O`ZBEKISTON RESPUBLIKASI

OLIY VA O`RTA MAXSUS TA`LIM VAZIRLIGI

ALISHER NAVOIY NOMIDAGI

SAMARQAND DAVLAT UNIVERSITETI

Mavzu:Tekislikda funksiyani tiklash masalasi

5A130101 «Matematika» mutaxassisligi

Matematika bo'yicha magistr akademik darajasini olish uchun

MAGISTRLIK DISSERTATSIYASI

Ilmiy rahbar: dots. Z.H.Ochilov

Magistrant: Ashurova Umida

Samarqand – 2014

Mundarija

KIRISH

Masalaning qo'yilishi. Siniq chiziq va parabolalar oilasi bo'yicha integrallar ma'lum bo'lsa, shu integrallar bo'yicha funksiyani tiklash masalasi qo'yilgan.

Mavzuning dolzarbligi. Integral geometriya masalasi –keng rivojlanayotgan sohalardan biridir. U matematik fizika va analizning nokorrekt masalalari nazariyasining eng yirik yo'nalishlaridan biri hisoblanadi. Uning masalalari ko'p sonli tadqiqotlarga bog'liq. Misol sifatida seysmik kuzatish, ma'lumotlarini interpretasiyalash masalalari, elektrik kuzatishlar, akustika va kompyuter tomografiya masalalarini ko'rsatishimiz mumkin.

Bu soha 1966 yildan yanada rivojlandi va ko'pchilik olimlar qiziqdilar.

M.M. Lavrent'ev va V.G. Romanovlar tomonidan bir qator giperbolik tenglamalar uchun teskari masalalar integral geometriya masalalaridan kelib chiqishini ko'rsatilgan.

Integral geometriya masalasining markaziy muammolaridan biri bu qandaydir ko'pxilliklarda aniqlangan funksiyani uning qandaydir kichik o'lchamdagi ko'pxilliklar oilasi bo'yicha integrali orqali topish masalasidir. Bu magistrlik dissertatsiasida ikki o'lchamli integral geometriya masalalari qaralgan.

Ishning maqsad va vazifalari. Magistrlik dissertatsiyasining maqsadi agar siniq chiziq va parabolalar oilasi bo'yicha integrallar ma'lum bo'lsa, funksiyani tiklash hisoblanadi.

Bu ishda siniq chiziq va parabolalar oilasi bo'yicha integral geometriya masalasi uchun yagonalik va mavjudlik teoremasi isbotlanadi.Turg'unlik bahosi va teskarilanish formulasi olinadi.

Ilmiy tadqiqot usullari. Integral geometriya masalasining yagonalik va mavjudlik teoremasi isbotlashda,turg'unlik va teskarilanish bahosi olishda analiz va matematik fizikaning usullaridan foydalaniladi.

Ishning ilmiy ahamiyati. Magistrlik dissertatsiyasida yuqori yarim tekislikda berilgan siniq chiziq va parabolalar oilasi uchun integral geometriya masalasining yangi qo'yilishi o'rganilgan. Qo'yilgan masala uchun yagonalik va mavjudlik teoremasi isbotlangan. Teskarilanish formulasi va turg'unlik bahosi olingan. Shuni inobatga olib ish nazariy xarakterga ega ekanligini aytish mumkin.

Ishning amaliy ahamiyati. Ishda ko'rilgan masalalarga ekvivalent bo'lgan masalalarning yechimini topishda olingan natijalardan foydalanish mumkin.

Ishning tuzilishi. Magistrlik dissertatsiyasi kirish qismi, uchta bob, 10 ta paragraf, xulosa va foydalanilgan adabiyotlardan iborat.

Kirish qismida masalaning qo'yilishi, dolzarbligi va olingan natijalar to'g'risida so'z yuritiladi.

Birinchi bob beshta paragrafdan iborat.

Birinchi paragrafda tekislikda integral geometriya masalasining umumiy ko'rinishi haqida so'z boradi.

Ikkinchi paragrafda tekislikda qo'zg'alishli integral geometriya masalasi keltirilgan.

Uchinchi paragrafda siniq chiziqlar oilasi bo'yicha integral geometriya masalasi yechimining yagonalik teoremasi isbotlangan,teskarilanish formulasi va turg'unlik bahosi olingan.

To'rtinchi paragrafda siniq chiziqlar oilasi bo'yicha integral geometriya masalasi yechimining mavjudlik teoremasi haqida so'z boradi.

Beshinchi paragrafda esa siniq chiziqlar oilasi bo'yicha qo'zg'alishli integral geometriya masalasi qaralgan.

Ikkinchi bob uchta paragrafdan tashkil topgan.

Birinchi paragrafda parabolalar oilasi bo'yicha integral geometriya masalasi yechimining yagonalik teoremasi isbotlangan.

Ikkinchi paragrafda parabolalar oilasi bo'yicha integral geometriya masalasi yechimining mavjudlik teoremasi va uning isboti keltirilgan.

Uchinchi paragrafda parabolalar oilasi bo'yicha qo'zg'alishli integral geometriya masalasi qaralgan.

Uchinchi bob ikkita paragrafdan tashkil topgan.

Birinchi paragrafda siniq chiziq va parabolalar oilasi bo'yicha integral geometriya masalasining qo'yilishi keltirilgan, yagonalik teoremasi isbotlangan, turg'unlik bahosi va teskarilanish formulasi olingan.

Ikkinchi paragrafda yechimning mavjudligi haqidagi teoremaning isboti keltirilgan.

Olingan natijalarning qisqacha mazmuni.

Bu magistrlik dissertatsiyasining asosiy masalasi yuqori yarim tekislikda siniq chiziq va parabolalar oilasi bo'yicha integrallari ma'lum bo'lib, unda uzluksiz finit funksiyalar sinfida yechimning yagonaligi haqidagi teorema keltirilib, bu teorema isbotlandi. Teskarilanish formulasi va turg'unlik bahosi olinib,yuqorida qo'yilgan masala uchun mavjudlik teoremasi isbotlandi.Olingan natijalar maqola sifatida tayyorlanib, Rossiyaning Novosibirsk shahrida 2013 yil 12-18 aprelda bo'lib o'tgan "Студент и научно-технический прогресс" nomli xalqaro konferensiyaga chop ettirildi, bundan tashqari Namangan shahrida 2013 yil 15-16 aprelda bo'lib o'tgan "Yosh matematiklarning yangi teoremalari" nomli xalqaro konferensiyada va Navoiy shahrida 2014-yil 13-14 iyunda bo'lgan'' 21-asr intellektual avlod asri" nomli konferensiyada chop ettirildi.

I-BOB. SINIQ CHIZIQLAR OILASI BO'YICHA INTEGRAL GEOMETRIYA MASALASI

1.1-§. Tekislikda integral geometriya masalasining umumiy ko'rinishi

(x, y) tekislikda Γ chegarali chegaralangan ochiq D bir bog'lamli sohani qaraymiz. Chegara $x = g(s)$, $y = p(s)$ ko'rinishdagi tenglama bilan berilgan bo'lsin, bu yerda s - Γ dagi tayinlangan nuqtadan Γ dagi yo'nalish bilan muvofiq keluvchi musbat yo'nalishda hisoblanuvchi uzunlik, $g(s)$, $p(s)$ - $C^1[0, l]$ sinf funksiyalari, $g(0) = g(l)$, $p(0) = p(l)$, l - Γ uzunligi D sohada quyidagi xossalarga ega ikki parametrli $L(t_1, t_2)$ egri chiziqlar oilasi berilgan bo'lsin.

1. D sohaning har bir juft nuqtasini oilaning yagona egri chizig'i tutashtiradi,har bir $L(t_1, t_2)$ egri chiziq Γ ni (x_1, y_1), (x_2, y_2) nuqtalarda kesib o'tadi, yani

$$x_1 = g(s_1), \quad x_2 = g(s_2)$$

$$y_1 = p(s_1), \quad y_2 = p(s_2)$$

$L(t_1, t_2)$ egri chiziqning boshqa nuqtalari D ga tegishli; $L(t_1, t_2)$ egri chiziqlar uzunliklari jamlanmasi bilan chegaralangan.

2. (x_0, y_0) nuqtadan $v^0 = (\cos\theta_0, \sin\theta_0)$ yo'nalishda o'tuvchi egri chiziq tenglamasi

$$\begin{aligned} x &= f_1(s, \theta_0, x_0, y_0) = x_0 + s\cos\theta_0 + s^2 \tilde{f}_1(s, \theta_0, x_0, y_0) \\ y &= f_2(s, \theta_0, x_0, y_0) = y_0 + s\sin\theta_0 + s^2 \tilde{f}_2(s, \theta_0, x_0, y_0) \end{aligned} \qquad (1.1.1)$$

tenglamalar bilan beriladi. Bularda $\tilde{f}_j(\cdot)$ (x_0, y_0) -nuqtada hisoblanuvchi s yoyi uzunligi va $\theta_0 \in [0, 2\pi]$ va $(x_0, y_0) \in \overline{D}$ parametrning uzluksiz differensiallanuvchi va hosilalari bilan chegaralangan funksiyalar, $f_j(s,0,x_0,y_0)=f_j(s,2\pi,x_0,y_0)$ va bundan tashqari (s,θ_0,x_0,y_0) o'zgaruvchilarning butun o'zgarish sohasida

$$\frac{1}{2}\frac{D(f_1,f_2)}{D(s,\theta_0)}\geq c_0>0 \qquad (1.1.2)$$

Ta'kidki, nolga yaqin s lar uchun (1.1.2) tengsizlikning bajarilishi ravshanki (1.1.1) dan kelib chiqadi, shuning uchun (1.1.2) tengsizlik chekli s lar uchun muhim. Bu holda u $D(f_1,f_2)/D(s_1,\theta_0)$ yakobianning musbatligiga teng kuchli.

L egri chiziqlar oilasiga nisbatan quyidagi lemma o'rinli.

1.1.1-Lemma. Egri chiziqlar oilasi yuqoridagi shartlarni qanoatlantirsin, bundan tashqari $f_j(\cdot)$ funksiyalar $m>1$ tartibgacha uzluksiz va chegaralangan hosilalarga ega bo'lsin. U holda (1.1.1) tenglik s,v^0 ni (x_0,y_0) nuqtalarning bir qiymatli va m marta uzluksiz differensiallanuvchi funksiyalari sifatida barcha (x_0,y_0), (x,y) lar $(x_0,y_0),\ (x,y)\in\overline{D}$ va $(x_0,y_0)\neq(x,y)$ lar uchun aniqlaydi, $(x,y)=(x_0,y_0)$ to'plam atrofida ular uchun

$$\begin{gathered}\left|D^\alpha s(x_0,y_0,x,y)\right|\leq\frac{c}{\left[(x-x_0)^2+(y-y_0)^2\right]^{(|\alpha|-1)/2}}\\ \left|D^\alpha v^0(x_0,y_0,x,y)\right|\leq\frac{c}{\left[(x-x_0)^2+(y-y_0)^2\right]^{|\alpha|/2}}\end{gathered} \qquad (1.1.3)$$

$|\alpha|\leq m$ baholar o'rinli.

Isbotlash uchun (1.1.1) dan quyidagi tengliklar kelib chiqishini ta'kidlaymiz:

$$s=\frac{\left[(x-x_0)^2+(y-y_0)^2\right]^{1/2}}{\left[(\cos\theta_0+s\tilde{f}_1)^2+(\sin\theta_0+s\tilde{f}_2)^2\right]^{1/2}}$$

$$\cos\theta_0=\frac{x-x_0}{\left[(x-x_0)^2+(y-y_0)^2\right]^{1/2}}\left[(\cos\theta_0+s\tilde{f}_1)^2+(\sin\theta_0+s\tilde{f}_2)^2\right]^{1/2}+$$

$$+\left[(x-x_0)^2+(y-y_0)^2\right]^{1/2}\frac{\tilde{f}_1}{\left[(\cos\theta_0+s\tilde{f}_1)^2+(\sin\theta_0+s\tilde{f}_2)^2\right]^{1/2}}$$

Bundan oshkormas holda berilgan funksiyalar haqidagi teoremani qo'llash bilan yetarlicha kichik $(x-x_0)^2+(y-y_0)^2<\delta^2$ sohada s va v funksiyalar quyidagi tuzilishga ega ekanligini topamiz:

$$s=\left[(x-x_0)^2+(y-y_0)^2\right]^{1/2}\left\{1+\left[(x-x_0)^2+(y-y_0)^2\right]^{1/2}\varphi(\cdot)\right\},$$

$$\cos\theta=\frac{x-x_0}{\left[(x-x_0)^2+(y-y_0)^2\right]^{1/2}}\left\{1+\left[(x-x_0)^2+(y-y_0)^2\right]^{1/2}\varphi(\cdot)\right\}+$$

$$+\left[(x-x_0)^2+(y-y_0)^2\right]^{1/2}\varphi(\cdot),$$

$$\sin\theta=\frac{y-y_0}{\left[(x-x_0)^2+(y-y_0)^2\right]^{1/2}}\left\{1+\left[(x-x_0)^2+(y-y_0)^2\right]^{1/2}\varphi(\cdot)\right\}+$$

$$+\left[(x-x_0)^2+(y-y_0)^2\right]^{1/2}\psi(\cdot),$$

$$\varphi=\varphi(x_0,y_0,x,y),\quad \psi=\psi(x_0,y_0,x,y)$$

Bu yerda φ,ψ –o'z argumentlarining m marta uluksiz differensiallanuvchi funksiyalari. Bundan, xususan (1.1.3) baholar kelib chiqadi.

$(x-x_0)^2+(y-y_0)^2>\delta^2$ uchun $s>\delta$ tengsizlikning demak, $D(f_1,f_2)/D(s,\theta_0)\geq c_0\delta>0$ tengsizlikning bajarilishi ravshan. Lekin bu holda lemma bilan postulatlanuvchi s,v^0 funksiyalar silliqligi $\tilde{f}(\cdot)$ funksiyaning silliqligidan kelib chiqadi, ularning mavjudligi va bir qiymatliligi egri chiziqlar oilasiga qo'yilgan (1.1.1) shartdan kelib chiqadi.

$L(x,y,x_0,y_0)$ $(x,y),(x_0,y_0)$ nuqtalar orqali o'tuvchi va ular orasida yotuvchi oilaning egri chiziqlar kesmasini va $v=(\cos\theta,\sin\theta)$ orqali (x,y)-nuqtada $L(x,y,x_0,y_0)$ egri chiziqqa o'tkazilgan urinma birlik vektorini belgilaymiz. v vektorni $(x,y),(x_0,y_0)$ nuqtalarning v^0 vektor kabi silliqlik xossalariga ega funksiya sifatida qarash mumkin. Haqiqatdan, agar $v^0 = h(x, y, x_0, y_0)$ bo'lsa, u holda $(x, y), (x_0, y_0)$ nuqtalar teng kuchliligidan ravshanki $v = -h(x_0, y_0, x, y)$.

1.1.2-Lemma.

$$\frac{\partial}{\partial s_1}\theta(g(s_1), p(s_1), x, y) \geq 0, \quad (x, y) \in D, \quad s_1 \in [0, l] \tag{1.1.4}$$

tengsizlik o'rinli.

Isbotlash uchun ta'kidlaymizki, tayinlangan $(x, y) \in D$ da qaralayotgan $\theta(x_0, y_0, x, y)$ funksiya o'zining chiziqlari sifatida (x_0, y_0) funksiyasi sifatida (x, y) nuqtadan o'tuvchi (x, y) va Γ chegara orasida joylashgan L oilaning egri chiziqlari kesmalarni oladi. Demak, $grad\ \theta(x_0, y_0, x, y)$, $L(x_0, y_0, x, y)$ egri chiziqqa (x_0, y_0) nuqtadagi normal bo'yicha θ ning o'sishi tomonga yo'nalgan. L chiziqlar tayinlangan (x, y) da bir-biri bilan faqat (x, y) nuqtada kesishadi, u holda $(x, y) \in D$ nuqtani o'z ichiga oladigan ixtiyoriy yopiq egri chiziqda, xususan Γ da, musbat yo'nalishiga θ ning oshishi mos keladi. $x_0 = g(s_1)$, $y_0 = p(s_1)$ deb faraz qilib, s_1 larga $\theta(g(s_1), p(s_1), x, y)$ katta qiymatlar mos kelishini olamiz. Bundan (1.1.4) tengsizlik kelib chiqadi.

Endi $u(x, y) \in C^1(\overline{D})$ va $\rho(x_0, y_0, x, y) \in C^1(\overline{D} \times \overline{D})$ funksiyalar uchun

$$w(x_0, y_0, x, y) = \int\limits_{L(x_0, y_0, x, y)} p(x_0, y_0, x_1, y_1) u(x_1, y_1) ds \tag{1.1.5}$$

formula bilan aniqlangan $w(x_0, y_0, x, y)$ formulani qaraymiz va quyidagi masalani qo'yamiz: $\overline{D}$ da berilgan $\rho(x_0, y_0, x, y)$ va $\upsilon(t_1, t_2)$ funksiyalarda $u(x, y)$ ni toping.

$$\upsilon(t_1, t_2) = w(g(t_1), p(t_1), g(t_2), p(t_2)), \qquad t_1, t_2 \in [0, l] \tag{1.1.6}$$

1.1.3-Teorema. Agar egri chiziqlar oilasi 1-2 xossalarga ega bo'lsa, $\rho(x_0, y_0, x, y) \in C^1(\overline{D} \times \overline{D})$ vazn funksiya

$$\rho(x_0, y_0, x, y) \geq \rho_0 > 0 \tag{1.1.7}$$

$$\left|\frac{\partial}{\partial t_1}\ln \rho(g(t_1), p(t_1), x, y)\right| \le q\frac{\partial}{\partial t_1}\theta(g(t_1), p(t_1), x, y) \qquad (1.1.8)$$

$$0 \le q < 1$$

shartlarni qanoatlantirsa, u holda qo'yilgan masalaning yechimi $u \in C^1(\overline{D})$ uchun yagona va uning uchun

$$\|u\|_{L_2(D)} \le \frac{1}{\rho_0\sqrt{1-q^2}}\frac{1}{2\sqrt{\pi}}\|grad_{t_1,t_2} v(t_1,t_2)\|_{L_2(Q)} \qquad (1.1.9)$$

turg'unlik bahosi o'rinli.

Dastlab (1.1.1) ga kiruvchi $u(\cdot), \rho(\cdot)$ va $\tilde{f}(\cdot)$ funksiyalar silliqligi teorema shartlarida bayon qilinganiga qaraganda bir birlik yuqori bo'lgan hol uchun (1.1.9) bahoning o'rinliligini isbotlaymiz. Bu holda $w(\cdot)$ funksiya uchun quyidagi lemma o'rinli.

1.1.4-Lemma. $\tilde{f}, \rho, u$ funksiyalar o'z argumentlari bo'yicha ikki marta uzluksiz differensiallanuvchi va o'z xususiy hosilalari bilan birgalikda chegaralangan. U holda $w(x_0, y_0, x, y)$ funksiya (x_0, y_0, x, y) bo'yicha atrofida

$$\left|D^\alpha w(\cdot)\right| \le c\left[(x-x_0)^2 + (y-y_0)^2\right]^{\frac{1-|\alpha|}{2}}, \qquad |\alpha| \le 2 \qquad (1.1.10)$$

baho o'rinli bo'lgan $(x_0, y_0) = (x, y)$ nuqtalardan tashqari barcha joyda ikki marta uzluksiz differensiallanuvchi bo'ladi.

Lemmaning o'rinliligi

$$w(x_0, y_0, x, y) = \int_0^{S(x_0, y_0, x, y)} \rho\left[x_0, y_0, f_1(s, \theta_0(x_0, y_0, x, y), x_0, y_0) f_2(\cdot)\right] u(f_1, f_2) ds$$

ifodadan va 1.1.1.lemmadan kelib chiqadi.

Endi $w(x_0, y_0, x, y)$ funksiya uchun differensial tenglama olamiz. Buning uchun

$$w(x_0, y_0, f_1(s, \theta_0, x_0, y_0), f_2(\cdot)) = \int_0^s \rho\left[x_0, y_0, f_1, f_2\right] u(f_1, f_2) ds_1$$

ekanligini ta'kidlaymiz.

Uni s bo'yicha differensiallab va

$$(f_1(s,\theta_0,x_0,y_0),\ f_2(\cdot)) = (x,y)\,,\ \left(f_{1S}(\cdot),\ f_{2S}(\cdot)\right) = v(x_0,y_0,x,y)$$

dan foydalanib

$$(\underset{(x,y)}{grad}\ w(x_0,y_0,x,y), v(x_0,y_0,x,y)) = \rho(x_0,y_0,x,y)u(x,y) \qquad (1.1.11)$$

ni topamiz.

Bu tenglikning ikkala tomonini $\rho(x_0,y_0,x,y)$ ga bo'lib, $x_0 = g(t_1)$, $y_0 = p(t_1)$ deb faraz qilib va t_1 bo'yicha topilgan tenglikni differensiallab

$$w(g(t_1),\ p(t_1),\ x,y) = \tilde{w}(t,x,y)$$

funksiya uchun

$$\frac{\partial}{\partial t_1}\left\{\frac{1}{\tilde{\rho}(t_1,x,y)}\left(\nabla_{xy}\tilde{w},\tilde{v}\right)\right\} = 0 \qquad (1.1.12)$$

ni olamiz, unda

$$\tilde{\rho}(\cdot) = \rho(g(t_1),\ p(t_1),\ x,y)\,,\quad \tilde{v}(\cdot) = v(g(t_1),\ p(t_1),\ x,y)$$

(1.1.12) tenglama aralash tipdagi tenglama, ya'ni giperbola-parabola tipdagi tenglamadan iborat. U $[0,l]\times\overline{D}$ silindrik sohada bajariladi. Bu sohaning chegarasida davriylik shartiga

$$\tilde{w}(0,x,y) = \tilde{w}(l,x,y), \qquad (x,y)\in\overline{D} \qquad (1.1.13)$$

va (1.1.6) formuladan kelib chiquvchi

$$\tilde{w}(t_1,g(t_2),\ p(t_2)) = v(t_1,t_2) \qquad (1.1.14)$$

shartga ega bo'lamiz.

Shunday qilib, $\tilde{w}(\cdot)$ funksiya (1.1.12)-(1.1.14) noklassik masalaning yechimidan iborat. (1.1.5), (1.1.6) formulalardan $v(t_1,t_2) = 0$ $\left(t_1\in[0,l]\right)$ kelib chiqadi. Bu shart bajarilganda (1.1.12)-(1.1.14) masala va yuqorida qo'yilgan integral geometriya masalasi teng kuchli. Shuning uchun $\tilde{w}(t_1,x,y)$ ni topish yetarli, keyin esa (1.1.11) formuladan $u(x,y)$ topiladi. (1.1.12)-(1.1.14) masalaning yagonaligi va turg'unligi masalalarni tadqiq qilish uchun energetik baholar usulini qo'llaymiz. $x \neq g(t_1)$, $y \neq p(t_1)$ va $\beta = (-\sin\tilde{\theta}, \cos\tilde{\theta})$, $\tilde{\theta} = \tilde{\theta}(t_1, x, y)$ lar uchun ikkita ravshan ayniyatlarga ega bo'lamiz.

$$\tilde{\rho}\left(\nabla_{xy}\tilde{w}, \beta\right)\frac{\partial}{\partial t_1}\left[\frac{1}{\tilde{\rho}}\left(\nabla_{xy}\tilde{w}, \tilde{v}\right)\right] = \left(\nabla_{xy}\tilde{w}, \beta\right)\left(\nabla_{xy}\tilde{w}, \tilde{v}_{t_1}\right) +$$

$$+\left(\nabla_{xy}\tilde{w}, \beta\right)\left(\nabla_{xy}\tilde{w}_{t_1}, \tilde{v}\right) - \left(\nabla_{xy}\tilde{w}, \beta\right)\left(\nabla_{xy}\tilde{w}, \tilde{v}\right)\frac{\partial}{\partial t_1}\ln\tilde{\rho},$$

$$\tilde{\rho}\left(\nabla_{xy}\tilde{w}, \beta\right)\frac{\partial}{\partial t_1}\left[\frac{1}{\tilde{\rho}}\left(\nabla_{xy}\tilde{w}, \tilde{v}\right)\right] = \frac{\partial}{\partial t_1}\left[\left(\nabla_{xy}\tilde{w}, \beta\right)\left(\nabla_{xy}\tilde{w}, \tilde{v}\right)\right] -$$

$$-\left(\nabla_{xy}\tilde{w}, \tilde{v}\right)\left(\nabla_{xy}\tilde{w}, \beta_{t_1}\right) - \left(\nabla_{xy}\tilde{w}, \tilde{v}\right)\left(\nabla_{xy}\tilde{w}, \beta\right) - \left(\nabla_{xy}\tilde{w}, \beta\right)\left(\nabla_{xy}\tilde{w}, \tilde{v}\right)\frac{\partial}{\partial t_1}\ln\tilde{\rho}$$

Bu ayniyatlarni qo'shib va

$$\tilde{v}_{t_1} = \beta\frac{\partial\tilde{\theta}}{\partial t_1}, \qquad \beta_{t_1} = -\tilde{v}\frac{\partial\tilde{\theta}}{\partial t_1},$$

$$\left(\nabla_{xy}\tilde{w}, \beta\right)\left(\nabla_{xy}\tilde{w}, \tilde{v}\right) - \left(\nabla_{xy}\tilde{w}, \tilde{v}\right)\left(\nabla_{xy}\tilde{w}_{t_1}, \beta\right) = \frac{\partial}{\partial x}(\tilde{w}_{t_1}, \tilde{w}_y) - \frac{\partial}{\partial y}(\tilde{w}_{t_1}, \tilde{w}_x)$$

ekanligini hisobga olib

$$2\tilde{\rho}\left(\nabla_{xy}\tilde{w}, \beta\right)\frac{\partial}{\partial t_1}\left(\frac{1}{\tilde{\rho}}\left(\nabla_{xy}\tilde{w}, \tilde{v}\right)\right) = \left\{\left[\left(\nabla_{xy}\tilde{w}, \tilde{v}\right)^2 + \left(\nabla_{xy}\tilde{w}, \beta\right)^2\right]\frac{\partial\tilde{\theta}}{\partial t_1} - 2\left(\nabla_{xy}\tilde{w}, \tilde{v}\right)\left(\nabla_{xy}\tilde{w}, \beta\right)\frac{\partial}{\partial t_1}\ln\tilde{\rho}\right\} +$$

$$+\frac{\partial}{\partial t_1}\left[\left(\nabla_{xy}\tilde{w}, \tilde{v}\right)\left(\nabla_{xy}\tilde{w}, \beta\right)\right] + \frac{\partial}{\partial x}\left(\tilde{w}_{t_1}, \tilde{w}\right) - \frac{\partial}{\partial y}\left(\tilde{w}_{t_1}, \tilde{w}_x\right) \qquad (1.1.15)$$

ni topamiz.

(1.1.12) tenglama yechimlarida bu ayniyatning chap qismi nolga aylanadi. (1.1.15) ayniyatni $\tilde{w}(t_1, x, y)$ -(1.1.12) tenglama yechimi deb faraz qilib $G = [0, l] \times \overline{D}$ sohadan

$$\left\{(t_1, x, y): \ t_1 \in [0, l], \ (x, y) \in \overline{D}, \ (x - g(t_1))^2 + (y - p(t_1))^2 \leq \varepsilon^2\right\}$$

to’plamni tashlab yuborish bilan hosil qilingan G_ε soha bo’yicha yetarlicha kichik ε son uchun integrallaymiz. U holda Gauss-Ostragradskiy formulasidan foydalanib

$$\int_{G_\varepsilon}\left\{\left[(\nabla_{xy}\tilde{w},\tilde{v})^2+(\nabla_{xy}\tilde{w},\beta)^2\right]\frac{\partial\tilde{\theta}}{\partial t_1}-2(\nabla_{xy}\tilde{w},\tilde{v})(\nabla_{xy}\tilde{w},\beta)\frac{\partial}{\partial t_1}\ln\tilde{\rho}\right\}dxdydt_1+$$

$$+\int_{S_\varepsilon}\left\{(\nabla_{xy}\tilde{w},\tilde{v})(\nabla_{xy}\tilde{w},\beta)\cos(\widehat{n,t_1})+\tilde{w}_{t_1}\left[\tilde{w}_y\cos(\widehat{n,x})-\tilde{w}_x\cos(\widehat{n,y})\right]\right\}dS=0$$

ni olamiz.Bu yerda S_ε - D_ε to’plam chegarasi, n - S_ε ga tashqi normal, dS - yuza elementi.

Bu tenglikda ε ni nolga intiltirib limitga o’tamiz. Bunda 1.1.3.lemma baholariga ko’ra G_ε va S_ε bo’yicha integrallar xosmas integrallar sifatida mos ravishda G va S bo’yicha integrallarga yaqinlashadi. S sirt bo’yicha integralni ikki qismga ajratish mumkin: G silindrning yuqori va quyi asosi bo’yicha va $[0,l]\times\Gamma$ yon sirti bo’yicha integral. Lekin silindrning quyi va yuqori asoslarida $\cos(\widehat{n,x})=\cos(\widehat{n,y})=0$,$\cos(\widehat{n,t_1})$ mos nuqtalarda qarama-qarshi ishoralarga ega. (1.1.13) davriylik sharti silindrning yuqori va quyi asoslari bo’yicha integrallar yig’indisi nolga aylanishiga olib keladi. Yon sirtda $\cos(\widehat{n,t_1})=0$ va $x=g(t_2)$, $y=p(t_2)$ lar uchun

$$\tilde{w}_{t_1}=v_{t_1}(t_1,t_2)\,,\quad \tilde{w}_y\cos(\widehat{n,x})-\tilde{w}_x\cos(\widehat{n,y})=\frac{\partial}{\partial t_2}\tilde{w}(t_1,g(t_2),p(t_2))=v_{t_2}(t_1,t_2)$$

$$dS=dt_1dt_2.$$

Shunday qilib

$$\int_{G}\left\{\left[(\nabla_{xy}\tilde{w},\tilde{v})^2+(\nabla_{xy}\tilde{w},\beta)^2\right]\frac{\partial\tilde{\theta}}{\partial t_1}-2(\nabla_{xy}\tilde{w},\tilde{v})(\nabla_{xy}\tilde{w},\beta)\frac{\partial}{\partial t_1}\ln\tilde{\rho}\right\}dxdydt_2=$$

$$=-\int_0^l\int_0^l v_{t_1}v_{t_2}\,dt_1dt_2 \qquad (1.1.16)$$

ni olamiz.

1.1.2.Lemmaga asosan $2\frac{\partial\tilde{\theta}}{\partial t_1}\geq 0$. $\frac{\partial\tilde{\theta}}{\partial t_1}=0$ bo’lgan barcha nuqtalarda o’rta qavslardagi ifoda (1.1.8) shart nolga aylanadi. $\frac{\partial\tilde{\theta}}{\partial t_1}>0$ bo’lgan nuqtalar uchun

$$\left[(\nabla_{xy}\tilde{w},\tilde{v})^2+(\nabla_{xy}\tilde{w},\beta)^2\right]\frac{\partial\tilde{\theta}}{\partial t_1}-2(\nabla_{xy}\tilde{w},\tilde{v})(\nabla_{xy}\tilde{w},\beta)\frac{\partial}{\partial t_1}\ln\tilde{\rho}=(\nabla_{xy}\tilde{w},\tilde{v})^2\times$$

$$\times\left[\frac{\partial\tilde{\theta}}{\partial t_1}-\left(\frac{\partial}{\partial t_1}\ln\tilde{\rho}\right)^2\left(\frac{\partial\tilde{\theta}}{\partial t_2}\right)^{-1}\right]+\left[(\nabla_{xy}\tilde{w},\tilde{v})\frac{\partial}{\partial t_2}\ln\tilde{\rho}\left(\frac{\partial\tilde{\theta}}{\partial t_2}\right)^{-1/2}-(\nabla_{xy}\tilde{w},\beta)\left(\frac{\partial\tilde{\theta}}{\partial t_2}\right)^{1/2}\right]\geq$$

$$\geq(\nabla_{xy}\tilde{w},\tilde{v})^2(1-q^2)\frac{\partial\tilde{\theta}}{\partial t_1}\geq\rho_0^2(1-q^2)u^2(x,y)\frac{\partial\tilde{\theta}}{\partial t_1}.$$

ga ega bo’lamiz.

Shuning uchun (1.1.16) tengsizlikni kuchaytirib

$$\rho_0^2(1-q^2)\int_D u^2(x,y)\,dxdy\cdot\int_0^{2\pi}\frac{\partial\tilde{\theta}}{\partial t_1}dt_1\leq\frac{1}{2}\int_0^l\int_0^l|\nabla v|^2\,dt_1dt_2$$

ni olamiz. Bundan (1.1.9) baho kelib chiqadi.

Teoremaning isbotini yakunlash uchun (1.1.9) tenglik o’ng qismidagi ifoda $u\in C^1(\overline{D})$, $\rho(x_0,y_0,x,y)\in C^1(\Gamma\times\overline{D})$ lar va 1-2 shartni qanoatlantiruvchi L egri chiziqlar oilasi uchun ma’noga ega. Shuning uchun teorema shartlarini qanoatlantiruvchi $u,\rho,\tilde{f}$ funksiyalarni (1.1.9) baho o’rinli bo’lgan $u_n,\rho_n,\tilde{f}_n$ funksiyalar bilan yaqinlashtirib unda $n\to\infty$ da limitga o’tib teorema shartlarida (1.1.9) bahoni olamiz. Bu bahodan xususan, integral geometriya masalasining $C^1(\overline{D})$ fazoda yagonaligi kelib chiqadi.

1.1.3- Teorema isbotlandi.

1.2-§. Tekislikda qo’zg’alishli integral geometriya masalasi

$$\int_0^y[u(x+h,\eta)+u(x-h,\eta)]\frac{d\eta}{\sqrt{y-\eta}}+\int_0^y\int_{x-h}^{x+h}K(x,y,\xi,\eta)u(\xi,\eta)\,d\xi\,d\eta=f(x,y)\quad h=\sqrt{y-\eta}\qquad(1.2.1)$$

$u(\xi,\eta)$. Funksiya ikki marta differensiallanuvchi finit va tashuvchisi $-l_o < \xi < l_o$, $0 < \eta < b$ to'rtburchakda К funksiya 2-tartibli uzluksiz hosilalarga ega hamda quydagi shartni qanoatlantiradi. К(x, y, ξ,η) = 0 agar $|\xi + x| < h$. bo'lsa. f(x, y) funksiya 0 < y < b da berilgan deb faraz qilinadi.

(1) tenglama integral geometriyaning qo'zg'alishli masalasiga to'g'ri keladi.

(1) ning chap tamonidagi birinchi qo'shiluvchi

$$\int_0^y [u(x+h,\eta)+u(x-h,\eta)]\frac{d\eta}{\sqrt{y-\eta}} = f_0(x,y)$$

(x, y) nuqtada parabolalar oilasida berilgan funksiyalardan olingan integrallar to'plamidir .

Ikkinchi qo'shiluvchi.

$$f_1(x,y) = f(x,y) - f_0(x,y)$$

— parabolalar bo'yicha chegaralangan yarim tekisliklar bo'yicha olingan К og'irlikli integraldir .

1.2.1-Teorema. (1) tenglamaning yechimi yagona. Faraz qilaylik (1) ning o'ng tomoni aynan nol:

$$f(x,y) \equiv 0$$

Demak, funksiyani qaraymiz

$$v(x,y,t) = \frac{\partial}{\partial}\int_0^y [u[x+gh,\eta]+u(x-gh,\eta)]\frac{d\eta}{\sqrt{y-\eta}},$$

$$g(t) = \sqrt{1-t},\ t\in[0,1].$$

U quydagi integral differensial tenglamani qanoatlantiradi:

$$\frac{\partial}{\partial y} v(x,y,t) = -\frac{1}{4}\int_0^t \frac{\partial^2}{\partial x^2} v(x,y,\tau)\,d\tau + \varphi(x,y),$$

$$\varphi(x,y) = -\frac{1}{4}\frac{\partial^2}{\partial x^2} f_0(x,y) = \frac{1}{4}\frac{\partial^2}{\partial x^2} f_1(x,y), \tag{1.2.2}$$

Faraz qlaylik v_v(x,y,t) = $e^{\sigma(b-y)^2}$ v(x,y,t). (2) dan v_σ tenglamani qanoatlantirishini topamiz.

$$\frac{\partial}{\partial y} v_0(x,y,t) = -\frac{1}{4}\int_0^t \frac{\partial^2}{\partial x^2} v_0(x,y,\tau)\,d\tau - 2\sigma(b-y)v_\sigma(x,y,t) + \varphi_\sigma(x,y),$$

$$\varphi(x,y) = e^{\sigma(b-y)^2}\varphi(x,y). \tag{1.2.3}$$

1.2.2 - Lemma . Shunday M o'zgarmas mavjudki,

$$\int_0^b \int_{-l}^{l} \varphi_\sigma^2(x,y)\,dx,dy \le M \int_0^b \int_{-l}^{l} \int_0^1 v_\sigma^2(x,y,t)\,dx\,dy\,dt, \tag{1.2.4}$$

bunda $l = l_0 + \sqrt{b}$.

v, f aniqlanishidan K ni xossasi quydagicha kelib chiqadi.

$$2\int_0^y u(x,\eta)\frac{d\eta}{\sqrt{y-\eta}} + f_1(x,y) = \int_0^1 v(x,y,t)\,dt,$$

$$u(x,y) + \int_0^y \int_{x-h}^{x+h} K_1(x,y,\xi,\eta)u(\xi,\eta)\,d\xi\,d\eta = \frac{1}{2\pi}\int_0^y \int_0^1 v'_\eta(x,y,t)\,dt\,\frac{d\eta}{\sqrt{y-\eta}} \tag{1.2.5}$$

$$K_1(x,y,\xi,\eta) = \frac{1}{2\pi}\int_{\eta+(x-\xi)^2}^{y} K'_{y1}(x,y,\xi,\eta)\frac{dy_1}{\sqrt{y-\eta}}$$

(5) ning chap qismi ikkinchi tur voltera operatorining u funksiyaning qo'llanishi oqibatidir. Ko'rsatilgan operatordan foydalanib quydagini hosil qilamiz.

$$u(x,y)=\frac{1}{2\pi}\int_0^y\int_0^1\int_0^1 v'_\eta(x,y,t)\,dt\,\frac{d\eta}{\sqrt{y-\eta}}+\int_0^y\int_{x-h}^{x+h}K_2(x,y,\xi,\eta)v'_\eta(\xi,\eta)\,d\xi\,d\eta \tag{1.2.6}$$

bunda K_2 — uzluksiz funksiya.

(6) dan v funksiya bo'yicha diferensiallash operatorini ko'chirish yordamida quyidagi tenglikka ega bo'lamiz.

$$\varphi(x,y)=\int_0^y\int_{x-h}^{x+h}\int_0^1 K_3(x,y,\xi,\eta)v(\xi,\eta,t)\,dt\,d\xi\,d\eta \tag{1.2.7}$$

bunda K_3 — uzluksiz funksiya.

(7) dan

$$\varphi_0(x,y)=\int_0^y\int_{x-h}^{x+h}\int_0^1 K_{3\sigma}(x,y,\xi,\eta)v(\xi,\eta,t)\,dt\,d\xi\,d\eta$$

$$K_{3\sigma}(x,y,\xi,\eta)=e^{\sigma(b-y)^2}K_3(x,y,\xi,\eta) \tag{1.2.8}$$

(4) tengsizlik (8) dan kelib chiqadi .

(2) va (3) tenglamalarga x bo'yicha Furye almashtirishlarni qo'llasak, tenglama quyidagi ko'rinishda bo'ladi.

$$\frac{\partial}{\partial y}w(\lambda,y,t)=\frac{\lambda^2}{4}\int_0^t w(\lambda,y,t)\,d\tau+\psi(\lambda,y), \tag{1.2.9}$$

$$w(\lambda,y,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{i\lambda x}v(x,y,t)\,dx,$$

$$\psi(\lambda,y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{i\lambda x}\varphi(x,y)\,dx,$$

$$\frac{\partial}{\partial y} w_\sigma(\lambda, y, t) = \frac{\lambda^2}{4}\int_0^t w_\sigma(\lambda, y, t) d\tau - 2\sigma(b-y) w_\sigma(\lambda, y, t) + \psi_\sigma(\lambda, y)$$

$$w_1(\lambda, t) = \begin{cases} w(\lambda, b, t) & t \le 1 \\ 0 & t > 1 \end{cases} \qquad (1.2.10)$$

belgilashni olamiz.

(9) tenglama uchun Koshi masalasini qaraymiz.

$$w(\lambda, b, t) = w_1(\lambda, t) \qquad (1.2.11)$$

$0 < y < b$, $0 < t < \infty$ yarim palosasida $w(\lambda, y, t)$ $t > 1$ bo'lganda Koshi masalasining yechimi sifatida qaraymiz.

$t < 1$ bo'lganda (9) tenglama uchun (11) Koshi masalasi yuqorida aniqlangan w funksiya bilan mos tushadi. (9) tenglama uchun (11) masalaning yechimi esa quyidagi formula bilan beriladi.

$$w(\lambda, y, t) = \int_0^t \frac{\cos\lambda\sqrt{(t-\tau)(b-y)}}{\sqrt{(t-\tau)}} w_2(\lambda, \tau) d\tau + \frac{1}{\lambda\sqrt{t}}\int_y^b \sin\lambda\sqrt{t(\eta - y)}\psi_1(\lambda, \eta) d\eta \qquad (1.2.12)$$

$$w_2(\lambda, t) = \frac{1}{\pi}\frac{\partial}{\partial t}\int_0^t w_1(\lambda, \tau)\frac{d\tau}{\sqrt{t-\tau}}$$

$$\psi_1(\lambda, t) = \frac{1}{\pi}\frac{\partial}{\partial y}\int_0^t \psi_1(\lambda, \eta)\frac{d\eta}{\sqrt{\eta - y}}$$

Haqiqatdan ham evalyutsion tenglamalar xossalariga ko'ra (9) tenglama uchun (11) masalaning yechimi yagona (12) tenglama esa bu masalaning yechimi bo'ladi . J orqali volterra integral operatorini belgilaymiz.

$$Jw = \int_0^t w(\lambda, y, t) d\tau$$

va quyidagi funksiyani qaraymiz

$$F(w_0)=\int_0^b\int_0^\infty e^{-\sigma t}\left[\left(\frac{\partial}{\partial y}-\frac{\lambda^2}{4}J+2\sigma(b-y)\right)w_\sigma(\cdot)\right]^2 dtdy =$$

$$=-\int_0^b\int_0^\infty e^{-\sigma t}\left[\left(\frac{\partial}{\partial y}+\frac{\lambda^2}{4}J*+2\sigma(b-y)\right)\times\left(\frac{\partial}{\partial y}-\frac{\lambda^2}{4}J+2\sigma(b-y)\right)w_\sigma(\cdot)w_\sigma(\cdot)\right]dtdy +$$

$$\int_0^\infty e^{-\sigma t}\left[\frac{\partial}{\partial y}w_\sigma^2(\lambda,b,t)+\frac{\lambda^2}{4}Jw_\sigma(\lambda,b,t)+2\sigma bw_\sigma^2(\lambda,b,t)\right]dt,$$

Bunda $J^* - 0<t<00$ aniqlangan funksiyalar gilbert fazosida J operatorning qo'shmasi $(g,h)=\int_0^\infty e^{\sigma t}g(t)h(t)dt$

(10) dan 5.6 bo'limdagi lemmalardan

$$F(w_0)=-\int_0^b\int_0^\infty e^{-\sigma t}\left[\left(\frac{\partial}{\partial y}+\frac{\lambda^2}{4}J*+2\sigma(b-y)\right)\times\left(\frac{\partial}{\partial y}-\frac{\lambda^2}{4}J+2\sigma(b-y)\right)w_\sigma(\cdot)w_\sigma(\cdot)\right]dtdy+$$
$$\int_0^\infty e^{-\sigma t}\left[\frac{\partial}{\partial y}w_\sigma^2(\lambda,b,t)+\frac{\lambda^2}{4}Jw_\sigma(\lambda,b,t)+2\sigma bw_\sigma^2(\lambda,b,t)\right]dt+$$
$$+\int_0^b\int_0^\infty e^{-\sigma t}\left[\left(2\sigma+\frac{\lambda^2}{16}(J*J-JJ*)\right)w_\sigma(\cdot)\right]dtdy=\int_0^b\int_0^\infty e^{-\sigma t}\left[\left(\frac{\partial}{\partial y}-\frac{\lambda^2}{4}J*+2\sigma(b-y)\right)w_\sigma(\cdot)\right]^2dtdy-$$
$$-\int_0^\infty e^{-\sigma t}\left[\frac{\lambda^2}{4}(J+J^*)w_\sigma(\cdot)w_\sigma(\cdot)\right]dt+\int_0^b\int_0^\infty e^{-\sigma t}\left[\left(2\sigma+\frac{\lambda^2}{16}(J*J-JJ*)\right)w_\sigma(\cdot)\right]w_\sigma(\cdot)dtdy=$$
$$\frac{1}{\sigma}\int_0^b\psi_\sigma^2(\lambda,y)dy. \qquad (1.2.13)$$

kelib chiqadi.

1.2.3 - Lemma . Faraz qilaylik g(t) —funksiya $0<t<\infty$ yarim to'g'ri chiziqda uzluksiz va chegaralangan $|g(t)|<g_o$. U holda quyidagi teglik o'rinli.

$$(J*J-JJ*)g=\frac{1}{\sigma}\int_0^\infty e^{-\sigma(t+\sigma)}g(\tau)dr, \qquad (1.2.14)$$

bunda J* quyidagicha aniqlanadi

$$J*g=e^{-\sigma t}\int_0^\infty e^{-\sigma t}g(\tau)dr.$$

J^* ning aniqlanishidan

$$J^* Jg = \int_0^\infty G(t,\tau) g(\tau) dr .$$

$$G(t,\tau) = \begin{cases} \frac{1}{\sigma} e^{-\sigma t}, & \tau \le t \\ \frac{1}{\sigma} e^{-\sigma \tau}, & \tau \ge t \end{cases}$$

$$(J^* J - JJ^*) g = \frac{1}{\sigma} \int_0^\infty e^{-\sigma(t+\sigma)} g(\tau) dr ,$$

(13) va (14) dan quyidagi tengsizlik kelib chiqadi

$$\begin{aligned} &2\sigma \int_0^b \int_0^\infty w^2(\lambda, y, t) dt, dy \le \\ &\le \int_0^\infty e^{-\sigma t} \left[\frac{\lambda^2}{4} (J + J^*) w_\sigma(\cdot) - 4\sigma b w_\sigma(\cdot) \right] w_\sigma(\cdot) dt + \frac{1}{\sigma} \int_0^\infty \psi^2{}_\sigma(\lambda, y) dy \end{aligned} \tag{1.2.15}$$

ω_σ, ψ_σ funksiyalarning obrazlaridan ularning proobrazlariga o’tib,

$$\begin{aligned} &2\sigma \int_0^b \int_0^\infty \int_{-\infty}^\infty e^{-2\sigma t} v_\sigma^2(x, y, t) dx, dy, dt \le \\ &\le \int_0^\infty \int_{-\infty}^\infty e^{-2\sigma t} \left[\frac{1}{4} (J + J^*) \frac{\partial^2}{\partial x^2} v_\sigma(x, b, t) - 4\sigma b v_\sigma(x, b, t) \right] v_\sigma^2(x, b, t) dx, dt + \\ &+ \frac{1}{\sigma} \int_0^\infty \int_{-\infty}^\infty \varphi_\sigma(x, y) dx, dy \end{aligned} \tag{1.2.16}$$

ni hosil qilamiz

(16) ning chap tomonidagi birinchi qo’shiluvchini qaraymiz . u(x,y,t) fazoda y = b tekislik bo’yicha olingan v_σ va $(J - J^*) v_{\sigma xx}^{11}$ funksiyalarni saqlovchi formani integralidir. u funksiyaga qo’yilgan shartlardan va v_σ funksiyaning aniqlanishidan shunday M_0, M_1 o’zgarmaslar mavjud

$$\int_0^\infty \int_{-\infty}^\infty e^{-2\sigma t}\left[\frac{1}{4}(J + J^*)\frac{\partial^2}{\partial x^2} v_\sigma(x,b,t) - 4\sigma b v_\sigma(x,b,t)\right] v_\sigma^2(x,b,t)dx, dt \le (M_0 + M_1\sigma)e^{-2\sigma b^2}. \quad (1.2.17)$$

to’g’ri bo’ladi.

(4), (16), (17) tengsizliklardan

$$(2\sigma - M_2)\int_0^b \int_0^\infty \int_{-\infty}^\infty v_0^2(x,y,t)dx, dy, dt \le (M_0 + M_1\sigma)e^{-2\sigma b^2 q}. \quad (1.2.18)$$

bunda M_2 — biror o’zgarmas . Osongina ko’rish mumkinki, agar u va v_a funksiyalar aynan nolga teng bo’lmasa, u holda shunday q topiladiki $0 < q < 1$ 0 , va

$$\int_0^b \int_0^\infty \int_{-\infty}^\infty v_0^2(x,y,t)dx, dy, dt \ge M_3 e^{-2\sigma b^2 q}. \quad (1.2.19)$$

bo’ladi

(18) va (19) dan quyidagini hosil qilamiz .

$$M_3(2\sigma - M_2)e^{-2\sigma b^2 q} \le (M_0 + M_1\sigma)e^{-2\sigma b^2} \quad (1.2.20)$$

Ko’rinib turibdiki bu o’zgarmaslardan yetarlicha katta qiymatlarida o’rinli emas , bu esa teoremaning o’rinli ekanligi kelib chiqadi.

1.2.1- Teorema isbotlandi.

1.3-§. Siniq chiziqlar oilasi bo’yicha integral geometriya masalasi yechimining yagonalik teoremasi

Keyinchalik foydalanishimiz kerak bo’lgan belgilashlarni kiritamiz.

$$(x,y) \in R^2, \quad (\xi,\eta) \in R^2, \quad \lambda \in R^1$$

$$\Omega = \{(x, y) : x \in R^1, \quad y \in (0, l), \quad l < \infty\}$$

$$\overline{\Omega} = \{(x, y) : x \in R^1, \quad y \in [0, l]\}$$

Masalaning qo'yilishi. $\overline{\Omega}$ polosada ya'ni yo'lakda uchining koordinatalari (x, y) yordamida bir qiymatli parametrlovchi siniq chiziqlar oilasini qaraymiz. Ixtiyoriy siniq chiziqlar oilasi $P(x, y)$ quyidagi munosabatlar bilan aniqlansin.

$$P(x, y) = \{(\xi, \eta) : (x - y) + (\eta - \xi) = 0, \ 0 \le \eta \le y, \ x - y \le \xi \le x\} \cup$$

$$\cup \{(\xi, \eta) : (x + y) - (\xi + \eta) = 0, \ \ 0 \le \eta \le y, \ x \le \xi \le x + y\}$$

Masala. Agar $\overline{\Omega}$ polosadagi barcha (x, y) uchun $u(x, y)$ funksiyaning $P(x, y)$ siniq chiziqlar oilasi bo'yicha

$$\int_{x-y}^{x} g(x - y) u(\xi, y - x) d\xi + \int_{x}^{x+y} g(x - y) u(\xi, x + y - \xi) d\xi = f(x, y) \tag{1.3.1}$$

integrali ma'lum bo'lsa ikki o'zgaruvchili $u(x, y)$ funksiyani topish talab qilinadi. Bu yerda

$$g(x - y) = \begin{cases} 1, & x - \xi > 0 \\ 0, & x - \xi < 0 \end{cases} \tag{1.3.2}$$

Xevisayd funksiyasi. Bu funksiya nol nuqtada uzilishga ega.

$u(x, y)$ funksiya ikkinchi tartibli xususiy hosilasigacha uzluksiz va tashuvchisi bilan R_+^2 finit bo'lgan u funksiyalar sinfidan olingan funksiya.

Tashuvchisi

$$Supp \ \ u \subset D = \{(x, y) : -a < x < a, 0 < a < \infty, 0 < y < l, \quad l < \infty\}$$

Quyidagi funksiyalarni kiritamiz.

$$I(\lambda, p) = \frac{p + i\lambda}{p^2 + \lambda^2} \tag{1.3.3}$$

$$I_1(\lambda, y) = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} e^{py} \frac{\lambda dp}{p^2 I(\lambda, p)} \tag{1.3.4}$$

$$I_2(x-\xi, y-\eta) = \int_{-\infty}^{+\infty} e^{i\lambda x} \frac{I_1(\lambda, y)}{1+\lambda^2} d\lambda \qquad (1.3.5)$$

Quyidagi teorema o'rinli.

1.3.1- Teorema $f(x,y)$ funksiya $\overline{\Omega}$ polosadan olingan ixtiyoriy (x,y) uchun aniqlangan bo'lsin. U holda (1.3.1) masalaning yechimi u sinfda yagonadir quyidagi ko'rinishga ega:

$$u(x,y) = \int_0^y \int_{-\infty}^{+\infty} I_2(x-\xi, y-\eta) \left(\frac{\partial^2}{\partial \eta^2} - \frac{\partial^2}{\partial \xi^2} \right) f(\xi, \eta) d\xi d\eta \qquad (1.3.6)$$

va quyidagi tengsizlik o'rinli.

$$\|u\|_{L_2(\Omega)} \le C_0 \|f\|_{W_2^2(\Omega)}$$

C_0 -o'zgarmas son

Isbot. Teoremani isbotlashdan oldin $f(x,y)$ funksiyani $y \ge l$ da aniqlaymiz. $y \ge l$ uchun $f(x,y) = 0$ bo'ladi. Bu holda $u(x,y)$ funksiya $y \ge l$ da (1.3.6) formula bo'yicha aniqlanadi. (1.3.1) tenglamani quyidagi almashtirish yordamida yanada qulayroq ko'rinishga keltiramiz.

1-chisidan

$$x - y + \eta - \xi = 0, \quad \eta = y - x + \xi$$

$$\xi = \eta - y + x, \quad d\xi = d\eta$$

2-chisidan

$$y + x - \eta - \xi = 0, \quad \eta = y + x - \xi$$

$$\xi = y + x - \eta, \quad d\xi = -d\eta$$

1- tenglamani η bo'yicha integrallashga o'tsak quyidagi integralga ega bo'lamiz.

$$\int_0^y g(x-h)[u(x-h,\eta) - u(x+h,\eta)] d\eta = f(x,y) \qquad (1.3.7)$$

Bu yerda $h = y - \eta$.

2 -ni (1.3.7) ga qo'yib quyidagiga ega bo'lamiz va uning ikkinchi qismi nolga teng.

$$\int_0^y u(x-h,\eta)d\eta = f(x,y) \tag{1.3.8}$$

(1.3.8) tenglamaning ikkala qismiga x o’zgaruvchi bo’yicha Furye almashtirishini qo’llaymiz.

$$\widehat{f}(\lambda,y) = \int_{-\infty}^{+\infty} e^{i\lambda x} \int_0^y u(x-h,\eta)d\eta dx = \int_0^y e^{i\lambda h}\left(\int_{-\infty}^{+\infty} e^{i(x-h)\lambda} u(x-h,\eta)dx\right)d\eta = \int_0^y e^{i\lambda h}\widehat{u}(\lambda,\eta)d\eta$$

$$\widehat{f}(\lambda,y) = \int_0^y e^{i\lambda h}\widehat{u}(\lambda,\eta)d\eta \tag{1.3.9}$$

$u(x,y)$ funksiyaning finitligiga ko’ra $P(x,y)$ siniq chiziqning uzunligi (1.3.1) integralda nol emas. Yuqoridan tekis chegaralangan bu yerda o’rta qiymat haqidagi teoremaga ko’ra $f(x,y)$ funksiya bo’yicha chegaralangan ya’ni o’sishini nolinchi darajada ekanligiga ega bo’lamiz. Shuning uchun (1.3.1) tenglamaning ikkala qismiga ham va shuning (1.3.9) tenglamaga ham bo’yicha Laplas almashtirishini qo’llash mumkin. (1.3.9) tenglamada ikkita tomoniga ham o’zgaruvchi bo’yicha Laplas almashtirishini qo’llaymiz va Fubini teoremasidan foydalanib quyidagiga ega bo’lamiz.

$$\tilde{\widehat{f}}(\lambda,p) = \int_0^\infty e^{-py}\int_0^y e^{i\lambda h}\widehat{u}(\lambda,h)d\eta dy = \int_0^\infty e^{-py}\int_0^y [\cos\lambda h + i\sin\lambda h]\widehat{u}(\lambda,h)d\eta dy =$$

$$= \int_0^\infty e^{-py}\int_0^y \cos\lambda h\widehat{u}(\lambda,h)d\eta dy + i\int_0^\infty e^{-py}\int_0^y \sin\lambda h\widehat{u}(\lambda,h)d\eta dy =$$

$$h = y-\eta, \quad y = h+\eta, \quad dy = d\eta$$

$$= \int_0^\infty e^{-ph}\widehat{u}(\lambda,h)d\eta\int_0^\infty e^{-ph}\cos\lambda h dh + i\int_0^\infty e^{-ph}\widehat{u}(\lambda,h)d\eta\int_0^\infty \sin\lambda h dh =$$

$$= \tilde{\widehat{u}}(\lambda,p)\int_0^\infty e^{-ph}\cos(\lambda,h)dh + i\tilde{\widehat{u}}(\lambda,p)\int_0^\infty e^{-ph}\sin(\lambda,h)dh =$$

$$\int_0^{\infty} e^{-ph}\cos(\lambda, h)dh = \frac{p}{p^2+\lambda^2}; \quad p>0$$

$$\int_0^{\infty} e^{-ph}\sin(\lambda, h)dh = \frac{\lambda}{p^2+\lambda^2}; \quad p>0$$

ni hisobga olib quyidagi tenglamaga ega bo'lamiz.

$$\tilde{\tilde{u}}(\lambda, p)\frac{p+i\lambda}{p^2+\lambda^2} = \tilde{\tilde{f}}(\lambda, p)$$

(1.3.10) $\tilde{\tilde{u}}(\lambda, p)\frac{p+i\lambda}{p^2+\lambda^2} = \tilde{\tilde{f}}(\lambda, p)$ funksiyaning nollar to'plami tekislikda nol

o'lchovga ega bo'ladi. Tenglamani ikkala qismini mos ravishda $\frac{p^2+\lambda^2}{p+i\lambda}$ ga

ko'paytirish mumkin.

$$\frac{p^2+\lambda^2}{p+i\lambda} = \frac{(p^2+\lambda^2)(p-i\lambda)}{(p+i\lambda)(p-i\lambda)} = \frac{(p^2+\lambda^2)(p-i\lambda)}{(p^2+\lambda^2)} = p-i\lambda$$

Bu yerda

$$\tilde{\tilde{u}}(\lambda, p) = \frac{p^2+\lambda^2}{p+i\lambda}\tilde{\tilde{f}}(\lambda, p) = (p-i\lambda)\tilde{\tilde{f}}(\lambda, p) \qquad (1.3.11)$$

$$\left|\frac{1}{I(\lambda, p)}\right| = |p-i\lambda| = \sqrt{p^2+\lambda^2} \le |p|+|\lambda| \qquad (1.3.12)$$

Bu yerda

$$I(\lambda, p) = \frac{p+i\lambda}{p^2+\lambda^2}$$

$$\tilde{\tilde{u}}(\lambda, p) = \frac{1}{I(\lambda, p)}\tilde{\tilde{f}}(\lambda, p) = \frac{p^2}{p^2 I(\lambda, p)}\tilde{\tilde{f}}(\lambda, p) \qquad (1.3.13)$$

(1.3.13) tenglamaga p o'zgaruvchi bo'yicha Laplasning teskari almashtirishini qo'llaymiz.

$$\hat{u}(\lambda,y)=\int_0^\infty e^{py}\frac{p^2}{p^2 I(\lambda,p)}\tilde{\hat{f}}(\lambda,p)dp=\int_0^\infty\frac{\partial^2\hat{f}(\lambda,y)}{\partial\eta^2}I_1(\lambda,y)d\eta \qquad (1.3.14)$$

yoki (1.3.14) ni ikkala tomoni λ ga ko'paytirib yozsak,quyidagi hosil bo'ladi.

$$\lambda\hat{u}(\lambda,y)=\int_0^\infty\frac{\partial^2\hat{f}(\lambda,y)}{\partial\eta^2}I_1(\lambda,y)\lambda d\eta$$

Bu tenglikning o'ng tomonini $(1+\lambda^2)$ ga ko'paytirib bo'lamiz.

$$\lambda\hat{u}(\lambda,y)=\int_0^\infty(1+\lambda^2)\frac{\partial^2\hat{f}(\lambda,y)}{\partial\eta^2}\frac{\lambda I_1(\lambda,y)}{(1+\lambda^2)}d\eta$$

(1.3.14) tenglamaga λ o'zgaruvchi bo'yicha teskari Furye almashtirishini qo'llaymiz.

Svertkalar haqidagi teoremadan va Furye almashtirishining differensiallash xossasidan foydalanib quyidagiga ega bo'lamiz.

$$u(x,y)=\int_{-\infty}^\infty e^{i\lambda x}\int_0^\infty(1+\lambda^2)\frac{\partial^2\hat{f}(\lambda,y)}{\partial\eta^2}\frac{\lambda I_1(\lambda,y)}{(1+\lambda^2)}d\eta dx=$$

$$=\int_0^\infty\int_{-\infty}^\infty I_2(x-\xi,y-\eta)\left(\frac{\partial^2}{\partial\eta^2}-\frac{\partial^2}{\partial\xi^2}\right)f(\xi,\eta)d\xi d\eta \qquad (1.3.15)$$

(1.3.15) formulada y o'zgaruvchi bo'yicha lokal xarakterga ega: $Supp\,u\in\Omega$ shartni hisobga olib (1.3.15),(1.3.1) tenglamani $l<\infty$ dagi yechimi bo'ladi.

U holda (1.3.11),(1.3.12) va (1.3.15) dan berilgan masalaning yechimi $C_0^2(\Omega)$ sinfda yagonaligi kelib chiqadi.

(1.3.11) tenglamadan (1.3.12) ni hisobga olib quyidagi tengsizlikni osongina olish mumkin.

$$\left|\tilde{\hat{u}}(\lambda,p)\right|\le\left(|p|+|\lambda|\right)\left|\tilde{\hat{f}}(\lambda,p)\right|$$

Bundan quyidagi baho kelib chiqadi.

$$\int_{-\infty}^\infty\int_{a-i\infty}^{a+i\infty}\left|\tilde{\hat{u}}(\lambda,p)\right|^2dpd\lambda\le\int_{-\infty}^\infty\int_{a-i\infty}^{a+i\infty}\left(|p|+|\lambda|\right)^2\left|\tilde{\hat{f}}(\lambda,p)\right|^2dpd\lambda \qquad (1.3.16)$$

Laplas va Furye almashtirishlarini differensiallash xossasidan foydalanib norma uchun uchburchak tengsizligidan va shuningdek (1.3.15) va (1.3.16) ni hisobga olib u funksiyaga qo'yilgan shartlardan quyidagi bahoga ega bo'lamiz.

$$\left\| u(x,y) \right\|_{L_2(\Omega)} \leq C_0 \left\| f(x,y) \right\|_{W_2^2(\Omega)}$$

Bu yerda C_0 -o'zgarmas.

Bu baho (1.3.1) masala yechimining yagonaligidan kelib chiqadi.

1.3.1- Teorema isbotlandi.

1.4-§. Siniq chiziqlar oilasi bo'yicha integral geometriya masalasi yechimining mavjudlik teoremasi

(1.3.1) integral geometriya masalasining yechimi mavjudligi haqidagi savolga javobni quyidagi teorema beradi.

1.4.1- Teorema. (1.3.1) tenglamaning o'ng tomoni quyidagi shartlarni qanoatlantirsin

1) $f(x,y)$ o'zgaruvchi x bo'yicha finit

2) $f(x,y)$ ikkinchi tartibgacha barcha uzluksiz xususiy hosilalarga ega

3) $\dfrac{\partial^m}{\partial y^m} f(x,y)|_{y=0} = \dfrac{\partial^m}{\partial y^m} f(x,y)|_{y=l} = 0 \qquad (m = 0,1,2)$

u holda (1.3.1) tenglamaning yechimi (1.3.6) formula bilan aniqlangan x argument bo'yicha finit uzluksiz funksiyalar sinfida mavjuddir.

Isbot. $I_2(x,y)$ funksiyaga va $u(x,y)$ funksiyaga qo'yilgan shartlardan (1.3.6) tenglamaning ikkala qismiga ham x o'zgaruvchi bo'yicha Furye almashtirishini qo'llash mumkin ekanligi kelib chiqadi.

Svertkalar haqidagi teorema va Furye almashtirishining xossalaridan foydalanib quyidagiga ega bo'lamiz.

$$i\lambda\widehat{u}(\lambda,y)=\int_0^y\left\{\int_{-\infty}^{+\infty}e^{i\lambda x}\left[\int_{-\infty}^{+\infty}I_2(x-\xi,y-\eta)\left(\frac{\partial^2}{\partial\eta^2}-\frac{\partial^2}{\partial\xi^2}\right)f(\xi,\eta)d\xi\right]dx\right\}d\eta=$$

$$=\int_0^y\left\{\int_{-\infty}^{+\infty}e^{-\lambda\xi}\left(\frac{\partial^2}{\partial\eta^2}-\frac{\partial^2}{\partial\xi^2}\right)f(\xi,\eta)d\xi\int_{-\infty}^{+\infty}e^{i\lambda(x-\xi)}I_2(x-\xi,y-\eta)dx\right\}d\eta=$$

$$=\int_0^y(1+\lambda^2)\frac{\partial^2}{\partial\eta^2}\widehat{f}(\lambda,\eta)\widehat{I}_2(\lambda,y-\eta)d\eta$$

Bu yerda $\widehat{I}_2(\lambda,y-\eta)$ funksiya $I_2(x-\xi,y-\eta)$ funksiyadan birinchi o'zgaruvchi bo'yicha olingan Furye almashtirishidir.

$I_2(x,y)$ funksiya uchun (1.3.5) munosabatdan quyidagiga ega bo'lamiz.

$$\widehat{u}(\lambda,y)=\int_0^y\frac{\partial^2}{\partial\eta^2}\widehat{f}(\lambda,\eta)I_1(\lambda,y-\eta)d\eta \qquad (1.4.1)$$

(1.3.5) formuladan va $u(x,y)$, $f(x,y)$ funksiyalarga qo'yilgan shartlardan (1.4.1) ning ikkala qismiga y o'zgaruvchi bo'yicha Laplas almashtirishini qo'llash mumkinligi kelib chiqadi.

Ko'paytmalar haqidagi teorema va originalni differensiallash xossasidan foydalanib quyidagiga ega bo'lamiz:

$$\tilde{\widehat{u}}(\lambda,p)=\int_0^{+\infty}e^{-py}\int_0^y\frac{\partial^2}{\partial\eta^2}\widehat{f}(\lambda,\eta)I_1(\lambda,y-\eta)d\eta dy=$$

$$=\int_0^{+\infty}e^{-py}\widehat{f}(\lambda,\eta)d\eta\int_0^{+\infty}e^{-ph}I_1(\lambda,h)dh=p^2\tilde{\widehat{f}}(\lambda,p)\int_0^{+\infty}e^{-ph}I_1(\lambda,h)dh$$

Bu yerda

$$\tilde{\hat{u}}(\lambda, p) = \int_0^{+\infty} e^{-ph} \hat{u}(\lambda, y) dy$$

$$\tilde{\hat{f}}(\lambda, p) = \int_0^{+\infty} e^{-ph} \hat{f}(\lambda, y) dy$$

(1.3.3) formula bilan aniqlangan $I(\lambda, p)$ funksiya barcha

$$\{(\lambda, p) : \lambda \in R^1, \ p \geq 0\}$$

larda o'rinli.

Oxirgi tenglik va (1.3.3) (1.3.4) lardan quyidagiga ega bo'lamiz.

$$\tilde{\hat{u}}(\lambda, p) = \tilde{\hat{f}}(\lambda, p) \frac{1}{I(\lambda, p)} \tag{1.3.2}$$

1.3.1-teoremaning isbotida biz $\frac{1}{p^2 I(\lambda, p)}$ funksiyani $p \geq 0$ yarim tekislikda $p \to \infty$ da nolga intilishini va

$$\int^{+\infty} e^{-ph} \frac{dp}{p^2 I(\lambda, p)}$$

integral absolyut yaqinlashuvchi ekanligini ko'rsatgan edik.

Demak,

$$\frac{1}{p^2 I(\lambda, p)}$$

funksiyada Laplasning teskari almashtirishini qo'llash mumkin. Bundan (1.3.2) tenglama Laplasning teskari almashtirishini qo'llash mumkinligi kelib chiqadi. (1.3.2) tenglamani quyidagi ko'rinishda yozish mumkin.

$$\tilde{\hat{u}}(\lambda,p)\frac{p+i\lambda}{p^2+\lambda^2}=\tilde{\hat{f}}(\lambda,p) \qquad (1.3.3)$$

(1.4.3) tenglamaning ikkala tomoniga ham p o'zgaruvchi bo'yicha Laplasning teskari almashtirishini qo'llaymiz. Svertkalar haqidagi teoremadan foydalanib quyidagiga ega bo'lamiz.

$$\hat{f}(\lambda,y)=\int_0^y e^{i\lambda h}\hat{u}(\lambda,\eta)d\eta \qquad (1.4.4)$$

Bu yerda $h=y-\eta$ ga teng.

Nihoyat (1.4.4) tenglamaning ikkala tomoniga ham λ o'zgaruvchi bo'yicha Furyening teskari almashtirishini qo'llaymiz. Svertkalar haqidagi teoremadan

$$\int_{-\infty}^{+\infty}e^{-i\lambda x}\int_0^y e^{i\lambda h}\hat{u}(\lambda,\eta)d\eta dx=\int_0^y e^{i\lambda h}\left\{\int_{-\infty}^{+\infty}e^{-i\lambda(x-h)}u(x-h,\eta)dx\right\}d\eta=f(x,y)$$

shuningdek (1.4.4) ni quyidagi ko'rinishda yozamiz.

$$\int_{x-y}^{x}g(x-\xi)u(\xi,y+\xi-x)d\xi+\int_x^{x+y}g(x-\xi)u(\xi,x+y-\xi)d\xi=f(x,y)$$

Bu yerdan 1.4.1-teorema tasdig'ining to'g'riligi kelib chiqadi.

1.5-§. Siniq chiziqlar oilasi bo'yicha qo'zg'alishli integral geometriya masalasi

Qo'zg'alishli integral geometriya masalasini o'rganamiz. $S(x,y)$ orqali R_+^2 dagi $P(x,y)$ siniq chiziq va $y=0$ o'q bilan chegaralangan qismini belgilaymiz.

1.5.1- Masala. Agar barcha $x,y\in\overline{\Omega}$ lar uchun $u(x,y)$ funksiyaning $P(x,y)$ siniq chiziq bo'yicha va $K(x,y;\xi,\eta)$ vazn funksiyali $S(x,y)$ yuza bo'yicha

$$\int_0^y u(x-h,\eta)d\eta+\int_0^y\int_{x-h}^{x+h}K(x,y;\xi,\eta)d\xi d\eta=F(x,y) \qquad (1.5.1)$$

integrallari ma'lum bo'lsa $u(x,y)$ ni aniqlang. Bu yerda $h = y - \eta$.

$K(x,y;\xi,\eta)$ funksiya ($P(x,y)$ siniq chiziqda) finit va ikkinchi tartibgacha barcha uzluksiz xususiy hosilalarga ega. O'zining xususiy hosilalari bilan birgalikda $P(x,y)$ siniq chiziqda nolga aylanadi.

$$P(x,y) = \{(\xi,\eta) : (x - y) + (\eta - \xi) = 0,\ 0 \le \eta \le y,\ x - y \le \xi \le x\} \cup$$

$$\cup \{(\xi,\eta) : (x + y) - (\xi + \eta) = 0,\ \ 0 \le \eta \le y,\ x \le \xi \le x + y\}$$

$F(x,y)$ funksiya butun yarim tekislikda berilgan.

(1.5.1) tenglama qo'zg'alishli integral geometriya masalasidir.

(1.5.1) ning chap tomonidagi integral

$$\int_0^y u(x - h,\eta)\,d\eta = f(x,y)$$

bu quyidagiga teng

$$f_0(x,y) = F(x,y) - f(x,y) = \int_0^y \int_{x-h}^{x+h} K(x,y;\xi,\eta)u(\xi,\eta)\,d\xi d\eta$$

$P(x,y)$ siniq chiziqlar oilasi bilan chegaralangan $K(x,y;\xi,\eta)$ vaznli integralni ifodalaydi.

1.5.1- Teorema. $F(x,y)$ Ω polosada aniqlangan bo'lsin. Vazn funksiya

$$K(x,y;\xi,\eta) \in C_0^2(\Omega \times \Omega)$$

o'zining ikkinchi tartibgacha hosilalari bilan birgalikda $P(x,y)$ siniq chiziqda nolga aylanadi, u holda 2-masalaning yechimi Ω polosada ikki marta uzluksiz differensiallanuvchi va finit funksiyalar sinfida yagonadir va quyidagi tengsizlik bajariladi.

$$\|u\|_{L_2(\Omega)} \le C_1 \|F\|_{W_2^2(\Omega)}$$

Bu yerda C_1-qandaydir o'zgarmas son. $u(x,y)$ va $K(x,y;\xi,\eta)$ funksiyalarga qo'yilgan shartlardan $f(x,y)$ va $F(x,y)$ funksiyalar ikkinchi tartibgacha barcha uzluksiz hosilalarga ega bo'ladi.

Isbot. $f_0(x,y) = F(x,y) - f(x,y)$ funksiyani qaraymiz.

$$f_0(x,y) = \int_0^y \int_{x-h}^{x+h} K(x,y;\xi,\eta)u(\xi,\eta)d\xi d\eta \qquad (1.5.2)$$

(1.5.1) tenglamaning chap qismi $K(x,y;\xi,\eta)$ vaznli $P(x,y)$ siniq chiziqlar oilasi va $y=0$ o'q bilan chegaralangan qismining integralini ifodalaydi. $u(x,y)$ va $K(x,y;\xi,\eta)$ funksiyalarga qo'yilgan shartlardan $f_0(x,y)$ funksiya o'zining ikkinchi tartibli xususiy hosilalari bilan birgalikda uzluksiz ekanligi kelib chiqadi. $K(x,y;\xi,\eta)$ funksiyani o'zining ikkinchi tartibli xususiy hosilasi bilan birgalikda $P(x,y)$ siniq chiziqda nolga aylanishini hisobga olib quyidagilarga ega bo'lamiz.

$$\frac{\partial f_0(x,y)}{\partial x} = \int_0^y \int_{x-h}^{x+h} K_x(x,y;\xi,\eta)u(\xi,\eta)d\xi d\eta \qquad (1.5.3)$$

$$\frac{\partial^2 f_0(x,y)}{\partial x^2} = \int_0^y \int_{x-h}^{x+h} K_{xx}(x,y;\xi,\eta)u(\xi,\eta)d\xi d\eta \qquad (1.5.4)$$

$y < y_0$ uchun bu yerda y_0-yetarlicha kichik.

(1.5.2) va (1.5.4) formulalardan $K(x,y;\xi,\eta)$ vazn funksiyaga qo'yilgan chegaranganlikni hisobga olib va funksiyaning mos hosilalari uchun ifodalardan foydalanib quyidagi bahoga ega bo'lamiz.

$$\|f_0(x,y)\|_{W_2^2(\Omega)} \le \varepsilon \|u(x,y)\|_{L_2(\Omega)} \qquad (1.5.5)$$

bu yerda ε -nolga intiladi agar $y_0 \to 0$ intilsa. 1-teoremadan olingan bahodan va norma uchun uchburchak tengsizligidan quyidagi tengsizlik kelib chiqadi.

$$\|u(x,y)\|_{L_2(\Omega)} \le C_0\left(\|F(x,y)\|_{W_2^2(\Omega)} + \|f_0(x,y)\|_{W_2^2(\Omega)}\right) \qquad (1.5.6)$$

(1.5.5) va (1.5.4) lardan foydalanib quyidagi tengsizlikni osongina olish mumkin.

$$\|u(x,y)\|_{L_2(\Omega)} \le C_1\|F(x,y)\|_{W_2^2(\Omega)} \qquad (1.5.7)$$

Bu yerda

$$C_1 = \frac{C_0}{1-\varepsilon C_0} \qquad (1.5.8)$$

(1.5.5), (1.5.7) va (1.5.8) dan (1.5.1) tenglamaning yechimi yagonaligi yetarlicha kichik larda kelib chiqadi.Shuningdek (1.5.1) tenglama Lavrent'ev ta'rifiga ko'ra Volter tenglamasidir, u holda yagonalik va turg'unlik bahosi nafaqat y lar uchun balki butun Ω polosa uchun o'rinlidir.

1.5.1- Teorema isbotlandi.

Birinchi bobga xulosa

Birinchi bob beshta paragrafdan iborat bo'lib, birinchi paragrafda tekislikda integral geometriya masalasining umumiy ko'rinishi keltirilgan. Ikkinchi paragrafda tekislikda qo'zg'alishli integral geometriya masalasi keltirilgan. Qolgan paragrafalarning asosiy natijalari quyidagilar hisoblanadi:

- polosada (yo'lakda) siniq chiziqlar oilasi bo'yicha tekislikda integral geometriya masalasining yangi sinfi o'rganilgan;
- normani aniqlashda chekli sondagi hosila qatnashgan va qo'yilgan masalaning kuchsiz nokorrekt ekanligini anglatuvchi fazolarda turg'unlik bahosi olingan;
- yechimni analitik ko'rinishi olingan;
- yechimning mavjudligi teoremasi isbotlangan;
- polosada (yo'lakda) siniq chiziqlar oilasi bo'yicha qo'zg'alishli integral geometriya masalasining yangi sinfi uchun yagonalik teoremasi isbotlangan va yechimning turg'unlik bahosi olingan.

II-BOB. PARABOLALAR OILASI BO'YICHA INTEGRAL GEOMETRIYA MASALASI

2.1-§. Parabolalar oilasi bo'yicha integral geometriya masalasi yechimining yagonalik teoremasi

Quyidagi belgilashlarni kiritamiz:

$$(x, y) \in R^2 \quad (\xi, \eta) \in R1^2 \quad \lambda \in R^1 \quad \mu \in R^1 \ ;$$

$$\Omega = \left\{(x, y) : x \in R^1 \quad y \in (0, l) \quad l < \infty\right\},$$

$$\overline{\Omega} = \left\{(x, y) : x \in R \quad y \in [0, l]\right\}.$$

Masalaning qo'yilishi: $\overline{\Omega}$ polosada uchining koordinatalari (x, y) yordamida bir qiymatli parametrlovchi egri chiziqlar oilasini qaraymiz . $P(x, y)$ oilaning ixtiyoriy egri chizig'i quyidagi munosabat bilan aniqlansin:

$$P(x, y) = \left\{(\xi, \eta) : (y, \eta) = (x - \xi)^2 \ 0 \le \eta \le y \quad y \le l \quad l < \infty\right\} \ .$$

2.1.1-Masala: Agar barcha $(x, y) \in \overline{\Omega}$ lar uchun $u(x, y)$ funksiyaning $P(x, y)$ egri chiziq bo'yicha integrallari ma'lum bo'lsa,

$$\int_{x-\sqrt{y}}^{x+\sqrt{y}} g(x - \xi) u(\xi, y - (x - \xi)^2) d\xi = f(x, y) \qquad (2.1.1)$$

bu yerda

$$g(x - \xi) = \begin{cases} 1, & x - \xi > 0 \\ 0 & x - \xi < 0 \end{cases}$$

Xevisayd funksiyasi

$u(x,y)$ funksiya V sinfdan olingan funksiya bo'lib, ikkinchi tartibgacha xususiy uzluksiz hosilalarga ega bo'lgan va R_+ da tashuvchisi bilan finit bo'lgan funksiya

$$\sup pu \subset D = \{(x,y): -a < x < a, 0 < y < l \;\; 0 < a < \infty \;\; l < \infty\} \tag{2.1.2}$$

(2.1.1) tenglamaning o'ng tomonini $y < 0$ da aniqlaymiz.

Quyidagi funksiyani kiritamiz:

$$f^*(x,y) = \begin{cases} f(x,y) & y \geq 0 \\ 0 & y < 0 \end{cases} \tag{2.1.3}$$

(2.1.1) masalaning qo'yilishidan va $u(x,y)$ funksiyaga qo'yilgan shartlardan $f^*(x,y)$ funksiyaga y o'zgaruvchi bo'yicha Fur'ye almashtirishini qo'llash mumkin. $f^*(x,y)$ funksiyaga y o'zgaruvchi bo'yicha Fur'ye almashtirishini qaraymiz. $f^*(x,y) = 0 \quad y < 0$ larni hisobga olib quyidagiga ega bo'lamiz.

$$\varphi(\lambda,\mu) = \int_{-\infty}^{+\infty} e^{i\mu y} \hat{f}(\lambda,y)dy$$

bu munosabatning o'ng tomonidagi $\int_0^{+\infty} e^{i\mu y} \hat{f}(x,y)dy$ shuningdek,

$f(x,y)$ funksiyaning pastki yarim tekislikda nolligini aniqlab, (1) tenglamaning ikkala qismiga ham y' o'zgaruvchi bo'yicha Fur'ye almashtirishini qo'llash mumkin.

U holda Furye integrali quyidagi ko'rinishga ega bo'ladi:

$$\int_0^{+\infty} e^{i\mu y} f(x,y)dy$$

Quyidagi funksiyalarni kiritamiz:

$$I(\lambda,\mu)=\int_0^{\infty}\frac{e^{i(\mu\tau-\lambda\sqrt{\tau})}}{\sqrt{\tau}}d\tau \tag{2.1.4}$$

$$I_1(\lambda,\mu)=\int_{-\infty}^{+\infty}e^{-i\mu y}\frac{d\mu}{(1+\mu^2)I(\lambda,\mu)} \tag{2.1.5}$$

$$I_2(\lambda,\mu)=\int_{-\infty}^{+\infty}e^{-i\lambda x}I_1(\lambda,y)d\lambda \tag{2.1.6}$$

Quyidagi teorema o'rinli:

2.1.1-Teorema : $f(x,y)$ funksiya $(x,y)\in\overline{\Omega}$ lar uchun aniqlangan bo'lsin. U holda (2.1.1) masalaning yechimi V sinfda yagonadir.

$$u(x,y)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}I_2(x-\xi,y-\eta)(E-\frac{\partial^2}{\partial\eta^2})f(\xi,\eta)d\xi d\eta \tag{2.1.7}$$

va quyidagi tengsizlik bajariladi:

$$\|U\|_{L_2(\Omega)}\le C_2\|f\|_{W_2^2(\Omega)}$$

Isbot: $f(x,y)$ funksiyani $y\ge l$ da aniqlaymiz. $y\ge l$ uchun $f(x,y)=0$ ni qo'yamiz.Bunda $u(x,y)$ funksiyamiz $y\ge l$ da (2.1.7) formula bo'yicha aniqlanadi.(2.1.1) tenglamani quyidagi almashtirishlar yordamida yanada qulayroq shaklga keltiramiz.

$$\eta=y-(x-\xi)^2 \quad \xi=x\pm\sqrt{y-\eta} \quad d\xi=\frac{d\eta}{\mp 2\sqrt{y-\eta}}$$

(2.1.1) tenglamani $d\eta$ bo'yicha integrallashga keltirsak, quyidagi integralni olamiz:

$$\frac{1}{2}\int_0^y g(x-\xi)\left[u(x+\sqrt{y-\eta})-u(x-\sqrt{y-\eta})\right]\frac{d\eta}{\sqrt{y-\eta}}=f(x,y) \tag{2.1.8}$$

bu yerda $h=\sqrt{y-\eta}$. (2.1.1) ni (2.1.8) ga qo'yib,quyidagiga ega bo'lamiz:

$$\int_0^y u(x+h,\eta)\frac{d\eta}{h} = -2f(x,y) \qquad (2.1.9)$$

Shuningdek, (2.1.1) ni quyidagi ko’rinishda yozib olamiz:

$$\int_0^y u(x-h,\eta)\frac{d\eta}{h} = f_0(x,y), \qquad (2.1.9')$$ bu

yerda $f_0(x,y) = -2f(x,y)$.

(2.1.9’) ning ikkala tomoniga ham x o’zgaruvchi bo’yicha Fur’ye almashtirishini almashtirishini qo'llaymiz.

$$\hat{f}_0(\lambda,y) = \int_{-\infty}^{+\infty} e^{i\lambda x}\int_0^y u(x-h,\eta)\frac{d\eta}{h}dx = \qquad (2.1.10)$$

$$\int_0^y \frac{1}{h}e^{i\lambda h}\int_{-\infty}^{+\infty} e^{i\lambda(x-h)}u(x-h,\eta)dxd\eta = \int_0^y \hat{u}(\lambda,h)\frac{e^{i\lambda h}}{h}d\eta = \hat{f}_0(x,y) = \int_0^y \hat{u}(\lambda,h)\frac{e^{i\lambda h}}{h}d\eta$$

(2.1.10) tenglamaga y o’zgaruvchi bo’yicha bir tomonli Fur’ye almashtirishini qo’llaymiz:

$$\hat{\hat{f}}_0(x,\mu) = \int_0^\infty e^{i\mu y}\int_0^y e^{i\lambda h}\hat{u}(\lambda,\eta)\frac{d\eta}{h}dy = \int_0^\infty \hat{u}(\lambda,\eta)\int_\eta^\infty \frac{e^{i\mu y}-e^{i\mu h}}{h}d\eta dy$$

Bunda $\tau = y-\eta$ almashtirish olsak, u holda

$$\hat{\hat{f}}_0(\lambda,\mu) = \int_0^\infty \hat{u}(\lambda,\eta)e^{i\mu\eta}d\eta\int_0^\infty \frac{e^{i\mu\tau}e^{i\lambda\sqrt{\tau}}}{\sqrt{\tau}}d\tau = \hat{\hat{u}}(\lambda,\eta)\int_0^\infty e^{i(\mu\tau+\lambda\sqrt{\tau})}\frac{d\tau}{\sqrt{\tau}} \qquad (2.1.11)$$

(2.1.11) quyidagi ko’rinishga ega:

$$v(\lambda,\mu)I(\lambda,\mu) = \varphi(\lambda,\mu) \qquad \textit{(2.1.11')}$$

bu yerda

$$v(\lambda,\mu) = \hat{\hat{u}}(\lambda,\mu) = \int_0^\infty e^{i\mu\eta}\hat{u}(\lambda,\eta)d\eta \qquad \textit{(2..112)}$$

$$\varphi(\lambda,\mu)=\int_0^\infty e^{i\tau\mu y}\hat{\hat{f}}_0(\lambda,y)dy=\hat{\hat{f}}(\lambda,y) \qquad (2.1.13)$$

$$I_1(\lambda,\mu)=\int_0^\infty \frac{e^{i(\mu t+\lambda\sqrt{\tau})}}{\sqrt{\tau}}d\tau \; .$$

$v(\lambda,\mu)$ funksiya uchun va undan keyin izlanayotgan $u(x,y)$ funksiya uchun bahoni olishda $I(\lambda,\mu)$ funksiyani modul bo'yicha pastdan baholashimiz kerak.(2.1.4)integral μ parametrning musbatligidan va λ va μ parametrlarga bog'liq bo'lib , tekis yaqinlashuvchi bo'lishini ko'rsatamiz. $t=\sqrt{\tau}$ $\quad dt=d\sqrt{\tau}$ almashtirish olsak, $d\sqrt{\tau}=\frac{1}{2}\frac{d\tau}{\sqrt{\tau}}$ ligini hisobga olib, (2.1.4) dan quyidagiga ega bo'lamiz:

$$I=2\int_0^\infty e^{i(\mu t^2+\lambda t)}dt=2(\int_0^\infty \cos(\mu t^2+\lambda t)+i\sin(\mu t^2+\lambda t))dt=2\int_0^\infty \cos(\mu t^2+\lambda t)+2i\int_0^\infty \sin\left(\mu t^2+\lambda t\right)$$

Quyidagi formulalardan foydalansak,

$$\int_0^\infty \cos(ax^2+2bx)dx=\frac{1}{2}(\cos\frac{b^2}{a}+\sin\frac{b^2}{a})\sqrt{\frac{\pi}{2a}}$$

$$\int_0^\infty \sin(ax^2+2bx)dx=\frac{1}{2}(\cos\frac{b^2}{a}-\sin\frac{b^2}{a})\sqrt{\frac{\pi}{2a}}$$

$$I=\sqrt{\frac{\pi}{2\mu}}\left[(\cos\frac{\lambda^2}{4\mu}+\sin\frac{\lambda^2}{4\mu})+i(\cos\frac{\lambda^2}{4\mu}-i\sin\frac{\lambda^2}{4\mu})\right]$$

ni olamiz. $I(\lambda,\mu)$ funksiyani moduli quyidagi ko'rinishga ega:

$$\left|I\right|=\left|\frac{\pi}{2\left|\mu\right|}\right|\sqrt{(\cos\frac{\lambda^2}{4\mu}+\sin\frac{\lambda^2}{4\mu})^2+(\cos\frac{\lambda^2}{4\mu}-\sin\frac{\lambda^2}{4\mu})^2}=\left|\frac{\pi}{2\left|\mu\right|}\right|\sqrt{2}=\left|\frac{\pi}{\left|\mu\right|}\right| \qquad (2.1.14)$$

U holda quyidagini osongina ko'rsatish mumkin: $\frac{1}{\left|I\right|}<\sqrt{\left|\mu\right|}$

(2.1.11') dan (2.1.14) ni hisobga olib quyidagiga ega bo'lamiz:

$$v(\lambda,\mu)=\varphi(\lambda,\mu)\frac{1}{I(\lambda,\mu)} \tag{2.1.15}$$

(2.1.15) ni o'ng tomonini $1+\mu^2$ ga ko'paytirsak,

$$v(\lambda,\mu)=(1+\mu^2)\varphi(\lambda,\mu)\frac{1}{(1+\mu^2)I(\lambda,\mu)} \tag{2.1.16}$$

$u(x,y)$ funksiyaga qo'yilgan shartlardan va (2.1.7) dan μ argument bo'yicha L_2 ga tegishli bo'lgan $\varphi(\lambda,\mu)$ funksiyani olamiz.

(2.1.14) va (2.1.15) dan kelib chiqadiki, $(1+\mu^2)^{-1}(I(\lambda,\mu))^{-1}$ formula (2.1.5) bo'yicha aniqlangan y o'zgaruvchi bo'yicha $I_1(\lambda,y)$ funksiyaning Fur'ye obrazi ekanligi kelib chiqadi. (2.1.16) tenglamaga μ o'zgaruvchi bo'yicha Fur'ye teskari almashtirishini qo'llaymiz.Teskarilanish teoremasi va svyortkalar haqidagi teoremadan foydalanib va shuningdek Fur'ye almashtirishining differensiallash xossasidan

$$\hat{u}(\lambda,y)=\int_{-\infty}^{+\infty} I_1(\lambda,y-\eta)\left(E-\frac{\partial^2}{\partial\eta^2}\right)\hat{f}(\lambda,\eta)\,d\xi\,d\eta \tag{2.1.17}$$

bu yerda $$I_1(\lambda,\mu)=\int_{-\infty}^{+\infty} e^{-i\mu y}\frac{d\mu}{(1+\mu^2)I(\lambda,\mu)}.$$

u funksiyaga qo'yilgan chegaralardan (cheklashlardan) λ argument bo'yicha L_2 ga tegishli bo'lgan $\hat{f}(\lambda,y)$ funksiya kelib chiqadi. (2.1.7) ga λ argument bo'yicha Fur'yening teskari almashtirishini qo'llaymiz. Teskarilanish teoremasi va svyortkalar teoremasidan foydalanib va shuningdek, Fur'ye almashtirishining xossalaridan (2.1.1) ning yechimi uchun quyidagi ifodani olamiz:

$$u(x,y)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} I_2(x-\xi,y-\eta)\left(E-\frac{\partial^2}{\partial\eta^2}\right)f(\xi,\eta)\,d\xi\,d\eta \tag{2.1.18}$$

bu yerda $$I_2(\lambda,\mu)=\int\limits_{-\infty}^{\infty} e^{-i\lambda x} I_1(\lambda,y)\,d\lambda$$

(2.1.18) formula y o'zgaruvchi bo'yicha lokal xarakterga ega. $\sup pu\subset\Omega$ shartdan (2.1.18) ifoda $h<\infty$ da ham (2.1.1) ning yechimi uchun o'rinli bo'ladi. U holda (2.1.11') (2.1.14) (2.1.18) dan $C_0^2(\Omega)$ sinfda izlanayotgan (2.1.1) masalaning yechimi yagonaligi kelib chiqadi. (2.1.11') tenglamadan

$$\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}|v|^2\,d\lambda\,d\mu\le\frac{1}{\sqrt{\pi}}\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}(1+|\mu|)^2\,|\varphi|^2\,d\lambda\,d\mu. \tag{2.1.19}$$

Fur'ye almashtirishining differensiallash xossasidan foydalanib norma uchun uchburchak tengsizligidan shuningdek (2.1.18) va (2.1.19) dan

$$\|U\|_{L_2(\Omega)}\le C_2\,\|f\|_{W_2^2(\Omega)}$$

C_2-qandaydir o'zgarmas son.

2.1.1−Teorema isbotlandi.

2.2-§. Parabolalar oilasi bo'yicha integral geometriya masalasi yechimining mavjudlik teoremasi

Integral geometriya masalasining yechimi mavjudligi haqidagi javobni quyidagi teorema beradi.

2.2.1 Teorema : (2.1.1) tenglamaning o'ng tomoni quyidagi shartlarni qanoatlantirsin:

1) $f(x,y)$ x o'zgaruvchi bo'yicha finit
2) $f(x,y)$ ikkinchi tartibgacha barcha uzluksiz xususiy hosilalarga ega.

3) $\frac{\partial^m}{\partial y^m} f(x,y)\Big|_{x=0} = \frac{\partial^m}{\partial y^m} f(x,y)\Big|_{x=l} = 0 \quad (m = 0,1,2,)$

U holda (2.1.7) formula bilan aniqlangan x argument bo'yicha finit uzluksiz funksiyalar sinfida (2.1.1) tenglamaning yechimi mavjuddir.

Isbot: $I_2(x,y)$ va $f(x,y)$ funksiyalarga qo'yilgan shartlardan (2.1.7) ning ikkala qismiga ham x o'zgaruvchi bo'yicha Fur'ye almashtirishini qo'llash mumkin.Svyortkalar haqidagi teoremani va Fur'ye almashtirishining xossalaridan foydalanib,quyidagiga ega bo'lamiz:

$$-i\lambda\hat{u}(\lambda,y) = \int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty} e^{i\lambda x}\int_{-\infty}^{+\infty} I_2(x-\xi,y-\eta)(E-\frac{\partial^2}{\partial\eta^2})f(\xi,\eta)d\xi dx\right]d\eta =$$

$$\int_{-\infty}^{+\infty}\left\{\int_{-\infty}^{+\infty} e^{i\lambda\xi}(E-\frac{\partial^2}{\partial\eta^2})f(\xi,\eta)d\xi\int_{-\infty}^{+\infty} e^{i\lambda(x-\xi)}I(x-\xi,y-\eta)dx\right\}d\eta =$$

$$\int_{-\infty}^{+\infty}(1+\lambda^2)(E-\frac{\partial^2}{\partial\eta^2})\hat{f}(\lambda,\eta)\hat{I}_2(\lambda,y-\eta)d\eta$$

Bu yerda $\hat{I}_2(\lambda,y-\eta)$ $I_2(x-\xi,y-\eta)$ funksiyaning birinchi o'zgaruvchi bo'yicha Fur'ye almashtiruvchi (2.1.6) munosatiga ko'ra $I_2(x,y)$ funksiya uchun quyidagiga ega bo'lamiz:

$$\hat{u}(\lambda,y) = \int_{-\infty}^{+\infty}(E-\frac{\partial^2}{\partial\eta^2})\hat{f}(\lambda,y)I_1(\lambda,y-\eta)d\eta \quad . \qquad (2.2.1)$$

(2.1.7) dan va $u(x,y)$ va $f(x,y)$ funksiyalarga qo'yilgan shartlardan (2.2.1) ning ikkala qismiga y o'zgaruvchi bo'yicha Fur'ye almashtirishini qo'llash mumkin. Svyortkalar haqidagi teorema va Fur'ye almashtirishi xossalaridan foydalanib, quyidagiga ega bo'lamiz:

$$\hat{\hat{u}}(\lambda,\mu)=\int_0^{+\infty}e^{i\mu y}\int_{-\infty}^{+\infty}(E-\frac{\partial^2}{\partial\eta^2})\hat{f}(\lambda,y)I_1(\lambda,y-\eta)d\eta\,dy=$$
$$\int_0^{+\infty}e^{i\mu\eta}(E-\frac{\partial^2}{\partial\eta^2})\hat{f}(\lambda,\eta)d\eta\int_0^{+\infty}e^{i\mu\tau}I_1(\lambda,\tau)d\tau=$$
$$\mu^2\hat{\hat{f}}(\lambda,\mu)\int_0^{+\infty}e^{i\mu\tau}I_1(\lambda,\tau)d\tau$$

Bu yerda $$\hat{\hat{u}}(\lambda,\mu)=\int_0^{+\infty}e^{i\mu y}\hat{u}(\lambda,y)dy$$

$$\hat{\hat{f}}(\lambda,\mu)=\int_0^{+\infty}e^{i\mu y}\hat{f}(\lambda,y)dy$$

(2.1.4) formula bilan aniqlangan $I(\lambda,\mu)$ funksiya barcha $\{(\lambda,\mu):\lambda\in R^1,\mu\geq 0\}$ larda noldan farqli. Oxirgi tenglikdan va (2.1.4) (2.1.5) formulalardan quyidagi kelib chiqadi:

$$\hat{\hat{u}}(\lambda,\mu)=\hat{\hat{f}}(\lambda,\mu)\frac{1}{I(\lambda,\mu)} \tag{2.2.2}$$

(2.1.1) teoremaning isbotida $\frac{1}{(1+\mu^2)I(\lambda,\mu)}$ $|\mu|\to\infty$ da nolga intilishini ko'rsatgan edik va

$$\int_{-\infty}^{+\infty}\frac{e^{-i\mu y}d\mu}{(1+\mu^2)I(\lambda,\mu)}$$ integral absolyut yaqinlashuvchi.

Demak, $\frac{1}{(1+\mu^2)I(\lambda,\mu)}$ funksiyaga Fur'yening teskari almashtirishini qo'llash mumkin. Bu yerdan (2.2.2) tenglikning ikkala tomoniga ham Fur'yening teskari almashtirishini qo'llash mumkinligi kelib chiqadi. (2.2.2) tenglamani quyidagi ko'rinishda yozish mumkin:

$$\hat{u}(\lambda,\mu)\int_0^{+\infty}\frac{e^{i(\mu\tau+\lambda\sqrt{\tau})}}{\sqrt{\tau}}d\tau = \hat{\hat{f}}(\lambda,\mu) \tag{2.2.3}$$

(2.2.3) ning ikkala qismiga ham μ o'zgaruvchi bo'yicha Fur'yening teskari almashtirishini qo'llaymiz. Svyortkalar haqidagi teoremani qo'llab, quyidagiga ega bo'lamiz:

$$\int_0^y e^{i\lambda h}\hat{u}(\lambda,h)\frac{d\eta}{h} = \widehat{f}(\lambda,\mu) \tag{2.2.4}$$

bu yerda $h=\sqrt{y-\eta}$.

(2.2.4) tenglamaning ikkala qismiga λ o'zgaruvchi bo'yicha Fur'yening teskari almashtirishini qo'llaymiz. Svyortkalar haqidagi teoremadan quyidagiga ega bo'lamiz:

$$\int_{-\infty}^{+\infty} e^{-i\lambda x}\int_0^y e^{i\lambda h}\hat{u}(\lambda,\eta)\frac{d\eta}{h} = \int_0^y \frac{1}{h}e^{i\lambda h}\int_{-\infty}^{+\infty} e^{i\lambda(x-h)}\hat{u}(x-h,\eta)dx d\xi = f(x,y)$$

bu yerda $h=\sqrt{y-\eta}$. Shuningdek, (2.2.4) ni

$$\int_{x-\sqrt{y}}^{x+\sqrt{y}} g(x-\xi)u(\xi,y-(x-\xi)^2)d\xi = f(x,y)$$

ko'rinishda yozamiz. Bundan 2.2.1 teorema tasdig'i to'g'ri ekanligi kelib chiqadi. **2.2.1 Teorema isbotlandi .**

2.3-§. Parabolalar oilasi bo'yicha qo'zg'alishli integral geometriya masalasi

Qo'zg'alishli integral geometriya masalasini o'rganamiz. $S(x,y)$ orqali R_+^2 dagi $P(x,y)$ to'g'ri chiziq va $y=0$ o'q bilan chegaralangan qismini belgilaymiz. $\overline{\Omega}$ polosada

$$\overline{\Omega} = \left\{(x, y) \in R^1, y \in [0, l]\right\}$$

2.3.1 -Masala . Agar barcha $x, y \in \overline{\Omega}$ lar uchun $u(x, y)$ funksiyaning $P(x, y)$ to'g'ri chiziq bo'yicha va $K(x, y; \xi, \eta)$ vazn funksiyali $S(x, y)$ yuza bo'yicha

$$\int_0^y u(x - h, \eta)\frac{d\eta}{h} + \int_0^y \int_{x-h}^{x+h} K(x, y; \xi, \eta)u(\xi, \eta)d\xi d\eta = F(x, y) \qquad (2.3.1)$$

integrallari ma'lum bo'lsa $u(x, y)$ ni aniqlang. Bu yerda $h = \sqrt{y - \eta}$.

$K(x, y; \xi, \eta)$ funksiya finit va ikkinchi tartibgacha barcha uzluksiz xususiy hosilalarga ega. O'zining xususiy hosilalari bilan birgalikda $P(x, y)$ parabolada nolga aylanadi.

$$P(x, y) = \left\{(\xi, \eta) : y - \eta = (x - \xi)^2, \quad \eta > 0, \quad y > 0.\right\}$$

$F(x, y)$ funksiya butun yarim tekislikda berilgan. (2.3.1) tenglama qo'zg'alishli integral geometriya masalasidir.

(2.3.1) ning chap tomonidagi integral

$$\int_0^y u(x - h, \eta)\frac{d\eta}{h} = f(x, y)$$

bu yerda $h = \sqrt{y - \eta}$.

Uchi (x, y) nuqtada bo'lgan yarim to'g'ri chiziqlar oilasi bo'yicha izlanayotgan funksiyaning integralini ifodalaydi. 2-integral esa quyidagiga teng:

$$f_0(x, y) = F(x, y) - f(x, y) = \int_0^y \int_{x-h}^{x+h} K(x, y; \xi, \eta)u(\xi, \eta)d\xi d\eta$$

$P(x, y)$ parabolalar oilasi bilan chegaralangan $K(x, y; \xi, \eta)$ vaznli integralni ifodalaydi.

2.3.1-Teorema. $F(x,y)$ Ω polosada aniqlangan bo'lsin. Vazn funksiya

$$K(x,y;\xi,\eta)\in C_0^2(\Omega\times\Omega)$$

o'zining ikkinchi tartibgacha hosilalari bilan birgalikda $P(x,y)$ parabolada nolga aylanadi, u holda 3.2.1 masalaning yechimi Ω polosada ikki marta uzluksiz differensiallanuvchi va finit funksiyalar sinfida yagonadir hamda quyidagi tengsizlik bajariladi.

$$\|u\|_{L_2(\Omega)}\leq C_3\|F\|_{W_2^2(\Omega)}$$

Bu yerda C_3-qandaydir o'zgarmas son.

$u(x,y)$ va $K(x,y;\xi,\eta)$ funksiyalarga qo'yilgan shartlardan $f(x,y)$ va $F(x,y)$ funksiyalar ikkinchi tartibgacha barcha uzluksiz hosilalarga ega bo'ladi.

Isbot. $f_0(x,y)=F(x,y)-f(x,y)$ funksiyani qaraymiz.

Bu
$$f_0(x,y)=\int_0^y\int_{x-h}^{x+h}K(x,y;\xi,\eta)u(\xi,\eta)d\xi d\eta \qquad (2.3.2)$$

ga teng. Bu yerda $h=\sqrt{y-\eta}$.

(2.3.1) tenglamaning chap qismi $K(x,y;\xi,\eta)$ vaznli $P(x,y)$ parabolalar oilasi va $y=0$ o'q bilan chegaralangan qismining integralini ifodalaydi. $u(x,y)$ va $K(x,y;\xi,\eta)$ funksiyalarga qo'yilgan shartlardan $f_0(x,y)$ funksiya o'zining ikkinchi tartibli xususiy hosilalari bilan birgalikda uzluksiz ekanligi kelib chiqadi.

$K(x,y;\xi,\eta)$ funksiyani o'zining ikkinchi tartibli xususiy hosilasi bilan birgalikda $P(x,y)$ parabolada nolga aylanishini hisobga olib quyidagilarga ega bo'lamiz.

$$\frac{\partial f_0(x,y)}{\partial x}=\int_0^y\int_{x-h}^{x+h}K_x(x,y;\xi,\eta)u(\xi,\eta)d\xi d\eta \qquad (2.3.3)$$

$$\frac{\partial^2 f_0(x,y)}{\partial x^2} = \int_0^y \int_{x-h}^{x+h} K_{xx}(x,y;\xi,\eta)u(\xi,\eta)d\xi d\eta \qquad (2.3.4)$$

$y < y_0$ uchun bu yerda y_0 -yetarlicha kichik.

(23.3) va (2.3.4) formulalardan $K(x,y;\xi,\eta)$ vazn funksiyaga qo'yilgan chegaranganlikni hisobga olib va funksiyaning mos hosilalari uchun ifodalardan foydalanib quyidagi bahoga ega bo'lamiz.

$$\|f_0(x,y)\|_{W_2^2(\Omega)} \leq \varepsilon \|u(x,y)\|_{L_2(\Omega)} \qquad (2.3.5)$$

bu yerda ε -nolga intiladi agar $y_0 \to 0$ intilsa.

2.3.1 teoremadan olingan bahodan va norma uchun uchburchak tengsizligidan quyidagi tengsizlik kelib chiqadi.

$$\|u(x,y)\|_{L_2(\Omega)} \leq C_2 \left(\|F\|_{W_2^2(\Omega)} + \|f_0\|_{W_2^2(\Omega)} \right). \qquad (2.3.6)$$

(2.3.5) va (2.3.6)lardan quyidagi tengsizlikni osongina olish mumkin.

$$\|u(x,y)\|_{L_2(\Omega)} \leq C_3 \|F(x,y)\|_{W_2^2(\Omega)}, \qquad (2.3.7)$$

bu yerda

$$C_3 = \frac{C_2}{1 - C_2\varepsilon} \qquad (2.3.8)$$

(2.3.6), (2.3.7) va (2.3.8) dan (2.3.1) tenglamaning yechimi yagonaligi yetarlicha kichik y larda kelib chiqadi.Shuningdek (2.3.1) tenglama Lavrent'ev ta'rifiga ko'ra Volterra tenglamasidir, u holda yagonalik va turg'unlik bahosi nafaqat y lar uchun balki butun Ω polosa uchun o'rinlidir.

2.3.1-Teorema isbotlandi.

Ikkinchi bobga xulosa

Ikkinchi bobda maxsuslikka ega bo’lgan vazn funksiyali parabolalar olisi bo’yicha polosada (yo’lakda) integral geometriya masalasining yangi sinfi o’rganilgan.

Asosiy natijalari quyidagilar hisoblanadi:

- maxsuslikka ega bo’lgan vazn funksiyali parabolalar olisi bo’yicha polosada (yo’lakda) integral geometriya masalasining yangi sinfi o’rganilgan;
- chekli silliq fazolarda turg’unlik bahosi olingan va shu bilan masalaning kuchsiz nokorrektligi ko’rsatilgan;
- yechimni analitik ko’rinishi olingan;
- yechimning mavjudligi teoremasi isbotlangan;
- polosada (yo’lakda) parabolalar oilasi bo’yicha qo’zg’alishli integral geometriya masalasining yangi sinfi uchun yagonalik teoremasi isbotlangan va yechimning turg’unlik bahosi olingan.

III - BOB. TEKISLIKDA FUNKSIYANI TIKLASH MASALASI

3.1-§. Siniq chiziq va parabolalar oilasi bo'yicha integral geometriya masalasi

Keyinchalik foydalanishimiz kerak bo'lgan belgilashlarni kiritamiz.

$$(x,y)\in R^2,\quad (\xi,\eta)\in R^2,\quad \lambda\in R^1$$

$$\Omega=\{(x,y): x\in R^1,\quad y\in(0,l),\quad l<\infty\}$$

$$\overline{\Omega}=\{(x,y): x\in R^1,\quad y\in[0,l]\}$$

Masalaning qo'yilishi. $\overline{\Omega}$ polosada ya'ni yo'lakda uchi (x,y) yordamida bir qiymatli parametrlovchi siniq chiziqlar oilasini qaraymiz. Ixtiyoriy siniq chiziqlar oilasi $P(x,y)$ quyidagi munosabatlar bilan aniqlansin.

$$P(x,y)=\{(\xi,\eta):(x-y)+(\eta-\xi)=0,\ 0\le\eta\le y,\ x-y\le\xi\le x\}\cup$$

$$\cup\{(\xi,\eta):(y-\eta)=(x-\xi)^2,\quad 0\le\eta\le y,\ x\le\xi\le x+\sqrt{y}\}$$

3.1.1-Masala. Agar $\overline{\Omega}$ polosadagi barcha (x,y) uchun $u(x,y)$ funksiyaning $P(x,y)$ siniq chiziqlar oilasi bo'yicha

$$\int_{x-y}^{x} u(\xi,\xi-x+y)\,d\xi+\int_{x}^{x+\sqrt{y}} u(\xi,y-(x-\xi)^2)\,d\xi=f_1(x,y) \qquad (3.1.1)$$

integrali ma'lum bo'lsa ikki o'zgaruvchili $u(x,y)$ funksiyani topish talab qilinadi.

$u(x,y)$ funksiya ikkinchi tartibli xususiy hosilasigacha uzluksiz va tashuvchisi bilan R_+^2 finit bo'lgan u funksiyalar sinfidan olingan funksiya.

Tashuvchisi

$$Supp\ \ u\subset D=\{(x,y): -a<x<a,\ 0<a<\infty,\ 0<y<l,\quad l<\infty\} \qquad (3.1.2)$$

Quyidagi funksiyalarni kiritamiz.

$$I=\frac{p+i\lambda}{p^2+\lambda^2}+\frac{1}{2}\sqrt{\frac{\pi}{p}}e^{\frac{-\lambda^2}{4p}}+\frac{\lambda}{2p}\sum_{k=1}^{\infty}\frac{1}{(2k-1)!!}\left(\frac{-\lambda^2}{2p}\right)^{k-1} \qquad (3.1.3)$$

$$I_1(\lambda, y-\eta) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{py}\frac{dp}{p^4 I(\lambda, p)}$$

$$I_2 = \int_{-\infty}^{+\infty} e^{-i\lambda x}\frac{I_1(\lambda, p)\,d\lambda}{1+p^4}$$

Quyidagi teorema o'rinli.

3.1.1-Teorema. $f(x,y)$ funksiya $\overline{\Omega}$ polosadan olingan ixtiyoriy (x,y) uchun aniqlangan bo'lsin. U holda (1) masalaning yechimi u sinfda yagonadir quyidagi ko'rinishga ega:

$$u(x,y) = \int_0^{+\infty}\int_{-\infty}^{+\infty} I_2(x-\xi, y-\eta)\left(\frac{\partial^4}{\partial\eta^4} - \frac{\partial^8}{\partial\eta^4\partial\xi^4}\right) f_1(\xi,\eta)\,d\xi\,d\eta \qquad (3.1.4)$$

va quyidagi tengsizlik o'rinli.

$$\|u\|_{L_2(\Omega)} \le C_0 \|f_1\|_{W_2^2(\Omega)}$$

C_0 -o'zgarmas son

Isbot. Teoremani isbotlashdan oldin $f_1(x,y)$ funksiyani $y \ge l$ da aniqlaymiz. $y \ge l$ uchun $f(x,y) = 0$ bo'ladi. Bu holda $u(x,y)$ funksiya $y \ge l$ da (3.1.6) formula bo'yicha aniqlanadi. (3.1.1) tenglamani quyidagi almashtirish yordamida yanada qulayroq ko'rinishga keltiramiz.

1-dan

$$x - y + \eta - \xi = 0, \qquad \eta = y - x + \xi$$

$$\xi = \eta - y + x, \qquad d\xi = d\eta$$

2-dan

$$y - \eta = (x-\xi)^2, \qquad \eta = y - (x-\xi)^2$$

$$\xi = x \mp \sqrt{y-\eta}, \qquad d\xi = \frac{1}{\pm 2\sqrt{y-\eta}}\,d\eta$$

(3.1.1)tenglamani η bo'yicha integrallashga o'tsak quyidagi integralga ega bo'lamiz.

$$\int_0^y [u(x-h^2, y-h^2) + u(x+h, y-h^2)\frac{1}{2h}](-2hdh) = f_1(x,y) \qquad (3.1.5)$$

Bu yerda $h = \sqrt{y-\eta}$.

$$2\int_0^y hu(x-h^2, y-h^2)dh + \int_0^y u(x+h, y-h^2)dh = f_1(x,y)$$

$$f_1(x,y) = -f(x,y)$$

(3.1.6)

(3.1.5) tenglamaning ikkala qismiga x o'zgaruvchi bo'yicha Fur'ye almashtirishini qo'llaymiz.

$$\widehat{f_1}(\lambda, y) = \int_{-\infty}^{+\infty} e^{i\lambda x}\left(2\int_0^y hu\left(x-h^2, y-h^2\right)dh + \int_0^y u\left(x+h, y-h^2\right)dh\right)dx =$$

$$= 2\int_{-\infty}^{+\infty}\int_0^y e^{i\lambda x}u(x-h^2, y-h^2)hdhdx + \int_{-\infty}^{+\infty}\int_0^y e^{i\lambda x}u(x+h, y-h^2)dhdx =$$

$$= 2\int_0^y e^{i\lambda h^2}\left(\int_{-\infty}^{+\infty} e^{i\lambda\left(x-h^2\right)}u\left(x-h^2, y-h^2\right)\right)hdxdh +$$

$$+\int_0^y e^{-i\lambda h}\left(\int_{-\infty}^{+\infty} e^{i\lambda(x+h)}u\left(x+h, y-h^2\right)\right)dxdh$$

$$\widehat{f_1}\left(\lambda, y\right) = 2\int_0^y he^{i\lambda h^2}\widehat{u}\left(\lambda, y-h^2\right)dh + \int_0^y e^{-i\lambda h}\widehat{u}\left(\lambda, y-h^2\right)dh =$$

$$\int_0^y \widehat{u}\left(\lambda, y-h^2\right)\left(2he^{i\lambda h^2} + e^{-i\lambda h}\right)dh =$$

$$= \int_0^y \hat{u}(\lambda, y-h^2)(2h\cos(\lambda h^2) + 2ih\sin(\lambda h^2) + \cos(\lambda h) - i\sin(\lambda h))dh =$$

$$= 2\int_o^y h\hat{u}\left(\lambda, y-h^2\right)\cos(\lambda h^2)dh + 2i\int_0^y h\hat{u}\left(\lambda, y-h^2\right)\sin\left(\lambda h^2\right)dh +$$

$$+ \int_0^y \hat{u}\left(\lambda, y-h^2\right)\cos\left(\lambda h\right)dh - i\int_0^y \hat{u}\left(\lambda, y-h^2\right)\sin\left(\lambda h\right)dh$$

Endi y o'zgaruvchi bo'yicha Laplas almashtirishini qo'llaymiz:

$$\tilde{\tilde{f}}_1(\lambda, p) = 2\int_0^\infty e^{-p\left(y-h^2\right)}\hat{u}\left(\lambda, y-h^2\right)dy\int_0^\infty e^{-ph^2} h\cos\left(\lambda h^2\right)dh +$$

$$+2i\int_0^\infty e^{-p\left(y-h^2\right)}\hat{u}\left(\lambda, y-h^2\right)dy\int_0^\infty e^{-ph^2} h\sin\left(\lambda h^2\right)dh +$$

$$+\int_0^\infty e^{-p\left(y-h^2\right)}\hat{u}\left(\lambda, y-h^2\right)dy\int_0^\infty e^{-ph^2} \cos\left(\lambda h\right)dh -$$

$$-i\int_0^\infty e^{-p\left(y-h^2\right)}\hat{u}\left(\lambda, y-h^2\right)dy\int_0^\infty e^{-ph^2} \sin\left(\lambda h\right)dh =$$

$$= \tilde{\tilde{u}}\left(\lambda, p\right)\left[2\int_0^\infty e^{-ph^2} h\cos\left(\lambda h^2\right)dh + 2i\int_0^\infty e^{-ph^2} h\sin\left(\lambda h^2\right)dh + \right.$$

$$\left. + \int_0^\infty e^{-ph^2} \cos\left(\lambda h\right)dh - i\int_0^\infty e^{-ph^2} \sin\left(\lambda h\right)dh \right]$$

$$\tilde{\tilde{f}}_1\left(\lambda, p\right) = \tilde{\tilde{u}}\left(\lambda, p\right) I\left(\lambda, p\right)$$

$$I\left(\lambda, p\right) = 2\int_0^\infty e^{-ph^2} h\cos\left(\lambda h^2\right)dh + 2i\int_0^\infty e^{-ph^2} h\sin\left(\lambda h^2\right)dh +$$

$$+ \int_0^\infty e^{-ph^2} \cos\left(\lambda h\right)dh - i\int_0^\infty e^{-ph^2} \sin\left(\lambda h\right)dh$$

Bu integralni hisoblashda quyidagilardan foydalanamiz:

$$2\int_0^\infty e^{-ph^2} h\cos\left(\lambda h^2\right)dh = \int_0^\infty e^{-ph^2}\cos\left(\lambda h^2\right)dh^2 = \frac{p}{p^2+\lambda^2}$$

$$2i\int_0^\infty e^{-ph^2} h\sin\left(\lambda h^2\right)dh = i\int_0^\infty e^{-ph^2}\sin\left(\lambda h^2\right)dh^2 = \frac{i\lambda}{p^2+\lambda^2}$$

$$\int_0^\infty e^{-ph^2}\cos\left(\lambda h\right)dh = \frac{1}{2}\sqrt{\frac{\pi}{p}}\,\mathrm{e}^{-\frac{\lambda^2}{4p}}$$

$$i\int_0^\infty e^{-ph^2}\sin\left(\lambda h\right)dh = \frac{\lambda}{2p}\sum_{k=1}^{\infty}\frac{1}{(2k-1)!!}\left(\frac{-\lambda^2}{2p}\right)^{k-1}$$

$$I\left(\lambda,p\right) = \frac{p+i\lambda}{p^2+\lambda^2} + \frac{1}{2}\sqrt{\frac{\pi}{p}}e^{\frac{-\lambda^2}{4p}} + \frac{\lambda}{2p}\sum_{k=1}^{\infty}\frac{1}{(2k-1)!!}\left(\frac{-\lambda^2}{2p}\right)^{k-1}$$

Funksiyaning nollar to’plami tekislikda nol o’lchovga ega bo’ladi.

Bu yerda

$$I = \frac{p+i\lambda}{p^2+\lambda^2} + \frac{1}{2}\sqrt{\frac{\pi}{p}}e^{\frac{-\lambda^2}{4p}} + \frac{\lambda}{2p}\sum_{k=1}^{\infty}\frac{1}{(2k-1)!!}\left(\frac{-\lambda^2}{2p}\right)^{k-1}$$

$$\left|\frac{1}{I(\lambda,p)}\right| = 6\left|p\right|^2 + \left|\lambda\right|^2 \tag{3.1.7}$$

Biz quyidagiga ega bo’lamiz:

$$\tilde{\tilde{u}}(\lambda,p) = \frac{1}{I(\lambda,p)}\tilde{\tilde{f}}_1(\lambda,p) \tag{3.1.8}$$

(3.1.8) ni o’ng tomonini p^4 ga ko’paytirib bo’lsak

$$\tilde{\tilde{u}}(\lambda,p) = \frac{1}{p^4 I(\lambda,p)}p^4\tilde{\tilde{f}}_1(\lambda,p) \tag{3.1.9}$$

(3.1.7) dan $\operatorname{Re} p \ge a$; $\left|p\right| \to \infty$ da $\frac{1}{p^4 I(\lambda,p)}$ analitik absolyut yaqinlashuvchi

$$I_1(\lambda, y-\eta) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{py}\frac{dp}{p^4 I(\lambda, p)} \tag{3.1.10}$$

Natijada $\left(p^4 I(\lambda, p)\right)^{-1}$ (3.1.10) funksiyaning Laplas almashtirishibo'ladi. (3.1.9) tenglamani ikkala tomonini p o'zgaruvchi bo'yicha teskari Laplas almashtirishini qo'llaymiz.

$$\hat{u}(\lambda, y) = \int_0^\infty I_1(\lambda, y-\eta)\frac{\partial^4}{\partial\eta^4}\hat{f}_1(\lambda,\eta)\,d\eta \tag{3.1.11}$$

(3.1.11) ning o'ng tomonini $(1+\lambda^4)$ ga bo'lamiz.

$$\hat{u}(\lambda, y) = \int_0^\infty \left(1+\lambda^4\right)\frac{I(\lambda, y-\eta)}{1+\lambda^4}\frac{\partial^4 \hat{f}_1(\lambda,\eta)}{\partial\eta^4}\,d\eta \tag{3.1.12}$$

(3.1.12) ga λ argument bo'yicha Fur'yening teskari almashtirishini qo'llaymiz.

Teskarilanish teoremasi va svyortkalar teoremasidan foydalanib va shuningdek, Fur'ye almashtirishining xossalaridan (3.1.1) ning yechimi uchun quyidagi ifodani olamiz:

$$u(x,y) = \int_0^{+\infty}\int_{-\infty}^{+\infty} I_2(x-\xi, y-\eta)\left(\frac{\partial^4}{\partial\eta^4} - \frac{\partial^8}{\partial\eta^4\partial\xi^4}\right) f_1(\xi,\eta)\,d\xi\,d\eta \tag{3.1.13}$$

$$I_2 = \int_{-\infty}^{+\infty} e^{-i\lambda x}\frac{I_1(\lambda, p)\,d\lambda}{1+p^4}$$

(3.1.13) formula y o'zgaruvchi bo'yicha lokal xarakterga ega.

$\sup u \subset \Omega$ shartdan (3.1.13) ifoda $h<\infty$ da ham (3.1.13) ning yechimi uchun o'rinli bo'ladi.

U holda (3.1.8) (3.1.13) dan $C_0^2(\Omega)$ sinfda izlanayotgan (3.1.1) masalaning yechimi yagonaligi kelib chiqadi.

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}|u|^2\,d\lambda\,d\mu \le \frac{6}{\sqrt{\pi}}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}p^4\varphi^2\,d\lambda\,d\mu \qquad (3.1.14)$$

Fur'ye almashtirishining differensiallash xossasidan foydalanib, norma uchun uchburchak tengsizligidan shuningdek, (3.1.13) va (3.1.14) dan

$$\|u\|_{L_2(\Omega)} \le C_0\,\|f_1\|_{W_2^2(\Omega)}$$

C_0 -qandaydir o'zgarmas son.

3.1.1-teorema isbotlandi.

3.2-§. Yechimning mavjudligi haqidagi teorema

(3.1.1) integral geometriya masalasining yechimi mavjudligi haqidagi savolga javobni quyidagi teorema beradi.

3.2.1-Teorema . (3.1.1) tenglamaning o'ng tomoni quyidagi shartlarni qanoatlantirsin

4) $f(x,y)$ o'zgaruvchi x bo'yicha finit

5) $f(x,y)$ ikkinchi tartibgacha barcha uzluksiz xususiy hosilalarga ega

6) $\frac{\partial^m}{\partial y^m}f(x,y)|_{y=0} = \frac{\partial^m}{\partial y^m}f(x,y)|_{y=l} = 0 \qquad (m = 0,1,2)$

u holda (3.1.1) tenglamaning yechimi (3.1.6) formula bilan aniqlangan x argument bo'yicha finit uzluksiz funksiyalar sinfida mavjuddir.

Isbot. $I_2(x,y)$ funksiyaga va $u(x,y)$ funksiyaga qo'yilgan shartlardan (3.1.6) tenglamaning ikkala qismiga ham x o'zgaruvchi bo'yicha Furye almashtirishini qo'llash mumkin ekanligi kelib chiqadi.

Svertkalar haqidagi teorema va Furye almashtirishining xossalaridan foydalanib quyidagiga ega bo'lamiz.

$$i\lambda \overset{\frown}{u}(\lambda,y)=\int_0^y\left\{\int_{-\infty}^{+\infty}e^{i\lambda x}\left[\int_{-\infty}^{+\infty}I_2(x-\xi,y-\eta)\left(\frac{\partial^4}{\partial\eta^4}-\frac{\partial^4}{\partial\eta^4\partial\xi^4}\right)f(\xi,\eta)d\xi\right]dx\right\}d\eta=$$

$$=\int_0^y\left\{\int_{-\infty}^{+\infty}e^{i\lambda\xi}\left(\frac{\partial^4}{\partial\eta^4}-\frac{\partial^4}{\partial\eta^2\partial\xi^4}\right)f(\xi,\eta)d\xi\int_{-\infty}^{+\infty}e^{i\lambda(x-\xi)}I_2(x-\xi,y-\eta)dx\right\}d\eta=$$

$$=\int_0^y(1+\lambda^4)\frac{\partial^2}{\partial\eta^2}\overset{\frown}{f}(\lambda,\eta)\overset{\frown}{I}_2(\lambda,y-\eta)d\eta$$

Bu yerda $\overset{\frown}{I}_2(\lambda,y-\eta)$ funksiya $I_2(x-\xi,y-\eta)$ funksiyadan birinchi o'zgaruvchi bo'yicha olingan Furye almashtirishidir.

$I_2(x,y)$ funksiya uchun (3) munosabatdan quyidagiga ega bo'lamiz.

$$\overset{\frown}{u}(\lambda,y)=\int_0^y\frac{\partial^4}{\partial\eta^4}\overset{\frown}{f}(\lambda,\eta)I_1(\lambda,y-\eta)d\eta \tag{3.2.1}$$

(3.1.3) formuladan va $u(x,y)$, $f(x,y)$ funksiyalarga qo'yilgan shartlardan (3.2.1) ning ikkala qismiga y o'zgaruvchi bo'yicha Laplas almashtirishini qo'llash mumkinligi kelib chiqadi.

Ko'paytmalar haqidagi teorema va originalni differensiallash xossasidan foydalanib quyidagiga ega bo'lamiz.

$$\overset{\approx}{u}(\lambda,p)=\int_0^{+\infty}e^{-py}\int_0^y\frac{\partial^4}{\partial\eta^4}\overset{\frown}{f}(\lambda,\eta)I_1(\lambda,y-\eta)d\eta dy=$$

$$= \int_0^{+\infty} e^{-p\eta} \hat{f}(\lambda,\eta) d\eta \int_0^{+\infty} e^{-ph} I_1(\lambda,h) dh = p^4 \tilde{\hat{f}}(\lambda,p) \int_0^{+\infty} e^{-ph} I_1(\lambda,h) dh$$

Bu yerda

$$\tilde{\hat{u}}(\lambda, p) = \int_0^{+\infty} e^{-ph} \hat{u}(\lambda, y) dy$$

$$\tilde{\hat{f}}(\lambda, p) = \int_0^{+\infty} e^{-ph} \hat{f}(\lambda, y) dy$$

(3.1.3) formula bilan aniqlangan $I(\lambda, p)$ funksiya barcha

$$\{(\lambda, p) : \lambda \in R^1, \ p \geq 0\}$$

larda o'rinli.

Oxirgi tenglik va (3.1.3) dan quyidagiga ega bo'lamiz.

$$\tilde{\hat{u}}(\lambda, p) = \tilde{\hat{f}}(\lambda, p) \frac{1}{I(\lambda, p)} \tag{3.2.2}$$

1-teoremaning isbotida biz $\dfrac{1}{p^4 I(\lambda, p)}$ funksiyani $p \geq 0$ yarim tekislikda $p \to \infty$ da nolga intilishini va

$$\int^{+\infty} e^{ph} \frac{dp}{p^4 I(\lambda, p)}$$

integral absolyut yaqinlashuvchi ekanligini ko'rsatgan edik.

Demak,

$$\frac{1}{p^4 I(\lambda, p)}$$

funksiyada Laplasning teskari almashtirishini qo'llash mumkin. Bundan (3.2.2) tenglama Laplasning teskari almashtirishini qo'llash mumkinligi kelib chiqadi. (3.2.2) tenglamani quyidagi ko'rinishda yozish mumkin.

$$\tilde{\tilde{u}}(\lambda,p)\left(\frac{p+i\lambda}{p^2+\lambda^2}+\frac{1}{2}\sqrt{\frac{\pi}{p}}e^{-\frac{\lambda^2}{4p}}+\frac{\lambda}{2p}\sum_{k=1}^{\infty}\frac{1}{(2k-1)!!}\left(-\frac{\lambda^2}{2p}\right)^{k-1}\right)=\tilde{\tilde{f}}(\lambda,p) \qquad (3.2.3)$$

(3.2.3) tenglamaning ikkala tomoniga ham p o'zgaruvchi bo'yicha Laplasning teskari almashtirishini qo'llaymiz. Svertkalar haqidagi teoremadan foydalanib quyidagiga ega bo'lamiz.

$$\widehat{f}(\lambda,y)=\int_0^y e^{i\lambda h}\widehat{u}(\lambda,\eta)\,d\eta \qquad (3.2.4)$$

Bu yerda $h=\sqrt{y-\eta}$ ga teng.Nihoyat (3.2.4) tenglamaning ikkala tomoniga ham λ o'zgaruvchi bo'yicha Furyening teskari almashtirishini qo'llaymiz. Svertkalar haqidagi teoremadan

$$\int_{-\infty}^{+\infty} e^{-i\lambda x}\left(2\int_0^y h\widehat{u}\left(\lambda,y-h^2\right)\cos\left(\lambda h^2\right)dh\right)+\int_{-\infty}^{\infty} e^{-i\lambda x}\left(2i\int_0^y h\widehat{u}\left(\lambda,y-h^2\right)\sin\left(\lambda h^2\right)dh\right)+$$

$$\int_{-\infty}^{+\infty} e^{-i\lambda x}\left(\int_0^y \widehat{u}\left(\lambda,y-h^2\right)\cos(\lambda h)dh\right)-\int_{-\infty}^{\infty} e^{-i\lambda x}\left(i\int_0^y \widehat{u}\left(\lambda,y-h^2\right)\sin(\lambda h)dh\right)=f(x,y)$$

shuningdek (3.2.4) ni quyidagi ko'rinishda yozamiz.

$$\int_{x-y}^{x} u(\xi,\xi-x+y)\,d\xi+\int_{x}^{x+\sqrt{y}} u(\xi,y-(x-\xi)^2)\,d\xi=f_1(x,y)$$

Bu yerdan 2-teorema tasdig'ining to'g'riligi kelib chiqadi.

3.2.2- Teorema isbotlandi.

Uchinchi bobga xulosa

Uchinchi bobda siniq chiziq va parabolalar oilasi bo'yicha polosada (yo'lakda) integral geometriya masalasining yangi sinfi o'rganilgan.

Asosiy natijalari quyidagilar hisoblanadi:

- Siniq chiziq va parabolalar oilasi bo'yicha polosada (yo'lakda) integral geometriya masalasining yangi sinfi o'rganilgan;
- Qo'yilgan masalaning yechimining yagonalik teoremasi isbotlangan;
- chekli silliq fazolarda turg'unlik bahosi olingan va shu bilan masalaning kuchsiz nokorrektligi ko'rsatilgan;
- yechimni analitik ko'rinishi olingan;
- yechimning mavjudligi teoremasi isbotlangan;

XULOSA

Bu ishda integral geometriya masalasining bitta yangi qo'yilishlari qaraldi. Bu qo'yilish yuqori yarim tekislikda markazi (x, y) nuqtada bo’lgan, siniq chiziq va parabolalar oilasi bo’yicha berilgan integralga asoslanib integral ostidagi noma’lum funksiyani tiklash masalasi edi. Bu qo'yilish uchun tashuvchisi yuqori yarim tekislikda yotgan silliq finit funksiyalar sinfida maxsus egri chiziq bo'yicha integral geometriya masalasi yechimining yagonaligi va mavjudligi haqidagi teorema isbotlandi. Bu qo'yilishdagi masala - kuchsiz nokorrekt masala ekanligi ko'rsatildi.

Masaladagi teskarilanish formulasining analitik ko'rinishga keltirilishi bu ishning katta ustunligidir va olingan natija konkret funksiyalar uchun ham foydalanish mumkin. Esdan chiqarmaslik kerakki, qaralgan masalalarga differensial tenglamalar uchun teskari masalalarning ekvivalent qo'yilishlari mavjud. Shunday qilib, bu masalalarga ekvivalent bo'lgan teskari masalalarning yechimini topishda dissertatsiyada olingan teskarilanish formulalardan foydalanish mumkin.

Volterra tipidagi integral tenglamalarni yechilishida paydo bo'ladigan bir qator muammolarga yuqorida keltirilgan usullarni qo'llash mumkin.

FOYDALANILGAN ADABIYOTLAR

1. Бегматов Акб.Х. Задачи интегралъной геометрии с возмущением в трехмерном пронстранстве//Сиб.мат.журнал.2000.Т.41.-№1. –С.3-14.
2. Бегматов Акб.Х. О единственности решения задачи интегралъной геометрии волътерровского типа на плоскости//Докл. РАН.-Москва.2009.Т.427.- №4.-С.439-441.
3. Бегматов Акр.Х. Слабо некорректные задачи интегралъной геометрии волътерровского типа // Докл. РАН.-Москва.1996.Т.349.-№3.-С.297-298.
4. Бегматов Акр.Х. Волътерровские задачи интегралъной геометрии на плоскости для кривых с особенностями // Сиб.мат.журнал.1997.Т.38.-№4. –С.723-737.
5. Бегматов Акр.Х. Задачи интегралъной геометрии по специалъным кривым и поверхностям с особенностями в вершине // Доклады. РАН.-Москва.1998.Т.358.- №2.-С.151-153.
6. Бегматов Акр.Х. Теоремы существования решения двух слабо некорректных задач интегралъной геометрии// Доклады. РАН.-Москва.2002.Т.386.- №1.-С.1-3.
7. Лаврентъев М.М. Математические задачи томографии и гиперболические отображения // Сиб.мат.журнал.2001.Т.42.-№5. –С.1094-1105.
8. Лаврентъев М.М. Задача интегралъной геометрии на плоскости с возмущением Сиб.мат.журнал.1996.Т.37.-№4. –С.851-857.
9. Лаврентъев М.М., авелъев Л.Я. Теория операторов и некорректные задачи. –Новосибирск: Изд-во Ин – та математики. 1999. –702с.

10. Лаврентъев А.М.,Шабат Б.В. Методы теории функций комплексного переменного. – М.: Наука, 1987.-688с.
11. Романов В.Г. Обратные задачи математической физики. – М.: Наука, 1984.-263с.
12. Lavrent'ev M/M/ I11-posed and inverse problems for hyperbolic equations // I11-Pozed and Non-Classical Problems of Mathematical Physics and Analysis,M.M.Lavrent'ev et al.,Eds.,Proceedings of International Converence , VSP.-Utrecht-Boston, 2003.-C.81-101.
13. Бегматов Акр. X.,Очилов З.Х. О ищсстаровлении функйии через интегралы по семейству кривых// Ш11-Posed and non classical problems of mathematical physics and analysis. Programme and book of abstracte.-Samarkand,Uzbekistan,2000.-C.25.
14. Бегматов Акр. X.,Очилов З.Х. О ищсстановлении функции по семейству парабол на плоскости //СамДУ Ilmiy tadqiqotlar axborotnomasi.-Самарканд. 2001.- №1. –С.116-119.
15. Бегматов Акр. Х., Очилов З.Х. Задачи интегральной геометрии с разрывной весовой функцией // Доклады РАН. –Москва, 2009.429.-№3.-С. 295-297.
16. Очилов З.Х. Задачи интегральной геометрии на плоскости и отображения Даламбера для симметричных областей // Узб. мат. жур.-Ташкент, 2011.-№3.- С.136-142.
17. Begmatov Akr. H., Ochilov Z.H. Recovering of function set by integrals along a curve in the plane //I11-Posed and Non-Classical Problems of mathematical Physics and Analysis,M.M.Lavrent'ev et al., Eds., Proceedings of International Converence, VSP.-Utrecht-Boston, 2003.-C.191-197.
18. Begmatov Akr. H., Ochilov Z.H. Integral Geometry Problem with a Discontinuous Weight Function//Doklady Mathematics,2009,Vol.80.

№3.Original Pussian Text Akr.H.Begmatov,Z.H.Ochilov, 2009Published in Doklady akademii Nauk,2009.Vol.429.-№3.-C.823-825.