

**TIQXMMI” MILLIY TADQIQOT UNIVERSITETI QOSHIDAGI  
FUNDAMENTAL VA AMALIY TADQIQOTLAR INSTITUTI  
HUZURIDAGI ILMIY DARAJALAR BERUVCHI  
DSc.03/31.03.2022.T/FM.10.04 RAQAMLI ILMIY KENGASH**

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**ASTRONOMIYA INSTITUTI  
FUNDAMENTAL VA AMALIY TADQIQOTLAR INSTITUTI**

**RAYIMBAYEV DJAVLANBEK RADJAPBAYEVICH**

**ASTROFIZIK KUZATUVLAR ASOSIDA QORA O’RALAR  
PARAMETRLARINING GRAVITATSIYA NAZARIYALARIDAGI  
TAHLILI**

**01.04.02 – Nazariy fizika**

**FIZIKA-MATEMATIKA FANLARI BO’YICHA FAN DOKTORI (DSc)  
DISSERTATSIYASI AVTOREFERATI**

**Toshkent – 2022**

**Fizika-matematika fanlari bo'yicha fan doktori (DSc) dissertatsiyasi  
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DARAJALAR BERUVCHI DSc.03/31.03.2022.T/FM.10.04 RAQAMLI  
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Fizika-matematika fanlari bo'yicha fan doktori (DSc) dissertatsiyasi mavzusi O'zbekiston Respublikasi Vazirlar Mahkamasi huzuridagi Oliy attestatsiya komissiyasida **B2022.3.DSc/FM203** raqami bilan ro'yxatga olingan.

O'zbekiston Respublikasi Fanlar akademiyasi Astronomiya instituti va Dissertatsiya Fundamental va amaliy tadqiqotlar institutida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o'zbek, ingliz, rus (rezyume)) Ilmiy kengashning veb-sahifasida ([www.ifar.uz](http://www.ifar.uz)) va «Ziyonet» axborot-ta'lim portalida ([www.ziyonet.uz](http://www.ziyonet.uz)) joylashtirilgan.

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Dissertatsiya bilan "TIQXMMI" Milliy tadqiqot universiteti huzuridagi fundamental va amaliy tadqiqotlar instituti Axborot-resurs markazida tanishish mumkin (\_\_\_\_ raqami bilan ro'yxatga olingan). (Manzil: 100000, Toshkent shahri, Qori Niyoziy ko'chasi, 39-uy, Fundamental va amaliy tadqiqotlar instituti, 108-katta majlislar zali; tel.: 71 237-09-61)

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## **KIRISH (fan doktori (DSc) dissertatsiyasi annotatsiyasi)**

**Mavzuning dolzarbligi.** Qora o‘ralar matematik nuqtai nazardan gravitatsion maydon tenglamalarining echimi bo‘lib, ilk bor Karl Shvartsschild tomonidan 1916 yil Eneynsheynning maydon tengamasi echimi, faqat massaga ega ob‘ekt sifatida olingan. Astrofizika tomondan, ular yulduzlar evolyutsiyasining eng oxirga bosqichida xosil bo‘luvchi relyativistik kompakt gravitatsion astrofizik ob‘ektlardir. Ular massalari ko‘ra o‘ta massiv qora o‘ralar, ular galaktika markazlarida joylashgan; o‘rtacha massali qora o‘ralar va ularning galaktikalarda tarqalganligi noaniq; va nihoyat, yulduz massali qora o‘ralar deb atalib asosan ular qo‘shaloq tizimlarida uchraydi. Yaqinda xodisalar gorizonti teleskopi yordamida ikkita o‘ta massiv, M87\* va O‘qotar A\* qora o‘ralarining soyalari tasvirini olishga va shu orqali ularning massalarini va aylanish parametrlari aniqlashga muvofiq bo‘lindi. Ammo, o‘rta va yulduz massali qora o‘ralar ko‘rish burchagi kichik bo‘lganligi sababli ularning tasvirini olish imkoni yo‘q. Ular asosan akkretsiya diskdagi nurlanishlar, xususan, kvazidavriy teblanish chastotalari orqaligina kuzatilishi va ularning massasi, zaryadi va spin parameterlarini aniqlash mumkin.

Ma‘lumki, qora o‘ra zaryadi ham xuddi aylanuvchi Kerr qora o‘rasining spini singari fotonsfera va akkretsiya diskini ichki orbitasi bo‘lgan ichki orbitasi radiusini kamaytiradi. Bu nuqtai nazardan, ularning qora o‘ra fazo-vaqt geometriyasiga ta‘siri o‘xshashdir. Umumiy nisbiylik nazariyasi ko‘plab astrofizik kuzatuvlarda tekshirilgan va tasdiqdan o‘tgan. Shunga qaramay, gravitatsiyaning boshqa, alternativ va modifikatsiyalangan nazariyalari ham kvazidavriy tebrashishlarning kuzatuv ma‘lumotlarini ishonchli turintira oladi. Bu esa umumiy nisbiylik nazariyasi va boshqa gravitatsiya nazariyalaridagi qora o‘ralar atrofidagi fazo-vaqt geometriyasida o‘xshashlik mavjudligini hamda ushbu o‘xshashliklarni kuzatuv ma‘lumotlari asosida farqlashni tadqiq etish nazariy astrofizikadagi dolzarb masalalardan biri ekanini ko‘rsatadi.

Yurtimizda ham qora o‘ralar va ular atrofidagi optik hamda energetik jarayonlar xususan, akkretsiya diskidagi nurlanish mexanizmlarini o‘rganishga, shu bilan birgalikda, kuzatuv ma‘lumotlari asosida ularning parametrlarini tadqiq etishga oid ilmiy tadqiqotlar jadal olib borilmoqda.

Ushbu dissertatsiya ishining vazifalari tasdiqlangan davlat normativ xujjatlari, O‘zbekiston Respublikasi Prezidentining 2017 yil 7 fevraldagi PQ-4947 “O‘zbekiston Respublikasini yanada rivojlantirish uchun chora-tadbirlar strategiyasi to‘g‘risida” farmoni va 2017 yil 18-fevraldagi PQ-2789 sonli «Fanlar akademiyasi faoliyati, ilmiy-tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari to‘g‘risida» qarori talablariga mos keladi.

**Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor**

**yo‘nalishlariga muvofiqligi.** Dissertatsiya tadqiqoti O‘zbekiston Respublikasi fan va texnologiyasining II ustuvor yo‘nalishi “Energetika, energiya va resurslarni tejash”ga muvofiq bajarildi.

### **Muammoning o‘rganilganlik darajasi.**

Qora o‘ralar akkretsiya diskidagi kvazidavriy chastotalarni paydo bo‘lish mexanizmlari, modellari va ularni turli gravitatsiya nazariyalari doirasida o‘rganish Yevropa (L.Rezolla, M.Abramovich, G. Torok, P. Bakala, Z. Stuklik, M va Kolosh), AQSh (S.Motta, L.Stella, M.Vietri, S.Spilberg, V.Uogoner) va dunyoning rivojlangan davlatlari (S.Kato, J. Fukue (Yaponiya), D.Vang, L.Cheng va M.Jang (Xitoy)) olimlari tomonidan o‘rganilgan.

Turli qora o‘ralar soyalarini nazariy tadqiq etish, ularni turli gravitatsiya nazariyalari doirasida o‘rganish ham bir qator dunyoning rivojlangan olimlari tomonidan o‘rganilgan (J.Schee, A.Ovgun, K. Jusufi, M.Jamil va hk.).

Respublikamizda ham umumnisbiylik nazariyasi, modifikatsiyalangan va muqobil gravitatsiya nazariyalari doirasida qora o‘ralar soyalarini nazariy tadqiq etish, qora o‘ralar atrofida zarralar tebranishlari B. Ahmedov, A.Abdujabbarov, B. Toshmatov, S.Shaymatov, A.Abdikamalovlar tomonidan tadqiq etilgan.

Ammo, qora o‘ralar soyalariga oid nazariy izlanishlar hamda kuzatuv ma‘lumotlari asosida qora o‘ralar va gravitatsiya modellarining parametrlariga cheklovlar olinmagan. Bundan tashqari, kuzatuvdagi kvazidavriy tebranishlarining kuzatuv ma‘lumotlari asosida muqobil gravitatsiya nazariyalarining parametrlari qiymatlariga, qora o‘ralar massalari va zaryadlariga chegaraviy qiymatlar olish usuli ishlab chiqilmagan.

**Dissertatsiya tadqiqotining dissertatsiya bajarilgan ilmiy-tadqiqot muassasasi ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi.** Dissertatsiya ishi 2022 yil davomida O‘zRFA Astronomiya insititi va Fundamental va amaliy tadqiqotlar institutlarining asosiy ilmiy tadqiqot yo‘nalishlari hamda O‘zRFA Yadro fizikasi insititudagi F-FA-2021-510 raqanli “Modifikatsiyalangan gravitatsiyalar doirasida neytron yulduzlari yadro muhitini tadqiq etish” ilmiy loyihasi doirasida bajarildi.

**Tadqiqotning maqsadi** qora o‘ralar atrofidagi kvazidavriy tebranish chastotalari va qora o‘ralarning soyalar o‘lchamlaridan olingan kuzatuv ma‘lumotlari asosida qora o‘ralar va gravitatsiya nazariyalari parametrlarini tahlil qilish.

### **Tadqiqotning vazifalari:**

- Aylanmaydigan qora o‘ralar atrofidagi kvazidavriy tebranishlar modellarini tekshirish;

- kvazidavriy tebranishlarni tadqiq etish orqali modifikatsiyalangan va kvantlangan gravitatsiya modellarini tekshirish;
- kvazidavriy tebranishlar kuzatuv ma'lumotlari asosida qora o'ra massasi, zaryadi va gravitatsiya nazariyalari parametrlariga cheklovlar olish;
- aylanadigan o'ta massiv qora o'ralar, jumladan M87 va o'qotar A\* soyalari o'lchamlari asosida ularning zaryadi va spin parametrlari cheklovlar olish.

**Tadqiqotning ob'ekti** sifatida kompakt relyativistik gravitatsion ob'ektlar – qora o'ralar tanlangan.

**Tadqiqotning predmeti** sifatida kichik massali qo'shaloq tizimlardagi qora tuynuklar akkretsiyon diskidagi kvazidavriy tebranishlari va o'ta yuroqi massali qora tuynuklardan tasvirlari tanlangan.

**Tadqiqotning usullari** sifatida umumiy nisbiylik nazariyasidagi matematik apparatlar, jarayonlarni matematik modellarshirish va ularni sonli echish usullaridan foydalanildi.

**Tadqiqotning ilmiy yangiligi** quyidagilardan iborat:

- ilk bor kvazidavriy tebranish chastotalari yordamida zaryadli aylanmaydigan qora o'ralarining massasi va zaryadlariga chekli qiymatlar olingan;
- ilk bor kuzatuv ma'lumotlari asosida topilgan M87\* va Sgr A\* soylarining o'lchamlaridan foydalangan holga, ularning massalari va zaryadlari orasidagi bog'lashinishni aniqladik;
- ilk bor kvant gravitatsiyasining non-kommutativ qora o'ralar atrofida kvazidavriy tebranishlar chastotalari va ularning soyalari o'lchamlari ta'sirini o'lchashdagi xatoliklardan ancha kichik ekanligi ko'rsatildi;
- ilk bor ikki cho'qqili kvazidavriy tebranishlar kuzatuvlar ma'lumotlari asosida modifikatsiyalangan gravitatsiyada regular va singular qora o'ralarini farq qilish usuli ko'rsatildi;
- ilk bor qora o'ralar atrofidagi kvazidavriy tebranishlar asosida ular atrofidagi fazo-vaqtning muhim hususiyatlaridan biri bo'lgan akkretsiya diskining kuyi chegarasi radiuslari baholash mumkinligi ko'rsatildi;
- ilk bor kuzatuvlarda Kerr qo'ra o'rasi spini va zaryadlari hosil qiluvchi o'xshash o'xshash gravitatsiya effektlarini farqlovchi chegaraviy qiymatlar ko'rsatildi.

**Tadqiqotning amaliy natijalari** quyidagilardan iborat:

- ilk bor kvazidavriy tebranishlar kuzatuv ma'lumotlaridan foydalangan holda

qora o‘ralarning zaryadi va massadari topilgan;

- ilk bor M 87\* va o‘qotar A\* o‘ta og‘ir qora o‘ralarining aylanish parametri va zaryadlari ularning soyalarining o‘lchamlari yordamida topilgan;
- ilk bor modifikatsiyalangan gravitatsiyasida regulyar va singulyar qora o‘ralarni kvazidavriy tebranishlar yordamida farqlash mumkinligi ko‘rsatilgan;
- ilk bor non-kommutativ kvant gravitatsiyasi effektlari qora o‘ralar atrofida kvazidavriy tebranishlar va ularning soyalarini o‘lchashdagi xatoliklardan kichik ekanligi ko‘rsatildi.

**Tadqiqot natijalarining ishonchliligi** quyidagilar bilan ta‘minlanadi:

- nisbiylik nazariyasining zamonaviy usullari va nazariy fizika hamda yuqori samarali sonli usul va algoritmlari qo‘llanildi;
- olingan nazariy natijalar kuzatuv ma‘lumotlari va boshqa mualliflarning natijalari bilan tekshirildi; xulosalar kompakt gravitatsion ob‘ektlar maydon nazariyasining asosiy qonuniyatlariga juda katta aniqlik bilan mos keladi.

**Tadqiqot natijalarining ilmiy va amaliy ahamiyati** shundan iboratki:

- olingan tadqiqot natijalari yulduz massali qora o‘ralar parametrlarini va ular atrofida fazo-vaqt hususiyatlarini shu bilan birgalikda kvazidavriy tebranishlarning akkretsiya diskida xosil bo‘lish fizik mexanizmlarini;
- qora o‘ralar parameterlariga olingan chekli qiymatlar ular atrofida fazo-vaqt gravitatsiyasini o‘rganish imkonini beradi hamda gravitatsiya effektlari muhim ahamiyatga ega ekanini izohlash imkonini berdi;
- kuzatuv ma‘lumotlari asosida modifikatsiyalan va kvantlangan gravitatsiya nazariyalarini tekshirishda qo‘llash mumkinligi bilan belgilanadi.

**Tadqiqot natijalarining joriy qilinishi:**

- Qora o‘ralar massalari va zaryadlarini kvazidavriy tebranishlar kuzatuv ma‘lumotlari asosida aniqlash bir qator mualliflar tomonidan qorong‘i materiya hususiyatlarini tekshirishda joriy etilgan (The Astrophysical Journal Vol.935,91, (2022), Progress of Physics, Volume 70, issue 9-10, 2200053, (2022), Journal of Cosmology and Astroparticle Physics, Volume 09, id 061, (2022), European Physics Journal C, Volume 82, id 636 (2022), Progress of Physics, Volume 70, Issue 9-10, id 2200053, (2022), Universe Volume 8, issue 3, id 182 (2022)). Olingan natijalar qora o‘ralar atrofida qorong‘i materiyaning parametrini kvazidavriy tebranishlar asosida tahlil qilish imkonini bergan.
- Qora o‘ralar soyalarini kuzatuv ma‘lumotlari asosida tahlil qilish va ularning parametrlarini aniqlashga oid natijalar bir qator mualliflar tomonidan turli

gravitatsiya modellari ularning soylari o'Ichamlaridan olingan kuzatuv ma'lumotlari asosida tahlil qilishda joriy etilgan (Chinese Journal of Physics Vol. 78, pp.141-154, (2022), Annals of Physics Vol.441, 168892,(2022), Universe, Volume 8, issue 10, id 536 (2022), Physics, Volume 4, issue 4, pp.1318-1330 (2022), European Physics Journal C, Volume 82, id 831 (2022), Annals of Physics, Volume 441, id 168892 (2022)). Olingan natijalar aylanuvchi zaryadlangan qora o'ralar atrofidagi foton orbitalarini va ularning soylarini tahlil qilishda ishlatilgan.

**Tadqiqot natijalarining approbatsiyasi:** Dissertatsiya natijalari 3 ta xalqaro, 2 ta mahalliy konferentsiyalarda va bir necha bor O'zbek-Qozoq haftalik ilmiy seminarlarda muhokama qilingan.

**Tadqiqot natijalarining e'lon qilinishi:** Dissertatsiya natijalari bo'yicha 20 dan ziyod ilmiy ish, shu jumladan OAK ro'yhatiga kiruvchi halqaro ilmiy jurnallarda 16 ta ilmiy maqola chop etilgan.

**Dissertatsiyaning tuzilishi va hajmi:** Dissertatsiya 117 varor bo'lib asosiy to'rt bob, xulosa va foydalanilgan adabiyotlar ro'yxatidan iborat.

## DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiyaning dolzarbligi va uning ahamiyati, ishning maqsad va vazifalari hamda undagi ilmiy va amaliy yangiliklar ko'rsatib o'tilgan bo'lib shu bilan birga olingan natijalarning ahamiyatlari muhokama etilgan.

I bob “**Kvazidavriy tebranish kuzatuv ma'lumotlari yordamida qora o'ralar zaryadlariga limitlar olish**” deb nomlangan bo'lib, unda zaryadli qora o'ralar atrofida sinov zarralari harakati va ularning turg'un aylana orbitalardagi kichik tebranishlari o'rganish hamda ularni kvazidavriy chastotalariga qo'llash, shu bilan birga kvazidavriy tebranishllar kuzatuv ma'lumotlari asosida qora o'ralar massalari va zaryadlarini aniqlasha bag'ishlangan.

Zaryadlangan statik qora o'ralar atrofidagi fazo-vaqt metrikasi quyidagicha yoziladi  $ds^2=f(r)dt^2 +dr^2 / f(r)+r^2 d\theta^2 +r^2 \text{Sin}^2\theta d\phi^2$  (1.1) bu yerda  $f(r)$  metrik funktsiya va u zaryadli qora o'ralar quyidagicha

|                                     |                                              |
|-------------------------------------|----------------------------------------------|
| Aylanmaydigan zaryadli qora o'ra    | $f(r)$                                       |
| Reissner-Nordstrom (RN) qora o'rasi | $1-2M/r +Q^2/r^2$                            |
| Bardeen qora o'rasi                 | $1-2M/r \Upsilon^{-3}$                       |
| Ayon-Beato-Garcia (ABG) qora o'rasi | $1-2M/r \Upsilon^{-3}+Q^2/r^2 \Upsilon^{-4}$ |
| Regulyar qora o'ra (model)1         | $1-2M/r \text{Exp}(-Q^2/(-2Mr))$             |
| Regulyar qora o'ra (model)2         | $1-2M/r (1+Q/r)^n$                           |

1-jadval: Zaryadlangan qora o'ralar metrik funksilarini

bu yerda  $\Upsilon^2=1+ Q^2/r^2$  , Q va M qora o'ra zaryadi va massasi.

Qora o'ralar ega bo'lishi mumkin bo'lgan eng katta zaryad va xodisalar gorizontining eng kichik qiymatining  $f(r)$  funksiyaning o'zi va uning radial koordinata bo'yicha olingan hosilasi nolga teng bo'ladigan qiymatlar orqali quyidagicha topiladi. Hodisalar gorizoniting eng kichik, ya'ni zaryad extremal bo'lgandagi eng kichik qaymati va zaryadning extramal qiymatlari bo'yicha olingan natijalarni yuqorida keltirilgan turli zaryadli qora o'ralar uchun quyida jadval ko'rinishida taqdim qilindi.

| Aylanmaydigan zaryadli qora o'ra    | $Q_{\text{extr}}/M$ | $(r_h)_{\text{min}}/M$ |
|-------------------------------------|---------------------|------------------------|
| Reissner-Nordstrom (RN) qora o'rasi | 1                   | 1                      |
| Bardeen qora o'rasi                 | 0.7698              | 1.08866                |
| Ayon-Beato-Garcia (ABG) qora o'rasi | 0.634181            | 1.00504                |
| Regulyar qora o'ra (model)1         | 1.21306             | 0.735759               |
| Regulyar qora o'ra (model)2 (n=3)   | 0.296296            | 0.592593               |
| Regulyar qora o'ra (model)3 (n=4)   | 0.210937            | 0.632812               |

2-jadval: Zaryadlangan qora o'ralarning maksimal zaryadi va mos hodisalar gorizonti radiusining qora o'ra massasiga normallashtirilgan qiymatlari Biz, sinov zarrasi uchun Lagrange zichligi

$$2L_p = mg_{\alpha\beta} x^\alpha x^\beta.$$

ko'rinishda ifodaladik. Uning harakatidagi energiyasi va burchak momentini Euler-Lagrange tenglamasi orqali quyidagicha aniqlaymiz

$$g_{tt} \dot{t} = -\varepsilon, g_{\phi\phi} \dot{\phi} = L.$$

Bunda zarralarning radial harakati uchun effektiv potentsial quyidagicha topildi:

$$V_{\text{eff}} = f(r) \left(1 + \frac{L^2}{r^2}\right)$$

Malumki, aylana orbitalar uchun  $V_{\text{eff}} = \varepsilon$  va  $V_{\text{eff}}' = 0$  shartlari bajarilishi kerak, va bunda, aylana orbitalar uchun zarralar energiya va burchak momenti quyidagi ifoda bilan topiladi:

$$\varepsilon^2 = \frac{2f(r)^2}{2f(r) - rf'(r)} \quad \text{va} \quad L^2 = \frac{r^3 f'(r)}{2f(r) - rf'(r)}$$

Aylana orbitalarning turg'unligi uchun  $V_{\text{eff}}'' \geq 0$  shart bajarilishi kerak va ichki turg'un aylana orbita (ITAO)larining radiusi quidagi tenglamaning echimi sifatida aniqlanadi:

$$f'(r) \left(2r \frac{f'(r)}{f(r)} - 3\right) - rf''(r) = 0.$$

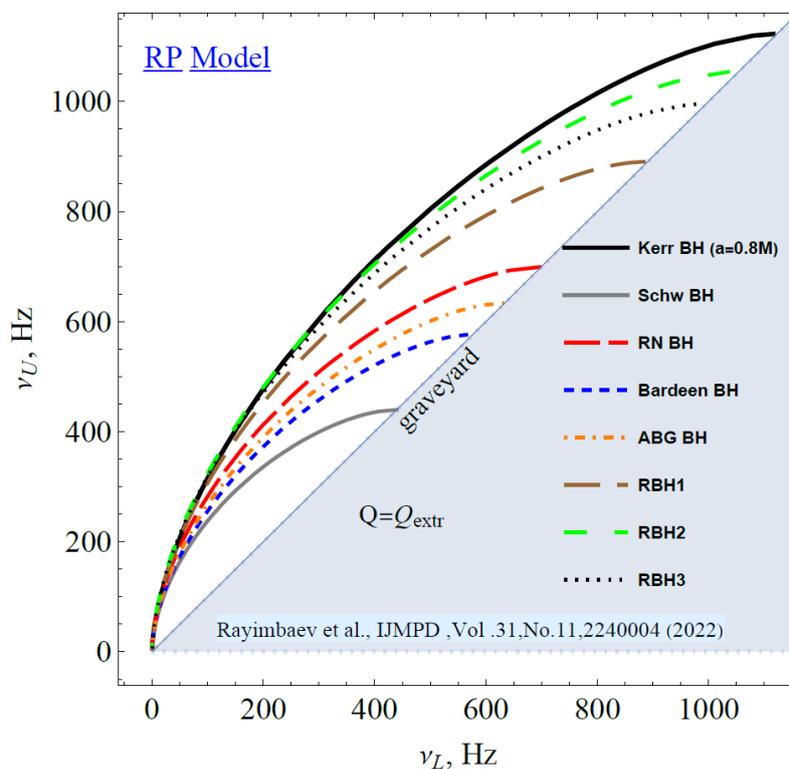
Zarralarning aylana orbitalaridagi burchak tezligi ya'ni Kepler chastotalari  $\Omega_K = d\phi/dt$  (1) ifodada berilgan metrik tenzor (1) uchun quyidagi ko'rinishga ega bo'ladi:

$$\Omega_K = \sqrt{\frac{f'(r)}{2r}}.$$

Ma'lumki, zarralarning turg'un aylana orbitalari atrofidagi kichik og'ishi uning ushbu orbitalar atrofida vertikal va radial yo'nalishlarda kichik tebranishlar xosil qilishiga olib keladi. Ushbu yo'nalishlardagi tebranishlar chastotalari uchun ifodalar sferik simmetrik bo'lgan fazo-vaqtda quyidagi sodda ko'rishishga keladi:

$$\Omega_r = \Omega_K \sqrt{\left(3 + \frac{rf''(r)}{f'(r)}\right)f(r) - 2rf'(r)}, \quad \Omega_\theta = \Omega_\phi = \Omega_K.$$

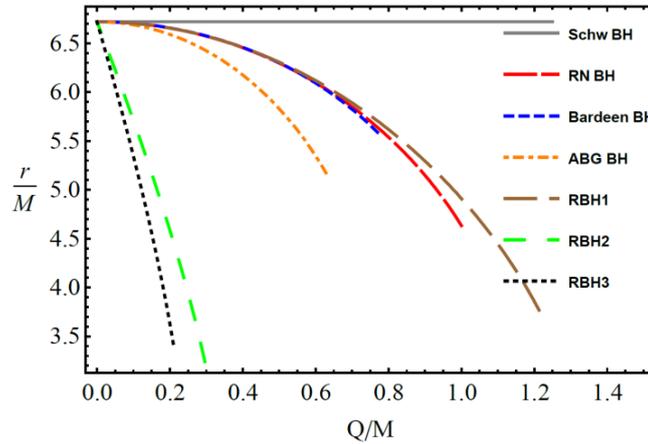
Ushbu fundamental chastotalarni Hz larda ifodalash uchun  $c^3/2\pi GM$  ko'paytiramiz (bu yerda  $c$  yorug'likni vacuumdagi tezligi va  $G$  gravitatsiya doimiysi). Ko'pchilik modellarda kvazidavriy tebranishlarning paydo bo'lish mexanizmi fundamental chastotalar bilan tushintiriladi. Masalan, relativistik presetsiya (RP) modeliga ko'ra ikki cho'qqili QPO ning uqori chastotasi Kepler chastotasi va quyisi esa vertical va radial chastotalarning farqi sifatida tushintiriladi. QPOLarning yuqori va quyi chastotalaring mumkin bo'lgan barcha qiymatlari to'plami matematik jihatdan radial koordinataning ISCO dan to cheksizlikkacha oraliqda topiladi. Quyida rasmda zaryadlangan qora o'ralar atrofida QPOLarning quyi va yuqori chastotalari uchun mumkin bo'lgan qiymatlari keltirilgan. Grafikdan shuni ko'rish mumkinki, masalan Bardeen qora o'rasi atrofida xosil bo'lishi mumkin bo'lgan chastotalar qiymatlar ko'k va kulrang chiziqlar orasida joylashadi. Undan tashqarida aniqlangan QPO manbai Bardeen qora o'rasi bo'la olmaydi. Xuddi shunday tahlilni boshqa zaryadli qora o'ralar uchun ham ko'rish mumkin.



1-rasm: Ikki cho'qqili QPOLarning yuqori va quyi chastotalari diagrammasi. Bu grafikda yuqorida keltirilgan maksimal zaryadli qora o'ralar uchun aniqlangan. Bunda qora o'ra massasi 10 Quyosh massasiga teng deb tanlab olingan.

Shu yo'l bilan biz zaryadli qora o'ralarni qaysi modeli kuzatuvdagi QPO manbaiga nomzod qora o'ra bo'lishi mumkinligi aniqlay olamiz. Muhim jihati shundaki, uning zaryadi, yuqorida aniqlangan kritik qiymatdan oshib ketmasligi lozim. Bundan tashqari, biz zaryad va spin parametrlarining gravitatsiyaviy

effektlerini taqqoslash maqsadida qora egri chiqiz orqali shunday najilarni spini 0.8 bo'lgan Kerr qora o'rasi uchun ham keltirdik. Bu shuni ta'kidlaydiki, qora o'ralar zaryadi uning modeliga bog'liq holda Kerr qora o'rasining spinini qandaydir qiymatigacha effektini bera oladi. Keyingi tahlillar uchun, misol tariqasida chastotalari  $168 \pm 5$  Hz va  $113 \pm 3$  Hz bo'lgan GRS 1915-105 mikrokvazardagi QPO obyektini tanlaymiz, va  $v_U(M; r, Q) = v_U^{ob}$  &  $v_L(M; r, Q) = v_L^{ob}$  munosabatlari yordamida ushbu QPO signallari mikrokvazar markazidan qancha masofada joylashganligini aniqlash mumkin.



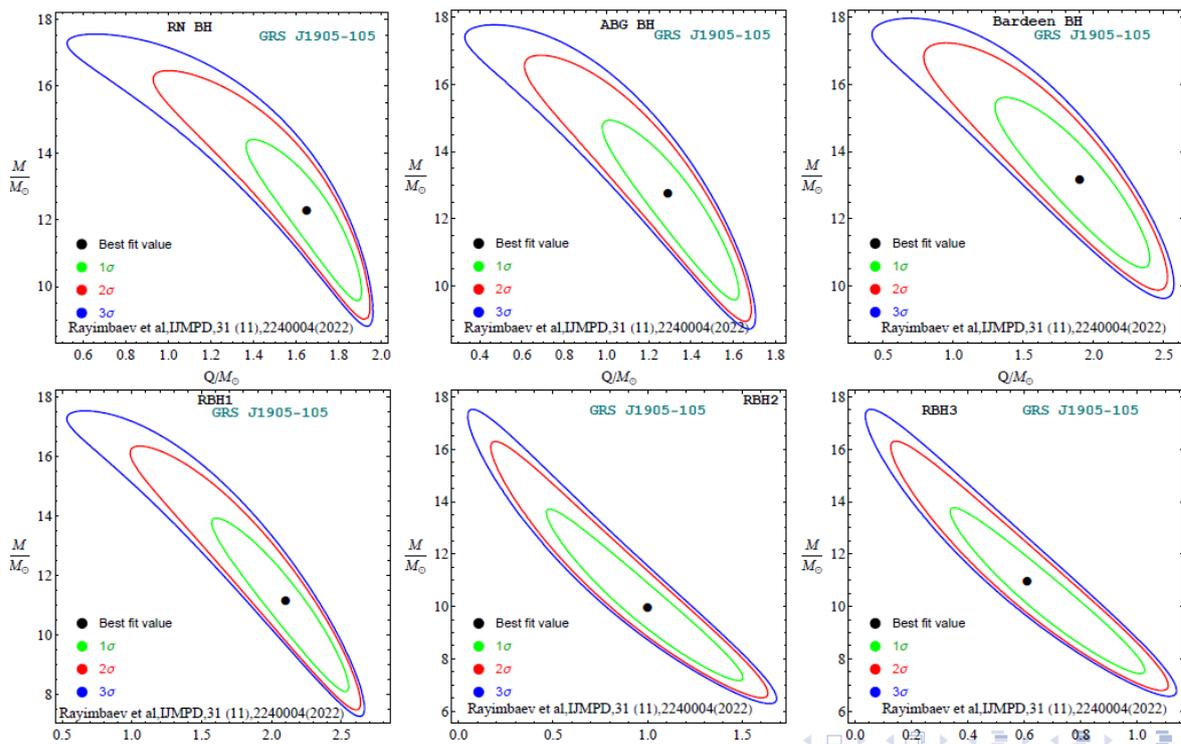
2-rasm: Zaryadlangan qora o'ralar atrofida hosil bo'luvchi yuqori va quyi chastotalari 3:2 nisbatda bo'lgan QPO larning orbita radiuslarining ularni zaryadlariga bog'lanish grafigi

Yuqoridagi rasmda QPO orbitasi radiusini markazdagi qora o'ra zaryadiga bog'lanishi ko'rsatilgan. Avvalo shuni takidlash kerakki, ushbu orbita radius  $Q=0$  da yani Shwarzschild qora o'rasi holida uning ISCO sidan uzoqroqda joylashadi. Ammo ular orasidagi masofa kuzatuvlarda aniqlanadigan hatoliklarga yaqinroq qiymat berishi mumkin. Qora o'ra zaryadi ortishi bilan QPO va ISC radiuslari kamayadi, jumladan ular masofa ham kamayib bu masofa aniqlangan xatolik tartibida bo'lib qoladi, va shu yo'l bilan qora o'ralar atrofidagi ISCO ya'ni akkretsiya diskining ichki chegarasini o'lchash mumkinligini ko'rsata olamiz. Endi, biz GRS 1915-105 mikrokvazardagi QPO kuzatuv ma'lumotlaridan foydalanib markazdagi qora o'ra massasi va ozaryadining qiymatini aniqlaymiz.

Bunda, xatoliklarni inobatga olishning  $\chi^2$  usulidan va unda Bayesian yaqinlashidan foydalanib xatolikni  $1 \sigma$ ,  $2 \sigma$  va  $3 \sigma$  aniqligidagi, ya'ni mos holda  $\chi^2 = 0.68$ ,  $0.86$  va  $0.95$  qiymatlardan foydalanamiz. Bunda quyidagi ifodadan foydalanamiz:

$$\chi^2(r, M, Q) = \frac{(v_U(r, M, Q) - v_U^{ob})^2}{\sigma_U^2} + \frac{(v_L(r, M, Q) - v_L^{ob})^2}{\sigma_L^2}$$

va natijalarni quyidagicha tasvirladik



3-rasm: GRS 1915-105 mikrokvazaridagi qora o'raning massasi va zaryadiga turli zaryadlandi qora o'ra modellarida  $\chi^2$  usuli bilan  $1\sigma$ ,  $2\sigma$  va  $3\sigma$  aniqligidagi olingan qiymatlar.

Grafiklardagi eng mos qiymatlar (grafijda “Best fit value” deb ko'rsatilgan qora nurta) ushbu yaninlashish qoidasiga binoan barcha parametrlarni minimumga olib ketgan holda quyidagicha topiladi:

| BHs        | $M/M_{\odot}$           | $Q/M_{\odot}$          | $Q/M$ |
|------------|-------------------------|------------------------|-------|
| RN BH      | $12.23^{+0.02}_{-0.05}$ | $1.64^{+0.07}_{-0.04}$ | 0.13  |
| Bardeen BH | $13.25^{+0.06}_{-0.04}$ | $1.93^{+0.05}_{-0.03}$ | 0.15  |
| ABG BH     | $12.56^{+0.06}_{-0.05}$ | $1.31^{+0.02}_{-0.07}$ | 0.104 |
| RBH1       | $11.19^{+0.07}_{-0.06}$ | $2.25^{+0.06}_{-0.05}$ | 0.2   |
| RBH2       | $10.06^{+0.05}_{-0.07}$ | $1.03 \pm 0.05$        | 0.102 |
| RBH3       | $10.92^{+0.08}_{-0.07}$ | $0.65^{+0.06}_{-0.05}$ | 0.06  |

3-jadval: GRS 1915-105 mikrokvazaridagi qora o'raning massasi va zaryadiga turli zaryadlandi qora o'ra modellarida  $\chi^2$  usuli bilan olingan eng mos qiymatlar.

Olingan natijalardan ko'rish mumkinki, GRS 1915-105 mikrokvazardagi joylashgan qora o'raning zaryadini o'zining massasiga nisbati kichik bo'lib uning qiymati 1-2 Quyosh massasi atrofida va massasi esa taxminan 10-13 Quyosh massasi oralig'ida.

Xuddi shunday hisoblashlarni extremal zaryadlangan qora o'ralar uchun

amalga oshiramiz, va ushbu zaryad miqdorining qora o'ra fazo-vaqtiga ko'rsatadigan gravitatsiya ta'siri bilan bir hil effekt ko'rsata oladigan Kerr qora o'rasining spin parametr qiymatlari ustida olib bordik va quyidagi jadvaldagi qiymatlarni oldik,

| Maksimal zaryadli qora o'ra         | $Q_{\text{extr}}/M$           | $(r_h)_{\text{min}}/M$ |
|-------------------------------------|-------------------------------|------------------------|
| Reissner-Nordstrom (RN) qora o'rasi | $17.0079^{+0.5217}_{-0.4916}$ | ~0.4884                |
| Bardeen qora o'rasi                 | $13.9495^{+0.4698}_{-0.4131}$ | ~0.2927                |
| Ayon-Beato-Garcia (ABG) qora o'rasi | $15.3169^{+0.4698}_{-0.4476}$ | ~0.3919                |
| Regulyar qora o'ra (model)1         | $21.1479^{+0.6847}_{-0.6117}$ | ~0.6671                |
| Regulyar qora o'ra (model)2 (n=3)   | $19.9994^{+0.6135}_{-0.5780}$ | ~0.7687                |
| Regulyar qora o'ra (model)3 (n=4)   | $23.6358^{+0.7250}_{-0.4831}$ | ~0.7392                |

4-jadval: Maksimal zaryadlangan GRS 1915-105 mikrokvazaridagi qora o'raning massasi va maksimal zaryad hosil qila oladigan gravitatsion effekti bera oladigan spin parameter

II bob “**Kuzatuv ma'lumotlari asosida zaryadlangan M87 va O'qotar A\* o'ta og'ir qora o'ralarining spini va zaryadlariga cheklovlar olish**” deb nomlanadi va ushbu bobda qora o'ralarning soyalari o'lchamlaridan olingan kuzatuv ma'lumotlari asosida ularning spin va zaryad parametrlari orasida bog'lanishlar Gauss Bonnet ( $\gamma$ ) va AdS parametrlari ( $l$ )ning turli hil qiymatlarida aniqlangan.

Glavan va Lin ilk bor massasiz graviton tarqala oladigan 4 o'lchovli fazo-vaqt uchun umumiy o'zgartirilgan gravitatsiya taklif qildilar. Ular GB bog'lanish parametri hadi koeffisientini  $d \rightarrow d/(d - 4)$  ko'rinishida o'zgartirib, so'ngra  $d \rightarrow 4$  ko'rinishda olib, statik va sferik simmetrik qora o'ra yechimini taklif qildilar. Natijada qora o'raning notrivial statik va sferik simmetrik yechimlari kashf etilishiga olib keladi. Bunday qora o'ralar GB gravitatsiya tabiatini teksirish uchun qulay platformani ta'minlaydi. Sferik koordinatalarda, qora o'raning metrikasi quyidagicha yoziladi:

$$ds^2 = -f(r)dt^2 + 1/f(r)dr^2 + r^2d\theta^2 + r^2\text{Sin}^2\theta d\phi^2$$

bu yerda

$$f(r) = 1 + \frac{r^2}{2\gamma} \left[ 1 - \sqrt{1 + 4\gamma \left( \frac{2M}{r^3} - \frac{Q^2}{r^4} - \frac{1}{l^2} \right)} \right]$$

Biz ushbu metrikadan Newman Janis algoritmi yordamida yangi AdS fazosidagi EGB nazariyasida aylanuvchi hamda zaryadlangan qora o'ra yechimini quyidagi ko'rinishda olamiz:

$$ds^2 = -\frac{\Delta}{\rho^2} (dt - a\text{Sin}^2\theta d\phi) + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \frac{\text{Sin}^2\theta}{\rho^2} ((r^2 + a^2)d\phi - a dt)^2$$

bu yerda  $\Delta = a^2 + r^2(1 + f(r))$  va  $\rho^2 = r^2 + a^2 \cos^2 \theta$

Ushbu aylanuvchi fazo-vaqtda fotonlarning harakatini biz quyidagi Hamilton-Jacobi tenglamasi yordamida o'rganamiz:

$$\frac{\partial H}{\partial \tau} + \frac{1}{2} g^{\mu\nu} \frac{\partial H}{\partial x^\mu} \frac{\partial H}{\partial x^\nu} = 0,$$

bu yerda biz H - Hamilton funksiyasini saqlanuvchi kattaliklarga nisbatan quyidagicha yozamiz:

$$H = H_r(r) - Et + L\phi + H_\theta(\theta) + \frac{1}{2} m^2 \tau.$$

Foton uchun saqlanuvchi harakat integralli: energiyasi  $E/m = g_{t\mu} \dot{x}^\mu$

va impulslari  $L/m = g_{\phi\mu} \dot{x}^\mu$  ga teng. Yuqoridagi H-J tenglamasidan foton uchun

harakat tenglamalarini va ekvatorial tekislik uchun effective potensial quyidagi ko'rinishda aniqlaymiz:

$$V_{eff}(r) = \Delta[(\xi - a)^2 + \eta] - [-a\xi + (a^2 + r^2)]^2$$

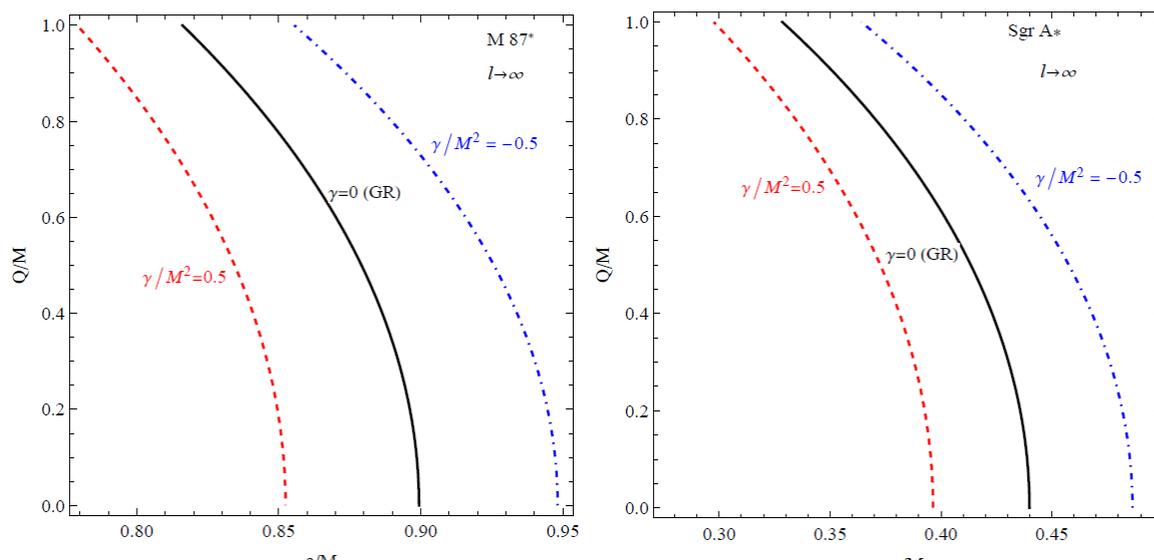
bu yerda  $\eta = \kappa/\varepsilon^2$  and  $\xi = L/\varepsilon$ .

Turgun aylana orbitalari uchun  $V_{eff}(r_{ph}) = V'_{eff}(r_{ph}) = 0$  shartini qanoatlantiradi va u orqali biz qora o'raning soyasini aniqlov tekislik tenglamalarini quyidagicha topiladi:

$$\alpha = \frac{4r\Delta - \Delta'(r^2 + a^2)}{a\Delta'} \quad \text{va} \quad \beta^2 = \frac{r^2[16\Delta(a^2 - \Delta) - r^2\Delta'^2 + 8r\Delta\Delta']}{a^2\Delta'^2}$$

Xosil bo'lgan soyaning radiusi  $R_{sh}^2 = \alpha^2 + \beta^2$  ifoda yordamida aniqlanadi.

Kuzatuvlarda o'ta og'ir qora o'ralar M87\* (EHT Collab. 2019) va Sgr A\* (GRAVITY Collab. 2022) larning tasvirlari olingan va ulardning burchak o'lchamlari mos holda quyidagicha aniqlangan:  $42 \pm 3$  mk arc sec va  $51.8 \pm 2.3$  mk arc sec. Biz ushbu o'ta og'ir qora o'ralarning tasvirlarining burchak o'lchamlari, ulargacha masofalar va ularng ko'rinish burchaklaridan foydalangan holda ushbu qora o'ralar massalariga normallashtirilgan zaryadi va aylanish momentlarini aniqlaymiz. Bunda biz kuzatuvdagi xatoliklarni inobatga olmagan holda kuzatuv ma'lumotlarining asosiy qiymatlaridan foydalandik.



4-rasm: O'ta o'gir qora o'ralar M87\* va Sgr A\* zaryadi va aylanish parametrlari orasidagi bog'lanish.

Ushbu rasmda biz GB parametrining musbat (qizil), manfiy (ko'k) va nol (GR) holatlari uchun M87\* (chapdagi panel) va Sgr A\* (o'ngdagi panel) qora o'ralarining zaryadi va spin parametrlari nisbati ko'rsatilgan. Rasmdan shuniki ko'rish mumkinki, qora o'ralar zaryadi ortishi bilan ularning spin parametrlari kamayadi. Bundan tashqari, GB parametrining musbat qiymatida uning gravitatsiya effekti spin effektini pasaytirishi, manfiy GB parametr esa oshirishini ko'rsatadi. Olingan qiymatlar, ularning spin parametrlarini boshqa astronomik kuzatuvlarda olingan qiymatlariga mos kelishi ko'rsatildi, ya'ni M87\* qora o'rasining aylanishi  $a \simeq 0.9M$  va Sgr A\*niki esa taxminan  $a \simeq 0.44M$

III bobda kvantlangan gravitatsiya nazariyasining kvazidavriy tebranishlar chastotalariga va qora o'ralari soyalari o'lchamlariga ta'siri tekshirilgan va "**Kvant qora o'ralarida kvazidavriy tebranishlar va qora o'ralar soyalari**" deb nomlanadi.

Biz ushbu bobda Non kommutatik (NC) gravitatsiyasida olingan qora o'ra nuqtaviy bo'lmagan statik sferik simmetrik yechim ilk bor quyidagi massa funksiyasi, ya'ni delta Dirac funksiyasi bilan Gauss funksiyasiga o'xshash yangi taqsimot funksiyasiga almashtirish orqali olingan:

$$\rho(r) = \frac{M}{(4\pi\theta)^{3/2}} e^{-r^2/4\theta}$$

ifodadagi  $\theta$  - NC parametri. Aylanmaydigan NC qora o'rasi atrofidagi fazo-vaqt geometriyasining metrik funksiyasi quyidagi ko'rinishda olingan:

$$f(r) = 1 - \frac{4M}{r\sqrt{\pi}} \gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right)$$

bunda, pastki to'liq bo'lmagan gamma funksiyasi

$$\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) = \int_0^{r^2/4\theta} \sqrt{h} e^{-h} dh.$$

Bu NC qora o'rasi regular qora o'ra bo'lib uning metrik funksiyasi nol qiymati

qora o'raning gorizontini belgilab beradi va uning birinchi hosilasi NC parametrining kritik va gorizont radiusining eng kichik qiymatini topishga yordan beradi:  $\Theta_{cr} = 0.2765M^2$  va  $r_{min} = 1.5873M$ .

Biz zarralarning NC qora o'rasi atrofidagi harakatini va ularning turg'ur orbitalari atrofida radial, orbital va vertial tebranishlarini 1-bobda ko'rsatilgan standard usulda o'rganamiz. Zarralarning Kepler orbitalaridagi burchak tezliklari

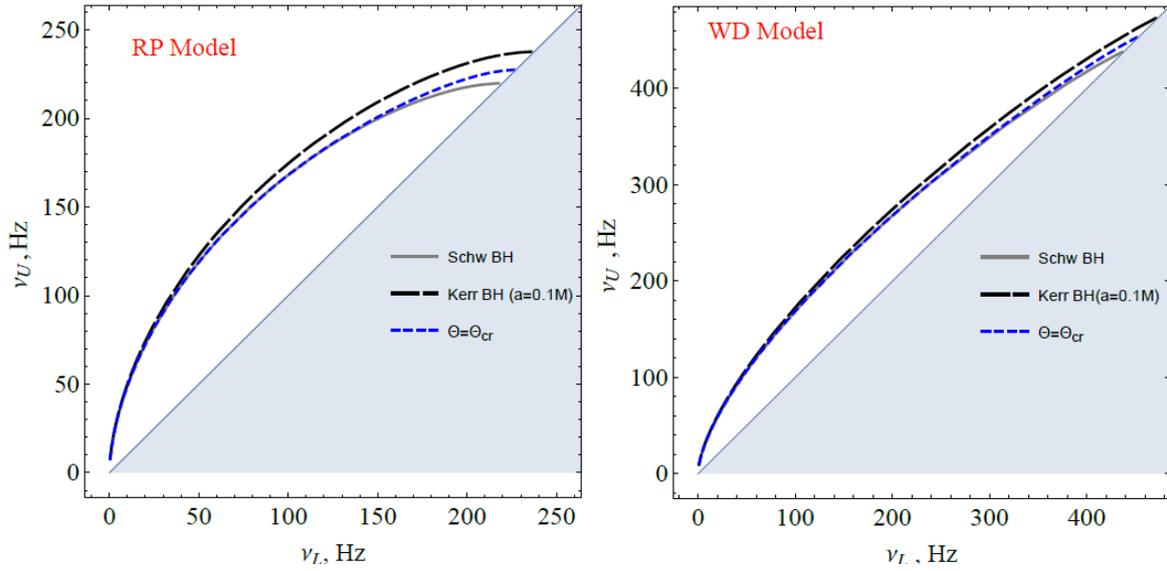
$$\Omega_K = \frac{\sqrt{M}}{\sqrt{2}\sqrt{\pi}r^{3/2}} \sqrt{4\gamma\left(\frac{3}{2}, \frac{r^2}{4\Theta}\right) - \frac{r^3}{\Theta^{3/2}} e^{-\frac{r^2}{4\Theta}}}.$$

ifoda orqali aniqlaniladi. Vertikal tebranishlar chastotalari ham Kepler chastotalariga teng bo'ladi, ammo radial chastotasi esa, ya'ni

$$\begin{aligned} \Omega_r = \Omega_K & \left\{ \left[ 4\pi\Theta^{\frac{5}{2}}r^2e^{\frac{r^2}{2\Theta}} \text{Erf}\left(\frac{r}{2\sqrt{\Theta}}\right) - 2\sqrt{\pi}\Theta r^3e^{\frac{r^2}{4\Theta}} \right. \right. \\ & \times (2\Theta + r^2) \left. \right]^{-1} \left[ r^5 \left( \sqrt{\pi}e^{\frac{r^2}{4\Theta}} (r^2 - 6\Theta) - \frac{4Mr^2}{\sqrt{\Theta}} \right) \right. \\ & - 4e^{\frac{r^2}{4\Theta}} \gamma\left(\frac{3}{2}, \frac{r^2}{4\Theta}\right) \left( 24\Theta^{5/2}Mr e^{\frac{r^2}{4\Theta}} \gamma\left(\frac{3}{2}, \frac{r^2}{4\Theta}\right) \right. \\ & \left. \left. + Mr^4 (r^2 - 14\Theta) - 2\sqrt{\pi}\Theta^{5/2}r^2e^{\frac{r^2}{4\Theta}} \right) \right] \left. \right\}^{\frac{1}{2}}, \end{aligned}$$

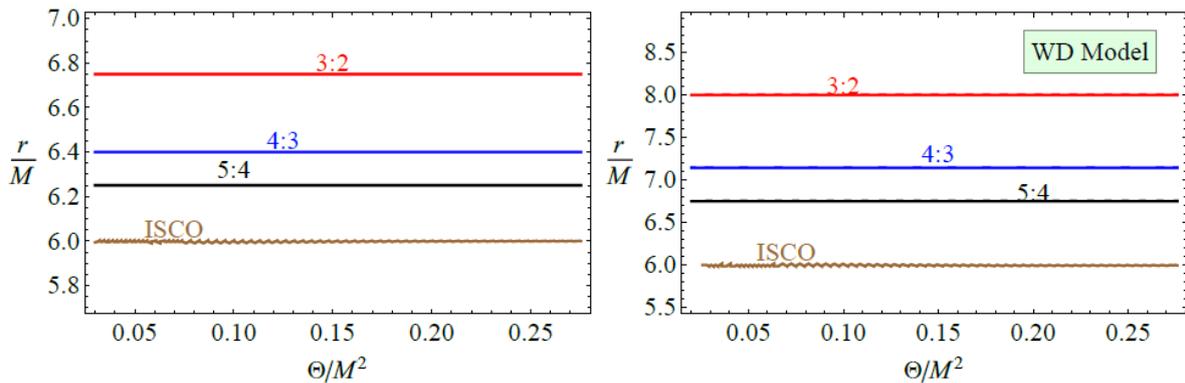
$$\Omega_\theta = \Omega_\phi = \Omega_K.$$

Ma'lumki, QPO signallarining paydo bo'lishi zarralarning radial va vertial tebranishlarining fundamental chastotalari bilan bog'liq bo'lib ularning hosil bo'lishida fazo-vaqt geometriyasi ham ro'li inobatga olish lozim. Ushbu maqsadda, biz NC parametrining QPO signalli chastotalariga ta'sirini kuzatuv ma'lumotlari asosida RP va WD (warped disk-qiymshiq disk) modellari doirasida tekshiramiz. Qora o'ra atrofida hosil bo'lishi mumkin bo'lgan ikki cho'qqili QPOlarning barcha yuqori va quyi chastotalari orasidagi bog'lanishlar NC parametri kritik qiymati, Schwarzschild qora o'rasi va spin parametri  $a=0.1M$  Kerr qora o'ralari uchun quyidagi grafikda ko'rsatilgan.



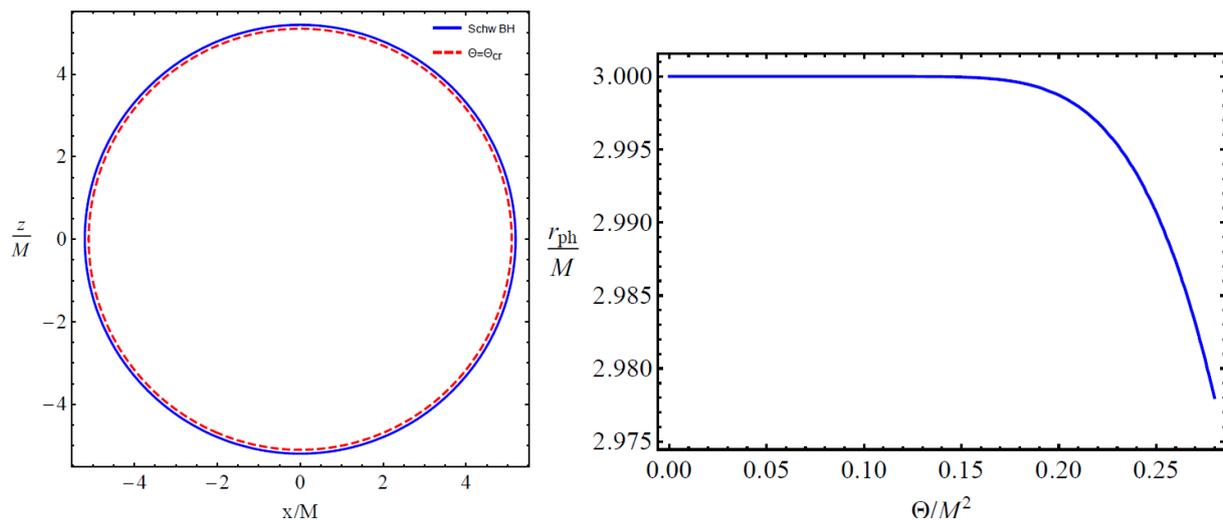
5-rasm. NC qora o'rasi atrofida hosil bo'luvchi ikki cho'qqili QPOLarning yuqori va quyi chastotalar diagrammasi NC parametrining  $\Theta=0.27M$  qiymatida.

Grafikdan ko'rish mumkinki har ikala modelda ham NC parametrining ta'sir faqatgina yuqori chastotalardagina sezilarli, ammo xosil bo'lgan chastotalarning Schwarzschild limitiga nisbatan eng katta farqi ham 5-10 Hzdan kichik bo'lib u esa o'lchashlardagi xatoliklardan kichik va yoki tartibida. Shu nuqtai nazardan, NC parametrining QPO chastotariga ta'siri deyarli yo'q deb hisoblash mumkin. Bundan tashqari, sonli usuldagi hisoblashlar shuni ko'rsatdiki, NC parametrning kritik qiymati Kerr qora o'rasining spinini  $a/M=0.052$  qiymatining ta'siricha QPO chastotalariga gravitatsion effekt ko'rsata oladi. Bunday spinli Kerr qora o'rasi astrofizik jihatda aylanmaydigan deb hisoblaniladi.



6-rasm. NC qora o'rasi atrofidagi QPO orbitalari radiusining NC parametriga bog'lanishi, yuqori va quyi chastotalarining turli qiymatlari uchun.

Bundan tashqari, sonli usullar yordamida shuni ko'rsatildiki, NC parametrining ISCO va QPO orbitalari radiusiga ta'siri ham inobatga olmasa ham bo'ladigan tartibda kichik, boshqacha aytganda deyarli yo'q.



7-rasm: Shwarzschild va NC kvant qora o'ralarining soyasi (chapda) va fonsfera radiusning NC parametriga bog'lanishi (o'ngda).

Kvanlangan gravitatsiyaning NC modelidagi fazo-vaqtning qora o'ra atrofidagi foton harakati va qora o'raning soyasiga ta'siri ham o'rganildi. Bunda NC parameterning nolga yaqin qiymatlarida fonsfera radiusiga ta'siri deyarli ko'rinmaydi, ammo kritik qiymatlarga yaqin qiymatlarda fonsfera biroz kamayadi. Biroq uning eng kichik qiymati ham Schwarzschild qora o'rasi fonsferasidan 0.8 % ga kichik. Kuzatuvlarda esa, M87\* qora orasi soyasini o'lchashdagi xatolik 7% va Sgr A\* niki esa 4% dan kichikni tashkil qiladi, ammo NC parametr qora o'ra soyasi o'lchamlarini 2% gagina kichraytira oladi va bu uning ta'sirini kuzatuv ma'lumotlari orqali ajratib bo'lmasligini ko'rsatadi.

IV bob “**Modifikatsiyalangan gravitatsiya nazariyasidagi regulyar va singulyar qora o'ralarni farqlashda kvazidavriy tebranish chastotalari orqali tahlil qilish**” deb nomlanadi va bunda QPO kuzatuv ma'lumotlari asosida regulyar va singulyar qora o'ralarni farqlash ko'rsatilgan.

Ma'lumki, Umumiy nisbirlik nazariyasi doirasi vakuum yechimlar fizik singulyarlikka ega ya'ni  $r=0$  da fazo-vaqt cheksiz egrilikka ega bo'ladi. Ammo, gravitatsiyani boshqa, jumladan kvant gravitatsiyasi yoki umum nisbiylik nazariyasida ham noxiziqli elektrodikamika mavjudligida regulyar qora o'ra olish mumkin. Modifikatsiyalangan gravitatsiya nazariyasi ham Einsteinning umumiy nisbiylik nazariyasining sklyar-vektor-tenzor maydon, boshqacha aytganda massiv skalyar va vektor maydon ishtirokidagi modifikatsiyasi bo'lib regulyar shu jumladan singulyar qora o'ra yechimlarini ham olish mumkin. Bu yechimlar Schwarzschild MOG va regulyar MOG qora o'ralari yechimlari deb ataladi. Bu yechimlarning metrik funksiyalari esa mos ravishda quyidagicha ifodalanadi:

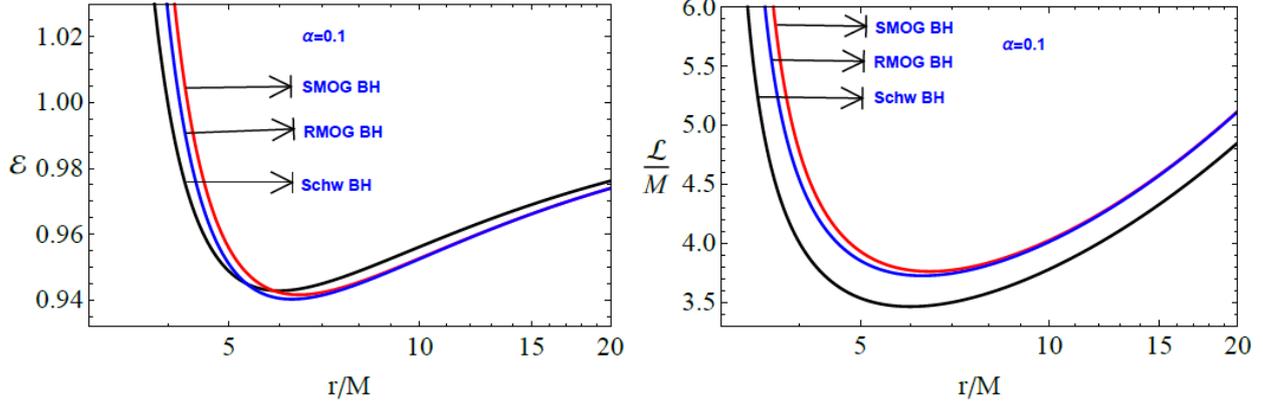
$$f(r) = 1 - \frac{2M}{r} \frac{\alpha + 1}{\left[1 + \alpha(\alpha + 1)\frac{M^2}{r^2}\right]^{3/2}} + \frac{\alpha(\alpha + 1)\frac{M^2}{r^2}}{\left[1 + \alpha(\alpha + 1)\frac{M^2}{r^2}\right]^2}$$

va

$$f(r) = 1 - \frac{2(1 + \alpha)M}{r} + \frac{\alpha(1 + \alpha)M^2}{r^2}$$

Avvalo, biz qora o'ralar atrofida zarralar harakatini ko'rib chiqamiz. Bunda, zarralarning ekvatorial tekislik uchun effektiv potensial yuqorida ko'rsatilgan standart Hamilton Jacobi metodi yordamida topiladi va zarraning aylana orbitalardagi burchak moment va energiyasi quyidagi ko'rinishda ega:

$$\mathcal{L}^2 = \frac{r^3 \partial_r \ln f(r)}{2 - r \partial_r \ln f(r)}, \quad \mathcal{E}^2 = \frac{2f(r)}{2 - r \partial_r \ln f(r)}$$



8-rasm. Regular va Schwarzschild MOG qora o'ralari atrofidagi zarralarning energiyasi va burchak momentlarining radial koordinaga bog'lanishi.

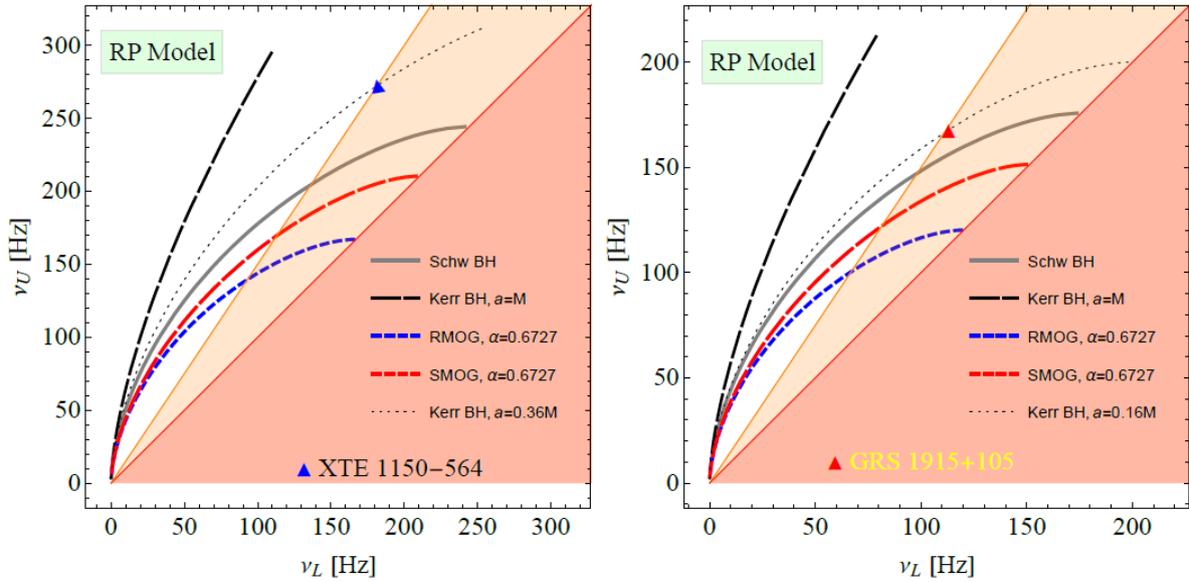
Yuqoridagi rasmdan ko'rinib turibdiki,  $\alpha$  parametr aylana orbitalargan mos keluvchi zarra energiyaning minimum qiymatini kamaytiradi, ammo burchak momentini oshiradi. Ushbu parametrning  $\alpha=0.1$  qiymatida modifikatsiyalangan gravitatsiyaning Schwarzschild va regular qora o'ra yechimlarida fazo-vaqt geometriyasining zarra energiyasi va burchak momentiga ta'siri deyarli bir hile ekenini ko'rish mumkin.

Oldingi boblarda ko'rsatilganidek, zarralarning turg'un aylana orbitalardagi Kepler chastotalari ifodasi quyidagicha topildi:

$$\Omega_K^S = \frac{\sqrt{(r - \alpha M)(\alpha + 1)M}}{r^2}$$

$$\Omega_K^R = \frac{\sqrt{\alpha + 1}\sqrt{M}}{(\alpha(\alpha + 1)M^2 + r^2)^{3/2}} \left\{ \alpha^2(\alpha + 1)M^3 - \alpha Mr^2 - \sqrt{1 + \frac{\alpha(\alpha + 1)M^2}{r^2}} r [2\alpha(\alpha + 1)M^2 - r^2] \right\}^{\frac{1}{2}},$$

Zarralarning radial tebranishlarini I bobda ko'rsatilgan ifoda yordamida hisoblaymiz va qora o'ralar atrofidagi ikki cho'qqili QPO larning yuqori va quyi chastotalariga MOG parametri, ya'ni skalyar-vektor-tensor maydonning ta'sirini tekshirishda ham RP modeldan foydalanamiz. Shu bilan birga ushbu natijalarni Kerr qora o'rasi uchun ham nazariy jihatdan tekshiramiz va natijalarni taqqoslaymiz.



9-rasm. Regular va Schwarzschild MOG qora o'raslari atrofida hosil bo'luvchi ikki cho'qqili QPOLarning yuqori va quyi chastotalar diagrammasi MOG parametrining  $\alpha=0.6727$  qiymatida. Bunda ikkita grafik ikki hil massali GRS 1915+105 va XTE 1150-564 mikrovazarlar markazidagi qora o'ralar uchun keltirilgan

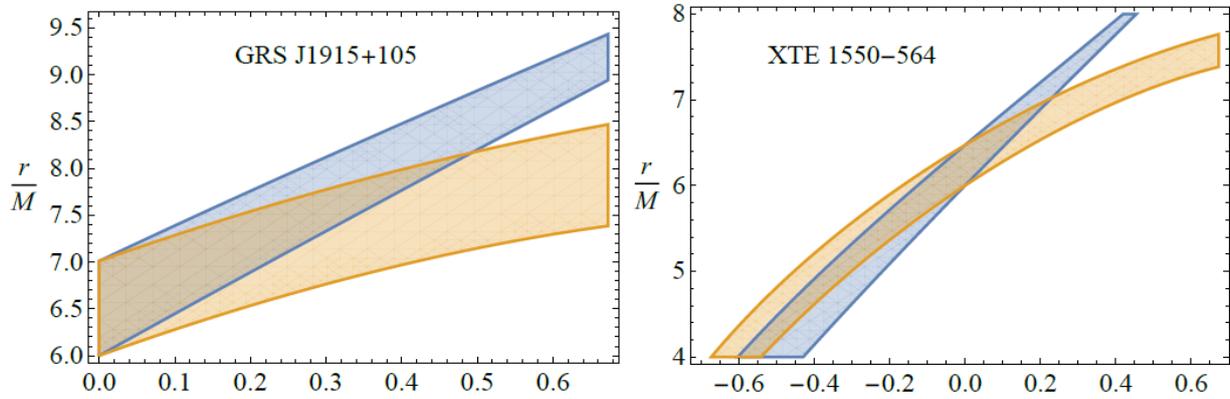
Yuqoridagi grafikdan shuni ko'rish mumkinki,  $\alpha$  parametrning musbat qiymatida yuqori va quyi chastotalarning egri chizig'i Schwarzschild qora o'rasinikiga nisbatan pastga siljiydi. Ammo, Kerr qora o'راسi spinining musbat qiymatlari ushbu egri chiziqni yuqoriga siljitadi. Bu shuni anglatadiki, Kerr qora o'rasining musbat spini va modifikatsilangan gravitatsiyaning manfiy parameterlari bir biriga o'xshash gravitatsiya xususiyatiga ega va bir hil chastotali QPOLar hosil qilishda bu ikki hil qora o'rani kuzatuvlarda bir biridan farqlash muammosi yuzaga kelishi mumkin.

Buning uchun biz ikkita QPO ob'yektlarining kuzatuv chastotalaridan foydalanamiz. Bular GRS 1915+105 (~ 12.5 Quyosh massali hamda yuqori va quyi chastotalari 113 va 168 Hz) va XTE 1150-564 (~ 9 Quyosh massali, chastotalari 179, 273 Hz) Kuzatuv ma'lumot natijalari va sonli usullar bilan olib borilgan nazariy hisob kitoblar shuni ko'rsatdiki, XTE 1150-564 mikrovazardagi markaziy qora o'ra agar aylanuvchi Kerr qora o'راسi bo'lsa uning spin parametri ~0.36M va GRS 1915-105 mikrovazardagi qora o'raniki esa ~0.16M ekanligi ko'rsatildi. Bundan tashqari, Singular va regular MOG qora o'ralar uchun bu natijalar esa mos holda XTE 1150-564 ob'yektida MOG parametr -0.33 va -0.48 ga tengligi, va bu qiymatlar GRS 1915-105 ob'yekt uchun -0.17 va -0.26 ga tengligi aniqladik. Har ikkala ob'yektlardagi QPO chastotalari nisbati 3:2 ga yaqinligini inobatga olsak ularning orbitalarining quyidagi munosabat orqali aniqlanadi:

$$3\nu_L(r/M; \alpha) = 2\nu_U(r/M; \alpha)$$

Eslatib o'tamiz QPO orbitalari deganda, biz zarralar tebranishlarining QPO modeliga bog'liq bo'lgan holda qandaydir resonans holatlari hisobiga nurlanish

hosil bo'ladigan orbitani nazarda tutamiz va bu orbitaning zarraning eng ichki stabil aylana orbitasi bilan taqqoslaymiz. Bu bilan biz astrofizik kuzatuvlar yordamida qora o'ra akkretsiya diskini ichki radiusini unda kuzatilayotgan QPO nurlanishlari orqali aniqlash mumkinligini ham ko'rsatamiz. Ya'ni ushbu orbitalar bir biri qanchalik yaqinligini tekshiramiz.



10-rasm: XTE 1150-564 va GRS 1915-105 mikrokvazarlarida kuzatilgan QPOlar hosil bo'luvchi orbita radiuslari egallashu mumkin bo'lgan sohalar modifikatsiyalangan gravitatsiyadagi regulyar va singulyar qora o'ra modellari uchun keltirilgan.

Biz ushbu rasmda har ikala qora o'ra modellari uchun QPO orbitalari va ISCO larni orasidagi mumkin bo'lgan masofala turli ranglarda (och-ko'k SMOG va och-jigar rang RMOG) keltirdik. Rasmdan ko'rish mumkinki, MOG parametrining kichik qiymatlarida kuzatuvda regular va singular MOG qora o'ralarini farqlab bo'lmaydi, yani gravitatsiya tabiati bir biriga o'xshash bo'ladi. Bundan shunday hulosaga kelish mumkinki, MOG parametrining kichikroq qiymatlarida singular va regular qora o'ralarning fazo-vaqtlari tabiati bir biriga o'xshash, ammo MOG parametрни ortishi bilan ularning hususiyatlari bir biridan farqlana boshlaydi.

## Xulosa

1. GRS 1905-105 mikrokvazarida QPO ma'lumotlarini tahlil qilish asosida qora o'ra

massasi va zaryadiga mos keladigan cheklovlar

2. QPO chastotalari tahlili asosida qora o'ra zaryadi Kerr qora o'ra spinini Bardin modelida  $a/M=0,2927$  gacha va regulyar qora o'ra modelida  $a/M=0,7687$  gacha taqlid qilishi aniqlandi.

3. M87\* va Sgr A\* supermassiv qora o'ralarning aylanish va zaryad munosabatlari uchun cheklovlar ularning soyalari o'lchamidan foydalangan holda olingan.

4. GRS 1915-105 mikrokvazaridagi qora o'ra massasi va QPO orbitalari RP va WD modellarida topilgan.

5. Modifikatsiyalangan gravitatsiya doirasida singulyar va regular qora o'ralarning farqlovchi xususiyatlari ikki cho'qqili QPOLardan olingan kuzatuv ma'lumotlardan foydalangan holda ko'rsatilgan.

6. Regulyar MOG qora o'rada XTE1150-564 uchun  $\alpha=-0,33$  va Schwarzschild MOG qora o'rasida  $\alpha=-0,48$ , GRS1915+105 uchun esa Schwarzschild MOG qora o'rasi  $\alpha=-0,17$  va regulyar MOG qora o'ra  $\alpha=-0,26$  olindi.

7. Ikki cho'qqi QPO orbitalari qora o'ralar atrofida ISCO dan tashqarida yaqinroq orbitalarda joylashgan. Agar QPO orbitasi ISCOga mos kelsa, ikkita cho'qqi bir-biriga tutashib bitta cho'qqi xosil qiladi. Ushbu natija astrofizik kuzatuvlarda ISCO radiusi o'lchash muammosini hal qilishda foydali bo'lishi mumkin.

9. Yuqori va past chastotali QPOLar ISCOga mos holda yaqin va uzoq orbitalarda hosil bo'lishi ko'rsatildi.

10. Non kommutativlik (NC) parametri egizak cho'qqili QPO chastotalariga ta'sir qilmasligi ko'rsatilgan.

11. Non kommutativ qora o'ralar atrofidagi fotonsfera atrofidagi fazo-vaqtda kvant gravitatsiyasi effektlarining roli deyarli ahamiyatsizligi aniqlandi.

**SCIENTIFIC COUNCIL DSc.03/31.03.2022.T/FM.10.04 ON AWARD OF  
SCIENTIFIC DEGREE AT INSTITUTE OF FUNDAMENTAL AND  
APPLIED RESEARCH “TIHAME” NATIONAL RESEARCH UNIVERSITY**

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**ASTRONOMICAL INSTITUTE  
INSTITUTE OF FUNDAMENTAL AND APPLIED RESEARCH**

**RAYIMBAEV DJAVLANBEK RADJAPBAEVICH**

**CONSTRAINING PARAMETERS OF BLACK HOLES IN GRAVITY  
THEORIES BASED ON ASTROPHYSICAL OBSERVATIONS**

**01.04.02 – Theoretical physics**

**DISSERTATION ABSTRACT OF THE DOCTOR OF SCIENCE (DSc)  
ON PHYSICAL AND MATHEMATICAL SCIENCES**

**Tashkent – 2022**

The theme of the dissertation of the doctor of science (DSc) on physical and mathematical sciences was registered by the Supreme Attestation Commission of the Cabinet of Ministers of the Republic of Uzbekistan under No. **B2022.3.DSc/FM203**

The doctoral (DSc) dissertation was carried out at the Astronomical Institute of the Academy of Sciences of the Republic of Uzbekistan and the Institute of Fundamental and applied research under "TIAMEE" National Research University.

The abstract of the dissertation was posted in three (Uzbek, English, and Russian (resume)) languages on the website of the Scientific Council at the address [www.ifar.uz](http://www.ifar.uz) and on the website of "Ziyonet" information and educational portal at [www.ziyonet.uz](http://www.ziyonet.uz).

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The defense of the dissertation will be held on "\_\_\_" \_\_\_\_\_ 2022 at \_\_\_ in the meeting of the Scientific Council No. DSc.03/31.03.2022.T/FM.10.04 at the Institute of Fundamental and Applied Research under the National Research University "TIAMEE" (Address: 100000, Tashkent city, Qori Niyazov Street 39, Institute of Fundamental and Applied Research, Hall 108; tel.: 71 237-09-61.; e-mail: [info@ifar.uz](mailto:info@ifar.uz))

The doctoral (DSc) dissertation can be looked through at the Information Resource Center of the Institute of Fundamental and Applied Research under the National Research University "TIAMEE" (registered under № \_\_\_\_). (Address: 100000, Tashkent city, 39 Qori Niyazov str., Institute of Fundamental and Applied Research, hall 108; ph.: 71 237-09-61)

The Abstract of the dissertation was distributed on "\_\_\_" \_\_\_\_\_, 2022.  
(Registry record № \_\_\_ dated "\_\_\_" \_\_\_\_\_, 2022)

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## INTRODUCTION (Annotation of DSc dissertation)

**Topicality and demand of the theme of dissertation.** Mathematical point of view black holes (BHs) are solutions of the gravitational field equations. For the first time, Carl Schwarzschild found a BH solution that has only mass, as a solution of Einstein's field equation in 1916. From an astrophysical point of view, they are relativistic compact gravitational astrophysical objects formed at the ending stage of stellar evolution. They are three types of BHs according to their mass: (a) supermassive BHs ( $10^6$ - $10^{10}$  solar masses), which are located in galactic centers; (b) medium-mass black holes whose mass  $10^2$ - $10^5$  solar masses), and their distribution in galaxies is still uncertain; and finally, (c) BHs with masses from 3 to  $10^2$  solar masses are called stellar mass black holes, and they are primarily found in binary systems. Recent observations of Event Horizon Telescope and GRAVITY collaborations have been obtained in the shadows of two supermassive black holes (SMBHs), M87\* and Sgr A\* which allowed us to determine their masses and spin parameters. However, black holes of medium and stellar mass cannot be observed due to their small angular size. Fortunately, they can be detected mainly by the radiation luminosity of the accretion disk, in particular, by quasi-periodic oscillation frequencies. It is possible to get information about spacetime around the BHs and constraints to their mass, charge, and spin parameters can be determined.

In fact, the BH charge or parameters of alternative and modified gravity (MOG) theories may provide similar gravitational effects around non-rotating BHs as the spin of rotating Kerr BHs, which are can reduce decreasing of inner radii of the photosphere and the accretion disk around the BHs. In this sense, their effects on the spacetime geometry around the BHs are similar, and these parameters can mimic each other. Another fact, Einstein's general theory of relativity has been well-tested in both weak and strong gravitational regimes and confirmed by many astrophysical observations so far. However, alternative and modified theories of gravity have also successfully passed through astrophysical tests where GR could not play a role. All the above-mentioned facts imply that test the theories of gravities and parameters of BHs using observational data from astrophysical BHs is one of the most actual and important issues in theoretical and observational astrophysics.

In our country, there is also much attention being paid to investigations of the radiation mechanisms in the accretion disk of BHs as well as optical and energetic processes around the BHs, and theoretical studies of gravity theories and testing them based on observational data.

This dissertation work corresponds to the tasks by the following state regulatory documents: Decree of the President of the Republic of Uzbekistan No. UP-4947 "On the Strategy of Actions for the Further Development of the Republic of Uzbekistan" dated February 07, 2017, Decree of the President of the Republic of Uzbekistan No. PP-2789 "On Measures for Further Improvement of Academy of Sciences, organization, management and financing of research activities "from 18.02.2017.

**Relevance of the research to the priority areas of science and technology development of the Republic of Uzbekistan.** The dissertation research has been carried out in accordance with the priority areas of science and technology in the Republic of Uzbekistan: II. "Power, energy and resource-saving".

**Degree of study of the problem.**

Radiation mechanisms in the accretion disk of BHs, in particular, generation of quasi periodic oscillations (QPOs) and their study within the framework of various gravity theories have been widely carried out in Europe (L. Rezzolla, M. Abramovich, G. Torok, P. Bakala, Z. Stucklik, and M. Kolos etc.), in the USA (S. Motta, L. Stella, M. Vietri, S. Spielberg, W. Wagoner) as well as scientists from developed countries in the world (S. Kato, J. Fukue (Japan), D. Wang, L. Cheng and M. Zhang (China)). The theoretical studies of the shadows of BHs in various gravity theories has also been studied by a number of scientists (Jan Schee, Ali Ovgun, Kimet Jusufi, Mubasher Jamil, and so on).

In our republic, the theoretical studies on optical and energetic processes around BHs in various gravity theories have also potentially been studied by B. Ahmedov, A. Abdujabbarov, F. Atamuratov, B. Toshmatov, S. Shaymatov, A. Abdikamalov etc. However, in their studies testing gravity theories and obtaining constrains on parameters of BHs in different theories of gravities based on observational data from BH shadow and quasi periodic oscillations have not been carried out. Also, the mimicking effects of BH parameters have not also been widely studied.

**Connection of the topic of dissertation with the scientific researches of the higher educational/research institutions, where the dissertation was carried out.** The dissertation was done in the framework of the scientific projects of Nuclear Physics Institute, Uzbek Academy of Sciences: F-FA-2021-510 "Investigations of nuclear matter of neutron stars in modified gravity" (2021-2026).

**The aim of the research of the dissertation** is testing gravity theories and constraining BH parameters using QPOs and shadow.

**The tasks of the research:**

- constraining the BH charge and mass using QPO data;
- to investigate mimicking values in black hole parameters providing the same value of QPO frequencies;
- to obtain limits of parameters of SMBH M87\* and SgrA\* using their shadow size;
- testing quantum gravity effects around BHs using QPO data.

**The objects of the research** are relativistic compact gravitating objects, BHs.

**The subjects of the research** are QPOs from low mass X-ray binaries and shadow of SMBHs.

**The methods of the research** are the mathematical apparatus of GR and numerical methods of statistics.

**The scientific novelty of the research** is as follows:

- for the first time, constraints on the BH parameters (mass and charge) have been obtained using QPO data;
- for the first time, it is shown that BH charge can mimic the spin of rotating Kerr BH providing the same values of upper and lower frequencies in twin-peak QPOs;
- for the first time, constraints on spin and charge parameters of BHs in 4D Einstein Gauss-Bonnet gravity have also been obtained using image size of M87\* and Sgr A\*;
- for the first time, it is shown that quantum gravity effects on QPOs around a NCi BH and its shadow are negligible;
- for the first time, a way to distinguish singular and regular BHs in MOG using QPO studies is formulated;
- for the first time, it is shown that innermost stable circular orbits (ISCO) radius around BHs can be estimated by QPO orbits.

**Practical results** of the research are as follows:

- for the first time, constraints for mass and charge of the BH in the microquasar GRS 1915-105 are obtained using observational data from twin peak QPOs;
- a new approach to determining ISCO radius around BHs have been developed using twin peak QPO frequencies;
- for the first time, constraints for the MOG parameter of Schwarzschild MOG and regular MOG BHs using observational data from QPOs GRS 1915-105 and XTE 1550-564 are obtained.

**Reliability of the research results** is provided by the following:

- modern methods of general relativity and the theoretical physics and highly effective numerical methods and algorithms are used;
- careful check of consistency of the received theoretical results with observational data and results of other authors is performed;
- conclusions are well consistent with the main provisions of the field theory of gravitational compact objects.

**The scientific and practical significance of the research** are:

- the obtained constraints for the parameters of BHs may allow getting information about the gravitational feature of the spacetime around the BHs, and it is possible to determine which gravitational effect plays dominant role;
- the obtained results may help to determine the parameters of stellar mass black holes and the space-time features around them based on observational data from QPOs and shadows of supermassive black holes, as well as the observation of QPOs and black hole accretion made it possible to explain the physical mechanisms in the accretion disc;

- the obtained results can be useful in determining the identifications of singular and regular BHs in modified and quantum theories of gravity.

### **Applications of the research results**

The results of the study of the QPOs from the BHs and shadow of the BHs in various gravity theories have been applied as follows:

- the formalism of determining the masses and charges of BHs based on observational data from QPOs have been used by a number of authors in the investigations of spacetime properties around BHs as well as gravitational models (The Astrophysical Journal Vol.935,91, (2022), Progress of Physics, Vol. 70, issue 9-10, 2200053, (2022), Journal of Cosmology and Astroparticle Physics, Volume 09, id 061, (2022), European Physics Journal C, Volume 82, id 636 (2022), Progress of Physics, Volume70, Issue 9-10, id 2200053, (2022), Universe Volume 8, issue 3, id 182 (2022)). The results have been provided to make possible analyses in various models of dark matter around BHs based on observational data of BHs;
- in obtaining constraints for the BH parameters using analyses of data from SMBH images have been used by a number of authors in the analysis of various gravity models and spacetime features nearby BHs based on the observational data from their shadows (Chinese Journal of Physics Vol. 78, pp.141-154, (2022), Annals of Physics Vol.441, 168892,(2022), Universe, Volume 8, issue 10, id 536 (2022), Physics, Volume 4, issue 4, pp.1318-1330 (2022), European Physics Journal C, Volume 82, id 831 (2022), Annals of Physics, Volume 441, id 168892 (2022)). The obtained results were used in the analysis of photon orbits and their shadows around rotating charged black holes;

### **Approbation of the research results**

The results of the dissertation have been discussed in 2 international and 1 local conferences, and regular weekly Uzbek-Kazakh seminars on theoretical physics and astrophysics.

### **Publication of the research results**

more than 20 scientific publications have been made on research results, 16 of them are research papers in refereed journals.

### **The volume and structure of the dissertation**

The dissertation consists of an introduction, four chapters, a conclusion, and a list of references all in 117 pages.

## **MAIN CONTENT OF THE DISSERTATION**

In **the introduction** topicality and significance of the dissertation are outlined, aims and tasks are defined, scientific novelty and practical results are shown and their theoretical and practical importance is discussed.

Chapter I entitled ‘**Constraining black hole charges using data from QPOs**’ is dedicated to studying the motion of test particles around charged BHs and their small oscillations along stable circular orbits to apply them to QPO frequencies, and determining the masses and charges of BHs based on observational data from QPOs. The spacetime metric around charged non-rotating BHs can be described as  $ds^2=f(r)dt^2 +dr^2 / f(r)+r^2 d\theta^2 +r^2 \text{Sin}^2\theta d\phi^2$  (1.1) where  $f(r)$  is the lapse function where  $\Upsilon^2=1+ Q^2/r^2$  ,  $Q$  and  $M$  are the charge and mass of the BH, respectively.

|                            |                                              |
|----------------------------|----------------------------------------------|
| Charged non-rotating BH    | $f(r)$                                       |
| Reissner-Nordstrom (RN) BH | $1-2M/r +Q^2/r^2$                            |
| Bardeen BH                 | $1-2M/r \Upsilon^{-3}$                       |
| Ayon-Beato-Garcia (ABG) BH | $1-2M/r \Upsilon^{-3}+Q^2/r^2 \Upsilon^{-4}$ |
| Regular BH (RBH1)          | $1-2M/r \text{Exp}(-Q^2/(-2Mr))$             |
| Regular BH (RBH2)          | $1-2M/r (1+Q/r)^n$                           |

Table 1: The metric function of charged BHs.

The extreme charge that BHs can have and the minimum value of the event horizon can be obtained using  $f(r)=f'(r)=0$  are found as follows:

| charged non-rotating BH    | $Q_{\text{extr}}/M$ | $(r_h)_{\text{min}}/M$ |
|----------------------------|---------------------|------------------------|
| Reissner-Nordstrom (RN) BH | 1                   | 1                      |
| Bardeen BH                 | 0.7698              | 1.08866                |
| Ayon-Beato-Garcia (ABG) BH | 0.634181            | 1.00504                |
| Regular BH (RBH1)          | 1.21306             | 0.735759               |
| Regular BH (RBH2)(n=3)     | 0.296296            | 0.592593               |
| Regular BH (RBH3) (n=4)    | 0.210937            | 0.632812               |

Table 2: Extreme value of BH charge and minimum value of event horizon radius.

We use, in order to describe the test particle's motion the Lagrangian density

$$2L_p = mg_{\alpha\beta}^{\cdot\alpha\cdot\beta} x^{\cdot\alpha} x^{\cdot\beta} .$$

The energy and angular momentum of the particle can be defined by the Euler-Lagrange equation  $g_{tt} \dot{t} = -\epsilon$ ,  $g_{\phi\phi} \dot{\phi} = L$  .

In this case, the effective potential for the radial motion of the particles was found as follows:  $V_{eff} = f(r)(1 + \frac{L^2}{r^2})$

In fact, the condition  $V_{eff} = \varepsilon$  and  $V_{eff}' = 0$  must be satisfied and, the energy and angular momenta of the particles

$$\varepsilon^2 = \frac{2f(r)^2}{2f(r) - rf'(r)} \quad \text{and} \quad L^2 = \frac{r^3 f'(r)}{2f(r) - rf'(r)}$$

For the stability of circular orbits, the condition  $V_{eff}'' \geq 0$  must be satisfied, and the radius of the internally stable circular orbit (ISCO) is defined as the solution of the following equation:

$$f'(r)(2r \frac{f'(r)}{f(r)} - 3) - rf''(r) = 0.$$

Angular velocity ( $\Omega_K = d\phi/dt$ ) in circular orbits of particles, i.e. Kepler frequencies, takes the form in the spacetime (1):  $\Omega_K = \sqrt{\frac{f'(r)}{2r}}$

It is known that the small deviation of particles around stable circular orbits causes them to generate small oscillations in vertical and radial directions around these stable orbits. Expressions for the frequencies of oscillations in these directions in a spherically symmetric space-time have the following simplified form:

$$\Omega_r = \Omega_K \sqrt{(3 + \frac{rf''(r)}{f'(r)})f(r) - 2rf'(r)}, \quad \Omega_\theta = \Omega_\phi = \Omega_K.$$

In order to express these fundamental frequencies in Hz, we multiply them by  $c^3/2\pi GM$  (where  $c$  is the speed of light in vacuum and  $G$  is the gravitational constant). In most models, the mechanism of QPOs can be explained by easily fundamental frequencies.

For example, according to the relativistic precession (RP) model, the upper frequency of the twin-peaked QPO is explained as Kepler's frequency, and the lower frequency is explained as the difference between the vertical and radial frequencies. The set of all possible values of the upper and lower frequencies of QPOs is mathematically found in the interval from the radial coordinate ISCO to infinity. The above figure shows possible values for lower and higher frequencies of QPOs around charged black holes. It can be seen from the graph that, for example, the frequency values that can be generated around the Bardeen black circle lie between the blue and gray lines. The QPO source identified outside of it cannot be the Bardeen black hole. See a similar analysis for other charged black holes possible

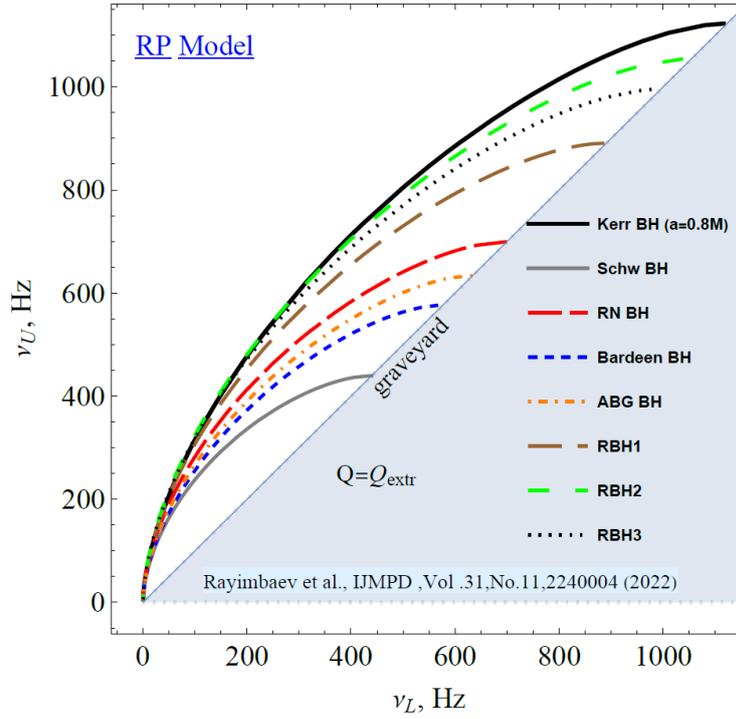
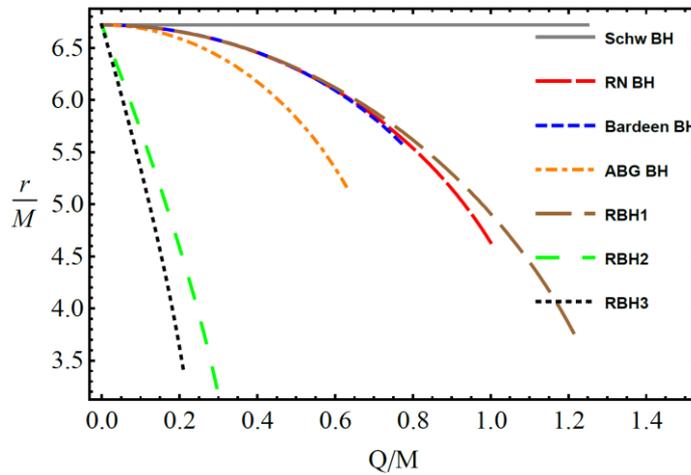


Figure 1: Relationships of upper and lower frequencies of twin peak QPOs around charged BHs with extreme charge.

In this way, we can determine which model of charged BHs can be a candidate to



be the central BH in a QPO source. In fact, the BH charge should not exceed its critical value defined above. In order to compare the gravitational effects of the charge and spin parameters, we also presented such mimickers for the Kerr BH with a spin of 0.8 through the black curve output. This means that the BH charge can affect the Kerr BH spin up to a certain value, depending on its model. For further analysis, we choose as an example the QPO object in the microquasar GRS 1915-105 with frequencies of  $168 \pm 5$  Hz and  $113 \pm 3$  Hz, and one may get relationships between QPO radius and BH charge using the following system of equations:  $\nu_U(M; r, Q) = \nu_U^{ob}$  &  $\nu_L(M; r, Q) = \nu_L^{ob}$ . The figure above shows the QPO orbit radius related to the central BH charge. Figure 2: Radius of QPO orbits around BHs as a function of the BH charge.

Firstly, it should be noted that this orbit is located far from its ISCO at radius  $Q=0$ , that is, in the case of a Schwarzschild black hole.

However, the distance between ISCO and QPO orbits may take a closer value to the errors in observations. As the charge of the BH increases, the radii of the QPO and the ISCO decrease, including their distance, which remains within the order of the specified error in observations of QPOs, thus measuring the inner boundary of the ISCO, i.e. the accretion disk, around the BHs. We can show that it is possible. Now, we use the QPO observational data from the microquasar GRS 1915-105 to estimate the mass and fraction of the central black hole. In this case, using the Bayesian approximation of the 2 methods of taking the errors into account, we get the results with the accuracy of errors  $1\sigma$ ,  $2\sigma$ , and  $3\sigma$  as follows.

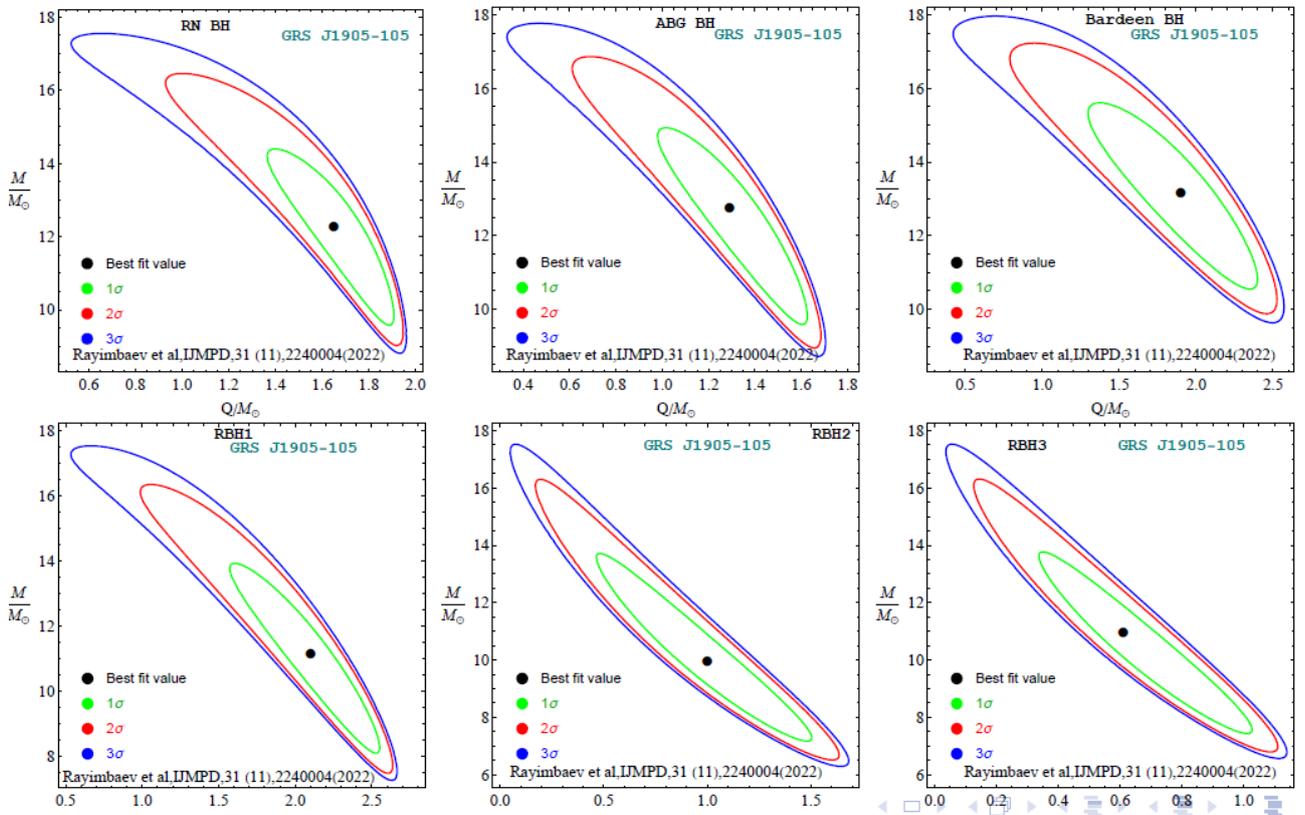


Figure 3: Constraints on mass and charge of BH in the mikroquasar GRS J1905-105 obtained using  $\chi^2$  method for the error bars  $1\sigma$ ,  $2\sigma$  va  $3\sigma$ .

The most accurate value ("Best fit value") in the graphs is found by minimizing all parameters according to the approximation rule, and they have the following value:

| BHs        | $M/M_{\odot}$           | $Q/M_{\odot}$          | $Q/M$ |
|------------|-------------------------|------------------------|-------|
| RN BH      | $12.23^{+0.02}_{-0.05}$ | $1.64^{+0.07}_{-0.04}$ | 0.13  |
| Bardeen BH | $13.25^{+0.06}_{-0.04}$ | $1.93^{+0.05}_{-0.03}$ | 0.15  |
| ABG BH     | $12.56^{+0.06}_{-0.05}$ | $1.31^{+0.02}_{-0.07}$ | 0.104 |
| RBH1       | $11.19^{+0.07}_{-0.06}$ | $2.25^{+0.06}_{-0.05}$ | 0.2   |
| RBH2       | $10.06^{+0.05}_{-0.07}$ | $1.03 \pm 0.05$        | 0.102 |
| RBH3       | $10.92^{+0.08}_{-0.07}$ | $0.65^{+0.06}_{-0.05}$ | 0.06  |

Table 3: Best fit values of mass and charge of the charged BHs, and their ratio.

It is seen from the table that charge to mass ratio of the BH in the microquasar GRS 1915-105 is about 0.1. It implies that astrophysical BHs can not accumulate too much charge or the effects of the BH charge on the upper and lower frequencies of QPOs is not as strong as the BH mass.

If we calculate the same calculations for the extremely charged state, and the gravitational effect of this amount of charge on the space-time of the black hole, we carried out the spin parameter values of the Kerr black hole, which can give such a gravitational effect, and we obtained the values in the table below.

| Extreme charged BHs        | $M/M_{\odot}$                 | $a/M$         |
|----------------------------|-------------------------------|---------------|
| RN BH                      | $17.0079^{+0.5217}_{-0.4916}$ | $\sim 0.4884$ |
| Bardeen BH                 | $13.9495^{+0.4698}_{-0.4131}$ | $\sim 0.2927$ |
| Ayon-Beato-Garcia (ABG) BH | $15.3169^{+0.4698}_{-0.4426}$ | $\sim 0.3919$ |
| RBH 1                      | $21.1479^{+0.6847}_{-0.6112}$ | $\sim 0.6671$ |
| RBH 2 (n=3)                | $19.9994^{+0.6135}_{-0.5780}$ | $\sim 0.7687$ |
| RBH 3 (n=4)                | $23.6358^{+0.7250}_{-0.4831}$ | $\sim 0.7392$ |

Table 4: Mass constraints of extreme charged BHs and mimicking values of the spin of rotating Kerr BH.

Chapter II is entitled "**Observational Constraints on the Spins and Charges of the Charged M87 and Sagittarius A\* supermassive Black Holes**" and in this chapter observational data from the sizes of black hole shadows on the basis of which the connections between their spin and charge parameters were determined at different values of Gauss-Bonnet and AdS parameters.

Glavan and Lin were the first to propose a general modified gravity for the four-dimensional situation in which massless gravitons propagate. They developed the static and spherically symmetric BH solution by varying the GB coupling parameter as  $d \rightarrow d/(d-4)$  and then taking the limit  $d \rightarrow 4$ . So, taking dimension  $d = 4$ , the GB term is gravitational dynamics. As a result, it leads to the discovery of nontrivial static and spherically symmetric solutions of BH. Such a BH GB provides a convenient platform for probing the nature of gravity. In spherical coordinates, its metric is written as:

$$ds^2 = -f(r)dt^2 + 1/f(r)dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2$$

here

$$f(r) = 1 + \frac{r^2}{2\gamma} \left[ 1 - \sqrt{1 + 4\gamma \left( \frac{2M}{r^3} - \frac{Q^2}{r^4} - \frac{1}{l^2} \right)} \right]$$

From this metric, using the Newman Janis algorithm, we obtain the solution of the rotating charged black hole in the EGB theory in the new AdS space in the following form:

where  $\Delta = a^2 + r^2(1 + f(r))$  va  $\rho^2 = r^2 + a^2 \cos^2\theta$  We study the motion of photons in this rotating space-time using the following Hamilton-Jacobi equation:

$$\frac{\partial H}{\partial \tau} + \frac{1}{2} g^{\mu\nu} \frac{\partial H}{\partial x^\mu} \frac{\partial H}{\partial x^\nu} = 0,$$

Here we write the Hamiltonian function H with respect to the conserved quantities as  $H = H_r(r) - Et + L\phi + H_\theta(\theta) + \frac{1}{2}m^2\tau$ .

The conservation of momentum for a photon is integral: its energy and momentum are defined as follows:  $E/m = g_{t\mu} \dot{x}^\mu$  and  $L/m = g_{\phi\mu} \dot{x}^\mu$

From the above H-J equation, we determine the equations of motion for the photon and the effective potential for the equatorial plane as follows:

$$V_{eff}(r) = \Delta[(\xi - a)^2 + \eta] - [-a\xi + (a^2 + r^2)]^2$$

Where  $\eta = \kappa/\varepsilon^2$  and  $\xi = L/\varepsilon$ .

The orbits of photons to be stable the following condition have to be satisfied:  $V_{eff}(r_{ph}) = V'_{eff}(r_{ph}) = 0$ . From the condition and effective potential for photon motion, we can get equations for the BH shadow in the form:

$$\alpha = \frac{4r\Delta - \Delta'(r^2 + a^2)}{a\Delta'} \quad \text{va} \quad \beta^2 = \frac{r^2[16\Delta(a^2 - \Delta) - r^2\Delta'^2 + 8r\Delta\Delta']}{a^2\Delta'^2}$$

The radius of the BH shadow defines by the equation  $R_{sh}^2 = \alpha^2 + \beta^2$  Recent observations of images of supermassive BHs M87\* (by EHT Collab. 2019) and Sgr A\* (GRAVITY Collab. 2022) have been obtained with the angular sizes  $42 \pm 3 \mu\text{as}$  and  $51.8 \pm 2.3 \mu\text{as}$ , respectively. One may determine normalized values of charge and its spin parameter of the supermassive BHs using observational data from the

angular sizes of their images and their distances. Below, we analyzed the mean values of the shadow size to estimate the relationship between the spin parameter and BH charges. In fact, the shape of the BH shadow strongly depends on the position of the distance observer. In other words, the shape can be affected by inclination angle between rotation axes of the BH and the direction of the distance observer. However, numerical calculations show that the size of the BH shadow is less sensitive with respect to the variation of the inclination angle. Therefore, we have considered simply that the observer is located at the equatorial plane and the rotation axis of the BH and radial direction are perpendicular each other.

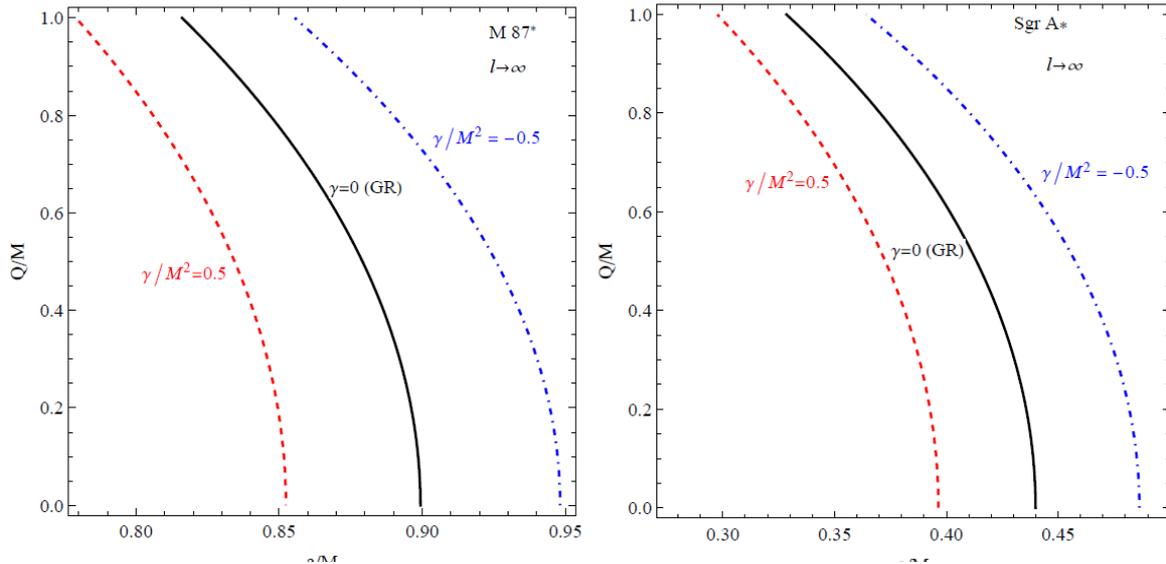


Figure 4: Spin and charge constraints of SMBHs M87\* (left panel) and Sgr A\* (right panel) in 4D EGB gravity for the different values of GB parameter.

In this figure, the relationships between charge and spin parameters of SMBHs M87\* (left panel) and Sgr A\* (right panel) for the positive (red), negative (blue), and zero (GR) values of the GB parameter are shown. It is seen from the figure that as the BH charge increases, the spin parameters decrease for the constant value of the shadow size. Moreover, the positive GB parameter decreases the gravitational effect of the spin parameter, while a negative GB parameter increases it. The values of the spin of the BHs as  $Q=0$  confirm by the values obtained in other astronomical observations, which are  $a \sim 0.9M$  for M87\* BH and  $a \sim 0.44M$  for Sgr A\* BH.

In Chapter III entitled "**Testing effects of quantum non-commutative black holes on QPOs and shadow size**" we test the effect of quantum non-commutative gravity on the frequencies of QPOs.

A non-point like BH static solution in NC gravity was first obtained by replacing the mass function from the delta Dirac function to the Gaussian-like distribution function as:

$$\rho(r) = \frac{M}{(4\pi\theta)^{3/2}} e^{-r^2/4\theta}$$

where  $\theta$  - is the non-commutativity parameter. The geometry of the spacetime around the static NCi BH can be described by

$$f(r) = 1 - \frac{4M}{r\sqrt{\pi}} \gamma\left(\frac{3}{2}, \frac{r^2}{4\Theta}\right)$$

With

$$\gamma\left(\frac{3}{2}, \frac{r^2}{4\Theta}\right) = \int_0^{r^2/4\Theta} \sqrt{h} e^{-h} dh .$$

is the lower incomplete gamma function.

In fact, NCi BHs are regular ones. The zeros of the metric function define the horizon around the BH, and its first derivative allows us to obtain the critical NC parameter and the minimum radius of the horizon:  $\theta_{cr} 0.2765M^2$  and  $r_{min} = 1.5873 M$ .

Furthermore, we study the motion of particles around the NC BH and their radial, orbital, and vertical oscillations along static orbits as described in Chapter 1. The angular velocities of particles in Kepler orbits

$$\Omega_K = \frac{\sqrt{M}}{\sqrt{2}\sqrt[4]{\pi}r^{3/2}} \sqrt{4\gamma\left(\frac{3}{2}, \frac{r^2}{4\Theta}\right) - \frac{r^3}{\Theta^{3/2}} e^{-\frac{r^2}{4\Theta}}} .$$

The frequencies of the vertical and azimuthal oscillations are equal to the Keplerian frequencies, but the radial frequency

$$\begin{aligned} \Omega_r = \Omega_K & \left\{ \left[ 4\pi\Theta^{\frac{5}{2}}r^2e^{\frac{r^2}{2\Theta}} \text{Erf}\left(\frac{r}{2\sqrt{\Theta}}\right) - 2\sqrt{\pi}\Theta r^3e^{\frac{r^2}{4\Theta}} \right. \right. \\ & \times (2\Theta + r^2) \left. \right]^{-1} \left[ r^5 \left( \sqrt{\pi}e^{\frac{r^2}{4\Theta}} (r^2 - 6\Theta) - \frac{4Mr^2}{\sqrt{\Theta}} \right) \right. \\ & - 4e^{\frac{r^2}{4\Theta}} \gamma\left(\frac{3}{2}, \frac{r^2}{4\Theta}\right) \left( 24\Theta^{5/2}Mr e^{\frac{r^2}{4\Theta}} \gamma\left(\frac{3}{2}, \frac{r^2}{4\Theta}\right) \right. \\ & \left. \left. + Mr^4 (r^2 - 14\Theta) - 2\sqrt{\pi}\Theta^{5/2}r^2e^{\frac{r^2}{4\Theta}} \right) \right] \left. \right\}^{\frac{1}{2}} , \end{aligned}$$

$$\Omega_\theta = \Omega_\phi = \Omega_K .$$

It is known that the generation of QPO signals around BHs is related to oscillations of particles, and the oscillations reflect the effects of spacetime geometry. We also investigate the effect of the NC parameter on the QPO signal frequencies using observational data using the RP and WD (warped disk) models (since these models are more appropriate in non-rotating spherically symmetric spacetime). The effect of the NC parameter on the upper and lower frequencies is shown in the graph below

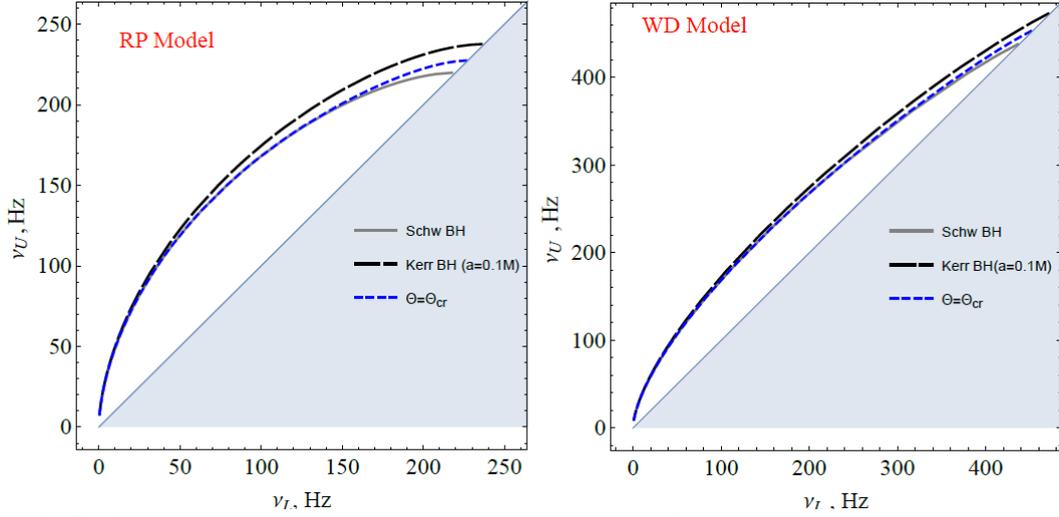


Figure 5: Relationships between upper and lower frequencies of twin peak QPOs around Schwarzschild BH, NCi BH with critic NC parameter and Kerr BH with the spin  $a=0.1 M$ .

It is seen from the figure that in both models the effect of the NC parameter is visible only at high frequencies, however the difference of frequencies with compare to the Schwarzschild limit is less than 5-10 Hz, and it is in the order of the measurement error. From this point of view, one may conclude that the effect of the NC parameter on the QPO frequencies is almost negligible. In addition, our numerical calculations have shown that the critical value of the NC parameter can provide the same gravitational effect on QPO frequencies as a Kerr BH with the spin  $a/M=0.052$  which is astrophysical non-rotating.

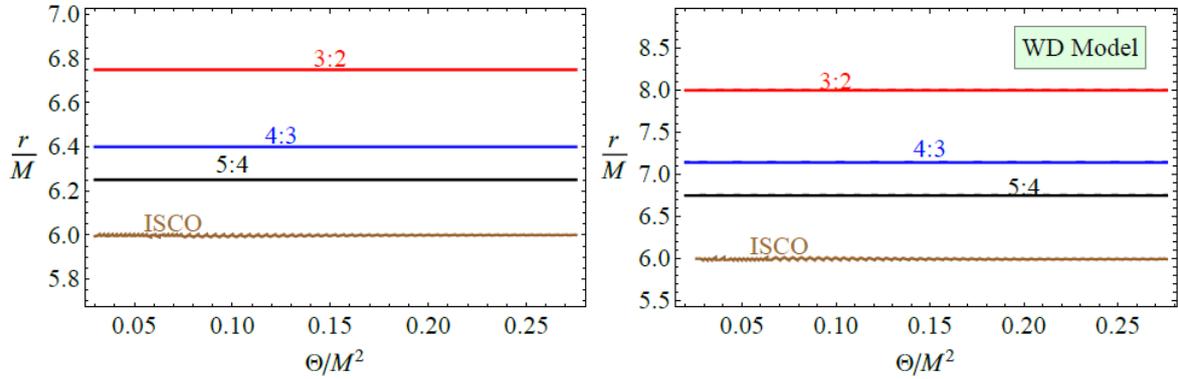


Figure 6:  $r$ - $\Theta$  dependence for the RP (left panel) and WD (right panel) models with the comparison of ISCO radius.

In addition, the influence of the NC parameter on the ISCO and QPO orbits is negligible or non-existent.

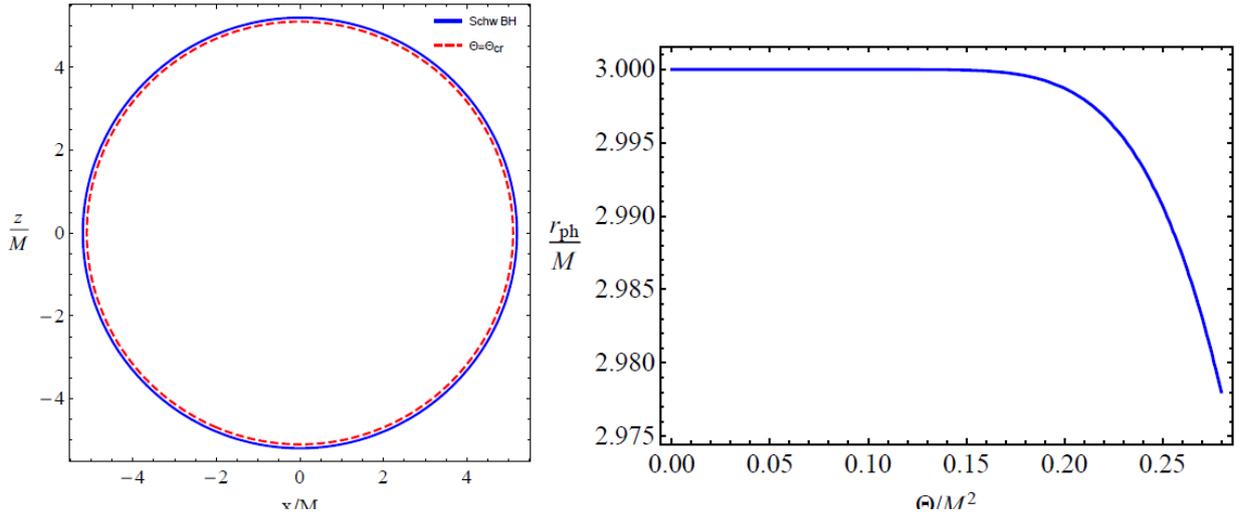


Figure 7: Shadow of Schwarzschild BH and NCi BH with critic parameter (left panel) and the dependence of photonsphere radius on the NC parameter.

The effect of quantum non-commutative gravity on photon motion around the NC i BHs and the shadow of the BH have also been studied. In this case, the influence of the NC parameter on the radius of the photosphere is almost invisible at values close to zero, but at values close to critical values, the photosphere slightly decreases, less than 0.8%. Observationally, the error in measurements of the shadow of the BH M87\* is about 7%, and it is in Sgr A\* case about 4%, however, the effects of the NC parameter may change the size of the BH shadow up to 2%. From this point of view, the effects of NC parameter on BH shadow are also negligible.

Chapter IV is entitled "**Distinguishing Regular and Singular Black Holes in Modified Gravitational Theory Using Quasi-periodic Oscillations**", and in this, we consider the distinction between regular and singular black holes based on QPO observational data.

It is known that vacuum solutions in general relativity have a singularity, which implies infinite curvature in the spacetime at  $r=0$ . However, regular BH solution can be obtained in quantum gravity, nonlinear electrodynamics coupled to general relativity, and scalar-vector-tensor fields (scalar and massive vector fields). The metric functions of Schwarzschild MOG and regular MOG BHs:

$$f(r) = 1 - \frac{2M}{r} \frac{\alpha + 1}{\left[1 + \alpha(\alpha + 1)\frac{M^2}{r^2}\right]^{3/2}} + \frac{\alpha(\alpha + 1)\frac{M^2}{r^2}}{\left[1 + \alpha(\alpha + 1)\frac{M^2}{r^2}\right]^2}$$

and

$$f(r) = 1 - \frac{2(1 + \alpha)M}{r} + \frac{\alpha(1 + \alpha)M^2}{r^2}$$

Firstly, we consider the motion of particles around the BHs. In this case, the effective potential of the particles at the equatorial plane is derived using the above-mentioned Hamilton Jacobi method. The angular momentum and energy of the particle in circular orbits have the following form:

$$\mathcal{L}^2 = \frac{r^3 \partial_r \ln f(r)}{2 - r \partial_r \ln f(r)}, \quad \mathcal{E}^2 = \frac{2f(r)}{2 - r \partial_r \ln f(r)}$$

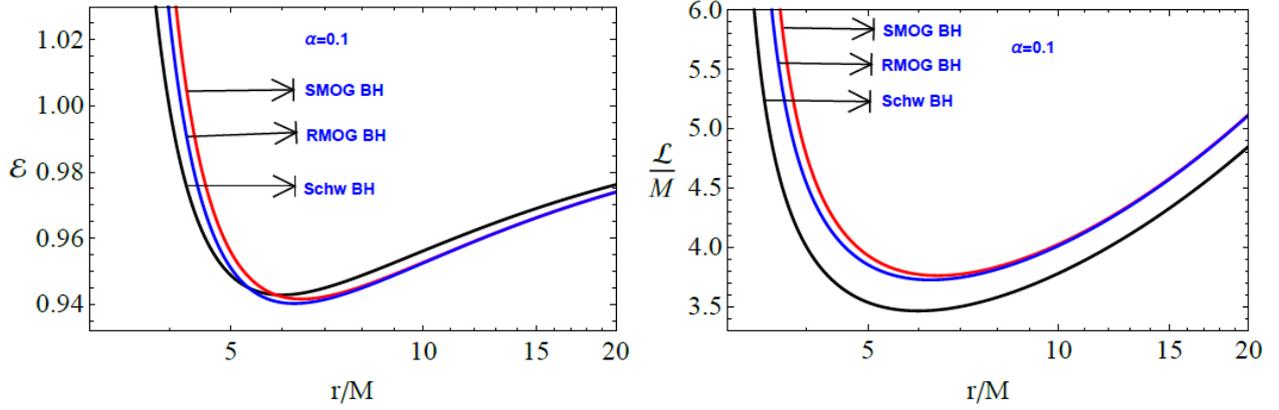


Figure 8: Specific energy and angular momentum for the circular motion of test particles around SMOG and RMOG BHs as functions of the radial coordinate.

The figure shows the radial dependence of the specific energy (left panel) and angular momentum (right panel) of test particles around regular and Schwarzschild BHs in MOG. One can see from the figure that the minimum energy in MOG slightly decreases, while the angular momentum increases and shifts outward of the central BH.

Expression for oscillations of particles in stable circular orbits

$$\Omega_K^S = \frac{\sqrt{(r - \alpha M)(\alpha + 1)M}}{r^2}$$

$$\Omega_K^R = \frac{\sqrt{\alpha + 1}\sqrt{M}}{(\alpha(\alpha + 1)M^2 + r^2)^{3/2}} \left\{ \alpha^2(\alpha + 1)M^3 - \alpha M r^2 - \sqrt{1 + \frac{\alpha(\alpha + 1)M^2}{r^2}} r [2\alpha(\alpha + 1)M^2 - r^2] \right\}^{\frac{1}{2}},$$

One can calculate frequencies of radial oscillations using the expression for the frequency shown in the first chapter. Due to the long form of the expression we will analyze the QPOs graphically.

Here also use the RP model in the investigation of the influence of the MOG field or in other words the scalar-vector-tensor field, on the upper and lower frequencies of twin-peaked QPOs around the MOG BHs. While, we have compared all the obtained results with results that obtained in the Kerr BH as shown in below figure.

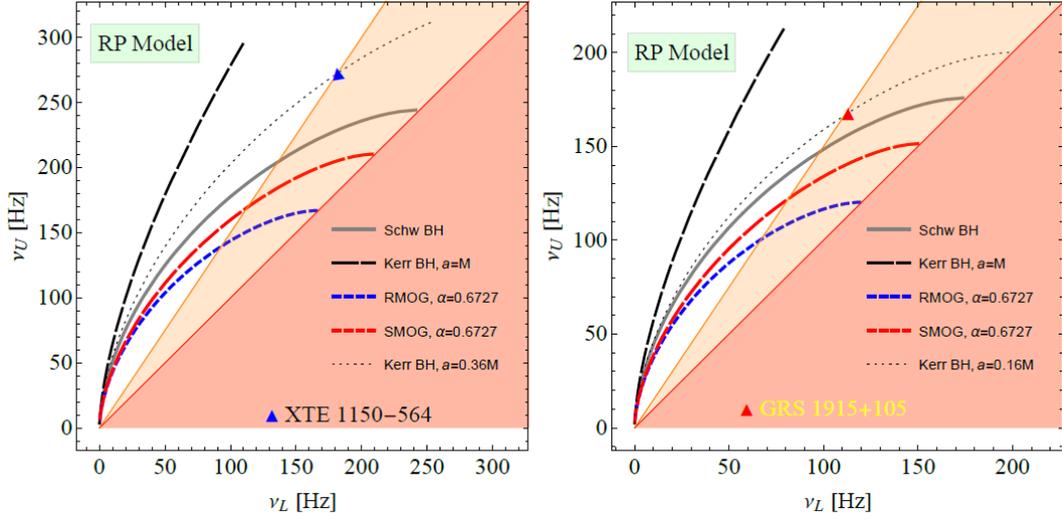


Figure 9: Upper and lower frequency diagram for twin peak QPOs around BHs in microquasars GRS 1915+105 and XTE 1150-564 with the masses  $12.5 M_{\odot}$  and  $9M_{\odot}$  respectively.

It is seen from the figure that the positive values of  $\alpha$  parameter shifts the frequency curve down with compare to the Schwarzschild case, while it is rotating Kerr BHs at positive spin parameter the curve shifts up. It implies that the positive values of spin and negative values of MOG parameter show similar gravitational effects on the spacetime around the BHs in providing the same upper and lower frequencies.

Here, we use the observed two QPO objects: mikroquasar GRS 1915+105 (mass  $\sim 12.5$  solar masses, and upper and lower frequencies of 113 and 168 Hz) and XTE 1150-564 (9 solar masses, frequencies 179, 273 Hz). The theoretical calculations using numerical methods and observational data by assuming the central BH in the microquasar XTE 1150-564 is a rotating Kerr BH, have shown that the spin parameter is  $\sim 0.36M$  and while it is for the BH in GRS 1915-105,  $a \sim 0.16M$ . Moreover, these results for singular and regular MOG BHs have shown that the MOG parameter is  $\sim -0.33$  and  $-0.48$  for XTE 1150-564, respectively, while it is  $\sim -0.17$  and  $-0.26$  for GRS 1915-105. In fact, the ratio of upper and lower frequencies in the QPO objects is approximately 3:2. Therefor one may determine QPO orbits by the following relationship:

$$3\nu_L(r/M; \alpha) = 2\nu_U(r/M; \alpha)$$

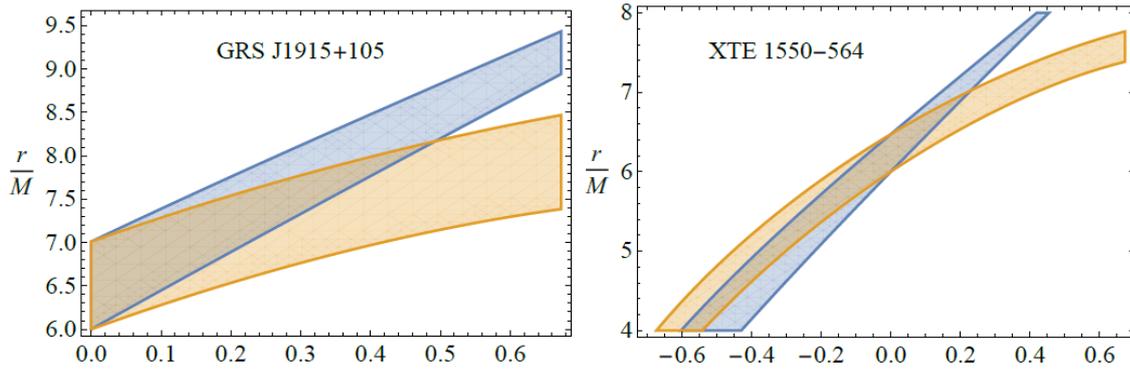


Figure 9: The range of orbits of twin peak QPOs called GRS J1915+105 and XTE 1550-564 shine around SMOG (light-blue area) and RMOG (light-orange area) BHs.

The ranges of radii of orbits where the object GRS J1915+105 and XTE 1550-564 are located around RMOG and SMOG BHs are located in light-blue and light-orange areas, respectively. One can see from the figure that at small MOG parameters the possible orbits of GRS J1915+105 and XTE 1550-564 take place across each other, and RMOG and SMOG BHs can not be distinguished. As the MOG parameter increase, the range of the orbits becomes larger and separable.

## Conclusion

Following conclusions have been obtained based on the results of the research for DSc dissertation “Constraining parameters of astrophysical black holes in various gravity theories based on observational data”:

1. Self-consistent constraints on the BH mass and charge based on data analysis from QPO in the microquasar GRS 1905-105
2. It is found that the BH charge can mimic the spin of the Kerr BH up to  $a/M=0.2927$  in the Bardeen model and to  $a/M=0.7687$  in the RBH1 one based on the analysis of QPO frequencies.
3. Constraints for the relationships of spin and charge of supermassive black holes M87\* and Sgr A\* have been obtained using their image size
4. The BH mass in the microquasar GRS 1915-105 and QPO orbits are found in the RP & WD models.
5. The distinguishing features of Schwarzschild and regular BHs in MOG are shown using data from twin peak QPOs.
6. It is obtained that  $\alpha=-0.33$  for XTE 1150-564 in the RMOG BH or  $\alpha=-0.48$  in SMOG one, while it is for GRS 1915+105 (SMOG BH  $\alpha=-0.17$  or RMOG BH  $\alpha=-0.26$ ).
7. Twin peak QPO orbits locates out of ISCO around BHs, and when QPO orbit matches with ISCO the two peaks coincide with each other.
8. This assumption may be helpful in solving the ISCO measurement problems in astrophysical observations
9. High (low) frequency QPOs generate at the orbits near to (far from) ISCO.
10. It is shown that the NC parameter does not affect the frequencies of twin peak QPOs.
11. The role of quantum effects in the region around photonsphere around NCi BHs is negligible.

**ФУНДАМЕНТАЛЬНЫХ И ПРИКЛАДНЫХ ИССЛЕДОВАНИЙ  
НАЦИОНАЛЬНОГО  
ИССЛЕДОВАТЕЛЬСКОГО УНИВЕРСИТЕТА “ТИИИМСХ”.**  

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**АСТРОНОМИЧЕСКИЙ ИНСТИТУТ  
ИНСТИТУТ ФУНДАМЕНТАЛЬНЫХ И ПРИКЛАДНЫХ  
ИССЛЕДОВАНИЙ**

**РАЙИМБАЕВ ДЖАВЛАНБЕК РАДЖАПБАЕВИЧ**

**ОЦЕНКА ПАРАМЕТРОВ ЧЕРНЫХ ДЫР В ТЕОРИЯХ ГРАВИТАЦИИ  
НА ОСНОВЕ АСТРОФИЗИЧЕСКИХ НАБЛЮДЕНИЙ**

**01.04.02 - Теоретическая физика**

**АВТОРЕФЕРАТ  
ДИССЕРТАЦИИ ДОКТОРА ФИЗИКО – МАТЕМАТИЧЕСКИХ НАУК (DSc)**

**Ташкент – 2022**

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## **ВВЕДЕНИЕ (аннотация диссертации доктора физико-математических наук DSc)**

### **Актуальность и востребованность темы диссертации.**

С математической точки зрения черные дыры (ЧД) являются решениями уравнений гравитационного поля. Впервые Карл Шварцшильд нашел решение ЧД, имеющее только массу, как точное решение вакуумных уравнений гравитационного поля Эйнштейна в 1916 году. С астрофизической точки зрения, эти релятивистские компактные гравитационные астрофизические объекты образовались на завершающей стадии звездной эволюции. По массе имеются три типа ЧД: (а) сверхмассивные ЧД ( $10^6$ - $10^{10}$  масс Солнца), которые расположены в центрах галактик; (б) черные дыры средней массы, масса которых  $10^2$ - $10^5$  масс Солнца) и их распределение в галактиках до сих пор остается неопределенным; и, наконец, (в) ЧД с массами от 3 до  $10^2$  масс Солнца, которые называются черными дырами звездной массы, и, в основном, встречаются в двойных системах. Недавние наблюдения коллаборациями Телескоп горизонта событий (Event Horizon Telescope) и GRAVITY привели к обнаружению теней двух сверхмассивных черных дыр, M87\* и Sgr A\*, что позволило оценить их массы и параметры вращения. Однако, изображения черных дыр средней и звездной массы не могут наблюдаться из-за их малого углового размера. К счастью, их можно обнаружить, в основном, по светимости излучения аккреционного диска, в частности, по частотам квазипериодических колебаний, что позволяет получить информацию о пространстве-времени вокруг ЧД и определить ограничения на их массу, заряд, спин и другие астрофизические параметры.

На самом деле, заряд ЧД и параметры альтернативных и модифицированных теорий гравитации могут привести к новым гравитационным эффектам вокруг ЧД, аналогично эффекту параметра вращения ЧД Керра, который уменьшает радиус фотонсферы и внутренней границы аккреционного диска вокруг ЧД. В этом смысле их влияние на геометрию пространства-времени вокруг ЧД практически неразличимо, и параметры могут имитировать друг друга. Другой факт заключается в том, что общая теория относительности Эйнштейна была хорошо проверена в режимах слабой и сильной гравитации и подтверждена многими астрофизическими процессами. Однако, многие альтернативные и модифицированные теории гравитации также успешно прошли астрофизические проверки, в которых ОТО не могло сыграть роль. Из всего вышеизложенного следует, что одной из наиболее актуальных задач теоретической астрофизики является проверка теорий гравитации и параметров ЧД по наблюдательным данным астрофизических ЧД.

В нашей стране также уделяется большое внимание изучению механизмов излучения в аккреционном диске ЧД, оптических и энергетических процессов вокруг ЧД, теоретическому изучению гравитационных теорий и их проверке на основе данных наблюдений.

Данная диссертационная работа соответствует задачам, утвержденным в

государственных нормативных документах: Указе президента Республики Узбекистан № УП-4947 “О Стратегии действий по дальнейшему развитию Республики Узбекистан” от 7 февраля 2017 года, Постановлении Президента Республики Узбекистан № ПП-2789 «О мерах по дальнейшему совершенствованию Академии Наук, организации, управления и финансирования научно-исследовательской деятельности» от 18.02.2017 года.

### **Соответствие исследования приоритетным областям научно-технического развития Республики Узбекистан.**

Диссертационное исследование выполнено в соответствии с приоритетным направлением развития науки и технологий в Республике Узбекистан П. “Энергетика, энерго- и ресурсосбережение”.

### **Степень изученности проблемы**

Механизмы излучения в аккреционном диске ЧД, в частности, генерация квазипериодических колебаний (КПК) и их изучение в рамках различных теорий гравитации широко проводились в Европе (Л. Реццолла, М. Абрамович, Г. Торок, П. Бакала, З. Стухлик, М. Колош и др.), в США (С. Мотта, Л. Стелла, М. Виетри, С. Спилберг, В. Вагонер), а также ученые из развитых стран в мире (С. Като, Дж. Фукуэ (Япония), Д. Ван, Л. Ченг и М. Чжан (Китай)).

Теоретические исследования теней ЧД в различных теориях гравитации также проводились рядом ученых (Ян Ше, Али Овгун, Кимет Джусуфи, Мубашер Джамиль и др.).

В нашей республике теоретическими исследованиями оптических и энергетических процессов вокруг ЧД в различных теориях гравитации также потенциально занимались Б. Ахмедов, А. Абдужаббаров, Ф. Атамуратов, Б. Тошматов, С. Шайматов, А. Абдикамалов и др.

Однако, в отмеченных исследованиях проверка теорий гравитации и получение ограничений на параметры ЧД в теориях гравитации по данным наблюдений теней ЧД и квазипериодических колебаний не проводились. Кроме того, имитирующие эффекты различных параметров ЧД также не были широко изучены.

### **Связь темы диссертации с научно-исследовательскими работами научно-исследовательского учреждения, где выполнена диссертация**

Диссертация выполнена в рамках научных проектов Института ядерной физики АН РУз: Ф-ФА-2021-510 «Исследования ядерной материи нейтронных звезд в модифицированной гравитации» (2021-2026 гг.).

**Целью исследования** является проверка теорий гравитации и ограничение параметров ЧД с использованием КПК и теней ЧД.

### **Задачи исследования:**

- Исследовать имитирующие значения параметров черных дыр, обеспечивающие одинаковые значения частот КПК;
- получить предельные значения параметров СМЧД M87\* и SgrA\* по размеру их теней;
- проверка эффектов квантовой гравитации вокруг ЧД с использованием данных КПК.

**Объектом исследования** являются релятивистские компактные гравитационные объекты - черные дыры.

**Предметом исследования** является КПК от маломассивных рентгеновских двойных систем и тени сверхмассивных черных дыр.

**Методами исследования** являются математический аппарат теорий гравитации и численные методы статистики.

**Научная новизна** исследования заключается в следующем:

- впервые получены ограничения на массу и заряда ЧД по наблюдательным данным КПК;
- впервые показано, что заряд ЧД может имитировать спин вращающейся ЧД Керра, при одинаковые значения верхней и нижней частот в двухпиковых КПК;
- впервые также были получены ограничения на спиновые и зарядовые параметры заряженных черных дыр в 4D гравитации Эйнштейна Гаусса-Бонне с использованием размеров изображения M87\* и Sgr A\*;
- впервые показано, что эффекты квантовой гравитации на КПК вокруг ЧД и ее тени пренебрежимо малы;
- впервые сформулирован способ различения стандартных и регулярных ЧД в МОГ с помощью исследований КПК;
- впервые показано, что радиус ВКСО вокруг ЧД можно оценить по орбитам КПК.

**Практические результаты исследования:**

- впервые получены ограничения на массу и заряд ЧД в микроквазарах GRS 1915-105 с использованием данных наблюдений двойных пиков КПК
- разработан новый альтернативный подход к определению радиуса ВКСО вокруг ЧД с использованием двойных пиков КПК
- впервые получены ограничения на параметр МОГ шварцшильдовской МОГ и обычных МОГ ЧД по данным наблюдений КПК GRS 1915-105 и XTE 1550-564

**Достоверность результатов исследования** обеспечена следующим:

- использование современных методов теории поля и эффективных численных методов и алгоритмов;

- тщательная проверка соответствия полученных теоретических результатов наблюдательным данным и результатам других авторов; согласованность выводов с основными положениями теории поля компактных гравитационных объектов.

**Научная и практическая значимость результатов исследования обуславливается возможностью:**

- полученные ограничения на параметры ЧД могут позволить получить информацию о гравитационных особенностях пространства-времени вокруг ЧД, а также выяснить, какие гравитационные эффекты играют доминирующую роль;

- Полученные результаты могут позволить в определении параметров черных дыр звездной массы и пространственно-временных особенностей вокруг них на основе данных наблюдений за квазарными ядрами и тенями сверхмассивных черных дыр, а также наблюдений квазикомплексов и аккреции черных дыр. Можно объяснить физические механизмы в аккреционном диске;

- кроме того, полученные результаты могут быть полезны при определении возможностей отождествления сингулярных и регулярных ЧД в модифицированной и квантовой теориях гравитации.

### **Внедрение результатов исследования**

- Формализм определения масс и зарядов ЧД по наблюдательным данным квазаров применялся рядом авторов при исследовании пространственно-временных свойств вокруг ЧД, а также в гравитационных моделях (The Astrophysical Journal Vol.935,91, (2022), Progress of Physics, Vol. 70, issue 9-10, 2200053, (2022), Journal of Cosmology and Astroparticle Physics, Volume 09, id 061, (2022), European Physics Journal C, Volume 82, id 636 (2022), Progress of Physics, Volume70, Issue 9-10, id 2200053, (2022), Universe Volume 8, issue 3, id 182 (2022)) , Результаты предоставлены для того, чтобы сделать возможным анализ в различных моделях темной материи вокруг ЧД на основе наблюдательных данных ЧД;

- Получение ограничений на параметры ЧД с помощью анализа данных изображений сверхмассивных ЧД использовалось рядом авторов при анализе различных моделей гравитации и пространственно-временных характеристик близлежащих ЧД на основе данных наблюдений их теней (Chinese Journal of Physics Vol. 78, pp.141-154, (2022), Annals of Physics Vol.441, 168892,(2022), Universe, Volume 8, issue 10, id 536 (2022), Physics, Volume 4, issue 4, pp.1318-1330 (2022), European Physics Journal C, Volume 82, id 831 (2022), Annals of Physics, Volume 441, id 168892 (2022)) Полученные результаты были использованы при анализе орбит фотонов и их теней вокруг вращающихся заряженных черных дыр;

### **Апробация результатов исследования**

Результаты, полученные в диссертационной работе, были доложены на 2 международных и 1 республиканских конференциях, а так же регулярные еженедельные узбекско-казахстанские семинары по теоретической физике и астрофизике.

### **Опубликованность результатов исследования**

По результатам диссертационного исследования опубликовано 20 научных работ, из них 16 статей в реферируемых журналах.

**Объём и структура диссертации** Диссертация состоит из введения, четырех глав, заключения и списка литературы.

- В первой главе мы изучаем квазипериодические колебания вокруг заряженных черных дыр, полученные в ОТО, объединяя линейную и нелинейную электродинамику, и получаем ограничения на заряд и массу черных дыр, используя данные квазипериодических колебаний, наблюдаемых в микроквазаре GRS 1915-105.

- Во второй главе мы впервые получили решение вращающейся заряженной черной дыры в четырехмерной гравитации Эйнштейна-Гаусса-Боннета с использованием алгоритма Ньюмана-Яниса. Затем мы рассмотрели движение фотона вокруг черной дыры и тени от нее. У нас также есть отношения между зарядом и параметром спина черной дыры, полученные для различных значений параметра Гаусса-Боннета с использованием данных наблюдений по размеру тени сумермассивных черных дыр M87\* и Sgr A\*.

- В третьей главе мы исследовали влияние квантовой некоммутативной гравитации на квазипериодические колебания и тени черных дыр. Показано, что роль квантовой некоммутативной теории незначительна на частотах квазипериодических колебаний вокруг черных дыр и размерах теней черных дыр.

- В последней четвертой главе мы показали новый подход к различению регулярных и сингулярных черных дыр в модифицированной гравитации с использованием данных наблюдений квазипериодических колебаний. Ограничения на параметр модифицированной гравитации для регулярных и сингулярных черных дыр в микро квазарах XTE 1150-564 и GRS 1915-105.

## Заклучение

По результатам исследований, проведенных по теме DSc диссертации “Оценка параметров черных дыр в теории гравитации на основе астрофизических наблюдений”, представлены следующие выводы:

1. Самосогласованные ограничения на массу и заряд черных дыр на основе анализа данных квазипериодических колебаний в микроквазаре GRS 1905-105.
2. Установлено, что заряд черных дыр может имитировать спин керровской черных дыр до  $a/M=0,2927$  в модели Бардина и до  $a/M=0,7687$  в модели RBH1 на основе анализа частот квазипериодических колебаний.
3. Ограничения на соотношения спина и заряда сверхмассивных черных дыр M87\* и Sgr A\*, полученные с использованием размера их изображений
4. Масса черного дыра находящие в центре микроквазара GRS 1915-105 и орбиты квазипериодических колебаний найдены в моделях релятивистской прецессии и искривленного диска.
5. Отличие в свойствах черных дыр Шварцшильда и регулярных черных дыр в модифицированной гравитации показаны с использованием данных квазипериодических колебаний двойного пика.
6. Получено, что параметр модифицированной гравитации  $\alpha=-0,33$  для XTE 1150-564 в регулярных черных дырах и  $\alpha=-0,48$  в черных дырах Шварцшильда, а также для черных дыр Шварцвальд в GRS 1915+105  $\alpha=-0,17$  и, а регулярных черных дыр  $\alpha=-0,26$ .
7. Двойной пик орбиты квазипериодических колебаний располагается вне ISCO вокруг черных дыр, и когда орбита квазипериодических колебаний совпадает с ISCO, два пика совпадают друг с другом.
8. Это допущение может оказаться полезным при решении задач измерения ISCO в астрофизические наблюдения
9. Высоко и низкочастотные квазипериодических колебаний генерируются на орбитах близких и далеких от ISCO, соответственно.
10. Показано, что параметр некоммутативности не влияет на частоты двухпиковых квазипериодических колебаний.
11. Показано, что роль квантовых эффектов в области фотонсферы вокруг некоммутативных черных дыр незначительна.

**ЭЪЛОН Қ ИЛИНГАН ИШЛАР РЎЙХАТИ**

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