

**V.I.ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI  
HUZURIDAGIILMIY DARAJA BERUVCHI  
DSC.02/30.12.2019.FM.86.01RAQAMLI ILMY KENGASH**

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**TOSHKENT DAVLAT TRANSPORT UNIVERSITETI**

**ABDULLAH KURUDIREK**

**TA'LIM JARAYONIDA NOYEVKLID GEOMETRIYASI  
ELEMANTLARIDAN FOYDALANISH METODIKASI**

**13.00.02 – Ta'lim va tarbiya nazariyasi va metodikasi (matematika)**

**PEDAGOGIKA FANLARI BO'YICHA FALSAFA DOKTORI (PhD)  
DISSERTATSIYASI AVTOREFERATI**

**TASHKENT-2022**

**Pedagogika fanlari bo'yicha falsafa doktori (PhD)  
dissertatsiyasi avtoreferati mundarijasi**

**Оглавление автореферата диссертации  
доктора философии (PhD) по педагогическим наукам**

**Content of thesis abstract of doctor of philosophy (PhD) on pedagogical  
sciences**

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Dissertatsiya bilan V.I.Romanovskiy nomidagi Matematika institutining Axborot-resurs markazida tanishish mumkin (\_\_\_\_-raqami bilan ro'yxatga olingan). (Manzil: 100174, Toshkent sh., Olmazor tumani, Unversitet ko'chasi, 9-uy. Tel.: (+99871)-207-91-40.

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## KIRISH (falsafa fanlari doktori (PhD) dissertatsiyasining izohi)

**Dissertatsiya mavzusining dolzarbligi va zarurligi.** Jahonda matematika ta'limining o'sish tendentsiyalari ko'rsatishicha, o'qitishning zamonaviy shakllari va texnologiyalari milliy, mintaqaviy va global miqyoslarda jadal rivojlanmoqda. Xususan, iqtidorli o'quvchilarni yoshligidan tanlash ularni maqsadli o'qitishga qaratilgan AQSH va Yevropa mamlakatlarida keng qo'llanilayotgan "Merit" (o'rta maktabda iqtidorli bolalarni tanlash), "Xed start" (akademik o'zlashtirishga yo'naltirilgan kompensator o'qitish) kabi loyihalar keng qo'llanilmoqda. Maktabda matematikani o'qitish, iqtidorli yoshlarni o'qishga jalb qilish har doim muammo bo'lgan. Bu o'rinda matematikaning bo'linmas qismi bo'lgan geometriyaning alohida o'rni bor. Geometriya fanini o'qitishda faqat hisoblash emas, fazoviy tasavvurni rivojlantirishga alohida e'tibor berish dolzarb ahamiyat kasb etmoqda.

Dunyoda bu loyihalarni qo'llash, iqtidorli yoshlarni ertaroq aniqlash, ularning iqtidorini maqsadli yo'nalishida rivojlanishiga erishish borasidagi tatqiqotlar muhim o'rin egallamoqda. Hozirgi davrda O'zbekistonda ham yoshlar tarbiyasiga katta e'tibor qaratilayotganini e'tirof etish zarur. Bunda respublikada yangi tashkil qilingan prezident maktablari, matematika yo'nalishida o'qituvchi maxsus maktablar, shuningdek, sa'nat va xunarmandchilik sohasiga yo'naltirilgan maktablarni misol sifatida keltirish mumkin. Bu maktablarning zamona talabiga javob beradigan binolar va kompyuter texnologiyalari bilan taminlanganligi davlatimizning yoshlarga yaratgan imkoniyatlarning namunasidir. Ayniqsa, iqtidorli yoshlarni qo'llab-quvvatlash, to'la va yaxshi bilim olishi uchun shart-sharoitlar yaratish, ularning matematika fani yo'nalishida jahon olimpiadalarida faol ishtirok etishini ta'minlash uchun barcha imkoniyatlar yaratilgan. Bu imkoniyatlar yoshlarni zamona talabiga javob beradigan qilib tarbiyalashga xizmat qiladi.

Mamlakatimizda matematikani zamonaviy talablarga mos ravishda o'qitish, nafaqat, yangi o'qitish usullarini joriy qilish, shuningdek, fan tarkibini yangi tushunchalar bilan boyitish, uning o'quvchi uchun qiziqarli bo'lishini ta'minlash borasida tizimli ishlar olib borilyapti. Xususan, ta'lim tizimining maktabgacha ta'lim, boshlang'ich sinflar va yuqori sinflarda fanlarning zamonaviy yutuqlari haqidagi ma'lumotlar uzliksiz va uzviylik bilan ta'lim jarayoniga kiritilib bormoqda. Matematikada konbinatorika elementlari va ehtimollar nazariyasining boshlang'iz tushunchalari kiritilishi ta'lim dasturining jahon standartlari darajasida bo'lishiga qaratilgandir. Bu borada geometriya faniga zamonaviy noyevklid geometriya tushunchalarni kiritish dolzarb ilmiy metodik ishlardan hisoblanadi. O'zbekiston Respublikasini yanada rivojlantirish bo'yicha "Harakatlar strategiyasi"da "Informatika, matematika, kimyo, biologiya" kabi boshqa muhim va talabi yuqori bo'lgan predmetlarni chuqurlashtirilgan tarzda o'rganish<sup>1</sup> ustuvor

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<sup>1</sup>O'zbekiston Respublikasi Vazirlar mahkamasi 2017 yil 18 maydagi «O'zbekiston Respublikasi Fanlar akademiyasining yangidan tashkil etilgan ilmiy tadqiqot muassasalari faoliyatini tashkil etish to'g'risida»gi 292-sonli qarori.

vazifa sifatida belgilangan. Shu nuqtai-nazardan maktabda geometriyani o‘qitishda yevklid bo‘lmagan geometriya tushunchalaridan foydalanish o‘quvchilarni fazoviy tasavurini rivojlantirish ilmiy izlanuvchanlik kompintensiyalarini oshirishda muhim ahamiyat kasb etmoqda.

O‘zbekiston Respublikasi Prezidentining 2017 yil 7 fevraldagi PF-4947-son “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha xarakteristik strategiyasi to‘g‘risida”gi Farmoni, 2019 yil 9 iyuldagi PQ-4387-son “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar Akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi va 2020 yil 7 maydagi PQ-4708-son “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

**Tadqiqotning respublika fan va texnologiyalar rivojlanishining ustuvor yo‘nalishlariga bog‘liqligi.** Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. «Matematika, mexanika va informatika» ustuvor yo‘nalishi doirasida bajarilgan.

**Muammoning o‘rganilayotganlik darajasi.** “Noyevklid geometriya” atamasi XX asr boshlarida paydo bo‘ldi va juda faol rivojlandi. Bu atamani paydo bo‘lishi o‘sha davrda va hozirda ham maktabda o‘qitiladigan geometriyani “yevklid geometriyasi” deb atalishiga sabab bo‘ldi. “Noyevklid geometriya” yo‘nalishining rivoji va undagi asosiy muammolar B.A.Rozenfeldning noyevklid geometriyaga bag‘ishlangan monografiyalarda to‘la keltirilgan.

O‘tgan asrning 60-yillarigacha noyevklid geometriya tushunchasi, asosan, ilmiy izlanishlarda yoki faqat oliy ta’lim tizimida ishlatilgan. Bunga sabab noyevklid geometriyasi tushunchasining asosan aniq fanlarning murakkab masalalarini yechishda amaliy ahamiyatga ega bo‘lganligidir. Ammo 1969-yillardan keyin noyevklid geometriyasining tekislikdagi holi o‘zining sodda va elementar matematikaga yaqin bayoniga ega bo‘ldi. Bu o‘z navbatida noyevklid geometriyaga doir ilmiy ommabop manbalarning paydo bo‘lishiga sabab bo‘ldi. Shuning bilan vaqtda maktab o‘quvchilari va talabalar uchun mo‘ljallangan I.M. Yaglomning “Galileyning nisbiylik nazariyasi va noyevklid geometriya” deb nomlangan kitobi chop etildi (1969). Bu kitob tekislikda mavjud bo‘lgan proyektiv metrikalik geometriyalar bayoniga bag‘ishlangan edi. Shundan so‘ng asrning 75-yillarga kelib, maktab o‘quvchilari uchun mo‘ljallangan o‘zbek tilidagi kitoblar paydo bo‘lgan. Bunga misol sifatida Naim G‘aybullayevning 1975-yilda chiqqan “Maktabda noyevklid geometriya elementlari” kitobini keltirish mumkin. Bu kitobda asosan elliptik va giperbolik ya’ni Labachevskiy geometriyasining aksiomatik qurulishi hamda asosiy tushunchalari berilgan.

V. Tyorston tamonidan uch o‘lchovli ko‘pxillar geometriyasi o‘rganilishi va barcha noyevklid geometriyalarni qamrab olishga qodir bo‘lgan geometriya fanining yangicha talqini umuman bu fanga, xususan tekislikdagi noyevklid

geometriyalarga yangicha nuqtai-nazardan qarashni paydo etdi. Shuning bilan noyevklid geometriyalarning aksiomatik qurilishi bilan bog'liq bo'lgan bir qator muommolarni xal qilinishiga olib keldi. Shu sababdan XX asr oxiri va XXI asr boshlaridan tekislikdagi noyevklid geometriya va uni sodda usulda o'quvchilarga o'rgatishga mos keladigan ko'plab adabiyotlar paydo bo'ldi. Ulardan, E.F.Krause (1986), A.B. Khachaturian (2005), A.Artikbayev, I. Hamdamovlarning "Tekislikda to'qqizta geometriya" (2021) kabi ommabop adabiyotlar tekislikdagi geometriyalarga bag'ishlangan.

**Tadqiqotning dissertatsiya bajarilgan oliy ta'lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog'liqligi.** Dissertatsiya tadqiqoti Toshkent davlat universiteti ilmiy tadqiqot ishlari rejasining "Funksiyalar nazariyasi, funksional analiz, differensial tenglamalar va ularning tadbqiqiga oid masalalarni tadqiq qilish" mavzusidagi ilmiy-amaliy loyihasi doirasida bajarildi.

**Tadqiqotning maqsadi** umumiy o'rta ta'lim maktablarida geometriya faniga noyevklid geometriyalari tushunchalarini kiritish yo'li bilan o'quvchilarni fanga qiziqishini orttirish.

**Tadqiqotning vazifalari:**

tekislikda mavjud noyevklid geometriyalarini o'rganish;  
noyevklid geometriyalarda maktab geometriya fanining asosiy tushunchalariga yaqin mavzularni aniqlash;

Yevklid va noyevklid geometriyalarining asosiy tushunchalarni taqqoslash;  
iqtidorli o'quvchilarni aniqlash, o'quvchilarni fanga qiziqtirishning noyevklid geometriyasi vositasida takomillashtirish usullarini ishlab chiqarish.

**Tadqiqotning obyekti** sifatida o'rta ta'lim maktablari geometriya fani o'qitiladigan yuqori sinf o'quvchilari.

**Tadqiqot predmeti** umumiy o'rta ta'lim maktablarida Evklid bo'lmagan geometriyalar orqali matematikani takomillashtirish mazmuni, shakl va medotlari.

**Tadqiqot usullari.** Adabiyotlar, darsliklar va o'quv qo'llanmalarni, maktab darsliklari va noyevklid geometriyasiga doir ilmiy maqolalardagi o'zgarishlarni tahlil qilish, darsdan tashqari to'garaklar tashkil etish, munozara guruhlar yoki guruh muhokamalari orqali tanlangan talabalar guruhi mavzuni muhokama qilish, tajriba-sinov ishlarini o'tkazish va ma'lumotlarni tizimlashtirish kabi metodlardan foydalanilgan.

**Tadqiqotning ilmiy yangiligi** quyidagilardan iborat:

tekislikda mavjud sodda noyevklid geometriyasiga doir tushunchalar maktab geometriya darsligi usullariga mos ravishda bayon etilgan hamda masofa va metrika atamalari orasidagi umumiylik va bir-biridan farqlilik sodda misollar bilan tushuntirilgan;

Galiley geometriyasidagi siklning Yevklid geometriyasidagi aylana urinmasi va kesuvchisi bilan bog'liq xossalari ko'rsatilgan;

uchburchak tengligiga doir birinchi va ikkinchi alomatning o'rinli hamda uchinchi alomatning bajarilmasligi yordamida o'quvchilarga teorema isbotini o'rgatish usuli takomillashtirilgan;

noyevklid geometriya elementlaridan foydalanib, o'quvchilarni matematikaga qiziqtirish usuli ishlab chiqilgan.

**Tadqiqotning amaliy natijasi** quyidagilarda o'z aksini topgan:

noyevklid geometriya elementlaridan foydalanib, iqtidorli talabalarni aniqlash usullari takomillashtirildi;

noyevklid geometriyasiga doir tushunchalarni o'rganish, o'quvchilarga yevklid geometriyasiga doir xossalarni chuqurroq tahlil qilish odatini hosil qilishga imkon berildi;

o'quvchilarni ilmiy-izlanish qilishga jalb qiladigan, darsdan tashqari o'qitiladigan loyihalar, to'garaklar tayyorlash usullari takomillashtirildi.

**Tadqiqot natijalarining ishonchliligi.** Tadqiqot muammosining ijobiy natijasini topishda ilg'or pedagogik, psixologik tajribalar, matematikaning zamonaviy usullari, metodikaning asoslangan qonunlariga tayanganligi va tadqiqot natijalarini matematik-statistik qayta ishlash va baholash bilan izohlanadi.

**Tadqiqot natijalarining ilmiy va amaliy ahamiyati.** Tadqiqotning ilmiy ahamiyati tekislikda Galiley geometriyasiga oid elementar geometrik xossalari isbot qilingan va bu xossalarning yevklid geometriyasi uchun ham ahamiyati mavjudligi ko'rsatilgan. Bu noyevklid geometriyasini o'rganish, yevklid geometriyasiga ham yangicha qarash yo'lini ko'rsatishi bilan izohlanadi.

Tadqiqotning amaliy ahamiyati noyevklid geometriyasi elementlari bilan tanishish o'quvchilarda fanga qiziqishni, ilmiy mulohaza qilish ko'nikmalarini paydo qilishi bilan amaliy ahamiyatga egaligi bilan izohlanadi.

**Tadqiqot natijalarini joriy qilinishi.** Ta'lim jarayonida noyevklid geometriyasi elementlaridan foydalanish metodikasi bo'yicha olingan natijalar asosida:

tekislikda mavjud sodda noyevklid geometriyasiga doir tushunchalar hamda masofa va metrika atamalari orasidagi umumiylik va bir-biridan farqlilikka oid sodda misollar Iraqning Sulaymaniya shaxridagi xalqaro kembrij tipidagi maktabda (<https://www.facebook.com/USGirls.H>) o'tkazilgan to'garaklarda o'quvchilarning tekislikda mavjud noyevklid geometriyalar haqidagi bilimlarni shakllantirishda foydalanilgan (Sulaymaniya shaxridagi maxsus qizlar o'rta maktabining 2021 yil 4 yanvardagi №02-sonli ma'lumotnomasi, Iraq respublikasi). Natijada 9-sinf o'quvchilarida noyevklid geometriyalarning maktab geometriya fanidagi asosiy tushunchalariga yaqin mavzularni aniqlashni imkonini bergan;

Galiley geometriyasidagi siklning Yevklid geometriyasidagi aylana urinmasi va kesuvchisi bilan bog'liq xossalari va Taxicab geometriyasiga oid tushunchalardan 2013-2015 yillarda ta'lim sohasida olib borilgan tadqiqot loyihalarida va maxsus ixtisoslashgan maktablarda o'tkazilgan "noyevklid geometriyasi" mavzusida tugaraklarda foydalanilgan (Tishk xalqaro universitetining 2022 yil 3 yanvardagi №58-sonli ma'lumotnomasi, Iraq respublikasi). Natijada yuqori sinf o'quvchilarida noyevklid geometriyalar haqida tasavvurini paydo qilish va bu geometriyani nazariyada va amaliyotda solishtirish imkonini bergan.

**Tadqiqot natijalarining aprobatsiyasi.** Mazkur tadqiqot natijalari 5 ta xalqaro va 2 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o'tkazilgan.

**Tadqiqot natijalarining e'lon qilinganligi.** Dissertatsiya mavzusi bo'yicha jami 22 ta ilmiy ish chop etilgan, shulardan, O'zbekiston Respublikasi Oliy attestatsiya komissiyasining doktorlik dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 10 maqola, shundan 2 tasi respublika va 8 tasi xorijiy jurnallarda chop etilgan.

**Dissertatsiyaning tuzulishi va hajmi.** Dissertatsiya kirish va 4 bobdan, xulosa, foydalanilgan adabiyotlar ro'yxatidan iborat bo'lib, xajmi 76 betni tashkil etadi.

## DISSERTATSIYANING ASOSIY MAZMUNI

**Kirish** qismida mavzuning dolzarbligi, muammoning o'rganilganlik darajasi, ishning maqsadi va vazifalari, obykti, predmeti, tadqiqot usullari keltirilgan. Dissertatsiyaning ilmiy yangiligi, amaliy natijalari, ularning ishonchligi, ishning ilmiy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy etilishi haqida ma'lumot berilgan.

Dissertatsiyaning "**Tekislikda noyevklid geometriyalar**" deb nomlangan birinchi bobida Geometriyaning zamonaviy ta'rifi, Minkovskiy geometriyasi, Galiley geometriyasi hamda chekli va taksikab geometriyasining asosiy tushunchalari keltirilgan bo'lib, bu tushunchalar maktab o'quvchilariga tushunarli darajada bayon etilgan. Shuningdek, keyingi boblarda zarur bo'lgan tushunchalar ta'rifi va ularning ba'zi xossalari referat shaklida berilgan.

Dissertatsiyaning "**Noyevklid va yevklid geometriyasi asosiy tushunchalarini taqqoslash**" deb nomlangan ikkinchi bobi maktab o'quvchilarini noyevklid geometriya tushunchalari bilan tanishtirish va ularni yevklid geometriyasidagi mos tushunchalar bilan solishtirish imkoni bo'lgan mavzularga bag'ishlangan. Bu bobda keltirilgan mavzular asosida maktabda qo'shimcha darslar yoki to'garaklar tashkil etish mumkin. Bu ishlarni qanday amalga oshirish usullari keltirilgan.

Ikkinchi bobning birinchi paragrafidametrika va masofa tushunchalarining ta'riflari keltirilib, ularning bir-biriga o'xshash va o'xshash bo'lmagan tomonlari misol bilan tushuntirilgan.

Masofa ikki nuqta funksiyasi sifatida turli qiymatlar qabul qilishi mumkinligi Minkovskiy geometriyasidagi masofa yordamida misollar bilan tushuntirilgan. Galiley geometriyasidagi masofaning geometrik ma'nosi tushuntirilgan.

Dissertatsiyaning Ikkinchi bobning ikkinchi paragrafida yevklid tekisligidagi harakat, parallel ko'chirish va burishdan iborat ekanligi va bu harakatda bir-biriga o'tadigan shakllar isometrik shakllar deb atalishi aytilgan.

Koordinatalari boshini saqlagan holda koordinatalar o'qini burish alohida keltirilgan. Koordinatalarni burishda tekislikdagi aylana o'zi-o'ziga o'tadi. Bu xossa noyevklid tekisliklarda ham o'rinli ekanligi Minkovskiy geometriyasi va

Galiley geometriyasi koordinatalarini burish formulalarida tushuntirilgan. Galiley geometriyasidagi ikkinchi xil aylana “sikl” tushunchasi berilgan.

Yevklid va noyevklid geometriyalarida burchak tushunchasi bir xil ta’riflanadi. Shuningdek, burchak kattaligini aniqlash usuli ham bir-biriga o’xshash. Bu masalalarni yevklid va noyevklid geometriyalarida aniqlash o’quvchilar uchun juda qiziqarli mashg’ulot bo’ladi. Shuning uchun, ushbu bobning uchinchi paragrafida burchak va uni o’lchash usullari Galeliy va Minkovski geometriyalarida xuddi yevklid geometriyasiga “o’xshash” ekanligi ko’rsatilgan.

Haqiqatan ham, galiley geometriyasida ham burchak kattaligi xuddi yevklid geometriyasidagidek markazi burchak uchida bo’lgan aylana yoyi uzunligiga teng qilib aniqlanadi.

Dissertatsiyaning uchinchi bobi **“Noyevklid geometriyalari tushunchasidan o’quv jarayonida foydalanish”** deb nomlangan.

Uchinchi bobning birinchi paragrafida uchburchaklarning tenglik alomatlariga doir tushunchalar berilgan. Ma’lumki, uchburchak tengligining uchta alomati yevklid geometriyasida asosiy, to’la isboti bilan beriladigan, bo’limni tashkil etadi.

Galiley geometriyasida masofa va burchak tushunchalari yevklid geometriyasidan boshqacha bo’lgani sababli, uchburchakning uchta tomoni bo’yicha tengligi alomati bajarilmaydi. Uchta tomonlari teng, ammo o’zaro teng bo’lmagan uchburchaklarni chizmada ko’rsatishimiz mumkin. Bu o’quvchilarda galiley geometriyasiga qiziqishni orttiradi.

Shu bilan bir vaqtda, uchburchak medianasi, bissektrisasi, balandligi tushunchalarini galiley geometriyasida ta’riflaymiz.

Affin tushuncha bo’lgani uchun, mediana bu geometriyalarda bir xil ta’riflanadi va bir xil xossaga ega.

Burchak galiley geometriyasida boshqacha bo’lganidan, bissektrisa ham boshqa geometrik shaklda bo’ladi.

Galiley geometriyasida balandlik tushunchasini ta’riflashga alohida e’tibor qaratishi zarur. Bu tushunchaning ta’rifi yevklid tekisligidagi balandlik xossasiga suyangan bo’lishi kerak. Bunda nuqtadan to’g’ri chiziqqacha masofa tushunchasi katta ahamiyatga ega.

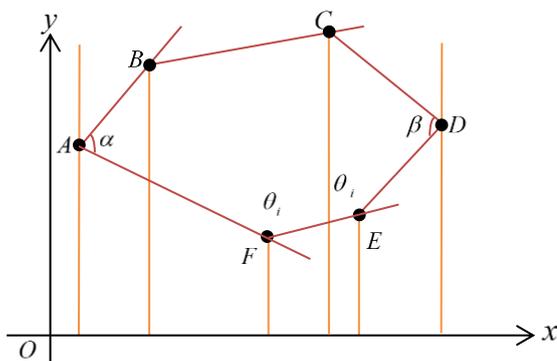
Yevklid va Galiley geometriyalari orasidagi bog’lanish, hamda bir-biridan farq qilishi o’quvchilarda har doim qiziqish uyg’otadi.

Uchinchi bobning ikkinchi paragrafida Galiley tekisligida ko’pburchaklar tushunchalarini umumlashtirishga harakat qilingan. Galiley geometriyasida parallelogramning ta’rifini yevklid geometriyasidagi kabi olsa bo’ladi. Ammo to’g’ri to’rtburchak kvadrat kabi xususiy hollar mavjud bo’lmaydi. Faqat parallelogramning qarama-qarshi uchlari maxsus to’g’ri chiziqqa tegishli bo’lganda, uni romb deb atash mumkin bo’ladi.

Qavariq ko’pburchaklarning ichki burchaklarini o’rganish, qiziqarli natijalarga olib keladi. Xususan, tekislikdagi qavariq ko’pburchaklar uchun ushbu teorema isbot qilingan.

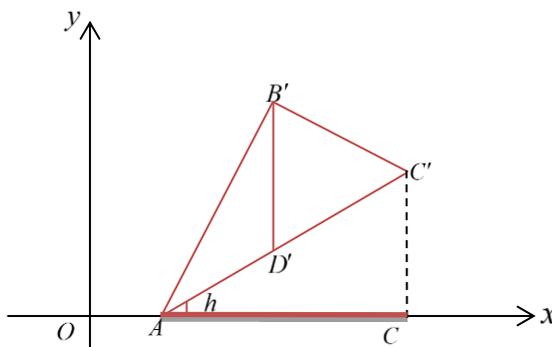
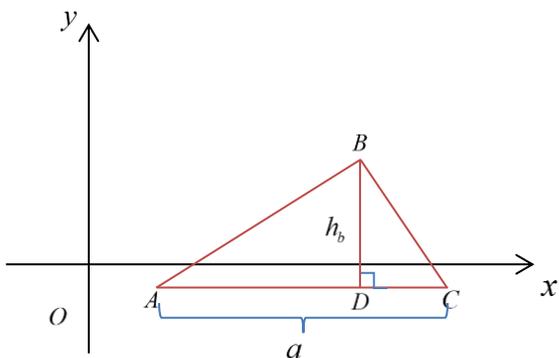
**1-teorema.** Qavariq ko'pburchaklarning ichki burchaklari yig'indisi, tayanch(asos) burchaklari yig'indisiga teng:  $\alpha + \beta = \sum_{i=1}^{n-2} \theta_i$ .

Bu teorema berilgan uchburchak ichki burchaklari yig'indisi tushunchasini ko'pburchak uchun umumlashgani hisoblanadi. O'quvchilarda juda katta qiziqish paydo bo'ladigan va o'zlari chizmalarda o'lchab ko'rib, aniq hisoblash yo'li bilan, haqiqatan ham, tengligiga ishonadigan tushuncha galiley geometrasidagi yuza tushunchasi.

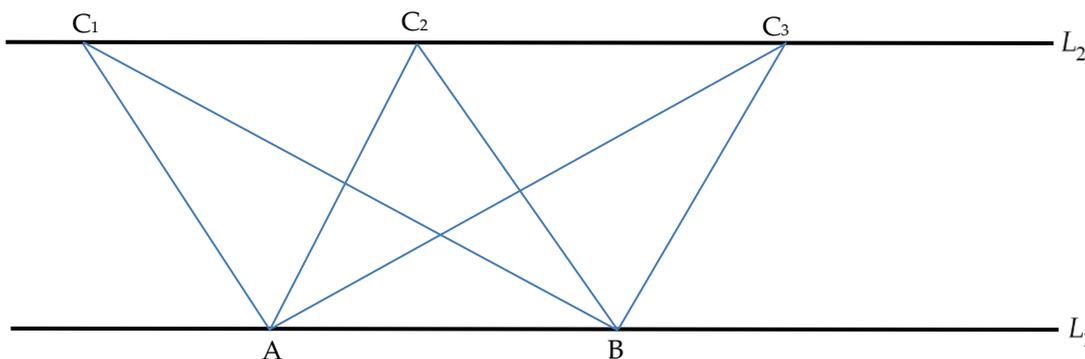


Shakl: Ko'pburchak

**2-teorema.** Uchburchakning yuzi uning asosi va balandligi uzunliklari ko'paytmasining yarmiga teng:  $S = \frac{1}{2} \cdot a \cdot h_b$ ,  $S_{ABC} = \frac{1}{2} AC \cdot BD = \frac{1}{2} AC' \cdot B'D'$



Bu formulaning geometrik jihatdan qiziqarli bo'lishini quyidagi shakldan ko'rish mumkin.

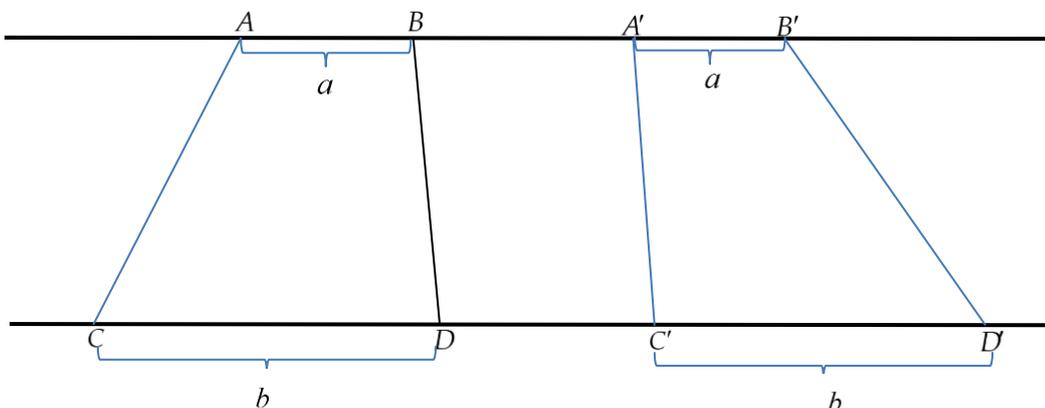


Bunda  $L_1 // L_2$  bolganda  $S_{ABC_1} = S_{ABC_2} = S_{ABC_3}$

Bu yarda ham  $S_{ABDC} = S_{A'B'D'C'}$

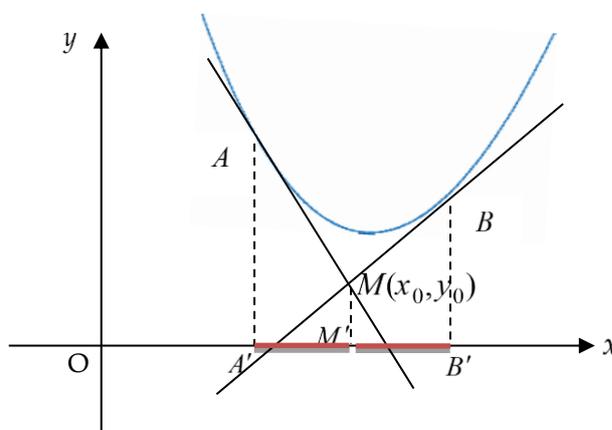
Yuza tushunchasidan foydalanib, Galiley harakati ta'sirida uchburchak (teorema 4) va trapetsiyaning (teorema 5) lar isbotlangan.

Uchinchi bobning uchinchi paragrafi galiley tekisligida sikl xossasiga bag'ishlangan. Sikl uchun quyidagi xossalar isbot qilingan.



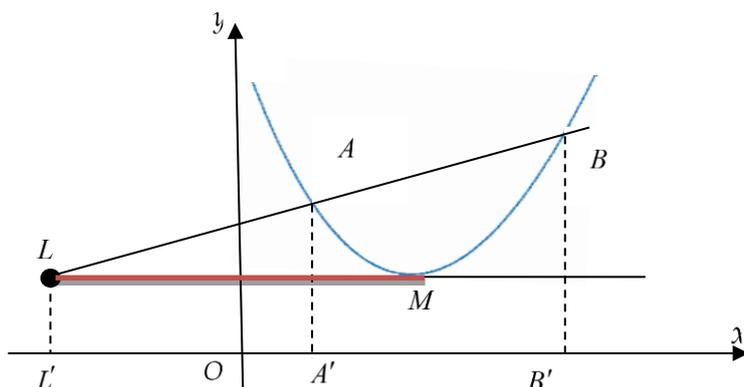
**1-xossa.** Sikldan tashqaridagi nuqtadan siklga ikkita urinma o'tkazish mumkin.

**2-xossa.** Siklga undan tashqarida yotgan nuqtadan urinmalar o'tkazilgan bo'lsa, u nuqtadan urinish nuqtalarigacha bo'lgan masofalar teng.



$$A'M' = M'B'$$

**3-xossa.** Sikldan tashqarida yotgan nuqta berilgan bo'lib, bu nuqtadan siklni A, B nuqtalarda kesuvchi to'g'ri chiziq o'tkazilgan bo'lsa,  $LA \cdot LB$  – ko'paytma faqat L nuqtaga bog'liq.



$$LA \cdot LB = LM^2$$

Galiley tekisligidagi sikl, yevklid tekisligidagi parabola bo'lgani sababli, yuqorida keltirilgan xossalar yevklid parabolasi uchun ham o'rinlidir.

Uchinchi bobning to'rtinchi paragrafda noyevklid geometriyasiga doir tushunchalardan foydalanib, matematik o'yin, proektlar tashkil qilish usuli berilgan. Bu usullarni biz taksikab geometriyasi tushunchalari yordamida berdik. Chunki, bu geometriyaning hayotiy ma'nosini o'quvchilar yaxshi tushunadilar. Biz keltirgan boshqa noyevklid geometriyalar uchun ham shu usulda matematik o'yinlar va praektlar tashkil etsa bo'ladi.

Bu matematik o'yinlar va proektlarni tuzishdan maqsad, noyevklid geometriyasining elementlarini maktab dasturiga kiritishga sabab bo'ladigan asoslarni yaratishdir.

Proekt oddiy moduldan iborat bo'lib, biz uni beshta qadamda amalga oshirishni taklif etdik.

Birinchi qadam geometriyaga doir asosiy tushunchalar bilan tanishtirish. Bu tushunchalar birinchi bobda har xil geometriyalar uchun berilgan.

Ikkinchi qadam masofa tushunchasini aniq hayotiy misollar bilan tushuntirish olish. Bunda biz taksikab geometriyasidagi masofa tushunchasini xonaning o'zida o'quvchilar ishtirokida qanday bajarish mumkin ekanligini amalga oshirish usulini berganmiz.

Uchinchi qadamda masofa va masofa yordamida aylana formulasini keltirib chiqarish usuliga bag'ishlanishi rejalashtirilgan.

To'rtinchi qadam, o'rganilayotgan noyevklid geometriyada to'g'ri chiziq, burchak, doira va uchburchak shakllarini hosil qilish, tassavur qilish va amalda ko'rsatish.

Beshinchi qadam, uchburchaklar tengligi va uchburchakka doir asosiy tushunchalarni tasovvur qilish. Noyevklid geometriyasidagi uchburchak va elementlarini yevklid holi.

Bu besh qadamda o'quvchilar noyevklid geometriyaga doir asosiy tushunchalar haqida yetarli darajada ma'lumotga ega bo'ladi.

O'rganish jarayonida bo'lishi mumkin bo'lgan murakkabliklar qanday bartaraf qilish usullari va modulni yana qanday davom ettirish imkoni paragraf oxirida berilgan.

Dissertatsiyaning to'rtinchi bobi "**Tadqiqot muhokamasi va xulosalar**" deb nomlangan. Bu bobning birinchi paragrafida "noyevklid geometriya" tushunchasi oid ba'zi natijalar keltirilgan. Ya'ni, bu paragrafda avvalo talabalarga Labochevskiy geometriyasini tushuntirishga bag'ishlangan.

Talabalar bilan galiley yoki taksikab geometriyasi bo'yicha ishlash juda qiziqarli bo'ldi. Bu geometriyadagi faktlarni evklit geometriyasidan tubdan farq qilishi, ammo o'quvchilar uchun tushunarli ekanligi, bu yo'nalishda olib borgan mashhulotlarimizga biz kutgandan ham ko'proq talabalar kelishiga sabab bo'ldi.

Tadqiqotlarni Qurdistonda olib bordik. Bu yerdagi maktablarning xilma-xilligi bizga o'qish jarayonida qo'shimcha "Noyevklid geometriya" bo'yicha mashg'ulotlar o'tkazish imkonini berdi.

Birinchi taklif Sulaymonidagi xalqaro kembrij tipidagi maktabdan bo'ldi, keyingi taklif qizlar uchun o'rta maktabdan bo'ldi. Bu maktablarda biz, asosan, taxsikab geometriyasiga doir uch oylik kurslarni o'tdik. "Birodar maktablar" proyeksi asosida bizni qilgan ishlarimiz boshqa maktablarga ham tarqaldi. Masalan, bu ishlarni Erbil shahridagi birodar maktablarimizdan biri bo'lgan Erbil o'g'il bolalar maktabida olib bordi.

Shuningdek, "Noevklid geometriya haqida suhbatlar" nomli mening o'quvchilarim tomonidan qilingan tadqiqotlar, yuzlab o'quvchilarni bu g'oyalardan xabardor qildi.

O'z ish faoliyatimda "Kengroq o'yla, yoshlikdan boshla, hoziroq boshla" degan so'zlarni qanchalar to'g'ri ekanligiga ishonдим. Bu so'zlar Iroq maktablarida o'quvchilar uchun shior sifatida ishlatiladi.

Faoliyatim davomida o'quvchilarni haqiqatan ham, geometriyaga qiziqтира oldim deb hisoblayman. Chunki ular o'rganib qolmay, o'z tengdoshlari o'rganishi uchun har xil taqdimotlar qila boshlashdi.

O'ylaymanki, "noevklid geometriya" tushunchasini maktab o'quv rejasiga kiritilishi nafaqat o'quvchilarni matematikaga qiziqishini kuchaytiradi, u hattoki o'qituvchilardan ham o'z kasbiy qobiliyatlarini kuchaytirishni talab qiladi.

To'rtinchi bobning ikkinchi paragrafi "Tadbirlar va musobaqalar" deb nomlangan bo'lib, paragrafda tadbir o'tkazish tartibi va unga zarur bo'lgan ko'rgazmali manbalar haqida fikr yuritilgan.

Biz 2014-yil Iroq milliy olimpiadasida qatnashdik va birinchi o'rinni egalladik. Albatta, bu olimpiadada qatnashish uchun avvalgi tanlovlarda qatnashish va g'olib chiqish zarur.

Bu g'oliblik bizga AQSHda o'tkazilgan "INTEL ISEF" – Los-Anjelesdagi loyihada qatnashish imkonini berdi.

### **Tadqiqotlarimiz natijasida quyidagi xulosalarga keldik:**

Ko'rsatish, asosiy tushunchalarni berish yo'li bilan o'rta maktabda noevklid geometriyasini o'rgatish mumkin;

Noyevklid geometriyasiga qiziquvchi o'quvchilarni rag'batlantiring, chunki bu ularning umuman fikrlashini va dunyoqarashini kengaytiradi;

Noyevklid geometriyani, xuddi an'anaviy yevklid geometriyasini tushuntirishning og'ir usulida emas, yangicha qiziqarli usulda berilishi zarur.

To'rtinchi bobning uchinchi paragrafidatavsiyalar va xulosalar keltirilgan. Asosiy tavsiyalardan biri, o'qituvchi maktabga geometriya fanini yaxshi o'qitishi uchun, uning o'zi noevklid geometriya tushunchalaridan xabardor bo'lishi zarur. Bu o'qituvchiga geometriya fanini o'qitishda o'ziga ishonch hosil qilish imkonini beradi.

Talabalarni kreativlik, erkinlik va o'ziga xoslik yo'qligi uchun ayblash kerak emas. Ularga bu xossalarni rivojlantirishga olib keladigan manbalarni bera olish kerak.

Bu borada noevklid geometriya elementlaridan xuddi "yangicha qarash" usuli sifatida foydalanish zarur. Olib borilgan ilmiy tadqiqotlarga asoslanib quyidagilarni tavsiya qilamiz:

– matematikaga ixtisoslashgan oʻrta maktablarda noyevklid geometriya fanidan sinov kurslari tashkil etish zarur;

– noyevklid geometriyasini mazmuni va pedagogik bayon usullariga oʻqituvchilarni tayyorlash kerak;

– avvalo, oʻqituvchilarni, keyin esa qiziquvchi oʻquvchilarni noyevklid geometriyasi bilan tanishish imkonlarini yaratish zarur;

– matematika oʻqituvchisi sifatida biz oʻquvchilarimizda noyevklid geometriyasining bu olamni oʻrganishdagi oʻrni va yangicha qarash usuli ekanligini koʻrsatishimiz kerak.

Oʻylaymanki, yoshlarning noyevklid geometriyasi tushunchalari bilan tanishishi, yevklid geometriyani yaxshiroq anglashiga va umuman, ularning matematikadan kompetensiyasini rivojlanishiga xizmat qiladi.

Shuningdek, noyevklid geometriyasi bilan ixtiyoriy darajadagi oʻquvchini tanishtirish, oʻquvchida yangi ixtirolarni ochishga sabab boʻlishi mumkin.

Bizning sikliga oid natijalarimiz maktabga geometriya kursi programmasida oʻziga mos qabul qilinishi lozim deb hisoblayman.

## **XULOSA**

Mazkur dissertatsiya umumiy oʻrta taʼlim maktablarida geometriya faniga noyevklid geometriyalari tushunchalarini kiritish yoʻli bilan oʻquvchilarni fanga qiziqishini orttirish bagʻishlangan.

Tadqiqot ishining asosiy natija va xulosalari quyidagilar:

1. Masofa va metrika atamalarini oʻrganish oʻquvchilarda umuman atamalarga ilmiy yondashuvni hosil qilishga olib keldi.
2. Burchak yuza tushunchalari oʻlchovlar nazariyasining xilma-xilligi kompetensiyalarini rivojlantirdi.
3. Noyevklid geometriyalarda uchburchak, aylana kabi tushunchalarni aniqlash aniq fanlardagi umumiylik va xilma-xillikni oʻrganish imkoniyatini berdi.
4. Noyevklid geometriyasini oʻrganish yevklid geometriyasiga ham oʻquvchilarda eʼtiborlik boʻlishini talab qiladi.
5. Noyevklid geometriyalardan foydalanish uchun maktab oʻqituvchilarini malakasini oshirish va qayta tayyorlash talab etiladi.

**SCIENTIFIC COUNCIL AWARDING OF THE SCIENTIFIC DEGREES  
DSc.02/30.12.2019.FM.86.01 INSTITUTE OF MATHEMATICS NAMED  
AFTER V.I. ROMANOVSKIY**

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**TASHKENT STATE TRANSPORT UNIVERSITY**

**ABDULLAH KURUDIREK**

**METHODS OF USING NON-EUCLIDEAN GEOMETRY CONCEPTS IN  
THE EDUCATIONAL PROCESS**

**13.00.02 – The methodology of teaching and upbringing (Mathematics)**

**ABSTRACT OF THESIS OF THE DOCTOR OF PHILOSOPHY (PhD)  
ON PEDAGOGICAL SCIENCES**

**TASHKENT-2022**

Thesis has been prepared at Tashkent State Transport University.

The abstract of the thesis is posted in three languages (Uzbek, English, Russian (summary)) on the website <http://kengash.mathinst.uz> and on the website of “ZiyoNet” Information and educational portal <http://www.ziynet.uz/>.

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**Leading organization:** **National University of Uzbekistan**

Defense will take place “\_\_\_” \_\_\_\_\_ 2022 at \_\_\_\_\_ at the meeting of Scientific Council number DSc.02/30.12.2019.FM.86.01 at Institute of Mathematics named after V.I. Romanovskiy. (Address: University str. 9, Almazar district, Tashkent city, 100174, Uzbekistan, Ph.: (99871) 207-91-40, e-mail: [uzbmath@umail.uz](mailto:uzbmath@umail.uz), Website: [www.mathinst.uz](http://www.mathinst.uz)).

Thesis is possible to review in Information-resource center at Institute of Mathematics named after V.I. Romanovskiy (is registered № \_\_\_\_\_). (Address: University str. 9, Almazar district, Tashkent city, 100174, Uzbekistan, Ph.: (99871)-207-91-40).

Abstract of the thesis sent out on «\_\_\_» \_\_\_\_\_ 2022 year.  
(Mailing report № \_\_\_ on «\_\_\_» \_\_\_\_\_ 2022 year).

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## INTRODUCTION (abstract of the doctoral (PhD) dissertation)

**Relevance and necessity of the dissertation.** Growing trends in mathematics education worldwide show that modern forms and technologies of teaching are developing rapidly at the national, regional, and global levels. Some example projects such as Merit (selection of gifted children in high school) and Head Start, widely used in the United States and Europe to select gifted students from an early age, are widely used. It has always been difficult to attract talented young people by teaching mathematics at school. In this case, geometry, which is an integral part of mathematics, has a special role. In the teaching of the science of geometry, special attention should be paid not only to calculation but also to the development of spatial imagination.

Research on the application of these projects globally, early identification of talented young people, and developing their talents in the targeted direction is essential. It should be noted that in Uzbekistan, a lot of attention is paid to the education of young people. For examples include the newly established presidential schools in the country, special schools for teachers of mathematics, as well as schools focused on arts and crafts. The fact that these schools are equipped with modern buildings and computer technology is an example of the opportunities created by our state for young people. There are all the conditions to support especially talented young people to gain in-depth knowledge and ensure their active participation in the World Mathematical Olympiads. These opportunities serve to educate young people to meet modern requirements.

In our country, systematic work is being carried out to teach mathematics under modern requirements to introduce new teaching methods and enrich the content of science with new concepts to make it interesting for students. Information on the current achievements of science in the education system in preschool, primary and upper grades is being continuously and continuously integrated into the educational process. The introduction of combinatorial elements in mathematics and the initial concepts of probability theory aim to ensure that the curriculum is at the level of world standards. In this regard, the introduction of modern non-Euclidean geometry concepts in the science of geometry is one of the current scientific and methodological works. The Action Strategy for the Further Development of the Republic of Uzbekistan identifies in-depth study of other essential and demanding subjects, such as computer science, mathematics, chemistry, and biology, as a priority. In this regard, non-Euclidean geometry concepts in the teaching of geometry in schools play an essential role in developing students' spatial imagination and increasing the competencies of scientific inquiry.

The subject and object of research of this dissertation are in line with tasks identified in the Decrees and Resolutions of the President of the Republic of Uzbekistan of February 7, 2017, PF-4947, «On the strategy of action for the further development of the Republic of Uzbekistan», PQ-4387 dated July 9, 2019

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<sup>1</sup>Decree of the President of the Republic of Uzbekistan dated February 7, 2017 No PF-4947 "On the Action Strategy for further development of the Republic of Uzbekistan". // Collection of Legislation of the Republic of Uzbekistan, 2017, No. 6, Article 70.

«On state support for the further development of mathematics education and science, as well as measures to radically improve the activities of the Institute of Mathematics named after V.I. Romanovskiy of the Academy of Sciences of the Republic of Uzbekistan», PQ-4708 of May 7, 2020 «On measures to improve the quality of education and research in the field of mathematics» as well as in other regulations related to basic sciences.

**The relevance of the research to the priorities of the development of science and technology in the country.** This research was performed as part of the priority in developing science and technology of the republic IV. "Mathematics, Mechanics and Informatics."

**The degree to which the problem is being studied.** The term "non-Euclidean geometry" appeared in the early twentieth century and developed very actively. The origin of the term led to the fact that the geometry taught in schools at that time and today is called Euclidean geometry. The development of non-Euclidean geometry and its main problems are described in detail in B.A. Rosenfeld's monographs on non-Euclidean geometry. Until the 1960s, the concept of non-Euclidean geometry was used mainly in scientific research or only in higher education. This is because the concept of Euclidean geometry is mainly of practical importance in solving complex problems of the exact sciences. But after 1969, the state of non-Euclidean geometry in the plane had its own explanation of simple and elementary mathematics. This, in turn, led to the emergence of popular scientific sources on non-Euclidean geometry. At the same time, I.M. Yaglom's book, Galileo's Theory of Relativity, and non-Euclidean Geometry, was published (1969). This book was devoted to the description of projective metric geometries in the plane. Then, in the 70s of the century, books in Uzbek for schoolchildren appeared. Another example of this is Naim Gaybullayev's book, Elements of Non-Euclidean Geometry at School (1975). This book is mainly elliptical and hyperbolic, i.e., the axiomatic structure and basic concepts of Lobachevsky geometry.

W.Thurston's study of three-dimensional polynomial geometry and his new interpretation of the science of geometry, which was able to cover all non-Euclidean geometries, gave rise to a new perspective on science in general, and in particular the non-Euclidean geometries on the plane. This led to the solution of several problems related to the axiomatic construction of non-Euclidean geometries. For this reason, since the end of the twentieth century and the beginning of the twenty-first century, there has been a great deal of literature on nonlinear geometry in the plane and its ability to be taught in a simple way. Of these, E.F. Krause (1986), A.B. Popular literature such as Khachatryan (2005), A. Artikbayev, I. Hamdamov's "Nine Geometries in the Plane" (2021) are devoted to plane geometries.

**The research is related to the research plans of the higher education institution where the dissertation was completed.** The dissertation's research was carried out within the framework of the research project of Tashkent State

University on "Theory of functions, functional analysis, differential equations and issues of their application."

**The aim of the research** is to engage and increase students' interest in science by introducing the concept of non-Euclidean geometries to the science of geometry in secondary schools.

**The tasks of the research:**

study of non-Euclidean geometries in the plane,  
identifying topics closer to the basic concepts of school geometry in non-Euclidean geometry,  
comparing the basic concepts of Euclidean and non-Euclidean,  
development of methods for identifying gifted students and improving their interest in science through non-Euclidean geometry.

**The object of the research** was high school students who trained in geometry in secondary schools.

**The subject of the research.** The content, form, and methods of improving mathematics through non-Euclidean geometries in general secondary schools.

**Methods of the research.** Analyzing research literature, textbooks, and manuals on the research problem, analyzing changes in school textbooks and scientific articles on non-Euclidean geometry, organizing extracurricular projects, clubs, group discussions through group disputation or group negotiation, conduct experiments. Methods such as data transfer and systematization, mathematical and statistical processing, and evaluation of research results were used.

**The scientific novelty of the research:**

the concepts of simple non-Euclidean geometry present in the plane are described in accordance with the methods of the school geometry textbook, and the commonalities and differences between the terms of distance and metric are explained by simple examples,

the properties of the "cycle" in Galilean geometry with respect to the circular motion and intersection in Euclidean geometry are shown,

the method of teaching the proof of the theorem to students has been improved by using the first and second signs of the triangle equality and the failure of the third sign,

a method of engaging students in mathematics was developed using elements of non-Euclidean geometry.

**Practical results of the research:**

methods of identifying gifted students have been improved using elements of non-Euclidean geometry,

the study of non-Euclidean geometry allowed students to develop a habit of in-depth analysis of Euclidean geometry properties,

the methods of preparing students for extracurricular projects and clubs have been improved by involving them in scientific research.

**The authenticity of the research results.** Finding a positive outcome of the research problem is explained by the fact that it is based on advanced pedagogical,

psychological experiments, modern mathematics methods, and the well-founded laws of methodology.

**The scientific and practical value of the research results.** The scientific importance of the study is that the elementary geometric properties of Galilean geometry have been proved in the plane, and these properties have also been shown to be essential for Euclidean geometry. This can be explained by the fact that the study of non-Euclidean geometry shows a new way of looking at Euclidean geometry.

The practical significance of the research is explained by the fact that acquaintance with the elements of non-Euclidean geometry is of practical importance as it develops students' interest in science and scientific reasoning skills.

**Implementation of the research results.** Based on the results of the study on the methods of using elements of non-Euclidean geometry in the educational process:

the generality between the concepts of simple non-Euclidean geometry in the plane and the terms distance and metrics and simple examples of differences were used to form students' knowledge of non-Euclidean geometries present in the plane held at the private girls' high school (<https://www.facebook.com/USGirls.H>) implementing a Cambridge curriculum in Sulaymaniyah, Iraq (Reference No. 02 of the Private Girls' High School in Sulaymaniyah, Republic of Iraq, January 4, 2022). As a result, it allowed 9<sup>th</sup> graders to identify topics close to the basic concepts of non-Euclidean geometry in school geometry.

The properties of the cycle in Galilean geometry with the tangent and intersecting lines of the circle in Euclidean geometry and the basic ideas of Taxicab geometry were benefited from the club activities related to non-Euclidean geometry carried out in schools with successful education and training research projects between 2013-2015 (Reference No.58 of Tishk International University, Republic of Iraq, January 3, 2022). As a result, it has allowed higher education students to unleash their imagination about non-Euclidean geometries and to compare these geometries in theory and practice.

**Approbation of the research results.** The results of this research have been presented at international conferences (International Conference "Applications and Problems of Modern Topology", Tashkent 2016. International Conference on Education and Teaching in K-12 Schools, London 2021) and have been discussed other 5 more conferences, online and another scientific seminar at the Institute of Mathematics of the Republic of Uzbekistan.

**Publication of the research results.** 22 scientific papers on the dissertation topic were published, including 10 articles in scientific journals recommended for publication of the leading scientific results of doctoral dissertations of the Higher Attestation Commission of the Republic of Uzbekistan, including 2 in national and 8 in foreign journals.

**Structure and volume of the dissertation.** The dissertation consists of an introduction part, 4 different chapters, a conclusion, and a list of references. The volume of the dissertation is 76 pages.

## **THE MAIN CONTENT OF THE DISSERTATION**

**The introductory** part describes the relevance of the topic, the level of study of the problem, the goals and objectives of the work, the object, the subject, the research methods. The scientific novelty of the dissertation, practical results, their reliability, the scientific and practical significance of the work are revealed, information on the implementation of research results are given.

The first chapter of the dissertation, entitled "**Non-Euclidean Geometries in the plane**" provides a modern definition of geometry, Minkowskian geometry, Galilean geometry, and the basic concepts of finite (limited) and Taxicab geometry, which are clearly explained to schoolchildren. In the following chapters, the definition of the required concepts and some of their properties are given in the form of abstracts.

The second chapter of the dissertation, entitled "**Comparison of basic concepts of non-Euclidean and Euclidean Geometry**," focuses on topics that can introduce students to the concepts of non-Euclidean geometry and compare them with relevant concepts in Euclidean geometry. Additional classes or clubs can be organized based on the topics covered in this chapter. Here are some ways to do this.

The first paragraph of the second chapter, entitled "Distance and Metrics," defines the concepts of metrics and distance, and explains their similarities and differences with examples.

The fact that distance can take different values as a function of two points is explained by examples using distance in Minkowskian geometry. The geometric meaning of distance in Galilean geometry is explained.

The second paragraph of the second chapter of the dissertation is entitled "Kinematic and isometry". This paragraph states that motion in the Euclidean plane consists of parallel displacement and rotation and that the shapes that intersect in this motion are called isometric shapes.

Rotating the coordinate axis while maintaining the coordinate head is specified separately. When you rotate the coordinates, the circle in the plane moves by itself. The fact that this property is also true in non-Euclidean planes is explained by the formulas for rotating the coordinates of Minkowskian's geometry and Galilean's geometry. The second type of circle in Galilean geometry is the concept of "cycle".

In Euclidean and non-Euclidean geometries, the concept of angle is defined in the same way. The method for determining the size of the angle is also similar. Identifying these problems in Euclidean and non-Euclidean geometries will be a very interesting activity for students. Therefore, the third paragraph of this chapter of the dissertation states that the angle and the methods of measuring it are "similar" to the Euclidean geometry in the Galilean and Minkowskian geometries.

Indeed, in Galilean geometry, as in Euclidean geometry, the magnitude of an angle is determined by the length of the arc of the circle at the end of the central angle.

The third chapter of the dissertation is entitled “Using non-Euclidean geometry concepts in the educational process”.

The first paragraph of the third chapter gives an overview of the equivalence signs of triangles. It is known that there is a section in Euclidean geometry with full proof of the three signs of the triangle.

Because the concepts of distance and angle in Galilean geometry are different from Euclidean geometry, the sign of equality on all three sides of a triangle is not fulfilled. We can show in the diagram triangles that are not equal but have equal sides. This arouses students’ interest in Galilean geometry.

At the same time, we define the concepts of median, bisector, and altitude of a triangle in Galilean geometry.

Affine is a concept, therefore that the median is defined the same in these geometries and has the same properties.

Because the angle is different in Galilean geometry, the bisector also has a different geometric shape.

Special attention should be paid to the definition of height in Galilean geometry. The definition of this concept should be based on the property of height in the Euclidean plane. The concept of distance from a point to a straight line is very important. The connection between Euclidean and Galilean geometries, as well as their differences, are always of interest to students.

The second paragraph of the third chapter entitled “Polygons and Surfaces”, seeks to generalize the concept of polygons in the Galilean plane. In Galilean geometry, the definition of a parallelogram is the same as in Euclidean geometry. However, there are no special cases, such as a rectangular square. A rhombus can only be called if the opposite ends of the parallelogram belong to a special straight line.

Exploring the interior angles of convex polygons leads to interesting results. Especially, this theorem is proved for convex polygons in a plane.

**Theorem 1.** The sum of the interior angles of a convex polygon is equal to the sum of the base angles:  $\alpha + \beta = \sum_{i=1}^{n-2} \theta_i$ .

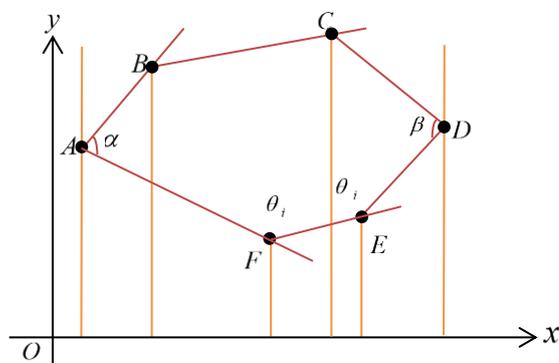


Figure 1

This theorem is a generalization of the concept of the sum of the interior angles of a given triangle to a polygon. The concept of surface in Galilean

geometry, which is of great interest to students, and that they themselves believe in equality through measurements and precise calculations.

**Theorem 2.** The area of a triangle is equal to half of the product of its base and height.  $S = \frac{1}{2} \cdot a \cdot h_b$ ,  $S_{ABC} = \frac{1}{2} AC \cdot BD = \frac{1}{2} AC' \cdot B'D'$

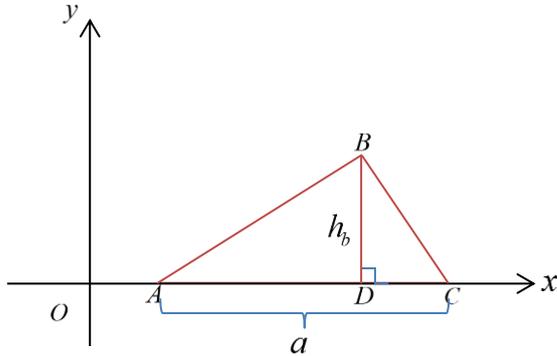


Figure 2

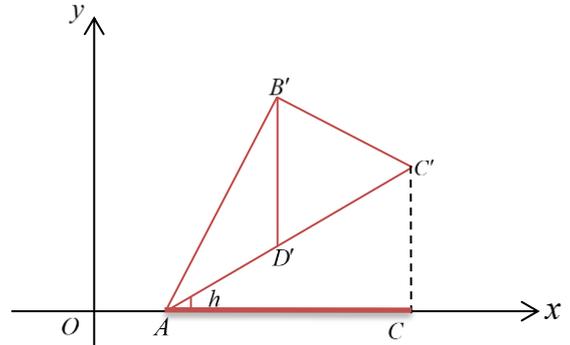


Figure 3

The geometric interestingness of this formula can be seen in the following figure.

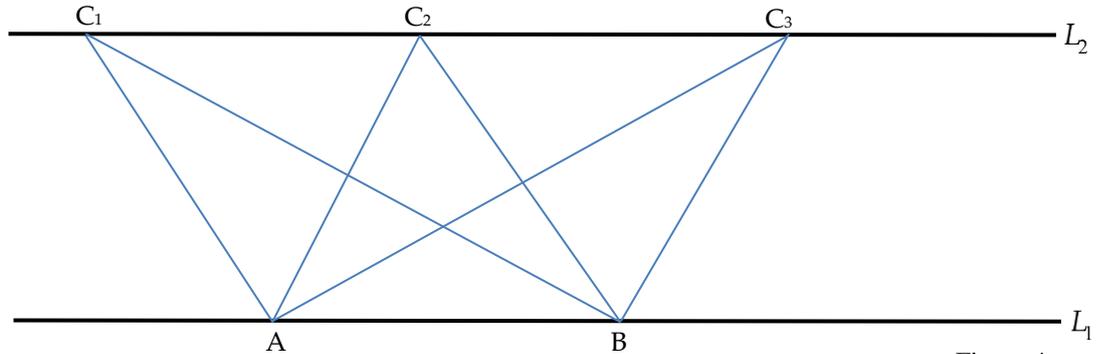


Figure 4

$$S_{ABC_1} = S_{ABC_2} = S_{ABC_3} \text{ where } L_1 // L_2$$

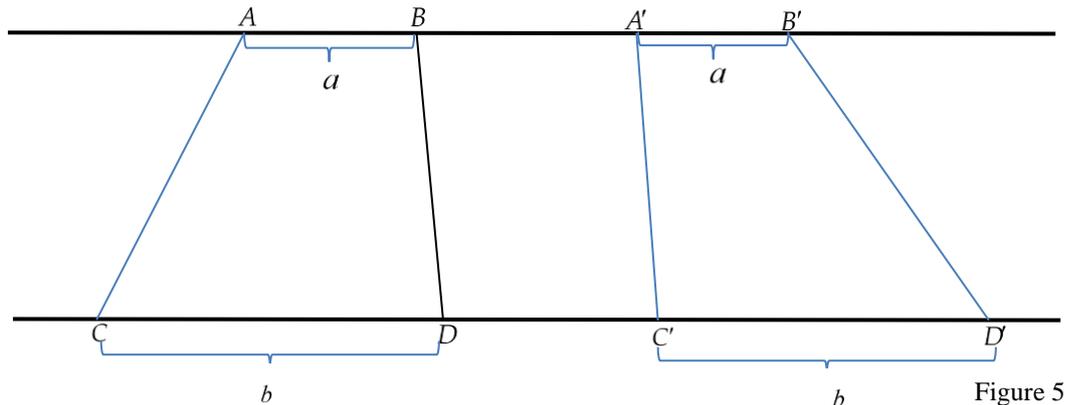


Figure 5

$$\text{Here also, } S_{ABDC} = S_{A'B'D'C'}$$

Using the concept of surface, triangles (theorem4) and trapezoids (theorem5) under the influence of Galilean motion were proved.

The third paragraph of the third chapter deals with the cyclic property in the Galilean plane. The following properties have been proven for the cycle.

**Property 1.** Two attempts can be made on a cycle from a point outside the cycle.

**Property 2.** If the cycle is attempted from a point outside it, the distances from that point to the point of attraction are equal.

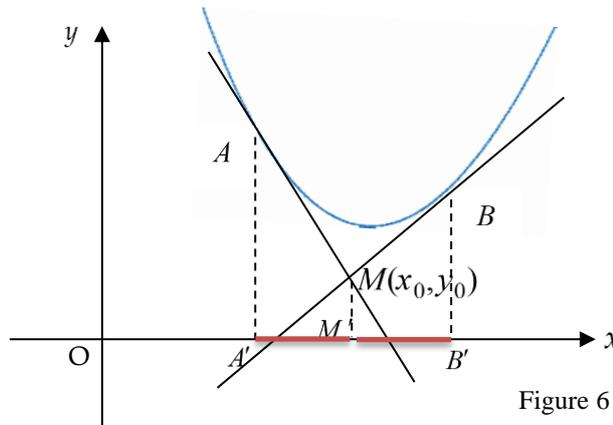


Figure 6

**Property 3.** Given a point lying outside the cycle, from which a straight line intersects the cycle at points  $A, B$  the multiplication  $LA \cdot LB$  depends only on the point  $L$ .

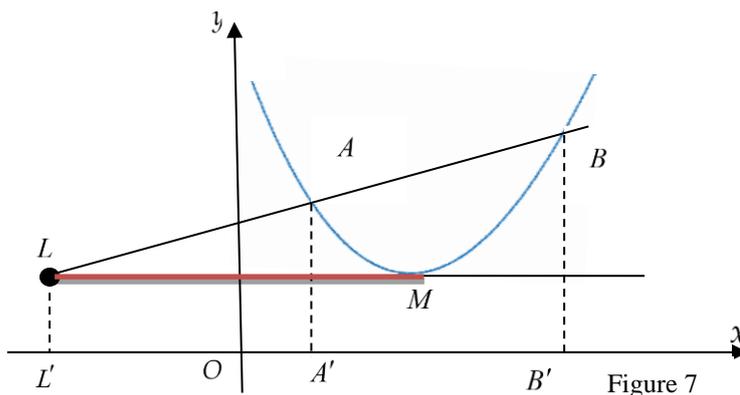


Figure 7

Since the cycle in the Galilean plane is a parabola in the Euclidean plane, the properties mentioned above also apply to the Euclidean parabola.

The section entitled “Mathematical Games & Projects” provides a mathematical game, a method of organizing projects, using the concepts of non-Euclidean geometry. We have given these methods using the concepts of Taxicab geometry. Because students understand the vital meaning of this geometry. For other non-Euclidean geometries, we can do math games and projects in the same way.

The purpose of these mathematical games & projects is to lay the groundwork for incorporating elements of non-Euclidean geometry into the school curriculum.

The project consists of a simple module, which we proposed to implement in five steps.

The first step is to get acquainted with the basic concepts of geometry. These concepts are given in the first chapter for different geometries.

The second step is to be able to explain the concept of distance with concrete life examples. In this case, we have given a way to implement the concept of distance in the Taxicab geometry with the participation of students in the room.

The third step is to focus on the method of deriving the formula for a circle using distance and distance.

The fourth step is to create, visualize, and practice straight lines, angles, circles, and triangles in the non-Euclidean geometry being studied.

The fifth step is to visualize the basic concepts of equality of triangles and with imagining basic ideas about triangles. Euclidean geometry state of the triangle and its elements in non-Euclidean geometry.

In these five steps, students will gain an in-depth understanding of the basic concepts of non-Euclidean geometry.

At the end of the paragraph, you will learn how to overcome the difficulties that may arise in the learning process and how to continue the module.

The fourth chapter of the dissertation is entitled "**Research Discussion and Conclusions**". The first paragraph of this chapter gives some conclusions about the concept of "non-Euclidean geometry." This section is primarily devoted to explaining Lobachevski geometry to students, however, we did not achieve good results in this area. It was very interesting to work with students on the geometry of Galilean or Taxicab. The fact that the facts in this geometry are radically different from Euclidean geometry, but understandable to students, has led to more students coming to our classes than we expected. We conducted the research in Kurdistan. The diversity of the schools here allowed us to take additional classes in non-Euclidean geometry during our studies.

The first offer has come from a girls' high school that implements Cambridge curricula in the Sulaymaniyah district, and the second from a private boys' high school in Kirkuk. In these schools, we took three-month courses, mainly on Taxicab geometry. Our work based on the "Brother Schools" project has spread to other schools. My friend Hussein, whom I work with, did this project at Erbil boys' high school, one of our brother schools in Erbil.

Also, a study by my students called on Non-Euclidean Geometry Talks informed hundreds of students of these ideas.

In my work, I have come to believe that the words "*Think big, start small, begin now*" are true. These words are used as slogans for students in Iraqi schools.

I think that during my teaching career my students were really interested in geometry. Because they not only learned but also began to make various presentations for their peers to learn.

I think that the inclusion of the concept of "non-Euclidean geometry" in the school curriculum will not only increase students' interest in mathematics, but it will also even require teachers to improve their professional skills.

The second paragraph of the fourth chapter entitled "Activities & Competitions", discusses the procedure for organizing the event and the visual aids required for it. We participated in Iraq National Project Olympiad and won first

place in 2014. Of course, to participate in these Olympiads, it is necessary to participate in previous competitions and win.

This victory gave us the opportunity to participate in the world's largest project Olympiad INTEL ISEF in Los Angeles, USA.

**As a result of our research, we came to the following conclusions.**

-Non-Euclidean geometry can be taught in secondary schools through demonstrations and basic concepts.

-Encourage students who are interested in non-Euclidean geometry, as this will broaden their thinking and outlook.

-Non-Euclidean geometry should be presented in a new and interesting way, not in a difficult way to explain traditional Euclidean geometry.

The third paragraph of the fourth chapter is entitled "Recommendations and Conclusion". One of the main recommendations is that for a teacher to teach geometry at school, they must be familiar with the concepts of non-Euclidean geometry. This allows the teacher to feel confident in teaching geometry.

Students should not be blamed for their lack of creativity, freedom, and originality. Teachers need to be able to provide them with the resources that lead to the development of these qualities.

In this regard, the elements of non-Euclidean geometry should be used as a method of "new perspective". Based on the research, we recommend the following:

-It is necessary to organize test courses on non-Euclidean geometry in secondary schools specializing in mathematics.

-It is necessary to train teachers in the content and methods of pedagogical expression of non-Euclidean geometry.

-It is necessary to create opportunities for teachers, and then for interested students to get acquainted with non-Euclidean geometry.

-As math teachers, we need to show our students that non-Euclidean geometry has a place in the study of this world and a new way of looking at it.

I think that getting acquainted with the concepts of non-Euclidean geometry will help young people to better understand Euclidean geometry and, in general, to develop their competence in mathematics.

Introducing students to non-Euclidean geometry at any level can also lead to new discoveries. I believe that the results of our cycle should be accepted in the school's geometry course program.

## **CONCLUSION**

Based on the theoretical analysis and experimental results of this research topic, the following conclusions were drawn:

1. Studying the terms distance and metrics has led students to develop a scientific approach to terms in general.

2. The concept of the angular surface has developed competencies in the diversity of measurement theory.

3. Defining concepts such as triangles and circles in neo-Euclidean geometries made it possible to study the generality and diversity of specific sciences.

4. The study of neo-Euclidean geometry also requires students to pay attention to Euclidean geometry.

5. Using non-Euclidean geometries requires in-service training and retraining of schoolteachers.

**НАУЧНЫЙ СОВЕТ DSc.02/30.12.2019.FM.86.01  
ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ  
ИНСТИТУТЕ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО**  

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**ТАШКЕНТСКИЙ ГОСУДАРСТВЕННЫЙ ТРАНСПОРТНЫЙ  
УНИВЕРСИТЕТ**

**АБДУЛЛАХ КУРУДИРЕК**

**МЕТОДИКА ПРИМЕНЕНИЯ ЭЛЕМЕНТОВ НЕЕВКЛИДОВОЙ  
ГЕОМЕТРИИ В ПРОЦЕССЕ ОБРАЗОВАНИЯ**

**13.00.02 –теория и методика преподавания и воспитания (математика)**

**АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD)  
ПО ПЕДАГОГИЧЕСКИМ НАУКАМ**

**ТАШКЕНТ-2021**

Диссертация выполнена в Ташкентском государственном транспортном университете.

Автореферат диссертации на трех языках (узбекский, английский, русский (резюме)) размещен на веб-странице по адресу <http://kengash.mathinst.uz> и на Информационно-образовательном портале «ZiyoNet» по адресу <http://www.ziyo.net.uz>.

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**Ведущая организация:** **Национальный Университет Узбекистана**

Защита диссертации состоится «\_\_\_» \_\_\_\_\_ 2022 года в \_\_\_\_\_ на заседании Научного совета DSc.02/30.12.2019.FM.86.01 при Институте Математики имени В.И.Романовского. (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 9.Тел.: (+99871) 207-91-40, e-mail: [uzbmath@umail.uz](mailto:uzbmath@umail.uz), Website: [www.mathinst.uz](http://www.mathinst.uz)).

С диссертацией можно ознакомиться в Информационно-ресурсном центре Института Математики имени В.И.Романовского (зарегистрирована за № \_\_\_\_). (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 9.Тел.: (+99871) 207-91-40).

Автореферат диссертации разослан «\_\_\_» \_\_\_\_\_ 2022 года.  
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## **ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))**

**Цель исследования.** Привлечь и выбрать одарённых учеников с внедрением элементов неевклидовой геометрии в предмет школьной геометрии.

**Объект исследования** совершенствование методов математики к определению одарённых учащихся.

**Научная новизна исследования** состоит в следующем:

основные понятия неевклидовой плоскости изложены в соответствии курсу школьной геометрии, в простых примерах объяснена общность и различие понятий метрика и расстояния;

доказана аналогия свойства элементов окружности для цикла в геометрии галилея;

совершенствованы методы обучения учеников доказательству теорем используя выполнение первого и второго признака неравенство треугольников и невыполнение третьего признака;

разработан метод привлечения учащейся к математике с использованием элементов неевклидовой геометрии.

**Внедрение результатов исследования.** Результаты исследования методику преподавания элементов неевклидовой геометрии в учебном процессе использованы:

основные понятия неевклидовой геометрии на плоскости и примеры связанные с общими свойствами метрики и расстояния и их различия использовались в неурочных занятиях школе кембриджского типа города Сулеймания в Ираке (<https://www.facebook.com/USGirls.H>) проведенные занятия были основой формированию знаний учащейся о неевклидовых геометриях (Справка специальной девичий школы города Сулаймания от 4-января 2022 года №02, Ирак). В результате ученики 9-классов ознакомились с понятиями неевклидовой геометрии близкие школьной геометрии;

Свойства цикла аналогичные свойствам касательной и секущей окружности евклидовой геометрии и геометрия Таксикаб в 2013-2015 годах была использована в специализированных школах в проектах и внеурочных занятиях по “неевклидовой геометрии” (Справка Международного университета ТИШК от 3-января 2022 года №58, Ирак). В результате у старшеклассников развивалось представление о неевклидовой геометрии и навыки сравнение этих результатов в науке и практике.

**Структура и объем диссертации.** Диссертация состоит из введения, четырёх глав, заключения и списка использованной литературы. Объем диссертации составляет 76 страниц.

**E'LON QILINGAN ISHLAR RO'YXATI**  
**LIST OF PUBLISHED WORKS**  
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