Development of Fractal Equations of National Design Patterns based on the Method of R-Function

Anarova Shahzoda, Narzulloev Oybek, Ibragimova Zulaykho

Abstract: The paper is dedicated to the development of fractal equations of national design patterns based on the method of R-function (RFM). Using the equation of the line, the equation of the circle, and the structural means of the method of R-functions R_0: R-conjunctions, R-disjunctions and R-reflections constructed various tree-like fractals, equations of fractals, consisting of intersections, tangents of circles. By setting the number of iterations n and the angle of inclination α various pre-fractals are generated based on these equations. These fractals are very beautiful, which can be used in creating computer landscapes, in various illustrations, telecommunications, the textile industry, when drawing patterns in ceramic and porcelain products, as well as developing patterns of modern design of Uzbek national carpets and costumes, etc.

Keywords: method of R–function, R-conjunctions, R-disjunctions, R-reflections, fractal, tree-like fractals, prefractal, Uzbek national carpets.

I. INTRODUCTION

The production of large volumes of cotton and cocoons, as well as fabrics from their fibers, is very important. It is very important for the buyer that the quality of the fabric and its patterns are exquisite. Moreover, color plays a special role in determining the price of fabric. Each nation has its own unique pattern of carpets and costumes. Carpets and costumes created by ancestors and decorated for centuries are a national treasure.

The patterns on the national carpets and costumes of people living in Uzbekistan perfectly match the features characteristic of all the peoples of the East, and unique features are not in the carpets and costumes of other countries.

The invention of fractals is the discovery of a new aesthetics in science and mathematics, in art and in the human perception of the universe [1]. Currently, the methods of fractal theory are widely used in the development of Uzbek national carpets and costumes. Fractals play a significant role in computer graphics. They help identify complex lines and surfaces using several factors. Computer art is a special creative activity that uses digital art and computer science. Fractal patterns in bright and unusual shapes are used for interior design, parquet, tablecloth, trays, vases, dress scarves, wood, ceramics, stained glass windows, lamps, glassware, furniture painting [2], [3], [4].

The development of equations of complex fractal structures of modern design patterns, the creation of a programming environment and the study of their capabilities is important for creating complex fractal structures of modern design patterns.

The results of the paper are widely used in light industry, for instance, in the creation of patterns of modern design on fabrics and carpets.

Let a complex domain $\Omega \subset R^2$ with boundary Γ be given as a combination of simple domains $\{\Omega_k\}_{k=1}^{m}$ using set-theoretic intersection, union, and complement operations.

If the implicit boundary equations of the boundaries of these regions $\{\omega_k(x,y)=0\}_{k=1}^{m}$, such that $\omega_k > 0$ for $(x,y)\in\Omega_i$ and $\omega_k < 0$ for $(x,y)\notin\overline{\Omega}_k = \Omega_k \cup \Gamma_k$, are known, then using the method R-functions it is possible to construct the boundary equation $\Gamma\ \omega(x,y)=0$, and the function ω is positive inside Ω, negative outside it and equal to zero on Γ.

The most common system of R-functions is the system $\mathfrak{R}_0$ which has an algebraic logical operation that has the form as follows:

$$f_1 \wedge f_2 \equiv f_1 + f_2 - \sqrt{f_1^2 + f_2^2},$$
$$f_1 \vee f_2 \equiv -(\overline{f}_1 \wedge \overline{f}_2),\ \ \overline{f} \equiv -f.$$

In [5], [6], [7], [8] and [9], fractal equations were constructed based on the R-function method of V. L. Rvachev [10], [11], [12], [13], [14], [15], such as the Koch curve, carpet and napkins Sierpinski, fractal antennas and other deterministic fractals. In [16], [17] and [18], the equation and fractal algorithms were constructed by the analytical method, such as star-shaped, fractals from a circle and rectangles.

Another method for constructing fractals based on the use of arithmetic relations, in particular, those proposed by academician B. A. Bondarenko [19], in which fractal structures from combinatorial numbers are studied, and based on them, software for constructing fractals of the Pascal triangle is developed.

Revised Manuscript Received on February 2, 2020.

Anarova Shahzoda*, Department of Audiovisual Technologies, Tashkent University of Information Technologies named after Muhammad al-Khwarizmi. E-mail: omon_shoira@mail.ru

Narzulloev Oybek, Department of Audiovisual Technologies, Tashkent University of Information Technologies named after Muhammad al-Khwarizmi. E-mail: oybek.88.13@gmail.com

Ibragimova Zulaykho, Department of Information Technologies, Samarkand Branch of Tashkent University of Information Technologies named after Muhammad al-Khwarizmi. E-mail: zuli117@mail.ru

This article is a continuation of [5], [6], [7], [8] and [9] and the article constructs fractal equations based on the R-function method of V. L. Rvachev [10], [11], [12], [13], [14] and [15].

II. DETERMINATION OF THE EQUATION OF THE GOSPER CURVE BASED ON THE METHOD OF R-FUNCTIONS

The initiator of the curve is a unit length segment, and the generator is shown in Figure 1a. It comprises 7 segments each $\frac{1}{\sqrt{7}}$ in length. The fractal dimension of this curve is 2.

Fig. 1a

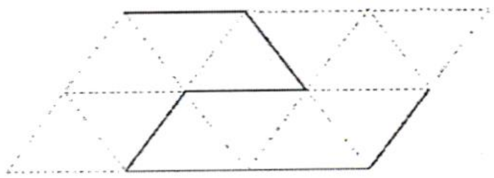


Fig. 1b

Fig. 1. Gosper Curve Initiators

The dotted line shows a triangular lattice that serves as a kind of generatrix for this generator (see. Fig 1b).

An interesting distinguishing feature of the Gosper curve is that the boundary of the region called the "Gosper island", which it fills in the limit of an infinite number of steps, is itself fractal with an integer dimension $D=\frac{\ln 3}{\ln\sqrt{7}}=1{,}1294$. This kind of islands can be used for a continuous covering of the plane, since it can be shown that they are ideally joined together. Moreover, seven such islands docked together (one in the center and six around it), again form the island of Gosper three times larger. It might be noticed that only a square has a similar property of regular polygons.

The curve, in fact, is similar to the Sierpinski curve, with the only difference being that the angles of the Gosper curve are oblique with respect to the axis OX and OY.

$$\omega_1(a,x,y)=\left(\frac{a^2}{4}-\left(x-\frac{a}{2}\right)^2\right)\wedge_0 (a^2_{11}-y^2)\geq 0,$$

where a_{11} is a sufficiently small number (line thickness).

Further, the values of the angle of inclination are calculated and the formulas for the transfer and rotation of the axes of the coordinate system relative to the fixed coordinate system are applied here.

$$\phi=\arctan(\frac{\sqrt{3}}{5});$$

$$a_{ky}=\frac{a}{\sqrt{7}};a_{mv}=a_{ky}\frac{\sqrt{3}}{2}$$

$$x_{ky}=x\cos(\phi)+y\sin(\phi);$$

$$y_{ky}=-x\sin(\phi)+y\cos(\phi)+a_{mv};$$

Further, applying the iteration (recursion) procedure, we obtain the following

$$\begin{aligned}
&\omega_n(a,x,y)=\omega_{n-1}(a_{ky},x_{ky}\ ,y_{ky}\ \text{-}\ a_{mv})\vee_0\\
&\vee_0\omega_{n-1}(a_{ky},(x_{ky}-\frac{3a_{ky}}{2})\cos(\frac{2\pi}{3})+y_{ky}\sin(\frac{2\pi}{3}),\\
&-(x_{ky}-\frac{3a_{ky}}{2})\sin(\frac{2\pi}{3})+y_{ky}\cos(\frac{2\pi}{3}))\vee_0\\
&\vee_0\omega_{n-1}(a_{ky},x_{ky}\ \text{-}\frac{a_{ky}}{2},y_{ky})\vee_0\\
&\vee_0\omega_{n-1}(a_{ky},(x_{ky}-\frac{a_{ky}}{2})\cos(\text{-}\frac{2\pi}{3})+y_{ky}\sin(\text{-}\frac{2\pi}{3}),\\
&-(x_{ky}-\frac{a_{ky}}{2})\sin(\text{-}\frac{2\pi}{3})+y_{ky}\cos(\text{-}\frac{2\pi}{3}))\vee_0\\
&\vee_0\omega_{n-1}(a_{ky},x_{ky}\ ,y_{ky}\ +\ a_{mv})\vee_0\ \omega_{n-1}(a_{ky},x_{ky}\text{-}a_{ky}\ ,y_{ky}\ +\ a_{mv})\vee_0\\
&\vee_0\omega_{n-1}(a_{ky},(x_{ky}-\frac{5a_{ky}}{2})\cos(\text{-}\frac{2\pi}{3})+y_{ky}\sin(\text{-}\frac{2\pi}{3}),\\
&-(x_{ky}-\frac{5a_{ky}}{2})\sin(\text{-}\frac{2\pi}{3})+y_{ky}\cos(\text{-}\frac{2\pi}{3}))\\
&n=2,3,4,...
\end{aligned}\tag{1}$$

The calculation results for various values are shown in Fig. 2.

III. DETERMINATION OF THE EQUATION OF FRACTAL TREES BASED ON THE R-FUNCTION METHOD

One application of fractal theory is the generation of fractal trees. The idea is simple. A tree trunk of a random length is built, several branches of a random length are also built from it, while the thickness decreases, then several more branches are built from each branch (although nothing is built from some), and the cycle repeats. Moreover, at each step, the length of the branch is checked, if it is less than a certain predetermined value, then a sheet is drawn instead of the branches, and for this branch the process stops. In this case, it might be changed a variety of parameters from branching, thickness of the trunk and branches to the angle of inclination of the branches and the color of the leaves.

However, the most important task is to develop universal methods for analytically describing the equations of the fractal domain geometry. Today, this can only be done on the basis of the algebraic logical method of R-functions V. L. Rvachev [10], [11], [12], [13], [14].

Consider a tree in a circle. Let $f(x_1,y_1,x_2,y_2,x,y)$ is a segment with limbs of (x_1,y_1) and (x_2,y_2) points.

We compose the equation of a line passing through arbitrarily given points (x_1,y_1) and (x_2,y_2).

$$f(x_1,y_1,x_2,y_2,x,y)=((\frac{1}{2}((x_2-x_1)\cos(\arctan(\frac{y_2-y_1}{x_2-x_1}))+$$
$$+(y_2-y_1)\sin(\arctan(\frac{y_2-y_1}{x_2-x_1}))))^2-((x-x_1)\cos(\arctan(\frac{y_2-y_1}{x_2-x_1}))+$$
$$+(y-y_1)\sin(\arctan(\frac{y_2-y_1}{x_2-x_1}))-\frac{1}{2}((x_2-x_1)\cos(\arctan(\frac{y_2-y_1}{x_2-x_1}))+$$
$$+(y_2-y_1)\sin(\arctan(\frac{y_2-y_1}{x_2-x_1}))))^2\geq 0)\wedge_0$$
$$\wedge_0(a^2-(-(x-x_1)\sin(\arctan(\frac{y_2-y_1}{x_2-x_1}))+$$
$$+(y-y_1)\cos(\arctan(\frac{y_2-y_1}{x_2-x_1})))^2\geq 0)$$

where a is the thickness of the segment (the thickness of the segment is 2a).

If k fuzzy, then $\phi_0=0$, else $\phi_0=\frac{\alpha}{2}$. For $n=1$, we get the followings:

$$\alpha=\frac{2\pi}{k}$$
$$\omega_1(x,y)=f(0,0,R\cos(\phi_0+0),R\sin(\phi_0+0),x,y)\vee_0$$
$$\vee_0 f(0,0,R\cos(\phi_0+\alpha),R\sin(\phi_0+\alpha),x,y)\vee_0$$
$$\vee_0 f(0,0,R\cos(\phi_0+2\alpha),R\sin(\phi_0+2\alpha),x,y)\vee_0\dots\vee_0$$
$$\vee_0 f(0,0,R\cos(\phi_0+(k-1)\alpha),R\sin(\phi_0+(k-1)\alpha),x,y)$$

For $n=2,3,4,\dots$

$$\alpha=\frac{2\pi}{k^{n-1}};k_1=-[k/2];\ R_{n-1}=2R(1-\frac{1}{2^{n-1}});\ R_n=2R(1-\frac{1}{2^n});$$

R_n is a radius boundary circle at n – iteration ($R_1=R$).

If k is fuzzy, then $k_2=[k/2]$, else $k_2=[k/2]-1$. It should be noted that *[x]* is the integer part of *x*.

Applying the iteration procedures, we obtain followings:

$$\omega_{nx1}(x,y)=f(R_{n-1}\cos(\varphi_0+\alpha),R_{n-1}\sin(\varphi_0+\alpha),$$
$$R_n\cos\left(\varphi_0+\alpha+\frac{(\varphi_0+k_1\alpha)}{k}\right),$$
$$R_n\sin\left(\varphi_0+\alpha+\frac{(\varphi_0+k_1\alpha)}{k}),x,y\right)\vee_0$$
$$\vee_0 f(R_{n-1}\cos(\varphi_0+\alpha),R_{n-1}\sin(\varphi_0+\alpha),$$
$$R_n\cos\left(\varphi_0+\alpha+\frac{(\varphi_0+(k_1+1)\alpha)}{k}\right),$$
$$R_n\sin\left(\varphi_0+\alpha+\frac{(\varphi_0+(k_1+1)\alpha)}{k}\right),x,y)\vee_0$$
$$\vee_0 f(R_{n-1}\cos(\varphi_0+\alpha),R_{n-1}\sin(\varphi_0+\alpha),$$
$$R_n\cos\left(\varphi_0+\alpha+\frac{(\varphi_0+(k_1+2)\alpha)}{k}\right),$$
$$R_n\sin\left(\varphi_0+\alpha+\frac{(\varphi_0+(k_1+2)\alpha)}{k}\right),x,y)\vee_0\dots\vee_0$$
$$\vee_0 f(R_{n-1}\cos(\varphi_0+\alpha),R_{n-1}\sin(\varphi_0+\alpha),$$
$$R_n\cos\left(\varphi_0+\alpha+\frac{(\varphi_0+k_2\alpha)}{k}\right),$$
$$R_n\sin\left(\varphi_0+\alpha+\frac{(\varphi_0+k_2\alpha)}{k}\right),x,y).\quad(2)$$

$$\omega_{nx2}(x,y)=f(R_{n-1}\cos(\varphi_0+2\alpha),R_{n-1}\sin(\varphi_0+2\alpha),R_n\cos\left(\varphi_0+2\alpha+\frac{(\varphi_0+k_1\alpha)}{k}\right),$$
$$R_n\sin\left(\varphi_0+2\alpha+\frac{(\varphi_0+k_1\alpha)}{k}),x,y\right)\vee_0$$
$$\vee_0 f(R_{n-1}\cos(\varphi_0+2\alpha),R_{n-1}\sin(\varphi_0+2\alpha),R_n\cos\left(\varphi_0+2\alpha+\frac{(\varphi_0+(k_1+1)\alpha)}{k}\right),$$
$$R_n\sin\left(\varphi_0+2\alpha+\frac{(\varphi_0+(k_1+1)\alpha)}{k}\right),x,y)\vee_0$$
$$\vee_0 f(R_{n-1}\cos(\varphi_0+2\alpha),R_{n-1}\sin(\varphi_0+2\alpha),R_n\cos\left(\varphi_0+2\alpha+\frac{(\varphi_0+(k_1+2)\alpha)}{k}\right),$$
$$R_n\sin\left(\varphi_0+2\alpha+\frac{(\varphi_0+(k_1+2)\alpha)}{k}\right),x,y)\vee_0\dots\vee_0$$
$$\vee_0 f(R_{n-1}\cos(\varphi_0+2\alpha),R_{n-1}\sin(\varphi_0+2\alpha),R_n\cos\left(\varphi_0+2\alpha+\frac{(\varphi_0+k_2\alpha)}{k}\right),$$
$$R_n\sin\left(\varphi_0+2\alpha+\frac{(\varphi_0+k_2\alpha)}{k}\right),x,y).$$

For $1\leq i\leq k^{n-1}$ we obtain followings:

$$\omega_{nxi}(x,y)=f(R_{n-1}\cos(\varphi_0+i\alpha),R_{n-1}\sin(\varphi_0+i\alpha),R_n\cos\left(\varphi_0+i\alpha+\frac{(\varphi_0+k_1\alpha)}{k}\right),$$
$$R_n\sin\left(\varphi_0+i\alpha+\frac{(\varphi_0+k_1\alpha)}{k}\right),x,y)\vee_0 f(R_{n-1}\cos(\varphi_0+i\alpha),R_{n-1}\sin(\varphi_0+i\alpha),$$
$$R_n\cos\left(\varphi_0+i\alpha+\frac{(\varphi_0+(k_1+1)\alpha)}{k}\right),R_n\sin\left(\varphi_0+i\alpha+\frac{(\varphi_0+(k_1+1)\alpha)}{k}\right),x,y)\vee_0$$
$$\vee_0 f(R_{n-1}\cos(\varphi_0+i\alpha),R_{n-1}\sin(\varphi_0+i\alpha),$$
$$R_n\cos\left(\varphi_0+i\alpha+\frac{(\varphi_0+(k_1+2)\alpha)}{k}\right),R_n\sin\left(\varphi_0+i\alpha+\frac{(\varphi_0+(k_1+2)\alpha)}{k}\right),x,y)\vee_0\dots\vee_0$$
$$\vee_0 f(R_{n-1}\cos(\varphi_0+i\alpha),R_{n-1}\sin(\varphi_0+i\alpha),$$
$$R_n\cos\left(\varphi_0+i\alpha+\frac{(\varphi_0+k_2\alpha)}{k}\right),R_n\sin\left(\varphi_0+i\alpha+\frac{(\varphi_0+k_2\alpha)}{k}\right),x,y)$$

(3)

$$\omega_n(x,y)=\omega_{n-1}(x,y)\vee_0\omega_{nx1}(x,y)\vee_0\omega_{nx2}(x,y)\vee_0\dots\vee_0$$
$$\omega_{nxi}(x,y)\vee_0\dots\vee_0\omega_{nxk^{n-1}}(x,y).$$

In previous formulas *k=2, 3, 4, 5,...* for all lines, might be drawn an outer circle of radius R_n (*n* is the iteration order)

The calculation results for various values of *n* and *k* are shown in Figure 3.

Pythagoras tree. Pythagoras, proving his famous theorem, built a figure where squares are located on the sides of a right triangle. If this process is continued, then the Pythagorean tree will turn out.

When preparing the equations, we use the equations of the square, i.e.

$$\omega_0(a,x,y)=((a^2-x^2\geq 0)\wedge_0((b^2-(y-a)^2\geq 0)\vee_0$$
$$\vee_0(b^2-(y+a)^2\geq 0)))\vee_0((a^2-y^2\geq 0)\wedge_0$$
$$\vee_0((b^2-(x-a)^2\geq 0)\vee_0(b^2-(x+a)^2\geq 0)))\geq 0$$

where $\omega_0(a,x,y)$ is a square with sides 2a and its thickness is 2b.

Next, applying recursion procedures, we obtain:

$$\begin{aligned}
&\omega_n(a,x,y) = \omega_0(a,x,y) \vee_0 \\
&\vee_0 \omega_{n-1}(a\cos(\alpha),(x+a-a\sqrt{2}\cos(\alpha)\sin(\frac{\pi}{4}-\alpha))\cos(\alpha)+ \\
&+(y-a-a\sqrt{2}\cos(\alpha)\cos(\frac{\pi}{4}-\alpha))\sin(\alpha), \\
&-(x+a-a\sqrt{2}\cos(\alpha)\sin(\frac{\pi}{4}-\alpha))\sin(\alpha)+ \\
&+(y-a-a\sqrt{2}\cos(\alpha)\cos(\frac{\pi}{4}-\alpha))\cos(\alpha)) \vee_0 \\
&\vee_0 \omega_{n-1}(a\sin(\alpha),-(x-a-a\sqrt{2}\sin(\alpha)\sin(\frac{\pi}{4}-\alpha))\sin(\alpha)+ \\
&+(y-a-a\sqrt{2}\sin(\alpha)\cos(\frac{\pi}{4}-\alpha))\cos(\alpha), \\
&(x-a-a\sqrt{2}\sin(\alpha)\sin(\frac{\pi}{4}-\alpha))\cos(\alpha)+ \\
&+(y-a-a\sqrt{2}\sin(\alpha)\cos(\frac{\pi}{4}-\alpha))\sin(\alpha))
\end{aligned} \quad (4)$$

Here, α is the angle when turning the tree branch to the left, takes values in the interval $0 < \alpha < \frac{\pi}{2}$, and when turning right, the angle of rotation is $\frac{\pi}{2} - \alpha$.

The calculation results for various values of n, α are shown in Fig. 4.

Tree-like fractals are considered the simplest fractals. Using the direct equation and constructive means of the method of *R*-functions R_0: *R*-conjunctions, *R*-disjunctions and *R*-reflections, various tree-like fractals can be constructed. Based on these equations, by setting the number of iterations and the angle of inclination α, various pre-fractals can be generated that can be used in creating computer landscapes, in various illustrations, in the textile industry, etc.

IV. DEFINITION OF THE EQUATION OF FRACTALS FROM CIRCLES BASED ON THE METHOD OF R-FUNCTIONS

One of the applications of fractal theory is the generation of fractals from circles. Currently, there are various methods for constructing fractals, such as the L-system method, systems of iterative functions, etc. [1, 12]. In contrast to these methods, the algebraic logical method of the R-function allows us to construct fractal equations. Then, using these equations, you can build visual representations of these fractals. Accordingly, the following is a method of constructing equations of fractals from circles on the basis of constructive means of the method of R-functions V. L. Rvachev. [10 - 15].

Connecting circles. The equation of the outer circle is defined as follows:

$$\omega_{00} = \omega_{00}(R,x,y) = (R^2 - x^2 - y^2 \geq 0)$$

and the equation of the connected circles has the following form:

$$\omega_0 = \omega_{00} \wedge_0 (x^2 + y^2 - (R-a)^2 \geq 0)$$

where a is a circle thickness (the thickness of the circle is *2a*), R is the radius of the outer circle, $\alpha = \frac{2\pi}{k}$; k id a number of inner circles after each iteration of $k=2,3,4,...$

Applying the iteration procedures here, we obtain:

$$\begin{aligned}
&\omega_n(R,x,y) = \omega_0(R,x,y) \vee_0 \omega_{n-1}(\frac{R}{3},x,y) \vee_0 \\
&\vee_0 \omega_{n-1}(\frac{R}{3}, x-\frac{2R}{3}\cos(0), y-\frac{2R}{3}\sin(0)) \vee_0 \\
&\vee_0 \omega_{n-1}(\frac{R}{3}, x-\frac{2R}{3}\cos(\alpha), y-\frac{2R}{3}\sin(\alpha)) \vee_0 \\
&\vee_0 \omega_{n-1}(\frac{R}{3}, x-\frac{2R}{3}\cos(2\alpha), y-\frac{2R}{3}\sin(2\alpha)) \vee_0 ... \vee_0 \\
&\vee_0 \omega_{n-1}(\frac{R}{3}, x-\frac{2R}{3}\cos((k-1)\alpha), y-\frac{2R}{3}\sin((k-1)\alpha)) \geq 0; \\
&n = 1,2,3,...
\end{aligned} \quad (5)$$

The results of a computational experiment are shown in Fig. 5.

Now we consider the case k = 6 for (1). In this case, we obtain fractal ring monopolies (Fig. 6)

It should be noted that if $k < 6$ the inner circles do not touch each other, if $k = 6$ the inner circles touch, if $k > 6$ then the inner circles intersect (Fig. 7).

Two circles. Now consider the case when inside a large circle we have two circles. These circles are tangent. In turn, in the inner circle there are two more circles, etc. We compose the equations of such a fractal.

In this case, in the first step of the equations of this fractal, we have the form:

$$\omega\ (R,x,y) = (R^2 - x^2 - y^2 \geq 0) \wedge_0 (x^2 + y^2 - (R-a)^2 \geq 0)$$

where a is a circle thickness (the thickness of the circle is *2a*), R is outer circle radius.

After applying the iteration procedure, we obtain:

$$\begin{aligned}
&\omega_n(R,x,y) = \omega_0(R,x,y) \vee_0 \omega_{n-1}(\frac{R}{2},x,y-\frac{R}{2}) \vee_0 \\
&\vee_0 \omega_{n-1}(\frac{R}{2},x,y+\frac{R}{2}) \geq 0; n = 1,2,3,...
\end{aligned} \quad (6)$$

The calculation results for various values of n are shown in Fig. 8.

Consider the case when the inner circles intersect and decrease. For this purpose, we introduce the reduction coefficient *l*.

As in the first problem, we define the equations of intersecting circles as follows:

$$\omega_0(R,x,y) = (R^2 - x^2 - y^2 \geq 0) \wedge_0 (x^2 + y^2 - (R-a)^2 \geq 0)$$

where a is a circle thickness (the thickness of the circle is *2a*) $\alpha = \frac{2\pi}{k}$;

k is the number of inner circles after each iteration k = *2,3,4,...*

l is the coefficient of reduction of the inner circles after each iteration, l = *2,3,4,...*

R is the radius of the outer circle.

Using the iteration procedure, we obtain followings:

ISSN: 2278-3075, Volume-9 Issue-4, February 2020

$$\omega_n(R,x,y)=\omega_0(R,x,y)\vee_0\omega_{n-1}(\frac{R}{l},x,y)\vee_0$$
$$\vee_0\omega_{n-1}(\frac{R}{l},x-\frac{(l-1)R}{l}\cos(0),y-\frac{(l-1)R}{l}\sin(0))\vee_0$$
$$\vee_0\omega_{n-1}(\frac{R}{l},x-\frac{(l-1)R}{l}\cos(\alpha),y-\frac{(l-1)R}{l}\sin(\alpha))\vee_0$$
$$\vee_0\omega_{n-1}(\frac{R}{l},x-\frac{(l-1)R}{l}\cos(2\alpha),$$
$$y-\frac{(l-1)R}{l}\sin(2\alpha))\vee_0\ldots\vee_0 \quad (7)$$
$$\vee_0\omega_{n-1}(\frac{R}{l},x-\frac{(l-1)R}{l}\cos((k-1)\alpha),$$
$$y-\frac{(l-1)R}{l}\sin((k-1)\alpha))\geq 0;$$
$$n=1,2,3,\ldots$$

The calculation results for various values are shown in Fig. 9.

It should be noted that when $l=3$ fractals are formed, composed of connecting circles. These results are shown in Fig. 10.

Iterative fractals are obtained with $k=8$, $l=2$ и $n=\overline{1,2,3}$ which are presented in fig. 11.

Calculation results with $k=15$, $l=4$ and various values of n are presented in fig. 12.

Using the equation of a circle and the constructiveness of the means of the algebraic logical method of R-functions, one can construct equations of fractals consisting of intersections, tangents of circles. These fractals are very beautiful, which can be applied in the textile industry, telecommunications, when drawing patterns in ceramic and porcelain, etc.

V. Results of Computational Experiments

The obtained equations of the Fractal model are based on the description of the shapes of trees and circles, and the application was developed in the Delphi programming environment.

For each iteration value using the R-function method, the program results obtained on the basis of model (1) are as follows:

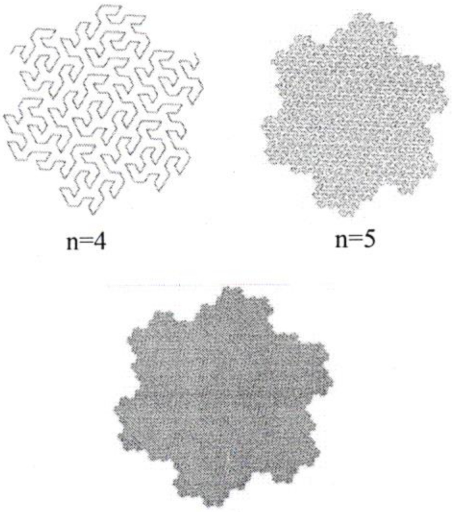


Fig. 2. Gosper curve for various values of iteration of n

Fig. 3 demonstrates the fractal structures in the tree structure for different iteration values k $(k=1,2,3,\ldots)$ and n $(n=1,2,3,\ldots)$ in accordance with formulas (2) and (3).

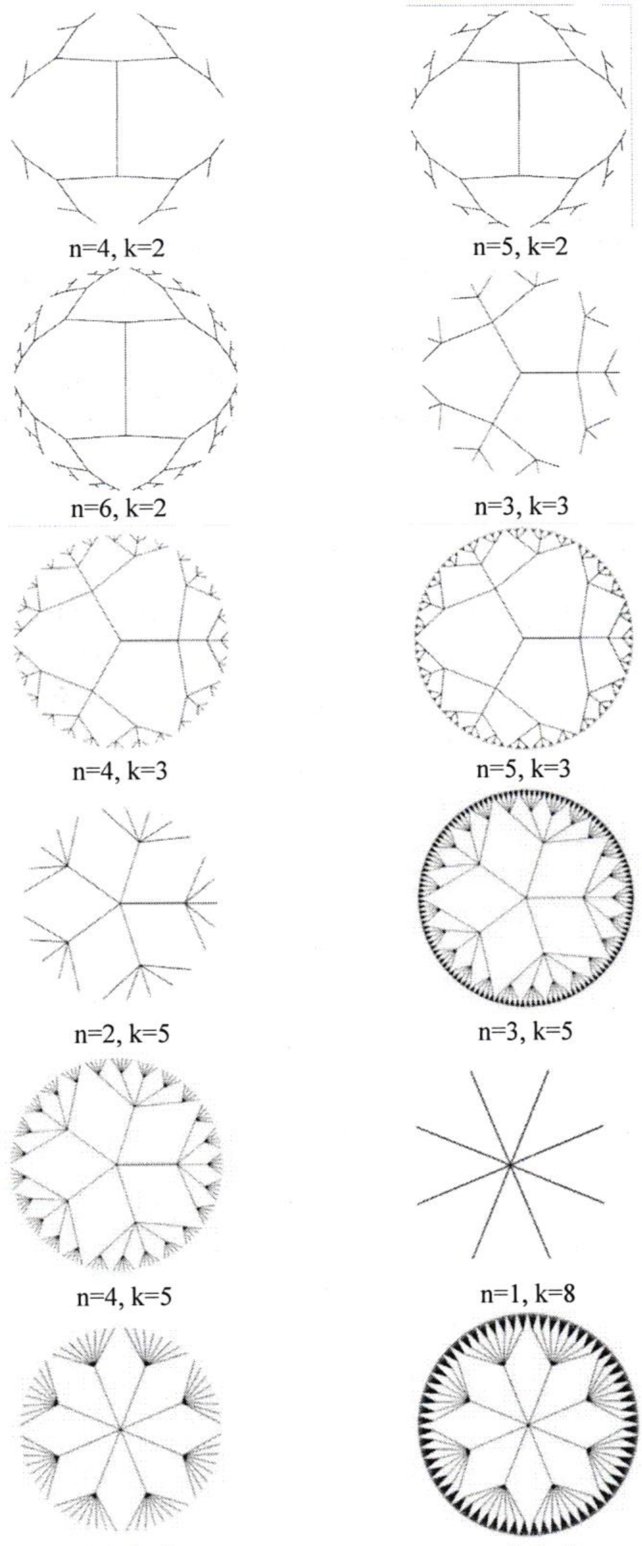
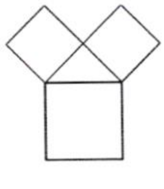
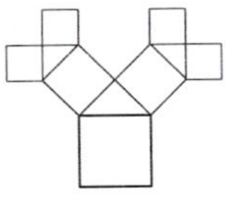


Fig. 3. Fractals in the form of trees

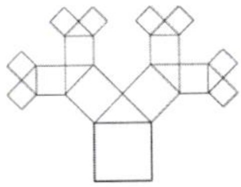
Fig. 4 illustrates the results for the Pythagorean fractals for different values of n $(n=1,2,3,\ldots)$ and α $(\alpha=\pi/8,\pi/4,\pi/2,\ldots)$, based on the model (4):



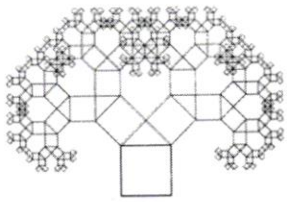
n=1, α=π/4



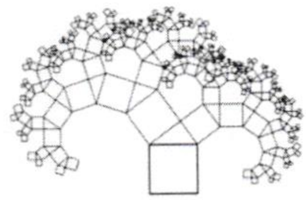
n=2, α=π/4



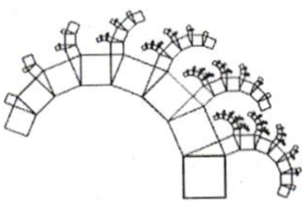
n=3, α=π/4



n=7, α=π/4



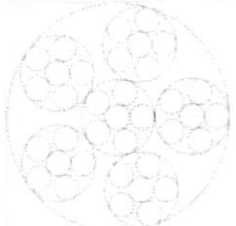
n=7, α=π/5



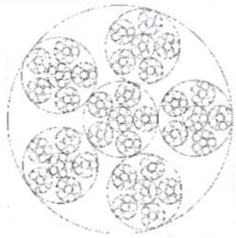
n=7, α=π/8

Fig. 4. Fractal of Pythagoras

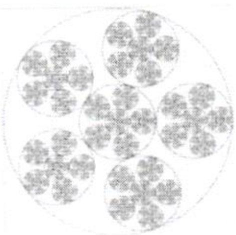
Fig. 5 shows the results of exclusive ring fractals with different values of and iterations based on model 5.



n=2, k=5

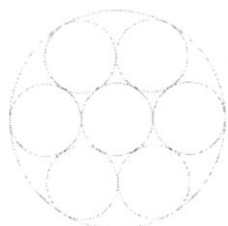


n=3, k=5

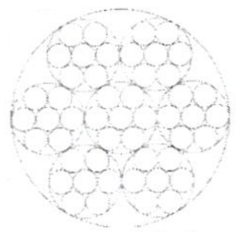


n=4, k=5

Fig. 5. Exclusive ring fractals



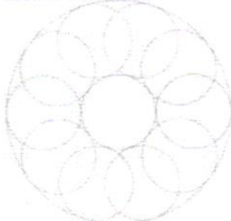
n=1, k=6



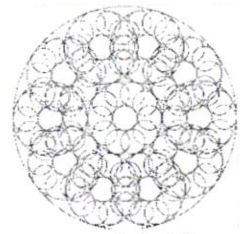
n=2, k=6

Fig. 6. Exclusive ring fractals

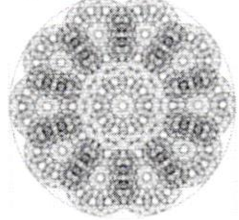
Fig. 7 shows the results of exclusive ring fractals with different values of $k\,(k>6)$ and $n\ (n=1,2,3)$ iterations based on model 5.



n=1, k=10



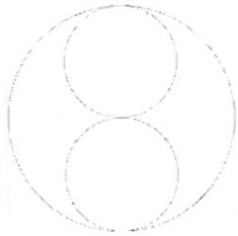
n=2, k=10



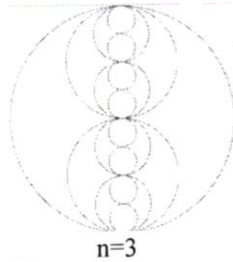
n=3, k=10

Fig. 7. Exclusive ring fractals

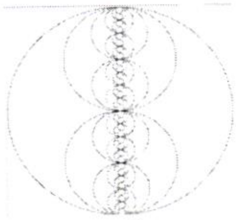
Fig. 8 illustrates the results of circular fractals for different values of $n\ (n=1,2,3,...)$ iterations based on model (6).



n=1



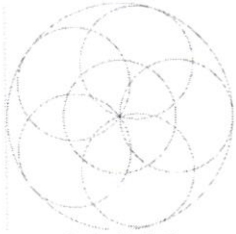
n=3



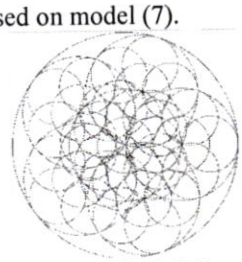
n=5

Fig 8. Fractals from circles

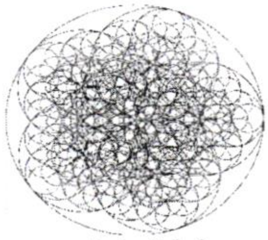
Fig. 9 demonstrates the result of fractals consisting of different values of $n\ (n=1,2,3)$ iterations and the intersection model for k = 5, l = 2, based on model (7).



n=1, k=5, l=2



n=2, k=5, l=2



n=3, k=5, l=2

Fig. 9. Fractals from intersecting circles

Retrieval Number: D1169029420/2020©BEIESP
DOI: 10.35940/ijitee.D1169.029420

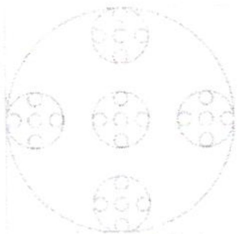
Published By:
Blue Eyes Intelligence Engineering

Fig. 10 shows the result of fractals consisting of a union of circles for different values of n ($n = 1, 2, 3, 4$) iteration, k ($k = 4, 5$) and l ($l = 4, 5$) in model (7).

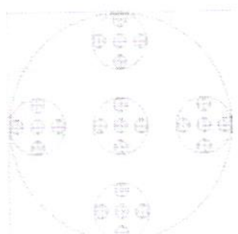
n=1, k=5, l=3

n=2, k=5, l=3

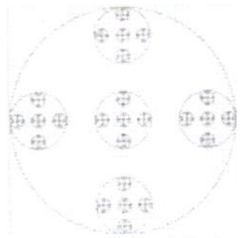
n=1, k=4, l=4



n=2, k=4, l=4



n=3, k=4, l=4

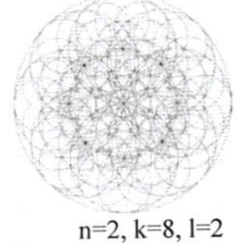


n=4, k=4, l=4

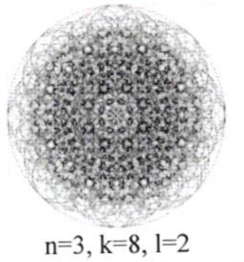
Fig. 10. Fractals from connecting circles

Fig. 11 shows the result of fractals consisting of different values of n ($n = 1, 2, 3$) iterations based on model 7 and intersections for k = 8, l = 2.

n=1, k=8, l=2



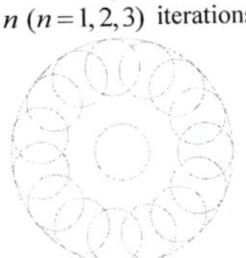
n=2, k=8, l=2

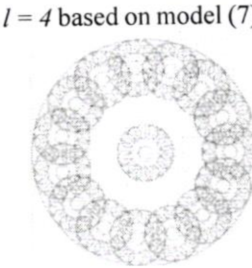


n=3, k=8, l=2

Fig. 11. Fractals from intersecting circles

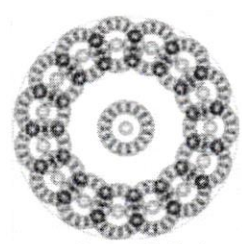
Fig. 12 illustrates the results of fractals consisting of intersecting and combining circles for various values of n ($n = 1, 2, 3$) iterations and $k = 15$, $l = 4$ based on model (7).





n=1, k=15, l=4

n=2, k=15, l=4



n=3, k=15, l=4

Fig. 12. Fractals from intersecting circles

VI. CONCLUSION

Recursive algorithms were developed using the method of R-function, based on this algorithm software is used to build fractals in 2D. Based on the method of R-function in 2D, an automated technology for describing the boundaries of complex forms has been developed. With the help of technology, a technique was developed to modernize the color design of Uzbek national carpets and costumes. A technology has been developed for geometric modeling of complex fractal structures using the capabilities of the method of R-function to create fractal designs. The design of Uzbek national carpets and costumes using this technology have been described in 2D. Geometric, algebraic fractals and their combinations are widely used in the development of modern design patterns of Uzbek national carpets and costumes.

REFERENCES

1. B. Mandel'brot, "Fraktal'naja geometrija prirody": per. s angl. – M.: Institut komp'juternyh issledovanij, 2002. – 656 s.
2. S. Lu, P. Y. Mok, X. Jin, "A new design concept: 3D to 2D textile pattern design for garments", Computer Aided Des. 89 (2017).
3. A. Bartle, D. M. Kaufman, et al., "Physics-driven pattern adjustment for direct 3D garment editing". ACM Trans. Graph. 35(4), 50 (2016).
4. S. Wang, X. Yang, "Generation of fractal image on complex plane and its application in textiles". Journal of Silk 54(8), 56–61 (2017).
5. Š. A. Nazirov, M. O. Èržanov, "Algebro-logičeskij metod postroenija ob"ektov fraktal'noj geometrii", Vestnik TUIT. Taškent, 2014. №1. c. 21-31.
6. Š. A. Nazirov, M. O. Èržonov, "Postroenie uravnenie determinirovannyh fraktalov", Uzb. žurnal Problemy informatiki i ènergetiki. Taškent, 2014. №1-2. c. 57-65.
7. Š. A. Anarova, T. Z. Tešabaev, F. M. Nuraliev, S. S .Abdikarimov, "Postroenie uravnenij kvadrata spiraleobraznyh fraktalov". Muhammad al-Xorazmiy avlodlari ilmiy-amaliy va axborot tahliliy jurnal. Taškent – 2019. № 1(7). - S. 121-124.
8. Sh. A. Anarova, F. M. Nuraliev, O. M. Narzulloev, "Construction of the equation of fractals structure based on the Rvachev R-functions theories" . Mechanical Science and Technology Update, IOP Publishing, IOP Conf. Series: Journal of Physics: Conf. Series 1260 (2019) 102002.
9. O. M. Narzulloev, M. N. Samidov, "Gazlama va gilamlar uchun zamonaviy dizajndagi naqshlarni fraktal ko'rinishdagi tuzilishlarini geometrik modellashtirish". Muhammad al-Horazmij avlodlari, № 3 (9), sentabr 2019. -16-20 b.
10. V. F. Kravčenko, V. L. Rvačev, "Algebra logiki, atomarnye funkcii i vejvlety v fizičeskih priloženijah". M.: Fizmatlit, 2006.-416s.
11. K. V. Maksimenko-Šejko, A. V. Tolok, T. I. Šejko, "R-funkcii v fraktal'noj geometrii", Informacionnye tehnologii. M.: Izdatel'stvo "Novye tehnologii", 2011. № 7. S.24-27.
12. K. V. Maksimenko-Šejko, "R-funkcii v matematičeskom modelirovanii geometričeskih ob"ektov i fizičeskih polej", Har'kov: IPMaš NAN Ukraina, 2009. 306 s.
13. K. V. Maksimenko-Šejko, A. V. Tolok, T. I. Šejko, "R-funkcii kak apparat v priloženijah fraktal'noj geometrii". Prikladnaja informatika. 2010, №6 (30). – S. 20-27.

14. K. V. Maksimenko-Šejko, T. I. Šejko, "R-funkcii v matematičeskom modelirovanie geometričeskih ob"ektov v 3D po informacii v 2D.", Vestnik Zaporožeskij nacional'nyj universitet. 2010, №1. – S. 98-104.
15. K. V. Maksimenko-Šejko, T. I. Šejko, "Matematičeskie modelirovanie geometričeskih fraktalov s pomoš'ju R-funkcij", Kibernetika i sistemnyj analiz. 2012, №4. – S. 155-162.
16. Sh. A. Anarova, N. G. Eshqoraeva, L. O'. Xaydarova, D. U. Sultonov, "Yulduzsimon fraktallarni qurishning geometrik modellari va algoritmlari", TATU xabarlari jurnali. 2016 №1(37). 40-43 b.
17. F. M. Nuraliev, Sh. A. Anarova., M. A. Mullamuxamedova, D. U. Sultonov, "Aylanalardan va to'rtburchaklardan iborat fraktallarni qurishning rekursiv algoritmlari", TATU xabarlari jurnali. 2015 №4(36). S. 82-88.
18. Sh. A. Anarova, F. M. Nuraliev, G. A. Qayumova, Z. M. Kurbanov, "Geometrik shakllardan iborat fraktallarni qurishning matematik va dasturiy ta'minoti", TATU xabarlari jurnali. 2017 №4 (44). –B. 48-62.
19. B. A. Bondarenko, "Generalized Pascal Triangles and Pyramids, their Fractals, Graphs, and Applications", USA, Santa Clara: Fibonacci Associations, The Third Edition. – 2010. – 296 p.

AUTHORS PROFILE

Anarova Shahzoda (Doctor of Science) is a professor of faculty "TV technologies" of Tashkent University of Information technologies. She has rich experience of 21 years in mathematical modeling of elasticity in Problems of Constraint Torsion and of the spatial loaded rods taking into account function of torsion and cross shifts and of fractal modeling of complex structures and has published more than 110 national and international publications in science journals and conferences. She is reviewer of International conferences.

Narzulloyev Oybek is an Assistant of Department of Audiovisual technologies of Tashkent University of Information Technologies named after Muhammad al-Khwarizmi. His research field is related to Computer Graphics. He has more than 30 national and international publications in scientific journals and conference proceedings.

Ibragimova Zulaykho is an assistant of professor at the Department of Information Technologies, Samarkand Branch of Tashkent University of Information Technologies. She is a junior researcher in the field of Computer Graphics. She has published several papers in national journals and conferences.