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Kinematic Nonuniformity of the Rotation of a Toothed Belt Transmission with a Composite Pulley

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Abstract—Existing drive transmissions are analyzed. A toothed belt transmission with a driven pulley that includes an elastic rubber bush is proposed. Formulas are presented for the kinematic nonuniformity of rotation of the driven pulley, its velocity, and its angular displacement as a result of bush deformation.

Keywords: toothed belt transmission, pulley, pulley velocity, kinematic nonuniformity, nonuniform rotation, torque, elastic modulus, rigidity, load

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In manufacturing, various types of transmissions are employed: in particular, gear, chain, and belt transmissions. Gear transmissions have a precise gear ratio, permit large gear ratios, and withstand high loads, but they are only used with small distances between the shafts. With large distances between the shafts, chain or belt transmissions are employed [1–5].

In chain transmissions, the chain engages with a sprocket. A significant deficiency of such transmissions is that impact loading appears at small gear ratios [6–9]. Therefore, they are suitable for low speeds. In a chain transmission, two rigid elements are in contact: the chain and a wheel. In a belt transmission, the belt engages with a pulley: in other words, a rigid element is in contact with a soft element.

In the case of two rigid elements, precise engagement is required, without a gap or tightness. However, if a rigid element and a flexible element are in tight engagement, their interaction is more effective and no impact loads appear [10].

As a rule, a belt transmission relies on friction (in a V belt) or simple engagement (in a toothed belt). Frictional transmissions are employed at large loads. The flexible element in such transmissions is made of an elastic material, such as rubber or plastic. In that case, elastic slip and elastic deformation are unavoidable.

In frictional transmissions, elastic slip in the frictional pair occurs at any load, as established in [11–13]. A refined method of determining a coefficient taking account of the load and operating conditions was proposed in [14]. The inverse problem was solved in [15]: given the specified belt life, to determine the maximum stress of the driving branch and the stress corresponding to the initial tension, the azimuthal force, and the cross sectional area ensuring specified life. Calculation of the life of belt transmissions was considered in [16–18].

A toothed belt transmission has a precise gear ratio and is designed so that precise engagement of the belt with the sprocket rules out slip of the belt and idle rotation of the pulley. However, this design is only suitable for small loads.

The reliability of a toothed belt transmission is determined by the reliability of the toothed drive belt. Industrial experience with such transmissions shows that, for toothed belts in which the working surface has a wear-resistant coating, the damage mainly (75–80%) takes the form of fatigue failure of the teeth [19].

In power transmission, the toothed drive belt is subject to impact loads, changing its stress–strain state. That determines the rate of destructive processes limiting the carrying capacity and life of the transmission [20, 21]. With little load variation, the use of ordinary toothed belt transmissions is not recommended.

We have developed a new design for a toothed belt transmission with a composite driven pulley (Fig. 1).

In ordinary toothed belt transmissions, where a tooth on the belt engages with a tooth of the drive pulley, its speed v_1 of remains constant in the course of rotation, but its direction changes. That alters the projection of v_1 onto the X and Y axes (Fig. 2a) [22, 23].

In position I

$$v_{x\max} = v_1 \cos\varphi_1;\ \ v_{y\max} = v_1 \sin\varphi_1.$$

In position II

$$v_{x\min} = v_1 \frac{\omega_1 t}{2\times 10^3 \sin\varphi_1};\ \ v_{y\min} = 0.$$

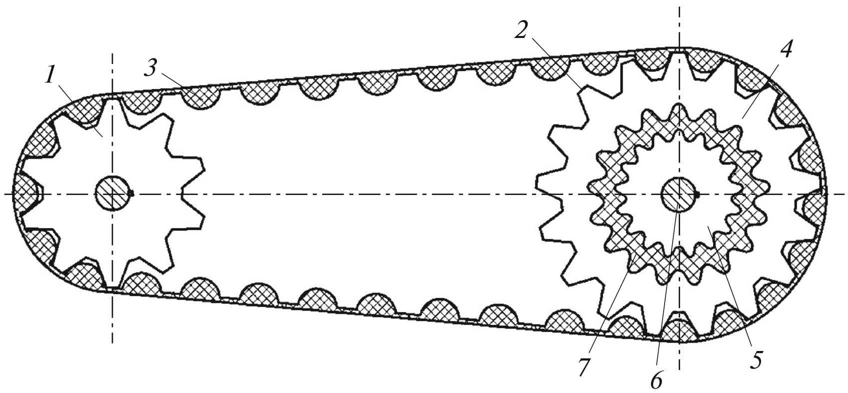

Fig. 1. Toothed belt transmission with a composite driven pulley: (*1*) drive pulley; (*2*) driven pulley; (*3*) toothed belt; (*4*) external gear rim; (*5*) pulley hub; (*6*) shaft; (*7*) rubber bush.

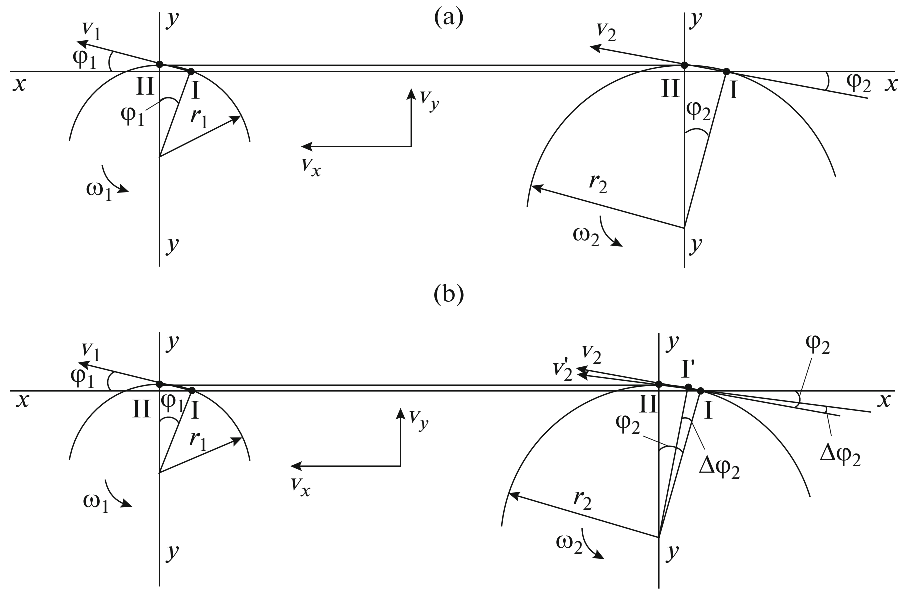

Fig. 2. Toothed belt transmission when belt tooth engages with tooth of drive pulley (a) and disengages from driven pulley (b).

Then the mean belt speed is

$$v_{x\,\min} = \frac{1}{2}(v_{x\,\min} + v_{x\,\max}) = \frac{\omega_1 t}{4\times 10^3}\left(\cot\varphi_1 + \frac{1}{\sin\varphi_1}\right).$$

The speed v_2 of a tooth disengaging from the driven pulley and its angular velocity ω_2 depend on the pulley motion (Fig. 2b).

In position I

$$v_{2\,\min} = v_1\frac{\cos\varphi_1}{\cos\varphi_2};\quad \omega_{2\,\min} = \omega_1\frac{r_1\cos\varphi_1}{r_2\cos\varphi_2}.$$

In position II

$$v_{2\,\max} = v_1;\quad \omega_{2\,\max} = \omega_1\frac{r_1}{r_2}.$$

The nonuniformity of the driven pulley's angular velocity is characterized by the coefficient δ_k, the kinematic nonuniformity of rotation

$$\delta_k = 2\frac{\omega_{2\,\max} - \omega_{2\,\min}}{\omega_{2\,\max} + \omega_{2\,\min}} = 2\frac{1 - \dfrac{\cos\varphi_1}{\cos\varphi_2}}{1 + \dfrac{\cos\varphi_1}{\cos\varphi_2}}.$$

We now consider the next stage in the motion of a toothed belt transmission with a composite driven pulley. In this stage, the mean velocity remains unchanged, while the azimuthal and angular velocities of the driven pulley depend on the additional angle $\Delta\varphi_2$ of pulley rotation.

In position I

$$v'_{2\,\min} = v_1 \frac{\cos\varphi_1}{\cos(\varphi_2 + \Delta\varphi_2)};\ \ \omega_{2\,\min} = \omega_1 \frac{r_1 \cos\varphi_1}{r_2 \cos(\varphi_2 + \Delta\varphi_2)}.$$

In position II

$$v'_{2\,\max} = v_1;\ \ \omega_{2\,\max} = \omega_1 \frac{r_1}{r_2}.$$

In this case, the kinematic nonuniformity of rotation is

$$\delta_k = 2\frac{1 - \dfrac{\cos\varphi_1}{\cos(\varphi_2 - \Delta\varphi_2)}}{1 + \dfrac{\cos\varphi_1}{\cos(\varphi_2 - \Delta\varphi_2)}}. \quad (1)$$

In Fig. 3, we show the cross section of the composite pulley. In theoretical analysis, it is regarded as a rubber–metal hinge. The torsional deformation of a uniform hollow cylinder was considered in [24]. For a particular case, the problem was reduced to a system of two transcendental equations for the Euler radial and azimuthal coordinates in a deformable configuration. The coaxial twist and axial shear of an incompressible cylinder were determined analytically on the basis of a single-parameter elastic potential in [25–28].

On the basis of elasticity theory, the deformational energy and tangential stress in a body of volume V are as follows [29, 30]

$$\Pi = \frac{\tau^2 V}{2G};\ \ \tau = \frac{M}{2\pi r_2^2 l},$$

where τ is the tangential stress in the material corresponding to the deformation; G is the elastic modulus of the material, Pa; and M is the external torque, N m.

Adopting an elementary cylinder as the deformable body and equating the deformation energy of the rubber bush to the work of the external torque M, we obtain

$$dV = 2\pi r l dr,\ \ \frac{1}{2} M \Delta\varphi_2 = \int_V \frac{\tau^2 dV}{2G}.$$

Substituting τ from Eq. (2) into this expression and integrating from r_2 to r_{bu}, we find that

$$\frac{1}{2} M \Delta\varphi_2 = \frac{M^2}{4\pi G l} \int_{r_2}^{r_{bu}} \frac{dr}{r^3} = \frac{M^2}{4\pi G l} \frac{r_{bu}^2 - r_2^2}{2 r_{bu}^2 r_2^2}.$$

Hence

$$\Delta\varphi_2 = \frac{M}{4\pi G l} \frac{r_{bu}^2 - r_2^2}{r_{bu}^2 r_2^2}. \quad (2)$$

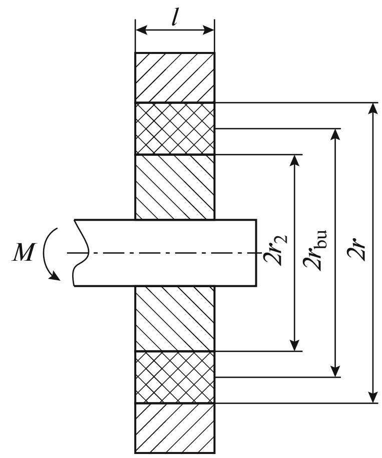

Fig. 3. Cross section of composite pulley.

Substituting $\Delta\varphi_2$ from Eq. (3) into Eq. (1), we obtain

$$\delta_k = 2\frac{1 - \dfrac{\cos\varphi_1}{\cos\left(\varphi_2 - \dfrac{M}{4\pi G l}\dfrac{r_{bu}^2 - r_2^2}{r_{bu}^2 r_2^2}\right)}}{1 + \dfrac{\cos\varphi_1}{\cos\left(\varphi_2 - \dfrac{M}{4\pi G l}\dfrac{r_{bu}^2 - r_2^2}{r_{bu}^2 r_2^2}\right)}}. \quad (3)$$

Solving Eq. (4), we may determine the limits of the pulley's angular velocity. Its parameters may be established by selecting its law of motion.

The kinematic problem for the toothed belt transmission is solved numerically with the following values: M = 80–120 N m; G = 1–3 MN/m^2; $\varphi_1 = 30°$; $\varphi_2 = 30°$; l = 0.03 m; r_{bu} = 1 m; and r_2 = 0.8 m.

EXAMPLE

Calculations show that, with increase in the external torque, the kinematic nonuniformity δ_k of rotation of the driven pulley increases linearly. With increase in the elastic modulus G of the pulley's rubber bush, we note decrease in δ_k (Fig. 4).

In Fig. 5, we plot δ_k against G. When M = 100 N m, δ_k declines nonlinearly with increase in G from 1 to 3 MN/m^2.

Analysis of Fig. 6 shows that, with increase in thickness $\Delta r = r_{bu} - r_2$ of the bush, δ_k increases nonlinearly.

Thus, we have obtained analytical expressions for the maximum and minimum linear and angular velocities of the pulleys and the kinematic nonuniformity of rotation of the driven pulley in a toothed belt transmis-

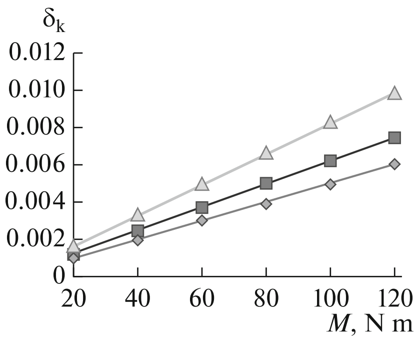

Fig. 4. Dependence of the kinematic nonuniformity δ_k of pulley rotation on the external torque M when G = 2.5 (–◆–), 2 (–■–), and 1.5 (–▲–) MN/m.

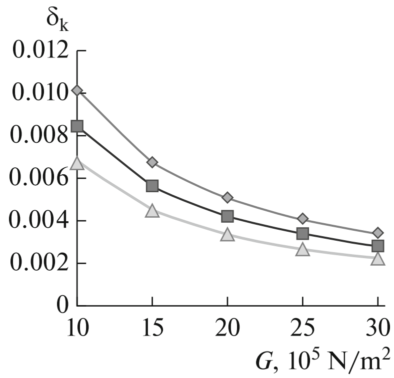

Fig. 5. Dependence of the kinematic nonuniformity δ_k of pulley rotation on the bush's elastic modulus G when M = 120 (–◆–), 100 (–■–), and 80 (–▲–) N m.

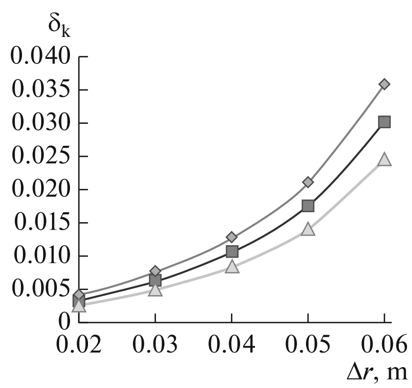

Fig. 6. Dependence of the kinematic nonuniformity δ_k of pulley rotation on the bush thickness Δr in the pulley when M = 120 (–◆–), 100 (–■–), and 80 (–▲–) N m.

sion. On the basis of those expressions, the design parameters ensuring the required performance may be determined.

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