

**“TIQXMMI” MILLIY TADQIQOT UNIVERSITETI QOSHDAGI  
FUNDAMENTAL VA AMALIY TADQIQOTLAR INSTITUTI  
HUZURIDAGI ILMIY DARAJALAR BERUVCHI  
DSc.03/31.03.2022.T/FM.10.04 RAQAMLI ILMIY KENGASH**

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**SHAROF RASHIDOV NOMIDAGI SAMARQAND DAVLAT  
UNIVERSITETI**

**MURODOV SARDOR NORMUMIN O‘G‘LI**

**YUQORI EGRILIK HADLARIGA EGA BO‘LGAN GRAVITATSIYA  
NAZARIYALARIDA YANGI SKALYARIZATSIYALANGAN  
YECHIMLAR**

**01.04.02 – Nazariy fizika**

**Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi  
AVTOREFERATI**

**Samarqand–2022**

**Fizika-matematika fanlari bo'yicha falsafa doktori (PhD)  
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**Content of dissertation abstract of doctor of philosophy (PhD) on physical-  
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**Оглавление автореферата диссертации  
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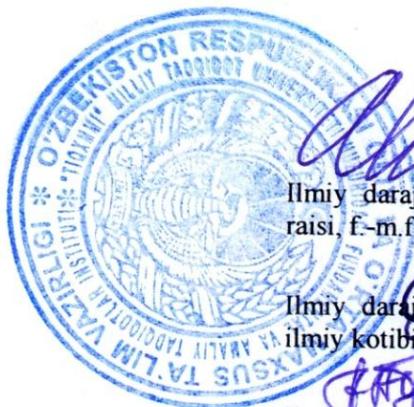
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## **Kirish (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)**

**Dissertatsiya mavzusining dolzarbligi va zarurati.** Jahonda so‘nggi yillarda olib borilayotgan tadqiqotlar modernizatsiyalangan Eynshteyn maydon tenglamalarining yangi yechimlari nafaqat qora tuynuklar, balki yumronqoziq inlari (wormholes) ham mavjudligini bashorat qilmoqda. Adabiyotlarda yumronqoziq inlari koinotning olis mintaqalarini bog‘laydigan “tunnellar” yoki turli koinotlarni bog‘laydigan “ko‘priklar” deb nomlanadi. Dastlab, 1935-yilda yumronqoziq ini (Eynshteyn-Rozen ko‘prigi) yechimini Eynshteynning o‘zi taklif qilgan. Bu sohada M.S.Morris, K.S.Thorne va U.Yurtseverlarning fundamental ishlari va J.A.Wheelerning geometrodinamikasi zamonaviy tadqiqotlarning asosini tashkil etadi. Yumronqoziq ini yechimlarini uzoq vaqtlardan buyon tadqiq qilinishining sabablaridan biri bu insoniyat uchun yulduzlararo va Galaktikalararo uzoq masofalarga sayohat qilish imkoniyating vujudga kelishidir. Shu sababdan yumronqoziq inlari tabiatini nazariy jihatdan o‘rganish muhim ahamiyatga ega ekanligini ko‘rsatmoqda.

Dunyo ilm-fanida so‘ngi o‘n yilliklarda yumronqoziq inlarini o‘rganish alohida dolzarblikka ega bo‘ldi. Bu materiyaning “ekzotik” shakllariga zamonaviy qiziqishlar bilan bog‘liq. Ma‘lumki, Yumronqoziq inlari mavjudligi uchun materiyaning qandaydir ekzotik shakli zarur bo‘lib, bu esa bir qator energetik shartlarni buzadi. Bugungi kunda, bu kabi materiya koinotda mavjud bo‘lishi mumkinligi haqidagi dalillar mavjud. Bu birinchi navbatda, “qora energiya” deb nomlangan yangi ekzotik moddani kiritishni talab qiladigan olamning tezlashtirilgan kengayishining kashf qilinishi bilan bog‘liq. Ammo, ayrim alternativ gravitatsiya nazariyalarida energetik shartlarni buzilishi gravitatsiyon o‘zaro ta’sir tufayli ham bo‘lishi mumkinligi ko‘rsatilgan. Chunki ushbu alternativ nazariyalarda hechqanday ekzotik materiyaning ishtirokisiz ham energetik shartlarini buzilishiga olib keluvchi effektktiv energiya-impuls tenzori hosil bo‘ladi. Ushbu alternativ gravitatsiya nazariyalardan eng diqqatga sazovvorlari, bu Eynshteyn-skalyar-Gauss-Bonnet va Eynshteyn-skalyar-Chern-Simons nazariyalari bo‘lib, hozirgi kunda ushbu nazariyalarda yumronqoziq inlarini o‘rganish dolzarb vazifaga aylanmoqda.

Mamlakatimizda so‘ngi yillarda fundamental va amaliy tadqiqotlarning dolzarb yo‘nalishlarini rivojlantirishga tobora ko‘proq e’tibor berilmoqda. Xususan, istiqbolli yo‘nalishlardan biri bo‘lgan astrofizik tadqiqotlarni rivojlantirish bugungi kunning muhim masalasidir. Yurtimizda ilm-fanning muvaffaqiyatli rivojlanishi uchun fundamental tadqiqotlar va ishlanmalarning asosiy yo‘nalishlari va ularni amaliy qo‘llash O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha 2022-2026-yillarga mo‘ljallangan Strategiyasida<sup>1</sup> o‘z aksini topgan. Shu sababli, fundamental tadqiqotlar sohasida yuqori egrilikli gravitatsiya nazariyalarida skalyarizatsiyalangan yechimlarni tadqiq qilish dolzarb masalalardan biri bo‘lib qolmoqda.

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<sup>1</sup> O‘zbekiston Respublikasi Prezidentining 2022-yil 1-yanvardagi № PF-60 son farmoni “2022-2026 yillarga mo‘ljallangan Yangi O‘zbekistonning taraqqiyot strategiyasi to‘g‘risida”.

O‘zbekiston Respublikasi Prezidentining 2019-yil 8-oktabrdagi PF-5847-sonli “O‘zbekiston Respublikasi oliy ta’lim tizimini 2030-yilgacha rivojlantirish konsepsiyasini tasdiqlash to‘g‘risida”gi Farmoni, O‘zbekiston Respublikasi Prezidentining 2021-yil 19-martdagi PQ-5032-sonli “Fizika sohasidagi ta’lim sifatini oshirish va ilmiy tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”, 2017-yil 17-fevraldagi PQ-2789-sonli “Fanlar akademiyasi faoliyati, ilmiy tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari to‘g‘risida”gi qarorlarini hamda mavzuga oid boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda mazkur dissertatsiya muayyan darajada xizmat qiladi.

**Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi.** Dissertatsiya respublika fan va texnologiyalar rivojlanishining II. “Fizika, astronomiya, energetika va mashinasozlik” ustuvor yo‘nalishlari doirasida bajarilgan.

**Muammoning o‘rganilganlik darajasi.** So‘nggi yillarda yuqori egrilik hadlariga ega bo‘lgan gravitatsiya nazariyalarida skalyarizatsiyalangan yechimlarni topish bo‘yicha ko‘plab fundamental izlanishlar olib borilgan, jumladan: G.Antoniou, A.Bakopoulos, P.Kanti, B.Kleihaus, va J.Kunz (Minnesota universiteti, AQSh; Ioannina universiteti, Gretsiya; Oldenburg universiteti, Germaniya) tadqiqotlari bir qancha bog‘lanish funksiyalari uchun Eynshteyn-skalyar-Gauss-Bonnet nazariyalarida yangi turdagi yumronqoziq ini yechimlariga olib kelgan. Bu yumronqoziq inlari bir va ikki bo‘g‘izdan iborat bo‘lib, hech qanday ekzotik materiyani talab qilmaydi. Shuningdek, skalyarizatsiyalangan qora tuynuk yechimlarini ham topishgan. P.Kanti, B.Kleihaus, and J.Kunz (Ioannina universiteti, Gretsiya; Oldenburg universiteti, Germaniya) ishlarida Eynshteyn-Gauss-Bonnet dilatoni nazariyalarda va to‘rt o‘lchamli fazo-vatda har qanday ekzotik materiyalarning ishtirokisiz o‘takazadigan yumronqoziq inlari va ularning mavjud bo‘lish sohalari aniqlangan. Shuningdek, ushbu yumronqoziq inlarining xususiyatlarini, ularning radial g‘alayonlanishlarga nisbatan chiziqli barqaror bo‘lishini va umumiy Smarr munosabatlarni aniqlashgan.

J.L. Blázquez-Salcedo, D. D. Doneva, S. Kahlen, J. Kunz, P. Nedkova, and S. S. Yazadjiev (Tubingen universiteti, Oldenburg universiteti, Germaniya; Sofiya universiteti, Matematika va informatika instituti, Bolgariya) izlanishlarida Eynshteyn-Gauss-Bonnet nazariyalarida spontan skalyarizatsiyalangan qora tuynuklarning qutbli kvazinormal modellari va aksial g‘alayonlanishlari o‘rganilgan. Ular qutbli kvazinormal modellarni hisoblashgan va yechimlar to‘plamini qutbli g‘alayonlanishlarga nisbatan stabil ekanligini ko‘rsatishgan.

Rossiyalik olimlar Kirill Bronnikov va Roman Konoplya hamda Chexiyalik olim Thomas Pappaslar birgalikda ixchamlashtirilgan radial koordinatada davomli fraksiya kengayishiga asoslangan metod yordamida gravitatsiyaning ixtiyoriy metrik nazariyasida statik, sferik, simmetrik o‘tkazadigan Lorents yumronqoziq ini geometriyalari uchun umumiy parametrlashni ishlab chiqishgan.

O‘zbekistonlik olimlar, Bobomurat Ahmedov va Sanjar Shaymatovlar Eynshteyn-Gauss-Bonnet nazariyasining to‘rt o‘lchovli chegarasida qora tuynuk yechimini qarab chiqishgan va tashqi magnit maydon mavjud bo‘lganda fazo-

vaqtda zaryadlangan zarrachalarning harakatini o'rganishgan. Gauss-Bonnet bog'lanishi yo'qolganda yechim Shvarsschild holatiga tushadi. Zaryadli zarralarning aylanma orbitalari uchun eng barqaror aylanma orbitadagi Gauss-Bonnet bog'lanishining effektini topishgan va gorizot yaqinida to'qnashuv energiyasini aniqlashgan.

So'nggi yillarda nashr etilgan ilmiy ishlarning tahlili shuni ko'rsatadiki, hozirgi vaqtda, modernizatsiyalangan gravitatsiya nazariyalarida qora tuynuklar va yumronqoziq inlarining xususiyatlarini to'laqonli tushuntira oladigan yagona nuqtai nazar yo'q, bu esa bayon etilayotgan ilmiy muammo chegarasida yangi nazariy tadqiqotlar o'tkazishni yanada dolzarb ekanligini ko'rsatadi.

**Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta'lim muassasasining ilmiy-tadqiqot ishlari rejaları bilan bog'liqligi.** Dissertatsiya Sharof Rashidov nomidagi Samarqand davlat universiteti ilmiy-tadqiqot ishlari rejasiga muvofiq, Fizika fakulteti Nazariy fizika va kvant elektronika kafedrasining 2017-2021 yillarga mo'ljallangan "Nazariy fizikaning hozirgi zamon muammolari bo'yicha izlanishlar" ilmiy-tadqiqot mavzusiga binoan olib borilgan. O'zbekiston Respublikasi Innovatsion rivojlanish vazirligining 2022-2026 yillarga mo'ljallangan № Ф3-20200929385 "Eynshteyn tenglamalarining Qora tuynuklar (Black holes) va Yumronqoziq inlari (Wormholes) uchun yangi yechimlari" loyihasi doirasida ham bajarildi.

**Tadqiqotning maqsadi** yuqori egrilik hadlariga ega bo'lgan Eynshteyn-skalyar-Gauss-Bonnet va Eynshteyn-skalyar-Chern-Simons gravitatsiya nazariyalarida skalyarizatsiyalangan yumronqoziq ini yechimlarini topishdan iborat.

**Tadqiqotning vazifalari:**

Massa hadi va o'zaro-ta'sirlashuv hadlarini o'z ichiga olgan skalyar maydon uchun Eynshteyn-skalyar-Gauss-Bonnet gravitatsiya nazariyalarida potentsialga ega bo'lgan yumronqoziq ini yechimlarini topish;

Gauss-Bonnet bog'lanish doimiysini o'zgartirib, Eynshteyn-skalyar-Gauss-Bonnet gravitatsiya nazariyalarida yumronqoziq inlarining mavjud bo'lish sohasi chegaralarini aniqlash;

bir bo'g'izli va ekvatorga ega ikki bo'g'izli yumronqoziq inlarining fizik xususiyatlarini, shu jumladan ularning massalarini, o'lchamlarini va geometriyalarini o'rganish;

yuqori egrilikli gravitatsiya nazariyalarida Newman-Unti-Tamburino zaryadiga ega skalyarizatsiyalangan yumronqoziq ini yechimlarini olish;

bog'lanish parametri va skalyar zaryadni o'zgartirish orqali Eynshteyn-skalyar-Gauss-Bonnet va Eynshteyn-skalyar-Chern-Simons gravitatsiya nazariyalarida skalyarizatsiyalangan yumronqoziq ini yechimlarini aniqlash va ularni Newman-Unti-Tamburino zaryadiga bog'liqligini o'rganish.

**Tadqiqotning ob'yekti** yuqori egrilikli gravitatsiya nazariyalari va yumronqoziq inlari.

**Tadqiqotning predmeti** yuqori egrilikli hadlar, effektiv ta'sir integrali, Eynshteyn va skalyar maydon tenglamalari, metrikalar, Israel bog'lanish shartlari, nol energetik shart.

**Tadqiqotning usullari.** Maydon tenglamalarini qisqartirish to'rtta bog'langan, chiziqli bo'lmagan, oddiy differensial tenglamalar sistemasiga olib keladi. Ushbu bog'langan oddiy differensial tenglamalar sistemasining raqamli bog'lanishi chekli element metodidan foydalanilib, COLSYS dasturlar paketi yordamida amalga oshirildi. Boshlang'ich qiymat masalasi sifatida oddiy differensial tenglamalar sistemasi uchun to'rtinchi tartibli Runge Kutta usulini qo'lladik.

**Tadqiqotning ilmiy yangiligi** quyidagilardan iborat:

Eynshteyn-skalyar-Gauss-Bonnet gravitatsiya nazariyalarida skalyar o'zaro ta'sirlashuv potensialga ega bir va ekvator bilan chegaralangan ikki bo'g'izli yumronqoziq ini yechimlari olindi va Gauss-Bonnet bog'lanish doimiysi o'zgartirilib, ushbu gravitatsiya nazariyalarida yumronqoziq inlarining mavjud bo'lish sohasi aniqlangan;

yuqori egrilik hadlariga ega Eynshteyn-skalyar-Gauss-Bonnet va ilk bor Eynshteyn-skalyar-Chern-Simons gravitatsiya nazariyalarida bog'lanish parametri va skalyar zaryad o'zgartirilib, yangicha skalyarizatsiyalangan Newman-Unti-Tamburino yumronqoziq inlarining mavjud bo'lishi, hamda ularning Newman-Unti-Tamburino zaryadiga bog'liqligi aniqlangan;

Gauss-Bonnet invarianti holatida va Newman-Unti-Tamburino zaryadining yo'qolish chegarasida skalyarizatsiyalangan yumronqoziq ini yechimlari to'plamiga erishildi. Chern-Simons invarianti holatida esa, Newman-Unti-Tamburino zaryadining kamayishi va bog'lanish doimiysing o'zgarishi sababli, ushbu yumronqoziq ini yechimlar chegaralari o'ziga xos bo'lgan ikki sohaga ajralishi aniqlangan.

**Tadqiqotning amaliy natijalari** quyidagilardan iborat: Amalga oshirilgan tadqiqotlar, umuman olganda, Eynshteyn-skalyar-Gauss-Bonnet va Eynshteyn-skalyar-Chern-Simons gravitatsiya nazariyalarida, yumronqoziq ini yechimlarining olinishi va o'rganilishi, ushbu astrofizik ob'ektlar haqidagi tushunchamizni kengaytiradi va chuqurlashtiradi, bu esa bunday ob'yektlarning tabiati va fizik xususiyatlarini yanada chuqurroq anglashga imkon beradi. Gravitatsiya nazariyasidagi zamonaviy tadqiqotlar bizga yumronqoziq inlarining fizik jihatlarini tushunishga imkon beradigan yangi modellarni taklif qilishga imkon yaratadi. Ushbu yumronqoziq inlarining xususiyatlari, ulardagi ta'sir etuvchi kuchlar va maydonlar insoniyatni kosmosda uzoq masofalarga harakatlanish muammolarini hal qilishga yaqinlashtiradi.

**Tadqiqot natijalarining ishonchliligi.** Ushbu ilmiy tadqiqot ishida o'rganilgan yumronqoziq ini yechimlari, gravitatsion nazariyalar sohasidagi fundamental tadqiqotlar bilan o'zaro bog'liqligi va uzluksizligi, ilmiy adabiyotlarda keltirilgan ma'lumotlarga zid bo'lmasligi, ba'zi hollarda olingan qiymatlarning umumiy qabul qilingan pozitsiyalarga o'xshashligi yoki to'liq mos kelishi, amalga oshirilgan matematik almashtirishlar va hisob kitoblarning to'g'riligi, asosiy natijalarning yetuk ilmiy nashirlarda chop etilishi, shuningdek, seminar va konferensiyalarda bir necha bor muhokama qilinganligi bilan izohlanadi.

**Tadqiqot natijalarining ilmiy va amaliy ahamiyati.** Tadqiqot natijalarining ilmiy ahamiyati shundan iboratki, erishilgan natijalar yuqori egrilikka ega bo'lgan gravitatsiya nazariyalarida skalyarizatsiyalangan yumronqoziq ini yechimlariga o'xshash astrofizik ob'yektlar haqidagi tushunchalarimizni yanada rivojlantirishga imkon beradi.

Tadqiqot natijalarining amaliy ahamiyati shundan iboratki, ushbu dissertatsiya ishi nazariy xarakterga ega bo'lib, olingan natijalardan yumronqoziq inlarini va qora tuynuklarni yanada chuqurroq o'rganish va kosmologiyadagi zamonaviy tadqiqotlarni tushuntirish uchun foydalanish mumkin. Demak, dissertatsiya ishining natijalarini nazariy fizika yo'nalishida tahsil olayotgan magistrantlar va doktorantlar uchun o'quv jarayoniga qo'llasa bo'ladi.

**Tadqiqot natijalarining joriy qilinishi.** Yuqori egrilik hadlariga ega bo'lgan gravitatsiya nazariyalarida yangi skalyarizatsiyalangan yechimlarni o'rganish bo'yicha olingan natijalar asosida:

Eynshteyn-skalyar-Gauss-Bonnet gravitatsiya nazariyalarida skalyar o'zaro ta'sirlashuv potensialga ega bir va ekvator bilan chegaralangan ikki bo'g'izli yumronqoziq ini yechimlarini tadqiq etish davomida olingan natijalar xalqaro ilmiy jurnallarda (Physical Review D 103, 124062, 2021; Physical Review D 104, 024048, 2021; Physical Review D 104, 044006, 2021; The European Physical Journal C, Vol.81, 285, 2021; The European Physical Journal C, Vol.81, 858, 2021; Journal of Cosmology and Astroparticle Physics, 10(2021)059; Journal High Energy Physics, 104, 2021; Nuclear Physics B, Vol.980, 115848, 2022; Journal of Cosmology and Astroparticle Physics, 05(2022)022) va DFG RTG 1620 "Models of Gravity" loyihasi doirasida foydalanilgan. Ilmiy natijalarning qo'llanilishi yuqori egrilikli gravitatsiya nazariyalarida qora tuynuk va o'tkazadigan yumronqoziq ini yechimlarining mavjud bo'lish sohalarini aniqlash imkonini bergan;

yuqori egrilik hadlariga ega bo'lgan gravitatsiya nazariyalarida yangi skalyarizatsiyalangan yechimlarini tadqiq etish davomida olingan natijalar Rossiya Federatsiyasi fan va oliy ta'lim vazirligining № 0723-2020-0041 "Elementar zarralar va kosmologiyaning asosiy xossalari" loyihasi, "RFBR 19-02-00346" loyihasi, va Malayziya texnologiya universitetining Q.J130000.2626.14J72 loyihasi doirasida foydalanilgan. Ilmiy natijalarning qo'llanilishi yuqori egrilikli gravitatsiya nazariyalarida qora tuynuk va yumronqoziq inlarini mavjudligi va xossalariга oid nazariy natijalarning muhim ilmiy asosli xulosalarini oqilona tushuntirishga imkon bergan;

Eynshteyn-skalyar-Gauss-Bonnet va Eynshteyn-skalyar-Chern-Simons gravitatsiya nazariyalarida skalyarizatsiyalangan yumronqoziq ini yechimlarini aniqlash va ularni Newman-Unti-Tamburino zaryadiga bog'liqligini o'rganish bo'yicha olingan natijalar xalqaro ilmiy jurnallarda (Journal High Energy Physics 104 (2021); The European Physical Journal C, Vol.82, 238, 2022; Journal of Cosmology and Astroparticle Physics, 05(2022)022) va "European COST action CA16104" loyihasida foydalanilgan. Ilmiy natijalarning qo'llanilishi massaga ega bo'lmagan zaryadli yumronqoziq ini yechimlarini va yumronqoziq ini yechimlarining katta sinfini aniqlash imkonini bergan.

**Tadqiqot natijalarining aprobatsiyasi.** Mazkur tadqiqotning asosiy natijalari 3 ta xalqaro va 10 ta Respublika miqyosidagi ilmiy-amaliy konferensiyalarda ma’ruzalar qilinib, tegishli muhokamalardan o’tkazilgan.

**Tadqiqot natijalarining e’lon qilinganligi.** Tadqiqot mavzusi bo’yicha jami 19 ta ilmiy ish, jumladan O’zbekiston Respublikasi Vazirlar Mahkamasi huzuridagi Oliy attestatsiya komissiyasining dissertatsiyalar asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlar ro’yxatida 6 ta ilmiy maqola (3 ta respublika va 3 ta xorijiy jurnallarda) chop etilgan.

**Dissertatsiyaning tuzilishi va hajmi.** Dissertatsiya tarkibi kirish, uchta bob, xulosa, foydalanilgan adabiyotlar ro’yxatidan iborat. Dissertatsiyaning hajmi 98 betni tashkil etadi.

## DISSERTATSIYANING ASOSIY MAZMUNI

Dissertatsiyaning **kirish** qismida mavzusining dolzarbligi, zarurati, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo’nalishlariga bog’liqligi, muammoning o’rganilganlik darajasi, dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejaları bilan bog’liqligi va tadqiqotning maqsadi, vazifalari, ob’yekti, predmeti, usullari, ilmiy yangiligi, amaliy natijasi, ishonchliligi, natijalarining ilmiy va amaliy ahamiyati, natijalarining amaliyotga joriy qilinishi, natijalarining aprobatsiyasi, natijalarining e’lon qilinishi hamda dissertatsiyaning tuzilishi va hajmi to’g’risida qisqacha ma’lumotlar keltirilgan.

Dissertatsiyaning birinchi bobi **“Umumiy nisbiylik va modernizatsiyalangan gravitatsiya nazariyalari”** deb nomlangan va unda Schwarzschild yechimlari, yumronqoziq inlari va Eynshteyn-Rosen ko’prigi, modernizatsiyalangan gravitatsiya nazariyalari va yuqori egrilikli nazariyalarga doir muhim ma’lumotlar ko’rib chiqildi. Shuningdek, asosiy e’tibor nazariy yo’l bilan olingan natijalarga qaratildi.

Dissertatsiyaning ikkinchi bobi **“Eynshteyn-skalyar-Gauss-Bonnet nazariyalarida skalyar o’zaro ta’sirlashuv potensialiga ega yumronqoziq inlari”** deb nomlangan va unda Massa hadi va o’zaro-ta’sirlashuv hadlarini o’z ichiga olgan skalyar maydon uchun Eynshteyn-skalyar-Gauss-Bonnet (EsGB) nazariyalarida potensialga ega bo’lgan yumronqoziq inlari ko’rib chiqilgan. Gauss-Bonnet (GB) bog’lanish doimiysi o’zgartirilib, ushbu nazariyalarda yumronqoziq inlarining mavjud bo’lish sohasining chegaralari aniqlangan. O’zaro ta’sirlashuvning ishtiroki mavjud bo’lish sohasini sezilarli darajada kengaytirdi. Bunda bir bo’g’izli yumronqoziq inlari va ikki bo’g’izli, ekvatorli yumronqoziq inlari hosil bo’ladi. Ushbu yumronqoziq inlarining fizik xususiyatlari, shu jumladan ularning massalari, o’lchamlari va geometriyalari tadqiq qilingan.

EsGB nazariyalarida quyidagi effektiv ta’sir integralidan foydalanilgan,

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - U(\phi) + F(\phi) R_{GB}^2 \right], \quad (1)$$

bu yerda  $R$  skalyar egrilik,  $\phi$  esa  $U(\phi)$  potensialga ega skalyar maydon va

$$R_{GB}^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2, \quad (2)$$

kvadratik Gauss-Bonnet hadi. Skalyar maydon  $F(\phi)$  bog‘lanish funksiyasi orqali  $R_{GB}^2$  kvadratik Gauss-Bonnet hadiga birlashtiriladi.

Eynshteyn tenglamalari (3) va skalyar maydon tenglamalari (4) ta’sir integralini metrika va skalyar maydon bo‘yicha variatsiyalashdan hosil bo‘ldi va nuqta  $\phi$  skalyar maydon bo‘yicha hosilani bildiradi.

$$G_{\mu}^{\nu} = T_{\mu}^{\nu}, \quad (3)$$

$$\nabla^{\mu}\nabla_{\mu}\phi + \dot{F}(\phi)R_{GB}^2 - \dot{U}(\phi) = 0. \quad (4)$$

Effektiv energiya-impuls tenzori quyidagi shakilni oladi

$$T_{\mu\nu} = -\frac{1}{4}g_{\mu\nu}(\partial_{\rho}\phi\partial^{\rho}\phi + 2U(\phi)) + \frac{1}{2}\partial_{\mu}\phi\partial_{\nu}\phi - \frac{1}{2}(g_{\rho\mu}g_{\lambda\nu} + g_{\lambda\mu}g_{\rho\nu})\eta^{\kappa\lambda\alpha\beta}\tilde{R}_{\alpha\beta}^{\rho\gamma}\nabla_{\gamma}\partial_{\kappa}F(\phi), \quad (5)$$

bu yerda  $\tilde{R}_{\alpha\beta}^{\rho\gamma} = \eta^{\rho\gamma\sigma\tau}R_{\sigma\tau\alpha\beta}$  va  $\eta^{\rho\gamma\sigma\tau} = \epsilon^{\rho\gamma\sigma\tau} / \sqrt{-g}$  almashtirishlar bajarildi.

Barqaror, sferik simmetrik yumronqoziq ini yechimlariga erishish uchun quyidagi chiziqli element shaklidan foydalanildi,

$$ds^2 = -e^{f_0}dt^2 + e^{f_1}\left[ d\eta^2 + h^2(d\theta^2 + \sin^2\theta d\varphi^2) \right]. \quad (6)$$

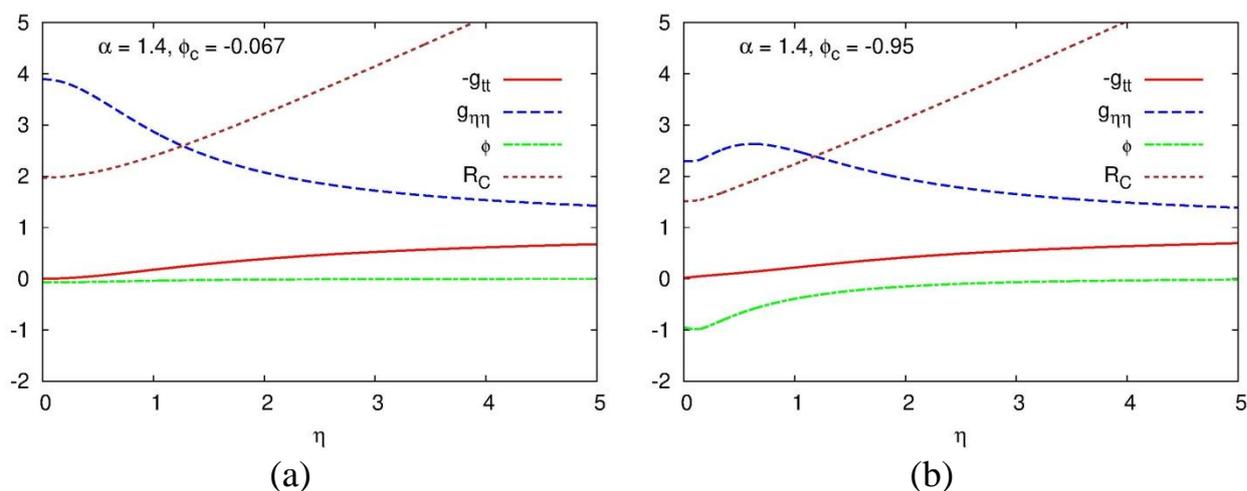
Bunda  $h^2 = \eta^2 + \eta_0^2$  yordamchi funksiya bo‘lib,  $\eta_0$  esa o‘lchamsiz parametr. Ikkita  $f_0$ ,  $f_1$  metrik funksiyalar va  $\phi$  skalyar maydon funksiyasi faqat  $\eta$  radial koordinataning funksiyalari.

Metrika va skalyar maydon uchun (6) tenglama orqali almashtirishlar o‘tkazildi. Natijada (4) skalyar maydon va (3) Eynshteyn tenglamalari to‘rtta bog‘langan, chiziqli bo‘lmagan, oddiy differensial tenglamalar (ODT) ga olib keldi. To‘rtta ODTlardan uchta ikkinchi tartibli va bittasi birinchi tartibli. Ammo, tenglamalardan faqat uchta mustaqil. Sonli tahlilimizda birinchi va ikkita ikkinchi tartibli ODTlar yechildi.

1-rasmda  $-g_{tt}$  va  $g_{\eta\eta}$  metrik komponentalarni,  $\phi$  skalyar maydonni va  $R_c$  sferik radiusni  $\eta$  radial koordinataga bog‘liqligi ko‘rsatilgan. Ikkita erkin parametrlar,  $\alpha$  bog‘lanish doimiysi va markazdagi  $\phi_c$  skalyar maydon qiymati uchun biz quyidagilarni tanladik: (a) qora tuynuk chegarasiga yaqin  $\phi_c$  ning qiymati uchun  $(1.4, -0.067)$ , (b)  $\phi_c$  ning kichik qiymati uchun  $(1.4, -0.95)$ .

Metrik funksiya  $-g_{tt}$  har doim monoton ravishda markazdagi kichik qiymatdan o‘zining asimptotik qiymatigacha ko‘tariladi. Odatda, metrik funksiya  $g_{\eta\eta}$  monoton emas, biroq o‘zining asimptotik qiymatiga yaqinlashishidan oldin, markazdagi minimumdan o‘zining maksimal qiymatiga oshadi. Ammo qora tuynuk

chegarasiga yaqinlashganda, u markazdagi o'zining maksimumi bilan monoton bo'ladi va  $g_{\eta\eta}(0)=4$  da qora tuynuk qiymatiga yaqinlashadi (1(b)-rasm). Qora tuynuk chegarasiga yaqinlashganda skalyar maydon juda kichik bo'ladi. Qora tuynuk chegarasiga yaqinlashganda,  $R_C$  ning qiymati  $R_C(0)=2$  ga yaqinlashadi (1(b)-rasm). Bir bo'g'izga ega yumronqoziq ini yechimlarida  $R_C$  monoton ravishda oshgan paytda, markazda ekvator va ikkita bo'g'izga ega yumronqoziq ini yechimlari uchun maksimum hosil bo'ladi.

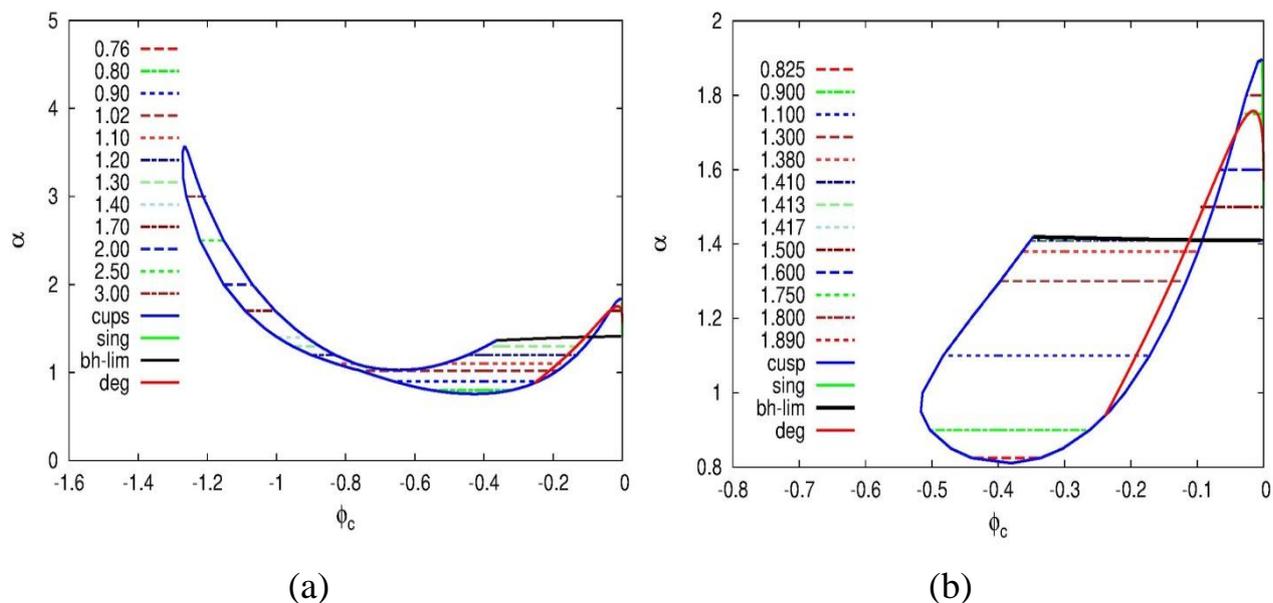


**1-rasm.** O'zaro ta'sirlashuv potensialiga ega yumronqoziq ini yechimlari uchun,  $-g_{tt}$  va  $g_{\eta\eta}$  metrik komponentalarni,  $\phi$  skalyar maydonni va  $R_C$  sferik radiusni  $\eta$  radial koordinataga bog'liqligi,  $(\alpha, \phi_c)$  parametrlar: (a)  $(\alpha, \phi_c) = (1.4, -0.067)$ , (b)  $(\alpha, \phi_c) = (1.4, -0.95)$ .

2-rasmda Massa va o'zaro-ta'sirga ega yumronqoziq inlari (chap ustun) va faqat massaga ega yumronqoziq inlari (o'ng ustun) uchun yumronqoziq ini yechimlarining mavjud bo'lish sohasi ko'rsatilgan. 2(a)-rasmda  $\alpha$  bog'lanish doimiysini markazda  $\phi_c$  skalyar maydon qiymatiga bog'lanish grafigi keltirilgan bo'lib, yumronqoziq ini yechimlari  $\alpha$  bog'lanish doimiysining  $\alpha_{m i n} \approx 0.7$  minimal qiymatidan boshlab paydo bo'ladi.  $\alpha$  oshgani sari mos ravishda yechimlar oilasi ham bog'lanish doimiysining  $\alpha_{cr} \approx 1.02$  kritik qiymatgacha kengayadi.

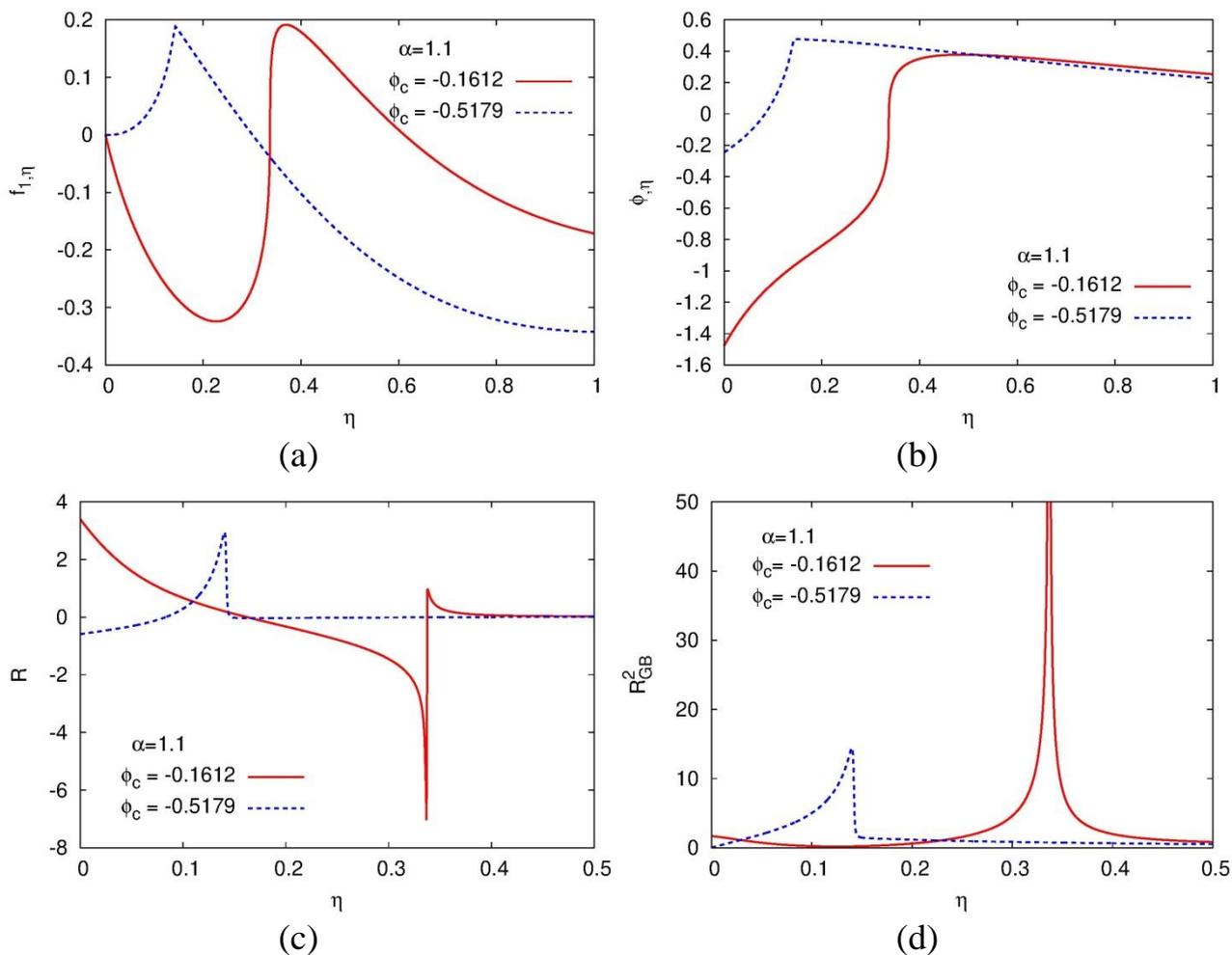
$\alpha$  ning navbatdagi oshishi yumronqoziq ini yechimlarining ikkita bog'lanmagan sohasiga olib keladi.  $\phi_c$  ning kichik qiymatlariga ega sohalarni  $\alpha_{max} \approx 3.56$  gacha davom etishi mumkin. Boshqa tarafdin  $\phi_c$  ning katta qiymatlariga ega sohalarda barcha uzoq davom etmaydi. Ular  $\alpha_{lim} \approx 1.83$  chegara qiymatida tugaydi. Mavjud bo'lish sohasi cho'qqi singulyarlik (ko'k) yechimlari, markazda singulyarlik yechimlari va skalyarizatsiyalangan EsGB qora tuynuk yechimlari orqali chegaralanadi. Bu Schwarzschild qora tuynugidan ularning ikkiga bo'linish nuqtasigacha bo'lgan barcha sohani o'z ichiga oladi.

Faqat massa hadiga ega yumronqoziq inlari uchun 2(b)-rasmda  $\alpha$  bog‘lanish doimiysining markazdagi  $\phi_c$  skalyar maydon qiymatiga bog‘lanish grafigi ko‘rsatilgan. Shuni ta’kidlash kerakki, mavjud bo‘lish sohasi asosan o‘zaro ta’sirga ega yumronqoziq inlarining mavjud bo‘lish sohasini faqat bir qismidan iborat bo‘ladi. Bu qism 2(a)-rasmdagi o‘ng tarafdagi sohaga mos tushadi, ya’ni  $\phi_c$  ning katta qiymatlariga mos tushadi. Shunday qilib, o‘zaro ta’sir natijasida  $\phi_c$  ning juda kichik qiymatlari va  $\alpha$  bog‘lanish doimiysining juda katta qiymatlariga ega bo‘lgan yumronqoziq ini yechimlari hosil bo‘ladi.



**2-rasm.**  $\alpha$  bog‘lanish doimiysining kiritilgan qiymatlar to‘plami uchun yumronqoziq ini yechimlarining mavjud bo‘lish sohasi. Markazdagi  $\phi_c$  skalyar maydonning har xil qiymatlarini  $\alpha$  bog‘lanish doimiysiga bog‘lanishi. Shuningdek, yechimlarning chegaralari, skalyarizatsiyalangan EsGB qora tuynuklari (qora), singulyar yechimlar (yashil), cho‘qqi singulyarliklari (ko‘k), buzilishga ega yumronqoziq ini yechimlari (qizil) ko‘rsatilgan.

3-rasmda mavjud bo‘lish sohasining chegaralari ko‘rsatilgan. Cho‘qqi singulyarliklarga ega bo‘lgan yechimlar ushbu chegaralarning katta qismini tashkil qiladi. Cho‘qqi singulyarliklarning mavjudligi determinantga bog‘liq bo‘lib, ushbu determinant, ikkinchi tartibli ODTlarni funksiyalarning ikkinchi tartibli hosilalariga nisbatan diagonallashtirishdan hosil bo‘ladi. Shuningdek, ushbu determinant  $\eta_*$  radial koordinataning qandaydir qiymatida tugunga ega bo‘lishi mumkin. Diagonalizatsiya determinant tomonidan taqsimlanishni o‘z ichiga olganligi sababli, harakat tenglamalari endi doimiy emas, balki o‘ziga xoslik xususiyatiga ega. Diagonalizatsiya determinant orqali qandaydir xususiyatni o‘z ichiga olganligi sababli, harakat tenglamalari  $\eta_*$  da uzoq regulyar bo‘lmaydi, biroq cho‘qqi singulyarligini o‘z ichiga oladi.

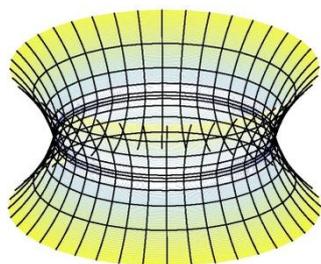


**3-rasm.** Mavjud bo‘lish sohasining chegarasi (o‘zaro ta’sirga ega namunalar):  $\eta_*$  ning ba’zi qiymatida cho‘qqi singulyarlikning paydo bo‘lishi:  $\alpha=1.1$  va markazdagi  $\phi_c$  skalyar maydonning ikkita qiymati uchun  $f_1$  (a) metrik funksiyaning va  $\phi$  (b) skalyar maydon funksiyasining birinchi darajali hosilasi:  $f_{1,\eta\eta}$  va  $\phi_{,\eta\eta}$  lar ba’zi  $\eta_*$  da sakrashni rivojlantiradi (ko‘k,  $\phi_c = -0.5179$ ),  $f_{1,\eta\eta}$  va  $\phi_{,\eta\eta}$  lar ba’zi  $\eta_*$  da ajraladi (qizil,  $\phi_c = -0.1612$ );  $R$  (c) skalyar egrilik va  $R_{GB}^2$  (d) Gauss-Bonnet hadini bir xil yechimlar uchun  $\eta$  radial koordinataga bog‘liqligi.

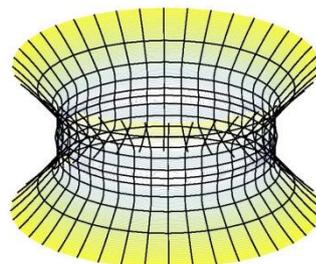
Yuqorida aytib o‘tilgan cho‘qqi singulyarliklar 3(a) va 3(b) rasmlarda keltirilgan bo‘lib, unda  $\alpha=1.0$  bog‘lanish doimiysi va markazdagi skalyar maydonning ikkita qiymatida,  $f_{1,\eta}$  metrik funksiyaning va  $\phi_{,\eta}$  skalyar funksiyaning birinchi tartibli hosilalarini  $\eta$  radial koordinataga nisbatan bog‘lanishi ko‘rsatilgan.  $\phi_c = -0.5179$  bo‘lganda  $f_{1,\eta\eta}$  va  $\phi_{,\eta\eta}$  ikkinchi tartibli hosilalar  $\eta_*$  radial koordinataning qandaydir qiymatida sakrashni vujudga keltiradi (ko‘k chiziq). Shuningdek,  $\phi_c = -0.1612$  bo‘lganda  $f_{1,\eta\eta}$  va  $\phi_{,\eta\eta}$  ikkinchi tartibli hosilalar  $\eta_*$  radial koordinataning qandaydir qiymatida ajraladi (qizil chiziq). Determinant  $(\eta - \eta_*)^g$  ga ega bo‘lib,  $g < 1$  bo‘lsa,  $f_{1,\eta\eta}$  va  $\phi_{,\eta\eta}$  ikkinchi tartibli

hosilalar ajraladi. Determinant  $(\eta - \eta_*)$ ga ega bo'lsa,  $f_{1,\eta\eta}$  va  $\phi_{,\eta\eta}$  larning sakrashi vujudga keladi. Bu bog'langan sakrashlar  $R$  (c) skalyar egrilikning va  $R_{GB}$  (d) GB skalyarining divergensiyalari deyiladi. Ular 3(c) va 3(d) rasmlarda bir xil yechimlar uchun ko'rsatilgan.

Biz 4(a)-rasmda bir bo'g'izli yumronqoziq ini geometriyasini vizuallashtirganmiz, bu yerda bog'lanish doimiysi uchun  $\alpha = 1.7$  qiymatni va markazdagi skalyar maydon uchun  $\phi_c = -0.0121$  qiymatni tanladik. 4(b)-rasmda ekvatorga va ikki bo'g'izga ega yumronqoziq ini tipining geometriyasi ko'rsatilgan, ushbu yumronqoziq iniga  $\alpha = 1.7$  va  $\phi_c = -0.03$  parametrlarni tanlash orqali erishildi. Rasmda ekvator va bo'g'izlar aniq ko'rinadi. Ikkala yechim ham o'zaro ta'sirlashuvchi yumronqoziq inlarining bitta oilasiga  $\alpha = 1.7$  tegishli.  $\phi_c$  oshgani sayin, markazdagi bir bo'g'iz (minimum) dastlab  $\phi_c$  ning kritik qiymatida degeneratsiyalangan bo'g'izga (egar nuqta) aylanadi va shundan so'ng barcha tarafdin bo'g'iz (minimum) bilan o'raladigan ekvator (maksimum) hosil bo'ladi.



(a)



(b)

**4-rasm.** (a) markazdagi  $\phi_c = -0.0121$  skalyar maydon qiymatida va  $\alpha = 1.7$  bog'lanish doimiysining qiymatida bir bo'g'izli yumronqoziq ini. (b) markazdagi  $\phi_c = -0.03$  skalyar maydon qiymatida va  $\alpha = 1.7$  bog'lanish doimiysining qiymatida ikki bo'g'izli yumronqoziq ini.

Dissertatsiyaning uchinchi bobi “**Skalyarizatsiyalangan Newman-Unti-Tamburino yumronqoziq inlari**” deb nomlangan va unda yuqori egrilikli gravitatsiya nazariyalarida Newman-Unti-Tamburino (NUT) zaryadiga ega skalyarizatsiyalangan yumronqoziq inlari o'rganilgan. Skalyar zaryad va bog'lanish parametrini o'zgartirish orqali, skalyarizatsiyalangan NUT yumronqoziq inlarining mavjud bo'lish sohalari va ularning NUT zaryadiga bo'g'liq bo'lishi ko'rsatilgan. Asosiy e'tibor bir bo'g'izli skalyarizatsiyalangan NUT yumronqoziq inlariga va ularni fizik xususiyatlarini o'rganishga qaratilgan.

Eynshteyn-skalyar-yuqori egrilikli nazariyalar uchun quyidagi effektiv ta'sir integralidan foydalandik,

$$S = \frac{1}{16\pi} \int \left[ R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + F(\phi) I(g) \right] \sqrt{-g} d^4 x. \quad (7)$$

$F(\phi)$  bog‘lanish funksiyasi uchun biz  $\alpha$  bog‘lanish doimiysiga ega kvadratik  $\phi$  skalyar maydon bog‘lanishini tanladik, ya’ni

$$F(\phi) = \alpha \phi^2, \quad (8)$$

$I(g)$  invariant uchun ikki xil tanlovni amalga oshirdik, (I) GB hadi uchun

$$I(g) = R_{\text{GB}}^2 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2, \quad (9)$$

(II) Chern-Simons (CS) hadi uchun

$$I(g) = R_{\text{CS}}^2 = {}^* R^\mu{}_\nu{}^{\rho\sigma} R^\nu{}_{\mu\rho\sigma}, \quad (10)$$

Bunda,  ${}^* R^\mu{}_\nu{}^{\rho\sigma} = \frac{1}{2} \eta^{\rho\sigma\kappa\lambda} R^\mu{}_{\nu\kappa\lambda}$  yulduzchali Riemann tenzori 4-o‘lchamli  $\eta^{\rho\sigma\kappa\lambda} = \epsilon^{\rho\gamma\sigma\tau} / \sqrt{-g}$  Levi-Civita tenzori bilan ifodalanadi. Ikkala invariant ham to‘rt o‘lchamli fazoda topologik bo‘lib,  $\phi$  skalyar maydonga va  $F(\phi)$  bog‘lanish funksiyasi orqali bog‘lanishi harakat tenglamalariga katta hissa qo‘shadi.

Agar (7) effektiv ta’sir integralini skalyar maydon va metrika bo‘yicha variatsiyalasak bog‘langan maydon tenglamalarining to‘plamiga erishamiz,

$$\nabla^\mu \nabla_\mu \phi + \frac{dF(\phi)}{d\phi} I = 0, \quad (11)$$

$$G_{\mu\nu} = \frac{1}{2} T_{\mu\nu}^{(\text{eff})}. \quad (12)$$

Bunda  $G_{\mu\nu}$  Eynshteyn tenzori va  $T_{\mu\nu}^{(\text{eff})}$  effektiv energiya-impuls tenzori,

$$T_{\mu\nu}^{(\text{eff})} = T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{(I)}. \quad (13)$$

Ushbu effektiv energiya-impuls tenzori skalyar maydonni,

$$T_{\mu\nu}^{(\phi)} = (\nabla_\mu \phi)(\nabla_\nu \phi) - \frac{1}{2} g_{\mu\nu} (\nabla_\rho \phi)(\nabla^\rho \phi), \quad (14)$$

va tegishli  $I(g)$  invariantni ifodalaydigan qismlardan tashkil topgan. Tanlangan (I) invariant uchun

$$T_{\mu\nu}^{(GB)} = (g_{\rho\mu} g_{\lambda\nu} + g_{\lambda\mu} g_{\rho\nu}) \eta^{\kappa\lambda\alpha\beta} R^{\rho\gamma}{}_{\alpha\beta} \nabla_\gamma \nabla_\kappa F(\phi), \quad (15)$$

va (II) invariant uchun esa

$$T^{(CS)\mu\nu} = -8 \left[ \nabla_\rho F(\phi) \right] \epsilon^{\rho\sigma\tau\mu} (\nabla_\tau R^\nu)_\sigma + \left[ \nabla_\rho \nabla_\sigma F(\phi) \right]^* R^{\sigma(\mu\nu)\rho}. \quad (16)$$

larga erishamiz.

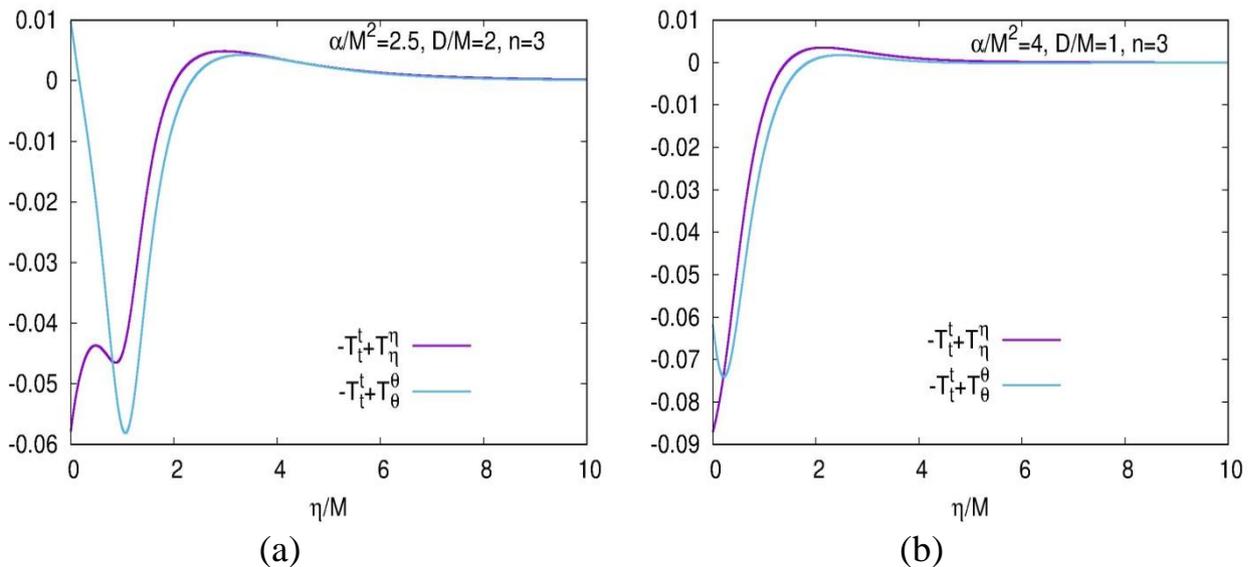
$N$ -NUT zaryadiga ega barqaror, sferik simmetrik yumronqoziq ini yechimlariga erishish uchun quydagi chiziqli element shaklidan foydalanildi,

$$ds^2 = -e^{f_0} (dt - 2N \cos \theta d\varphi)^2 + e^{f_1} \left[ dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right]. \quad (17)$$

Bunda ikkita  $f_0$  va  $f_1$  metrik funksiyalar va  $\phi$  skalyar maydon funksiyasi faqat  $r$  radial koordinataga bog'liq.

Metrik va skalyar maydon uchun yuqoridagi (17) tenglamani (11) skalyar maydon tenglamasiga va (13) effektiv energiya-impuls tenzoriga ega (12) Eynshteyn tenglamalariga qo'yganimizda, beshta bog'langan, chiziqli bo'lmagan ODTlarga ega bo'lamiz.

5-rasmda GB invarianti (a) va CS invarianti (b) uchun mos ravishda,  $\alpha/M^2 = 2.5$ ,  $D/M = 2$ ,  $n = N/M = 3$  va  $\alpha/M^2 = 4$ ,  $D/M = 1$ ,  $n = N/M = 3$  parametrlar tanlanib,  $-T_t^t + T_\eta^\eta \geq 0$  va  $-T_t^t + T_\theta^\theta \geq 0$  nol energetik holatlarning radial koordinataga bog'lanishi ko'rsatilgan. Ushbu figuralar ikkala invariant uchun ham nol energetikli holat buzilishini aniq ko'rsatib beradi.

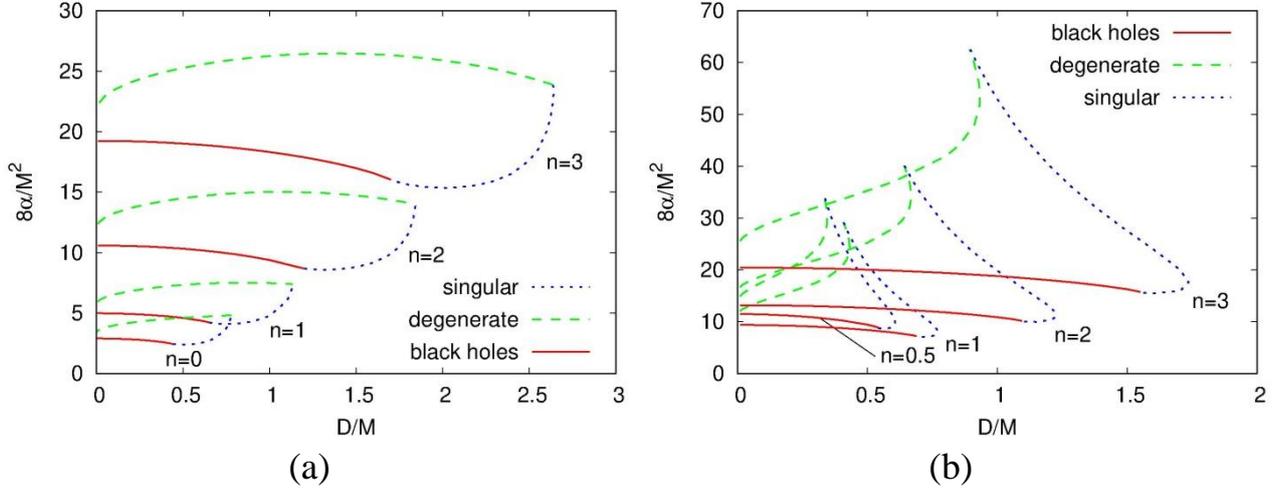


**5-rasm.** NUT yumronqoziq ini yechimlarining namunalari: (a)  $\alpha/M^2 = 2.5$ ,  $D/M = 2$  va  $n = N/M = 3$  parametrlar bilan Gauss-Bonnet; (b)  $\alpha/M^2 = 4$ ,  $D/M = 1$  va  $n = N/M = 3$  parametrlar bilan Chern-Simons;  $-T_t^t + T_\eta^\eta \geq 0$  va  $-T_t^t + T_\theta^\theta \geq 0$  nol energetik holatlarning  $\eta/M$  radial koordinataga bog'liqligi.

6-rasmda NUT yumronqoziq ini yechimlarining mavjud bo'lish sohalari GB invariant (a) va CS invariant (b) uchun,  $n = N/M$  NUT zaryadining qiymatlar to'plami uchun ko'rsatilagn. Xususan, mavjud bo'lishning tegishli sohalaridan tashqari chegaralarni,  $\alpha/M^2$  bog'lanish doimiysining  $D/M$  skalyar zaryadga nisbatan bog'lanishi orqali ko'rsatamiz.

Yumronqoziq ini yechimining mavjud bo'lishi skalyar maydon mavjudligini talab qiladi. Tenglamalar  $\phi \rightarrow -\phi$  almashtirishga nisbatan invariant bo'lgani uchun, mavjud bo'lish sohasi  $D \rightarrow -D$  ga nisbatan simmetrik bo'ladi va u faqat

$D \geq 0$  ko‘rinish uchun yetarli.  $D=0$  skalyar maydon yo‘qoladi, bu esa haqiqiy umumiy nisbiylik nazariyasi yechimlari hosil bo‘ladigan  $D=0$  mavjud bo‘lish sohasining chegarasini tasvirlaydi. Birinchi chegara skalyarizatsiyalangan NUT qora tuynuklar chegarasiga mos keladi va u uzluksiz qizil chiziqlar bilan ko‘rsatilgan.



**6-rasm.**  $n = N/M$  NUT zaryadining bir qancha qiymatlari uchun mavjud bo‘lish sohasi (chap ustun: Gauss-Bonnet, o‘ng ustun: Chern-Simons):  $\alpha/M^2$  bog‘lanish doimiysining  $D/M$  skalyar zaryad. Uzluksiz qizil chiziqlar qora tuynuk chegarasini, uzlukli yashil chiziqlar degenerativ yumronqoziq ini chegarasini va nuqtali ko‘k chiziqlar singulyarlik chegarasini ko‘rsatadi.

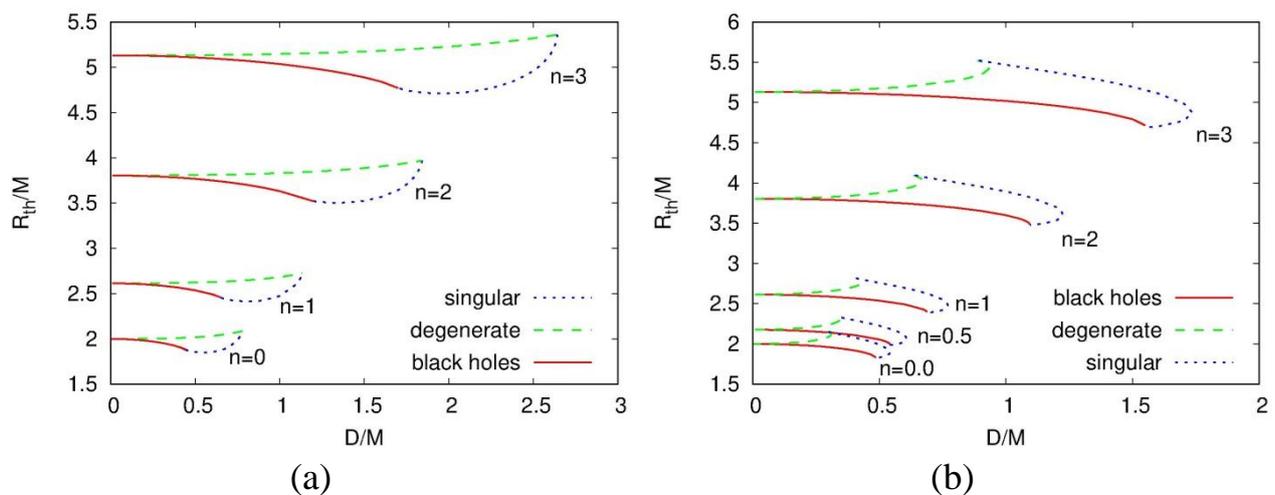
Rasmlardagi ikkinchi chegara degeneratsiya deb ataladi va uzlukli yashil egri chiziqlar bilan ko‘rsatilgan. Ushbu chegarani tushunish uchun yechimlarning sonli konstruksiyasidan foydalanamiz. Hisoblash ekstremumga erishilganda tugaydi. Ammo, prinsip jihatdan, ushbu ekstremumdan keyin ham hisoblashni davom ettirishimiz va bu yerda biz ikkinchi ekstremumni ham topishimiz mumkin. Shundan so‘ng birinchisi bo‘g‘izga, ikkinchisi esa ekvatorga to‘g‘ri keladi. Bog‘lanish doimiysi o‘zgarganligi sababli, ikkita ekstremma bir-biriga yaqinlashib, oxir-oqibat degenerativ ekstremumga erishiladi. Ushbu degenerativ ekstremumga erishilganda, uchinchi chegara bog‘lanish doimiysining aniq qiymatlarini ifodalaydi.

Oxirgi chegara singulyar deb nomlangan va nuqtali ko‘k egri chiziqlar bilan ko‘rsatilgan. Ushbu chegaradagi hisoblashlar vazo-vaqtning biror joyida cho‘qqi singulyarlikning paydo bo‘lishini ko‘rsatadi. Ularning mavjudligi ODTning diagonalizatsiyasida paydo bo‘ladigan determinantidagi  $\eta_*$  radial koordinataning ma‘lum bir qiymatida tugunning paydo bo‘lishi bilan bog‘liq. Ushbu cho‘qqi singulyarliklar skalyarizatsalangan yumronqoziq inlarining umumiy xususiyatlariga juda yaqindek ko‘rinadi. Bu yerda ular nafaqat GB nazariyalari uchun, balki CS nazariyalari uchun ham mavjudligini ko‘ramiz.

Shuningdek ushbu figuralar NUT zaryadining oshishi bilan yumronqoziq ini yechimlarining mavjud bo‘lish sohasining kuchli oshishini ko‘rsatadi. GB

bog‘lanishida NUT zaryadining yo‘qolish chegarasi skalyarizatsiyalangan yumronqoziq ini yechimlariga imkon beradi. CS bog‘lanishida esa NUT zaryadining yo‘qolishiga alohida e‘tibor berilishi kerak, chunki bu bog‘lanishda yechimlarning ikkita sohasi paydo bo‘ladi. Birinchi soha GB holatiga o‘xshash, chunki bu soha bo‘ylab NUT zaryadining kamayishi bilan bog‘lanish doimiysi kamayadi. Ammo ikkinchi soha bo‘ylab NUT zaryadining kamayishi bilan bog‘lanish doimiysi oshadi. 6(b)-rasmdan ko‘rish mumkinki, NUT zaryadining  $n = N/M = 3, 2$  va  $1$  qiymatlari uchun yechimlari birinchi sohaga mos keladi, NUT zaryadining  $n = N/M = 0.5$  qiymati uchun esa yechimlari ikkinchi sohaga mos keladi. Qizig‘i shundaki, NUT zaryadining qiymatlari qanchalik kichik bo‘lsa, skalyarizatsiyalangan yechimlarni olish uchun zarur bo‘lgan bog‘lanish doimiysining qiymatlari shunchalik katta bo‘lar ekan.

Yumronqoziq ini markazining xususiyatlarini ko‘rib chiqamiz. Bularga nafaqat ekvatorlar, balki bo‘g‘izlar kirganligi uchun bu xususiyatlarni bo‘g‘iz xususiyatlari deb qaraymiz. Ushbu xususiyatlar uchun tegishli sohalarning chegaralarini chizish orqali 7-rasmda ularning mavjud bo‘lish sohasini aniqlaymiz. Yuqorida aytib o‘tilgani kabi uzluksiz qizil chiziq orqali qora tuynuk chegarasini, uzlukli ko‘k chiziq orqali degeneratsiyani va nuqtali ko‘k chiziq orqali singulyarlik chegarasini ko‘rsatamiz.



**7-rasm.**  $n = N/M$  NUT zaryadining bir qancha qiymatlari uchun bo‘g‘izdagi xususiyatlar: (a) va (b)  $R_{th}/M$  sferik radiusni  $D/M$  skalyar zaryadga bog‘liqligi. Uzluksiz qizil chiziqlar qora tuynuk chegarasini, uzlukli yashil chiziqlar degenerativ yumronqoziq ini chegarasini va nuqtali ko‘k chiziqlar singulyarlik chegarasini ko‘rsatadi.

7-rasmda GB invariant (a) va CS invariant (b) uchun,  $n = N/M$  NUT zaryadining bir nechta qiymati uchun  $R_{th}/M$  qiymatining sohasini  $D/M$  skalyar zaryadga nisbatan grafigi berilgan. Shuni ta‘kidlash kerakki,  $n = N/M$  NUT zaryadining berilgan qiymatlari uchun skalyar zaryadning yo‘qolish chegarasida ( $D/M \rightarrow 0$ ) ikkala invariant uchun ham bir xil yechimlarga erishiladi. Ushbu chegarada skalyar maydonning bir xilda yo‘qolishi, yumronqoziq ini yechimlarini

hosil qilmaydi. Shu sababdan  $R_{th}/M$  ning qiymati faqatgina  $n=N/M$  NUT zaryadining qiymatiga bog‘liq bo‘lgani uchun ushbu chegaradagi yechimlar Schwarzschild-NUT qora tuynuk yechimiga to‘g‘ri keladi.

NUT zaryadi nolga intilganda skalyarizatsiyalangan qora tuynuklardan iborat chegara cheklangan bo‘lib qoladi. GB holatida skalyarizatsiyalangan Schwarzschild yechimlariga erishiladi. CS holatida bog‘lanish doimiysi ajradi, NUT zaryadi nolga intilganidek, skalyar maydonining asosiy hadi sababli cheklangan qiymat qoldiriladi.

GB holi uchun  $n=N/M \rightarrow 0$  bo‘lgan chegarada barcha mavjud bo‘lish sohasining ko‘rib chiqilishi chegaralangan, skalyarizatsiyalangan yumronqoziq ini yechimlariga olib keladi. CS holida  $n=N/M \rightarrow 0$  chegara yechimlarning chegaralangan sohasiga imkon beradi. Shunga qaramay,  $R_{th}/M$  sferik radiusining mavjud bo‘lish sohasining o‘zgarishi chegarada butunlay silliqdir, chunki  $n=N/M$  ning bir nechta kichik qiymatlari uchun hisoblashlar shuni ko‘rsatdi. chegaralanadigan soha 7(b)-rasmda ko‘rsatilgan. Ushbu rasmda yechimlar birinchi (katta  $n=N/M$ ) yoki ikkinchi (kichik  $n=N/M$ ) sohada bo‘lsa ham bir biridan farq qilmaydi.

## XULOSALAR

“Yuqori egrilik hadlariga ega bo‘lgan gravitatsiya nazariyalarida yangi skalyarizatsiyalangan yechimlar” mavzusidagi falsafa doktori (PhD) dissertatsiyasi bo‘yicha olib borilgan tadqiqotlar asosida quyidagi xulosalar taqdim etildi:

1. EsGB nazariyalarida potentsialdan foydalanilib yumronqoziq ini yechimlari olindi. Energiya-impuls tenzori energetik holatlarning buzilishiga olib kelishi ko‘rsatildi.

2. Markazda skalyar maydon qiymati va GB bog‘lanish doimiysini o‘zgartirib, yumronqoziq ini yechimlarining mavjud bo‘lish sohasini topdik.

3. Mavjud bo‘lish sohasi, faqatgina massa hadi bilan taqqoslaganda, o‘zaro ta’sirning mavjudligi tufayli sezilarli darajada oshishi ko‘rsatildi. O‘zaro ta’sir tufali mavjud bo‘lgan maydon parametrlarining yangi sohasida ko‘plab yumronqoziq inlarining yechimlari olindi.

4. Ko‘pgina yumronqoziq inlari markazda bir bo‘g‘izga ega bo‘lishini ko‘rsatdik. Biroq, ba’zi yumronqoziq inlari har tomondan minimal bilan o‘ralgan markazda radiusning maksimumni hosil qiladi. Bunday holatda, yumronqoziq ini yechimlari bo‘g‘iz orqali har bir asimptotik tekis fazolarni bog‘laydigan ekvator va ikki bo‘g‘izga ega bo‘ladi.

5. GB va CS invariantlari uchun skalyarizatsiyalangan NUT zaryadli yumronqoziq ini yechimlarini oldik. Ushbu skalyarizatsiyalangan yumronqoziq inlarining mavjud bo‘lish sohalari ikkala invariantlar uchun ham bir xil turdagi chegaralardan iborat bo‘lib, ushbu chegaralar qora tuynuk va yumronqoziq ini chegaralaridan tashkil topgan.

Автореферат “Фан ва инновациялар” халқаро илмий журнали (International scientific journal “Science and Innovation”) таҳририятида таҳрирдан ўтказилиб, ўзбек, инглиз ва рус тилларидаги матнлари ўзаро мувофиқлаштирилди (20.09.2022).

**SCIENTIFIC COUNCIL DSc.03/31.03.2022.T/FM.10.04 ON AWARD OF  
SCIENTIFIC DEGREE AT INSTITUTE OF FUNDAMENTAL AND  
APPLIED RESEARCH “TIAME” NATIONAL RESEARCH UNIVERSITY**  

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**SAMARKAND STATE UNIVERSITY NAMED AFTER SHAROF  
RASHIDOV**

**MURODOV SARDOR NORMUMIN UGLI**

**NOVEL SCALARIZED SOLUTIONS IN GRAVITY THEORIES WITH  
HIGHER CURVATURE TERMS**

**01.04.02 – Theoretical physics**

**ABSTRACT**

**of the dissertation of Doctor of Philosophy (PhD) on physical and mathematical sciences**

**Samarkand – 2022**

**The theme of the dissertation of the doctor of philosophy (PhD) on physical and mathematical sciences was registered by the Supreme Attestation Commission of the Cabinet of Ministers of the Republic of Uzbekistan under No.B2021.1.PhD/FM570.**

The doctoral (PhD) dissertation was carried out at Samarkand State University named after Sharof Rashidov.

The abstract of the dissertation was posted in three (Uzbek, English, and Russian (resume)) languages on the website of the Scientific Council ([www.tiame.uz](http://www.tiame.uz)) and on the website of "Ziyonet" informational and educational portal ([www.ziyonet.uz](http://www.ziyonet.uz)).

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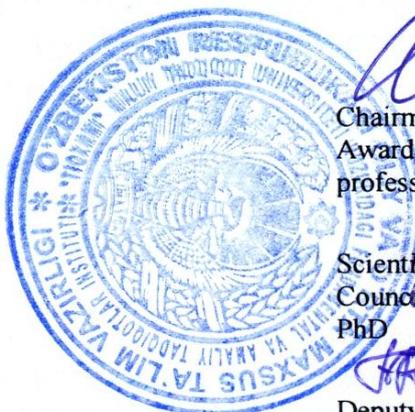
**Leading organization:**

**Al-Farabi Kazakh National University**

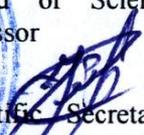
The defense of the dissertation will be held on "14" October 2022, at the meeting of the Scientific Council DSc.03/31.03.2022.T/FM.10.04 at institute of fundamental and applied research "TIAME" National Research University (Address: 39, Kari-Niyaziy street, Tashkent 100000 Uzbekistan. Phone: (+99871) 237-19-31; fax: (+99871) 237-38-79, e-mail: ifar@tiame.uz)

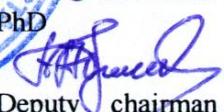
The doctoral (PhD) dissertation can be looked through in the Information Resource Center of Samarkand State University named after Sharof Rashidov (registered under No. 95). (Address: 15, University boulevard, 140104, Samarkand city. Phone: (+99866) 239-11-51).

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## INTRODUCTION (Annotation of PhD dissertation)

**The Topicality and demand of the dissertation topic.** Recent researches around the world suggest that new solutions of modified Einstein's field equations predict the existence of not only black holes, but also wormholes. In the kinds of literature, wormholes are referred to as "tunnels" that connect distant regions of the universe or "bridges" that connect different universes. Initially, in 1935, a solution to the wormhole (Einstein-Rosen bridge) was proposed by Einstein himself. In this field, The fundamental work of M.S. Morris, K.S. Thorne, and U. Yurtsever and the geometrodynamics of J.A.Y. Wheeler, form the basis of modern research. One of the reasons why wormhole solutions have been studied for so long is that it allows humanity to travel long distances between stars and galaxies. For this reason, a theoretical study of the nature of wormholes is important.

In world science, the study of wormholes has become particularly relevant in recent decades. This is due to the modern interest in "exotic" forms of matter. It is known that the presence of wormholes requires some exotic form of matter, which violates several energy conditions. At present, there is evidence that such matter may exist in the universe. This is primarily due to the discovery of the accelerated expansion of the Universe, which requires the introduction of a new exotic matter called "dark energy". However, in some alternative theories of gravity, it is shown that the violation of energy conditions can also be due to gravitational interaction. Indeed, alternative theories of gravity can give rise to an effective stress-energy tensor that leads to violation of the energy conditions without the need for exotic matter. The most notable of these alternative theories of gravitation are the Einstein-scalar-Gauss-Bonnet and Einstein-scalar-Chern-Simons theories, and the study of wormholes is now a topical issue in these theories.

In recent years, in our country, more and more attention has been paid to the development of current directions of fundamental and applied research. In particular, the development of astrophysical research, which is one of the promising areas, is an important issue today. The main directions of fundamental research and development and their practical application for the successful development of science in our country are reflected in the Strategy<sup>12</sup> for the further development of the Republic of Uzbekistan from 2022-2026. Therefore, the research of scalarized solutions in higher curvature gravity theories remains one of the urgent issues in the field of fundamental research.

This dissertation serves to a certain extent in the implementation of the Decree of the President of the Republic of Uzbekistan No. PF-5847 of October 8, 2019 "On approval of the Concept of development of the higher education system of the Republic of Uzbekistan until 2030", of the decisions of the President of the Republic of Uzbekistan No. PQ-5032 of March 19, 2021 "On measures to improve the quality of education and development of scientific research in the field of physics", No. PQ-2789 of February 17, 2017 "On measures to further improve the

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<sup>1</sup> Decree No. PF-60 of the President of the Republic of Uzbekistan dated January 1, 2022 "On the Development Strategy of New Uzbekistan for 2022-2026"

activities of the Academy of Sciences, the organization, management, and funding of research” and tasks set out in other normative legal acts on the subject.

**Conformity of the research to the main priorities of science and technology development of the Republic.** The dissertation research has been carried out in accordance with the priority areas of science and technology in the Republic of Uzbekistan: II. “Physics, astronomy, energy and mechanical engineering”.

**Degree of study of the problem.** In recent years, many fundamental studies have been conducted to find scalarized solutions in gravitational theories with high curvature terms, such as the researches of G.Antoniou, A.Bakopoulos, P.Kanti, B.Kleihaus, and J.Kunz (University of Minnesota, USA; University of Ioannina, Greece; University of Oldenburg, Germany), novel wormholes were obtained in EsGB theory for several coupling functions. The wormholes may feature a single-throat or a double-throat geometry and do not demand any exotic matter. Scalarized black hole solutions have also been found. Furthermore, in the works of P.Kanti, B.Kleihaus, and J.Kunz (University of Ioannina, Greece; University of Oldenburg, Germany), traversable wormholes have been found in dilatonic Einstein-Gauss-Bonnet theory in four spacetime dimensions, without needing any form of exotic matter. They determined their domain of existence and showed that these wormholes satisfy a generalized Smarr relation. They also demonstrated linear stability with respect to radial perturbations for a subset of these wormholes.

In the researches of J.L. Blázquez-Salcedo, D. D. Doneva, S. Kahlen, J. Kunz, P. Nedkova, and S. S. Yazadjiev (the University of Tübingen, University of Oldenburg, Germany; Sofia University, Institute of Mathematics and Informatics, Bulgaria) the polar quasinormal modes of spontaneously scalarized black holes were studied in Einstein-Gauss-Bonnet theory. They calculated the polar quasinormal modes and showed that the set of solutions is stable against the polar perturbations.

Russian scientists Kirill Bronnikov and Roman Konoplya and Czech scientist Thomas Pappas used a method based on a continued-fraction expansion in terms of a compactified radial coordinate to develop a general parametrization for spacetimes of spherically symmetric Lorentzian, traversable wormholes in an arbitrary metric theory of gravity.

Uzbek scientists Bobomurat Ahmedov and Sanjar Shaymatov considered a black hole solution in the 4D limit of Einstein-Gauss-Bonnet theory and studied the motion of charged particles in spacetime when an external magnetic field is present. The solution reduces to the Schwarzschild case when the Gauss-Bonnet (GB) coupling vanishes. They found the effect of the GB coupling on the innermost stable circular orbit for circular orbits of charged particles and determined the collision energy in the vicinity of the horizon.

An analysis of recent scientific studies showed that currently there is no single point of view in modernized gravitational theories that can fully explain the properties of black holes and wormholes, which makes it more relevant to conduct new theoretical research within the scope of the scientific problem.

**Connection of the topic of the dissertation topic to the scientific works of higher education and research institutions, where the dissertation is carried out.** The dissertation work was carried out based on the research plan of Samarkand state university, Department of Theoretical Physics and Quantum Electronics, Faculty of Physics for 2017-2021 on the topic “Research on modern problems of theoretical physics”. Also, It has been carried out under the project of the Ministry of Innovative Development of the Republic of Uzbekistan for 2022-2026 № FZ-20200929385 “New solutions of Einstein’s equations for black holes and Wormholes”.

**The aim of the research** is to find scalarized wormhole solutions in Einstein-scalar-Gauss-Bonnet and Einstein-scalar-Chern-Simons gravitational theories with high curvature terms.

**The tasks of the research:**

to find wormhole solutions in Einstein-scalar-Gauss-Bonnet gravitational theories with a potential for the scalar field that includes a mass term and self-interaction terms;

to delimit the domain of the existence of wormholes in Einstein-scalar-Gauss-Bonnet gravitational theories by varying the Gauss-Bonnet coupling constant;

to study the physical properties of wormholes with a single throat and wormholes with an equator and a double throat including their mass, their size, and their geometry;

to construct scalarized wormholes with a Newman-Unti-Tamburino charge in higher curvature gravitational theories;

to find the scalarized wormhole solutions by varying the coupling parameter and the scalar charge in Einstein-scalar-Gauss-Bonnet and Einstein-scalar-Chern-Simons gravitational theories, and to study their dependence on the Newman-Unti-Tamburino charge.

**The object of the research** is higher curvature gravitational theories and wormholes.

**The subject of the research** is higher curvature terms, the effective action, the Einstein and the scalar field equations, the line elements, the Israel junction conditions, and the null energy condition.

**The methods of the research.** Reducing the field equations leads to four coupled, nonlinear, ordinary differential equations. The numerical integration of the system of coupled ordinary differential equations was done with the help of COLSYS program package, using the finite element method. We have employed the fourth-order Runge Kutta method for the system of ordinary differential equations as an initial value problem.

**The scientific novelty of the research** is the following:

in Einstein-scalar-Gauss-Bonnet gravitational theories, wormholes with a single throat and wormholes with an equator and a double throat with a scalar self-interaction potential have been obtained. By varying the Gauss-Bonnet coupling constant the domain of existence of wormholes has been determined in these theories;

in Einstein-scalar-Gauss-Bonnet and for the first time in Einstein-scalar-Chern-Simons gravitational theories with high curvature terms, novel scalarized Newman-Unti-Tamburino wormholes have been found by varying the coupling parameter and the scalar charge, as well as their dependence on the Newman-Unti-Tamburino charge;

in the Gauss-Bonnet case, the set of scalarized wormhole solutions has been reached in the limit of vanishing the Newman-Unti-Tamburino charge. In the Chern-Simons invariant case, however, it has been found that the limits of these wormhole solutions are inherently separated into two branches since the Newman-Unti-Tamburino charge vanishes and the coupling constant diverges.

**The practical results of the research** are the following:

The research carried out, in general, in Einstein-scalar-Gauss-Bonnet and Einstein-scalar-Chern-Simons gravitational theories, the discovery and study of wormhole solutions, expands and deepens our understanding of these astrophysical objects, allowing a deeper understanding of the nature and physical properties of such objects. Modern research in the theory of gravity allows us to propose new models that allow us to understand the physical aspects of wormholes. The properties of these wormholes, and the forces, and fields that affect them, bring humanity closer to solving the problem of long-distance movement in space.

**The reliability of the research results.** The wormhole solutions studied in this research work are interrelated and continuous with fundamental research in the field of gravitational theories, not contradict the information given in the scientific literature, in some cases, the values obtained are similar or completely consistent with generally accepted positions, mathematical substitutions, and the accuracy of the calculations is explained by the fact that the main results have been published in leading scientific publications, as well as discussed several times at seminars and conferences.

**The scientific and practical significance of the research results.** The scientific significance of the research results is that the results obtained contribute to the development of our understanding of astrophysical objects similar to scalarized wormhole solutions in gravitational theories with high curvature.

The practical significance of the research results is that this dissertation work is of a theoretical nature and the results obtained can be used to study wormholes and black holes in more depth and to explain modern research in cosmology. Therefore, the results of the dissertation can be used in the educational process for master's and doctoral students studying physics.

**Application of the research results.** Based on the results of the study of novel scalarized solutions in gravity theories with higher curvature terms:

in Einstein-scalar-Gauss-Bonnet gravitational theories, the results obtained during the research of wormholes with a single throat and wormholes with an equator and a double throat with a scalar self-interaction potential were used in international scientific journals (Physical Review D 103, 124062, 2021; Physical Review D 104, 024048, 2021; Physical Review D 104, 044006, 2021; The European Physical Journal C, Vol.81, 285, 2021; The European Physical Journal C, Vol.81, 858, 2021; Journal of Cosmology and Astroparticle Physics,

10(2021)059; Journal High Energy Physics, 104, 2021; Nuclear Physics B, Vol.980, 115848, 2022; Journal of Cosmology and Astroparticle Physics, 05(2022)022; arXiv:2010.00197, 2020; arXiv:2204.08232, 2022; arXiv:2204.11851, 2022) and within the frame of the DFG RTG 1620 “Models of Gravity” project. The application of scientific results made it possible to determine the domain of the existence of solutions of the black hole and the travel wormhole in the high-curvature gravity theories;

the results obtained during the research on Novel scalarized solutions in gravity theories with higher curvature terms have been used in the framework of the Ministry of Science and Higher Education of the Russian Federation, project “Fundamental properties of elementary particles and cosmology” N 0723-2020-0041, RFBR Project No 19-02-00346 and project Q.J130000.2626.14J72 of the Universiti Teknologi Malaysia. The use of scientific results could make it possible to rationally explain the important scientifically based conclusions of the theoretical results regarding the existence and properties of black holes and wormholes in the high-curvature gravity theories;

in Einstein-scalar-Gauss-Bonnet and Einstein-scalar-Chern-Simons gravitational theories, the results obtained during the research of scalarized wormhole solutions and studying their dependence on the Newman-Unti-Tamburino charge were used in international scientific journals (Journal High Energy Physics 104 (2021); The European Physical Journal C, Vol.82, 238, 2022; Journal of Cosmology and Astroparticle Physics, 05(2022)022; arXiv:2204.11851, 2022) and within the frame of the European COST action CA16104. The application of scientific results made it possible to determine Massless charged wormhole solutions and a large class of wormhole solutions;

**Testing of the research results.** The research results were reported and discussed at 3 international conferences and 10 local scientific conferences.

**Publication of the research results.** In the field of PhD topic, 19 scientific works were published, of which 6 papers (3 in national and 3 in foreign journals) in scientific journals recommended by the Supreme Attestation Commission of the Republic of Uzbekistan for publishing basic scientific results of PhD theses.

**Volume and structure of the dissertation.** The PhD dissertation consists of an introduction, three chapters, a conclusion, and a bibliography. The volume of the dissertation is 98 pages.

## THE MAIN CONTENTS OF THE DISSERTATION

**In the introduction** the topicality and demand of the dissertation topic, the main aims and tasks of the dissertation are set out, the objects and subjects of the dissertation are formulated, the scientific novelty and the practical results were emphasized, and the reliability of the research results and their scientific and practical significance where specified, the application, testing, publication of the research results and the dissertation structure were given.

The first chapter of the dissertation is entitled “**General relativity and modified gravitational theories**” and it covers essential information on The Schwarzschild solutions, wormholes and the Einstein-Rosen bridge, modified

gravitational theories and Higher curvature theories. Also, primary attention was paid to the theoretical results.

The second chapter of the dissertation is entitled **“Wormholes in Einstein-scalar-Gauss-Bonnet theories with a scalar self-interaction potential”** and wormholes were considered in Einstein-scalar-Gauss-Bonnet (EsGB) theories with a potential for the scalar field that includes a mass term and self-interaction terms. By varying the Gauss-Bonnet (GB) coupling constant we delimit the domain of the existence of wormholes in these theories. The presence of the self-interaction enlarges the domain of existence significantly. There arise wormholes with a single throat and wormholes with an equator and a double throat. We determine the physical properties of these wormholes including their mass, their size and their geometry.

The following effective action is used in EsGB theories,

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - U(\phi) + F(\phi) R_{GB}^2 \right], \quad (1)$$

where  $R$  is the curvature scalar,  $\phi$  is the scalar field potential  $U(\phi)$ , and

$$R_{GB}^2 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2, \quad (2)$$

is the quadratic Gauss-Bonnet term. With coupling function  $F(\phi)$ , scalar field is coupled to the quadratic Gauss-Bonnet term  $R_{GB}^2$ .

The Einstein equations and the scalar field equation are obtained from the variation of the action with respect to the metric and the scalar field,

$$G_\mu^\nu = T_\mu^\nu, \quad (3)$$

$$\nabla^\mu \nabla_\mu \phi + \dot{F}(\phi) R_{GB}^2 - \dot{U}(\phi) = 0. \quad (4)$$

The effective stress-energy tensor is given by the expression,

$$T_{\mu\nu} = -\frac{1}{4} g_{\mu\nu} \left( \partial_\rho \phi \partial^\rho \phi + 2U(\phi) \right) + \frac{1}{2} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \left( g_{\rho\mu} g_{\lambda\nu} + g_{\lambda\mu} g_{\rho\nu} \right) \eta^{\kappa\lambda\alpha\beta} \tilde{R}_{\alpha\beta}^{\rho\gamma} \nabla_\gamma \partial_\kappa F(\phi). \quad (5)$$

Here, we have defined  $\tilde{R}_{\alpha\beta}^{\rho\gamma} = \eta^{\rho\gamma\sigma\tau} R_{\sigma\tau\alpha\beta}$  and  $\eta^{\rho\gamma\sigma\tau} = \epsilon^{\rho\gamma\sigma\tau} / \sqrt{-g}$ , and the dot denotes the derivative with respect to the scalar field  $\phi$ .

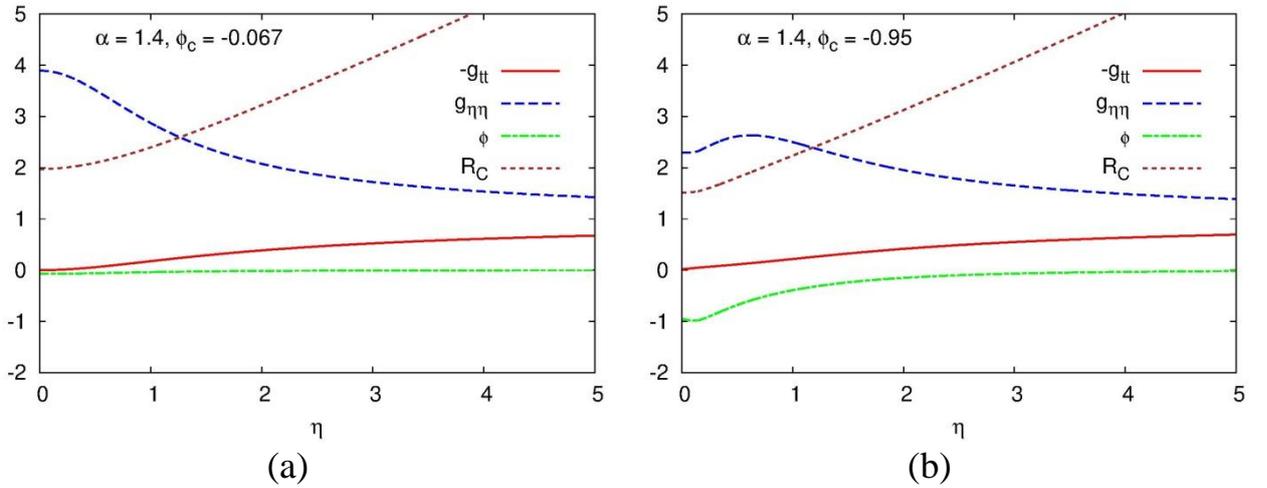
To obtain static, spherically symmetric wormhole solutions we assume the line element in the form

$$ds^2 = -e^{f_0} dt^2 + e^{f_1} \left[ d\eta^2 + h^2 \left( d\theta^2 + \sin^2 \theta d\varphi^2 \right) \right], \quad (6)$$

with the auxiliary function  $h^2 = \eta^2 + \eta_0^2$ , where  $\eta_0$  is a scaling parameter. The two metric functions  $f_0$  and  $f_1$  and the scalar field function  $\phi$  are functions of the radial coordinate  $\eta$  only.

Substitution of the above ansatz (6) for the metric and the scalar field in the Einstein equations and in the scalar-field equation leads to four coupled, nonlinear, ordinary differential equations (ODEs). Out of the four ODEs, three ODEs are of second order, and one ODE is of the first order. But only three of the equations are independent. In our numerical analysis, we solve the first order and two of the second order ODEs.

We exhibit in Figure 1 the profile functions for a set of wormholes solutions with self-interaction potential. Shown are the metric components  $-g_{tt}$  and  $g_{\eta\eta}$ , the scalar field  $\phi$ , and the circumferential radial coordinate  $R_C$  vs the radial coordinate  $\eta$ . For the two free parameters, the coupling constant  $\alpha$  and the value of the scalar field  $\phi_c$  at the center, we have selected the values (a)  $(1.4, -0.067)$ , a value of  $\phi_c$  close to the black hole limit; (b)  $(1.4, -0.95)$ , a small value of  $\phi_c$ .

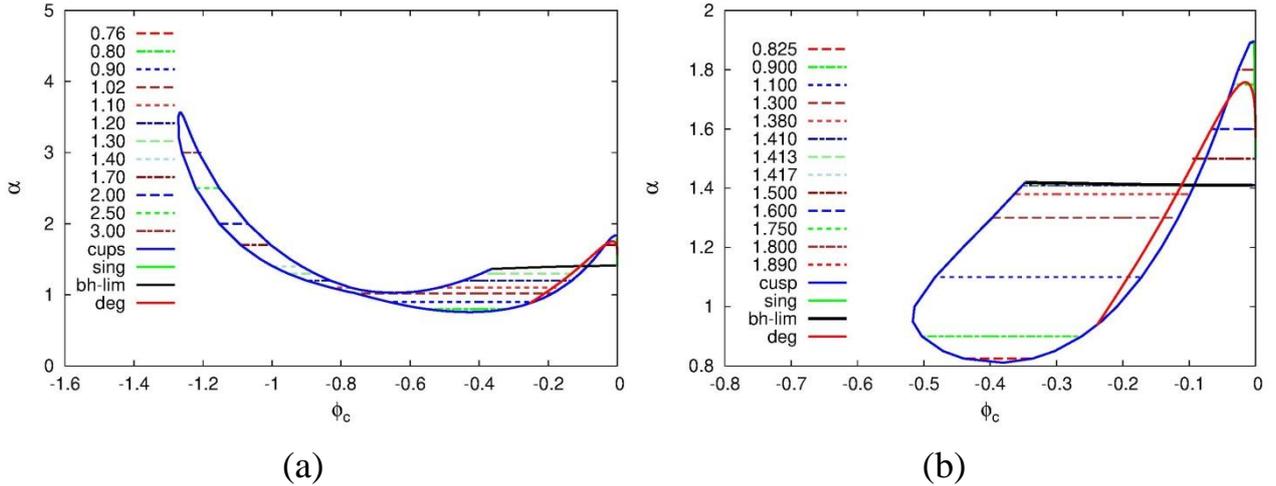


**Figure 1.** Metric components  $-g_{tt}$  and  $g_{\eta\eta}$ , scalar field  $\phi$ , and circumferential radial coordinate  $R_C$  vs radial coordinate  $\eta$  for a set of self-interacting wormhole solutions with parameters  $(\alpha, \phi_c)$ : (a)  $(1.4, -0.067)$ , (b)  $(1.4, -0.95)$ .

The metric function  $-g_{tt}$  is always monotonically rising from a small value at the center to its asymptotic value. The metric function  $g_{\eta\eta}$  is typically not monotonic, but exhibits a minimum at the center, from where it rises to its maximum, before approaching its asymptotic value. When the black hole limit is approached, however, it becomes monotonic with its maximum at the center approaching the black hole value of  $g_{\eta\eta}(0) = 4$  [see Figure 1(b)]. The scalar field becomes very small as the black hole limit is approached. Of interest is also the circumferential coordinate  $R_C$ . As the black hole limit is approached, the value of  $R_C$  at the center approaches  $R_C(0) = 2$  [see Figure 1(b)]. While  $R_C$  is monotonically increasing in the case of wormhole solutions with a single throat, it has a maximum at the center for wormhole solutions with an equator and a double throat.

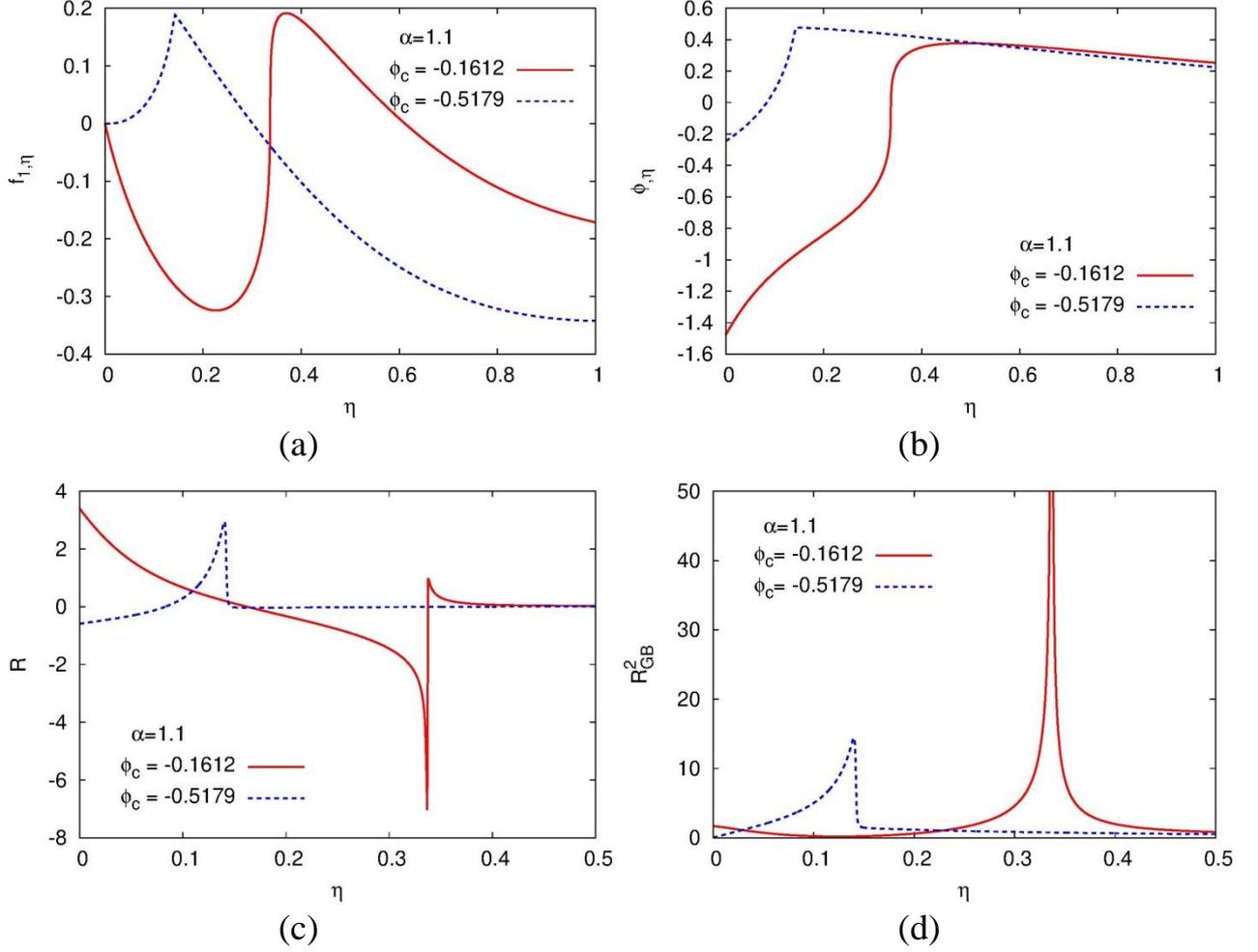
The domain of the existence of wormhole solutions is illustrated in Figure 2 for wormholes with mass and self-interaction (left column) and wormholes with a mass term only (right column). As seen in Figure 2(a), where the coupling constant  $\alpha$  is shown versus the value of the scalar field at the center  $\phi_c$ , wormhole solutions arise for a minimal value of the coupling constant  $\alpha$ ,  $\alpha_{\min} \approx 0.76$ . As  $\alpha$  increases, the corresponding families of solutions expand, until a critical value of  $\alpha$ ,  $\alpha_{\text{cr}} \approx 1.02$ , is reached.

Further increase of  $\alpha$  then leads to two disconnected branches of wormhole solutions. The branches with the smaller values of  $\phi_c$  can be continued up to  $\alpha_{\max} \approx 3.56$ . The branches with the larger values of  $\phi_c$ , on the other hand, cannot be continued that far. They end at a limiting value of  $\alpha_{\text{lim}} \approx 1.83$ . The domain of existence is delimited by solutions with cusp singularities (blue), solutions with singularities at the center (green), and scalarized EsGB black hole solutions (black), reaching all the way up to their bifurcation point from the Schwarzschild black hole.



**Figure 2.** The domain of existence of wormhole solutions for a set of fixed values of the coupling constant  $\alpha$ . Various quantities are shown vs the value of the scalar field at the center  $\phi_c$  and coupling constant  $\alpha$ . Also shown are the limiting solutions, scalarized EsGB black holes (black), singular solutions (green), cusp singularities (blue), and the degenerate wormhole solutions (red).

For wormholes with a mass term only, the coupling constant  $\alpha$  is shown versus the value of the scalar field at the center  $\phi_c$  in Figure 2(b). We immediately note, that the domain of existence basically only consists of one part of the domain of existence of wormholes with self-interaction. This part more or less agrees with the right-hand side of the domain shown in Figure 2(a), and thus the larger values of  $\phi_c$ . Thus the effect of the self-interaction is to allow for wormhole solutions with much smaller values of  $\phi_c$  and much larger values of the coupling constant  $\alpha$ .

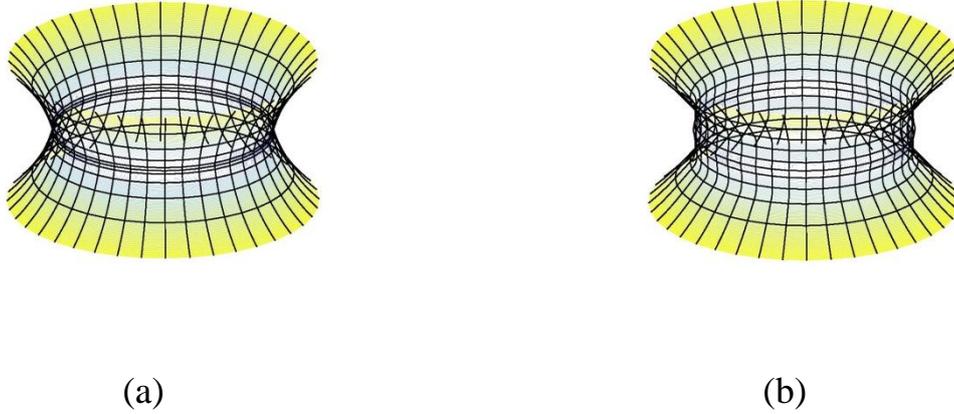


**Figure 3.** The boundary of the domain of existence (examples with self-interaction): the emergence of a cusp singularity at some value  $\eta_*$  the first derivative of the metric function  $f_1$  (a) and of the scalar field function  $\phi$  (b) for  $\alpha=1.1$  and two values of the scalar field at the center  $\phi_c$ :  $f_{1,\eta\eta}$  and  $\phi_{,\eta\eta}$  develop a jump at some  $\eta_*$  (blue,  $\phi_c = -0.5179$ ),  $f_{1,\eta\eta}$  and  $\phi_{,\eta\eta}$  diverge at some  $\eta_*$  (red,  $\phi_c = -0.1612$ ); scalar curvature  $R$  (c) and Gauss-Bonnet term  $R_{GB}^2$  (d) vs radial coordinate  $\eta$  for the same solutions.

In Figure 3, the boundary of the domain of existence are shown. Solutions with cusp singularities form the largest part of the boundary. The emergence of cusp singularities is related to the determinant that is encountered upon diagonalization of the set of second-order ODEs with respect to the second derivatives of the functions. This determinant may possess a node at some value  $\eta_*$  of the radial coordinate. Since diagonalization involves division by the determinant, the equations of motion are then no longer regular at  $\eta_*$ , but feature a cusp singularity.

We demonstrate how such cusp singularities form in Figs. 3(a) and 3(b), where we show for the metric function  $f_1$  (a) and the scalar function  $\phi$  (b) the

first derivative  $f_{1,\eta}$  and  $\phi_{,\eta}$  versus the radial coordinate  $\eta$  for the coupling constant  $\alpha=1.0$  and two values of the scalar field at the center  $\phi_c$ : for  $\phi_c = -0.5179$  the second derivatives  $f_{1,\eta\eta}$  and  $\phi_{,\eta\eta}$  develop a jump at some value  $\eta_*$  (blue curves), whereas for  $\phi_c = -0.1612$  the second derivatives  $f_{1,\eta\eta}$  and  $\phi_{,\eta\eta}$  diverge at some value  $\eta_*$  (red curves). The second derivatives  $f_{1,\eta\eta}$  and  $\phi_{,\eta\eta}$  diverge at  $\eta_*$ , when the determinant behaves as  $(\eta - \eta_*)^g$  with  $g < 1$ . In contrast, when the determinant behaves as  $(\eta - \eta_*)$ , a jump of  $f_{1,\eta\eta}$  and  $\phi_{,\eta\eta}$  arises. Associated with these jumps/divergences are divergences of the curvature scalar  $R$  and the GB scalar  $R_{\text{GB}}$ , as seen in Figs. 3(c) and 3(d), where the curvature scalar  $R$  (c) and the GB scalar  $R_{\text{GB}}$  (d) are shown for the same solutions.



**Figure 4.** Embeddings of the equatorial plane: (a) single throat wormhole with coupling constant  $\alpha = 1.7$  and value of the scalar field at the center  $\phi_c = -0.0121$ ; (b) double throat wormhole with  $\alpha = 1.7$  and  $\phi_c = -0.03$ .

We visualize the geometry of a typical single wormhole in Figure 4(a), where we have chosen for the coupling constant the value  $\alpha = 1.7$  and for the scalar field at the center  $\phi_c = -0.0121$ . Figure 4(b) shows the geometry of a typical wormhole with an equator and a double throat, obtained with the parameter choice  $\alpha = 1.7$  and  $\phi_c = -0.03$ . The equator and the throats are clearly visible in the figure. Both solutions belong to the same family ( $\alpha = 1.7$ ) of self-interacting wormholes. As  $\phi_c$  is increased, the single throat (minimum) at the centre first turns into a degenerate throat (saddle point) at a critical value of  $\phi_c$ , and then becomes an equator (maximum), concealed by a throat (minimum) on each side.

The third chapter of the dissertation is entitled “**Scalarized nutty wormholes**” and scalarized wormholes with a Newman-Unti-Tamburino (NUT) charge were studied in higher curvature theories. By varying the coupling parameter and the scalar charge we determine the domain of existence of the

scalarized nutty wormholes, and their dependence on the NUT charge. We focus on scalarized nutty wormholes with a single throat and study their properties.

We consider the effective action for Einstein-scalar-higher curvature invariant theories

$$S = \frac{1}{16\pi} \int \left[ R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + F(\phi) I(g) \right] \sqrt{-g} d^4x, \quad (7)$$

For the coupling function  $F(\phi)$  we choose a quadratic  $\phi$ -dependence with coupling constant  $\alpha$ ,

$$F(\phi) = \alpha \phi^2, \quad (8)$$

the simplest choice leading to curvature-induced spontaneous scalarization of black holes.

For the invariant  $I(g)$  we make two choices, (i) the GB term

$$I(g) = R_{\text{GB}}^2 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2, \quad (9)$$

and (ii) the Chern-Simons (CS) term

$$I(g) = R_{\text{CS}}^2 = {}^* R^\mu{}_\nu{}^{\rho\sigma} R^\nu{}_{\mu\rho\sigma}, \quad (10)$$

where the Hodge dual of the Riemann-tensor  ${}^* R^\mu{}_\nu{}^{\rho\sigma} = \frac{1}{2} \eta^{\rho\sigma\kappa\lambda} R^\mu{}_{\nu\kappa\lambda}$  is defined with the 4-dimensional Levi-Civita tensor  $\eta^{\rho\sigma\kappa\lambda} = \epsilon^{\rho\gamma\sigma\tau} / \sqrt{-g}$ . While both invariants are topological in four dimensions, the coupling to the scalar field  $\phi$  via the coupling function  $F(\phi)$  provides significant contributions to the equations of motion.

We obtain the coupled set of field equations by varying the action (7) with respect to the scalar field and to the metric,

$$\nabla^\mu \nabla_\mu \phi + \frac{dF(\phi)}{d\phi} I = 0, \quad (11)$$

$$G_{\mu\nu} = \frac{1}{2} T_{\mu\nu}^{(\text{eff})}, \quad (12)$$

where  $G_{\mu\nu}$  is the Einstein tensor and  $T_{\mu\nu}^{(\text{eff})}$  denotes the effective stress-energy tensor

$$T_{\mu\nu}^{(\text{eff})} = T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{(I)}, \quad (13)$$

which consists of the scalar field contribution

$$T_{\mu\nu}^{(\phi)} = (\nabla_\mu \phi)(\nabla_\nu \phi) - \frac{1}{2} g_{\mu\nu} (\nabla_\rho \phi)(\nabla^\rho \phi), \quad (14)$$

and a contribution from the respective invariant  $I(g)$ . For the chosen invariants we obtain (i)

$$T_{\mu\nu}^{(GB)} = (g_{\rho\mu}g_{\lambda\nu} + g_{\lambda\mu}g_{\rho\nu})\eta^{\kappa\lambda\alpha\beta*}R^{\rho\gamma}_{\alpha\beta}\nabla_{\gamma}\nabla_{\kappa}F(\phi), \quad (15)$$

and (ii)

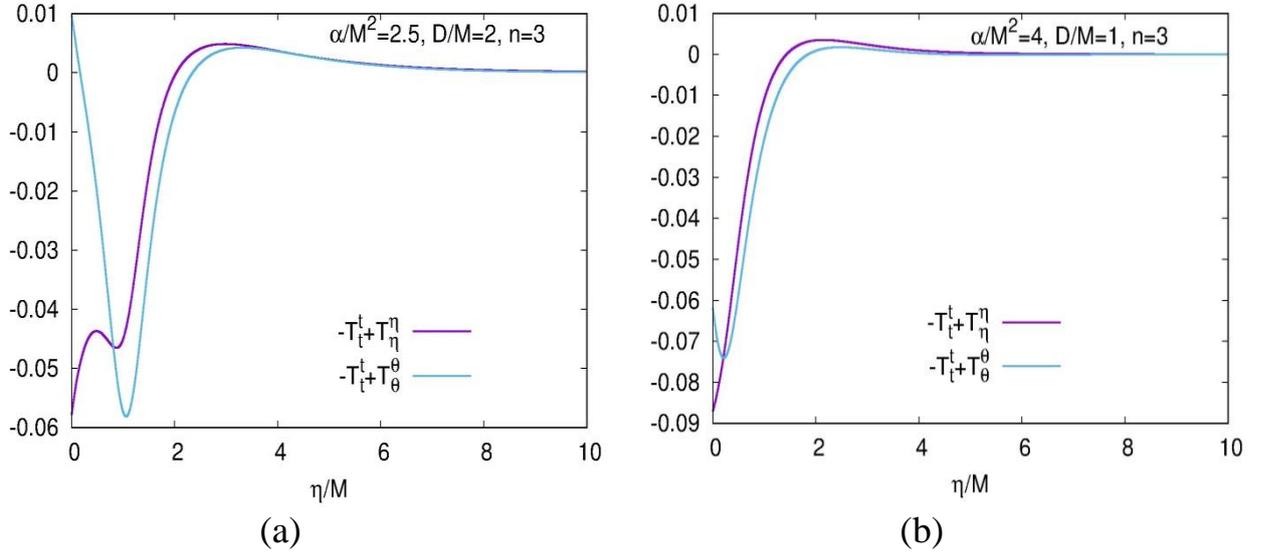
$$T^{(CS)\mu\nu} = -8[\nabla_{\rho}F(\phi)]\epsilon^{\rho\sigma\tau\mu}(\nabla_{\tau}R^{\nu})_{\sigma} + [\nabla_{\rho}\nabla_{\sigma}F(\phi)]^*R^{\sigma(\mu\nu)\rho}. \quad (16)$$

To obtain static, spherically symmetric wormhole solutions with a NUT charge  $N$  we assume the line element to be of the form

$$ds^2 = -e^{f_0}(dt - 2N\cos\theta d\varphi)^2 + e^{f_1}[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)]. \quad (17)$$

All three functions, the two metric functions  $f_0$  and  $f_1$  and the scalar field function  $\phi$ , depend only on the radial coordinate  $r$ .

When we insert the above ansatz (17) for the metric and the scalar field into the scalar-field equation (11) and the Einstein equations (12) with effective stress-energy tensor (13), we obtain five coupled, nonlinear ODEs.



**Figure 5.** Examples of nutty wormhole solutions: (a) Gauss-Bonnet with parameters  $\alpha/M^2 = 2.5$ ,  $D/M = 2$  va  $n = N/M = 3$ ; (b) Chern-Simons with  $\alpha/M^2 = 4$ ,  $D/M = 1$  and  $n = N/M = 3$ ; NEC conditions  $-T_t^t + T_n^\eta \geq 0$  and  $-T_t^t + T_\theta^\theta \geq 0$  vs radial coordinate  $\eta/M$ .

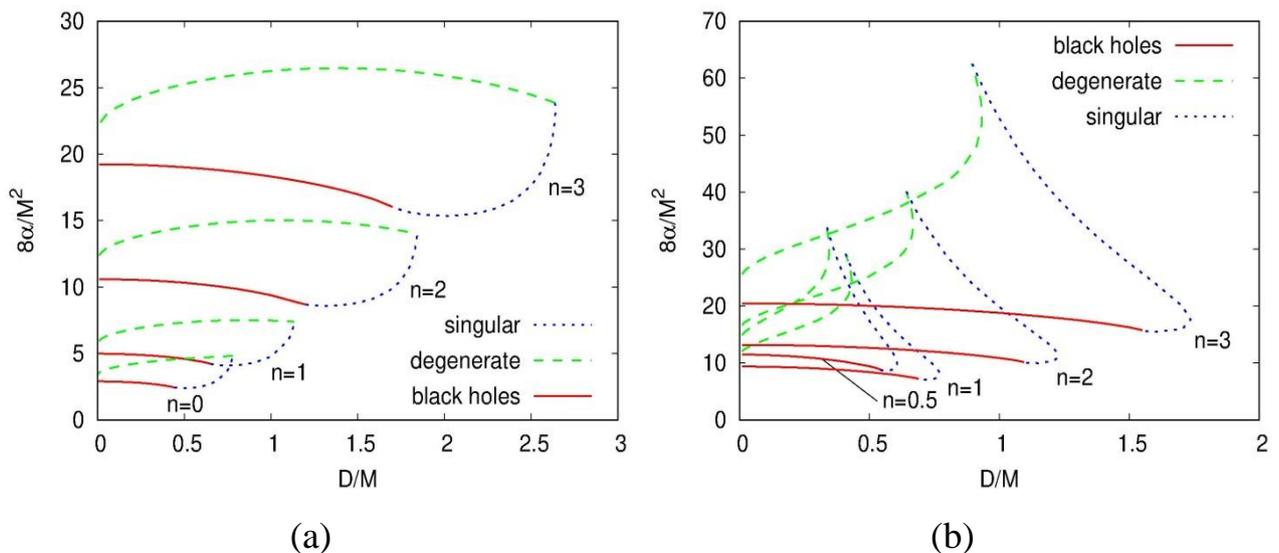
In Figure 5, the NEC conditions  $-T_t^t + T_n^\eta \geq 0$  and  $-T_t^t + T_\theta^\theta \geq 0$  versus the radial coordinate  $\eta/M$  for the GB invariant (a) and the CS invariant (b), choosing parameters  $\alpha/M^2 = 2.5$ ,  $D/M = 2$  and  $n = N/M = 3$ , and  $\alpha/M^2 = 4$ ,  $D/M = 1$  and  $n = N/M = 3$ , respectively. The figures clearly demonstrate the NEC violation for both invariants.

We exhibit the domain of existence in Figure 6 for the GB (a) and CS (b) invariants, for a set of values of the scaled NUT charge  $n = N/M$ . In particular,

we show the outer boundaries of the respective domains of existence, by presenting the scaled coupling constant  $\alpha / M^2$  versus the scaled scalar charge  $D / M$ .

The existence of the wormhole solutions requires the presence of a non-trivial scalar field. Since the equations are invariant under the transformation  $\phi \rightarrow -\phi$ , the domain of existence is symmetric with respect to  $D \rightarrow -D$ , and it is sufficient to only exhibit  $D \geq 0$ . For  $D = 0$  the scalar field vanishes, and thus  $D = 0$  represents the trivial boundary of the domain of existence, where pure GR solutions reside. The first non-trivial boundary corresponds to scalarized nutty black holes and is shown by the solid red curves. Here the center/throat turns into a black hole horizon.

The second non-trivial boundary in the figures is termed degenerate and is shown by the dashed green curves. To understand this boundary, we recall the numerical construction of the solutions. The calculation is ended, when an extremum is reached. However, in principle, we can continue the calculation beyond the extremum, where we might find a second extremum. The first one then corresponds to a throat while the second one corresponds to an equator. As the coupling constant is varied, the two extrema will approach each other until finally a degenerate extremum is reached. This third boundary represents precisely the values of the coupling constant, where such a degenerate extremum is reached.

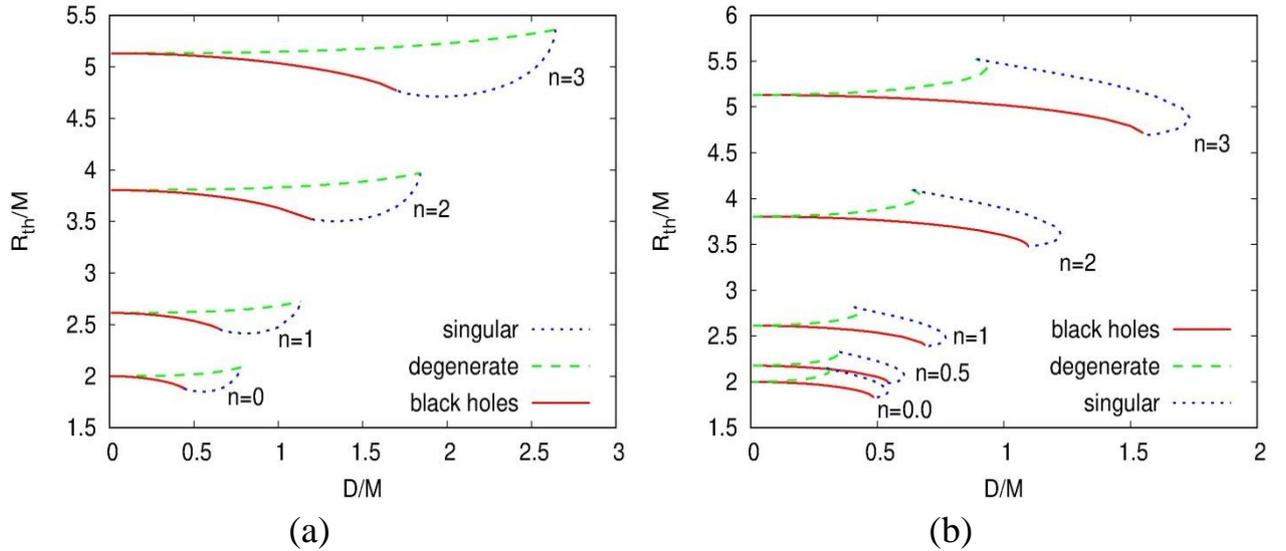


**Figure 6.** The domain of existence (left plot: Gauss-Bonnet, right plot: Chern-Simons) for several values of the scaled NUT charge  $n = N / M$ : scaled coupling constant  $\alpha / M^2$  vs scaled scalar charge  $D / M$ . The solid red curves represent the black hole limit, the dashed green curves the degenerate wormhole limit, and the dotted blue curves the singular limit.

The last boundary has been labelled singular and is shown by the dotted blue curves. At this boundary the calculations reveal the appearance of a singularity somewhere in the spacetime, that is of the cusp type. Their presence is due to the emergence of a node at some value of the radial coordinate  $\eta_*$  in the determinant, that arises upon diagonalisation of the ODEs. These cusp singularities seem to be a

rather common feature of scalarized wormholes. Here we see, that they do not only arise for GB theories but are also present for CS theories.

The figures also show that the domain of existence of wormhole solutions increases strongly with increasing NUT charge. For the GB coupling, the limit of vanishing NUT charge leads to the scalarized wormhole solutions. For the CS coupling, the vanishing of the NUT charge needs special attention, since in this case two branches of solutions arise, as shown for the Schwarzschild-NUT solutions. Along the first branch the coupling constant decreases with decreasing NUT charge, analogous to the GB case. However, along the second branch the coupling constant increases with decreasing NUT charge. This is seen in Figure 6(b), where the solutions for  $n = N/M = 3, 2$  and  $1$  are on the first branch, while the solutions for  $n = N/M = 0.5$  are already on the second branch. Interestingly, the smaller the values of the NUT charge, the larger the values of the coupling constant that are necessary to obtain scalarized solutions. In fact, both conspire in such a way, that a finite domain arises also in the CS case in the limit  $N \rightarrow 0$  as further discussed below.



**Figure 7.** Properties at the throat (left plots: Gauss-Bonnet, right plots: Chern-Simons) for several values of the scaled NUT charge  $n = N/M$ : (a) and (b) scaled circumferential radius  $R_{\text{th}}/M$  vs scaled scalar charge  $D/M$ . The solid red curves represent the black hole limit, the dashed green curves the degenerate wormhole limit, and the dotted blue curves the singular limit.

We next address properties of the wormhole center, and since we do not consider equators here, but only throats, we refer to these properties as throat properties. For these properties, we delimit their domains of existence in Figure 7, by extracting the boundaries of the respective domains. As before we show the black hole limit by solid red curves, the degenerate limit by dashed green curves, and the singular limit by dotted blue curves.

We exhibit the domain of the scaled value  $R_{\text{th}}/M$  versus the scaled scalar charge  $D/M$  in Figure 7 for the GB invariant (a) and the CS invariant (b) for several values of the NUT charge  $n = N/M$ . We note that for a fixed value of the NUT charge  $n = N/M$ , in the limit of vanishing scalar charge ( $D/M \rightarrow 0$ ) the same solution is approached for both the GB invariant (a) and the CS invariant (b). In this limit, the scalar field vanishes identically, and thus there are no wormhole solutions. The limiting solution, therefore, corresponds to a Schwarzschild-NUT black hole solution, for which the value of  $R_{\text{th}}/M$  depends only on the value of the NUT charge  $n = N/M$ .

When taking the NUT charge to zero, the boundary composed of scalarized black holes remains finite. In the GB case, the scalarized Schwarzschild solutions are approached. In the CS case, the coupling constant diverges, as the NUT charge goes to zero, leaving a finite value for the source term of the scalar field.

Considering the full domain of existence, in the GB case in the limit  $n = N/M \rightarrow 0$  the known finite domain of scalarized wormhole solutions is approached. In the CS case, the limit  $n = N/M \rightarrow 0$  leads to a finite domain of solutions. Nevertheless, the change of the domain of existence of the circumferential radius  $R_{\text{th}}/M$  is completely smooth in the limit  $n = N/M \rightarrow 0$ , as calculations for several small values of  $n = N/M$  have shown. The limiting domain is seen in Figure 7(b). In this figure, it does not make a difference whether the solutions are on the first (large  $n = N/M$ ) or second (small  $n = N/M$ ) branch.

## CONCLUSIONS

The following conclusions were presented on the basis of research carried out on the topic of ‘‘Novel scalarized solutions in gravity theories with higher curvature terms’’ for the Doctor of Philosophy (PhD) dissertation:

1. Wormhole solutions have been constructed within EsGB theories using potential. It has been shown that the effective stress-energy tensor provides the violation of the energy conditions.
2. We have mapped out the domain of existence of these wormhole solutions, varying the GB coupling constant and the value of the scalar field at the center.
3. It has been shown that the domain of existence is significantly increased by the presence of the self-interaction as compared to the case with a mass term only. Massive wormholes have been obtained in the new region in parameter space available due to the self-interaction.
4. We have also shown that most of the wormholes possess a single throat at the center. However, some of the wormholes develop a maximum of the radius at the center surrounded by a minimum on each side. In this case, the wormhole

solutions possess an equator and a double throat, that is connected to each asymptotically flat region via a throat.

5. We have obtained scalarized nutty wormhole solutions for GB and CS invariants. The domains of existence of the scalarized nutty wormholes contain the same type of boundaries for both invariants. These boundaries consist of the black hole and wormhole boundaries.



Автореферат “Фан ва инновациялар” халқаро илмий журнали (International scientific journal “Science and Innovation”) таҳририятида таҳрирдан ўтказилиб, ўзбек, инглиз ва рус тилларидаги матнлари ўзаро мувофиқлаштирилди (20.09.2022).

**НАУЧНЫЙ СОВЕТ DSc.03/31.03.2022.T/FM.10.04 ПО ПРИСУЖДЕНИЮ  
УЧЁНОЙ СТЕПЕНИ ПРИ ИНСТИТУТЕ ФУНДАМЕНТАЛЬНЫХ И  
ПРИКЛАДНЫХ ИССЛЕДОВАНИИ НАЦИОНАЛЬНОГО  
ИССЛЕДОВАТЕЛЬСКОГО УНИВЕРСИТЕТА «ТИИИМСХ»**  

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**САМАРКАНДСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ ИМЕНИ  
ШАРОФ РАШИДОВА**

**МУРОДОВ САРДОР НОРМУМИН УГЛИ**

**НОВЫЕ СКАЛЯРИЗОВАННЫЕ РЕШЕНИЯ В ТЕОРИЯХ  
ГРАВИТАЦИИ С ЧЛЕНАМИ ВЫСОКОЙ КРИВИЗНЫ**

**01.04.02 – Теоретическая физика**

**АВТОРЕФЕРАТ**  
**диссертации доктора философии (phd) по физико-математическим наукам**

**Самарканд – 2022**

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Кабинете Министров Республики Узбекистан под номером B2021.1.PhD/FM570.

Диссертация выполнена в Самаркандском государственном университете имени Шароф Рашидова.

Автореферат диссертации на трех языках (узбекский, английский, русский (резюме)) размещён на веб-странице Научного совета ([www.tiiame.uz](http://www.tiiame.uz)) и Информационного-образовательном портале «Ziyonet» ([www.ziyonet.uz](http://www.ziyonet.uz)).

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**Ведущая организации:**

**Казахский национальный университет имени**

**Аль-Фараби**

Защита диссертации состоится «14» Октябрь 2022 года в 16<sup>00</sup> часов на заседании Научного совета DSc.03/31.03.2022.T/FM.10.04 при Институте фундаментальных и прикладных исследований Национального исследовательского университета «ТИИИМСХ». (Адрес: 100000, Узбекистан, г.Ташкент, ул. Кори Ниязов, дом 39. Тел.: (+99871) 237-19-31; факс: (+99871) 237-38-79; e-mail: ifar@tiiame.uz)

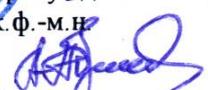
С диссертацией можно ознакомиться в Информационно-ресурсном центре Самаркандского государственного университета имени Шароф Рашидова (регистрационный номер 95). (Адрес: Университетский бульвар, 15, 140104, г. Самарканд. Тел.: (+99866) 239-11-51).

Автореферат диссертации разослан «30» Сентябрь 2022 г.  
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## **ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))**

**Целью исследования** является нахождения скалярных решений кротовых нор в теориях гравитации Эйнштейна-скаляра-Гаусса-Боннета и Эйнштейна-скаляра-Черна-Саймонса с членами высокой кривизны.

**Объектом исследования** является теории гравитации с высокой кривизны и кротовые норы.

**Научная новизна исследования** заключается в следующем:

в гравитационных теориях Эйнштейна-скаляра-Гаусса-Бонне были получены кротовые норы с одинарным горлом и кротовые норы с экватором и двойным горлом со скалярным потенциалом самодействия. Варьируя константу связи Гаусса-Бонне, в этих теориях определялась область существования кротовых нор;

в теориях гравитации Эйнштейна-скаляра-Гаусса-Бонне и впервые в Эйнштейна-скаляра-Черна-Саймонса с членами высокой кривизны новые скалярные кротовые норы Ньюмана-Унти-Тамбурино были найдены путем варьирования параметра связи и скалярного заряда, а также их зависимости от заряда Ньюмана-Унти-Тамбурино;

было получено множество решений скаляризованных кротовых нор на пределе потери заряда Ньюмана-Унти-Тамбурино и в случае инварианта Гаусса-Бонне. А в инвариантном случае Черна-Саймонса, из-за изменения константы связи и уменьшение заряда Ньюмана-Унти-Тамбурино, было определено, что предел решений кротовых нор разделяется на характерные две области.

**Внедрение результатов исследования.** На основе результатов исследования новых скаляризованных решений в теориях гравитации с членами более высокой кривизны:

в гравитационных теориях Эйнштейна-скаляра-Гаусса-Бонне результаты, полученные при исследовании кротовых нор с одинарным горлом и кротовых нор с экватором и двойным горлом со скалярным потенциалом самодействия, использовались в международных научных журналах (Physical Review D 103, 124062, 2021; Physical Review D 104, 024048, 2021; Physical Review D 104, 044006, 2021; The European Physical Journal C, Vol.81, 285, 2021; The European Physical Journal C, Vol.81, 858, 2021; Journal of Cosmology and Astroparticle Physics, 10(2021)059; Journal High Energy Physics, 104, 2021; Nuclear Physics B, Vol.980, 115848, 2022; Journal of Cosmology and Astroparticle Physics, 05(2022)022) и в рамках проекта DFG RTG 1620 « Models of Gravity ». Применение научных результатов позволило определить область существования решений черной дыры и проходимой кротовой норы в теориях гравитации с высокой кривизной;

результаты, полученные в ходе исследований новых скаляризованных решений в теориях гравитации с членами высокой кривизны, были использованы в проекте “Элементарные частицы и основные свойства космологии” № 0723-2020-0041 Министерства науки и высшего образования

РФ, в проекте “РФБР 19-02-00346” и в рамках проекта Q.J130000.2626.14J72 Малазийского технологического университета. Применение научных результатов позволило рационально объяснить важные научно обоснованные выводы теоретических результатов о существовании и свойствах черных дыр и кротовых нор в теориях гравитации с высокой кривизной;

в теориях гравитации Эйнштейна-скаляра-Гаусса-Бонне и Эйнштейна-скаляра-Черна-Саймонса результаты, полученные при исследовании скаляризованных решений кротовых нор и изучении их зависимости от заряда Ньюмена-Унти-Тамбурино, использовались в международных научных журналах (Journal High Energy Physics 104 (2021); The European Physical Journal C, Vol.82, 238, 2022; Journal of Cosmology and Astroparticle Physics, 05(2022)022) и в рамках “the European COST action CA16104”. Применение научных результатов позволило определить большой класс решений кротовых нор и безмассовых заряженных кротовых нор.

**Апробация результатов исследования.** Результаты диссертационного исследования доложены на 3 международных и 10 республиканских конференциях.

**Опубликованность результатов исследования.** По теме исследования опубликована 19 научных работ, в том числе 6 статей (3 в республиканских и 3 в зарубежных журналах), в научных изданиях, рекомендованных Высшей аттестационной комиссией при Кабинете Министров Республики Узбекистан для публикации основных научных результатов диссертационных работ.

**Структура и объем диссертации.** Диссертация состоит из введения, четырёх глав, заключения и списка литературы. Объем диссертации 98 страниц.



Автореферат “Фан ва инновациялар” халқаро илмий журнали (International scientific journal “Science and Innovation”) таҳририятида таҳрирдан ўтказилиб, ўзбек, инглиз ва рус тилларидаги матнлари ўзаро мувофиқлаштирилди (20.09.2022).

**E'LON QILINGAN ISHLAR RO'YXATI**  
**LIST OF PUBLISHED WORKS**  
**СПИСОК ОПУБЛИКОВАННЫХ РАБОТ**

**I bo'lim (I part; I часть)**

1. R.Ibadov, B.Kleihaus, J.Kunz, S.Murodov. Wormholes in Einstein-scalar-Gauss-Bonnet theories with a scalar self-interaction potential // Phys. Rev. D 2020, 102, 064010 (№1.Web of Science; Scopus; IF=5.29).
2. R.Ibadov, B.Kleihaus, J.Kunz, S.Murodov. Scalarized Nutty Wormholes // Symmetry 2021, 13, 89 (№1.Web of Science; Scopus; IF=2.7).
3. R.Ibadov, B.Kleihaus, J.Kunz, S.Murodov. Wormhole solutions with NUT charge in higher curvature theories // Arabian Journal of Mathematics 2021, 11 (№1.Web of Science; Scopus; IF=1.29).
4. S.Murodov. Eynshteyn-skalyar-Gauss-Bonnet nazariyalarida “yumronqoziq ini” yechimlari // Scientific Journal of Samarkand University. 2021, No. 1(125) (01.00.00.№2).
5. R.M.Ibadov, S.N.Murodov. Wormholes with a NUT charge in higher curvature theories // Electronic journal of actual problems of modern science, education and training, Urgench 2021, ISSN 2181-9750 Vol. 10 (01.00.00.№1).
6. S.N. Murodov, Q.A.Badalov, De Sitter impuls fazosi va uning giperboloidi // Scientific journal of Samarkand State University. 2018, No. 5(111) (01.00.00.№2).

**II bo'lim (II part; II часть)**

7. R.M.Ibadov, S.N.Murodov. Koinotda “yumronqoziq inlari” muammosi // Ёш олимлар республика илмий амалий конференцияси 2016 йил 29-30 январ 2 қисм. 696-698 бетлар. ТерДУ
8. У.Д. Тўлиев С.Н. Муродов. Eynshteyn-Yang-Mills-Higgs tenglamalari va “mathematica” dasturi to'g'risida. Ёш олимлар республика илмий амалий конференцияси 2016 йил 29-30 январ 2 қисм. 41-42 бетлар. ТерДУ
9. С.Н. Муродов. Eynshteyn-Yang-Mills-Higgs tenglamalari va higgs potentsiali. Bir va ikki bo'g'izli yumronqoziq inlari (inglizcha „wormhole“) // Худудий илмий амалий конференцияси. Самарқанд 2016 йил 2-3 июн. 320 – 322 бетлар.
10. У.Д. Тўлиев С.Н. Муродов. Bir va ikki bo'g'izli yumronqoziq inlari geometriyasi va fizikasi yechimiga olib keluvchi Eynshteyn-Yang-Mills-Higgs tenglamalari va higgs potentsiali // Ёш олимлар республика илмий амалий конференцияси 2017 йил 30 март-1 апрель 2 қисм. 211-213 бетлар, ТерДУ.
11. R.M.Ibadov, S.N.Murodov. Modernizatsiyalangan Eynshteyn tenglamalarida yumronqoziq inlari yechimlari // Наманган муҳандислик-технология институти, физикани ўқитишнинг долзарб муаммолари, республика илмий-амалий анжумани, 2018- йил 11-июл, 48-51 бетлар.

- 12.S.N.Murodov. Gravitatsion maydon uchun Eynshteyn tenglamalari, Qora tuynuklar va yumronqoziq inlari // Ўзбекистон Республикаси фанлар академияси Навоий филиали, фан ва талимни ривожлантиришда ёшларнинг ўрни. 2018-йил 23-ноябр, 38-39 бетлар.
- 13.R.M.Ibadov, S.N.Murodov. Fantom yumronqoziq inlarining (wormholes) simmetrik va assimetrik ko‘rinishlari // Нукус давлат педагогика институти “Фан ва талим-тарбиянинг долзарб масалалари” Республика илмий-назарий анжумани, 2019-йил, 263-265 бетлар.
- 14.R.M.Ibadov, S.N.Murodov, M.A.Javkanov. Hozirgi zamon astrofizikasida fantom maydonining ahamiyati // Ilm-fan va texnikaning rivojlanishida innovatsion yondashuvlar, ilmiy-amaliy onlayn konferensiya materiallari, 20.11.2020, 151-153 b.
- 15.P.M. Ибадов, А.А.Яхшимуродов С.Н. Муродов. Yang-mills maydoni bilan bog‘langan eynshteyn tenglamalarining yumronqoziq inlariga olib keluvchi yechimlari // Science and education, Scientific journal, илм-фан ва талимнинг ривожланиш истиқболлари илмий конференция тўплами, 27 апрел 2020 йил, 186-188 бетлар.
- 16.R.Ibadov, J.Kunz, S.Murodov. Nazariy fizika va astrofizikada qora tuynuklar va yumronqoziq inlari muammolari // Инновационное развитие науки и образования, международная научно-практическая конференция, июн 27, 2020, 357-360 бетлар.
- 17.R.M.Ibadov, S.N.Murodov. Eynshteyn tenglamalarining sferik simmetrik va statik yumronqoziq inlariga olib keluvchi yechimlari // science and education, Scientific journal, ISSN 2181-0842, 2020, Vol. 1 (No. 2), 33-38p.
- 18.R.Ibadov, J.Kunz, S.Murodov. Wormholes in Einstein-scalar-Gauss-Bonnet theories // Euro Asia 8th. International congress on applied sciences march 15-16, 2021, Tashkent, Uzbekistan 559-564 p.
- 19.S.Murodov. Wormhole solutions in higher curvature theories // International Workshop on Relativistic Astrophysics and Gravitation, Ulugh Beg Astronomical Institute, Tashkent, Uzbekistan , 12-14 may 2021.



“Fan va innovatsiyalar” xalqaro ilmiy jurnali (International scientific journal “Science and Innovation”) tahririyatida tahrirdan o‘tkazilib, o‘zbek, ingliz va rus tillaridagi matnlari o‘zaro muvofiqlashtirildi (20.09.2022).

Nashr lits. AI<sup>1</sup> 305. Bosishga ruxsat etildi 27.09.2022.  
Qog‘oz bichimi 60x84 1/16. Shartli bosma tabog‘i, 3.  
Hisob-nashr tabog‘i 3. Adadi 100.  
75-buyurtma.

«IQTISOD-MOLIYA» nashriyotida tayyorlandi.  
100000, Toshkent, Amir Temur ko‘chasi, 60<sup>a</sup>.

«DAVR MATBUOT SAVDO» bosmaxonasida chop etildi.  
100198, Toshkent, Qo‘yliq, 4-mavze, 46.