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Mavzu: Elliptik tipli tenglamalar sistemasi uchun to’rt o’lchovli fazoda chegaralanmagan sohada Koshi masalasi

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MAGISTRLIK DISSERTATSIYASI

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MUNDARIJA

Kirish

Bu dissertatsiya ishiga to‘rt o‘lchovli elliptik sistema uchun qo‘yilgan Koshi masalasini chegaralanmagan sohada yechishdan iborat. Chegaralanmagan sohada o‘suvchi vektor funksiyaning qiymati soha chegarasining bir qismida berilgan bo‘lsa uning qiymatini sohaning ichida topish masalasi qaralgan.

Dissertatsiya ishida elliptik tipdagi tenglamalar sistemasi uchun qo’yilgan Koshi masalasining yechimi haqida bag’ishlanganda Adamar misoli ko’satadiki elliptik tipdagi tenglama va tenglamalar sistemasi uchun qo’yilgan Koshi masalasi turg’un bo’lmaydi. Bunday masalalarni yechish uchun yechimlar to’plamini kompakt to’plamgacha qisqartirsak u holda Tixonov manosida shartli korrekt masala bo’ladi. Analitik funksiyalarni sohaning chegarasining bir qismidagi qiymatidan foydalanib sohada topish masalasi Karleman, Goluzin Krilov va boshqalar tomonidan qaralgan. Xuddi shunday turdagi masalalar Laplas tenglamasi uchun ham qo’yish mumkin. Bunday masalalarga Laplas tenglamasi uchun qo’yilgan Koshi masalasi deyiladi. Bu masalalarni akademik M.M.Lavrentev, akademik Ivanov, prof Ayzenberg, Tarxanov va boshqalar shug’ullangan.

Laplas tenglamasiga qo’yilgan Koshi masalasini yechishda sohaning chegarasining bir qismida noma’lum funksiya va uning normal bo’yicha hosilasi berilgan bo’ladi. Demak shunday funksiyalar yechimini aniqlash mumkinki, u ikkita garmonik funksiyalarning yig’indisidan iborat bo’lib, chegaraning qolgan qismidan olingan integral cheksiz kichik bo’lsin. Bunday funksiyalarga Karleman funksiyasi deyiladi.

1.Dissertatsiyaning maqsadi va vazifasi.

To‘rt o‘lchovli fazoda elliptik sistema uchun qo‘yilgan Koshi masalasini chegaralanmagan sohada yechishni regulyarizasiyasini topish masalasi dissertatsiyaning asosiy maqsadidan iborat. Chegaralanmagan sohada o‘suvchi vektor funksiyaning chegaraning bir qismidagi vektor funksiyani qiymatini bilgan holda qolgan qismida qiymati nolga yaqin bo’lgan Karleman matrisasini tuzish ishning asosiy fazifasidir.

2.Masalaning dolzarbligi.

Chegaralangan sohada integral formulani bilgan holda chegaralanmagan sohada o‘suvchi vektor funksiyaning sistemasining yechimlarida integral formulani o‘rinliligini ko‘rsatish, hamda vektor funksiyani soha chegarasining bir qismidagi qiymatidan foydalanib sohaning ichidagi qiymatini aniqlash masalasi ishning dolzarbligini ifodalaydi. Chunki bunday masalani yechish uchun regulyarizasiyalanuvchi yechimlar oilasini tuzishga to‘g’ri keladi. Bu yechimlar oilasi parametrga bog’liq bo‘lib parametrning cheksizlikga intilishiga yechimlar oilasi aniq yechimga intilishini ko‘rsatish lozim.

3.Muammoning ishlab chiqarish darajasi.

Elliptik tenglamalar sistemasi uchun qo‘yilgan Koshi masalasi korrekt bo‘lmagan masalalar qatoriga kiradi, ya’ni bunday masalalarni yechimi turg’un emas. Agar yechimlar sinfini kompaktlar to‘plamiga qisqartirsak u holda bunday masalalar shartli korrekt masalasiga kiradi.

4.Tadqiqotning ilmiy yangiligi.

Koshi – Riman sistemasi uchun qo‘yilgan Koshi masalasining korrekt bo‘lmasligini isbotlovchi Adamar misoli mavjud. Elliptik sistemalar uchun ham qo‘yilgan Koshi masalasi nokorrekt bo‘ladi. O‘suvchi vektor funksiya uchun ham chegaralanmagan sohada integral formula asosiy rol o‘ynaydi. Integral formula ma’lum bo‘lsa u holda Koshi masalasi regulyarizasiyasini tuzish mumkin.

5.Tadqiqot predmeti.

Matematik fizik tenglamalari va sistemalariga qo‘yilgan chegaraviy masalalar asosan elliptik tipli tenglamalar qatoriga kiradi. Demak dissertatsiyaning tadqiqot predmeti sifatida chegaraviy masalalarni yechish hisoblanib uning tipini aniqlash va unga to‘g’ri keladigan yechish usuli hisoblanadi.

6.Tadqiqotning obyekti.

Dissertatsiyaning asosiy tadqiqot obyekti elliptik tipli tenglamalar sistemasi bo‘lib bu sistemaning xaraktristik matritsasi shunday matritsadan iboratki uning ermitli qo‘shmasi bilan ko‘paytmasi Laplas operatori bilan faktorizasiyalanadi. Bu sistema uchun integral formulaning fundamental yechimlar sistemasi Laplas operatorining fundamental yechimlari orqali ifodalanadi.

7.Ilmiy-amaliy ahamiyati.

Elliptik tipli tenglamalar sistemasi uchun qo‘yilgan Koshi masalasini yechishga Karleman matritsasi tuzishda u orqali yechim oshkor ravishda ifodalanadi. Bunday fundamental yechimlar sistemasi orqali ifodalanuvhi yechimlar sistemasi regulyarizasiyalovchi yechimlar deyiladi. Bu ishda shunday yechimlar sistemasi tuziladi. U yechimlar parametrga bog’liq bo‘ladi. Parametr esa taqribiy yechimning bahosi orqali ifodalanadi. Shuning uchun bu amaliy axamiyatga egadir.

8.Ishning tuzilishi.

Magistrlik dissertatsiyasi uch bob va o‘n bitta paragraf, xulosa, adabiyotlar ruyxatidan iborat.

Birinchi bobda elliptik tipdagi tenglamalar va uning xossalari keltirilgan.

Birinchi paragrafda elliptik tipdagi tenglamalar uchun asosiy chegaraviy masalalar.

Asosan stasionar bo‘lgan, ya’ni vaqtga bog’liq bo‘lmagan fizik prosesslarini, ya’ni issiqlikning tarqalishi to‘lqinning harakati, maydon potensialining tortilish kuchi kabilarni tekshirishda asosan elliptik tipdagi tenglamalarga keltiriladi. Ya’ni yuqoridagi qaralgan barcha tenglamalarda qatnashayotgan noma’lum funksiya vaqt t ga bog’liq bo‘lmaydi. [1]

Bu xildagi tenglamalarga Laplas $\Delta u = 0$ va Puasson $\Delta u = f$ tenglamalari misol bo'ladi. Agar Laplas tenglamasida qatnashayotgan noma'lum funksiyaning ko'rinishi

$$u = u(x_1, x_2, ..., x_n) \quad \Delta u = \frac{\partial^2 x}{\partial x_1^2} + \frac{\partial^2 x}{\partial x_2^2} + ... + \frac{\partial^2 x}{\partial x_n^2}$$

Laplas tenglamasining qanoatlantiradigan funksiyaga umuman olganda garmonik funksiya deb yuritiladi.

Ikkinchi paragrafda Grin formulalari va Garmonik funksiyalarning integral munosabatlari.

Garmonik funksiyalarning xossalarini o'rganish uchun avvalo ularning integral munosabatlarini qarab chiqamiz. Bu integral munosabatlarini keltirish uchun esa. Grin formulalaridan foydalanishga to'g'ri keladi. Grin formulalarining o'zlari esa Ostogradskiy formulasining xususiy hollaridan iboratdir. Ostogradskiy formulasining ma'nosi esa quyidagichadir.

Faraz qilaylik P, Q, R funksiyalar $S + T$ yopiq sohada o'zining bir tartibli hosilalari bilan uzluksiz funksiyalardan iborat bo'lsin. Agarda S sirtda o'tkazilgan tashqi normal n ni Ox, Oy, Oz o'qlar bilan tashkil etgan burchaklari mos holda

$$\alpha = (n, x); \quad \beta = (n, y); \quad \gamma = (n, z)$$

lardan iborat bo'lsin. Bu holda Ostogradskiy formulasi quyidagicha edi.

$$\iiint_T \left[\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right] d\tau = \iint_S (P\cos\alpha + Q\cos\beta + R\cos\gamma) ds \quad (1.3.1)$$

Endi quyidagi formuladan foydalanib,

$$\iiint_T [u\Delta v - v\Delta u] d\tau = \iint_S \left[u\frac{\partial v}{\partial n} - v\frac{\partial v}{\partial n} \right] ds \qquad (1.4.1)$$

Garmonik funksiyalarning qaralayotgan soha ichidagi ixtiyoriy nuqtadagi qiymatlarini integral orqali ifodalash mumkinligini qaraymiz. Buning uchun esa faraz qilaylik (1.4.1) tenglamada qatnashayotgan $u = u(M)$ funksiya quyidagi shartlarni qanoatlantirsin. Ya'ni bu funksiya $T+S$ yopiq sohada o'zining birinchi tartibli hosilasi bilan birgalikda uzluksiz hamda T sohaning ichida barcha argumentlari bo'yicha ikkinchi tartibgacha differensiallanuvchi garmonik funksiyadan iborat bo'lsin.

Uchinchi paragrafda doira uchun Dirixlening ichki va tashqi masalalari. Puasson integrali.

Endi $\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ Lappas tenglamasi uchun Dirixle masalasini qarab chiqamiz.

MASALA: R radiusli doirada (ichkarida yoki tashqarida) shunday bir $u = u(x, y)$ funksiyani topish kerakki natijada bu funksiya quyidagi chegaraviy shartlarni $U|_R = f$ qanoatlantirsin, ya'ni: quyidagi masalaga keladi

$$\left.\begin{aligned} &\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \\ &u / R = f \end{aligned}\right\} = 0 \qquad (1.5.A)$$

Bundagi f doiraning konturida berilgan funksiyadan iboratdir.

Endi (1.5.A) ko'rinishidagi masalaning yechimini topamiz. Bu uchun esa Lappas tenglamasining qutb koordinata, sistemasidan foydalanamiz,

ya'ni:

$$\Delta u(p, \varphi) = \frac{1}{p^2} \frac{\partial}{\partial p}\left(p \frac{\partial}{\partial p}\right) + \frac{1}{p^2} \frac{\partial^2 u}{\partial \varphi^2} = 0 \qquad (1.5.1)$$

To 'rtinchi paragrafda garmonik funksiyalarning sodda xossalari.

Endi yuqoridagi garmonik funksiyalarga tegishli bo'lgan formulalardan foydalangan holda shu funksiyalarning ba'zi soda xossalarini qarab chqamiz. Buning uchun esa quyidagi xossalarni alohida – alohida qaraymiz.

Xossa – 1: Agar $u = u(M)$ va $v = v(M)$ o‘zining funksiyalar S+T yopiq sohada o‘zining 1 – tartibli xosilasi bilan birga uzluksiz, hamda T sohaning ichida garmonik funksiyadan iborat bo‘lsa, u holda quyidagi tengliklar o‘rinlidir.

$$\iint_S \frac{\partial u}{\partial n} d\tau = 0; \quad \iint_S \frac{\partial v}{\partial n} d\tau = 0$$

Ikkinchi bobda to‘rt o‘lchovli Koshi-Riman sistemasi uchun sophada Koshi masalasi keltirilgan.

Birinchi paragrafda matrisalar sinfi keltirilgan.

Agar $R^n - n$ o‘lchovli $[5]$

$$x = \begin{Vmatrix} x_1 \\ .... \\ x_n \end{Vmatrix} \qquad \text{va} \qquad y = \begin{Vmatrix} y_1 \\ .... \\ y_n \end{Vmatrix},$$

$C^n - n$ o‘lchovli, kompleks maydonlar fazosidan olingan bo‘lsin.

$$z = \begin{Vmatrix} z_1 \\ .... \\ z_n \end{Vmatrix}, \quad \text{o‘zgaruvchilar kompleks fazosi;}$$

$$\frac{\partial}{\partial x} = \begin{Vmatrix} \frac{\partial}{\partial x_1} \\ .... \\ \frac{\partial}{\partial x_n} \end{Vmatrix}, \qquad \partial x = \begin{Vmatrix} dx_1 \\ .... \\ dx_n \end{Vmatrix} \qquad \text{va} \qquad E(x) = \begin{Vmatrix} x_1 ... 0 \\ ..... \\ 0 .... x_n \end{Vmatrix}$$

Agar $I = (i_1 ... i_q), \quad 0 \le q \le n$ multindeks.

Ikkinchi paragrafda birinchi tartibli elliptik tipli tenglamalar sistemasi uchun integral formula keltirilgan.

Xususiy hosilalardagi birinchi tartibli

$$D(\frac{\partial}{\partial x}) f(x) = 0 \qquad (2.2.1)$$

chiziqli differensial tenglamalar sistemasini qaraymiz, uning xarakteristik matrisasi

$$D(x) \in A_{k\times l}(p(x), x) \qquad (2.2.2)$$

$$f(x) = \begin{vmatrix} f_1(x) \\ \dots \\ f_l(x) \end{vmatrix} \qquad (2.2.3)$$

esa kompleks qiymatli umumiy holda vektor funksiya .

2.2.1-teorema. $G \in R^n$ - silliq bo‘lakli chegarali chegaralangan soha va (2.2.5) $f(x) \in C^1(G) \cap C(\bar{G})$ vektor funksiya (2.2.2) shartda (2.2.1) sistemaning yechimi bo‘lsin , u holda

$$\int_{\partial G} M(y-x) f(y) = \begin{cases} f(x), \ x \in G, \\ 0, \quad x \notin \bar{G} \end{cases} \qquad (2.2.10)$$

(2.2.7) formuladan va [3] natijadan xususan, (2.2.1) sistemaning $C^1(G)$ sinfdagi yechimi (2.2.2) shart bajarilganda haqiqatdan, G sohada analitik funksiyalar bo‘ladi. Bundan va (2.2.7) misoldan ko‘rinadiki, (2.2.1) sistemaga (2.2.2) shartning qo‘yilishi bu sistema elliptik bo‘lishiga teng kuchli.

Uchinchi paragrafda birinchi tartibli elliptik tipli tenglamalar sistema uchun misollar keltirilgan.

2.3.1-misol.

$(n\times n)$ matrisa $\|a_{ij}\| a_{ij} + a_{ji} = 0, \quad i = j, \quad a_{ij} = 1$ (2.3.1)

shartni qanoatlantiruvchi $a_{ij}, \quad i, j = \overline{1,n}$ kompleks sonlardan iborat bo‘lsin.

U holda $(n\times 1)$ matrisa

$$D(x)=\begin{Vmatrix}\sum_{j=1}^{n} a_{ij}x_j \\ ..... \\ \sum_{j=1}^{n} a_{jn}x_j\end{Vmatrix}\in A_{n\times 1}(|x|^2,x) \qquad (2.3.2)$$

chunki (2.3.1) shartga ko‘ra $\Gamma D(x)=\|x_1...x_n\|$ deb olish mumkin.

To‘rtinchi paragrafda birinchi tartibli elliptik tipli tenglamalar sistema uchun Morere teoremasi keltirilgan.

Agar :

1) $(l\times l)-$ vektor $f(x)$ funksiya $G\in R^n$ sohada uzluksiz;

2) $D(x)\in A_{k\times l}(p(x),x)-$ matrisa;

3) ixtiyoriy yopiq bo‘lakli silliq σ bilan chegaralangan chekli sohasi bilan kiruvchi S sirt uchun

$$\int_{\sigma}(*D(\partial y))f(y)=0$$

tenglik o‘rinli bo‘lsa , u holda $f(x)$ (2.4.1) sistemaning har bir $x\in G$ nuqtada yechimi bo‘ladi.

Uchinchi bobda asosiy bob hisoblanib, elliptik tipli tenglamalar sistemasi uchun to‘rt o‘lchovli fazoda chegaralanmagan sohada Koshi masalasi keltirilgan.

Birinchi paragrafda To‘rt o‘lchovli elliptik sitema va unga qo‘yilgan Koshi masalasi ko‘rsatilgan.

Quyidagi belgilashlarni kiritamiz.

R^n-n o‘lchovli Evklid fazosidan olingan bo’lib, bunda

$$x=\begin{Vmatrix}x_1\\ ....\\ x_4\end{Vmatrix} \quad \text{va} \quad y=\begin{Vmatrix}y_1\\ ....\\ y_4\end{Vmatrix}$$, vektorlar bo‘lsin.

C^n-n o‘lchovli, kompleks maydon bo’lib,

$$z = \begin{Vmatrix} z_1 \\ .... \\ z_4 \end{Vmatrix},$$ o‘zgaruvchilar kompleks fazosidagi vektor bo‘lsin.

$$\frac{\partial}{\partial x} = \begin{Vmatrix} \frac{\partial}{\partial x_1} \\ .... \\ \frac{\partial}{\partial x_n} \end{Vmatrix}, \qquad \partial x = \begin{Vmatrix} dx_1 \\ .... \\ dx_n \end{Vmatrix} \qquad \text{va} \qquad E(x) = \begin{Vmatrix} x_1 ...0 \\ ..... \\ 0....x_n \end{Vmatrix}$$

Bu yerda $E(x)$ dioganal matritsa.

Ikkinchi paragrafda chegaralanmagan sohada to‘rt o‘lchovli elliptik sistema uchun integral formula ko‘rsatilgan.

Faraz qilaylik $G \subset R^{(4)}$ - 4- o‘lchovli fazodan olingan chegaralangan soha bo‘lib chegarasi silliq sirtdan iborat bo‘lsin.

G sohada quyidagi tenglamalar sistemasini qaraymiz

$$D\left(\frac{\partial}{\partial x}\right)u(x) = 0 \qquad (3.2.1)$$

Bu yerda $D(x^T)$ xaraktristik matritsa $D(x^T) \in A_{cx_n}(x)$ ya’ni $D^*(x^T)D(x^T) = E(|x|^2 \cdot u^0)$

Bu qaralayotgan sistemani qanoatlantiruvchi $U(x)$ vektor funksiya garmonik funksiya bo‘lib, (3.2.1) sistemaning yechimidan iboratdir.

Uchinchi paragrafda to‘rt o‘lchovli elliptik tipli tenglamalar sistemasi uchun chegaralanmagan sohoda Koshi masalasining regulyarizasiyasi ko‘rsatilgan.

Faraz qilamiz G soha chegarasi S silliq sirt cheksizlikka intiluvchi va $y_4 = 0$ tekislikdan iborat bo‘lib $0 < y_n < h, \quad h = \frac{\pi}{\rho}, \quad \rho > 0$ qatlamda joylashgan chegaralanmagan soha bo‘lsin.

Teorema 2.1.2. Faraz qilamiz $u(y)$ vektor funksiya bo’lib G_ρ - ning ichida to‘rt o‘lchovli elliptik sistemasini qanoatlantirib, uzluksiz bo’lsin.

Agar:

$$|u(y)| \le 1, \quad y \in T = \{y_u = 0\}$$

bo‘lib

$$|u(y) - u_\sigma(y)| \le \delta, \quad 0 < \delta < 1$$

uchun uzluksiz yaqinlashishidan $y \in S$ - lar uchun

$$u(x) = \int_{\partial G_\rho} N_\sigma(y,x) u(y) ds_y, \quad x \in G_\rho$$

bo‘lsa, u holda

$$|u_\sigma(x) - u(x)| \le C(x) \bar{C}(\sigma) \sigma^{\frac{x_u}{y_u}}$$

Tengsizlik bajariladi. Bu yerda

$$y_u^0 = \max_{y \in s} y_u$$

Natija:

$$\lim_{\sigma \to 0} u_\sigma(x) = u(x) : \quad x \in G_\rho$$

G_ρ - ning har bir kompaktida tekis bajariladi.

I – BOB. Elliptik tipdagi tenglamalar va uning xossalari.

1.1-§. Elliptik tipdagi tenglamalar uchun asosiy chegaraviy masalalar.

Asosan stasionar bo‘lgan, ya’ni vaqtga bog’liq bo‘lmagan fizik prosesslarini, ya’ni issiqlikning tarqalishi to‘lqinning harakati, maydon potensialining tortilish kuchi kabilarni tekshirishda asosan elliptik tipdagi tenglamalarga keltiriladi. Ya’ni yuqoridagi qaralgan barcha tenglamalarda qatnashayotgan noma’lum funksiya vaqt t ga bog’liq bo‘lmaydi. [1]

Bu xildagi tenglamalarga Laplas $\Delta u = 0$ va Puasson $\Delta u = f$ tenglamalari misol bo‘ladi. Agar Laplas tenglamasida qatnashayotgan noma’lum funksiyaning ko‘rinishi

$$u = u(x_1, x_2, ..., x_n) \quad \Delta u = \frac{\partial^2 x}{\partial x_1^2} + \frac{\partial^2 x}{\partial x_2^2} + ... + \frac{\partial^2 x}{\partial x_n^2}$$

Laplas tenglamasining qanoatlantiradigan funksiyaga umuman olganda garmonik funksiya deb yuritiladi.

Quyidagi belgilashlar kiritamiz.

T n-o‘lchovli fazoda olingan sohadan iborat bo‘lsin. $\bar{n}$ – tashqi birlik normal.

Ta’rif: S sirt bilan chegaralangan T sohada $u = u(x, y, z)$ funksiya o‘zining barcha argumentlari bo‘yicha 2 tartibgacha differensiallanuvchi funksiyadan iborat bo‘lib, bu funksiya shu sohaning barcha nuqtalarida Laplas tenglamasini qanoatlantirsa, u funksiyaga garmonik funksiya deyiladi. Bu funksiya $u = u(x, y, z)$ funksiya chegaralanmagan T sohada garmonik deb yuritiladi, agarda shu chegaralanmagan sohaning barcha nuqtalarida ikkinchi tartibgacha differensiallanuvchi bo‘lib, shu chegaralanmagan sohadagi nuqtaning cheksizcha

M intilganda funksiyaning shu nuqtadagi qiymati nolga tekis intilsa, ya'ni $\forall E \succ 0 \quad \exists A \succ 0$ bo'lsaki, buning natijasida koordinata boshida shu M nuqtagacha bo'lgan masofa $\tau \geq A$ sondan katta bo'lganda funksiyaning shu nuqtadagi qiymati $|u(M)| \leq E$ bo'lsa, endi garmonik funksiyalarga tegishli quyidagi sodda lemmalarni qaraymiz.

1-Lemma:

$$u = u(x,y,z) = \frac{1}{(x-x_0)^2 + (y-y_0)^2 + (z-z)^2} = \frac{1}{r}$$
$$r = (x-x_0)^2 + (y-y_0)^2 + (z-z)^2$$

Funksiya uch o'lchovli fazodagi M_0 va M nuqtalar orasida masofani bildiradi. Funksiya uch o'lchovli fazodagi $M_0 = (x_0, y_0, z_0)$ nuqtalardan tashqari barcha nuqtalarda garmonik funksiyadan iboratdir. Ya'ni

$$\Delta(1/r) = \partial^2(1/r)\partial x^2 + \partial^2(1/r)\partial y^2 + \partial^2(1/r)\partial z^2 = 0$$

Shu sababli $u(x,y,z) = 1/r$ funksiyaga Laplas tenglamasining fundamental yechimi deyiladi.

Isbot: Haqiqatan $M_0 \neq M$ bo'lganligi sababli

$$r = (x-x_0)^2 + (y-y_0)^2 + (z-z)^2$$

dan tenglik xossalarini hisoblasak quyidagilarga kelamiz.

$$\partial r / \partial x = (x-x_0)/r, \quad \partial r / \partial y (y-y_0)/r, \quad \partial r / \partial z = (z-z_0)/r$$

Endi $u = 1/r$ funksiyadan tegishli xossalarni hisoblaymiz.

$$\frac{\partial\left(\frac{1}{r}\right)}{\partial x} = \frac{\partial\left(\frac{1}{r}\right)}{\partial r} \cdot \frac{\partial r}{\partial x} = -\frac{1}{r^2} \cdot \frac{x-x_0}{r} = \frac{x-x_0}{r^2}$$
$$\frac{\partial\left(\frac{1}{r}\right)}{\partial y} = \frac{x-x_0}{r^3}; \quad \frac{\partial\left(\frac{1}{r}\right)}{\partial y} = \frac{y-y_0}{r^3}; \quad \frac{\partial\left(\frac{1}{r}\right)}{\partial z} = \frac{z-z_0}{r^3};$$

$$\frac{\partial^2\left(\frac{1}{r}\right)}{\partial x^2}=\frac{\partial}{\partial x}\left[-\frac{\partial\left(\frac{1}{r}\right)}{\partial x}\right]=-\frac{\partial}{\partial x}\left[\frac{x-x_0}{r}\right]=$$

$$=\frac{r^3-(x-x_0)3r^2\frac{\partial r}{\partial x}}{r^6}=\frac{r^3-3(x-x_0)r^2\left(\frac{x-x_0}{r}\right)}{r^6}=-\frac{1}{r^3}+\frac{3(x-x_0)^2}{r^5}$$

Xuddi shuningdek, quyidagilarga kelamiz.

$$\frac{\partial^2\left(\frac{1}{r}\right)}{\partial y^2}=-\frac{1}{r^3}+\frac{3(y-y_0)^2}{r^5}$$

$$\frac{\partial^2\left(\frac{1}{r}\right)}{\partial z^2}=-\frac{1}{r^3}+\frac{3(z-z_0)^2}{r^5}$$

Demak,

$$\Delta\left(\frac{1}{r}\right)=\frac{\partial^2\left(\frac{1}{r}\right)}{\partial x^2}+\frac{\partial^2\left(\frac{1}{r}\right)}{\partial y^2}+\frac{\partial^2\left(\frac{1}{r}\right)}{\partial z^2}=-\frac{3}{r^3}+$$

$$+\left[\frac{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}{r^5}\right]=\frac{3}{r^3}-\frac{3}{r^3}=0$$

Haqiqatan ham $u=\frac{1}{r}$ qaralayotgan sohadagi M_0 nuqtadan tashqari barcha nuqtalarga garmonik funksiyadan iborat ekan.

2-Lemma.

$$u(x,y)=\ln\frac{1}{r}=\ln\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2}}$$

Funksiya tekislikdagi $U_0(x_0,y_0)$ nuqtadan tashqari barcha nuqtalarda garmonik funksiyadan iborat.

Ya’ni

$$\Delta\left(\ln\frac{1}{r}\right)=\frac{\partial^2\left(\ln\frac{1}{r}\right)}{\partial x^2}+\frac{\partial^2\left(\ln\frac{1}{r}\right)}{\partial y^2}=0$$

Isbot: Haqiqatan ham $M_0 \neq M$ bo'lganligi sababli

$$r=\sqrt{(x-x_0)^2+(y-y_0)^2}$$

$$\frac{\partial r}{\partial x}=\frac{x-x_0}{r}; \quad \frac{\partial r}{\partial y}=\frac{y-y_0}{r}$$

ga kelamiz.

Endi $u=\ln\frac{1}{r}$ funksiyaning tegishli xossalarini hisoblaymiz. Ya'ni

$$\frac{\partial\left(\ln\frac{1}{r}\right)}{\partial x}=\frac{\partial\left(\ln\frac{1}{r}\right)}{\partial r}\cdot\frac{\partial r}{\partial x}=\frac{1}{\frac{1}{r}}\left(-\frac{1}{r^2}\right)\frac{x-x_0}{r}=-\frac{x-x_0}{r^2}$$

$$\frac{\partial\left(\ln\frac{1}{r}\right)}{\partial x}=\frac{x-x_0}{r^3}; \quad \frac{\partial\left(\ln\frac{1}{r}\right)}{\partial y}=\frac{y-y_0}{r^3};$$

$$\frac{\partial^2\left(\ln\frac{1}{r}\right)}{\partial x^2}=\frac{\partial}{\partial x}\left[-\frac{\partial\left(\ln\frac{1}{r}\right)}{\partial x}\right]=\frac{\partial}{\partial x}\left[\frac{x-x_0}{r}\right]=\frac{-r^2-(x-x_0)/r\cdot\frac{\partial}{\partial x}}{r^4}=$$

$$=\frac{-r^2-2(x-x_0)^2}{r^6}=\frac{1}{r^2}+\frac{2(x-x_0)^2}{r^4}\frac{\partial^2\left(\ln\frac{1}{r}\right)}{\partial x^2}=-\frac{1}{r^2}+\frac{2(x-x_0)^2}{r^4}$$

oxirgi tengliklarni qo'shamiz:

$$\Delta\left(\ln\frac{1}{r}\right)=\frac{\partial^2\left(\ln\frac{1}{r}\right)}{\partial x^2}+\frac{\partial^2\left(\ln\frac{1}{r}\right)}{\partial y^2}=-\frac{2}{r^2}+\frac{2[(x-x_0)^2+(y-y_0)^2]}{r^4}=-\frac{2}{r^2}+-\frac{2}{r^2}=0$$

3 – Lemma: $u(x,y)=\ln r$ funksiya $M_0(x_0,y_0)$ nuqtadan tashqari barcha nuqtalarda garmonik funksiyadan iborat. Ya'ni

$$\Delta\ln r=\frac{\partial^2(\ln r)}{\partial x^2}+\frac{\partial^2(\ln r)}{\partial y^2}=0$$

Isbot: $M_0 \neq M$ bo'lganligidan $\frac{\partial r}{\partial x} = \frac{x - x_0}{r}$; $\frac{\partial r}{\partial y} = \frac{y - y_0}{r}$ ga kelamiz.

Endi $\ln r$ funksiyaning tegishli xosilalarini hisoblaymiz.

$$\frac{\partial(\ln r)}{\partial x} = \frac{\partial(\ln r)}{\partial r}; \quad \frac{\partial r}{\partial x} = \frac{1}{r} \cdot \frac{x - x_0}{r} = \frac{x - x_0}{r^2}$$

$$\frac{\partial(\ln r)}{\partial r} = \frac{y - y_0}{r^2}$$

$$\frac{\partial^2(\ln r)}{\partial x^2} = \frac{\partial}{\partial x}\left[\frac{\partial(\ln r)}{\partial x}\right] = \frac{\partial}{\partial x}\left[\frac{x - x_0}{\partial x}\right] = \frac{r^2 - (x - x_0)^2 / r \frac{\partial r}{\partial x}}{r^4} =$$

$$= \frac{r^2 - 2(x - x_0)^2}{r^4} = -\frac{1}{r^2} - \frac{2(x - x_0)^2}{r^4}$$

Demak,

$$\frac{\partial^2(\ln r)}{\partial^2 x} = -\frac{1}{r^2} - \frac{2(x - x_0)^2}{r^4}$$

$$\frac{\partial^2(\ln r)}{\partial^2 x} = -\frac{1}{r^2} - \frac{2(y - y_0)^2}{r^4}$$

Demak bularni qo'shamiz

$$\Delta \ln r = \frac{\partial^2(\ln r)}{\partial x^2} + \frac{\partial^2(\ln r)}{\partial y^2} = 0$$

Yuqorida ko'rdikki, vaqtga bog'liq bo'lmagan stasionar chegaraviy masalalarni tekshirishda Laplas va Puasson tenglamalarini

$$\Delta u = 0 \qquad (1.1.1)$$

$$\Delta u = -f(u) \qquad (1.1.2)$$

ko'rgan edik.

Qaralayotgan masala stasionar chegaraviy masalalardan iborat bo‘lganligi uchun bu masalalarda boshlang’ich shartlar ishtirok etmaydi. Xuddi yuqoridagilar kabi chegaraviy masalalarning ko‘rinishlarini qarab chiqamiz.

Ma’lumki,

$$\Delta u = 0 \qquad (1.1.3)$$

$$\Delta u = -f(u) \qquad (1.1.4)$$

(1.1.1) va (1.1.2) ko‘rinishdagi tenglamalar vaqtga bog’liq bo‘lmagan prosesslarning to‘liq tarqalishi qonuniyatlarni aniqlashga yetarli emas. Shu sababli bu qonuniyatlarning to‘la aniqlash uchun topilishi kerak bo‘lgan funksiyaning qaralayotgan sohaning sirtidagi qiymati ma’lum bo‘lishi kerak.

1) S sirt bilan chegaralangan T sohada shunday bir funksiyani $u = u(x, y, z)$ topish kerakki, bu natijada (1.1.1) yoki (1.1.2) shu sohaning sirtida $u / s = f_1(x, y, z)$ berilgan funksiyaga aylansin. Ya’ni quyidagi masalalarga kelamiz.

$$\left.\begin{array}{l} \Delta u = 0 \\ u / s = f_1(x, y, z) \end{array}\right\} \qquad (1.1.5)$$

$$\left.\begin{array}{l} \Delta u = -f \\ u / s = f_1(x, y, z) \end{array}\right\} \qquad (1.1.6.)$$

Bu keltirgan masalalarga elliptik tipdagi tenglamalar uchun qo‘yilgan birinchi chegaraviy masala deb yoki Dirixle masalasi deyiladi.

2) S sirt bilan chegaralangan T sohada shunday bir $u = u(x, y, z)$ funksiyani topish kerakki, natijada bu funksiya (1.1.1) yoki (1.1.2) ni qanoatlantirib quyidagi chegaraviy shartni qanoatlantirsin.

$$\frac{\partial u}{\partial n} / s = f_2(x, y, z)$$

Bundagi

$$\frac{\partial u}{\partial n}/s=\left[\frac{\partial \varphi}{\partial x}\cos\alpha+\frac{\partial \varphi}{\partial y}\cos\beta+\frac{\partial \varphi}{\partial z}\right]/s$$

Tashqi normal bo‘yicha olingan hosilaning S sirtdagi qiymati, ya’ni quyidagi masalaga kelamiz.

$$\left.\begin{aligned}&\Delta u=0\\&\frac{\partial \varphi}{\partial \bar{n}}/s=f_2(x,y,z)\end{aligned}\right\} \tag{1.1.7}$$

$$\left.\begin{aligned}&\Delta u=-f\\&\frac{\partial u}{\partial \bar{n}}/s=f_2(x,y,z)\end{aligned}\right\} \tag{1.1.8}$$

Bu masalalarga elliptik tipli tenglama uchun qo‘yilgan ikkinchi chegaraviy masala yoki Neyman masalasi deyiladi.

3) S sirt bilan chegaralangan T sohada shunday bir funksiyani, ya’ni $u=u(x,y,z)$ ni topish kerakki, natijada bu funksiya shu qaralayotgan sohada (1.1.1) yoki (1.1.2) tenglamani qanoatlantirib, quyidagi chegaraviy shart bajarilsin.

$$\left[\frac{\partial \varphi}{\partial \bar{n}}+ku\right]/s=f_3(x,y,z)$$

ya’ni quyidagi masalaga kelamiz.

$$\left.\begin{aligned}&\Delta u=0\\&\left[\frac{\partial \varphi}{\partial \bar{n}}+Hu\right]/s=f_3(x,y,z)\end{aligned}\right\} \tag{1.1.9}$$

Bu keltirgan masalalargga elliptik tipdagi tenglamalar uchun qo‘yilgan uchinchi chegaraviy masala deb yuritiladi. Bunda ham uchinchi chegaraviy masala umumlashgan chegaraviy masaladan iboratdir. Bu masalalarning yechish jarayonida agar yechimi qaralayotgan T sohaning S sirtga nisbatan sohaning ichkarisida yoki tashqarisida topishni talab etiliishi mumkin. Shu sababli bu xildagi masalalarga ichki yoki tashqi masalalar deyiladi. Bu keltirgan masalalardagi f_1, f_2, f_3 lar sohaning sirtida berilgan funksiyalardan bo‘lib hisoblanadilar.

1.2-§. Grin formulalari va Garmonik funksiyalarning integral munosabatlari

Garmonik funksiyalarning xossalarini o‘rganish uchun avvalo ularning integral munosabatlarini qarab chiqamiz. Bu integral munosabatlarini keltirish uchun esa. Grin formulalaridan foydalanishga to‘g’ri keladi. Grin formulalarining o‘zlari esa Ostogradskiy formulasining xususiy hollaridan iboratdir. Ostogradskiy formulasining ma’nosi esa quyidagichadir. [6]

Faraz qilaylik P,Q,R funksiyalar $S+T$ yopiq sohada o‘zining bir tartibli hosilalari bilan uzluksiz funksiyalardan iborat bo‘lsin. Agarda S sirtda o‘tkazilgan tashqi normal n ni Ox,Oy,Oz o‘qlar bilan tashkil etgan burchaklari mos holda

$$\alpha=(n,x); \quad \beta=(n,y); \quad \gamma=(n,z)$$

lardan iborat bo‘lsin. Bu holda Ostogradskiy formulasi quyidagicha edi.

$$\iiint_T \left[\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}\right]d\tau=\iint_S (P\cos\alpha+Q\cos\beta+R\cos\gamma)ds \qquad (1.2.1)$$

Bu tenglikning chap tomonidagi qavs ichidagi ifoda:

$$\bar{a}(x,y,z)=iP+jQ+kR$$

ko‘rinishdagi vektorlar maydonining yo‘nalishi yoki divergensiyasini bildiradi.

$$div\,\bar{a}=div(iP+jQ+\bar{k}\,R)=\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}$$

skalyardan iborat.

$$\bar{n}=i\cos\alpha+j\cos\beta+k\cos\gamma$$

Bularni hisobga olganda (1) tenglikning o‘ng tomonidagi qavs ichidagi ifoda

$$(\bar{a},\bar{n}) = (iP + jQ + kR; \quad i\cos\alpha + j\cos\beta + k\cos\gamma) =$$
$$= P\cos\alpha + Q\cos\beta + R\cos\gamma = a_n$$

Shu ko‘rinishda (1.2.1) ning ko‘rinishi quyidagicha bo‘ladi.

$$\iiint_T div a d\tau = \iint_S a_n d\tau \qquad (1.2.2)$$

Endi keltirilgan formulalardan foydalanib Grin formulasini hosil qilamiz. Buning uchun faraz qilaylik: S sirt bilan chegaralangan T sohada

$$\left.\begin{matrix} u = u(x,y,z) \\ v = v(x,y,z) \end{matrix}\right\}$$

Funksiyalar o‘zlarining argumentlari bo‘yicha ikkinchi tartibgacha differensiallanuvchi funksiyalardan iborat bo‘lsin. Quyidagicha belgilashlar kiritamiz:

$$\left.\begin{matrix} P = u\dfrac{\partial v}{\partial x} \\ Q = u\dfrac{\partial v}{\partial y} \\ Z = u\dfrac{\partial v}{\partial z} \end{matrix}\right\} \qquad (1.2.3)$$

(1.2.3) ni (1.2.1) ga qo‘yamiz.

$$\iiint_T \left[\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + u\frac{\partial^2 v}{\partial x^2} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial y} + u\frac{\partial^2 v}{\partial y^2} + \frac{\partial u}{\partial z}\frac{\partial v}{\partial z} + u\frac{\partial^2 v}{\partial z^2}\right] d\tau =$$
$$\iint_S \left[\frac{\partial v}{\partial x}\cos\alpha + \frac{\partial v}{\partial y}\cos\beta + \frac{\partial v}{\partial z}coz\gamma\right] ds = d\iint_S u\frac{\partial v}{\partial \bar{n}} ds.$$

$$\frac{\partial u}{\partial \bar{n}}\Big|_S \left[\frac{\partial u}{\partial x}\cos\alpha + \frac{\partial u}{\partial y}\cos\beta + \frac{\partial u}{\partial z}\cos\gamma\right] / S$$

$$\iiint_T n\Delta V d\tau = \iint_S u\frac{\partial u}{\partial \bar{n}} d\tau$$

$$\left.\begin{aligned}\nabla u = gradu = i\frac{\partial u}{\partial x} + j\frac{\partial u}{\partial y} + k\frac{\partial u}{\partial z}\\ \nabla v = gradv = i\frac{\partial v}{\partial x} + j\frac{\partial v}{\partial y} + k\frac{\partial v}{\partial z}\end{aligned}\right\}$$

$$(\nabla u, \nabla v) = \frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial y} + \frac{\partial u}{\partial z}\frac{\partial v}{\partial z}$$

Demak quyidagi masalaga kelamiz.

$$\iiint_T u\Delta v d\tau = \iint_S u\frac{\partial v}{\partial \bar{n}}d\sigma - \iiint_T (\nabla u, \nabla v)d\tau \qquad (1.2.4)$$

(1.2.4) dagi u va v larning o‘rinlarini almashtirib, quyidagilarga kelamiz.

$$\iiint v\Delta u d\tau = \iint_S v\frac{\partial u}{\partial \bar{n}}d\sigma - \iiint_T (\nabla u, \nabla v)d\tau \qquad (1.2.5)$$

(1.2.4) va (1.2.5) ga Grinning birinchi formulasi deb yuritiladi. Endi (1.2.4) va (1.2.5) larni mos ravishda ayirib quyidagiga kelamiz:

$$\iiint_T (u\Delta v - v\Delta u)d\gamma = \iint_S \left[u\frac{\partial v}{\partial n} - v\frac{\partial v}{\partial n}\right]d\tau \qquad (1.2.6)$$

Bunga Grinning ikkinchi formulasi deyiladi. Agarda bu keltirilgan formulalarda qatnashayotgan funksiyalar

$$\left.\begin{aligned}u = u(x, y)\\ v = v(x, y)\end{aligned}\right\}$$

ko‘rinishda bo‘lsa, u holda (1.2.4) va (1.2.5) hamda (1.2.6) ning ko‘rinishi quyidagicha bo‘ladi.

$$\iint_S u\Delta v ds_1 = \int_{C_1} u\frac{\partial v}{\partial \bar{n}}dc_1 - \iint_S (\nabla u, \nabla v)dS_1 \qquad (1.2.7)$$

$$\iint_S v\Delta I ds_1 = \int_{C_1} v\frac{\partial v}{\partial \bar{n}}dc_1 - \iint_S (\nabla v, \nabla n)dS_1 \qquad (1.2.8)$$

$$\iint_S [u\Delta v - v\Delta u] ds_1 = \int_{C_1} \left[u\frac{\partial v}{\partial n} - v\frac{\partial v}{\partial \bar{n}} \right] dS_1 \qquad (1.2.9)$$

$$\begin{cases} \Delta u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 n}{\partial y^2}; \quad \dfrac{\partial v}{\partial \bar{n}} = \dfrac{\partial u}{\partial x}\cos\alpha + \dfrac{\partial u}{\partial y}\cos\beta \\ \nabla u = gradu = i\dfrac{\partial u}{\partial x} + j\dfrac{\partial u}{\partial y} \end{cases}$$

Ya’ni keltirilgan (1.2.4), (1.2.5) va (1.2.6) formulalar tekislik uchun Grin formulalaridan iboratdir. Endi shu funksiyaning ba’zi integral munosabatlarini hosil qilish masalasini qaraymiz.

Endi quyidagi formuladan foydalanib, [3]

$$\iiint_T [u\Delta v - v\Delta u] d\tau = \iint_S \left[u\frac{\partial v}{\partial n} - v\frac{\partial v}{\partial n} \right] ds \qquad (1.2.10)$$

Garmonik funksiyalarning qaralayotgan soha ichidagi ixtiyoriy nuqtadagi qiymatlarini integral orqali ifodalash mumkinligini qaraymiz. Buning uchun esa faraz qilaylik (1.2.10) tenglamada qatnashayotgan $u = u(M)$ funksiya quyidagi shartlarni qanoatlantirsin. Ya’ni bu funksiya $T + S$ yopiq sohada o‘zining birinchi tartibli hosilasi bilan birgalikda uzluksiz hamda T sohaning ichida barcha argumentlari bo‘yicha ikkinchi tartibgacha differensiallanuvchi garmonik funksiyadan iborat bo‘lsin.

Endi sirt bilan chegaralangan T sohaning ichida qo‘zg’almas $M_0(x_0, y_0, z_0)$ va ixtiyoriy $M(x, y, z)$ nuqtalarni olamiz. Bu nuqtalar orasidan masalani esa quyidagicha belgilaymiz.

$$R_{mom} = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$$

endi (1.2.1) formulada qatnashayotgan funksiyani esa quyidagicha belgilaymiz.

$$V = \frac{1}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}} \qquad (1.2.A)$$

Yuqorida ko‘rdikki (1-lemma) agar $M_0 \neq M$ bo‘lsa (1.2.A) ko‘rinishdagi funksiya garmonik funksiyadan iborat bo‘lar edi.

Agarda $M_0 = M$ bo‘lsa, u holda $M_{mom} = 0$ bo‘lib, u holda (1.2.A) ko‘rinishdagi funksiya M_0 nuqtada cheksizga aylanadi. Bu holda (1.2.A) ko‘rinishdagi funksiyani (1.2.10) formulaga tadbiq qilish mumkin emas. (1.2.A) ko‘rinishdagi funksiyaga Grin formulasini tadbiq qilish uchun M_0 nuqtani markaz qilib E radiusli u K_E sfera bilan o‘rab olamiz. Bu sferaning sirti S_E bo‘lsin. T sohadagi shu sferani ajratib olamiz. Ya’ni sohada $T - K_E$ sirti esa $S + S_E$ sirtlar bilan chegaralangan. Demak (A) ko‘rinishdagi funksiya $T - K_E$ sohaning barcha nuqtalarda garmonik funksiyadan iborat bo‘ladi. Shu sababli (1) ko‘rinishdagi Grin formulasini

$V = \frac{1}{R}$ va $u = u(M)$ funksiyalarga tadbiq etamiz: ya’ni

$$\iiint\limits_{T-K}\left[u\Delta\left(\frac{1}{R}\right) - \frac{1}{R}\Delta u\right]d\tau = \iint\limits_{S+S_E}\left[u\frac{\partial}{\partial \bar{n}}\left(\frac{1}{R}\right) - \frac{1}{R}\frac{\partial}{\partial \bar{n}}\right]d\sigma =$$
$$\iint_S\left[u\frac{\partial}{\partial \bar{n}}\left[u\frac{\partial}{\partial \bar{n}}\left(\frac{1}{R}\right) - \frac{1}{R}\frac{\partial}{\partial n}\right]d\sigma\right] + \iint_{S_E} u\frac{\partial}{\partial \bar{n}}\left[u\frac{\partial}{\partial \bar{n}}\left(\frac{1}{2}\right) - \frac{1}{R}\frac{\partial u}{\partial \bar{n}}\right]d\tau \tag{1.2.11}$$

Endi ε ga bog’liq bo‘lgan integrallarni alohida – alohida qaraymiz.

1) Ya’ni $V = \frac{1}{R}$ funksiya $T - K_E$ sohaning barcha nuqtalarda garmonik funksiyadan iboratdir.

2) S_ε sirtga o‘tkazilgan tashqi normal $\bar{n}$ ning yo‘nalishni shu sferaning radiusi yo‘nalishiga teskari deb qabul qilamiz, ya’ni

$$\frac{\partial}{\partial n}\left(\frac{1}{R}\right)/S_\varepsilon = -\frac{\partial}{\partial R}\left(\frac{1}{R}\right)/S_\varepsilon = \frac{1}{R^2}/S_\varepsilon = \frac{1}{\varepsilon^2}$$

3) $\iint\limits_S u\frac{\partial}{\partial \bar{n}}\left(\frac{1}{R}\right)d\tau = \frac{1}{\varepsilon^2} = \iint\limits_{S_E} u d\tau = \frac{u*}{\varepsilon^1}\iint\limits_{S_E} d\delta$

($u*$ - esa $u = u(M)$ funksiya S_ε dagi o‘rtacha qiymati sferaning sirtining bildiradi.)

$$\iint_{S_\varepsilon} d\tau = 4\pi\varepsilon^2 = \frac{4\pi\varepsilon^2 \cdot u*}{\varepsilon^2} = 4\pi u*$$

4) $\iint_{S_\varepsilon} \frac{1}{R}\frac{\partial u}{\partial \bar{n}} d\tau = \frac{1}{\varepsilon} \cdot \iint_{S_\varepsilon} \frac{\partial u}{\partial \bar{n}} d\tau = \frac{1}{\varepsilon}\left(\frac{\partial u}{\partial \bar{n}}\right)\iint_{S_\varepsilon} d\tau = \frac{1}{\varepsilon}\left(\frac{\partial u}{\partial \bar{n}}\right)4\pi\varepsilon^2 = 4\pi\varepsilon\left(\frac{\partial u}{\partial \bar{n}}\right)$

(esa $\left(\frac{\partial u}{\partial \bar{n}}\right)$ ning S_ε dagi o‘rtacha qiymati)

Bu topilgan ifodalarni (1.2.11) ga qo‘yamiz. Ya’ni (1.2.11) ning davomi

$$\iiint_{T-S_\varepsilon}\left[u\Delta\left(\frac{1}{R}\right) - \frac{1}{R}\Delta u\right]d\tau = \iint_S \left[u\frac{\partial}{\partial \bar{n}}\left(\frac{1}{R}\right) - \frac{1}{R}\frac{\partial u}{\partial n}\right]d\tau + 4\pi u* - 4\pi\varepsilon\left(\frac{\partial u}{\partial n}\right) \qquad (1.2.12)$$

Bu tenglikdan $\varepsilon \to 0$ da limitga o‘tkazamiz.

$$\lim_{\varepsilon\to 0}(t - K_E) = T$$

1) $\lim_{\varepsilon\to 0} u* = u(M_0)$

$$\lim_{\varepsilon\to 0} \varepsilon\left(\frac{\partial u}{\partial \bar{n}}\right)^* = 0$$

Bularni hisobga olganda (1.2.12) ning ko‘rinishi quyidagidan iborat bo‘ladi.

$$-\iiint_T \frac{1}{R_{M_0M}} \cdot \Delta u(M) d\tau_\mu =$$

$$= \iint_S \left[u(P)\frac{\partial}{\partial n_p}\left(\frac{1}{R_{M_0p}}\right) - \frac{1}{R_{M_0p}}\frac{\partial u}{\partial \bar{n}_p}\right]d\tau_p + 4\pi u(M_0)$$

$$4\pi u(M_0) = \iint_S \left[\frac{1}{R_{M_0p}}\frac{\partial}{\partial \bar{n}} - u(p)\frac{\partial}{\partial \bar{n}_p} \cdot \frac{1}{R_{M_0p}}\right] - \iiint \frac{\Delta u(\mu)}{R_{M_0p}} d\tau_M \qquad (1.2.13)$$

yoki

$$U(M_0) = \frac{1}{4\pi}\iint_S \left[\frac{1}{R_{M_0p}}\frac{\partial}{\partial \bar{n}} - u(P)\frac{\partial}{\partial \bar{n}_p} \cdot \frac{1}{R_{M_0p}}\right]d\tau - \frac{1}{4\pi}\iiint_T \frac{\Delta u(\mu)}{R_{M_0p}} d\tau_M \qquad (1.2.14)$$

Bunga Grinning asosiy integral formulasi deb yuritiladi. Yoki garmonik funksiyalarning integral munosabatlaridan iboratdir. Agarda oxirgi ifoda U funksiyaning T sohadagi garmonikligini hisobga olsak (1.2.14) ning ko‘rinishi quyidagidan iborat bo‘ladi.

$$U(M_0)=\frac{1}{4\pi}\iint_S\left[\frac{1}{R_{M_0p}}\frac{\partial}{\partial\bar{n}}-u(P)\frac{\partial}{\partial\bar{n}_p}\cdot\frac{1}{R_{M_0p}}\right]d\sigma_p \tag{1.2.15}$$

Bu ham garmonik funksiyaning integral munosabatidan iborat bo‘lib undan quyidagini tushunamiz. Ya’ni T sohadagi garmonik bo‘lgan funksiyaning shu sohaning M_0 nuqtasidagi qiymati shu funksiyaning hamda shu funksiyadan tashqi normal bo‘yicha hosilaning S sirtidagi qiymati orqali ifodalanar ekan. Buning uchun esa $u=u(M)$ funksiya hamda uning birinchi tartibli hosilasining T sohadagi uzluksizligi talab qilinar ekan.

Endi (1.2.15) ko‘rinishdagi integral munosabatlariga yuqoridagi Lemma 2 va Lemma 3 ni tadbiq etamiz. Ya’ni Lemma 2 ni tadbiq qilsak,

$$V=\ln\frac{1}{R_{M_0p}}=\ln\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2}}$$

funksiya $M_0(x_0,y_0)\neq M(x,y)$ bo‘lsa, u holda (1.4.6) ning ko‘rinishi quyidagicha bo‘ladi.

$$U(\mu_0)=\frac{1}{2\pi}\int_c\left[\ln\frac{1}{R_{M_0p}}\cdot\frac{\partial u}{\partial n_p}-u(P)\frac{\partial}{\partial\bar{n}_p}\left(\ln\frac{1}{R_{M_0p}}\right)\right]dC_1 \tag{1.2.16}$$

Lemma 3 ni tadbiq etsak

$$V=\ln R_{M_0p}=\ln\sqrt{(x-x_0)^2+(y-y_0)^2}$$

$$M_0(x_0,y_0)\neq M(x,y)$$

Bo‘lganda garmonik funksiyadan iborat bo‘ladi.

$\Delta(\ln R_{M_0p})=0$ bu funksiyani hisobga olganda (1.2.15) funksiyaning ko‘rinishi quyidagidan iborat bo‘ladi.

$$U(\mu_0)=\frac{1}{2\pi}\int_c\left[\ln R_{M_0p}\cdot\frac{\partial\mu}{\partial n_p}-u(P)\frac{\partial}{\partial\bar{n}_p}\left(\ln R_{M_0p}\right)\right]dC_1 \qquad (1.2.17)$$

Bu formulalar ham garmonik funksiyalarning tekislik uchun integral munosabatlarni bildiradi.

1.3-§. Doira uchun Dirixlening ichki va tashqi masalalari. Puasson integrali.

Endi $\Delta u=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ Lappas tenglamasi uchun Dirixle masalasini qarab chiqamiz. [4]

MASALA: R radiusli doirada (ichkarida yoki tashqarida) shunday bir $u=u(x,y)$ funksiyani toppish kerakki natijada bu funksiya quyidagi chegaraviy shartlarni $U|_R=f$ qanoatlantirsin: ya’ni: quyidagi masalaga keladi

$$\left.\begin{aligned}&\Delta u=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\\&u/R=f\end{aligned}\right\}=0 \qquad (1.3.A)$$

Bundagi f doiraning konturida berilgan funksiyadan iboratdir.

Endi (1.5.A) ko‘rinishidagi masalaning yechimini topamiz. Bu uchun esa Lappas tenglamasining qutb koordinata, sistemasidan foydalanamiz,

Ya’ni:

$$\Delta u(p,\varphi)=\frac{1}{p^2}\frac{\partial}{\partial p}\left(p\frac{\partial}{\partial p}\right)+\frac{1}{p^2}\frac{\partial^2 u}{\partial\varphi^2}=0 \qquad (1.3.1)$$

MASALA: Markazi koordinata boshida joylashgan, radiusi R dan iborat bo‘lgan doiraning ichkarisida yoki tashqarisida shunday bir funksiyani toppish kerakki, $u=u(p,v)$ natijada bu funksiya bir qiymatli bo‘lib, ya’ni $u(p,\varphi+2\pi)=u(p,\varphi)$ bo‘lib, u quyidagi masalaning yechimini bildiradi.

$$\Delta u = \frac{1}{p^2}\frac{\partial}{\partial p}\left(p\frac{\partial}{\partial p}\right) + \frac{1}{p^2}\frac{\partial^2 u}{\partial \varphi^2} = 0 \qquad (1.3.2)$$

$$u(0,\varphi) = chekli$$
$$u(R,\varphi) = f(\varphi)$$
$$u(p,\varphi + 2\pi) = u(p,\varphi)$$

Bu (1.3.1) masalaning yechimini ham xuddi yuqoridagilar kabi o‘zgaruvchilarni ajratishni metodidan foydalanib topamiz.

$$u(p,\varphi) = w(p)\phi(\varphi)(2)w(p) \neq 0$$
$$\phi(\varphi \quad 2\varphi) \quad \phi(p) \neq 0$$

(1.3.2) ni (1.3.1) ga qo‘yamiz.

$$p\phi(\varphi)\frac{d}{dp}\left[p\frac{dw}{dp}\right] + \phi^{\pi}(\varphi)w(p) = 0$$

$$p\phi(\varphi)\frac{d}{dp}\left[p\frac{dw}{dp}\right] + \phi^{\pi}(\varphi)w(p) = 0 \left| \frac{1}{w(p)\phi(\varphi)} \right.$$

$$p\frac{d}{dp}\frac{\left[p\frac{dw}{dp}\right]}{w(p)} = -\frac{\phi^{\pi}(\varphi)}{\phi(\varphi)} = \lambda$$

Bundan quyidagi Shturm – Livull masalalariga kelamiz.

$$\begin{cases} \Phi''(\varphi) + \lambda(\varphi) = 0 \\ \Phi(\varphi + 2\pi) = (\varphi) \end{cases} \qquad (1.3.3)$$

$$p\frac{d}{dp}\left[p\frac{dw}{dp}\right] - \lambda w(p) = 0 \qquad (1.3.4)$$

Endi bu masalalarning yechimlarini topamiz. (1.3.3) tenglama oddiy 2-tartibli differensial tenglama bo‘lganligi uchun uning yechimining ko‘rinishi quyidagidan iborat bo‘lib.

$$\Phi(\varphi) = \alpha\cos\sqrt{2\varphi} + \sin\sqrt{\lambda\varphi} \qquad (1.3.5)$$

Bu funksiya bir qiymatli bo‘lishi kerak, ya’ni.

$\Phi(y + 2\pi) = \alpha\cos\sqrt{\lambda}(\varphi + 2\pi) = \Phi(\varphi)$ bo‘lishi uchun $\sqrt{\lambda = n \in Z}$ shart bajarilishi kerak. U holda $\lambda_n = n^2$ bo‘ladi (1.3.2) (1.3.3) ni (1.3.4) ga qo‘yamiz.

Bu (1.3.3) masalaning yechimidan iboratdir. Endi (1.3.4) tenglamaning yechimini topdamiz. Ya’ni

$$p\frac{d}{dp}\left[p\frac{dw}{dp}\right]-\lambda w(p)=0$$

(1.3.3) ga asosan bu tenglamaning ko‘rinishi quyidagidan iborat bo‘ladi.

$$p\frac{d}{dp}\left[p\frac{dw_n}{dp}\right]-n^2 w_n(p)=0 \qquad (1.3.6)$$

Bu tenglama 2 – tartibli o‘zgaruvchi koeffiesientli oddiy differensial tenglamadan iboratdir. Bu tenglamani yechimini toppish uchun uni o‘zgarmas koeffisiyentli tenglamaga keltiramiz.

Shu maqsadda quyidagi almashtirishlarni bajaramiz;

a) $p=e^t$ almashtirishdan foydalanib, tenglamani o‘zgarmas koeffisiyentli tenglamaga keltiramiz.

b) Hosil bo‘lgan o‘zgarmas koeffisiyentli tenglamada $\omega_n(p)=e^{k!}$ almashtirishdan foydalanib, xarakteristik tenglama tuziladi.

Bularni hisobga olganda (1.3.4) tenglamada quyidagi ko‘rinishli almashtirishdan foydalanamiz:

$W_n(p)=e^{kt}=(e^t)^t=p^k$ (1.5.4) (1.3.5) dan quyidagini $\omega_n(p)=e^k$ topamiz

$$\frac{dw_n}{dp_1}=kp^{k-1} \qquad (1.3.7)$$

(1.3.7) ni va (1.3.6) ni 1.3.5) ga qo‘yamiz.

$$p\frac{d}{dp}[p\cdot k\cdot p^{k-1}]-n^2p^k=0$$

$$\frac{d}{dp}[kp^k]-n^2p^{k-1}=0$$

$$k^2p^{k-1}-n^2p^{k-1}=0$$

$$k^2-n^2=0\Rightarrow k_1=n,k_1=-n$$

Bularni hisobga olishdan (1.3.6) ni ko‘rinishi quyidagicha bo‘ladi.

$$W_{n1}(p)=p^n,\quad W_{n2}(p)=p^{-n} \qquad (1.3.8)$$

Bular (1.3.8) tenglamaning xususiy yechimlaridan iboratdir. U holda (1.3.4) tenglamaning umumiy yechimi quyidagicha bo‘ladi.

$$W_n(p) = c_1 W_{n1}(p) + c_2 W_{n2}(p) = c_1 p^n + c_1 p^{-n}$$
$$W_n(p) = c_1 p^n + c_1 p^{-n} \qquad (1.3.9)$$

(1.3.9) tenglama (1.3.8) ning umumiy yechimidir.

Endi (1.3.9) dan doira uchun qo‘yilgan Dirixlening ichki va tashqi masalalarning yechimini ajratib olamiz.

a) Doira uchun ichki Dirixle masalasi, ya’ni $p < R$ bo‘lsin. Bu holda topilgan yechim doiraning markazida ham chekli bo‘lishi kerak. Shu sababli $p = 0$ da (1.3.9) da quyidagicha kelamiz. $w_n(0) - c_1 0 + \frac{c_2}{0}$ chekli bo‘lishi kerak.

$$w_n(p) = p^n \quad c_2 = 0, \quad c_1 = 1 \neq 0 \quad p < R \,(1.5.10)$$

ichki Dirixle masalasi uchun

b) $(p < R)$ tashqi Dirixle masalasi.

Bu holda topilgan yechim $p = \infty$ da ham cheksiz bo‘lishi kerak. U holda (1.3.10) ning ko‘rinishi quyidagidan iborat bo‘ladi.

$$w_n(\infty) = c_1 \infty + \frac{c_2}{\infty}$$

chekli bo ‘lishi kerak (1.3.10) ning ko‘rinishi quyidagicha bo‘ladi.

$$w_n(p) = \frac{1}{p_n} \qquad (1.3.7) \qquad c_1 = 0, \quad c_2 = 1 \neq 0$$

$(p > R)$ tashqi Dirixle masalasi uchun topiladi (1.3.8) va (1.3.7) larni (1.3.2) ga qo‘yib, quyidagi ko‘rilishiga kelamiz.

$$u_n(p,\varphi) = [a_n \cos n\varphi + \beta_n \sin\varphi] p^n \qquad (1.3.11)$$

$p > R$ ichki Dirixle masalasining yechimiga kelamiz.

Xuddi shuningdek (1.3.11) va (1.3.2) qo‘yib, quyidagicha kelamiz,

$$u_n(p,\varphi) = \frac{[a_n \cos n\varphi + \beta_n \sin\varphi]}{p^n} \qquad (1.3.12)$$

$p > R$ tashqi Dirixle masalasining yechimiga kelamiz.

Yoki bularning yig’indilarini olib quyidagilarga ega bo‘lamiz.

$$u_n(p,\varphi)=\sum_{n=\infty}^{\infty}[a_n\cos n\varphi+\beta_n\sin\varphi]p^n=$$
$$=a_0+\sum_{n=1}^{\infty}[a_n\cos n\varphi+\beta_n\sin\varphi]p^n$$

$p<R$ da.

$$u(p,\varphi)=\sum_{n=1}^{\infty}\frac{[a_n\cos n\varphi+\beta_n\sin\varphi]}{p^n}=$$
$$=a_0+\sum_{n=1}^{\infty}\frac{[a_n\cos n\varphi+\beta_n\sin\varphi]}{p^n},\qquad p>R\quad da$$

Bulardan ba’zi hisoblashlarni qulaylashtirish maqsadida a_0 ni $R_0=\frac{a_0}{2}$ bilan almashtiramiz.

$$u(p,\varphi)=\frac{a_0}{2}+\sum_{n=1}^{\infty}[a_n\cos n\varphi+\beta_n\sin\varphi]\cdot p^n \qquad (1.3.13)$$

$p<R$ - ichki masalaning yechimi

$$u(p,\varphi)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\frac{[a_n\cos n\varphi+\beta_n\sin\varphi]}{p^n} \qquad (1.3.14)$$

$p>R$ - tashqi masalaning yechimi.

Bu hosil bo‘lgan qatorlar mos ravishda R radiusli doiraning ichkarisida garmonik bo‘lgan funksiyalardan iboratdir. Ulardan birini qarash kifoyadir.

Ya’ni (1.3.14) qator a_n va b_n koeffisiyentlarning chekli qiymatlarida doiraning ichkarisida tekis yaqinlashuvchi qatoridan iboratdir.

Chunki bu qator $0<p<1$ bo‘lganda bu qatorni $0<p<p<1$ shart bajarilganda quyidagi ko‘rinishli

$$M(1+p_1+p_1^2+p_1^3+\ldots+p_1^n+\ldots) \qquad (1.3.15)$$

Qator yordamida takrorlash mumkin. Ya’ni (1.3.14) qator (1.3.10) qator uchun mojarant qator rolini bajaradi. Chunki (1.3.14) cheksiz kamayuvchi geometric progressiyadan iboratdir. Demak bu qator yaqinlashuvchi u holda (1.3.10) qatorning tekis yaqinlashuvchiligiga kelamiz.

Ikkinchi tomonidan esa (1.3.10) qatorning har bir qadi garmonik funksiyadan iboratdir. Bunga ishonch hosil hosil qilish uchun uni x,y koordinatalar yerdamida quyidagicha yozamiz.

$$\operatorname{Re}al\left[(a_n - i\beta_n)(x+iy)^n\right] = p^n(a_n \cos n\varphi + \beta_n \sin n\varphi)$$

$$u(x,y) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \operatorname{Re}al\left[(a_n - i\beta_n)(x+iy)^n\right] \qquad \text{(A)}$$

$$\left.\begin{aligned} x &= p\cos\varphi \\ y &= p\cos\varphi \end{aligned}\right\}$$

$$\operatorname{Re}al[(a_n - i\beta_n)p^n[\cos\varphi + i\sin]^n] =$$

$$p^n \operatorname{Re}al[(a_n - i\beta_n)(\cos n\varphi + \beta_n \sin n\varphi)] =$$

$$p^n \operatorname{Re}al[a_n \cos n\varphi + ia\sin n\varphi - i\beta_n \cos n\varphi] =$$

$$= p^n(a_n \cos n\varphi + \beta_n \sin n\varphi)$$

Endi (A) ko‘rinishidagi funksiyaning garmonik funksiya ekanligiga ishonch hosil qilish mumkin.

$$u(x,y) = \frac{a_0}{2} + \sum A_n(x+iy)^n$$

Bundan tegishli hosilalarni hisoblaymiz:

$$\frac{\partial u}{\partial x} = \sum A_n n(x+iy)^{n-1}$$

$$\frac{\partial^2 u}{\partial x^2} = \sum A_n n(n-1)(x+iy)^{n-2}$$

$$\frac{\partial u}{\partial y} = \sum A_n in(x+iy)^{n-1}$$

$$\frac{\partial^2 u}{\partial x^2} = -\sum A_n(n-1)n(x+iy)^{n-2}$$

Bulardan esa $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ ga kelamiz.

Bundan ko‘rinadiki (1.3.10) qatorning har bir hadi garmonik funksiyadan iborat. Endi (1.3.10) va (1.3.12) qatorlardagi koeffisiyentlarni hisoblaymiz. Buning uchun esa chegaraviy shartdan foydalanamiz. Ya’ni (1.3.10) va (1.3.12) da *p=R* da quyidagilarga kelamiz.

$$u(R,\varphi) = f(\varphi) = \frac{a_0}{2} + \sum_{n=1}\left[a_n R^n \cos n\varphi + \beta_n R^n \sin n\varphi\right] \qquad (1.3.16)$$

$p<R$ ichki Dirixle masalasi uchun:

$$u(R,\varphi) = f(\varphi) = \frac{a_0}{2} + \sum_{n=1}\left[\frac{a_n^n}{R^n}\cos n\varphi + \frac{b_n}{R^n}\sin n\varphi\right] \qquad (1.3.17)$$

p>R tashxis Dirixle masalasi uchun.

(1.3.16) va (1.3.17) da quyidagi almashtirishlarni kiritamiz.

$$\left.\begin{aligned} a_0 = \alpha_0;\ a_n R^n = \alpha_n,\ b_n R^n = \beta_n p < R \quad uchun \\ a_0 = \alpha_0;\ \frac{a_n}{R^n} = \alpha_n,\ \frac{b_n}{R^n} = \beta_n p < R \quad uchun \end{aligned}\right\} \qquad (B)$$

Bu belgilashlarni hisobga olganda, (1.3.2) va (.1.3.14) lar quyidagidan iborat bo‘ladi.

$$f(\varphi) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos n\varphi + \beta_n \sin n\varphi] \qquad (V)$$

Endi (B) dagi koeffisiyentlarni hisoblaymiz.

$$a_0 = \alpha_0;\ \alpha_n = \frac{a_n}{R^n},\ b_n = \frac{\beta_n}{R^n},\ \ p < R \quad uchun$$

$$a_0 = \alpha_0;\ \alpha_n = a_n R^n,\ b_n = R^n \beta_n,\ \ p < R \quad uchun$$

Bu koeffisiyentlarni (1.5.10) va (1.5.12) larga qo‘yamiz. Ya’ni

$$u(\rho,\varphi) = \frac{\alpha_0}{9} + \sum_{n=1}^{\infty}\left(\frac{P}{R}\right)^n [\alpha_n \cos n\varphi + \beta_n \sin\varphi], \qquad (1.3.18) \quad \rho < R \quad uchun$$

$$u(\rho,\varphi) = \frac{\alpha_0}{2} + \sum_{n=1}^{\infty}\left(\frac{R}{P}\right)^n [\alpha_n \cos n\varphi + \beta_n \sin\varphi], \qquad (1.3.19) \quad \rho > R \quad uchun$$

Bu keltirilgan (1.3.18) va (1.3.19) qatorlar doira uchun qo‘yilgan ichki va tashqi masalalarning yechimlaridan iborat.

Endi (1.3.18) va (1.3.19) dagi koeffisiyentlarni hisoblaymiz. Buning uchun esa (1.3.5) tenglikning 2 la tomonini 1 ga, $\cos n\varphi$ ga ko‘paytirib, hosil bo‘lgan tengliklarni $[0,2\pi]$ oraliqda integrallaymiz.

$$\int_0^{2\pi} f(\psi)d\psi = \frac{\alpha_0}{2}\cdot\int_0^{2\pi} d\psi + \sum_{n=1}^{\infty}\left[L_n \int_0^{2\pi}\cos n\psi d\psi + \beta_n \int_0^{2\pi}\sin\varphi d\psi\right]$$

$$\int_0^{2\pi} f(\psi)d\psi = \frac{\alpha_0}{2}\psi\Big|^{2\pi} 0 = \frac{\alpha_0 \cdot 2\pi}{2} = \alpha_0 T$$

$$\left.\begin{aligned}
&\alpha_0 = \frac{1}{\pi}\int_0^{2\pi} f(\psi)d\psi \\
&\textit{xuddi shuningdek} \\
&f(\psi)\cos n\psi d\psi \\
&\alpha_0 = \frac{1}{\pi}\int_0^{2\pi} f(\psi)\cos n\psi d\psi \\
&\beta_0 = \frac{1}{\pi}\int_0^{2\pi} f(\psi)\sin n\psi d\psi
\end{aligned}\right\}$$

Demak, doira uchun qo‘yilgan ichki va tashqi masalalarning yechimlari mos ravishda (1.3.18) va (1.3.19) lardan iborat bo‘lib, ulardagi koeffisiyentlar esa (1.3.10) ko‘rinishdagi formulalar yordamida topiladi.

Endi yuqorida ko‘rilgan doira uchun Dirixli masalasining yechimlari bo‘lgan quyidagi (1.3.1) va (1.3.2) qatorlarning soddaroq holini ko‘rib chiqamiz.

$$u(\rho,\varphi) = \frac{\alpha_0}{2} + \sum_{n=1}^{\infty}\left(\frac{P}{R}\right)^n \left[\alpha_n \cos n\varphi + \beta_n \sin\varphi\right], \qquad \rho < R \qquad (1.3.20)$$

$$u(\rho,\varphi) = \frac{\alpha_0}{2} + \sum_{n=1}^{\infty}\left(\frac{P}{R}\right)^n \left[\alpha_n \cos n\varphi + \beta_n \sin\varphi\right], \qquad \rho > R \qquad (1.3.21)$$

Ma’lumki bu qatorlardagi koeffisiyentlar quyidagilardan iborat edilar;

$$\begin{aligned}
&\alpha_0 = \frac{1}{\pi}\int_0^{2\pi} f(\psi)d\psi \\
&\alpha_n = \frac{1}{\pi}\int_0^{2\pi} f(\psi)\cos n\psi d\psi \qquad (1.3.22) \\
&\beta_0 = \frac{1}{\pi}\int_0^{2\pi} f(\psi)\sin d\psi
\end{aligned}$$

Bu qatorlarni soda holda keltirish maqsadida ulardan biridan foydalanamiz, ya’ni, doira uchun Dirixle masalasining ichki yechimini qaraymiz. Buning uchun esa (1.3.22) ni (1.3.20) ga qo‘yamiz.

$$u(\rho,\varphi) = \frac{L_0}{2} + \sum_{n=1}^{\infty}\left(\frac{P}{R}\right)^n \left[L_n \cos\varphi + \beta_n \sin\varphi\right] = \frac{1}{\pi}\int_0^{2\pi} f(\psi)\left[\frac{1}{2} + \sum_{n=1}^{\infty}\left(\frac{P}{R}\right)^n\right]$$

$$\left[\cos n\varphi \cos n\psi + \sin\varphi \sin\psi\right]d\psi =$$

$$\frac{1}{\pi}\int_0^{2x} f(\psi)\left[\frac{1}{2} + \sum_{n=1}^{\infty}\left(\frac{P}{R}\right)^n \cos(\psi-\varphi)d\psi = \frac{1}{T}\int_0^{2x} f(\psi) =\right.$$

$$\frac{1}{2}+\sum_{n=1}^{\infty}\left(\frac{P}{R}\right)^{n}\cdot\frac{e^{in(\psi-\varphi)}+e^{-in(\psi-\varphi)}}{2}\Bigg]d\varphi=\frac{1}{\pi}\int_{0}^{2x}f(\psi)$$

$$\left[1+\sum_{n=1}^{\infty}\left(\frac{P}{R}\cdot e^{1(\psi-\varphi)}\right)^{n}+\sum_{4=1}^{\infty}\left(\frac{P}{R}\cdot e^{-1(\psi-\varphi)}\right)^{n}\right]d\varphi=\frac{1}{2\pi}\int_{0}^{2x}f(\psi)$$

$$\left[\frac{1+p/\mathrm{Re}^{i(\psi-\varphi)}+p/\mathrm{Re}^{-in(\psi-\varphi)}}{1-p/\mathrm{Re}^{\pm i(\psi-\varphi)}}\right]d\varphi=\frac{1}{2\pi}\int_{0}^{2x}f(\psi).$$

$$\left[\frac{1-p/\mathrm{Re}^{i(\psi-\varphi)}-p/\mathrm{Re}^{-i(\psi-\varphi)}+p^{2}/R^{2}+p/\mathrm{Re}^{i(\psi-\varphi)}-p^{2}/R^{2}+p/\mathrm{Re}^{-i(\psi-\varphi)}-p^{2}/R^{2}}{1-p/\mathrm{Re}^{i(\psi-\varphi)}-p/\mathrm{Re}^{-i(\psi-\varphi)}+p^{2}/R^{2}}\right]d\varphi=$$

$$=\frac{1}{2\pi}\int_{0}^{2x}f(\psi)\frac{1-\frac{P^{2}}{R^{2}}}{1-p/R(e^{i(\psi-\varphi)}+e^{-i(\beta\psi-\varphi)}+p^{2}/R^{2})}d\psi=$$

$$=\frac{1}{2T}\int_{0}^{2x}f(\psi)\frac{R^{2}-P^{2}}{R^{2}+p^{2}-pR\left(\frac{e^{i(\psi-\varphi)}+e^{-i(\psi-\varphi)}}{2}\right)}d\psi=$$

$$=\frac{1}{2T}\int_{0}^{2x}f(\psi)\frac{R^{2}-P^{2}}{R^{2}+p^{2}-2pR\cos(\psi-\varphi)}d\psi;$$

Shunday qilib;

$$u(p,\varphi)=\frac{1}{2T}\int_{0}^{2x}f(\psi)\frac{R^{2}-P^{2}}{R^{2}+p^{2}-2pR\cos(\psi-\varphi)}d\psi \qquad (1.3.23)$$

$p< R$ ichki masala uchun. (1.3.23) dagi

$$K(p,\varphi,R,\psi)=\frac{R^{2}-P^{2}}{R^{2}+p^{2}-2pR\cos(\psi-\varphi)} \qquad (1.3.24)$$

ga Puasson yadrosi deyiladi.

Agar $p<R$ bo‘ladi, ya’ni

$$k(p,\varphi,R,\psi)>0 \text{ bo'lsa, } /\cos(\psi-\varphi)/=1$$

$$(R-p)^{2}=R^{2}+p^{2}-2pR>0$$

$$(MN)^{2}=R^{2}+p^{2}-2pR\cos(\psi-\varphi) \text{ bo'ladi.}$$

Agar (1.3.23) formulada $p=R$ bo‘lsa u holda bu formula o‘z ma’nosini yo‘qotadi. Bu holda Puasson integralini hosil qilishdagi qatorlarini yig’indisi yopiq sohada uzluksizdir. Shu sababli quyidagiga ega bo‘lamiz.

$$\lim u(p,\varphi)=u(R,\varphi)=F(\varphi) \quad p\to R$$

Demak, doira uchun qo‘yilgan Dirixle ichki masalasining yechimi quyidagidan iborat bo‘ladi.

$$u(p,\varphi)=\begin{cases}\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} f(\psi)\dfrac{R^2-P^2d\psi}{R^2+p^2-2pR\cos(\psi-\varphi)}, & p<R,\\ f(\varphi), \quad p=R \qquad bo'lsa\end{cases}$$

Xuddi shuningdek doira uchun qo‘yilgan Dirixle tashqi masalasining yechimi quyidagidan iborat.

$$u(p,\varphi)=\begin{cases} f(\varphi), \quad agar \quad p=R \qquad bo'lsa\\ \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} f(\psi)\dfrac{R^2-P^2d\psi}{R^2+p^2-2pR\cos(\psi-\varphi)}, & p<R,\end{cases}$$

Quyidagi masalaning yechimini toping

$$\left.\begin{array}{l}\Delta u=\dfrac{1}{p}\dfrac{\partial}{\partial p}\left[p\dfrac{\partial u}{\partial p}+\dfrac{\partial^2 u}{\partial\varphi^2}\dfrac{1}{p^2}=0\right]\\ u(0,\varphi)=chekli\\ u(R,\varphi)=\cos u=f(\varphi)\\ u(r,\varphi+2\pi)=u(p,\varphi)\end{array}\right\}$$

Bu masalaning yechimi quyidagidan iborat.

$$u(p,\varphi)=\begin{cases}\dfrac{1}{2\pi}\displaystyle\int_0^{2x}\cos+\dfrac{R^2-P^2d\psi}{R^2+p^2-2pR\cos(\psi-\varphi)}, & p<R\\ \cos\varphi, \quad agar, \quad p=R \quad bo'lsa\end{cases}$$

$$u(p,\varphi)=\begin{cases}\dfrac{1}{2\pi}\displaystyle\int_0^{2x}\cos+\dfrac{R^2-P^2d\psi}{R^2+p^2-2pR\cos(\psi-\varphi)}, & p>R\\ \cos\varphi, \quad agar, \quad p=R \quad bo'lsa\end{cases}$$

1.4-§. Garmonik funksiyalarning soda xossalari

Endi yuqoridagi garmonik funksiyalarga tegishli bo‘lgan formulalardan foydalangan holda shu funksiyalarning ba’zi soda xossalarini qarab chqamiz. Buning uchun esa quyidagi xossalarni alohida – alohida qaraymiz. [5]

Xossa – 1: Agar $u = u(M)$ va $v = v(M)$ o‘zining funksiyalar S+T yopiq sohada o‘zining 1 – tartibli xosilasi bilan birga uzluksiz, hamda T sohaning ichida garmonik funksiyadan iborat bo‘lsa, u holda quyidagi tengliklar o‘rinlidirlar.

$$\iint_S \frac{\partial u}{\partial n} d\tau = 0; \quad \iint_S \frac{\partial v}{\partial n} d\tau = 0$$

Isbot: bunga ishonch hosil qilish Grinning (1.4.1) formulasidan foydalanamiz.

$$\iiint_T u\Delta v d\tau = \iint_S u \frac{\partial v}{\partial n} d\tau - \iint_T (\Delta u, \Delta v) ds$$

$$\iiint_T v\Delta u d\tau = \iint_S v \frac{\partial u}{\partial n} d\tau - \iint_T (\Delta u, \Delta v) ds$$

Agar bu formulalarning 1- sida

$v = c = const$ (hamma vaqt garmonik funksiyadan iborat) deb olsak,

$$\Delta(c) = i\frac{\overset{=0}{\partial c}}{\partial x} + j\frac{\overset{=0}{\partial c}}{\partial y} + k\frac{\overset{=0}{\partial c}}{\partial z} = 0, \quad \Rightarrow \Delta(c) = 0$$

bo‘ladi.

Bu holda formulalarning ko‘rinishlari quyidagicha bo‘ladi.

$$\left.\begin{matrix} 0 = \iint c\dfrac{\partial u}{\partial \bar{n}} \partial\tau - 0 \\ 0 = \iint c\dfrac{\partial v}{\partial \bar{n}} \partial\tau - 0 \end{matrix}\right\} \Rightarrow \quad \left\{\begin{matrix} \iint_S \dfrac{\partial u}{\partial \bar{n}} \partial\tau = 0 \\ \iint_S \dfrac{\partial v}{\partial \bar{n}} \partial\tau = 0 \end{matrix}\right.$$

Sizga ma’lumki, elliptic tipdagi tenglamalar uchun 2 chegaraviy masalani qo‘yilishi quyidagidan iborat edi.

$$\left.\begin{matrix} \Delta u = 0 \\ \dfrac{\partial u}{\partial \bar{n}} / S = f \end{matrix}\right\} \qquad \text{(A)}$$

Bu masalaning yechimini topishda xossa 1-ni tadbiq etamiz. Xossa 1-ni asosan (A) masalaning chegaraviy sharti quyidagidan iborat bo‘lidi

$$\iint_S \frac{\partial u}{\partial \bar{n}} d\tau = \iint_S f d\tau = 0 \qquad \text{(B)}$$

Demak (A) ko‘rinishidagi Neyman masalasi yechimiga ega bo‘lishi uchun (B) ko‘rinishidagi shart bajarilishi kerak. (B) shart qaralayotgan prosessdagi manbani yo‘qotish sharti deyiladi.

Bu shartga garmonik funksiyaning xossasini chegaraviy masalani yechishdagi tadbiqi deb yuritiladi.

Xossa – 2: (garmonik funksiya uchun o‘rta qiymat haqidagi Gauss tebranmasi).

Teorema.1: agar $u = u(M)$ funksiya $S+T$ yopiq sohada o‘zining birinchi tartibli hosilasi bilan uzluksiz hamda T sohada garmonik funksiyadan iborat bo‘lsa va $M_0(x_0, y_0, z_0)T$ sohadagi ixtiyoriy nuqtadan iborat bo‘lsa, g holda garmonik funksiya U ning M_0 nuqtadagi qiymati quyidagi formula yordamida aniqlanadi.

$$u(M_0) = \frac{\iint_{SR} u(p) d\tau}{4TR^2}$$

Bunga garmonik funksiya uchun o‘rta qiymat haqidagi Gaucc formulasi deb yuritiladi.

Isbot: Bunga ishonch hosil qilish uchun T sohaning ichida M_0 nuqtadagi olib, shu nuqtani markaz hisoblab, K radiusli T_R sferani hosil qilamiz.

Sferaning sirti S_R dan iborat bo‘lsin. Bizga ma’lumki, garmonik funksiyaning M_0 nuqtadagi qiymati Grinning integral formulasi yordamida quyidagicha aniqlanar edi.

$$u(\mu_0) = \frac{1}{4\pi} \iint_{S_R} \left[\frac{1}{r} \frac{\partial u}{\partial \bar{n}} - u(p) \frac{\partial}{\partial \bar{n}} \left(\frac{1}{r} \right) \right] d\sigma \qquad \text{(A)}$$

Bundagi integrallarning alohida – alohida hisoblaymiz.

a) S_R sirtga o‘tkazilgan tashqi normal $\bar{n}$ ning yo‘nalishini sfera radiusining yo‘nalishi bilan bir xil qilib yo‘naltiramiz. Ya’ni

$$\frac{\partial}{\partial \bar{n}}\left(\frac{1}{r}\right)\Bigg| S_R = \frac{\partial}{\partial r}\Bigg| S_R = -\frac{1}{r^2}|S_R = -\frac{1}{R^2}$$

b) $\frac{1}{R}\frac{\partial u}{\partial \bar{n}} S_R = \frac{1}{R}\frac{\partial u}{\partial \bar{n}}|S_R$

$$\iint\limits_{S_R} \frac{1}{R}\frac{\partial u}{\partial \bar{n}} d\sigma = \frac{1}{R}\iint\limits_{S_R} \frac{\partial u}{\partial \bar{n}}\partial\sigma = 0$$

Xossa 1 ga asosan.

Bularga asosan (A) ning ko‘rinishi quyidagicha bo‘ladi:

$$u(\mu_0) = -\frac{1}{4\pi}\iint\limits_{S_R} M(p)\frac{\partial}{\partial \bar{n}}\left(\frac{1}{r}\right)d\tau = \frac{\iint_{S_R} u(p)d\tau}{4RT^2}$$

Isbot bo‘ldi.

Xossa – 3: (garmonik funksiyalar uchun maksimal qiymat prinsipi haqidagi teorema).

Teorema.2: Agar $u = u(M)$ funksiya $T + S$ yopiq sohada o‘zining 1 tartibli hosilasi bilan uzluksiz, hamda T sohada garmonik funksiyadan iborat bo‘lsa, u holda u funksiya o‘zining eng katta yoki eng kichik quymatlariga shu T sohaning S sirtida erishadi.

Bunga ishonch hosil qilish uchun T sohaning ichidagi M_0 nuqtani markaz qilib, K radiusli T_R sferani chizamiz. Bu sferaning sirti S_R bo‘lsin. Yuqoridan ma’lumki, T sohanng ichida yotuvchi M_0 nuqtadagi garmonik funksiyaning qiymati Gauss formulasi yordamida aniqlanadi.

Isbot: Teskarisini faraz qilaylik:

Faraz qilaylik: Teorema shartini qanoatlantiruvchi $u = u(M)$ funksiya o‘zining eng katta qiymatini sferani S_R sirtida emas, balki uning ichki biror M_0 nuqtasida erishsin. Ma’lumki garmonik funksiyaning M_0 nuqtadagi erishgan qiymati quyidagicha bo‘ladi:

$$U_0 = u(M_0) \geq u(M) \qquad \text{(k)}$$

Ikkinchi tomonidan ma’lumki, bu funksiyaning M_0 dagi qiymati quyidagi formula yordamida aniqlanar edi.

$$u(M_0)=\frac{\iint_{S_R}u(p)d\tau}{4\pi R^2}$$

Agar bu formulaga (k) tengsizlikni tadbiq etsak, quyidagiga kelamiz.

$$u(M_0)=\frac{\iint_{S_R}u(p)d\tau}{4\pi R^2}=\frac{u(M_0)}{4\pi R^2}\iint_{S_R}u4\pi R^2 d\tau=\frac{4\pi R^2 u(M)}{4\pi R^2}=u(M_0)$$

Demak,

$$u(M_0)\geq u(M_0) \qquad (k_1)$$

Agar qaralayotgan T_R sferaning birorta nuqtasida (k) munosabatdagi katta ishora o‘rniga kichik ishora o‘rinli bo‘lsa, u holda (k_1) dagi kichik ishoraga kelamiz. Ya’ni $u(M_0)\prec u(M_0)$

Buning bo‘lishi mumkin emas. Bu natijaga esa garmonik funksiya o‘zining eng katta qiymatiga sohaning ichki nuqtasiga erishadi, degan farazimiz tefayli kelinadi.

Demak, bu funksiya o‘zining eng katta qiymatini shu sohaning sirtida qabul qiladi, degan xulosaga kelamiz. Xuddi yuqoridagilar kabi garmonik funksiyalarning xossasidan foydalanib, eliptik tipdagi tenglamalar uchun qo‘yilgan chegaraviy masalalar yechimining yagonaligini, hamda chegaraviy shartlardan uzluksiz bog’langan ko‘rsatish mumkin.

Agar Dirixle masalasining yechimi mavjud bo‘lsa, u holda u yagonadir.

Ya’ni

$$\left.\begin{array}{l}\Delta u_1=0\\ u_1/s=f\end{array}\right\}(a) \qquad \left.\begin{array}{l}\Delta u_2=0\\ u_2/s=f\end{array}\right\}(b)$$

Teskarisini faraz qilamiz: $u_1\neq u_2$ bo‘lsin. Mos ravishda yuqoridagilarni ayiramiz.

$$\left.\begin{array}{l}\Delta(u_1-u_2)=0\\ (u_1-u_2)/s=0\end{array}\right\}\Rightarrow u_1-u_2\equiv 0\Rightarrow u_1\equiv u_2$$

Xuddi shuningdek masala yechimining chegaraviy shartlardan uzluksiz bog’liqligini ham ko‘rsatish mumkin.

I Bob bo’yicha xulosa.

Birinchi bobda elliptik tipdagi tenglamalar uchun asosiy chegaraviy masalalar o’rganilgan. Grin formulalari va Garmonik funksiyalarning integral munosabatlari keltirilgan.

Bu bobda garmonik funksiyalar va garmonik funksiyalarning xossalari keltirilgan. Bu xossalardan biri integral formula va o’rta qiymat haqidagi teorema. Garmonik funksiyalar uchun yagonalik teoremasi hamda Drixle masalalarining chegaralanmagan sohalarda yechimning yagonaligini qanoatlantiruvchi shartlardan iborat.

Garmonik funksiyalar uchun integral formula quyidagilardan iborat. Uch o’lchovli fazoda garmonik funksiyalar uchun integral formula quyidagicha

$$U(M_0)=\frac{1}{4\pi}\iint_S\left[\frac{1}{R_{M_0P}}\frac{\partial}{\partial\bar{n}}-u(P)\frac{\partial}{\partial\bar{n}_p}\cdot\frac{1}{R_{M_0P}}\right]d\sigma_p$$

bo’ladi.

II- Bob. To‘rt o‘lchovli Koshi-Riman sistemasi uchun sohada Koshi masalasi.

2.1-§ Matrisalar sinfi.

Agar $R^n - n$ o‘lchovli [2]

$$x = \begin{Vmatrix} x_1 \\ .... \\ x_n \end{Vmatrix} \qquad \text{va} \qquad y = \begin{Vmatrix} y_1 \\ .... \\ y_n \end{Vmatrix},$$

$C^n - n$ o‘lchovli, kompleks maydonlar fazosidan olingan bo‘lsin.

$$z = \begin{Vmatrix} z_1 \\ .... \\ z_n \end{Vmatrix}, \quad \text{o‘zgaruvchilar kompleks fazosi;}$$

$$\frac{\partial}{\partial x} = \begin{Vmatrix} \frac{\partial}{\partial x_1} \\ .... \\ \frac{\partial}{\partial x_n} \end{Vmatrix}, \qquad \partial x = \begin{Vmatrix} dx_1 \\ .... \\ dx_n \end{Vmatrix} \qquad \text{va} \qquad E(x) = \begin{Vmatrix} x_1 ...0 \\ ..... \\ 0....x_n \end{Vmatrix}$$

Agar $I = (i_1 ... i_q)$, $0 \le q \le n$ multindeks, u holda

$$dx_I = \bigwedge_{m=1}^{z} dx_{im}$$

$$dx[J] = \bigwedge_{i=1}^{n} dx_i \quad ; \; i \notin J$$

$$dx = \bigwedge_{i=1}^{n} dx_i \quad ;$$

$\sigma(I) -$ konstanta, $\sigma(I) dx_j \Lambda dx[J] = dx$ tenglik bilan aniqlanadi, ψ operator esa tashqi differensial formada aniqlangan, $q -$darajali forma.

$$\psi(x) = \sum\nolimits_{|J|=q}^{'} \psi(x)dx$$

(shtirix yig'indi multindekslar o'smasligi bo'yicha olmasligini bildiradi.)

$$*\psi(x) = \sum_{|J|=q} \psi_1(x)\sigma(I)dx[I]$$

tenglik bilan aniqlanadi.

*operatorni $\psi^1,...\psi^n$ formalardan tuzilgan $D(\psi^1,...\psi^n)$ matrisalarga

$$*D(\psi^1,...\psi^n) = D(*\psi^1,...*\psi^n)$$

deb faraz qilib qo'llaymiz.

$R(x)$ va $C(x)$ orqali koeffisienlari mos ravishda R yoki C maydondan bo'lgan $x_1,...,x_n$ larning ko'phadlar chiziqli algebralarni belgilaymiz. $LR(x)$ va $LC(x)$ mos ravishda $R(x)$ yoki $C(x)$ ning qism to'plamlari bo'lib , chiziqli formalardan iborat.

k va l - natural sonlar, $p_i(x)...p_l(x) \in R(x)$

$$p(x) = \begin{Vmatrix} p_i(x) \\ ..... \\ p_l(x) \end{Vmatrix} p_i(x) = 0 \text{ yoki } x \in R^n x \neq 0 \ \ i = \overline{1,l} \, .$$

(bu shart $p_i = (\frac{\partial}{\partial x})$ operatorlar elliptik ekanligini bildiradi). U holda $A_{k\times l}(p(x), x)$ elementlarni $LC(x)$ dan bo'lgan $(k \times l)$ – matrisa $\Gamma D(x)$ mavjudki,

$$\Gamma D(x) \cdot D(x) = E(p(x)) \qquad (2.1.1)$$

o'rinli bo'lib, uning elementlari $LC(x)$ dan bo'lgan $(k \times l)$ tartibli $D(x)$ matrisalar sinfini bildiradi. Belgilashlar qulay bo'lishi uchun ya'ni ,

$$1_l = \begin{Vmatrix} 1 \\ ... \\ 1 \end{Vmatrix} = l \quad \text{marta}$$

vektorni kiritamiz. $O_{k\times l}(x)$

$$D^*(x)\cdot D(x)=E(|x|^2 1_l)$$

o‘rinli bo‘ladigan $D(x)$ matrisalardan iborat.

$A_{k\times l}(|x|^2 1_l, x)$ qism sinfni bildiradi, bu yerda

$$|x|^2=\sum_{i=1}^{n} x_i^2,\quad D^*(x)-D(x)$$

matrisaga ermit qo‘shma.

Agar $p(x)-R(x)$ dagi ko‘phad, $p(x)\neq 0$, $x\neq l$, $x\neq 0$ bo‘lsa, u holda

$$A_{k\times l}(p(x),x)$$

$$FD(x)D(x=E(p(x)1_l) \qquad (2.1.2)$$

$$D(x)FD(x=E(p(x)1_k) \qquad (2.1.3)$$

shartlarni qanoatlantiruvchi elementi $C(x)$ dan bo‘gan $(k\times l)$ matrisa mavjud bo‘ladiki, $D(x)$ matrisalar

$$A_{k\times l}(p(x)1_l, x)$$

qism sinfi bo‘ladi. $O_{k\times l}(x)$

$$\{D^*(x)D(x)-E(|x|^2 1_l)\}$$

$$\{D(x)D^*(x)=E(|x|^2 1_k)\}\ .$$

shartni qanoatlantiruvchi $D(x)$ matrisalarning $O_{k\times l}(x)$ qism sinfi.

Nihoyat $C^m(Q)$ orqali bu $Q-R^n$ dagi to‘plam $m=0,1,2,...,\infty$ Q atrofda m marta uzluksiz differensiallanuvchi ko‘phadlar sinflari.

Agar $f_i(x)\in C^m(Q),\quad i=\overline{1,e}$ bo‘lsa,

$$f(x)=\begin{vmatrix} f_1(x) \\ \dots \\ f_l(x) \end{vmatrix} \in C^m(Q)$$

ga vektor funksiya deymiz.

2.2-§ Birinchi tartibli elliptik tipli tenglamalar sistemasi uchun integral formula.

Xususiy hosilalardagi birinchi tartibli $[4]$

$$D(\frac{\partial}{\partial x})f(x)=0 \qquad (2.2.1)$$

chiziqli differensial tenglamalar sistemasini qaraymiz, uning xarakteristik matrisasi

$$D(x)\in A_{k\times l}(p(x),x) \qquad (2.2.2)$$

$$f(x)=\begin{vmatrix} f_1(x) \\ \dots \\ f_l(x) \end{vmatrix} \qquad (2.2.3)$$

esa kompleks qiymatli umumiy holda vektor funksiya .

$g(_1x)...g_l(x)$ - mos ravishda

$$p_1(\frac{\partial}{\partial x})....p_l(\frac{\partial}{\partial x})$$

operatorlarning fundamental yechimlari va

$$g(x)=\begin{vmatrix} g_1(x) \\ \dots \\ g_l(x) \end{vmatrix}$$

$$M(x)=\left(E(g(x))\cdot \Gamma D\left(\frac{\partial}{\partial x}\right)\right)\cdot\left(*D\left(\frac{\partial}{\partial x}\right)\right) \qquad (2.2.4)$$

$(l\times l)$ – matrisa differensial operator bu yerda formal ravishda ,

$$N(x)=\left(E(g(x))\cdot\Gamma D\left(\frac{\partial}{\partial x}\right)\right)\cdot\left(*D\left(\frac{\partial}{\partial x}\right)\right) \qquad (2.2.5)$$

$$*D\left(\frac{\partial}{\partial x}\right)=D\left(*\frac{\partial}{\partial x}\right)=D\left(\frac{\partial}{\partial x}dx\right)=D\left(\frac{\partial}{\partial x}\right)dx$$

deb faraz qilamiz.

(2.2.2) va (2.2.1) dan

$$dM(x)=*E(g(x))\left(D\cdot\Gamma D\left(\frac{\partial}{\partial x}\right)\right)=*E(g(x))E\left(p\left(\frac{\partial}{\partial x}\right)\right)=*(E(\delta(x)1_l)=\delta(x)Edx \qquad (2.2.6)$$

kelib chiqadi, bu erda $E=E(1_l)$ birlik $(l\times l)$ – matrisa , $\delta(x)$ Dirak δ -funksiyasi.

Agar (2.2.3) vektor funksiya C^l sinfga tegishli bo‘lsa, u holda (2.2.4) va (2.2.5) dan

$$M(x)\Lambda df(x)=(-1)^{n-1}N(x)f(x) \qquad (2.2.7)$$

kelib chiqadi.

(2.2.5) va (2.2.6) dan birdaniga tayinlangan x uchun ayniyat kelib chiqadi:

$$d(M(y-x)f(y))-N(y-x)f(y)=\delta(y-x)f(y)dy \qquad (2.2.8)$$

Endi R^n dagi elliptik operatorlarning fundamental yechimlari xossalari sohadagi [3] dagi natijani e’tiborga olib, (2.2.5) ni integrallash bilan silliq bo‘lakli ∂G chegarali $G\in R^n$ chegaralangan soha va $C^1(\overline{G})$ sinfning (2.2.1) vektor funksiya uchun quyidagi integral formula o‘rinli.

$$\int_{\partial G_y} M(y-x)f(y)-\int_G N(y-x)f(y)=\begin{cases} f(x),\ x\in G,\\ 0,\quad x\notin\overline{G}\end{cases} \qquad (2.2.9)$$

(2.2.6) ni bu sistematik $D(x)$ xarakteristik matrisasiga (2.2.2) shartga esa (2.2.3) sistema yechimlari ucnun qurilgan Grin formulasi sifatida qarash mumkin.

(2.2.3) ni hisobga olib, (2.2.5) dan standart usul bilan quyidagi teorema olinadi.

2.2.1-teorema. $G \in R^n$ - silliq bo‘lakli chegarali chegaralangan soha va (2.2.5) $f(x) \in C^1(G) \cap C(\bar{G})$ vektor funksiya (2.2.2) shartda (2.2.1) sistemaning yechimi bo‘lsin , u holda

$$\int_{\partial G} M(y-x) f(y) = \begin{cases} f(x), \; x \in G, \\ 0, \quad x \notin \bar{\bar{G}} \end{cases} \qquad (2.2.10)$$

(2.2.7) formuladan va [8] natijadan xususan, (2.2.1) sistemaning $C^1(G)$ sinfdagi yechimi (2.2.2) shart bajarilganda haqiqatdan, G sohada analitik funksiyalar bo‘ladi. Bundan va (2.2.7) misoldan ko‘rinadiki, (2.2.1) sistemaga (2.2.2) shartning qo‘yilishi bu sistema elliptik bo‘lishiga teng kuchli. (2.2.1) elliptik sistemalar yechimlari uchun soha chegarasi bo‘yicha integral tasvirlashlarning mavjudligi [7] da isbotlangan.

2.2.1 eslatma. 2.2.1-teoremada $A_{k\times l}(p(x), x)$ determenantda $p_i(x), i = \overline{1, l}$ ko‘phadlarni faqat β gipoelliptikligini talab qilsak, o‘zida saqlaydi.

Bunda (2.2.1) sistemaning $C^1(G)$ sinfga tegishli barcha yechimlari G sohagan tegishli bo‘ladi . Bundan tashqari yuqoridagi barcha hisoblashlar unchalik katta bo‘lmagan o‘zgarishlar bilan ixtiyoriy sistemalarda, shu sistemalarga o‘tkaziladi, faqat qo‘shma $*D\left(\frac{\partial}{\partial x}\right)y(x) = 0$ sistemaning tegishli xossalarga ega fundamental yechimlar matrisasi mavjud bo‘lishi lozim.

2.3-§ Birinchi tartibli elliptik tipli tenglamalar sistema uchun misollar.

2.3.1-misol.

$(n\times n)$ matrisa $\quad \|a_{ij}\| a_{ij}+a_{ji}=0, \quad i=j, \quad a_{ij}=1$ (2.3.1)

shartni qanoatlantiruvchi $a_{ij}, \quad i,j=\overline{1,n}$ kompleks sonlardan iborat bo‘lsin.

U holda $(n\times 1)$ matrisa

$$D(x)=\begin{Vmatrix} \sum_{j=1}^{n} a_{ij}x_j \\ \ldots\ldots \\ \sum_{j=1}^{n} a_{jn}x_j \end{Vmatrix} \in A_{n\times 1}(|x|^2, x) \qquad (2.3.2)$$

chunki (2.3.1) shartga ko‘ra $\Gamma D(x)=\|x_1 \ldots x_n\|$ deb olish mumkin. [7]

2.3.2- misol.

[10] da R^3 fazoda C^1dagi Koshi-Riman sistemasining uning yechimlari ma’nosidagi analitik o‘xshash Koshi integralini tuzish mumkin. (4×4) – matrisa

$$\begin{Vmatrix} 0 & x_1 & x_2 & x_3 \\ x_1 & 0 & x_3 & x_2 \\ x_2 & x_3 & 0 & -x_1 \\ x_3 & -x_2 & x_1 & 0 \end{Vmatrix} \qquad (2.3.3)$$

(2.3.1) ko‘rinishidagi Montel-Teodoresko sistemasi hisoblanadi.

(2.3.3) matrisa $O_{4\times 4}(x), \quad x\in R^3$ sinfga tegishli bo‘lishi oson tekshiriladi. [6]

2.3.3-misol.

[11] da [10] ning umumlashmasi sifatida

$$D(f)\frac{\partial}{\partial x}=0 \qquad (2.3.4)$$

sistemaning yechimlari uchun R^n da Koshi integrali analogi o‘rnatilgan, bu yerda $D(x)-$ giperkompleks $n\times n$ matrisa (bunda matrisalar faqat $n=2^m,\quad m=1,2,...$[11] da mavjud).

Haqiqatdan giperkompleks $n\times n$ matrisa $O_{n\times n}(x),\quad x\in R^n$ sinfga tegishli (2.3.) esa [11] dan formulaning boshqacha ko‘rinishi.

2.3.4-misol.

[12] da [10] natijaning boshqa umumlashmasi $(n-2)$ darajali forma (komponentlariga) va

$$\begin{cases}\delta\psi=0,\\ d\psi=*du\end{cases}\quad \left(\begin{Vmatrix}\delta & 0\\ d & -*d\end{Vmatrix}\begin{Vmatrix}\psi\\ u\end{Vmatrix}=0\right) \tag{2.3.5}$$

u differensiallarga nisbatan birinchi tartibli chiziqli differensial tenglamalar sistemasi qaralgan. δ operator R^n dagi $q-$ chi darajali formalarda

$$\delta\psi=(-1)^{n(q+1)+1}*d*\psi \tag{2.3.6}$$

tenglik bilan aniqlanadi.

Bu sistemaning xarakteristik matrisasi

$O^c{}_{(c_n^1+c_n^3)\times(c_n^2+1)}(x)$ sinfga tegishli (2.1.10) formula [12] dagi integral tasvirlashlarning boshqacha ko‘rinishini beradi.

2.3.5- misol.

[14] da

$$(D(x))^2=E()|x|^2 1_2 n \tag{2.3.7}$$

sharti esa $(2^n\times 2^n)-D(x)$ matrisalar qaralgan.

Bunday matrisalar Klifford [14] algebraik tasvirlashlarni tuzishda paydo bo‘ladi. Ko‘rish qiyin emaski, (2.3.7) matrisalar

$A_{2^n\times 2^n}(|x|^2 1_{2^n}, x)$ sinfga tegishli (bu yerda $F = E(1_{2^n})$ (2.1.10) –[13] dagi integral formulasi boshqacha ko‘rinishi).

2.3.6-misol.

$$\begin{cases} \delta\psi = 0, \\ d\psi = 0 \end{cases} \quad \left(\left\| \begin{matrix} \psi \\ d \end{matrix} \right\| \psi = 0 \right) \tag{2.3.8}$$

sistemaning xarakteristik matrisasi ∂ operator (3.6) da $q, 0 \le q \le n$ darajali ψ forma (komponentlari) nisbati R^n da aniqlangan.

$$O^c{}_{(c_n^{q-1}+c_n^{q+1})\times(c_n^q)}(x)$$

sinfga tegishli bo‘ladi.

$\psi \in C^1(G)$ - (2.3.8) sistema yechimlari G da garmonik maydonlar deb ataladi.[12]

2.3.7-natija.

Agar $GCCR^n$ - bo‘lakli silliq chegarali soha va $\psi \in C^1(\overline{G})$ forma G da q, $0 \le q \le n$ darajali garmonik maydon hisoblanadi, u holda quyidagi integral formula o‘rinli.

$$\int_{\partial G_y} \psi(y)\Lambda u_q(y-x) + *\psi(y)\Lambda *_x u_{n-q}(y-x) + \begin{cases} \psi(x), \, x \in G \\ 0, \, x \notin G \end{cases} \tag{2.3.9}$$

$$U_q = (y-x) = \sum_{|j|=q}' \sum_{\substack{k=1 \\ k \notin j}}^{n} \sigma(j,k) \frac{\partial g(y-x)}{\partial y_k} dy[j,k] dx_j$$

$0 \le q \le n-1, U_n(y-x)$, $d_{yk}\Lambda dy[j,k] = \sigma(j,k) dyg(x) - R^n$ da Laplas tenglamasining fundamental yechimi $*_x$ operator esa ψ o‘zgaruvchi bo‘yicha olinadi.

(3.9) formula (3.8) sistema yechimlari uchun qurilgan (2.10) formulaning boshqacha ko‘rinishi.

2.3.8-misol.

C^n fazoda $(p \neq q)$, $0 \leq p \leq q \leq n$ tipdagi ψ forma (komponentlari) nisbatan

$$\begin{cases} \bar{\partial}\psi = 0 \\ \bar{\partial}^* \psi = 0 \end{cases} \left(\begin{Vmatrix} \bar{\partial} \\ \bar{\partial}^* \end{Vmatrix} \psi = 0 \right) \tag{2.3.10}$$

sistemani qaraymiz. Bu yerda $\bar{\partial}$ - Koshi-Riman operatori unga metrik qo‘shma $\bar{\partial}^* = -*\bar{\partial}*$, $*$-operator (p,q) tipdagi

$$\sum_{|J|=p,|I|=q} \psi_{JI}(z) dz_J \Lambda d\bar{\bar{z}}_I$$

formalardagi operator.

$$*\psi(z) = \left[\left(\frac{2}{i}\right)^{p+q-n} i^{q-p} (-1)^{p(n-p)} \right] \sum_{|J|=p,|I|=q} \sigma(J)\sigma(I)\psi_{JI} dz[J]\Lambda d\bar{z}[I]$$

tenglik bilan aniqlanadi.

Bu sistemaning xarakteristik matrisasi

$$O_{C_n^p (C_n^{q-1} + C_n^{q+1}) \times C_n^p C_n^q (x)}$$

sinfga tegishli bo‘ladi , chuning uchun (3.10) yechimlari uchun (2.1.10) formulaning boshqacha ko‘rinishini olamiz.

2.3.9-natija.

Agar $G \in C^n$ - silliq bo‘lakli chegaraga ega chegaralangan soha va (p,q) tipdagi $\psi \in C^1(\bar{G})$ forma (2.3.10) sistemani qanoatlantirsa, u holda

$$\int_{\partial G\zeta} \psi(\zeta)\Lambda U_{p.q}(\zeta - z) + *\psi(\zeta)\Lambda *_z | V_{n-q,n-p}(\zeta - z) = \begin{cases} \psi(z), & z \in G \\ 0, & z \notin \bar{G} \end{cases} \tag{2.3.11}$$

$$U_{p,q}(\zeta - z) = \frac{(-1)^{p(n-q-1)}(n-1)!}{(2\pi i)^n} \sum_{\substack{|I|=q \\ |J|=q}}' \sum_{\substack{k=1 \\ k\notin j}}^{n} \sigma(j,k)\sigma(I) \frac{\zeta_k - z_k}{|\zeta - z|^{2n}} d\zeta[j,k] \Lambda d\zeta[I] d\zeta_j \Lambda dz_I$$

$$V_{p,q}(\zeta - z) = \frac{(-1)^{p(n-q-1)}(n-1)!}{(2\pi i)^n} \sum_{\substack{|I|=q \\ |J|=q}}' \sum_{\substack{k=1 \\ k\notin j}}^{n} \sigma(I,k)\sigma(J) \frac{\zeta_k - z_k}{|\zeta - z|^{2n}} d\zeta[I,k] \Lambda d\zeta[J] d\zeta_I \Lambda dz_J$$

$0 \le p, q \le n, U_{p,n} = 0, V_{n,q} = 0$ - mos ravishda Martinelli-Boxnera-Koppel'man yadrosi.

[16] va uning ∂ operatori uchun analogi $*_x$ operator z o‘zgaruvchi bo‘yicha olinadi. Xususan (2.3.11) $B_{p,n-1}(\bar{G})$ tipidagi formalarning soha chegarasi bo‘yicha integral ifodasini beradi.

2.3.10-misol.

Ixtiyoriy $(k \times l) - D(x)$ matrisa uchun

$$adj(D^*(x)D(x))(D^*(x)D(x)) - E(|D^*(x)D(x)|1_l) \qquad (2.3.12)$$

bu yerda $adj(D^*D)$ - matrisaga biriktirilgan matrisa $(D^*D) - D^*D$ ning ditermenanti (2.3.12) tenglikdan $|D^*(x)D(x)| \ne 0, \quad x \in R^n, \quad x \ne 0$ shartni qanoatlantiruvchi elementar $LC(x)$ dan bo‘lgan har bir $(k \times l) - D(x)$ matrisa $A_{k\times l}(|D^*(x)D(x)|1_l, x)$ sinfga tegishli bo‘lishi kelib chiqadi.

Bundan tashqari (2.3.1), (2.3.3) va (2.3.2) tengliklardan $(k \times l) - D(x)$ matrisa $|D^*(x)D(x)| \ne 0, \quad x \in R^n, \quad x \ne 0$ elementlari $LC(x)$ dan bo‘lgani uchun $A_{k\times l}(|D^*(x)D(x)|1_l, x)$ sinfga tegishli bo‘lishining etarli sharti.

$$D(x)adj(D^*(x)D(x))(D^*(x)D(x)) = E(|D^*(x)D(x)|1_k) \qquad (2.3.13)$$

(2.3.1) sistemaning $D(x)$ matrisasi $A^c_{k\times l}(p(x)1_l, x)$ sinfga tegishli bo‘lsin ,u holda

$$M(x)\left(FD\left(\frac{\partial}{\partial x}\right)g(x)\right)(*D(\partial x)) \qquad (2.3.14)$$

yadroli (2.3.10) integral tasvirlashni (2.3.1) sistema uchun Koshi integrali deb ataladi.

Agar $S-R^n$ dagi chekli yuzali silliq bo‘lakli giper sirt va $(l\times 1)-f(x)\in C(S)$ vektor funksiya bo‘lsa, u holda

$$\Phi(x)=\int_{S_y} M(y-x)f(y) \qquad (2.3.15)$$

Ifoda S ga tegishli bo‘lmagan har bir $x\in R^n$ da ma’noga ega va [8] natijaga $(l\times 1)$ vektorga ko‘ra komponentlari $R^n\setminus S$ da haqiqatda analitik funksiyalardan iborat.

$$D\left(\frac{\partial}{\partial x}\right)M(y-x)=-\delta(y-x)E(1_k)(*D(\partial y))$$

kelib chiqadi, (2.3.15) dan $x\notin S$ da

$$D\left(\frac{\partial}{\partial x}\right)\Phi(x)=0 \qquad (2.3.16)$$

ni olamiz va (2.3.15.) integralni (2.2.1) sistema uchun Koshi tipidagi integral deb atash mumkin.

2.3.11-misol.

Quyidagi tasdiqlarni bayon etishga imkon beradi.

2.3.1- mulohaza.

Agar (2.1) sistemaning $(k\times l)-D(x)$ xarakteristik matrisasi (2.3.13) sharti va

$$|D^*(x)D(x)|\neq 0,\quad x\in R^n,\quad x\neq 0$$

ni qanoatlantirsa, u holda bu sistema uchun

$$M(x)\left\{\left[\left(adj\left(D^*D\right)D^*\right)\left(\frac{\partial}{\partial x}\right)\right]g(x)\right\}(*D(\partial x))$$

yadroli Koshi tipdagi integralni tuzish mumkin, bu yerda $g(x)-|D*D|\left(\frac{\partial}{\partial x}\right)$ operatorning fundamental yechimi.

2.3.2- mulohaza.

Agar (2.3.1)- ma’lum xarakteristik $(l\times l)-D(x)$ matrisani R^n dagi xususiy hosilasi birinchi tartibli chiziqli differensial tenglamalar elliptik sistemasi bo‘lsa, u holda

$$M(x)=\left[adj(D)\left(\frac{\partial}{\partial x}\right)g(x)\right](*D(\partial x))$$

yadroli Koshi tipidagi integralni ko‘rish mumkin, bu yerda

$$|D|\left(\frac{\partial}{\partial x}\right)g(x)=\delta(x)$$

2.4-§ Birinchi tartibli elliptik tipli tenglamalar sistema uchun Morere teoremasi.

$C\in R^n$- bo‘lakli silliq ∂G chegarasi bilan chegaralangan soha bo‘lsin. $D(x)-(k\times l)$ matrisa $LC(x)$ elementlaridan iborat.

$f(x)-C^1(\overline{G})$ sinfning $(l\times 1)$ vektor Stoks formulasi bo‘yicha bevosita ,

$$\int_G\left(*D\left(\frac{\partial}{\partial y}\right)\right)f(y)=\int_{\partial G}(*D(\partial y)f(y)) \tag{2.4.1}$$

ni olamiz.

Agar $f(x)(*D)\left(\frac{\partial}{\partial y}\right)f(x)$ (2.1) sistemaning yechimi bo‘lsa, (5.1) formuladan

$$\int_{\partial G}(*D(\partial y))f(y)=0$$

kelib chiqadi. Yuqoridagi teoremaning fazoviy analogi bo‘lib, quyidagi tasdiq hisoblanadi.

24.1- mulohaza.

Agar :

1) $(l \times l) -$ vektor $f(x)$ funksiya $G \in R^n$ sohada uzluksiz;

2) $D(x) \in A_{k \times l}(p(x), x) -$ matrisa;

3) ixtiyoriy yopiq bo‘lakli silliq σ bilan chegaralangan chekli sohasi bilan kiruvchi S sirt uchun

$$\int_{\sigma} (*D(\partial y)) f(y) = 0$$

tenglik o‘rinli bo‘lsa , u holda $f(x)$ (2.4.1) sistemaning har bir $x \in G$ nuqtada yechimi bo‘ladi.

$f \in C^1(G)$ ushbu mulohazani isbotlash (2.4.1) formulaning to‘g’ri natijasi, bunda 2)- shart faqat va faqat $D(x)$ matrisa $LC(x)$ elementlaridan tuzilgan bo‘lsa.

$f \in C(G)$ uchun (bu mulohazani) [8] natijadan foydalanib dastlab ([17]244-245b) da taklif etilgan soha bo‘yicha haqiqiy o‘lchov uchun har bir $x \in G$ nuqtasini biror atrofida $f(x)$ (2.1.10) formula bilan tasvirlanadi, bu yerda integrallash bu atrofning chegarasi bo‘yicha olinadi. Demak, $f(x) - G$ da haqiqiy analitik funksiya.

II Bob bo'yicha xulosa.

Ikkinchi bobda Matrisalar sinfi o'rganilgan. Birinchi tartibli elliptik tipli tenglamalar sistemasi uchun integral formula keltirilgan.

Bu bobda elliptik sistemalar uchun umumiy holda ta'rifi keltirilgan bo'lib bu ta'rif orqali ifodalanuvchi elliptik sistemalar elliptik tenglamalar orqali faktorizasiyalanadi. Bulardan eng sodda holi Laplas tenglamasi bilan faktorizasiyalanuvchi sistemalar misol bo'la oladi. Bunday sistemalarga bu bobda bir nechta misollar keltirilgan. Har bir elliptik sistema uchun unga mos bo'lgan integral formulaning o'rinliligi ko'rsatilgan va bu integral formuladan foydalanib Morere teoremasining o'rinliligi ham ko'rsatilgan. Bu yerda integral formulaning ko'rinishi umumiy holda quyidagi ko'rinishda

$$\int_{\partial G} M(y-x)f(y) = \begin{cases} f(x), x \in G, \\ 0, \quad x \notin \overline{G} \end{cases}$$

bo'ladi.

III BOB. Elliptik tipli tenglamalar sistemasi uchun to‘rt o‘lchovli fazoda chegaralanmagan sohada Koshi masalasi

3.1-§ To‘rt o‘lchovli elliptik sitema va unga qo‘yilgan Koshi masalasi .

Quyidagi belgilashlarni kiritamiz. [2]

$R^n - n$ o‘lchovli Evklid fazosidan olingan bo’lib, bunda

$$x = \begin{Vmatrix} x_1 \\ .... \\ x_4 \end{Vmatrix} \qquad \text{va} \qquad y = \begin{Vmatrix} y_1 \\ .... \\ y_4 \end{Vmatrix}$$, vektorlar bo‘lsin.

$C^n - n$ o‘lchovli, kompleks maydon bo’lib,

$$z = \begin{Vmatrix} z_1 \\ .... \\ z_4 \end{Vmatrix},$$ o‘zgaruvchilar kompleks fazosidagi vektor bo‘lsin.

$$\frac{\partial}{\partial x} = \begin{Vmatrix} \dfrac{\partial}{\partial x_1} \\ .... \\ \dfrac{\partial}{\partial x_n} \end{Vmatrix}, \qquad \partial x = \begin{Vmatrix} dx_1 \\ .... \\ dx_n \end{Vmatrix} \qquad \text{va} \qquad E(x) = \begin{Vmatrix} x_1 ... 0 \\ ..... \\ 0 .... x_n \end{Vmatrix}$$

Bu yerda $E(x)$ dioganal matritsa

$$r = |y - x| = \sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2 + (y_3 - x_3)^2 + (y_4 - x_4)^2}$$

$$\alpha^2 = (y_1 - x_1)^2 + (y_2 - x_2)^2 + (y_3 - x_3)^2, \quad S = \alpha^2$$

$$\mathrm{r} = \sqrt{\alpha^2 + (y_4 - x_4)^2}$$

$$w = i\sqrt{u^2 + \alpha^2} + y_4 \, u \geq 0$$

$x^T = (x_1, x_2, x_3, x_4)^T$ $x-$vektorning transpozitsiyasi

$u = \{u_1(x) u_2(x) .... u_m(x)\}^T$ nomalum funksiya

$u^0 = (1,1.....1) \in R^n$ vektor.

$R(x)$ va $C(x)$ lar haqiqiy va kompleks sonlar maydonidan olingan koefitsentlari haqiqiy yoki kompleks sonlardan iborat bo‘lgan ko‘phadlarni belgilaymiz.

G soha 4-o‘lchovli fazoda chegrasi silliq S sirt bilan va $\partial G / S$ sirtdan iborat bo‘lgan soha bo‘lsin.

Bu sohada $A_{exm}(p(x), x)$- matrisalar sinfini belgilaymiz, unda $D(x^T)$ elementlari chiziqli formadan ya’ni $P_n(x)$ dan iborat bo‘lgan matritsa bo‘sin, koefitsentlari C maydondan olingan bo‘lib, quyidagi shartni qanoatlantirsin.

$$D*(x^T)D(x^T) = E\left\| x \right|^2 u^0\right| \qquad (3.1.1)$$

Bu yerda $D*(x^T)$ matritsa $D(x^T)$ matrisaga ermetli qo‘shma matritsadan iboratdir.

G sohada quyidagi tenglamalar sistemasini qaraymiz

$$D(\frac{\partial}{\partial x})u(x) = 0 \qquad (3.1.2)$$

Bu yerda $D(x^T) \in A_{exm}(p(x), u)$ sinfga qarashli bo‘lgan xaraktristik matrisaga ega bo‘lgan sistemadan iborat. 2-o‘lchovli fazoda (3.1.2) ko‘rinishdagi sistema Koshi – Riman sistemasi misol bo‘la oladi.

3-o‘lchovli fazoda esa Moisel – Teodorosku sistemasi misol bo‘la oladi.

(3.1.1) sistema uchun Koshi masalasi quyidagicha qo‘yiladi.

Faraz qilaylik $U(y)$ vektor funksiya $C^1(\overline{G}) \cap C(\overline{G})$ sinfga qarashli bo‘lib (3.1.1) sistemani qanoatlantirsin. $U(y)$ vektor funksiyani qiymati chegaraning S qismida berilgan bo‘lsin, ya’ni

$$U(y)\big|_s = f(y) \qquad (3.1.3)$$

Bu yerda $f(y)$ berigan uzluksiz vektor funksiyadan iborat.

$U(y)$ vektor funksiyani G sohada topish masalasiga to‘rt o‘lchovli fazoda elliptik sistema uchun Koshi masalasi deyiladi. Ma’lumki bunday masalalar nokorretk masalalar qatoriga kiradi. Ya’ni bunday masalaning yechimida turg’unlik sharti buziladi. Uning turg’unmasligi Laplas tenglamasi uchun qo‘yilgan Koshi masalasidek xarakterga egadir. Demak bunday masalalarni yechish uchun Karleman matretsasini tuzish lozimdir. [12]

3. 2-§. Chegaralanmagan sohada to‘rt o‘lchovli elliptik sistema uchun integral formula

Faraz qilaylik $G \subset R^{(4)}$ - 4- o‘lchovli fazodan olingan chegaralangan soha bo‘lib chegarasi silliq sirtdan iborat bo‘lsin. [13]

G sohada quyidagi tenglamalar sistemasini qaraymiz

$$D\left(\frac{\partial}{\partial x}\right)u(x) = 0 \qquad (3.2.1)$$

Bu yerda $D(x^T)$ xaraktristik matrisa $D(x^T) \in A_{cx_n}(x)$ ya’ni $D*(x^T)D(x^T) = E(|x|^2 \cdot u^0)$

Bu qaralayotgan sistemani qanoatlantiruvchi $U(x)$ vektor funksiya garmonik funksiya bo‘lib, (3.2.1) sistemaning yechimidan iboratdir.

Agar $U(x) \in C^{'}(G) \cap C(\overline{G})$ sinfdan bo‘lsa, ya’ni G sohada (3.2.1) sistemani qanoatlantirib $\overline{G}$ sohada uzluksiz bo‘lsa, u holda quyidagi integral formula o‘rinli bo‘ladi.

$$U(x) = \int_{\partial G} M(x,y)U(y)ds_y \qquad x \in G \qquad (3.2.2)$$

Bu yerda

$$M(x,y) = \left(E\left(\frac{C_4}{r^2}u^0\right)D*\left(\frac{\partial}{\partial x}\right)\right)D(t^T), \; C_4 = \frac{1}{2w}$$ w_4 – 4-o‘lchovli fazodagi birlik sferaning sirt yuzasi.

Bu integral formulani o‘rinliligini N.N. Tarxanov o‘zining ishlarida isbotlagan .

Faraz qilaylik $K(w)$ funksiya analitik funksiya bo‘lib $(w = v + iu)$ w - ning haqiqiy qiymatida haqiqiy funksiya bo‘lib quyidagi shartni qanoatlantirsin

$$\begin{aligned} &K(u) \neq 0 \quad \sup_{p \geq 1} \left| v^p k^p (w) \right| = M(p,u) \prec \infty \\ &m = 0,1,2,3 \quad -\infty < u < \infty \end{aligned} \tag{3.2.3}$$

Quyidagi funksiyani kiritamiz.

$$\Phi_\sigma (y,x) = \operatorname{Im} \frac{k(w_0)}{\alpha (w_0 - x_4)}, \quad x \neq y$$

$$\alpha = \sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2 + (y_3 - x_3)^2} \quad w_0 = i\alpha + x_4$$

Sh. Yarmuxammedovning ishlarida $\Phi_\gamma (y,x)$ funksiyaning tuzulish strukturasi va uning xossalarini isbotlagan.

Bu xossalardan biri $\Phi_\sigma (y,x) = \frac{C_4}{r^2} + g_\sigma (y,x)$ ko‘rinishda tasvirlangan. Bu yerda $g_\sigma (y,x)$ barcha y lar uchun garmonik funksiyadan iboratdir. Ya’ni bu funksiya Grin funksiyasiga o‘xshash bo‘lar ekan. (3.2.1) sistema shunday xossalarga ega. Agar $\frac{C_4}{r^2}$ funksiyaning o‘rniga $\Phi_\sigma (y,x) = \frac{C_4}{r^2} + g_\sigma (y,x)$ ni ham qo‘ysak, u holda (2) integral formula o‘zgarmaydi, ya’ni quyidagi integral formula o‘rinli bo‘ladi.

$$U(x) = \int_{\partial G} N_\sigma (y,x) u(y) ds_y \quad x \in D \tag{3.2.4}$$

Bu yerda $N_\sigma (y,x)$ quyidagicha aniqlanadi

$$N_\sigma (y,x) = \left(E \left(\Phi_\sigma (y,x) u^0 \right) D * \left(\frac{\partial}{\partial x} \right) \right) D(t^T)$$

Faraz qilaylik G_ρ soha chegaralanmagan soha bo‘lsin, u holda T_R deb markazi koordinata boshida bo‘lgan radusi R ga teng bo‘lgan sharni belfgilaymiz $G_{\rho R}$ deb $\overset{\infty}{G}_{\rho r} = G_\rho / T_R$ ni olamiz.

Agar $\lim\limits_{R\to\infty}\int\limits_{G_{\rho R}^{\infty}} N_r(y,x)u(y)ds=0$ shart bajarilsa u holda (3.2.4) integral forma ham chegaralanmagan sohalarda o‘rinli bo‘ladi.

Faraz qilaylik G_ρ soha qalinligi $h=\dfrac{\pi}{\rho},\quad \rho>0$ ga teng bo‘lgan qatlamdan iborat bo‘lsin, ya’ni uning chegarasi $y=0$ va $y=h$ tengliklardan iborat bo‘lib chegaralanmagan soha bo‘lsin.

Bu holda $u(y)$ funksiyaning uzluksizligini kompoktda G_ρ dan olingan sohada uzluksizligi talab etiladi.

Agar $u(y)$ vektor funksiya $U(y)=\exp\left(\exp\rho(x-\pi)\right)\ldots$ o‘sishga ega bo‘lsa, u holda (3.2.4) integral formula o‘rinli bo‘ladi.

Buning uchun $\Phi_\sigma(y,x)$ dagi $K(w)$ funksiyani quyidagicha tanlaymiz.

$$K(w)=(w_0-x_4+2h)^{-2}\exp\left\{-b_0chi\rho\left(w_0-\frac{h}{2}\right)-b_1chi\rho_0\left(w_0-\frac{h}{2}\right)+\partial w\right\}$$

$$w=y_4+i\alpha\quad 0<\rho_1<\rho\quad 0<x_m<h$$

$$b_1=\frac{b_2}{\cos\rho_0\dfrac{h}{2}}+\frac{b_0}{\cos\rho_0\dfrac{h}{2}},\quad b_2>0$$

k - butun son .

Integral formulani o‘rinli ekanligini ko‘rsatish uchun $\dfrac{\partial\Phi_\sigma}{\partial x_i}\quad (i=1,n)$ va $\Phi_\sigma(y,x)$ larni baholashimiz kerak

$$\varphi_\sigma(y,x)=\operatorname{Im}\frac{K(i\alpha+y_4)}{\alpha(i\alpha+y_4-x_4)},\quad i\alpha+y_4=w_0$$

$$\varphi_\sigma(y,x)=\operatorname{Im}\frac{(w_0-x_4+2h)^2\exp\left\{-b_0chi\rho\left(w_0-\dfrac{h}{2}\right)-b_1chi\rho_0\left(w_0-\dfrac{h}{2}\right)+\partial w_0\right\}}{\alpha(w_0-x_4)}$$

$|\varphi_\sigma(y,x)|$ ni baholash uchun quyidagi tengsizlikdan foydalanamiz.

$\mathrm{Im}e^{\alpha+i\beta}=e^\alpha\cos\beta$ $\quad -\frac{\pi}{2}<\alpha<\frac{\pi}{2}$ bo‘lsa, u holda $\cos\alpha>0$.

$$-\frac{\pi}{2}<-\frac{\tau h}{2}\le\tau\left(y_4-\frac{h}{4}\right)\le\frac{\tau h}{4}<\frac{h}{\rho},\qquad \rho<\tau<2\rho$$

Tengsizlikdan foydalanib $\cos\tau\left(y_4-\frac{h}{4}\right)\ge\delta_0>0$ ekanligiga ega bo‘lamiz.

$\frac{\partial\Phi_\sigma}{\partial y_i}=\frac{\partial\Phi}{\partial s}(y_i-x_i)$ formuladan foydalanib $\Phi_\sigma(y,x)$ va $\frac{\partial\Phi}{\partial y_i}$ larni baholashda quyidagilarga ega

bo‘lamiz.

$$|\Phi_\sigma(y,x)|+\left|\frac{\partial\Phi_\sigma}{\partial y_i}\right|\le O\left(\exp\left[-\varepsilon_1 ch\rho_1|y'|-b_0 ch\rho_0|y'|\right]\right)$$

$y\to\infty,\quad \Phi_\sigma(y,x)$ va $\frac{\partial\Phi_\sigma}{\partial y_i}$ larning qiymati nolga intiladi. Bu esa

$\lim\limits_{R\to\infty}\int\limits_{G_{\rho\sigma}^{\infty}}N_\sigma(y,x)u(y)ds=0$ tenglik bajariladi.

3. 3-§.To‘rt o‘lchovli elliptik tipli tenglamalar sistemasi uchun Koshi masalasining regulyarizatsiyasi.

Faraz qilamiz G soha chegarasi S silliq sirt cheksizlikka intiluvchi va $y_4 = 0$ tekislikdan iborat bo‘lib $0 < y_n < h, \quad h = \frac{\pi}{\rho}, \quad \rho > 0$ qatlamda joylashgan chegaralanmagan soha bo‘lsin. [14]

S sirtning tenglamasi $y_4 = F(y')$ bo‘lib, u quyidagi shartlarni qanoatlantirsin.

$$\partial < y_4 = F(y') < h, \quad \left|\frac{\partial F}{\partial y_i}\right| \le \mu \prec \infty, \quad y' \in R^3, \quad i = 1,2,3. \qquad (3.1.1)$$

$K(G_\rho)$ deb $(3.3.1)$ sistemani qanoatlantiruvchi $u(y)$ vektor funksiya bo‘lib, u $U(y) \subset C^2(G_\rho) \cap C(\overline{G}_\rho)$ sinfga qarashli bo‘lsin. $u(y)$ vektor funksiyaning S dagi qiymati ma’lum bo‘lsa, ya’ni

$$u(y)\big|_s = f(y) \qquad (3.3.2)$$

$$f(y) = \begin{Bmatrix} f_1(y) \\ f_2(y) \\ \vdots \\ f_n(y) \end{Bmatrix} \qquad n \ge 4$$

(3.3.1), (3.3.2) masalaga G_ρ soha uchun Koshi masalasi deyiladi. Bu masalani yechishni aniqlash uchun $\Phi_\sigma(y,x)$ funksiyani kiritamiz va u quyidagicha aniqlanadi.

$$C_4 k(x_4)\Phi_\sigma(y,x) = \mathrm{Im} = \frac{k(i\alpha + y_4)}{\alpha(i\alpha + y_4 - x_4)}$$

$K(w)$ ni quyidagi ko‘rinishda tanlaymiz

$$K(w) = (w_0 - x_4 + 2h)^2 \exp \sigma w_0 \qquad w_0 = i\alpha + y_4$$

Agar $u(y) \subset K'(G_\rho)$ dan iborat bo‘lsa quyidagi formula o‘rinli bo‘ladi.

$$u(x)=\int_{\partial G_\rho} N_\sigma(y,x)u(y)ds_y,\quad x\in G_\rho \qquad (3.3.3)$$

Bu yerda

$$N_\sigma(y,x)=\left(E\left(\Phi_\sigma(y,x)u^0\right)D*\left(\frac{\partial}{\partial x}\right)\right)D(t^T)\ \ t=(t_1,t_2,t_3,t_4)$$

birlik tashqi normalning koordinatalari.Quyidagi teorema o‘rinli.

Teorema 3.3.1. Faraz qilamiz, $u(x)\in K(G_\rho)$ bo‘lsin. Agar $|u(y)|\le 1\quad y\in S$ bo‘lsa, u holda quyidagi tenglik o‘rinli bo‘ladi.

$$|u(x)-u_\sigma(x)|\le C\rho(x)\sigma^2\exp(-\sigma x_4);$$

bunda:

$$U_\sigma=\int_S N_\sigma(x,y)uds_y\quad x\in S$$

$$\sigma\ge\sigma>1\quad x\in G_\rho$$

Isbot:

$$I=u(x)-u_\sigma=\int_{\partial G_\rho} N_\sigma(y,x)u(y)ds_y$$

$$[I]\le c(x),\quad \bar{C}(\sigma)\exp(-\sigma x_4);\quad x\in G_\rho$$

$$\varphi_\delta(y,x)=\exp\sigma y_u\ \mathrm{Im}\frac{(i\alpha+B_1)^{-2}\exp(i\sigma\alpha)}{\alpha(i\alpha+B)}=$$

$$\exp\sigma y_u\ \mathrm{Im}\frac{(\cos\sigma\alpha+i\sin\sigma\alpha)(B-i\alpha)(B_1-i\alpha)^{-2}}{\alpha(B^2+\alpha^2)(B_1^2+\alpha^2)}= \qquad (3.3.4)$$

$$\exp\sigma y_u\ \mathrm{Im}\frac{(\cos\sigma\alpha+i\sin\sigma\alpha)(B+i\alpha)(B_1^2+\alpha^2-2B_1\alpha)}{\alpha^2r^2r_1^2}=$$

$$\exp\sigma y_u\ \mathrm{Im}\frac{(\cos\sigma\alpha+i\sin\sigma\alpha)((B)B_1^2+\alpha^2)+2B\alpha^2-2(B_1^2+\alpha^2)i}{\alpha^2r^2r_1^2}=$$

$$\exp\sigma y_u\left(-\frac{\cos\sigma\alpha}{1}\left(\frac{1}{r^2}+\frac{2BB_1}{r^2r_1^2}\right)+\frac{\sin\sigma\alpha}{\alpha}\right)+\frac{\sin\sigma\alpha}{\alpha}\left(\frac{B}{r^2}+\frac{2B\alpha}{r^2r_1^2}\right). \qquad (3.3.5)$$

Quyidagi formulalardan foydalanamiz.

$$\left|\frac{\partial^p}{\partial S^p}\cdot\frac{\sin\sigma\alpha}{\alpha}\right|\le C_\rho\frac{\partial^p}{\alpha^{p+1}},\qquad \alpha\ge 1;\quad p=0,1,\ldots.$$
$$\left|\frac{\partial^p}{\partial S^p}\cdot\frac{\sin\sigma\alpha}{\alpha}\right|\le C_\rho\frac{\partial^p}{\alpha^{2p}},\qquad 0<\alpha\ge 1;\quad p=0,1,\ldots.$$
$$\left|\frac{\partial^p}{\partial S^p}\cdot\frac{\cos\sigma\alpha}{1}\right|\le C_\rho\frac{\partial^p}{\alpha^{p}},\qquad \alpha\ge 1;\quad p=0,1,\ldots. \tag{3.3.6}$$
$$\left|\frac{\partial^p}{\partial S^p}\cdot\frac{\cos\sigma\alpha}{1}\right|\le C_\rho\frac{\partial^p}{\alpha^{2(p-1)}},\qquad 0<\alpha\ge 1;\quad p=2,3,\ldots.$$
$$\left|\frac{\partial}{\partial S^p}\cdot\frac{\cos\sigma\alpha}{1}\right|\le C_\rho\frac{C_\rho}{\alpha}$$

$\alpha>1$ bo‘lganda (3.3.5) – tenglikni baholaymiz, buning uchun (3.3.6) – dan foydalanamiz.

$$|\varphi_\sigma(y,x)|\le\exp\sigma y_4\left\{|\cos\sigma\alpha|\left(\frac{1}{r^2}+\frac{BB_1}{r^2r_1^2}\right)+\frac{\sin\sigma\alpha}{\alpha}\left(\frac{1}{r^2}+\frac{2B_1\alpha}{r^2r_1^2}\right)\right\}\le c\exp\sigma y_4\times$$
$$\times\left\{\frac{1}{r^2}+\frac{2}{rr_1}+\frac{1}{\alpha r}+\frac{2}{rr_1}\right\}$$

$$|\varphi_\sigma(y,x)|\le\exp\sigma y_4\left\{|\cos\sigma\alpha|\left(\frac{1}{r^2}+\frac{2BB_1}{r_1^2r^2}\right)+\frac{\sin\sigma\alpha}{\alpha}\left(\frac{B}{r^2}+\frac{2B\alpha}{r_1^2r^2}\right)\right\}\le c\exp\sigma y_4\times$$
$$\times\left\{\frac{1}{\alpha r^2}+\frac{2}{\alpha r_1 r}+\frac{1}{\alpha r}+\frac{2}{rr_1}\right\}$$

(3.3.5) ni $0<\alpha<1$ bo‘lganda baholaymiz. (3.3.6) – dan foydalanib quyidagiga ega bo‘lamiz:

$$r=\sqrt{(x_1-y_1)+(x_2-y_2)^2+(x_3-y_3)^2(x_4-y_4)^2}$$

(3.3.6)– ning hosilasini olamiz.

$$\alpha=\sqrt{(x_1-y_1)^2+(x_2-y_2)^2+(x_3-y_3)^2}$$
$$r_1=\sqrt{(x_1-y_1)^2+(x_2-y_2)^2+(x_3-y_3)^2+(x_3-2h)^2}$$

U holda

$$\frac{\partial\alpha}{\partial y_4}=0;\quad \frac{\partial r}{\partial y_4}=\frac{y_4-x_4}{r};$$
$$\frac{\partial\alpha}{\partial y_4}=\frac{y_4-x_4-2h}{r}.$$

(3.3.6) – ning hosilalaridan foydalanib quyidagiga ega bo‘lamiz.

$$\begin{aligned}\frac{\partial\varphi_\sigma}{\partial y_4}&=\sigma\exp\sigma y_4\left[-\frac{\cos\sigma\alpha}{1}\left(\frac{r^2}{1}+\frac{BB_1}{r^2r_1^2}\right)+\frac{\sin\sigma\alpha}{\alpha}\left(\frac{B}{r^2}+\frac{2B\alpha}{r^2r_1^2}\right)\right]+\\&+\exp\sigma y_4\left[r^2+\frac{2B\alpha}{r^2r_1^2}+\frac{\sin\sigma\alpha}{\alpha}\frac{\partial}{\partial y_4}\left(\frac{B}{r^2}+\frac{2B\alpha}{r^2r_1^2}\right)\right]\end{aligned}\tag{3.3.7}$$

(3.3.6) – dan foydalanib, $\alpha\geq 1$ bo‘lganda (3.3.7) – ni baholaymiz

$$\left|\frac{\partial\varphi_\sigma(y,x)}{\partial y_4}\right|\leq C\sigma\exp\sigma y_4\left\{\frac{1}{\alpha r^2}+\frac{2}{\alpha r_1 r}+\frac{1}{\alpha r}+\frac{2}{rr_1}\right\}$$

Bu yerda C - har bir tengsizlikda har xil o‘zgarmasdir.

(3.3.7) – ni $0<\alpha<1$ bo‘lganda baholaymiz:

$$\left|\frac{\partial\varphi_\sigma(y,x)}{\partial y_4}\right|\leq C\sigma\exp\sigma y_4\left\{\frac{1}{\alpha r^2}+\frac{2}{\alpha r_1 r}+\frac{1}{\alpha r}+\frac{2}{rr_1}\right\}$$

$\varphi_\sigma(y,x)$ dan y_i bo‘yicha hosila olamiz.

$$\frac{\partial\varphi\sigma(y,x)}{\partial y_i}=\frac{\partial\varphi\sigma(y,x)}{\partial s}\cdot\frac{\partial s}{\partial y_i}=2(y_i-x_i)\frac{\partial\varphi\sigma(y,x)}{\partial s}$$

bu yerda $i=1,2,3,..\qquad s=\alpha^2$.

Xuddi shunday,

$$\frac{\partial}{\partial S}=\frac{\partial}{\partial\alpha}\cdot\frac{\partial\alpha}{\partial S}=\frac{\partial}{\partial\alpha}\cdot\frac{1}{\partial\alpha}$$

$$\frac{\partial\varphi_\sigma}{\partial y_1}=2(y_1-x_1)\frac{\partial\varphi_\sigma}{\partial S}\frac{(y_1-x_1)}{\alpha}\cdot\frac{\partial\varphi_\sigma}{\partial\alpha}=\frac{y_1-x_1}{\alpha}.$$

$$\exp\sigma y_4\left[-\frac{\cos\sigma\alpha}{1}\left(\frac{1}{r^2}+\frac{2BB_1}{r^2r_1^2}\right)+\frac{\sin\sigma\alpha}{\alpha}\left(\frac{B}{r^2}+\frac{2B\alpha}{r^2r_1^2}\right)\right] \quad (3.3.8)$$

(3.3.8) ni $\alpha \geq 1$ bo‘lganda (3.3.6) dan foydalanib baholaymiz.

$$\left|\frac{\partial\varphi_\sigma(y,x)}{\partial y_u}\right| \leq C\exp\sigma y_u\left\{\frac{1}{\alpha r^2}+\frac{2}{\alpha r_1^2}+\frac{1}{\alpha^2 r}+\frac{2}{\alpha r r_1}\right\}$$

Xuddi shunday (3.3.6) – dan foydalanib, $0<\alpha<1$ bo‘lganda baholaymiz.

$$\left|\frac{\partial\varphi_\sigma(y,x)}{\partial y_u}\right| \leq C\exp\sigma y_u\left\{\frac{1}{\alpha^2 r^2}+\frac{2}{\alpha^2 r_1 r}+\frac{1}{\alpha^2 r}+\frac{2}{\alpha r r_1}\right\}$$

Baholarni integral formulaga qo‘yib va xususiy integrallarning yaqinlashishidan teoremaning isboti hosil bo‘ladi. [11]

Natija . quyidagi tenglik $\lim\limits_{\sigma\to\infty} u_\sigma(x)=u(x), \quad x\in G_\rho$

G_ρ - ning ixtiyoriy kompaktida tekis bajariladi.

Teorema 2.1.2. Faraz qilamiz $u(y)$ vektor funksiya bo’lib G_ρ - ning ichida to‘rt o‘lchovli elliptik sistemasini qanoatlantirib, uzluksiz bo‘lsin.

Agar:

$$|u(y)|\leq 1, \quad y\in T=\{y_u=0\}$$

bo‘lib

$$|u(y)-u_\sigma(y)|\leq\delta, \quad 0<\delta<1 \qquad y\in S$$

uchun uzluksiz yaqinlashishidan $y\in S$ - lar uchun

$$u(x)=\int\limits_{\partial G_\rho} N_\sigma(y,x)u(y)ds_y, \quad x\in G_\rho$$

bo‘lsa, u holda

$$\left|u_\sigma(x)-u(x)\right| \le C(x)\bar{C}(\sigma)\sigma^{\frac{x_u}{y_u}}$$

Tengsizlik bajariladi. Bu yerda

$$y_u^0 = \max_{y\in s} y_u$$

Natija:

$$\lim_{\sigma\to 0} u_\sigma(x) = u(x): \quad x\in G_\rho$$

G_ρ - ning har bir kompaktida tekis bajariladi.

III Bob bo’yicha xulosa.

Uchichi bob asosiy bob hisoblanib to‘rt o‘lchovli elliptik sitema va unga qo‘yilgan Koshi masalasi o’rganilgan. Chegaralanmagan sohada to‘rt o‘lchovli elliptik sistema uchun integral formula keltirilgan. To‘rt o‘lchovli elliptik tipli tenglamalar sistemasi uchun chegaralanmagan sohoda Koshi masalasining regulyarizasiyasi o’rganilgan.

Bu bobda Laplas operatori bilan faktorizasiyalanuvchi elliptik sistemalar uchun chegaralanmagan sohada quyidagi integral formulaning o’rinliligi ko’rsatilgan.

$$U(x)=\int_{\partial G} N_{\sigma}(y,x)u(y)ds_y \qquad x\in D$$

Bu yerda $N_{\sigma}(x,y)$ quyidagicha aniqlanadi.

$$N_{\sigma}(y,x)=\left(E\left(\Phi_{\sigma}(y,x)u^{0}\right)D*\left(\frac{\partial}{\partial x}\right)\right)D(t^{T})$$

Bu integral formuladan foydalanib vektor funksiyaning chegaradagi qiymati berilgan bo’lsa sohaning ichida topish masalasi ya’ni Koshi masalasining yechimi keltirilgan.

Xulosa.

Magistrlik dissertatsiyasi to‘rt o‘lchovli fazoda elliptik sistema uchun qo‘yilgan Koshi masalasini chegaralanmagan sohada yechishni regulyarizasiyasini topish masalasidan iborat. Elliptik tipli tenglamalar sistemasi uchun qo‘yilgan Koshi masalasini yechishga Karleman matritsasi tuzishda u orqali yechim oshkor ravishda ifodalanadi. Integral formula ma’lum bo‘lsa u holda Koshi masalasi regulyarizasiyasini tuzish mumkin.

Laplas tenglamasiga qo’yilgan Koshi masalasini yechishda sohaning chegarasining bir qismida noma’lum funksiya va uning normal bo’yicha hosilasi berilgan bo’ladi. Demak shunday funksiyalar yechimini aniqlash mumkinki, u ikkita garmonik funksiyalarning yig’indisidan iborat bo’lib, chegaraning qolgan qismidan olingan integral cheksiz kichik bo’lsin. Bunday funksiyalarga Karleman funksiyasi deyiladi.

Karleman funksiyasini oshkor ravishda topish va ularni Koshi masalalariga qo’llashni prof Yarmuxammedov tomonidan keltirilgan. Shu karleman funksiyasidan foydalanib chegaralanmagan sohalarda ham o’suvchi vektor funksiyalar uchun integral formulaning o’rinliligi ko’rsatilgan.

Teorema. Faraz qilamiz, $u(x) \in K(G_\rho)$ bo‘lsin. Agar $|u(y)| \le 1 \quad y \in S$ bo‘lsa, u holda quyidagi tenglik o‘rinli bo‘ladi.

$$|u(x) - u_\sigma(x)| \le C\rho(x)\sigma^2 \exp(-\sigma x_4);$$

bunda:

$$U_\sigma = \int_S N_\sigma(x,y) u ds_y \quad x \in S$$

$$\sigma \ge \sigma > 1 \quad x \in G_\rho \text{ bo'ladi.}$$

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