ЎЗБЕКИСТОН МИЛЛИЙ УНИВЕРСИТЕТИ ҲУЗУРИДАГИ ИЛМИЙ ДАРАЖАЛАР БЕРУВЧИ DSc.03/30.12.2019.FM.01.02 РАҚАМЛИ ИЛМИЙ КЕНГАШ

ЎЗБЕКИСТОН МИЛЛИЙ УНИВЕРСИТЕТИ

ЭШКУВАТОВ ЗАЙНИДИН КАРИМОВИЧ

СИНГУЛЯР ВА ИНТЕГРО-ДИФФЕРЕНЦИАЛ ТЕНГЛАМАЛАР УЧУН АВТОМАТИК КВАДРАТУР СХЕМАСИ ВА ГОМОТОПИК ТАҲЛИЛ ҚИЛИШ УСУЛИ

01.01.03 – Ҳисоблаш математикаси ва дискрет математика (физика-математика фанлари)

ФИЗИКА-МАТЕМАТИКА ФАНЛАРИ ДОКТОРИ (DSc) ДИССЕРТАЦИЯСИ АВТОРЕФЕРАТИ

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Докторлик (DSc) диссертацияси автореферати мундарижаси

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Оглавление автореферата докторской (DSc) диссертации

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ТОШКЕНТ – 2022

Фан доктори (Doctor of Science) диссертацияси мавзуси Ўзбекистон Республикаси Вазирлар Маҳкамаси ҳузуридаги Олий аттестация комиссиясида B2022.2.DSc/FM194 рақам билан рўйхатга олинган

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КИРИШ (докторлик диссертацияси аннотацияси)

Диссертация мавзусининг долзарблиги ва зарурати. Жаҳон миқёсида турли тартибдаги сингуляр интеграллар ва интеграл тенгламаларни сонли ечиш учун квадратур формулалар қуриш, ярим аналитик усуллар ишлаб чиқиш ва уларни тадқиқ этиш долзарб ҳисобланиб, аэродинамика, гидродинамика, электрон оптика, тўлқин тарқалиши ва суюқликлар механикаси каби соҳалардаги масалаларни ечишда кенг қўлланилади. Бу соҳалардаги қатор жараёнларнинг математик моделлари классик бўлмаган муайян сингуляр интеграллар ва гиперсингуляр интеграллар билан ифодаланиб, уларнинг аналитик ечимини топиш мураккаб ҳисобланади. Шу сабабли, чегараланган ёки чегараланмаган соҳаларда регуляр ва сингуляр интегралларни сонли ҳисоблаш учун самарали ва юқори аниқликдаги квадратур формулаларни қуриш ҳамда турли тартибдаги сингуляр интеграл тенгламаларни тақрибий ечиш усулларини ишлаб чиқиш ҳисоблаш математикасининг муҳим вазифаларидан бири бўлиб қолмоқда.

Ҳозирги кунда дунёда акустика, суюқликлар механикаси, эластиклик, аэродинамика, гидродинамика ва синиш механикаси каби соҳаларга замонавий технологияларни жорий этиш, хусусан, илмий-техник имкониятлар суръатини ошириш, сингуляр интеграллар ва гиперсингуляр интегралларни ҳисоблашда интерполяцион типдаги юқори аниқликка эга бўлган квадратур формулаларни қуриш бўйича кўплаб илмий тадқиқотлар олиб борилмоқда. Бу борада сингуляр ва гиперсингуляр интегралларни тақрибий ҳисоблаш учун самарали квадратур формулалар қуриш, силлиқ функциялар фазоси ва вазнли Гильберт фазоларида квадратур формуланинг хатолигини баҳолаш муҳим рол ўйнайди. Шу сабабли, ночизиқли ва каср тартибли интегро-дифференциал тенгламаларни ечишнинг янги алгоритмларини ишлаб чиқиш, шу билан бирга ночизиқли интегро-дифференциал тенгламалар ечимининг мавжудлиги ва ягоналигини текшириш ҳамда турғун ҳисоблаш алгоритмларини яратиш ва уларнинг яқинлашувини таъминлаш мақсадли илмий тадқиқотлардан ҳисобланади.

Мамлакатимизда фундаментал фанларнинг илмий ва амалий тадбиқига эга бўлган эластиклик назарияси, квант механикаси, аэродинамика, электрон оптика, тўлқин тарқалиши ва суюқликлар механикаси масалаларининг сонли-аналитик ечиш усулларини ишлаб чиқиш каби долзарб йўналишларга катта эътибор қаратилмоқда. Хусусан, квазичизиқли гиперболик системалар учун адекват ҳисоблаш моделларини қуриш бўйича муҳим натижаларга эришилди. «Функционал анализ, алгебра, дифференциал тенгламалар, математик физика, математик моделлаштириш, ҳисоблаш математикаси ва дискрет математика » устувор йўналишлар бўйича халқаро стандартлар даражасидаги илмий изланишлар олиб бориш ЎзР ФА В.И.Романовский номидаги Математика институти фаолиятининг асосий вазифаларидан бири ҳисобланади[1]. Қарор ижросини таъминлашда ночизиқли интеграл тенгламалар системаси ва кўп

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[1] Ўзбекистон Республикаси Президентининг 2020 йил 7 майдаги “Математика соҳасидаги таълим сифатини ошириш ва илмий - тадқиқотларни ривожлантириш чора-тадбирлари тўғрисида”ги ПҚ-4708-сон қарори.

ўлчовли интеграл тенгламаларни ечиш алгоритмларини ишлаб чиқиш ва ечимнинг ягоналигини исботлаш ҳамда турғун ҳисоблаш алгоритмларини яратиш муҳим аҳамиятга эга.

Ўзбекистон Республикаси Президентининг 2017 йил 7 февралдаги ПҚ-4947-сон «Ўзбекистон Республикасини янада ривожлантириш бўйича ҳаракатлар стратегияси тўғрисида»ги, 2017 йил 17 февралдаги ПҚ-2789-сон «Фанлар академияси фаолияти, илмий-тадқиқот ишларини ташкил этиш, бошқариш ва молиялаштиришни янада такомиллаштириш чора-тадбирлари тўғрисида»ги, 2017 йил 20 апрелдаги ПҚ-2909-сон «Олий таълим тизимини янада ривожлантириш чора чора-тадбирлари тўғрисида»ги, 2018 йил 27 апрелдаги ПҚ-3682-сон «Инновацион ғоялар, технологиялар ва лойиҳаларни амалиётга жорий қилиш тизимини янада такомиллаштириш чора-тадбирлари тўғрисида»ги, 2020 йил 7 майдаги ПҚ-4708-сон «Математика соҳасидаги таълим сифатини ошириш ва илмий-тадқиқотларни ривожлантириш чора-тадбирлари тўғрисида»ги қарорлари ҳамда мазкур фаолиятга тегишли бошқа норматив-ҳуқуқий ҳужжатларда белгиланган вазифаларни амалга оширишда ушбу диссертация тадқиқоти муайян даражада хизмат қилади.

Тадқиқотнинг республика фан ва технологиялари ривожланишининг устувор йўналишларига боғлиқлиги. Мазкур тадқиқот республика фан ва технологиялар ривожлантиришнинг IV. «Математика, механика ва информатика» устувор йўналиши доирасида бажарилган.

Диссертация мавзуси бўйича хорижий илмий-тадқиқотлар шарҳи[2].

Вазнли сингуляр интегралларнинг сонли ечимини ҳисоблаш ва турли типдаги интеграл тенгламаларнинг тақрибий ечимини топиш, турғунлигини текшириш ва яқинлашишини таъминлаш бўйича илмий изланишлар дунёнинг етакчи илмий марказлари ва олий ўқув юртларида жумладан; Россия Фанлар Академиясининг С.Л.Соболев номидаги Математика институти, Г.И.Марчук номидаги ҳисоблаш Математикаси институти, Пенза давлат университети (Россия), Сербия Фанлар ва санъат академиясининг Математика институти (Сербия), Ўзбекистон Фанлар академияси, В.И.Романовский номидаги Математика институти, Мирзо Улугбек номидаги Ўзбекистон миллий университети (Ўзбекистон), Корея фан ва технология институтининг Мия фанлари институти (Жанубий Корея), Мериленд университетининг Математика институти, (Вашингтон, АҚШ), Гарвард университетининг Математика кафедраси (Кембриж, АҚШ), Фукуи университетининг Математика кафедраси (Япония), Токио университетининг Математика ва информатика маркази (Япония), Эрон фан ва технология университети, Фундаментал фанлар тадқиқоти институти (Эрон), Малайзия технология университети, Путра Малайзия университетининг Математик тадқиқотлар институти, Малая университетининг Математика фанлари институти

[2] Диссертация мавзуси бўйича хорижий илмий тадқиқотлар: Journal of Approximation Theory, Applied Mathematics and Computation, Applied Mathematics Letters, Applied Numerical Mathematics, Journal of Computational and Applied Mathematics, Numerical Algorithms, Journal of Integral Equations and Applications, SIAM Journal on Numerical Analysis (SINUM), BIT Numerical Mathematics, Сибирский журнал вычислительной математики, Журнал вычислительной математики и математической физики, манбалар асосида ишлаб чиқилган.

(Малайзия), Ҳиндистон фан институтининг физика-математика фанлари бўлими, Алигарҳ муслим университети (Ҳиндистон) ва бошқаларда олиб борилмоқда.

Коши типидаги сингуляр интеграллар ва Адамар типидаги гиперсингулар интегралларни тақрибий ҳисоблашнинг эффектив усуллари ва уларнинг хатолигини баҳолаш бўйича бир қатор муҳим илмий натижалар олинган: сингуляр интеграл тенгламларни ечиш учун дискрет уюрмали метод ишлаб чиқилган ва бу метод билан юқори ўлчовли сингуляр интеграл тенгламларнинг ечими олинган (Москва давлат университети, Россия); умумлашган ҳаво плёнкаси (airfoil) тенгламасини ечиш учун тезкор алгоритм (квадратур формула) қурилган ва оптимал яқинлашиш тартиби аниқланган (Чемниз техника университети, Германия); Бернштейн полиномлари асос қилиб олган ҳолда интеграл тенгламаларнинг айрим синфлари (иккинчи турдаги Фредгольм сингуляр интеграл тенгламалар, характеристик ГСИТлар ва иккинчи турдаги ГСИТлар)нинг тақрибий сонли ечимлари топилган (Ҳиндистон статистика институти, Ҳиндистон); бир ўлчовли ГСИТлар, ярим-ГСИТ ва биринчи турдаги кўп ўлчовли ГСИТларни ечиш учун сплайн-коллокация усули таклиф қилинган ва у асосланган (Пенза давлат университети, Россия); биринчи турдаги ГСИларни ечиш учун модификацияланган гомотопия силжитиш усули (ГСУ) таклиф қилинган, таклиф этилган усул стандарт ГСУ билан таққосланган ва модификацияланган ГСУнинг тез яқинлашиши ҳамда аниқ ечимларни бериши кўрсатилган, таклиф қилинган усул тебраниш ва фаол назорат масаласига қўлланилган (Islamic Azad University, Эрон); интеграл тенгламалар учун дискрет проекциялаш усуллари ишлаб чиқилган ва кўп турдаги (регуляр, сингуляр ва гиперсингуляр) тенгламаларга қўллаб хатоликларни баҳолаш формаси ишлаб чиқилган (Доира Университети, АҚШ); Коши типидаги сингуляр интеграллар учун оптимал квадратур формула қурилган, Соболев фазосида хатолик баҳолаб, таклиф этилган методнинг яқинлашиш исбот қилинган (ЎзРФА, Математика институти).

Дунёда сингуляр ва гиперсингуляр интегралларни тақрибий ҳисоблаш учун самарали квадратур формулалар қуриш, силлиқ функциялар фазоси ва вазнли Гильберт фазоларида квадратур формуланинг хатолигини баҳолаш бўйича қатор устувор йўналишларда илмий тадқиқот ишлари олиб борилмоқда, жумладан, сингуляр ва гиперсингуляр, интеграл ва интегро-дифференциал тенгламалар, ночизиқли интеграл тенгламалар системаси ва кўп ўлчовли интеграл тенгламалар ҳамда ночизиқли ва каср тартибли интеграл тенгламаларни тақрибий ечишнинг эффектив усулларини яратиш, турли функционал фазоларда уларнинг яқинлашишини таъминлаш, ечим мавжудлиги ва ягоналигини текшириш, янги турғун ҳисоблаш алгоритмларини ишлаб чиқиш ва уларнинг яқинлашишини таъминлаш.

Муаммонинг ўрганилганлик даражаси. Назарий ва амалий математикада энг муҳим математик воситалардан бири биринчи ва иккинчи тартибли сингуляр операторлар мавзусидир. 20-аср ўрталарида (1950 йиллар)

Коши типидаги бош қийматли интеграллар ва (1970 йиллар) Адамар типидаги СИлар яни ГСИларни сонли баҳолашга алоҳида эътибор берилди. СИларни ва ГСИларни тақрибий яқинлаштириш бўйича адабиётларда кўплаб муҳим натижаларга эришилди. Хусусан, вазнли Коши типидаги сингуляр интегралларни ва гиперсингуляр интегралларни тақрибий ҳисоблаш учун квадратур формулалар (КФ), оптимал КФлар, сплайн функциялар ёрдамида қурилган КФлар ва модификацияланган Симпсон қоидалари билан яқинлаштириш ва турли синфларда баҳолаш ишлари И.М.Лонгман, С.Э.Стеварт, Р.Пиессенс, Д.Б.Ҳунтер, Д.Ф.Пагет, Д.Эллиотт, Д.Г.Саникидзе, М.М.Чавла, Ш.Кумар, Т.Р.Рамакришнан, Н.И.Иоакимидис, Г.Монегато, П.С.Цеокарис, С.Л.Соболев, И.В.Бойков, А.И.Бойкова, А.Н.Тунда, С.Дагнино, П.Ламберти, Э.Санти, С.Амари, К.Диетселм, Т.Ҳасегава, Т.Тории, М.И.Исроилов, Х.М.Шадиметов, А.Р.Ҳаётов, З.К.Эшкуватов, С.А.Бахромов, Д.М.Ахмедов, Ҳ.Р.Кутт, Г.Монегато, Э.Лутз, П.А.Мартин, Ф.Ж.Риззо, И.К.Лифанов, А.С.Ненашев, Л.Н.Полтавскии, Ф.Ким, С.Й.Ҳуи, Д.Шиа, Самко, Й.Ш.Чан, Б.Ф.Фенг, З.Зҳанг, Й.Ш.Чан, Г.Ҳ.Паулино, А.Сиди ва бошқалар, каби илмий мақолаларда батафсил ёритилган. Масон ва Ҳандскомб монографиясида, Чебышев кўпҳадларининг барча турларининг батафсил хоссалари ва уларнинг қолдиқ ҳадлари берилган.

СИТлар, айниқса ГСИТлар ҳозирда чизиқли бўлмаган математик моделларнинг кенг доираларида, хусусан, математик физиканинг аралаш чегаравий муаммолари, изотропик эластик, суюқликлар механикаси, эластиклик ва синиш механикасида учрайди. Сўнгги бир неча ўн йилликларда адабиётларда СИТлар ва ГСИТларни сонли ечиш усулларининг кенг доираси ишлаб чиқилган. Хусусан, бир қанча монографиялар: Н.И.Мусхелишвили, Ф.Д.Гаҳов, Ф.Эрдоген ва б., С.М. Белоцерковский ва И.К. Лифанов, М.А.Голберг, Б.Н.Мандал ва А.Чакрабарти томонидан чоп этилган бўлиб, бу соҳаларда кўплаб мақолалар ҳам нашр этилган, жумладан, Б.Н.Мандал, С.Бҳаттачаря, Г.Ҳ.Бера, А.Чакрабарти, П.А.Мартин, Ф.Ж.Риззо, Г.Монегато, С.Мондал, Б.Н.Мандал, Р.Новин, И.К.Лифанов, И.Э.Полонскии, Э.Г.Ладополос, Л.Н.Полтавскии, А.С.Ненашев, Г.М.Ваиникко, М.Ҳ.Салеҳ, С.М.Амер, М.А.Абдоу, А.А.Насер, Р.П.Сривастав, И.В.Бойков, А.И.Бойкова, В.А. Роуднев, З.К.Эшкуватов, Э.С.Венцел, Ҳ.Чай, З.Занг, С.Банержеа, З.Чен, Й.Ф.Зоу, Ҳ.Фенг, Х.Занг, ва бошқалар шулар жумаласидандир.

Маълумки, чизиқли бўлмаган ҳодисалар фан оламининг кўпгина илмий соҳаларида учрайди. Хусусан физика, муҳандислик ва фан соҳаларидаги кўплаб муаммолар чизиқли бўлмаган ИДТлар билан моделлаштирилади ва ночизиқли ИДТнинг аналитик ечими камдан-кам ҳолатларда топилади. Олинган чизиқли бўлмаган ИДТларни кўпгина сонли усуллар билан ечиш мумкин ва бу йўналишда кўплаб илмий мақолалар чоп этилган ва чоп этилмоқда. Бу соҳада жавлон урган олимлар: А.Вазваз, С.Аббасбандий, К.Малекнежад, И.Подлубний, С.Ж.Лиао, Ж.Ҳ.Ҳе, Э.Баболиан, А.С. Батайнеҳ, М.С.М. Ноорани, З.Аяти, Ж.Биазар, К.Ал-Кҳалед, А.Авудаинаягам, М.Деҳган, Ф.Шакери, С.М.Эл-Саед, М.Р.Абдел-Азиз, А.Голбабай, М.Жавиди, Ж.Сабери-Наджаф, З.Б.Жафари, З.К.Эшкуватов, Ш.Н.Ҳусен, М.АЭл-Тawил, С.Кумар ва бошқалар.

Диссертация мавзусининг диссертация бажарилаётган олий таълим муассасасининг илмий-тадқиқот ишлари билан боғлиқлиги. Диссертация тадқиқоти Ўзбекистон Миллий университети илмий-тадқиқот ишлари режасига мувофиқ ОТ-Ф4-28 «Гиперболик системалар учун адекват ҳисоблаш моделларини қуриш» мавзусидаги илмий-тадқиқот лойиҳаси доирасида бажарилган.

Тадқиқотнинг мақсади вазнли сингуляр ва гипер-сингуляр интеграл-ларни ҳисоблаш учун автоматик квадратур схемасини қуриш ва хатоликларини баҳолаш шунингдек сингуляр ва гипер-сингуляр интеграл тенгламаларнинг тақрибий ечимларини топиш методини ишлаб чиқиш ҳамда ночизиқли интегро-дифференциал тенгламалар учун гомотопик таҳлил қилиш усулининг янги ишланмасини яратиш ва ночизиқли интеграл тенгламалар системаси ҳамда кўп ўлчовли интеграл тенгламани тақрибий ечиш учун Ньютон-Канторович усулини ривожлантиришдан иборат.

Тадқиқотнинг вазифалари қуйидагилардан иборат:

ўзгарувчан ва қўзғалмас оралиқлар бўйича вазнли сингуляр ва гипер-сингуляр интегралларнинг тақрибий ҳисоблаш учун автоматик квадратур схемасини қуриш ва турли синфларда автоматик квадратур схеманинг қолдиқ ҳадини баҳолаш;

Коши типидаги вазнли сингуляр интеграллар учун дискрет уюрмали методни модификация қилиш ва турли синфларда унинг баҳосини олиш;

биринчи ва иккинчи турдаги сингуляр ва гиперсингуляр интеграл тенгламаларни ечиш учун чекли Чебышев кўпҳадлар қаторининг тенглама ечимига яқинлашувларини асослаш ҳамда ядроли интеграллар учун Гаусс-Лежандр квадратур формуласини қуриш ва унинг қолдиқ ҳадини баҳолаш;

гомотопик таҳлил қилиш усулининг янги ишланмасини яратиш ва уни ночизиқли бошланғич қийматли интегро-дифференциал тенгламаларни ҳамда аралаш чегаравий шартларга эга каср тартибли интегро-дифференциал тенгламаларни тақрибий ечиш учун қўллаш ва унинг яқинлашишини таъминлаш;

такомиллаштирилган Ньютон-Канторивич усулинини 2×2 чизиқли бўлмаган интеграл тенгламалар системасига ва чизиқли бўлмаган кўп ўлчовли интеграл тенгламани тақрибий ечишга татбиқ этиш, шу билан бирга унинг вазнли Гильберт фазосида қолдиқ ҳадини баҳолаш.

Тадқиқотнинг объекти сингуляр ва гиперсингуляр интеграллар ва интеграл тенгламалар, ночизиқли интегро-дифференциал тенгламалар, интеграл тенгламалар системаси ва юқори ўлчовли интеграл тенгламалардан иборат.

Тадқиқотнинг предмети операторлар, квадратур формулалар, интерполяцион кўпҳадлар ва функциялар фазосидан иборат.

Тадқиқотнинг усуллари: Тадқиқот ишида, операторлар назарияси, гомотопия назарияси, квадратур формулалар назарияси, интерполяцион кўпҳадлар назарияси, функциялар фазоси назарияси, дискрет аргументли функциялар ва каср тартибли ҳосилалар назариясидан фойдаланилган.

Тадқиқотнинг илмий янгилиги қуйидагилардан иборат:

ўзгарувчан ва қўзғалмас оралиқларда вазнли сингуляр ва гипер-сингуляр интеграллар учун автоматик квадратур схемаси қурилган ва қўзғалмас оралиқда вазнли сингуляр интеграллар учун модификацияланган дискрет уйирмали усул ишлаб чиқилган;

вазнли Гильберт ва силлиқ функциялар фазоларида чегараланган ва чегараланмаган ечим ҳолатлари учун автоматик квадратур схемасининг яқинлашуви исботланган ва модификацияланган дискрет уйирмали усулнинг яқинлашувчанлиги Гёльдер ва силлиқ функциялар синфида исботланган;

биринчи турдаги сингуляр ва гиперсингуляр интеграл тенгламалар учун чекли Чебышев кўпҳадлари қаторининг аниқ ечимга яқинлашуви кўрсатилган ва 2-турдаги сингуляр интеграл тенгламалар учун чегарада чекли бўлган ечимлар ҳолати учун чекли Чебышев кўпҳадлари қаторининг аниқ ечимга яқинлашуви кўрсатилган.

чекли ечим ҳолатида, гиперсингуляр интеграл тенгламалар учун ечимнинг мавжудлиги, ягоналиги ва ҳосиласи Гёльдер синфида бўлган тақрибий ечимнинг меъёрий яқинлашуви исботланган;

гомотопияни таҳлил қилиш усулининг янги турдаги ишланмаси ишлаб чиқилган ва у чизиқли бўлмаган бошланғич шартли интегро-дифференциал тенгламаларни ва каср тартибли чегаравий масалали интегро-дифференциал тенгламаларни ечиш учун қўлланилган;

чизиқли бўлмаган интегро-дифференциал тенгламаларни ва каср тартибли интегро-дифференциал тенгламаларни ечимининг мавжудлиги ва ягоналиги исботланган ҳамда гомотопияни таҳлил қилиш усули янги ишланмасининг яқинлашиши исботланган;

2×2 ночизиқли интеграл тенгламалар системаси учун такомиллаштирилган Ньютон-Канторович усули ишлаб чиқилган ва янги мажорант функциялар топилган ҳамда ечимнинг мавжудлиги, ягоналиги ва Ньютон-Канторович усулининг яқинлашиши исботланган;

ўзлаштирилган Ньютон усули ёрдамида ночизиқли кўп ўлчовли интеграл тенгламалар ечилган ҳамда вазнли Гильберт фазосида, ечимнинг мавжудлиги ва ягоналиги ҳамда таклиф этилган методнинг яқинлашиши исботланган.

Тадқиқотнинг амалий натижалари қуйидагилардан иборат:

вазнли регуляр ядроли интеграллар, сингуляр ва гипер-сингуляр интегралларни юқори аниқликда тақрибий ҳисоблаш учун қурилган автоматик квадратур схемаси жисмлар ёрилишида стресс интенсивлиги омилини юқори аниқликда ҳисоблашда фойдаланиш мумкин ҳамда сингуляр ва гипер-сингуляр интеграл тенгламаларни тақрибий ечиш учун чекли сондаги Чебышев қаторининг яқинлашувини таминлаш кўплаб моделли масалаларни ечишда муҳим рол ўйнаши мумкин;

ночизиқли интегро-дифференциал тенгламалар ва каср тартибли интегро-дифференциал тенгламаларни тақрибий ечиш учун гомотопияни таҳлил қилиш усулининг янги ишланмаси кўплаб ночизиқли муаммаларни ечишда асқотиши мумкин ҳамда ночизиқли интеграл тенгламалар системаси ва кўп ўлчовли интеграл тенгламалар учун ишлаб чиқилган ўзлаштирилган Ньютон-Канторович усули кўплаб амалий аҳамиятга эга бўлган муаммоларда ва илдиз топилмалари назариясида қўлланилиши мумкин.

Тадқиқот натижаларининг ишончлилиги ҳисоблаш математикаси, математик таҳлил, функционал таҳлил, шунингдек, математик ҳисоблашнинг қатъийлиги ёрдамида асосланади. Барча хулосалар ва сонли натижалар дастурий ҳисоблаш томонидан тасдиқланган.

Тадқиқот натижаларининг илмий ва амалий аҳамияти. Диссертацияда олинган натижаларнинг илмий аҳамияти амалиётда сингуляр ва гипер-сингуляр интеграллар, сингуляр ва гипер-сингуляр интеграл тенгламаларни тақрибий ҳисоблаш учун самарали квадратур формуласини қуриш алгоритми ишлаб чиқилганлиги билан изоҳланади.

Диссертациянинг амалий аҳамияти самарали квадратур формула, гомотопияни таҳлил қилиш усули ва Ньютон-Канторович усулининг эластиклик назарияси, квант механикаси, аэродинамика, электрон оптика, тўлқин тарқалиши масалаларини ечишда қўлланилиши билан изоҳланади.

Тадқиқот натижаларининг жорий қилиниши. Тадқиқот натижалари қуйидаги йўналишларда амалиётга жорий этилган:

эгри ёриқлар муаммоси учун формулировка қилинган гипер-сингулар интеграл тенгламалар бўйича олинган натижалар (Journal of Eng. Math., Vol. 126(1), 4, 24 pages, 2021; Gongcheng Lixue/Engin. Mech., 37(6), pp. 34-41, 2020; Journal of Computational and Applied Mathematics, 343, pp. 520-538, 2018) хорижий илмий журналларда сув тўлқинининг тарқалиши, плиталарнинг ихтиёрий нуқтасида нормал тезликларнинг ифодаларини олишда ҳамда эгри ёриқ ичидаги стресс интенсивлиги омилини топишда қўлланилган. Илмий натижаларнинг қўлланилиши ёриқнинг чегарасини баҳолаш имконини берган.

1-турдаги сингулар интеграл тенгламаларни тақрибий ечиш учун таклиф қилинган Чебишев яқинлаштириш методи бўйича олинган натижалар (Superconductor Science and Technology, 34(6), paper ID: 065006, 2021; Studies in Systems, Decision and Control, Vol. 340, pp. 63-101, 2021; Journal of Mathematical Analysis and Application, 482(1), ID: 123530, 2020) хорижий илмий журналларда юқори ҳароратли ўта ўтказувчан динамолар моделида ва фазода Чебишев кўпҳадлари ёрдамида вақт бўйича интеграллаш учун чизиқлаштириш усулидан фойдаланиб, интегралнинг аниқ сонли усулини олишда ҳамда таклиф этилган рақамли усул ёрдамида транспорт оқими ва критик оқим зичлиги билан боғлиқ муаммоларни ечишда қўлланилган. Илмий натижаларнинг қўлланилиши оқимларнинг зичлигини баҳолаш ва чегарасини аниқлаш имконини берган.

1-турдаги гипер-сингулар интеграл тенгламаларни тақрибий ечиш учун таклиф қилинган Галёркин-Чебишев методи бўйича олинган натижалар

(Bulletin of the Iranian Mathematical Society, 46(3), pp. 799-814, 2020; Journal of Low-Frequency Noise Vibration and Active Control, 38(2), pp. 706-727, 2019; Journal of Computational and Applied Mathematics, Vol. 343, pp. 619-634, 2018) хорижий илмий журналларда турли типдаги гиперсингуляр интеграл тенгламаларни юқори аниқликда ечишда ҳамда сув тўлқинининг тарқалиши билан боғлиқ масалаларни ечишда қўлланилган. Илмий натижаларнинг қўлланилиши сув тўлқинининг тарқалиши масаласини ечиш имконини берган.

Тадқиқот натижаларининг апробацияси: Мазкур диссертация тадқиқот натижалари 30 та халқаро илмий-амалий анжуманларда ва 3 та Ўзбекистон Республика илмий-амалий анжуманларида маъруза қилинган.

Тадқиқот натижаларининг эълон қилиниши: Диссертация мавзуси бўйича жами 57 та илмий иш чоп этилган, шулардан, Ўзбекистон Республикаси Олий Аттестация комиссиясининг диссертацияларга доир илмий натижаларини чоп этиш учун тавсия этилган илмий нашрларда 37 та мақола чоп этилган.

Диссертациянинг ҳажми ва тузилиши: Диссертация кириш қисми, тўртта боб, хулоса ва фойдаланилган адабиётлар рўйхатидан ташкил топган. Диссертацияning асосий қисми 197 бетдан иборат.

ДИССЕРТАЦИЯНИНГ АСОСИЙ НАТИЖАЛАРИ

Кириш қисмида, диссертация мавзусининг долзарблиги ва зарурати асосланган, тадқиқотнинг республика фан ва технологиялари ривожланишининг устивор йўналишларига мослиги кўрсатилган. Бундан ташқари, диссертация мавзуси бўйича хорижий илмий тадқиқотларнинг натижалари шарҳи, муаммонинг ўрганилганлик даражаси, ишнинг мақсад ва вазифалари, тадқиқот объекти ва предмети, тадқиқотнинг илмий янгилиги ва тадбиқий натижалари, олинган натижаларнинг назарий ва амалий аҳамияти, тадқиқот натижаларининг жорий қилиниши, нашр этилган ишлар ва диссертация тузилиши ҳақидаги маълумотлар берилган.

1-БОБ (асосий натижалар): Вазнли юқори тартибли сингуляр интеграллар учун автоматик қуадратур схема ва унинг қолдиқ ҳадлари.

Ушбу бобда, Коши ва Адамар типидаги вазнли сингулар интегралларни юқори аниқликда тақрибий ечиш учун метод (Автоматик квадратур схема) таклиф қилинган ва унинг қолдиқ ҳадларининг турли синфда баҳоланиши кўрсатилган. Диссертациянинг 1-боби учта катта қисмдан иборат.

- **1-қисми** (1.3.1-1.3.6) Автоматик квадратур схема (АКС) ёрдамида Коши типидаги вазнли сингуляр интеграллар(СИ)ни яқинлаштириш ва уларнинг хатоликларини баҳолашга бағишланган.
- **2-қисми** (1.4.1-1.4.5) эса, асосан, Адамар типидаги вазнли сингуляр интеграллар (яъни гипер-сингуляр интеграллар (ГСИлар))ни АКС ёрдамида ҳисоблаш ва қолдиқ ҳадларини баҳолашга тааллуқлидир.

- **3-қисми** (1.5.1-1.5.4) Модификацияланган дискрет уюрмали метод ёрдамида Коши типидаги вазнли сингуляр интегралларни яқинлаш-тириш ва уларнинг хатоликларини баҳолашга қаратилган.

Энди вазнли СИлар ва ГСИларни умумий ҳолда кўриб чиқайлик (1.1-1.2):

$$H_r^{(p)}(h,y,z,c) = \frac{w_r(c)}{\pi}\int_y^z \frac{h(t)}{w_r(t)(t-c)^p}dt\,, \quad r = \{0,1,2,3,4\},\ p = \{1,2\} \tag{1.1}$$

бу ерда $c \in (-1,1)$ – сингуляр нуқта, $h(t)$ берилган $[-1,1]$ оралиқда етарлича силлиқ функция, y,z параметерлар $-1 \le y < z \le 1$ ва $w_r(t)$ вазн функцияси бўлиб қуйидагича аниқланади.

$$w_0(t) = 1, \quad w_r(t) = \frac{\lambda_r(t)}{\sqrt{1-t^2}}, \quad \lambda_r(t) = \begin{cases} 1-t^2, & r = 1, \\ 1, & r = 2, \\ 1-t, & r = 3, \\ 1+t, & r = 4. \end{cases} \tag{1.2}$$

Ушбу (1.1) да, $p = 1$ бўлса, $H_r^{(1)}(h,y,z,c)$ функция Коши типидаги сингуляр интеграл деб аталади, ва $p = 2$ да $H_r^{(2)}(h,y,z,c)$ функция Адамар типидаги сингуляр интеграл деб аталади. Агар $p > 2$ бўлса, у ҳолда у интеграл супер-сингуляр интеграллар деб аталади.

1-боб, 1.3-1.4 параграфларнинг асосий мақсади, ҳар бир $p = 1,2$ ва $r = 0,1,2,3,4$ қийматлари учун $[y,z]$ ўзгарувчи ва $[-1,1]$ ўзгармас оралиқлар учун автоматик квадратур схемани (АКС) қуриш ва унинг хатологини турли синфларда баҳолашдир.

1.5 параграф эса Коши типидаги СИларни ўзгармас оралиқда тақрибий ҳисоблаш учун модификацияланган дискрет уюрмали методни қуриш ва унинг хатолигини баҳолашга бағишланган.

1.3.1 параграфда қуйидаги кўринишдаги Коши типидаги вазнли сингуляр интегралларни қарайлик

$$C_r(h,y,z,c) = \frac{w_r(c)}{\pi}\int_y^z \frac{h(t)}{w_r(t)(t-c)}dt\,, \quad r = \{0,1,2,3,4\}, \tag{1.3}$$

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$$P_{r,j}(t) = \begin{cases} T_j(t) = cos(j\theta),\ r = 1, \\ U_j(t) = \frac{sin((j+1)\theta)}{sin(\theta)},\ r = 2, \\ V_j(t) = \frac{cos((j+1/2)\theta)}{cos(\theta/2)},\ r = 3, \\ V_j(t) = \frac{sin((j+1/2)\theta)}{sin(\theta/2)},\ r = 4. \end{cases} \tag{1.4}$$

ва чекли сондаги Чебышев қаторини аниқлаб олайлик

$$S_{N,0}(t) = \sum_{k=0}^{N}{}' a_{k,0}T_k(t)\,,\ S_{N,r}(t) = \sum_{k=0}^{N}{}' a_{k,}P_{r,j}(t),\ r = 1,2,3,4, \tag{1.5}$$

бунда, штрих йиғиндидаги 1-ҳад иккига бўлинганини билдиради

Ушбу параграфда (1.3) сингуляр интеграллар учун ўзгарувчан оралиқ $[y,z]$ да, ҳар бир $r=0,1,2,3,4$ нинг қийматида АКС қурилган. Хусусан:

0 - ҳолат: $r=0$ да СИлар (1.3) учун АКС қуйидаги кўринишга эга:

$$Q_{N,0}(f,y,z,c)=\frac{1}{\pi}\left[\frac{1}{2}\sum_{k=0}^{N} B_{k,0}[T_k(z)-T_k(y)]+h(c)\ln\frac{z-c}{c-y}\right], \qquad (1.6)$$

бу ерда $T_k(t)$ (1.4) билан аниқланган бўлиб, $B_{k,0}$ коэффициентлар қуйидагича аниқланади:

$$B_{0,0}=\frac{b_{1,0}}{4},\ B_{k,0}=\frac{b_{k-1,0}-b_{k+1,0}}{2k},\ k=\{1,\dots,N\},\ b_{N,0}=b_{N+1,0}=0. \qquad (1.7)$$

ва $b_{k,0}$ учун уч диагоналли алгебраик тенгламалар системасига эга бўламиз:

$$b_{k-1,0}-2cb_{k,0}+b_{k+1,0}=2a_{k,0},\ \ k=1,2,\dots,N, b_{N,0}=b_{N+1,0}=0. \qquad (1.8)$$

(1.8) тенгламани ечиб $b_{k,0}(c)$ топилади, (1.7) дан $B_{k,0}(c)$ коэффициентлар топилгач (1.3) нинг тақрибий ечимларини (1.6) орқали ҳисоблаймиз.

1 - ҳолат: $r=1$ да (1.3) СИ учун қурилган АКС қуйидагича бўлади:

$$Q_{N,1}(h,y,z,c)=\frac{\sqrt{1-c^2}}{\pi}\cdot\left\{\left[\frac{b_{0,1}}{2}\text{arcsin}t\Big|_y^z-\sum_{k=1}^{N-1}\frac{b_{k,1}}{k}\sqrt{1-t^2}U_{k-1}(t)\Big|_y^z\right]\right.$$
$$\left.+\frac{h(c)}{\sqrt{1-c^2}}\ln\left|\frac{t\sqrt{1-c^2}-c\sqrt{1-t^2}}{\sqrt{1-c^2}+\sqrt{1-t^2}}\right|_y^z\right\} \qquad (1.9)$$

2 - ҳолат: $r=2$ учун АКС қуйидагича олинади

$$Q_{N,2}=\frac{1}{\pi\sqrt{1-c^2}}\cdot\left\{\frac{b_{0,2}}{2}\text{arcsin}t\Big|_y^z+\sum_{k=1}^{N-1}\left[\frac{b_{k-1,2}}{k+1}-\frac{b_{k+1,2}}{k+1}\right]\sqrt{1-t^2}U_k(t)\Big|_y^z\right.$$
$$\left.+h(c)\left[\left[\sqrt{1-t^2}-c\cdot arcsin t\right]_y^z\sqrt{1-c^2}\ln\left|\frac{t\sqrt{1-c^2}-c\sqrt{1-t^2}}{\sqrt{1-c^2}+\sqrt{1-t^2}}\right|_y^z\right]\right\}. \qquad (1.10)$$

3 - ҳолат: $r=3$ учун биз қуйидаги кўринишдаги АКС га эга бўламиз:

$$Q_{N,3}(h,c,y,z)=\frac{1}{\pi}\sqrt{\frac{1-c}{1+c}}\cdot\left\{b_{0,3}\text{arcsin}t\Big|_y^z-\sum_{k=0}^{N-1}\frac{1}{k+1}\left(b_{k,3}+b_{k+1,3}\right)\sqrt{1-t^2}U_k(t)\Big|_y^z\right.$$
$$\left.+h(c)\left[\text{arcsin}t\Big|_y^z+\sqrt{\frac{1+c}{1-c}}\ln\left|\frac{t\sqrt{1-c^2}-c\sqrt{1-t^2}}{\sqrt{1-c^2}+\sqrt{1-t^2}}\right|_y^z\right]\right\}. \qquad (1.11)$$

4 - ҳолат: $r=4$ учун АКС қуйидаги формада қурилган:

$$Q_{N,4}(h,c,y,z)=\frac{1}{\pi}\sqrt{\frac{1+c}{1-c}}\cdot\left\{b_{0,4}\text{arcsin}t\Big|_y^z-\sum_{k=0}^{N-1}\frac{1}{k+1}\left(b_{k,4}-b_{k+1,4}\right)\sqrt{1-t^2}U_k(t)\Big|_y^z\right.$$
$$\left.+h(c)\left[-\text{arcsin}t\Big|_y^z+\sqrt{\frac{1-c}{1+c}}\ln\left|\frac{t\sqrt{1-c^2}-c\sqrt{1-t^2}}{\sqrt{1-c^2}+\sqrt{1-t^2}}\right|_y^z\right]\right\}, \qquad (1.12)$$

бу ерда номаълум $b_{k,r}=b_{k,r}(c)$ коэффициентлар қуйидагича аниқланади:

$$b_{k-1,r}(c)-2cb_{k,r}(c)+b_{k+1,r}(c)=2a_{k,r}, \qquad k=1,2,\dots,N,$$
$$b_{N,r}(c)=b_{N+1,r}(c)=0, \quad r=1,2,3,4 \qquad (1.13)$$

бунда $a_{k,r}$ лар интерполяция шартларидан фойдаланган ҳолда топилади.

1.3.2 параграфда қўзғалмас $[-1,1]$ оралиқда қуйидаги кўринишдаги

$$C_r(h,c)=\frac{w_r(c)}{\pi}\int_{-1}^{1}\frac{h(t)}{w_r(t)(t-c)}dt\,,\ r=0,1,2,3,4,\ c\in(-1,1), \qquad (1.14)$$

бу ерда $w_r(t)$ (1.2) орқали аниқланган, вазнли СИлар учун АКС қурилган.

0 - ҳолат: ($r=0$) учун АКС қуйидаги кўринишда қурилган:

$$Q_{N,0}(h,c)=\frac{1}{\pi}\left[b_{0,0}(c)-\sum_{k=1}^{\left[\frac{N+1}{2}\right]-1}b_{2k,0}(c)\frac{2}{(2k)^2-1}+h(c)\ln\left|\frac{1-c}{1+c}\right|\right], \qquad (1.15)$$

(1.15) дан келиб чиқадики [-1,1] да АКСни ҳисоблаш жараёни йиғиндининг ярмига қисқаради ва бу ҳисоблаш вақтини тежашга ёрдам беради.

1-4 ҳолат: Бундай ҳолларда АКС қуйидаги кўринишда қурилган

$$C_r(h,c)=\frac{w_r(c)}{\pi}\int_{-1}^{1}\frac{h(t)}{w_r(t)(t-c)}dt\approx\frac{w_r(c)}{\pi}\int_{-1}^{1}\frac{S_{N,r}(t)}{w_r(t)(t-c)}dt=Q_{N,r}(h,c)$$

$$=\begin{cases}\sqrt{1-c^2}\sum_{k=1}^{N}a_{k,1}U_{k-1}(c), & r=1,\\ -\frac{1}{\sqrt{1-c^2}}\sum_{k=0}^{N}a_{k,2}T_{k+1}(c), & r=2,\\ \sqrt{\frac{1-c}{1+c}}\sum_{k=0}^{N}a_{k,3}W_k(c), & r=3,\\ -\sqrt{\frac{1+c}{1-c}}\sum_{k=0}^{N}a_{k,4}V_k(c), & r=4.\end{cases} \qquad (1.16)$$

1.3.3 параграфда таклиф этилган усулнинг афзалликларини кўрсатувчи кўплаб сонли натижалар олинган.

1.3.4 параграфда Коши типидаги вазнли сингуляр интеграллар учун қурилган АКС нинг қолдиқ ҳадини $r=\{0,1\}$ ҳолатлари учун баҳоланган. Чекли Чебышев қатори яқинлашиш хатолигини кўрсатиш учун функцияларнинг қуйидаги синфларини киритамиз.

- $H^{\alpha}([a,b],K)$ шундай функциялар синфики, $[a,b]$ оралиқда Гёльдер шартини, α индекси ва K доимийси билан қаноатлантиради.
- $C^{m,\alpha}[a,b]=\{f(t)\colon\ f^{(m)}\in H^{\alpha}([a,b],A_m)\}$.
- $L_{p,\mathrm{w}}=\left\{f(t)\colon\ \int_y^z w(t)|f(t)|^p dt<\infty\right\}$.
- Xatolikni baholashda quyidagi normalarni ishlatamiz. Фараз қилайлик $e_{N,r}(t)=h(t)-S_{N,r}(t)$, у ҳолда нормалар қуйидагича аниқланади

  L_p норма: $\|e_{N,r}\|_p=\left[\int_a^b w(t)\left|e_{N,r}(t)\right|^p dt\right]^{1/p}$,

  Чебышев норма: $\|e_{N,r}\|_C=max_{t\in[a,b]}\left|h(t)-S_{N,r}(t)\right|$.

Ушбу параграфда қурилган АКС учун қуйидаги теоремаларни исботладик.

Теорема 1.1 *Фараз қилайлик,* $h\in L_{2,w_0}[-1,1]$ *ва* $S_{N,0}(t)$ (1.5) *билан берилган бўлиб ушбу* $\omega_{N+1}(t)=2(t^2-1)U_{N-1}(t)$ *функциянинг нолларида яъни* $t_j=cos\left(\frac{\pi j}{N}\right), j=0,1,\cdots,N$ *аниқланган бўлсин. У ҳолда* (1.6) *билан аниқланган*

$Q_{N,0}(h,y,z,c)$ *квадратур қоида,* (1.3) *орқали аниқланган* $C_0(h,y,z,c)$ *сингуляр интегралга* L_{q,w_0} *нормада яқинлашади*

$$\left\|R_{N,0}(h,c)\right\|_q = \left\|C_0(h,y,z,c) - Q_{N,0}(h,y,z,c)\right\|_q \underset{N\to\infty}{\to} 0,$$

бу ерда $1 < q < \frac{p}{p_0}$ ва $p > p_0 > 1$.

Теорема 1.2. Фараз қилайлик, $h \in C[-1,1]$ *ва* $S_{N,1}(t)$ (1.5) *билан берилган бўлиб, ушбу* $\omega_{N+1}(t) = (1-t^2)U_{n-1}(t)$ *функциянинг нолларида аниқланган бўлсин. У ҳолда* (1.9) *билан аниқланган* $Q_{N,1}(h,y,z,c)$ *квадратур қоида,* (1.3) *билан аниқланган* $C_1(h,y,z,c)$ *га* L_{q,w_1} *нормада яқинлашади.*

$$\|R_{N,1}(h,c)\|_q = \|C_1(h,y,z,c) - Q_{N,1}(h,y,z,c)\|_q \to 0_{N\to\infty},$$

бу ерда $1 < q < \frac{p}{p_0}$ ва $p > p_o > 1$.

Теорем 1.3. *Фараз қилайлик,* $f(t) \in C^{N+1,\alpha}[-1,1]$, *ва* $S_{N,1}(t)$ (1.15) *билан берилган бўлиб, ушбу* $\omega_{N+1}(t) = (1-t^2)U_{n-1}(t)$ *функциянинг нолларида аниқланган бўлсин. У ҳолда* (1.16) билан аниқланган *АКС* ($r = 1$) *учун хатолик чегараси қуйидаги кўринишда бўлади:*

$$\left\|R_{N,1}(f,c)\right\|_C \le \frac{8A_1}{2^{N-1}N!}\left[1 + \frac{3.529}{\alpha ln(N+1)}\right]\frac{ln(N+1)}{N+1}.$$

Шундай қилиб, биз ўзгарувчан [*y,z*] ва қўзғалмас [-1,1] оралиқда Коши типидаги вазнли СИлар (1.14) учун АКС ни қурдик ва таклиф қилинган усулнинг турли функциялар синфларида яқинлашишини исботладик.

1.4.1-1.4.2 параграфларда умумий тушунчалар киритилгандан сўнг қуйидаги вазнли гипер-сингуляр интеграллар

$$H_r^{(2)}(h,y,z,c) = \frac{w_r(c)}{\pi}\int_y^z \frac{h(t)}{w_r(t)(t-c)^2}dt, \quad r = 0,1,2,3,4, \qquad (1.17)$$

учун АКС ўзгарувчан оралиқ $[y,z]$ да, ҳар бир $r = 0,1,2,3,4$ нинг қийматида қурилган. Хусусан:

0 - ҳолат, $r = 0$. Бундай ҳолда, АКС қуйидаги шаклда қурилган:

$$H_{N,0}(h,y,z,c) = \frac{1}{\pi}\left[\sum_{k=0}^{N}\frac{d}{dc}B_{k,0}(c)T_k(t)\Big|_y^z + h'(c)\ln\left|\frac{z-c}{y-c}\right|\right] + \frac{h(c)}{\pi}\frac{z-y}{(z-c)(y-c)} + R_{N,0}(c),$$

бу ерда $B_{k,0}(c)$ (1.5) орқали аниқланади.

1-4 ҳолатлар: ҳар бир $r = \{1,2,3,4\}$ қиймтида (1.9)-(1.12) тенгламаларни c га нисбатан дифференциаллаб, АКС қуйидаги шаклда қурилган:

$\boldsymbol{r = 1}$ **да**

$$H_{N,1}(h,y,z,c) = \frac{\sqrt{1-c^2}}{\pi}\frac{d}{dc}\left\{\left[\frac{b_{0,1}(c)}{2}\text{arcsin}t\Big|_y^z - \sum_{k=1}^{N-1}\frac{b_{k,1}(c)}{k}\sqrt{1-t^2}U_{k-1}(t)\Big|_y^z\right] + \frac{h(c)}{\sqrt{1-c^2}}\ln\left|\frac{t\sqrt{1-c^2}-c\sqrt{1-t^2}}{\sqrt{1-c^2}+\sqrt{1-t^2}}\right|_y^z\right\}.$$

$\boldsymbol{r=2}$ **да**

$$H_{N,2}(h,y,z,c)=\frac{1}{\pi\sqrt{1-c^2}}\frac{d}{dc}\cdot\left\{\frac{b_{0,2}(c)}{2}\text{arcsin}t\Big|_y^z\right.$$
$$+\sum_{k=1}^{N-1}\left[\frac{b_{k-1,2}(c)}{k+1}-\ \frac{b_{k+1,2}(c)}{k+1}\right]\sqrt{1-t^2}U_k(t)\Big|_y^z$$
$$+h(c)\left[\left[\sqrt{1-t^2}-c\cdot\text{arcsin}t\right]_y^z+\sqrt{1-c^2}\ln\left|\frac{t\sqrt{1-c^2}-c\sqrt{1-t^2}}{\sqrt{1-c^2}+\sqrt{1-t^2}}\right|_y^z\right]$$

$\boldsymbol{r=3,4}$ да (1.11) ва (1.12) ларни мос равишда дифференциаллаб АКС қурилган, бу ерда $b_{k,r}(c)$ коэффициентлар (1.13) орқали аниқланади.

1.4.3 параграфда қўзғалмас $[-1,\ 1]$ оралиқда вазнли ГСИлар (1.13) ва (1.14) ни мос равишда дифференциаллаб АКС қурилган.

Эслатма: ГСИлар учун АКСни қуриш жуда оддий, лекин [-1,1] оралиқнинг чегарасидаги ГСИнинг қиймати СИларнинг қийматидан кескин фарқ қилади.

1.4.4 параграфда қурилган АКС лар учун бир қанча сонли натижалар келтирилган сўнг, **1.4.5 параграфда** ГСИларнинг чегараланган ва чегараланмаган ечими учун хатоликлар баҳоланган. Бунинг учун хатолик функцияси керак бўлади. Фараз қилайлик,

$$E_{N,r}(h)=H_r(h,x)-H_{N,r}(h,x),\ \ r=\{1,2,3,4\}$$

бўлсин. Асосий натижалар қуйидаги теорема келтирилган.

Теорема 1.4. Фараз қилайлик, $0<\alpha\le 1$ учун $h(t)\in C^{N+1,\alpha}[-1,1]$ *ва Чебышев полиномларининг қирқилган қатори* $S_{N,r}(t)$, $r=\{1,2\}$ *бўлсин. Уҳолда АКС (1.17) хатолиги қуйигича бўлади.*

$$\|E_{N,r}(h)\|_C\le\begin{cases}\dfrac{2.12Mln(N)}{2^{N-2}(N-2)!}\left[1+\dfrac{3.12}{6ln(N)}\right], & r=1,\\[2mm] \dfrac{3.38M}{2^{N-2}(N-2)!}\left[1+\dfrac{0.63}{N}L_{1N}+\dfrac{4.19}{N+1}\right], & r=2.\end{cases}$$

бунда $L_{1N}=1+\frac{1.21}{N}+\frac{3.35}{N+1}$, $M=max\{M_1,M_2\}$ *бунда* M_1,M_2 *лар ўзгармаслар.*

1.5.1-1.5.2 параграфда Коши типидаги вазнли СИлар учун дискрет уюрмали усулининг янги модификацияси (МДУМ) ишлаб чиқилган. Бунинг учун қуйидаги вазнли сингуляр интегралларни кўриб чиқилган.

$$I_1(f,x)=\frac{w_1(x)}{\pi}\int_{-1}^{1}\frac{f(t)dt}{w_1(t)(t-x)},$$
$$I_i(f,x)=I_1(f,x)+Q_i(f,x),\ \ \ i=\{2,3,4\}$$

бу ерда

$$Q_i(f,x)=\frac{(-1)^i}{\pi}w_i(x)\int_{-1}^{1}\frac{f(t)dt}{\sqrt{1-t^2}},\ i=\{2,3\},$$
$$Q_4(f,x)=-\frac{1}{\pi}w_4(x)\int_{-1}^{1}\frac{g(x,t)dt}{\sqrt{1-t^2}},\ g(x,t)=(x+t)f(t),$$

ва $w_1(x)=\sqrt{1-x^2},\ w_2(x)=\sqrt{\frac{1-x}{1+x}},\ w_3(x)=\sqrt{\frac{1+x}{1-x}},\ w_4(x)=\frac{1}{\sqrt{1-x^2}}$.

Ушбу параграфда, МДУМ ва чизиқли интерполяцион сплайн бирлаштириб, иккита янги КФлар қурилган. Биринчи гуруҳ КФлар $Q_i(f,x)$ лар учун ва иккинчи гуруҳ КФлар $I_i(f,x)$ лар учун. Бунинг учун фараз қилайлик, $t_k=-1+kh, h=2/(N+1)$ ва $E=\{t_k,k=1,\dots,N\}$ лар $[-1,1]$ оралиқнинг каноник бўлиниши бўлсин, у ҳолда $x=t_j+\varepsilon, j=1,\dots,N-1$ да $I_1(f,x)$ функция учун КФлар қуйидаги кўринишда қурилган:

$$I_1(f,x)\cong\widetilde{I_{N,1}}(f,x)=\left(\sum_{k=1}^{j-2}+\sum_{k=j+3}^{N}\right)A_k^{(1)}(x)\varphi(t_k)+\sum_{\nu=j-1}^{j+2}A_\nu^{(1)}(x)f(x_\nu)$$
$$+A_0^{(1)}(x)f(-1)+A_{N+1}^{(1)}(x)f(1), \qquad (1.18)$$

бу ерда $\varphi(t)=f(t)-\frac{1}{2}[(1-t)f(-1)+(1+t)f(1)$, $A_k^{(1)}$ va $B_k^{(1)}$ коэффициентлар (1.19) даги СИларни баҳолаш орқали ҳисобланади. Бинобарин, $x=t_j+\varepsilon, j=1,\dots N-1$ да СИлар (1.20) учун КФлар қуйидагича

$$I_i(f,x)\cong\widetilde{I_{N,\iota}}(f,x)=\left(\sum_{k=1}^{j-2}+\sum_{k=j+3}^{N}\right)\left(A_k^{(1)}(x)\varphi(t_k)+\frac{(-1)^i}{\pi}w_i(x)a_i(x,t_k)C_kf(t_k)\right)$$
$$+\left(A_0^{(1)}(x)+\frac{(-1)^i}{\pi}w_i(x)a_i(x,-1)C_0\right)f(-1)+\left(A_{N+1}^{(1)}(x)+\frac{(-1)^i}{\pi}w_i(x)a_i(x,1)C_{N+1}\right)f(1)$$
$$+\sum_{\nu=j-1}^{j+2}\left(A_\nu^{(1)}(x)+\frac{(-1)^\nu}{\pi}w_\nu(x)a_i(x,t_\nu)C_\nu\right)f(t_\nu),\ i=\{2,3,4\}, \qquad (1.19)$$

бу ерда $a_2(x,t)=a_3(x,t)=1,\ a_4(x,t)=-(x+t)$, ва C_k коэффициентлар $Q_i(f,x)$ ни ҳисоблаш йўли орқали топилади.

1.5.3 параграфда модификацияланган дискрет айирмали усулнинг хатолиги қуйидаги теоремаларда исботланган.

Теорема 1.5. Фараз қилайлик, $f(t)\in C([-1,1])$, *ва* E $[-1,1]$ *оралиқнинг каноник қисмлар тўплами бўлсин, у ҳолда* (1.17) *ва* (1.18) *нинг хатоси*

$$\left\|R_N^{(1)}(f,x)\right\|_C\le\begin{cases}L_1h^\alpha\, ln(N+1), & f(t)\in H^\alpha([-1,1],A),\\ L_2h\, ln(N+1), & f(t)\in C^1([-1,1]),\end{cases}$$

бу ерда L_1, L_2 *ўзгармас сонлар.*

Теорема 1.6. Фараз қилайлик, $f(t)\in C([-1,1])$ *ва* E $[-1,1]$ *оралиқнинг каноник қисмлар тўплами бўлсин, у ҳолда* (1.17) *ва* (1.18) *КФларнинг хатоси қуйидагича:*

$$R_N^{(i)}(f,x\le\begin{cases}L_1h^\alpha\, ln(N+1)+\dfrac{d_i\bar{A}_i}{2^\alpha}w_i(x)h^\alpha,\ \ when\ \ f(t)\in H^\alpha([-1,1],A),\ \ i=\{2,3,4\},\\ `L_2h\, ln(N+1)+\dfrac{d_i\bar{M}_i}{2}w_i(x)h,\ \ when\ \ f(t)\in C^1([-1,1]),\ \ i=\{2,3,4\},\end{cases}$$

бу ерда $d_2=d_3=1, d_4=3, \underline{A}_2=\underline{A}_3, \underline{M}_2=\underline{M}_3$, va L_1,L_2 доимийдир.

1.5.4 параграфда модификацияланган ДУМнинг афзаллигини кўрсатувчи кўплаб сонли натижалар олинган.

2-БОБ (асосий натижалар): Биринчи ва иккинчи тур сингуляр ва гипер-сингуляр интеграл тенгламамлар учун тақрибий методлар.

Ушбу бобда, p-тартибли сингуляр интеграл тенгламаларнинг умумий тарифидан сўнг, биринчи тур Коши ва Адамар типидаги сингулар интеграл тенгламаларни юқори аниқликда ечиш учун тақрибий метод (Чебышев аппрокцимация методи) таклиф қилинган ва унинг қолдиқ ҳадларининг турли синфда баҳоланиши кўрсатилган. Диссертациянинг 2-боби асосан учта катта қисмдан иборат.

- **1-қисми** (2.1-2.3) p-тартибли сингуляр интеграл тенгламалар (СИТлар) нинг умумий тафсифидан сўнг, (2.4.1-2.4.2) 1-тур СИТларнинг тақрибий ечимини топиш, таҳлил этиш ва сонли натижаларга бағишланган.
- **2-қисми** (2.4.3-2.4.6) эса, асосан, Адамар типидаги гипер-сингуляр интеграл тенгламалар (ГСИТлар)нинг тақрибий ечимини топиш, таҳлил этиш, хатолигини баҳолаш ва сонли натижаларга бағишланган.
- **3-қисми** (2.5.1-2.5.2) асосан 2-тур СИТларни тақрибий ечиш, таҳлил этиш ва сонли натижаларга бағишланган.

2.1-2.3 параграфда p-тартибли биринчи тур СИТларнинг умумий тавсифи, қўшимчалар ва янги қурилган КФлар ҳақида малумотлар берилган.

2.4 параграфда эса қуйидаги p-тартибли 1-тур умумий СИТлар кўриб чиқилган ва умумий кўринишдаги ечимлари тавсиф қилинган

$$\frac{1}{\pi}\int_{-1}^{1}\varphi(t)\left[\frac{K(x,t)}{(t-x)^p}+L_1(x,t)\right]dt=f(x),\ \ p=\{1,2,\cdots\},-1<x<1, \quad (2.1)$$

бу ерда x сингуляр нуқта, $K(x,t), L_1(x,t)$ ва $f(x)$ ҳақиқий қийматли узлуксиз функциялар бўлиб, $\varphi(t)$ аниқланиши керак бўлган функциядир.

Фараз қиламиз, (2.1) тенгламадаги $K(x,t)$ ядро D=[-1,1]×[-1,1] соҳанинг диагоналида доимий бўлсин. У ҳолда, ядрони қуйидаги кўринишда оламиз:

$$K(x,x)=c_0\neq 0. \quad (2.2)$$

(2.2) шартни ҳисобга олиб, (2.1) тенгламани қуйидаги кўринишда ёзамиз:

$$\frac{c_0}{\pi}\int_{-1}^{1}\frac{\varphi(t)}{(t-x)^p}dt+\frac{1}{\pi}\int_{-1}^{1}\frac{Q_1(x,t)\varphi(t)}{(t-x)^{p-1}}dt+\frac{1}{\pi}\int_{-1}^{1}L_1(x,t)\varphi(t)dt=f(x), \quad (2.3)$$

бу ерда $-1<x<1$ va $Q_1(x,t)=\frac{K(x,t)-K(x,x)}{t-x}$.

Асосий мақсад $p=\{1,2\}$ қийматлари учун (2.3) тенгламанинг тўрт хил ечимини топишдир. Бунинг учун ечимни қуйидаги кўринишда қидирамиз.

$$\varphi(x)=w_r(x)u(x),\quad r=\{1,2,3,4\}, \quad (2.4)$$

бу ерда $w_i(x), i=\{1,2,3,4\}$ (1.2) орқали аниқланган. (2.4) ни (2.3) га қўйиш орқали қуйидаги оператор кўринишдаги тенгламани оламиз.

$$H_{p,r}u+C_{p,r}u+L_ru=f,\quad p=\{1,2\},\ \ r=\{1,2,3,4\}, \quad (2.5)$$

бу ерда $f\in L_{2\rho},\quad u\in L_{1\rho}$, ва $L_{2\rho}$ ($L_{1\rho}$) фазолари 2.4.4 - параграфда аниқланган ва операторлар қуйидагича аниқланади:

$$H_{p,r}u=\frac{c_0}{\pi}\int_{-1}^{1}\frac{w_r(t)}{(t-x)^p}\,u(t)dt,$$
$$C_{p,r}u=\frac{1}{\pi}\int_{-1}^{1}\frac{w_r(t)Q_1(x,t)}{(t-x)^{p-1}}\,u(t)dt, \tag{2.7}$$
$$L_ru=\frac{1}{\pi}\int_{-1}^{1}w_r(t)L_1(x,t)u(t)dt.$$

Шундай қилиб, келгуси параграфларда $p=\{1,2\}$ нинг қиймати учун $r=\{1,2,3,4\}$ нинг ҳар бир ҳолатида (2.6) тенгламанинг тақрибий ечимини топамиз.

2.4.1 параграфда биринчи тур сингуляр интеграл тенгламаларнинг тақрибий ечими топилган. Бунинг учун $p=1$ бўлсин, у ҳолда (2.6)-(2.7) нинг оператор кўринишдаги тенгламаси қуйидагича бўлади:

$$C_{1,r}u+L_ru=f,\quad r=\{1,2,3,4\},\quad f\in L_{2\rho},\quad u\in L_{1\rho}, \tag{2.8}$$

бу ерда

$$C_{1,\mathrm{r}}u=\frac{c_0}{\pi}\int_{-1}^{1}\frac{w_r(t)}{(t-x)}u(t)dt,\ L_ru=\frac{1}{\pi}\int_{-1}^{1}w_r(t)L(x,t)u(t)dt$$
$$L(x,t)=Q_1(x,t)-L_1(x,t),\ Q_1(x,t)=\frac{K(x,t)-K(x,x)}{t-x} \tag{2.9}$$

(2.8) тенгламанинг тақрибий ечимини топиш учун номаълум функция $u(t)$ ни қуйидаги кўринишда яқинлаштирамиз:

$$u(t)\cong u_{n,r}(t)=\sum_{j=0}^{n}b_{j,r}P_{j,r}^*(t),\quad r=\{1,2,3,4\}, \tag{2.10}$$

бу ерда $P_{j,r}^*(t)$ қуйидагича аниқланади.

$$P_{j,r}^*(t)=\begin{cases}U_j(t)=\cos(j\theta), & r=1,\\ T_j(t)=\frac{\sin((j+1)\theta)}{\sin(\theta)}, & r=2,\\ W_j(t)=\frac{\sin((j+1/2)\theta)}{\sin(\theta/2)}, & r=3,\\ V_j(t)=\frac{\cos((j+1/2)\theta)}{\cos(\theta/2)}, & r=4.\end{cases}$$

(2.10) тенгламани (2.8) га қўйсак, қуйидаги тенглама ҳосил бўлади.

$$\sum_{j=1}^{n}b_{j,r}\left[\frac{c_0}{\pi}\int_{-1}^{1}\frac{w_r(t)}{(t-x)}P_{j,r}^*(t)dt+\frac{1}{\pi}\int_{-1}^{1}w_r(x)L(x,t)P_{j,r}^*(t)dt\right]$$
$$+b_{0,r}\left[\frac{c_0}{\pi}\int_{-1}^{1}\frac{w_r(t)}{(t-x)}\,dt+\frac{1}{\pi}\int_{-1}^{1}w_r(x)L(x,t)\,dt\right]=f(x). \tag{2.11}$$

Ушбу белгилашни киритиб:

$$G_{j,r}^*(x)=\frac{1}{\pi}\int_{-1}^{1}\frac{w_r(t)}{(t-x)}P_{j,r}^*(t)dt=\begin{cases}U_{j-1}(x), & r=1,\\ -T_{j+1}(x), & r=2,\\ W_j(x), & r=3,\\ -V_j(x), & r=4.\end{cases}\quad h_r(x)=\frac{1}{\pi}\int_{-1}^{1}\frac{w_r(t)}{(t-x)}\,dt=\begin{cases}0, & r=1,\\ -x, & r=2,\\ 1, & r=3,\\ 1, & r=4\end{cases}$$

қуйидаги тенгламани оламиз:

$$\sum_{j=1}^{n}b_{j,r}\left[c_0\,G_{j,r}^*(x)+\psi_{j,r}^*(x)\right]+b_{0,r}\left[c_0h_r(x)+\psi_{0,r}^*(x)\right]=f(x), \tag{2.12}$$

бунда $r=\{1,2,3,4\}$ ва $\psi_{j,r}^*(x)$ қуйидагича бўлади

$$\psi_{j,r}^*(x)=\frac{1}{\pi}\int_{-1}^{1}w_r(x)L(x,t)P_{j,r}^*(t)\,dt.$$

Номаълум $b_{j,r}$ ларни топиш учун (2.12) тенгламада $r = \{1,2,3,4\}$ нинг ҳар бир қиймати учун аниқ схема қурилган ва ядро $L(x,t)$ ни ёйиш схемаси билан бирга коллокация усули қўлланилган. $b_{j,r}(c)$ қийматларини топилганидан кейин уни (2.10) га қўйиб (2.4) тенгламанинг ярим аналитик ечими олинган.

2.4.2 параграфда таклиф этилган методнинг афзаллигини кўрсатиш учун кўплаб сонли натижалар ишлаб чиқилган.

2.4.3 параграфда биринчи тур ГСИТ учун тақрибий ечим тавсифи батафсил ёритилган. $p = 2$ бўлганда (2.6)-(2.7) оператор тенгламасидан қуйидаги кўринишдаги тенглама олинган:

$$H_{2,r}u + C_{2,r}u + L_r^*u = f, \quad r = \{1,2,3,4\}, \tag{2.13}$$

бу ерда

$$\begin{aligned} H_{2,r}u &= \frac{c_0}{\pi}\int_{-1}^{1} \frac{w_r(t)}{(t-x)^2}\, u(t)dt, \\ C_{2,r}u &= \frac{Q_1(x,x)}{\pi}\int_{-1}^{1} \frac{w_r(t)}{t-x}\, u(t)dt, \\ L_r^*u &= \frac{1}{\pi}\int_{-1}^{1} w_r(t)L^*(x,t)u(t)dt, \end{aligned} \tag{2.14}$$

бунда $Q_1(x,t)$ ядроси (2.9) орқали аниқланади ва $L^*(x,t)$ қуйидагича бўлади:

$$L^*(x,t) = Q_2(x,t) + L_1(x,t), Q_2(x,t) = \frac{Q_1(x,t)-Q_1(x,x)}{t-x} \tag{2.15}$$

(2.13) тенгламанинг тақрибий ечимини топиш учун (2.11) ни (2.13) га қўйиб қуйидаги тенглама ҳосил қилинган.

$$\begin{aligned} \sum_{j=1}^{n} b_{j,r} \Big[\frac{c_0}{\pi}\int_{-1}^{1} \frac{w_r(t)}{(t-x)^2}\, P_{j,r}^*(t)dt + \frac{Q_1(x,x)}{\pi}\int_{-1}^{1} \frac{w_r(t)}{(t-x)}\, P_{j,r}^*(t)dt \\ + \frac{1}{\pi}\int_{-1}^{1} w_r(x)L^*(x,t)P_{j,r}(t)dt\Big] + b_{0,r}\Big[\frac{c_0}{\pi}\int_{-1}^{1} \frac{w_r(t)}{(t-x)^2}\, dt \\ + \frac{Q_1(x,x)}{\pi}\int_{-1}^{1} \frac{w_r(t)}{(t-x)}\, dt + \frac{1}{\pi}\int_{-1}^{1} w_r(x)L^*(x,t)\, dt\Big] = f(x) \end{aligned} \tag{2.16}$$

(2.16) тенгламада гипер-сингуляр, сингуляр интеграллар ва $b_{0,r}$ коэффициент олдидаги вазнли интегралларни аниқ ҳисоблаш қуйидагиларга олиб келади:

$$\begin{aligned} \sum_{j=1}^{n} b_{j,r} \Big[c_0\frac{d}{dx}G_{j,r}^*(x) + Q_1(x,x)G_{j,r}^*(x) + \psi_{j,r}^*(x)\Big] \\ + b_{0,r}\Big[c_0\frac{d}{dx}h_r(x) + Q_1(x,x)h_r(x) + \psi_{0,r}^*(x)\Big] = f(x), \end{aligned} \tag{2.17}$$

бу ерда $L^*(x,t)$ (2.17) орқали аниқланади ва $\psi_{j,r}^*(x)$ қуйидагича бўлади:

$$\psi_{j,r}^*(x) = \frac{1}{\pi}\int_{-1}^{1} w_r(x)L^*(x,t)P_{j,r}^*(t)dt, \;\; r = \{1,2,3,4\}.$$

(2.17) тенглама $r = \{1,2,3,4\}$ нинг ҳар бир қиймати учун ядрони ёйиш схемаси билан биргаликда номаълум $b_{j,r}$ параметрларни топиш учун коллокация усулидан фойдаланилди ва $b_{j,r}$ қийматларини топиб, уни (2.11) га қўйиб, (2.4) тенгламадан (2.15) тенгламанинг ярим аналитик тақрибий ечими топилган.

2.4.4-2.4.5 параграфларда ечимнинг мавжудлиги ва тавсия этилган усулнинг меъёр яқинлашуви чегараланган ечим ҳолати учун батафсил ёритилган. Қисқача баёни қуйидагича

$$\left(H_{2,r} + C_{2,r} + L_r^*\right)u = f, \quad r = \{1,2\},\ f \in L_{2\rho}, \quad u \in L_{1\rho}, \qquad (2.18)$$

бу ерда $H_{2,r}u, C_{2,r}u, L_r^*u$ операторлар (2.14) орқали аниқланади.
Дастлаб икки турдаги фазоларни киритилган:

- **1-фазо,** вазнли Гильберт фазоси яни $L_{2\rho}(-1,1)$ ҳақиқий қийматли ρ вазнга нисбатан квадрати билан интегралланувчи функциялар фазоси.
- **2-фазо** эса $L_{2\rho}$ Гильберт фазосининг ($L_{1\rho} \subseteq L_{2\rho}$) қисм фазоси бўлиб, барча $u \in L_{1\rho}$ лар учун қуйидаги тенглик ўринлидир:
  $||u||_1^2 = \sum_{k=0}^{\infty} (k+1)^2 \langle u, \phi_{k,1}\rangle^2 < \infty.$

Агар ички кўпайтма аниқланса, бу қисм фазосини Гилберт фазосига айлантириш мумкин. Бунинг учун, агар $u \in L_{1\rho}$ бўлса,

$$u = \sum_{k-0}^{\infty} (k+1)\langle u, \phi_{k,1}\rangle \phi_{k,1}, \ \rightarrow \|u\|_{1=}^2 \sum_{k=0}^{\infty} (k+1)^2 \langle u, \phi_{k,1}\rangle^2 \qquad (2.19)$$

Фараз қилайлик $\boldsymbol{C}$ комплекс сонлар фазоси бўлсин. $\boldsymbol{C_1}$ тўплам $\boldsymbol{C}$ нинг дискрет кичик тўплами дейилади агарда $\boldsymbol{C_1}$ тўплам $\boldsymbol{C}$ да лимит нуқталарга эга бўлмаса. Ушбу параграфда қуйидаги асосий леммалар ва теоремалар исботланган:

Лемма 2.1. $H_{2,1}^{-1}: L_{1\rho} \rightarrow L_{2\rho}$ *операторнинг нормаси* $\| H_{2,1}^{-1} \| = \frac{1}{|c_0|}$.

Лемма 2.2. $C_{2,1}: L_{1,\rho} \rightarrow L_{2\rho}$ *ва* $H_{2,1}^{-1}C_{2,1}: L_{1\rho} \rightarrow L_{2\rho}$ *операторлар зичдир.*

Лемма 2.3. $\lambda = 1$, $\boldsymbol{C_1}$ *га тегишли эмас яъни унинг нол фазасида ётмайди.*

$$N(I + \lambda H_{2,1}^{-1}(C_{2,1} + L_1^*)) = \{0\}.$$

Лемма 2.4. *Фараз қилайлик,* $f(x) \in C^r[-1,1]$ *ва* $K(x,t) \in C^{r+2}[-1,1]$, $L(x,t)^* \in C^r[-1,1]$, $r \geq 1$ *ядролар* $D = [-1,1] \times [-1,1]$ *соҳада аниқланган бўлсин, у ҳолда* $H_{2,1}u \in C^r[-1,1]$ *бўлади.*

Теорема 2.5. *Фараз қилайлик* $u \in L_{1\rho}$ *ва* $\lambda = 1$ *қиймат* $\boldsymbol{C_1}$ тўпламга тегишли бўлмасин *у ҳолда* $H_{2,1} + C_{2,1} + L_1^*$ *тескариланувчи бўлиб, асосий тенглама* (2.18)*,* $r = 1$ *да ягона ечимга эга бўлади.*

Теорема 2.6. *Фараз қилайлик,* $f(x) \in C^r[-1,1]$ *ва* $D = [-1,1] \times [-1,1]$ *соҳада ядролар* $K(x,t) \in C^{r+2}$, $L(x,t) \in C^r$, $r \geq 1$ *бўлсин, у ҳолда қуйидагича баҳо ўринли:*

$$||u - u_n||_\rho \leq |a| c \frac{12^{r+1}}{n^r} w_r\left(\frac{1}{n}\right),$$

бу ерда $w_r(\delta)$ *узлуксизлик модули ва ўзгармас с (2.20) тенгсизликни қаноатлантиради.*

2.4.4 параграфда таклиф қилинган усулнинг афзаллигини кўрсатувчи кўплаб сонли натижалар олинган.

2.5 параграфда 2-тур Фредгольм типидаги сингуляр интеграл тенгламалар масаласи қаралган ва таклиф этилган методнинг аниқлигини ифода этувчи леммалар исботланган.

Лемма 2.7. *Ихтиёрий* $i, j, k = \{0,1, \dots.\}$ *лар учун қуйидаги тенглик ўринли*

$$\int_{-1}^{1} \sqrt{1-t^2}\, U_i(t) T_i(t) T_k(t) dt = \frac{1}{4}[h_1(i,j,k) + h_2(i,j,k) + h_3(i,j,k) + h_4(i,j,k)],$$

бу ерда $h_r(i,j,k), r = \{1,2,3,4\}$ *ички* (i,j,k) *ларга боғлиқ бўлган ўзгармаслар.*

Лемма 2.8. *Ихтиёрий* $i, j, k = \{0,1, \dots.\}$ *лар учун қуйидаги тенглик ўринли*

$$\int_{-1}^{1} T_k(t) T_{i+1}(t) T_j(t) dt = \frac{1}{4}[h_5(i,j,k) + h_6(i,j,k) + h_7(i,j,k) + h_8(i,j,k)],$$

бу ерда $h_r(i,j,k), r = \{5,6,7,8\}$ *ички* (i,j,k) *ларга боғлиқ бўлган ўзгармаслар.*

3-БОБ (асосий натижалар): Ночизиқли интегро-дифференциал тенгламалар учун гомотопия таҳлил қилиш усули ва унинг янги ишланмаси.

Ушбу бобда, чизиқли бўлмаган бошланғич қийматли интегро-дифференциал тенгламалар (ИДТлар) ва аралаш чегаравий масала бўлган, чизиқсиз, кўп ҳадли, каср тартибли ИДТларни тақрибий ечиш учун гомотопия таҳлил қилиш усули (ГТУ) ва унинг янги ишланмаси қўлланилган шу билан бирга ечимнинг ягоналиги ва таклиф этилган методнинг яқинлашиши исботланган. Диссертациянинг 3-боби иккита қисмдан иборат.

- **1-қисми** (3.1.1-3.1.3) чизиқли бўлмаган бошланғич қийматли ИДТларнинг тақрибий ечимини топиш, таҳлил этиш, хатолигини баҳолаш ва сонли натижаларга бағишланган.
- **2-қисми** (3.2.1-3.2.6) аралаш чегаравий масала бўлган, чизиқсиз, кўп ҳадли, каср тартибли ИДТларнинг тақрибий ечимини топиш, таҳлил этиш, хатолигини баҳолаш ва сонли натижалар олишга бағишланган.

3.1.1 параграфда ночизиқли ИДТлар учун машҳур гомотопия таҳлил қилиш усули (ГТУ) ривожлантирилган ва ГТУ нинг янги ишланмаси (ЯИ-ГТУ) таклиф қилинган ва бошланғич шартли ночизиқли ИДТларга қўлланилган ҳамда ГТУ, модификацияланган ГТУ (МГТУ), янги модификацияланган ГТУ (мГТУ), ГТУ нинг умумлаштириш усули (q-ГТУ) каби ярим аналитик усуллар билан солиштирилган.

Қуйидаги ночизиқли ИДТларни кўриб чиқайлик.

3.1.1-масала. *p-тартибли чизиқли бўлмаган Вольтерра-Фредгольм ИДТ*лар *қуйидагича берилган бўлсин.*

$$u^{(p)}(t) + \sum_{j=1}^{p-1} a_j(t) u^{(j)}(t) = f(t)$$

$$+\lambda_1 \int_a^t K_1(t,s) F_1\big(u(s)\big) ds + \lambda_2 \int_a^b K_2(t,s) F_2\big(u(s)\big) ds, \qquad (3.1)$$

бошланғич шартлар: $u^{(k)}(a) = \alpha_k, \quad k = 0, \dots, p-1, p \in N, (p \geq 2),$ (3.2) бу ерда $t \in \Omega = [a, b]$, $K_1, K_2 \colon \Omega \times \Omega \longrightarrow R, f \colon \Omega \longrightarrow R$ ва $a_j \colon \Omega \longrightarrow R$ маълум функциялар, λ_1, λ_2 – параметрлар ва $F_1, F_2 \colon C(\Omega, R) \longrightarrow R$- чизиқсиз функциялар шунингдек $u(t)$ аниқланиши керак бўлган номаълум функциядир.

ГТУнинг асосий ғояси. Чизиқсиз тенглама оператор кўринишида қуйидагича берилган бўлсин:

$$N[u(t)] = 0. \quad (3.3)$$

ГТУнинг асосчиси Лаё (1992 йилда) **нолинчи тартибли** деформация тенгламасини қуйидагича қурган.

$$(1 - q)£[\phi(t; q) - u_0(t)] = q\hbar H(t)[N[\phi(t; q)]],$$

бу ерда £ - чизиқли оператор, $q \in [0,1]$-ички параметр, $\hbar \neq 0$– контрол параметр, $H(t)$– қўшимча функция, $u_0(t)$ бошланғич функция ва $\phi(t; q)$ топилиши зарур бўлган номалум функция ва m**!-тартибли** деформация тенгламасини қуйидаги кўринишда топган:

$$£[u_m(t) - \chi_m u_{m-1}(t)] = \hbar H(t) \Re_m(\overleftarrow{u}_{m-1}(t)), \quad (3.4)$$

бу ерда

$$\Re_m(\overleftarrow{u}_{m-1}(t)) = \frac{1}{(m-1)!} \frac{\partial^{m-1}[N[\phi(t;q)]]}{\partial q^{m-1}}\bigg|_{q=0}, \quad (3.5)$$

ва χ_m қуйидагича топилади

$$\chi_m = \begin{cases} 0, & m \leqslant 1, \\ 1, & m > 1. \end{cases} \quad (3.6)$$

(3.4)-(3.6) орқали аниқланган итерацион ечим **стандарт ГТУ** деб аталади. q ни сохта ўзгарувчи сифатида олиб, $\phi(t; q)$ функцияни Тейлор қаторига ёйамиз.

$$\phi(t; q) = u_0(t) + \sum_{m=1}^{+\infty} u_m(t) q^m, \quad (3.7)$$

ёрдамчи параметр $\hbar$ тўғри танланган бўлса, (3.7) даги қатор $q = 1$ да яқинлашувчи бўлади, демак, (3.3) тенгламанинг $u(t)$ ечими қуйидагича бўлади.

$$u(t) = u_0(t) + \sum_{m=1}^{+\infty} u_m(t). \quad (3.8)$$

Бу ерда $u_m(t)$ (3.4)-(3.6) деформация тенгламалари орқали аниқланади.
ЯИ-ГТУ ни қуриш учун, биз (3.3) ни қуйидаги кўринишда қайта ёзилган

$$N[u(t)] = f(t), \quad (3.9)$$

ва $f(t)$ функцияни n та ҳадга ажратилган деб ҳисобланган, яъни:

$$f(t) = x_0(t) + x_1(t) + \ldots + x_n(t), \quad (3.10)$$

ва $g(t, q)$ ни $(q\hbar)$ параметрларнинг даражаси бўйича қурилган

$$g(t; q) = x_0(t) + x_1(t) + x_2(t)(q\hbar) + \ldots + x_n(t)(q\hbar)^{n-1}. \quad (3.11)$$

Ниҳоят (3.8)-(3.9) тенгламаларнинг ЯИ-ГТУ ни қуйидагича қурилган.

$$£[u_0(t)] = x_0(t), \tag{3.12}$$
$$£[u_m(t) - \chi_m u_{m-1}(t)] = \hbar H(t)\Re_m(\overleftarrow{u}_{m-1}(t)), \tag{3.13}$$

бу ерда χ_m (3.8) орқали аниқланади ва

$$\Re_m(\overleftarrow{u}_{m-1}(t)) = \frac{1}{(m-1)!}\frac{\partial^{m-1}[N[\phi(t;q)] - g(t;q)]}{\partial q^{m-1}}\bigg|_{q=0}. \tag{3.14}$$

ЯИ-ГТУ да биз қуйидаги афзалликларга эгамиз:

- айрим ҳолларда берилган $f(t)$ функцияга қараб $x_0(t)$ ни танлаш ва (3.12) тенгламани ечишнинг 1-итерацияси (3.9) нинг $u(t) = u_0(t)$ аниқ ечимини бериши мумкин. Бу ҳолда (3.13) бўйича олинган кейинги итерациялар тўлиқ нол ечимни беради $u_i(t) = 0, i = 1,2, ...$;
- агар (3.12) тенглама аниқ ечимини бермаса, у чегаравий шартларни қаноатлантирувчи бошланғич функция $u_0(t)$ сифатида хизмат қилади.

3.1.2 параграфда ночизиқли Вольтерра-Фредгольм ИДТлар (3.1)-(3.2) учун стандард ГТУ ва ЯИ-ГТУнинг схемаси қурилган ва қаралаётган масаланинг ечмини топиш учун деталлари кўрсатилган.

3.1.3 параграфда (3.1)-(3.2) тенглама ечимининг ягоналигини ва Банах фазосида ЯИ-ГТУ яқинлашишини исботланган. (3.1)-(3.2) тенгламанинг ягоналик ечимини исботлаш учун биз қуйидаги гипотезаларни киритдик:

- **(H1):** Фараз қилайлик $C(J, R), J = [a, b]$ узлуксиз функциянинг класси бўлиб, $a_j(t)$ ва $f(t)$ узлуксиз бўлиб, $J \to R$ га акслантирсин.
- **(H2):** Ҳар қандай $u_1, u_2 \in C(J, R)$ учун $L_1, L_2 > 0$ ва $\gamma_j > 0$ қуйидаги тенгсизликларни қаноатлантирувчи Липшиц ўзгармаси мавжуд:

$$|F_1(u_1(t)) - F_1(u_2(t))| \le L_1|u_1 - u_2|,$$
$$|F_2(u_1(t)) - F_2(u_2(t))| \le L_2|u_1 - u_2|,$$
$$|D^j(u_1(t)) - D^j(u_2(t))| \le \gamma_j|u_1 - u_2|, j = \{0,1, \dots, p-1\}.$$

бу ерда D^j ҳосила оператори.

- **(H3):** $D = \{(t, s) \in R \times R: a \le s \le t \le b\}$ соҳада узлуксиз ва мусбат бўлган функциялар $K_1^*, K_2^* \in C(D, R^+)$ мавжуд бўлиб қуйидагича аниқлансин.

$$K_1^* = \sup_{t\in[a,b]} \int_a^t |K_1(t,s)ds| < \infty, K_2^* = \sup_{t\in[a,b]} \int_a^b |K_2(t,s)ds| < \infty.$$

Дастлаб қуйидаги лемма исботланди:

Лемма 3.1. *Фараз қилайлик,* $\varphi(t) \in C(J, R^+)$ *бўлсин, у ҳолда* $u(t) \in C(J, R^+)$ (3.1)-(3.2) *масалаларнинг ечими бўлиши учун қуйидаги тенгламани қаноатлантириши зарур ва етарлидур.*

$$u(t) = \varphi(t) - \sum_{j=1}^{p-1} \frac{1}{(p-1)!}\int_a^t (t-s)^{p-1}(a_j(s)D^j u(s))ds$$
$$+\frac{1}{(p-1)!}\int_a^t (t-s)^{p-1}\left[\int_a^s K_1(s,r)F_1\big(u(r)\big)dr\right]ds$$
$$+\frac{1}{(p-1)!}\int_a^t (t-s)^{p-1}\left[\int_a^s K_2(s,r)F_2(u(r))dr\right]ds,$$

бунда $t \in J = [a, b]$ ва $\varphi(t)$ қуйидагича аниқланган.

$$\varphi(t) = \sum_{k=0}^{p-1} \frac{\alpha_k}{k!}(t-a)^k + \frac{1}{(p-1)!}\int_a^t (t-s)^{p-1} f(s)ds.$$

Лемма 3.1.3 ва Банах қисқариш принципи асосида, ушбу теорема исботланди:

Теорема 3.2. *(Ягоналик теоремаси)***:** *фараз қилайлик (H1)-(H3) гипотезалар ўринли бўлсин* ва агар

$$\delta^* = \left(\frac{\gamma^* a^* p}{p!} + \frac{\lambda_1 K_1^* L_1}{p!} + \frac{\lambda_1 K_2^* L_2}{p!}\right)(b-a)^p < 1,$$

бу ерда $\gamma^* = \max_{1 \le j \le p-1} \gamma_j$ *ва* $a^* = \max_{1 \le j \le p-1} |a_j(t)|$, *тенгсизлик бажарилса, у ҳолда* $u(x) \in C(J)$ *функция* (3.1)-(3.2) *ни қаноатлантирувчи ягона ечим бўлади.*

Теорема 3.3. *(Яқинлашиш теоремаси): Фараз қилайлик,* (3.8) *орқали аниқланган* $\sum_{m=0}^{\infty} u_m(t)$ *қатор* $u(t)$ *функцияга яқинлашсин,* ва $u_m \in C(\Omega, R)$ *итератив функциялар ЯИ-ГТУнинг* (3.12)-(3.14) *юқори-тартибли деформация тенгламаси билан аниқлансин. У ҳолда* (3.8) *билан аниқланган* $u(t)$ *функция қатори* (3.1)-(3.2) *нинг аниқ ечими бўлади.*

3.1.4 параграфда таклиф этилган усул(ЯИ-ГТУ)нинг афзаллигини кўрсатиш учун бир қанча сонли натижалар солиштирма сифатида тақдим этилди.

3.2 (3.2.1-3.2.4) параграфларда чизиқсиз аралаш кўп ҳадли каср тартибли Вольтерра-Фредгольм ИДТлар учун янги ишланмали ГТУнинг қўлланилиши ва схемаси батафсил ёритилган. Қисқача баёни қуйидагича. Ушбу чизиқсиз аралаш кўп ҳадли каср тартибли ИДТларни қарайлик

$$\left({}^cD_{0^+}^{\beta_p} + \sum_{j=1}^{p-1} \xi_j {}^cD_{0^+}^{\beta_j}\right)u(t) = \varphi(t) + \lambda\int_0^t\int_0^T K(x,s)F(u(s))dxds, \quad (3.15)$$

бошланғич шарти $u^{(k)}(0) = \alpha_k, \quad k = 0, \dots, p-1,$ (3.16)

ва чегаравий шартлари $u^{(k)}(0) = \alpha_k, \quad k = 0, \dots, p-2, \ u(T) = B,$ (3.17)

бу ерда $t \in \Omega = [0, T], K: \Omega \times \Omega \longrightarrow R, \ \varphi: \Omega \longrightarrow R$ маълум функциялар, $F: C(\Omega, R) \longrightarrow R$ чизиқсиз функция, $\xi_j, \lambda, \ B$ ўзгармаслар, $p - 1 < \beta_p \le p$ ва $p \in N$ тенгламаниинг тартиби ва ${}^cD_{0^+}^{\beta_j}$ - β_j каср тартибли Капуто ҳосиласидир.

ЯИ-ГТУ ёрдамида чизиқсиз, аралаш Вольтерра-Фредгольм ИДТлар (3.15)-(3.17) ни ечиш учун қуйидаги чизиқли бўлмаган операторни киритамиз.

$$N[\phi(t;q)] = \left({}^cD_{0^+}^{\beta_p} + \sum_{j=1}^{p-1} \xi_j {}^cD_{0^+}^{\beta_j}\right)\phi(t;q) - \lambda\int_0^t\int_0^T K(x,s)F\big(\phi(s;q)\big)dsdx. \quad (3.18)$$

сўнгра ЯИ-ГТУнинг схемасини (3.18) ёрдамида қурдик.

3.2 (3.2.5-3.2.6) параграфда каср тартибли интегро-дифференциал тенгламаларнинг ягона ечими ва ГТУ нинг яқинлашуви исботланди.

Теорема 3.4*: Фараз қилайлик,* $\sum_{m=0}^{\infty} u_m(t)$ *қатор* $u(t)$ *функцияга яқинлашсин, бунда* $u_m \in C(\Omega, R)$ *итератив функциялар ГТУ нинг* (3.6)-(3.8) *аниқлансин. У ҳолда* (3.8) *билан аниқланган* $u(t)$ *функцияси* (3.15)-(3.16) *нинг* *аниқ ечими бўлади.*

Қисқартириш учун қуйидаги ўзгармас константани киритамиз:

$$\varrho = \frac{(p-1)!}{T^{p-1}}\left\{B - \sum_{k=0}^{p-2}\frac{\alpha_k T^k}{k!} - \sum_{j=1}^{p-1}\left(\sum_{k=0}^{j-1}\frac{\xi_j \alpha_k T^{\beta_p-\beta_j+p-1}}{\Gamma(\beta_p-\beta_j+k)}\right)\right\}.$$

Теорема 3.5. *Фараз қилайлик, $F: R \to R$ бўлиб, барча $(x_1, x_2) \in R^2$ лар учун $\| F(x_1) - F(x_2) \| \leq L \| x_1 - x_2 \|$ бўлсин. У ҳолда* (3.15)–(3.16) *муаммонинг ягона ечими мавжуд агар Δ қуйидаги*

$$\Delta = \sum_{j=1}^{p-1}\frac{|\xi_j| T^{\beta_p-\beta_j}}{\Gamma(\beta_p-\beta_j+1)} + |\lambda||L|\|K\|\frac{T^{\beta_p+2}}{\Gamma(\beta_p+2)} < \frac{1}{2},$$

шартни қаноатлантирса, бу ерда $\|K\| = sup_{t,s\in 0,T]}|K(t,s)|$.

3.3 параграфда сонли мисоллар келтирилиб таклиф этилган методни бошқа усуллар билан солиштирилди ва афзаллиги кўрсатилди.

4-БОБ (асосий натижалар): Чизиқсиз Вольтерра типидаги интеграл тенгламалар системаси ва кўп ўлчамли интеграл тенгламалар учун Ньютон-Канторович усули.

Ушбу бобда, чизиқсиз 2×2 интеграл тенгламалар системаси учун Ньютон-Канторович усули (НКУ) қўлланилган шу билан бирга ечимнинг мавжудлиги ва ягоналиги ҳамда таклиф этилган методнинг яқинлашиши исботланган. Бундан ташқари чизиқсиз кўп ўлчамли Вольтерра интеграл тенгламалар учун ўзлаштирилган Ньютон усули (ЎНУ) ишлаб чиқилган ва ечимнинг мавжудлиги ва ягоналиги ҳамда таклиф этилган методнинг яқинлашиши исботланган. Диссертациянинг 4-боби асосан иккита катта қисмдан иборат.

- **1-қисми** (4.1.1-4.1.6), чизиқсиз 2×2 интеграл тенгламалар системаси (ИнТС) учун НКУ, янги мажорант функцияси ва НКУнинг хатолигини баҳолаш ва сонли натижаларга бағишланган.
- **2-қисми** (4.2.1-4.2.5), чизиқсиз кўп ўлчамли Вольтерра интеграл тенгламалар (ВИТлар)ни ўзлаштирилган Ньютон усули (ЎНУ) билан ечиш ва ЎНУнинг хатолигини баҳолаш ва сонли натижаларга бағишланган.

4.1 (4.1.1) параграфда чизиқсиз 2×2 интеграл тенгламалар системаси учун НКУ қўлланилган ва янги мажорант функцияси киритилган. Бунинг қисқача баёни қуйидагича, ночизиқли 2×2 ИТСини қарайлик:

$$\begin{cases} a(t)x(t) - \int_{y(t)}^{t} H(t,\tau)F(x(\tau))d\tau = g(t), \\ b(t)x(t) + \int_{y(t)}^{t} K(t,\tau)F(x(\tau))d\tau = f(t), \end{cases} \tag{4.1}$$

бу ерда $0 < t_0 \leq t \leq T$, $y(t) < t$ бўлиб, $H(t,\tau), K(t,\tau) \in C_{[t_0,T]\times[t_0,T]}$ ядролар, ва $a(t), b(t), f(t), g(t) \in C_{[t_0,T]}$ функциялар берилган мос оралиқларида узлуксиз функциялардир ва $x(t) \in C_{[t_0,T]}$, $y(t) \in C^1_{[t_0,T]}$ топилиши керак бўлган номаълум функциялардир ва ниҳоят $F(x(t))$ турли типидаги чизиқсиз ифода.

Ушбу параграфда (4.1) тенгламанинг ечимини НКУ ёрдамида, Гаусс-Лежандр КФлар ва Ньютоннинг олдинга интерполяцион формуласи ёрдамида топдик. Бунинг учун операторларни киритамиз.

$$\begin{cases} P_1(x(t),y(t)) = a(t)x(t) - \int_{y(t)}^{t} H(t,\tau)F(x(\tau))d\tau - g(t), \\ P_2(x(t),y(t)) = b(t)x(t) + \int_{y(t)}^{t} K(t,\tau)F\big(x(\tau)\big)d\tau - f(t), \end{cases} \tag{4.2}$$

сўнгра (4.1) ни оператор кўринишда қуйидагича ёзамиз.

$$P(X) = \big(P_1(X), P_2(X)\big) = (0{,}0),\ X = \big(x(t), y(t)\big),$$

НКУнинг бошланғич итерациясини

$$P'(X_0)(X - X_0) + P(X_0) = 0,$$

(4.2) га қўллаб, қуйидагини оламиз

$$\left.\begin{aligned} &\Delta x(t) - \frac{1}{c(t)}\int_{y_0(t)}^{t} K_1(t,\tau)F'(x_0(\tau))\Delta x(\tau)d\tau = \psi_0(t), \\ &\Delta y(t) = \frac{1}{d(t)}\Big[\int_{y_0(t)}^{t} K(t,\tau)F'(x_0(\tau))\Delta x(\tau)d\tau \\ &\qquad + \int_{y_0(t)}^{t} K(t,\tau)F(x_0(\tau))d\tau + b(t)x_0(t) - f(t)\Big], \end{aligned}\right\} \tag{4.3}$$

бунда $\Delta x(t) = x_1(t) - x_0(t)$ ва $\Delta y(t) = y_1(t) - y_0(t)$ ҳамда

$$\begin{aligned} &c(t) = \mathrm{a}(t) + \mathrm{b}(t)G(t) \neq 0,\ d(t) = \mathrm{H}\big(t, y_0(t)\big)F\Big(x_0\big(y_0(t)\big)\Big) \neq 0, \\ &K_1(t,\tau) = \mathrm{H}(t,\tau) - G(t)\mathrm{K}(t,\tau),\ G(t) = \frac{\mathrm{H}\big(t,y_0(t)\big)}{\mathrm{K}\big(t,y_0(t)\big)} \\ &\psi_0(t) = \frac{1}{c(t)}\Big[\int_{y_0(t)}^{t} K_1(t,\tau)\,F\Big(x_0\big(y_0(t)\big)\Big)\,d\tau - c(t)x_0(t) + \mathrm{g}(t) + f(t)G(t)\Big], \end{aligned} \tag{4.4}$$

Дастлаб (4.3) ни Δx ва Δy, га нисбатан ечиб $(x_1(t), y_1(t))$ ни топамиз. Ньютон-Канторовичнинг қуйидаги формасини

$$P'(X_0)(X_m - X_{m-1}) + P(X_{m-1}) = 0,$$

(4.2) га қўллаб ушбуга эга бўламиз

$$\begin{aligned} &\Delta x_m(t) - \frac{1}{c(t)}\int_{y_0(t)}^{t} K_1(t,\tau)\mathrm{F}'\big(x_0(\tau)\big)\Delta x_m(\tau)d\tau = \psi_{m-1}(t), \\ &\Delta y_m(t) = \frac{1}{d(t)}\Big[\int_{y_0(t)}^{t} K(t,\tau)\mathrm{F}'\big(x_0(\tau)\big)\Delta x_m(\tau)d\tau \\ &\qquad + \int_{y_{m-1}(t)}^{t} K(t,\tau)\mathrm{F}\big(x_{m-1}(\tau)\big)d\tau + b(t)x_m(t) - f(t)\Big], \end{aligned} \tag{4.5}$$

бунда $\Delta x_m = x_m - x_{m-1}$ ва $\Delta y_m = y_m - y_{m-1}$. Қолган параметерлар (4.4) нинг умумий ҳолида аниқланади.

(4.5) дан тақрибий ечим $(x_m(t), y_m(t))$ кетма-кетлигини олинади.

4.1.2 параграфда Гаусс-Лежандр квадратур формуласи (КФси) ядроли интеграллар учун ривожлантирилди ва хатолик баҳоси олинди**.** Бунинг учун ўзгарувчи $[y(t),t]$ ораликни қисм ораликларга $[y(t_i),t_i], i=1,2,\dots,n$ ажратилди, бунда $t_i=t_0+ih, h=\frac{T-t_0}{n}, i=0,1,\dots,n$ ва Гаусс-Лежандр КФласи қуйидагича ривожлантирилди.

$$\int_{y(t_i)}^{t_i} K(t_i,\tau)x(\tau)d\tau=\frac{t_i-y(t_i)}{2}\sum_{j=1}^{l} W_j(t_i)x(\tau_j^i)+R_{n+1}(Kx), \qquad (4.6)$$

$$W_j(t_i)=K(t_i,\tau_j^i)w_j,\quad \tau_j^i=\frac{t_i-y(t_i)}{2}s_j+\frac{t_i+y(t_i)}{2},\quad j=1,2,\dots,l,$$

бу ерда $\tau_j^i\neq t_i$ ва $[y(t_i),t_i]\in[t_0,T]$ ва l қисм оралиқ $[y(t_i),t_i]$ нинг бўлишлари сонини билдиради, w_j ва s_j эса мос равишда интеграл вазнни ва Лежандр кўпҳадларининг илдизларини билдиради. Ривожлантирилган Гаусс-Лежандр КФласи учун қуйидаги теорема исботланди.

Теорема 4.1. *Фараз қилайлик, $K(t,\tau)$ ва $x(t)$ функциялар ушбу $C^{(2n+2)}[t_0,T]$ синфида тегишли бўлсин, у ҳолда (4.6) ривожлантирилган Гаусс-Легендре КФласи ҳатолиги қуйидаги кўринишга эга бўлади.*

$$|R_{n+1}(Kx)|\leq\frac{(T-t_0)^{2n+3}}{(2n+3)}\left[\frac{1\cdot2\cdot3\cdots(n+1)}{(n+1)\cdot(n+2)\cdots(2n+2)}\right]^2\frac{T^{(2n+2)}}{1\cdot2\cdots(2n+2)},$$

бу ерда параметрлар қуйидагича аниқланади.

$$T^{(q)}=X^{(0)}M_t^{(q)}+b_1X^{(1)}M_t^{(q-1)}+\dots+b_{q-1}X^{(q-1)}M_t^{(1)}+X^{(q)}M_t^{(0)}$$

бунда $b_i=\frac{q!}{i!(q-i)!},\ i=1,\dots,q-1$ *бином коэффициентлари.*

4.1.3-4.1.4 параграфда квадратур қоиданинг турғунлиги ва Ньютон-Канторович усули (НКУ)нинг дискретизацияси батафсил баёнидан сўнг **4.1.5 параграфда** янги мажорант функцияси қурилди ва шу функция асосида Ньютон-Канторович усулининг яқинлашувини кўрсатувчи бир нечта теоремалар исботланди. Бунинг учун бизга қуйидаги функциялар синфлари керак бўлади.

- $C_{[t_0,t]\times[t_0,T]}$ - $[t_0,T]\times[t_0,T]$ фазода аниқланган барча узлуксиз $S(t,\tau)$ функциялар тўплами,
- $\underline{C}=\{X: X=(x(t),y(t)): x(t),y(t)\in C_{[t_0,T]}\}$;
- $\mathcal{C}_{[t_0,T]}=\{y(t)\in C^1_{[t_0,T]}: y(t)<t\}$.

Ҳақиқий қийматли янги мажорант функцияни қуйидагича киритган ҳолда

$$\psi(t)=(t-t_0)^2-(\zeta+\eta)(t-t_0)+\zeta\eta, \qquad (4.7)$$

бу ерда $\zeta,\eta>0$ ҳақиқий қийматли коэффициентлар, қуйидаги параметрик тенгламани қараймиз:

$$X=S(X), \qquad (4.8)$$

$$t=\phi(t). \qquad (4.9)$$

Бошлангич $X_0=(x_0(t),y_0(t))$ функцияни $\Omega_0=(\| X-X_0 \|\leq r)$ сферадан олган ҳолда, бунда параметер r қуйидаги тенгсизликни қаноатлантирсин:

$$min\{\xi+t_0,\eta+t_0\}\leq r\leq max\{\xi+t_0,\eta+t_0\},$$

Канторович ва Акилов мажорант функция таърифини киритган ва унинг ёрдамида қуйидаги теоремалар исботланди.

Теорема 4.2. *Фараз қилайлик,* (4.3) *даги чизиқсиз оператор* $P(X) = 0$ *ушбу* $\Omega = \{X \in C([t_0, T]) : \| X - X_0 \| < R\}$ *очиқ тўпламда аниқланган бўлсин ва ёпиқ тўплам* $\Omega_0 = \{X \in C([t_0, T]) : \| X - X_0 \| \le r\}$ *да иккинчи тартибли узлуксиз ҳосилага эга бўлсин,* ва r *ушбу* $T = t_0 + r \le t_0 + R$ *ни қаноатлантирсин. Бундан ташқари фараз қиламиз қуйидаги шартлар бажарилсин:*

$$\| \Gamma_0 P(X_0) \| \le \frac{\zeta\eta}{\zeta+\eta}, \text{ ва } \| \Gamma_0 P''(X) \| \le \frac{2}{\zeta+\eta}, \text{ қачонки } \| X - X_0 \| \le t - t_0 \le r,$$

у ҳолда (4.7) *даги* $\psi(t)$ *функция,* (4.3) *орқали аниқланган чизиқсиз* $P(X)$ *оператор учун бошқарувчи функция бўлади.*

Таклиф этилган усул учун қуйидаги асосий теорема исботланди:

Теорема 4.3. *Фараз қилайлик* $f(t)$, $g(t) \in C_{[t_0,T]}$, $x_0(t) \in C^1[t_0, T]$ *узлуксиз функция,* $H(t,\tau), K(t,\tau) \in C^1_{[t_0,T]\times[t_0,T]}$ *узлуксиз ядролар ва бошлангич итерация* $X_0 = (x_0(t), y_0(t)) \in \Omega_0$ *бўлсин. Агар қуйидаги шартлар бажарилса:*

1. $\Gamma_0 = [P'(X_0)]^{-1}$ *мавжуд ва* $\| \Gamma_0 \| \le s_0 M e^{M(T-H_3)}$ *қаноатлантирса;*
2. $\| \Delta X \| \le \frac{\zeta\eta}{\zeta+\eta}$, *бунда* ζ *ва* η *(4.7) орқали берилган;*
3. $\| \Gamma_0 P''(X) \| \le \frac{2}{\zeta+\eta}$, *бунда* $\| P''(X) \| \le \eta_1$;
4. *(4.3) тенглама* $\underline{t} \in [t_0, t']$, $t' = t_0 + r$ *оралиқда ягона илдизга эга бўлсин ва* $min\{\zeta + t_0, \eta + t_0\} < r < max\{\zeta + t_0, \eta + t_0\}$, *ва* $\phi(t') \le t'$.

У ҳолда (4.1) *система ягона ечим* $X^* = (x^*, y^*) \in \Omega_0$ *эга бўлади ва яқинлашишлар* $X_m(t) = (x_m(t), y_m(t))$ (4.4) *кетма-кетлиги мавжуд ва* X^* *ечимга яқинлашади. Яқинлашишлар даражаси қуйидагича:*

$$\| X^* - X_m \| \le \left(\frac{2\zeta}{\zeta+\eta}\right)^m \zeta,$$

бу ерда $\zeta + t_0$ *(4.3) нинг минимал нолидир, яъни* $\zeta < \eta$.

4.2 параграфда иккинчи тур, чизиқсиз кўп ўлчамли Вольтерра интеграл тенгламалар (КЎ-ВИТлар)ни ўзлаштирилган Ньютон усули билан ечилди. Бунинг усун қуйидаги чизиқли бўлмаган КЎ-ВИТларни қараймиз:

$$u(\mathbf{t}) - \int_{a_1}^{b_1(t)} \int_{a_2}^{b_2(t)} \cdots \int_{a_n}^{b_n(t)} K(\mathbf{t}, \mathbf{x}) G(u(\mathbf{x})) dx_n dx_{n-1} \cdots dx_1$$
$$= f(\mathbf{t}), \ \mathbf{t} = (t_1, t_2, \cdots, t_n), \ \mathbf{x} = (x_1, x_2, \cdots, x_n), \qquad (4.10)$$

бу ерда, $u(\mathbf{t}) \in \Omega_1 = C_{\prod_{i=1}^n [a_i, b_i]}$ номаълум функция, $G(u(t))$ эса чизиқсиз узлуксиз функция, $f(t) \in \Omega_1$ ва $K(t, x) \in \Omega_1 \times \Omega_2$, $\Omega_2 = C_{\prod_{i=1}^n [c_i, d_i]}$ берилган силлиқ функциялар, ва $b_i(t)$, $i = 1, 2, \cdots, n$ берилган узлуксиз функциялардир.

КЎ-ВИТлар (4.10) ни ечишда ЎНУ қўллаш учун (4.10) масалани оператор тенгламаси кўринишда ёзамиз.

$$\mathbf{Q}(u(\mathbf{t})) = 0, \quad \mathbf{t} = (t_1, t_2, \cdots, t_n), \tag{4.11}$$

бунда

$$\mathbf{Q}(u(\mathbf{t})) = u(\mathbf{t}) - f(\mathbf{t}) - \int_{a_1}^{b_1(t)} \cdots \int_{a_n}^{b_n(t)} K(\mathbf{t}, \mathbf{x}) G(u(\mathbf{x})) dx_n dx_{n-1} \cdots dx_1 \tag{4.12}$$

Сўнгра тақрибий ечимнинг кетма-кетлигини $u_r(t), (r = 2,3, \cdots)$ ни қуйидаги тенгламадан оламиз.

$$\mathbf{Q}'(u_0(\mathbf{t}))\Delta u_r(\mathbf{t}) + \mathbf{Q}(u_{r-1}(\mathbf{t})) = 0, \tag{4.13}$$

(4.12) дан

$$\Delta u_r(\mathbf{t}) - \int_{a_1}^{b_1(t)} \int_{a_2}^{b_2(t)} \cdots \int_{a_n}^{b_n(t)} [\Psi_0(\mathbf{t}, \mathbf{x}; u_0)\Delta u_r(\mathbf{x})] dx_n \cdots dx_1 = \Phi_{r-1}(\mathbf{t}) \tag{4.14}$$

бу ерда $t = (t_1, t_2, \cdots, t_n)$, $x = (x_1, x_2, \cdots, x_n)$ ва

$$\Delta u_r(\mathbf{t}) = u_r(\mathbf{t}) - u_{r-1}(\mathbf{t}), \quad r = 2,3, \cdots, \ \Psi_0(\mathbf{t}, \mathbf{x}; u_0) = K(\boldsymbol{t}, \boldsymbol{x})G'(u_0(\boldsymbol{x}))$$

$$\Phi_{r-1}(\mathbf{t}) = f(\mathbf{t}) - u_{m-1}(\mathbf{x}) + \int_{a_1}^{b_1(t)} \int_{a_2}^{b_2(t)} \cdots \int_{a_n}^{b_n(t)} \left[K(\mathbf{t}, \mathbf{x}) G\big(u_{r-1}(\mathbf{x})\big)\right] dx_n dx_{n-1} \cdots dx_1$$

(4.13) ни $\Delta u_r(t)$ га нисбатан ечиб, $u_r(t)$ тақрибий аппроксимациа кетма-кетлигини оламиз. Ушбу можарант функциядан

$$Z(t) = \eta(t - t_0)^2 - (1 + \eta\beta)(t - t_0) + \beta,$$

бунда η ва β манфий бўлмаган ҳақиқий сонлар, фойдаланиб, қуйидаги теорема исботланди.

Теорема 4.4. *Фараз қилайлик* (4.11) *орқали аниқланган оператор тенглама* $Q(u(t)) = 0$ *ушбу* $\Omega_1 = \left\{u \in C_{\prod_{i=1}^n [a_i, b_i]} : \| u - u_0 \| \leq R\right\}$ *соҳада аниқланган бўлиб,* $\Omega_0 = \left\{u \in C_{\prod_{i=1}^n [a_i, b_i]} : \| u - u_0 \| \leq r \leq R\right\}$ *да иккинчи тартибли узлуксиз ҳосилага эга бўлсин. Агар қуйидаги шартлар бажарилса:*

1. *Чизиқли КЎ-ВИТлар* (4.14) *бош ядрога* $\Gamma(\boldsymbol{t}, \boldsymbol{x})$ *эга бўлиб,* $\| \Gamma \| \leq R_3 R_5 e^{R_3 R_5 \prod_{i=1}^n (b_i - a_i)}$ *қаноатлантирса*
2. $|\Delta \mathbf{t}| \leq \frac{\zeta}{1+\eta\zeta}$, $|\mathbf{Q}''(\mathbf{t})| \leq \eta_1$.

У ҳолда, (4.9) *ёпиқ сфера* Ω_0 *да ягона* $u^*(t)$ *ечимга эга бўлади ва аппроксимация кетма-кетлиги* $u_r(t), r \geq 0$

$$\Delta u_r\big(\mathbf{t}_j\big) - \int_{a_i}^{b_1(t_j)} \cdots \int_{a_n}^{b_n(t_j} \Psi_0\big(\mathbf{t}_j, \mathbf{x}; u_0\big) \Delta u_r(\mathbf{x}) dx_n dx_{n-1} \cdots dx_1 = \Phi_{r-1}\big(\mathbf{t_j}\big),$$

бунда $j = 1,2, \cdots, m_i$ *ва* $u_r(t) = u_{r-1}(t) + \Delta u_r(t)$, *аниқ* $u^*(t)$ *ечимга яқинлашади ва яқинлашиш даражаси қуйидагича бўлади.*

$$\| u^* - u_r \| \leq \left(\frac{2}{1 + \eta\beta}\right)^r \left(\frac{1}{\eta}\right), \quad r = 1,2, \cdots$$

ХУЛОСА

Ушбу диссертация ўзгарувчи ва ўзгармас оралиқларда вазнли сингулар ва гиперсингулар инегралларни тақрибий ҳисоблаш учун эффектив қуадратур формула қуриш ва уни турли синфларда баҳолаш, шу билан бирга сингуляр ва гиперсингуляр интеграл тенгларларни тақрибий ечиш учун самарали сонли усулларини ишлаб чиқиш, чизиқли бўлмаган интегро-дифференциал тенгламаларни тақрибий ечиш учун ярим аналитик методларнинг янги ишланмасини ишлаб чиқиш, чизиқли бўлмаган интеграл тенгламалар системаси ҳамда юқори ўлчовли интеграл тенгламамларни тақрибий ечиш учун Ньютон методининг мадификациясини ишлаб чиқиш ва уни амалда қўллашдан иборат.

Олинган асосий натижалар қуйидагидан иборат:

1. Ўзгарувчан ва қўзғалмас оралиқларда вазнли сингуляр ва гипер-сингуляр интеграллар учун автоматик квадратур схемаси қурилди ва қўзғалмас оралиқда вазнли сингуляр интеграллар учун модификацияланган дискрет уйирмали усули ишлаб чиқилди;
2. Вазнли Гильберт, Гёльдер ва силлиқ функциялар фазоларида чегараланган ва чегараланмаган ечим ҳолатлари учун квадратур формулаларининг яқинлашуви исботланди ва барча (4 ҳолат) учун МДУМнинг яқинлашувчилиги Гёльдер синфида таминланди;
3. Биринчи турдаги сингуляр ва гипер-сингуляр интеграл тенгламалар учун чекли Чебишев кўпҳадлари қаторининг аниқ ечимга яқинлашуви (4 та ҳолат учун) кўрсатилди ва 2- турдаги СИТлар учун чегарада чекли бўлган ечимлар таҳлил қилинди;
4. Чекли ечим ҳолатида, гипер-сингуляр интеграл тенгламалар учун ечимнинг мавжудлиги, ягоналиги ва вазнли Гёльдер синфида тақрибий ечимнинг меъёрий яқинлашуви исботланди;
5. Гомотопияни таҳлил қилиш усули (ГТУ)нинг янги ишланмаси (ЯИ-ГАУ) ишлаб чиқилди ва уни чизиқли бўлмаган бошланғич шартли ИДТларни ва каср тартибли чегаравий масалали ИДТларни ечиш учун қўлланилди;
6. Чизиқли бўлмаган ИДТларни ва каср тартибли ИДТларни ечимининг мавжудлиги ва яноналиги исботланди ҳамда ЯИ-ГАУning яқинлашиши таминланди;
7. 2×2 ночизиқли интеграл тенгламалар системаси учун ўзлаштирилган Ньютон-Канторович усули (НКУ) ишлаб чиқилди ва янги мажорант функциялар топилди ҳамда ечимнинг мавжудлиги, ягоналиги ва НКУнинг яқинлашиши исботланди;
8. Ўзлаштирилган Ньютон усули ёрдамида ночизиқли кўп ўлчовли интегро-дифференциал тенгламалар ечилди ҳамда вазнли Гильберт фазосида, ечимнинг мавжудлиги ва ягоналиги ҳамда таклиф этилган методнинг яқинлашиши исботланди.

SCIENTIFIC COUNCIL AWARDING OF THE SCIENTIFIC DEGREES DSc.03/30.12.2019.FM.01.02 AT NATIONAL UNIVERSITY OF UZBEKISTAN

NATIONAL UNIVERSITY OF UZBEKISTAN NAMED AFTER MIRZO ULUGBEK

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AN AUTOMATIC QUADRATURE SCHEME AND HOMOTOPY ANALYSIS METHOD FOR SINGULAR AND INTEGRO-DIFFERENTIAL EQUATIONS

01.01.03 – Computational and discrete mathematics
(Physical and Mathematical Sciences)

ABSTRACT OF DSc DOCORAL DISSERTATION

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INTRODUCTION (annotation of doctoral dissertation)

Relevance and demand of the theme of the dissertation. On the worldwide, the research devoted to the construction of effective quadrature formulas and the development of semi-analytical methods for the numerical solution of singular integrals and integral equations of various orders are relevant and important as well as are widely used in solving problems in areas of aerodynamics, hydrodynamics, electron optics, wave propagation, and fluid mechanics. Mathematical models of a series of processes in these areas are described by certain non-classical singular integrals and hypersingular integrals, and it is difficult to find their analytical solution. Constructing effective and highly accurate quadrature formulas for the numerical evaluation of singular and hypersingular integrals in bounded or unbounded regions and the development of methods for the approximate solution of singular integral equations of various oder is one of the main problems of computational mathematics.

Currently in the world, many scientific research works are being conducted in the area of acoustics, fluid mechanics, elasticity, aerodynamics, hydrodynamics and fracture mechanics to introduce of modern technologies and increase the pace of scientific and technical opportunities. In this regard, the construction of effective quadrature formulas for the approximate evaluation of singular and hypersingular integrals and estimation of its error terms in the space of smooth functions and weighted Hilbert spaces play an important role. Moreover, many problems in mathematical physics such as the theory of elasticity, fluid dynamics, physics of solids, mathematical biology, quantum mechanics, mathematical economics and mechanics of continuous media are reduced to the integro-differential equations of the first or the second kind. Therefore, the development of new algorithms for solving nonlinear and fractional integro-differential equations, establishing the existence and uniqueness of the solution, as well as creating stable computational algorithms and providing the convergence are considered the targeted scientific research.

In our country, great attention is paid to fundamental, applied and innovative sciences, in particular to areas with practical applications, such as computing and discrete mathematics. Research in such priority areas are designated as the main tasks and areas of scientific research at the level of international standards in the priority areas of the theory of differential and integral equations, functional analysis, algebra, mathematical physics, applied mathematics, mathematical modeling, computational and discrete mathematics is one of the main tasks of the Institute of Mathematics named after V. I. Romanovsky of the Academy of Sciences of the Republic of Uzbekistan. It is important to develop algorithms for solving systems of nonlinear integral equations and multidimensional integral equations, and to prove the uniqueness of the solution, as well as to create stable calculation algorithms in order to ensure the implementation of the decision.

In this dissertation, many results were obtained on the construction of effective quadrature formulas and their error estimations for the approximate evaluation of weighted singular (hypersingular) integrals and integral equations, as well as the development of approximate analytical methods for the solution of nonlinear integro-differential equations, system of integral equations, and multi-dimensional integral equations which are of practical importance. This corresponds to the development of current trends in our country, which have scientific and practical applications of fundamental sciences such as engineering, fluid dynamics, solid-state physics, plasma physics, mathematical biology, aerodynamics, quantum mechanics, mathematical economics, and elasticity theory.

This dissertation work, to a certain degree, serves the implementation of the tasks specified in the Decree of the President of the Republic of Uzbekistan dated February 7, 2017, PQ-4947 "On the strategy of action for further development of the Republic of Uzbekistan", PQ-2789 dated February 17, 2017 "On measures to further improve the activities, organization, management, and financing of research activities of the Academy of Sciences", PQ-2909, dated April 20, 2017 "On measures for further development of the higher education system ", ПҚ-3682, dated April 27, 2018 "On measures to further improve the system of practical implementation of innovative ideas, technologies and projects", PQ-4387 dated July 09, 2019. "On measures of state support for the further development of mathematical education and science, as well as the radical improvement of the activities of the Institute of Mathematics named after V.I. Romanovsky of the Academy of Sciences of the Republic of Uzbekistan", PQ-4708 dated May 7, 2020 "On measures to improve the quality of education and research in the field of mathematics" as well as in the implementation of the tasks set out in other normative and legal documents related to this activity.

Connection of the research to priority areas of development of science and technology of the republic: This study was carried out in accordance with the priority direction of the development of science and technology in the Republic of Uzbekistan IV-"Mathematics, Mechanics, and Informatics".

Review of international scientific research on the topic of dissertation[1]:

Scientific research on obtaining interpolation quadrature and cubature formulas and its error estimations in different classes for weighted singular (hypersingular) integrals and developing approximate method for singular (hyper-singular) integral equations as well as integro-differential equations are carried out in large scientific centers and higher educational institutions of the world, in particular: at the Institute of Mathematics named after S.L. Sobolev of Russian Academic of Science (RAS), Russia. Institute of Computational Mathematics named after V.L. G.I.Marchuk of RAS (Russia), Department of higher and applied mathematics, Penza State University (Russia), Institute of Serbian Academy of Sciences and Arts (Serbia), Institute of Mathematics named after V.I.Romanovsky of Academy of Science of Uzbekistan, Faculty of applied mathematics and intellectual technology, National University of Uzbekistan named after Mirzo Ulugbek (Uzbekistan), Mathematical school of mathematics, Korea Institute for Advanced Study (South Korea), Brain Science Institute, Korean Institute of Science and Technology (South Korea),

[1] Review of foreign research on the topic of the dissertation: Journal of Approximation Theory, Applied Mathematics and Computation, Applied Mathematics Letters, Applied Numerical Mathematics, Journal of Computational and Applied Mathematics, Numerical Algorithms, Journal of Integral Equations and Applications, SIAM Journal on Numerical Analysis (SINUM), BIT Numerical Mathematics, Siberian Journal of Computational Mathematics.

Institute of mathematics, University of Meryland (Washington, USA), Department of Mathematics, Harvard University (Cambridge, USA), Department of Mathematics, Fukui University (Japan), Mathematics and informatics center, University of Tokyo (Japan), School of mathematics, Iran University of Science and Technology (Iran), Institute for Research in Fundamental Sciences (Iran), Department of Mathematical Sciences Universiti Teknologi Malaysia (Malaysia), Institute for mathematical research, Universiti Putra Malaysia (Malaysia), Institute of mathematical sciences, University of Malaya (Malaysia), Department of Mathematics, Indian Institute of Science (India), Department of Math, Aligarh Muslim University and so on.

A number of non-traditional scientific results have been obtained on the development of effective quadrature methods for approximating Cauchy-type singular integrals (singular integral equations) and Hadamard-type singular integrals (hypersingular integral equations) and their error estimation. In particular, developed a discrete vortex method and obtained the solution of singular integral equations from one dimentional to high-dimensional (Moskow State University, Russia): constructed a fast algorithm (quadrature formula) for solving the generalized airfoil equation and determined the optimal order of approximation (Technical University of Chemnitz, Germany); for the classes of integral equations (Fredholm singular integral equations of the second kind, characteristic and the second kind of hyper-singular integral equations) obtained the approximate solution based on Bernstein polynomials (Indian Statistical Institute, India); spline-collocation method were proposed and substantiated for solving one-dimensional hyper-singular integral equations and first-kind multidimensional hyper-singular integral equations (Penza State University, Russia); proposed and applied a modified homotopy perturbation method to solve hyper-singular integral equations of the first kind and compared with the standard homotopy perturbation method and finally the modified homotopy perturbation method was applied to the vibration and active control problem (Islamic Azad University, Iran); developed a discrete projection method and its error estimations for many types of (regular, singular, and hypersingular) equations (Circle University, USA); proposed an optimal quadrature formula for Cauchy-type singular integrals and estimated its error terms in Sobolev space and proved the convergence of the proposed method (Institute of Mathematics of Academy of Science of Uzbekistan); developed hybrid homotopy perturbation method to find approximate solution of the first and second kind singular (hypersingular) integral equations and proved convergence of the method (National university of Uzbekistan, Uzbekistan).

At the global level, scientific research works are being carried out in a number of priority directions of computational mathematics for instance on the construction of effective quadrature formulas for the approximate evaluation of singular and hypersingular integrals and its error of estimation in the space of smooth functions and weighted Hilbert spaces, including to find effective methods of approximate solution for singular and hypersingular integral equations, integro-differential equations, fractional integro-differential equations, system of nonlinear integral equations and multidimensional integral equations. In addition, to provide convergence of the method in various functional spaces, to establish the existence and uniqueness of the solution, as well as to develop new stable computational algorithms to ensure the convergence.

The degree of scrutiny of the problem. One of the most valuable mathematical tools is the subject of singular integral operators of order one and two in both pure and applied mathematics. Special attention has been paid to the numerical evaluation of Cauchy principal value integrals in the middle of the 20-th century (1950 year) and Hadamard type singular integral (1970 year). There are a lot of significant results have been achieved in the literature on approximating singular (hypersingular) integrals. In particular, Gaussian quadrature formula, interpolation type of quadrature formula, optimal quadrature formula, spline approximation, and modified Simpson's rules for approximation and evaluation of weighted Cauchy-type singular integrals and hyper-singular integrals are well described in the scientific articles: I.M. Longman, C.E. Stewart, R. Piessens, D.B.Hunter, D.F.Paget, D.Elliott, D.G.Sanikidze, M.M. Chavla, S. Kumar, T. R. Ramakrishnan, N. I. Ioakimidis, G. Monegato, P.S. Tseokaris, S.L. Sobolev, I.V. Boykov, A.I. Boykova, A.N. Tunda, C. Dagnino, P. Lamberti, E. Santi, S. Amari, K. Dietselm, T. Hasegawa, T.Torii, M.I. Israelov, H.M. Shadimetov, A.R. Khayotov, Z.K. Eshkuvatov, S.A.Bakhromov, D.M.Akhmedov, H.R.Kutt, G.Monegato, E.Lutz, P.A.Martin, F.J.Rizzo, I.K.Lifanov, A.S.Nenashev, L.N. Poltavskii, F.Kim, C.Y.Hui, D.Shia, Samko, Y.Sh.Chan, B.F.Feng, Z.Zhang, Y.Sh.Chan, G.H.Paulino, A.Sidi and others. In the Mason and Handscomb monograph, detailed properties of all types of Chebyshev polynomials and their residual terms are given.

Singular and hypersingular integral equations are found in a wide range of nonlinear mathematical models, particularly mixed boundary value problems in mathematical physics, isotropic elastic, fluid mechanics, elasticity, and fracture mechanics. A number of quadrature methods have been developed to calculate kernel integrals. In the last few decades, a wide range of numerical solutions for singular and gipersingular integral equations have been developed in the literature. In particular, several monographs: N.I.Musxelishvili, F.D.Gakhov, F.Erdogen et al., S.M. Belotserkovskiy va I.K. Lifanov, M.A.Golberg, B.N.Mandal va A.Chakrabarti, many articles in this field have also been published, including B.N.Mandal, S.Bhattacharya, G.H.Bera, A.Chakrabarti, P.A.Martin, F.J.Rizzo, G.Monegato, S.Mondal, B.N.Mandal, R.Novin, I.K.Lifanov, I.E.Polonskii, L.N.Poltavskii, G.M.Vainikko, A.S.Nenashev, M.H.Saleh, S.M.Amer, R.P.Srivastav, Z.Fenggang, Boykov I.V., A.I.Boykova, V.A. Roudnev, Z.K.Eshkuvatov, E.S.Ventsel, H.Chay, Zh.Zhang, S.Banerjea, Zh.Chen, Y.F.Zhou, H.Feng, X.Zhang, and others are among them.

It is known that non-linear phenomena occur in many scientific fields of science. In particular, many problems in physics, engineering, and science are modeled by nonlinear integro-differential equations, and the analytical solution of nonlinear IDTs is rarely found. The resulting nonlinear integro-differential equations can be solved by many numerical methods, and many scientific papers have been published and are being developed in this direction. Leading scientists in this field:

A.Wazwaz, S. Abbasbandi, K. Maleknejad, I.Podlubniy, S.J.Liao, J.H. He, E.Babolian, A.S.Batayneh, M.S.M.Noorani, Z.Ayati, J. Biazar, K. Al-Khaled, A. Avudainayagam, M. Dehgan, F.Shakeri, S.M.El-Saed, M.R.Abdel-Aziz, A.Golbabay, M.Javidi, J.Saberi-Nadjaf, Z. B.Jafari, Sh.N.Husen, M.AEl-Tawil, S. Kumar and others.

Connection of the topic of the dissertation to the research work of the higher education institution where the dissertation is performed. The research of the dissertation was carried out within the research project of the research plan of the National University of Uzbekistan, OT-F4-28 scientific research project on the topic "Construction of adequate computational models for hyperbolic systems".

The objective of the thesis. The purpose of the research is to construct effective quadrature formulas for the evaluation of weighted singular and hyper-singular integrals and their error estimations, as well as to find approximate solutions of singular and hyper-singular integral equations, and to create a new development of the homotopy analysis method for nonlinear integro-differential equations, and develop the Newton-Kantorovich method for the approximate solution of the system of nonlinear IEs and multidimensional IEs.

Tasks of research. To achieve these goals the following tasks are formulated:

to construct automatic quadrature scheme for the approximating weighted singular and hypersingular integrals on the variable and fixed intervals as well as estimate the residual terms of automatic quadrature scheme in different classes of functions;

to develop discrete vortex method for approximate evaluation of weighted Cauchy type singular integrals and estimates the error terms in in different classes of functions;

to develop Chebyshev polynomial approximations for solving singular and hyper-singular integral equations of the first and second kind as well as develop Gauss-Legendre QFs for the kernel integration and estimate its residual terms;

to establish new development of homotopy analysis method (named ND-HAM) and implement it to the non-linear integro-differential equations with initial value problems and for non-linear fractional integro-differential equations with mixed boundary conditions and provide the convergence of ND-HAM as well as proof unique solution of the problems;

to develop Newton-Kantorivich method and implement it to solve the 2×2 system of non-linear integral equations and multi-dimensional integral equations and obtain its residual in the Hilbert space as well as establish existence and uniqueness solutions.

The research object. Singular and hypersingular integrals and integral equations, nonlinear integro-differential equations and fractional order integro-differential equations, system of nonlinear integral equations and multi-dimensional integral equations.

Research subject consists of operators, quadrature formulas, interpolation polynomials, and functional spaces.

Research methods. In the dissertation, operator theory, homotopy theory, the theory of quadrature formulas and interpolation polynomials, function space theory and functions with discrete arguments, the theory of fractional derivatives were used.

The scientific novelty of the research are as follows:

constructed automatic quadrature scheme for the weighted singular and hypersingular integrals on the variable and fixed intervals and developed discrete vortex method for the weighted singular integrals on the fixed interval;

proved convergence of automatic quadrature scheme in the cases of bounded and unbounded solutions in the weighted Hilbert spaces and provided convergence of modified discrete vortex method in Holder and differentiable class spaces for all the cases;

constructed the truncated series of Chebyshev polynomial approximations to the exact solution for singular and hypersingular integral equations of the first kind and for second kind singular integral equations the bounded solution is investigated;

proved the existence and solvability of the hypersingular integral equations in the case of bounded solution and obtained norm convergence of the approximate solution in the differentiable Holder class;

developed a new homotopy analysis method and applied it to solve nonlinear integro-differential equations with initial value problems and fractional-order of integro-differential equations with boundary-value problems;

proved the convergence of new homotopy analysis method to the exact solution and established existence and uniqueness solutions of the nonlinear integro-differential equations and fractional integro-differential equations.

developed Newton-Kantorovich method for 2×2 nonlinear system of integral equations and proved the existence and uniqueness of the solutions and obtained the convergence of the proposed method in the weighed Hilbert space.

developed Newton method for nonlinear multi-dimensional integral equations and proved the existence and uniqueness of the solutions and obtained the convergence of the proposed method in the weighed Hilbert space.

Practical results of the research are:

Constructed AQS for weighted singular integrals and hyper-singular integrals can be used for high-accuracy calculation of stress intensity factor in cracks as wel as established convergence of a truncated Chebyshev series for the approximate solution of singular and hyper-singular integral equations can play a useful role in solving many modeling problems.

New developed of HAM for approximating nonlinear integro-differential equations and fractional-order of integro-differential equations can be implemented for many other nonlinear problems which has practical importance and developed NKM for system of IEs and multi-dimensional IEs can be applied in many problems of practical importance and in the theory of root findings.

The reliability of the research results is substantiated using methods of computational mathematics, mathematical analysis, functional analysis, theory of functions of a discrete argument, as well as the rigor of mathematical computations. All conclusions and experiments are certified by computer programming.

The scientific and practical significance of the research results. The **scientific significance** of the obtained results consists in the fact that an algorithm for constructing effective quadrature formula to evaluate SIs and HSIs as well as

SIEs and HSIEs approximately which is important in practical application.

The practical significance of the thesis is determined by the wide range of applicability of effective quadrature formulas (AQS), the HAM and NKM.

Implementation of the research results. The results obtained in the dissertation were used in the reference papers and research projects viz:

Results obtained on "Hyper-singular integral equations formulated for the problem of curved cracks" were used in international scientific articles (Journal of Eng. Math., Vol. 126(1), 4, 24 pages, 2021; Gongcheng Lixue/Engin. Mech., 37(6), pp. 34- 41, 2020; Journal of Computational and Applied Mathematics, 343, pp. 520-538, 2018) for the water wave propagation, obtaining expressions of normal velocities at arbitrary point of plates, and finding the stress intensity factor on the curved cracks. In addition, it helped to estimate the boundary of the cracks.

The results obtained by Chebyshev approximation method for the approximate solution of the singular integral equations of the first kind were used in international scientific articles (Superconductor Science and Technology, 34(6), paper ID: 065006, 2021; Studies in Systems, Decision and Control, Vol. 340, pp. 63-101, 2021; Journal of Mathematical Analysis and Application, 482(1), ID: 123530, 2020), for the modelling of high-temperature superconducting dynamos as well as for linearization method in integration over time in space, and in obtaining an exact solution of the integral equations, at the end the proposed numerical method were applied for finding transport flow and critical flow density problems.

The results obtained by Galerkin-Chebyshev method for the approximate solution of hyper-singular integral equations of the first kind were used in international scientific articles (Bulletin of the Iranian Mathematical Society, 46(3), pp. 799-814, 2020; Journal of Low-Frequency Noise Vibration and Active Control, 38(2), pp. 706 -727, 2019; Journal of Computational and Applied Mathematics, Vol. 343, pp. 619-634, 2018) to solve hypersingular integral equations of the first kind with high accuracy and problems related to water wave propagation. In addition it helped to get estimation of error of the proposed method

Approbation of the research results. The main results of the research have been presented at more than 30 international scientific conferences and 3 local scientific conference

Publications of the research results. On the topic of the dissertation 57 research papers have been published in the scientific journals. All of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the DSc thesis, among them 37 have published in oversees journals.

Structure and volume of the dissertation. The dissertation consists of an introduction, four chapters, a conclusion, a list of used literature, and appendices. The main volume of the dissertation is 197 pages

MAIN RESULTS OF THE DISSERTATION

In the introductory part of the dissertation, the relevance and necessity were based, as well as the relevance of the research to the priorities and directions of the science and technology of the republic were also shown. Moreover, the review of the results of foreign research, the level of study, goals, and objectives of the work, the object and subject of research, scientific novelty and applied results of the research, and the theoretical and practical significance of the obtained results, the introduction of research results, published works and dissertation structure were given.

Chapter 1 (Main Results): Automatic Quadrature Scheme and Error Estimations For Weighted Higher Order Singular Integrals

In this chapter, a method (Automatic quadrature scheme) was proposed for the approximate solution of weighted singular integrals of Cauchy and Hadamard type, and the evaluation of its residual terms in different classes were shown. Chapter 1 of the dissertation consists of three major parts.

- Part 1 (1.3.1-1.3.6) is devoted to the approximation of Cauchy-type weighted singular integrals (SI) using an automatic quadrature scheme (AQS) and their error estimation.
- Part 2 (1.4.1-1.4.5) mainly concerns the calculation of Hadamard-type weighted hyper-singular integrals (HSIs) using AQS and the estimation of error terms.
- Part 3 (1.5.1-1.5.4) focuses on the approximation of Cauchy-type weighted singular integrals and their error estimation using the modified discrete vortex method (MMDV).

Let us consider general weighted SIs and HSIs of the form (1.1-1.2 section)

$$H_r^{(p)}(h,y,z,c)=\frac{w_r(c)}{\pi}\int_y^z\frac{h(t)}{w_r(t)(t-c)^p}dt,\quad r=\{0,1,2,3,4\},\ p=\{1,2\} \tag{1.1}$$

where $c\in(-1,1)$ is a singular point and $h(t)$ is a given smooth enough function on the interval $[-1,1]$ and y,z are the parameters with $-1\le y<z\le 1$ and $w_r(t)$ are the weights defined by

$$w_0(t)=1,\quad w_r(t)=\frac{\lambda_r(t)}{\sqrt{1-t^2}},\quad \lambda_r(t)=\begin{cases}1-t^2, & r=1,\\ 1, & r=2,\\ 1-t, & r=3,\\ 1+t, & r=4.\end{cases} \tag{1.2}$$

when $p=1$, the function $H_1^{(1)}(h,y,z,c)$ is called SIs in the sense of Cauchy, and $p=2$, the function $H_2^{(2)}(h,y,z,c)$ is named hypersingular integrals (or Hadamard type singular integrals). If $p>2$ then it is called super-singular integrals which are aimed to be considered as **future problems**.

The main aim of paragraph 1.3-1.4 in Chapter 1, is to construct automatic quadrature scheme (AQS) for each value of $p=\{1,2\}$ and $r=\{0,1,2,3,4\}$, on the variable $[y,z]$ and fixed intervals $[-1,1]$.

Paragraph 1.5 devotes the development of the discrete vortex method for weighted SIs on the fixed interval.

In paragraph 1.3.1, AQS is constructed for Cauchy type weighted singular integrals on the variable interval $[y,z]$. Let us consider Cauchy type weighted SIs of the form

$$C_r(h,y,z,c) = \frac{w_r(c)}{\pi}\int_y^z \frac{h(t)}{w_r(t)(t-c)}dt, \quad r = \{0,1,2,3,4\}. \tag{1.3}$$

where all the parameters and functions are defined as in Eq. (1.1). Particularly,
Case 0: The AQS for SIs (1.5) at $r = 0$ has the form

$$Q_{N,0}(f,y,z,c) = \frac{1}{\pi}\left[\frac{1}{2}\sum_{k=0}^{N} B_{k,0}[T_k(z) - T_k(y)] + h(c)\ln\frac{z-c}{c-y}\right], \tag{1.4}$$

where coefficients $B_{k,0}$ are defined by

$$B_{0,0} = \frac{b_{1,0}}{4},\ B_{k,0} = \frac{b_{k-1,0}-b_{k+1,0}}{2k},\ k = \{1,\dots,N\},\ b_{N,0} = b_{N+1,0} = 0\,, \tag{1.5}$$

and for finding $b_{k,0}$, we arrived at three diagonal systems of an algebraic equation

$$\begin{aligned} &b_{k-1,0} - 2cb_{k,0} + b_{k+1,0} = 2a_{k,0},\ \ k = 1,2,\dots,N, \\ &b_{N,0} = b_{N+1,0} = 0. \end{aligned} \tag{1.6}$$

Once solve Eq. (1.6) and found $b_{k,0}$ then we can compute the coefficients $B_{k,0}(c)$ through the Eq. (1.5), and approximate solutions of (1.3) is computed by (1.4).
Case 1: Constructed AQS for SIs (1.5) at $r = 1$ is of the form

$$\begin{aligned} Q_{N,1}(h,y,z,c) = \frac{\sqrt{1-c^2}}{\pi}\cdot\Bigg\{&\left[\frac{b_{0,1}}{2}\mathrm{arcsin}t\Big|_y^z - \sum_{k=1}^{N-1}\frac{b_{k,1}}{k}\sqrt{1-t^2}U_{k-1}(t)\Big|_y^z\right] \\ &+\frac{h(c)}{\sqrt{1-c^2}}\ln\left|\frac{t\sqrt{1-c^2}-c\sqrt{1-t^2}}{\sqrt{1-c^2}+\sqrt{1-t^2}}\right|_y^z\Bigg\}. \end{aligned} \tag{1.7}$$

Case 2: For $r = 2$, AQS were obtained as follows

$$\begin{aligned} Q_{N,2}(h,y,z,c) = \frac{1}{\pi\sqrt{1-c^2}}\Bigg\{&\frac{b_{0,2}}{2}\mathrm{arcsin}t\Big|_y^z + \sum_{k=1}^{N-1}\left[\frac{b_{k-1,2}}{k+1} - \frac{b_{k+1,2}}{k+1}\right]\sqrt{1-t^2}U_k(t)\Big|_y^z \\ &+h(c)\left[\left[\sqrt{1-t^2} - c\cdot\mathrm{arcsin}t\right]_y^z + \sqrt{1-c^2}\ln\left|\frac{t\sqrt{1-c^2}-c\sqrt{1-t^2}}{\sqrt{1-c^2}+\sqrt{1-t^2}}\right|_y^z\right]\Bigg\}. \end{aligned} \tag{1.8}$$

Case 3: For $r = 3$, we have obtained AQS in the following form

$$\begin{aligned} Q_{N,3}(h,c,y,z) = \frac{1}{\pi}\sqrt{\frac{1-c}{1+c}}\Bigg\{&b_{0,3}\mathrm{arcsin}t\Big|_y^z - \sum_{k=0}^{N-1}\frac{1}{k+1}\left(b_{k,3} + b_{k+1,3}\right)\sqrt{1-t^2}U_k(t)\Big|_y^z \\ &+h(c)\left[\mathrm{arcsin}t\Big|_y^z + \sqrt{\frac{1+c}{1-c}}\ln\left|\frac{t\sqrt{1-c^2}-c\sqrt{1-t^2}}{\sqrt{1-c^2}+\sqrt{1-t^2}}\right|_y^z\right]\Bigg\}. \end{aligned} \tag{1.9}$$

Case 4: $r = 4$, AQS is constructed by the following form

$$\begin{aligned} Q_{N,4}(h,c,y,z) = \frac{1}{\pi}\sqrt{\frac{1+c}{1-c}}\Bigg\{&b_{0,4}\mathrm{arcsin}t\Big|_y^z - \sum_{k=0}^{N-1}\frac{1}{k+1}\left(b_{k,4} - b_{k+1,4}\right)\sqrt{1-t^2}U_k(t)\Big|_y^z \\ &+h(c)\left[-\mathrm{arcsin}t\Big|_y^z + \sqrt{\frac{1-c}{1+c}}\ln\left|\frac{t\sqrt{1-c^2}-c\sqrt{1-t^2}}{\sqrt{1-c^2}+\sqrt{1-t^2}}\right|_y^z\right]\Bigg\} \end{aligned} \tag{1.10}$$

where unknown coefficients $b_{k,\mathrm{r}} = b_{k,\mathrm{r}}(c)$ are defined by

$$\begin{aligned} &b_{k-1,r}(c) - 2cb_{k,r}(c) + b_{k+1,r}(c) = 2a_{k,r},\ k = 1,2,\dots,N, \\ &b_{N,r}(c) = b_{N+1,r}(c) = 0, \quad r = \{1,2,3,4\}, \end{aligned} \tag{1.11}$$

where $a_{k,\mathrm{r}}$ are defined by using the interpolation conditions.

In the subsection 1.3.2, we consider the Cauchy type integral of the form

$$C_r(h,c) = \frac{w_r(c)}{\pi}\int_{-1}^{1}\frac{h(t)}{w_r(t)(t-c)}dt,\ r \in \{0,1,2,3,4\},\ c \in (-1,1) \tag{1.12}$$

where $w_r(t)$ are defined by (1.2). We have constructed AQS in two cases:

Case 0 ($r=0$): We have obtained AQS in the form

$$Q_{N,0}(h,c)=\frac{1}{\pi}\left[b_{0,0}(c)-\sum_{k=1}^{\left[\frac{N+1}{2}\right]-1} b_{2k,0}(c)\frac{2}{(2k)^2-1}+h(c)\ln\left|\frac{1-c}{1+c}\right|\right], \quad (1.13)$$

From (1.13), it follows that on $[-1,1]$ the process of calculation of AQS is reduced by half of the summation and it helps us to economize the computational time.

Case 1-4: In this cases, we have obtained the AQS in the form of

$$C_r(h,c)=\frac{w_r(c)}{\pi}\int_{-1}^{1}\frac{h(t)}{w_r(t)(t-c)}dt\approx\frac{w_r(c)}{\pi}\int_{-1}^{1}\frac{S_{N,r}(t)}{w_r(t)(t-c)}dt$$

$$\approx\begin{cases}\sqrt{1-c^2}\sum_{k=1}^{N} a_{k,1}U_{k-1}(c), & r=1,\\ -\frac{1}{\sqrt{1-c^2}}\sum_{k=0}^{N} a_{k,2}T_{k+1}(c), & r=2,\\ \sqrt{\frac{1-c}{1+c}}\sum_{k=0}^{N} a_{k,3}W_k(c), & r=3,\\ -\sqrt{\frac{1+c}{1-c}}\sum_{k=0}^{N} a_{k,4}V_k(c), & r=4,\end{cases} \quad (1.14)$$

$$=Q_{N,r}(h,c).$$

In the section 1.3.3, there are many examples to show the validity and efficiency of the proposed method.

In section 1.3.4, we present two types of error terms of AQS for the cases $r=\{0,1\}$. To show the error term of truncated Chebyshev series approximation, let us introduce the following classess of functions

- $H^\alpha([a,b],K)$ is a class function, that satisfies the Holder condition on the interval $[a,b]$ with the index α and constant K.
- $C^{r+1}[-1,1]=\{h(t): h^{r+1}(t)\in C([-1,1])\}$, where $C[-1,1]$ is a class of continuous function
- $C^{m,\alpha}[a,b]=\{f(t):\ f^{(m)}\in H^\alpha([a,b],A_m)\}$.
- $L_{p,w}=\left\{f(t):\ \int_y^z \rho(t)|f(t)|^p dt<\infty\right\}$
- Let truncated Chebyshev polynomials of the 1-kind be given by

$$S_{N,0}(t)=S_{N,1}(t)=\sum_{k=0}^{N}{}' a_{k,0}T_k(t), \quad (1.15)$$

where coefficients $a_{k,0}$ are found by interpolation conditions

Theoretical results of the Section 1.1-1.2 are summurised in the following theorems.

Theorem 1.1. *Let* $h\in L_{2,w_0}[-1,1]$ *and* $S_{N,0}(t)$ *be interpolating polynomials (1.9) at the zeros of* $\omega_{N+1}(t)=2(t^2-1)U_{N-1}(t)$ *which are* $t_j=cos\left(\frac{\pi j}{N}\right), j=0,1,\cdots N$. *Then the constructed quadrature rule* $Q_{N,0}(h,y,z,c)$ *defined by (1.13) converges to original singular integral* $C_0(h,y,z,c)$ *defined by (1.14) in the sense of* L_{q,w_0} *norm*

$$\|R_{N,0}(h,c)\|_q=\|C_0(h,y,z,c)-Q_{N,0}(h,y,z,c)\|_q \underset{N\to\infty}{\to} 0,$$

where $1<q<\frac{p}{p_0}$ and $p>p_0>1$.

Theorem 1.2. *Let* $h\in C[-1,1]$ *and* $S_{N,1}(t)$ *be an interpolating polynomial at the zeros of* $\omega_{N+1}(t)=(1-t^2)U_{n-1}(t)$. *Then the quadrature rule* $Q_{N,1}(h,y,z,c)$

defined by (1.16) converges to $C_1(h,y,z,c)$ *in the sense of* L_{q,w_1} *norm*

$$\|R_{N,1}(h,c)\|_q = \|C_1(h,y,z,c) - Q_{N,1}(h,y,z,c)\|_q \underset{N\to\infty}{\to} 0,$$

where $1 < q < \frac{p}{p_0}$ and $p > p_o > 1$.

Theorem 1.3. *Let* $f(t) \in C^{N+1,\alpha}[-1,1]$ *and* $t_k, k = 0,\ldots,N$ *be the roots of* $\omega_{N+1}(t)$. *Then the error bound for an AQS (1.16) is of the form*

$$R_{N,1}(f,c) \le \frac{8A_1}{2^{N-1}N!}\left[1 + \frac{3.529}{\alpha\ln(N+1)}\right]\frac{\ln(N+1)}{N+1}.$$

Thus, we have constructed AQS for Cauchy type weighted SIs (1.5) on the variable interval $[y,z]$ and on the fixed interval $[-1,1]$ as well as proved convergence of the proposed method in different classes of functions.

In section 1.4.1-1.4.2, constructed the AQS for weighted hypersingular integrals (HSIs) of the form

$$H_r^{(2)}(h,y,z,c) = \frac{w_r(c)}{\pi}\int_y^z \frac{h(t)}{w_r(t)(t-c)^2}dt, \quad r = \{0,1,2,3,4\}, \tag{1.16}$$

on the variable intervals $[y,z]$ in the following form

Case 0, $r = 0$. In this case, we have constructed AQS in the following form

$$\begin{aligned} H_0(h,y,z,c) &= \frac{1}{\pi}\left[\sum_{k=0}^{N}\frac{d}{dc}B_{k,0}(c)T_k(t)|_y^z + h'(c)\ln\left|\frac{z-c}{y-c}\right|\right] \\ &+ \frac{h(c)}{\pi}\frac{z-y}{(z-c)(y-c)} + R_{N,0}(c) + R_0(h,c), \end{aligned} \tag{1.17}$$

where $B_{k,0}(c)$ are defined by (1.5).

Cases 1-4: Let $r = \{1,2,3,4\}$. Differentiating Eqs. (1.9)-(1.12) with respect to c, and taking into account that the coefficients $b_{k,1}(c)$ is a function of c, we arrived at

For $r = 1$,

$$\begin{aligned} H_{N,1}(h,y,z,c) &= \frac{\sqrt{1-c^2}}{\pi}\frac{d}{dc}\left\{\left[\frac{b_{0,1}(c)}{2}\text{arcsin}t|_y^z\right.\right. \\ &\left.\left. - \sum_{k=1}^{N-1}\frac{b_{k,1}(c)}{k}\sqrt{1-t^2}U_{k-1}(t)\Big|_y^z\right] + \frac{h(c)}{\sqrt{1-c^2}}\ln\left|\frac{t\sqrt{1-c^2}-c\sqrt{1-t^2}}{\sqrt{1-c^2}+\sqrt{1-t^2}}\right|_y^z\right\}. \end{aligned}$$

For $r = 2$,

$$\begin{aligned} H_{N,2}(h,y,z,c) &= \frac{1}{\pi\sqrt{1-c^2}}\cdot \\ &\cdot\frac{d}{dc}\left\{\frac{b_{0,2}(c)}{2}\text{arcsin}t|_y^z + \sum_{k=1}^{N-1}\left[\frac{b_{k-1,2}(c)}{k+1} - \frac{b_{k+1,2}(c)}{k+1}\right]\sqrt{1-t^2}U_k(t)\Big|_y^z\right. \\ &\left. + h(c)\left[\left[\sqrt{1-t^2} - c\cdot\text{arcsin}t\right]_y^z + \sqrt{1-c^2}\ln\left|\frac{t\sqrt{1-c^2}-c\sqrt{1-t^2}}{\sqrt{1-c^2}+\sqrt{1-t^2}}\right|_y^z\right]\right\}. \end{aligned}$$

In the case $\boldsymbol{r} = \{\mathbf{3},\mathbf{4}\}$, Eqs. (1.9) and (1.10) are differentiated respectively to construct AQS, where the coefficients $b_{k,r}(c)$ are defined by (1.11).

In section 1.4.3, AQS were constructed for HSIs by differentiating (1.13) - (1.14) on the fixed interval [−1, 1].

Notes: Derivation of AQS for HSIs is very simple but in terms of changing values of the solution at the boundary of $[-1,1]$ is quite different than the singular integrals.

In section 1.4.4, once provided many examples for AQS then **in** section **1.4.5**

the error estimation of AQS for the approximate solution of HSIs in the bounded and unbounded cases are presented. Let $e_{N,r}(t) = h(t) - S_{N,r}(t),\ r = \{1,2,3,4\}$, with maximum norm and

$$E_{N,r}(h) = H_r(h,x) - H_{N,r}(h,x),\ \ r = \{1,2,3,4\}$$

Main results are given in the following theorems and lemmas

Theorem 1.4. *Let* $h(t) \in C^{N+1,\alpha}[-1,1]$ *for* $0 < \alpha \le 1$ *and the truncated series of Chebyshev polynomials* $S_{N,r}(t),\ r = \{1,2\}$ *be defined by (1.3). Then the AQS defined by (1.17) has an error bound given by*

$$\|E_{N,r}(h)\|_C \le \begin{cases} \dfrac{2.12M ln(N)}{2^{N-2}(N-2)!}\left[1+\dfrac{3.12}{6ln(N)}\right], & r=1,\\ \dfrac{3.38M}{2^{N-2}(N-2)!}\left[1+\dfrac{0.63}{N}L_{1N}+\dfrac{4.19}{N+1}\right], & r=2. \end{cases}$$

where $L_{1N} = 1 + \frac{1.21}{N} + \frac{3.35}{N+1}$ and $M = \max\{M_1, M_2\}$ where M_1, M_2 are constants

In section 1.5.1-1.5.2, developed discrete vortex method (MDV) for weighted Cauchy type SIs on fixed interval.

$$I_i(f,x) = \frac{w_1(x)}{\pi}\int_{-1}^{1}\frac{f(t)dt}{w_1(t)(t-x)},$$

$$I_i(f,x) = I_1(f,x) + Q_i(f,x),\ \ i = \{2,3,4\}$$

where

$$Q_i(f,x) = \frac{(-1)^i}{\pi}w_i(x)\int_{-1}^{1}\frac{f(t)dt}{\sqrt{1-t^2}},\ i = \{2,3\},$$

$$Q_4(f,x) = -\frac{1}{\pi}w_4(x)\int_{-1}^{1}\frac{g(x,t)dt}{\sqrt{1-t^2}},\ g(x,t) = (x+t)f(t),$$

and $w_1(x) = \sqrt{1-x^2},\ w_2(x) = \sqrt{\frac{1-x}{1+x}},\ w_3(x) = \sqrt{\frac{1+x}{1-x}},\ w_4(x) = \frac{1}{\sqrt{1-x^2}}$.

In this section, we have constructed two groups of new QFs by combining the modified discrete vortex method and linear spline interpolation. First group of QFs are for $Q_i(f,x)$, and the second group QFs are for $I_i(f,x)$. Let $t_k = -1 + kh,\ \mathrm{h} = 2/(\mathrm{N}+1)$ and $E = \{t_k, k = 1, \ldots, N\}$ be a canonic partition of the interval $[-1,1]$. Then new QFs for $I_1(f,x)$ at $x = t_j + \varepsilon, j = 1, \ldots, N-1$ are construted to be

$$I_1(f,x) \cong \widetilde{I_{N,1}}(f,x) = \left(\sum_{k=1}^{j-2} + \sum_{k=j+3}^{N}\right)A_k^{(1)}(x)\varphi(t_k) + \sum_{v=j-1}^{j+2} A_v^{(1)}(x)f(x_v) + A_0^{(1)}(x)f(-1) + A_{N+1}^{(1)}(x)f(1), \quad (1.17)$$

where $\varphi(t) = f(t) - \frac{1}{2}[(1-t)f(-1) + (1+t)f(1)]$ and the coefficients $A_k^{(1)}$ and $B_k^{(1)}$ are computed by evaluating SIs in (1.27) exactly. Consequently, the QFs (Eshkuvatov et al. [241]) for SIs (1.28) at $x = t_j + \varepsilon, j = 1, \ldots N-1$ are of the form

$$I_i(f,x) \cong \widetilde{I_{N,\iota}}(f,x) = \left(\sum_{k=1}^{j-2} + \sum_{k=j+3}^{N}\right)\left(A_k^{(1)}(x)\varphi(t_k) + \frac{(-1)^i}{\pi}w_i(x)a_i(x,t_k)C_kf(t_k)\right) + \left(A_0^{(1)}(x) + \frac{(-1)^i}{\pi}w_i(x)a_i(x,-1)C_0\right)f(-1)\left(A_{N+1}^{(1)}(x) + \frac{(-1)^i}{\pi}w_i(x)a_i(x,1)C_{N+1}\right)f(1) + \sum_{v=j-1}^{j+2}\left(A_v^{(1)}(x) + \frac{(-1)^v}{\pi}w_v(x)a_i(x,t_v)C_v\right)f(t_v),\ i = \{2,3,4\}, \quad (1.18)$$

where $a_2(x,t) = a_3(x,t) = 1,\ a_4(x,t) = -(x+t)$, and C_k are calculated by evaluating $Q_i(f,x)$.

In section 1.5.3, error estimation of modified MDV were given in the following theorems

Theorem 1.5. *Let* $f(t) \in C([-1,1])$, *and* E *be a set of canonic partition of the* *interval* $[-1,1]$. *Then the error of the QFs (1.29) and (1.30) are*

$$R_N^{(1)}(f,x) \leq \begin{cases} L_1 h^{\alpha}\, ln(N+1), & when \quad f(t) \in H^{\alpha}([-1,1], A), \\ L_2 h\, ln(N+1), & when \quad f(t) \in C^1([-1,1]), \end{cases}$$

where L_1, L_2 are defined as in Theorem 1.5.1.

Theorem 1.6. *Let* $f(t) \in C([-1,1])$ *and* E *be a set of canonic partition of the interval* $[-1,1]$. *Then the error of the QFs defined by (1.29) and (1.30) are*

$$R_N^{(i)}(f,x \leq \begin{cases} L_1 h^{\alpha}\, ln(N+1) + \frac{d_i \bar{A}_i}{2^{\alpha}} w_i(x) h^{\alpha}, & when \ f(t) \in H^{\alpha}([-1,1], A), \ i = \{2,3,4\}, \\ L_2 h\, ln(N+1) + \frac{d_i \bar{M}_i}{2} w_i(x) h, & when \ f(t) \in C^1([-1,1]), \ i = \{2,3,4\}, \end{cases}$$

where $d_2 = d_3 = 1, d_4 = 3$, $\bar{A}_2 = \bar{A}_3 = A$, $\bar{M}_2 = \bar{M}_3 = M_1$, and L_1, L_2 are consts

In section 1.5.4, the advantages of modified MDV were shown over standard discrete vortex method

Chapter 2 (Main Results): Approximate Solution of Singular and Hypersingular Integral Equations of the First and Second Kind.

In this chapter, after the general treatment of p-order singular integral equations, an approximate method (Chebyshev approximation method) is proposed for solving the first type Cauchy and Adamard type singular integral equations with has a high accuracy, and the evaluation of its residual terms in different classes were shown. Chapter 2 of the dissertation consists of three major parts.

- Part 1 (2.1-2.3) devotes to a general description of p-order SIEs of the first kind, (2.4.1-2.4.2) deals with finding an approximate solution, analysis of the solution, and numerical experiments of SIEs of the first kind.
- Part 2 (2.4.3-2.4.6) is mainly dedicated to finding the approximate solution of hyper-singular integral equations (HSIEs) of the first kind, its analysis, error estimations and numerical experiments.
- Part 3 (2.5.1-2.5.2) is mainly devoted to approximate solution of SIEs of the second kind, analysis of the solution, and numerical experiments.

Section 2.1-2.3, provide a general description of p-order SIEs of the first kind, preliminaries and construction on new QFs for kernel integration.

In section 2.4, general SIEs of the first kind of order p of the form is investigated

$$\frac{1}{\pi}\int_{-1}^{1} \varphi(t)\left[\frac{K(x,t)}{(t-x)^p} + L_1(x,t)\right] dt = f(x), \ \ p = \{1,2,3,\cdots\}, -1 < x < 1, \quad (2.1)$$

where x is the singular point, $K(x,t), L_1(x,t)$ and $f(x)$ are given real valued continues functions and $\varphi(t)$ is to be determined.

Let kernel $K(x,t)$ in Eq. (2.1) be constant on the diagonal of the region $D = [-1,1] \times [-1,1]$. Hence, we can assume that

$$K(x,x) = c_0 \neq 0. \quad (2.2)$$

Taking into account Eq. (2.2), we can write Eq. (2.1) in the form

$$\frac{c_0}{\pi}\int_{-1}^{1} \frac{\varphi(t)}{(t-x)^p} dt + \frac{1}{\pi}\int_{-1}^{1} \frac{Q_1(x,t)\varphi(t)}{(t-x)^{p-1}} dt + \frac{1}{\pi}\int_{-1}^{1} L_1(x,t)\varphi(t) dt = f(x), \quad (2.3)$$

where $-1 < x < 1$ and $Q_1(x,t) = \frac{K(x,t) - K(x,x)}{t-x}$.

Main aim is to find four type of solutions of Eq. (2.3) for the values of $p = \{1,2\}$. Hence, we search solution in the form

$$\varphi(x) = w_r(x)u(x), \quad r = \{1,2,3,4\}, \tag{2.4}$$
where $w_i(x), i = \{1,2,3,4\}$ are defined by (1.2). Substituting (2.4) into (2.3) yields

$$\frac{c_0}{\pi}\int_{-1}^{1} \frac{w_r(t)}{(t-x)^p}\, u(t)dt + \frac{1}{\pi}\int_{-1}^{1} \frac{w_r(t)Q_1(x,t)}{(t-x)^{p-1}}\, u(t)dt$$
$$+\frac{1}{\pi}\int_{-1}^{1} w_r(x)L_1(x,t)u(t)dt = f(x),\ p = \{1,2\},\ r = \{1,2,3,4\}. \tag{2.5}$$

Operator equations of (2.5) is

$$H_{p,r}u + C_{p,r}u + L_r u = f, \quad p = \{1,2\},\ r = \{1,2,3,4\}, \tag{2.6}$$

where $f \in L_{2\rho},\ \ u \in L_{1\rho}$, the spaces $L_{2\rho}$ and $L_{1\rho}$ are defined in Section 2.4. and

$$\begin{aligned} H_{p,r}u &= \frac{c_0}{\pi}\int_{-1}^{1} \frac{w_r(t)}{(t-x)^p}\, u(t)dt,\\ C_{p,r}u &= \frac{1}{\pi}\int_{-1}^{1} \frac{w_r(t)Q_1(x,t)}{(t-x)^{p-1}}\, u(t)dt,\\ L_r u &= \frac{1}{\pi}\int_{-1}^{1} w_r(t)L_1(x,t)u(t)dt. \end{aligned} \tag{2.7}$$

Thus, we will find approximate solution of Eq. (2.6) in each cases of $r = \{1,2,3,4\}$ for two values of $p = \{1,2\}$.

In section 2.4.1, approximate solution of SIEs of the first kind were found. Let $p = 1$, then operator form of the Eq. (2.6) becomes

$$C_{1,\mathrm{r}}u + L_r u = f, \quad r = \{1,2,3,4\}, \quad f \in L_{2\rho}, \quad u \in L_{1\rho}, \tag{2.8}$$

where

$$C_{1,\mathrm{r}}u = \frac{c_0}{\pi}\int_{-1}^{1} \frac{w_r(t)}{(t-x)} u(t)dt,\ L_r u = \frac{1}{\pi}\int_{-1}^{1} w_r(t)L(x,t)u(t)dt$$
$$L(x,t) = Q_1(x,t) - L_1(x,t),\ Q_1(x,t) = \frac{K(x,t)-K(x,x)}{t-x} \tag{2.9}$$

To find an approximate solution of Eq. (2.8), the unknown function $u(t)$ is approximated by

$$u(t) \cong u_{n,r}(t) = \sum_{j=0}^{n} b_{j,r}P_{j,r}^*(t), \quad r = \{1,2,3,4\}, \tag{2.10}$$

where $P_{j,r}^*(t)$ are defined by

$$P_{j,r}^*(t) = \begin{cases} U_j(t) = \cos(j\theta), & r = 1,\\ T_j(t) = \frac{\sin((j+1)\theta)}{\sin(\theta)}, & r = 2,\\ W_j(t) = \frac{\sin((j+1/2)\theta)}{\sin(\theta/2)}, & r = 3,\\ V_j(t) = \frac{\cos((j+1/2)\theta)}{\cos(\theta/2)}, & r = 4. \end{cases}$$

Substituting Eq. (2.10) into (2.8), yields

$$\sum_{j=1}^{n} b_{j,r}\left[\frac{c_0}{\pi}\int_{-1}^{1} \frac{w_r(t)}{(t-x)} P_{j,r}^*(t)dt + \frac{1}{\pi}\int_{-1}^{1} w_r(x)L(x,t)P_{j,r}^*(t)dt\right]$$
$$+b_{0,r}\left[\frac{c_0}{\pi}\int_{-1}^{1} \frac{w_r(t)}{(t-x)}\, dt + \frac{1}{\pi}\int_{-1}^{1} w_r(x)L(x,t)\, dt\right] = f(x), \tag{2.11}$$

Introducing notations

$$G_{j,r}^*(x) = \frac{1}{\pi}\int_{-1}^{1} \frac{w_r(t)}{(t-x)} P_{j,r}^*(t)dt = \begin{cases} U_{j-1}(x), & r = 1,\\ -T_{j+1}(x), & r = 2,\\ W_j(x), & r = 3,\\ -V_j(x), & r = 4. \end{cases} \quad h_r(x) = \frac{1}{\pi}\int_{-1}^{1} \frac{w_r(t)}{(t-x)}\, dt = \begin{cases} 0, & r = 1,\\ -x, & r = 2,\\ 1, & r = 3,\\ 1, & r = 4 \end{cases}$$

leads to

$$\sum_{j=1}^{n} b_{j,r}\left[c_0\, G_{j,r}^*(x) + \psi_{j,r}^*(x)\right] + b_{0,r}\left[c_0 h_r(x) + \psi_{0,r}^*(x)\right] = f(x), \tag{2.12}$$

where $r = \{1,2,3,4\}$, and

$$\psi^*_{j,r}(x) = \frac{1}{\pi}\int_{-1}^{1} w_r(x)L(x,t)P^*_{j,r}(t)\,dt.$$

The collocation method together with the kernel expansion scheme are used to determine the unknown parameters $b_{j,r}$ in (2.12) for each value of $r = \{1,2,3,4\}$. Once we found the values of $b_{j,r}$ then substitute it into the Eq. (2.11) and from (2.4), semi-analytical approximate solution of Eq. (2.8) are obtained.

In section 2.4.2, many numerical results are presented to show the advantage of the proposed method.

In section 2.4.3, a detail description of the approximate solution for the first kind of HSIEs was presented. Let $p = 2$, then from the operator equation (2.6)-(2.7) it follows that

$$H_{2,r}u + C_{2,r}u + L^*_r u = f, \quad r = \{1,2,3,4\}, \tag{2.13}$$

where

$$H_{2,r}u = \frac{c_0}{\pi}\int_{-1}^{1} \frac{w_r(t)}{(t-x)^2}\,u(t)dt,$$
$$C_{2,r}u = \frac{Q_1(x,x)}{\pi}\int_{-1}^{1} \frac{w_r(t)}{t-x}\,u(t)dt, \tag{2.14}$$
$$L^*_r u = \frac{1}{\pi}\int_{-1}^{1} w_r(t)L^*(x,t)u(t)dt.$$

where $Q_1(x,t)$ is defined by (2.10) and

$$L^*(x,t) = Q_2(x,t) + L_1(x,t), Q_2(x,t) = \frac{Q_1(x,t)-Q_1(x,x)}{t-x} \tag{2.15}$$

To find approximate solution of Eq. (2.13) substitute Eq. (2.10) into (2.13) to yield

$$\sum_{j=1}^{n} b_{j,r}\left[\frac{c_0}{\pi}\int_{-1}^{1} \frac{w_r(t)}{(t-x)^2}\,P^*_{j,r}(t)dt + \frac{Q_1(x,x)}{\pi}\int_{-1}^{1} \frac{w_r(t)}{(t-x)}\,P^*_{j,r}(t)dt \right.$$
$$\left. + \frac{1}{\pi}\int_{-1}^{1} w_r(x)L^*(x,t)P_{j,r}(t)dt\right] + b_{0,r}\left[\frac{c_0}{\pi}\int_{-1}^{1} \frac{w_r(t)}{(t-x)^2}\,dt \right.$$
$$\left. + \frac{Q_1(x,x)}{\pi}\int_{-1}^{1} \frac{w_r(t)}{(t-x)}\,dt + \frac{1}{\pi}\int_{-1}^{1} w_r(x)L^*(x,t)\,dt\right] = f(x) \tag{2.16}$$

Exact calculation of hypersingular, singular integrals and weighted integrals in front of $b_{0,r}$ coefficients leads to

$$\sum_{j=1}^{n} b_{j,r}\left[c_0\frac{d}{dx}G^*_{j,r}(x) + Q_1(x,x)G^*_{j,r}(x) + \psi^*_{j,r}(x)\right]$$
$$+ b_{0,r}\left[c_0\frac{d}{dx}h_r(x) + Q_1(x,x)h_r(x) + \psi^*_{0,r}(x)\right] = f(x) \tag{2.17}$$

where $L^*(x,t)$ is determined by (2.15) and

$$\psi^*_{j,r}(x) = \frac{1}{\pi}\int_{-1}^{1} w_r(x)L^*(x,t)P^*_{j,r}(t)dt, \quad r = \{1,2,3,4\}.$$

Eq. (2.17) were solved for each value of $r = \{1,2,3,4\}$ by collocation method together with kernel expansion scheme to determine the unknown parameters $b_{j,r}$. Once we find the values of $b_{j,r}$ then substitute it into the Eq. (2.10) and from the Eq. (2.4), semi-analytical approximate solution of Eq. (2.13) were obtained.

Sections 2.4.4-2.4.5, discuss in detail the existence of the solution and the norm convergence of the proposed method for the bounded solution case. A brief description is as follows

$$\left(H_{2,\mathrm{r}} + C_{2,\mathrm{r}} + L^*_r\right)u = f, \quad r = f \in L_{2\rho}, \quad u \in L_{1\rho}, \tag{2.18}$$

where $H_{2,\mathrm{r}}u, C_{2,\mathrm{r}}u, L^*_r u$ are defined by (2.19).

First of all, we introduce two types of spaces.

- **The first one** is weighted Hilbert space i.e. $L_{2\rho}(-1,1)$ denote the space of real-valued square integrable functions with respect to the weight ρ.
- **Second space for the bounded case** is a subspace of the Hilbert space denoted by $L_{1\rho} \subseteq L_{2\rho}$ which is consisting of all $u \in L_{1\rho}$ such that
$||u||_1^2 = \sum_{k=0}^{\infty} (k+1)^2 \langle u, \phi_{k,1} \rangle^2 < \infty.$ (2.19)

This subspace can be made into Hilbert space if we define an inner product and hence if $u \in L_{1\rho}$ then

$$u = \sum_{k-0}^{\infty} (k+1) \langle u, \phi_{k,1} \rangle \phi_{k,1}, \to \|u\|_{1=}^2 \sum_{k=0}^{\infty} (k+1)^2 \langle u, \phi_{k,1} \rangle^2 \quad (2.20)$$

Let **C** be the space of complex numbers. A set $\boldsymbol{C_1}$ is called a discrete subset of **C** if $\boldsymbol{C_1}$ has no limit points in **C**. In this section, we have proved the following results:

Lemma 2.1. *The norm of operator* $H_{2,1}^{-1}: L_{1\rho} \to L_{2\rho}$ *is* $\| H_{2,1}^{-1} \| = \frac{1}{|c_0|}$.

Lemma 2.2. *The operators* $C_{2,1}: L_{1,\rho} \to L_{2\rho}$ *and* $H_{2,1}^{-1} C_{2,1}: L_{1\rho} \to L_{2\rho}$ *are dence.*

Let C_1 be a discrete subset of $\mathbb{C}$ (i.e. a set C_1 has no limit points in $\mathbb{C}$)

Lemma 2.3. $\lambda = 1$ *does not belong to* $\boldsymbol{C_1}$ *i.e. it does not lie the null space of*

$$N(I + \lambda H_{2,1}^{-1}(C_{2,1} + L_1^*)) = \{0\}$$

Lemma 2.4. *Let* $f(x) \in C^r[-1,1]$ *and* $K(x,t) \in C^{r+2}$, *and* $L(x,t)^* \in C^r, r \geq 1$ *defined on the region* $D = [-1,1] \times [-1,1]$, *then* $H_{2,1}u \in C^r[-1,1]$.

Theorem 2.5. *Let* $u \in L_{1\rho}$ *and* $\lambda = 1$ *value not belong to* $\boldsymbol{C_1}$, *then the operator* $H_{2,1} + C_{2,1} + L_1^*$ *is invertible, and the main equation (2.18) for the case* $r = 1$ *has a unique solution.*

Theorem 2.6. *Let* $f(x) \in C^r[-1,1]$ *and* $K(x,t) \in C^{r+2}$, $L(x,t) \in C^r$, $r \geq 1$ *on the region* $D = [-1,1] \times [-1,1]$. *Then*

$$u||u - u_n||_\rho \leq |a| c \frac{12^{r+1}}{n^r} w_r \left(\frac{1}{n}\right),$$

where $w_r(\delta)$ is the modulus of continuity and c satisfies the inequality (2.37).

In section 2.4.4, a large number of numerical results were obtained, to show the advantage of the proposed method.

In section 2.5, the Fredholm IDEs of the 2nd kind were considered and proved two lemmas to show the accuracy of the proposed method.

Lemma 2.7. For any value of $i, j, k = \{0,1, \ldots.\}$ *the following inequalities hold*

$$\int_{-1}^{1} \sqrt{1-t^2}\, U_i(t) T_i(t) T_k(t) dt = \frac{1}{4} [h_1(i,j,k) + h_2(i,j,k) + h_3(i,j,k) + h_4(i,j,k)],$$

where $h_r(i,j,k), r = \{1,2,3,4\}$ *are constants.*

Lemma 2.8. For any value of $i, j, k = \{0,1, \ldots.\}$ *the following inequalities hold*

$$\int_{-1}^{1} T_k(t) T_{i+1}(t) T_j(t) dt = \frac{1}{4} [h_5(i,j,k) + h_6(i,j,k) + h_7(i,j,k) + h_8(i,j,k)],$$

where $h_r(i,j,k), r = \{5,6,7,8\}$ *are constants.*

Chapter 3 (Main Results): Homotopy Analysis Method and New Development of HAM foLr Non-Linear Integro-Differential Equations

In this chapter, the homotopy analysis method (HAM) and its new development were used for the approximate solution of nonlinear initial-value integro-differential equations (IDEs) and nonlinear, fractional-order IDEs with mixed boundary value problems. In addition, the uniqueness of the solution and the the convergence of the proposed method were proven. Chapter 3 of the dissertation consists of two main parts.

- Part 1 (3.1.1-3.1.3) is devoted to finding an approximate solution of nonlinear initial value problems of integro-differential equations (IDEs), analysis of solution, error estimation and numerical experiments.
- Part 2 (3.2.1-3.2.6) deals with finding the approximate solution of mixed boundary value problems of nonlinear fractional-order IDEs, analysis of the solution, error estimation, and obtaining numerical results.

In this Chapter, famous homotopy analysis method (HAM) is developed and named ND-HAM and applied to initial value problem of non-linear IDEs and compared with other methods such as HAM, modified HAM (MHAM), new modified HAM (mHAM), general development of HAM (q-HAM).
Let us consider the following nonlinear IDEs with initial conditions.

Problem 3.1. *Let non-linear Volterra-Fredholm integro-differential equations (VF-IDEs) of order* p *be given by*

$$u^{(p)}(t) + \sum_{j=1}^{p-1} a_j(t)u^{(j)}(t) = f(t)$$
$$+\lambda_1 \int_a^t K_1(t,s)F_1\big(u(s)\big)ds + \lambda_2 \int_a^b K_2(t,s)F_2\big(u(s)\big)ds, \quad (3.1)$$

with initial conditions

$$u^{(k)}(a) = \alpha_k, \quad k = 0,\dots,p-1, p \in \mathbb{N}, (p \geq 2), \quad (3.2)$$

where $t \in \Omega = [a,b]$ and $K_1, K_2: \Omega \times \Omega \longrightarrow \mathbb{R}, f: \Omega \longrightarrow \mathbb{R}$ and $a_j: \Omega \longrightarrow \mathbb{R}$, $j = 1, \cdots p-1$ are known functions, λ_1, λ_2 are parameters and $F_1, F_2: C(\Omega, \mathbb{R}) \longrightarrow \mathbb{R}$ are non-linear functions as well as $u(t)$ is an unknown function to be determined.

Basic idea of the HAM is as follows. Let non-linear equation be given by

$$N[u(t)] = 0. \quad (3.3)$$

Liao has constructed the **zero-order** deformation equation in the form

$$(1-q)£[\phi(t;q) - u_0(t)] = q\hbar H(t)[N[\phi(t;q)]],$$
$$\phi^{(i)}(t;0) = u_0^{(i)}(t), \; i = 0,1,2,\dots$$

where £ is the linear and N is nonlinear operator, $q \in [0,1]$ is the embedding parameter, $\hbar \neq 0$ is an auxiliary parameter, $H(t)$ is auxilary function, $u_0(t)$ is an initial guess and $\phi(t;q)$ is an unknown function to be determined and $\boldsymbol{m}$**!th-order** deformation equation has the form

$$£[u_m(t) - \chi_m u_{m-1}(t)] = \hbar H(t)\Re_m(\overleftarrow{u}_{m-1}(t)), \quad (3.4)$$

where

$$\Re_m(\overleftarrow{u}_{m-1}(t)) = \frac{1}{(m-1)!}\frac{\partial^{m-1}[N[\phi(t;q)]]}{\partial q^{m-1}}\bigg|_{q=0}, \quad (3.5)$$

and

$$\chi_m = \begin{cases} 0, & m \leqslant 1, \\ 1, & m > 1. \end{cases} \quad (3.6)$$

The iteration defined by (3.4)-(3.6) is called standard HAM. Using the parameter q as a dummy variable, the function $\phi(t;q)$ can be expanded in the Taylor series

$$\phi(t;q) = u_0(t) + \sum_{m=1}^{+\infty} u_m(t)q^m. \; (3.7)$$

Assuming that the auxiliary parameter $\hbar$ is properly selected so that the series (3.4) is convergent when $q = 1$, hence the solution $u(t)$ of Eq, (3.3) is

$$u(t) = u_0(t) + \sum_{m=1}^{+\infty} u_m(t). \quad (3.8)$$

where $u_m(t)$ is determined from the deformation equation (3.6)-(3.8).

To derive the ND-HAM, we rewrite Eq. (3.3) in the form

$$N[u(t)] = f(t), \quad (3.9)$$

and assume that the function $f(t)$ is split into n terms, namely

$$f(t) = x_0(t) + x_1(t) + \ldots + x_n(t) \quad (3.10)$$

and construct $g(t, q)$ into powers of the parameters $(q\hbar)$

$$g(t; q) = x_0(t) + x_1(t) + x_2(t)(q\hbar) + \ldots + x_n(t)(q\hbar)^{n-1}. \quad (3.11)$$

For ND-HAM we rewrite Eq. (3.8)-(3.9), in the form

$$£[u_0(t)] = x_0(t), \quad (3.12)$$

$$£[u_m(t) - \chi_m u_{m-1}(t)] = \hbar H(t) \Re_m(\overleftarrow{u}_{m-1}(t)), \quad (3.13)$$

where χ_m is defined by (3.10) and

$$\Re_m(\overleftarrow{u}_{m-1}(t)) = \frac{1}{(m-1)!} \frac{\partial^{m-1}[N[\phi(t;q)] - g(t;q)]}{\partial q^{m-1}}\bigg|_{q=0}. \quad (3.14)$$

In ND-HAM, we have the following advantages:

- In some cases, suitable choice of $x_0(t)$ depending on given function $f(t)$ and solving Eq. (3.12), can give exact solution $u(t) = u_0(t)$ of the Eq. (3.9) from the first iteration. In this case, the next iterations obtained by Eq. (3.13) gives exactly zero solution $u_i(t) = 0, i = 1,2, \ldots$
- If Eq. (3.12) does not give exact solution, then it will serve as a choice of initial guess $u_0(t)$ satisfying initial or boundary conditions.

In section 3.1.2, the standard HAM and ND-HAM were constructed for nonlinear Volterra-Fredholm IDEs (3.1)-(3.2) and detailed solution of the considered problem was shown.

In the section 3.1.3, we have proven uniqueness solution of the Eq. (3.1)-(3.2) and the convergence of ND-HAM in Banach space. To prove the uniqueness solutions of Eq. (3.1)-(3.2), we introduced the following hypotheses:

- **(H1):** Let $C(J, R), J = [a, b]$ be the class of continuous function and functions $a_j(t), j = \{1,2, \ldots, p-1\}$ and $f(t)$ mapping $J \to R$ are continuous functions.
- **(H2):** There exist Lipschitz constants constants $L_1, L_2 > 0$ and $\gamma_j > 0, j = \{0,1, \ldots, p-1\}$ such that for any $u_1, u_2 \in C(J, R)$ the following inequality holds

$$|F_1(u_1(t)) - F_1(u_1(t))| \leq L_1|u_1 - u_2|,$$
$$|F_2(u_1(t)) - F_2(u_2(t))| \leq L_2|u_1 - u_2|,$$
$$|D^j(u_1(t)) - D^j(u_2(t))| \leq \gamma_j|u_1 - u_2|, j = \{0,1, \ldots, p-1\}.$$

where D^j is a derivative operator.

- **(H3):** There exist two functions $K_1^*, K_2^* \in C(D, R^+)$ the set of all positive function continuous on $D = \{(t, s) \in R \times R: a \leq s \leq t \leq b\}$ such that

$$K_1^* = \sup_{t \in [a,b]} \int_a^t |K_1(t, s)ds| < \infty, \quad K_2^* = \sup_{t \in [a,b]} \int_a^b |K_2(t, s)ds| < \infty.$$

We first have proved the following lemma.

Lemma 3.1. *Let $\varphi(t) \in C(J, R^+)$ then $u(t) \in C(J, R^+)$ is a solution of the problem (3.1)-(3.2) iff u is satisfying*

$$u(t) = \varphi(t) - \sum_{j=1}^{p-1} \frac{1}{(p-1)!} \int_a^t (t-s)^{p-1} (a_j(s) D^j u(s)) ds$$
$$+ \frac{1}{(p-1)!} \int_a^t (t-s)^{p-1} \left[\int_a^s K_1(s,r) F_1(u(r)) dr\right] ds$$
$$+ \frac{1}{(p-1)!} \int_a^t (t-s)^{p-1} \left[\int_a^s K_2(s,r) F_2(u(r)) dr\right] ds,$$

for $t \in J = [a,b]$ and

$$\varphi(t) = \sum_{k=0}^{p-1} \frac{\alpha_k}{k!} (t-a)^k + \frac{1}{(p-1)!} \int_a^t (t-s)^{p-1} f(s) ds.$$

Based on the results of Lemma 3.1 and Banach contraction principle, we are able to prove the following theorems

Theorem 3.2: *Assume that the hypothesises (H1)-(H3) hold. If*

$$\delta^* = \left(\frac{\gamma^* a^* p}{p!} + \frac{\lambda_1 K_1^* L_1}{p!} + \frac{\lambda_1 K_2^* L_2}{p!}\right) (b-a)^p < 1,$$

where $\gamma^* = \max_{1 \le j \le p-1} \gamma_j$ and $a^* = \max_{1 \le j \le p-1} |a_j(t)|$, then there exists a unique solution $u(x) \in C(J)$ to (3.1)-(3.2).

Theorem 3.3. (Convergence theorem): *Suppose that the series* $\sum_{m=0}^{\infty} u_m(t)$ *defined by (3.8) converges to a function* $u(t)$*, where the functions* $u_m \in C(\Omega, \mathbb{R})$ *are governed by the high-order deformation equation* (3.12)- (3.14) *of ND-HAM. Then,* $u(t)$ *defined by (3.8) is the exact solution of the problem (3.1)–(3.2).*

In section 3.1.4, a number of comparative results were presented to show the advantage of the proposed method (ND-HAM).

In section 3.2(3.2.1-3.2.4), the mixed nonlinear Volterra-Fredholm integro-differential equations (VF-IDEs) of multi-term fractional orders are detailed presented. Short discription is the following consider IDE of the form

$$\left({}^cD_{0^+}^{\beta_p} + \sum_{j=1}^{p-1} \xi_j {}^cD_{0^+}^{\beta_j}\right) u(t) = \varphi(t) + \lambda \int_0^t \int_0^T K(x,s) F(u(s)) dx ds, \quad (3.15)$$

with initial conditions

$$u^{(k)}(0) = \alpha_k, \quad k = 0, \dots, p-1, \quad (3.16)$$

and boundary conditions

$$u^{(k)}(0) = \alpha_k, \quad k = 0, \dots, p-2, \; u(T) = B, \quad (3.17)$$

where $t \in \Omega = [0,T], K: \Omega \times \Omega \longrightarrow \mathbb{R}, \varphi: \Omega \longrightarrow \mathbb{R}$ are known functions, $F: C(\Omega, \mathbb{R}) \longrightarrow \mathbb{R}$ is a nonlinear function, ξ_j, λ and B are constants, β_p is an order of Eq. (3.35) with $p-1 < \beta_p \le p, \; p = 1,2,\cdots$, with $p \in \mathbb{N}$ $(p \ge 2)$ and ${}^cD_{0^+}^{\beta_j}$ is the Caputo fractional derivative of order β_j.

To solve mixed non-linear VF-IDEs (3.15)–(3.16) using the HAM and the ND-HAM, we introduce the following non-linear operator

$$N[\phi(t;q)] = \left({}^cD_{0^+}^{\beta_j} + \sum_{j=1}^{p-1} \xi_j {}^cD_{0^+}^{\beta_j}\right) \phi(t;q) - \lambda \int_0^t \int_0^T K(x,s) F(\phi(s;q)) ds dx. \quad (3.18)$$

Next, with the help of Eq. (3.18) we were able to construct ND-HAM scheme to solve the initial value problems (3.15) - (3.16).

In section 3.2 (3.2.5-3.2.6) the unique solution of fractional order integro-differential equations and the convergence of HAM were proved.

Theorem 3.4. (Convergence theorem) *Suppose that the series* $\sum_{m=0}^{\infty} u_m(t)$ *converges to a function* $u(t)$*, where the functions* $u_m \in C(\Omega, \mathbb{R})$ *are governed by the high-order deformation equation* (3.8) *of HAM. Then,* $u(t)$ *is the exact solution*

of problem (3.15)–(3.16).
For the sake of brevity, let us define the following constant:

$$\varrho = \frac{(p-1)!}{T^{p-1}}\left\{B - \sum_{k=0}^{p-2} \frac{\alpha_k T^k}{k!} - \sum_{j=1}^{p-1} \left(\sum_{k=0}^{j-1} \frac{\xi_j \alpha_k T^{\beta_p-\beta_j+p-1}}{\Gamma(\beta_p-\beta_j+k)}\right)\right\}.$$

Theorem 3.5. *Let* $F: \mathbb{R} \to \mathbb{R}$ *such that* $\| F(x_1) - F(x_2) \| \leq L \| x_1 - x_2 \|$ *for some* $L > 0$ *and for all* $(x_1, x_2) \in \mathbb{R}^2$. *Then,* (3.15)–(3.16) *has a unique solution if*

$$\Delta = \sum_{j=1}^{p-1} \frac{|\xi_j| T^{\beta_p-\beta_j}}{\Gamma(\beta_p-\beta_j+1)} + |\lambda| L \|K\| \frac{T^{\beta_p+2}}{\Gamma(\beta_p+2)} < \frac{1}{2},$$

with $\|K\| = \sup_{t,s\in 0,T]} |K(t,s)|$.

In section 3.3, the proposed method was compared with other methods and its advantage was shown by presenting several examples.

Chapter 4 (Main Results): Newton-Kantorovich Method For the System of Nonlinear Integral Equations From One Dimensional to Higher Dimensional

In this chapter, the Newton-Kantorovich method (NKU) was applied to the system of nonlinear 2×2 integral equations, and the existence and uniqueness of the solution together with the convergence of the proposed method were proved.
Chapter 4 of the dissertation consists of two main parts.

- Part 1 (4.1.1-4.1.6) deals with the Newton-Kantorovich method (NKM) for nonlinear 2×2 systems of integral equations (sysIEs), the new majorant function and the error estimation of NKM and numerical experiments.
- Part 2 (4.2.1-4.2.5) devoted with the approximate solution of nonlinear multi-dimensional Volterra integral equations (VIEs) by the modified Newton method (MNM) and its error estimations and numerical experiments.

In paragraph 4.1 (4.1.1), NKM was used for the system of nonlinear 2×2 integral equations and a new majorant function was introduced. Here's a quick summary of this topic. The aim of this section is to investigate the general 2×2 system of NIEs of the form

$$\begin{cases} a(t)x(t) - \int_{y(t)}^{t} H(t,\tau)F(x(\tau))d\tau = g(t), \\ b(t)x(t) + \int_{y(t)}^{t} K(t,\tau)F(x(\tau))d\tau = f(t), \end{cases} \tag{4.1}$$

where $0 < t_0 \leq t \leq T,\ y(t) < t,$ kernels $H(t,\tau), K(t,\tau) \in C_{[t_0,T]\times[t_0,T]}$, and functions $a(t), b(t), f(t), g(t) \in C_{[t_0,T]}$ are given continuous functions and $x(t) \in C_{[t_0,T]}, y(t) \in C^1_{[t_0,T]}$ are unknown functions, finally $F(x(t))$ is a nonlinear terms.

In this section, we have solved Eq. (4.1) by using NKM together with Gauss-Legendre QFs and Newton's forward interpolation formula. To do this end let us introduce operators

$$\begin{cases} P_1(x(t),y(t)) = a(t)x(t) - \int_{y(t)}^{t} H(t,\tau)F(x(\tau))d\tau - g(t), \\ P_2(x(t),y(t)) = b(t)x(t) + \int_{y(t)}^{t} K(t,\tau)F\big(x(\tau)\big)d\tau - f(t), \end{cases} \tag{4.2}$$

then rewrite (4.1) in the operator form

$$P(X) = (P_1(X), P_2(X)) = (0,0),\ X = (x(t), y(t)).$$

Applying the initial iteration of the NKM

$$P'(X_0)(X - X_0) + P(X_0) = 0,$$
to (4.2), we get

$$\left.\begin{aligned} &\Delta x(t) - \frac{1}{c(t)}\int_{y_0(t)}^{t} K_1(t,\tau)F'(x_0(\tau))\Delta x(\tau)d\tau = \psi_0(t), \\ &\Delta y(t) = \frac{1}{d(t)}\Big[\int_{y_0(t)}^{t} K(t,\tau)F'(x_0(\tau))\Delta x(\tau)d\tau \\ &\qquad + \int_{y_0(t)}^{t} K(t,\tau)F(x_0(\tau))d\tau + b(t)x_0(t) - f(t)\Big], \end{aligned}\right\} \quad (4.3)$$

where $\Delta x(t) = x_1(t) - x_0(t)$ and $\Delta y(t) = y_1(t) - y_0(t)$.
By solving (4.3) in terms of Δx and Δy, we obtain $(x_1(t), y_1(t))$. Applying the modified Newton method of the form

$$P'(X_0)(X_m - X_{m-1}) + P(X_{m-1}) = 0,$$
to (4.2), we obtain

$$\begin{aligned} &\Delta x_m(t) - \frac{1}{c(t)}\int_{y_0(t)}^{t} K_1(t,\tau)\mathrm{F}'\big(x_0(\tau)\big)\Delta x_m(\tau)d\tau = \psi_{m-1}(t), \\ &\Delta y_m(t) = \frac{1}{d(t)}\Big[\int_{y_0(t)}^{t} K(t,\tau)\mathrm{F}'\big(x_0(\tau)\big)\Delta x_m(\tau)d\tau \\ &\qquad + \int_{y_{m-1}(t)}^{t} K(t,\tau)\mathrm{F}\big(x_{m-1}(\tau)\big)d\tau + b(t)x_m(t) - \phi(\tau)\Big], \end{aligned} \quad (4.4)$$

where $\Delta x_m = x_m - x_{m-1}$ and $\Delta y_m = y_m - y_{m-1}$. Since (4.4) is a system of linear Volterra type integral equations (sysLVIEs), it can easily be solved in terms of Δx_m and Δy_m. Once solved (4.4) with respect to Δx_m and Δy_m, we obtain a sequence of approximate solutions $(x_m(t), y_m(t))$.

In section 4.1.2, developed the Gauss-Legendre QFs and its estimation were obtained. Gauss-Legendre QFs to the kernel integral on the variable interval $[y(t_i), t_i], i = 1,2,\ldots,n$ with $t_i = t_0 + ih, h = \frac{T-t_0}{n}$ were developed as follows

$$\int_{y(t_i)}^{t_i} K(t_i,\tau)x(\tau)d\tau = \frac{t_i - y(t_i)}{2}\sum_{j=1}^{l} W_j(t_i)x(\tau_j^i) + R_{n+1}(Kx), \quad (4.5)$$

$$W_j(t_i) = K(t_i,\tau_j^i)w_j, \quad \tau_j^i = \frac{t_i - y(t_i)}{2}s_j + \frac{t_i + y(t_i)}{2}, \quad j = 1,2,\ldots,l,$$

where $\tau_j^i \neq t_i$ with $[y(t_i), t_i] \in [t_0, T]$ and l refers to the number of sub partitions of the interval $[y(t_i), t_i]$ and w_j and s_j are the weights of integral and roots of Legendre polynomials. The following theorem is proven.

Theorem 4.1.3 *Let kernel $K(t,\tau)$ and $x(t)$ be in the class of $C^{(2n+2)}[t_0, T]$ then the error term of developed Gauss-Legendre QF (4.5) has the form*

$$|R_{n+1}(Kx)| \le \frac{(T-t_0)^{2n+3}}{(2n+3)}\left[\frac{1\cdot 2\cdot 3\cdots(n+1)}{(n+1)\cdot(n+2)\cdots(2n+2)}\right]^2 \frac{T^{(2n+2)}}{1\cdot 2\cdots(2n+2)},$$

where

$$T^{(q)} = X^{(0)}M_t^{(q)} + b_1 X^{(1)}M_t^{(q-1)} + \cdots + b_{q-1}X^{(q-1)}M_t^{(1)} + X^{(q)}M_t^{(0)}$$

with $b_i = \frac{q!}{i!(q-i)!}$, $i = 1,\ldots,q-1$ *binomial coefficients.*

In section 4.1.3-4.1.4, after showing the stability of the QFs and detailed description of the Newton-Kantorovich method (NKU) **in section 4.1.5,** a new majorant function was constructed. Based on majorant function the convergence of the NKM were proved with regard to the successive approximations. Let us introduce the following classes of functions.

- $C_{[t_0,T]}$ the set of all continuous functions $f(t)$ defined on the interval $[t_0, T]$,

- $C_{[t_0,t]\times[t_0,T]}$ the set of all continuous functions $S(t,\tau)$ defined on the region $[t_0,T]\times[t_0,T]$,
- $\overline{C}=\{X: X=(x(t),y(t)): x(t),y(t)\in C_{[t_0,T]}\}$,
- $\breve{C}_{[t_0,T]}=\{y(t)\in C^1_{[t_0,T]}: y(t)<t\}$.

Introducing the new majorant real-valued function

$$\psi(t)=(t-t_0)^2-(\zeta+\eta)(t-t_0)+\zeta\eta, \tag{4.7}$$

where $\zeta,\eta>0$ are real coefficients and considering the following equations

$$X=S(X),\ t=\phi(t). \tag{4.8}$$

Taking initial guess $X_0=(x_0(t),y_0(t))$ in $\Omega_0=(\| X-X_0 \|\leq r)$, where radius r satifying the inequality

$$min\{\xi+t_0,\eta+t_0\}\leq r\leq max\{\xi+t_0,\eta+t_0\},$$

the following majorant function definition is formulated.

Theorem 4.1. *Let the nonlinear operator $P(X)=0$ be defined in an open set $\Omega=\{X\in C([t_0,T]): \| X-X_0 \|< R\}$ and has continuous second derivative in a closed set $\Omega_0=\{X\in C([t_0,T]): \| X-X_0 \|\leq r\}$ such that $T=t_0+r\leq t_0+R$. Assume the following conditions are satisfied*

- $\| \Gamma_0 P(X_0) \|\leq \frac{\zeta\eta}{\zeta+\eta}$,
- $\| \Gamma_0 P''(X) \|\leq \frac{2}{\zeta+\eta}$, when $\| X-X_0 \|\leq t-t_0\leq r$,

then $\psi(t)$ in (4.7) *is a majorant function for the nonlinear operator $P(X)$.*

Main theorem for the proposed method is as follows:

Theorem 4.2. *Let the functions $f(t),\ g(t)\in C_{[t_0,T]}$, $x_0(t)\in C^1[t_0,T]$, and the kernels $H(t,\tau),K(t,\tau)\in C^1_{[t_0,T]\times[t_0,T]}$ and $(x_0(t),y_0(t))\in\Omega_0$. If*

1. $\Gamma_0=[P'(X_0)]^{-1}$ exists and satisfies $\| \Gamma_0 \|\leq s_0 M e^{M(T-H_3)}$
2. $\| \Delta X \|\leq \frac{\zeta\eta}{\zeta+\eta}$, ζ and η are given in (4.13)
3. $\| \Gamma_0 P''(X) \|\leq \frac{2}{\zeta+\eta}$, with $\| P''(X) \|\leq \eta_1$,
4. Equation (4.3) has a root $\bar{t}\in[t_0,t']$, $t'=t_0+r$ where $min\{\zeta+t_0,\eta+t_0\}<r<max\{\zeta+t_0,\eta+t_0\}$, with $\phi(t')\leq t'$.

Then the system (4.1) has a unique solution $X^*=(x^*,y^*)\in\Omega_0$ and the sequence $X_m(t)=(x_m(t),y_m(t))$, $m\geq 0$ of successive approximations (4.4) converges to the solution X^*. The rate of convergence is given by

$$\| X^*-X_m \|\leq\left(\frac{2\zeta}{\zeta+\eta}\right)^m\zeta,$$

when $\zeta+t_0$ is the minimum zero of (4.7), i.e $\zeta<\eta$.

In section 4.2, the second kind, nonlinear multi-dimensional Volterra integral equations (VIEs) were solved by Newton's method. For this point, we consider the following non-linear multi-dimensional VIEs:

$$u(\mathbf{t})-\int_{a_1}^{b_1(t)}\int_{a_2}^{b_2(t)}\cdots\int_{a_n}^{b_n(t)}K(\mathbf{t},\mathbf{x})G(u(\mathbf{x}))dx_n dx_{n-1}\cdots dx_1 = f(\mathbf{t}),\ \mathbf{t}=(t_1,t_2,\cdots,t_n),\mathbf{x}=(x_1,x_2,\cdots,x_n), \tag{4.9}$$

where, $u(\mathbf{t})\in\Omega_1=C_{\prod_{i=1}^{n}[a_i,b_i]}$ is unknown function, $G(u(\mathbf{t}))$ is nonlinear continuous function, $f(\mathbf{t}) \in \Omega_1$ and $K(\mathbf{t},\mathbf{x}) \in \Omega_1 \times \Omega_2$, $\Omega_2 = C_{\prod_{i=1}^{n}[c_i,d_i]}$ are given smooth functions, and $b_i(t)$, $i = 1,2,\cdots,n$ are real valued continuous functions.

In solving (4.9), the modified Newton method (MNM) were applied to linearize nonlinear multi-dimensional VIEs (4.9) into linear multi-dimensional VIEs. Then the developed Gauss-Legendre QFs was used to find the approximate solution. To do this end let us use the operator equation for the problem (4.9)

$$\mathbf{Q}(u(\mathbf{t})) = 0, \quad \mathbf{t} = (t_1, t_2, \cdots, t_n), \tag{4.10}$$

where

$$\mathbf{Q}(u(\mathbf{t})) = u(\mathbf{t}) - f(\mathbf{t}) - \int_{a_1}^{b_1(t)} \cdots \int_{a_n}^{b_n(t)} K(\mathbf{t},\mathbf{x}) G(u(\mathbf{x})) dx_n dx_{n-1} \cdots dx_1 \tag{4.11}$$

Next, approximate solution of the (4.10) is obtained from the equation

$$\mathbf{Q}'(u_0(\mathbf{t}))\Delta u_r(\mathbf{t}) + \mathbf{Q}(u_{r-1}(\mathbf{t})) = 0(4.12)$$

From the (4.12), we have

$$\Delta u_r(\mathbf{t}) - \int_{a_1}^{b_1(t)} \int_{a_2}^{b_2(t)} \cdots \int_{a_n}^{b_n(t)} [\Psi_0(\mathbf{t},\mathbf{x}; u_0)\Delta u_r(\mathbf{x})] dx_n \cdots dx_1 = \Phi_{r-1}(\mathbf{t}) \tag{4.13}$$

where $\mathbf{t} = (t_1, t_2, \cdots, t_n)$, $\mathbf{x} = (x_1, x_2, \cdots, x_n)$ and

$$\Delta u_r(\mathbf{t}) = u_r(\mathbf{t}) - u_{r-1}(\mathbf{t}), \quad r = 2,3,\cdots,$$

$$\Phi_{r-1}(\mathbf{t}) = f(\mathbf{t}) - u_{m-1}(\mathbf{x}) + \int_{a_1}^{b_1(t)} \int_{a_2}^{b_2(t)} \cdots \int_{a_n}^{b_n(t)} \left[K(\mathbf{t},\mathbf{x}) G\big(u_{r-1}(\mathbf{x})\big)\right] dx_n dx_{n-1} \cdots dx_1 \tag{4.14}$$

Solving Eq.(4.14) w.r.t. $\Delta u_r(\mathbf{t})$, we obtain a sequence of approx. solution $u_r(\mathbf{t})$.
To proof the convergence analysis of modified Neton method we assume that

$$|f(\mathbf{t})| \le R_1, \quad |u_0(\mathbf{t})| \le R_2, \quad |K(\mathbf{t},\mathbf{x})| \le R_3,$$

$$|G(u_0(\mathbf{t}))| \le R_4, \quad |G'(u_0(\mathbf{t}))| \le R_5, \quad |G''(u_0(\mathbf{t}))| \le R_6.$$

Next, we use the following majorant function

$$Z(t) = \eta(t - t_0)^2 - (1 + \eta\beta)(t - t_0) + \beta,$$

where η and β are nonnegative real number.

Theorem 4.3. *Let the operator equation $\boldsymbol{Q}(u) = 0$ in* (4.10) *is defined in* $\Omega_1 = \{u \in C_{\prod_{i=1}^{n}[a_i,b_i]} : \| u - u_0 \| \le R\}$ *and has a continuous second derivative in* $\Omega_0 = \{u \in C_{\prod_{i=1}^{n}[a_i,b_i]} : \| u - u_0 \| \le r \le R\}$. *If the following conditions hold:*

3. *Linear MD-VIEs in Eq. (4.14) has a resolvent kernel $\Gamma(\boldsymbol{t},\boldsymbol{x})$ with* $\| \Gamma \| \le R_3 R_5 e^{R_3 R_5 \prod_{i=1}^{n}(b_i - a_i)}$,
4. $|\Delta \mathbf{t}| \le \frac{\zeta}{1+\eta\zeta}$,
5. $|\mathbf{Q}''(\mathbf{t})| \le \eta_1$.

Then Eq. (4.13) *has a unique solution $u^*(\boldsymbol{t})$ in the closed ball Ω_0 and the sequence* $u_r(\boldsymbol{t}), r \ge 0$ *of successive approximation*

$$\Delta u_r(\mathbf{t}_j) - \int_{a_i}^{b_1(t_j)} \int_{a_2}^{b_2(t_j)} \cdots \int_{a_n}^{b_n(t_j} \Psi_0(\mathbf{t}_j,\mathbf{x}; u_0)\Delta u_r(\mathbf{x}) dx_n dx_{n-1} \cdots dx_1 = \Phi_{r-1}(\mathbf{t}_j),$$

where $j = 1,2,\cdots,m_i$ and $u_r(\boldsymbol{t}) = u_{r-1}(\boldsymbol{t}) + \Delta u_r(\boldsymbol{t})$, converges to the solution $u^*(\mathbf{t})$. The rate of convergence is given by

$$\| u^* - u_r \| \le \left(\frac{2}{1+\eta\beta}\right)^r \left(\frac{1}{\eta}\right), \quad r = 1,2,\cdots$$

CONCLUSION

This dissertation aims is to construct an effective quadrature formula for the approximate calculation of weighted singular and hypersingular integrals in variable and constant intervals and to evaluate it in different classes, at the same time to develop effective numerical methods for the approximate solution of singular and hypersingular integral equations, for the approximate solution of nonlinear integro-differential equations, a new development of semi-analytical methods were proposed, as well as for the 2x2 system of nonlinear integral equations modified Newton method are presented and a modification of Newton's method for the approximate solution of high-dimensional integral equations is proposed which has practical importance.

The main results obtained are as follows:

1. Constructed automatic quadrature scheme (AQS) for the weighted singular and hypersingular integrals on the variable and fixed intervals and developed discrete vortex method (modified MDV) for the weighted singular integrals on the fixed interval;
2. Proved convergence of AQS in the cases of bounded and unbounded solutions in the weighted Hilbert and smooth function spaces and provided convergence of modified MDV in Holder and differentiable class spaces for all the cases;
3. Constructed the truncated series of Chebyshev polynomial approximations to the exact solution (for 4 cases) for SIEs and HSIEs of the first kind and for second kind SIEs the bounded solution is investigated;
4. Proved the existence and solvability of the HSIEs in the case of bounded solution and obtained norm convergence of the approximate solution in the differentiable Holder class;
5. Developed a new homotopy analysis method (ND-HAM) and applied it to solve nonlinear IDEs with initial value problems and fractional-order of IDEs with boundary-value problems;
6. Proved the convergence of ND-HAM to the exact solution and established existence and uniqueness solutions of the nonlinear IDEs and fracIDEs;
7. Developed NKM for 2×2 nonlinear system of integral equations and proved the existence and uniqueness of the solutions and obtained the convergence of the proposed method in the weighed Hilbert space.
8. Developed Newton method for nonlinear multi-dimensional IEs and proved the existence and uniqueness of the solutions and obtained the convergence of the proposed method in the weighed Hilbert space.

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ЭШКУВАТОВ ЗАЙНИДИН КАРИМОВИЧ

АВТОМАТИЧЕСКАЯ КВАДРАТУРНАЯ СХЕМА И МЕТОД ГОМОТОПИЧЕСКОГО АНАЛИЗА ДЛЯ СИНГУЛЯРНЫХ И ИНТЕГРО-ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ

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ВВЕДЕНИЕ (аннотация докторской диссертации)

Целью исследования: Построение эффективных квадратурных формул, называемых автоматической квадратурной схемой для вычисления взвешенных сингулярных и гиперсингулярных интегралов, оценка их погрешности в различных классах функций, а также нахождение приближенных решений систем сингулярных интегральных уравнений и систем гиперсингулярных интегралных уравнений. Далее создание новой разработки метода гомотопического анализа для нелинейных интегро-дифференциальных уравнений. Также разработка метода Ньютона-Канторовича (МНК) для приближенного решения системы нелинейных интегральных уравнений и многомерного интегрального уравнения. Научная новизна исследования состоит в следующем:

Объект исследования: Сингулярные и гиперсингулярные интегралы и интегральное уравнение, нелинейное интегро-дифференциальное уравнение, система нелинейных интегральных уравнений и многомерные интегральные уравнения.

Научная новизна исследования состоит в следующем:

построена автоматическая квадратурная схема на основе конечного ряда Чебышева для взвешенных сингулярных и гиперсингулярных интегралов в переменных и фиксированных интервалах, также разработан модифицированный метод дискретных вихрей (МДВ) взвешенных сингулярных интегралов на фиксированном интервале;

доказана сходимость (АКС и МДВ) квадратурных формул для случаев ограниченных и неограниченных решений в пространствах Гильберта, Гольдера и гладких функций;

показана сходимость ряда конечных полиномов Чебышева к точному решению (для 4 случаев) для систем сингулярных интегральных уравнений и гиперсингулярных интегралных уравнений первого типа;

в случае конечного решения доказано существование и единственность решения для истем гиперсингулярных интегралных уравнений и нормативная сходимость приближенного решения в классе Гёльдера;

разработано и применено новое развитие метода гомотопического анализа для решения интегро-дифференциальных уравнений с нелинейными начальными условиями и краевых задач интегро-дифференциальных уравнений с дробным порядком, и доказана его сходимость;

для системы нелинейных интегральных уравнений 2×2 развит метод Ньютона-Контаровича, найдены новые мажорантные функции, также доказано существование, единственность и сходимость решения;

нелинейные многомерные интегро-дифференциальные уравнения решались с помощью метода Ньютона, а в весовом пространстве Гильберта доказаны существование и единственность решения и сходимость предложенного метода.

Структура и объем диссертации. Диссертация состоит из введения, четырех глав, заключения и списка использованных литератур. Объем диссертации составляет 197 страницы.

ЭЪЛОН ҚИЛИНГАН ИШЛАР РЎЙХАТИ
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

1-бўлим (Part 1, Часть 1)

1. Eshkuvatov Z.K., Nik Long N.M.A., Muminov Z. I., Khaldjigitov A.A., Mixed Method for the Product Integral on the Infinite Interval. Special Issue of Malaysian Journal of Mathematical Sciences, vol. 8, 2014, 71-82. (3, Scopus IF=0.39)
2. Eskhuvatov Z.K., Ahmedov A., Nik Long N.M.A., Amalina N.J., Approximating Cauchy-type singular integral by an automatic quadrature scheme. Journal of Computational and Applied Mathematics, vol. 235, 2011, 4675-4686. (3, Scopus IF=1.292)
3. Eshkuvatov Z.K., Nik Long N.M.A., Approximating the singular integrals of Cauchy type with weight function on the interval. J. Comp. and Appl. Math. vol. 235(16), 2011, 4742-4753. (3, Scopus IF=1.292)
4. Eshkuvatov Z.K., Nik Long N.M.A., Abdulkawi M., Numerical evaluation for Cauchy type singular integrals on the interval. J. Comput. Appl. Math., vol. 233(8), 2010, 1995-2001. (3, Scopus IF=1.176)
5. Eshkuvatov Z.K., Nik Long N.M.A., Abdulkawi M., Quadrature formula for approximating the singular integral of Cauchy type with unbounded weight function on the edges, J. Comput. Appl. Math. vol. 233, 2009, 334-345. (3, Scopus IF=1.176)
6. S.J. Obaiys., Z.K. Eshkuvatov., N.M.A. Nik Long, On Error Estimation of Automatic Quadrature Scheme for the Evaluation of Hadamard Integral of Second Order Singularity. U.P.B. Sci. Bull., Series A, Vol. 75, 2013, 85-98. (3, Scopus IF=0.39)
7. Bichi S.L., Eshkuvatov Z.K., Nik Long N.M.A., An Automatic Quadrature Schemes and Error Estimates for Semibounded Weighted Hadamard Type Hypersingular Integrals. Abstract and Applied Analysis. Volume 2014, Article ID 387246, 13 pages. (3, Scopus IF=1.274)
8. Bichi S.L., Eshkuvatov Z.K., Nik Long N.M.A. A note on Evaluating Weighted Hadamard Integral of Second Order Singularity Using Airfoil Polynomials. Uzbek Math. Journal, Vol. 4, 2013, pp. 94-105. (01.00.00; № 6)
9. Sirajo Lawan Bichi, Z. K. Eshkuvatov, Nik Long N.M.A., Abdurahim Okhunov. Quadrature formula for evaluating left bounded Hadamard type hypersingular integrals. AIP Conference Proceedings 1635, 2014, 395. (3, Scopus IF=0.189)
10. Zainidin Eshkuvatov, Hasham Alhawamda, Bachok M. Taib, and Rose Irnawaty Ibrahim. Bachok-Hasham polynomials for solving a special class of singular integral equations. AIP Conference Proceedings Vol. 1974, 2018, 020006-1-11. (3, Scopus IF=0.189)

11. Eshkuvatov Z.K., Nik Long N.M.A., Abdulkawi M., Approximate solution of singular integral equations of the first kind with Cauchy kernel. Applied Mathematics Letters, Vol. 22(5), 2009, 651- 657. (3, Scopus IF=0.948)

12. Eshkuvatov Z.K., M. Abdulkawi., Nik Long N.M.A.,. Numerical Solution of FIE of the second kind with Cauchy kernel. AIP Conf. Proc, 1450, 2012, 230-233. (3, Scopus IF=0.189)

13. Abdulkawi.M., Eshkuvatov Z.K., Nik Long N.M.A. A method for the numerical solution of Cauchy singular integral equations. International Journal of Appl. Math., Vol. 22(2), 2009, 287-297. (3, Scopus IF=0.27).

14. Abdulkawi M., Eshkuvatov Z.K., Nik Long N.M.A. A Note on the Numerical Solution of Singular Integral Equations of Cauchy type. International Journal of Applied Mathematics and Computer Sciences Vol. 5(2), 2009, 90-93. (3, Scopus IF=0.55).

15. Nik Long N.M.A., Eshkuvatov Z.K., Abdulkawi M. Semi-Bounded Solutions of Singular Integral Equations of Cauchy type. Int. J. Contemp. Math. Sciences, Vol. 4(22), 2009, 1059-1066. (Indexed in Google Scholar).

16. Eshkuvatov Z.K., Kammuji M., Shahrina Ismail., Nik Long N.M.A. Super-singular IEs of the First Kind and its approximate solutions. AIPCP Vol. 2266, 2020, 070001 doi.org/10.1063/5.0018071. (3, Scopus IF=0.189).

17. Eshkuvatov Z.K., Zulkarnain F.S., Nik Long N.M.A., Muminov Z. Homotopy Perturbation Method for the Hypersingular Integral Equations of the First Kind. Ain Shams Engineering Journal (ASEJ), Vol.9(4), 2018, 3359–3363. (3, Scopus IF=2.801).

18. Eshkuvatov Z.K. Homotopy Perturbation Method and Chebyshev Polynomials for Solving a Class of Singular and Hypersingular Integral Equations. Numerical Algebra, Control and Optimization (NACO), Vol.8(3), 2018, 337-350. (3, Scopus IF=0.312).

19. Eshkuvatov Z.K., Zulkarnain F.S., Nik Long N.M.A., Muminov Z. Modified Homotopy Perturbation Method for Solving HSIEs of the First Kind. SpringerPlus 5:1473. 2016, 1-21. (3, Scopus IF=0.9543)

20. Hamzah K.B., Nik Long N.M.A., Senu N., Eshkuvatov Z.K. Numerical solution for the thermally insulated cracks in bonded dissimilar materials using hypersingular integral equations. Applied Mathematical Modelling, Vol. 91, 2021, 358–373. (3, Scopus IF=2.234)

21. Nik Long N.M.A., Eshkuvatov Z.K., Hypersingular intergral equations for multiple curved cracks in plane elasticty. International Journal of Solids and Structures. Vol. 46(13), 2009, 2611–2617. (3, Scopus IF=2.617).

22. Eskhuvatov Z.K. New Development of Homotopy Analysis Method for a Non-linear Integro-Differential Equations and Integral Equations. Mathematical Modelling and Computing(MMC), Vol. 9, No. 4, 2022, 842–859. (3, Scopus, IF=0.26).

23. Eshkuvatov Z.K., Laadjal Z., Ismail Sh., Numerical treatment of nonlinear mixed Volterra-Fredholm integro-differential equations of fractional order. AIP Conference Proceedings 2365, 020006 (2021); doi.org/10.1063/5.0057120. (3,

Scopus IF=0.189).

24. Eshkuvatov Z.K., Khadijah M.H., Taib B.M., Modified HPM for high-order linear fractional integro-differential equations of Fredholm-Volterra type. IOP Journal of Physics: Conf. Series Vol. 1149, pp. 012019-1-10, 2018. (3, Scopus IF=0.112).

25. Zainidin K. Eshkuvatov, Fatimah Samihah Zulkarnain, Zahriddin Muminov, Nik Mohd Asri Nik Long. Convergence of modified homotopy perturbation method for Fredholm-Volterra integro-differential equation of order m. Malaysian Journal of FAS (MJFAS), 2017, 340-345. (3, Scopus IF=0.16).

26. Eshkuvatov Z.K, Kammuji M., Nik long N.M.A., Arif A. M. Yunus. Effective quadrature formula in solving linear integro-differential equations of order two. AIP Conference Proceedings 1870, 040077 (2017); doi: 10.1063/1.4995909, (3, Scopus IF=0.189).

27. Kammuji M., Eshkuvatov Z.K., Muminov Z. Approximate solution for linear IDEs of order one by Legendre polynomials. Indian Journal of Mathematics, Vol. 60(3), 2018, 415-438. (3, Scopus IF=0.26).

28. Zulkarnain F.S., Eshkuvatov Z.K., Nik long N.M.A., Ismail F. Modified Homotopy Perturbation Method for Fredholm–Volterra Integro-Differential Equation. Springer, Recent Advances in Math. Sciences, 2016, 37-44. (3, Scopus IF=0.16).

29. Atabakan Z.P., Nasab A.K., Kiliçman A., Eshkuvatov Z.K. Numerical Solution of Nonlinear Fredholm IDEs Using Spectral Homotopy Analysis Method. Mathematical Problems in Engineering, Volume 2013, Article ID 674364, 9 pages, (3, Scopus IF=1.057).

30. Eshkuvatov Z.K, Hameed Husam Hameed., Taib B.M., Nik long N.M.A. General 2×2 System of Nonlinear Integral Equations and Its Approximate Solution. Journal of Computational and Applied Mathematics, Vol. 361, 2019, 528-546. (3, Scopus IF=2.872).

31. Eshkuvatov Z.K, Kammuji M., Bachok M. Taib., Nik long N.M.A. Effective Approximate Method for Solving Linear Fredholm-Volterra Integral Equations. Numerical Algebra, Control and Optimization (NACO), Vol. 7(1), 2017, 77-88. (3, Scopus IF=0.312).

32. Eshkuvatov Z.K., Hameed H.H., Nik Long N.M.A. One dimensional nonlinear integral operator with NKM. J. of King Saud Univ. Science. Vol. 28. 2016, 172-177. (3, Scopus IF=3.829).

33. Eshkuvatov Z.K., Ahmedov A., Nik Long N.M.A., O. Shafiq. Approximate solution of the system of nonlinear integral equation by Newton–Kantorovich method. Applied Mathematics and Computation, Vol. 217(8), 2010, 3717–3725. (3, Scopus IF=1.1.24).

34. Hameed H.H., Eshkuvatov Z.K., Nik Long N.M.A. On the Solution of Multi-Dimensional Nonlinear Integral Equation with Modified Newton Method. Journal of Comput. and Theor. Nanoscience, Vol. 14, 2017, 5298-5303. (3, Scopus IF=2.134).

35. Hameed H.H., Eshkuvatov Z.K., Nik Long N.M.A. On solving an $n \times n$ system of nonlinear Volterra integral equations by the Newton-Kantorovich method. ScienceAsia Vol. 42(S), 2016, 11–18. (3, Scopus IF=0.23).

36. Hameed Husam Hameed., Eshkuvatov Z.K., Nik Long N.M.A. An Approximate Solution of Two Dimensional Nonlinear Volterra Integral Equation Using Newton-Kantorovich Method. Malaysian Journal of Science Vol. 35 (1): 2016, 40–47. (3, Scopus IF=0.16).

37. Hameed H.H., Eshkuvatov Z. K., Ahmedov A., and Nik Long N.M.A. On Newton-Kantorovich Method for Solving the Nonlinear Operator Equation. Abstract and Applied Analysis, Vol. 2015, Article ID 219616, 2015, 12 pages. (3, Scopus IF=0.29).

2-бўлим (Part II, Часть II)

38. Eshkuvatov Z.K., Narzullaev A. Projection Method for Bounded and Unbounded Solution of Hypersingular Integral Equations of the First Kind. Indian Journal of Industrial and Applied Mathematics (IJIAM), Vol. 10(1), 2019, 11-37.

39. Yaghobifar M., Nik Long N.M.A., Eshkuvatov Z.K. Analytical Solutions of Characteristic Singular Integral Equations in the Class of Rational Functions. Int. J. Contemp. Math. Sciences, Vol. 5(56), 2010, 2773-2779.

40. Abdulkawi M.M., Nik Long N.M.A., Eshkuvatov Z.K., Numerical solution of hypersingular integral equations. International Journal of Pure and Applied Mathematics., Vol. 69(3), 2011, 265-274.

41. Obaiys S.J., Eskhuvatov Z.K., Nik long N.M.A., Jamaludin M.A. Galerkin Method for the Numerical Solution of Hypersingular Integral Equations Based Chebyshev Polynomials. Int. Journal of Math. Analysis, Vol. 6(54), 2012, 2653-2664.

42. Eskhuvatov Z.K. New development of homotopy analysis method for non-linear integro-differential equations with initial conditions. Presented to 16th International Scientific Conference “which was held on December 01–04, 2021, Penza, Russia. Published in ANM21 Conference Proceedings, January 2022, 22-27.

42. Z. K. Eshkuvatov, N.M.A. Nik Long. A Quadrature formula for the approximation of Cauchy type singular integrals on the interval. Int. J. Math. Analysis, vol 1(8), 2007, 355-368.

43. M. Abdulkawi, N.M.A. Nik Long, Z. K. Eshkuvatov. A note on the numerical solution for Fredholm integral equation of the second kind with Cauchy kernel. Journal of Mathematics and Statistics (JMS), Vol. 7(1), pp. 68-72, 2011.

44. Z. K. Eshkuvatov, Approximate solution of Cauchy Type Singular Integral on the Variable Interval. Presented to 16th International Scientific Conference “ which was held on December 01–04, 2021, Penza, Russia. Published in ANM21 Conf. Proceedings, Jan 2022, pp. 86-90.

45. Z.K. Eshkuvatov, N.M.A. Nik Long. Numerical solution of Singular Integral on closed smooth contour. Science Putra. (2007). Vol. 15(1), pp. 27-32. Malaysia.

43. S.J. Obaiys, Z.K. Eshkuvatov, N.M.A. Nik Long, Automatic Quadrature Scheme for Evaluating Hypersingular Integrals. International Journal of Modern

Physics: Conference Series Vol. 9, 2012, pp. 581–585.

46. Mathematical and Computer Modelling of Natural Science and Social Problems (MCM-2022) Proceedings of the Sixteenth Russian Conference with international participation MCM-2022 Penza, Russian Federation, 1–4 June, 2022. Semi-Bounded Solution of Hypersingular Integral Equations of the First Kind, pp. 15-21.

47. Proceedings of the XVI-th International Conference ANM–2021 Penza, Russian Federation, 1–4 Dec., 2021. Analytical and Numerical Methods of Modelling of Natural Science and Social Problems (ANM–2021) Eshkuvatov Z. K. New development of homotopy analysis method for a non-linear IDEs with initial conditions, pp. 86-90.

48. The 15th Universiti Malaysia Terengganu Annual Symposium 2021 (UMTAS 2021) which was held on 21-23 November 2021, UMT, Malaysia. Title: The Automatic Quadrature Scheme for the First Order weighted Singular Integrals, pp. 89.

49. The 15th Universiti Malaysia Terengganu Annual Symposium 2021 (UMTAS 2021) which was held on 21-23 November 2021, UMT, Malaysia. Title: Application of HAM and ND-HAM for Nonlinear IDEs of order two, pp. 336.

50. International Conference on Recent Advances in Applied Mathematics 2020 (ICRAAM2020) was held 4-6 February 2020, Universiti Putra Malaysia (UPM), Malaysia. Title: Modified HAM for solving a linear system of Fredholm-Volterra Integral Equations, pp 92.

51. Uzbekistan-Malaysia international conference on Computational Models and Technologies CMT2020 was held August 24-25, 2020. Title: Numerical treatment of nonlinear mixed Volterra-Fredholm integro-differential equations of fractional order, pp. 22.

52. Uzbekistan-Malaysia international conference on Computational Models and Technologies CMT2020 was held August 24-25, 2020. Title: Numerical approach for nonlinear system of Fredholm-Volterra Integral equations, pp. 23.

53. ”The 4th International Conference on Computing, Mathematics, and Statistics (iCMS2019), Universiti Teknologi MARA, Kedah, Malaysia” which was held on 23-24 April 2019. Title: Semi-Bounded Solution of HSIEs of the First Kind on the Rectangle, pp. 108.

54. 27 th National Symposium on Mathematical Sciences (SKSM27), which was hold in Tenera Hotel, Bangi, Selangor on 26- 27 November 2019. Super-singular Integral Equations of the First Kind and its Approximate Solutions, pp. 31.

55. Third International Conference on Mathematical Sciences and Statistics (ISMSS2018), Putrajaya, Malaysia”, which was held on 6-8 February 2018. Title: Modified HPM for high-order linear fractional integro-differential equations of Fredholm-Volterra type, pp. 6.

56. The 14 th International Symposium on Geometric Function Theory and Applications (GFTA 2018). Which was hold 3 – 6 December 2018. Extended Chebyshev Polynomials for Solving Bounded and Unbounded Singular Integral Equations, pp. 16.

57. International Conference on Analysis and its Applications (ICAA2017) which was held on 20-22 November 2017. Aligarh Muslim University, Aligarh, India. Title: Projection Method for Bounded and Unbounded Solution of HSIEs of the First Kind, pp. 24.

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