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ON THE DYNAMIC STRESSED-DEFORMED STATE OF ISOTROPIC RECTANGULAR PLATES ON AN ELASTIC BASE WITH VIBRATION LOADS

Abstract: *The problem of calculating the dynamic stress-strain state of viscoelastic rectangular isotropic plates on a deformed base, including freely lying on the ground medium under the influence of vibration loads, is solved. Several models of the dynamic reaction of the base are considered and a qualitative comparison of the results is carried out. In the calculations, the Gauss method, the Mueller method and the smallest residuals were used.*
Key words: *rectangular plate, deformed base, stress-strain state, Gauss method, Mueller method and least residuals.*
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Introduction.

Rectangular plates with variable geometric and mechanical parameters located under dynamic vibration loads are used in various industries and construction. A rectangular transversely dynamically loaded plate can rest on a deformable (elastic or viscoelastic) base. For example, in the coverings of roads, bridges or runways of airfields. To study the dynamic strength and bearing capacity of such structures, knowledge of their dynamic stress-strain state under vertical loads is required. The problem of bending vibrations of viscoelastic plates on an elastic base is an urgent problem in the mathematical theory of viscoelasticity. In a closed analytical form, its solution, to simplify the elastic formulation, manages to obtain a limited number of boundary value problems. An alternative approach to finding an approximate or semi-analytical solution of an elastic problem is to present the solution in the form of a series [1,2,3]. The authors of [4] propose, using the variation method for elastic problems, to reduce the resolving equations to a system of ordinary differential equations. The disadvantage of these methods is their explicit dependence on the methods for setting the boundary conditions and patterns of loading. In [4], a finite-difference approach is used for statically loading, which in turn leads to difficulties in the implementation of boundary conditions. For high-order differential equations, a large template is used. All of the above reasoning necessitates the development of effective methods for solving boundary value problems of plate theory operating on a deformable base.

Problem statement and solution methods In this paper, for the numerical solution of the problem of plate bending, the method of collocations and least residuals (KNI) is used. The KNN method has proven itself in solving ordinary differential equations and partial differential equations for hydrodynamic problems [6]. It is used for the first time to calculate the VAT of plates. Consider a rectangular plate on an elastic base.
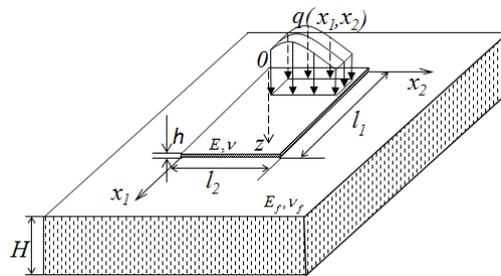

Fig.1. Plate on an elastic base

Problem statement and solution methods In this paper, for the numerical solution of the problem of plate bending, the method of collocations and least residuals (KNI) is used. The KNN method has proven itself in solving ordinary differential equations and partial differential equations for hydrodynamic problems [6]. It is used for the first time to calculate the VAT of plates. Consider a rectangular plate on an elastic base. The elastic base reaction will be considered using a one-parameter model based on the Winkler hypothesis (hereinafter referred to as the Winkler model) [5-10], and two more complex two-parameter models of Vlasov [4] and Pasternak [8]. Winkler's hypothesis suggests that the reaction of the base is proportional to the deflection of the slab

$$p = k_m(w - \int_0^t R(t-\tau)w(\tau)d\tau), \qquad (1)$$

Where p - is the reaction of the base, w-is the deflection of the slab, k_m-is the instantaneous bed coefficient (proportionality coefficient), determined experimentally for each type of soil. Despite its simplicity, in many cases the use of this model is sufficient to obtain practical results. However, this representation of the reaction of the soil has several disadvantages. For example, external loads are distributed on the soil only within the area of the bottom of the slab. This position does not correspond to real observations, according to which the soil settles, and therefore is stressed outside the plate. Another disadvantage k -is the difficulty in determining the value of the bed coefficient, which depends on the size and shape of the test stamp. A more complex model of soil reaction is embedded in two-parameter models.

In [8] it C_1 is proposed to obtain coefficients from the following considerations. connects the intensity of the vertical rebound of the soil with its sediment, and the second independent coefficient allows you to determine the intensity of the vertical shear force. The following possible parameter values are also given

$$p = C_{m1}(w - \int_0^t R_1(t-\tau)w(\tau)d\tau) - $$

$$- C_{m2}(\Delta w - \int_0^t R_2(t-\tau)\Delta w(\tau)d\tau) \qquad (2)$$

Where Δ-is the Laplace operator, C_{m1}, C_{m2}-are the soil parameters. Here, in addition to the work of the base for compression (Winkler hypothesis), the work of the base for shear or shear is additionally taken into account. In [4], the authors present the base as a medium in which there are no longitudinal (along the plane of the resting plate) displacements. Then the coefficients can be determined by the following formulas

$$C_{m1} = \frac{E_0}{1-v_0^2} \int_0^H (\phi'(z))^2 dz,$$

$$C_{m2} = \frac{E_0}{2(1+v_0)} \int_0^H (\phi(z))^2 dz$$

$$E_0 = \frac{E_f}{1-v_f^2}, \quad v_0 = \frac{v_f}{1-v_f}, \qquad (3)$$

E_f, v_f - the instantaneous Young's modulus and the Poisson's ratio of the elastic base, $\phi(z)$-is the transverse distribution function of the elastic base, which characterizes the extinction of the soil tension with increasing depth H. In that work

$$\phi(z) = \frac{1}{2i}(e^{\gamma(z-H)} - e^{\gamma(H-z)})\sin(i\gamma H)$$

Where. $\gamma = 1.5$ In [8] it is proposed to obtain coefficients from the following considerations. Connects the intensity of the vertical rebound of the soil with its sediment, and the second independent coefficient allows you to determine the intensity of the vertical shear force. The following possible parameter values are also given

$$C_{m1} = \frac{E_0 H^{-1}}{1-2v_0^2}, \quad C_{m2} = \frac{E_0 H}{6(1+v_0)} \qquad (4)$$

Let's move on to the mathematical formulation of the problem. In a rectangular area, we consider a boundary value problem that describes the bending of the plate taking into account the reaction of the elastic base (Fig. 1) [1,4].

$$D(\Delta\Delta w(x_1,x_2,t) - \int_{-\infty}^t R(t-\tau)\Delta\Delta w(x_1,x_2,\tau)d\tau) + $$

$$+ \rho H \frac{\partial^2 w}{\partial t^2} = q(x_1,x_2,t) - p(x_1,x_2,t)$$

where $w(x_1,x_2,t)$-deflection of the plate; $q(x_1,x_2,t)$ - external load; $p(x_1,x_2,t)$ - reaction of the elastic base; $D = E_0 h^2/(12(1-v^2))$ - cylindrical stiffness; l_1, l_2, h - length, width, thickness of the plate; E_0, v - moments young's modulus and Poisson's ratio of the

plate. The el astic base reaction is determined for each model from the corresponding formulas (1), (2) with coefficients (3) or (4). We $p(x_1, x_2) \equiv 0$ obtain the classical equation of plate bending [1].

On the edges of the plate, we will use the known boundary conditions [1]. For $x_1 = 0$ example, when there may be a free edge:

$$\left(\frac{\partial^2 w}{\partial x_1^2} + v\frac{\partial^2 w}{\partial x_2^2}\right) = 0, \qquad \left(\frac{\partial^3 w}{\partial x_1^3} + (2-v)\frac{\partial^3 w}{\partial x_1\partial x_2^2}\right) = Q^f.$$

Special attention should be paid to the size . This function can be interpreted as the influence of the soil outside the plate on its edges [4,8]. Since the Winkler model does not account for this effect, then for her . For two-parameter models, it takes the following form [4]

$$Q^\Phi = C_2\left(\alpha w + \frac{\partial w}{\partial x_1} - \frac{1}{2\alpha}\left(\frac{\partial^2 w}{\partial x_2^2}\right)\right), \quad \alpha = \sqrt{C_{m1}/C_{m2}}.$$

Similarly, you can write conditions on other edges of the plate. Let's cover the Ω area with a rectangular grid uniform in each direction with cells Ω ($i=1,..., N$). To determine the solution in each cell, we will use the domain decomposition method-the method of iterations on subdomains (the alternating Schwartz method), in which the subdomain is a cell. In each cell, a local coordinate system is entered, associated with the source variables by the following formulas $y_1 = (x_1 - x_1^*)/h_1$, $y_2 = (x_2 - x_2^*)/h_2$, where, $2h_1, 2h_2$ - cell dimensions in the direction, x_1, x_2 respectively; (x_1^*, x_2^*) - cell center coordinate. In each cell, we present the approximate solution as a fourth-degree polynomial and write a local system of linear algebraic equations to determine the unknown coefficients. This system includes.

collocation equations

$$D_0\left(\frac{h_2^2}{h_1^2}\frac{\partial^4 w_i^k}{\partial y_1^4} + 2\frac{\partial^4 w_i^k}{\partial y_1^2\partial y_2^2} + \frac{h_1^2}{h_2^2}\frac{\partial^4 w_i^k}{\partial y_2^4} - $$

$$- \int_{-\infty}^t R_i^k(t-\tau)\left(\frac{h_2^2}{h_1^2}\frac{\partial^4 w_i^k(\tau)}{\partial y_1^4} + \frac{\partial^4 w_i^k(\tau)}{\partial y_1^2\partial y_2^2} + \frac{h_1^2}{h_2^2}\frac{\partial^4 w_i^k(\tau)}{\partial y_2^4}\right)d\tau - $$

$$- C_{m2}\left(h_2^2\frac{\partial^2 w_i^k}{\partial y_1^2} + h_1^2\frac{\partial^2 w_i^k}{\partial y_2^2}\right) + $$

$$+ h_2^2 h_1^2 C_{m1} w_i^k + \rho H\frac{\partial^2 w}{\partial t^2} = h_2^2 h_1^2 q,$$

where w_i^k - решение в ячейке Ω_i на k-ой итерации;

terms of agreement

$$w_i^k + \frac{1}{h_n}\frac{\partial w_i^k}{\partial n} = w_j + \frac{1}{h_n}\frac{\partial w_j}{\partial n}$$

$$\frac{\partial^2 w_i^k}{\partial n^2} + \frac{1}{h_n}\frac{\partial^3 w_i^k}{\partial n^3} = \frac{\partial^2 w_j}{\partial n^2} + \frac{1}{h_n}\frac{\partial^3 w_j}{\partial n^3};$$

Where w_j is the solution from the neighboring cell on the k-th iteration if Ω_j "calculated" and $(k-1)$- th

otherwise; n - external normal to the boundary Ω_i;

boundary condition $w_i^k = 0$, $\dfrac{\partial w_i^k}{\partial n} = 0$ in case of pinching.

The local system of linear algebraic equations consists of 9 Integra-differential collocation equations written at the inner points of the cell. Also, at each cell boundary, depending on whether this boundary is adjacent to the boundary of the source region, three matching conditions or three boundary conditions are written. The resulting SLOUGH will be redefined. Its solution will be understood in the sense of least squares.

Numerical results.

Consider a rectangular plate on an elastic base under the action of a uniform dynamic load $q = Q_0 e^{-ipt}$.
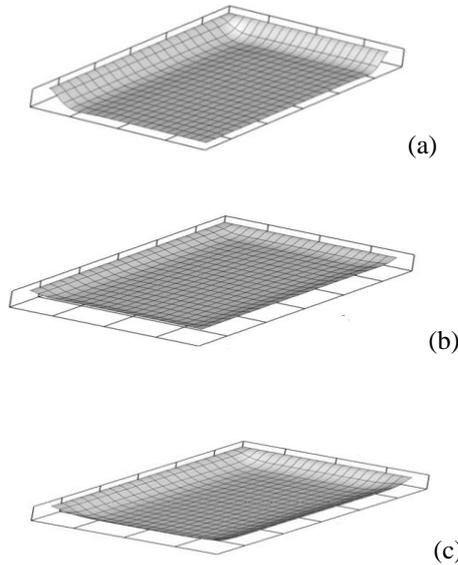
(a)

(b)

(c)

Fig. 2. The shape of the amplitude of a uniformly loaded plate whose two edges are pinched for the Winkler (a), Vlasov (b), and Pasternak (c) models.

Two adjacent sides of the plate are pinched, the other two are free. In the experiment, calculations are given for three models of the base (Fig. 2, 3) for the parameters $l_1 = 2l_2 = 20$ m, $h = 0.1$ m, $H = 2$ m, $E_0 = 200$ GPa, $v = 0.28$, $E_f = 0.4$ GPa, $v_f = 0.4$, $k = 0.3$ GPA/m, $Q_0 = 1$ MPa.
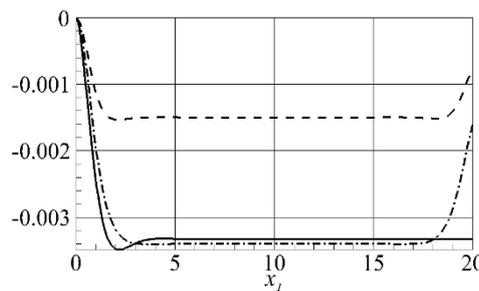
Fig. 3. Section of the amplitude of the deflection of the plate at, two edges of which are pinched, for the models of Winkler (solid), Vlasov (dashed), Pasternak (dashed).

The figures show that for two Q^f-parameter models, taking into account the function on the free edge leads to its lifting, which from the point of view of real experience is more logical than for the case of the Winkler model, when the free edge is deformed without bending.
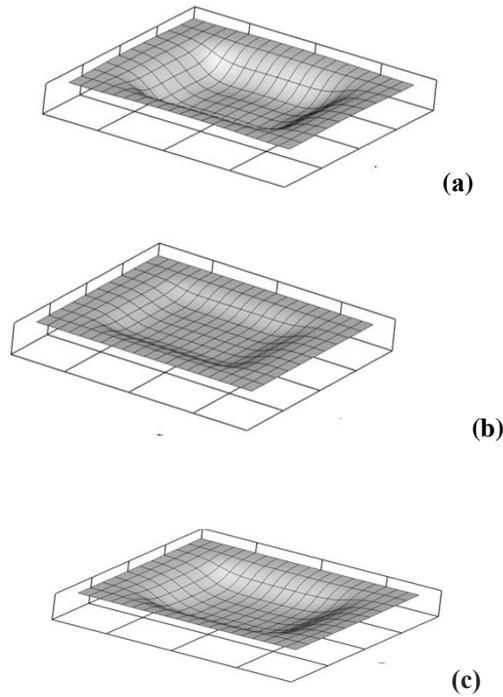
(a)

(b)

(c)

Fig. 4. The shape of the amplitude of the free-lying plate on the ground under a special load for the models of Winkler (a), Vlasov (b), Pasternak (c).
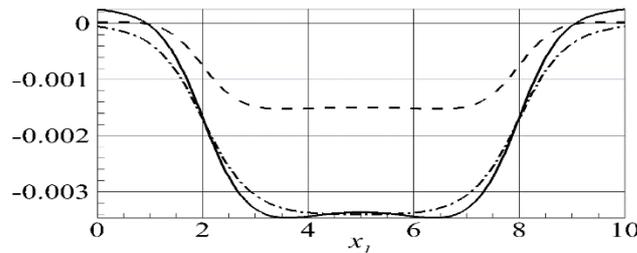

Fig. 5. Section of the amplitudes of the deflection of the free-lying plate on the ground for the models of Winkler (solid), Vlasov (dashed), Pasternak (dashed).

Consider a square plate, free-lying on the ground, simulating, for example, the Foundation of the bridge support. The plate is under the action of a uniform dynamic (harmonic) load applied to the region $[2,8] \times [2,8]$ (Fig.4,5), $l_1 = l_2 = 10 \, \text{m}$, $h = 0.1 \, \text{m}$, $H = 2$ m, $E = 200 \, \text{GPa}$, $v = 0.28$, $E_f = 0.4 \, \text{GPa}$, $v_f = 0.4$,

$k = 0.3$, Q_0=1 MPa.

In this case, the deflection of the plates does not depend qualitatively on the choice of the base reaction model, since the values of deflections on the contour are small.
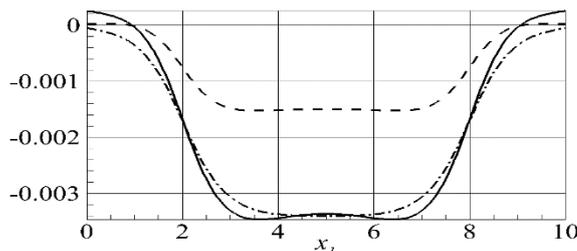

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