O‘ZBEKISTON RESPUBLIKASI

OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

ALISHER NAVOIY NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI

Mexanika – matematika fakulteti

«Hisoblash usullari» kafedrasi

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«5480100 – Amaliy matematika va informatika» ta’lim yo‘nalishi bo‘yicha bakalavr darajasini olish uchun

BITIRUV MALAKAVIY ISH

Nochiziqli chegaraviy shartli bir o’lchovli issiqlik o’tkazuvchanlik tenglamasini sonly echish.

Ilmiy rahbar____________dots. Sh.S.Mamatov

2014 y. «____»__________

Bitiruv malakaviy ish «Hisoblash usullari» kafedrasida bajarildi, kafedraning 2014 yil «__» _________dagi majlisida muhokama qilindi va himoyaga tavsiya etildi (____ -son bayonnoma).

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Bitiruv malakaviy ish YaDAKning 2014 yil «__»__________dagi majlisida himoya qilindi va ____ ball bilan baholandi (____-son bayonnoma).

YaDAK raisi: ________________

A’zolari: ________________

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Samarqand - 2014

Mundarija

KIRISH

Issiqlik almashinuvi jarayonlarni sonli modullashtirishning keskin oshishi hozirgi paytda hozirgi zamon fan va texnikasi uchun bunday jarayonlarni ishonchli bashorat qila bilishning zarurligi bilan bog'liqdir. Bu jarayonlarni eksperimental va natural sharoitlarda tekshirish juda qiyin va qimmatga tushadi, ko'p hollarda esa umuman mumkin emas. Issiqlik uzatish jarayonlarini sonli modellashtirish har xil ilmiy tekshirish, proekt-konstruktorlik va ishlab chiqarish korxonalarining faoliyat sohalariga aylanib bormoqda.

Ushbu ish issiqlik o'tkazuvchanlik hollariga bag'ishlanadi.

Issiqlik o'tkazuvchanlik deb issiqlikni muhitda molekulyar uzatishga aytiladi.Bu jarayon temperaturaning tekis taqsimlanmagan holatida ro'y beradi. Bu holda issiqlik har xil temperaturali zarrachalarning bevosita tutashtirish hisobiga uzatiladi va molekulalar, atomlar va ozod elektronlar orasida energiya almashinuviga olib keladi.

Issiqlik o'tkazuvchanlik moddaning agregat holatiga, uning tarkibiga, temperaturasiga, bosimiga va boshqa xarakteristikalariga bog'liq. Ko'p hollarda suyuq holdagi moddaning issiqlik o'tkazuvchanligi gaz holatdagi moddaning issiqlik o'tkazuvchanligidan taxminan o'n marta ko'pbo'ladi.Qattiq jism uchun issiqlik o'tkazuvchanlik erish nuqtasi atrofida suyuqlikka qaraganda(suyuq vismut, olova va tellurdan tashqari) ancha yuqori bo'ladi.

Amaliyotda jism ichidagi va uning chegarasi yaqinidagi issiqlik o'tkazuvchanlik har xil bo'lishi tez-tez uchrab turadi. Bu farqlanish issiqlik uzatish jarayoning borish shartlarining o'zgarishi bilan, hamda modda strukturasining o'zgarishi bilan(termikqayta ishlash tekshirish, ko'p ishlatish va hakazo natijasida) sodir bo'ladi.

Issiqlik o'tkazuvchanlikka tashqi shart-sharoitlar, masalan, nurlanish, bosimning o'zgarishi, magnit maydon, sezilarli ta'sir qilishi mumkin.

Xiralashgan muhitlarda issiqlik o'tkazuvchanlik radiatsion issiqlik o'tkazish bilan birga ro'y beradi. Bunday muhitlardagi kuzatiladigan issiqlik o'tkazuvchanlik xususiy issiqlik o'tkazuvchan va radiatsion issiqlik o'tkazishning yig'indisidan iborat bo'ladi. Radiatsion issiqlik uzatishning ulushi temperaturaning ko'tarilishi bilan oshib boradi va temperatura bir necha yuz selziy gradusga yetganda muhim ta'sir o'tkazadi.

Masalaning qo’yilishi.

Cheksiz plastinkadagi issiqlik uzatishni qaraymiz. Buni natijasida bittagina yo’nalishi bo’yicha issiqlik uzatish qaralib bir o’lchovli issiqlik o’tkazish tenglamasini tahlil qilinadi. Yechimlar sohasining chegarasida konveksiya va nurlanish hisobiga issiqlik uzatish modellashtiriladi. Nurlanish bilan issiqlik uizatishni Stefan-Bolsman qonuni asosida qaraymiz. Shunday qilib bayon qilingan masala matematik nuqtai nazaridan quydagi ko’rinishni oladi:

B ka $= \frac{2}{2}, \quad 0 < \; < ;$ (44)

$= 0: \; = {}_0, 0 \leq \; \leq ;$

$= 0: - \; = {}_1({}^1 -) + {}_1(({}^1)^4 - {}^4), \quad > 0, \; {}_1 > 0;$ (45)

$= : \; = {}_2({}^2 -) + {}_2(({}^2)^4 - {}^4), \quad > 0, \quad {}_2 > 0;$

Bo’ $_{1\,2}$–qorayish darajasi $= 5{,}669 \cdot 10^{-8} / ({}^2 \cdot {}^4)$Bo’ qo’yilgan chegaraviy masalada nochiziqli chegaraviy shartlar bo;lganda bizni asosan qiziqtiradi.

III Tur chochiziq li chegaraviy shartlar bo’l aniqlikda diskretlashtiramiz.

$_1 \quad _1$ birinchi progonka koeffisentlarini

$_1 = {}_1 \cdot {}_2 + {}_1$ shrtlardan aniqlaymiz. Shunday qilib (45) munosabatning ikkinchisidan

$-\Big|_{=0} = {}_1({}^1 - |_{=0}) + {}_1(({}^1)^4 - (|_{=0})^4);$

$-\frac{{}_2 - {}_1}{} = {}_1({}^1 - {}_1) + {}_1(({}^1)^4 - ({}_1)^4);$ kelib chiqadi.

$\frac{1}{} \equiv {}_1$ belgilashni kiritamiz U holda

$_1 - {}_2 \equiv {}_1 \cdot {}^1 - {}_1 \cdot {}_1 + \frac{1}{}({}^1)^4 - \frac{1}{}({}_1)^4;$

$_1 = \frac{1}{1 + {}_1} \cdot {}_2 + \frac{1}{1 + {}_1} \cdot {}^1 + \frac{1}{(1 + {}_1)}(({}^1)^4 - ({}_1)^4);$

$$\begin{cases} {}_1 = \dfrac{1}{1 + {}_1} \\ {}_1 = \dfrac{1}{1 + {}_1} \cdot {}^1 + \dfrac{1}{(1 + {}_1)}(({}^1)^4 - ({}_1)^4) \end{cases}$$

$$\begin{cases} {}_1 = \dfrac{}{+ \cdot {}_1} \\ {}_1 = \dfrac{\cdot 1}{+ \cdot 1} \cdot {}^1 + \dfrac{1}{+ \cdot 1}(({}^1)^4 - ({}_1)^4) \end{cases} \qquad (46)$$

Ko'rinib turibdiki progonkaning ${}_1$- birinchi koefisenti chap chegaradagi temperaturaga bog'langan . U holda temperatura maydonini aniqlash uchuin masalan, oddiy iteratsiya usulidan foydalanish zarur. Asosiy g'oya shundan iboratki har bir vaqt qatlamida tenperatura maydonini hisoblash $\left| {}_1^{+1} - {}_1 \right| \leq$ bu yerda S-iteratsiya nomiri

-xisoblash xatoligi,shart bajarilgancha davom ettiriladi.

T_N temperaturani aniqlash uchun o'ng chegaraviy shartdan foydalaniladi.

$$+ \Big|_{=} = {}_2({}^2 - |_{=}) + {}_2(({}^2)^4 - (|_{=})^4);$$

$$+ \frac{- \ {}_{-1}}{} = {}_2({}^2 -) + {}_2(({}^2)^4 - ()^4);$$

$${}_{-1} = {}_{-1} \cdot + {}_{-1},$$

$$- {}_{-1} \cdot - {}_{-1} = {}_2({}^2 -) + {}^2_{}(({}^2)^4 - ()^4);$$

$$= \frac{{}_{-1} + {}_2 {}^2 + {}^2(({}^2)^4 - ()^4)}{1 + {}_2 - {}_{-1}} \quad \text{yoki}$$

$$= \frac{\cdot {}_{-1} + \cdot {}_2 {}^2 + {}_2(({}^2)^4 - ()^4)}{\cdot {}_2 + (1 - {}_{-1})} \qquad (47)$$

Natijada o'ng chegarada temperaturani aniqlash uchun (47)tenglamani hosil qilishimiz. Bu tenglamani ham eng sodda metod oddiy iteratsiya usuli bilan yechish mumkin.

(45) chegaraviy shartlarni O(h') anqlikda diskretlashtiramiz. Faraz qilaylik chegarada (44) issiqlik o'tkazuvchanlik tenglamasi bajarilsin T(x) funksiyasi x=0 no'qta atrofida h ga nisbatan 2-tartibli hadgacha Teylor qatoriga yoyamiz.

$${}_2^{+1} = {}_1^{+1} + \cdot \Big|_{=0}^{+1} + \frac{2}{2} \cdot \frac{2}{2} \Big|_{=0}^{+1}.$$

(44) munosabatdan foydalanib topamiz

$$_2^{+1} = _1^{+1} + \cdot \Big|_{=0}^{+1} + \frac{2}{2\cdot} \cdot \Big|_{=0}^{+1};$$

$$\Big|_{=0}^{+1} = \frac{_2^{+1} - _1^{+1}}{} - \frac{}{2\cdot} \cdot \Big|_{=0}^{+1} = \frac{_2^{+1} - _1^{+1}}{} - \frac{}{2\cdot} \frac{_1^{+1} - _1}{}.$$

(45) ning 2-munosabatdan

$$\Big|_{=0}^{+1} = \tfrac{1}{}\left(_1^{+1} - {}^1\right) - \tfrac{1}{}\left(\left({}^1\right)^4 - \left(_1^{+1}\right)^4\right).$$

Oxirgi ikki munosabatlarni tenglashtirib topamiz:

$$\frac{_2^{+1} - _1^{+1}}{} - \frac{}{2\cdot} \cdot \frac{_1^{+1} - _1}{} = \tfrac{1}{}\left(_1^{+1} - {}^1\right) - \tfrac{1}{}\left(\left({}^1\right)^4 - \left(_1^{+1}\right)^4\right);$$

$$_2^{+1} - _1^{+1} - \frac{2}{2..} \cdot _1^{+1} + \frac{2}{2..} \cdot _1 = \tfrac{1}{} {}_1^{+1} - {}^{1\,1} - \tfrac{1}{}\left(\left({}^1\right)^4 - -\left(_1^{+1}\right)^4\right);$$

$$_1^{+1}\left(1 + \frac{2}{2..} + \tfrac{1}{}\right) = _2^{+1} + \frac{2}{2..} \cdot _1 + {}^{1\,1} + \tfrac{1}{}\left(\left({}^1\right)^4 - -\left(_1^{+1}\right)^4\right);$$

$$_1^{+1} = \frac{2\cdot\cdot}{{}^2 + 2\cdot(+{}_1)^2} {}_2^{+1} + \frac{2}{{}^2 + 2\cdot(+{}_1)} \cdot {}_1 +$$

$$+ \frac{2_1}{{}^2+2\cdot(+_1)} {}^1 + \frac{2_1}{{}^2+2\cdot(+_1)} \left(\left({}^1\right)^4 - \left(_1^{+1}\right)^4\right).$$

$$\begin{cases} _1 = \frac{2..}{{}^2+2\cdot(+_1)}; \\ \quad _1 = \frac{2}{{}^2+2\cdot(+_1)} \cdot {}_1 + \frac{2_1}{{}^2+2\cdot(+_1)} {}^1 + \\ \quad + \frac{2_1}{{}^2+2\cdot(+_1)} \left(\left({}^1\right)^4 - \left(_1^{+1}\right)^4\right) \end{cases} \qquad (48)$$

O'ng chegaradan foydalanib T_N ni topamiz

$$_{-1}^{+1} = {}^{+1} - \cdot \Big|_{=}^{+1} + \frac{2}{2} \cdot \frac{2}{2} \Big|_{=}^{+1} = {}^{+1} - \cdot \Big|_{=}^{+1} + \frac{2}{2\cdot} \cdot \Big|_{=}^{+1};$$

$$_{-1}^{+1} = {}^{+1} - \tfrac{2}{}\left({}^2 - _1^{+1}\right) - \tfrac{2}{}\left(\left({}^2\right)^4 - \left(_1^{+1}\right)^4\right) + \frac{2}{2\cdot} \cdot \frac{{}^{+1} -}{}.$$

shunday qilib $\quad _{-1} = _{-1} \cdot + _{-1}$

$$_{-1} \cdot {}^{+1} + _{-1} = {}^{+1} + \tfrac{2}{}{}^2 + \tfrac{2}{}{}^{+1} + \frac{2}{2\cdot\cdot} {}^{+1} - \frac{2}{2\cdot\cdot} -$$

$$- \tfrac{2}{}\left(\left({}^2\right)^4 - \left(_1^{+1}\right)^4\right);$$

$${}^{+1}\left(1 - _{-1} + \tfrac{2}{} + \frac{2}{2..}\right) = _{-1} + \tfrac{2}{}{}^2 + \frac{2}{2..} + \tfrac{2}{}\left(\left({}^2\right)^4 - \left(_1^{+1}\right)^4\right);$$

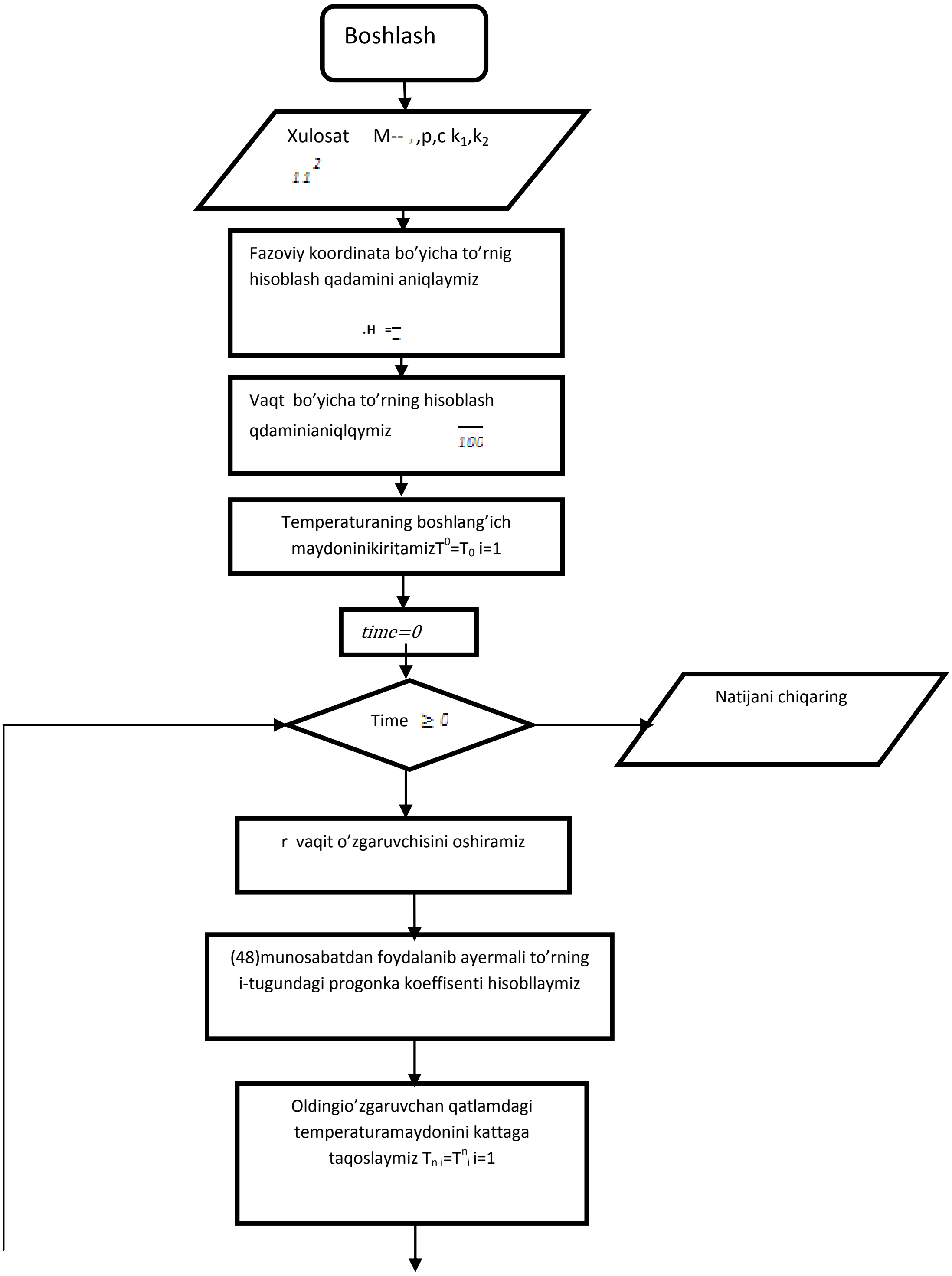
$$^{+1} = \frac{2_{-1}}{2(1 - {}_{-1}) + 2_2 + {}^2} + \frac{2_2{}^2}{2(1 - {}_{-1}) + 2_2 + {}^2} +$$

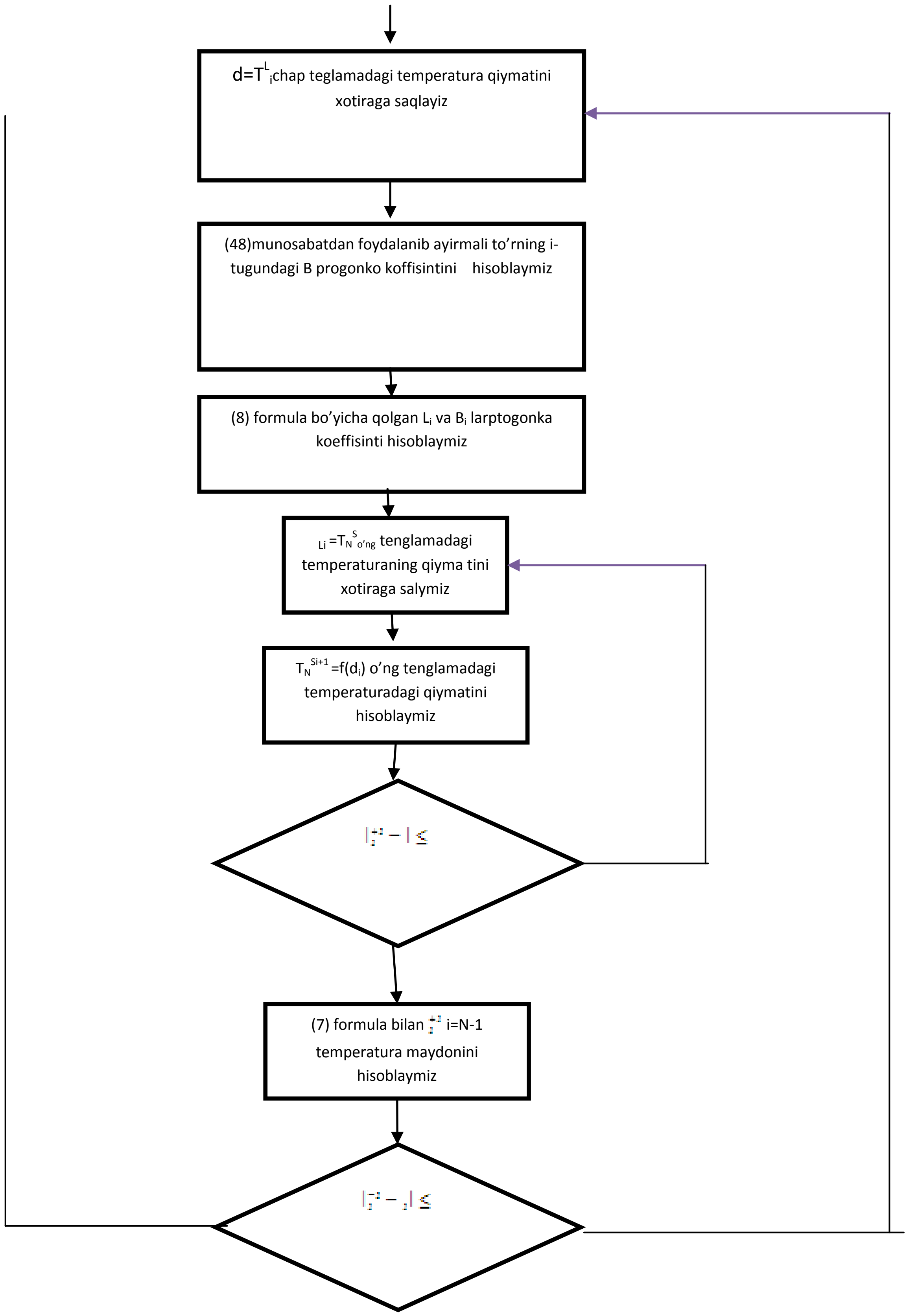
$$+\frac{{}^2}{2(1-{}_{-1})+2_2+{}^2} + \frac{2_2}{2(1-{}_{-1})+2_2+{}^2}\left(({}^2)^4 - \left({}^{+1}_{1}\right)^4\right). \quad (49)$$

Beton plastinkada 600,1800,3600,va 7200 sekunddan keyinm tenperatura naydonini aniqlaymiz. Plastina qalinligi L=93 m Boshlang'ich temperatura T_0=50C . Beton quyidagi teplo fizik xarakteristiklarga ega. $= 0.9 /(\ *\ )$ p=2000kg/m^3

c=920Dj/(kg*c)x=0 va x=L chegaralarg plastinka tashqi muhit bilan aloqada bo'ladi.

(K=1000 vt/m^2*c) $=30^0 {}_1 = 0.5,$ K_2=500vt/m^2*c) , $=70^0 {}_2 = 0.2$


Boshlash
Xulosat M-- , ,p,c k_1,k_2
Fazoviy koordinata bo'yicha to'rnig hisoblash qadamini aniqlaymiz
Vaqt bo'yicha to'rning hisoblash qdaminianiqlqymiz
Temperaturaning boshlang'ich maydoninikiritamiz T^0=T_0 i=1
time=0
Time ≥ 0
Natijani chiqaring
r vaqit o'zgaruvchisini oshiramiz
(48)munosabatdan foydalanib ayermali to'rning i-tugundagi progonka koeffisenti hisobllaymiz
Oldingio'zgaruvchan qatlamdagi temperaturamaydonini kattaga taqoslaymiz $T_{n\,i}$=T^n_i i=1



Qaralayotgan masalaning listing dasturi

```
uses;crt;
const mf=500;
            sigma=5.669e-8;
            eps=1e-5;
type
vector=array[1..mf] of real;

var {dasturda foydalaniladigan o’zgaruvchilarning belgilash bo’limi}
i, j, N                                         :integer;
t,tn,alfa, beta                      :vector;
ai, bi, ci, fi, d, dl                :real;
lamda, ro, c, h, tau                            :real;
kapa1, kapa2, te1, te2               :real;
eps1, eps2                                      :real;
T0, L, t_end, time                              :real;
f,g                                             :text;
begin
clrser;
{Klaviatura bilan barcha kirish parametrni keitamiz}
Writeln ();
Readln(N);
Writeln(vaqt bo’yicha tugatishni kiritng,t-end);
Readln(Plastina qalinligini kiriting);
Writeln(‚L’);
Readln(L);
Writeln(Plastena materialining issiqlik o’tkazuvchanlik koeffisintini kiriteng lamda);
Readln(lamda);
Writeln(Plastena materialining zichligini  kiriteng);
Readln(ro);
```

```
Writeln(Plastena materialining issiqlik sig'imini kiriteng);
Readln(c);
Writeln(x=0, chegarada issiqlik almashinivi koeffisintini kiriting);
Readln(kapa 1);
Writeln(x=L, x=0, chegarada issiqlik almashinivi koeffisintini kiriting);
Readln(kapa2);
Writeln(x=0 chegaraga nisbatan tashqi muhit temperaturasini kiriteng);
Readln(Tel);
Writeln(x=L, chegaraga nisbatan tashqi muhit temperaturasini kiriteng);
Readln(Te2);
Writeln(x=0, chegaradagi qavarig'lik darajasini kiriteng);
Readln(eps1);
Writeln(x=L, chegaradagi qavarig'lik darajasini kiriteng);
Readln(eps2);
Writeln(boshlang'ich temperatuani keriteng);
Readln(T0);
{Fazoviy kordinata bo'yicha turining hisoblash qadamini aniqlaymiz}
h:L/(N-1);
{ vaqt bo'yechato'rning hisoblash qadamini aniqlaymiz }
Tau:=t_end/100.0;
{ vaqtning boshlang'ich momentida temperatura maydonini aniqlaymiz}
For i:=1 to N do
T[i]=T0;
{nostasionar issiqlik o'tkazish tenglamasini integrallaymiz}
time:=0;
while time<t_end do {shartli siklda foydalanamiz}
begin
{vaqt o'zgaruvchisini r qadamga oshiramiz}
time:=time+tau;
```

{(48)muinosabatdan foydalanib chap chetidagi boshlang’ich progonka koiffisintini aniqlaymiz}

alfa[1]:=2.0*tau*(2.0*tau*(lamda+kapal*h)+ro*c*sqr(h));

{oldingi vaqt maydonidagi temperatura maydonini xoteraga saqlaymiz}

for i:=1 to N do

Tn[i]=T[1];

{chap chigaraviy shartning chiziqlimasligidan temperatura maydonini iterasion hisoblavchi shartli sikl}.

Repeat

{(48) formuladan foydalanib chap chegaraviy shartlardan boshlang’ich progonka koeffisintni aniqlqymiz .Bunda iterasion sikl chap chegara shart boshlanadi.}

d:=T[1];

beta[1]:=(ro*c*sqr(h))*Tn[1]+2.0*tau*kapal*h*Tel+2.0*tau*eps1*sigma*h*(sqr(sqr(Tel))-sqr(sqr(d))))/(2.0*tau*lamda+kapal*h)+ro*c*sqr(h));

{(18) formula yordamida progonka koeffisintni aniqlavchi parametrli sikl}

for i:=2 to n-1 do

begin

{ai, bi, ci, fi}-uch dioganalli matrisali tenglamalar sistemasini kanonik tasvirlash koeffisintlari}

ai:=lamda/sqr(h);

bi:=2.0*lamda/sqr(h)+ro*c/tau;

ci:=lamda/sqr(h);

fi:=-ro*c*Tn[i]/tau;

{alfa[i], beta{i}} {progonka koeffisintlari}

alfa[i]:=ai/(bi-ci*alfa[i-1]);

beta[i]:=(ci*beta[i-1]-fi)/(bi-ci*alfa[i-1]);

end;

{o’ng chetdagi chegaraviy shartlarning chiziqlimasligidan foydalanib temperatura qiymatini iteratsion hisoblash imkonini beradigan shartli sikl}

repeat

d1:=T[N];

{ (49) formulada foydalanib o’ng chegaraviy shartlardan o’ng chetidagi temperaturani hisoblaymiz}

```
T[N]:=(ro*c*sqr(h)*Tn[N]+2.0*tau*(lamda*beta[N-1]+kapa2*h*Te2+eps2*sigma*h*(sqr(sqr(Te2))-sqr(sqr(d1)))))/(ro*c*sqr(h)+2.0*tau*(lamda*(1-alfa[N-1])+kapa2*h));

until abs(d1-T[N])<=eps; {(7) munosabatdan noma'lum vaqt maydonini aniqlaymiz}

for i:=N-1 downto 1do

T[i]:=alfa[i]*T[i+1]+beta[i];

Until abs(d-T[1])<=eps; {shartli sikl tugadi}

end; {natijalarni fayelga chiqaramiz}

Assign(f,'res.txt');

Rewrite(f);

Writeln(f,'Plastina qalinligi L=',L:6:4);

Writeln(f,'N= koordinata bo'yicha tugunlar soni);

Writeln (Plastina materyalning issiqlik o'tkazish koeffisinti)

Writeln(f' plastinka materyalning zichligi ro=,'ro:6:4);

Writeln(f,'plastinka materyalning issiqlik sig'imi c=',v;6;4);

Writeln(f,boshlang'ich tempratura To=To;6;4);

Writeln

Writeln(f,'kapa2=,'kapa2:6:4);

Writeln(f,'Te1=,'Te1:6:4);

Writeln(f,'Te2=,'Te2:6:4);

Writeln

Writeln(f,'qavariqlik darajasi eps= ,eps4:1:6:4);

Writeln(f,''qavariqlik darajasi eps2=eps2:6:4);

Writeln(f,'koordinata bo'yicha qadambilan nartijalandi);

Writeln(f,'vaqt bo'yicha qadam bilan natija olinadi);

Writeln(f,' vaqt momintidagi temperatura maydoni);

close(f);

Assign(g,'tempr.txt');

Rewrite(g);

for i:=1 to N do
```

```
writeln(g,' ',h*(i-1):10:8,' ',T[i]:8:5);

close(g);

end.
```

Temperaturaning tasimlanish quydgicha bo'ladi.

Mavzuning dolzarbligi.

Oxirgi o'n yilliklarda tezkor jadal suratlar bilan tekshirishlar va issiqlik almashinuvi hodisalarining tadbiqlari juda kengayib ketdi. U o'zida texnika (kimyo texnalogiyasi, metallurgiya, qurilish ishlari, neftni qayta ishlash, mashinasozlik, agrotexnika va hakozo) hamda asosiy tabbiy fanlar (biologiya, geologiya, atmosfera va atom fizikasi va boshqalar) ning barcha sohalarini qamrab oldi. Issiqlik almashinuvi jarayonini nazariy o'rganish hozirgi paytda sezilarli darajada EHMlardan foydalanilgan holda ularni matematik modellashtirish atrofida olib borilmoqda.Bu natijalarga xususiy hosilali sonli yechish metodlarining sezilarli tarzda rivojlanishi va zamonaviy hisoblash mashinalari quvvatining oshishi hisobiga erishilmoqda.

Ishning maqsadi va vazifalari. Ushbu malakaviy bitiruv ishida qo'zg'aluvchan chegarali bir o'zgaruvchili issiqlik o'tkazuvchanlik tenglamasini sonli yechish maqsad qilib qo'yilgan.

Tadqiqot metodlari. Bayon qilingan masalani fazoviy to'rning tugunlarida issiqlik tarqalishi chegarasini tutish metodi bilan echiladi.

Xususiy hosilali differensial tenglamalarni yechishda chekli ayirmalar usuli va oddiy iteratsiya usullaridan foydalaniladi.

Chekli ayirmalar usulini issiqlik o'tkazuvchanlik masalalariga qo'llashda qattiq jismni tugunlar to'plami sifatida qaraladi. (1) tenglamaning xususiy hosilalarini chekli ayirmalar bilan almashtirish to'rning har bir tugunidagi temperaturaning local xarekteristikasi sifatida algebraik tenglamalar sistemasi hosil qilinadi.Qatlamda, yopiq chiziqli algebraik tenglamalar sistemasini EHM yordamida sonli usullar bilan yechiladi.

Ilmiy ahamiyati. Qo'zg'aluvchan chegarali bir o'zgaruvchili issiqlik o'tkazuvchanlik tenglamasini sonli yechish.Masalani yeshish uchun zarur bo'lgan dasturlar, blok sxemalar tuzuldi.

Amaliy ahamiyati. Ishning natijalaridan hisoblash matematikasi va hisoblash usullari fanlari bo'yicha otkaziladigan amaliy va laboratoriya mashg'ulotlarida foydalanish mumkin.

Ishnig tuzilishi. Bitiruv malakaviy ishi kirish, asosiy qisim (4 ta paragraf), xulosa, foydalanilgan adabiyotlar ro'yxatidan iboratdir. Ishning umumiy hajmi 48 bet.

Olingan natijalarning qisqacha mazmuni. Yuqorida aytilganlarga asosan masalaning matematik ko'rinishi quydagi ko'rinishni oladi.

$$\begin{cases} \dfrac{\partial T_1}{\partial t} = a_1 \dfrac{\partial^2 T_1}{\partial x^2}, \quad 0 < x < \xi(t), t > 0; \\ \dfrac{\partial T_2}{\partial t} = a_2 \dfrac{\partial^2 T_2}{\partial x^2}, \quad \xi(t), < x < l, t > 0; \end{cases} \qquad (59)$$

$$t = 0: T(x) = T_0; \; 0 \le x \le L;$$

$$x = 0: T(t) = T_c; \; t > 0;$$

$$x = L: \frac{\partial T}{\partial x} = 0; \; t > 0;$$

$$x = \xi(t): \begin{cases} T_1 = T_2 = T \\ \lambda_1 \dfrac{\partial T_1}{\partial x} - \lambda_2 \dfrac{\partial T_2}{\partial x} = Q_f \rho w \dfrac{d\xi}{dt}; \end{cases} \qquad (60)$$

Bu yerda $\rho -$ tuproqning zichligi (kg/m^3) ,$\omega -$ tuproqning namligi (absalyut quruq tuproqning birlik massasisdagi namlik massasi) (kg/kg).

Masalaning qo'yilishida qatnashuvchi o'zgarmas temperaturalar

$$T_c < T_3 < T_0$$

Bayon qilingan masalani fazoviy turning tugunlarida issiqlik tarqalishi chegarasini tutish metodi bilan echamiz. Buning uchun teng qadamli fazoviy to'rni qurib olamiz

$$x_i = (i-1)\cdot h, \quad i = 1, \dots, N;$$

$$x_1 = 0, \dots, x_N = L;$$

$$h = \frac{L}{N-1}$$

Bundan tashqari teng qadamli bo'lmagan vaqt to'ri tuzamiz.

$$t_{n+1} = t_n + \tau_{n+1}, n = 0,1, \dots, M-1;$$

$$t_0 = 0, \dots, t_M = t_{koneshne};$$

$$\tau_{n+1} > 0;$$

Vaqt bo'yicha τ_{n+1}, n =0,1,…, M -1 qadamni shunday tanlash kerakki, bu vaqt oralig'ida $(t_n \text{ dan } t_{n+1} \text{ gacha})$fazoviy o'tish chegarasi fazoviy to'rning toppa to'g'ri bitta qadamga o'tsin.

Uxolda $\frac{d\xi}{dt} \approx \frac{h}{\tau_{n+1}}$; bo'ladi.

Tuproqning birinchi qismida ayirmali sxemani qaraymiz.(59) sistemaning birinchi tenglamasini diskretlashtirish uchun oshkormas to'rt nuqtali sxemadan foydalanamiz.

$$\frac{\partial T_1}{\partial t} = a_1 \frac{\partial^2 T_1}{\partial x^2}, 0 < x < \xi(t), t > 0);$$

$$\frac{T_{1,i}^{n+1} - T_{1,i}^{n}}{\tau_{n+1}} = a_1 \frac{T_{1,i+1}^{n+1} - 2T_{1,i}^{n+1} + T_{1,i-1}^{n+1}}{h^2}, \ i = 2, \dots, i^* - 1;$$

$$T_1 \Big|_{i=1} = T_c;$$

$$T_1 \Big|_{i=i^*} = T_c;$$

Bu yerda $i = i^*$ fazoviy o’tish chegarasi.

Xosil qilingan sistemani quydagi umumiy ko’rinishga kelishi mumkin.

$$A_i \cdot T_{1,i+1}^{n+1} - B_1 \cdot T_{1,i}^{n+1} + C_i \cdot T_{1,i-1}^{n+1} = F_i;$$

$$A_i = C_i = \frac{a_1}{h^2},\ B_i = \frac{2 \cdot a_1}{h^2} + \frac{1}{\tau_2},\ \ F_i = -\frac{T_{1,i}^n}{\tau_{n+1}};$$

Progonka koeffisientlari (8) fo’rmuladan topiladi. So’ngra nomalum temperatura maydoni (7) ifodadan topiladi. Tuproqning ikkinchi qismida ayirmali sxemani qaraymiz (59) sistemaning ikkinchi tenglamasini diskretlashtirish uchun ham to’t nuqtali oshkormas sxemadan foydalanamiz.

$$\frac{\partial T_2}{\partial t} = a_2 \frac{\partial^2 T_2}{\partial x^2}, \xi(t) < x < L, t > o;$$

$$\frac{T_{2,i}^{n+1} - T_{2,i}^n}{\tau_{n+1}} = a_2 \frac{T_{2,i+1}^{n+1} - 2T_{2,i}^{n+1} + T_{2,i-1}^{n+1}}{h^2}, i^* + 1, \dots, N - 1;$$

$$T_2 \Big|_{i=i^*} = T_3;$$

$$\frac{\partial T_2}{\partial x} \Big|_{i=N} = 0;$$

Xosil qilingan sistemani quydagi umumiy ko’rinishga keltirish mumkin

$$A_i \cdot C_{2,i+1}^{n+1} - B_i \cdot T_{2,i}^{n+1} + C_i \cdot T_{2,i+1}^{n+1} = F_i\ ;$$

Bu yerda

$$A_i = C_i = \frac{a_2}{h^2}, B_i = \frac{2 \cdot a_2}{h_2} + \frac{1}{\tau_{n+1}}, F_i = -\frac{T_{2,i}^n}{\tau_{n+1}};$$

Progonka koeffisientlarini (8) fo'rmulalar yordamida topiladi. So'ngra nomalum temperatura maydoni (7) ifodadan topiladi. Chegaraviy shartni $x = \xi(t)$ da diskretlashtiramiz.

$$\lambda_1 \frac{\partial T_1}{\partial x} - \lambda_2 \frac{\partial T_2}{\partial x} = Q_{ф} \rho w \frac{d\xi}{dt} \qquad (61)$$

Bu shart xar bir vaqt qatlamida vaqt bo'yicha qadamni aniqlash uchun zarur.

$O(h)$ xatolik bilan diskretlashtirish o'tkazamiz.

$$\lambda_1 \frac{\partial T_1}{\partial x}\Big|_{x=\xi} - \lambda_2 \frac{\partial T_2}{\partial x}\Big|_{x=\xi} = Q_{ф} \rho w \frac{d\xi}{dt};$$

$$\lambda_1 \frac{T_{1,i^*} - T_{1,i^*-1}}{\square} - \lambda_2 \frac{T_{2,i^*+1} - T_{2,i^*}}{\square} = Q_{ф} \rho w \frac{\square}{\tau_{n+1}};$$

So'ngra $i < i^*$ bo'lganda tuproqning 1-qismi ,$i < i^*$ da tuproqning

2-qismida qaralgani uchun tuproqning qaralayotgan qismini xarakterlovchi indekslarni tushirib

$$\lambda_1 \frac{T_3 - T_{1,i^*-1}}{\square} - \lambda_2 \frac{T_{i^*+1} - T_3}{\square} = Q_{ф} \rho w \frac{\square}{\tau_{n+1}};$$

Va natiyjada

$$\tau_{n+1} = \frac{Q_{ф} \rho w \square^2}{\lambda_1 (T_3 - T_{i^*} - 1) - \lambda_2 (T_{i^*+1} - T_3)}; \qquad (62)$$

Ko'ramizki vaqt bo'yicha qadam temperaturaga bog'liq ekan. Uxolda temperatura maydonini aniqlash uchun oddiy iteratsiya metodidan foydalanish zarur. Bu metodning asosiy g'oyasi avval bayon qilingan edi. (61) chiziqlimas chegaraviy shartlarni $O(\square^2)$ xatolik bilan diskretlashtirish o'tkazamiz.(61) chegaraviy shartni

$O(\square^2)$ xatolik bilan diskretlashtiramiz. $T(x)$ funksiyani $x = \xi$ nuqta atrofida Teylor qatoriga $\square$ ga nisbatan ikkinchi tartibgacha yoyamiz.

$$T_{i^*+1}^{n+1} = T_{i^*}^{n+1} + \square \cdot \frac{\partial T_2}{\partial x}\Big|_{x=\xi}^{n+1} + \frac{\square^2}{2} \cdot \frac{\partial^2 T_2}{\partial x^2}\Big|_{x=\xi}^{n+1};$$

$$T_{i^*-1}^{n+1} = T_{i^*}^{n+1} - \square \cdot \frac{\partial T_2}{\partial x}\Big|_{x=\xi}^{n+1} + \frac{\square^2}{2} \cdot \frac{\partial^2 T_1}{\partial x^2}\Big|_{x=\xi}^{n+1};$$

(59) munosabatdan quydagini xosil qilamiz.

$$\frac{\partial^2 T_1}{\partial x^2}\Big|_{x=\xi}^{n+1} = \frac{1}{a_1}\frac{\partial T_1}{\partial t}\Big|_{x=\xi}^{n+1} = \frac{T_{i^*}^{n+1} - T_{i^*}^{n+1}}{a_1 \tau_{n+1}};$$

$$\frac{\partial^2 T_2}{\partial x^2}\Big|_{x=\xi}^{n+1} = \frac{1}{a_2}\frac{\partial T_2}{\partial t}\Big|_{x=\xi}^{n+1} = \frac{T_{i^*}^{n+1} - T_{i^*}^{n+1}}{a_2 \tau_{n+1}};$$

U xolda

$$\lambda_1 \frac{\partial T_1}{\partial x}\Big|_{x=\xi}^{n+1} = \frac{\lambda_1}{\square}\left(T_{i^*}^{n+1} - T_{i^*}^{n+1}\right) + \frac{\lambda_1 \square}{2a_1 \tau_{n+1}}\left(T_{i^*}^{n+1} - T_{i^*}^{n}\right);$$

$$\lambda_2 \frac{\partial T_2}{\partial x}\Big|_{x=\xi}^{n+1} = \frac{\lambda_2}{\square}\left(T_{i^*}^{n+1} - T_{i^*}^{n+1}\right) - \frac{\lambda_2 \square}{2a_2 \tau_{n+1}}\left(T_{i^*}^{n+1} - T_{i^*}^{n}\right);$$

(61) chegaraviy shartlarning $O(\square^2)$ xatolikdagi aproksimatsiyasi

$$\frac{\lambda_1}{\square}\left(T_{i^*}^{n+1} - T_{i^*-1}^{n+1}\right) + \frac{\lambda_1 \square}{2a_1 \tau_{n+1}}\left(T_{i^*}^{n+1} - T_{i^*}^{n}\right) - \frac{\lambda_2}{\square}\left(T_{i^*+1}^{n+1} - T_{i^*}^{n+1}\right) + \frac{\lambda_2 \square}{2a_2 \tau_{n+1}}\left(T_{i^*}^{n+1} - T_{i^*}^{n}\right) = Q_{\phi} \rho w \frac{\square}{\tau_{n+1}};$$

ko’rinishni oladi.

$$\tau_{n+1} = \frac{2a_1 a_2 \square^2 Q_{\phi} \rho w - \square^2 (\lambda_1 a_2 + \lambda_2 a_1)(T_{i^*}^{n+1} - T_{i^*}^{n})}{2a_1 a_2 [\lambda_1 \left(T_{i^*}^{n+1} - T_{i^*-1}^{n+1}\right) - \lambda_2 (T_{i^*+1}^{n+1} - T_{i^*}^{n+1})]};$$ yoki

$$\tau_{n+1} = \frac{2a_1 a_2 \square^2 Q_{\phi} \rho w - \square^2 (\lambda_1 a_2 + \lambda_2 a_1)(T_3 - T_{t^*}^{n})}{2a_1 a_2 [\lambda_1 \left(T_3 - T_{t^*-1}^{n+1}\right) - \lambda_2 (T_{i^*+1}^{n+1} - T_3)]}; \quad (63)$$

Tuproq qatlamining chuqurligi $L = 0.3m$ Tuproqning muzlagan qismining teplofizik xarakteristikalari

$$\lambda_1 = 2.3 \quad V/(m \cdot K), \rho_1 = 917\, kg/m^3, c_1 = 2090 \; Dj/(kg \cdot K)$$

Tuproqning yonbosh qismining teplofizik xarakteristikalari

$$\lambda_2 = 0.6\; Vt/(m \cdot K), \;\; \rho_2 = 1000\; kg/m^3, c_2 = 4220\, Dj/(kg \cdot K)$$

Xarakterli temperaturalar $T_0 = 293K,\; T_3 = 273K,\; T_c = 268K$

Fazoviy o'tishning issiqligi $Q_{\phi} = 3.32 \cdot 10^5\, Dj/kg$

Tuproqning namligi $w = 1\; kg/kg$.

I-BOB.

Chekli ayirmali sxemalar (ChAS) to`g`risida tushunchalar. Differentsial operatorning chekli ayirmali (ChA) approksimatsiyasi. ChA masalaning qo`yilishi.

1.1 To`rlar va to`r funktsiyalar

Berilgan differentsial tenglamani taqribiy ifodalovchi ayirmali sxemalarni yozish uchun quyidagi ikkita amal bajarilishi kerak.

1. Argumentning uzluksiz o`zgarish sohasini uning diskret o`zgarish sohasiga almashtirish kerak;

2. Differentsial operatorni qandaydir chekli ayirmali operatorga almashtirish, bundan tashqari chegaraviy shartlar va boshlang`ich ma`lumotlar uchun ayirmali almashtirishlar tuzish kerak.

Bu jarayon amalga oshirilgandan keyin algebraik tenglamalar sistemasiga o`tamiz.

Uzluksiz argumentning barcha qiymatlari uchun ayirmali masalani echib bo`lmaydi. SHuning uchun bu sohada qandaydir chekli sondagi nuqtalar to`plami olinadi va faqat shu nuqtalarda taqribiy echim izlanadi. Bunday nuqtalar to`plamiga to`r deyiladi. Nuqtalarning o`zi esa to`r tugunlari deb ataladi.

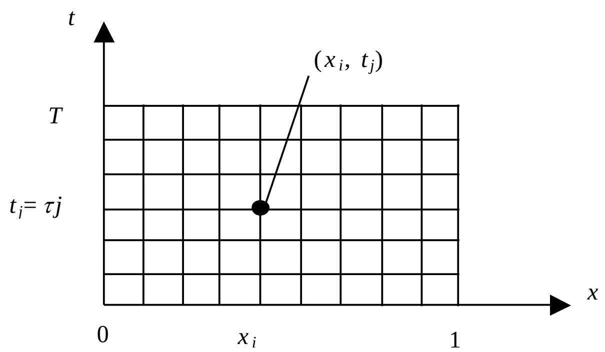
To`r tugunlarida aniqlangan funktsiyaga to`rli funktsiya deyiladi.

SHunday qilib differentsial tenglama echimlari fazosini to`r funktsiyalar fazosi bilan almashtirdik.

<u>1 misol.</u> <u>Kesmada tekis to`r.</u> $[0,1]$ kesmani $N$ ta teng bo`lakga bo`lamiz. $h = x_i - x_{i-1} = 1/N$ to`r qadami deb ataladi. Bo`linish nuqtalari $x_i = ih$ - to`r tugunlari deyiladi. Barcha tugunlar to`plami $\omega_h = \{x_i = ih, \, i = \overline{1, N-1}\}$ to`rni tashkil qiladi. Bu to`plamga $x_0 = 0, \, x_n = 1$ chegaraviy nuqtalarni qo`shish mumkin, ya`ni $\overline{\omega}_h = \{x_i = ih, \, i = \overline{0, N}\}$ deb belgilaymiz.

<u>2 misol.</u> <u>Tekislikda tekis to`r</u>. $\overline{D} = \{0 \le x \le 1, \, 0 \le t \le T\}$ sohada aniqlangan ikki o`zgaruvchili $u(x,t)$ funktsiyalar to`plamini qaraymiz.

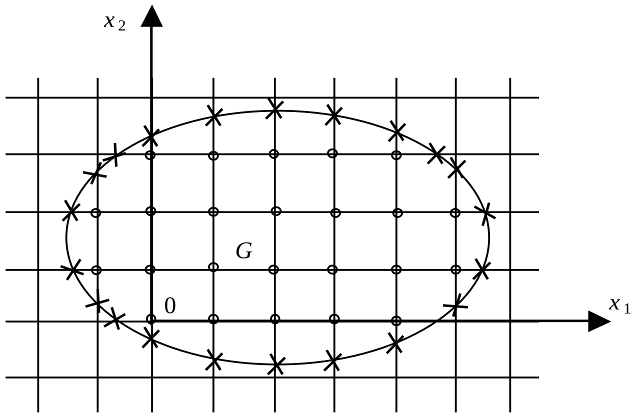
x o`qining $[0,1]$ va $t$ o`qining $[0,T]$ kesmalarini mos ravishda N_1 va N_2 ta bo`laklarga bo`lamiz. $h = 1/N_1, \, \tau = T/N_2$ bo`lsin. Bo`linish nuqtlaridan o`qlarga parallel to`g`ri chiziqlar o`tkazamiz. Natijada bu to`g`ri chiziqlar kesishishidan (x_i, t_j) tugunlarni hosil qilamiz, ular $\overline{\omega}_{h\tau} = \{(x_i, t_j) \in \overline{D}\}$ to`rni tashkil qiladi.



Bu to`r $x$ va $t$ yo`nalishlar bo`yicha h va τ qadamlarga ega.

SHunga o`xshash kesmada yoki tekislikda notekis to`rni qurish mumkin.

$x = (x_1, x_2)$ tekislikda Γ chegarali murakkab ko`rinishli $G$ soha berilgan bo`lsin.



$x_{1i} = ih_1, x_{2j} = jh_2, \; i, j = 0, \pm 1, \pm 2, ..., \quad h_1, h_2 > 0$ to`g`ri chiziqlar o`tkazamiz. U holda $(ih_1, jh_2)$ to`rni hosil qilamiz. «$\circ$» bilan ichki, «$\times$» bilan esa tashqi nuqtalar belgilangan. Ichki nuqtalar to`plamini $\omega_h$ bilan, chegaraviy nuqtalar to`plamini γ_h bilan belgilaymiz. Shunday qilib ω_h to`r $ox_1$, $ox_2$ yo`nalishlar bo`yicha tekis, ammo $\overline{G}$ soha uchun $\overline{\omega_h} = \omega_h + \gamma_h$ to`r esa chegara yaqinida notekis.

Uzluksiz $x \in \overline{G}$ argumentli $u(x)$ funktsiyalar o`rniga $y(x_i)$ to`r funktsiya olinadi. $y(x_i)$ to`r funkiyani vektor ko`rinishda berish mumkin.

Odatda $\{\omega_h\}$ to`r to`plami h qadamga bog`liq bo`ladi. Mos ravishda $y_h(x)$ to`r funktsiyalar ham $h$ parametrdan bog`liq bo`ladi. Agar to`r notekis bo`lsa h sifatida $h = (h_1, h_2, ..., h_n)$ vektor qaraladi.

Uzluksiz argumentli $u(x), \; x \in G$ funktsiyalar qandaydir H_0 funktsional fazo elementlaridan iborat. $y_h(x)$ to`r funktsiyalar esa $H_h$ fazoning elementlari. SHunday qili chekli ayirmalar usuli $H_0$ fazoni $y_h(x)$ to`r funktsiyalarning H_h fazosiga o`tkazadi.

H_0 fazodagi $\|\cdot\|_0$ norma kabi H_h chiziqli fazoda $\|\cdot\|_h$ norma kiritiladi.

Bir qator normalarni keltiramiz

1) C da normaning to`r ko`rinishi:

$$\|y\|_c = \max_{x \in \overline{\omega}_h} |y(x)| \text{ yoki } \|y\|_c = \max_{0 \le i \le N} |y_i|.$$

2) L_2 da normaning to`r ko`rinishi:

$$\|y\| = \left(\sum_{i=1}^{N-1} y_i^2 h \right)^{1/2} \text{ yoki } \|y\| = \left(\sum_{i=1}^{N} y_i^2 h \right)^{1/2}.$$

Taqribiy usullar nazariyasining asosiy e`tibori y_h ning $u(x)$ ga yaqinligini baholashga qaratiladi. Biroq y_h va $u(x)$ lar turli fazolarning vektorlaridir.

Baholashning ikkita imkoniyati mavjud:

1. $\omega_h(G)$ da berilgan y_h funktsiya G sohaning barcha nuqtalarida aniqlanadi (masalan, chiziqli interpolyatsiya yordamida). Natijada $x \in G$ uzluksiz argumentli $\tilde{y}(x,h)$ funktsiyani olamiz. $\tilde{y}(x,h)-u(x)$ ayirma H_0 ga tegishli bo`ladi. y_h ning u ga yaqinligi $\left\|\tilde{y}(x,h)-u(x)\right\|_0$ bilan xarakterlanadi, bunda $\|\cdot\|_0$ - H_0 dagi norma.
2. H_0 fazo H_h ga akslantiriladi. Har bir $u(x) \in H_0$ funktsiyaga mos $u_h(x),\ x \in \omega_h$ to`r funktsiyaga o`tkaziladi, ya`ni $u_h = \Re_h u \in H_h$. Bunda $\Re_h$ - $H_0$ dan $H_h$ ga o`tkazuvchi chiziqli operator. Bu moslikni turli yo`llar bilan amalga oshirish mumkin ($\Re_h$ turli operatorlarni tanlash bilan). Agar $u(x)$ uzluksiz funktsiya bo`lsa, $u_h(x)=u(x)$ deyish mumkin, bu erda $x \in \omega_h$. Ba`zan $x_i \in \omega_h$ tugunda $u_h(x_i)$ berilgan $x_i \in \omega_h$ tugunning qandaydir atrofi bo`yicha $u(x)$ ning o`rta integral qiymati bilan aniqlanadi. Bundan keyin $u(x)$ - uzluksiz funktsiya va barcha $x_i \in \omega_h$ lar uchun $u_h(x_i)=u(x_i)$ bo`ladi deb faraz qilamiz.

u_h to`r funktsiyaga ega bo`lib, H_h fazoning vektori bo`lgan $y_h - u_h$ ayirmani hosil qilamiz. y_h ning u ga yaqinligi $\left\|y_h - u_h\right\|_h$ bilan xarakterlanadi, bunda $\|\cdot\|_h$ - H_h dagi norma. Bunda H_h fazodagi norma $\|\cdot\|_0$ normani barcha $u \in H_0$ vektor uchun approksimatsiyalaydi

$$\lim_{h\to 0}\left\|u_h\right\|_h = \left\|u\right\|_0$$

deb faraz qilish tabiiydir. Bu shartni H_h va H_0 fazodagi normalarning o`zaro kelishganlik sharti deb ataymiz.

Biz bundan keyin ikkinchi yo`ldan foydalanamiz.

1.2 Oddiy differentsial operatorlarning ayirmali approksimatsiyasi

Lv chiziqli differentsial operator bo`lsin. $Lv$ ga kiruvchi hosilalarni ayirmali munosabatlar bilan almashtiramiz, $Lv$ o`rniga shablon deb ataluvchi biror to`r tugunlari to`plamida v_h to`r funktsiya qiymatlarining chiziqli kombinatsiyasidan iborat $L_h v_h$ ni hosil qilamiz:

$$L_h v_h(x) = \sum_{\xi \in Ш(x)} A_h(x,\xi) v_h(\xi)$$

yoki

$$(L_h v_h)_i = \sum_{x_j \in Ш(x_i)} A_h(x_i, x_j) v_h(x_j),$$

bu erda $A_h(x,\xi)$ - koeffitsientlar, h - to`r qadami, $Ш(x)$ - x nuqtadagi shablon. Lv ni $L_h v_h$ ga bunday taqribiy almashtirish differentsial operatorni ayirmali operator bilan approksimatsiyalash deyiladi (yoki L operatorning ayirmali approksimatsiyasi deyiladi).

L operatorni ayirmali approksimatsiyaga keltirishda shablon tanlash zarur, ya`ni $L$ operatorni approksimatsiyalash uchun qo`llash mumkin bo`lgan $v(x)$ to`r funktsiyaning qiymatlaridan bog`liq bo`lgan x tugun bilan qo`shni tugunlar to`plamini ko`rsatish kerak.

Quyidagi misollarni qaraymiz.

1. $Lv = dv/dx$.

x nuqtani fiksirlaymiz va $x-h$, $x+h$ nuqtalarni olamiz, bu erda $h>0$. Lv approksimatsiya qilish uchun quyidagi ifodalardan biridan foydalanish mumkin

$$L_h^+ v \equiv \frac{v(x+h)-v(x)}{h} \equiv v_x , \tag{1}$$

$$L_h^- v \equiv \frac{v(x)-v(x-h)}{h} \equiv v_{\bar{x}} . \tag{2}$$

(1) ifoda o`ng ayirmali hosila, (2) esa chap ayirmali hosilani tasvirlaydi. Ayirmali ifodalar ikki nuqtada aniqlangan.

Yana quyidagi ifodani olish mumkin

$$L_h^\sigma v \equiv \sigma v_x + (1-\sigma) v_{\bar{x}} ,$$

bunda σ - ixtiyoriy haqiqiy son. Xususiy holda $\sigma = 0,5$ bo`lganda markaziy ayirmali hosilani olamiz

$$v_{\overset{\circ}{x}} \equiv \frac{1}{2}\left(v_x + v_{\bar{x}}\right) = \frac{v(x+h)-v(x-h)}{2h} . \tag{3}$$

(1), (2), (3) approksimatsiyalarda qanday xatolikga yo`l qo`yildi va x nuqtada $h \to 0$ da $\psi(x) = L_h v(x) - Lv(x)$ ayirma qay holda o`zini tutadi kabi savollarga javob berish muhimdir. $\psi(x)$ miqdor x nuqtada Lv ayirmali approksimatsiya xatoligi deyiladi.

$v(x)$ ni Teylor qatoriga yoyamiz

$$v(x \pm h) = v(x) \pm h v'(x) + \frac{h^2}{2} v''(x) + O\left(h^3\right).$$

Bu ifodani (1), (2), (3) larga qo`yamiz

$$v_x = v'(x) + \frac{h}{2} v''(x) + O(h^2), \quad v_{\bar{x}} = v'(x) - \frac{h}{2} v''(x) + O(h^2), \quad v_{\overset{\circ}{x}} = v'(x) + O(h^2).$$

Bundan ko`rinib turibdiki

$$\psi = v_x - v'(x) = O(h), \quad \psi = v_{\bar{x}} - v'(x) = O(h), \quad \psi = v_{\overset{\circ}{x}} - v'(x) = O(h).$$

V - L_h ayirmali operatorning $h < h_0$ o`lganda $Ш(x,h)$ shablondan iborat $x$ nuqtaning $Ш(x,h_0)$ atrofida berilgan etarlicha silliq funktsiyalar $v \in V$ sinfi bo`lsin.

Agar

$$\psi(x) = L_h v(x) - Lv(x) = O(h^m)$$

bo`lsa L_h operator L differentsial operatorni x nuqtada $m > 0$ tartib bilan approksimatsiyalaydi deymiz.

<u>2 misol</u>. $Lv = v'' = \dfrac{d^2 v}{dx^2}$.

Uch nuqtali shablonda

$$L_h v = \frac{v(x+h) - 2v(x) + v(x-h)}{h^2}.$$

$$v_x(x) = v_{\bar{x}}(x+h), \quad L_h v = \frac{v_x(x) - v_{\bar{x}}(x)}{h} = \frac{1}{h}\left[v_{\bar{x}}(x+h) - v_{\bar{x}}(x)\right] = v_{\bar{x}x}(x).$$

Ushbu holda approksimatsiya tartibi ikkiga teng bo`ladi, ya`ni $v_{\bar{x}x}(x) - v''(x) = O(h^2)$.

Besh nuqtali shablonda

$$(x - 2h, x - h, x, x + h, x + 2h) \tag{4}$$

$Lv = v^{(4)}$ hosila uchun

$$L_h v = v_{\bar{x}x\bar{x}x} = \frac{1}{h^2}\left[v_{\bar{x}x}(x+h) - 2v_{\bar{x}x}(x) + v_{\bar{x}x}(x-h)\right] =$$

$$= \frac{1}{h^4}\left[v(x+2h) - 4v(x+h) + 6v(x) - 4v(x-h) + v(x-h)\right]$$

olinadi.

Approksimatsiya tartibi ikkiga teng, ya`ni

$$\psi = v_{\bar{x}x\bar{x}x} - v^{(4)} = \frac{h^2}{6}v^{(6)} + O(h^4).$$

h daraja bo`yicha $\psi = L_h v - Lv$ approksimatsiya xatoligi yoyilmasidan approksimatsiya tartibini oshirish uchun foydalanish mumkin. SHunga binoan

$$v_{\bar{x}x} - v'' = \frac{h^2}{12}v^{(4)} + O(h^4) \le \frac{h^2}{12}v_{\bar{x}x\bar{x}x} + O(h^4)$$

ga ega bo`lamiz.

Bundan (4) shablonda aniqlangan

$$L_h v = v_{\bar{x}x} - \frac{h^2}{12}v_{\bar{x}x\bar{x}x}$$

operator $Lv = v''$ ni to`rtinchi tartib bilan approksimatsiya qilishi kelib chiqadi.

Lemma. Agar $v \in C^{(2)}[x-h, x+h]$ bo`lsa

$$v_{\bar{x}x} = \frac{v(x+h) - 2v(x) + v(x-h)}{h^2} = v''(\xi), \quad \xi = x + \theta h, \quad |\theta| \le 1,$$

va agar $v \in C^{(4)}[x-h, x+h]$ bo`lsa

$$v_{\bar{x}x} = v''(x) + \frac{h^2}{12} v^{(4)}(\xi), \quad \xi = x + \theta_1 h, \quad |\theta_1| \le 1,$$

formulalar o`rinli bo`ladi.

Isbot. Integral shakldagi qoldiq hadi bilan olingan Teylor formulasidan foydalanamiz

$$v(x) = v(a) + (x-a)v'(a) + \dots + \frac{(x-a)^r}{r!} v^{(r)}(a) + R_{r+1}(x), \tag{5}$$

bunda

$$R_{r+1}(x) = \frac{1}{r!}\int_a^x (x-\xi)^r v^{r+1}(\xi)d\xi = \frac{(x-a)^{r+1}}{r!}\int_0^1 (1-s)^r v^{r+1}(a + s(x-a))ds .$$

Integral uchun o`rta qiymat haqidagi teoremani qo`llaymiz

$$R_{r+1}(x) = \frac{(x-a)^{r+1}}{(r+1)!} v^{(r+1)}(\xi),$$

bunda ξ - $[a, x]$ kesmada x ning o`rta qiymati,

$$\xi = a + \theta(x-a), \quad 0 \le \theta \le 1, \quad \int_0^1 (1-s)^r ds = \frac{1}{r+1}.$$

(5) da x ni $x+h$ va a ni x bilan almashtirib, $r = 1$ va $r = 3$ uchun mos ravishda quyidagilarni olamiz

$$v(x+h) = v(x) + hv'(x) + h^2\int_0^1 (1-s)v''(x+sh)ds, \tag{6}$$

$$v(x+h) = v(x) + hv'(x) + \frac{h^2}{2}v''(x) + \frac{h^3}{6}v'''(x) + \frac{h^4}{6}\int_0^1 (1-s)^3 v^{(4)}(x+sh)ds. \tag{7}$$

Bu erda h ni $-h$ ga, s ni $-s$ almashtirib

$$v(x-h)=v(x)-hv'(x)+h^2\int_{-1}^{0}(1+s)v''(x+sh)ds\ , \tag{8}$$

$$v(x-h)=v(x)-hv'(x)+\frac{h^2}{2}v''(x)-\frac{h^3}{6}v'''(x)+\frac{h^4}{6}\int_{-1}^{0}(1+s)^3v^{(4)}(x+sh)ds \tag{9}$$

formulalarni olamiz.

(6), (8) dan quyidagini olamiz

$$v_{\bar{x}x}=\frac{v(x+h)-2v(x)+v(x-h)}{h^2}=\int_{-1}^{1}g_2(s)v''(x+sh)ds\ ,$$

bunda

$$g_2(s)=\begin{cases}1+s & \text{агар} \quad -1\le s\le 0 \quad \text{да}\ ,\\ 1-s & \text{агар} \quad 0\le s\le 1 \quad \text{да}\ .\end{cases}$$

$g_2(s)\ge 0$ bo`lganligidan o`rta qiymat haqidagi teoremadan foydalanish mumkin, natijada

$$v_{\bar{x}x}=v''(x+\theta h)\int_{-1}^{1}g_2(s)ds=v''(x+\theta x)=v''(\xi),\quad -1\le\theta\le 1\ ,$$

bu erda ξ - $[x-h,x+h]$ kesmada o`rta nuqta.

(7) va (9) dan

$$v_{\bar{x}x}=v''(x)+\frac{h^2}{6}\int_{-1}^{1}g_4(s)v^{(4)}(x+sh)ds$$

hosil qilamiz, bu erda

$$g_4(s)=\begin{cases}(1+s)^3 & \text{агар} \quad -1\le s\le 0 \quad \text{да}\ ,\\ (1-s)^3 & \text{агар} \quad 0\le s\le 1 \quad \text{да}\ ,\end{cases}$$

$$\int_{-1}^{1}g_4(s)ds=\frac{1}{2}\ .$$

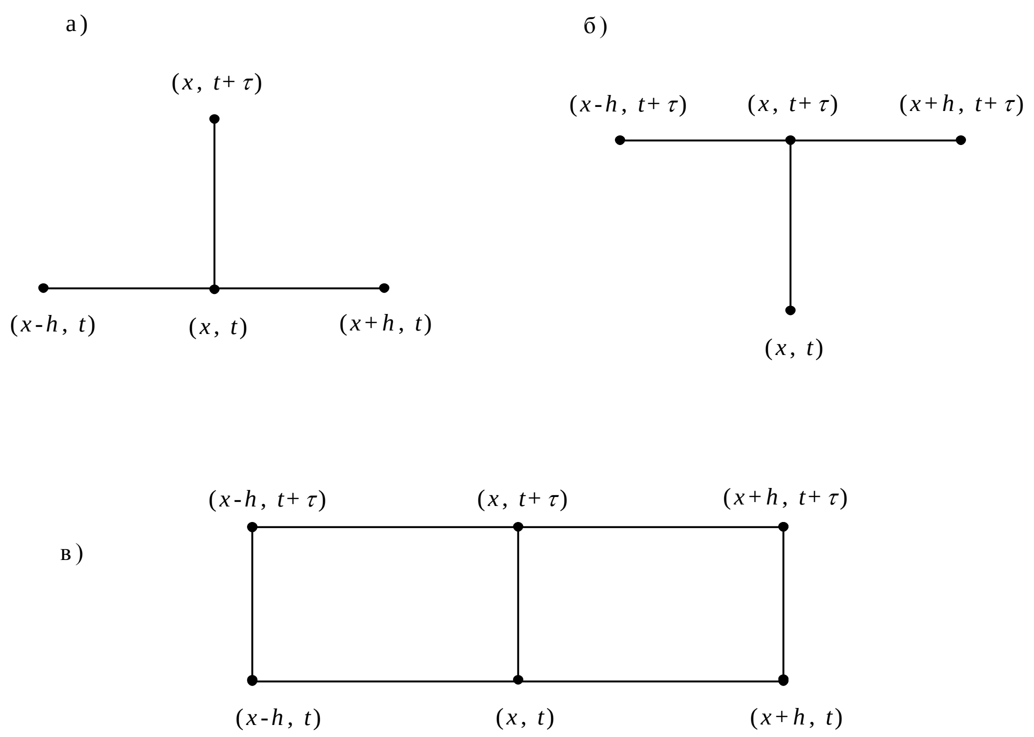
$g_4(s)\ge 0$ va $v^{(4)}(x)$ uzluksizligidan, o`rta qiymat haqidagi teoremani qo`llab

$$v_{\bar{x}x}=v''(x)+\frac{h^{(4)}}{12}(x+\theta h)\ ,\ |\theta|\le 1$$

ni olamiz va shu bilan lemma isbotlanadi.

<u>4 misol</u>. $Lv=\dfrac{\partial v}{\partial x}-\dfrac{\partial^2 v}{\partial x^2}\ ,\ v=v(x,t)\ .$

(x, t) - tekislikda nuqta bo`lsin. SHablonni aniqlaymiz. U to`rtta nuqtadan tashkil topgan bo`lsin (a rasm).



$L_{h\tau}$ ni quyidagicha aniqlaymiz

$$L_{h\tau}^{(0)} v = \frac{v(x, t+\tau) - v(x,t)}{\tau} - \frac{v(x+h, t) - 2v(x,t) + v(x-h,t)}{h^2}.$$

Quyidagi belgilashlarni kiritamiz

$$v = v(x,t), \quad \hat{v} = v(x, t+\tau), \quad \check{v} = v(x, t-\tau).$$

Unda

$$v_t = \frac{v(x, t+\tau) - v(x,t)}{\tau} = \frac{\hat{v} - v}{\tau}$$

va

$$L_{h\tau}^{(0)} = v_t - v_{\bar{x}x}. \tag{10}$$

b rasmdagi shablondan foydalanilganda, $t+\tau$ momentda $v_{\bar{x}x}$ olinsa, u holda

$$L_{h\tau}^{(1)} = v_t - \hat{v}_{\bar{x}x}. \tag{11}$$

(10) va (11) larning chiziqli kombinatsiyasini olib, $\sigma \neq 0$ va $\sigma \neq 1$ bo`lganda oltinuqtali shablonda (v rasm) aniqlangan chekli operatorlarning bir parametrli oilasini hosil qilamiz

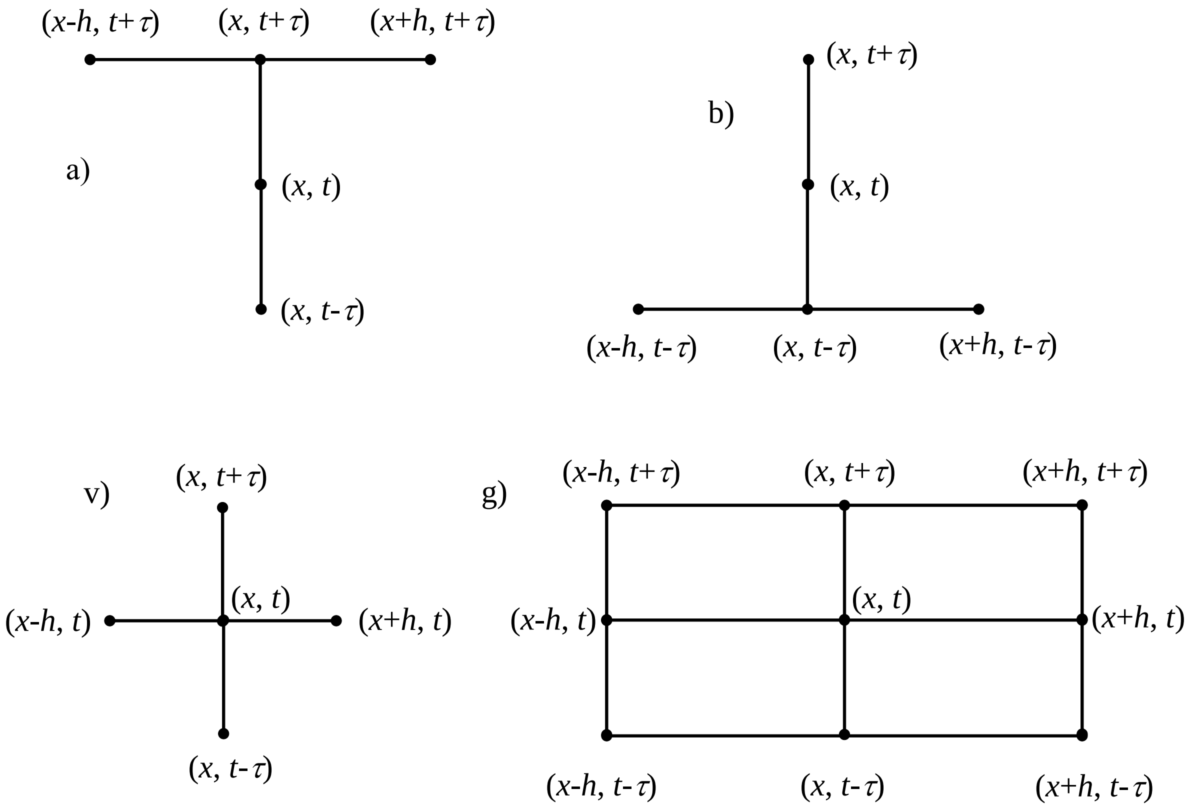
$$L^{(\sigma)}_{h\tau} v = v_t - \left(\sigma \hat{v}_{\bar{x}x} + (1-\sigma) v_{\bar{x}x}\right). \tag{12}$$

$L^{(0)}_{h\tau}$ operatorlar $L^{(1)}_{h\tau}$ ning approksimatsiya tartibiga ega, (12) esa $\sigma < 0{,}5$ bo`lganda $O\left(h^2 + \tau\right)$, $\sigma = 0{,}5$ bo`lganda $O\left(h^2 + \tau^2\right)$ approksimatsiya tartibiga ega.

<u>5 misol</u>.

$$Lv = \frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 v}{\partial x^2}.$$

Quyidagi shablonlardan foydalanilamiz



Approksimatsiyalardan biri (v rasm)

$$L_{h\tau} = v_{\bar{t}t} - v_{\bar{x}x}, \tag{13}$$

bunda

$$v_{\bar{t}t} = \left(v(x, t+\tau) - 2v(x,t) + v(x, t-\tau)\right) / \tau^2 .$$

a) shablonda:

$$L_{h\tau} v = v_{\bar{t}t} - \hat{v}_{\bar{x}x}. \tag{14}$$

To`qqiznuqtali shablonda (g rasm) ayirmali operatorlarning ikkiparametrli oilasini yozish mumkin

$$L_{h\tau}^{(\sigma_1,\sigma_2)} v = v_{\bar{t}t} - \left(\sigma_1 \hat{v}_{\bar{x}x} + (1-\sigma_1-\sigma_2) v_{\bar{x}x} + \sigma_2 \check{v}_{\bar{x}x}\right). \tag{15}$$

(15) dan $\sigma_1 = \sigma_2 = 0$ bo`lganda (13), $\sigma_2 = 0, \sigma_1 = 1$ bo`lganda esa (14) kelib chiqadi.

(13), (14), (15) ayirmali operatorlar $O\left(h^2 + \tau^2\right)$ approksimatsiya tartibiga ega.

1.3 Approksimatsiya, korrektlik, turg`unlik, yaqinlashish va ular o`rtasida bog`lanish.

1.3.1 To`rda approksimatsiya xatoligi

Biz xozirgacha lokal ayirmali approksimatsiyani qaradik. Odatda to`rda ayirmali approksimatsiya tartibini baholash talab qilinadi.

ω_h - $\{x = (x_1, x_2, \ldots, x_p)\}$ to`r funktsiyalarning biror $G$ Evklid fazosidagi to`r, H_h - ω_h da berilgan to`r funktsiyalarning chiziqli fazosi, $H_0$ - $v(x)$ silliq funktsiyalar fazosi bo`lsin. Faraz qilaylik, 1) ixtiyoriy $u \in H_0$ uchun $\Re_h u = u_h \in H_h$ bo`ladigan $\Re_h$ operator mavjud, 2) $\|\cdot\|_h$ va $\|\cdot\|_0$ normalar quyidagicha bo`lsin, ya`ni

$$\lim_{|h|\to 0} \|\Re_h u\|_h = \|u\|_0,$$

bunda $|h|$ - h vektorning normasi.

H_0 da berilgan qandaydir L operatorni va ω_h da berilgan v_h to`r funktsiyani $L_h v_h$ to`r funktsiyaga akslantiruvchi L_h operatorni qaraymiz (ya`ni $H_h$ dan $H_h$ ga ta`sir qiluvchi).

L operatorni L_h ayirmali operator bilan approksimatsiyalash xatoligi deb

$$\psi_h = L_h v_h - (Lv)_h,$$

to`r funktsiyaga aytiladi, bunda $v_h = \Re_h v$, $(Lv)_h = \Re_h (Lv)$, v - H_0 dagi ixtiyoriy funktsiya (vektor, element).

$|h| \to 0$ da $\|\psi_h\|_h \to 0$ intilsa L differentsial operatorni L_h ayirmali operator approksimatsiyalaydi deymiz.

$$\|\psi_h\|_h = \|L_h v_h - (Lv)_h\|_h = O\left(|h|^m\right), \tag{1}$$

yoki

$$\left\| L_h v_h - (Lv)_h \right\|_h \le M \cdot |h|^m$$

bo`lsa $m > 0$ tartib bilan $L$ differentsial operatorni $L_h$ ayirmali operatori approksimatsiyalaydi deb ataymiz, bunda $M$ - $|h|$ dan bog`liq bo`lmagan musbat o`zgarmas son.

$\Re_h$ opeartorni tanlashga misollar:

1) agar $v(x)$ - uzluksiz funktsiya bo`lsa, u holda

$$v_h = \Re_h v(x) = v(x), \quad x \in \omega_h ;$$

2) $$v_h = \Re_h v(x) = \frac{1}{2h}\int_{x-h}^{x+h} v(t)dt = \frac{1}{2}\int_{-1}^{1} v(x+sh)ds ,$$

bunda $v(x)$ - integral funktsiya va h.k.

<u>1 eslatma</u>. Agar $h = (h_1, h_2, ..., h_p)$ - vektor bo`lsa, $|h|$ ni $|h| = \left(h_1^2 + h_2^2 + ... + h_p^2\right)^{1/2}$ uzunlik deb tushunish mumkin. $\alpha = 1,2,..., p$ tartib bilan turli $h_\alpha$ bo`yicha approksimatsiya qilish mumkin. U holda (16) o`rniga

$$\left\| L_h v_h - (Lv)_h \right\|_h \le M \sum_{\alpha=1}^{p} h_\alpha^{m_\alpha} \text{, bunda } m_\alpha > 0 .$$

$m_1, m_2, ..., m_p$ lar orasida eng kichik sonni olamiz va uni m bilan belgilab (1) baholashni olamiz.

1. Agar ω_h notekis to`r, ya`ni $h = (h_1, h_2, ..., h_n)$ bo`lsa, misol uchun $|h| = \max\limits_{1 \le i \le n} h_i$ yoki o`rta kvadratik qiymat $|h|$ ni olish mumkin, bunda n - tugunlar soni.

<u>Misol</u>. *Notekis to`rda ayirmali approksimatsiya*. $0 \le x \le 1$ kesmada berilgan $H_0 = C^{(4)}[0,1]$ funktsiyalar fazosida $Lv = d^2 v / dx^2$ ni qaraymiz. Quyidagi to`rni olamiz

$$\hat{\bar{\omega}}_h = \{x_i , i = 0,1,..., n , x_0 = 0 , x_n = 1\}.$$

Lv operator (x_{i-1}, x_i, x_{i+1}) noregulyar shablonda x_i tugunda aniqlangan

$$(L_h v)_i = \frac{1}{\bar{h}_i}\left[\frac{v_{i+1} - v_i}{h_{i+1}} - \frac{v_i - v_{i-1}}{h_i}\right], \quad v_i = v(x_i), \quad \bar{h}_i = 0,5(h_i + h_{i+1}),$$

ayirmali operatorga mos keladi. Quyidagi belgilashlarni kiritamiz

$$v_{\bar{x},i} = \frac{v_i - v_{i-1}}{h_i}, \quad v_{x,i} = v_{\bar{x},i+1} = \frac{v_{i+1} - v_i}{h_{i+1}}, \quad v_{\hat{x},i} = \frac{v_{i+1} - v_i}{\bar{h}_i} .$$

$L_h v$ operatorni quyidagi ko`rinishda yozish mumkin

$$(L_h v)_i = v_{\bar{x}\hat{x},i} = v_{\bar{x}\hat{x}} .$$

Approksimatsiyaning lokal xatoligi

$$\psi_i = (L_h v)_i - (Lv)_i = \frac{h_{i+1} - h_i}{3} v_i''' + O(h_i^2)$$

ga teng.

Demak $L_h v$ opeator to`r normada

$$\|\psi\|_C = \max_{1 \le i \le n-1} |\psi_i| = O(h), \quad h = \max_{1 \le i \le n-1} h_i$$

birinchi tartibli approksimatsiyaga ega.

L_2 to`r normada quyidagicha

$$\|\psi\| = \left(\sum_{i=1}^{n-1} \hbar_i \psi_i^2 \right)^{1/2} = O(h)$$

birinchi tartibli approksimatsiyani ham olishimiz mumkin.

Biroq

$$\|\psi\|_{(-1)} = \left[\sum_{i=1}^{n-1} h_i \left(\sum_{k=1}^{i} \hbar_k \psi_k \right)^2 \right]^{1/2}$$

normada ψ ikkinchi tartibga ega, ya`ni

$$\|\psi\|_{(-1)} = O(h^2), \text{ bunda } h = \max_{1 \le i \le n} h_i .$$

Bu tasdiqni isbotlaymiz. ψ ni

$$\psi_i = \frac{h_{i+1}^2 - h_i^2}{6 \hbar_i} v_i''' + O(\bar{h}_i^2)$$

ko`rinishda yozamiz.

$v_i''' = v_{i+1}''' + O(h_{i+1})$ ni inobatga olib

$$\psi_i = \frac{h_{i+1}^2 v_{i+1}''' - h_i^2 v_i'''}{6 \hbar_i} + \psi_i^* = \overset{\circ}{\psi}_i + \psi_i^* ,$$

topamiz, bu erda $\psi_i^* = O(h^2)$ ixtiyoriy normada.

$\overset{\circ}{\psi}_i$ bosh had divergent ko`rinishga ega. SHuning uchun

$$S_i = \sum_{k=1}^{i} h_k \overset{\circ}{\psi}_k = \sum_{k=1}^{i} \left(h_{k+1}^2 v_{k+1}''' - h_k^2 v_k''' \right) / 6 = \left(h_{i+1}^2 v_{i+1}''' - h_1^2 v_1''' \right) / 6 .$$

Bundan $\left|S_i\right| \le Mh^2$ ekanligi ko`rinib turibdi va haqiqatdan

$$\left\|\overset{\circ}{\psi}\right\|_{(-1)} = \left(\sum_{i=1}^{n-1} h_i S_i^2\right)^{1/2} = O(h^2) .$$

Bundan

$$\left\|\psi\right\|_{(-1)} \le \left\|\overset{\circ}{\psi}\right\|_{(-1)} + \left\|\psi^*\right\|_{(-1)} \le Mh^2 ,$$

ya`ni $\|\cdot\|_{(-1)}$ normada approksimatsiya xatoligi ikkinchi tartibga ega.

$y(x, t) = y_{h\tau}(x,t)$ to`r funktsiyani

$$\omega_{h\tau} = \omega_h \times \omega_\tau = \{(x,t), \quad x \in \omega_h , \ t \in \omega_\tau\}$$

to`rda aniqlaymiz.

U $x \in \omega_h$ argumentning funktsiyasi bo`lib $\|\cdot\|_h$ norma bilan $H_h$ fazoning vektori hisoblanadi. $\omega_{h\tau}$ to`rda $y(x,t)$ ni baholash uchun odatda

$$\|y\|_{h\tau} = \max_{t\in\omega_\tau} \|y(t)\|_h$$

normadan yoki quyidagilarning biridan foydalaniladi

$$\|y\|_{h\tau} = \sum_{t\in\omega_\tau} \tau \|y(t)\|_h , \quad \|y\|_{h\tau} = \left[\sum_{t\in\omega_\tau} \tau \|y(t)\|_h^2\right]^{1/2} .$$

$L_{h\tau} v_{h\tau}$ - Lu $(u = u(x,t))$ opeatorning ayirmali approksimatsiyasi bo`lsin. $L_{h\tau}$ operator $\omega_{h\tau}$ to`rda berilgan $v_{h\tau}(x,t)$ to`r funktsiyalarda aniqlangan. $v(x,t) \in H_0$ bo`lsin. Agar $v(x,t)$ t bo`yicha uzluksiz bo`lsa, barcha $t \in \omega_\tau$ lar uchun $v_{h\tau}(x,t) = v_h(x,t)$ bo`lishi mumkin. SHunday qilib, $\omega_{h\tau}$ to`rda berilgan $v_{h\tau}(x,t)$ va approksimatsiya xatoligini aniqlash uchun

$$\psi_{h\tau}(x,t) = L_{h\tau} v_{h\tau}(x,t) - (Lv)_{h\tau}(x,t), \quad (x,t) \in \omega_{h\tau} .$$

Bu erda $v_h(x,t) = \Re_h v(x,t)$.

L ni $L_{h\tau}$ x bo`yicha $m > 0$ va $t$ bo`yicha $l > 0$ tartib bilan approksimatsiya qiladi deymiz, agar $v(x,t)$ etarli silliq funktsiyalar sinfida

$$\|\psi_{h\tau}(x,t)\|_{h\tau} = O\left(|h|^m + \tau^l\right) \text{ yoki } \|\psi_{h\tau}\|_{h\tau} \le M\left(|h|^m + \tau^l\right)$$

baholash bajarilsa, bunda M - $|h|$ va l dan bog`liq bo`lmagan musbat o`zgarmas.

1.3.2 Sxemalar yaqinlashishi va aniqligi

Chekli to`rda biror masalani taqribiy echishda dastlab quyidagicha muloxazaga ega bo`lish kerak, ya`ni bu usul yordamida masala echilaganda masalaning aniq echimiga qanday aniqlikda yaqinlashishi mumkin. SHuning uchun ayirmali sxemalarning yaqinlashishi va aniqligi to`g`risidagi savolni qarab o`tish kerak.

Γ chegarali G sohada quyidagi chiziqli differentsial tenglama echimini topish talab qilingan bo`lsin

$$Lu = f(x), \quad x \in G . \tag{2}$$

Echim quyidagi qo`shimcha (chegaraviy va boshlang`ich) shartni qanoatlantirsin

$$lu = \mu(x), \quad x \in \Gamma , \tag{3}$$

bunda $f(x), \mu(x)$ - berilgan funktsiyalar, l - chiziqli differentsial operator.

$\overline{G}$ soha to`r bilan almashtiriladi.

h - to`r tugunlarining joylashish zichligini xarakterlaydigan vektor parametr bo`lsin, γ_h - chegaraviy tugunlar to`plami.

(2), (3) masalaga quyidagi ayirmali masalani mos qo`yamiz

$$\begin{aligned} L_h y_h &= \varphi_h, \quad x \in \omega_h, \\ l_h y_h &= \chi_h, \quad x \in \gamma_h, \end{aligned} \tag{4}$$

bunda $\varphi_h(x)$, $\chi_h(x)$ - ma`lum to`r funktsiyalar, L_h, l_h - $x \in \overline{\omega}_h = \omega_h + \gamma_h$ uchun berilgan to`r funktsiyalarga ta`sir qiluvchi operatorlar.

h ni o`zgartirib $\{y_h\}$ echimlar oilasini olamiz. SHunday qilib turli h lar uchun (4) ayirmali masalalar oilasi qaralishi kelib chiqadi. Bu (4) ayirmali masalalar oilasini ayirmali sxemalar deb ataymiz.

(2), (3) masalaning yechimiga, $h(\varepsilon)$ qadamni tanlashdan bog`liq ixtiyoriy berilgan $\varepsilon > 0$ aniqlikda (4) masalaning y_h echimi yaqinlashishini tushuntirish uchun y_h va $u(x)$ larni taqqoslash zarur.

Bu taqqoslashni H_h to`r funktsiyalar fazosida o`tkazamiz. u_h - ω_h to`rda $u(x)$ ning qiymati bo`lsin, bundan $u_h \in H_h$. Ayirmali sxema xatoligi

$$z_h = y_h - u_h$$

ni qaraymiz.

z_h uchun shartni yozamiz. $y_h = z_h + u_h$ ni (4) ga qo`yib $z_h$ uchun (4) ga o`xshash quyidagi masalani olamiz

$$
\begin{aligned}
&L_h z_h = \psi_h, \quad x \in \omega_h, \\
&l_h z_h = \nu_h, \quad x \in \gamma_h,
\end{aligned}
\tag{5}
$$

bunda ψ_h va ν_h - tafovutlar, ular $\psi_h = \varphi_h - L_h u_h, \quad \nu_h = \chi_h - l_h u_h$ teng.

(5) ning o`ng tomonlari (2) tenglamani (4) ayirmali tenglama bilan va (3) qo`shimcha shartlarni $L_h y_h = \chi_h$ ayirmali shart bilan approksimatsiyalashdagi xatolik deyiladi. Qisqasi ψ_h - (2) tenglamaning $u(x)$ echimida $L_h y_h = \varphi_h$ tenglama uchun approksimatsiya xatoligi, ν_h - (2), (3) masalani echishda $l_h y_h = \chi_h$ approksimatsiya xatoligi deymiz.

Sxemaning z_h xatoligi va ψ_h, ν_h approksimatsiya xatoliklarini baholash uchun, mos ravishda $\|\cdot\|_{(1h)}$, $\|\cdot\|_{(2h)}$, $\|\cdot\|_{(3h)}$ to`r funktsiyalar normalarini kiritamiz.

(4) ayirmali masala yechimi (2), (3) masala echimiga yaqinlashadi ((4) sxema yaqinlashadi) deymiz, agar

$$|h| \to 0 \text{ da } \|z_h\|_{(1h)} = \|y_h - u_h\|_{(1h)} \to 0,$$

yoki

$$\|z_h\|_{(1h)} = \|\rho(|h|)\|,$$

bunda

$$h \to 0 \text{ da } \rho(|h|) \to 0.$$

(4) ayirmali sxema $O(|h|^m)$ tezlik bilan yaqinlashadi yoki m -nchi tartibli aniqlikga ega deyiladi, agar etarlicha $|h| \le h_0$ da

$$\|z_h\|_{(1h)} = \|y_h - u_h\|_{(1h)} \le M|h|^m,$$

tengsizlik bajarilsa, bunda $M > 0$ - $|h|$ dan bog`liq bo`lmagan o`zgarmas, $m > 0$.

Quyidagicha savol tug`iladi, ya`ni sxema aniqligining tartibi approksimatsiya tartibidan bog`liqmiq $z_h = y_h - u_h$ xatolik $\psi_h$ (va $\nu_h$) o`ng qismli (5) masalaning echimi. Approksimatsiya tartibi bilan aniqlik tartibi o`rtasidagi aloqa ayirmali masala echimining o`ng tomondan bog`liqligi bilan xarakterlanadi. Agar $z_h$ $\psi_h$ va $\nu_h$ lardan uzluksiz bog`liq bo`lsa, aniqlik tartibi approksimatsiya tartibi bilan mos tushadi.

1.3.3 Ayirmali sxemalar turg`unligi

<u>Misollar</u>. Tenglamalarning o`ng tomonlarini, chegaraviy va boshlang`ich ma`lumotlarni chekli approksimatsiya qilishda biz bundan keyin bir atama bilan – muayyan xatolik bilan berilgan boshlang`ich qiymatlar deb ataymiz. Algebraik tenglamalar sistemasini sonli echish jarayonida ham xatolik ro`y beradi.

Boshlang`ich ma`lumotlardan bog`liq kichik xatoliklar hisoblash jarayonida oshmasligi va izlanayotgan echimni olishni buzmasligi sxemadan talab qilinadi.

Agarda boshlang`ich xatoliklar hisoblash jarayonida oshib ketsa sxemalarga turg`unmas sxemalar deyiladi va amalda ulardan foydalanib bo`lmaydi.

Misollar keltiramiz.

1 misol. *Turg`un sxema*.

$$u' = -\alpha u, \quad x > 0, \quad u(0) = u_0, \quad \alpha > 0 \tag{6}$$

bo`lsin.

Masalaning aniq echimi quyidagicha

$$u(x) = u_0 e^{-\alpha x}.$$

Bu echim uchun $\alpha > 0$ da $|u(x)| < |u_0|$ va haqiqatdan, $u(x)$ u_0 dan uzuluksiz bog`liq.

(6) masalani $\omega_h = \{x_i = ih, \quad i = 0,1,...\}$ tekis to`rda ayirmali masala approksimatsiyalaydi

$$(y_i - y_{i-1})/h + \alpha y_i = 0, \quad y_0 = u_0, \quad i = 1,2,...$$

yoki

$$y_i = sy_{i-1}, \quad s = \frac{1}{1+\alpha h}, \quad y_0 = u_0.$$

Bundan

$$y_i = s^i y_0$$

kelib chiqadi.

$\bar{x}$ fiksirlangan nuqtani qaraymiz va h qadamlar ketma-ketligini shunday tanlaymizki, $\bar{x}$ hamma vaqt $\bar{x} = i_0 h$ tugun nuqta bo`lsin. U holda $h \to 0$ da to`rni kichiklashtirganda tanlangan nuqta $\bar{x}$ ga mos keluvchi i_0 nomer cheksiz o`sadi. SHu nuqtada y ning qiymatini hisoblaymiz

$$y(\bar{x}) = y_{i_0} = s^{i_0} y_0.$$

$\alpha > 0$ va ixtiyoriy h da $|s| < 1$ bo`lganidan, ixtiyoriy $h$ da $|y(\bar{x})| \le s^{i_0}|y_0| < |y(0)|$ bo`ladi.

Oxirgi tengsizlikdan ko`rinib turibdiki, (6) ayirmali masala echimi boshlang`ich qiymatlardan uzluksiz bog`liq.

2 misol. Turg`unmas sxema. (6) masala uchun quyidagi sxemani qaraymiz

$$\sigma \frac{y_i - y_{i-1}}{h} + (1-\sigma)\frac{y_{i+1} - y_i}{h} + \alpha y_i = 0, \qquad (7)$$
$$y_0 = u_0, \quad y_1 = \bar{u}_0, \quad i = 1,2,\ldots,$$

bu erda $\sigma > 1$ - sonli parametr.

Sxema uch nuqtali bo`lganligi uchun, $y_0$ dan tashqari $y_1$ ning ham berilishi talab qilinadi. Agar $\bar{u}_0 = (1-\alpha h)u_0$ deb olinsa, u holda $\bar{u}_0 - u(h) = O(h^2)$ bo`ladi. (6) masalaning ayirmali echimini $y_i = s^i$ ko`rinishda izlaymiz. U holda (7) dan

$$(\sigma - 1)s^2 - (2\sigma - 1 + \alpha h)s + \sigma = 0,$$

kelib chiqadi. Bu tenglamaning 2 ta turli echimlari mavjud

$$s_{1,2} = \frac{2\sigma - 1 + \alpha h \pm \sqrt{(2\sigma - 1 + \alpha h)^2 - 4\sigma(\sigma - 1)}}{2(\sigma - 1)} =$$
$$= \frac{2\sigma - 1 + \alpha h \pm \sqrt{1 + 2(2\sigma - 1)\alpha h + \alpha^2 h^2}}{2(\sigma - 1)}.$$

(6) ning umumiy echimi quyidagi ko`rinishda

$$y_i = As_1^i + Bs_2^i. \qquad (8)$$

$i = 0, i = 1$ larni qo`yib va $y_0 = u_0, \quad y_1 = \bar{u}_0$ larni hisobga olib $A$ va $B$ o`zgarmaslarni topamiz

$$A = \frac{\bar{u}_0 - s_2 u_0}{s_1 - s_2}, \quad B = \frac{s_1 u_0 - \bar{u}_0}{s_1 - s_2}.$$

$\sigma > \sigma - 1 > 0$ dan, $s_1 s_2 > 1$ bo`ladi. Ixtiyoriy $\alpha h$ da $s_2 < 1$ bo`lishini ko`rsatamiz. O`z navbatida $\sigma > 1$ da

$$2(\sigma - 1) - \left(2\sigma - 1 + \alpha h - \sqrt{1 + 2(2\sigma - 1)\alpha h + \alpha^2 h^2}\right) =$$
$$= \sqrt{1 + 2(2\sigma - 1)\alpha h + \alpha^2 h^2} - (1 + \alpha h) > 0.$$

Ixtiyoriy αh ning qiymatida $s_1 > 1$ bo`lishidan, $s_1 s_2 > 1$ kelib chiqadi.

Agar $A \neq 0$ bo`lganda $y_i = As_1^i + Bs_2^i$ formuladan $i \to \infty$ da $y_i \to \infty$ bo`ladi. $y_1 = \bar{u}_0$ ni shunday tanlash kerakki, $A = 0$ bo`lsin. Buning uchun $\bar{u}_0 = s_2 u_0$ deb olish kifoya. $s_1^i$ echim atrofidagi xatolikdan hisoblash jarayonida qochib bo`lmaydi hamda turg`unmas sxemaga keladi.

h fiksirlangan nuqtada bu sxemada $x_i = ih$ oshishi bilan echimning oshishi kelib chiqadi. h ning kamayishi $\bar{x}_i = i_0 h$ fiksirlangan nuqtada xatolikning oshishga olib keladi, ya`ni $h$ ning kamayishi bilan $i_0 = \bar{x}/h$ oshadi. $h \to 0$ da boshlang`ich qiymatlarning kam o`zgarishi ixtiyoriy $\bar{x}$ fiksirlangan nuqtada masala echimining cheksiz o`sshiga olib keladi.

1.3.4 Ayirmali masala korrektligi

y_h biror ayirmali masalaning echimi, φ_h esa boshlang`ich qiymatlari bo`lsin. Ular h parametrdan bog`liq. $h$ ni o`zgartirib $\{\varphi_h\}$ boshang`ich qiymatlarga mos $\{y_h\}$ echimlar ketma-ketligini olamiz. Shunday qilib, nafaqat bir ayirmali masalani, balki $h$ parametrdan bog`liq masalalar oilasini qaraymiz. $|h| \to 0$ da ayirmali masalalar oilasi uchun korrektlik tushunchasi kiritiladi.

Barcha etarlicha kichik $|h| \le h_0$ larda masala korrekt deyiladi, agar:

1) Qandaydir mumkin bo`lgan oiladan barcha $\varphi_h$ boshlang`ich qiymatlar uchun ayirmali masala echimi y_h mavjud va yagona bo`lsa;
2) y_h echim φ_h dan uzluksiz bog`liq hamda bu bog`liqlik h ga nisbatan tekis bo`lsa.

2-nchi shart yanada aniqroq shuni bildiradiki, etarlicha kichik $|h| \le h_0$ da h dan bog`liq bo`lmagan $M > 0$ o`zgarmas mavjud bo`lib

$$\left\|\tilde{y}_h - y_h\right\|_{(1h)} \le M \left\|\tilde{\varphi}_h - \varphi_h\right\|_{(2h)} \tag{9}$$

tengsizlik bajariladi, bunda $\tilde{y}_h$ - $\tilde{\varphi}_h$ boshlang`ich qiymatli masala echimi, $\|\cdot\|_{(1h)}$ va $\|\cdot\|_{(2h)}$ lar esa $\omega_h$ to`rda berilgan to`r funktsiya to`plamidagi normalar.

(8) tengsizlik bilan ifodalangan ayirmali masala echimining boshlang`ich qiymatlardan uzluksiz bog`liqlik xossasiga boshlang`ich qiymat bo`yicha sxema turg`unligi deyiladi.

1.4 Ikkinchi tartibli ODT uchun chegaraviy masalalarni o`q otish va chekli ayirmalar usuli bilan yechish. Progonka usulining turg`unligi

Chegaraviy masalalarni echishning sonli usullarini qaraymiz. Ularni ikkita guruiga ajratish mumkin:

1) Chegaraviy masala echimini ketma-ket Koshi masalalarini echishga keltirish;

2) Chekli ayirmalar usullarini qo`llash.

Birinchi gurui usullariga, xususan, o`q otish usuli kiradi.

1.4.1 O`q otish usuli

[0,1] kesmada ikkinchi hosilaga nisbatan echilgan ikkinchi tartibli tenglama uchun chegaraviy masalani qaraymiz:

$$y'' = f(x, y, y'). \tag{1}$$

Iar qanday kesmani

$$t = \frac{x - a}{b - a}$$

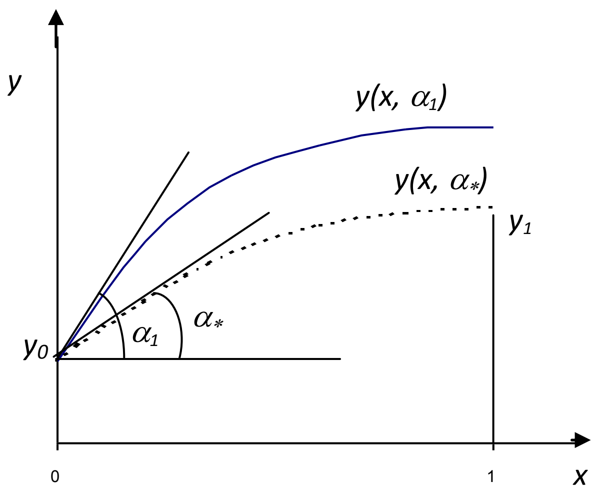
almashtirish yordamidamojno [0,1] kesmaga keltirish mumkin.

CHegaraviy shartni quyidagi oddiy ko`rinishda olamiz

$$y(0) = y_0, \quad y(1) = y_1 . \tag{2}$$

O`q otish usulining moiiyati (1), (2) chegaraviy masalani echishni (1) tenglama uchun

$$y(0) = y_0, \qquad y'(0) = k = tg\,\alpha , \tag{3}$$



boshlang`ich shartli masala echimiga keltirishdan iborat, bunda $\alpha$ - parametr $x = 0$ nuqtada integral chiziqga o`tkazilgan urinmaning $0x$ o`qi bilan hosil qilgan burchagidir.

(1), (3) Koshi masalasini α dan boғliq deb hisoblaylik, ya`ni *y=y(x,α)*, shunday *y=y(x,α_*)* integral chiziqni izlaymizki, u *(0,y_0)* nuqtadan chiqib

$(1, y_1)$ nuqtaga tushsin.

SHunday qilib, agar $\alpha=\alpha_*$ bo`lsa, u holda $y(x,\alpha)$ Koshi masalasi echimi $y(x)$ chegaraviy masala echimi bilan ustma-ust tushadi. $x=1$ da (2) ni hisobga olib $y(1,\alpha)=y_1$ ni hosil qilamiz

$$y(1,\alpha)-y_1=0. \tag{4}$$

Demak $F(\alpha)=0$ ko`rinishdagi tenglamani hosil qildik, bunda $F(\alpha)=y(1,\alpha)-y_1$.

(4) tenglamani echish uchun chiziqlimas tenglamlarni yechishning birorta usulini qo`llash mumkin.

1.4.2 Chekli ayirmalar usuli

Quyidagi

$$Lu = u'' + p(x)u' + q(x)u = f(x), \tag{5}$$

tenglamaning

$$\begin{aligned} l_0 y &= c_1 y(a) + c_2 y'(a) = c, \\ l_1 y &= d_1 y(b) + d_2 y'(b) = d. \end{aligned} \tag{6}$$

shartlarni qanoatlantiruvchi echimini topish talab etilgan bo`lsin.

Masalani sonli yechish izlanayotgan $u(x)$ haqiqiy echimning $x_0, x_1, x_2,\dots, x_n$ nuqtalardagi $y_0, y_1,\dots y_n$ taqribiy qiymatlarini topishdan iborat. x_i, nuqtalar to`r tugunlari deb ataladi. Bir-biridan bir xil uzoqlikda joylashgan tugunlar sistemasidan hosil bo`lgan quyidagi tekis to`rni qo`llaymiz

$$x_i=x_0+ih, \qquad i=0,1,2,\dots,n.$$

Bundan

$$x_0=a,\ \ x_n=b,\ \ h=(b-a)/n.$$

h – kattalik to`r qadami.

Quyidagi belgilashlarni kiritamiz

$$p(x_i)=p_i, \qquad q(x_i)=q_i, \qquad f(x_i)=f_i,$$

$$y(x_i)=y_i, \qquad y'(x_i)=y_i', \qquad y''(x_i)=y_i''.$$

$y'(x_i)$ va $y''(x_i)$ larni har bir ichki tugunda ayirmali markaziy hosilalar yordamida approksimatsiyalaymiz

$$y'(x_i)=\frac{y_{i+1}-y_{i-1}}{2h}+O\left(h^2\right),\qquad y''(x_i)=\frac{y_{i+1}-2y_i+y_{i-1}}{h^2}+O\left(h^2\right).$$

Kesma oxirilarida bir tomonlama ayirmali iosilalarni qo`llaymiz

$$y_0'=\frac{y_1-y_0}{h}+O(h),\qquad y_n'=\frac{y_n-y_{n-1}}{h}+O(h).$$

Bu formulalarni qo`llab (5), (6) berilgan masala ayirmali approksimatsiyasini hosil qilamiz:

$$\begin{cases}\dfrac{y_{i+1}-2y_i+y_{i-1}}{h^2}+p_i\dfrac{y_{i+1}-y_{i-1}}{2h}+q_i\,y_i=f_i, & i=\overline{1,n-1},\\[2mm] c_1\,y_0+c_2\dfrac{y_1-y_0}{h}=c,\\[2mm] d_1y_n+d_2\dfrac{y_n-y_{n-1}}{h}=d.\end{cases}\tag{7}$$

Izlanayotgan echimning y_0, y_1,..., y_n taqribiy qiymatlarini topish uchun (7) *n+1* noma`lumli *n+1* ta chiziqli tenglamalar sistemasini echish zarur. Bu sistemani CHATS ni echishning biron bir standart usullari yordamida echish mumkin. Ammo (7) tenglamalar koeffitsientlaridan tuzilgan matritsa uch dioganallidir, shuning uchun uni echishda *progonka usuli* deb ataluvchi maxsus usulni qo`llaymiz.

(7) sistemani quyidagi tarzda yozamiz

$$\begin{cases}\beta_0y_0+\gamma_0y_1=\varphi_0\\ \alpha_i\,y_{i-1}+\beta_iy_i+\gamma_iy_{i+1}=\varphi_i, & i=1,2,\ldots,n-1,\\ \alpha_ny_{n-1}+\beta_ny_n=\varphi_n,\end{cases}\tag{8}$$

bunda $\beta_0=c_1h-c_2$, $\gamma_0=c_2$, $\gamma_0=s_2$, $\varphi_0=hs$, $\varphi_I=f_ih^2$,

$$\alpha_i=1-\frac{1}{2}p_ih,\qquad \beta_i=-2+q_ih^2,\qquad \gamma_i=1+\frac{p_ih}{2},\quad i=1,2,\ldots,n-1,$$

$$\alpha_n=-d_2,\qquad \beta_n=hd_1+d_2,\quad \varphi_n=hd.$$

(8) sistema echimini quyidagi ko`rinishda izlaymiz

$$y_i=u_i+v_iy_{i+1},\quad i=0,1,\ldots,n-1,\tag{9}$$

bu erada u_i, v_i, $i=0,1,\ldots,(n-1)$ lar *progonka koeffitsientlari* deb ataladi.

(9) ni (8) ga qo`yib u_i, v_i lar uchun quyidagi rekkurent formulani hosil qilamiz:

$$v_i = -\frac{\gamma_i}{\beta_i + \alpha_i v_{i-1}}, \qquad u_i = \frac{\varphi_i - \alpha_i u_{i-1}}{\beta_i + \alpha_i v_{i-1}}, \qquad i = \overline{1, n}. \tag{10}$$

Hisoblash sxemasini bir jinsli qilish uchun

$$\alpha_0=0, \qquad \gamma_n=0,$$

deb olamiz.

Progonka usuli ikki bosqichdan iborat.

1) *Progonkaning to`g`ri yo`li.* (10) bo`yicha *i* indes o`zgarishining o`sib borish tartibida ketma-ket u_i, v_i koeffitsientlar

$$v_0 = -\frac{\gamma_0}{\beta_0}, \qquad u_0 = \frac{\varphi_0}{\beta_0},$$

qiymatlar yordamida hisoblanadi.

2) *Progonkaning teskari yo`li.* (9) formula bo`yicha *i* indeksning kamayish tartibida ketma-ket $y_n, y_{n-1},\ldots,y_0$ kattaliklar aniqlanadi.

SHunday qilib $\gamma_n=0$, u holda $v_n=0$ va $y_n=u_n$, ya`ni progonkaning to`ғri yo`lida v_i , u_i kattaliklar yordami bilan y_n echim hisoblanadi.

SHunday qilib, progonka usuli bilan (9) sistemaning aniq echimini topa olamiz, bu esa (5), (6) chegaraviy masala echimi xatoligi faqat berilgan masala ayirmali approksimatsiya xatoligi bilan aniqlanishini va xatolik *O(h)* ga teng ekanligini ko`rsatadi.

(9) sistemani

$$\begin{aligned} &\alpha_i y_{i-1} - \beta_i y_i + \gamma_i y_{i+1} = \varphi_i, \quad i = \overline{1, n-1}, \\ &y_0 = \chi_1 y_1 + \mu_1, \quad y_n = \chi_2 y_{n-1} + \mu_2, \end{aligned} \tag{11}$$

ko`rinishda yozamiz, bu erda

$$\chi_1 = -\frac{\gamma_0}{\beta_0}, \quad \mu_1 = \frac{\varphi_0}{\beta_0}, \quad \chi_2 = -\frac{\alpha_n}{\beta_n}, \quad \mu_1 = \frac{\varphi_n}{\beta_n}, \quad \beta_i = 2 - q_i h^2,$$

$\alpha_i \neq 0, \quad \beta_i \neq 0$.

U holda (10) formula quyidagi ko`rinishga ega bo`ladi

$$v_i = \frac{\gamma_i}{\beta_i - \alpha_i v_{i-1}}, \quad u_i = \frac{\alpha_i u_{i-1} - \varphi_i}{\beta_i - \alpha_i v_{i-1}}. \tag{12}$$

$x = b$ nuqtada (ya`ni $i = n$ da) y_n

$$\begin{cases} y_n = \chi_2 y_{n-1} + \mu_2 \\ y_{n-1} = u_{n-1} + v_{n-1} y_n, \end{cases}$$

sistemadan y_n

$$y_n = \chi_2 (u_{n-1} + v_{n-1} y_n) + \mu_2, \qquad (1 - \chi_2 v_{n-1}) y_n = \chi_2 u_{n-1} + \mu_2$$

$$y_n = \frac{\chi_2 u_{n-1} + \mu_2}{1 - \chi_2 v_{n-1}} \tag{13}$$

kabi aniqlanadi.

(12), (13) formulalar ma`noga ega bo`ladigan etarlilik shartlarini isbotlaymiz:

$$|\beta_i| \geq |\alpha_i| + |\gamma_i|, \quad i = \overline{1, n-1}, \quad |\chi_i| \leq 1, \quad i = 1,2, \quad |\chi_1| + |\chi_2| < 2. \tag{14}$$

Bu shartlarda $i = \overline{0, n-1}$ uchun $|v_i| \leq 1$ bo`lishini ko`rsatamiz.

$|v_{i-1}| \leq 1$ bo`lsin. Bundan $|v_i| \leq 1$ bo`lishini ko`rsatamiz. SHunday qilib $|v_0| = |\chi_1| \leq 1$, u holda bundan barcha $i = 1,2,..., n-1$ lar uchun $|v_i| \leq 1$ bo`lishligi kelib chiqadi.

(14) qo`llab quyidagi ayirmani baholaymiz

$$|\beta_i - \alpha_i v_{i-1}| - |\gamma_i| \geq |\beta_i| - |\alpha_i| \cdot |v_{i-1}| - |\gamma_i| \geq$$

$$\geq |\alpha_i| + |\gamma_i| - |\alpha_i| \cdot |v_{i-1}| - |\gamma_i| = |\alpha_i| - |\alpha_i| \cdot |v_{i-1}| = |\alpha_i| \cdot (1 - |v_{i-1}|) \geq 0 .$$

Bundan $|\beta_i - \alpha_i v_{i-1}| \geq |\gamma_i|$.

SHunday qilib $\gamma_i \neq 0$, u holda $|\beta_i - \alpha_i v_{i-1}| > 0$, ya`ni $|v_i| = \dfrac{|\gamma_i|}{|\beta_i - \alpha_i v_{i-1}|} \leq 1$.

Bundan ko`rinadiki agar $|v_{i-1}| < 1$ bo`lsa, u holda $|v_i| < 1$ bo`ladi. $|v_0| = |\chi_1| < 1$ da barcha $|v_i| < 1$ bo`ladi.

(10) ning maxrajini baholaymiz:

$$|1 - \chi_2 v_{n-1}| \geq 1 - |\chi_2| \cdot |v_{n-1}| \geq 1 - |\chi_2| > 0 \ ,$$

bundan $|\chi_2| < 1$ yoki $|v_{n-1}| < 1$ ($|\chi_1| < 1$ da), ya`ni $|1 - \chi_2 v_{n-1}| > 0$.

Agar $|\beta_{i_0}| > |\alpha_{i_0}| + |\gamma_{i_0}|$ hech bo`lmaganda bitta $i = i_0$ nuqtada bajarilsa, u holda barcha $i > i_0$ uchun $|v_i| < 1$ bajariladi va jumladan $i = n - 1$ da $|v_{n-1}| < 1$ ga ega bo`lamiz. Bu holda $|\chi_1| + |\chi_1| < 2$ shart ortiqcha hisoblanadi, chunki $|\chi_1| = 1$ va $|\chi_2| = 1$ da

$$|1 - \chi_2 v_{n-1}| \geq 1 - |\chi_2| \cdot |v_{n-1}| > 0$$

bo`ladi.

1.4.3 Turg`unlik, approksimatsiya, yaqinlashish

$$x \in G \text{ da } Lu = f(x), \ x \in \Gamma \quad \text{да} \quad lu = \mu(x) \tag{10}$$

uzluksiz masala berilgan bo`lsin va $\overline{\omega}_h = \varpi_h + \gamma_h$ to`rda uni quyidagi ayirmali masala approksimatsiya qilsin

$$x \in \omega_h \text{ da } L_h y_h = \varphi_h, \quad x \in \gamma_h \text{ da } l_h y_h = \tilde{\mu}_h . \tag{11}$$

$z_h = y_h - u_h$ xatolik uchun masala (bunda u_h - ω_h to`rda (10) masala echimining qiymatlari) quyidagi ko`rinishda bo`ladi

$$L_h z_h = \psi_h, \quad x \in \omega_h, \quad l_h z_h = \nu_h, \ x \in \gamma_h, \tag{12}$$

bu erda ψ_h, ν_h - tenglama va qo`shimcha shartlarning approksimatsiya xatoligi. (12) ning o`rninga

$$\tilde{L}_h z_h = \tilde{\psi}_h$$

ni yozamiz.

Agar $\widetilde{L}_h$ operator chiziqli va ayirmali sxema korrekt bo`lsa, (9) o`rniga quyidagiga ega bo`lamiz

$$\|z_h\|_{(1h)} \le M\|\widetilde{\psi}_h\|_{(2h)} \quad \text{yoki} \quad \|z_h\|_{(1h)} \le M\left(\|\psi_h\|_{(2h)} + \|\nu_h\|_{(3h)}\right). \tag{13}$$

Bu erdan ko`rinib turibdiki, agar sxema turg`un va masalani approksimatsiya qilsa, u holda yaqinlashuvchi bo`ladi (odatda "approksimatsiya va turg`unlikdan yaqinlashish kelib chiqadi" deyiladi), sxemaning aniqlik tartibi uning approksimatsiya tartibi bilan aniqlanadi.

YUqorida aytib o`tilganlardan shunday xulosa chiqadiki sxema yaqinlashishi va aniqlik tartibini o`rganish approksimatsiya xatoligi va turg`unligini o`rganishga olib keladi, ya`ni aprior baholash deb ataluvchi (13) ko`rinishdagi baholash olinadi.

II-Bob

Nochiziqli chegaraviy shartli bir o'lcho'vli issiqlik o'tkazuvchanlik tenglamasini sonli yechish.

2.1 Bir o`lchamli issiqlik o`tkazuvchanlik tenglamasini sonli yechish

2.1.1 Masalaning berilishi

Bir o`lchamli nostatsionar issiqlik o`tkazuvchanlik tenglamasi quyidagicha bo`ladi

$$c\rho\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial u}{\partial x}\right) + \overline{f}, \tag{1}$$

bunda $u = u(x,t)$ - temperatura, *s* – birlik massa issiqlik sig`imi, *ρ* - zichlik, *k* – issiqlik o`tkazuvchanlik koeffitsienti, $\bar{f}$ - issiqlik manbalari zichligi, ya`ni birlik vaqtda birlik uzunlikdan ajralib chiquvchi issiqlik. Agar $c = c(x,t,u)$, $k = k(x,t,u)$ bo`lsa tenglama kvazichiziqli deb ataladi. Agar *s*=const*, k*=const bo`lsa tenglama quyidagicha bo`ladi

$$\frac{\partial u}{\partial t} = a^2\frac{\partial^2 u}{\partial x^2} + \tilde{f}, \quad a^2 = \frac{k}{\rho c}, \quad \tilde{f} = \frac{\bar{f}}{\rho c}, \tag{2}$$

bu erda a^2 - temperatura o`tkazuvchanlik koeffitsienti.

Umumiylikdan ajralmagan holda $a = 1$ deb hisoblash mumkin, u holda (2) dan quyidagini hosil qilamiz

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + f. \tag{3}$$

Birinchi chegaraviy masala (I): $\overline{D} = \{0 \le x \le 1,\ 0 \le t \le T\}$ da uzluksiz bo`lgan quyidagi masalaning $u(x,t)$ yechimini topamiz

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + f(x,t), \quad 0 < x < 1, 0 < t \le T,$$
$$u(x,0) = u_0(x), \quad 0 \le x \le 1,$$
$$u(0,t) = u_1(t), \quad u(1,t) = u_2(t), \quad 0 \le t \le T.$$

2.1.2 Olti nuqtali sxemalar oilasi

Quyidagi to`rni kiritamiz

$$\overline{\omega}_h = \{x_i = ih, \quad i = 0,1,\dots, I\}, \quad \omega_\tau = \{t_j = j\tau, \quad j = 0,1,\dots, J\}$$

va $\overline{D}$ to`rni

$$\overline{\omega}_{h\tau} = \overline{\omega}_h \times \omega_\tau = \{(ih, j\tau), \quad i = 0,1,\dots, I, \quad j = 0,1,\dots, J\}$$

ko`rinishda $h = 1/I$, $\tau = T/J$ qadamlar bilan kiritamiz, bunda $y_i^j$ – $\overline{\omega}_{h\tau}$ da aniqlangan bo`lib, *y* funktsiyaning (x_i, t_j) tugundagi qiymati.

Bir parametrli ayirmali sxemalar oilasini qaraymiz

$$\frac{y_i^{j+1} - y_i^j}{\tau} = \Lambda\left(\sigma y_i^{j+1} + (1-\sigma) y_i^j\right) + \varphi_i^j, \quad 0 < i < I, \quad 0 \le j < J, \tag{4}$$

bunda $\Lambda y_i^j = y_{\bar{x}x} = \dfrac{y_{i-1}^j - 2y_i^j + y_{i+1}^j}{h^2}$, σ – haqiqiy parametr.

(4) sxema ba`zan *vaznli sxema* deb ataladi.

CHegaraviy va boshlang`ich shartlar quyidagicha aniq approksimatsiyalanadi:

$$y_0^j = u_1^j, \quad y_I^j = u_2^j, \tag{5}$$

$$y_i^0 = y(x_i, 0) = u_0(x_i). \tag{6}$$

Bunda φ_i^j – (3) tenglama o`ng tarafi f ni approksimatsiyalovchi funktsiya, masalan

$$\varphi_i^j = f\left(x_i, t_{j+0,5}\right), \quad t_{j+0,5} = t_j + 0,5\tau .$$

(4)-(6) ni (II) ayirmali masala deb ataymiz.

(4) AS quyidagi olti nuqtali shablonda yozilgan

$$\left(x_{i\pm1}, t_{j+1}\right), \quad \left(x_i, t_{j+1}\right), \quad \left(x_{i\pm1}, t_j\right), \quad \left(x_i, t_j\right).$$

(4) tenglama *ichki tugunlar* deb ataluvchi $\left(x_i, t_{j+1}\right)$ $\quad i = 0,1, \ldots, I-1,$ $j + 1 = 1,\ldots, J$ tugunlarda echiladi. $\overline{\omega}_{h\tau}$ dagi barcha ichki tugunlar to`plamini $\omega_{h\tau} = \{(x_i, t_j), \ 1 \le i \le I-1, \ 1 \le j \le J\}$ ko`rinishida belgilaymiz.

(5), (6) boshlang`ich va chegaraviy shartlar $\overline{\omega}_{h\tau}$ ning chegaraviy nuqtalarida yoziladi. $t=t_j$ to`g`ri chiziqda yotuvchi $\overline{\omega}_{h\tau}$ to`r tugunlari odatda *qatlamlar* deb ataladi. (4) da y_i^j qiymatlar ikkita qatlamda yotadi va shuning uchun bunday sxemalar *ikki qatlamli* sxemalar deb ataladi.

$\sigma=0$ da (x_i,t_{j+1}), (x_i,t_j), $(x_{i\pm1},t_j)$ shablonda aniqlanuvchi to`rt nuqtali $\dfrac{y_i^{j+1} - y_i^j}{\tau} = \Lambda y_i^j + \varphi_i^j$ sxemani hosil qilamiz yoki uni quyidagicha yoza olamiz

$$y_i^{j+1} = (1 - 2\gamma) y_i^j + \gamma(y_{i-1}^j + y_{i+1}^j) + \tau\varphi_i^j, \quad \gamma = \frac{\tau}{h^2}. \tag{7}$$

$t=t_{j+1}$ qatlamning har bir nuqtasidagi y_i^{j+1} qiymat (7) formula yordamida $t=t_j$ qatlamdagi y_i^j qiymatlar orqali oshkor ko`rinishda ifodalanadi. SHunday qilib $t=0$ da $y_i^0 = u_0(x_i)$ berilsa, u holda (7) formula bo`yicha ketma-ket ixtiyoriy qatlamdagi *y* ning qiymatlarini aniqlay olamiz. (7) sxema *oshkor* sxema deb ataladi.

Agar $\sigma \neq 0$ bo`lsa, u holda (7) sxema *oshkormas ikki qatlamli sxema* deb ataladi. $\sigma \neq 0$ da y_i^{j+1} larni aniqlash uchun quyidagi chegaraviy shartlarni qanoatlantiruvchi

$$y_0^{j+1} = u_i^{j+1}, \quad y_l^{j+1} = u_2^{j+1},$$

algebraik tenglamalar sistemasini hosil qilamiz:

$$\sigma \Lambda y_i^{j+1} - \frac{1}{\tau} y_i^{j+1} = -F_i^j, \tag{8}$$

$$F_i^j = \frac{1}{\tau} y_i^{j+1} + (1-\sigma)\Lambda y_i^{j+1} + \varphi_i^j,$$

$$i=1,\dots,l-1.$$

(8) ayirmali tenglama echimi progonka usuli bilan topiladi.

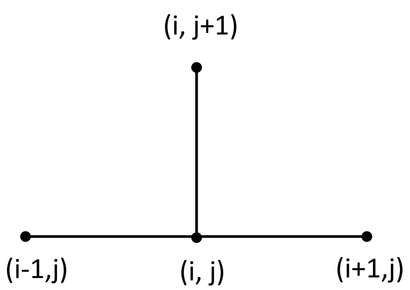
$\sigma = 1$ da *sof oshkormas sxemaga* ega bo`lamiz

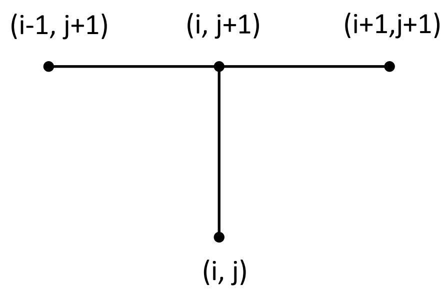
$$\frac{y_i^{j+1} - y_i^j}{\tau} = \Lambda y_i^{j+1} + \varphi_i^j. \tag{9}$$

$\sigma = 0{,}5$ da *olti nuqtali simmetrik sxemani* hosil qilamiz

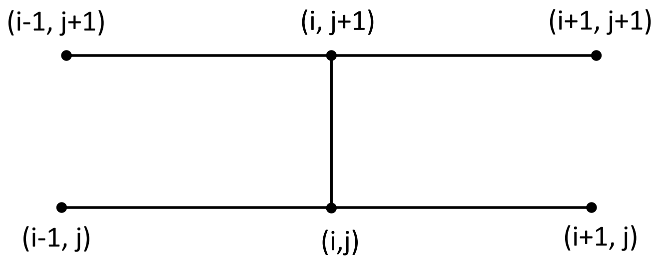
$$\frac{y_i^{j+1} - y_i^j}{\tau} = \frac{1}{2}\Lambda\left(y_i^{j+1} + y_i^j\right) + \varphi_i^j, \tag{10}$$

ba`zan bu sxema *Krank-Nikol'son sxemasi* deb ataladi.





(7) oshkor sxema shabloni (9) sof oshkormas sxema shabloni



(8) ikki qatlamli oshkormas va (10) Krank-Nikol'son sxemalari shabloni

2.1.3 Approksimatsiya aniqligi

(4)-(6) sxemalar aniqligi haqidagi savolga javob berish uchun (4)-(6) masala echimi $y = y_i^j$ ni (I) masala echimi *u=u(x,t)* bilan taqqoslash kerak. Shunday qilib *u(x,t)* (I) masalaning uzluksiz yechimi bo`lsin, u holda $u_i^j = u(x_i, t_j)$ qo`yamiz va $z_i^j = y_i^j - u_i^j$ ayirmani qaraymiz.

z_i^j ni baholash uchun quyidagi normalardan birini tanlaymiz

$$\|z\| = \|z\|_C = \max_{0 \le i \le I} |z_i|, \qquad \|z\| = \left(\sum_{i=1}^{I-1} z_i^2 h \right)^{1/2}.$$

$y_i^j = y, \quad y_i^{j+1} = \hat{y}, \quad y_t = (\hat{y} - y)/\tau$ indekssiz belgilashlar yordamida (4)-(6) masalani quyidagi ko`rinishda yozamiz

$$y_t = \Lambda(\sigma \hat{y} + (1 - \sigma) y) + \varphi, \quad (x,t) \in \omega_{h\tau},$$

$$y(0,t) = u_1(t), \quad y(1,t) = u_2(t), \quad t \in \omega_\tau, \qquad \text{(II)}$$

$$y(x,0) = u_0(x), \quad x \in \overline{\omega}_h, \quad \Lambda y = y_{\bar{x}x}.$$

$y = z + u$ ni (II) ga qo`yib va *u* ni berilgan funktsiya deb z uchun quyidagi masalani hosil qilamiz

$$y_z = \Lambda(\sigma\hat{y} + (1-\sigma)y) + \varphi, \quad (x,t) \in \omega_\tau,$$

$$z(0,t) = z(1,t) = 0, \quad t \in \omega_\tau,$$

$$z(x,0) = 0, \quad x \in \overline{\omega}_h,$$

bunda $\psi = \Lambda(\sigma\hat{u} + (1-\sigma)u) - u_t + \varphi$ – (I) tenglama *u(x,t)* yechimida (II) sxemaning *approksimatsiya xatoligi*.

Ta`rif. (II) sxema (I) tenglamani *(m,n)* tartib bilan approksimatsiyalaydi yoki (I) tenglama *u=u(x,t)* yechimda $O(h^m + \tau^n)$ approksimatsiyaga ega deyiladi, agar $\|\psi(x,t)\|_2 = O(h^m + \tau^n)$ yoki $\|\psi\|_2 \le M(h^m + \tau^n)$ tengsizliklar barcha $t \in \omega_\tau$ lar uchun bajarilsa, M *e*sa h va τ dan bog`liq bo`lmagan musbat o`zgarmas, $\|\cdot\|_2$ – $\omega_h$ to`rdagi qandaydir norma.

u=u(x,t) dan *x* va *t* bo`yicha kerakli hosilalarni qo`yib, (II) ning approksimatsiya tartibini baholaymiz. Quyidagi belgilashlardan foydalanamiz

$$\dot{u} = \partial u / \partial t, \quad u' = \partial u / \partial x, \quad \overline{u} = u(x_i, t_{j+0,5}), \quad \bar{t} = t_{j+0,5} = t_j + 0,5\tau .$$

u(x,t) ni $(x_i, t_{j+0.5})$ nuqta atrofida Teylor qatoriga yoyamiz.

Ushbu formulalarni qo`llab

$$\hat{u} = 0,5(\hat{u} + u) + 0,5(\hat{u} - u) = 0,5(\hat{u} + u) + 0,5\tau u_t,$$

$$u = 0,5(\hat{u} + u) - 0,5\tau u_t,$$

$$\sigma\hat{u} + (1+\sigma)u = 0,5(\hat{u} + u) + (\sigma - 0,5)\tau u_t$$

ψ ni quyidagicha yozamiz

$$\psi = 0,5\,\Lambda(\hat{u} + u) + (\sigma - 0,5)\tau \Lambda u_t - u_t + \varphi .$$

Yuqoridagi ifodalarni bu erga qo`yib hamda

$$\Lambda u = u'' + \frac{h^2}{12}u^{(4)} + O(h^4) = Lu + \frac{h^2}{12}L^2 u + O(h^4), \quad Lu = \frac{\partial^2 u}{\partial x^2},$$

$$\hat{u} = \bar{u} + 0,5\tau\,\bar{\dot{u}} + \frac{\tau^2}{8}\bar{\ddot{u}} + O(\tau^3),$$

$$u = \bar{u} - 0,5\tau\,\bar{\dot{u}} + \frac{\tau^2}{8}\bar{\ddot{u}} + O(\tau^3),$$

$$0,5(\hat{u} + u) = \bar{u} + \frac{\tau^2}{8}\bar{\ddot{u}} + O(\tau^3),$$

$$u_t = \bar{\dot{u}} + O(\tau^2)$$

ifodalardan foydalanib

$$\psi = (L\bar{u} - \bar{\dot{u}} + \varphi) + (\sigma - 0,5)\tau L\bar{\dot{u}} + \frac{h^2}{12}L^2\bar{u} + O(\tau^2 + h^4) \tag{12}$$

ni hosil qilamiz.

Bundan ko`rinadiki $\varphi = \bar{f} = f(x_i, t_{j+0,5})$ da

$$\psi = (\sigma - 0,5)\tau L\bar{\dot{u}} + O(\tau^2 + h^2)$$

bunda faqat $\dot{u} = Lu + f$. $L\dot{u} = L^2u + Lf = u^{(IV)} + f''$ va $L^2u = L\dot{u} - Lf$ ekanini hisobga olib (12) dan quyidagini hosil qilamiz

$$\psi = (\varphi - \bar{f}) + \left[(\sigma - 0,5)\tau + \frac{h^2}{12}\right]L\bar{\dot{u}} - \frac{h^2}{12}L\bar{f} + O(h^4 + \tau^2). \tag{13}$$

(13) da o`rta qavs ichidagi ifodani nolga tenglab ushbu tenglikka kelamiz

$$\sigma = \frac{1}{2} - \frac{h^2}{12\tau} = \sigma_*. \tag{14}$$

$\sigma = \sigma_*$ qiymatda va φ esa $\varphi = \bar{f} + \frac{h^2}{12}L\bar{f}$ bo`lganda sxema (II) $O(\tau^2 + h^4)$ approksimatsiyaga ega. Agar biz $f''$ ni $f_{\bar{x}x} = \Lambda f$ ifodaga almashtirsak sxema approksimatsiya tartibi buzilmaydi, ya`ni $\varphi = \bar{f} + \frac{h^2}{12}\Lambda\bar{f}$ yoki quyidagiga kelamiz

$$\varphi_i^j = f_i^{j+1/2} + \frac{1}{12}\left(f_{i-1}^{j+1/2} - 2f_i^{j+1/2} + f_{i+1}^{j+1/2}\right) = \frac{5}{6} f_i^{j+1/2} + \frac{1}{12}\left(f_{i-1}^{j+1/2} + f_{i+1}^{j+1/2}\right).$$
(15)

$C_n^m(\overline{D})$ – shunday funktsiyalar sinfi bo`lsinki, ularning *x* bo`yicha *m* va *t* bo`yicha *n* tartibli hosilalari $\overline{D}$ da uzluksiz bo`lsin. (13) va (14) formulalardan ko`rinadiki (II) sxema quyidagi approksimatsiyalarga ega:

1. $\sigma = 0{,}5$, $\varphi = \bar{f}$ yoki $\overline{\varphi} = \bar{f} + O(h^2 + \tau^2)$ da $O(h^2 + \tau^2)$ bo`ladi, agar $u \in C_3^4$ bo`lsa;
2. $\sigma \neq 0{,}5$, $\varphi = \bar{f} + O(h^2 + \tau)$ da $O(h^2 + \tau)$ bo`ladi, masalan, $\varphi = \hat{f}$ yoki $\varphi = f$ bo`lganda, agar $u \in C_2^4$ bo`lsa;
3. $\sigma = \sigma_*$ da va φ esa (15) formula bilan berilsa, $O(h^4 + \tau^2)$ bo`ladi, agar $u \in C_3^6$ bo`lsa.

(II) sxema $\sigma = \sigma_*$ va $\varphi = \bar{f} + \frac{h^2}{12}\Lambda\bar{f}$ da odatda *yuqori tartibli aniqlikdagi sxema* deb ataladi. φ o`ng tarafni tanlash berilgan $\sigma$ da approksimatsiya tartibiga qo`yilgan talablarga bo`ysungan bo`lishi kerak.

Shunday qilib $\sigma = 0{,}5$ da φ ni $\varphi = 0{,}5(\hat{f} + f)$, $\varphi = \bar{f}$ deb olish mumkin va i.k.

(13) dan ko`rinadiki $O(h^2 + \tau^2)$ xatolikka $\sigma \neq 0{,}5$ da ham erishishi mumkin. Masalan $\sigma = 0{,}5 + h^2\alpha/\tau$ deb olish mumkin, bunda $\alpha$ - *h* va $\tau$ dan bog`liq bo`lmagan ixtiyoriy o`zgarmas. α ni tanlash sxema turg`unligi sharti bilan chegaralangan.

2.1.4 Issiqlik o`tkazuvchanlik tenglamasi uchun uch qatlamli sxemalar

Ba`zan uch qatlamli sxemalar qo`llaniladi. Bunday sxemalardan bittasi Richardson sxemasidir:

$$\frac{y^{j+1} - y^{j-1}}{2\tau} = \Lambda y^j \quad \text{yoki} \quad y_{\overset{0}{t}} = \Lambda y^j, \tag{16}$$

bunda $y_{\overset{0}{t}} = \dfrac{\hat{y} - \check{y}}{2\tau}$, $\hat{y} = y^{j+1}$, $\check{y} = y^{j-1}$, $y = y^{j}$, $\Lambda y = y_{\bar{x}x}$. Bu sxema τ va h bo`yicha ikkinchi tartibli approksimatsiga ega $\psi = \Lambda u - u_{\overset{0}{t}} = O(\tau^2 + h^2)$. Ammo u absalyut turg`unmas sxemadir.

(16) ni quyidagi ko`rinishda yozamiz

$$\frac{y_i^{j+1} - y_i^{j-1}}{2\tau} = \frac{y_{i-1}^{j} - 2y_i^{j} + y_{i+1}^{j}}{h^2}. \tag{17}$$

Agar (17) ning o`ng tarafidagi $2y_i^j$ ni $y_i^{j+1} + y_i^{j-1}$ ga almashtirsak, u holda uch qatlamli «romb» sxemaga (*Dyuffort-Frenkel sxemaga*) kelamiz:

$$\frac{y_i^{j+1} - y_i^{j-1}}{2\tau} = \frac{y_{i-1}^{j} - y_i^{j+1} - y_i^{j-1} + y_{i+1}^{j}}{h^2}, \tag{18}$$

bu sxema y_i^{j+1} ga nisbatan oshkor qoladi va absolyut turg`un hisoblanadi. «Romb» sxemani ushbu ko`rinishda yozish mumkin

$$y_{\overset{0}{t}} + \frac{\tau^2}{h^2} y_{\bar{t}t} = \Lambda yy \tag{19}$$

bunda $y_{\bar{t}t} = (y_i^{j+1} - 2y_i^{j} + y_i^{j-1})/\tau^2$.

Haqiqatdan

$$\frac{y_{i-1} - \hat{y}_i - \check{y}_i + y_{i+1}}{h^2} = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} - \frac{\hat{y}_i - 2y_i + \check{y}_i}{h^2} = y_{\bar{x}x} - \frac{\tau^2}{h^2} y_{\bar{t}t}.$$

Bu ifodani (18) ga qo`yib (19) ni hosil qilamiz. Demak Richardson sxemasi «romb» sxemaning xususiy holi hisoblanadi. $\dfrac{\tau^2}{h^2} y_{\bar{t}t}$ had turg`unlikni ta`minlaydi.

(19) ning approksimatsiya xatoligi quyidagicha

$$\psi = \Lambda u - u_{\overset{0}{t}} - \frac{\tau^2}{h^2} u_{\bar{t}t} = u'' - \dot{u} - \frac{\tau^2}{h^2}\ddot{u} + O(\tau^2 + h^2) = -\frac{\tau^2}{h^2}\ddot{u} + O(\tau^2 + h^2).$$

Bundan ko`rinadiki «romb» sxema shartli approksimatsiyaga ega bo`ladi

$$\psi = O\left(\tau^2 + h^2 + \frac{\tau^2}{h^2}\right) = O(h^2), \quad \tau = O(h^2) \text{ da.}$$

Agar $\tau = \alpha h(1 + O(h))$ deb olsak, u holda (19) sxema

$$\frac{\partial u}{\partial t} + \alpha^2 \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

ko`rinishdagi tenglamani approksimatsiyalaydi, bunda $\alpha = const$.

Odatda (3) uchun vaznli oshkormas uch qatlamli sxemalar qo`llaniladi

a) simmetrik sxemalar

$$y_{\overset{0}{t}} = \Lambda\left(\sigma\hat{y} + (1 - 2\sigma)y + \sigma \check{y}\right) + \varphi, \tag{20}$$

b) simmetrik bo`lmagan sxemalar

$$y_t + \sigma\tau y_{\bar{t}t} = \Lambda \check{y} + \varphi. \tag{21}$$

(20), (21) tenglamalar uchta (t_{j-1}, t_j, t_{j+1}) qatlamga ega. Shuning uchun ular $t_j \geq \tau, \quad j \geq 1$ qatlamlarda yoziladi. $y(x,\tau)$ qiymatini qo`shimcha ravishda berish kerak, masalan $y_t(x,0) = \bar{u}_0(x)$ yoki $y(x,\tau) = y(x,0) + \tau\,\bar{u}_0(x)$, bunda $\bar{u}_0(x) = u_0''(x) + f(x,0)$ ifoda $y(x,\tau) - u(x,\tau) = O(\tau^2)$ shartdan tanlanadi.

Ba`zan $y(x,\tau)$ ni aniqlash uchun ikki qatlamli sxemalar qo`llaniladi.

2.2 Chiziqli bo`lmagan issiqlik o`tkazuvchanlik tenglamasi uchun chekli ayirmali sxemalar tuzish

2.2.1 Kvazichiziqli issiqlik o`tkazuvchanlik tenglamasi

Yuqori temperaturada o`tuvchi jarayonlar uchun, masalan plazmada, issiqlik o`tkazuvchanlik koeffitsienti temperaturaning chiziqlimas funktsiyasi bo`ladi (zichligi ham), bir qator masalalarda esa temperatura gradienti funktsiyasi ham chiziqlimas funktsiya bo`ladi. Bundan tashqari yana issiqlik manbalari (issiqlik o`tkazuvchanlik tenglamasining o`ng tarafi) temperaturadan bog`liq bo`lishi mumkin, masalan issiqlik ximyaviy reaktsiya natijasida ajralsa. Muhitning issiqlik sig`imi ham temperaturadan bog`liq bo`lishi mumkin.

SHu tarzda biz chiziqlimas issiqlik o`tkazuvchanlik tenglamasiga kelamiz:

$$\frac{\partial \varepsilon(x,t,u)}{\partial t} = -\frac{\partial w}{\partial x} + f(x,t,u),$$

bunda issiqlik oqimi (ε - ichki energiya) quyidagicha bo`ladi

$$w = w\left(x, t, u, \frac{\partial u}{\partial x}\right)$$

va u ham *i* temperatura va uning hosilasining chiziqlimas funktsiyasi bo`ladi. Agar issiqlik oqimi *di/dx* hosiladan chiziqli bog`liq bo`lsa va Fur'e qonuni bajarilsa

$$w = -k(x, t, u)\frac{\partial u}{\partial x},$$

u holda biz kvazichiziqli issiqlik o`tkazuvchanlik tenglamasini hosil qilamiz

$$c(x,t,u)\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(k(x,t,u)\frac{\partial u}{\partial x}\right) + f(x,t,u),$$
$$c(x,t,u) > 0, \quad k(x,t,u) > 0.$$

Bu holda issiqlik sig`imi *s*, issiqlik o`tkazuvchanlik koeffitsienti *k* va o`ng taraf *f* (issiqlik manbalari zichligi) *u(x,t)* temperaturadan bog`liq. Birjinslimas muhitda *k, s, f* lar *x* va *t* ning uzulishli funktsiyalari bo`lishi mumkin (har xil moddalar uchun *k, s, f* lar *u* temperaturadan har xil bog`liq bo`lishi mumkin).

k=k(u), c=c(u), f=f(u) funktsiyalar faqat *u* temperaturadan bog`liq hol tipik hol sanaladi:

$$c(u)\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(k(u)\frac{\partial u}{\partial x}\right) + f(u). \tag{1}$$

Yangi o`zgaruvchi kiritsak

$$v = \int_0^u k(\xi)d\xi,$$

(1) tenglama ushbu ko`rinishga keladi

$$\frac{\partial \varphi(v)}{\partial t} = \frac{\partial^2 v}{\partial x^2} + \hat{f}(v), \quad \varphi(v) = \int_0^u c(\xi)d\xi.$$

Agar o`zgaruvchini

$$v = \int_0^u c(\xi)d\xi,$$

deb kiritsak, u holda (1) o`rniga quyidagi tenglamani hosil qilamiz

$$\frac{\partial v}{\partial t} = \frac{\partial}{\partial x}\left(\chi(v)\frac{\partial v}{\partial x}\right) + \tilde{f}(v),$$

shunday qilib

$$\int_0^v \chi(v)dv = \int_0^u k(u)du$$

tenglik bajariladi (bu x bo`yicha differentsiallab tekshiriladi).

Ko`pincha *s(i)* va *k(i)* temperaturaning darajali funktsiyasi ko`rinishida bo`ladi

$$c(u) = c_0 u^{\alpha}, \quad k(u) = k_0 u^{\beta}.$$

Bu holda

$$v = \int_0^u c(\xi)d\xi = c_0 \frac{u^{\alpha+1}}{\alpha+1}$$

o`zgaruvchini kiritamiz va

$$k\frac{\partial u}{\partial x} = \frac{k}{c_0 u^{\alpha}}\frac{\partial v}{\partial x} = \frac{k_0}{c_0}u^{\beta-\alpha}\frac{\partial v}{\partial x} = \frac{k_0}{c_0}\left(\frac{\alpha+1}{c_0}\right)^{\frac{\beta-\alpha}{\alpha+1}} v^{\frac{\beta-\alpha}{\alpha+1}}\frac{\partial v}{\partial x},$$

ekanligini hisobga olib (1) tenlamani quyidagi ko`rinishga keltiramiz

$$\frac{\partial v}{\partial t} = \frac{\partial}{\partial x}\left(\chi_0 v^{\sigma}\frac{\partial v}{\partial x}\right) + \tilde{f}(v), \quad \sigma = \frac{\beta-\alpha}{\alpha+1},$$

$$\chi_0 = \frac{k_0}{c_0}\left(\frac{\alpha+1}{c_0}\right)\frac{\beta-\alpha}{\alpha+1}.$$

2.2.2 Ayirmali sxema. N'yuton usuli

Agar *k(u), s(u), f(u)* lar temperaturaning tez o`zgaruvchi funktsiyalari bo`lsa kvazichiziqli issiqlik o`tkazuvchanlik tenglamasi uchun oshkor sxemalarni qo`llash maqsadga muvofiq emas.

Oshkor sxemaning turg`unlik sharti

$$\frac{\tau}{h^2} \le \frac{1}{2} \frac{\min \ c(u)}{\max \ k(u)}$$

bo`lib, vaqt bo`yicha kichik qadamni talab etadi, *k, s* funktsiyalar qiymatlari ko`pincha katta bo`lmagan tugunlar sonida aniqlanadi. Shuning uchun shartsiz turg`un oshkormas sxemalar qo`llaniladi. Avval ushbu tenglamani

$$\frac{\partial \varphi(u)}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \tag{2}$$

quyidagi boshlang`ich va chegaraviy shartlar bilan qaraymiz

$$u(x,0) = u_0(x), \quad u(0,t) = \mu_1(x), \quad u(1,t) = \mu_2(t).$$

y^{j+1} ga nisbatan chiziqsiz ayirmali sxemani qo`llaymiz

$$\frac{\varphi\left(y^{j+1}\right) - \varphi\left(y^{j}\right)}{\tau} = y_{\bar{x}x}^{j+1}, \quad x = x_i = ih, \quad 0 < i < N, \quad hN = 1. \tag{3}$$

y^{j+1} echimni yangi qatlamda topish uchun biz chiziqlimas tenglamaga ega bo`lamiz

$$\varphi\left(y^{j+1}\right) - \tau y_{\bar{x}x}^{j+1} = \varphi\left(y^{j}\right).$$

Uni yechish uchun N'yuton iteratsion usulini qo`llaymiz

$$\varphi\left(\overset{k}{y}\right) + \varphi'\left(\overset{k}{y}\right)\left(\overset{k+1}{y} - \overset{k}{y}\right) - \tau \overset{k+1}{y}_{\bar{x}x} = \varphi\left(y^{j}\right). \tag{4}$$

$$F = \varphi(x) + x - f = 0, \quad \varphi(x) = \varphi(y^{j+1}), \quad x = -\tau y_{\bar{x}x}^{j+1}, \quad f = \varphi(y^{j}),$$

$$x_{k+1} = x_k - \frac{F(x_k)}{F'(x_k)}, \qquad x_{k+1} - x_k = -\frac{\varphi + x_k - f}{\varphi' + 1},$$

$$(x_{k+1} - x_k)(\varphi' + 1) = -\varphi - x_k + f,$$

$\varphi + (x_{k+1} - x_k)\varphi' + x_{k+1} = f$ - (4) bilan solishtiring.

Bu erda $\overset{k+1}{y}$ ni aniqlash uchun

$$\overset{k+1}{y}_0 = \mu_1(t_{j+1}), \quad \overset{k+1}{y}_N = \mu_2(t_{j+1}),$$

chegaraviy shartlarda progonkani qo`llash mumkin va progonka

$$\varphi'(\overset{k}{y}) \geq 0$$

shart bajarilsa turg`un bo`ladi.

Agar tenglamalarni quyidagi ko`rinishda yozsak

$$\frac{\tau}{h^2}\overset{k+1}{y}_{i-1} - \left(\varphi'(\overset{k}{y}) + \frac{2\tau}{h^2}\right)\overset{k+1}{y}_i + \frac{\tau}{h^2}\overset{k+1}{y}_{i+1} = \overset{k}{F}_i,$$

$$i = 1, 2, \ldots, N-1,$$

$$\overset{k}{F}_i = \varphi(\overset{k}{y}) - \varphi(y) - \varphi'(\overset{k}{y})\overset{k}{y}, \quad y = y^j.$$

Turg`unlik sharti haqiqatdan ham yuqoridagi kabi bo`lishini ko`rishimiz mumkin.

2.2.3 Kvazichiziqli issiqlik o`tkazuvchanlik tenglamasi uchun har xil oshkormas sxemalar

Endi oddiy ko`rinishdagi kvazichiziqli issiqlik o`tkazuvchanlik tenglamasi uchun ikki tipdagi sof oshkormas sxemani qaraymiz ($\sigma = 1$ holdagi)

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(k(u)\frac{\partial u}{\partial x} \right) + f(u),\ 0 < x < 1,\ 0 < t \le T,$$

$$u(x,0) = u_0(x), \quad u(0,t) = u_1(x), \quad u(1,t) = u_2(t),$$

bunda *k(i)* > 0.

a) sxema:

$$\frac{\hat{y}_i - y_i}{\tau} = \frac{1}{h}\left[a_{i+1}(y)\frac{\hat{y}_{i+1} - \hat{y}_i}{h} - a_i(y)\frac{\hat{y}_i - \hat{y}_{i-1}}{h} \right] + f(y_i). \tag{5}$$

b) sxema:

$$\frac{\hat{y}_i - y_i}{\tau} = \frac{1}{h}\left[a_{i+1}(\hat{y})\frac{\hat{y}_{i+1} - \hat{y}_i}{h} - a_i(\hat{y})\frac{\hat{y}_i - \hat{y}_{i-1}}{h} \right] + f(\hat{y}_i). \tag{6}$$

bunda

$$\hat{y}_i = y_i^{j+1}, \quad y_i = y_i^j, \quad a_i(v) = a_i(v_{i-1}, v_i),$$

masalan,

$$a_i(v) = 0{,}5\left[k(v_{i-1}) + k(v_i)\right].$$

$$a_i(v) = k\left(\frac{v_{i-1} + v_i}{2} \right),$$

$$a_i(v) = \frac{2k(v_{i-1})k(v_i)}{k(v_{i-1}) + k(v_i)}.$$

$a_i(v)$ ni hisoblash yo`lidan temperatura to`lqinini hisoblash aniqligi kuchli bog`liq.

(5) va (6) sxemalarni taqqoslaymiz. Bu sxemalarningapproksimatsiya xatoligi $O(\tau + h^2)$. Ularning ikkalasi ham absolyut turg`un.

a) sxema y^{j+l} ga nisbatan chiziqli va uning qiymatlarini y^j funktsiyaning qiymati bo`yicha topish mumkin, masalan progonka usuli bilan. $\tau$ qadam faqat aniqlikni talablari asosida tanlanadi. b) sxema $y^{j+l}$ ga nisbatan chiziqlimas va uni topish uchun iteratsiya usuli qo`llaniladi. Iteratsiya jarayoni quyidagi tarzda quriladi:

$$\frac{\overset{(s+1)}{y_i} - y}{\tau} = \frac{1}{h}\left[a_{i+1}\left(\overset{(s)}{y}\right)\frac{\overset{(s+1)}{y}_{i+1} - \overset{(s+1)}{y_i}}{h} - a_i\left(\overset{(s)}{y}\right)\frac{\overset{(s+1)}{y_i} - \overset{(s+1)}{y}_{i-1}}{h}\right] + f\left(\overset{(s)}{y_i}\right). \qquad (7)$$

$\overset{(s+1)}{y_i}$ ga nisbatan ayirmali sxema chiziqlidir. Boshlang`ich iteratsiya sifatida vaqt bo`yicha avvalgi qadamdagi *u* funktsiya olinadi: $u^{(0)}=y^j$.

(6), (7) iteratsion sxema bo`yicha hisoblashda yoki iteratsiya soni yoki iteratsiya yaqinlashishi aniqligi ε beriladi va quyidagi shart bajarilishi talab etiladi

$$\max_i \left|\overset{(s+1)}{y}_{i+1} - \overset{(s)}{y_i}\right| \le \varepsilon .$$

(6), (7) sxemaning kamchiligi shundan iboratki a) sxema bilan taqqoslaganda iteratsiyani hisoblashda mashina xotirasi yacheykalarini egalagan sonlar ikki marta oshadi, shunday qilib $\overset{(s+1)}{y}$ ni hisoblash uchun $\overset{(s)}{y}$ va *u* larni «eslab qolish» kerak. a) sxema bo`yicha y^{j+l} ning qiymati birdaniga topiladi.

Madomiki ikkala sxema ham absalyut turg`un va bir xil tartibli approksimatsiyaga ega ekan, u holda bu munosabatdan a) sxema b) sxemadan afzal ko`rinadi. Ammo bunday emas. a) va b) sxemalar bo`yicha bir xil aniqlikda olib borilgan hisoblash natijalari ko`rsatdiki b) sxemani vaqt bo`yicha etarlicha yirik qadamlar bilan qo`llash mumkin ekan. Bu esa b) sxemada iteratsiya zarurligiga qaramasdan hisoblash ishlari hajmining kamayishiga olib keladi.

Tenglama kvazichiziqliligi kuchsiz bo`lgan holda, masalan *k=k(x,t), f=f(u), c=c(x,t)* bo`lganda, ba`zan aniqligi $O\left(\tau^2 + h^2\right)$ bo`lgan *prediktor-korrektor* deb ataluvchi sxemalar qo`llaniladi. *c=k=1, f=f(u)* bo`lganda bunday sxemaga misol keltiramiz:

$$\frac{\bar{y} - y}{0{,}5\,\tau} = \bar{y}_{\bar{x}x} + f(y), \quad \bar{y} = y\left(t_{j+\frac{1}{2}}\right), \quad \frac{\hat{y} - y}{\tau} = \frac{1}{2}\left(\bar{y}_{\bar{x}x} + y_{\bar{x}x}\right) + f(y).$$

SHuni ta`kidlaymizki (6) masalani echish uchun N’yuton usulini qo`llash mumkin.

2.3 Nochiziqli chegaraviy shartli bir o’lcho’vli issiqlik o’tkazuvchanlik tenglamasini sonli yechish.

Misol sifatida cheksiz plastinkadagi issiqllek uzatishni qaraymiz. Bbuni natijasida bittagina yo’nalishi bo’yicha issiqlik uzatish qaralib bir o’lchavli issiqlik o’tkazish tenglamasini tahlil qilinadi. Yechimlar sohasining chegarasidakonveksiya va nurlanish hisobiga issiqlik uzatish modellashtirikdi Nurlanish bilan issiqlik uizatishni Stefan-bolsman qonuni asosida qaraymiz. Shunday qilib bayon qilingan masala matematik nuqtai nazaridan quydagi ko’rinishni oladi.

B ka $= \frac{2}{2}, \quad 0 < \ < ;$ (44)

$= 0: \ = {}_{0}, 0 \leq \ \leq ;$

$= 0: - \ = {}_{1}({}^{1} - \ ) + {}_{1}(({}^{1})^{4} - {}^{4}), \quad > 0, \ {}_{1} > 0;$ (45)

$= : \ = {}_{2}({}^{2} - \ ) + {}_{2}(({}^{2})^{4} - {}^{4}), \quad > 0, \quad {}_{2} > 0;$

Bo’$_{1\,2}$–qorayish darajasi $= 5{,}669 \cdot 10^{-8} / ({}^{2} \cdot {}^{4})$Bo’ qo’yilgan chegaraviy masalada nochiziqli chegaraviy shartlar bo;lganda bizni asosan qiziqtiradi.

III Tur chochiziq li chegaraviy shartlar bo’l aniqlikda diskretlashtiramiz.

$_{1} \quad _{1}$ birinchi progonka koeffisentlarini

$_{1} = {}_{1} \cdot {}_{2} + {}_{1}$ shrtlardan aniqlaymiz. Shunday qilib (45) munosabatning ikkinchisidan

$- \Big|_{=0} = {}_{1}({}^{1} - |_{=0}) + {}_{1}(({}^{1})^{4} - (|_{=0})^{4});$

$- \frac{2-1}{} = {}_{1}({}^{1} - {}_{1}) + {}_{1}(({}^{1})^{4} - ({}_{1})^{4});$ kelib chiqadi.

$\frac{1}{} \equiv {}_{1}$ belgilashni kiritamiz U holda

$_{1} - {}_{2} \equiv {}_{1} \cdot {}^{1} - {}_{1} \cdot {}_{1} + \frac{1}{}({}^{1})^{4} - \frac{1}{}({}_{1})^{4};$

$$_{1} = \frac{1}{1 + {}_{1}} \cdot {}_{2} + \frac{1}{1 + {}_{1}} \cdot {}^{1} + \frac{1}{(1 + {}_{1})}(({}^{1})^{4} - ({}_{1})^{4});$$

$$\begin{cases} {}_{1} = \dfrac{1}{1 + {}_{1}} \\ {}_{1} = \dfrac{1}{1 + {}_{1}} \cdot {}^{1} + \dfrac{1}{(1 + {}_{1})}(({}^{1})^{4} - ({}_{1})^{4}) \end{cases}$$

$$\begin{cases} {}_1 = \frac{}{+\cdot_1} \\ {}_1 = \frac{\cdot_1}{+\cdot_1}\cdot{}^1 + \frac{_1}{+\cdot_1}\left(({}^1)^4 - ({}_1)^4\right) \end{cases} \quad (46)$$

Ko'rinib turibdiki progonkaning ${}_1$- birinchi koefisenti chap chegaradagi temperaturaga bog'langan . U holda temperatura maydonini aniqlash uchuin masalan, oddiy iteratsiya usulidan foydalanish zarur. Asosiy g'oya shundan iboratki har bir vaqt qatlamida tenperatura maydonini hisoblash $\left|{}_1^{+1} - {}_1\right| \leq$ bu yerda S-iteratsiya nomiri

-xisoblash xatoligi,shart bajarilgancha davom ettiriladi.

T_N temperaturani aniqlash uchun o'ng chegaraviy shartdan foydalaniladi.

$$+\Big|_{=} = {}_2({}^2 - |_=) + {}_2(({}^2)^4 - (|_=)^4);$$

$$+\frac{- \; _{-1}}{} = {}_2({}^2 -) + {}_2(({}^2)^4 - ()^4);$$

$$_{-1} = {}_{-1}\cdot \; + {}_{-1},$$

$$- {}_{-1}\cdot \; - {}_{-1} = {}_2({}^2 -) + \frac{2}{}(({}^2)^4 - ()^4);$$

$$= \frac{_{-1} + {}_2{}^2 + {}^2(({}^2)^4 - ()^4)}{1 + {}_2 - {}_{-1}} \quad \text{yoki}$$

$$= \frac{\cdot_{-1} + \cdot_2{}^2 + {}_2(({}^2)^4 - ()^4)}{\cdot_2 + (1 - {}_{-1})} \quad (47)$$

Natijada o'ng chegarada temperaturani aniqlash uchun (47)tenglamani hosil qilishimiz. Bu tenglamani ham eng sodda metod oddiy iteratsiya usuli bilan yechish mumkin.

(45) chegaraviy shartlarni O(h') anqlikda diskretlashtiramiz. Faraz qilaylik chegarada (44) issiqlik o'tkazuvchanlik tenglamasi bajarilsin T(x) funksiyasi x=0 no'qta atrofida h ga nisbatan 2-tartibli hadgacha Teylor qatoriga yoyamiz.

$${}_2^{+1} = {}_1^{+1} + \cdot\Big|_{=0}^{+1} + \frac{2}{2}\cdot\frac{2}{2}\Big|_{=0}^{+1}.$$

(44) munosabatdan foydalanib topamiz

$${}_2^{+1} = {}_1^{+1} + \cdot\Big|_{=0}^{+1} + \frac{2}{2\cdot}\cdot\Big|_{=0}^{+1};$$

$$\Big|_{=0}^{+1} = \frac{{}_2^{+1} - {}_1^{+1}}{} - \frac{}{2\cdot}\cdot\Big|_{=0}^{+1} = \frac{{}_2^{+1} - {}_1^{+1}}{} - \frac{}{2\cdot}\frac{{}_1^{+1} - {}_1}{}.$$

(45) ning 2-munosabatdan

$$\Big|_{=0}^{+1} = \frac{1}{}\left({}_1^{+1} - {}^1\right) - \frac{1}{}\left(({}^1)^4 - ({}_1^{+1})^4\right).$$

Oxirgi ikki munosabatlarni tenglashtirib topamiz:

$$\frac{{}_{2}^{+1} - {}_{1}^{+1}}{} - \frac{}{2\cdot}\cdot\frac{{}_{1}^{+1} - {}_{1}}{} = {}^{1}\left({}_{1}^{+1} - {}^{1}\right) - {}^{1}\left(\left({}^{1}\right)^{4} - \left({}_{1}^{+1}\right)^{4}\right);$$

$${}_{2}^{+1} - {}_{1}^{+1} - \frac{2}{2\cdot\cdot}\cdot{}_{1}^{+1} + \frac{2}{2\cdot\cdot}\cdot{}_{1} = {}^{1}{}_{1}^{+1} - {}^{1\,1} - {}^{1}\left(\left({}^{1}\right)^{4} - -\left({}_{1}^{+1}\right)^{4}\right);$$

$${}_{1}^{+1}\left(1 + \frac{2}{2\cdot\cdot} + {}^{1}\right) = {}_{2}^{+1} + \frac{2}{2\cdot\cdot}\cdot{}_{1} + {}^{1\,1} + {}^{1}\left(\left({}^{1}\right)^{4} - -\left({}_{1}^{+1}\right)^{4}\right);$$

$${}_{1}^{+1} = \frac{2\cdot\cdot}{{}^{2} + 2\cdot(+{}_{1})}{}_{2}^{+1} + \frac{2}{{}^{2} + 2\cdot(+{}_{1})}\cdot{}_{1} +$$

$$+\frac{2_{1}}{{}^{2}+2\cdot(+{}_{1})}{}^{1} + \frac{2_{1}}{{}^{2}+2\cdot(+{}_{1})}\left(\left({}^{1}\right)^{4} - \left({}_{1}^{+1}\right)^{4}\right).$$

$$\begin{cases} {}_{1} = \frac{2\cdot\cdot}{{}^{2}+2\cdot(+{}_{1})}; \\ {}_{1} = \frac{2}{{}^{2}+2\cdot(+{}_{1})}\cdot{}_{1} + \frac{2_{1}}{{}^{2}+2\cdot(+{}_{1})}{}^{1} + \\ \quad + \frac{2_{1}}{{}^{2}+2\cdot(+{}_{1})}\left(\left({}^{1}\right)^{4} - \left({}_{1}^{+1}\right)^{4}\right) \end{cases} \quad (48)$$

O’ng chegaradan foydalanib T_N ni topamiz

$${}_{-1}^{+1} = {}^{+1} - \cdot\Big|_{=}^{+1} + \frac{2}{2}\cdot\frac{2}{2}\Big|_{=}^{+1} = {}^{+1} - \cdot\Big|_{=}^{+1} + \frac{2}{2\cdot}\cdot\Big|_{=}^{+1};$$

$${}_{-1}^{+1} = {}^{+1} - {}^{2}\left({}^{2} - {}_{1}^{+1}\right) - {}^{2}\left(\left({}^{2}\right)^{4} - \left({}_{1}^{+1}\right)^{4}\right) + \frac{2}{2\cdot}\cdot\frac{{}^{+1} -}{}.$$

shunday qilib $\;{}_{-1} = {}_{-1}\cdot + {}_{-1}$

$${}_{-1}\cdot{}^{+1} + {}_{-1} = {}^{+1} + {}^{2\,2} + {}^{2\,+1} + \frac{2}{2\cdot\cdot}{}^{+1} - \frac{2}{2\cdot\cdot} -$$

$$-{}^{2}\left(\left({}^{2}\right)^{4} - \left({}_{1}^{+1}\right)^{4}\right);$$

$${}^{+1}\left(1 - {}_{-1} + {}^{2} + \frac{2}{2\cdot\cdot}\right) = {}_{-1} + {}^{2\,2} + \frac{2}{2\cdot\cdot} + {}^{2}\left(\left({}^{2}\right)^{4} - \left({}_{1}^{+1}\right)^{4}\right);$$

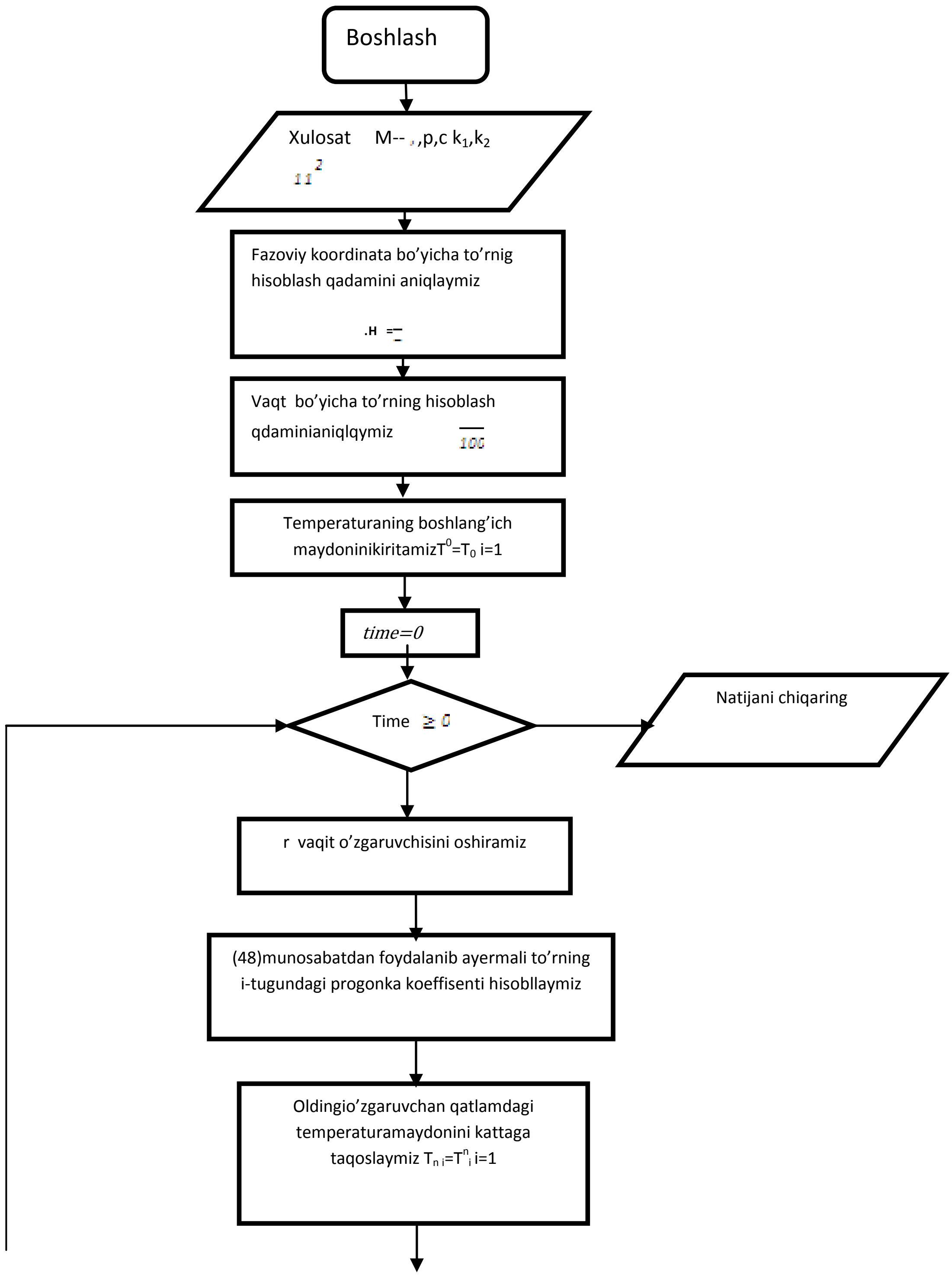
$${}^{+1} = \frac{2_{-1}}{2(1 - {}_{-1}) + 2_{2} + {}^{2}} + \frac{2_{2}{}^{2}}{2(1 - {}_{-1}) + 2_{2} + {}^{2}} +$$

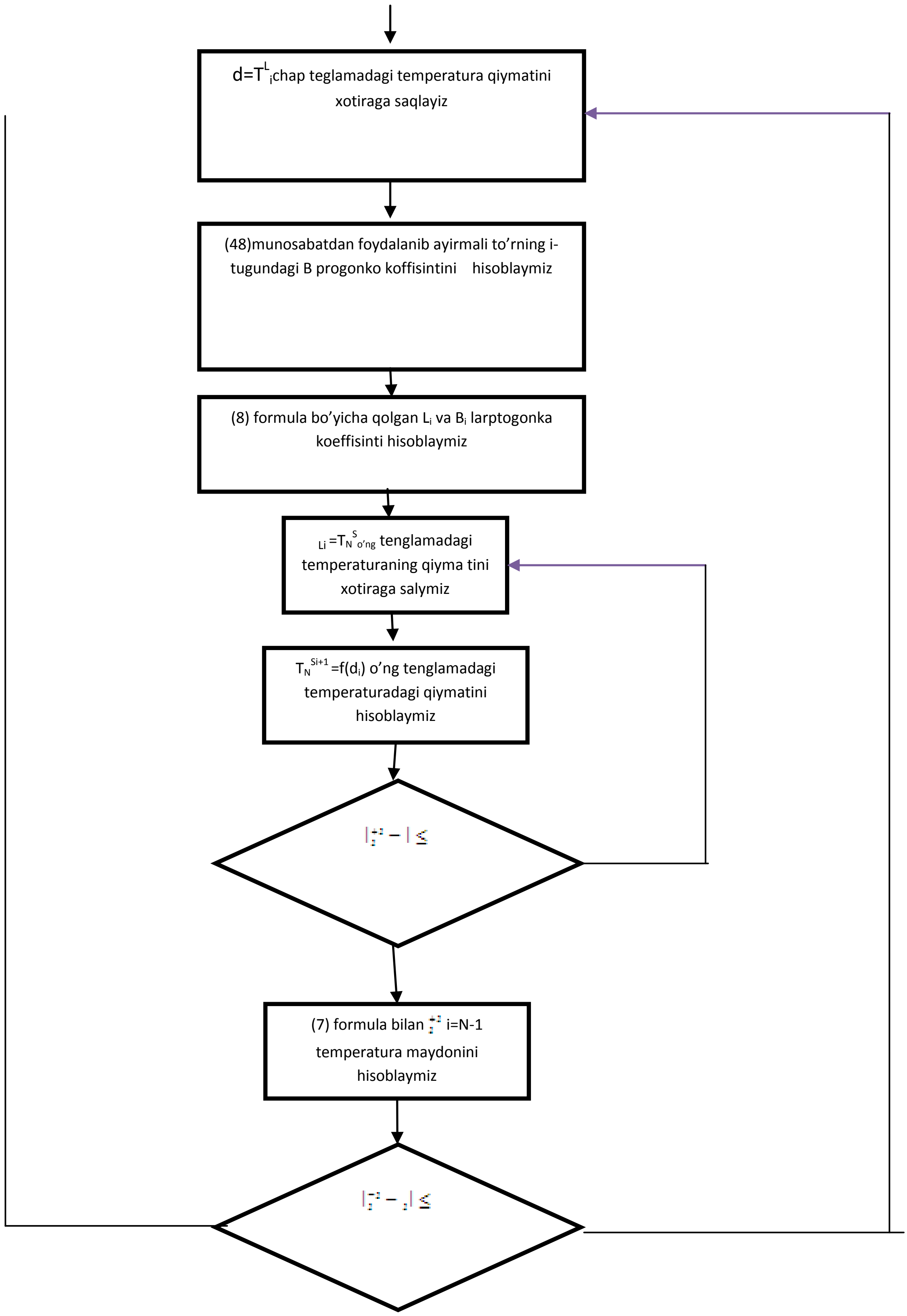
$$+\frac{2}{2(1-{}_{-1})+2_{2}+{}^{2}} + \frac{2_{2}}{2(1-{}_{-1})+2_{2}+{}^{2}}\left(\left({}^{2}\right)^{4} - \left({}_{1}^{+1}\right)^{4}\right). \quad (49)$$

Beton plastinkada 600,1800,3600,va 7200 sekunddan keyinm tenperatura naydonini aniqlaymiz. Plastina qalinligi L=93 m Boshlang’ich temperatura T_0=50C . Beton quyidagi teplo fizik xarakteristiklarga ega.
$= 0.9\,/(\,*\,)$ p=2000kg/m^3

c=920Dj/(kg*c)x=0 va x=L chegaralarg plastinka tashqi muhit bilan aloqada bo’ladi.

(K=1000 vt/m^2*c) $\;=30^0{}_{1} = 0.5,\;$ K$_2$=500vt/m^2*c) , $\;=70^0{}_{2} = 0.2$


Boshlash
Xulosat M-- , ,p,c k1,k2
Fazoviy koordinata bo'yicha to'rnig hisoblash qadamini aniqlaymiz
.H =
Vaqt bo'yicha to'rning hisoblash qdaminianiqlqymiz
100
Temperaturaning boshlang'ich maydoninikiritamiz T0=T0 i=1
time=0
Time ≥ 0
Natijani chiqaring
r vaqit o'zgaruvchisini oshiramiz
(48)munosabatdan foydalanib ayermali to'rning i-tugundagi progonka koeffisenti hisobllaymiz
Oldingio'zgaruvchan qatlamdagi temperaturamaydonini kattaga taqoslaymiz Tn i=Tn i i=1



Qaralayotgan masalaning listing dasturi

```
uses;crt;

const mf=500;

      sigma=5.669e-8;

      eps=1e-5;

type

vector=array[1..mf] of real;


var {dasturda foydalaniladigan o’zgaruvchilarning belgilash bo’limi}

i, j, N                              :integer;

t,tn,alfa, beta               :vector;

ai, bi, ci, fi, d, dl         :real;

lamda, ro, c, h, tau                 :real;

kapa1, kapa2, te1, te2        :real;

eps1, eps2                           :real;

T0, L, t_end, time                   :real;

f,g                                  :text;

begin

clrser;

{Klaviatura bilan barcha kirish parametrni keitamiz}

Writeln ();

Readln(N);

Writeln(vaqt bo’yicha tugatishni kiritng,t-end);

Readln(Plastina qalinligini kiriting);

Writeln(‚L’);

Readln(L);

Writeln(Plastena materialining issiqlik o’tkazuvchanlik koeffisintini kiriteng lamda);

Readln(lamda);

Writeln(Plastena materialining zichligini  kiriteng);

Readln(ro);
```

```
Writeln(Plastena materialining issiqlik sig'imini kiriteng);
Readln(c);
Writeln(x=0, chegarada issiqlik almashinivi koeffisintini kiriting);
Readln(kapa 1);
Writeln(x=L, x=0, chegarada issiqlik almashinivi koeffisintini kiriting);
Readln(kapa2);
Writeln(x=0 chegaraga nisbatan tashqi muhit temperaturasini kiriteng);
Readln(Tel);
Writeln(x=L, chegaraga nisbatan tashqi muhit temperaturasini kiriteng);
Readln(Te2);
Writeln(x=0, chegaradagi qavarig'lik darajasini kiriteng);
Readln(eps1);
Writeln(x=L, chegaradagi qavarig'lik darajasini kiriteng);
Readln(eps2);
Writeln(boshlang'ich temperatuani keriteng);
Readln(T0);
{Fazoviy kordinata bo'yicha turining hisoblash qadamini aniqlaymiz}
h:L/(N-1);
{ vaqt bo'yechato'rning hisoblash qadamini aniqlaymiz }
Tau:=t_end/100.0;
{ vaqtning boshlang'ich momentida temperatura maydonini aniqlaymiz}
For i:=1 to N do
T[i]=T0;
{nostasionar issiqlik o'tkazish tenglamasini integrallaymiz}
time:=0;
while time<t_end do {shartli siklda foydalanamiz}
begin
{vaqt o'zgaruvchisini r qadamga oshiramiz}
time:=time+tau;
```

{(48)muinosabatdan foydalanib chap chetidagi boshlang'ich progonka koiffisintini aniqlaymiz}

alfa[1]:=2.0*tau*(2.0*tau*(lamda+kapal*h)+ro*c*sqr(h));

{oldingi vaqt maydonidagi temperatura maydonini xoteraga saqlaymiz}

for i:=1 to N do

Tn[i]=T[1];

{chap chigaraviy shartning chiziqlimasligidan temperatura maydonini iterasion hisoblavchi shartli sikl}.

Repeat

{(48) formuladan foydalanib chap chegaraviy shartlardan boshlang'ich progonka koeffisintni aniqlqymiz .Bunda iterasion sikl chap chegara shart boshlanadi.}

d:=T[1];

beta[1]:=(ro*c*sqr(h))*Tn[1]+2.0*tau*kapal*h*Tel+2.0*tau*eps1*sigma*h*(sqr(sqr(Tel))-sqr(sqr(d))))/(2.0*tau*lamda+kapal*h)+ro*c*sqr(h));

{(18) formula yordamida progonka koeffisintni aniqlavchi parametrli sikl}

for i:=2 to n-1 do

begin

{ai, bi, ci, fi}-uch dioganalli matrisali tenglamalar sistemasini kanonik tasvirlash koeffisintlari}

ai:=lamda/sqr(h);

bi:=2.0*lamda/sqr(h)+ro*c/tau;

ci:=lamda/sqr(h);

fi:=-ro*c*Tn[i]/tau;

{alfa[i], beta{i}} {progonka koeffisintlari}

alfa[i]:=ai/(bi-ci*alfa[i-1]);

beta[i]:=(ci*beta[i-1]-fi)/(bi-ci*alfa[i-1]);

end;

{o'ng chetdagi chegaraviy shartlarning chiziqlimasligidan foydalanib temperatura qiymatini iteratsion hisoblash imkonini beradigan shartli sikl}

repeat

d1:=T[N];

{ (49) formulada foydalanib o'ng chegaraviy shartlardan o'ng chetidagi temperaturani hisoblaymiz}

```
T[N]:=(ro*c*sqr(h)*Tn[N]+2.0*tau*(lamda*beta[N-1]+kapa2*h*Te2+eps2*sigma*h*(sqr(sqr(Te2))-sqr(sqr(d1)))))/(ro*c*sqr(h)+2.0*tau*(lamda*(1-alfa[N-1])+kapa2*h));

until abs(d1-T[N])<=eps; {(7) munosabatdan noma’lum vaqt maydonini aniqlaymiz}

for i:=N-1 downto 1do

T[i]:=alfa[i]*T[i+1]+beta[i];

Until abs(d-T[1])<=eps; {shartli sikl tugadi}

end; {natijalarni fayelga chiqaramiz}

Assign(f,’res.txt’);

Rewrite(f);

Writeln(f,’Plastina qalinligi L=’,L:6:4);

Writeln(f,’N= koordinata bo’yicha tugunlar soni);

Writeln (Plastina materyalning issiqlik o’tkazish koeffisinti)

Writeln(f’ plastinka materyalning zichligi ro=,’ro:6:4);

Writeln(f,’plastinka materyalning issiqlik sig’imi c=’,v;6;4);

Writeln(f,boshlang’ich tempratura To=To;6;4);

Writeln

Writeln(f,’kapa2=,’kapa2:6:4);

Writeln(f,’Te1=,’Te1:6:4);

Writeln(f,’Te2=,’Te2:6:4);

Writeln

Writeln(f,’qavariqlik darajasi eps= ,eps4:1:6:4);

Writeln(f,’’qavariqlik darajasi eps2=eps2:6:4);

Writeln(f,’koordinata bo’yicha qadambilan nartijalandi);

Writeln(f,’vaqt bo’yicha qadam bilan natija olinadi);

Writeln(f,’ vaqt momintidagi temperatura maydoni);

close(f);

Assign(g,’tempr.txt’);

Rewrite(g);

for i:=1 to N do
```

writeln(g,’ ‘,h*(i-1):10:8,’ ‘,T[i]:8:5);

close(g);

end.Temperaturaning tasimlanish quydgicha bo’ladi.