumiy yechimini toping.

$$y - x\frac{dy}{dx} = x + y\frac{dy}{dx}$$

Yechish: Bu tenglamadan $\frac{dy}{dx}$ ni topamiz:

$$(y+x)\frac{dy}{dx} = y - x \quad \text{yoki}$$

$$\frac{dy}{dx} = \frac{y-x}{y+x}$$

Bu tenglama birinchi tartibli bir jinsli tenglamadir. Tenglamani $y = x\cdot u$ almashtirish yordamida yechamiz. $\frac{dy}{dx} = u + x\frac{du}{dx}$. Bularni yuqoridagi tenglamaga qo`yamiz.

$$x\frac{du}{dx} + u = \frac{ux-x}{ux+x} = \frac{x(u-1)}{x(u+1)} = \frac{u-1}{u+1}$$

yoki

$$x\frac{du}{dx} + u = \frac{u-1}{u+1}$$

$$x\frac{du}{dx} = \frac{u-1}{u+1} - u = \frac{u-1-u^2-u}{u+1}$$

$$x\frac{du}{dx} = \frac{u^2+1}{u+1}$$

o`zgaruvchilarni ajratamiz.

$\frac{u+1}{u^2+1}du = -\frac{dx}{x}$ ikkala tomonini integrallaymiz.

$$\int\frac{u}{u^2+1}du + \int\frac{1}{u^2+1}du = -\int\frac{dx}{x}$$

bundan

$$\frac{1}{2}\ln\left|u^2+1\right| + arctgu = -\ln|x| + \ln|c|$$

bundan

$$arctgu = \ln\left|\frac{c}{x\sqrt{u^2+1}}\right|$$

Endi $u=\frac{y}{x}$ ni qo`ysak tenglamani umumiy yechimini topamiz:

$$arctg\frac{y}{x}=\ln\left|\frac{c}{\sqrt{x^2+y^2}}\right|.$$

3) Chiziqli differentsial tenglamani yeching

$\frac{dy}{dx}=\frac{y}{8x-y^3}$ tenglamani yeching.

Yechish. Bu tenglamada *u* – noma'lum funksiya bo'lib, *x* argument. Ravshanki bu tenglama *u* ga nisbatan chiziqli emas.

Agar *x* va *u* larning rollarini almashtirsak, ushbu

$$\frac{dx}{dy}=\frac{8}{y}x-y^2,\quad x'(y)=a(y)x+b(y)$$

chiziqli tenglamaga kelamiz. Buning umumiy yechimini, quyidagi

$$x(y)=c\cdot e^{\int a(y)dy}+e^{\int a(y)dy}\cdot\int b(y)\cdot e^{-\int a(y)dy}dy$$

formula orqali topish mumkin, tenglamamizda $a(y)=\frac{8}{y},\ b(y)=-y^2$.

Natijada, bo'ladi:

$$X(y)=c\cdot e^{\int\frac{8}{y}dy}-e^{\int\frac{8}{y}dy}\cdot\int y^2\cdot e^{-\int\frac{8}{y}dy}dy=c\cdot e^{8\ln|y|}-e^{8\ln|y|}\cdot\int y^2\cdot e^{8\ln|y|}dy=c\cdot y^8-y^8\cdot\int y^2\cdot y^{-8}dy$$

Demak, tenglamaning umumiy yechimi

$X=cy^8+\frac{1}{5}y^8\cdot y^{-5}$ ёки $X=cy^8+\frac{1}{5}y^3$, bunda *s* – ixtiyoriy o'zgarmas.

4) Koshi shartlari berilgan differentsial teglamaning ko'rsatilgan boshlang'ich shartini qanoatlantiruvchi yechimini toping.

$dy-e^{-x}dx+ydx-xdy=xydx$ ***differentsial tenglamani*** $y(0)=\ln 5$ ***boshlang`ich shartni qanoatlantiruvchi xususiy yechimini toping.***

Yechish: Berilgan tenglamada shakl almashtiramiz:

$$(1-x)dy=(xy+e^{-x}-y)dx$$

$$\frac{dy}{dx}=\frac{xy+e^{-x}-y}{1-x}=\frac{-(1-x)y+e^{-x}}{1-x}=-y+\frac{e^{-x}}{1-x}\qquad\frac{dy}{dx}+y=\frac{e^{-x}}{1-x}$$

Bu tenglama birinchi tartibli chiziqli tenglamadir. Tenglamani yechish uchun $y=u(x)\cdot v(x)$ almashtirish olamiz. U holda $\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$ ekanidan foydalansak, tenglamani ko`rinishi:

$$u\frac{dv}{dx}+v\frac{du}{dx}+uv=\frac{e^{-x}}{1-x}$$ yoki

$$u\frac{dv}{dx}+v\frac{du}{dx}+uv=\frac{e^{-x}}{1-x}\qquad(*)$$

$v(x)$ funktsiyaning qiymatini $\frac{dv}{dx}+v=0$ tenglamadan topamiz:

$$\frac{dv}{dx} = -v$$

$$\frac{dv}{v} = -dx$$

integrallaymiz.

$$\int \frac{dv}{v} = -\int dx \qquad \ln|v| = -x \qquad v = e^{-x}$$

topilgan qiymatni (*) tenglamaga qo`yamiz

$$\frac{du}{dx} e^{-x} = \frac{e^{-x}}{1-x} \qquad \text{yoki}$$

$$\frac{du}{dx} = \frac{1}{1-x}$$

buni integrallamiz

$$\int du = \int \frac{1}{1-x} dx \, ,$$

$$u = -\ln|1-x| + \ln C \, ,$$

$$u = \ln \frac{C}{|1-x|} \, ,$$

$y = uv$ ekanidan foydalansak tenglamani umumiy yechimi

$$y = uv = e^{-x} \cdot \ln \frac{C}{|1-x|}$$

$y(0) = \ln 5$ boshlang`ich shartdan foydalansak

$$\ln 5 = e^{-0} \cdot \ln C \, ,$$

$$\ln C = \ln 5 \, ,$$

$$C = 5 \, ,$$

U holda tenglamani xususiy yechimi

$$y = e^{-x} \cdot \ln \frac{5}{|1-x|}$$

bo`ladi.

5) **Egri chiziqlar oilasining differensial tenglamasini tuzing**

Berilgan egri chiziqlar oilasining differensial tenglamasini tuzing. $(x-c)^2 + y^2 = c^2$

$$F(x, y, c) \equiv x^2 - 2xc + y^2 - c^2 = 0 \qquad (1)$$

ga ega bo’lib, bundan

$$2x - 2c + 2yy' = 0$$

$$2c = 2x + 2yy' \qquad (2)$$

(1) va (2) dan

$$x^2 - x(2x + 2yy') + y^2 = 0$$

yoki $x^2 + 2xyy' - y^2 = 0$

6) **Bernulli tenglamasini yeching.**

a) **(Chiziqli tenglamaga keltirib)**

1-Misol. $xy' + y = y^2 \ell nx$ Tenglamani yeching.

Yechish: Tenglamaning ikkala tomonini x-ga bo’lamiz:

$$y'+\frac{y}{x}=y^2\frac{\ell nx}{x} \qquad y^{-2}y'+\frac{1}{x}y^{-1}=\frac{\ell nx}{x}$$

$$z=y^{-1} \qquad z'=-y^{-2}y'$$

$$-z'+\frac{1}{x}z=\frac{\ell nx}{x} \qquad z'-\frac{1}{x}z=-\frac{\ell nx}{x}$$

$$z=\ell^{\int\frac{dx}{x}}\left[-\int \ell^{-\int\frac{dx}{x}}\frac{\ell nx}{x}dx+c\right]=\ell^{\ell n|x|}\cdot\left[(-\int \ell^{-\ell n|x|}\frac{\ell nx}{x}dx+c\right]=x\left[-\int\frac{\ell nx}{x^2}dx+c\right]=$$

$$u=\ell nx \qquad du=\frac{dx}{x} \qquad \int\frac{\ell nx}{x^2}dx=-\frac{\ell nx}{x}-\frac{1}{x}$$

$$d\vartheta=\frac{dx}{x^2} \qquad \vartheta=-\frac{1}{x}$$

$$=x\left[\frac{\ell nx+1}{x}+c\right]=cx+1-\ell nx$$

$$y^{-1}=cx+1-\ell nx, \qquad y=\frac{1}{cx+1-\ell nx}$$

b) (Ozgarmaslarni variatsiyalash usuli bilan)

2-Misol $y'+2y=y^2e^x$ tenglamaning umumiy yechimini toping

Yechimi. Tenglamaning ikkala tomonini y^2 ga bo’lamiz.

$$\frac{y'}{y^2}+\frac{2}{y}=e^x, \quad y\neq 0, \quad n=2, \quad z=y^{1-n}=y^{-1},$$

$$z'=-1\cdot y^{-2}y'=\frac{y'}{y^2}, \quad -z'+2z=e^x, \quad z'-2z=e^x;$$

$$z'-2z=0, \quad \frac{dz}{dx}=2z, \quad \frac{dz}{z}=2dx, \quad \ln|z|=2x+\ln|C_1|, \quad C_1\neq 0,$$

$$z=C^{2x}=0, \quad C\in R; \quad z=C(x)e^{2x}, \quad z'=C'(x)e^{2x}+2C(x)e^{2x},$$

$$C'(x)e^{2x}+2C(x)e^{2x}-2C(x)e^{2x}=e^x, \quad C'(x)=-e^{-x},$$

$$C(x)=e^{-x}+C_2, \quad z=\left(C_2+e^{-x}\right)e^{2x}, \quad \frac{1}{y}=\left(C_2+e^{-x}\right)e^{2x},$$

$$y=\frac{1}{\left(C_2+e^{-x}\right)e^{2x}}, \quad y=0.$$

Oxirgi $y=0$ yechimni tenglamaning har ikkala qismini y^2 ga bo’linganda yo’qotilgan, shuni qo’shib qo’ydik.

7) Quyidagi tenglamalar to’liq difrentsiali tenglamalar ekanligini tekshiring va integrallang.

$(2xy+3y^2)dx+(x^2+6xy-3y^2)dy=0$ tenglama integrallansin.

Yechish. Bu yerda

$$M(x,y) = 2xy + 3y^2$$

$$N(x,y) = x^2 + 6xy - 3y^2$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 2x + 6y .$$

Demak, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ shart bajariladi. Berilgan tenglama to'liq differensialli tenglama ekan, ya'ni

$$\left(2xy + 3y^2\right)dx + \left(x^2 + 6xy - 3y^2\right)dy \equiv \frac{\partial M}{\partial y}dx + \frac{\partial N}{\partial x}dy .$$

Bundan,

$$\frac{\partial u}{\partial x} = 2xy + 3y^2, \quad \frac{\partial u}{\partial y} = x^2 + 6xy - 3y^2$$

oxirgi ikki tengliklardan birinchi tenglikni u ni o'zgarmas deb, x bo'yicha integrallaymiz

$$u(x,y) = \int (2xy + 3y^2)dx + \varphi(y) = x^2 y + 3xy^2 + \varphi(y) ,$$

bunda $\varphi(y) \in c'$ funksiya u ning hozircha noma'lum funksiyasi. Bu munosabatni u bo'yicha differensiallab va

$$\frac{\partial u}{\partial y} = x^2 + 6xy - 3y^2$$

ekanini e'tiborga olib,

$$x^2 + 6xy + \varphi'(y) = x^2 + 6xy - 3y^2$$

bo'lishini topamiz. Demak,

$$\varphi'(y) = -3y^2, \quad \varphi(y) = -y^3 + c_1 ,$$

bunda s_1- ixtiyoriy o'zgarmas.

Shunday qilib, dastlabki tenglamaning umumiy integrali

$$x^2 y + 3xy^2 - y^3 = C .$$

8) Qulay almashtirish yoki integrallovchi ko'paytuvchi yordamida quyidagi differentsial tenglamalarni integrallang.

$(2xy^2 - y)dx + (y^2 + x + y)dy = 0$ tenglama integrallansin.

Yechish. $M(x,y) = 2xy^2 - y, \quad N(x,y) = y^2 + x + y$

$$\frac{\partial M}{\partial y} = 4xy - 1, \quad \frac{\partial N}{\partial x} = 1, \quad \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

Integrallovchi ko'paytuvchini topamiz

$$\psi(y) = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M} = \frac{4xy - 1 - 1}{-y(2xy - 1)} = \frac{2(2xy - 1)}{-y(2xy - 1)} = -\frac{2}{y} .$$

Demak, berilgan tenglama faqat y ga bog'liq bo'lgan integrallovchi ko'paytuvchiga ega ekan. Bu misol uchun

$$\mu = e^{\int \psi(y)dy} = e^{-\int \frac{2}{y}dy} = e^{-2\,iny} = y^{-2} = \frac{1}{y^2}$$

$$\mu = \frac{1}{y^2}$$

Berilgan tenglamani $\mu = \frac{1}{y^2}$ ga ko'paytirsak,

$$\left(2x - \frac{1}{y}\right)dx + \left(1 + \frac{x}{y^2} + \frac{1}{y}\right)dy = 0$$

to'liq differensialli tenglama hosil bo'ladi.

U holda $u(x,y) = \int\left(2x - \frac{1}{y}\right)dx + \varphi(y)$

yoki $u(x,y) = x^2 - \frac{x}{y} + \varphi(y)$,

bu yerdan $\frac{\partial u}{\partial y} = \frac{x}{y^2} + \varphi'(y)$

Ikkinchi tomondan $\frac{\partial u}{\partial y} = N(x,y)$, ya'ni $\frac{\partial u}{\partial y} = 1 + \frac{x}{y^2} + \frac{1}{y}$.

U holda

$$\frac{x}{y^2} + \varphi'(y) = 1 + \frac{x}{y^2} + \frac{1}{y} \Rightarrow \varphi'(y) = 1 + \frac{1}{y}$$

yoki

$$\varphi(y) = y + \ln y + C_1 .$$

Demak, $x^2 - \frac{x}{y} + y + \ln y = C$ tenglamaning umumiy integrali ekan, bunda C - ixtiyoriy o'zgarmas.

MUSTAQIL YECHISH UCHUN MASHQLAR

1-Variant

1. $x^3 dy - y^3 dx = 0$.
2. $(3x^2 + 2xy - y^2)dx + (x^2 - 2xy - 3y^2)dy = 0$.
3. $y' + 2y = e^{-x}$.
4. $y' + 2xy = 2xe^{-x^2}$, $y(0) = 0$.
5. $x = cy + \cos y$.
6. $y - y' = +xy'$.
7. $\frac{2x}{y^3}dx + \frac{y^2 - 3x^2}{y^4}dy = 0.$
8. $(x^2 + y^2 + 2x)dx + 2ydy = 0$

2-Variant

1. $tgx \sin^2 ydx + \cos^2 xctgydy = 0$.
2. $xdx + (x+y)dy = 0.$

3. $y' - 2xy = 2xe^{x^2}$.

4. $y' + \frac{3}{x}y = \frac{2}{x^3}$, $y(1) = 1$.

5. $x = cy^2 - \sin y$.

6. $\frac{dx}{x} = \left(\frac{1}{y} - 2x\right)dy$.

7. $e^y dx + (xe^y - 2y)dy = 0$.

8. $\frac{y}{x}dx + (y^2 - \ln x)dy = 0$.

3-Variant

1. $(xy^2 + x)dx + (y - x^2 y)dy = 0$.
2. $(x^2 + y^2)dx - 2xydy = 0$.
3. $y' + 2xy = e^{-x^2}$.
4. $y' - 2xy = 1$, $y(0) = 0$.
5. $x = \frac{c}{y^3} + 5y$.
6. $2x^2 y = y^2(2xy' - y)$.
7. $(4x^3 + 5x^4y^2)dx + (2x^5y + 6y^3)dy = 0$.
8. $(x^2 + y)dx + xdy = 0$.

4-Variant

1. $(xy^2 + x)dx + (x^2 y - y)dy = 0$.
2. $(x^3 - 3x^2 y)dx + (y^3 - x^3)dy = 0$.
3. $xy' - 2y = x^3 \cos x$.
4. $xy' - 2y = x$. $y(0) = 0$.
5. $x = cy^3 + \ln y$.
6. $y = (xy' + 2y)^2$.
7. $(5x + y - 7)dx + (8y + x - 9)dy = 0$.
8. $\left(\frac{x}{y} + 1\right)dx + \left(\frac{x}{y} - 1\right)dy = 0$.

5-Variant

1. $x \cdot \frac{dy}{dx} - y = y^3$.
2. $(xye^{\frac{x}{y}} + y^2)dx - x^2 e^{\frac{x}{y}} dy = 0$.

3. $y'x\ln x - y = 3x^3\ln^2 x.$

4. $xy' = x + \frac{1}{2}y,$ $y(0) = 0.$

5. $x = y(c + \cos y).$

6. $xy^2y'' + x^2 + y^3 = 0.$

7. $\frac{y-7}{x^3}dx - \frac{1}{2x^2}dy = 0.$

8. $(1 - y\sin x)dx - \cos xdy = 0.$

6-Variant

1. $x \cdot \frac{dy}{dx} - y = y^3.$

2. (x+y-1)dy+(2x+2y-3)dy=0.

3. $(2x - y^2)y' = 2y.$

4. $xy' = x + y,$ $y(0) = 0.$

5. $x = ctgy - \frac{1}{\sin y}.$

6. $xy^2y'' + x^2 + y^3 = 0.$

7. $\left(\frac{1}{x^2} - y\right)dx + (y - x)dy = 0.$

8. $x^2y^3 + y + (x^3y^2 - x)y^1 = 0.$

7-Variant

1. $tgx \cdot \frac{dy}{dx} - y = a.$

2. $y' = \frac{x}{y} + \frac{y}{x}.$

3. $y' = \frac{y}{2y\ln y + y - x}.$

4. $x^2 + xy' = y,$ $y(1) = 0.$

5. $x = ce^{y^2} + y.$

6. $(x + 1)(y' + y^2) = -y.$

7. $(x+y)dx+(x+\frac{1}{y})dy=0.$

8. $(1 - y^2)dx + (3 - 4xy)dy = 0.$

8-Variant

1. $\frac{dy}{dx} = \cos(x + y).$

2. $2xydx+(y^2-x^2)dy=0$.

3. $\left(e^{\frac{y^2}{2}}-xy\right)dy-dx=0$.

4. $y'-y=-2e^{-x}$, $y\to 0$, $agar$ $x\to +\infty$.

5. $x=c\cos y+\dfrac{\cos y}{x}$.

6. $xy\cdot dy=(y^2+x)dx$.

7. $(1+\frac{2}{x}+\frac{y}{x^2})dx+(3y-\frac{1}{y})dy=0$,

8. $(1-y\sin x)dx-\cos xdy=0$.

9-Variant

1. $\dfrac{dy}{dx}=(1+y^2)x$.

2. $\dfrac{dy}{dx}=\dfrac{y}{x}+\sqrt{1\dfrac{y}{x}}$.

3. $y'-y.e^x=2xe^{ex}$.

4. $x^2y'\cos\dfrac{1}{x}-y\sin\dfrac{1}{x}=-1$, $y\to 1$, $agar$ $x\to\infty$

5. $x=c\ln^2 y-\ln y$.

6. $xy'-2x^2\sqrt{y}=4y$.

7. $(1-\frac{y^2}{x^2}+\frac{y}{x^2})dx+\frac{2y-1}{x}dy=0$.

8. $(2x^3y^2-y)dx+(2x^2y^3-x)dy=0$.

10-Variant

1. $\dfrac{dy}{dx}=y\cdot\sin x$.

2. $2xydx-(x^2-y^2)dy=0$.

3. $(x^2+2x+1)y'-(x+1)y=x-1$.

4. $y'\cdot\sin x-y\cos x=-\dfrac{\sin^2 x}{x^2}$, $y\to 0$, $agar$ $x\to\infty$

5. $x=c\cdot e^{y^2}+y^2-1$.

6. $xy'+2y+x^5y^3e^x=0$.

7. $\left(2x-\frac{1}{x^2y}\right)dx+\left(2y-\frac{1}{xy^2}\right)dy=0$.

8. $(x^2+y)dy+(x-xy)dx=0$.

11-Variant

1. $y \cdot \cos xdx - \sin x \cdot dy = 0$.
2. $xy' = y(\ln y - \ln x)$.
3. $y'x - y = x\cos x - \sin x$.
4. $2xy' - y = 1 - \frac{2}{\sqrt{x}}$, .
5. $x = cctgy - \frac{1}{\cos y}$.
6. $2y' - \frac{x}{y} = \frac{x \cdot y}{x^2 - 1}$.
7. $\left(xy^2 + \frac{1}{x}\right)dx + \left(x^2y - \frac{1}{y}\right)dy = 0.$
8. $(1 - 2xy)dy - y(y-1)dx = 0$.

12-Variant

1. $xy' = 1 + y^2$.
2. $(y^2 - 3x^2)y' + 2xy = 0$.
3. $2(x - y^2)dy = y \cdot dx$.
4. $2xy' + y = 2x$,
5. $x = ce^y + \sin y$.
6. $x(x-1) \cdot y' + y^3 = x \cdot y$.
7. $(xy^2 + x)dx + \left(x^2y - \frac{1}{y}\right)dy = 0.$
8. $(1 - x^2y)dx + x^2(y - x)dy = 0$.

13-Variant

1. $1 + y' = e^y$.
2. $(2xy + 3y^2)dx + (x^2 + 6xy - 3y^2)dy = 0$.
3. $xy' + 3y = 15$.
4. $y'\sin x + y\cos x = 1$.
5. $y \cdot y' + y^2 ctgx = \cos x$.
6. $(x^2 + y^3 + 7)dx + 3xy^2dy = 0.$
7. $(\sqrt{x^2 - y} + 2x)dx - dy = 0$.
8. $3y'^3 - xy + 1 = 0.$

14-Variant

1. $y' = 3^{x+y}$.
2. $(y^2 + 2xy)dx + (2x^2 + 3xy)dy = 0$.
3. $xy' - 3y = 3\ln x - 1$.

4. $y' \cos x - y \sin x = -\sin 2x$, $y \to 0$ agar $x \to \frac{\pi}{2}$

5. $x = \frac{c}{y} + tgy$.

6. $y' + y = x \cdot y^3$.

7. $(x^2 cosy - ysin)dy + xsinydx = 0.$

8. $(x^2 + y^2 + x)dx + ydy = 0$.

15-Variant

1. $2x \cdot \sqrt{1 - y^2} dx = (1 + x^2) dy$.
2. $xy' - y = \sqrt{x^2 + y^2}$.
3. (1-2xy)y$' = y(y-1)$.
4. $y' \cos x - y \sin x = 2x$, .
5. $x \cdot y' = 2\sqrt{y} \cos x - 2y$.
6. $(x^2 + y)dx + (x + 7)dy = 0.$
7. $y\sqrt{1-y^2} dx + (x\sqrt{1-y^2} + y)dy = 0$ $\mu = \mu(y)$.

16-Variant

1. $(1 + 2y - y^2)dx + x(1 - y)dy = 0$.
2. $(x - \sqrt{x^2 + y^2})dx + ydy = 0$.
3. $y' \cdot \sin 2x = 2(y + \cos x)$.
4. $y' + y \cos x = \cos x$,
5. $x = \frac{1}{ce^y + y + 5}$.
6. $\frac{x \cdot y'}{y} + 2xy \ln x + 1 = 0$.
7. $(y^2 + x + y)dx + (2y + 1)xdy = 0.$
8. $(x^2 - y)dx + xdy = 0$ $\mu = \mu(x)$.

17-Variant

1. $y(y^2 + 1)dx + x(y^2 - 1)dy = 0$.
2. $(x^3 - 3x^2 y)dx + (y^3 - x^3)dy = 0$.
3. $x - \frac{y}{y'} = \frac{2}{y}$.
4. $y' - y = \sin x - \cos x$,
5. $x = c \ln^2 y - \ln y$.

6. $(y' - x\sqrt{y})(x^2 - 1) = x \cdot y$.

7. $(2xlny - 7x^2)dx + \left(\frac{x^2}{y} + 4\right)dy = 0$.

8. $(x^2 \sin^2 y)dx + x \sin 2y dy = 0; \ \sin y = z$.

18-Variant

1. $(xy^2 + x)dx + y(y - x^2 y)dy = 0$.
2. $x^2 y^2 - 2xyy' = x^2 + 3y^2$.
3. $y' + y = 4x^2 + 8x$.
4. $y' - ytgx = y^2 \sin x \cos x$,
5. $x = y(c + \sin x)$.
6. $xy \cdot dy = (y^2 + x)dx$.
7. $\left(\frac{1}{2}x^2 y + x + 1\right)dx + \left(\frac{1}{6}x^3 - y^2\right)dy = 0$.
8. $(y - \frac{1}{x})dx + \frac{dy}{y} = 0$.

19-Variant

1. $(y^2 + 1)dx - x(y + 1)dy = 0$.
2. $yy' = 4x + 3y - 2$.
3. $(2x + y)dy = ydx + 4\ln ydy$.
4. $y' \cos x + y \sin x = 2\cos^2 x$.
5. $x^{-2} = y^4(2e^4 + c)$.
6. $2y' - \frac{x}{y} = \frac{x \cdot y}{x^2 - 1}$.
7. $2ysin2xdx - (9 + 2cos^2 x)dy = 0$.
8. $(x\cos y - y\sin y)dy + (x\sin y + y\cos y - \sin y)dx = 0, \ \mu = \mu(x)$.

20-Variant

1. $y' = \frac{1}{1 - x}$.
2. $(x - y\cos\frac{y}{x})dx + x \cdot \cos\frac{y}{x}dx = 0$.
3. $x(-1)y' + 2xy = 1$.
4. $xy' - 2y = x^2\sqrt{y}$.
5. $x(e^y + ce^{2x}) = 1$.
6. $y' = y^4 \cos x + ytgx$.
7. $(e^y - e^x)dx + xe^y dy = 0$.

8. $(3y^2 - x)dx + (2y^3 - 6xy)dy = 0, \quad \mu = \mu(x + y^2)$.

Uy ishlari, auditoriyaga oddiy differensial tenglamalar bo‘yicha joriy nazoratlar o‘tkazish uchun toshiriqlar variantlari:

I. O‘zgaruvchilari ajraladigan differensial tenglamalarni yeching.

1. $x^3 dy - y^3 dx = 0$
2. $tgx \sin^2 ydx + \cos^2 xctgydy = 0$
3. $(xy^2 + x)dx + (y - x^2 y)dy = 0$
4. $(xy^2 + x)dx + (x^2 y - y)dy = 0$
5. $x \cdot \frac{dy}{dx} - y = y^3$
6. $x \cdot \frac{dy}{dx} + y = y^2$
7. $tgx \cdot \frac{dy}{dx} - y = a$
8. $\frac{dy}{dx} = \cos(x + y) \qquad 1 + sosz = 0 \ z = (2\pi + 1)\pi$
9. $\frac{dy}{dx} = (1 + y^2)x$
10. $\frac{dy}{dx} = y \cdot \sin x$
11. $y \cdot \cos dx - \sin x \cdot dy = 0 \qquad y\left(\frac{\pi}{2}\right) = 1$
12. $xy' = 1 + y^2$
13. $1 + y' = e^y$
14. $y' = 3^{x+y}$
15. $2x \cdot \sqrt{1 - y^2} dx = (1 + x^2)dy$
16. $(1 + 2y - y^2)dx + x(1 - y)dy = 0$
17. $y(y^2 + 1)dx + x(y^2 - 1)dy = 0$
18. $(xy^2 + x)dx + y(y - x^2 y)dy = 0$
19. $(y^2 + 1)dx - x(y + 1)dy = 0$
20. $y' = \frac{1}{1 - x}$

II. Bir jinsli va unga keltiriladigan differensial tenglamalar.

1. $(3x^2 + 2xy - y^2)dx + (x^2 - 2xy - 3y^2)dy = 0$
2. $xdx + (x + y)dy = 0$
3. $(x^2 + y^2)dx - 2xydy = 0$
4. $(x^3 - 3x^2 y)dx + (y^3 - x^3)dy = 0$
5. $(xye^{\frac{x}{y}} + y^2)dx - x^2 e^{\frac{x}{y}} dy = 0$
6. $(x + y - 1)dy + (2x + 2y - 3)dy = 0$

7. $y' = \frac{x}{y} + \frac{y}{x}$

8. $2xydx + (y^2 - x^2)dy = 0$

9. $\frac{dy}{dx} = \frac{y}{x} + \sqrt{1\frac{y}{x}}$

10. $2xydx - (x^2 - y^2)dy = 0$

11. $xy' = y(\ln y - \ln x)$

12. $(y^2 - 3x^2)y' + 2xy = 0$

13. $(2xy + 3y^2)dx + (x^2 + 6xy - 3y^2)dy = 0$

14. $(y^2 + 2xy)dx + (2x^2 + 3xy)dy = 0$

15. $xy' - y = \sqrt{x^2 + y^2}$

16. $(x - \sqrt{x^2 + y^2})dx + ydy = 0$

17. $(x^3 - 3x^2 y)dx + (y^3 - x^3)dy = 0$

18. $x^2 y^2 - 2xyy' = x^2 + 3y^2$

19. $yy' = 4x + 3y - 2$

20. $(x - y\cos\frac{y}{x})dx + x \cdot \cos\frac{y}{x}dx = 0$

III. Chiziqli differenssial tenglamani quydagi usullar bilan yeching.

a) O‘zgarmasni variasiya usuli.
b) Bernulli almashtirish orqali.
c) Umumiy yechim formulasidan foydalanib.
d) Integrallovchi ko‘paytuvchi orqali.

1. $y' + 2y = e^{-x}$

2. $y' - 2xy = 2xe^{x^2}$

3. $y' + 2xy = e^{-x^2}.$

4. $xy' - 2y = x^3 \cos x.$

5. $y' x \ln x - y = 3x^3 \ln^2 x.$

6. $(2x - y^2)y' = 2y.$

7. $y' = \frac{y}{2y\ln y + y - x}.$

8. $\left(e^{\frac{y^2}{2}} - xy\right)dy - dx = 0$

9. $y' - ye^{x} = 2xe^{ex}$

10. $(x^2 + 2x + 1)y' - (x + 1)y = x - 1$

11. $y' x - y = x\cos x - \sin x.$

12. $2(x - y^2)dy = y \cdot dx.$

13. $xy' + 3y = 15$

14. $xy' - 3y = 3\ln x - 1.$

15. $(1 - 2xy)y' = y(y - 1).$

16. $y' \cdot \sin 2x = 2(y + \cos x).$

17. $x - \frac{y}{y'} = \frac{2}{y}$

18. $y' + y = 4x^2 + 8x$

19. $(2x + y)dy = ydx + 4\ln ydy$

20. $x(-1)y' + 2xy = 1$

IV. Berilgan differensial teglamaning ko‘rsatilgan boshlang‘ich shartini qanotlantiruvchi yechimini toping. Chiziqli tenglamaning umumiy yechim formulasidan foydalaning.

1. $y' + 2xy = 2xe^{-x^2}$ $\quad y(0) = 0$
2. $y' + \frac{3}{x}y = \frac{2}{x^3}$ $\quad y(1) = 1$
3. $y' - 2xy = 1$ $\quad y(0) = 0$
4. $xy' - 2y = x$ $\quad y(0) = 0$
5. $xy' = x + \frac{1}{2}y$ $\quad y(0) = 0$
6. $xy' = x + y$ $\quad y(0) = 0$
7. $x^2 + xy' = y$ $\quad y(1) = 0$
8. $y' - y = -2e^{-x}$ $\quad y \to 0$ agar $x \to +\infty$
9. $x^2 y' \cos\frac{1}{x} - y\sin\frac{1}{x} = -1$ $\quad y \to 1$ agar $x \to \infty$
10. $y' \cdot \sin x - y\cos x = -\frac{\sin^2 x}{x^2}$ $\quad y \to 0$ agar $x \to \infty$
11. $2xy' - y = 1 - \frac{2}{\sqrt{x}}$ $\quad y \to -1$ agar $x \to \infty$
12. $2xy' + y = 2x$ $\quad y$ chegaralangan agar $x \to 0$
13. $y'\sin x + y\cos x = 1$ $\quad y$ chegaralangan agar $x \to 0$
14. $y'\cos x - y\sin x = -\sin 2x$ $\quad y \to 0$ agar $x \to \frac{\pi}{2}$
15. $y'\cos x - y\sin x = 2x$ $\quad y(0) = 0$
16. $y' + y\cos x = \cos x$ $\quad y(0) = 1$
17. $y' - y = \sin x - \cos x$ $\quad y$ chegaralangan agar $x \to +\infty$
18. $y' - ytgx = y^2 \sin x \cos x$ $\quad y\left(\frac{\pi}{3}\right) = \frac{4}{3}$
19. $y'\cos x + y\sin x = 2\cos^2 x$ $\quad y(0) = 0$
20. $xy' - 2y = x^2\sqrt{y}$ $\quad y(0) = 0$

V. Egri chiziqlar oilasining differensial tenglamasini tuzing va turini aniqlang.

1. $x = cy + \cos y$	1. $y = c \cdot \sin x + \frac{\sin x}{x}$
2. $x = cy^2 - \sin y$	2. $y = c \cdot tgx - \frac{1}{\cos x}$

3. $x = \frac{c}{y^3} + 5y$

4. $x = cy^3 + \ln y$

5. $x = y(c + \cos y)$

6. $x = ctgy - \frac{1}{\sin y}$

7. $x = ce^{y^2} + y$

8. $x = c\cos y + \frac{\cos y}{x}$

9. $x = c\ln^2 y - \ln y$

10. $x = c \cdot e^{y^2} + y^2 - 1$

11. $x = cctgy - \frac{1}{\cos y}$

12. $x = ce^y + \sin y$

13. $x = cy^3 + y$

14. $x = \frac{c}{y} + tgy$

15. $xy = (x^3 + c)e^x$

16. $x = \frac{1}{ce^y + y + 5}$

17. $x = c\ln^2 y - \ln y$

18. $x = y(c + \sin x)$

19. $x^{-2} = y^4(2e^4 + c)$

20. $x(e^y + ce^{2x}) = 1$

3. $y = c \cdot e^{x^2} + x^3$

4. $y = x \cdot (c + \sin x)$

5. $y = c \cdot \ln^2 x - \ln x$

6. $y = x^4 \ln^2 cx$

7. $x = e^y + c \cdot e^{-y}$

8. $x = 2\ln y - y + 1 + cy^2$

9. $xy = (x^3 + c)e^{-x}$

10. $y^{-2} = x^4(2e^x + c)$

11. $y(e^x + c \cdot e^{2x}) = 1$

12. $y = x(ce^{-x} - 1)$

13. $y^2 = c(x+1)^2 - 2(x+1)$

14. $y = ce^{x^2} - x^2 - 1$

15. $x = (c - \cos y) \cdot \sin y$

16. $y = \frac{1}{ce^x + x + 2}$

17. $x = cy + y^3$

18. $y^2 = c \cdot (xy - 1)$

19. $x = y^2(c - 2\ln y)$

20. $y(xy - 1) = c \cdot x$

VI. Bernulli tenglamasini quyidagi usullar bilan yeching.

a) Chiziqli tenglamaga keltirib;

b) O‘zgarmasni variatsiya usuli bilan

1. $y - y' = xy'$.

2. $\frac{dx}{x} = \left(\frac{1}{y} - 2x\right)dy$.

3. $2x^2y = y^2(2xy' - y)$.

4. $y = (xy' + 2y)^2$.

5. $(1 - x^2)y' - 2xy^2 = xy$.

6. $xy^2y' + x^2 + y^3 = 0$.

7. $(x+1)(y' + y^2) = -y$

8. $xy \cdot dy = (y^2 + x)dx$.

9. $xy' - 2x^2\sqrt{y} = 4y$

10. $xy' + 2y + x^5y^3e^x = 0$

11. $2y' - \frac{x}{y} = \frac{x \cdot y}{x^2 - 1}$

12. $x(x-1)\cdot y'+y^{3}=x\cdot y.$

13. $y\cdot y'+y^{2}ctgx=\cos x.$

14. $y'+y=x\cdot y^{3}$

15. $x\cdot y'=2\sqrt{y}\cos x-2y$

16. $\frac{xy'}{y}+2xy\ln x+1=0$

17. $(y'-x\sqrt{y})(x^{2}-1)=x\cdot y$

18. $xy\cdot dy=(y^{2}+x)dx$

19. $2y'-\frac{x}{y}=\frac{x\cdot y}{x^{2}-1}$

20. $y'=y^{4}\cos x+ytgx$

VII. Quyidagi tenglamalar to‘liq differensial tenglamalar ekanligini tekshiring va integrallang.

1. $\frac{2x}{y^3}dx+\frac{y^2-3x^2}{y^4}dy=0$
2. $e^y dx+(xe^y-2y)dy=0$
3. $(4x^3+5x^4y^2)dx+(2x^5y+6y^3)dy=0$
4. $(5x+y-7)dx+(8y+x-9)dy=0$
5. $\frac{y-7}{x^3}dx-\frac{1}{2x^2}dy=0$
6. $\left(\frac{1}{x^2}-y\right)dx+(y-x)dy=0$
7. $(x+y)dx+\left(x+\frac{1}{y}\right)dy=0$
8. $1-\frac{2}{x}+\frac{y}{x^2})dx+\left(3y-\frac{1}{y}\right)dy=0$
9. $(1-\frac{y^2}{x^2}+\frac{y}{x^2})dx+\frac{2y-1}{x}dy=0$
10. $\left(2x-\frac{1}{x^2y}\right)dx+\left(2y-\frac{1}{xy^2}\right)dy=0$
11. $\left(xy^2+\frac{1}{x}\right)dx+\left(x^2y-\frac{1}{y}\right)dy=0$
12. $(xy^2+x)dx+\left(x^2y-\frac{1}{y}\right)dy=0$
13. $(x^2+y^3+7)dx+3xy^2dy=0$
14. $(x^2cosy-ysin)dy+xsinydx=0$
15. $(x^2+y)dx+(x+7)dy=0$
16. $(y^2+x+y)dx+(2y+1)xdy=0$
17. $(2xlny-7x^2)dx+\left(\frac{x^2}{y}+4\right)dy=0$
18. $\left(\frac{1}{2}x^2y+x+1\right)dx+\left(\frac{1}{6}x^3-y^2\right)dy=0$

19. $2y\sin 2x dx-(9+2\cos^2 x)dy=0$
20. $(e^y-e^x)dx+xe^y dy=0$

VIII. Qulay almashtirish yoki integrallovchi ko‘paytuvchi yordamida quyidagi differensial tenglamalarni integrallang.

1. $(x^2+y^2+2x)dx+2ydy=0$
2. $\frac{y}{x}dx+(y^2-\ln x)dy=0$
3. $(x^2+y)dx+xdy=0$
4. $\left(\frac{x}{y}+1\right)dx+\left(\frac{x}{y}-1\right)dy=0$
5. $(2xy^2-y)dx+(y^2+x+y)dy=0$
6. $x^2y^2+y+(x^2y^2-x)y'=0$
7. $(1-y^2)dx+(3-4xy)dy=0$
8. $(1-y\sin x)dx-\cos xdy=0$
9. $(2x^3y^2-y)dx+(2x^2y^3-x)dy=0$
10. $(x^2+y)dy+(x-xy)dx=0$
11. $(1-2xy)dy-y(y-1)dx=0$
12. $(1-x^2y)dx+x^2(y-x)dy=0$
13. $\left(\sqrt{x^2-y}+2x\right)dx-dy=0$
14. $(x^2+y^2+x)dx+ydy=0$
15. $y\sqrt{1-y^2}dx+\left(x\sqrt{1-y^2}+y\right)dy=0$
16. $(x^2-y)dx+xdy=0$
17. $(x^2\sin^2 y)dx+x\sin 2ydy=0$
18. $\left(y-\frac{1}{x}\right)dx+\frac{dy}{y}=0$
19. $(x\cos y-y\sin y)dy+(x\sin y+y\cos y-\sin y)dx=0$
20. $(3y^2-x)dx+(2y^2-6xy)dy=0, \qquad \mu=\mu(x+y^2)$

Test topshiriqlari

Birinchi tartibli differensial tenglamalar mavzulari bo’yicha testlar

1. $e^{x+3y}dy=xdx$

A) $e^{3y}=3(C-xe^{-x}-e^{-x})$, B) $\ln y=Ctg\frac{x}{2}$, C) $\ln|\cos y|=x-x^2+C$, D) $C=tgxtgy$.

2. $y'\sin x=y\ln y$

A) $y = C\sin x - 2$, B) $\ln y = Ctg\frac{x}{2}$, C) $C = \frac{\cos x}{\cos y}$, D) $tgy = \frac{C}{e^x - 1}$.

3. $y' = (2x-1)ctgy$

A) $e^{3y} = 3(C - xe^{-x} - e^{-x})$, B) $\ln y = Ctg\frac{x}{2}$, C) $\ln\left|\cos y\right| = x - x^2 + C$, D) $tgy = \frac{C}{e^x - 1}$.

4. $\sec^2 x \cdot tgydy + \sec^2 y \cdot tgxdx = 0$

A) $tgy = \frac{C}{e^x - 1}$, B) $\ln y = Ctg\frac{x}{2}$, C) $arctgy = C + \frac{1}{2}e^{x^2}$, D) $C = tgxtgy$.

5. $(1+e^x)ydy - e^y dx = 0$

A) $-e^{-y}(y+1) = \ln\frac{e^x}{e^x+1} + C$, B) $\ln y = Ctg\frac{x}{2}$, C) $arctgy = C + \frac{1}{2}e^{x^2}$, D) $C = \frac{\cos x}{\cos y}$.

6. $(y^2+3)dx - \frac{e^x}{x}ydy = 0$

A) $tgy = \frac{C}{e^x - 1}$, B) $\ln(y^2+3) = 2(C - xe^{-x} - e^{-x})$, C) $C = tgxtgy$, D) $tgy = \frac{C}{e^x - 1}$.

7. $\sin y\cos xdy = \cos y\sin xdx$

A) $tgy = \frac{C}{e^x - 1}$, B) $arctgy = C + \frac{1}{2}e^{x^2}$, C) $C = \frac{\cos x}{\cos y}$, D) $\sin y = C(e^x - 1)^3$.

8. $y' = (2y+1)tgx$

A) $C = tgxtgy$, B) $\ln y = Ctg\frac{x}{2}$, C) $\sin 2y = tgx + C$, D) $\sqrt{2y+1} = \frac{C}{\cos x}$.

9. $(\sin(x+y) + \sin(x-y))dx + \frac{dy}{\cos y} = 0$

A) $tgy = C + \cos 2x$, B) $\ln y = Ctg\frac{x}{2}$, C) $\sqrt{2y+1} = \frac{C}{\cos x}$, D) $C = \frac{\cos x}{\cos y}$.

10. $((1+e^x)yy' = e^x$

A) $C(e^y - 1) = e^{-x}$, B) $y^2 = 2\ln C(e^x+1)$, C) $\ln\left|tg\left(\frac{\pi}{4} + \frac{y}{2}\right)\right| = C - e^x$, D) $tgy = \frac{C}{e^x - 1}$.

11. $(xy + x^3y)y' = 1 + y^2$

A) $\sqrt[3]{y^3+1} = \frac{C}{\sqrt{x^2-1}}$, B) $y = C\sqrt{x^2-1}$, C) $Cx = \sqrt{(1+x^2)(1+y^2)}$, D) $\frac{C\sqrt[3]{x}}{\sqrt[3]{x+3}} + 3$.

12. $\frac{y'}{7^{y-x}} = 3$

A) $\frac{C\sqrt[3]{x}}{\sqrt[3]{x+3}} + 3$, B) $y = C\sqrt{x^2-1}$, C) $y = \frac{Cx}{x+1} + 1$, D) $7^{-y} = 3\cdot 7^{-x} + C\ln 7$.

13. $y - xy' = 2(1 + x^2y')$

A) $y = \frac{Cx}{\sqrt{1+2x^2}} + 2$, B) $\sqrt{\frac{y-2}{y}} = Ce^x$, C) $y = \frac{Cx}{x+1} + 1$, D) $\frac{y}{y+1} = C - x$.

14. $y - xy' = 1 + x^2y'$

A) $\frac{y}{y+1} = C - x$, B) $y = \frac{Cx}{x+1} + 1$, C) $\sqrt{\frac{y-2}{y}} = Ce^x$, D) $\frac{y}{y+1} = Cx$.

15. $(x+4)dy - xydx = 0$

A) $y = C\sqrt{x^2-1}$, B) $\sqrt{y^2+1} = \ln Cx$, C) $y = \frac{Ce^x}{(x+4)^4}$, D) $\frac{C\sqrt[3]{x}}{\sqrt[3]{x+3}} + 3$.

16. $y' + y + y^2 = 0$

A) $\frac{1}{y} + \ln y = C + \frac{1}{2}\ln^2 x$, B) $y = \frac{Cx}{\sqrt{1+2x^2}} + 2$, C) $y = C\sqrt{x^2-1}$, D) $\frac{y}{y+1} = C - x$.

17. $y^2 \ln xdx - (y-1)xdy = 0$

A) $y + \ln\frac{(y-1)^2}{y} = C + \ln x$, B) $\frac{C\sqrt[3]{x}}{\sqrt[3]{x+3}} + 3$, C) $C(e^y - 1) = e^{-x}$, D) $\ln\left|\frac{y}{y+2}\right| = C + x^2$.

18. $(x + xy^2)dy + ydx - y^2dx = 0$

A) $y = C\sqrt{x^2-1}$, B) $y + \ln\frac{(y-1)^2}{y} = C + \ln x$, C) $arctgy = C + arctgx$, D) $\sin\frac{y}{x} = \ln\frac{C}{|x|}$.

19. $y' + 2y - y^2 = 0$

A) $\frac{C\sqrt[3]{x}}{\sqrt[3]{x+3}} + 3$, B) $\frac{y}{y+1} = C - x$, C) $\sqrt{\frac{y-2}{y}} = Ce^x$, D) $y^3 = 3(C - x + \ln|x+1|$.

20. $(x^2 + x)ydx + (y^2+1)dy = 0$

A) $\frac{y}{y+1} = C - x$, B) $y = \frac{Cx}{x+1} + 1$, C) $C = \frac{\cos x}{\cos y}$, D) $\frac{y^2}{2} + \ln y = C - \frac{x^3}{3} - \frac{x^2}{2}$.

21. $y - xy' = x\sec\frac{y}{x}$

A) $\sin\frac{y}{x} = \ln\frac{C}{|x|}$, B) $y = -\frac{x}{\ln(Cx)}$, C) $y^2 = x^2\ln(Cx)^2$, D) $y = xe^{\frac{C}{x}}$.

22. $(y^2 - 3x^2)dy + 2xydx = 0$

A) $\frac{y}{y+1} = C - x$, B) $(y^2 - x^2)^2 = Cx^2y^3$, C) $C = \frac{\cos x}{\cos y}$, D) $\sqrt{\frac{y}{x}} - \frac{y}{x} = \ln Cx$.

23. $(x + 2y)dx - xdy = 0$

A) $\frac{y}{y+1} = C - x$, B) $y = xe^{\frac{C}{x}}$, C) $y = Cx^2 - x$, D) $y^2 = x^2\ln(Cx)^2$.

24. $(x - y)dx + (x+y)dy = 0$

A) $y = xe^{\frac{C}{x}}$, B) $y = \frac{Cx}{x+1} + 1$, C) $y = -\frac{x}{\ln(Cx)}$, D) $arctg\frac{y}{x} + \frac{1}{2}\ln\frac{y^2+x^2}{x^2} = \ln\frac{C}{x}$.

25. $(y^2 - 2xy)dx + x^2dy = 0$

A) $\frac{y}{x-y} = Cx$, B) $y = \frac{Cx}{x+1} + 1$, C) $C = \frac{\cos x}{\cos y}$, D) $y = -\frac{x}{\ln(Cx)}$.

26. $y^2 + x^2y' = xyy'$

A) $\ln\left|1 + \frac{y}{x}\right| = Cx$, B) $e^{\frac{y}{x}} = Cy$, C) $y = \frac{C}{x} - \frac{x}{2}$, D) $\frac{y^2}{2} + \ln y = C - \frac{x^3}{3} - \frac{x^2}{2}$.

27. $xy' - y = xtg\frac{y}{x}$

A) $\frac{y}{y+1} = C - x$, B) $\ln\left|1+\frac{y}{x}\right| = Cx$, C) $\sin\left(\frac{y}{x}\right) = Cx$, D) $y = \frac{C}{x} - \frac{x}{2}$.

28. $xy' = y - xe^{\frac{y}{x}}$

A) $y = \frac{C}{x} - \frac{x}{2}$, B) $y = \frac{Cx}{x+1} + 1$, C) $\sqrt{\frac{y}{x}} - \frac{y}{x} = \ln Cx$, D) $e^{-\frac{y}{x}} = \ln Cx$.

29. $xy' - y = (x+y)\ln\left(\frac{x+y}{x}\right)$

A) $\ln\left|1+\frac{y}{x}\right| = Cx$, B) $\ln\left|1+\frac{y}{x}\right| = Cx$, C) $-e^{-\frac{y}{x}} = \ln Cx$, D) $y = x\ln\left(\frac{C}{x}\right)$.

30. $xy' = y\cos\ln\frac{y}{x}$

A) $y = \frac{x}{4}\ln^2 Cx$, B) $ctg\left(\frac{1}{2}\ln\frac{y}{x}\right) = \ln Cx$, C) $arc\sin\frac{y}{x} = \ln Cx$, D) $-e^{-\frac{y}{x}} = \ln Cx$.

31. $(x^2+1)y' + 4xy = 3,\ y(0) = 0.$

A) $y = \frac{x^3+3x}{(x^2+1)^2}$, B) $y = x^2 - 1$, C) $y = (\sin x - 1)x$, D) $x = y^2 - y$.

32. $y' + ytgx = \sec x,\ y(0) = 0.$

A) $y = (\sin x - 1)x$, B) $y = \sin x$, C) $x = y^2 - y$, D) $-e^{-\frac{y}{x}} = \ln x$.

33. $(1-x)(y'+y) = e^{-x},\ y(0) = 0.$

A) $x = y^2 - y$, B) $ctg\left(\frac{1}{2}\ln\frac{y}{x}\right) = \ln x$, C) $y = e^{-x}\ln\frac{1}{1-x}$, D) $y = (\sin x - 1)x$.

34. $xy' - 2y = 2x^4,\ y(1) = 0.$

A) $y = x^2 - 1$, B) $y = e^x\ln x$, C) $arc\sin\frac{y}{x} = \ln x$, D) $y = x^4 - x^2$.

35. $y' = 2x(x^2+y),\ y(0) = 0.$

A) $y = x^2 + 1 - e^{x^2}$, B) $y = \ln x$, C) $x = e^y - e^{-y}$, D) $-e^{-\frac{y}{x}} = \ln x$.

36. $y' - y = e^x,\ y(0) = 1.$

A) $y = \frac{x}{4}\ln^2 x$, B) $y = (x+1)e^x$, C) $arc\sin\frac{y}{x} = \ln x$, D) $y = \ln x$.

Test javoblari

Birinchi tartibli differensial tenglamalar											
1	A	7	A	13	C	19	D	25	C	31	B
2	D	8	A	14	A	20	D	26	C	32	B
3	D	9	C	15	B	21	D	27	D	33	D
4	B	10	B	16	D	22	A	28	B	34	D
5	A	11	B	17	D	23	C	29	D	35	C
6	C	12	B	18	D	24	D	30	C	36	A

Sinov savollari

1. Differensial tenglamaning xususiy va umumiy yechimlari deb nimaga aytiladi?
2. Koshi masalasi qanday qo'yiladi?
3. O'zgaruvchilari ajraladigan differensial tenglama ta'rifi.
4. Umumiy integral deb nimaga aytiladi?
5. Qanday funksiya bir jinsli funksiya deyiladi?
6. Bir jinsli differensial tenglama deb nimaga aytiladi?
7. Bir jinsli differensial tenglama qanday integrallanadi?
8. Qanday tenglamaga umumlashgan bir jinsli differensial tenglama deyiladi?
9. Qanday tenglamaga chiziqli differensial tenglama deyiladi?
10. Chiziqli differensial tenglamalarni qanday usullarda yechish mumkin?
11. Bernulli tenglamasini ko'rinishi qanday?
12. Rikkati tenglamasining ko'rinishi qanday?
13. To'liq differensialli tenglama deb qanday tenglamaga aytiladi?
14. Integrallovchi ko'paytuvchi deb qanday funksiyaga aytiladi?

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