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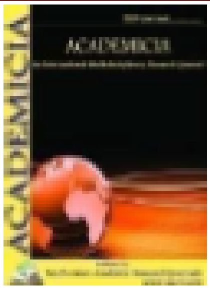

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# THE METHOD OF APPROXIMATION OF THE SEQUENCE FOR SOME NONELINEAR BOUNDARY VALUE PROBLEMS

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**ABSTRACT**

*In this paper, we consider a system of simple differential equations for boundary-value nonlinear problems. Nonlinear boundary conditions of the considered interval for the condition of the solution of the studied function of this side the considered boundary value problem is solved using the numerical analytical method of A.M. Samoylenko and is erroneously estimated. Therefore together with this nonlinear boundary condition for considering the right side interval instructions are given for solving the value of the function for investigation.*

**KEYWORDS:** *Simple Differential Equations, Nonlinear Boundary Value Problems Numerical-Analytical Method For Solving Approximations.*

## INTRODUCTION

For the first nonlinear ordinal differential equation it will be

$$\frac{dx}{dt} = f(t,x) \ , \qquad (1)$$

$$\Phi(x(0), x(T)) = 0 \qquad (2)$$

the boundary value problem will be given, here is the function $f$ function in the area $[0,T]\times D$ , and $\Phi$ function will be $D\times D$ defined in the area $n$ measurement functions of the vector. Let be nonlinear (1), (2) numerical-analytical method for solving boundary value problems of approximation in turn [1,2] using this method, let's consider the task of calculating the approximation.

ISSN: 2249-7137 Vol. 10, Issue 6, June 2020 Impact Factor: SJIF 2020 = 7.13

If from (2) $\det A \neq 0$ will be $x(0)$ can be divided , then (2) will be

$$Ax(0) + Bx(T) = \varphi(x(T)) \quad (3)$$

written like this , here $A$ and $B$ matrixes $n \times n$ measuring constants of the square matrixes. In this case (1),(3) boundary task for this $\alpha = 0$ will be equivalent

$$x(t) = x_T - \int_t^T f(s, x(s))ds + \alpha(T - t), \quad (4)$$

we can change with integral equation, here $x_T = x(T)$.

$\alpha$ to choose parameter (4) to (3) put to boundary condition

$$A\left[x_T - \int_0^T f(s, x(s))ds + \alpha T\right] + Bx_T = \varphi(x_T)$$

from that $\alpha$ will have the meaning.

This meaning of $\alpha$ will put instead of this (4)

$$x(t) = x_T - \int_t^T f(s, x(s))ds + \frac{T-t}{T}\int_0^T f(s, x(s))ds + \frac{T-t}{T}A^{-1}\left[\varphi(x_T) - (A+B)x_T\right]$$

or

$$x(t) = x_T + \int_0^t \left[f(s, x(s))ds - \frac{1}{T}\int_0^T f(s, x(s))ds\right]ds + \frac{T-t}{T}A^{-1}\left[\varphi(x_T) - (A+B)x_T\right]$$

we will have such equation .

(4) if you use the method of approximation to the integral equation in turn

$$x_{m+1}(t, x_T) = x_T + \int_0^t \left[f(s, x_m(s, x_T)) - \frac{1}{T}\int_0^T f(s, x_m(s, x_T))ds\right]ds + \\ + \frac{T-t}{T}A^{-1}\left[\varphi(x_T) - (A+B)x_T\right], \; m = 0,1,2,... \quad (5)$$

this will be, this order is defined as $x_m(t, x_T)$ functions any $m = 0,1,2,...$ and consistently $x_T$ parameter (3) satisfies the boundary conditions, here will be $x_0(t, x_T) = x_T$. That's why our next aim is (5) determine the conditions for the collected sequence order of the function and limitation of these sequences (1) to satisfy the system of differential equations is to choose parameter $x_T$ .

ISSN: 2249-7137 Vol. 10, Issue 6, June 2020 Impact Factor: SJIF 2020 = 7.13

From (5) any $m=0,1,2,...$ will be this

$$|x_{m+1}(t,x_T)-x_T|\le\left(1-\frac{t}{T}\right)\int_0^t|f(s,x_m(s,x_T))|ds+\frac{t}{T}\int_t^T|f(s,x_m(s,x_T))|ds+ \\ +\left|A^{-1}\left[\varphi(x_T)-(A+B)x_T\right]\right|=M\alpha_1(t)+N\le M\frac{T}{2}+N \qquad (6)$$

can take inequalities, here $M$ and $N$ constancies, $|f(t,x)|\le M,\ \ |\varphi(x_T)-(A+B)x_T|\le N$

will be taken the conditions of satisfaction of the inequality, here

$$\alpha_1(t)=\left(1-\frac{t}{T}\right)\int_0^t ds+\frac{t}{T}\int_t^T ds=2t\left(1-\frac{t}{T}\right).$$

From (6) as can be seen (5) each approximation defined by the formula $D$ to avoid leaving the area includes $x_T$ taking this point and its $M\frac{T}{2}+N$ circle with $D$ located in the area $D_1$ multitude should be, in other words the empty set should not be :

$$D_f\ne\varnothing. \qquad (7)$$

As we know [1,2], (5) solved with the formula $x_m(t,x_T)$ functions any $m=0,1,2,...$ and $j=1,2,...$ is

$$\left|x_{m+j}(t,x_T)-x_m(t,x_T)\right|\le\left(E+Q+Q^2+...+Q^{j-1}\right)Q^m\left(M+\frac{\pi}{T}N\right)\tilde{\alpha}_1(t)$$

satisfies the inequality, here $Q=K\frac{T}{\pi},\ \tilde{\alpha}_1(t)=\frac{\pi}{3}\alpha_1(t)$ and with this matrix $K$ of $f$ function is

$$|f(t,x_2)-f(t,x_1)|\le K|x_2-x_1| \qquad (8)$$

defined by the condition of satisfying the Lipschitz condition.

Let $Q$ matrix eigenvalues are the same small, that is

$$\lambda_i(Q)<1,\ i=1,2,...,n. \qquad (9)$$

Then from (8) $j\to\infty$ limit of efforts

$$\left|x^*(t,x_T)-x_m(t,x_T)\right|\le\left(E-Q\right)^{-1}Q^m\left(M+\frac{\pi}{T}N\right)\tilde{\alpha}_1(t)$$

ISSN: 2249-7137 Vol. 10, Issue 6, June 2020 Impact Factor: SJIF 2020 = 7.13

can take this inequality, here $x^*(t,x_T)$ as said (5) defined with the formula $x_m(t,x_T)$ sequence of functions $m \to \infty$ limit of efforts.

But this limit (1),(3) to boundary value problem $\alpha = 0$ equivalent for (4) tries to find a solution to integral equations $\alpha$ equalize to zero,

$$\alpha = \frac{1}{T}A^{-1}\left[\varphi(x_T) - (A+B)x_T\right] + \frac{1}{T}\int_0^T f(s, x^*(s, x_T))ds = 0 \qquad (10)$$

From equality $x_T$ to $x_T^*$ if we define, then $x^*(t,x_T^*)$ function (1),(3) it is an exact solution to the boundary value problem.

When these obtained results are generalized the following theorem is given.

**Theorem.** Let $f(t,x)$ function $(t,x) \in [0,T] \times D$ defined in this area, is a continuous function and in this area (7)-(9) conditions will be given.

There (5) defined with formula $\{x_m(t,x_T)\}$ sequence of functions $m \to \infty$ (4) solution of the integral equation $x^*(t,x_T)$ efforts to equal function dimensions and $x_T$ parameter (8) defined with formula (10) if you select equality turning to zero then the function (1), (3) it will be a solution to the boundary value problem.

Exact $x^*(t,x_T^*)$ with solution $x_m(t,x_T)$ error between approximation solution

$$\left|x^*(t,x_T) - x_m(t,x_T)\right| \le \tilde{\alpha}_1(t) Q^m (E-Q)^{-1}\left(M + \frac{\pi}{T}N\right)$$

Estimated with inequality.

If from (2) $\det A \neq 0$ will be $x(0)$ not this , but $\det B \neq 0$ will be $x(T)$ if you can divide this, then (2)

$$Ax(0) + Bx(T) = \varphi(x(0)) \qquad (11)$$

can write like this. In this case (1),(11) boundary value problem to $\alpha = 0$ equivalent will be

$$x(t) = x_0 + \int_0^t f(s, x(s))ds + \alpha t \qquad (12)$$

Change with integral equation , there $x_0 = x(0)$. (12) put to (11), define $\alpha$

$$\alpha = \frac{1}{T}B^{-1}\left[\varphi(x_0) - (A+B)x_0\right] - \frac{1}{T}\int_0^T f(s, x(s))ds$$

this will be , in this case (5) sequence of functions will be like this

$$x_{m+1}(t,x_0)=x_0+\int_0^t\left[f(s,x_m(s,x_0))-\frac{1}{T}\int_0^T f(s,x_m(s,x_0))ds\right]ds+$$

$$+\frac{t}{T}B^{-1}\left[\varphi(x_0)-(A+B)x_0\right],\ m=0,1,2,...$$

The sequence of collecting these functions is proved similar to the previous case.

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