

**QARSHI DAVLAT UNIVERSITETI HUZURIDAGI
ILMIY DARAJALAR BERUVCHI
PhD.03/30.06.2020.FM.70.04 RAQAMLI ILMIY KENGASH**

BUXORO DAVLAT UNIVERSITETI

XUDAYAROV SA’NAT SAMADOVICH

$(\sigma | \mu)$ -TIPLI KVADRATIK STOXASTIK DINAMIK SISTEMALAR

01.01.01 – Matematik analiz

**Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi
AVTOREFERATI**

Qarshi – 2023

**Fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi
avtoreferati mundarijasi**

**Оглавление автореферата диссертации доктора философии (PhD) по
физико-математическим наукам**

**Contents of dissertation abstract of doctor of philosophy (PhD) on
physical-mathematical sciences**

Xudayarov Sa'nat Samadovich

$(\sigma | \mu)$ -tipli kvadratik stoxastik dinamik sistemalar..... 5

Khudayarov Sanat Samadovich

Quadratic stochastic dynamical systems of the type $(\sigma | \mu)$ 23

Худаяров Саънат Самадович

Квадратичные стохастические динамические системы типа $(\sigma | \mu)$ 43

E'lon qilingan ishlar ro'yxati

List of published works

Список опубликованных работ..... 47

**QARSHI DAVLAT UNIVERSITETI HUZURIDAGI
ILMIY DARAJALAR BERUVCHI
PhD.03/30.06.2020.FM.70.04 RAQAMLI ILMIY KENGASH**

BUXORO DAVLAT UNIVERSITETI

XUDAYAROV SA’NAT SAMADOVICH

$(\sigma | \mu)$ -TIPLI KVADRATIK STOXASTIK DINAMIK SISTEMALAR

01.01.01 – Matematik analiz

**Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi
AVTOREFERATI**

Qarshi – 2023

Fizika-matematika fanlari bo'yicha falsafa doktori (Doctor of Philosophy) dissertatsiyasi mavzusi Oliy attestatsiya komissiyasida B2020.4.PhD/FM531 raqam bilan ro'yxatga olingan.

Dissertatsiya Buxoro davlat universitetida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o'zbek, rus, ingliz (rezyume)) Ilmiy kengash veb-sahifasi (www.qarshidu.uz) va "Ziyonet" axborot ta'lim tarmog'ida (www.ziyonet.uz) joylashtirilgan.

Ilmiy rahbar:

Rozikov Utkir Abdulloyevich
fizika-matematika fanlari doktori, professor

Rasmiy opponentlar:

Jamilov Uygun Umurovich
fizika-matematika fanlari doktori

Usmonov Javoxir Baxodir o'g'li
fizika-matematika fanlari bo'yicha
falsafa doktori (PhD)

Yetakchi tashkilot:

Namangan davlat universiteti

Dissertatsiya himoyasi Qarshi davlat universiteti huzuridagi PhD.03/30.06.2020.FM.70.04 raqamli Ilmiy kengashning 2023 yil "____" _____ soat ____ dagi majlisida bo'lib o'tadi (Manzil: 180103, Qarshi sh., Ko'chabog' ko'chasi, 17-uy. Tel.: (+998 75) 225-34-13; faks: (+998 75) 221-00-56, e-mail: qarshidu@umail.uz). Qarshi davlat universiteti 2-binosi, 202-xona.

Dissertatsiya bilan Qarshi davlat universitetining Axborot-resurs markazida tanishish mumkin (____ raqami bilan ro'yxatga olingan). (Manzil: 180103, Qarshi sh., Ko'chabog' ko'chasi, 17-uy. Tel.: (+998 75) 225-34-13; faks: (+998 75) 221-00-56, e-mail: qarshidu@umail.uz).

Dissertatsiya avtoreferati 2023 yil "____" _____ kuni tarqatildi.
(2023 yil "____" _____ dagi ____ raqamli reestr bayonnomasi).

B.A.Shoimqulov
Ilmiy darajalar beruvchi
Ilmiy kengash raisi,
f.-m.f.d., professor

Sh.D.Nodirov
Ilmiy darajalar beruvchi
Ilmiy kengash ilmiy kotibi,
f.-m.f.f.d. (PhD)

A.A.Imomov
Ilmiy darajalar beruvchi
Ilmiy kengash qoshidagi
Ilmiy seminar raisi
f.-m.f.d. (DSc), dotsent

KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahon miqiyosida olib borilayotgan ko‘plab amaliy ishlar aksariyat hollarda stoxastik va nostoxastik kubik matritsalar orqali aniqlangan evalyutsion operatorlar dinamikasini aniqlashda samarali qo‘llanilmoqda. Kubik matritsalarining vaqtga bog‘liq oilasi biologiya va fizika masalalarini matematik modelini ifodalashda foydalaniladi. Shu sababli kubik matritsalar yordamida qurilgan operatorlar va simpleksni saqlovchi stoxastik va nostoxastik kubik matritsalar yordamida qurilgan noxiziqli operatorlar dinamikasi muhim masalalaridan biri bo‘lib qolmoqda.

Dunyoda xaotik dinamik sistema hosil qiluvchi stoxastik va nostoxastik kvadratik operatorlar hamda kvadratik nostoxastik operatorlar hosil qilgan dinamik sistemalarni aniqlashga doir ilmiy izlanishlar olib borilmoqda. Shu jumladan, biror fiksirlangan ko‘paytma va stoxastiklikka nisbatan Kolmogorov-Chapman tenglamasini qanoatlantiruvchi matritsalar oilasi tadqiq qilish, kubik matritsalar hosil qilgan kvadratik stoxastik va nostoxastik jarayonlarning dinamikasining tuzilishini taqiq qilish, stoxastik va nostoxastik kvadratik operator xaotik dinamik sistema hosil qilishi uchun uning parametrlari qanday shartlarni qanoatlantirishini topish masalalarini o‘rganishga alohida e‘tibor qaratildi.

Mamlakatimizda so‘nggi yillarda fundamental fanlarning ilmiy va amaliy tadbiqiga ega bo‘lgan statistik fizika, biologiya va geologiyaning dolzarb yo‘nalishlariga e‘tibor qaratilmoqda. Jumladan, genitika, populyatsion biologiyada uchraydigan asosiy obyektlardan bo‘lgan simpleksni saqlovchi stoxastik va nostoxastik kubik matritsalar yordamida qurilgan noxiziqli operatorlar dinamikasi topishga oid salmoqli natijalarga erishildi. “Funksional analiz, matematik fizika va statistik fizika” fanlarining ustuvor yo‘nalishlari bo‘yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish matematika fanining asosiy vazifalari va faoliyat yo‘nalishlari etib belgilandi¹. Qaror ijrosini ta‘minlashda ilmiy natijalardan ilm-fanning turdosh sohalaridan foydalanish maqsadida noxiziqli operatorning dinamik sistemalari nazariyasini tadqiq etishni rivojlantirish muhim ahamiyat ega.

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PF-4947-son “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha Harakatlar strategiyasi to‘g‘risida”gi Farmoni, 2019-yil 9-iyuldagi PQ-4387-son “Matematika ta‘limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar Akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi Farmoni, 2020-yil 7-maydagi PQ-4708-son “Matematika sohasidagi ta‘lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy

¹ O‘zbekiston Respublikasi Vazirlar mahkamasi 2017-yil 18-maydagi “O‘zbekiston Respublikasi Fanlar akademiyasining yangidan tashkil etilgan ilmiy tadqiqot muassasalari faoliyatini tashkil etish to‘g‘risida”gi 292-sonli qarori.

hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalar rivojlanishining ustuvor yo'nalishlariga bog'liqligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. "Matematika, mexanika va informatika" ustuvor yo'nalishi doirasida bajarilgan.

Muammoning o'rganilganlik darajasi. Fizika yoki biologiyadagi matematik modellarni o'rganish uzluksiz yoki diskret vaqtli kvadratik tasodifiy jarayonlarni o'rganishga keltiriladi. Bunday holda Kolmogorov-Chapman tenglamasini qanoatlantiradigan kubik matritsalar oilasini qurish muhim masala hisoblanadi.

Maksimov tomonidan odatdagi matritsani ko'paytirishni umumlashiruvchi kub matritsalarini assotsiativ ko'paytirish kiritildi. U kubik stoxastik matritsalarining analoglarini kiritdi va ularning ehtimollik talqinlarini berdi. Stoxastik matritsalarining kubik statsionarligi tavsiflandi va kubik stoxastik matritsalarining statsionarga yaqinlashishi haqidagi tasdiqni isbotladi. Markov jarayoni tushunchasini umumlashiruvchi Markov o'zaro ta'sir jarayoni tushunchasini kiritdi va bunday jarayonning ergodikligi tushunchasi tabiiy ravishda kub matritsalarini assotsiativ ko'paytirish bilan bog'liqligini ko'rsatdi.

J.M.Kasas, M.Ladra va U.A.Rozikovlar tomonidan kubik stoxastik matritsalarining Markov jarayoni kiritildi, bu jarayon kvadratik stoxastik jarayon (KSJ) deb ham ataladi. KSJ uzluksiz vaqtli dinamik sistemalarning xususiy holi bo'lib, Kolmogorov-Chapman tenglamasining (KChT) analogini qanoatlantiradigan stoxastik kubik matritsalar yordamida hosil qilinadi. Kubik matritsalar o'rtasida ko'paytirishning bir nechta turlari mavjud bo'lganligi sababli, dastavval kubik matritsalar uchun ko'paytirish amalini kiritish lozim, so'ngra belgilangan ko'paytirish amaliga nisbatan KChTni qanoatlantiruvchi matritsalar oilasini ajratish lozim. J.M.Kasas, M.Ladra, U.A.Rozikovlar maqolalarida KSJ ni qurish uchun maxsus tanlangan ikkita turdagi stoxastik kubik matritsalar qaralib, bunday matritsalar uchun ikki turdagi ko'paytma amali (Maksimov ko'paytmasi sifatida ma'lum bo'lgan) qaralgan. KSJlarning keng sinfi va vaqtga bog'liq bo'lgan bunday jarayonlarning ba'zi holatlari berilgan. Bundan tashqari, egizaklar tug'ilishi mumkin bo'lgan populyatsiyaning vaqtinchalik xatti-harakatlarini (dinamikasini) tavsiflovchi KSJni biologiyaga qo'llanilishiga doir misollar keltirilgan.

U.A.Rozikov va Sh.N.Murodov ishlarida ikki o'lchovli evolyutsion algebralarni zanjirlariga 25 ta turdagi misollar qurilgan. Qurilgan 25 ta zanjirlarning barchasini barik xususiyati, absolyut nilpotent elementlar to'plami va idempotent elementlar to'plamining vaqt funksiyasi sifatida dinamikasi o'rganilgan.

R.N.G'anixo'jaevning ishlarida Volterra tipidagi kvadratik stoxastik operatorlar nazariyasi o'rganilgan. Keyinchalik chiziqli va nochiziqli stoxastik operatorlar U.A.Rozikov, X.Akin, U.U.Jamilov, A.Zada, O.N.Hakimov, F.M.Muhamedov, A.Yu.Hamrayev, F.A.Shahidi, N.N.G'anixo'jaev, J.P.Tian, S.K.Shoyimardonov, M.V.Velasco, R.Varrolarning ilmiy ishlarida o'rganilgan.

U.A.Rozikov tomonidan 2020-yilda yozilgan “Populatsiya dinamikasi: algebraik va ehtimolli yondashuv” nomli kitobining ehtimolli yondoshuv qismida uzluksiz dinamik sistemalarni hosil qiluvchi va Kolmogorov-Chapman tenglamasining analogini qanoatlantiruvchi kubik stoxastik (belgilangan ma’noda) matritsalarining Markov jarayonlari (KSMMJ) keltirilgan. Ikkita maxsus tanlangan ikki tipdagi stoxastik kubik matritsalar qaralib, bunday matritsalarda Maksimovning ikki turdagi ko‘paytmasi uchun KSMMJi bayon qilingan. Bunday jarayonlarning vaqtga bog‘liq xatti-harakatlari egizak tug‘ilish ehtimoli bo‘lgan populyatsiyaga qo‘llanilishi taqiq etilgan.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya tadqiqoti Buxoro davlat universiteti ilmiy tadqiqot ishlari rejasining 2017-2022-yillarga mo‘ljallangan M.01.2017-raqamli “Chiziqli operatorlarning spektral nazariyasi” ilmiy tadqiqot loyihalari doirasida bajarilgan.

Tadqiqotning maqsadi turli stoxastik kubik matritsalar uchun aniqlangan ko‘paytmalar yordamida Kolmogorov-Chapman tenglamasi o‘rinli bo‘lish shartlarini topish. Kvadratik operator simpleksni saqlashi uchun kvadrat operatorning koeffitsiyentlariga zarur va etarli shartni topishdan iborat.

Tadqiqotning vazifalari:

kvadratik operator simpleksni saqlashi uchun uning koeffitsiyentlariga zaruriy va yetarli shartlarni topish;

kvadratik nostoxastik operatorlar hosil qilgan dinamik sistemalarni o‘rganish; xaotik dinamik sistema hosil qiluvchi nostoxastik kvadratik operatorlarni aniqlash;

berilgan ko‘paytma va stoxastiklikka nisbatan kvadratik stoxastik jarayonni hosil qiladigan kubik matrisalar yordamida qurilgan dinamik sistemalarni o‘rganish;

kiritilgan ko‘paytmalarga nisbatan Kolmogorov-Chapman tenglamasini qanoatlantiradigan kubik matritsalar oilasini qurish. Hosil qilingan dinamik sistemalarning vaqt bo‘yicha evolyutsiyasini o‘rganish.

Tadqiqotning obyekt Maksimov ko‘paytmasi yordamida Kolmogorov-Chapman tenglamasini qanoatlantiruvchi kubik matritsalar yordamida aniqlangan kvadratik stoxastik operatorlar dinamikasi olingan.

Tadqiqotning predmetini haqiqiy o‘zgaruvchili funksiyalar nazariyasi, dinamik sistemalar, tasodifiy jarayonlar nazariyasi, kubik matritsalar nazariyasi.

Tadqiqot usullari. Bitiruv ishida matematik tahlil, funksional tahlil, stoxastik jarayonlar va dinamik sistemalar nazariyasi usullaridan foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

kvadratik operator simpleksni saqlashi, ya’ni simpleksni o‘zini-o‘ziga akslantirishi uchun uning koeffitsiyentlariga zarur va yetarli shartlari topilgan;

ixtiyoriy bir o‘lchovli kvadratik nostoxastik operator dinamikasi to‘liq o‘rganilgan hamda operator parametrlarining ba’zi qiymatlarida xaotik dinamik sistema hosil qilishi shartlari logistik akslantirish xossalardan foydalangan holda aniqlangan;

ikki o'lovli kvadratik nostoxastik operatorlar orasida xaotik dinamik sistema hosil qiluvchi va xaotik dinamik sistema hosil qilmaydigan sinflari qurilgan bo'lib, ushbu operatorlarning dinamikasi to'la tasniflangan;

kubik matritsalar uchun bir nechta turdagi maxsus ko'paytma kiritilgan, hamda bu ko'paytmalarga nisbatan kvadratik stoxastik jarayonlarni hosil qiluvchi matritsalar oilasi Kolmogorov-Chapman tenglamasini qanoatlantirishi isbotlangan. Ushbu kvadratik stoxastik jarayonlarning vaqt o'zgargandagi limit nuqtalari topilgan.

Tadqiqotning amaliy natijasi quyidagilardan iborat:

Matematik biologiyada populatsiya jarayonlarining matematik modellarini kubik matritsalar yordamida tuzilgan noxiziqli stoxastik va nostoxastik kvadratik operatorlar bilan ifodalash taklif etilib, ushbu operatorlar yordamida qurilgan dinamik sistemalarda traektoriyaning limit nuqtalari to'plamini aniqlash usullaridan foydalanilgan.

Tadqiqot natijalarining ishonchliligi. matematik va funksional analiz, dinamik sistemalar nazariyasi usullaridan foydalanilgani hamda olingan natijalar matematik jihatdan qat'iy isbotlangani bilan asoslanadi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Dissertatsiya natijalarining ilmiy ahamiyati dinamik sistemalardagi traektoriyalarning limit nuqtalari to'plami tavsifi diskret vaqtli chiziqli bo'lmagan dinamik sistemalar nazariyasida qo'llanilishi mumkinligi bilan izohlanadi.

Dissertatsiyaning amaliy ahamiyati chiziqli bo'lmagan operator hosil qilgan dinamik sistemalarda traektoriyaning limit nuqtalari to'plamini aniqlash orqali matematik biologiyadagi populyatsiya jarayonlariga qo'llanilishi bilan izohlanadi.

Tadqiqot natijalarini joriy qilinishi. Dissertatsiya tadqiqoti jarayonida olingan ilmiy natijalar quyidagi yo'nalishlarda amaliyotga joriy qilingan:

nostoxastik kvadratik operatorlar hosil qilgan dinamik sistemalar va ularning limit nuqtalari haqidagi natijalardan Malayziya Xalqaro islom universitetining FRGS21-230-0839-raqamli "Dynamics of finite dimensional orthogonality preserving cubic stochastic operators" nomli loyihasida foydalanilgan (Malayziya Xalqaro islom universitetining 2023 yil 15-yanvardagi ma'lumotnomasi). Ilmiy natijalarni qo'llash biologik sistemalar, matematik biologiya va boshqa tabiiy fanlarda uchraydigan evolyutsion operatorlarni tasniflash imkonini bergan;

berilgan ko'paytmaga nisbatan Markov jarayonlarini tashkil qiluvchi nostoxastik kubik matritsalar hosil qilgan kvadratik operatorlar oilasining dinamikasiga oid natijalardan Birlashgan Arab Amirliklari universitetining G00003447-raqamli "Quantum genetic algebras and their applications" loyihasida foydalanilgan (Birlashgan Arab Amirliklari Universitetining 2023-yil 06-yanvardagi ma'lumotnomasi). Ilmiy natijani qo'llash standart simpleksda aniqlangan bir nechta kvadratik stoxastik operatorlar hosil qilgan dinamik sistemalarni tekshirish imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Tadqiqotning asosiy natijalari 3 ta xalqaro va 5 ta Respublika ilmiy-amaliy anjumanlarida muhokamadan o'tgan.

Tadqiqot natijalarining e'lon qilinganligi. Dissertatsiya mavzusi bo'yicha jami 13 ta ilmiy ish chop etilgan, shulardan, O'zbekiston Respublikasi Oliy attestatsiya komissiyasining falsafa doktori dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 5 ta maqola, jumladan, 2 tasi xorijiy va 3 tasi respublika jurnallarida nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro'yxatidan tashkil topgan. Dissertatsiyaning hajmi 94 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiyada tanlangan mavzuning dolzarbligi va zarurati asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo'nalishlariga mosligi yoritilgan, mavzu bo'yicha xorijiy va mahalliy ilmiy-tadqiqot ishlari harhi, muammoning o'rganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, obyekt va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan maqolalar va dissertatsiya tuzilishi bo'yicha ma'lumotlar keltirilgan.

Dissertatsiyaning "**Dastlabki tushunchalar. Kvadratik operatorning dinamik sistemalari**" deb nomlanuvchi birinchi bobida dissertatsiya mavzusini to'la yoritish uchun zarur bo'lgan asosiy ta'riflar va muhim tushunchalar keltirilgan. Shuningdek ikki o'lchovli nostoxastik operatorning qo'zg'almas nuqtalari, invariant to'plamlari, traektoriyasi haqidagi natijalar bayon qilingan.

Aytmalik $I = \{1, 2, \dots, m\}$ bo'lsin. $x = (x_1, \dots, x_m)$ - I to'plamning ehtimollik taqsimoti bo'lsin, ya'ni x

$$S^{m-1} = \left\{ x \in \mathbb{R}^m : x_i \geq 0, \sum_{i=1}^m x_i = 1 \right\}. \quad (1)$$

simpleks elementi bo'lsin.

Kvadratik stoxastik operator deb S^{m-1} simpleksni o'zini-o'ziga akslantiruvchi quyidagi akslantirishga aytiladi:

$$V : x'_k = \sum_{i,j=1}^m P_{ij,k} x_i x_j, \quad k = 1, \dots, m \quad (2)$$

bu yerda $P_{ij,k}$ koeffitsiyentlar

$$P_{ij,k} \geq 0, P_{ij,k} = P_{ji,k}, \sum_{k=1}^m P_{ij,k} = 1, i, j, k = 1, \dots, m \quad (3)$$

shartlarni qanoatlantiradi.

Har bir (3) shart bilan aniqlangan kvadratik stoxastik operator V biror $\mathbf{P} = (P_{ij,k})_{i,j,k=1}^m$ kubik matritsa yordamida bir qiymatli aniqlanadi.

(2) ko‘rinishida aniqlangan kvadratik stoxastik operator ta’sirida $x^{(0)} \in S^{m-1}$ boshlang‘ich nuqtaning traektoriyasi (orbitasi) $\{x^{(n)}\}$ quyidagicha aniqlanadi:

$$x^{(n+1)} = V(x^{(n)}), \quad n = 0, 1, 2, \dots$$

Umuman olganda \mathbf{P} kubik matritsaga mos V , $V: x \in \mathbb{R}^m \rightarrow x' = V(x) \in \mathbb{R}^m$ kvadratik operator (2) tenglik bilan aniqlanadi. Umumiylikka ziyon yetkazmagan holda $P_{ij,k} = P_{ji,k}$ deb olamiz. Haqiqatdan ham, agar bu tenglik bajarilmasa, u holda $\bar{P}_{ij,k} = \frac{1}{2}(P_{ij,k} + P_{ji,k})$ deb olamiz.

\mathcal{C} orqali haqiqiy sonlar maydoni ustida aniqlangan m^3 -o‘lchamli kubik matritsalar to‘plamini belgilaymiz. E_{ijk} , $i, j, k \in I$ matritsa \mathcal{C} to‘plamdagi birlik matritsa bo‘lsin, ya’ni E_{ijk} matritsa m^3 -o‘lchamli matritsa bo‘lib, (i, j, k) -o‘rinda turgan element 1 ga, qolgan elementlar 0 ga teng.

Birlik matritsalar o‘rtasida ko‘paytirish amalini quyidagicha kiritamiz:

$$E_{ijk} *_a E_{lnr} = \delta_{kl} E_{ia(j,n)r} \quad (4)$$

bu yerda $a: I \times I \rightarrow I$, $(j, n) \mapsto a(j, n) \in I$ ixtiyoriy assotsiativ binar amal, δ_{kl} esa Kroneker simvoli.

O_m orqali I dagi barcha binar amallar to‘plamini belgilaymiz.

Ko‘paytirishning umumiy formulasi (4) ni bichiziqilik bo‘yicha davom ettirish orqali topiladi, ya’ni ikkita $A = (a_{ijk}), B = (b_{ijk}) \in \mathcal{C}$ kubik matritsalar uchun $A *_a B = (c_{ijk})$ ko‘paytma

$$c_{ijr} = \sum_{l, n: a(l, n) = j} \sum_k a_{ilk} b_{knr}$$

tenglik bilan aniqlanadi. Bundan ko‘rish mumkinki $\{l, n: a(l, n) = j\} = \emptyset$ shartni qanoatlantiruvchi j uchun $c_{ijr} = 0$ bo‘ladi.

Agar har qanday $u, v \in I$ uchun $a(x, u) = v$ ($a(u, x) = v$) tenglama yagona yechimga ega bo‘lsa, a amal I da o‘ng (mos ravishda chap) yagona yechiluvchan deyiladi.

1-lemma. Agar a amal I da o‘ng yoki chap yagona yechiluvchan bo‘lsa, u holda

$$\sum_{d \in I} \sum_{j, m: a(j, m) = d} \gamma_{j, m} = \sum_{j \in I} \sum_{m \in I} \gamma_{j, m}$$

munosabat o‘rinli bo‘ladi.

Stoxastiklik. Bir nechta turdagi kubik stoxastik matritsalar ta’rif berib o‘tamiz. Aytaylik $\mathbf{P} = (P_{ij,k})_{i, j, k=1}^m$ kubik matritsa berilgan bo‘lsin.

• agar $P_{ij,k} \geq 0$, $i, j, k = \overline{1, m}$ va har bir k uchun $\sum_{i, j=1}^m P_{ij,k} = 1$ bo‘lsa, \mathbf{P} kubik

matritsa (1, 2) - stoxastik deyiladi;

• agar $P_{ij,k} \geq 0, i, j, k = \overline{1, m}$ va har bir j uchun $\sum_{i,k=1}^m P_{ij,k} = 1$ bo'lsa, \mathbf{P} kubik matritsa (1,3)- *stoxastik* deyiladi;

• agar $P_{ij,k} \geq 0, i, j, k = \overline{1, m}$ va har bir i uchun $\sum_{j,k=1}^m P_{ij,k} = 1$ bo'lsa, \mathbf{P} kubik matritsa (2,3)- *stoxastik* deyiladi.

• agar $P_{ij,k} \geq 0, i, j, k = \overline{1, m}$ va har bir i va j uchun $\sum_{k=1}^m P_{ijk} = 1$ bo'lsa, \mathbf{P} kubik matritsa 3- *stoxastik* deyiladi.

Xuddi shu kabi birinchi va ikkinchi indekslar bo'yicha ham mos ta'riflarni berish mumkin. Maksimovning ishlarida juft stoxastiklik ta'rifi ham keltirilgan:

agar (2,3)-stoxastik kubik matritsa uchun barcha j, k da $\sum_{i=1}^m P_{ij,k} = \frac{1}{m}$ bo'lsa, \mathbf{P} *juft stoxastik* matritsa deyiladi.

S orqali barcha mumkin bo'lgan stoxastikliklar to'plamini va M orqali esa kubik matritsalar o'rtasidagi barcha ko'paytmalar to'plamini belgilaymiz.

Aytaylik, $s \geq 0, t \geq 0$ parametrlar vaqtni ifodalasin. s va t parametrlarga bog'liq kubik matritsani $M^{[s,t]} = \left(P_{ij,k}^{[s,t]} \right)_{i,j,k=1}^m$ ko'rinishida aniqlaymiz.

1-ta'rif. Agar har bir s va t vaqt uchun $M^{[s,t]}$ kubik matritsa $\sigma \in S$ stoxastik bo'lib, $\mu \in \mathbf{M}$ ko'paytmaga nisbatan

$$M^{[s,t]} = M^{[s,\tau]} *_{\mu} M^{[\tau,t]}, \text{ har bir } 0 \leq s < \tau < t \quad (5)$$

Kolmogorov-Chapman tenglamasini qanoatlantirsa, $\{M^{[s,t]} : s, t \in R_+\}$ kubik matritsalar oilasiga $(\sigma | \mu)$ -tipli Markov jarayoni (yoki kvadratik stoxastik jarayon) deyiladi.

2-ta'rif. S^{m-1} simpleksni saqlovchi (2) kvadratik stoxastik operatorning hech bo'lmaganda bitta $P_{ij,k}, i \neq j$ koeffitsiyenti manfiy bo'lsa, bu operatorga nostoxastik kvadratik operator deyiladi.

Ikki o'lchamli S^2 simpleksda aniqlangan nostoxastik kvadratik operatorga misol qaraymiz:

$$V_0 : \begin{cases} x' = 1/2 \cdot (z - y)^2 + 3/2 \cdot x(y + z) \\ y' = 1/2 \cdot (x - z)^2 + 3/2 \cdot y(x + z) \\ z' = 1/2 \cdot (y - x)^2 + 3/2 \cdot z(x + y) \end{cases} \quad (6)$$

$$a_1 = (0, 1/2, 1/2), a_2 = (1/2, 0, 1/2), a_3 = (1/2, 1/2, 0), a_4 = (1/3, 1/3, 1/3)$$

nuqtalar V_0 operatorning qo'zg'almas nuqtalari bo'ladi.

Bunda a_1, a_2 va a_3 nuqtalar egar, a_4 esa tortuvchi qo'zg'almas nuqta bo'ladi.

Quyidagicha to‘plamlarni aniqlaymiz:

$$\begin{aligned} M_1 &= \{(x, y, z) \in S^2 : x > y > z > 1/6\}, M_2 = \{(x, y, z) \in S^2 : x > z > y > 1/6\}, \\ M_3 &= \{(x, y, z) \in S^2 : y > x > z > 1/6\}, M_4 = \{(x, y, z) \in S^2 : y > z > x > 1/6\}, \\ M_5 &= \{(x, y, z) \in S^2 : z > x > y > 1/6\}, M_6 = \{(x, y, z) \in S^2 : z > y > x > 1/6\}, \\ l_1 &:= \{(x, y, z) \in S^2 : x = y, x + y + z = 1\}, l_2 := \{(x, y, z) \in S^2 : x = z, x + y + z = 1\}, \\ l_3 &:= \{(x, y, z) \in S^2 : y = z, x + y + z = 1\}. \end{aligned}$$

1-teorema. $M_i, i = 1, 2, 3, 4, 5, 6, l_j, j = 1, 2, 3$ to‘plamlar V_0 operatorga nisbatan invariant to‘plamlar bo‘ladi.

$v^{(0)} \in S^2$ boshlang‘ich nuqtaning V_0 operatorga nisbatan $\{v^{(n)}\}_{n=0}^\infty$ traektoriyani $v^{(n+1)} = V_0(v^{(n)})$, $n = 0, 1, 2, \dots$ kabi aniqlaymiz.

2-teorema. Har bir $v^{(0)} \in S^2 \setminus \{a_i, e_i : i = 1, 2, 3\}$ boshlang‘ich nuqta uchun quyidagi munosabat o‘rinli

$$\lim_{n \rightarrow \infty} V_0^{(n)}(v^{(0)}) = \lim_{n \rightarrow \infty} v^{(n)} = \lim_{n \rightarrow \infty} (x^{(n)}, y^{(n)}, z^{(n)}) = (1/3, 1/3, 1/3).$$

Dissertatsiya ishining ikkinchi bobi “**Nostoxastik matritsalarining kvadratik operatorlarida bo‘lingan xaos**” deb nomlanadi. Bu bobda kvadratik operator koeffitsiyentlari uchun operator simpleksni saqlashini ta‘minlovchi zaruriy va yeratli shartlar topilgan.

Ikkinchi bobning asosiy natijasi quyidagi teoremadan iborat.

3-teorema. V ((2) tenglik bilan aniqlangan) kvadratik operator S^{m-1} simpleksni saqlashi uchun quyidagi shartlarning bajarilishi yetarli

$$\text{i) } \sum_{k=1}^m P_{ij,k} = 1, \quad i, j = 1, \dots, m,$$

$$\text{ii) } 0 \leq P_{ii,k} \leq 1, \quad i, k = 1, \dots, m,$$

$$\text{iii) } -\frac{1}{m-1} \sqrt{P_{ii,k} P_{jj,k}} \leq P_{ij,k} \leq 1 + \sqrt{(1 - P_{ii,k})(1 - P_{jj,k})}$$

va (i), (ii) hamda iii’) $-\sqrt{P_{ii,k} P_{jj,k}} \leq P_{ij,k} \leq 1 + \sqrt{(1 - P_{ii,k})(1 - P_{jj,k})}$ shartlarning bajarilishi zarur.

1-eslatma. 1. A.T.Sarimsoqovning ishlarida $m = 2$ bo‘lgan holda kvadratik operator simpleksni saqlashi uchun (i), (ii) va (iii’) shartlarning bajarilishi zarur va yetarli ekanligi isbotlangan.

2. $m \geq 3$ bo‘lgan holda (i), (ii) va (iii’) shartlar kvadratik operator simpleksni saqlashi uchun yetarli emas. Quyidagicha misolni qaraymiz (misolda qaraladigan operator (i), (ii) va (iii’) shartlarni qanoatlantiradi):

$$P_{ii,1} = 1, \quad P_{ii,k} = 0, \quad \forall i = 1, \dots, m; \quad \forall k = 2, \dots, m;$$

$$P_{ij,1} = -\sqrt{P_{ii,1} P_{jj,1}} = -1, \quad \forall i \neq j; \quad P_{ij,k} \in [0, 2], \quad \forall i \neq j, k \geq 2 \text{ bunda } \sum_{k=2}^m P_{ij,k} = 2.$$

V operator uchun $x'_1 = (2x_1 - 1)^2 - 4 \sum_{2 \leq i < j \leq m} x_i x_j$ tenglikni hosil qilamiz.

Quyidagi $x_1 = \frac{1}{2}$ va $x_j > 0, j \geq 2$ bunda $\sum_{j=2}^m x_j = \frac{1}{2}$ shartlarni qanoatlantiruvchi

$x \in S^{m-1}$ nuqtani olamiz. U holda $x'_1 < 0$ bo'ladi, ya'ni $x' = V(x) \notin S^{m-1}$.

S^1 da kvadratik nostoxastik operatorni umumiy holda qaraymiz:

$$\begin{aligned} x' &= ax^2 + 2bxy + cy^2 \\ y' &= (1-a)x^2 + 2(1-b)xy + (1-c)y^2, \end{aligned} \quad (7)$$

bu yerda $a, c \in [0, 1], b \in [-\sqrt{ac}, 1 + \sqrt{(1-a)(1-c)}]$.

$x + y = 1$ tenglikni inobatga olib, (7) operatorni

$$x' = ax^2 + 2bx(1-x) + c(1-x)^2 = (a - 2b + c)x^2 + 2(b - c)x + c$$

kabi yozib olamiz va quyidagi funksiyani qaraymiz:

$$f(x) = (a - 2b + c)x^2 + 2(b - c)x + c. \quad (8)$$

Biz traektoriyasi xaotik bo'ladigan kvadratik stoxastik operatorlarni bilmaymiz. Ammo quyidagicha muhim holni ko'rib o'tamiz.

$a = c = 1, b \in [-1, 0)$ bo'lgan holda (7) operator

$$\begin{aligned} x' &= x^2 + 2bxy + y^2 \\ y' &= 2(1-b)xy. \end{aligned} \quad (9)$$

ko'rinishga keladi.

$P_{12,1} = b < 0$ bo'lganligi uchun bu operator nostoxastik bo'ladi. Ixtiyoriy $(1 - y_0, y_0) \in S^1$ boshlang'ich nuqta uchun uning traektoriyasi $(1 - y_n, y_n)$ kabi ko'rinishda bo'ladi, bunda $y_n = \mu y_{n-1}(1 - y_{n-1}), \mu = 2(1 - b), n = 1, 2, \dots, m$.

$f(x) = \mu(1 - x)$ logistik akslantirishning ma'lum xossalardan² foydalanib, $-1 \leq b < -0.784975$ bo'lganda (9) operator bir o'lchamli simpleksda xaotik dinamik sistema tashkil qilishini ko'rsatish mumkin.

Yuqorida biz bir o'lchamli kvadratik nostoxastik bo'lgan operatorlarni ko'rib chiqdik va bir o'lchamli kvadratik nostoxastik operatorlarni to'liq o'rganish mumkinligini ko'rdik, hatto ular orasida parametrlarining ba'zi qiymatlarida xaotik dinamik sistema tashkil qiladiganlari ham bor ekan. Ammo ikki o'lchamli kvadratik nostoxastik operatorlarni umumiy holda o'rganish qiyin, shuning uchun biz quyida ikkita aniq qiziqarli misolni qarab o'tamiz.

1. S^2 simpleksda quyidagicha kvadratik nostoxastik operatorni qaraymiz

$$\begin{aligned} x' &= x^2 + y^2 + z^2 - axy - axz + 2yz \\ y' &= (2 + a)xy \\ z' &= (2 + a)xz, \end{aligned} \quad (10)$$

bu yerda $a \in [0, 2]$. Ta'kidlash joizki $P_{12,1} = P_{13,1} = -a/2$.

² [https://en.wikipedia.org/wiki/Logistic map](https://en.wikipedia.org/wiki/Logistic_map)

$M_0 = \{(x, y, z) \in S^2 : y = 0\}$, $M_1 = \{(x, y, z) \in S^2 : z = 0\}$,
 $M_\omega = \{(x, y, z) \in S^2 : y = \omega z\}$, $\omega \in [0, +\infty)$ to‘plamlar (10) operatorga nisbatan invariant to‘plamlar bo‘ladi.

Bundan tashqari, $S^2 = M_0 \cup M_1 \cup \left(\bigcup_{\omega \in (0, +\infty)} M_\omega \right)$ tenglik o‘rinli bo‘ladi.

2-eslatma. (10) operator a parametr uchun $1.56995 < a \leq 2$ tengsizlik o‘rinli bo‘lganida xaotik bo‘ladi, ammo u simpleksda Devaniy ma’nosida xaos emas. Chunki, u topologik tranzitiv emas. Bu operator bo‘lingan xaos bo‘ladi, ya’ni simpleks sanoqsizta invariant qism to‘plamlarga ajralgan bo‘lib, (10) operator har bir invariant to‘plamda chegaralangan va Devaniy ma’nosida xaos.

2. S^2 simpleksda koeffitsiyentlari

$$P_{ii,1} = 1, P_{ii,k} = 0, \forall i = 1, 2, 3; \forall k = 2, 3; P_{ij,1} = -\frac{1}{2} \sqrt{P_{ii,1} P_{jj,1}} = -\frac{1}{2}, \forall i \neq j$$

$$P_{ij,k} \in [0, \frac{3}{2}], \forall i \neq j, k = 2, 3 \quad \text{bunda} \quad P_{ij,2} + P_{ij,3} = \frac{3}{2}$$

shartlar bilan aniqlangan kvadratik nostoxastik operatorni qaraymiz.

Ba’zi parametrlarni nolga tenglashtirib, quyidagicha V operatorni hosil qilamiz:

$$\begin{aligned} x' &= x^2 + y^2 + z^2 - xy - xz - yz \\ y' &= 3xy + ayz \\ z' &= 3xz + (3 - a)yz, \end{aligned} \quad (11)$$

bu yerda $a \in (0, 3)$.

3-eslatma. (11) operator uchun $a = 0$ va $a = 3$ hollarni ham qarash mumkin. Bu hollar $a \in (0, 3)$ bo‘lgan holga nisbatan ancha sodda. Misol uchun agar $a = 0$ bo‘lsa, u holda $z' = 3z(x + y) = 3z(1 - z)$ bo‘ladi, ya’ni z o‘zgaruvchi qolgan o‘zgaruvchilarga bog‘liq bo‘lmagan dinamikaga ega bo‘ladi. Shu sababli biz $a \neq 0$, $a \neq 3$ bo‘lgan holni qaraymiz.

(11) operatorning qo‘zg‘almas nuqtalari quyidagicha bo‘ladi:

$$s_1 = (1, 0, 0), s_2 = \left(\frac{1}{3}, 0, \frac{2}{3} \right), s_3 = \left(\frac{1}{3}, \frac{2}{3}, 0 \right), s_4 = \left(\frac{a^2 - 3a + 3}{a^2 - 3a + 9}, \frac{2a}{a^2 - 3a + 9}, \frac{2(3 - a)}{a^2 - 3a + 9} \right). \quad (12)$$

$T = \{(y, z) \in [0, 1]^2 : y + z \leq 1\}$ bo‘lsin.

1-tasdiq. s_1 qo‘zg‘almas nuqta itaruvchi. s_2 va s_3 qo‘zg‘almas nuqtalar giperbolik emas (ammo yarim-itaruvchi³). s_4 qo‘zg‘almas nuqtalar giperbolik emas (ammo yarim-tortuvchi⁴).

³ Ikkinchi xos qiymatning moduli birdan katta ekanligini anglatadi.

⁴ Ikkinchi xos qiymatning moduli birdan kichik ekanligini anglatadi.

Quyidagicha to‘plamlarni kiritamiz:

$$M_1 = \{(y, z) \in T : y = 0\}, \quad M_2 = \{(y, z) \in T : z = 0\}, \quad M_3 = \{(y, z) \in T : z = \frac{3-a}{a} y\},$$

$$M_4 = \{(y, z) \in T : z < \frac{3-a}{a} y\}, \quad M_5 = \{(y, z) \in T : z > \frac{3-a}{a} y\},$$

$$\hat{M}_i = \{(x, y, z) \in S^2 : (y, z) \in M_i\}, \quad i = 1, 2, 3, 4, 5.$$

U holda $S^2 = \bigcup_{i=1}^5 \hat{M}_i$ bo‘ladi.

4-teorema. Agar biror $i = 1, 2, 3, 4, 5$ uchun $(x^{(0)}, y^{(0)}, z^{(0)}) \in \hat{M}_i$ bo‘lsin, u holda (11) operator uchun quyidagi munosabat o‘rinli bo‘ladi

$$\lim_{n \rightarrow \infty} V^n(x^{(0)}, y^{(0)}, z^{(0)}) = \begin{cases} s_1, & \text{agar } x^{(0)} = 1 \text{ bo'lsa} \\ s_2, & \text{agar } i = 1, z^{(0)} > 0 \text{ bo'lsa} \\ s_3, & \text{agar } i = 2, y^{(0)} > 0 \text{ bo'lsa} \\ s_4, & \text{agar } i = 3, y^{(0)} > 0 \text{ bo'lsa} \\ \in \hat{M}_3, & \text{agar } i = 4, 5 \text{ bo'lsa} \end{cases}$$

bu yerda $s_i, i = 1, 2, 3, 4$ nuqtalar (12) tengliklar bilan aniqlangan.

Sonli usullardan foydalangan holda quyidagicha gipotezani qurishimiz mumkin

Gipoteza. Agar $(x^{(0)}, y^{(0)}, z^{(0)}) \in \hat{M}_4 \cup \hat{M}_5$ bo‘lsa, u holda (11) tenglik bilan aniqlangan operator uchun $\lim_{n \rightarrow \infty} V^n(x^{(0)}, y^{(0)}, z^{(0)}) = s_4$ munosabat o‘rinli bo‘ladi.

Shunday qilib, (11) operator xaotik dinamik sistema hosil qilmaydi.

Dissertatsiyaning “ $(\sigma | \mu)$ -**tipli kvadratik stoxastik jarayonlar**” deb nomlangan uchinchi bobida $(\sigma | \mu)$ -tipli kvadratik stoxastik jarayonlar qurilgan, ya’ni (5) tenglamani qanoatlantiruvchi kubik matritsalar oilasi topilgan. Bundan tashqari $t - s \rightarrow +\infty$ da bu kvadratik stoxastik jarayonlarning dinamikasi o‘rganilgan.

$Q = (q_{ij})_{i,j=1}^m$ kvadratik matritsa berilgan bo‘lib, uning elementlari uchun

$$q_{ij} \geq 0, \quad \forall i, j = 1, \dots, m; \quad \sum_{j=1}^m q_{ij} = 1, \quad \forall i = 1, \dots, m.$$

munosabat o‘rinli bo‘lsa, Q ga o‘ng stoxastik matritsa deyiladi.

Xuddi shu kabi elementlari manfiymas bo‘lgan matritsaning har bir ustun bo‘yicha elementlari yig‘indisi 1 ga teng bo‘lsa, bu matritsaga *chap stoxastik matritsa* deyiladi.

I da o‘ng (chap) yagona yechiluvchan bo‘lgan a Maksimov ko‘paytmasini qaraymiz. U holda (5) tenglama

$$P_{ijr}^{[s,t]} = \sum_{l,n:a(l,n)=j} \sum_{k=1}^m P_{ilk}^{[s,\tau]} P_{knr}^{[\tau,t]}, \quad \forall i, j, r \in I \quad (13)$$

ko‘rinishiga keladi.

Dastlab kubik matritsalar uchun ma’lum stoxastik tushunchasini kiritib, bunday matritsalar sinfida (13) tenglamani yechimini topamiz.

Quyidagicha belgilash olamiz

$$q_{ir}^{[s,t]} = \sum_{j=1}^m P_{ijr}^{[s,t]}, \quad Q^{[s,t]} = \left(q_{ir}^{[s,t]} \right)_{i,r=1}^m. \quad (14)$$

3-lemma. $M^{[s,t]} = \left(P_{ijk}^{[s,t]} \right)_{i,j,k=1}^m$ (bunda $P_{ijk}^{[s,t]} \geq 0$) kubik matritsa

• (1,2)-stoxastik (mos ravishda (2,3)-stoxastik) bo‘lishi uchun unga mos $Q^{[s,t]}$ kvadratik matritsa chap (mos ravishda o‘ng) stoxastik bo‘lishi zarur va yetarli;

• (1,3)-stoxastik bo‘lishi uchun unga mos $Q^{[s,t]}$ kvadratik matritsa elementlari

$\sum_{i,r=1}^m q_{ir}^{[s,t]} \equiv m$ shartni qanoatlantirishi zarur va yetarli;

• 1-stoxastik (mos ravishda 3-stoxastik) uchun unga mos $Q^{[s,t]}$ kvadratik matritsa elementlari $\sum_{i=1}^m q_{ir}^{[s,t]} \equiv m$ (mos ravishda $\sum_{r=1}^m q_{ir}^{[s,t]} \equiv m$) shartni qanoatlantirishi zarur va yetarli;

• 2-stoxastik bo‘lishi uchun $q_{ir}^{[s,t]} \equiv 1$ tenglik bajarilishi zarur va yetarli.

2-tasdiq. Agar $m > 1$ va a amal I da o‘ng yoki chap yagona yechiluvchan bo‘lsa, u holda (5) tenglama i -stoxastik (har bir $i = 1, 2, 3$ uchun) kubik matritsalar sinfida yechimga ega emas.

1-shart. Aniqlik uchun $I = \{0, 1, 2, \dots, m-1\}$ deb, bu to‘plamda a amalni $a(i, j) = (i + j) \pmod{m}$ kabi aniqlaymiz. Bu holda a ning yagona yechiluvchan ekanligini ko‘rish mumkin.

Bu holda $M^{[s,t]}$ matritsani $M^{[s,t]} = \left(P_{ijk}^{[s,t]} \right)_{i,j,k=0}^{m-1}$ kabi aniqlash mumkin. U holda (13) tenglamani

$$P_{ijr}^{[s,t]} = \sum_{k=0}^{m-1} \left(\sum_{l=0}^j P_{ilk}^{[s,\tau]} P_{k(j-l)r}^{[\tau,t]} + \sum_{l=1}^{m-j-1} P_{i(j+l)k}^{[s,\tau]} P_{k(m-l)r}^{[\tau,t]} \right), \quad \forall i, j, r \in I. \quad (15)$$

kabi yozish mumkin.

Bu m^3 ta noma’lumli, $P_{ijr}^{[s,t]}$ ikki o‘zgaruvchili funksional tenglamalar sistemasi bo‘lib, $P_{ijr}^{[s,t]} = 1/m^2$, $\forall i, j, r \in I$, $0 \leq s < t$ trivial yechimga ega. (15) tenglamalar sistemasini to‘la tahlil qilsih birmuncha qiyin bo‘lib, biz faqat $m = 2$ holni qarash bilan cheklanamiz.

(1,3)-stoxastik yechimlari. (13|a)-tipli kvadratik stoxastik jarayonni quramiz. Bunda 13 deganda (1,3)-stoxastikni, a deganda (4) ko‘paytmani tushunamiz.

$M^{[s,t]}$ kubik matritsani

$$M^{[s,t]} = \begin{pmatrix} P_{000}^{[s,t]} & P_{001}^{[s,t]} & P_{100}^{[s,t]} & P_{101}^{[s,t]} \\ P_{010}^{[s,t]} & P_{011}^{[s,t]} & P_{110}^{[s,t]} & P_{111}^{[s,t]} \end{pmatrix} \quad (16)$$

kabi qulay shaklda yozib olamiz.

Quyidagi matritsalarini qaraymiz:

$$M_1^{[s,t]} = \begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix}; \quad (17)$$

$$M_2^{[s,t]} = \begin{pmatrix} f(s,t) & f(s,t) & f(s,t) & 1-3f(s,t) \\ 1/2-f(s,t) & 1/2-f(s,t) & f(s,t) & 3f(s,t)-1/2 \end{pmatrix} \quad (18)$$

bu yerda $f(s,t) = 1/4 \cdot (\Phi(t)/\Phi(s) + 1)$ bo'lib, Φ ixtiyoriy $\Phi(s) \neq 0$ shartni qanoatlantiruvchi funksiya.

3-tasdiq. (17) tenglik bilan aniqlangan $M_1^{[s,t]}$ matritsa (13|a)-tipli kvadratik stoxastik jarayonni hosil qiladi. (18) tenglik bilan aniqlangan $M_2^{[n,m]}$, $n, m \in \mathbb{N}$, $n < m$ matritsalar diskret vaqtli (13|a)-tipli kvadratik stoxastik jarayonni hosil qiladi.

(1,2)-stoxastik yechimlari. (12|a)-tipli kvadratik stoxastik jarayonni quramiz.

Quyidagi kubik matritsalarini qaraymiz:

$$M_3^{[s,t]} = \begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix}; \quad (19)$$

$$M_4^{[s,t]} = \begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 + \frac{\psi(t)}{2\psi(s)} \\ 1/4 & 1/4 & 1/4 & 1/4 - \frac{\psi(t)}{2\psi(s)} \end{pmatrix} \quad (20)$$

bu yerda ψ uchun $-1/2 \leq \psi(t)/\psi(s) \leq 1/2$ shart bajariladi. Bu shart ψ funksiya uchun vaqt diskret ya'ni, $t \in \mathbb{N}$ bo'lganda bajariladi. Masalan $\psi(n) = 2^{-n}$ deb olish mumkin;

$$M_5^{[s,t]} = \begin{pmatrix} g(s,t) & g(s,t) & g(s,t) & g(s,t) \\ 1/2-g(s,t) & 1/2-g(s,t) & 1/2-g(s,t) & 1/2-g(s,t) \end{pmatrix}; \quad (21)$$

bu yerda $g(s,t) = \frac{1}{4} \left(\frac{\varphi(t)}{\varphi(s)} + 1 \right)$ bo'lib, $-1 \leq \frac{\varphi(t)}{\varphi(s)} \leq 1$ munosabat o'rinli;

$$M_6^{[s,t]} = \begin{cases} \begin{pmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ 0 & 0 & 0 & 0 \end{pmatrix}, & \text{agar } s \leq t < c \\ \begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix}, & \text{agar } t \geq c. \end{cases} \quad (22)$$

4-tasdiq. Yuqorida keltirilgan $M_i^{[s,t]}$, $i=3,5,6$ matritsalar $(12|a)$ -tipli kvadratik stoxastik jarayonni hosil qiladi. $M_4^{[n,m]}$, $n,m \in \mathbb{N}$, $n < m$ matritsalar diskret vaqtli $(12|a)$ -tipli kvadratik stoxastik jarayonni hosil qiladi.

1-shart o‘rinli bo‘lmaydigan holga misol. I da yagona yechiluvchan bo‘lmagan $a(i, j) = \max\{i, j\}$ amalni qaraymiz. Haqiqatdan ham $m \geq 2$ bo‘lganda $\max\{x, m\} = m$ tenglama $x = 1, 2, \dots, m$ yechimlarga ega.

σ kubik matritsalarining biror stoxastikligi bo‘lsin. \max amaliga mos $(\sigma | \max)$ -tipli kvadratik stoxastik jarayonga misol keltiramiz.

Soddalik uchun $m = 2$ bo‘lgan holni qaraymiz va $M^{[s,t]} = (a_{ij}^{[s,t]})_{i,j,k=1}^2$ uchun (5) tenglamaning yechimini topamiz.

Quyidagicha belgilash kiritamiz:

$$b_{ij}^{[s,t]} = a_{i1j}^{[s,t]} + a_{i2j}^{[s,t]}, \quad B^{[s,t]} = (b_{ij}^{[s,t]}); \quad (23)$$

$$c_{ij}^{[s,t]} = a_{i1j}^{[s,t]}, \quad C^{[s,t]} = (c_{ij}^{[s,t]}). \quad (24)$$

Agar $B^{[s,t]} = (b_{ij}^{[s,t]})$

$$B^{[s,t]} = B^{[s,\tau]} B^{[\tau,t]} \quad (25)$$

tenglamaning yechimi, $C^{[s,t]} = (c_{ij}^{[s,t]})$ esa

$$C^{[s,t]} = C^{[s,\tau]} C^{[\tau,t]} \quad (26)$$

tenglamaning yechimi bo‘lsa, u holda (5) tenglamaning mos yechimi quyidagicha bo‘ladi:

$$M_7^{[s,t]} = \left(\begin{array}{cc|cc} c_{11}^{[s,t]} & c_{12}^{[s,t]} & c_{21}^{[s,t]} & c_{22}^{[s,t]} \\ b_{11}^{[s,t]} - c_{11}^{[s,t]} & b_{12}^{[s,t]} - c_{12}^{[s,t]} & b_{21}^{[s,t]} - c_{21}^{[s,t]} & b_{22}^{[s,t]} - c_{22}^{[s,t]} \end{array} \right). \quad (27)$$

5-teorema. $B^{[s,t]} = (b_{ij}^{[s,t]})$ matritsa (23) tenglamaning yechimi, $C^{[s,t]} = (c_{ij}^{[s,t]})$ esa (24) tenglamaning yechimi bo‘lib, barcha $i, j = 1, 2$, $0 \leq s < t$ lar uchun $c_{ij}^{[s,t]} \in [0, 1]$ va $b_{ij}^{[s,t]} - c_{ij}^{[s,t]} \in [0, 1]$ bo‘lsin. U holda (27) tenglik bilan aniqlangan $M_7^{[s,t]}$ matritsalar oilasi

- $(12|\max)$ -tipli kvadratik stoxastik jarayonni hosil qilishi uchun $B^{[s,t]}$ ning har qanday $0 \leq s < t$ uchun chap stoxastik bo‘lishi zarur va yetarli;

- $(13|\max)$ -tipli bo‘lishi uchun $B^{[s,t]}$ (mos ravishda $C^{[s,t]}$) barcha elementlari nomanfiy va barcha elementlari yigindisi 2 (mos ravishda 1) ga teng bo‘lishi zarur va yetarli.

- $(23|\max)$ -tipli bo‘lishi uchun $B^{[s,t]}$ ning barcha $0 \leq s < t$ lar uchun o‘ng stoxastik bo‘lishi zarur va yetarli.

- $(1|\max)$ -tipli bo‘lishi uchun $B^{[s,t]}$ (mos ravishda $C^{[s,t]}$) matritsaning barcha elementlari nomanfiy va har bir ustunidagi elementlari yig‘indisi 2 ga teng (mos ravishda chap stoxastik) bo‘lishi zarur va yetarli.

- $(2|\max)$ -tipli bo'lmaydi.

- $(3|\max)$ -tipli bo'lishi uchun $B^{[s,t]}$ (mos ravishda $C^{[s,t]}$) ning har bir elementi nomanfiy va har bir satridagi elementlari yig'indisi 2 ga teng (mos ravishqa o'ng stoxastik) bo'lishi zarur va yetarli.

KSJning dinamik sistemalari. $M_i^{[s,t]}$, $i=1,\dots,7$ lar hosil qilgan kvadratik stoxastik jarayonlar uchun $x^{(t)} = (x_0^{(t)}, x_1^{(t)}) \in S^1$ taqsimotning vaqt xarakteristikasini aniqlaymiz. Tayin $s \geq 0$ va $x^{(s)} = (x_0^{(s)}, x_1^{(s)}) \in S^1$ vektorni olamiz.

$M_1^{[s,t]}$ va $M_3^{[s,t]}$ **bo'lgan hol.** $x^{(s)}$ vektorga bog'liq bo'lmagan holda, har qanday $t > s$ uchun $x_0^{(t)} = \frac{1}{2}$, $x_1^{(t)} = \frac{1}{2}$ ni hosil qilamiz.

Shunday qilib, $x^{(t)}$ ning vaqt bo'yicha o'zini tutishi aniq: jarayon s vaqtida $x^{(s)}$ boshlang'ich taqsimot vektori bilan boshlansa, so'ngra vaqt o'tishni boshlaganda tizim $(1/2, 1/2)$ taqsimotga o'tadi va bu taqsimot barcha vaqt davomida barqaror bo'lib qoladi.

$M_2^{[s,t]}$ **bo'lgan hol.** Tayin $s \geq 0$ uchun $x^{(s)}$ vektor va ixtiyoriy $t > s$ ni olamiz hamda

$$x_0^{(t)} = \left(\frac{1}{2} + \frac{\Phi(t)}{4\Phi(s)} \right) x_0^{(s)} + \left(\frac{1}{2} - \frac{\Phi(t)}{4\Phi(s)} \right) x_1^{(s)},$$

$$x_1^{(t)} = \left(\frac{1}{2} - \frac{\Phi(t)}{4\Phi(s)} \right) x_0^{(s)} + \left(\frac{1}{2} + \frac{\Phi(t)}{4\Phi(s)} \right) x_1^{(s)}$$

munosabatlarni hosil qilamiz.

$x^{(t)}$ ning vaqt harakati Φ funksiyaga bog'liq (bizning farazimizga ko'ra $-1/3 \leq \Phi(t)/\Phi(s) \leq 1/3$).

$M_4^{[s,t]}$ **bo'lgan hol.** Bu holda biz quyidagi munosabatlarni hosil qilamiz

$$x_0^{(t)} = \frac{1}{2} x_0^{(s)} + \left(\frac{1}{2} + \frac{\psi(t)}{4\psi(s)} \right) x_1^{(s)}, \quad x_1^{(t)} = \frac{1}{2} x_0^{(s)} + \left(\frac{1}{2} - \frac{\psi(t)}{4\psi(s)} \right) x_1^{(s)}.$$

Yuqoridagi hol kabi, $x^{(t)}$ ning vaqt harakati ψ funksiyaga bog'liq (bizning farazimizga ko'ra $-1/2 \leq \psi(t)/\psi(s) \leq 1/2$).

$M_5^{[s,t]}$ **bo'lgan hol.** Bu holda boshlang'ich $x^{(s)}$ vektorga bog'liq bo'lmagan holda $x_0^{(t)} = \frac{1}{2} + \frac{\varphi(t)}{4\varphi(s)}$, $x_1^{(t)} = \frac{1}{2} - \frac{\varphi(t)}{4\varphi(s)}$ ni hosil qilamiz.

Bu holda qiziqarli dinamik sistema hosil bo'ladi. Chunki, har bir boshlang'ich (tayin) s vaqtda sistema dastlabki holat $x^{(s)}$ ga bog'liq emas. Traektoriya boshlang'ich vaqtga va $x^{(t)}$ vaqt bo'yicha φ funksiyagagina bog'liq (bizning farazimizga ko'ra $-1 \leq \varphi(t)/\varphi(s) \leq 1$ munosabat o'rinli).

$M_6^{[s,t]}$ **bo‘lgan hol.** Bu holda boshlang‘ich $x^{(s)}$ vektorga bog‘liq bo‘lmagan

$$x_0^{(t)} = 1 - x_1^{(t)} = \begin{cases} 3/4, & \text{agar } 0 \leq s < t < a \\ 1/2, & \text{agar } t \geq a. \end{cases}$$

tenglikni hosil qilamiz.

Shunday qilib biz uzluksiz bo‘lmagan (vaqt bo‘yicha), traektoriyasining limiti $1/2$ bo‘lgan dinamik sistemani hosil qildik.

$M_7^{[s,t]}$ **bo‘lgan hol.** $(3 | \max)$ -tipli kvadratik stoxastik jarayonni qaraymiz (boshqa hollarni shu kabi ko‘rib chiqish mumkin). 5-teoremadan foydalanib

$$x_0^{(t)} = c_{11}^{[s,t]}(x_0^{(s)})^2 + (b_{11}^{[s,t]} - c_{11}^{[s,t]} + c_{21}^{[s,t]})x_0^{(s)}x_1^{(s)} + (b_{21}^{[s,t]} - c_{21}^{[s,t]})(x_1^{(s)})^2,$$

$$x_1^{(t)} = c_{12}^{[s,t]}(x_0^{(s)})^2 + (b_{12}^{[s,t]} - c_{12}^{[s,t]} + c_{22}^{[s,t]})x_0^{(s)}x_1^{(s)} + (b_{22}^{[s,t]} - c_{22}^{[s,t]})(x_1^{(s)})^2$$

munosabatni hosil qilamiz. Bu uzluksiz vaqtli dinamik sistemani ifodalaydi. $x^{(t)}$ ning harakati $M_7^{[s,t]}$ matritsaga bog‘liq. Dinamik sistemaning harakatini tadqiq qilish lozim bo‘lganda bu matritsani olish mumkin.

M.Ladra va U.A.Rozikovlarning ishlarida quyidagicha teorema isbotlangan⁵.

6-teorema. *Har bir t uchun $\{A^{[t]} = (a_{ij}^{[t]}, t \geq 0)\}$ teskarilantiruvchi $m \times m$ kvadrat matritsalar oilasi, $(A^{[t]})^{-1} = (b_{ij}^{[t]})$ esa $A^{[t]}$ matritsaning teskarisi bo‘lsin.*

$B^{(s)} = (\beta_{ijk}^{(s)})_{i,j,k=1}^m$ kubik matritsaning $\beta_{ijk}^{(s)}, i, j, k = 1, \dots, m$ elementlari barcha i, k va s uchun $\sum_{j=1}^m \beta_{ijk}^{(s)} = a_{ik}^{[s]}$, shartni qanoatlantiruvchi funksiyalar bo‘lsin. U holda

$$M^{[s,t]} = \left(\sum_{k=1}^m \beta_{ijk}^{(s)} b_{kr}^{[t]} \right)_{i,j,r=1}^m \quad (28)$$

kubik matritsa algebralar oqimini hosil qiladi (ya’ni (5) tenglikni qanoatlantiradi).

Umuman olganda (28) matritsa kvadratik stoxastik jarayonni hosil qilmaydi. Bu yerda bizning maqsadimiz (28) matritsa kvadratik stoxastik jarayonni hosil qiladigan $A^{[t]}$ va $B^{(s)}$ (6-teorema shartlarini qanoatlantiruvchi) uchun shartlar topishdan iborat.

4-lemma. $A^{[t]} = (a_{ij}^{[t]})_{i,j=1}^m$ matritsa o‘ng stoxastik va teskarilantiruvchi bo‘lib, $(A^{[t]})^{-1} = (b_{ij}^{[t]})_{i,j=1}^m$ bo‘lsin. Agar $(A^{[t]})^{-1}$ matritsa stoxastik bo‘lmasa, u holda uning hech bo‘lmaganda bitta elementi manfiy bo‘ladi.

5-tasdiq. 6-teorema shartlari yordamida aniqlangan $A^{[t]}$ va $B^{(s)} = (\beta_{ijk}^{(s)})_{i,j,k=1}^m$ matritsalar uchun quyidagi shartlar o‘rinli bo‘lsin:

⁵Ladra M., Rozikov U.A. Construction of flows of finite-dimensional algebras, Jour. Algebra, 492 (2017) 475-489.

- $(A^{[t]})^{-1}$ matritsa chap stoxastik;
- $B^{(s)}$ matritsa (1,2)-stoxastik.

U holda (28) tenglik bilan aniqlangan $M^{[s,t]}$ matritsa $(12|D)$ tipli kvadratik stoxastik jarayonni hosil qiladi.

7-teorema. Agar 6-teoremada biror $t > 0$ uchun $A^{[t]}$ va $(A^{[t]})^{-1}$ matritsalarining har ikkalasi ham stoxastik, biror $s < t$ uchun $B^{(s)}$ matritsa 3-stoxastik bo'lsa, u holda $M^{[s,t]}$ matritsa 3-stoxastik emas..

4-eslatma. 7-teoremadan kelib chiqadiki, $M^{[s,t]}$ kvadratik stoxastik jarayonni hosil qilishi uchun $A^{[t]}$ matritsa almashtirish matritsasi bo'lishi kerak.

6-tasdiq. Aytaylik, 6-teoremada $A^{[t]}$ va $B^{(s)}$ matritsalar quyidagi shartlarni qanoatlantirsin:

- $(A^{[t]})^{-1}$ matritsa o'ng stoxastik;
- $B^{(s)}$ matritsa (1,3)-stoxastik.

U holda (28) tenglik bilan aniqlangan $M^{[s,t]}$ matritsa $(13|D)$ tipli kvadratik stoxastik jarayonni ifodalaydi.

7-tasdiq. Aytaylik, 6-teoremada $(A^{[t]})^{-1}$ va $B^{(s)}$ matritsalar quyidagi shartlarni qanoatlantirsin:

- $(A^{[t]})^{-1}$ matritsa o'ng stoxastik;
- $B^{(s)}$ matritsa (2,3)-stoxastik.

U holda (28) tenglik bilan aniqlangan $M^{[s,t]}$ matritsa $(23|D)$ tipli kvadratik stoxastik jarayonni hosil qiladi.

XULOSA

Ushbu dissertatsiya ishi stoxastik va nostoxastik kubik matritsalar hosil qilgan dinamik sistemalarni o'rganishga bag'ishlangan.

Tadqiqotning asosiy natijalari quyidagilardan iborat:

1. V kvadratik operator S^{m-1} simpleksni o'zini o'ziga o'tkazishi uchun uning koeffitsiyentlariga zaruriy va yetarli shartlarni beruvchi teorema ishbotlangan;

2. Simpleksda aniqlangan diskret va uzluksiz vaqtli kvadratik stoxastik va nostoxastik operatorlarning barcha qo'zg'almas nuqtalari topilgan hamda traektoriyaning limit nuqtalari tadqiq etilgan.

3. Bir o'lchovli kvadratik nostoxastik operator umimiy holda dinamikasi o'rganilgan. Bunday operator xaotik dinamik sistema hosil qilishi uchun uning parametrlariga shartlar topilgan.

4. Ikki o'lchovli kvadratik nostoxastik operatorlarning bir sinfi uchun xaotik dinamik sistema hosil qiladiganlari hamda xaotik dinamik sistema hosil qilmaydiganlari mavjudligi ko'rsatilgan.

5. Kubik matritsalar uchun ko'paytirish amallari kiritilib, bu ko'paytmalar uchun Kolmogorov-Chapman tenglamasini qanoatlantiruvchi matritsalar oilasi aniqlangan.

6. $(\sigma | D)$ -tipi kvadratik stoxastik jarayonlarning bir nechta misollari qurilgan.

**SCIENTIFIC COUNCIL AWARDING SCIENTIFIC
DEGREES PhD.03/30.06.2020.FM.70.04**

**KARSHI STATE UNIVERSITY
BUKHARA STATE UNIVERSITY**

KHUDAYAROV SANAT SAMADOVICH

**QUADRATIC STOCHASTIC DYNAMICAL SYSTEMS
OF THE TYPE $(\sigma | \mu)$**

01.01.01 – Mathematical analysis

**ABSTRACT OF DISSERTATION
OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND
MATHEMATICAL SCIENCES**

Karshi – 2023

The theme of dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Commission under number B2020.4.PhD/FM531.

The dissertation was performed at the Bukhara State University.

The abstract of dissertation is posted in three languages (Uzbek, Russian and English (resume)) on the website (www.qarshidu.uz) and in the “ZiyoNet” Information and educational portal (www.ziynet.uz).

Scientific supervisor:

Rozikov Utkir Abdulloevich

doctor of physical and mathematical sciences, professor

Official opponents:

Jamilov Uygun Umurovich

doctor of physical and mathematical sciences

Usmonov Javokhir Bakhodir ogli

doctor of philosophy (PhD) on physical and mathematical sciences

Leading organization:

Namangan State University

Defense will take place “_____” _____2023 at _____ at the meeting of Scientific Council number PhD.03/30.06.2020.FM.70.04 at Karshi State University (Address: Kuchabag street, 17, Karshi city, 180103, Uzbekistan. Ph.: (+998 75) 225-34-13; fax: (+998 75) 221-00-56; e-mail: qarshidu@umail.uz). Karshi State University, Building№2, Room №202.

Dissertation is possible to review in Information-resource centre at Karshi State University (is registered №_____). (Address: Kuchabag street, 17, Karshi city, 180103, Uzbekistan. Ph.: (+998 75) 225-34-13; fax: (+998 75) 221-00-56; e-mail: qarshidu@umail.uz).

Abstract of dissertation sent out on “_____” _____2023 year
(Mailing report №_____ on “_____” _____2023 year).

B.A.Shoimkulov

Chairman of Scientific council
on award of scientific degree,
Dr.Phys.-Math.Sci., professor

Sh.D.Nodirov

Scientific secretary of Scientific council
on award of scientific degree, (PhD)

A.A.Imomov

Chairman of scientific seminar under
Scientific council on award of scientific
degree, DSc., associate professor

INTRODUCTION (abstract of PhD thesis)

Actuality and demand of the theme of the dissertation. In the world, many scientific and practical problems in dynamical systems are modeled by evolutionary operators of stochastic and non-stochastic cubic matrices. In many biological and physical systems their behaviour can be given by dynamics of quadratic (non-)stochastic operators defined by cubic matrices. A quadratic stochastic process is defined by a family of cubic matrices which satisfy Kolmogorov-Chapman equation with respect to a fixed multiplication and stochasticity of such matrices. Therefore, the study of the dynamics of nonlinear operators constructed using stochastic and non-stochastic cubic matrices that preserve simplex remains one of the important and urgent tasks.

Nowadays in the world, the theory of nonlinear dynamical systems of mathematics is used as the main tool for solving many practical problems. The study of the dynamics of quadratic stochastic and non-stochastic operators generated by cubic matrices leads to many problems due to the non-linearity of the operator. In particular, targeted scientific research includes finding fixed points and determining their types, describing the set of periodic points and determining their type, and describing the set of limit points of trajectories.

In recent years, in our republic, much attention has been paid to mathematics, physics, biology and geology, which have scientific and practical application of fundamental sciences. In particular, special attention was paid to the study of the dynamics of nonlinear operators, which are the main objects of genetics and population biology.

The study of the dynamics of modern quadratic operators is reduced to the multiplication of cubic matrices, the main problem here is that the general rule for the multiplication of cubic matrices has not yet been determined. Nevertheless, many significant results have been achieved today. As the main tasks and areas of activity of mathematics, the conduct of scientific research at the level of international standards in the priority areas of functional analysis, mathematical physics and statistical physics are determined¹. It is important to develop the theory of piecewise smooth dynamical systems in order to use scientific results in related fields of science to ensure the implementation of the solution.

The subject and object of research of this dissertation are in line with tasks identified in the Decrees and Resolutions of the President of the Republic of Uzbekistan of February 7, 2017, PF-4947, “On the strategy of action for the further development of the Republic of Uzbekistan”, PQ-4387 dated July 9, 2019 “On state support for the further development of mathematics education and science, as well as measures to radically improve the activities of the Institute of

¹ Decree of Cabinet of Ministers of the Republic of Uzbekistan at the 2017 year 18 May “On measures on the organization of activities of the first created scientific research institutions of the Academy of Sciences of the Republic of Uzbekistan” № 292 dated May 17, 2017.

Mathematics named after V.I. Romanovskiy of the Academy of Sciences of the Republic of Uzbekistan”, PQ-4708 of May 7, 2020 “On measures to improve the quality of education and research in the field of mathematics” as well as in other regulations related to basic sciences.

Connection of research to priority directions of development of science and technologies of the Republic. This study was performed in accordance with the priority areas of science and technology of the Republic of Uzbekistan IV, “Mathematics, Mechanics and Computer Science”.

The degree of scrutiny of the problem. The study of mathematical models in physics or biology is reduced to the study of continuous or discrete time quadratic random processes. In this case, an important issue is the construction of a family of cubic matrices that satisfy the Kolmogorov-Chapman equation.

Maksimov consider associative multiplications of cubic matrices, which generalize the usual matrix multiplication. Cubic analogues of stochastic matrices are introduced and their probabilistic interpretations are given. Cubic stationary of stochastic matrices are described and a proposition is proved on the convergence of a cubic stochastic matrix to a stationary one. The concept of a Markov interaction process is introduced, which generalizes the concept of a Markov process, and it is shown that the concept of ergodicity of such a process is naturally related to the associative multiplication of cubic matrices.

By J.M.Casas, M.Ladra and U.A.Rozikov a Markov process of cubic stochastic matrices is introduced, which is also called quadratic stochastic process (QSP). QSP is a special case of a dynamic system with continuous time, the states of which are stochastic cubic matrices that satisfy an analog of the Kolmogorov-Chapman equation (KCE). Since there are several kinds of multiplication between cubic matrices, one has to first fix the multiplication and then consider the KCE with respect to the fixed multiplication. In the paper J.M.Casas, M.Ladra, U.A.Rozikov to construct a QSP two specially chosen concepts of stochastic cubic matrices and two multiplications of such matrices (known as Maksimov multiplications) are considered. A wide class of QSPs and some behavior of such processes depending on time are given. Moreover, an example with applications to biology of QSP is given which describes the temporal behavior (dynamics) of a population with the possibility of giving birth to twins.

In the work of U.A.Rozikov and Sh.N.Murodov 25 distinct examples of chains of two-dimensional evolutionary algebras are constructed. For all these 25 chains the behavior of the baric property, the behavior of the set of absolute nilpotent elements and the dynamics of the set of idempotent elements as a function of time are studied.

The theory of quadratic stochastic operators of the Volterra type was developed in the works of R.N. Ganikhodzhaev. Later, results related to linear and nonlinear stochastic operators were studied in the works of U.A.Rozikov, X.Akin,

U.U.Zhamilov, A.Zada, O.N.Khakimov, F.M.Mukhamedov, A.Yu.Khamraev, F.A.Shahidi, N.N.Ganikhodzhaev, J.P.Tian, S.K.Shoyimardonov, M.V.Velasco, R.Varro.

In the book entitled “Population dynamics: algebraic and probabilistic approach” which is written by U. A.Rozikov in 2020, the probabilistic approach part presents Markov processes of cubic stochastic (in a fixed sense) matrices (MPCSM), which are continuous-time dynamical systems, whose states are stochastic cubic matrices satisfying an analogue of the Kolmogorov-Chapman equation. MPCSM considered for two specially chosen notions of stochastic cubic matrices and two Maksimov’s multiplications of such matrices. Time-dependent behavior of such processes is given with applications to a population with the possibility of twin births.

The connection of the theme of the thesis with the research plans of the higher education institute, where the research on the thesis is carried out. The dissertation research was carried out in accordance with the plan of scientific research of the scientific direction M.01.2017 “Spectral theory of linear operators” of Bukhara state university for 2017-2022.

The aim of the research work Determining the conditions for the validity of the Kolmogorov-Chapman equation using multiplications defined for various stochastic cubic matrices, and studying the dynamics of nonlinear operators constructed using such matrices. Finding conditions for the coefficients of quadratic operators to preserve simplex. Finding the limit points of trajectories for a class of such operators.

Research problems:

to find necessary and sufficient conditions on coefficients of a quadratic operator under which it preserves simplex;

study dynamical systems generated by quadratic non-stochastic operators;

determination of non-stochastic quadratic operators forming a chaotic dynamical system;

study the dynamical systems generated by quadratic operators defined by cubic matrices with some fixed multiplication and stochasticity;

construct a family of cubic matrices satisfying the Kolmogorov-Chapman equation with respect to some multiplications. Give time depending evolution of corresponding dynamical system.

The research object. Quadratic (non-)stochastic operators defined by cubic matrices that satisfy the Kolmogorov-Chapman equation using Maksimov multiplication.

The research subject. Theory of functions with real variables, dynamical systems, theory of random processes, theory of cubic matrices.

Research methods. The thesis used methods of mathematical analysis, functional analysis, stochastic processes and the theory of dynamical systems.

Scientific novelty of the research work consists of the following:

a necessary and sufficient condition is proved for the preservation by the coefficients of the quadratic operator of the simplex, i.e., for the reflection of the simplex into itself;

the dynamics of an arbitrary one-dimensional quadratic non-stochastic operator is completely studied and the conditions for the formation of a chaotic dynamical system are determined for certain values of the operator parameters using the properties of logistic reflection;

among the two-dimensional quadratic non-stochastic operators, classes are constructed that create a chaotic dynamical system and those that do not create a chaotic dynamical system, and the dynamics of these operators is fully studied;

several types of special multiplications for cubic matrices are introduced and it is proved that the family of matrices generating quadratic random processes with respect to these multiplications satisfies the Kolmogorov-Chapman equation. Time-varying limit points of these quadratic random processes are found.

Practical results of the research. Mathematical models of population processes in mathematical biology are proposed to be represented by nonlinear stochastic and non-stochastic quadratic operators built using cubic matrices, and methods for determining the set of limit points of the trajectory in a dynamic system created using these operators are described.

The reliability of the results of the study. The results have been obtained by using the methods of functional analysis, theory of discrete time dynamical systems and stochastic processes. The obtained results are mathematically strongly proved.

Scientific and practical significance of the research results. The scientific significance of the research results is explained by the fact that the description of the set of trajectory limit points in the dynamical systems can be used in the theory of nonlinear discrete-time dynamical systems.

The practical significance of the research is determined by applications to population processes in mathematical biology by determining the set of limit points of trajectory in a dynamical system generated by non-linear operator.

Implementation of the research results. The results related to quadratic stochastic dynamical system of the type $(\sigma | \mu)$ were used in the following research projects:

the results about dynamical systems generated by non-stochastic quadratic operators and their limit points, were used in a foreign grant research project “Dynamics of Finite Dimensional Orthogonality Preserving Cubic Stochastic Operators” No. FRGS21-230-0839 (Reference of International Islamic University Malaysia dated January 15, 2023). The application of scientific results made it possible to classify evolutionary operators of biological systems, mathematical biology and other natural sciences;

the results on dynamics of a family of quadratic operators generated by non-stochastic cubic matrices, for which the Markov process is suitable with respect to a certain multiplication, were used in a foreign grant research project “Quantum Genetic Algebras and their applications” No. G00003447 (Reference of United Arab Emirates University dated January 06, 2023). The application of the scientific result made it possible to check the dynamical systems generated by several quadratic stochastic operators defined on standard simplex.

Approbation of the research results. The main results of the research have been discussed at 3 international and 5 national scientific conferences.

Publications of the research results. On the topic of the dissertation 13 research papers have been published in the scientific journals, 5 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the PhD thesis, in addition 2 of them were published in international journals and 3 papers published in national mathematical journals.

The structure and volume of the thesis. The dissertation consists of an introduction, three chapters, conclusion and bibliography. The general volume of the thesis is 94 pages.

THE MAIN CONTENT OF THE THESIS

The **introduction** of the thesis includes the motivation of the research, the relevance of the research to the priorities of science and technology, the review of foreign research on the topic, the degree of scrutiny of the problem, the aim, research problems, object and subject of research, scientific novelty and practical results, theoretical and practical significance of the results obtained, the statement of research results, published works and information on the structure of the thesis.

In the first chapter of the thesis, entitled “**Preliminaries. Dynamical systems of a quadratic operator**”, contains the basic definitions and important concepts necessary for a complete coverage of the topic of the dissertation. We also describe results on fixed points, invariant sets, and the trajectory of a two-dimensional non-stochastic operator

Let $I = \{1, 2, \dots, m\}$. A distribution (or state) of the set I is a probability measure $x = (x_1, \dots, x_m)$, i.e. an element of the simplex:

$$S^{m-1} = \left\{ x \in \mathbb{R}^m : x_i \geq 0, \sum_{i=1}^m x_i = 1 \right\}. \quad (1)$$

The quadratic stochastic operator (QSO) is a mapping of the simplex S^{m-1} into itself, of the form

$$V : x'_k = \sum_{i,j=1}^m P_{ij,k} x_i x_j, \quad k = 1, \dots, m \quad (2)$$

where $P_{ij,k}$ are coefficients of heredity and

$$P_{ij,k} \geq 0, P_{ij,k} = P_{ji,k}, \sum_{k=1}^m P_{ij,k} = 1, i, j, k = 1, \dots, m, \quad (3)$$

Thus, each quadratic stochastic operator V can be uniquely defined by a cubic matrix $\mathbf{P} = (P_{ij,k})_{i,j,k=1}^m$ with conditions (3).

For a given $x^{(0)} \in S^{m-1}$ the trajectory (orbit) $\{x^{(n)}\}$ of $x^{(0)}$ under the action of QSO (2) is defined by

$$x^{(n+1)} = V(x^{(n)}), \text{ where } n = 0, 1, 2, \dots$$

In general, a quadratic operator $V, V: x \in \mathbb{R}^m \rightarrow x' = V(x) \in \mathbb{R}^m$ corresponding to a cubic matrix \mathbf{P} is defined by (2). Without loss of generality we assume $P_{ij,k} = P_{ji,k}$. Indeed, if this equality is not satisfied then we can introduce $\bar{P}_{ij,k} = \frac{1}{2}(P_{ij,k} + P_{ji,k})$.

Let \mathcal{C} be the set of all m^3 -dimensional cubic matrices over the field of real numbers. Denote by $E_{ijk}, i, j, k \in I$ the basis cubic matrices in \mathcal{C} , i.e., E_{ijk} is a m^3 -dimensional cubic matrix whose (i, j, k) th entry is equal to 1 and all other entries are equal to 0.

Define multiplication:

$$E_{ijk} *_a E_{lnr} = \delta_{kl} E_{ia(j,n)r}, \quad (4)$$

where $a: I \times I \rightarrow I, (j, n) \mapsto a(j, n) \in I$, is an arbitrary associative binary operation.

Where δ_{kl} is the Kronecker symbol.

Denote by O_m the set of all associative binary operations on I .

The general formula for the multiplication is the extension of (4) by bilinearity, i.e. for any two cubic matrices $A = (a_{ijk}), B = (b_{ijk}) \in \mathcal{C}$ the matrix $A *_a B = (c_{ijk})$ is defined by

$$c_{ijr} = \sum_{l, n: a(l, n) = j} \sum_k a_{ilk} b_{knr}.$$

Note that $c_{ijr} = 0$ for j such that $\{l, n: a(l, n) = j\} = \emptyset$.

If the equation $a(x, u) = v$ (resp. $a(u, x) = v$) is uniquely solvable for any $u, v \in I$ then the operation a on I has right (resp. left) unique solvability.

Lemma 1. *If the operation a on I has right or left unique solvability, then*

$$\sum_{d \in I} \sum_{j, m: a(j, m) = d} \gamma_{j, m} = \sum_{j \in I} \sum_{m \in I} \gamma_{j, m}.$$

Stochasticity. *Define several kinds of cubic stochastic matrices. A cubic matrix $\mathbf{P} = (P_{ij,k})_{i,j,k=1}^m$ is called*

- (1,2)-stochastic if $P_{ijk} \geq 0, \sum_{i,j=1}^m P_{ijk} = 1, \text{ for all } k.$
- (1,3)-stochastic if $P_{ijk} \geq 0, \sum_{i,k=1}^m P_{ijk} = 1, \text{ for all } k.$
- (2,3)-stochastic if $P_{ijk} \geq 0, \sum_{j,k=1}^m P_{ijk} = 1, \text{ for all } i.$
- 3-stochastic if $P_{ijk} \geq 0, \sum_{k=1}^m P_{ijk} = 1, \text{ for all } i, j.$

The last one can be also given with respect to first and second index.

In the work Maksimov also defined a twice stochastic matrix: a (2,3)-stochastic cubic matrix is called *twice stochastic* if $\sum_{i=1}^m P_{ijk} = \frac{1}{m}, \text{ for all } j, k$

Denote by \mathbf{S} the set of all possible kinds of stochasticity and denote by \mathbf{M} the set of all possible multiplication rules of cubic matrices.

Let parameters $s \geq 0, t \geq 0,$ are considered as time.

Denote by $M^{[s,t]} = \left(P_{ijk}^{[s,t]} \right)_{i,j,k=1}^m$ a cubic matrix with two parameters.

Definition 1. A family $\{M^{[s,t]} : s, t \in R_+\}$ is called a Markov process of cubic matrices (or a quadratic stochastic process (QSP)) of type $(\sigma | \mu)$ if for each time s and t the cubic matrix $M^{[s,t]}$ is stochastic in sense $\sigma \in \mathbf{S}$ and satisfies the Kolmogorov-Chapman equation (for cubic matrices):

$$M^{[s,t]} = M^{[s,\tau]} *_{\mu} M^{[\tau,t]}, \text{ for all } 0 \leq s < \tau < t \quad (5)$$

with respect to the multiplication $\mu \in \mathbf{M}$.

Definition 2. A quadratic operator (2), preserving a simplex, is called non-stochastic (QnSO) if at least one of its coefficients $P_{ij,k}, i \neq j$ is negative.

Now we study the following example of QnSO on the 2D-simplex S^2 :

$$V_0 : \begin{cases} x' = \frac{1}{2}(z - y)^2 + \frac{3}{2}x(y + z) \\ y' = \frac{1}{2}(x - z)^2 + \frac{3}{2}y(x + z) \\ z' = \frac{1}{2}(y - x)^2 + \frac{3}{2}z(x + y). \end{cases} \quad (6)$$

The following points are fixed points of the operator V_0 :

$$a_1 = \left(0, \frac{1}{2}, \frac{1}{2}\right), a_2 = \left(\frac{1}{2}, 0, \frac{1}{2}\right), a_3 = \left(\frac{1}{2}, \frac{1}{2}, 0\right), a_4 = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right).$$

Thus a_1, a_2 and a_3 are saddle points, but a_4 is an attracting fixed point.

Introduce the following sets:

$$\begin{aligned} M_1 &= \{(x, y, z) \in S^2 : x > y > z > 1/6\}, M_2 = \{(x, y, z) \in S^2 : x > z > y > 1/6\}, \\ M_3 &= \{(x, y, z) \in S^2 : y > x > z > 1/6\}, M_4 = \{(x, y, z) \in S^2 : y > z > x > 1/6\}, \\ M_5 &= \{(x, y, z) \in S^2 : z > x > y > 1/6\}, M_6 = \{(x, y, z) \in S^2 : z > y > x > 1/6\}, \\ l_1 &:= \{(x, y, z) \in S^2 : x = y, x + y + z = 1\}, l_2 := \{(x, y, z) \in S^2 : x = z, x + y + z = 1\}, \\ l_3 &:= \{(x, y, z) \in S^2 : y = z, x + y + z = 1\}. \end{aligned}$$

Theorem 1. *The sets $M_i, i=1,2,3,4,5,6, l_j, j=1,2,3$ are invariant with respect to the operator V_0 .*

For $v^{(0)} \in S^2$ define its trajectory $\{v^{(n)}\}_{n=0}^\infty$, with respect to operator V_0 , as

$$v^{(n+1)} = V_0(v^{(n)}), \quad n = 0, 1, 2, \dots$$

Theorem 2. *For the operator V_0 , for any $v^{(0)} \in S^2 \setminus \{a_i, e_i : i=1,2,3\}$ the following holds*

$$\lim_{n \rightarrow \infty} V_0^{(n)}(v^{(0)}) = \lim_{n \rightarrow \infty} v^{(n)} = \lim_{n \rightarrow \infty} (x^{(n)}, y^{(n)}, z^{(n)}) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right).$$

In the second chapter of the dissertation “**Splitted chaos in quadratic operators of non-stochastic matrices**” necessary and sufficient conditions are found for the coefficients of a quadratic operator to transform a simplex into a simplex.

The main result of the chapter two is the following theorem.

Theorem 3. *For a quadratic operator V (given by (2)), to preserve a simplex S^{m-1} it is sufficient that*

$$\text{i) } \sum_{k=1}^m P_{ij,k} = 1, \quad i, j = 1, \dots, m;$$

$$\text{ii) } 0 \leq P_{ii,k} \leq 1, \quad i, k = 1, \dots, m;$$

$$\text{iii) } -\frac{1}{m-1} \sqrt{P_{ii,k} P_{jj,k}} \leq P_{ij,k} \leq 1 + \sqrt{(1-P_{ii,k})(1-P_{jj,k})}$$

and necessary that the conditions (i), (ii) and

$$\text{iii') } -\sqrt{P_{ii,k} P_{jj,k}} \leq P_{ij,k} \leq 1 + \sqrt{(1-P_{ii,k})(1-P_{jj,k})}$$

are satisfied.

Remark 1.

1. In the work A.T Sarymsakov for $m=2$, it is proven that the conditions (i), (ii) and (iii') are sufficient and necessary to preserve the simplex.

2. In case $m \geq 3$ the conditions (i), (ii) and (iii') are not sufficient to preserve the simplex. Indeed, consider the example (satisfying (i), (ii) and (iii')):

$$P_{ii,1} = 1, \quad P_{ii,k} = 0, \quad \forall i = 1, \dots, m; \quad \forall k = 2, \dots, m;$$

$$P_{ij,1} = -\sqrt{P_{ii,1} P_{jj,1}} = -1, \quad \forall i \neq j; \quad P_{ij,k} \in [0, 2], \quad \forall i \neq j, k \geq 2 \text{ with } \sum_{k=2}^m P_{ij,k} = 2.$$

Then for the first coordinate of the corresponding quadratic operator V we have

$$x'_1 = \sum_{i=1}^m x_i^2 - 2 \sum_{1 \leq i < j \leq m} x_i x_j = \left(x_1 - \sum_{i=2}^m x_i \right)^2 - 4 \sum_{2 \leq i < j \leq m} x_i x_j = (2x_1 - 1)^2 - 4 \sum_{2 \leq i < j \leq m} x_i x_j.$$

Take $x \in S^{m-1}$ such that $x_1 = \frac{1}{2}$ and $x_j > 0$, $j \geq 2$ with $\sum_{j=2}^m x_j = \frac{1}{2}$. Then $x'_1 < 0$, i.e. $x' = V(x) \notin S^{m-1}$.

We consider arbitrary QnSO on S^1 :

$$\begin{aligned} x' &= ax^2 + 2bxy + cy^2 \\ y' &= (1-a)x^2 + 2(1-b)xy + (1-c)y^2, \end{aligned} \tag{7}$$

where $a, c \in [0, 1]$, $b \in [-\sqrt{ac}, 1 + \sqrt{(1-a)(1-c)}]$.

Using $x + y = 1$, the operator (7)

$$x' = ax^2 + 2bx(1-x) + c(1-x)^2 = (a - 2b + c)x^2 + 2(b - c)x + c$$

can be reduced to the function

$$f(x) = (a - 2b + c)x^2 + 2(b - c)x + c. \tag{8}$$

We do not know any quadratic stochastic operator with chaotic behavior of trajectories.

In the (above considered) case: $a = c = 1$, $b \in [-1, 0)$, the operator (7) has the form

$$\begin{aligned} x' &= x^2 + 2bxy + y^2 \\ y' &= 2(1-b)xy. \end{aligned} \tag{9}$$

Since $P_{12,1} = b < 0$, this operator is non-stochastic. For arbitrary initial point $(1 - y_0, y_0) \in S^1$, its trajectory has the form $(1 - y_n, y_n)$, where x_n is defined by

$$y_n = \mu y_{n-1} (1 - y_{n-1}), \quad \mu = 2(1 - b), \quad n = 1, 2, \dots, m.$$

Therefore, from known properties of the logistic map², $f(x) = \mu(1 - x)$, it follows that when $-1 \leq b < -0.784975$ the operator (9) generates a chaotic dynamical system on the one-dimensional simplex.

Above, we considered one-dimensional quadratic non-stochastic operators and saw that one-dimensional quadratic non-stochastic operators lend themselves to complete study, even among them there are those that form a chaotic dynamical system for some values of their parameters. But it is difficult to understand the generality of a two-dimensional quadratic non-stochastic operator, so below we give two specific interesting examples.

1. Consider the following quadratic non-stochastic operator in the S^2 simplex

² https://en.wikipedia.org/wiki/Logistic_map

$$\begin{aligned}
x' &= x^2 + y^2 + z^2 - axy - axz + 2yz \\
y' &= (2+a)xy \\
z' &= (2+a)xz,
\end{aligned} \tag{10}$$

where $a \in [0, 2]$. Note that $P_{12,1} = P_{13,1} = -a/2$.

It is easy to see that the following sets are invariant with respect to (10):

$$\begin{aligned}
M_0 &= \{(x, y, z) \in S^2 : y = 0\}, \quad M_1 = \{(x, y, z) \in S^2 : z = 0\}, \\
M_\omega &= \{(x, y, z) \in S^2 : y = \omega z\}, \quad \omega \in [0, +\infty).
\end{aligned}$$

Moreover, $S^2 = M_0 \cup M_1 \cup \left(\bigcup_{\omega \in (0, +\infty)} M_\omega \right)$.

Remark 2. *The operator (10) is chaotic for $1.56995 < a \leq 2$, but it is not chaos on the simplex in the sense of Devaney. Because, it is not topologically transitive. It is splitted chaos meaning that the simplex is partitioned into uncountably many invariant subsets and the restriction of the operator (10) on each invariant set is chaos in the sense of Devaney.*

2. Consider the following example:

$$\begin{aligned}
P_{ii,1} &= 1, \quad P_{ii,k} = 0, \quad \forall i = 1, 2, 3; \quad \forall k = 2, 3 \\
P_{ij,1} &= -\frac{1}{2} \sqrt{P_{ii,1} P_{jj,1}} = -\frac{1}{2}, \quad \forall i \neq j \\
P_{ij,k} &\in [0, \frac{3}{2}], \quad \forall i \neq j, k = 2, 3 \quad \text{with} \quad P_{ij,2} + P_{ij,3} = \frac{3}{2}.
\end{aligned}$$

Then taking some parameters equal to zero we get the following quadratic operator V :

$$\begin{aligned}
x' &= x^2 + y^2 + z^2 - xy - xz - yz \\
y' &= 3xy + ayz \\
z' &= 3xz + (3-a)yz,
\end{aligned} \tag{11}$$

where $a \in (0, 3)$.

Remark 3. *In the operator (11) one can also consider the cases $a = 0$ and $a = 3$. These cases are more simple than the case $a \in (0, 3)$. Because, for example, if $a = 0$ then $z' = 3z(x+y) = 3z(1-z)$, i.e., the variable z has dynamics independent from other variables. Therefore, below we consider the case $a \neq 0$, $a \neq 3$.*

Fixed points of the operator (11) are

$$\begin{aligned}
s_1 &= (1, 0, 0), \quad s_2 = \left(\frac{1}{3}, 0, \frac{2}{3} \right), \quad s_3 = \left(\frac{1}{3}, \frac{2}{3}, 0 \right), \\
s_4 &= \left(\frac{a^2 - 3a + 3}{a^2 - 3a + 9}, \frac{2a}{a^2 - 3a + 9}, \frac{2(3-a)}{a^2 - 3a + 9} \right).
\end{aligned} \tag{12}$$

Let $T = \{(y, z) \in [0, 1]^2 : y + z \leq 1\}$.

Proposition 1. *Fixed point s_1 is repeller. Points s_2 and s_3 are non hyperbolic (but semi-repeller³). The fixed point s_4 is non-hyperbolic (but semi-attracting⁴).*

Introduce the following sets:

$$M_1 = \{(y, z) \in T : y = 0\}, M_2 = \{(y, z) \in T : z = 0\}, M_3 = \{(y, z) \in T : z = \frac{3-a}{a} y\},$$

$$M_4 = \{(y, z) \in T : z < \frac{3-a}{a} y\}, M_5 = \{(y, z) \in T : z > \frac{3-a}{a} y\};$$

$$\hat{M}_i = \{(x, y, z) \in S^2 : (y, z) \in M_i\}, i = 1, 2, 3, 4, 5.$$

Then

$$S^2 = \bigcup_{i=1}^5 \hat{M}_i.$$

Theorem 4. *If $(x^{(0)}, y^{(0)}, z^{(0)}) \in \hat{M}_i$ for some $i = 1, 2, 3, 4, 5$ then for the operator (11) the following holds*

$$\lim_{n \rightarrow \infty} V^n(x^{(0)}, y^{(0)}, z^{(0)}) = \begin{cases} s_1 & \text{if } x^{(0)} = 1 \\ s_2 & \text{if } i = 1, z^{(0)} > 0 \\ s_3 & \text{if } i = 2, y^{(0)} > 0 \\ s_4 & \text{if } i = 3, y^{(0)} > 0 \\ \in \hat{M}_3 & \text{if } i = 4, 5, \end{cases}$$

where $s_i, i = 1, 2, 3, 4$ are defined in (12).

Based on numerical analysis we make the following

Conjecture. *If $(x^{(0)}, y^{(0)}, z^{(0)}) \in \hat{M}_4 \cup \hat{M}_5$ then for the operator (11) the following holds*

$$\lim_{n \rightarrow \infty} V^n(x^{(0)}, y^{(0)}, z^{(0)}) = s_4.$$

Thus the operator (10) does not generate a chaotic dynamical system.

In the third chapter of the dissertation “**Quadratic stochastic processes of type $(\sigma | \mu)$** ” we consider to construct QSPs of type $(\sigma | \mu)$, i.e. to solve (5). To study the dynamics of such system when $t - s \rightarrow +\infty$.

Recall that a square matrix $Q = (q_{ij})_{i,j=1}^m$ is called *right stochastic* if

$$q_{ij} \geq 0, \quad \forall i, j = 1, \dots, m; \quad \sum_{j=1}^m q_{ij} = 1, \quad \forall i = 1, \dots, m.$$

Similarly one can define a *left stochastic* matrix being a non-negative real square matrix, with each column summing to 1

³ meaning that the second eigenvalue is greater than 1 in absolute value.

⁴ meaning that the second eigenvalue is less than 1 in absolute value.

Consider Maksimov's multiplication corresponding to arbitrary operation a on I which has right (resp. left) unique solvability.

Then the equation (5) has the following form

$$P_{ijr}^{[s,t]} = \sum_{l:n:a(l,n)=j} \sum_{k=1}^m P_{ilk}^{[s,\tau]} P_{knr}^{[\tau,t]}, \quad \forall i, j, r \in I. \quad (13)$$

We have to fix a stochasticity of cubic matrices first and solve (13) in class of such matrices.

Denote

$$q_{ir}^{[s,t]} = \sum_{j=1}^m P_{ijr}^{[s,t]}, \quad Q^{[s,t]} = \left(q_{ir}^{[s,t]} \right)_{i,r=1}^m. \quad (14)$$

Lemma 3. *The matrix $M^{[s,t]} = \left(P_{ijk}^{[s,t]} \right)$, with $P_{ijk}^{[s,t]} \geq 0$, is*

- (1,2)-stochastic (resp. (2,3)-stochastic) if and only if the corresponding matrix $Q^{[s,t]}$ is left (resp. right) stochastic;

- (1,3)-stochastic if and only if the corresponding matrix $Q^{[s,t]}$ satisfies

$$\sum_{i,r=1}^m q_{ir}^{[s,t]} \equiv m;$$

- 1-stochastic (resp. 3-stochastic) if and only if the corresponding matrix

$$Q^{[s,t]} \text{ satisfies } \sum_{i=1}^m q_{ir}^{[s,t]} \equiv m \text{ (resp. } \sum_{r=1}^m q_{ir}^{[s,t]} \equiv m);$$

- 2-stochastic iff $q_{ir}^{[s,t]} \equiv 1$.

Proposition 2. *If $m > 1$ and the operation a on I has right or left unique solvability, then equation (5) does not have any solution in class of i -stochastic (for any $i = 1, 2, 3$) cubic matrices.*

Condition 1. For definiteness let us take $I = \{0, 1, 2, \dots, m-1\}$ as a group with respect to operation a , defined by $a(i, j) = (i + j)(\text{mod } m)$. Then it is easy to see that a is uniquely solvable.

Under this condition the elements of the matrix $M^{[s,t]}$ can be renumbered as $M^{[s,t]} = \left(P_{ijk}^{[s,t]} \right)_{i,j,k=0}^{m-1}$.

The equation (13) can be written as

$$P_{ijr}^{[s,t]} = \sum_{k=0}^{m-1} \left(\sum_{l=0}^j P_{ilk}^{[s,\tau]} P_{k(j-l)r}^{[\tau,t]} + \sum_{l=1}^{m-j-1} P_{i(j+l)k}^{[s,\tau]} P_{k(m-l)r}^{[\tau,t]} \right), \quad \forall i, j, r \in I. \quad (15)$$

This is a non-linear system of functional equations with m^3 unknown two-variable functions $P_{ijr}^{[s,t]}$. Trivial solution is $P_{ijr}^{[s,t]} = \frac{1}{m^2}$, $\forall i, j, r \in I$, $0 \leq s < t$. The analysis of the system (15) is difficult. Therefore below we shall mainly consider the case $m = 2$.

(1,3)-stochastic solutions. Now we construct QSPs of type (13|a), where 13 means (1,3)-stochasticity and a means that we are considering multiplication (4).

For simplicity let us consider the case $m = 2$. Write a cubic matrix $M^{[s,t]}$ in the following convenient form:

$$M^{[s,t]} = \left(\begin{array}{cc|cc} P_{000}^{[s,t]} & P_{001}^{[s,t]} & P_{100}^{[s,t]} & P_{101}^{[s,t]} \\ P_{010}^{[s,t]} & P_{011}^{[s,t]} & P_{110}^{[s,t]} & P_{111}^{[s,t]} \end{array} \right) \quad (16)$$

We denote following matrices

$$M_1^{[s,t]} = \left(\begin{array}{cc|cc} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{array} \right) \quad (17)$$

$$M_2^{[s,t]} = \left(\begin{array}{cc|cc} f(s,t) & f(s,t) & f(s,t) & 1-3f(s,t) \\ \frac{1}{2}-f(s,t) & \frac{1}{2}-f(s,t) & f(s,t) & 3f(s,t)-\frac{1}{2} \end{array} \right) \quad (18)$$

where , $f(s,t) = \frac{1}{4} \left(\frac{\Phi(t)}{\Phi(s)} + 1 \right)$ and Φ is an arbitrary function with $\Phi(s) \neq 0$.

Proposition 3. *The matrices $M_1^{[s,t]}$ defined in (17) generate a QSP of type (13|a). The matrices $M_2^{[n,m]}$, $n, m \in \mathbb{N}$, $n < m$ defined in (18) generate a discrete-time QSP of type (13|a).*

(1,2)-stochastic solutions. Now we construct QSPs of type (12|a).

We consider the following cubic matrices:

$$M_3^{[s,t]} = \left(\begin{array}{cc|cc} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{array} \right); \quad (19)$$

$$M_4^{[s,t]} = \left(\begin{array}{cc|cc} 1/4 & 1/4 & 1/4 & \frac{1}{4} + \frac{\psi(t)}{2\psi(s)} \\ 1/4 & 1/4 & 1/4 & \frac{1}{4} - \frac{\psi(t)}{2\psi(s)} \end{array} \right) \quad (20)$$

where ψ is such that $-\frac{1}{2} \leq \frac{\psi(t)}{\psi(s)} \leq \frac{1}{2}$. This condition can be satisfied for a function

ψ when time is discrete, i.e., $t \in \mathbb{N}$. Then for example we can take $\psi(n) = 2^{-n}$;

$$M_5^{[s,t]} = \left(\begin{array}{cc|cc} g(s,t) & g(s,t) & g(s,t) & g(s,t) \\ \frac{1}{2}-g(s,t) & \frac{1}{2}-g(s,t) & \frac{1}{2}-g(s,t) & \frac{1}{2}-g(s,t) \end{array} \right); \quad (21)$$

where $g(s,t) = \frac{1}{4} \left(\frac{\varphi(t)}{\varphi(s)} + 1 \right)$ with $-1 \leq \frac{\varphi(t)}{\varphi(s)} \leq 1$;

$$M_6^{[s,t]} = \begin{cases} \left(\begin{array}{cc|cc} 1/2 & 1/2 & 1/2 & 1/2 \\ 0 & 0 & 0 & 0 \end{array} \right), & \text{if } s \leq t < c \\ \left(\begin{array}{cc|cc} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{array} \right), & \text{if } t \geq c. \end{cases} \quad (22)$$

Proposition 4. *The matrices $M_i^{[s,t]}$, $i = 3, 5, 6$ defined above generate QSPs of type $(12|a)$. The matrices $M_4^{[n,m]}$, $n, m \in \mathbb{N}$, $n < m$ generate a discrete-time QSP of type $(12|a)$.*

An example when Condition 1 is not satisfied. We consider an operation a on I which is not uniquely solvable. Consider binary operation $a(i, j) = \max\{i, j\}$. It is not uniquely solvable, in general. Indeed, for $m \geq 2$, the equation $\max\{x, m\} = m$ has many solutions: $x = 1, 2, \dots, m$.

Let σ is a fixed stochasticity of cubic matrices then the QSP corresponding to \max operation is denoted as type $(\sigma | \max)$. Here we give some examples of such QSP.

For simplicity we take $m = 2$ and solve the equation (5) for matrix $M^{[s,t]} = \left(a_{ijk}^{[s,t]} \right)_{i,j,k=1}^2$.

Denote

$$b_{ij}^{[s,t]} = a_{i1j}^{[s,t]} + a_{i2j}^{[s,t]}, \quad B^{[s,t]} = \left(b_{ij}^{[s,t]} \right); \quad (23)$$

$$c_{ij}^{[s,t]} = a_{i1j}^{[s,t]}, \quad C^{[s,t]} = \left(c_{ij}^{[s,t]} \right). \quad (24)$$

If $B^{[s,t]} = \left(b_{ij}^{[s,t]} \right)$ is a solution to

$$B^{[s,t]} = B^{[s,\tau]} B^{[\tau,t]} \quad (25)$$

and $C^{[s,t]} = \left(c_{ij}^{[s,t]} \right)$ is a solution to

$$C^{[s,t]} = C^{[s,\tau]} C^{[\tau,t]} \quad (26)$$

then corresponding solution to the equation (5) for $M^{[s,t]} = \left(a_{ijk}^{[s,t]} \right)_{i,j,k=1}^2$ is

$$M_7^{[s,t]} = \left(\begin{array}{cc|cc} c_{11}^{[s,t]} & c_{12}^{[s,t]} & c_{21}^{[s,t]} & c_{22}^{[s,t]} \\ b_{11}^{[s,t]} - c_{11}^{[s,t]} & b_{12}^{[s,t]} - c_{12}^{[s,t]} & b_{21}^{[s,t]} - c_{21}^{[s,t]} & b_{22}^{[s,t]} - c_{22}^{[s,t]} \end{array} \right) \quad (27)$$

Theorem 5. *Let $B^{[s,t]} = \left(b_{ij}^{[s,t]} \right)$ be a solution to (23) and $C^{[s,t]} = \left(c_{ij}^{[s,t]} \right)$ be a solution to (24) with $c_{ij}^{[s,t]} \in [0, 1]$ and $b_{ij}^{[s,t]} - c_{ij}^{[s,t]} \in [0, 1]$ for any $i, j = 1, 2$, $0 \leq s < t$ then the family of matrices $M_7^{[s,t]}$ given in (27) is a QSP of type*

- (12|max) iff $B^{[s,t]}$ is left stochastic for any $0 \leq s < t$.

- (13|max) iff $B^{[s,t]}$ (resp. $C^{[s,t]}$) with non negative elements with sum of all elements equals to 2 (resp. 1).

- (23 | max) iff $B^{[s,t]}$ is right stochastic for any $0 \leq s < t$.
- (1 | max) iff $B^{[s,t]}$ (resp. $C^{[s,t]}$) with non negative elements with sum of all elements of each column equals to 2 (resp. left stochastic).
- (2 | max) never.
- (3 | max) iff $B^{[s,t]}$ (resp. $C^{[s,t]}$) with non negative elements with sum of all elements of each row equals to 2 (resp. right stochastic).

Dynamical systems of QSPs. For QSPs generated by $M_i^{[s,t]}$, $i = 1, \dots, 7$ let us give the time behavior of the distribution $x^{(t)} = (x_0^{(t)}, x_1^{(t)}) \in S^1$. Fix $s \geq 0$ and by take a vector $x^{(s)} = (x_0^{(s)}, x_1^{(s)}) \in S^1$.

Case. $M_1^{[s,t]}$ and $M_3^{[s,t]}$, independently on the vector $x^{(s)}$, for any $t > s$, we get

$$x_0^{(t)} = \frac{1}{2}, \quad x_1^{(t)} = \frac{1}{2}.$$

Thus the time behavior of $x^{(t)}$ is clear: start process at time s with an arbitrary initial distribution vector $x^{(s)}$ then as soon as the time t turns on the distribution of the system goes to the distribution $(1/2, 1/2)$ and this distribution remains stable during all time $t > s$.

Case. $M_2^{[s,t]}$. For fixed $s \geq 0$, given vector $x^{(s)}$ and any $t > s$, we get

$$x_0^{(t)} = \left(\frac{1}{2} + \frac{\Phi(t)}{4\Phi(s)} \right) x_0^{(s)} + \left(\frac{1}{2} - \frac{\Phi(t)}{4\Phi(s)} \right) x_1^{(s)},$$

$$x_1^{(t)} = \left(\frac{1}{2} - \frac{\Phi(t)}{4\Phi(s)} \right) x_0^{(s)} + \left(\frac{1}{2} + \frac{\Phi(t)}{4\Phi(s)} \right) x_1^{(s)}.$$

The time behavior of $x^{(t)}$ depends on function Φ (which by our assumption satisfies $-1/3 \leq \Phi(t)/\Phi(s) \leq 1/3$).

Case. $M_4^{[s,t]}$. In this case we have

$$x_0^{(t)} = \frac{1}{2} x_0^{(s)} + \left(\frac{1}{2} + \frac{\psi(t)}{4\psi(s)} \right) x_1^{(s)},$$

$$x_1^{(t)} = \frac{1}{2} x_0^{(s)} + \left(\frac{1}{2} - \frac{\psi(t)}{4\psi(s)} \right) x_1^{(s)}.$$

As previous case, the time behavior of $x^{(t)}$ depends on function ψ (which by our assumption satisfies $-1/2 \leq \psi(t)/\psi(s) \leq 1/2$).

Case. $M_5^{[s,t]}$. In this case independently on the initial state vector $x^{(s)}$ we obtain

$$x_0^{(t)} = \frac{1}{2} + \frac{\varphi(t)}{4\varphi(s)},$$

$$x_1^{(t)} = \frac{1}{2} - \frac{\varphi(t)}{4\varphi(s)}.$$

This is an interesting dynamical system, because at each initial (fixed) time s the system does not depend on the initial state of the system. The trajectory only depends on the initial time itself and the time behavior of $x^{(t)}$ depends on function $\varphi(x^{(s)})$ which by our assumption satisfies $-1 \leq \varphi(t) / \varphi(s) \leq 1$.

Case. $M_6^{[s,t]}$. In this case independently on the initial state vector $x^{(s)}$ we obtain

$$x_0^{(t)} = 1 - x_1^{(t)} = \begin{cases} \frac{3}{4}, & \text{if } 0 \leq s < t < a \\ \frac{1}{2}, & \text{if } t \geq a. \end{cases}$$

Thus we get a discontinuous (with respect to time) dynamical system, the trajectory has limit $1/2$.

Case. $M_7^{[s,t]}$. Consider (3|max) type (other cases can be considered similarly). By Theorem 5 we get

$$\begin{aligned} x_0^{(t)} &= c_{11}^{[s,t]}(x_0^{(s)})^2 + (b_{11}^{[s,t]} - c_{11}^{[s,t]} + c_{21}^{[s,t]})x_0^{(s)}x_1^{(s)} + (b_{21}^{[s,t]} - c_{21}^{[s,t]})(x_1^{(s)})^2, \\ x_1^{(t)} &= c_{12}^{[s,t]}(x_0^{(s)})^2 + (b_{12}^{[s,t]} - c_{12}^{[s,t]} + c_{22}^{[s,t]})x_0^{(s)}x_1^{(s)} + (b_{22}^{[s,t]} - c_{22}^{[s,t]})(x_1^{(s)})^2. \end{aligned}$$

This is a quadratic continuous time dynamical system. The behavior of $x^{(t)}$ depends on the matrix $M_7^{[s,t]}$. One can choose this matrix to make the behavior of the dynamical system as reach as needed.

In section 3.3 we have several examples of quadratic stochastic processes (QSP) of type $(\sigma|D)$, where the type of σ stochastic cubic matrices, and D stands for a specific multiplication between cubic matrices.

In the work M. Ladra and U.A Rozikov proved the following theorem⁵.

Theorem 6. Let $\{A^{[t]} = (a_{ij}^{[t]}, t \geq 0)\}$ be a family of invertible (for all t), $m \times m$ square matrices and let $(A^{[t]})^{-1} = (b_{ij}^{[t]})$ denote the inverse of $A^{[t]}$. Let $B^{(s)} = (\beta_{ijk}^{(s)})_{i,j,k=1}^m$, be a cubic matrix, where $\beta_{ijk}^{(s)}, i, j, k = 1, \dots, m$, are arbitrary functions such that

$$\sum_{j=1}^m \beta_{ijk}^{(s)} = a_{ik}^{[s]}, \text{ for any } i, k \text{ and } s.$$

Then cubic matrix

$$M^{[s,t]} = \left(\sum_{k=1}^m \beta_{ijk}^{(s)} b_{kr}^{[t]} \right)_{i,j,r=1}^m \quad (28)$$

generates an flow of algebras (i.e. satisfies equation (5) of type (D)).

In general, the matrix (28) does not generate a QSP. Here our aim is to find conditions on matrices $A^{[t]}$ and $B^{(s)}$ (mentioned in Theorem 6) ensuring that the matrix (28) generates a QSP.

⁵Ladra M., Rozikov U.A., *Construction of flows of finite-dimensional algebras*, Jour. Algebra, 492 (2017) 475-489.

Lemma 4. Let $A^{[t]} = (a_{ij}^{[t]})_{i,j=1}^m$ be a right stochastic and invertible matrix and let $(A^{[t]})^{-1} = (b_{ij}^{[t]})_{i,j=1}^m$ be its inverse matrix. If $(A^{[t]})^{-1}$ is not a stochastic matrix, then it has a least one negative element.

Proposition 5. If $A^{[t]}$ and $B^{(s)} = (\beta_{ijk}^{(s)})_{i,j,k=1}^m$ in Theorem 6 satisfies the following conditions:

- $(A^{[t]})^{-1}$ is a left stochastic;
- $B^{(s)}$ is a (1,2)-stochastic

then the matrix $M^{[s,t]}$ (given by (28)) generates a QSP of type (12|D).

Theorem 7. If in Theorem 6 both $A^{[t]}$ and $(A^{[t]})^{-1}$ are stochastic for some $t > 0$ and $B^{(s)}$ is 3-stochastic for some $s < t$, then $M^{[s,t]}$ is not 3-stochastic.

Remark 4. It follows from the Theorem 7 that in order for $M^{[s,t]}$ to represent a quadratic stochastic process, the matrix $A^{[t]}$ must be a permutation matrix.

Proposition 6. If $A^{[t]}$ and $B^{(s)} = (\beta_{ijk}^{(s)})_{j,k=1}^m$ in Theorem 6 satisfies the following conditions:

- $(A^{[t]})^{-1}$ is a right stochastic;
- $B^{(s)}$ is a (1,3)-stochastic.

Then the matrix $M^{[s,t]}$ (given by (28)) generates a QSP of type (13|D)

Proposition 7. If $(A^{[t]})^{-1}$ and $B^{(s)} = (\beta_{ijk}^{(s)})_{j,k=1}^m$ in Theorem 6 satisfies the following conditions:

- $(A^{[t]})^{-1}$ is a right stochastic;
- $B^{(s)}$ is a (2,3)-stochastic.

Then the matrix $M^{[s,t]}$ (given by (28)) generates a QSP of type (23|D)

CONCLUSION

The thesis is devoted to investigation of the dynamical systems generated by stochastic and non-stochastic cubic matrices.

Basic results of the research are as follows:

1. Necessary and sufficient conditions to coefficients of the of quadratic operator to map the simplex S^{m-1} to itself are found.

2. All fixed points of discrete and continuous-time quadratic stochastic and non-stochastic operators defined in the simplex were found and the limit points of the trajectory were studied.

3. The dynamics of a one-dimensional quadratic non-stochastic operator is studied in general. Conditions for the parameters of the one-dimensional quadratic nonstochastic operator to form a chaotic dynamical system are given.

4. For a class of two-dimensional quadratic non-stochastic operators it is shown that among two-dimensional quadratic non-stochastic operators in

particular, there are those that form a chaotic dynamical system and those that do not form a chaotic dynamical system.

5. Some multiplication operations for cubic matrices are introduced, and a family of matrices satisfying the Kolmogorov-Chapman equation for these multiplications is determined.

6. Several examples of quadratic stochastic processes of type $(\sigma|D)$ have been constructed.

**НАУЧНЫЙ СОВЕТ PhD.03/30.06.2020.FM.70.04
ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ
КАРШИНСКОМ ГОСУДАРСТВЕННОМ УНИВЕРСИТЕТЕ**

БУХАРСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

ХУДАЯРОВ САЪНАТ САМАДОВИЧ

**КВАДРАТИЧНЫЕ СТОХАСТИЧЕСКИЕ ДИНАМИЧЕСКИЕ
СИСТЕМЫ ТИПА $(\sigma | \mu)$**

01.01.01 – Математический анализ

**АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD) ПО
ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ**

Карши – 2023

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии за В2020.4.PhD/FM531.

Диссертация выполнена в Бухарском государственном университете.

Автореферат диссертации на трех языках (узбекский, русский, английский (резюме)) размещен на веб-странице Научного совета (www.qarshidu.uz) и на Информационно-образовательном портале «Ziyonet» (www.ziyonet.uz).

Научный руководитель:	Розиков Уткир Абдуллоевич доктор физико-математических наук, профессор
Официальные оппоненты:	Жамилов Уйгун Умурувич доктор физико-математических наук Усмонов Жавохир Баходир угли доктора философии (PhD) по физико-математическим наукам
Ведущая организация:	Наманганский государственный университет

Защита диссертации состоится «___» _____ 2023 года в ___ на заседании Научного совета PhD.03/30.06.2020.FM.70.04 при Каршинском государственном университете (Адрес: 180103, г. Карши, ул. Кучабег, 17. Тел.: (+998 75) 225-34-13; факс: (+998 75) 221-00-56; e-mail: qarshidu@umail.uz). Каршинский государственный университет 2-корпус (аудитория № 202).

С диссертацией можно ознакомиться в Информационно-ресурсном центре Каршинского государственного университета (зарегистрирована за №___). (Адрес: 180103, г. Карши, ул. Кучабег, 17. Тел.: (+998 75) 225-34-13; факс: (+998 75) 221-00-56; e-mail: qarshidu@umail.uz).

Автореферат диссертации разослан «___» _____ 2023 года.
(протокол рассылки №___ от «___» _____ 2023 года).

Б.А.Шоимкулов
Председатель Научного совета
по присуждению научных
степеней, д.ф.-м.н., профессор

Ш.Д.Нодиров
Ученый секретарь Научного совета
по присуждению научных степеней,
д.ф.ф.-м.н. (PhD)

А.А.Имомов
Председатель Научного семинара
при Научном совете по
присуждению научных степеней,
д.ф.-м.н. (DSc), доцент

ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Цель исследования. Определение условий справедливости уравнения Колмогорова-Чепмена с использованием умножений, определенных для различных стохастических кубических матриц, и изучение динамики нелинейных операторов, построенных с использованием таких матриц. Нахождение условий сохранения симплекса коэффициентами квадратичных операторов. Нахождение предельных точек траекторий для класса таких операторов.

Объект исследования. Квадратичные (не)стохастические операторы, определяемые кубическими матрицами, удовлетворяющими уравнению Колмогорова-Чепмена с использованием умножения Максимова.

Научная новизна исследования заключается в следующем:

доказывается необходимое и достаточное условие сохранения коэффициентами квадратичного оператора симплекса, т. е. отражения симплекса в себя;

полностью изучена динамика произвольного одномерного квадратичного нестохастического оператора и определены условия формирования хаотической динамической системы при некоторых значениях параметров оператора с использованием свойств логистического отражения;

Среди двумерных квадратичных нестохастических операторов построены классы, создающие хаотическую динамическую систему и не создающие хаотическую динамическую систему, и полностью изучена динамика этих операторов;

вводятся несколько видов специальных умножений для кубических матриц и доказывается, что семейство матриц, порождающих квадратичные случайные процессы относительно этих умножений, удовлетворяет уравнению Колмогорова – Чепмена. Найдены изменяющиеся во времени предельные точки этих квадратичных случайных процессов.

Внедрение результатов исследования. Результаты, относящиеся к квадратичной стохастической динамической системе типа, были использованы в следующих исследовательских проектах:

результаты о динамических системах, порожденных нестохастическими квадратичными операторами и их предельными точками, были использованы в зарубежном грантовом исследовательском проекте «Динамика конечномерной ортогональности, сохраняющей кубические стохастические операторы» № FRGS21-230-0839 (Справочник Международного исламского университета Малайзии от 15 января 2023 г.). Применение научных результатов позволило классифицировать эволюционные операторы биологических систем, математической биологии и других естественных наук;

результаты по динамике семейства квадратичных операторов, порожденных нестохастическими кубическими матрицами, для которых марковский процесс пригоден относительно некоторого умножения, были использованы в зарубежном грантовом исследовательском проекте «Квантовые генетические алгебры и их приложения» №1. G00003447 (справка Университета Объединенных

Арабских Эмиратов от 06 января 2023 г.). Применение научного результата позволило проверить динамические системы, порожденные несколькими квадратичными стохастическими операторами, определенными на стандартном симплексе.

Структура и объем диссертации. Диссертация состоит из введения, трех глав, заключения и списка литературы. Общий объем диссертации составляет 94 страницы.

E'LON QILINGAN ISHLAR RO'YXATI
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ
LIST OF PUBLISHED WORKS

I bo'lim (part I; часть I)

1. Mamurov B.J, Rozikov U.A., Xudayarov S.S. Quadratic stochastic processes of type $(\sigma | \mu)$ // Markov Processes And Related Fields, 26:5 (2020). – P. 915-933. (Scopus).
2. Rozikov U.A., Xudayarov S.S. Quadratic non-stochastic operators: examples of splitted chaos // Annals of Functional Analysis. 13:17 (2022). – P. 1-17. (Scopus, IF=0.885).
3. Xudayarov S.S. Quadratic stochastic processes of permutation matrix // Bulletin of the Institute of Mathematics. 5:2 (2022). – P 33-39. (01.00.00. №17).
4. Xudayarov S.S. A quadratic operator corresponding to a non-stochastic matrix on 2D-simplex // Uzbek Mathematical Journal. 66:3 (2022). – P. 140-150. (01.00.00. №6).
5. Xudayarov S.S. A quadratic non-stochastic operator on 2D-simplex // Doklady Akad. Nauk. Uzbekistan. 4 (2022). – P. 27-30. (01.00.00. №7).

II bo'lim (part II; часть II)

1. Rozikov U.A, Xudayarov S.S., A class of quadratic stochastic progresses // Republican scientific and practical conference “Problems of Mathematics, physics and information technologies”. – Bukhara, 15 april, 2020 y. – P. 64-66.
2. Худаяров С.С. О квадратическом стохастическом процессом типа $(12|a)$ // Республиканской научно-практической конференции «Актуальные вопросы математики и прикладной математики в эпоху глобализации». – Ташкент, 1-2 июня 2021 г. – С. 129-133.
3. Rozikov U.A, Xudayarov S.S. A quadratic non-stochastic operator // Republican scientific conference with the participation of foreign scientists “Sarymsakov readings”. – Tashkent, 21-22 september, 2021 y. – P. 265-267.
4. Худаяров С.С. Решение квадратически стохастически процесс типа $(13|a)$ // Международной научно-практической конференции «Современные проблемы прикладной математики и информационных технологий». – Бухара, 15-апреля, 2021 г. – С. 239-241.
5. Xudayarov S.S. Dynamical systems of QSPs // Republican scientific conference with the participation “of foreign scientists “Differential equations and related problems of analysis” – Bukhara, 04-05 november, 2021 y. – P. 69-71.
6. Xudayarov S.S. On invariant sets of a quadratic non-stochastic operator // International scientific and practical conference “Modern problems of applied mathematics and information technologies”. – Bukhara, 11-12 may, 2022 y. – P. 52-53.
7. Xudayarov S.S. On invariant sets of a quadratic non-stochastic operator // International conference “Mathematical analysis and its applications in modern

mathematical physics”. – Samarkand, 23-24-sentabr, 2022 г. – С. 124-125.

8. Xudayarov S.S. A quadratic operator corresponding to a non-stochastic matrix on 2D-simplex // Republican scientific and practical conference “Operator algebras, non-associative structures and related problems”. – Tashkent, 14-15-sentabr, 2022 y. -- P. 307-308.

Avtoreferat Qarshi davlat universitetining “QarDU xabarları” ilmiy-nazariy,
uslubiy jurnali tahririyatida tahrirdan o‘tkazildi (08.04.2023 yil).

Guvohnoma № 14-061

08.04.2023. Bosishga ruxsat etildi.

Ofset bosma qog‘ozi. Qog‘oz bichimi 60x84 1/16.

“Times” garniturası. Ofset bosma usuli.

Hisob-nashriyot t. 3.2. shartli b.t. 3,7.

Adadi 60 nusxa. Buyurtma № 45.

Qarshi davlat universiteti

Kichik bosmaxonasida chop etildi.