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«HISOBLASH USULLARI» KAFEDRASI

KURS ISHI

CHIZIQLI BO’LMAGAN TENGLAMALAR SISTEMASINI YECHISH USULLARINI TAQQOSLASH

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Ko'plab amaliy masalalar nochiziqli tenglamalar sistemasini yechishga olib kelinadi. Umumiy holda n noma'limli n ta nochiziqli algebraik yoki transendent tenglamalar sistemasi quyidagicha yoziladi:

$$\left.\begin{array}{l} f_1(x_1, x_2, \cdots, x_n) = 0 \\ f_2(x_1, x_2, \cdots, x_n) = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ f_n(x_1, x_2, \cdots, x_n) = 0 \end{array}\right\}. \tag{1}$$

Ushbu (1) sistemani vektor shaklida quyidagicha yozish mumkin:

$$f(x) = 0. \tag{1'}$$

bu yerda $x = (x_1, x_2, \cdots, x_n)^T$ – argumentlarning vektor ustuni; $(f_1, f_2, \cdots, f_n)^T$ – funksiyalarning vektor ustuni; $(\ldots)^T$ – transponirlash operatsiyasi belgisi.

Nochiziqli tenglamalar sistemasi yechimini izlash – bu bitta nochiziqli tenglamani yechishga nisbatan ancha murakkab masala. Bitta tenglamani yechish uchun qo'llanilgan usullarni nochiziqli tenglamalar sistemasini yechishga umumlashtirish juda ko'p hisoblashlarni talab qiladi yoki uni amaliyotda qo'llab bo'lmaydi. Xususan, bu oraliqni teng ikkiga bo'lish usuliga tegishli. Shunga qaramasdan nochiziqli tenglamani yechishning bir qator iteratsion usullarini nochiziqli tenglamalar sistemasini yechishga umumlashtirish mumkin.

1. Nyuton usuli

(1′) tenglamalar sistemasini yechish uchun ketma-ket yaqinlashish usulidan foydalanamiz. Faraz qilaylik, (1′) vektor tenglamaning izolyatsiyalangan $x = \left(x_1, x_2, \cdots, x_n\right)$ ildizlaridan bittasi bo'lgan ushbu k -chi yaqinlashish

$$x^{(k)} = \left(x_1^{(k)}, x_2^{(k)}, \cdots, x_n^{(k)}\right)$$

topilgan bo'lsin. U holda (1′) vektor tenglamaning aniq ildizini ushbu

$$x = x^{(k)} + \varepsilon^{(k)}, \tag{2}$$

ko'rinishda ifodalash mumkin, bu yerda $\varepsilon^{(k)} = \left(\varepsilon_1^{(k)}, \cdots, \varepsilon_n^{(k)}\right)$ – xatolikni tuzatuvchi had (ildizning xatoligi).

(2) ifodani (1′) ga qo'yib, qyidagi tenglamani hosil qilamiz:

$$f\left(x^{(k)}+\varepsilon^{(k)}\right)=0\,. \qquad (3)$$

Faraz qilaylik, $f(x)$ – bu x va $x^{(k)}$ larni o'z ichiga olgan biror qovariq D sohada uzluksiz differensiallanuvchan funlsiya bo'lsin. (3) tenglamaning o'ng tarafini $\varepsilon^{(k)}$ – kichik vektor darajalari bo'yicha qatorga yoyamiz va bu qatorning chiziqli hadlari bilangina cheklanamiz:

$$f\left(x^{(k)}+\varepsilon^{(k)}\right)=f\left(x^{(k)}\right)+f'\left(x^{(k)}\right)\varepsilon^{(k)}=0\,. \qquad (4)$$

(4) formuladan kelib chiqadiki, $f'(x)$ hosila deb $x_1, x_2, \cdots, x_n$ – o'zgaruvchilarga nisbatan $f_1, f_2, \cdots, f_n$ – funksiyalar sistemasining quyidagi Yakob matritsasi tushuniladi:

$$f'(x)=W(x)=\begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} & \cdots & \dfrac{\partial f_1}{\partial x_n} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} & \cdots & \dfrac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \dfrac{\partial f_n}{\partial x_1} & \dfrac{\partial f_n}{\partial x_2} & \cdots & \dfrac{\partial f_n}{\partial x_n} \end{bmatrix},$$

yoki uni qisqacha vektor shaklida yozsak,

$$f'(x)=W(x)=\left[\frac{\partial f_i}{\partial x_j}\right], \quad i, j=\overline{1, n}\,.$$

(4) sistema bu xatolikni tuzatuvchi had $\varepsilon_i^{(k)}$ $\left(i=\overline{1, n}\right)$ larga nisbatan $W(x)$ matritsali chiziqli sistema. Bundan (4) formulani quyidagicha yozish mumkin:

$$f\left(x^{(k)}\right)+W\left(x^{(k)}\right)\varepsilon^{(k)}=0\,.$$

Bu yerdan, $W\left(x^{(k)}\right)$ – maxsus bo'lmagan matritsa deb faraz qilib, quyidagiga ega bo'lamiz:

$$\varepsilon^{(k)}=-W^{-1}\left(x^{(k)}\right)f\left(x^{(k)}\right).$$

Natijada ushbu

$$x^{(k+1)}=x^{(k)}-W^{-1}\left(x^{(k)}\right)f\left(x^{(k)}\right), \quad k=0,1,2, \cdots \qquad (5)$$

Nyuton usuli formulasiga kelamiz, bunda $x^{(0)}$ – nolinchi yaqinlashish sifatida izlanayotgan ildizning qo'pol qiymatini olish mumkin.

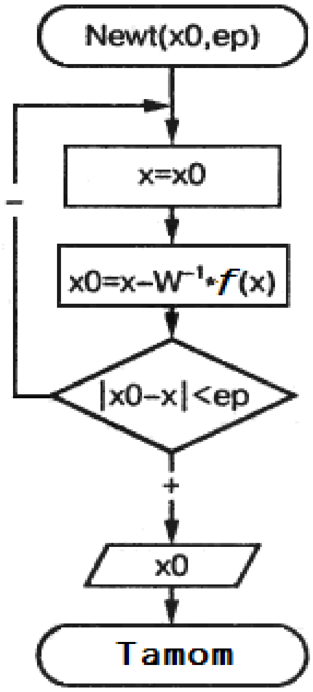
Amaliyotda (1′) nochiziqli tenglamalar sistemasini bu usul bilan yechish uchun hisoblashlar (5) formula bo'yicha quyidagi shart bajarilgunga qadar davom ettiriladi:

$$\left| x^{(k+1)} - x^{(k)} \right|_{\infty} < \varepsilon \, . \qquad (6)$$

Yuqoridagilardan kelib chiqib, Nyuton usulining algoritmini quyidagicha yozamiz:

1. $x^{(0)}$ – boshlang'ich yaqinlashish aniqlanadi.
2. Ildizning qiymati (5) formula bo'yicha aniqlashtiriladi.
3. Agar (6) shart bajarilsa, u holda masala yechilgan bo'ladi va $x^{(k+1)}$ – (1′) vektor tenglamaning ildizi deb qabul qilinadi, aks holda esa 2-qadamga o'tiladi.

Hisoblashlarda (1′) nochiziqli tenglamalar sistemasining $f(x)$ funksiyalari va ularning hosila-lari matritsasi $W(x)$ aniq berilgan geymiz, u holda bu sistemani yechishning blok-sxemasi 1-rasmdagi ko'rinishda bo'ladi.



1-rasm. Nochiziqli tenglamalar sistemasini yechish uchun Nyuton usulining algoritmi

f(*x*) vektor-funksiya *x* ildizi atrofida ikki martagacha uzluksiz differensiallanuvchi va Yakob matritsasi $W(x)$ maxsus bo'lmagan (aynimagan), ko'p o'lchovli Nyuton usuli kvadratik yaqinlashishga ega:

$$\left| x^{(k+1)} - x \right| < C \left| x^{(k)} - x \right|^2 .$$

Shuni ta'kidlaymizki, usulning yaqinlashishini ta'minlash uchun boshlang'ich yaqinlashishni muvaffaqiyatli tanlash muhim ahamiyatga ega. Tenglamalar sonining oshishi va ularning murakkabligi ortishi bilan yaqinlashish sohasi torayib boradi.

Xususiy hol. Hisoblash amaliyotida n=2 bo'lgan hol ko'p uchraydi. Buni, masalan, $f(z)$=0 nochiziqli tenglamaning kompleks ildizlarini topishda ham ko'rish mumkin. Haqiqatan ham, agar ushbu

$$f_1(x, y) = \mathrm{Re}\left(f\left(x + jy\right)\right) \text{ va } f_2(x, y) = \mathrm{Im}\left(f\left(x + jy\right)\right)$$

funksiyalarni kiritsak, z - kompleks ildizning x – haqiqiy qismi va y – mavhum qismi quyidagi ikki noma'lumli ikkita nochiziqli tenglamalar sistemasini taqribiy yechishdan hosil bo'ladi:

$$\begin{cases} f_1(x, y) = 0; \\ f_2(x, y) = 0, \end{cases} \tag{7}$$

bu taqribiy hisoblashni Nyuton usuli yordamida ε aniqlik bilan bajaraylik.

D sohaga tegishli $X_0 = (x_0, y_0)$ - nolinchi yaqinlashishni tanlab olamiz. (4) dan quyidagi chiziqli algebraik tenglamalar sistemasini tuzib olamiz:

$$\begin{aligned} \frac{\partial f_1}{\partial x}(x - x_0) + \frac{\partial f_1}{\partial y}(y - y_0) = -f_1(x_0, y_0) \\ \frac{\partial f_2}{\partial x}(x - x_0) + \frac{\partial f_2}{\partial y}(y - y_0) = -f_2(x_0, y_0) \end{aligned} \tag{8}$$

Quyidagi belgilashlarni kiritamiz:

$$x - x_0 = \Delta x_0, \qquad y - y_0 = \Delta y_0 \tag{9}$$

(8) sistemani Δx_0, Δy_0 larga nisbatan, masalan, Kramer usuli yordamida yechamiz. Kramer formulalarini quyidagicha yozamiz:

$$\Delta x_0 = \frac{\Delta_1}{J}, \quad \Delta y_0 = \frac{\Delta_2}{J}, \tag{10}$$

bu yerda (8) sistemaning asosiy determinanti quyidagicha:

$$J = \begin{vmatrix} \frac{\partial f_1(x_0, y_0)}{\partial x} & \frac{\partial f_1(x_0, y_0)}{\partial y} \\ \frac{\partial f_2(x_0, y_0)}{\partial x} & \frac{\partial f_2(x_0, y_0)}{\partial y} \end{vmatrix} \neq 0, \tag{11}$$

(8) sistemaning yordamchi determinantlari esa quyidagicha:

$$\Delta_1 = \begin{vmatrix} -f_1(x_0, y_0) & \frac{\partial f_1(x_0, y_0)}{\partial y} \\ -f_2(x_0, y_0) & \frac{\partial f_2(x_0, y_0)}{\partial y} \end{vmatrix}; \qquad \Delta_2 = \begin{vmatrix} \frac{\partial f_1(x_0, y_0)}{\partial x} & -f_1(x_0, y_0) \\ \frac{\partial f_2(x_0, y_0)}{\partial x} & -f_2(x_0, y_0) \end{vmatrix}.$$

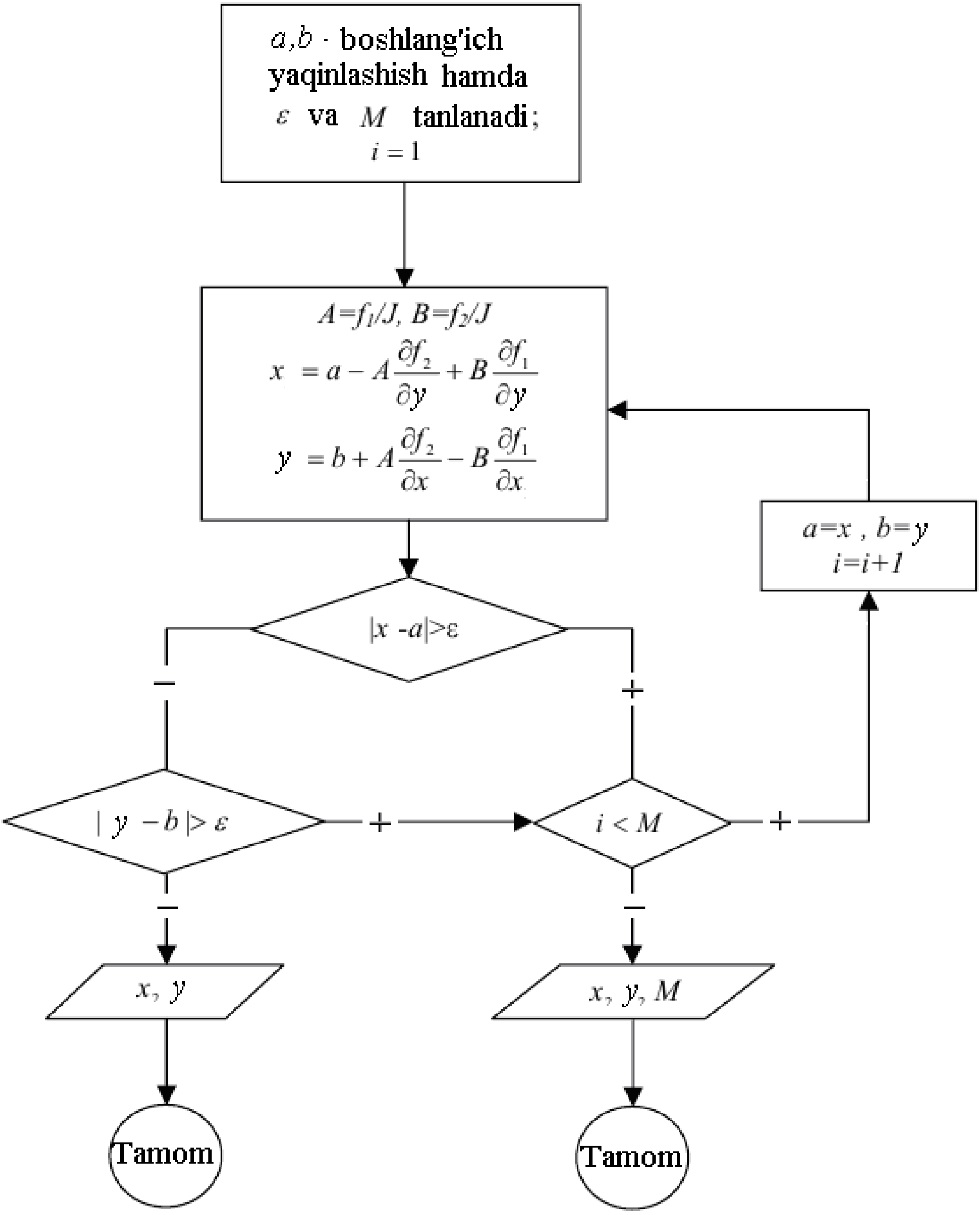
Δx_0, Δy_0 larning topilgan qiymatlarini (9) ga qo'yib, (8) sistemaning $X_1 = (x_1, y_1)$ - birinchi yaqinlashishi komponentalarini topamiz:

$$x_1 = x_0 + \Delta x_0, \qquad y_1 = y_0 + \Delta y_0. \tag{12}$$

Quyidagi shartning bajarilishini tekshiramiz:

$$\max(|\Delta x_0|, |\Delta y_0|) \leq \varepsilon , \qquad (13)$$

agar bu shart bajarilsa, u holda $X_1 = (x_1, y_1)$ birinchi yaqinlashishni (8) sistemaning taqribiy yechimi deb, hisoblashni to'xtatamiz. Agar (13) shart bajarilmasa, u holda $x_0 = x_1$, $y_0 = y_1$ deb olib, yangi (8) chiziqli algebraik tenglamalar sistemasini tuzamiz. Uni yechib, $X_2 = (x_2, y_2)$ - ikkinchi yaqinla-shishni topamiz. Topilgan yechimni ε ga nisbatan (13) bo'yicha tekshiramiz. Agar bu shart bajarilsa, u holda (8) sistemaning taqribiy yechimi deb $X_2 = (x_2, y_2)$ ni qabul qilamiz. Agar (13) shart bajarilmasa, u holda $x_1 = x_2$, $y_1 = y_2$ deb olib, $X_3 = (x_3, y_3)$ ni topish uchun yangi (8) sistemani tuzamiz va hokazo. Bu sistemani yechishning blok-sxemasi 2-rasmda tasvirlangan.



2-rasm. Ikki noma'lumli ikkita nochiziqli tenglamalar sistemasini taqribiy yechishning Nyuton usuli blok-sxemasi.

1-Misol. Quyidagi

$$\left.\begin{aligned} F(x, y) &= 2x^3 - y^2 - 1 = 0 \\ G(x, y) &= xy^3 - y - 4 = 0 \end{aligned}\right\} \qquad (14)$$

sistemaning yechimini Nyuton usulida taqribiy hisoblang.

Yechish. Grafik usulda yoki tanlov yo'li bilan dastlabki yaqinlashish $x_0 = 1,2 \quad y_0 = 1,7$ aniqlangan bo'lsin. U holda

$$J(x_0, y_0) = \begin{vmatrix} 6x^2 & -2y \\ y^3 & 3xy^2 - 1 \end{vmatrix}, \text{ demak } \quad J(1,2;\, 1,7) = \begin{vmatrix} 8,64 & -3,40 \\ 4,91 & 9,40 \end{vmatrix} = 97,910$$

(12) formulaga ko'ra

$$\left.\begin{aligned} x_1 &= 1{,}2 - \frac{1}{97{,}91}\begin{vmatrix} -0{,}434 & -3{,}40 \\ 0{,}1956 & 9{,}40 \end{vmatrix} = 1{,}2 + 0{,}0349 = 1{,}2349 \\ y_1 &= 1{,}7 - \frac{1}{97{,}91}\begin{vmatrix} 8{,}64 & -0{,}434 \\ 4{,}91 & 0{,}1956 \end{vmatrix} = 1{,}7 - 0{,}0390 = 1{,}6610 \end{aligned}\right\}$$

Hisoblashlarni shu singari davom qilib,

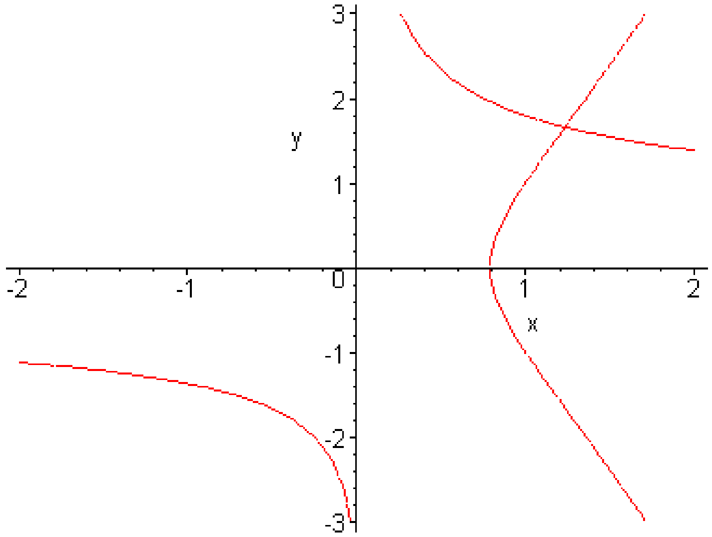
$$x_2 = 1{,}2343 \qquad y_2 = 1{,}6615$$

ni topamiz va hisoblashlarni talab qilingan aniqlikkacha davom ettiramiz.

Ushbu misolda berilgan tenglamalar sistemasi bitta haqiqiy yechimga ega ekanligini quyidagi Maple dastur va grafiklardan ko'rish mumkin (3-rasm):

```
> plots[implicitplot]({2*x^3-y^2-1=0,x*y^3-y-4=0},x=-2..2,y=-3..3);
solve({2*x^3-y^2-1=0,x*y^3-y-4=0},{x,y});
allvalues(%);
evalf(%);
```

$\{x = 1.234274484, y = 1.661526467\}$



3-rasm. Misolda berilgan tenglamalar sistemasidagi funksiyalarning Maple dasturida chizilgan grafiklari.

2. Iteratsiyalar usuli (ketma-ket yaqinlashishlar usuli)

Yuqoridagi (1) nochiziqli tenglamalar sistemasi ushbu

$$\left.\begin{aligned} x_1 &= \varphi_1(x_1,\ x_2,\ ...,\ x_n), \\ x_2 &= \varphi_2(x_1,\ x_2,\ ...,\ x_n), \\ &...\ ...\ ...\ ...\ ...\ ...\ ...\ ... \\ x_n &= \varphi_n(x_1,\ x_2,\ ...,\ x_n) \end{aligned}\right\} \tag{15}$$

ko'rinishga keltirilgan bo'lsin, bu yerda $\varphi_1, \varphi_2,\ \cdots,\ \varphi_n$ - haqiqiy funksiyalar bo'lib, ular bu sistema izolyatsiyalangan $\left(x_1^*, x_2^*,\ \cdots,\ x_n^*\right)$ yechimining biror atrofida aniqlangan va uzluksiz.

Qulaylik uchun quyidagi vektorni kiritamiz:

$$x = \left(x_1, x_2,\ \cdots,\ x_n\right) \text{ va } \varphi(x) = \left(\varphi_1(x), \varphi_2(x),\ \cdots, \varphi_n(x)\right).$$

U holda (15) ni quyidagi vektor shaklida yozish mumkin:

$$x = \varphi(x). \tag{16}$$

(16) tenglamaning $x^* = \left(x_1^*, x_2^*,\ \cdots,\ x_n^*\right)$ vector-ildizini topish uchun ko'pincha quyidagi *iteratsiyalar usuli*ni qo'llash juda qulay:

$$x^{(k+1)} = \varphi\left(x^{(k)}\right), \quad k = 0, 1, 2,\ \cdots, \tag{17}$$

yoki

$$\left.\begin{aligned} x_1^{(k+1)} &= \varphi_1(x_1^{(k)},\ x_2^{(k)},\ ...,\ x_n^{(k)}), \\ x_2^{(k+1)} &= \varphi_2(x_1^{(k)},\ x_2^{(k)},\ ...,\ x_n^{(k)}), \\ &...\ ...\ ...\ ...\ ...\ ...\ ...\ ... \\ x_n^{(k+1)} &= \varphi_n(x_1^{(k)},\ x_2^{(k)},\ ...,\ x_n^{(k)}), \end{aligned}\right\} \quad k = 0, 1, 2,\ \cdots,$$

bu yerda yuqoridagi indeks iteratsiyalar yaqinlashishi nomerini bildiradi; $x^{(0)} \approx x^*$ - boshlang'ich yaqinlashish. Usulning blok-sxemali algoritmi 4-rasmda tasvirlangan.

Agar (17) iteratsion jarayon yaqinlashivchan bo'lsa, u holda ushbu

$$\xi = \lim_{k \to \infty} x^{(k)} \tag{18}$$

limitik qiymat (17) tenglamaning ildizi bo'ladi.

Haqiqatdan ham, agar (18) munosabat bajarilgan desak, u holda (17) tenglikda $k \to \infty$ bo'yicha limitga o'tib, $\varphi(x)$ funksiyalarning uzluksizligidan quyidagiga ega bo'lamiz:

$$\lim_{k \to \infty} x^{(k+1)} = \varphi\left(\lim_{k \to \infty} x^{(k)}\right),$$

ya'ni $\xi = \varphi(\xi)$.

Shunday qilib, ξ – bu (16) vektor tenglamaning ildizi.

Agar, bundan tashqari, barcha $x^{(k)}$ $(k = 0,1, \cdots)$ yaqinlashishlar biror Ω - sohaga tegishli bo'lsa, u holda $\xi = x^*$ ekanligi yaqqol ko'rinadi.

Soddaroq qilib aytganda, (17) iteratsion jarayon $x^{(0)} = \left(x_1^{(0)}, x_2^{(0)}, ..., x_n^{(0)}\right)$ – boshlang'ich yaqinlashishdan boshlanib, bitta iteratsiyadan keyin barcha argumentlar orttirmasining moduli berilgan ε miqdordan kichik bo'lmagincha davom ettiriladi, ya'ni

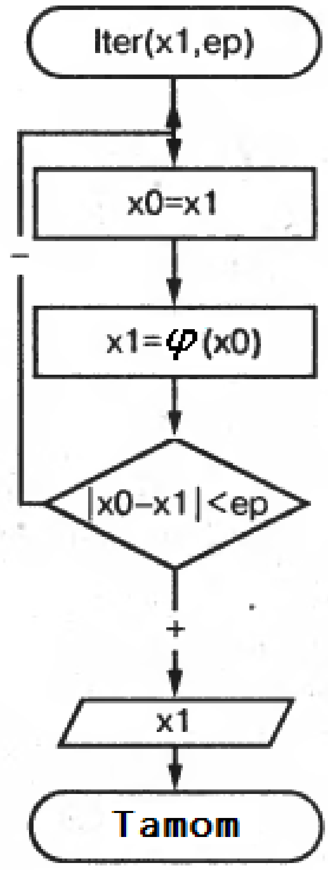
$$\left\|x^{(k+1)} - x^{(k)}\right\|_\infty = \max_{1 \le i \le n} \left\{\left|x_i^{(k+1)} - x_i^{(k)}\right|\right\} < \varepsilon .$$

Bu shartga teng kuchli bo'lgan quyidagi shartdan ham foydalanish mumkin:

$$\left\|x^{(k+1)} - x^{(k)}\right\|_2 = \left|x^{(k+1)} - x^{(k)}\right| =$$

$$\max_{1 \le i \le n} \left\{\sqrt{\left(x_i^{(k+1)} - x_i^{(k)}\right)^2}\right\} < \varepsilon$$

Oddiy iteratsiya usuli dasturlash uchun juda qulay, ammo u quyidagi muhim kamchiliklarga ega:



4-rasm. Chiziqli bo'lmagan tenglamalar sistemasini yechish uchun iteratsiyalar usulining blok-sxemali algoritmi.

a) $\|\varphi'(x)\|_\infty \le q < 1$, bu yerda φ' - vektor-funksiya φ ning Yakob matritsasi, $\|\cdot\|_\infty$ belgi bilan esa matritsa normasi kiritilgan: $\|\varphi'(x)\|_\infty = \max_{1 \le i \le n} \left\{\sum_{j=1}^{n} \frac{\partial \varphi_i}{\partial x_j}\right\}$;

b) $\|\varphi'(x)\|_l \le q < 1$, bu yerda φ' - vektor-funksiya φ ning Yakob matritsasi, $\|\cdot\|_l$ belgi bilan esa matritsa normasi kiritilgan: $\|\varphi'(x)\|_l = \max\limits_{1\le j\le n}\left\{\sum\limits_{i=1}^{n}\dfrac{\partial\varphi_i}{\partial x_j}\right\}$;

c) agar boshlang'ich yaqinlashish aniq yechimdan uzoqroq tanlangan bo'lsa, u holda a) shart bajarilishiga qaramasdan, usulning yaqinlashishiga kafolat yo'q; demak, boshlang'ich yaqinlashishni tanlashning o'zi ham sodda emas ekan;

d) iteratsion jarayon juda sekin yaqinlashadi.

3. Oddiy iteratsiya usuli

Iteratsiyalar usulini dastlabki $f(x)=0$ umumiy sistemaga ham qo'llash mumkin, bu yerda $f(x)$ – vektor-funksiya bo'lib, izolyatsiyalangan x^* - vektor-ildiz atrofida aniqlangan va uzluksiz.

Masalan, bu sistemani quyidagi ko'rinishda yozaylik:

$$x = x + \Lambda f(x)$$

bu yerda Λ – xosmas matritsa. Quyidagi belgilashni kiritamiz:

$$x + \Lambda f(x) = \varphi(x). \tag{19}$$

U holda quyidagiga ega bo'lamiz:

$$x = \varphi(x). \tag{20}$$

Agar $f(x)$ funksiya $f'(x)$ – uzluksiz hosilaga ega bo'lsa, u holda (19) formuladan quyidagi tenglik kelib chiqadi:

$$\varphi'(x) = E + \Lambda f'(x).$$

Agar $\varphi'(x)$ o'zining normasi bo'yicha kichik bo'lsa, u holda (20) tenglama uchun iteratsiyalar jarayoni tez yaqinlashadi. Bu holatni e'tiborga olib, A matritsani shunday tanlaymizki, ushbu

$$\varphi'\left(x^{(0)}\right) = E + \Lambda f'\left(x^{(0)}\right) = 0$$

tenglik bajarilsi. Bu yerdan, agar $f'\left(x^{(0)}\right)$ - xosmas bo'lsa, u holda quyidagi tenglikka ega bo'lamiz:

$$\Lambda = -\left|f'\left(x^{(0)}\right)\right|^{-1}.$$

Shuni ta’kidlash muminki, bu mazmunan Nyuton modifikatsion jarayonining (19) tenglamaga qo’llanilishi demakdir.

Xususan, agar $\det f'\left(x^{(0)}\right) = 0$ bo’lsa, u holda boshqa $x^{(0)}$ - boshlang’ich yaqinlashishni tanlash lozim bo’ladi.

Oddiy iteratsiya usuli nafaqat haqiqiy ildizlarni, balki kompleks ildizlarni ham topish imkonini beradi. Oxirgi holda kompleks boshlang’ich yaqinlashishni tanlash lozim bo’ladi.

Iteratsiyalar jatayoni yaqinlashishining yetarli sharti quyidagicha.

Faraz qilaylik, shunday $D \subset R^n$ yopiq soha mavjud bo’lsinki, bunda ixtiyoriy $x \in D$ uchun $\varphi(x) \in D$ bo’lsin. Xuddi shunday, ixtiyoriy x_1 va $x_2 \in D$ lar uchun quyidagi shart bajarilsin:

$$\left\|\varphi(x_1) - \varphi(x_2)\right\| \le q\left\|x_1 - x_2\right\|, \quad q < 1, \tag{*}$$

bu yerda $\|\cdot\|$ - R^n dagi biror norma. U holda osongina ko’rsatish mumkinki, D sohada (16) tenglamaning x^* yechimi mavjud bo’lib, (17) iteratsion jarayon tanlangan ixtiyoriy $x_0 \in D$ uchun shu yechimga yaqinlashadi. Bunda quyidagi yaqinlashish tezligini baholash o’rinli: $\left\|x^m - x^*\right\| \le \dfrac{cq^m}{1-q}$, bu yerda c – biror o’zgarmas.

Yuqoridagi (*) shartni qanoatlantiruvchi φ funksiya *siqiluvchan akslanish* deb, (18) tenglamaning yechimi esa φ funksiyaning *qo’zg’almas nuqtasi* deb ataladi.

Shuni ta’kidlaymizki,

$$\left\|x^{m+1} - x^*\right\| = \left\|\varphi(x^m) - \varphi(x^*)\right\| \le q\left\|x^m - x^*\right\|,$$

shuning uchun, oddiy iteratsiya usulining yaqinlashish tartibi 1 ga teng.

Agar $\varphi(x)$ funksiya D sohada uzluksiz va differensiallanuvchan bo’lsa, u holda (*) shartning bajarilishi uchun ixtiyoriy $x \in D$ lar uchun $\left\|\varphi'(x)\right\| \le q < 1$ shartning bajarilishi yetarli.

Izoh. $\varphi(x)$ funksiya $f(x)$ funksiya orqali bir qiymatli aniqlanmaydi. $\left\|\varphi'(x)\right\| \le q < 1$ shartning bajarilishi uchun $\varphi(x)$ funksiyani qanday tanlash lozim? Agar $x^{(0)}$ nuqta atrofida $f(x)$ uzluksiz differensiallanuvchan va $f'(x)$ matritsa-funksiya aynimagan bo’lsa, unda umumiy holda quyidagicha yozish mumkin:

$$\varphi(x) = x - \frac{f(x)}{f'(x_0)}$$

Xususiy hollarda $\varphi(x)$ funksiyani tanlash va ushbu $\left\|\varphi'(x)\right\| \le q < 1$ shartning bajarilishini tekshirish ancha sodda bo’lishi mumkin.

Xususiy hol. Hisoblashlarni amaliyot uchun qulay bo'lgan n=2 bo'lgan holda ko'rib chiqaylik. (15) sistemani

$$\left.\begin{aligned} x &= \varphi_1(x, y), \\ y &= \varphi_2(x, y) \end{aligned}\right\} \tag{21}$$

ko'rinishda yozib olamiz. $\varphi_1(x, y), \varphi_2(x, y)$ funkiyalar iterasiyalovchi funksiyalar deb yuritiladi. Taqribiy yechimni topish algoritmi

$$\left.\begin{aligned} x_{n+1} &= \varphi_1(x_n, y_n), \\ y_{n+1} &= \varphi_2(x_n, y_n) \end{aligned}\right\} \; n = 0,1,2,3,..... \tag{22}$$

ko'rinishda beriladi. Bu yerda (x_0, y_0) - birinchi yaqinlashish qiymatlari.

(22) iterasion hisoblash jarayoni yaqinlashuvchi bo'ladi, agarda

$$\left.\begin{aligned} \left|\frac{\partial \varphi_1}{\partial x}\right| + \left|\frac{\partial \varphi_1}{\partial y}\right| &\le q_1 < 1, \\ \left|\frac{\partial \varphi_2}{\partial x}\right| + \left|\frac{\partial \varphi_2}{\partial y}\right| &\le q_2 < 1 \end{aligned}\right\} \tag{23}$$

tengsizliklar bajarilsa. Xususiy va umumiy holda $\varphi(x)$ funksiyaning grafigini qurish va iteratsion jarayonning R^2 dagi yaqinlashishini ta'minlovchi shartni tekshirishga oid misollar qaraylik.

2-Misol. Quyidagi

$$\left.\begin{aligned} x^3 + y^3 - 6x + 3 &= 0 \\ x^3 - y^3 - 6y + 2 &= 0 \end{aligned}\right\}$$

sistemaning yechimini oddiy iterasiya usulida 0,001 aniqlikda taqribiy hisoblang.

Yechish. Iterasiya usulini qo'llash uchun berilgan sistemani

$$\left.\begin{aligned} x &= \frac{x^3 + y^3}{6} - \frac{1}{2} \\ y &= \frac{x^3 - y^3}{6} + \frac{1}{3} \end{aligned}\right\}$$

ko'rinishda yozib olamiz.

$0 \le x \le 1, \; 0 \le y \le 1$ kvadrat sohani qaraylik. Agar (x_0, y_0) shu sohaga qarashli bo'lsa, u holda $0 < \varphi_1(x_0, y_0) < 1, \; 0 < \varphi_2(x_0, y_o) < 1$ o'rinli bo'ladi. Demak shu sohadan (x_0, y_0) ixtiyoriy tanlaganimizda ham (x_n, y_n) ham o'sha sohaga tegishli bo'ladi. Bundan esa (23) yaqinlashish shartining bajarilishi kelib chiqadi, ya'ni

$$\left.\begin{aligned}\left|\frac{\partial \varphi_1}{\partial x}\right| + \left|\frac{\partial \varphi_1}{\partial y}\right| = \frac{x^2}{2} + \frac{y^2}{2} < 1 \\ \left|\frac{\partial \varphi_2}{\partial x}\right| + \left|\frac{\partial \varphi_2}{\partial y}\right| = \frac{x^2}{2} + \left|-\frac{y^2}{2}\right| < 1\end{aligned}\right\}$$

o'rinli bo'ladi. Demak, qaralayotgan kvadrat sohada yagona yechim mavjud va uni iterasiya usuli yordamida taqribiy hisoblash mumkin. Dastlabki yaqinlashishni $x_0 = \frac{1}{2},\ y_0 = \frac{1}{2}$ deb olaylik.

$$x_1 = \frac{1}{2} + \frac{\frac{1}{8} + \frac{1}{8}}{6} = 0{,}542\ ;\quad y_1 = \frac{1}{3} + \frac{\frac{1}{8} - \frac{1}{8}}{6} = 0{,}333\ ;$$

$$x_2 = \frac{1}{2} + \frac{0{,}19615}{6} = 0{,}533\ ;\quad y_2 = \frac{1}{3} + \frac{0{,}1233}{6} = 0{,}354\ ;$$

Hisoblashlarni shu singari davom ettirib,

$$x_3 = 0{,}533\ ;\quad y_3 = 0{,}351\ ;\quad x_4 = 0{,}532\ ;\quad y_4 = 0{,}351\ ;$$

bo'lishini aniqlaymiz. $q_1 = q_2 = \frac{34}{72} < 0{,}5$ bo'lganligidan va uchinchi va to'rtinchi taqribiy yechimlarning kasr qismidagi uchta raqamining mos kelishi talab qilingan aniqlikka erishilganini bildiradi. Taqribiy yechim sifatida $x = 0{,}532\ ;\ y = 0{,}351\ ;$ qiymatlarni olish mumkin.

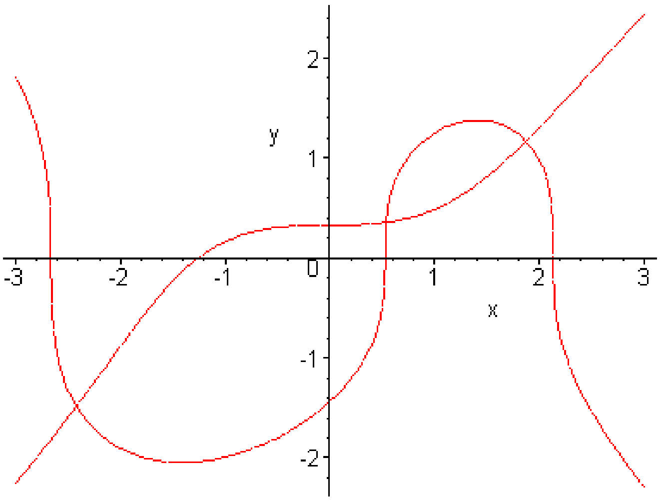
Ushbu misolda berilgan tenglamalar sistemasi 3 ta haqiqiy yechimga ega ekanligini quyidagi Maple dastur va grafiklardan ko'rish mumkin (8-rasm):

```
> plots[implicitplot]({x^3+y^3-6*x+3=0,x^3-y^3-6*y+2=0},x=-3..3,y=-3..3);
solve({x^3+y^3-6*x+3=0,x^3-y^3-6*y+2=0},{x,y});
allvalues(%);
evalf(%);
```

$\{ y = .3512574476,\ x = .5323703724 \}$

$\{ x = 1.882719112,\ y = 1.175129224 \}$

$\{ y = -1.489322079,\ x = -2.423800711 \}$



5-rasm. Misolda berilgan tenglamalar sistemasidagi funksiyalarning Maple dasturida chizilgan grafiklari.

Misolning Nyuton usuli bo'yicha Maple dasturidagi yechilishi

```
> with(linalg):
F:=(x,y)->[x^3+y^3-6*x+3,x^3-y^3-6*y+2];
FP:=jacobian(F(x,y),[x,y]); FPINV:=inverse(FP);
 xx:=[0.5,0.5]; eps:=0.0001; Err:=1000; v:=xx; v1:=[1e10,1e10];
j:=0: for i while Err>eps do
   v1:=eval(v);
   M:=eval(eval(FPINV),[x=v[1],y=v[2]]):
   v:=evalm(v-M&*F(v[1],v[2]));
Err:=max(abs(v1[1]-v[1]),abs(v1[2]-v[2]));
j:=j+1; end do;
```

$$F := (x, y) \rightarrow [x^3 + y^3 - 6x + 3, x^3 - y^3 - 6y + 2]$$

$$FP := \begin{bmatrix} 3x^2 - 6 & 3y^2 \\ 3x^2 & -3y^2 - 6 \end{bmatrix}$$

$$FPINV := \begin{bmatrix} \frac{1}{6}\frac{y^2+2}{y^2x^2+x^2-y^2-2} & \frac{1}{6}\frac{y^2}{y^2x^2+x^2-y^2-2} \\ \frac{1}{6}\frac{x^2}{y^2x^2+x^2-y^2-2} & -\frac{1}{6}\frac{x^2-2}{y^2x^2+x^2-y^2-2} \end{bmatrix}$$

$$xx := [.5, .5]$$

$$eps := .0001$$

$$Err := 1000$$

$$v := [.5, .5]$$

$$v1 := [.1\ 10^{11}, .1\ 10^{11}]$$

$$v1 := [.5, .5]$$

$$M := \begin{bmatrix} -.1935483872 & -.02150537635 \\ -.02150537635 & -.1505376344 \end{bmatrix}$$

$$v := [.5268817204, .3548387097]$$

$$Err := .1451612903$$

$$j := 1$$

$$v1 := [.5268817204, .3548387097]$$

$$M := \begin{bmatrix} -.1953940827 & -.01157253256 \\ -.02551483073 & -.1583067195 \end{bmatrix}$$

$$v := [.5323579817, .3512504710]$$

$$Err := .0054762613$$

$$j := 2$$

$$v1 := [.5323579817, .3512504710]$$

$$M := \begin{bmatrix} -.1960636703 & -.01139210219 \\ -.02616842480 & -.1585031434 \end{bmatrix}$$

$$v := [.5323703724, .3512574476]$$

$$Err := .0000123907$$

$$j := 3$$

Misolning iteratsiya usuli bo'yicha Maple dasturidagi yechilishi

```
> with(linalg):
 G:=(x,y)->[(x^3+y^3)/6+1/2,(x^3-y^3)/6+1/3];
v:=evalf(G(v[1],v[2]));
eps:=0.0001;  Err:=1000;
  v:=[1/2,1/3];    j:=0;
for i while Err>eps do
   v1:=evalf(v):      v:=evalf(G(v[1],v[2]));
Err:=max(abs(v1[1]-v[1]),abs(v1[2]-v[2])):
j:=j+1;  end do;
```

$$G := (x, y) \to \left[\frac{1}{6}x^3 + \frac{1}{6}y^3 + \frac{1}{2}, \frac{1}{6}x^3 - \frac{1}{6}y^3 + \frac{1}{3}\right]$$

$$v := [.4815158646, .3053580466]$$

$$eps := .0001$$

$$Err := 1000$$

$$v := \left[\frac{1}{2}, \frac{1}{3}\right]$$

$$j := 0$$

$$v1 := [.5000000000, .3333333333]$$

$$v := [.5270061728, .3479938272]$$

$$Err := .0270061728$$

$$j := 1$$

$$v1 := [.5270061728, .3479938272]$$

$$v := [.5314183793, .3507043961]$$

$$Err := .0044122065$$

$$j := 2$$

$$v1 := [.5314183793, .3507043961]$$

$$v := [.5322016428, .3511568471]$$

$$Err := .0007832635$$

$$j := 3$$

$$v1 := [.5322016428, .3511568471]$$

$$v := [.5323402650, .3512397490]$$

$$Err := .0001386222$$

$$j := 4$$

$$v1 := [.5323402650, .3512397490]$$

$$v := [.5323650143, .3512542731]$$

$$Err := .0000247493$$

$$j := 5$$

Hisoblashlar shuni ko'rsatadiki, har ikkala usul bo'yicha ham analitik va ham sonly hamda kompyuter hisobi natijalari bir xil: Nyuton usulida 3 ta iteratsiyada va iteratsion usulda 5 ta iteratsiyada bir xil natijaga kelindi. Shunday qilib, taqribiy yechim sifatida $x = 0{,}532$; $y = 0{,}351$; qiymatlarni olish mumkin.

Xulosa

Kurs ishida olingan natijalar shuni ko'rsatadiki, dastur to'g'ri ishlayapti va nochiziqli tenglamalar sistemasining yechimlari to'g'ri topilgan va ular bir xil. Aniqlikning oshirish bilan iteratsiyalar soni ham oshib boradi. Agar boshlang'ich yaqinlashish aniq yechimga yaqinroq olinsa yaqinlashish tezligi ortadi va, tabiiyki, iteratsiyalar soni ham kamayadi.

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