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Mayer Problem Application in Optimal Control of the Hitching Systems of Cotton-Harvesting Machines

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ABSTRACT: Based on the obtained equations of motion, the models and optimal control algorithm for the hitching system of cotton harvesting machine (CHM) MX-1.8 are developed in the paper. The necessary conditions for optimal control of the CHM MX-1.8 motion are investigated using the Mayer problem and applying the Pontryagin maximum principle. The operation parameters of the CHM MX-1.8 hitching system under vertical vibrations are determined. The optimal angle of rotation of the rockshaft which predetermines the uniform oscillation of the picking unit of the harvester hitching system is determined.

KEY WORDS: Ccotton picker, Hitching mechanism, Modeling, Optimal control, Mayer problem

I.INTRODUCTION

Control effectiveness of cotton-picking machines with hitching systems depends on technical features, on how fully they correspond to the established design requirements. For the maximum effectiveness, the operation of control system should take place according to a certain regularity described by the control algorithm, where all current factors and all known dependences are taken into account. Here, one of the most important roles is played by the characteristics of the cotton picker as an object of control.

Accurate mathematical description of these characteristics is a rather difficult task. However, from the point of view of creating an effective (that is, optimal in a practical sense) control system, a complete and accurate solution to this problem is not necessary. It is enough to single out those characteristics that have the greatest impact on a particular type of control. But here it turns out that it is necessary to take into account many of the interconnections and links that are inherent in the cotton picker operation.

As practice has shown, at motion of a cotton picker with hitching systems, random uncontrolled disturbances cause natural oscillations. As a result, steady state oscillations can be established in the system.

The system under consideration, like any other self-oscillating system, is a nonlinear one. Amplitudes, frequencies, and modes of oscillations, as well as the existence of a stationary vibrational mode, can be determined by solving nonlinear equations, which is very difficult for a system with many degrees of freedom. An important factor for assessing steerability is the reaction to a rapid change in control position of the hitching system of the cotton picker, i.e., of the rockshaft.

When using the classical variation calculus for dynamical systems described by ordinary differential equations, the definition of optimal control is reduced to solving Mayer, Lagrange, or Boltza problems depending on the specified quality criterion. The solution to these problems is based on the method of Lagrange multipliers and the study of variations of the extended function. The necessary conditions for the extremum of the quality functional are obtained from the vanishing of the first variation of the extended function, which implies the Euler equations written for all coordinates and controls included in the functional. The missing boundary conditions are determined from the transversality condition [1].

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II. SIGNIFICANCE OF THE SYSTEM

The main attention in the article is paid to how the methods of mathematical modeling and optimal control are used to determine the optimal parameters of the hitching systems of cotton-harvesting machines. The study of literature survey is presented in section III, Methodology is explained in section IV, section V covers the experimental results of the study, and section VI discusses the future study and Conclusion.

III. LITERATURE SURVEY

The development and improvement of the processes of information systems and technologies predetermine the principles and approaches to the formation and formalization of ever new classes of models for analysis, prediction of productivity and testing of cotton-picking machines.

In the process of research and analysis of many scientific works in the field of developing object-oriented mathematical models for evaluating their use in the operation of machine-tractor units and cotton-picking machines under various driving conditions, the corresponding features were established.

Tilloev S proposed the equation of motion for the machine assembly of cotton-picking machines and the root cultivators of cotton stalks, and obtained their analytical solutions.

Yuldashev S et al proposed well-known and new models of the productivity of cotton-picking machines, as well as the results of calculating various tractors for their productivity.

Van der Sluijs and Marinus H. J. suggested some aspects regarding the design and working process of the press wheel of agricultural machinery for planting and sowing. In addition, they presented theoretical studies of the working bodies of agricultural machinery related to rolling resistance, level of compaction and assessment of the depth of compaction achieved by the pressure wheel. They provided an analysis of the finite elements for the press wheel and support for its stability during the working process, for the normal nature of the movement of the wheel on the ground.

Mitsyn G.P. investigated and solved the problem of evaluating the control of wheeled tractors with mounted systems, consisting of a linkage mechanism with executive power cylinders, together with a hydraulic control system, which is an integral part of the wheeled tractor.

Tayanovsky G established that if the distribution coefficient of the sprung masses is close to unity, then the vibrations of the front and back parts of the core become unconnected.

Lurie Z I et al. Proposed a method for studying the dynamic modeling of the tractor's technical system - seeders, using specialized Inventor software. They studied the process of designing and modeling three different models: a dynamic model of a seeder cross-section, a dynamic model of a mechanical transmission of a seeder and a dynamic model of a tractor.

Alshaer B.J et al proposed a methodology for modeling and driving vehicles. The developed methodology for determining the trajectory takes into account the dynamics and performance characteristics of elements of vehicles of heavy construction, and also takes into account restrictions on movement. The criterion for optimizing the trajectory is based on minimizing the distance and inclination without taking into account the restrictions imposed on the vehicle and turning radius.

IV. METHODOLOGY

Based on the analysis of design scheme of a cotton harvesting machine with hitching system, a generalized mathematical model of vertical vibrations of CHM MX-1.8 in the process of motion along the headland roughness of a cotton field was obtained in the form of Lagrange equations of the second kind [2-5]:

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$$\left.\begin{aligned}
&m_м\ddot{y}_м = F_y - b_1(\dot{y}_м - \dot{y}_{к_1}) - c_1(y_м - y_{к_1}) - b_2(\dot{y}_м - \dot{y}_{к_2}) - c_2(y_м - y_{к_2}),\\
&m_1\ddot{y}_{к_1} = b_1(\dot{y}_м - \dot{y}_{к_1}) + c_1(y_м - y_{к_1}) - m_1\frac{2\pi^2V_{к_1}^2}{l_5^2}h_n(1-\cos\frac{2\pi V_{к_1}}{l_5}t),\\
&(m_2 - m_3)\ddot{y}_{к_2} = b_2(\dot{y}_м - \dot{y}_{к_2}) + c_2(y_м - y_{к_2}) - (m_2 - m_3)\frac{2\pi^2V_{к_2}^2}{l_5^2}h_n(1-\cos\frac{2\pi V_{к_2}}{l_5}t),\\
&j_{гц}\ddot{\varphi}_{гц} = F_{гц}\cdot l_6 - b_3(\dot{\varphi}_{гц} - \dot{\varphi}_{вк}) - c_3(\varphi_{гц} - \varphi_{вк}) - l_7\cdot m_a\ddot{y}_м,\\
&j_{вк}\ddot{\varphi}_{вк} = b_3(\dot{\varphi}_{гц} - \dot{\varphi}_{вк}) + c_3(\varphi_{гц} - \varphi_{вк}) - l_7\cdot m_a\ddot{y}_м,\\
&m_{гц}\ddot{y}_{гц} = \frac{j_{гц}\ddot{\varphi}_{гц}}{l_7 - l_6},\\
&m_{вк}\ddot{y}_{вк} = \frac{j_{вк}\ddot{\varphi}_{вк}}{l_7}.
\end{aligned}\right\} \quad (1)$$

$$J(q_0, u(t), q(t)) = \int_{t_0}^{T} f^0(q(t), u(t), t)dt + g^0(q_0, g(T)) \to \min$$

$$q_i(0) = q_0(0), \qquad \dot{q}_i(0) = \dot{q}_0(0), \qquad V_i(0) = V_0(0)$$

$$q_i(t) = q_0(t),\ \ \dot{q}_i(t) = \dot{q}_0(t),\ \ V_i(t) = V_0(t)\ \ (i = \overline{1,n}),\quad 0 \le t \le T,$$

Where $\dot{y}_м$ *and* $\ddot{y}_м$ – are the linear speed and acceleration of machine; $\dot{y}_{к_1}$ and $\ddot{y}_{к_1}$ - are the linear speed and front wheels acceleration; $\dot{y}_{к_2}$ and $\ddot{y}_{к_2}$ - are the linear speed and rear wheels acceleration; $\dot{\varphi}_{гц}$ *and* $\ddot{\varphi}_{гц}$ - are the rotational speed and acceleration of torsional vibrations of the hydraulic cylinder lever; $\dot{\varphi}_{вк}$ *and* $\ddot{\varphi}_{вк}$ - are the rotational speed and acceleration of torsional vibrations of the rockshaft lever; b_3 is the coefficient of viscous resistance of the rockshaft of the hitching mechanism of the harvesting unit; c_3 –is the stiffness coefficient of the rockshaft of the hitching mechanism of the harvesting unit; m_a is the distributed mass of the harvesting apparatus; $m_{гц}$ is the distributed mass along the hydraulic cylinder; $m_{вк}$ is the distributed mass along the rockshaft; $F_{гц}$ is the force in the hydraulic cylinder of the harvester hitching mechanism; l_1, l_2, l_3,l_4 and l_5 are the distances between supports and roughness; $l_6$is the lever length of the hydraulic cylinder; l_7 is the lever length of the hitching mechanism of the harvester; $j_{гц}$ and j_a are the moments of inertia of the linking levers of hydraulic cylinder and the harvester hitching mechanism.

The Pontryagin maximum principle [7]was used in the study of necessary conditions for optimal control. A structure for optimal control of the guide wheels motion of cotton picker was obtained from the principle of Pontryagin maximum. To determine the auxiliary functions by a numerical method, the link system with a variation of design parameters was studied. It was determined that left and right picking mechanisms fluctuate unevenly when the machine swings vertically. The main reason for the uneven fluctuations of harvesting machines is the installation of a lever for connecting the hydraulic cylinder on the left edge, and not in the middle of the rockshaft [3-5].

In the problem to be solved, a necessary condition is the optimal, from practical point of view, evaluation of the angle of rotation of a rockshaft, which can provide the smallest fluctuations in the CHM hitching system.

The control model is described by the equation

$$j_{гц}\ddot{\varphi}_{гц} = F_{гц}\cdot l_6 - b_3(\dot{\varphi}_{гц} - \dot{\varphi}_{вк}) - c_3(\varphi_{гц} - \varphi_{вк}) - l_7\cdot m_a\ddot{y}_м \quad \text{or}$$

$$\ddot{\varphi}_{гц} = j_{гц}^{-1}\,[\,F_{гц}\cdot l_6 - b_3(\dot{\varphi}_{гц} - \dot{\varphi}_{вк}) - c_3(\varphi_{гц} - \varphi_{вк}) - l_7\cdot m_a\ddot{y}_м\,] \qquad (2)$$

Here $\varphi_{вк} = \gamma$ is the angle of rotation of the rockshaft; $j_{вк}$ - the moment of inertia of the linking levers of the rockshaft.
To ensure the stability of the rotation angle for equation (2), the following conditions are additionally required:

$$\dot{\varphi}_{гц}(0) = 0, \dot{\varphi}_{гц}(T) = 0,\ \ \varphi_{гц}(T) = \max\,. \qquad (3)$$

The above equation is written in normal form, for this the following notation is introduced:

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$$y_{\text{м}}(t) = x_1(t), \dot{y}_{\text{м}}(t) = x_2(t), y_{k_1}(t) = x_3(t), \dot{y}_{k_1}(t) = x_4(t), y_{k_1}(t) = x_5(t), \dot{y}_{k_1}(t) = x_6(t),$$

$$\varphi_{\text{гц}}(t) = x_7(t), \dot{\varphi}_{\text{гц}}(t) = x_8(t)\ , \varphi_{\text{вк}}(t) = x_9(t), \dot{\varphi}_{\text{вк}}(t) = x_{10}(t).$$

$$\bar{x} = (x_1(t),\ x_2(t),\ x_3(t),\ x_4(t),\ x_5(t),\ x_6(t), x_7(t),\ x_8(t),\ x_9(t),\ x_{10}(t));$$

$$\dot{x}_1 = x_2;$$

$$\dot{x}_2 = \frac{F_y}{m_M} - \left(\frac{b_1+b_2}{m_M}\right)x_2 - \left(\frac{c_1+c_2}{m_M}\right)x_1 + \frac{b_1}{m_M}x_4 + \frac{c_1}{m_M}x_3 + \frac{b_2}{m_M}x_6 + \frac{c_2}{m_M}x_5;$$

$$\dot{x}_3 = x_4;$$

$$\dot{x}_4 = \frac{b_1}{m_1}x_2 - \frac{b_1}{m_1}x_4 + \frac{c_1}{m_1}x_1 - \frac{c_1}{m_1}x_3 - Q_1;$$

$$\dot{x}_5 = x_6;$$

$$\dot{x}_6 = \frac{b_2}{m_2-m_3}x_2 - \frac{b_2}{m_2-m_3}x_6 + \frac{c_2}{m_2-m_3}x_1 - \frac{c_2}{m_2-m_3}x_5 - Q_2;$$

$$\dot{x}_7 = x_8;$$

$$\dot{x}_8 = \frac{F_{\text{гц}} \cdot l_6}{j_{\text{гц}}} - \frac{b_3}{j_{\text{гц}}}x_8 + \frac{b_3}{j_{\text{гц}}}x_{10} - \frac{c_3}{j_{\text{гц}}}x_7 + \frac{c_3}{j_{\text{гц}}}x_9 - \frac{l_7 m_a F_y}{j_{\text{гц}} m_M} + \frac{l_7 m_a (b_1+b_2)}{j_{\text{гц}} m_M}x_2 +$$
$$+\frac{l_7 m_a (c_1+c_2)}{j_{\text{гц}} m_M}x_1 - \frac{l_7 m_a b_1}{j_{\text{гц}} m_M}x_4 - \frac{l_7 m_a c_1}{j_{\text{гц}} m_M}x_3 - \frac{l_7 m_a b_2}{j_{\text{гц}} m_M}x_6 - \frac{l_7 m_a c_2}{j_{\text{гц}} m_M}x_5$$

$$\dot{x}_9 = x_{10},$$

$$\dot{x}_{10} = \frac{b_3}{j_{\text{вк}}}x_8 - \frac{b_3}{j_{\text{вк}}}x_{10} + \frac{c_3}{j_{\text{вк}}}x_7 - \frac{c_3}{j_{\text{вк}}}x_9 - -\frac{l_7 m_a F_y}{j_{\text{вк}} m_M} + \frac{l_7 m_a (b_1+b_2)}{j_{\text{вк}} m_M}x_2 +$$
$$+\frac{l_7 m_a (c_1+c_2)}{j_{\text{вк}} m_M}x_1 - \frac{l_7 m_a b_1}{j_{\text{вк}} m_M}x_4 - \frac{l_7 m_a c_1}{j_{\text{вк}} m_M}x_3 - \frac{l_7 m_a b_2}{j_{\text{вк}} m_M}x_6 - \frac{l_7 m_a c_2}{j_{\text{вк}} m_M}x_5$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{C_1+C_2}{m_M} & -\frac{b_1+b_2}{m_M} & \frac{C_1}{m_M} & \frac{b_1}{m_M} & \frac{C_2}{m_M} & \frac{b_2}{m_M} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{C_1}{m_1} & \frac{b_1}{m_1} & -\frac{C_1}{m_1} & -\frac{b_1}{m_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{C_2}{m_2-m_3} & \frac{b_2}{m_2-m_3} & 0 & 0 & -\frac{C_2}{m_2-m_3} & -\frac{b_2}{m_2-m_3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{l_7 m_a (C_1+C_2)}{j_{\text{гц}} m_M} & \frac{l_7 m_a (b_1+b_2)}{j_{\text{гц}} m_M} & \frac{-l_7 m_a C_1}{j_{\text{гц}} m_M} & \frac{-l_7 m_a b_1}{j_{\text{гц}} m_M} & \frac{-l_7 m_a C_2}{j_{\text{гц}} m_M} & \frac{-l_7 m_a b_2}{j_{\text{гц}} m_M} & \frac{-C_3}{j_{\text{гц}}} & \frac{-b_3}{j_{\text{гц}}} & \frac{C_3}{j_{\text{гц}}} & \frac{b_3}{j_{\text{гц}}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{l_7 m_a (C_1+C_2)}{j_{\text{вк}} m_M} & \frac{l_7 m_a (b_1+b_2)}{j_{\text{вк}} m_M} & \frac{-l_7 m_a C_1}{j_{\text{вк}} m_M} & \frac{-l_7 m_a b_1}{j_{\text{вк}} m_M} & \frac{-l_7 m_a C_2}{j_{\text{вк}} m_M} & \frac{-l_7 m_a b_1}{j_{\text{вк}} m_M} & \frac{C_3}{j_{\text{вк}}} & \frac{b_3}{j_{\text{вк}}} & \frac{-C_3}{j_{\text{вк}}} & \frac{-b_3}{j_{\text{вк}}} \end{bmatrix} \quad (3)$$

$$\bar{B} = (0,0,0,0,0,0,0, -\frac{b_3}{j_{\text{вк}}}, 0,0)^T;$$

$$\bar{q} = (0,\ \frac{F_y}{m_M},\ 0,\ -\frac{2\pi^2 V^2 x_1}{l_5^2} h_n (1-\cos\frac{2\pi V x_1}{l_5} t),\ 0,\ -\frac{2\pi^2 V^2 x_2}{l_5^2} h_n (1-\cos\frac{2\pi V x_2}{l_5} t),\ 0, u,\ 0,\ -\frac{l_7 m_a F_y}{j_{\text{вк}} m_M}).$$

Then problem (1) - (3) is reduced to solving the following problem:

In general form, the equation of motion of an object in vector form is:

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$$\dot{\bar{x}}(t) = A\bar{x}(t) + \bar{B}\,u + \bar{q} \quad (4)$$

At boundary conditions

$$\bar{x}(0) = 0 \quad (5)$$

At control conditions

$$0 \le F_{гц} \le 2225, t \in [o, T], \; T = 10.$$

Quality criterion is

$$J(u) = x_8(T) \to min, \quad (6)$$

Here $u=F_{гц}$, is the set time of the turn end. It is required to choose control *u(t)* so that at time *T* the rotational speed of hydraulic cylinder is equal to zero, and the rotation angle is maximum. Control tasks were reduced to Mayer's problem.

To solve the Mayer problem, the Pontryagin maximum principle was used.

The Hamiltonian in a vector form is:

$$H = \psi(Ax + Bu + q). \quad (7)$$

To compose the Pontryagin function, we introduce the following vector functions:

$$u^* = signB\psi = \begin{cases} o, & B\psi = 0 \\ 2225, & B\psi > 0 \end{cases} \quad (8)$$

Linked system in a vector form has the form

$$\dot{\psi(t)} = \frac{\partial H}{\partial x} = A\psi$$

Initial conditions in a linked system are:

$$\psi_1(T) = -\frac{\partial J}{\partial x_1(T)} = 0, \psi_6(T) = -\frac{\partial J}{\partial x_1(T)} = 0,$$
$$\psi_2(T) = -\frac{\partial J}{\partial x_2(T)} = 0, \psi_7(T) = -\frac{\partial J}{\partial x_7(T)} = 0,$$
$$\psi_2(T) = -\frac{\partial J}{\partial x_3(T)} = 0, \psi_8(T) = -\frac{\partial J}{\partial x_8(T)} = -1,$$
$$\psi_4(T) = -\frac{\partial J}{\partial x_4(T)} = 0, \psi_9(T) = -\frac{\partial J}{\partial x_9(T)} = 0,$$
$$\psi_5(T) = -\frac{\partial J}{\partial x_5(T)} = 0, \;\; \psi_{10}(T) = -\frac{\partial J}{\partial x_{10}(T)} = 0,$$

Linked system solution

$$\dot{\psi(t)} = A\psi(t)$$

has the form

$$\psi(t) = ce^{At} \; or \; \psi(t) = -e^{A(t-10)}.$$

The maximum condition is satisfied at the ends of domain at $F_{гц}=0$, $F_{гц}=2225$.

At $F_{гц}=0$ we have

$$\dot{x} = Ax + q,$$

At $F_{гц}$=*2225* we have

$$\dot{x} = Ax + B \cdot 2225 + q.$$

V. EXPERIMENTAL RESULTS

A computational experiment was carried out with the following parameter values:

c_1=1254208.5 N/m; b_1=105634.2 Nf/m; c_2=637650 N/m; b_2=53705.3 Nf/m; c_3=263377.3 Nm/rad; b_3=22182.643 Nmf/m; $m_м$=7714 kg;m_1=5114 kg; m_2=2600 kg;m_3=1262 kg; m_a=675 kg; $j_{гц}$=552.96 Nmf^2; $j_{вк}$=276.48 Nmf^2; r_1=0.785 m; r_2=0.43 m; $h_п$=0.07 m; $h_ш$=0.03 m; $V_м$=1.21 m/s; $F_м = 18050\ N$, $F_y = F_м \sin\alpha = 18050 \sin 45^0 = 12763.277 H$; $F_{гц} = 2225\ N$

Table1.Operation parameters of the CHM MX-1.8 hitching system under vertical vibrations

t	$\dot{y}_м$, m/s	$\dot{y}_{к_1}$, m/s	$\dot{y}_{к_2}$, m/s	$\varphi_{вк}$
0	0	0	0	0
1	0.171	0.169	0.173	1.69
2	0.319	0.316	0.322	3.16
3	0.469	0.475	0.464	4.75
4	0.642	0.639	0.645	6.39
5	0.789	0.787	0.79	7.87
6	0.943	0.948	0.938	9.48
7	1.116	1.112	1.12	11.12
8	1.262	1.261	1.263	12.61
9	1.421	1.427	1.416	14.27
10	1.596	1.59	1.6	15.9

VI. CONCLUSION AND FUTURE WORK

As the results of solving the Mayer problem demonstrate, a change in the angle of rotation of the rockshaft by 15.9 degrees determines the steady state motion of a cotton picker and uniform oscillation of the harvester hitching system, showing the correctness of developed model.

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