Certificate

09 December2020

EF: 000118

SCIENTIFIC JORNAL «EUROPEAN SCIENCE». № 7(56) 2020

НАУЧНЫЙ ЖУРНАЛ «EUROPEAN SCIENCE»

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JOURNAL ARCHIVE

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http://scientific-publication.com/arkhiv-zhurnala.html

ISSN 2410-2865 (Print)
ISSN 2541-786X (Online)

EUROPEAN SCIENCE

2020. № 7 (56)

PUBLISHING HOUSE «PROBLEMS OF SCIENCE»
EDITORIAL OFFICE ADDRESS: 153008, RUSSIAN FEDERATION, IVANOVO, LEZHNEVSKAYA ST., H.55, 4TH FLOOR, PHONE: +7 (910) 690-15-09

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HTTPS://SCIENTIFIC-PUBLICATION.COM

ISSN 2410-2865 (Print)
ISSN 2541-786X (Online)

EUROPEAN SCIENCE

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Российский импакт-фактор: 0,17

Подписано в печать:
18.12.2020
Дата выхода в свет:
21.12.2020

Формат 70x100/16.
Бумага офсетная.
Гарнитура «Таймс».
Печать офсетная.
Усл. печ. л.
Тираж 1 000 экз.
Заказ № 3000

ИЗДАТЕЛЬСТВО
«Проблемы науки»

Территория распространения: зарубежные страны, Российская Федерация

Журнал зарегистрирован Федеральной службой по надзору в сфере связи, информационных технологий и массовых коммуникаций (Роскомнадзор)
Свидетельство
ПИ № ФС77 - 60218
Издается с 2014 года

Свободная цена

Содержание

PHYSICO-MATHEMATICAL SCIENCES

ABOUT THE ISSUES OF GEOMETRICAL INEQUALITIES AND THE METHODS OF THEIR SOLUTION

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Abstract: *in practice, in the process of solving mathematical problems, we are faced with many professions related to geometric inequalities. Solving them requires students to have a thorough enough mathematical knowledge. It is one of the important tasks to teach mathematics teachers methods of proving geometrical inequalities in the process of forming the ability to apply their mathematical knowledge to solving various issues. This article explores some issues regarding geometric inequalities and the pure analytical and geometric ways to solve them.*
Keywords: *geometric inequalities, triangular, inequality, analytical method, geometric method, median, perimeter, angl, restangl, circle, radius.*

О ВОПРОСАХ ГЕОМЕТРИЧЕСКОГО НЕРАВЕНСТВА И СПОСОБАХ ИХ РЕШЕНИЯ

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Аннотация: *в процессе решения математических задач на практике мы сталкиваемся со многими задачами, относящимися к геометрическим неравенствам. Их решение требует от учащихся достаточно основательных математических знаний. Одной из важных задач является обучение способам доказательства геометрических неравенств в процессе формирования навыков одаренных обучающихся применять математические знания к решению различных задач. В этой статье изучены задачи, относящиеся геометрическим неравенствам, а также чисто аналитические и геометрические методы их решения.*
Ключевые слова: *геометрические неравенства, неравенство треугольника, аналитический метод, геометрический метод, медиана, периметр, угол, треугольник, четырехугольник, окружность, радиус.*

We are faced with geometrical inequalities in several issues related to proof. Teaching them in elective classes or circle classes will help to formulate the skills of strengthening and applying mathematical knowledge to the students. Several geometrical issues related to the joint application of the basic theorem and formulas of igeometrianin [1] and many algebraic inequalities [2-3] were studied in the study. Below are some of the issues related to geometrical inequalities and we aimed to study the methods of their solution. The

simplest view of geometrical inequalities is the inequality of triangles [6], in solving many issues, this fundamental inequality is used.

Issue 1. The sum of the medians of the triangle is equal to S , its perimeter is $2p$. $\frac{3}{2}p < s \le 2p$ prove inequality.

Proof. Let's assume $BD = m_c$ ABC mediana, which is lowered to the side of the triangle AC, triangular icons $BC = a,\ CA = b,\ AB = c$ and F location BC let it be the middle of the side (**figure 1**).

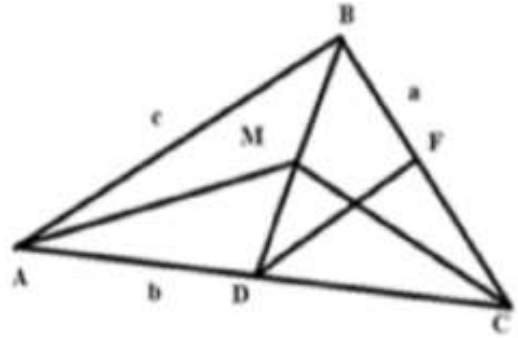

Fig. 1. m-ABC is the mediana lowered to the side of the triangle AC,triangle tomz a,b, c and f are the middle of the sides of the point BC

Thus BFD for a triangle, according to the inequality of the triangle $m_b < BF + FD = \frac{1}{2}(a+c)$ tensile fitting and m_a va m_c for medians, too, we write about similar inequalities:

$m_a < \frac{1}{2}(b+c)$, $m_c < \frac{1}{2}(a+b)$. In the result of the addition of these inequalities

$$m_b + m_a + m_c < \frac{1}{2}(a+c+b+c+a+b) = \frac{1}{2}(2a+2b+2c) = a+b+c$$

much we form. According to the condition of the matter $m_a + m_b + m_c = c$ and $a+b+c = 2p$ from that $s < 2p$ it turns out.

Let's assume that the point M is the point at the intersection of the medians of the ABC triangle. In that case, as a result of applying the inequality of triangles to ABM, BCM and CAM triangles, we will have the following:

$$\frac{2}{3}(m_a + m_b) > c,\ \frac{2}{3}(m_b + m_c) > a,\ \frac{2}{3}(m_c + m_a) > b.$$

If we add these inequalities to the limit, $\frac{2}{3}(m_a + m_b + m_b + m_c + m_c + m_a) > a+b+c$ or

$\frac{4}{3}(m_b + m_a + m_c) > 2p$ or we will have. From the condition of matter $\frac{4}{3}s > 2p$ or $s > \frac{3}{2}p$. And so on, $\frac{3}{2}p < s < 2p$ inequality proved.

Issue 2. If *BC=a, AC=b, AB=c*-the sides of the acute-angled triangle, *R*-the outer drawn circle to it, then *a)* $a^2 + b^2 + c^2 > 8R^2$; *b)* $a + b + c > 4R$ prove that you are.

Proof. a) let's assume $m_c = CD$, ABC let it be the median of the triangle (**figure 2**).

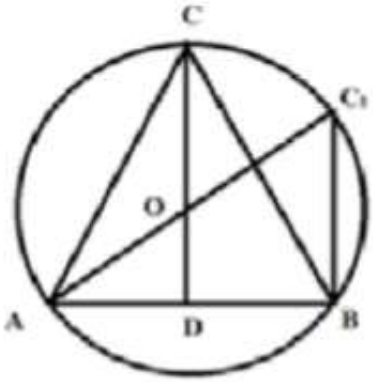

Fig. 2. BC=a, AC=b, AB=c-the sides of an acute-angled triangle,R – the outer drawn circle to it, m-the median of an ABC triangle

According to the formula of the length of the mediana $m_c^2 = \frac{1}{4}(2a^2 + 2b^2 - 2c^2)$. As a result, we get $a^2 + b^2 + c^2 = 2m_c^2 + \frac{3}{2}c^2$. ΔABC since it is a sharp angle, the center of the circle lies in a triangle with *O* points. ΔAC_1B we look at a triangle with an acute angle.

$\angle COD > \angle C_1OD$ because it is $C_1D < CD$ we find that it is. As a result, $a^2 + b^2 + c^2 = 2m_c^2 + \frac{3}{2}c^2 > 2C_1D^2 + \frac{3}{2}c^2 = AC_1^2 + C_1B^2 + BA^2 = 8R^2$

b) Since a, b and c are smaller $2R$ than $2R(a+b+c) > a^2 + b^2 + c^2 > 8R^2$. All in all, $a + b + c > 4R$.

Issue 3. Prove inequality if *a, b ,c* and *S* are triangular integers and surfaces, $a^2 + b^2 + c^2 \geq 4S\sqrt{3} + (a-b)^2 + (b-c)^2 + (c-a)^2$ respectively.

Proof. On the right side of the given inequality, we leave only the first suffixes, we shift the rest to the left and group them in the form of a pair of square brackets. Then we divide into multipliers in pairs and $x = a + b - c$, $y = a - b + c$, $z = -a + b + c$ we will add new variables. Thus $xy + xz + yz = (a^2 - (b-c)^2) + (b^2 - (c-a)^2) + (c^2 - (a-b)^2)$

Now $p=\frac{1}{2}(a+b+c)=\frac{1}{2}(x+y+z)$ va $p-a=\frac{1}{2}z$, $p-b=\frac{1}{2}y$, $p-c=\frac{1}{2}x$ if we take into account the equations, we form $S=\frac{1}{4}\sqrt{(x+y+z)xyz}$ according to the formula of Geron. As a result, our perceived inequality will have $xy+xz+yz\geq\sqrt{3(x+y+z)xyz}$ appearance. We divide both sides of this inequality into xyz and get $u=\frac{1}{x}$, $v=\frac{1}{y}$, $w=\frac{1}{z}$ new variables. We will have $u+v+w\geq\sqrt{3(uv+vw+wu)}$ without it. After the last inequality is squared and simplified, we come to a certain $u^2+v^2+w^2\geq uv+uw+wv$ inequality.

Comments. it is always useful to switch from *a,b,c* triangular integers to *x,y,z* variables by formulas $x=a+b-c$, $y=a-b+c$, $z=-a+b+c$. The main thing is that *a,b, c* are provided that together with the variables being positive, the triangle satisfies the inequality, in this case the new variables will also be positive.

Issue 4. Prove that *R* and *r* are $R\geq 2r$, if there are radiuses of the outer and inner drawn circles to the triangle respectively.

Proof. Suppose, A_1, B_1 and C_1 are the middle of the *ABC* triangle integers (**figure 3**).

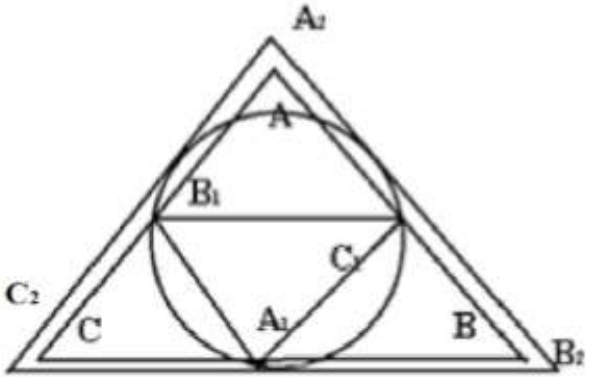

Fig. 3. Radius of the outer and inner drawn circles R and r respectively into the triangle, A_1, B_1 and C_1 are the middle of the triangle integers ABC

$A_1B_1C_1$ is equal to the radius $\frac{R}{2}$ of the outer drawn circle to the triangle, and this circle in general "goes out" from the border of the triangle *ABC*. Therefore $\frac{R}{2}\geq r$ place. We pass the parallel strokes to the sides of the *ABC* triangle into the circle we are looking at. As a result, we get an inner drawn circle with $\frac{R}{2}$ radi, $A_2B_2C_2$ triangle, similar to the *ABC* triangle, which contains the *ABC* triangle itself.

Issue 5. On the AC side of the ABC triangle, the points K and M are obtained so that $AK=MC$. If $AB > BC$. then prove that there will be $\angle ABK < \angle MBC$ (**figure 4**).

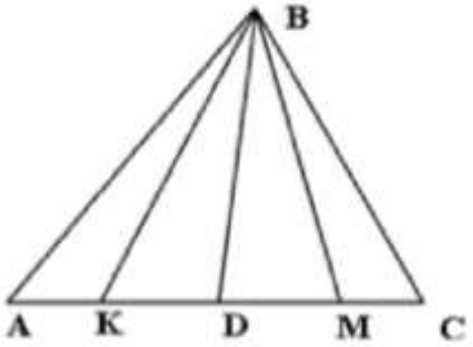

Fig. 4. On the AC side of the ABC triangle, the points K and M are obtained so that AK=MC. BD – mediana

Proof. We can assume that $AK = MC \le \frac{1}{2}AC$. We pass the BD media. It does not pass from $AB > BC$ to $\angle BDA$, and the projection of point B to AC lies in the DC light. So, the projection of KB is larger than the projection of BM and $BK > BM$. But, triangles of equal size ΔABK and ΔCBM, that is $AB \cdot BK \sin\angle ABK = CB \cdot BM \sin\angle CBM$. As a result, $\sin\angle MBC > \sin\angle KBA$. However $\angle ABK$ sharp angle. All in all, $\angle ABK < \angle MBC$.

Issue 6. a, b, c, d – the lengths of the sides of the rectangle, and S –let it be his face, then prove $S \le \frac{1}{2}(ac + bd)$ inequality.

Proof. If a, c and b, d were the bases of the quadrilateral army, then the inequality is seen to be reasonable (the face of the triangle does not exceed half the multiples of its two integers). That's why we make them an army. To do this, we change the $ABCD$ rectangle to the ABC_1D rectangle (**drawing** 5).

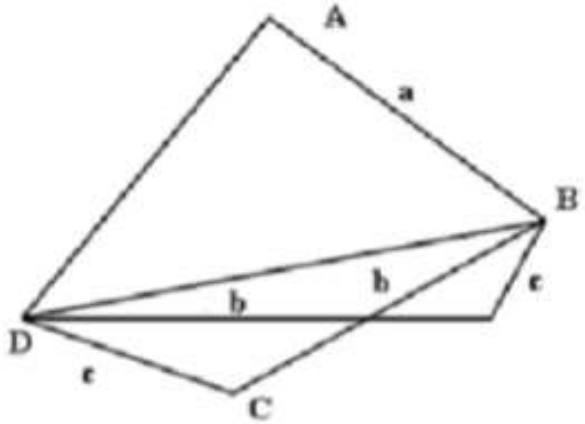

Fig. 5. a, b, c, d – the lengths of the sides of the rectangle, and S –its face

There $\Delta BC_1D = \Delta BCD$. Now the ABC_1D rectangle is equal in size with the $ABCD$ rectangle. In it we pass AC_1 dioganal and get two triangles, the sides of which are a, c and b, d are equal, respectively.

Conclusion. Problems with geometric inequalities are also common in mathematics. By solving the above problems, the applications of triangular inequalities, as well as pure analytical and geometric methods of proving inequalities were studied. Issues like these can be used as a ready-made material in a circle for gifted students in mathematics.

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