

**“TIQXMMI” MILLIY TADQIQOT UNIVERSITETI, FUNDAMENTAL VA
AMALIY TADQIQOTLAR INSTITUTI HUZURIDAGI FAN DOKTORI
ILMIY DARAJASINI BERUVCHI DSc.03/31.03.2022.T/FM.10.04
RAQAMLI ILMIY KENGASH**

FUNDAMENTAL VA AMALIY TADQIQOTLAR INSTITUTI

MARDONOV SHUXRAT NUMONJONOVICH

**BOZE-EYNSHTEYN KONDENSATLARIDA NOCHIZIQLI DINAMIKA,
POLARONLAR VA O'Z-O'ZINI TASHKIL ETISH MEXANIZMLARI**

01.04.02 – Nazariy fizika

**Fizika-matematika fanlari doktori (DSc) dissertasiyasi
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Mardonov Shuxrat Numonjonovich

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KIRISH (fan doktori (DSc) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Dunyoda ilk bor kvant fizikasidagi bashorat qilingan makroskopik hodisa Boze-Eynshteyn kondensati (BEK) hisoblanadi. Boze-Eynshteyn kondensati materiyaning ajoyib va g'ayrioddiy holati bo'lib, unda bir guruh bozonlar, subatomik zarralar bir xil kvant holatni egallaydi. Bu hodisa 1924 yilda hind fizigi Satendra Nas Boze va Albert Eynshteyn tomonidan bashorat qilingan va 70 yildan ko'proq vaqt o'tgach, birinchi marta 1995 yilda eksperimental ravishda kuzatilgan. Ushbu tajribalar 2001 yilda fizika bo'yicha Nobel mukofotiga sazovor bo'ldi. BEKning kashfiyoti fizika uchun, jumladan, o'ta o'tkazuvchanlik, o'tasuyuqlik va kvant hisoblashlarni o'rganish uchun muhim ahamiyatga ega bo'ldi. Shuningdek, u atom va kondensirlangan muhitlar fizikasida yangi tadqiqot yo'nalishlarini ochdi.

Boze-Eynshteyn kondensati nazariyasidagi fizikasining eng qiziqarli mavzularidan biri bu o'zaro ta'sir qiluvchi zarralardan tashkil topgan Boze-Eynshteyn kondensatlarini tushunishdir. Kondensatni tashkil qiluvchi zarralar o'zaro ta'siri ikki xil xususiyatga ega bo'lishi, yani o'zaro itaruvchan yoki tortishuvchan bo'lishi mumkin. Sintetik spin-orbital bog'lanish (SOB) bilan Boze-Eynshteyn kondensatining fizik xususiyatlarini o'rganish Boze-Eynshteyn kondensati to'g'risidagi bilimlarni yanada kengaytiradi. Spin-orbital bog'lanish holatida optik tarzda yaratilgan atom psevdospini-1/2, atom impulsi va sintetik magnit maydon bilan birlashadi. Spin-orbital bog'lanish qattiq jismlar fizikasida mavjud bo'lgan simmetrik Rashba va Dresselhaus kabi, ko'p shakllarga ega bo'lishi mumkin. Nochiziqli fotonik panjaralar kabi boshqa tizimlarda soliton formadagi Boze-Eynshteyn kondensati tushunchasini kengaytirish bo'yicha harakatlar davom etmoqda. Bundan tashqari, eksperimental tarzda yaratilishi mumkin bo'lgan tartibsiz potentsiallardagi Boze-Eynshteyn kondensati xususiyatlarini o'rganish nochiziqli dinamika va kvant lokalizatsiyasi (mahalliylashuvi) o'rtasidagi munosabatlarini ko'rsatib beradi. Bundan tashqari, Boze-Eynshteyn kondensatida holatida solitonning harakatini o'rganish muhim ahamiyatga ega bo'lib, uning dinamikasiga tartibsiz potensial va spin-orbital bog'lanish sezilarli darajada ta'sir qilishi mumkin, hatto yarim klassik rejimda ham. So'nggi yillarda Boze-Eynshteyn kondensati koinotdagi ba'zi hodisalarni bashorat qilish uchun astronomiya sohasidagi olimlarning e'tiborini tortdi. Katta materiya to'lqini sifatida Boze-Eynshteyn kondensati qorong'u materiyaning xususiyatlarini taqlid qilishi mumkin va ularni o'rganish qorong'u va ko'rinadigan materiyaning dinamikasi va xususiyatlari haqida tushuncha berishi mumkin. Umuman olganda, Boze-Eynshteyn kondensatining dinamik xususiyatlarini o'rganish bizning dunyo haqidagi tushunchamizga sezilarli ta'sir ko'rsatishi va muhim texnologik yutuqlarga olib kelishi mumkin. Bu esa ushbu ilmiy tadqiqot ishini global darajada dolzarbligini asoslaydi.

Mamlakatimizda so'ngi yillarda fundamental va amaliy tadqiqotlarning dolzarb yo'nalishlarini rivojlantirishga tobora ko'proq e'tibor berilmoqda. Xususan, istiqbolli yo'nalishlardan biri bo'lgan nazariy fizikani rivojlantirish bugungi kunning muhim muammosidir. Yurtimizda ilm-fanning muvaffaqiyatli rivojlanishi

uchun fundamental tadqiqotlar va ishlanmalarning asosiy yo‘nalishlari va ularni amaliy qo‘llash O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha 2022-2026-yillarga mo‘ljallangan Strategiyasida¹ o‘z aksini topgan. Shuning uchun Boze-Eynshteyn kondensatlarining makroskopik dinamikasini o‘rganish fundamental tadqiqotlar sohasidagi dolzarb muammolardan biri bo‘lib qolmoqda.

Mazkur dissertatsiya ishi davlat me‘yoriy hujjatlariga, O‘zbekiston Respublikasi Prezidentining 2020-yil 29-oktabrdagi “Ilm-fanni 2030-yilgacha rivojlantirish konsepsiyasini tasdiqlash to‘g‘risida”gi PF-6097-sonli Farmonida, shuningdek, O‘zbekiston Respublikasi Prezidentining 2021-yil 19-martdagi “Fizika sohasida ta’lim sifatini oshirish va ilmiy tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi PQ-5032-sonli qarorlari hamda ushbu sohadagi boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda mazkur dissertatsiya tadqiqoti muayyan darajada xizmat qilaqdi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishi-ning ustuvor yo‘nalishlariga bog‘liqligi. Mazkur tadqiqot respublika fan va texnologiyalari rivojlanishining II. «Energetika, energiya va resurs tejamkorligi» ustuvor yo‘nalishi doirasida bajarilgan.

Dissertatsiya mavzusi bo‘yicha xorijiy ilmiy-tadqiqotlar sharhi.² Vaqtga bog‘liq bo‘lgan Gross-Pitayevskiy tenglamasi (GPT) bilan tavsiflangan Boze-Eynshteyn kondensatining nochiziqli dinamikasi, kvazi-ikki o‘lchovli Boze-Eynshteyn kondensatining kollapsi, tashqi tasodifiy potentsiallardagi Boze-Eynshteyn kondensati dinamikasi dunyoning yetakchi olimlari tomonidan eksperimental va nazariy jihatdan o‘rganilgan, jumladan, Ispaniya (G. Muga) Portugaliya (V.V. Konotop), Amerika Qo‘shma Shtatlari, Kolorado universiteti va NIST (E.A. Kornell, C.E. Wieman, V. Ketterle, M. Edvards, D. Kleppner, A. J. Leggett), Germaniya (T. V. Hänsch, I. Bloch, D. Jaksh, M. Weidemüller), Fransiya: (J. Dalibard, Ch. Salomon, T. Giamarchi), Buyuk Britaniya (S.L. Kornish, K. Burnett), Kanada: (R.G. Hulet), Italiya (M. Inguscio, S. Stringari, L. Pitaevskii, L. Salasnich), Isroil (B.A. Malomed, N. Davidson), Rossiya (A.Fetter, E.I. Rashba, V.V. Kartashov) va boshqalar.

Boze-Eynshteyn kondensatlarda kollaps (qulash) nazariyasi doirasida mahalliy olim F.X. Abdullayev va Y. Kogon, A.E. Muryshev, G.V. Shlyapnikov kabi xorijiy mualliflar tomonidan o‘rganilgan. Bir guruh E.A. donli, N.R. Klaussen, S.L. Kornish, J.L. Roberts, E.A. Kornell va C.E. Vieman kabi olimlar tomonidan Bose–Eynshteyn kondensatlarining kollapsi va portlash dinamikasi eksperimental ravishda olingan.

Boze-Eynshteyn kondensatlarida Spin-orbital bog‘lanish va Zeeman ta’sirini immitatsiya qilish imkoniyati mavjudligida Y.-J Lin, R.L.Kompton, K.Ximenes-Garsiya, J.V.Portu, I.B. Spielman, X. Zhai, V. Galitski kabi olimlar tomonidan katta

¹ O‘zbekiston Respublikasi Prezidentining 2022-yil 1-yanvardagi № PF-60 son farmoni “2022-2026 yillarga mo‘ljallangan Yangi O‘zbekistonning taraqqiyot strategiyasi to‘g‘risida”.

²Dissertatsiya mavzusi bo‘yicha xorijiy ilmiy-tadqiqotlar sharhi: <http://arxiv.org>; <https://webofknowledge.com>; <https://scholar/google.com>. J. Nature, J. Science, J. Reviews of Modern Physics, J. Physical Review Letters; J.; va boshqa manbalar asosida ishlab chiqilgan.

hissa qo'shildi va bu nohiziqli hodisalarning o'zaro ta'siri va tizimlarning spin dinamikasiga katta qiziqish uyg'otdi.

Polaron tushunchasi, ya'ni biror-bir o'rnatilgan muhitda bog'liq holatni hosil qiladigan tashqi zarracha, fizikadagi eng qiziqarli g'oyalardan biri hisoblanadi. Bu L.D.Landau va R.P.Feynmanning muhim asarlarida paydo bo'lib, qattiq jism va kondensirlangan muhit fizikasining turli sohalarida, shu jumladan magnit hodisalarida (F.C Zhang and T.M. Rice tomonidan o'rganilgan) bundan tashqari panjara deformatsiyasi effektlarining o'zgaishida (V.V. Konotop, J.T. Devreese, M.A. Semina, M.M. Glazov and E.Ya. Sherman tomonidan o'rganilgan) yetakchi va samarali tushunchalardan biriga aylandi.

Muammoning o'rganilganlik darajasi.

So'nggi bir necha o'n yilliklardan beri Bose–Eynshteyn kondensatining kollaps hodisasi keng miqyosda o'rganilib kelinmoqda va uni nazorat qilishning ko'plab turli usullari taklif qilindi va tekshirildi. Bu yondashuvlardan biri kondensatni kollapsiga qarshi barqarorlashtirish uchun tashqi potentsiallardan foydalanish, garmonik potentsialni o'rnatish yoki atomlararo o'zaro ta'sirlarni sozlash uchun Feshbax rezonanslaridan foydalanishdir. Yana bir yondashuv - tizimning nohiziqli dinamikasini kollaps uchun ishlatish va keyin uni oldini olish uchun teskari aloqa nazorat qilish usullarini foydalanish hisoblanadi.

Umuman olganda, Boze-Eynshteyn kondensatining kollapsini nazorat qilishni o'rganish ko'plab eksperimental va nazariy o'rganishlarning faol tadqiqot yo'nalishi hisoblanadi. Ba'zi tegishli nashrlar quyidagilarni o'z ichiga oladi: M. Edwards and K. Burnett, Phys. Rev. A 51, 1382 (1995); J. Javanainen and M. Mackie, Phys. Rev. A 66, 013607 (2002); S. T. Karpiuk, et al., Phys. Rev. Lett. 109, 205302 (2012); K. Adhikari, J. Phys. B 49, 170201 (2016).

Spin-orbital bog'langan Boze-Eynshteyn kondensatlari so'ngi yillarda keng qamrovli nazariy va eksperimental tadqiqotlar mavzusiga aylandi. Spin-orbital bog'langan Boze-Eynshteyn kondensatlarni o'rganish spin Hall effektlari, topologik izolyatorlar va Abel bo'lmagan o'lchov maydonlari kabi yangi hodisalarni kashf etishga olib kelgan turli nazariy ishlarda o'rganilgan (Zhai, H. Reports on Progress in Physics, 78(2), 026001 (2015); N. Goldman, G. Juzeliūnas, P. Öhberg, & I.B. Spielman, Reports on Progress in Physics, 77(12), 126401 (2014)). Spin-orbital bog'langan Boze-Eynshteyn kondensatlar, shuningdek, yuqori haroratli o'ta o'tkazgichlar va topologik izolyatorlar kabi murakkab kondensatsiyalangan moddalar tizimlarini simulyatsiya qilish uchun platforma sifatida ishlatilgan (Y. Xu, F. Zhang, & C. Zhang, Reports on Progress in Physics, 79(6), 066501 (2016); M. Aidelsburger, et al. Physical Review X, 8(3), 031027 (2018)). Bundan tashqari, Spin-orbital bog'langan Boze-Eynshteyn kondensatlar atom interferometri, kvant simulyatsiyasi va kvant ma'lumotlarini qayta ishlash kabi turli xil eksperimental ilovalarda qo'llanilgan (Y.J. Lin, et al. Nature, 471(7336), 83 (2011); P. Wang, et al. Physical Review Letters, 117(21), 215301 (2016)).

Boze-Eynshteyn kondensatlarida polaron hosil bo'lishi ham keng o'rganilgan, chunki u tashqi aralashma va atrofdagi kondensat o'rtasidagi o'zaro ta'sirni tushunishda muhim rol o'ynaydi. Polaron hosil bo'lishi aralashma va atrofdagi

kondensat o‘rtasidagi kuchli bog‘lanish tufayli yuzaga keladi va bu aralashmani bozonlar bilan yopishiga olib keladi. Boze-Eynshteyn kondensatlarda polaronni o‘rganish Bose polaroni, Fermi polaroni va Fröhlich polaroni kabi turli hodisalarning kashf etilishiga olib keldi, ular turli nazariy ishlarda o‘rganilgan (T. Yin, D. Cocks, and W. Hofstetter, Phys. Rev. A 92, 063635 (2015); N.B. Jørgensen, L. Wacker, K.T. Skalmstang, M.M. Parish, J. Levinsen, R.S. Christensen, G.M. Bruun, and J.J. Arlt, Phys. Rev. Lett. 117, 055302 (2016); Z.Z. Yan, Y. Ni, C. Robens, M.W. Zwierlein, Science 368, 190 (2020); J. Takahashi, R. Imai, E. Nakano, and K. Iida, Phys. Rev. A 100, 023624 (2019); A. Schirotzek, C.-H. Wu, A. Sommer, and M.W. Zwierlein, Phys. Rev. Lett. 102, 230402 (2009)). Boze-Eynshteyn kondensatlarda polaronlarning shakllanishi kondensirlangan holatlar tizimlarida aralashmalar dinamikasini o‘rganish uchun, shuningdek, Boze-Eynshteyn kondensatlarda o‘ta sovuq aralashmalarining xatti-harakatlarini o‘rganish uchun platforma sifatida ishlatilgan (M. Cetina, et al. Science, 354 (6316), 96-99 (2016); C. Kohstall, M. Zaccanti, M. Jag, et al. Nature 485, 615–618 (2012)).

Dissertatsiya mavzusining dissertatsiya bajarilgan oliy ta’lim va ilmiy-tadqiqot muassasasi ilmiy-tadqiqot ishlari bilan bog‘liqligi. Dissertatsiya ishi quyidagilar doirasida amalga oshirildi: O‘zbekiston Innovatsion rivojlanish vazirligining F-FA-2021-432 (2021-2025) raqamli ilmiy loyihasi; O‘zbekiston Respublikasi Innovatsion rivojlanish vazirligi huzuridagi Fanni moliyalashtirish va innovatsiyalarni qo‘llab-quvvatlash jamg‘armasi tomonidan moliyalashtiriladigan “Yosh olimlarning xorijiy ilmiy tashkilotlarda qisqa muddatli ilmiy stajirovkalarini tashkil etish va moliyalashtirish to‘g‘risida”gi 2022-yil 19-yanvardagi 2-sonli shartnomasi; O‘zbekiston Respublikasi Vazirlar Mahkamasi huzuridagi “El-yurt umidi” Mutaxassislarni xorijda tayyorlash va vatandoshlar bilan muloqot qilish jamg‘armasining 2019 yil stipendiyasi;

Tadqiqot maqsadi nochiziqli Gross-Pitayevskiy tenglamasi asosida turli ichki va tashqi tizimlar ta’siridagi Boze-Eynshteyn kondensatining yangi nochiziqli dinamik xususiyatlarini rivojlantirish, Boze-Eynshteyn kondensatini makroskopik hodisa sifatida dinamik xususiyatlarini klassik fizika (Nyuton mexanikasi) ba’zi qonunlari yordamida tavsiflashdan iborat.

Tadqiqotning vazifalari:

Boze-Eynshteyn kondensatining kollapsiga spin-orbital bog‘lanish ta’sirini va kollapsga to’sqinlik qiluvchi kritik shartlarni aniqlash;

Gross-Pitayevskiy tenglamasida diagonal va ichki spin-komponentlarning nochiziqliklari bo‘lgan holda, kondensat spiniga tashqi magnit maydoni qo‘llash orqali kondensat kollapsiga ta’sirini o‘rganish;

tartibsiz potensial va spini kuchli o‘zaro ta’siri ostidagi Boze-Eynshteyn kondensatida solitonning harakat dinamikasini tahlil qilish;

Gross-Pitayevskiy tenglamasi va klassik fizika qonunlari bilan tavsiflangan bir va ikki o‘lchovli Boze-Eynshteyn kondensatida polaron effekli o‘rnatilgan tashqi zarrachaning kondensat bilan o‘zaro bog‘liqlik dinamikasini o‘rganish;

bir o'lovli Boze-Eynshteyn kondensatida "gravitatsion" o'zaro ta'sirga ega ikkita zarrachalar dinamikasi bilan modellashtirilgan qorong'u materiyaning bir hil bo'lmagan zichligi modelini tahlil qilish.

Tadqiqot ob'ektlari Boze-Eynshteyn kondensati, spin, soliton, tasodifiy potentsial, tashqi o'rnatilgan zarralar, polaronlardir.

Tadqiqot predmeti Spin-orbital bog'langan Boze-Eynshteyn kondensatining kollapsi; tasodifiy potentsialda spin-orbital bog'langan soliton dinamikasi; Boze-Eynshteyn kondensatidagi polaronlar va tashqi zarralar.

Taqdidot usullari. Boze-Eynshteyn kondensati nazariyasining matematik apparati, klassik fizika ba'zi qonunlari va xususiy differensial tenglamalar sistemasi yechishning analitik va raqamli usullarini qo'llash bilan hisoblanadi.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

ilk bor spin-orbital bog'lanish va tashqi magnit maydon tomonidan boshqariladigan kondensat spinining kvazi-ikki o'lovli kondensat kollapsiga ta'siri o'rganildi va spinga bog'liq anomal tezlik kondensat kollapsini to'liq nazorat qilishi ko'rsatildi;

ilk bor spinga bog'liq anomal tezlik tufayli tasodifiy potentsialda soliton dinamikasining lokalizatsiyasi yoki delokalizatsiyasi uchun Zeeman bog'lanishi muhim rol o'ynashi ko'rsatildi;

ichki tortishuvchan spin komponentalariga ega Boze-Eynshteyn kondensatining siljishi Zeeman maydonidagi spin presessiyasi va tasodifiy potentsial mavjudligining birgalikdagi effekti tufayli yuzaga kelishi mumkinligi ko'rsatilgan;

ilk bor Gross-Pitayevskiy tenglamasi va klassik Nyuton mexanikasi qonunlari bilan tavsiflangan bir va ikki o'lovli Boze-Eynshteyn kondensatlari va polaronlarning o'zaro bog'liqlik dinamikasi ko'rsatildi;

Ilk bor bir o'lovli Boze-Eynshteyn kondensatida ikkita tashqi o'rnatilgan zarralarning "gravitatsion" o'zaro ta'siri dinamikasi bilan modellashtirilgan qorong'u materiyaning bir hil bo'lmagan zichligining soddalashtirilgan modeli taqdim etildi. Polaron, ya'ni o'rnatilgan zarracha yaqinidagi Boze-Eynshteyn kondensati zichligining o'zgarishi zarralar orasidagi "gravitatsion" kuchlari ta'sirida yuzaga keladigan dinamikani sezilarli darajada o'zgartirishi va gravitatsiyaviy qulash vaqtining o'zgarishiga olib kelishi aniqlandi.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

Kvazi-ikki o'lovli Boze-Eynshteyn kondensatining kollapsni boshqarish uchun spin-orbital bog'lanishidan kelib chiqadigan anomal tezlikning kollaps tezligiga nisbatan kritik qiymatlarini aniqlanishi tajriba o'tkazishga ahamiyatliligi bilan asoslanadi.

spin-orbital bog'lanish va Zeeman ta'siriga proporsional bo'lgan spinga bog'liq anomal tezlik tufayli solitonning lokalizatsiyasi yoki delokalizatsiyasining analitik va sonli shartlari aniqlandi. Yetarlicha kuchli Zeeman maydoni potentsialning tasodifiy minimallari yaqinida yorqin solitonning lokalizatsiyasiga olib kelishi ko'rsatildi;

Gross-Pitayevskiy va klassik Nyuton tenglamalari sistemasida tasvirlangan polaron va kondensatning o'zaro bog'liqgan birjinsli bo'lmagan dinamikasi ko'rsatildi. Boze-Eynshteyn kondensati polaronlar uchun o'zini yumshoq kvant materiya sifatida tutishi aniqlandi;

polaron effekti zarralar orasidagi Nyuton "gravitatsiyasi" kuchlari ta'sirida yuzaga kelgan dinamikani sezilarli darajada o'zgartirishi va gravitatsiyaviy qulash vaqtining o'zgarishiga olib kelishi ko'rsatildi;

statsionar bo'lmagan potentsialdagi kondensat dinamikasining "gravitatsiyalanuvchi" tashqi zarrachalar harakatiga ta'siri tahlil qilinadi.

Tadqiqot natijalarining ishonchliligi Tadqiqot natijalarining ishonchliligi dissertatsiya ishida matematik va nazariy fizikaning standart usuli, jumladan, yuqori samarali sonli usullar va dasturlardan foydalanilganligi; boshqa mualliflarning nazariy natijalarining muvofiqligi sinchkovlik bilan solishtirilganligi; xulosalar Boze-Eynshteyn kondensatlari nazariyasining asosiy qoidalariga juda mos kelishi bilan izoxlanadi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati shundan iboratki, dissertatsiyada ishlab chiqilgan formalizm Gross-Pitayevskiy tenglamasi bilan tavsiflangan makroskopik modda sifatida Boze-Eynshteyn kondensatining nochiziqli dinamik xususiyatlarini tahlil qilish asosida Boze-Eynshteyn kondensatini kvant hisoblashlar va kvant ma'lumotlar sohalarda keng qo'llash bilan belgilanadi. Shuningdek, kvant materiyaning makroskopik xususiyatlarini chuqurroq tushunishni yoritib berishi, atom kondensatlarining turli xil ta'sirlarini o'rganish orqali bugungi kunda hatto asosiy xususiyatlari ham yoritilmagan qorong'u materiyaning xususiyatlari va dinamikasini yoritib berishi mumkin bo'ladi.

Tadqiqot natijalarining amaliy ahamiyati shundan iboratki, ulardan Boze-Eynshteyn kondensatining nochiziqli dinamikasini, masalan, kollapslanadigan materiyani, spin dinamikasini, deformatsiya parametrlarini, siljishlarni o'rganishda, shuningdek Nyutonning gravitatsion modelidagi tuzatishlardan qorong'u materiya modellarini baholash uchun foydalanish mumkin. Natijalar kondensirlangan holatning tabiati va dinamikasini tahlil qilish, kuzatuv tajribalarini ishlab chiqishda ham foydali bo'lishi mumkin.

Tadqiqot natijalarining joriy qilinishi.

Sharqiy Yevropadagi hamkorlar bilan institutsional hamkorlikni qo'llab-quvvatlash uchun SCOPES dasturi doirasida Shveytsariya Milliy jamg'armasi ("Schweiz. Nationalfonds") granti IZ74Z0_160527/1.

Spin-orbital bo'glangan Boze-Eynshteyn kondensatlarining kollapsi bo'yicha chop etilgan maqolalarga yuqori impakt faktorli jurnallarda chop etilgan 40 dan ortiq ilmiy maqolalar tomonidan havola qilingan (H. Sakaguchi, B. Li va B.A. Malomed, Phys. Rev. E 89), 032920; Y. Chjan, M.E. Mossman, T. Busch va boshqalar. Front. Fizika 11, 118103; H. Sakaguchi, E.Ya. Sherman va B.A. Malomed, Phys. Rev. E 94, 032202; B.A. Malomed 2018 EPL 122, 36001 va boshqalar).

Turli tizimlarda spinga bog'liq anomal tezliklarning dinamik xususiyatlarini ishlab chiqish uchun tasodifiy potentsialdagi solitonning spin orqali kiritilgan

dinamik jarayonlar bo'yicha chop etilgan maqolalarga 10 dan ortiq ilmiy maqolalar tomonidan havola qilingan (J. Sun, Y. Chen, X. Chen va Y. Zhang, Phys. Rev. A 101, 053621; J. Yang va Y. Chjan, Phys. Rev. A 107, 023316; J. Fan, G. Chen va S. Jia, Phys. Rev. A 102, 063311 va boshqalar).

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqotning asosiy natijalari 4 ta xalqaro va 4 ta Respublika miqyosidagi ilmiy-amaliy konferensiyalarda ma'ruzalar qilinib, tegishli muhokamalardan o'tkazilgan.

Tadqiqot natijalarining e'lon qilinganligi. Dissertatsiya mavzusi bo'yicha jami 19 ta ilmiy ishlar chop etilgan, shu jumladan, O'zbekiston Respublikasi Oliy ta'lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasining dissertatsiyalar asosiy ilmiy natijalarini chop etish uchun tavsiya etgan ilmiy nashrlar ro'yxatida 11 ta ilmiy maqola (1 ta respublika va 10 ta xorijiy jurnallarda) chop etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya tarkibi kirish, to'rtta bob, xulosa, bitta ilova va foydalanilgan adabiyotlar ro'yxatidan iborat. Dissertatsiyaning hajmi 165 betni tashkil etadi.

DISSERTATSIYANING ASOSIY MAZMUNI

Dissertatsiyaning **kirish** qismida o'tkazilgan tadqiqotlarning dolzarbligi va zarurati asoslangan, tadqiqotning maqsadi va vazifalari, ob'ekt va predmetlari tavsiflangan, respublika fan va texnologiyalari rivojlanishining ustuvor yo'nalishlariga mosligi ko'rsatilgan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning ilmiy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarini amaliyotga joriy qilish, nashr etilgan ishlar va dissertatsiya tuzilishi bo'yicha ma'lumotlar keltirilgan.

Dissertatsiyasining birinchi bobi "**Boze-Eynshteyn kondensatining kollapsi**" deb nomlangan kvazi-ikki o'lchovli BEKning kollaps jarayonini, shu jumladan SOB spinning o'zaro ta'sirini o'rganishga bag'ishlangan. Bundan tashqari, Rabi chastotasi bilan spin komponentasining ichki va o'zaro ta'sirlar ko'rib chiqiladi. Ushbu bobda biz SOB bilan BEKning kollaps jarayonini stabillashtirish (barqarorlashtirish) sharti ko'rsatiladi va Rabi chastotasiga asoslangan spin komponentasining kollaps maydonini taqdim etladi. Bu yerda nazariy va eksperimental kondensatsiyalangan moddalar fizikasida keng o'rganilayotgan ikki komponentali to'lqin funksiyasi va sintetik SOB va magnit maydonlarni tavsiflovchi psevdospin-1/2 ni ko'rib chiqamiz.

Faraz qilaylik, nol Kelvin haroratda kondensatning dastlabki holati kvant garmonik potentsialning asosiy holati quyidagi shaklda tayyorlangan bo'lsin:

$$\psi(\mathbf{r}, t=0) \equiv \frac{\sqrt{N/\pi}}{a(0)} \exp\left[-\frac{r^2}{2a^2(0)}\right] \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \quad (1)$$

Bu erda $[1,0]^T$ - z o'qi bo'ylab yo'naltirilgan spinning boshlang'ich holati. Spin-orbital bog'langan, psevdospin-1/2 bog'langan kondensatning keyingi dinamikasi $\psi = [\psi_1(\mathbf{r}, t), \psi_2(\mathbf{r}, t)]^T$ to'lqin funksiyasi bilan tavsiflanadi, bunda $\mathbf{r} \equiv (x, y)$, to'lqin funksiyasi zarrachalar soni N ga normallashtirilgan. To'lqin funksiyasining evolyutsiyasi quyidagi GPT tomonidan tasvirlanadi

$$i\hbar \frac{\partial \psi}{\partial t} = \left[-\frac{\hbar^2}{2m} \Delta + H_{so} - g|\psi|^2 \right] \psi, \quad (2)$$

bu yerda m zarracha massasi, H_{so} SOB Gamiltoniani. (2)-tenglamada o'zaro ta'sir konstantasi $g = -4\pi\hbar^2 a_s / ma_z$ ifoda bilan aniqlanadi, bunda a_z kondensatning z o'qi bo'ylab tarqalish uzunligi va a_s manfiy. Quyida, $\hbar \equiv m \equiv 1$ birliklar va $\tilde{g} \equiv -4\pi a_s / a_z$ o'lchovsiz birligisiz o'zaro ta'sir kuchi ishlatiladi. Uzunlik birligi sifatida $a(0)$ ixtiyoriy ravishda tanlanadi va mos keladigan vaqt birligi $a^2(0)$ bilan olinadi.

Kondensatning spindan bog'liq bulmagan kollapsi uchun umumiy energiya quyidagicha aniqlanadi

$$E = -\frac{1}{2} \int [\psi^\dagger \Delta \psi + \tilde{g} |\psi|^4] dx dy. \quad (3)$$

Kollaps evolyutsiyasini ko'rsatish uchun biz variatsion yondashuvga asoslangan Gauss ansatz funksiyasidan foydalanamiz va (3)-tenglamaga asosan umumiy energiyani quyidagi ko'rinishda olamiz

$$E = \frac{N}{2a^2} \left(1 - \frac{\tilde{g}N}{2\pi} \right). \quad (4)$$

(4) ifoda shuni ko'rsatadiki, agar parametr $\tilde{g}N$ kritik qiymat $\lambda = 2\pi$ dan ohsa, kondensat kollapslanishi mumkin, chunki energiya manfiy bo'ladi va kondensat diametric nolga intilganda energiya minus cheksizga intiladi. Kondensatning vaqtdan bog'liq diametri $a(t)$ quyidagicha aniqlanadi

$$a(t) = a(0) \sqrt{1 - \frac{\Lambda t^2}{a^4(0)}}, \quad (5)$$

bunda $\Lambda = (\tilde{g}N - \lambda) / 2$. (5) ifodadan kelib chiqadiki kollaps vaqti $T_c \equiv a^2(0) / \sqrt{\Lambda}$ va kollaps tezligi $v_c \equiv a(0) / T_c = \sqrt{\Lambda} / a(0)$ ga teng.

Endi SOBning kollaps jarayoniga ta'sirini ko'rib chiqamiz. Anomal spinga bog'liq o'zgartirilgan tezlikni quyidagicha yozamiz:

$$\mathbf{v} = \mathbf{k} + \frac{\partial}{\partial \mathbf{k}} H_{so}, \quad (6)$$

(6) ifodaning oxirgi qism to'g'ridan-to'g'ri kondensatning spiniga bog'liq qo'shimcha hisoblanadi. Bu erda oqim (doimiy hyarakat) zichligi evolyutsiyasini ko'rish uchun quyidagi formulani keltiramiz

$$\mathbf{J}(\mathbf{r}, t) = \frac{i}{2} [\psi \nabla \psi^\dagger - \psi^\dagger \nabla \psi] + \psi^\dagger \left[\frac{\partial}{\partial \mathbf{k}} H_{so} \right] \psi. \quad (7)$$

Kondensatning kollapsini ko'rsatish uchun biz kondensatning diametrini aniqlovchi quyidagi formuladan foydalanamiz

$$a(t) = \frac{N}{\sqrt{2\pi}} \left[\int |\psi|^4 dx dy \right]^{-1/2}. \quad (8)$$

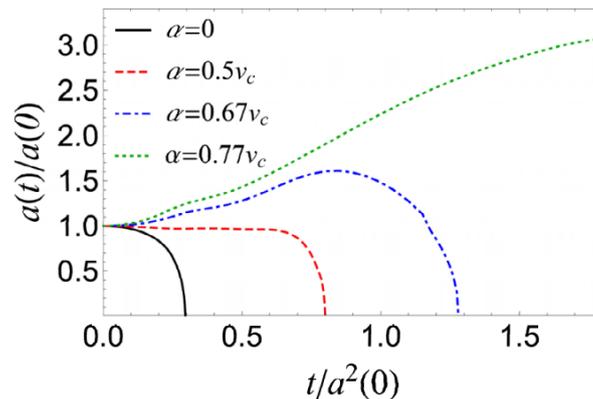
Quyidagi shaklda Dresselhaus SOB simmetriyalangan Gamiltonianini tanlaymiz

$$H_{so} = \alpha (k_x \sigma_x + k_y \sigma_y), \quad (9)$$

bu yerda α SOB doimiysi, σ_x , σ_y - mos ravishdagi Pauli matritsalar. (9)-ifodadan tezlikning mos koordinatalar bo'yicha komponentlari quyidagicha aniqlanadi

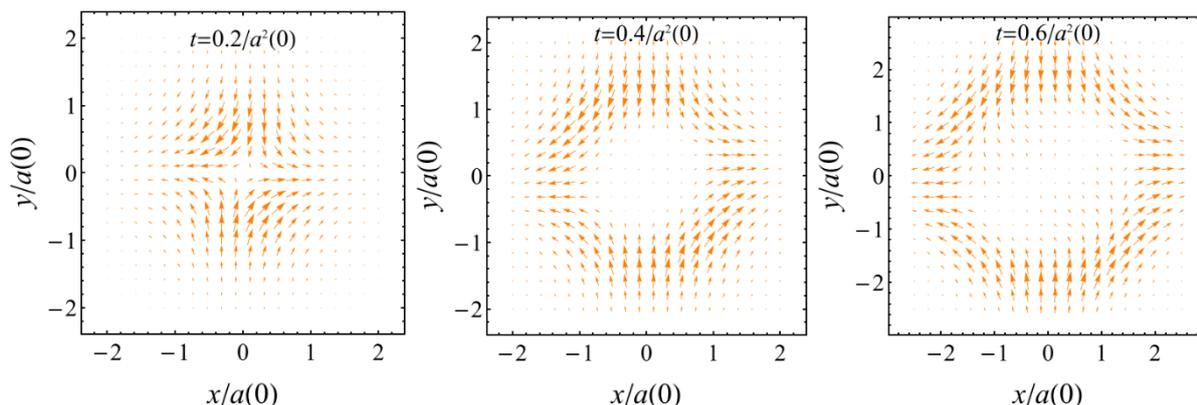
$$v_x = k_x + \alpha \sigma_x, \quad v_y = k_y + \alpha \sigma_y. \quad (10)$$

Spin tezligiga mos bir aylanishga ten xarakterli masofa $L_{so} = 1 / \alpha$ bilan belgilanadi.



1-rasm. (8)-tenglama va (2)-tenglamaning to'g'ridan-to'g'ri sonli yechimi bilan aniqlangan kondensat diametrining dinamikasi. Egri chiziqlar SOB konstantasi α ning rasmda berilgan qiymatlariga mos keladi va o'zaro ta'sir parametri $\tilde{g}N = 8\pi$.

1-rasm shuni ko'rsatadiki, qisqa vaqt davomida kondensat diametri SOB ning barcha qiymatlar uchun doimiy bo'lib, shundan keyin zarralar orasidagi tortishish va spinga bo'g'liq anomal tezlik o'rtasidagi o'zaro ta'sir boshlanadi. Ko'rinib turibdiki, SOBning ma'lum bir qiymatidan so'ng, kondensat kollaps bo'lishdan to'xtaydi. Shunday qilib, BEKning kollapsini oldini olish uchun $\alpha = v_c$ olish etarli, shunda 2-rasmda ko'rsatilgandek Desselhaus SOB BEK markazidan qochuvchi zichlik oqimini keltirib chiqaradi.



2-rasm. Panellarda $\alpha = v_c$, $\tilde{g}N = 8\pi$ va mos ravishda turli vaqt qiymatlari uchun (7)-ifoda bilan aniqlangan zichlik oqimi chizmalari belrilgan.

Dissertatsiya ishining ikkinchi bobi “**Tasodifiy potentsialdagi solitonning nochiziqli dinamikasi**” deb nomlangan bo'lib, tashqi sun'iy magnit maydonda va tasodifiy potentsialga joylashgan ichki o'zaro ta'sirga ega kvazi-bir-o'lchovli BEK tomonidan hosil bo'lgan spin-orbital bog'langan solitonlar dinamikasini o'rganishga bag'ishlangan. Kvazi-bir o'lchovli tizim sifatida koordinatali fazo mavjud deb taxmin qilinadi va tizim vaqtga bog'liq GPT yordamida namoyish etiladi. Kvazi-bir o'lchovli tizim sifatida koordinatalar maydoni $\mathbf{r} \equiv \mathbf{x} \equiv (x, t)$ deb qabul qilinadi va tizim vaqtga bog'liq quyidagi GPT tomonidan namoyish etiladi

$$i\partial_t \psi = \left[\frac{\hat{k}^2}{2} + \alpha \sigma_z \hat{k} + \frac{\Delta}{2} \sigma_x + U(x) + g |\psi_\kappa|^2 + \tilde{g} |\psi_{\kappa'}|^2 \right] \psi, \quad (11)$$

bunda $\kappa, \kappa' = 1, 2$ ($\kappa \neq \kappa'$). Bu yerda $\hat{k} = -i\partial / \partial x$ - impuls operatori, Δ - Zeeman bo'linishi, $U(x)$ - tasodifiy potentsial. Spin komponentasi o'zaro-ichki ta'siri g manfiy, ($g < 0$), va ikkala komponent uchun bir xil deb hisoblanadi. Shunda komponentalararo bog'lanish ta'siri \tilde{g} , $g = \tilde{g}$ shaklida berilgan deb taxmin qilinadi, ya'ni kondensat o'zaro ta'sir qilish energiyasi butun spin aylanishiga nisbatan o'zgarmasdir. Faraz qilaylik, (11) tenglamada $U(x) = 0$ va $\Delta = \alpha = 0$ bo'lmasin, u holda solitonning asosiy holati quyidagi ifoda bilan aniqlanadi,

$$\psi_{gr} = \frac{\sqrt{-g}}{2 \cosh[2(x - x_0) / g]} \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad (12)$$

bu erda x_0 - solitonning massa markazining boshlang'ich nuqtasi.

Silliq tartibsizlik katta L oraliqda o'zaro bog'liq bo'lmagan tasodifiy tuqtalar x_j va o'rtacha chiziqli zichlik $\bar{n} = N_{\text{im}} / L$ bo'lgan "aralashmalar" $N_{\text{im}} \gg 1$ ta taqsimot orqali quyidagicha hosil qilinadi

$$U(x) = U_0 \sum_{j=1}^{j=N_{\text{im}}} s_j u(x - x_j). \quad (13)$$

Bu erda $s_j = \pm 1$ - j ning tasodifiy funktsiyasi, o'rtacha qiymatlari $\langle s_j \rangle = 0$, shuning uchun $\langle U(x) \rangle = 0$. Bunda biz aralashmalarni $u(y) = \exp(-y^2 / \xi^2)$ ko'rinishida olamiz, bu erda ξ potensial kengligidir.

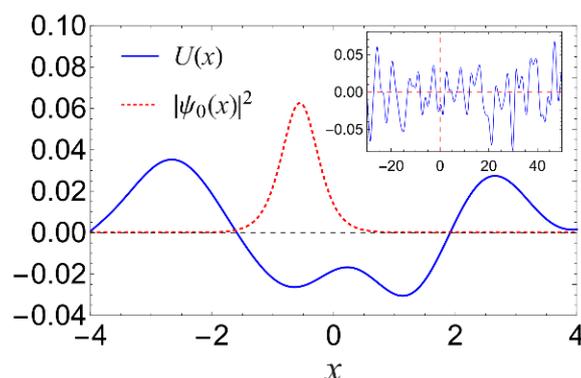
Solitonning tasodifiy potentsialdagi dinamikasini tavsiflash uchun biz har bir kuzatiladigan \mathcal{O} bilan bog'langan va quyiagi formula bilan aniqlangan $\mathcal{O}(t)$ integral miqdorlarni o'rganamiz

$$\mathcal{O}(t) = \int_{-\infty}^{\infty} \psi^\dagger(\mathbf{x}) \mathcal{O} \psi(\mathbf{x}) dx. \quad (14)$$

Xususan, soliton umumiy impulslari $k(t)$ ni va kuchi $F(t)$ ni aniqlash uchun (14) ifodadagi \mathcal{O} o'rniga \hat{k} va $\hat{F} \equiv -dU(x)/dx$ larni qo'yib va (11) tenglamadan foydalanib, Erenfest munosabatiga o'xshash quyudagi ifodaga ega bulamiz

$$\frac{dk(t)}{dt} = F(t). \quad (15)$$

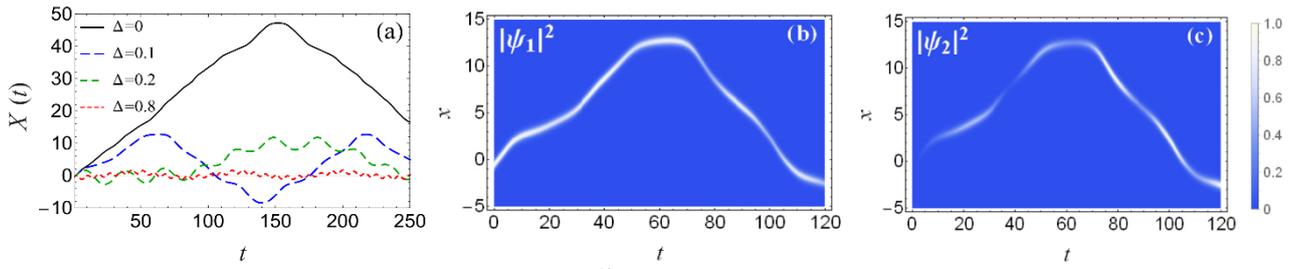
Biz boshlang'ich soliton statsionar va tasodifiy potentsial bilan muvozanat holatda deb faraz qilamiz. 3-rasmda $U(x)$ ning ko'rinishi va shu protokol yordamida tayyorlangan solitonning zichligi ko'rsatilgan. Ushbu soliton potentsialning minimumga yaqin joyda lokalizatsiyalangan va keyingi dinamika SOB va Zeeman maydonini kiritish orqali yuzaga keladi.



3-rasm. Qizil nuqtali chiziq $g = \tilde{g} = -5$ uchun solitonning boshlang'ich holati va ko'k rangli chiziq $U_0 = 0.01$, $\bar{n} = 10$ va $\xi = 1$ uchun hosil qilingan tasodifiy potentsial keltirilgan. Ichki qismdagi rasmda katta masofada ko'rsatilgan tasodifiy potentsial keltirilgan.

(11) - tenglama bilan berilgan soliton tezligi, $v(t) = dX(t)/dt$ sifatida aniqlanadi, va quyidagi ifoda bilan beriladi.

$$v(t) = k(t) + \alpha \sigma_z(t). \quad (16)$$



4-rasm. Δ ning har xil qiymatlari va $g = \tilde{g} = -5$ va $\alpha = 0.4$ uchun solitonning traektoriyasi keltirilgan. (a) paneldan ko'rish mumkinki, $\Delta = 0$ uchun soliton uzoq masofani bosib o'tadi, lekin Zeeman maydonini yoqish uning harakat trayektoriyasini cheklaydi. (b, c) panellar $\Delta = 0.1$ uchun (t, x) - tekislikdagi ikkita spinor komponentining zichliklar dinamikasi keltirilgan.

4-rasmda $\alpha = 0.4$ va Δ ning turli qiymatlari uchun soliton dinamikasi ko'rsatilgan. (a) panelda $\Delta = 0$ bo'lganda, soliton $U(x)$ ning katta tebranishga urilib teskari tezlik olguncha tartibsiz potentsial bo'ylab katta masofani bosib o'tishi keltirilgan. Zeeman maydonining mavjudligi soliton harakatini cheklaydi va oxir-oqibat Δ ning etarlicha katta qiymatlarida joyiga ushlab qoladi (bu grafikda $\Delta = 0.8$).

Biz spinning to'liq aylanishi uchun saqlanadigan o'z-o'ziga ta'sirlashuvchi energiyaga ega ingichka solitonni ko'rib chiqayotganimiz uchun adiabatik soliton $\psi_{ad}(x)$ uchun (11) chizikli qismining Gamiltonianni o'rtachasi saqlangan "kam energiya" miqdorini sifatida $\epsilon_0 = k^2(t)/2 + \alpha\sigma_z(t)k(t) + \Delta\sigma_x(t)/2 + U(X(t))$ ni kiritish mumkin, ya'ni $\sigma_z(t) = \cos\theta(t)$, $\sigma_x(t) = \sin\theta(t)\cos\phi(t)$ hisobga olgan holda ϵ_0 saqlanishni (adiabatik yaqinlashuvda tasdiqlangan) quyidagicha ifodalash mumkin.

$$v^2(t) - \alpha^2\sigma_z^2(t) + \Delta\sigma_x(t) = 2[U(X(0)) - U(X(t))], \quad (17)$$

bunda $v(t)$ tezlik (16) ifodada bilan berilgan.

Dissertatsiyaning uchinchi bobini **“Polaronlar o'z-o'zini tashkil etishi va Boze-Eynshteyn kondensatlarining”** garmonik potentsial bilan chegaralangan bir va ikki o'lchovli BEKda polaronning kogerent bog'langan harakatining turli rejimlarini o'rganishga bag'ishlangan. Bir-biri bilan chambarchas bog'liq bo'lgan kondensat shakli, evolyutsiyasi, uning massa markazi va polaron koordinatasi uchun nochizikli bo'lmagan polaron-kondensat bog'langan tebranishlari o'rganiladi. Faraz qilaylik, o'zaro ta'sir qiluvchi BEK $N \gg 1$ zarrachalar tomonidan qurilgan bo'lsin, u holda bir o'lchovli fazoda tizim quyidagi shakldagi GPT tomonidan tasvirlanishi mumkin:

$$i \frac{\partial}{\partial t} \psi(x, t) = \left(-\frac{1}{2} \frac{d^2}{dx^2} + \frac{x^2}{2} \right) \psi(x, t) + g |\psi(x, t)|^2 \psi(x, t) + V(x - X(t)) \psi(x, t). \quad (18)$$

(18)- tenglamada yangi ko'paytuvchi $V(x - X(t))$ o'rnatilgan tashqi zarracha potentsialini ifodalaydi. Biz o'rnatilgan tashqi zarrachaning potentsialini $V(x - X(t)) = V_0 \exp(-(x - X(t))^2 / 2\delta^2)$ ko'rinishida tanlaymiz, bu erda V_0 amplituda musbat yoki manfiy bulishi mumkin, va δ kondensat uzunligidan ancha kichik bolgan uzunligi. Shuning uchun, zarracha va kondensat o'rtasidagi o'zaro ta'sir energiyasini quyidagicha aniqlaymiz:

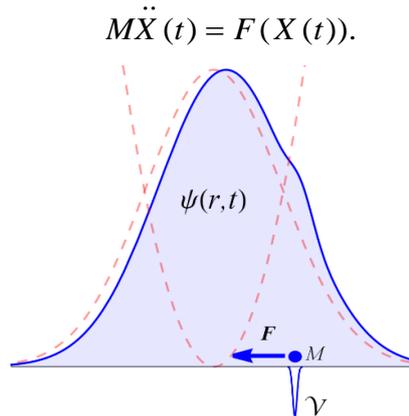
$$V(X(t)) = V_0 \int_{-\infty}^{\infty} |\psi(x)|^2 \exp\left(-\frac{(x - X(t))^2}{2\delta^2}\right) dx \approx \sqrt{2\pi\tilde{V}} |\psi(X(t))|^2, \quad (19)$$

bu erda ($\tilde{V} \equiv V_0\delta$). Yana, (19)- tenglama sifat tahlili va sonli hisoblar uchun juda mos keladi. Biz kondensat va tashqi zarralar o'rtasidagi kuchga bog'liq yondashuvni umumlashtirish uchun quyidagi aniq vaqtga bog'liq kuchdan foydalanamiz:

$$F(X(t)) \equiv -\partial_{X(t)} \int_{-\infty}^{\infty} V(x - X(t)) |\psi(x, t)|^2 dx. \quad (20)$$

Biz tashqi o'rnatilgan zarrachalarni kondensat zarrachalariga nisbatan juda og'ir deb olamiz, shuning uchun ularning trayektoriyasi quyidagi klassik Nyuton tenglamasi bilan tavsiflanadi:

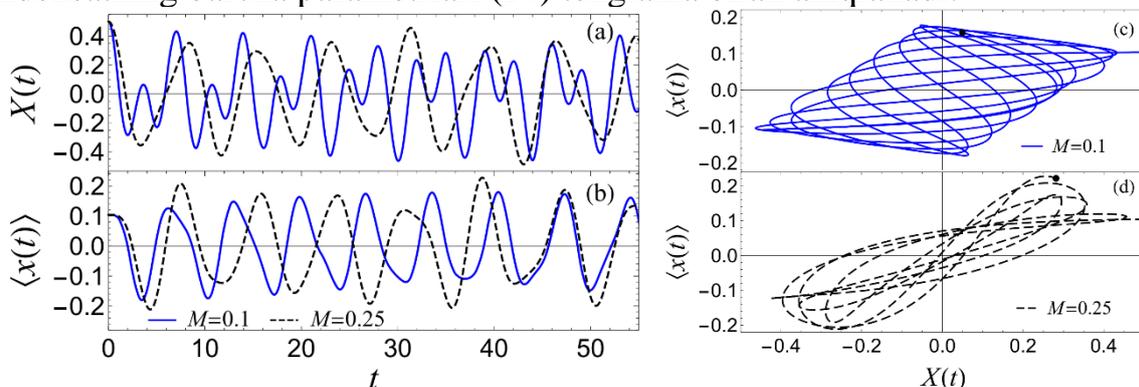
$$M\ddot{X}(t) = F(X(t)). \quad (21)$$



5-rasm. BEKda polaron hosil qilish modelining sxematik shakli.

5-rasmda BEKda hosil bo'ladigan polaron modelining sxematik shakli ko'rsatilgan. Bunda $\psi(r, t)$ funktsiya $t = 0$ da BEK zichligi taqsimotini, $V < 0$ - tashqi zarrachaning tortishishuvchan potentsialini, M - massasi bildiradi va zarracha kondensat massa markazidan ma'lum masofada joylashgan. Qizil chiziqchali chiziqlar bilan parabolik potentsiali va uning asosiy holatidagi kondensat sizchlik taqsimoti, ko'k chiziq esa polaron hosil qiluvchi zarralar tufayli deformatsiyalangan kondensat zichlik taqsimoti ko'rsatilgan. Polaron va kondensatning barcha dinamiklari $t > 0$ vaqtda taqdim etiladi.

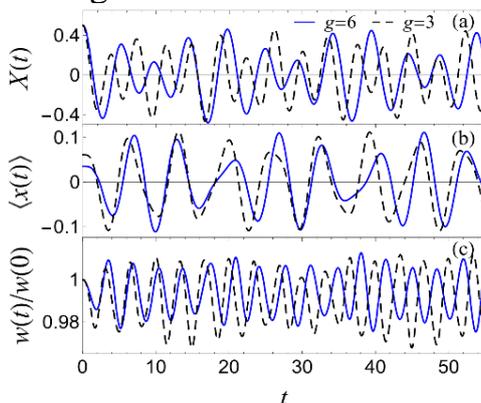
O'rnatilgan tashqi zarracha va kondensatning barcha dinamika parametrlari (18), (20) va (21) tenglamalar sistemasining sonli yechimi bilan hisoblanadi. Kondensatning barcha parametrlari (14) tenglama bilan aniqlanadi.



6-rasm. Grafiklarda berilgan massa qiymatlari uchun polaron harakatining traektoriyalari (a), kondensatning massa markazi traektoriyalari (b) va massa markazining $X(t)$ ga nisbatan holati

($t \leq 40$ vaqt uchun) (c), (d) grafiklari keltirilgan. O'zaro ta'sir qilmaydigan BEK ($g = 0$)da polaronning dastlabki parametrlari quyidagicha olingan: $X(0) = 0.5$, $\delta = 0.1$, $V_0 = -1$.

6-rasmda polaron va kondensatning massa markazining o'zaro bog'liq tartibsiz dinamikasi keltirilgan. Natija shuni ko'rsatadiki, polaron massasining ortishi polaron tebranishlarida tartibsiz impulslarning kamayishiga olib keladi, chunki polaronning massasi kondensatning umumiy massasiga yaqin bo'lsa, u holda sistema kvazi muvozanat holatida kelib qoladi. (c) va (d) chizmalarda qizil (qora) nuqta polaronning dastlabki (yakuniy) holatiga mos keladi. E'tibor beraylik, (c) chizmadagi boshlang'ich nuqtasi ($(X(t), \langle x(t) \rangle)$) -traektoriya bilan to'ldirilgan parallelogram strukturasidan tashqarida joylashgan. Bu tebranishlarning nochiziqli tabiatining kelib chiqib, tashqi zarra dastlab kondensat zichligi yuqori bo'lgan hududga tortiladi va keyin ulangan tebranishlar boshlanadi.



7-rasm. Polaron trayektoriyasi (a), massa markazi trayektoriyasi (b) va kondensatning kengligi (c) harakatining chizmalari keltirilgan. Bu erda polaron boshlang'ich parametrlari $M = 0.05$, $X(0) = 0.5$, $\delta = 0.3$, $V_0 = -0.5$ ko'rinishida olingan bulib kondensat o'zaro ta'sir parametri qiymatlari chizmada keltirilgan.

Ichki o'zaro ta'sirga ega BEKning polaron va kondensatning dinamikasiga ta'siri 7-rasmda ko'rsatilgan. E'tibor beraylik, kutilganidek, $g = 3$ uchun $X(t)$ ning ba'zi dastlabki tebranishlari $g = 6$ ga qaraganda qisqaroq davrga ega. Biroq, vaqt o'tishi bilan nochiziqli ta'sir effekti kuchayadi va va farqni aniqlash qiyin bo'ladi. Tomas-Fermi yaqinlashuvidagi BEK chegaralari mos ravishda $g = 6$ uchun $x_{\max} = 2.08$ va $g = 3$ uchun $x_{\max} = 1.65$ ga teng.

Shuningdek, parabolik potentsial bilan chegaralangan o'zaro ichi ta'sirga ega bo'lmagan kvazi-ikki o'lchovli BEKda polaronning o'zabor bog'langan dinamikasi o'rganildi. Ikki o'lchovli BEK uchun (18) tenglamada x koordinata $\mathbf{r} = (x, y)$ bilan almashtiriladi. Ikki o'lchovli tizim uchun polaron kondensatning massa markaziga nisbatan perpendikulyar bo'lgan boshlang'ich v_0 tezlikka ega deb faraz qilinadi. Bunda barcha dinamik (18)-(21) tenglamalar ikki o'lchovli koordinatali tekisligida ifodalanishi va tenglamalarni sonli yechish orqali tahlil qilinishi mumkin.

Kondensatdagi o'rnatilgan tashqi zarracha dinamikasini tushunish uchun biz statik BEC holatidan boshlaymiz. Kondensat va polaron o'rtasidagi o'zaro ta'sir nisbatan juda kichik ($\tilde{v} \ll 1$) deb faraz qilamiz va bu kondensat zichligi

deformatsiyalanmaydi va u doimo tinch holatda bo'ladi. Natijada zarrachalar dinamikasi faqat (21) tenglama yechimi bilan aniqlanadi. (18)- GPTning statik yechimi bilan (19)- energiya taqribiy echimidan foydalanib, biz (21) tenglamani quyidagi shaklda qayta yozamiz

$$M \ddot{\mathbf{R}}(t) = -2\pi\tilde{V} \frac{\partial |\psi(\mathbf{R}(t))|^2}{\partial \mathbf{R}(t)} \equiv 4\tilde{V}\mathbf{R}(t)\exp(-R^2(t)), \quad (22)$$

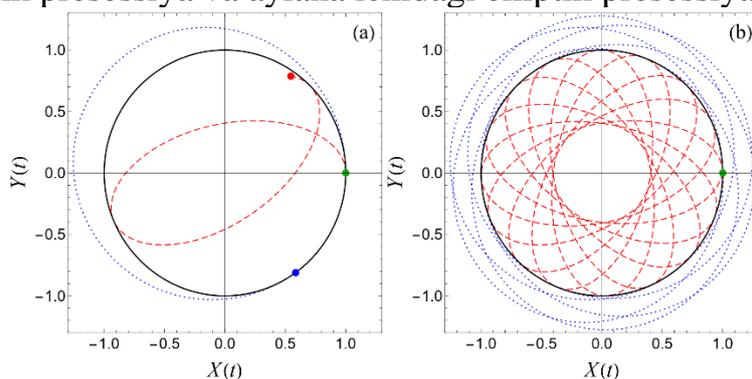
(19)- energiya polaron va kondensat o'rtasidagi tortishishuvchanlikni ifodalagani sababli, (20) tenglama zarracha uchun markazga tortuvchi kuchini ta'minlashi mumkin. Bundan kelib chiqadiki, $\mathbf{R}(0)$ vektorga perpendikulyar bo'lgan har qanday boshlang'ich tezlik $\mathbf{v}(0)$ uchun zarrachaning aylana traektoriyasining tenglamasi quyidagi ifoda bilan aniqlanadi

$$M \frac{v_0^2}{R_0} = F(R_0), \quad (23)$$

bu erda $v_0 \equiv |\mathbf{v}(0)|$ va $R_0 \equiv |\mathbf{R}(0)|$. Aslida, aylanma traektoriya tufayli bizda tezlik $v(t) \equiv v_0 = \text{const}$, bundan kelib chiqadiki $R(t) \equiv R_0 = \text{const}$. (22) va (23) tenglamalardan foydalanib, quyidagi tenglamani olamiz

$$Mv_0^2 = 4|\tilde{V}|R_0^2 \exp(-R_0^2). \quad (24)$$

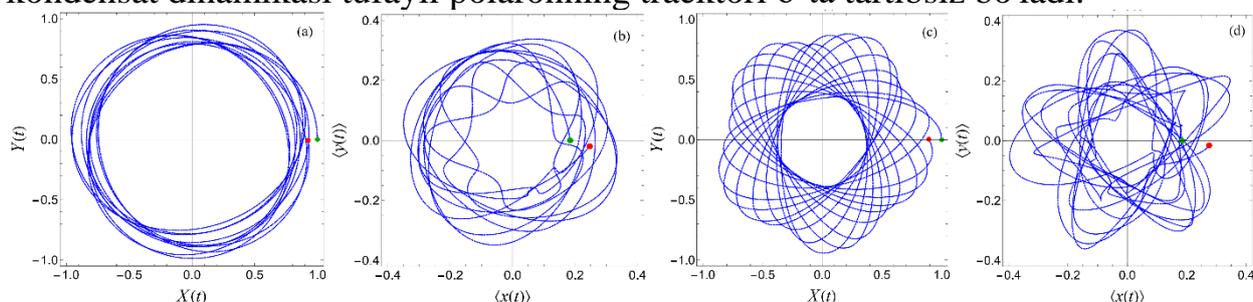
Natijada (24) tenglama statik BEKda o'rnatilgan zarrachaning aylana traektoriyasini aniqlaydi. Endi faraz qilaylik, berilgan dastlabki tezlik $\mathbf{v}(0)$ va joylashuv nuqtasi $\mathbf{R}(0)$ (24)- tenglamani qanoatlantirmasin. (24). Bunda, agar $Mv_0^2 < 4|\tilde{V}|R_0^2 \exp(-R_0^2)$ bo'sa, u holda zarracha traektoriyasi aylana orbita ichida kvazi-apsidal presessiya bo'yicha harakatlanadi yoki agar $Mv_0^2 > 4|\tilde{V}|R_0^2 \exp(-R_0^2)$ bo'lsa, u holda zarracha traektoriyasi aylana orbitadan tashqarida kvazi-apsidal presessiya bo'ylab harakatlanadi. 9-rasmda har xil boshlang'ich tezlikka mos keladigan zarrachalarning uchta traektoriyasi ko'rsatilgan aylana, : aylanadan tashqarida elliptik presessiya va aylana ichidagi elliptik presessiya.



9-rasm. Statik BEK uchun o'rnatilgan zarrachaning (x, y) tekislikdagi traektoriyasining grafigi. Zarrachaning dastlabki parametrlari $M = 1$, $\mathbf{R}(0) = (1, 0)$, $\tilde{V} = -10^{-2}$ ko'rinishida olingan. Chiziqlar turli xil quyidagi dastlabki tezliklarga mos keladi: $\mathbf{v}(0) = (0, 0.5v_0)$ - qizil kesmali chiziq, $\mathbf{v}(0) = (0, v_0)$ - qora uzluksiz chiziq va $\mathbf{v}(0) = (0, 1.1v_0)$ - ko'k nuqtali chiziq.

9-rasmda zarracha traektoriyasi aylana bo'ylab harakatlanishi uchun tezlik $v_0 = 2(|\tilde{V}| R_0^2 \exp(-R_0^2))^{1/2} \approx 0.121$ ga va aylanish davri $t_0 = 2\pi R_0 / v_0 \approx 51.8$ ga teng. (a) grafikda traektoriyalar vaqti t_0 tent, (b) grafikda traektoriyalar zarrachaning dastlabki holatiga (ko'k nuqta) qaytgan vaqti t_f ga to'g'ri keladi ($\mathbf{R}(t_f) \approx \mathbf{R}(0)$). (a) chizmadan ko'rinib turibdiki, garchi boshlang'ich tezligi $0.5v_0$ ga kamaygan bo'lsa ham, qizil nuqtali traektoriya zarrachaning to'liq bir aylanishdan ko'proq masofani bosib o'tgan. Mos ravishda, ko'k nuqta bilan belgilangan traektoriya tezligi $1.1v_0$ ga oshirilsa ham, trayektoriya to'liq bir aylanani tashkil etmagan. Bu xususiyatlar, tevlilik trayektoriya bo'lab o'zgaruvchanligini, yani markazga yaqin joyda oshishini va uzoqlashgan sari kamayishini ko'rsatadi.

Polaron hosil qiladidan zarralar uchun (22)-(24) tenglamalar o'rinli bo'lmaydi, chunki BEK ning boshlang'ich holatining deformatsiyasi tufayli polanlar uchun tashqi potentsial rolini o'ynaydigan BEKning dinamikasi hosil bo'ladi. Shu sababli, polaron potentsialining dastlabki parametrlari asosida deformatsiyalangan kondensat dinamikasi tufayli polaronning traektoriyasi o'ta tartibsiz bo'ladi.



10-rasm. (a), (c)- polaron va (b), (d)- kondensat massa markazi traektoriyalarining (x, y) tekislikdagi chizmalari berilgan. Dastlabki parametrlar $M = 1$, $\mathbf{R}(0) = (1, 0)$, $\tilde{V} = -10^{-1}$ ko'rinishida olingan. (a, b) chizmalar $\mathbf{v}(0) = (0, v_0)$ va (c, d) chizmalar $\mathbf{v}(0) = (0, 0.5v_0)$ boshlang'ich tezliklarga mos keladi.

10-rasmda polaron va kvazi ikki o'lchovli BECning massa markazi traektoriyalari keltirilgan. Polaron boshlang'ich parametrlari uchun aylana orbitasiga mos keladigan $v_0 = (4|\tilde{V}| R_0^2 \exp(-R_0^2))^{1/2} \approx 0.384$ tezlik olingan. (a), (b) chizmalar uchun traektoriya vaqti $t_f = 133$ ga va (c), (d) chizmalar uchun $t_f = 187.7$ ga teng bo'lib, bu vaqtlarda $\mathbf{R}(t_f)$ - qizil nuqta boshlang'ich $\mathbf{R}(0)$ ko'k nuqtaga eng yaqin keladi. Grafiklar polaron ta'siridan kelib chiqqan kondensat dinamikasi tufayli zarrachaning va kondensat massa markazining notekis traektoriyasini ko'rsatadi.

Dissertatsiyaning to'rtinchi bobi "**Boze-Eynshteyn kondensatlarida gravitatsiyalanayotgan zarralar**" deb nomlangan bo'lib, bir o'lchovli BEKda joylashtirilgan, o'zaro Nyutonning "gravitatsion" modeli bilan ta'sirlashadigan tashqi zarralar o'rtasidagi dinamikani o'rganishga bag'ishlangan. Bunda BEK tashqi garmonik osilator potentsial bilan chegaralangan deb qaraladi. Biz BEKdagi oddiy polaron modeli asosida qorong'u materiya bilan bog'liq bo'lgan o'zaro ta'sirlarning

namoyon bo'lishi sifatida ko'rib chiqamiz, ya'ni biz asosan "markazga tushish" dinamikasining analitik va raqamli hisob-kitoblariga e'tibor qaratamiz. Bundan tashqari, kondensat evolyutsiyasining ta'sirini tushunish uchun biz tebranuvchi kondensat yordamida gravitatsiyalanayotgan zarrachalarning dinamikadasini o'rganamiz. Tizim GPTning quyidagi shakli bilan tasvirlangan:

$$i \frac{\partial}{\partial t} \psi(x,t) = \left(-\frac{1}{2} \frac{\partial^2}{\partial x^2} + \frac{\omega^2(t)}{2} x^2 \right) \psi(x,t) + g |\psi(x,t)|^2 \psi(x,t) + \mathcal{V}_{\text{ext}}(x) \psi(x,t), \quad (25)$$

bunda

$$\mathcal{V}_{\text{ext}}(x) = \mathcal{V}_1(x - X_1(t)) + \mathcal{V}_2(x - X_2(t)). \quad (26)$$

(25)- tenglamada $\omega(t)$ - statsionar bo'lmagan garmonik potensial chastotasi. (25)-tenglamadagi $\mathcal{V}_1(x - X_1(t))\psi(x,t)$ va $\mathcal{V}_2(x - X_2(t))\psi(x,t)$ qismlar vaqtga bog'liq $X_1(t)$ va $X_2(t)$ nuqtalarda joylashgan harakatlanuvchi zarralar bilan BEKning mahalliy o'zaro ta'sirini tavsiflaydi. Bu zarrachalarning massalari mos ravishda M_1 va M_2 ga teng. Shunda (19) va (20) tenglamalar ikkala zarra uchun ham taqdim etilishi mumkin. Kondensat va tashqi zarralar orasidagi vaqtga bog'liq kuch ikkala zarrachaning holatiga bog'liq bo'lib quyidagicha aniqlanadi

$$\mathcal{F}_c^{[i]}(X_i(t), X_j(t)) = -\partial_{X_i(t)} \int_{-\infty}^{\infty} \mathcal{V}_{\text{ext}}(x) |\psi(x,t)|^2 dx, \quad (27)$$

Bu erda $(i, j) = (1, 2)$ indekslar ikkita tashqi o'rnatilgan zarralarni ifodalaydi.

Biz o'rnatilgan tashqi zarralar orasidagi asosiy o'zaro ta'sir quyidagi formada berilgan Nyutonning "gravitatsion" kuchiga o'xshash model bilan tasvirlangan deb faraz qilamiz

$$\mathcal{F}_G^{[i]}(X_i(t), X_j(t)) = GM_1 M_2 \frac{X_j(t) - X_i(t)}{|X_i(t) - X_j(t)|^3}, \quad (28)$$

bu yerda G Nyutonning "gravitatsion" o'zaro ta'sir konstantasi. U holda zarrachalarning trayektoriyasi quyidagicha berilgan klassik Nyuton tenglamasi bilan aniqlangan

$$M_i \frac{d^2 X_i(t)}{dt^2} = \mathcal{F}_c^{[i]}(X_i(t), X_j(t)) + \mathcal{F}_G^{[i]}(X_i(t), X_j(t)). \quad (29)$$

Bizni qiziqtirgan muammoning parametrik fazosi juda katta bo'lganligi sababli, biz koordinatalar boshida joylashgan nisbatan juda og'ir zarracha ($M_2 \gg M_1$) va koordinata boshiga yiqiladigan nisbatan engil zarracha M_1 bilan yulduz shaklidagi model sifatida ko'rib chiqamiz. Shunday qilib, biz $X_2(t) \equiv 0$ deb olib, koordinatalar boshida joylashgan zarrachani turg'un holatda deb qaraymiz va (4) $X_1(t)$ - dinamikasiga e'tibor beramiz.

Biz muammoni $\omega(t) \equiv \text{const}$ bo'lgandagi statsionar garmonik potensialdan boshlaymiz. Bizni qiziqtirgan dinamika haqida tasavvurga ega bo'lish uchun biz klassik mexanikaning muammosidan boshlaymiz, ya'ni zarrachaning quyida keltirilgan bir o'lchovli gravitatsion va garmonik potentsiallar yig'indisidagi harakatlanadigan

$$U(x) = -G \frac{M_1 M_2}{x} + M_1 \frac{\Omega^2}{2} x^2, \quad (30)$$

bu erda $M_1 \Omega^2 x^2 / 2$ ifoda BEK bilan zarraning o'zaro ta'siriga to'g'ri keladi va $M_2 \gg M_1$, shart M_2 zarrachani doimo tinch holatda ekanligini bildiradi deb faraz qilamiz. $\Omega = 0$ bo'ganda vaqt va koordinataning o'zaro bog'liqligi quyidagicha aniqlanadi

$$t(x) = \sqrt{\frac{x(0)}{2\gamma M_2}} \left(\sqrt{x(x(0) - x)} + x(0) \arccos \frac{x}{x(0)} \right), \quad (31)$$

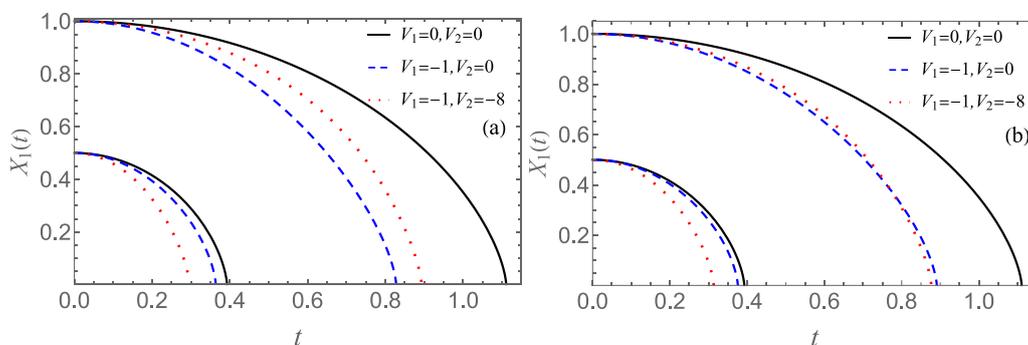
bu erda $x(0)$ boshlang'ich koordinata va $dx/dt|_{t \rightarrow 0} = 0$ boshlang'ich tezligi. Shunday qilib, tizim ikkita asosiy vaqt shkalasi bilan tavsiflanadi, masalan, gravitatsion maydonidagi harakat uchun Keplerning uchinchi qonuniga mos keladigan qulash vaqti T_G :

$$T_G = \frac{\pi}{2\sqrt{2}} \sqrt{\frac{1}{GM_2}} x^{3/2}(0) \quad (32)$$

va $T_\Omega = 2\pi / \Omega$ - tebranish davri. Bu yerda koordinata boshiga yetarli darajada yaqin bolgan M_1 zarracha uchun $T_G \ll T_\Omega$ deb faraz qilamiz. Ushbu holatda, $x(T_f) = 0$ bo'lgan qulash vaqti T_f , gravitatsion qulash T_G ga yaqin bo'ladi va mos keladigan tuzatish quyidagicha topiladi

$$\frac{T_f - T_G}{T_G} = -\frac{11}{2\pi^2} (T_G \Omega)^2. \quad (33)$$

Polaron hosil qiluvchi $M_1 \Omega^2 x^2 / 2$ potentsial ta'sir tufayli qulash vaqtini kiritilgan tuzatish BEKning gravitatsion dinamikadagi roli bizni qiziqtiradigan moammolardan biridir.

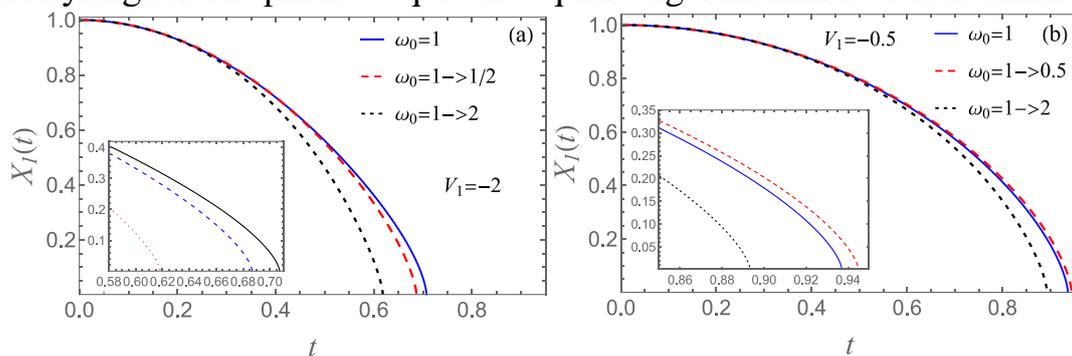


11-rasm. Turli boshlang'ich shartlar va (a) $g = 0$, (b) $gN = 2$ lar uchun $X_1(t)$ zarrachaning qulash trayektoriyasi keltirilgan. Qora chiziq sof gravitatsion ta'sirga mos keladi va (31) tenglama bilan xarakterlanadi. Ko'k chiziq kondensat ta'siriga mos keladi va qizil chiziq kondensat orqali o'zaro ta'sirning qo'shimcha effektini ko'rsatadi. Biz barcha sonli hisoblashlarni ushbu doimiy $M_2 = 10^9$, $M_1 = 10^3$, $G = 10^{-9}$ va $\delta = 0.1$ boshlang'ich parametrlarda amalga oshiramiz.

11-rasmda har xil turdagi o'zaro ta'sirlar va boshlang'ich shartlar uchun harakatlanuvchi $X_1(t)$ zarracha trayektoriyasi ko'rsatilgan. Ushbu effektlar bizning sifat tahlilimizga mos keladi va dastlabki pozitsiyaga kuchli bog'liqlikni ko'rsatadi. $X_1(0) = 0.5$ uchun qo'shimcha potentsial taxminan parabolik bo'lib, ortib borayotgan tortuvchi kuchga mos keladi. Shuning uchun qulash vaqti T_f , $T_f < T_G$ bilan

kamayadi. $X_1(0) = 1$ uchun boshlang'ich pozitsiyasi kondensatning kamayib ketgan zichligiga mos keladi va shuning uchun qulash vaqti ortadi.

Endi biz $t = 0$ da potentsial chastota ω ni to'satdan $\omega_- \equiv \omega(t < 0)$ dan boshqa $\omega_+ \equiv \omega(t > 0)$ ga o'zgarishini ko'rib chiqamiz. Potensial chastotasining buday o'zgarishi kondensatning siqilishiga ($\omega_+ > \omega_-$) yoki kengayishiga ($\omega_+ < \omega_-$) olib bo'ladi. Shundan so'ng, kondensat shaklidagi tebranishlar boshlanadi va natijada vaqt va pozitsiyaga bog'liq bo'lgan kondensat zichlik tashqi zarrachaning $X_1(t)$ trayektoriyasiga ta'sir qilishi va qulash vaqtini o'zgartirishini kutish mumkin.



12-rasm. Potentsial chastotasi ω o'zgarishini $X_1(t)$ trayektoriyasi vaqulash vaqtiga ta'siri keltirilgan. (a) chizma tortilishga qarshi ta'sirini ko'rsatadi, bu erda M_1 - zarrachaning kuchli polaron ta'siri tufayli kondensatning kengayishi va siqilishi uchun qulash vaqti kamayadi. (b) chizma tortishish ta'sirini ko'rsatadi, bu erda qulash vaqti siqilish uchun kamayadi va kengaygan kondensat uchun ortadi. Bunda M_1 - zarrachaning polaron ta'siri nisbatan kichik va asosiy ta'sir BEK dinamikasi bilan bog'liq. Aytish kerakki, (a) paneldagi $|T_f - T_G|$ farq (b) paneldagi farqdan ancha katta. Ichki chizmalar qulash vaqti T_f ga yaqin vaqt oralig'ida kattalashtirishni trayektoriyani ko'rsatadi. Bu rasmda $V_2 = g = 0$ deb olingan.

Olingan sonli natijalar 12-rasmda keltirilgan. Ko'rinib turibdiki, tebranuvchi kondensat shakli zarrachaning markazga qulashini o'zgartiradi. 12 (a)-rasmda polaron effecti tufayli tortishish va teskari oqim (anti-drag) effectlarini ko'rsatilgan. Oqim effecti ta'siri 12(b) -rasmda aniq ko'rinadi, bu erda kondensatning kengayishi polaronni tashqariga tortadi (qulash vaqti $T_f > T_G$), BEKning siqilishi polaronni ichkariga tortadi ($T_f < T_G$).

XULOSA

“Boze-Eynshteyn kondensatlarida noxiziqli dinamika, polaronlar va o‘z-o‘zini tashkil etish mexanizmlari” doktorlik dissertatsiyasi mavzusi bo‘yicha olib borilgan tadqiqot natijalariga ko‘ra quyidagi xulosalar keltirildi:

1. Dresselhaus SOB ning mavjudligi, kollaps bo‘layotgan kondensatning qisqarishiga yo‘l qo‘ymasligi, "anomal" aylanishga bog‘liq tezlikni keltirib chiqarishi ko‘rsatildi. Kuchsiz SOB uzoqdagi vaqtinchalik siqilishga imkon berdi, kuchli SOB esa zichlikning BEK markazidan chiqib ketishiga olib keldi, natijada halqali zichlik paydo bo‘ldi. Bu kondensatning siqilishiga yo‘l qo‘ymadi va zarrachalarning

tortilishini kamaytiradi. Bu natijalar shuni ko'rsatadiki, SOB barqaror kondensat hosil bo'lishiga olib keladigan BEK GPT ning kollapsiga yo'l qo'ymaydi.

2. Rabi magnit maydoni har bir spin komponentasidagi atomlar sonini o'zgartirib, intra-spin (spin komponentasining ichki) ta'siri bilan kondensatning qulashiga ta'sir qilishi ko'rsatildi. Bu o'zaro ta'sir energiyasini tebranishiga olib keladi, bu esa kondensatni barqarorlashtiradi. Cross-spin (diagonal spin komponentalari) ta'siri holida, spinning aylanishi o'zaro tortishish ta'sirini kuchaytiradi, bu esa kondensatning kollapsiga olib keladi. Biz ikkala spin ta'siri uchun kollaps va barqarorlik diagrammalarini aniqladik. Intraspin bog'lanishi atomlarning kritik sonini o'zgartiradi, va kollapsga olib keladi, cross-spin bo'lgan esa spin komponentining ikkalasini bir vaqtda kollapsiga olib keladi.

3. Berilgan SOB uchun soliton harakati Zeeman bo'linishiga va kondensatning o'z-o'zidan o'zaro ta'siriga kuchli bog'liqligini ko'rsatdik. Xususan, Zeeman o'zaro ta'siri SOB ga proporsional spinga bog'liq bo'lgan anomal tezlik tufayli solitonning lokalizatsiyasi yoki delokalizatsiyasiga olib kelishi mumkin. Etarli darajada kuchli Zeeman maydoni solitonning tasodifiy potentsial minimaliga yaqin joylashishiga olib kelishi mumkin. Agar Zeeman chastotasi tasodifiy potentsialdagi soliton tebranishlarining odatiy chastotasiga yaqin bo'lsa, bu rezonans uning delokalizatsiyasiga olib kelishi mumkin.

4. Intra-spin tortuvchan ta'sirga ega BEKning massa markazining harakati Zeeman maydonidagi spin aylanishi va tashqi tasodifiy potentsial mavjudligining qo'shma effekti fayli yuzaga kelishi mumkinligini ko'rsatildi. Spin aylanishi tufayli kondensatning kengayishi aniq kuch hosil qiladi, bu esa SOBsiz ham harakatlanishiga olib keladi.

5. Biz BEK yumshoqligi polaronlarga asoslangan tashqi zarralar dinamikasida kritik rol o'ynashini ko'rsatdik. Kuchsiz o'zaro ta'sir rejimida kondensat zarracha uchun statik potentsialni ta'minlaydi, bu esa aylana va kvazi-davriy traektoriyalarga olib keladi. Biroq, nisbatan kuchli o'zaro ta'sir rejimida kondensat polaronlarni hosil qiluvchi zarralar uchun vaqtga bog'liq potentsialga aylanadi, bu esa kondensat massa markazining parabolik potentsial manbasidan uzoqlashishiga olib keladi. BEK o'zini yumshoq kvant moddasi kabi tutadi va tashqi zarralar bilan o'zaro ta'sirlar zarrachalar va kondensatning yuqori tartibid dinamikasiga olib kelishi mumkin.

6. Biz BEKda ikkita polaron effektga ega gravitatsiyalanuvchi zarralarni modellashtirdik. Bunda polaron effekti zarralar orasidagi Nyuton "gravitatsiya" kuchlari ta'sirida yuzaga kelgan dinamikani o'zgartiradi va gravitatsion qulash vaqtining o'zgarishiga olib keladi.

**SCIENTIFIC COUNCIL on AWARD of SCIENTIFIC DEGREE of
DOCTOR of SCIENCES DSc.03/31.03.2022.T/FM.10.04 at the INSTITUTE
OF FUNDAMENTAL AND APPLIED RESEARCH UNDER
“TIAME” NATIONAL RESEARCH UNIVERSITY**

INSTITUTE OF FUNDAMENTAL AND APPLIED RESEARCH

MARDONOV SHUKHRAT NUMONJONOVICH

**NONLINEAR DYNAMICS, POLARONS AND MECHANISMS OF
SELF-ORGANIZATION IN BOSE-EINSTEIN CONDENSATES**

01.04.02 – Theoretical physics

**ABSTRACT OF THE DISSERTATION
Doctor of science (DSc) on physical and mathematical sciences**

Tashkent – 2023

The theme of the dissertation of doctor of physical and mathematical sciences (DSc) was registered by the Supreme Attestation Commission at the Ministry of Higher Education, Science and Innovations of the Republic of Uzbekistan under number B2022.2.DSc/FM195.

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The doctoral (DSc) dissertation can be looked through at the Information Resource Center of the Institute of Fundamental and Applied Research under the "TIAMEE" National Research University (registered under № ____). (Address: 100000, Tashkent city, 39 Qori Niyazov str., ph.: 71 237-09-62)

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INTRODUCTION (Annotation of doctoral dissertation)

Topicality and demand of the theme of the dissertation. In the world, the first predicted macroscopic phenomenon in quantum physics is the Bose-Einstein condensation. Bose-Einstein condensate (BEC) is a fascinating and unusual state of matter in which a group of bosons, subatomic particles, occupy the same quantum state. The phenomenon was predicted by Albert Einstein and Indian physicist Satyendra Nath Bose in 1924 and was first observed experimentally in 1995, over 70 years later. These experiments earned them the Nobel Prize in Physics in 2001. The discovery of BEC has had significant implications for physics, including the study of superconductivity, superfluidity, and quantum computing. It has also opened up new areas of research in the fields of atomic and condensed matter physics.

One of the most fascinating topics in condensed matter physics is comprehending Bose-Einstein condensates consisting of interacting particles. This interaction can be either repulsive or attractive. The study of BEC physics becomes significantly more diverse with synthetic spin-orbit coupling (SOC). In the case of SOC, an optically produced atomic pseudospin $1/2$ is joined to atomic momentum and a synthetic magnetic field. The SOC can take on various forms, simulating the Rashba and Dresselhaus symmetries present in solid-state physics. Efforts to expand the comprehension of soliton-formed BEC in other systems, such as nonlinear photonic lattices, are ongoing. Moreover, investigations into disorder potentials, which can be created experimentally, have displayed a robust qualitative interplay between nonlinearity and quantum localization. In addition, the study of the motion of a bright soliton in a BEC with attractive interactions is noteworthy, as its dynamics can be substantially impacted by the disorder and the SOC, even in the semiclassical regime, where quantum effects are insufficiently strong to induce Anderson localization. In recent years, the BEC has attracted the attention of scientists in the field of astronomy to predict some phenomena in the universe. BEC as a giant matter wave can simulate the properties of dark matter, and their study may provide insights into the dynamics and properties of both dark and visible matter. Overall, the study of the dynamical properties of BECs has the potential to significantly impact our understanding of the natural world and lead to important technological breakthroughs. These objectives justify the topicality of the global level of scientific research.

In recent years, in our country, more and more attention has been paid to the development of current directions of fundamental and applied research. In particular, the development of theoretical physics, which is one of the promising areas, is an important issue today. The main directions of fundamental research and development and their practical application for the successful development of science in our country are reflected in the Strategy³ for the further development of the Republic of Uzbekistan from 2022-2026. Therefore, the research of macroscopic

¹ Decree No. PF-60 of the President of the Republic of Uzbekistan dated January 1, 2022 “On the Development Strategy of New Uzbekistan for 2022-2026”

dynamics of Bose-Einstein condensation theories remains one of the urgent issues in the field of fundamental research.

This dissertation work is included in the state regulatory documents, in the Decrees of the President of the Republic of Uzbekistan "On approval of the concept of development of science until 2030" No. PF-6097, dated October 29, 2020, as well as Decrees of the President of the Republic of Uzbekistan dated March 19, 2021 No. PQ-5032 "On measures to improve the quality of education in the field of physics and develop scientific research" and this dissertation research serves to a certain extent in the implementation of tasks defined in other normative legal documents in this field.

Conformity of the research to the main priorities of science and technology development of the republic. The dissertation research has been carried out in accordance with the priority areas of science and technology in the Republic of Uzbekistan: II. "Power, energy and resource-saving".

Review of international scientific researches on dissertation subject⁴. Nonlinear dynamics of BECs described by the time-dependent Gross-Pitaevskii equations (GPEs) such as the quasi-two-dimensional collapse of BEC, dynamics of BEC in the external random systems, as well as nonstatic impurity characterizing heavy particle or polarons have been explored both experimentally and theoretically by the world's leading scientists, including Spain (G. Muga) Portugal (V.V. Konotop), United States of America, University of Colorado and NIST (E.A. Cornell, C.E. Wieman, W. Ketterle, M. Edwards, D. Kleppner, A.J. Leggett), Germany (T.W. Hänsch, I. Bloch, D. Jaksch, M. Weidemüller), France: (J. Dalibard, Ch. Salomon, T. Giamarchi), United Kingdom (S.L. Cornish, K. Burnett), Canada: (R.G. Hulet), Italy (M. Inguscio, S. Stringari, L. Pitaevskii, L. Salasnich), Israel (B.A. Malomed, N. Davidson), Russia (A.Fetter, E.I. Rashba, V.V. Kartashov) and others.

Within the framework of the theory of collapse in BECs were studied by domestic scientist F.X. Abdullayev, and foreign authors such as Yu. Kagan, A.E. Muryshev, G.V. Shlyapnikov. Experimentally obtained dynamics of collapsing and exploding Bose-Einstein condensates by group of scientists E.A. Donley, N.R. Claussen, S.L. Cornish, J.L. Roberts, E.A. Cornell and C.E. Wieman.

The ability to emulate SOC and Zeeman interaction in Bose-Einstein condensates made a big contribution by scientists Y.-J. Lin, R.L. Compton, K. Jiménez-García, J.V. Porto, I.B. Spielman, H. Zhai, V. Galitski and has raised a great interest in the interplay of nonlinear phenomena and spin dynamics of these systems.

The concept of polaron, which is an external particle forming a bound state in the medium where it is embedded, belongs to the most interesting ideas in physics. Originated in seminal works of L.D. Landau and R.P. Feynman it became one of the leading and most fruitful concepts in various domains of solid-state and condensed matter physics, including magnetic phenomena (studied by F.C Zhang and T.M.

⁴ Review of international scientific researches on dissertation subject composed on the basis of the following sources: <http://arxiv.org>; <https://webofknowledge.com>; <https://scholar/google.com>. J. Nature, J. Science, J. Reviews of Modern Physics, J. Physical Review Letters; J.; and others.

Rice) in addition to the variety of lattice deformation effects (studied by V.V. Konotop, J.T. Devreese, M.A. Semina, M.M. Glazov and E.Ya. Sherman).

Degree of study of the problem.

The phenomenon of the collapse of BECs has been extensively studied in the past few decades, and many different methods for controlling it have been proposed and investigated. One approach is to use external potentials to stabilize the condensate against collapse, such as by imposing a harmonic trapping potential or using Feshbach resonances to tune the interatomic interactions. Another approach is to use the nonlinear dynamics of the system to induce collapse and then apply feedback control techniques to prevent it.

Overall, the study of controlling the collapse of Bose-Einstein condensates is an active area of research with numerous experimental and theoretical studies. Some relevant publications include: M. Edwards and K. Burnett, "*Numerical simulation of the dynamics of Bose-Einstein condensates*," Phys. Rev. A 51, 1382 (1995); J. Javanainen and M. Mackie, "*Stabilization of a collapsing Bose-Einstein condensate by feedback control*," Phys. Rev. A 66, 013607 (2002); S. T. Karpiuk, et al., "*Collapse and revival of a Bose-Einstein condensate in a time-dependent double-well potential*," Phys. Rev. Lett. 109, 205302 (2012); K. Adhikari, "*Nonlinear dynamics of Bose-Einstein condensates*," J. Phys. B 49, 170201 (2016).

Spin-orbit coupled Bose-Einstein condensates have been the subject of extensive theoretical and experimental research in recent years. The study of SOC BECs has led to the discovery of new phenomena such as spin Hall effects, topological insulators, and non-Abelian gauge fields, which have been investigated in various theoretical works (Zhai, H. *Degenerate quantum gases with spin-orbit coupling: a review*. Reports on Progress in Physics, 78(2), 026001 (2015); N. Goldman, G. Juzeliūnas, P. Öhberg, & I.B. Spielman, *Light-induced gauge fields for ultracold atoms*. Reports on Progress in Physics, 77(12), 126401 (2014)). SOC BECs have also been used as a platform for simulating complex condensed matter systems, such as high-temperature superconductors and topological insulators (Y. Xu, F. Zhang, & C. Zhang, *Topological states in two-dimensional Bose-Einstein condensates with spin-orbit coupling*. Reports on Progress in Physics, 79(6), 066501 (2016); M. Aidelsburger, et al. *Realization of the Hofstadter Hamiltonian with ultracold atoms in optical lattices*. Physical Review X, 8(3), 031027 (2018)). Moreover, SOC BECs have been utilized in various experimental applications such as atom interferometry, quantum simulation, and quantum information processing (Y.J. Lin, et al. *Spin-orbit-coupled Bose-Einstein condensates*. Nature, 471(7336), 83 (2011); P. Wang, et al. *Quantum simulation of helical edge states in a spin-orbit-coupled Bose-Einstein condensate*. Physical Review Letters, 117(21), 215301 (2016)).

Polaron formation in Bose-Einstein condensates has also been extensively studied, as it plays a crucial role in understanding the interaction between impurities and the surrounding condensate. Polaron formation arises due to the strong coupling between the impurity and the surrounding condensate, leading to the dressing of the impurity by the bosons. The study of polarons in BECs has led to the discovery of

various phenomena such as the Bose polaron, the Fermi polaron, and the Fröhlich polaron, which have been investigated in various theoretical works (T. Yin, D. Cocks, and W. Hofstetter, *Polaronic effects in one- and two-band quantum systems*, Phys. Rev. A 92, 063635 (2015); N.B. Jørgensen, L. Wacker, K.T. Skalmstang, M.M. Parish, J. Levinsen, R.S. Christensen, G.M. Bruun, and J.J. Arlt, *Observation of Attractive and Repulsive Polarons in a Bose-Einstein Condensate*, Phys. Rev. Lett. 117, 055302 (2016); Z.Z. Yan, Y. Ni, C. Robens, M.W. Zwierlein, *Bose polarons near quantum criticality*, Science 368, 190 (2020); J. Takahashi, R. Imai, E. Nakano, and K. Iida, *Bose polaron in spherical trap potentials: Spatial structure and quantum depletion*, Phys. Rev. A 100, 023624 (2019); A. Schirotzek, C.-H. Wu, A. Sommer, and M.W. Zwierlein, *Observation of Fermi Polarons in a Tunable Fermi Liquid of Ultracold Atoms*, Phys. Rev. Lett. 102, 230402 (2009)). Polaron formation in BECs has also been used as a platform for studying impurity dynamics in condensed matter systems, as well as for investigating the behaviour of ultracold impurities in BECs (M. Cetina, et al. *Ultrafast many-body interferometry of impurities coupled to a Fermi sea*. Science, 354 (6316), 96-99 (2016); C. Kohstall, M. Zaccanti, M. Jag, et al. *Metastability and coherence of repulsive polarons in a strongly interacting Fermi mixture*. Nature 485, 615–618 (2012)).

Connection of the topic of dissertation with the scientific works of scientific research organizations, where the dissertation was carried out. The dissertation work was carried out in the framework of a scientific project of the Astronomical Institute F-FA-2021-432 of the Ministry for Innovative Development of Uzbekistan (2022-2023); Contract № 2 from 19 January 2022 year on “the organization and financing of short-term scientific internship of young scientists in foreign scientific organizations” financed by Fund for Financing Science and Innovation Support under Ministry of Innovative Development of the Republic of Uzbekistan; 2019 year scholarship of the “El-Yurt Umidi” Foundation for training specialists abroad and communication with compatriots under the Cabinet of Ministers of the Republic of Uzbekistan.

The aim of the research is the development of new nonlinear dynamic properties of the Bose-Einstein condensate based on the GPE under various internal and external nonlinear systems, exploring the dynamic properties of the Bose-Einstein condensate as a macroscopic matter using the laws of classical physics (Newton’s mechanics).

The tasks of the research:

determine the effect of SOC on the collapse of the Bose-Einstein condensate and defined the critical value of SOC that prehends collapsing condensate;

study the effect of an external magnetic field applied to the spin of the condensate with inter- and intra-spin-components nonlinearities in GPE to the collapse of the condensate;

analyze the motion of a bright soliton in a BEC with attractive interactions, the dynamics of which can be strongly affected by the disorder and the spin on condensate;

study the coupled dynamics of the embedded particle with the polaron effect in one and two-dimensional BECs characterized by GPE and laws of classical physics; analyze a model of the nonuniform density of the dark matter simulated by dynamics of “gravitational” interacting two particles in one-dimensional BEC.

The objects of the research are the Bose-Einstein condensate, spin, soliton, random potential, external embedded particles, polarons.

The subjects of the research are the collapse of the spin-orbit coupled Bose-Einstein condensate, spin-orbit-coupled soliton in a random potential, polarons and gravitating particles in Bose–Einstein condensate.

The methods of the research. On the theoretical level, the research methods are the mathematical apparatus of the theory of BEC with the application of classical physics laws and analytical and numerical methods for solving systems of partial differential equations.

The scientific novelty of the research is the follows:

for the first time, the effect of condensate spin driven by SOC and external magnetic field on the collapse of quasi-two-dimensional BEC has been studied, and it has been shown that anomalous spin-induced velocities fully control the collapse of the condensate;

for the first time, it was shown that due to the anomalous spin-dependent velocity, the synthetic Zeeman coupling can play a critical role in the soliton dynamics by causing its localization or delocalization in the random potential;

it was shown that the displacement of a BEC with self-attractive spin-component can be caused by the joint effect of the spin precession in a Zeeman field and the presence of a random potential;

for the first time, it was demonstrated the coupled dynamics of the polarons in one and two-dimensional BECs characterized by GPE and laws of classical Newtonian mechanics;

for the first time, it was presented a simplified model of the nonuniform density of the dark matter simulated by dynamics of “gravitational” interacting two embedded particles in one-dimensional BEC;

it was found that the polaronic effect, that is a modification of the BEC density in the vicinity of an embedded particle, considerably modifies the dynamics caused by the “gravitational” forces between the particles and leads to the modification of the gravitational fall time.

Practical results of the research are as follows:

numerically obtained critical value of anomalous spin-dependent velocity caused by symmetrized Dresselhaus coupling that can prevent collapse velocity and stabilize the quasi-two-dimensional Bose-Einstein condensate. It has been set collapse field of the condensate vs external magnetic field intensity and interaction parameter of the condensate for corresponding intra- and inter-spin component interactions;

an analytical and numerical criterion for the localization or delocalization of a soliton due to the spin-dependent anomalous velocity, which is proportional to the SOC and Zeeman interaction, has been obtained. It was shown that a strong enough

Zeeman field results in the localization of a bright soliton near random minima of the potential;

it was shown that the coupled heterogeneous dynamics of polaron and condensate are described by a system of Gross-Pitaevskii and classical Newton equations. It has been found the BEC behaves as a soft quantum matter for polarons;

it was shown that the polaron effect significantly modifies the dynamics caused by Newton's "gravitational" forces between particles and leads to a modification of the gravitational fall time;

analyzed the effects of the condensate dynamics in a time-dependent potential on the motion of "gravitating" embedded particles.

Reliability of the research results is ensured by the fact that in the dissertation work standard method of mathematical and theoretical physics were used, including highly efficient numerical methods and programs; careful check of consistency of the received theoretical results of other authors was performed; conclusions are well consistent with the main provisions of the theory of Bose-Einstein condensates.

Scientific and practical significance of the research results. The scientific significance of the research results is determined by the ability of the developed formalism in the dissertation to analyze the nonlinear dynamic properties of Bose-Einstein condensate as a macroscopic matter characterized by GPE for a wide application of the BEC in fields such as quantum computing and quantum information, as well as shedding light on the deeper understanding of the macroscopic properties of quantum matter. In addition, studies of various effects in atomic condensates can shed light on the properties and dynamics of dark matter with a variety of yet unknown even basic characteristics.

The practical significance of the results of the research lies in the fact that they can be used to obtain estimates of nonlinear dynamics of Bose-Einstein condensate, such as collapsing matter, spin-related dynamics, deformation parameters, displacement, as well as dark matter model that appears due to the corrections in Newton's gravity model. Results can also be useful for the analysis of the nature and dynamics of the condensed matter, in the development of observational experiments.

Application of the research results. The dissertation work was carried out in the framework of a grant of the Swiss National Foundation ("Schweiz. Nationalfonds") within the SCOPES program to support institutional partnerships with partners from Eastern Europe IZ74Z0_160527/1 (2016-2017); 2019 year scholarship of the "El-Yurt Umidi" Foundation for training specialists abroad and communication with compatriots under the Cabinet of Ministers of the Republic of Uzbekistan; 2022 year scholarship (Contract № 2 from 19 January) on the program "the organization and financing of short-term scientific internship of young scientists in foreign scientific organizations" financed by Fund for Financing Science and Innovation Support under Ministry of Innovative Development of the Republic of Uzbekistan.

Obtained results by the collapse of spin-orbit-coupled Bose-Einstein condensates have been used by more than 40 scientific papers published in journals with high impact factors (H. Sakaguchi, B. Li, and B.A. Malomed, Phys. Rev. E 89,

032920; Y. Zhang, M.E. Mossman, T. Busch, et al. Front. Phys. 11, 118103; H. Sakaguchi, E.Ya. Sherman, and B.A. Malomed, Phys. Rev. E 94, 032202; B.A. Malomed 2018 EPL 122, 36001, etc.). Results of spin-induced dynamics of soliton in a random potential used by more than 10 papers (J. Sun, Y. Chen, X. Chen, and Y. Zhang, Phys. Rev. A 101, 053621; J. Yang and Y. Zhang, Phys. Rev. A 107, 023316; J. Fan, G. Chen, and S. Jia, Phys. Rev. A 102, 063311, etc.) to develop dynamical properties of anomalous spin-dependent velocities in various systems.

The results on the nonlinear coupled dynamics of externally embedded "gravitating" particles and BECs, provide opportunity to shed light to model dark matter with laws of classical Newtonian mechanics.

Approbation of the research results. The research results were reported in the form of reports and tested at 8 international and local scientific conferences.

The main results of the study were tested at the scientific seminars of the Institute of Fundamental and Applied Research (2023), Astronomical Institute (2020-2022), of the Department of Theoretical Physics of Samarkand State University (2019), Department of Chemistry-Physics in Basque Country University (Spain, 2017-2018), Department of Theoretical Physics in University of Zurich (Switzerland, 2016) etc.

Publication of the research results. On the dissertation theme there were published 19 scientific works, including 11 scientific papers in international journals with high impact factors and recommended by the Supreme Attestation Commission of the Republic of Uzbekistan for publishing basic scientific results of doctoral theses.

Volume and structure of the dissertation. The dissertation consists of an introduction, four chapters, a conclusion, one appendix and a bibliography. The size of the dissertation is 165 pages.

THE MAIN CONTENT OF THE DISSERTATION

In the introduction the topicality and relevance of the dissertation theme were justified, the aims and objectives were formulated, the scientific novelty and the practical results of the study were set out, the reliability of the obtained results was proved and their theoretical and practical significance were disclosed, a summary of the application of the research results and the structure of the dissertation were given.

The first chapter of the thesis entitled “**Collapse of Bose-Einstein condensate**” is devoted to studying the collapse process of quasi-two-dimensional BEC including global-spin-interaction with Dresselhaus SOC. In addition, considered intra- and cross-spin-interactions with Rabi frequency. In this chapter, we have demonstrated the stabilization condition of the collapsing BEC with SOC and presented the collapse field of the spin components driven by Rabi frequency. Here we consider pseudospin-1/2 which characterizes two-component wave function and synthetic SOC and magnetic fields that are widely explored in theoretical and experimental condensed matter physics nowadays.

We assume at zero temperature an initial state of the condensate is prepared in a ground state form of the quantum harmonic potential:

$$\psi(\mathbf{r}, t = 0) \equiv \frac{\sqrt{N/\pi}}{a(0)} \exp\left[-\frac{r^2}{2a^2(0)}\right] \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \quad (1)$$

Here $[1, 0]^T$ is the initial state of the spin directed along the z axis. The subsequent dynamics of the spin-orbit coupled condensate with pseudospin-1/2 is described by a wave function $\psi = [\psi_1(\mathbf{r}, t), \psi_2(\mathbf{r}, t)]^T$, where $\mathbf{r} \equiv (x, y)$, normalized to the total number of particles N . The evolution of the wavefunction is described by GPE

$$i\hbar \frac{\partial \psi}{\partial t} = \left[-\frac{\hbar^2}{2m} \Delta + H_{so} - g|\psi|^2 \right] \psi, \quad (2)$$

where m is the particle mass, H_{so} is the SOC Hamiltonian. In Eq. (2) the interaction constant is given by $g = -4\pi\hbar^2 a_s / ma_z$, where a_z is the condensate scattering length along the z axis, and a_s is negative. Below, we use the units $\hbar \equiv m \equiv 1$ and the dimensionless interaction $\tilde{g} \equiv -4\pi a_s / a_z$. As a unit of length is chosen $a(0)$ arbitrarily, and the corresponding unit of time is $a^2(0)$.

For the spin-independent collapse of the condensate a total energy is defined by

$$E = -\frac{1}{2} \int [\psi^\dagger \Delta \psi + \tilde{g} |\psi|^4] dx dy. \quad (3)$$

To demonstrate the evolution of the collapse we use a variational approach based on the Gaussian ansatz function and from equation (3) and one can find the total energy

$$E = \frac{N}{2a^2} \left(1 - \frac{\tilde{g}N}{2\pi} \right). \quad (4)$$

The equation (4) clearly shows that, the condensate can collapse if $\tilde{g}N$ parameter exceeds the critical value $\lambda = 2\pi$. The width $a(t)$ is defined by

$$a(t) = a(0) \sqrt{1 - \frac{\Lambda t^2}{a^4(0)}}, \quad (5)$$

where $\Lambda = (\tilde{g}N - \lambda) / 2$. From Eq. (5) it follows the collapse time is $T_c \equiv a^2(0) / \sqrt{\Lambda}$, and the characteristic collapse velocity is $v_c \equiv a(0) / T_c = \sqrt{\Lambda} / a(0)$.

Now, we to consider effect of SOC to the collapse process. We write the modified velocity including the anomalous spin-dependent term in the form:

$$\mathbf{v} = \mathbf{k} + \frac{\partial}{\partial \mathbf{k}} H_{\text{so}}. \quad (6)$$

In equation (6) the last term is directly related to the condensate spin. It is interesting to demonstrate the evolution of the flux density

$$\mathbf{J}(\mathbf{r}, t) = \frac{i}{2} [\psi \nabla \psi^\dagger - \psi^\dagger \nabla \psi] + \psi^\dagger \left[\frac{\partial}{\partial \mathbf{k}} H_{\text{so}} \right] \psi. \quad (7)$$

To characterize the collapse of the condensate we demonstrate the width of the condensate defined by

$$a(t) = \frac{N}{\sqrt{2\pi}} \left[\int |\psi|^4 dx dy \right]^{-1/2}. \quad (8)$$

The symmetrized Dresselhaus coupling Hamiltonian we choose in the form

$$H_{\text{so}} = \alpha (k_x \sigma_x + k_y \sigma_y), \quad (9)$$

where α is coupling constant σ_x, σ_y are corresponding Pauli matrices. From Eq. (9) the corresponding spatial components of the velocity become

$$v_x = k_x + \alpha \sigma_x, \quad v_y = k_y + \alpha \sigma_y. \quad (10)$$

Spin velocity induced the characteristic distance to flip the spin is defined by $L_{\text{so}} = 1 / \alpha$.

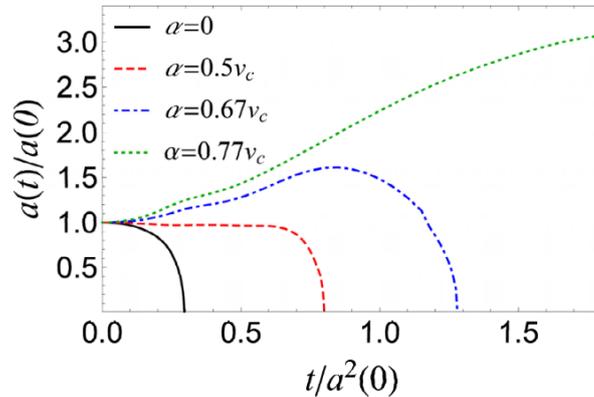


Fig. 1. Dynamics of width of the condensate defined by Eq. (8) and direct numerical solution of Eq. (2). The curves correspond to marked values of coupling constant α and interaction parameter is $\tilde{g}N = 8\pi$.

Fig.1 presents that, at a short time width is constant for all values of α and after starts interplay between attraction and anomalous velocities. One can see, after critical value $\alpha_c \approx 0.75v_c$ of SOC collapse of the condensate is prevented. To prevent the collapse of BEC, it is sufficient to take $\alpha = v_c$ and Dresselhaus SOC with the

present initial spin state causing a density flux leaving the center of the BEC presented in Fig.2.

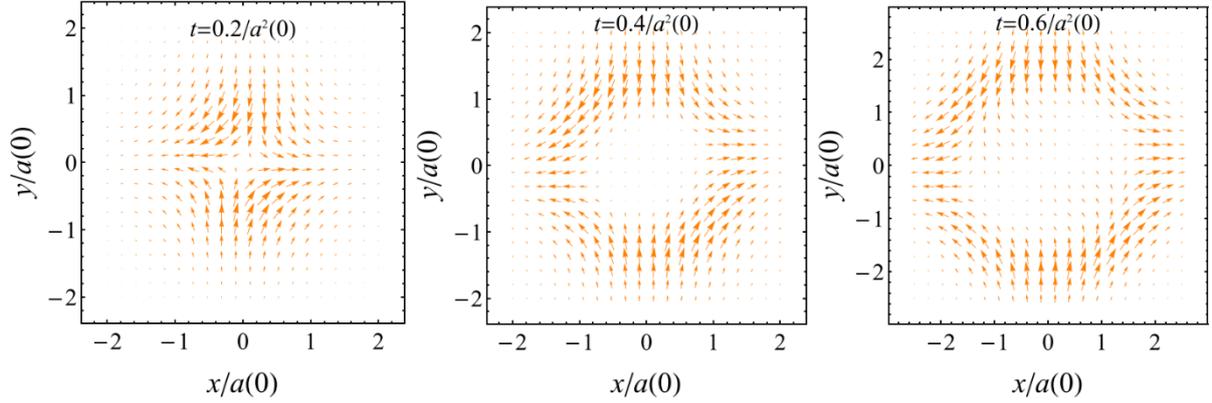


Fig. 2. Plots of the density flux defined by Eq.7 for $\alpha = v_c$, $\tilde{g}N = 8\pi$ and different time values marked on the panels respectively.

The second chapter of the thesis entitled “**Nonlinear dynamics of soliton in a random potential**” is devoted to studying the dynamics of a spin-orbit coupled soliton formed by a self-interacting quasi-one-dimensional BEC immersed in a random potential, in the presence of an artificial magnetic field. As a quasi-one-dimensional system, it is assumed coordinate space is $\mathbf{r} \equiv \mathbf{x} \equiv (x, t)$ and the system will be demonstrated by the time-dependent GPE

$$i\partial_t \psi = \left[\frac{\hat{k}^2}{2} + \alpha \sigma_z \hat{k} + \frac{\Delta}{2} \sigma_x + U(x) + g |\psi_\kappa|^2 + \tilde{g} |\psi_{\kappa'}|^2 \right] \psi, \quad (11)$$

where $\kappa, \kappa' = 1, 2$ ($\kappa \neq \kappa'$). Here $\hat{k} = -i\partial / \partial x$ is the momentum operator, Δ is the Zeeman splitting, $U(x)$ is the random potential. The intra-component coupling g is assumed to be negative, $g < 0$, and equal for the two components. Here we assume the inter-component coupling \tilde{g} is given as $g = \tilde{g}$, that is, the system self-interaction energy is invariant with respect to the global spin rotations. In this case and the absence of potential $U(x)$ and spin-related interactions with being $\Delta = \alpha = 0$, the ground state of soliton is given by

$$\psi_{\text{gr}} = \frac{\sqrt{-g}}{2 \cosh[2(x - x_0) / g]} \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad (12)$$

where x_0 is a position of center of mass of the soliton.

A smooth disorder is produced at a long interval L by a distribution of $N_{\text{im}} \gg 1$ “impurities” with uncorrelated random positions x_j and mean linear density $\bar{n} = N_{\text{im}} / L$ as

$$U(x) = U_0 \sum_{j=1}^{j=N_{\text{im}}} s_j u(x - x_j). \quad (13)$$

Here $s_j = \pm 1$ is a random function of j with mean values $\langle s_j \rangle = 0$, so that $\langle U(x) \rangle = 0$. Here we model the impurities as $u(y) = \exp(-y^2 / \xi^2)$, where ξ is the corresponding width.

In order to describe the dynamics of a soliton in the random potential we explore the integral quantities $\mathcal{O}(t)$ associated with each observable \mathcal{O} and defined by

$$\mathcal{O}(t) = \int_{-\infty}^{\infty} \psi^\dagger(x) \mathcal{O} \psi(x) dx. \quad (14)$$

In particular, defining the total soliton momentum $k(t)$ and the force $F(t)$, for which \mathcal{O} is substituted by \hat{k} and by $\hat{F} \equiv -dU(x)/dx$, respectively, and using Eq. (11), it is straightforward to verify the Ehrenfest-like relation

$$\frac{dk(t)}{dt} = F(t). \quad (15)$$

We assume that initial soliton is in a stationary state and at equilibrium with the random potential. Fig.3 shows a realization of $U(x)$ and the density of the soliton prepared with this protocol. This soliton is localized near a potential minimum and subsequent dynamics is induced by switching on the SOC and the Zeeman field.

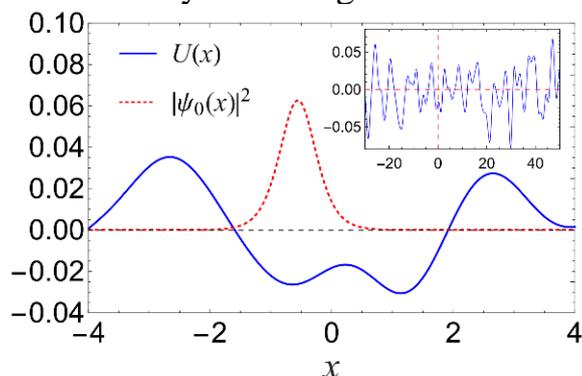


Fig. 3. Red dashed line is the soliton initial state for $g = \tilde{g} = -5$, and blue solid line is a realization of random potential with $U_0 = 0.01$, $\bar{n} = 10$, and $\xi = 1$, that shown in the inset with long distance.

The velocity of the soliton, described by Eq. (11), defined as $v(t) = dX(t)/dt$, is given by the relation

$$v(t) = k(t) + \alpha \sigma_z(t). \quad (16)$$

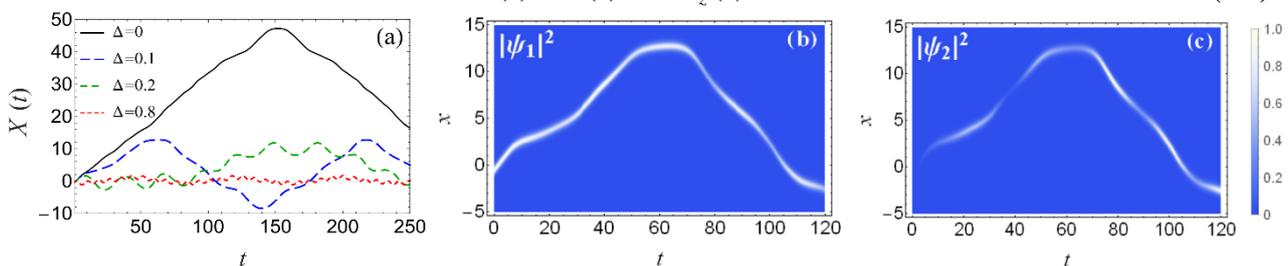


Fig. 4. Position of the soliton as a function of time for different value of Δ (marked in the plot), $g = \tilde{g} = -5$ and $\alpha = 0.4$. Panel (a) shows that, for $\Delta = 0$ the soliton travels a long distance, whereas switching on the Zeeman field eventually traps it. (b, c) Density plots of the two spinor components in the (t, x) - plane for $\Delta = 0.1$.

Figure 4 is presenting the dynamics of the soliton for $\alpha = 0.4$ at different values of Δ . The panel (a) shows that, whereas for $\Delta = 0$ the soliton moves a long distance through the disordered potential until it is reflected by a large fluctuation of $U(x)$.

The presence of a Zeeman field inhibits the propagation, and eventually traps it, for sufficiently large values of Δ ($= 0.8$ in this plot).

Since we consider a narrow soliton with the self-interaction energy conserved under the total spin rotations, we can introduce a conserved “low-energy” quantity ϵ_0 obtained as the average (11) of the linear part of the Hamiltonian for the adiabatic soliton $\psi_{\text{ad}}(\mathbf{x})$ i.e., $\epsilon_0 = k^2(t)/2 + \alpha\sigma_z(t)k(t) + \Delta\sigma_x(t)/2 + U(X(t))$. Taking into account that $\sigma_z(t) = \cos\theta(t)$, $\sigma_x(t) = \sin\theta(t)\cos\phi(t)$, the conservation of ϵ_0 (verified in the adiabatic approximation) can be presented as

$$v^2(t) - \alpha^2\sigma_z^2(t) + \Delta\sigma_x(t) = 2[U(X(0)) - U(X(t))], \quad (17)$$

with the velocity $v(t)$ given by Eq. (16).

The third chapter of the thesis, “**Self-organization of polarons and Bose-Einstein condensates**” is devoted to studying various regimes of coherent coupled motion of a polaron with one- and two-dimensional BEC confined by a harmonic potential. Strongly mutually related evolution of the condensate shape, its center of mass position, and polaron coordinate are studied for coupled nonlinear polaron-condensate oscillations. We assume a self-interacting BEC, built by $N \gg 1$ particles, then in a one-dimensional space the system can be described by the GPE in the form:

$$i\frac{\partial}{\partial t}\psi(x,t) = \left(-\frac{1}{2}\frac{d^2}{dx^2} + \frac{x^2}{2}\right)\psi(x,t) + g|\psi(x,t)|^2\psi(x,t) + V(x - X(t))\psi(x,t). \quad (18)$$

In Eq. (18) new factor $V(x - X(t))$ describes the polaron effect produced by interaction with an embedded external particle. We choose the potential of the embedded particle in the form of a local interaction $V(x - X(t)) = V_0 \exp(-(x - X(t))^2 / 2\delta^2)$, where the amplitude V_0 is either positive or negative, and δ is the narrow width, much less than the condensate extension. Therefore, we define in the saddle-point approximation the interaction energy between the particle and the condensate as:

$$V(X(t)) = V_0 \int_{-\infty}^{\infty} |\psi(x)|^2 \exp\left(-\frac{(x - X(t))^2}{2\delta^2}\right) dx \approx \sqrt{2\pi}\tilde{V} |\psi(X(t))|^2, \quad (19)$$

where $\tilde{V} \equiv V_0\delta$. Although Eq. (19) is well-suitable for qualitative analysis and for numerical calculations we will use exact time-dependent force for generalization of the force-related approach between condensate and external particle defined as:

$$F(X(t)) \equiv -\partial_{X(t)} \int_{-\infty}^{\infty} V(x - X(t)) |\psi(x,t)|^2 dx. \quad (20)$$

We consider sufficiently heavy embedded particles, with respect to the condensed particles, such that evolution of their positions is described by the classical Newton equation:

$$M\ddot{X}(t) = F(X(t)). \quad (21)$$

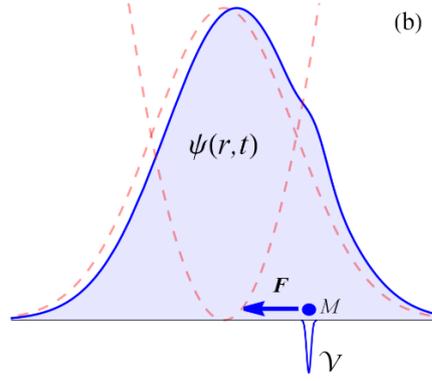


Fig. 5. The schematic form of a model of polaron forming in a BEC.

Figure 5 is presenting schematic form of a model of polaron forming in a BEC, $\psi(r,t)$ is characterising the BEC density profile at $t=0$, $V < 0$ is an attractive potential of embedded particles with mass M and the particle is located in some distance from the center of mass of the condensate. Red dashed lines are presenting parabolic potential and ground state condensate profile of its, blue line is presenting a deformed condensate density profile due to polaron-forming particles. All dynamics of polaron and condensate will be presented at time $t > 0$.

All dynamics of embedded polaron and condensate are demonstrated by numerical solution of the system of equations (18), (20) and (21). All condensate parameters will be defined by equation (14).

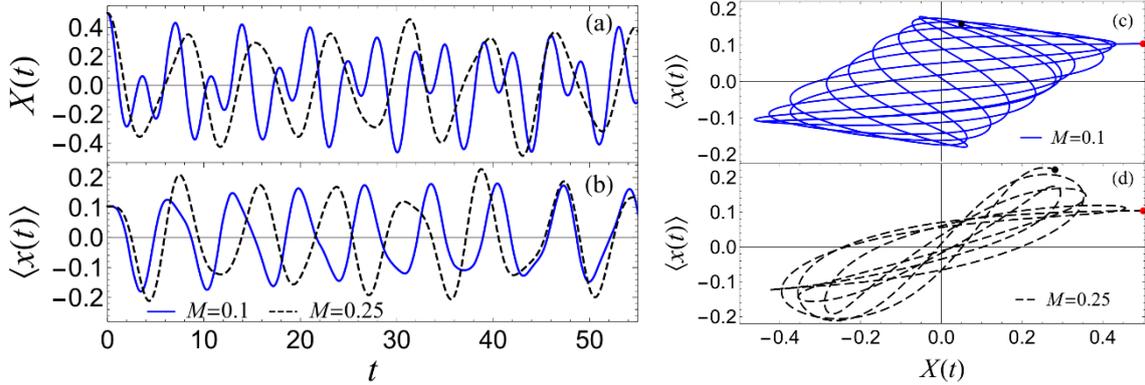


Fig. 6. Plots of motion of the polaron (a), center of mass of the condensate (b) and center of mass position vs $X(t)$ (for time $t \leq 40$) (c), (d) for given masses pointed a panels and initial parameters of the polaron are $X(0) = 0.5$, $\delta = 0.1$, and $V_0 = -1$, in the noninteracting BEC $g = 0$.

Coupled irregular dynamics of polaron and center of mass of the condensate presented in Fig. 6. The result is presenting that, increasing of mass of the polaron leads to decreasing in irregular pulses in the oscillation of the polaron. Because if the polaron mass is close to the total mass of the condensate then the system is in a quasi-equilibrium state. In panels (c) and (d) the red (black) point corresponds to the initial (final) position of the polaron, respectively. Note that the initial point in (c) is outside of the parallelogram-like structure filled by the $(X(t), \langle x(t) \rangle)$ - trajectory. This is a consequence of nonlinear character of the oscillations, when the external particle is initially attracted to the region with a higher density of the condensate, and the coupled oscillations begin then.

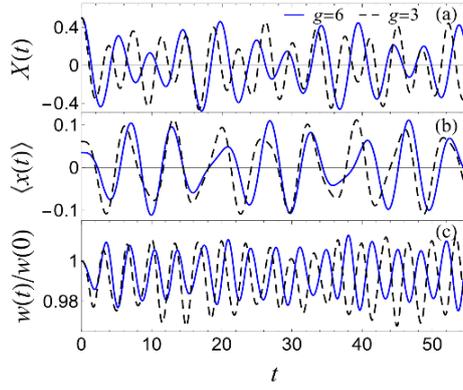


Fig. 7. Plots of motion of the polaron (a), center of mass position (b) and width (c) of the condensate. Here $M = 0.05$, $X(0) = 0.5$, $\delta = 0.3$, $V_0 = -0.5$, and $g = 6$ and $g = 3$.

Effect of self-interacting BEC to the coupled dynamics of polaron and condensate are given in Fig.7. Note that, as expected, several initial oscillations of $X(t)$ for $g = 3$ have a smaller period than those for $g = 6$. However, with the course of time, nonlinear effect prevails, and the difference becomes less pronounced. The BEC boundaries in the Thomas-Fermi approximation are $x_{\max} = 2.08$ for $g = 6$ and $x_{\max} = 1.65$ for $g = 3$, respectively.

Also, explored coupled dynamics of polaron in a non-self-interacting quasi-two-dimensional BEC confined by parabolic potential. For two dimensional BEC in the GPE (18) coordinate “ x ” will be substituted by $\mathbf{r} = (x, y)$. For two-dimensional system it is assumed that the polaron has initial velocity presented in Fig. 8.

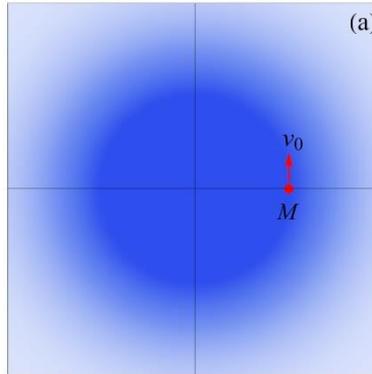


Fig. 8. The schematic form of a model in two-dimensional BEC density and polaron located in some distance, with mass M and initial velocity v_0 , that is perpendicular to the force F shown in Fig 5.

All dynamic equations (18)-(21) can be represented in two-dimensional coordinate space and analyzed by numerical solution of the equations.

To understand the dynamics of external particles in the condensate, we start with the condition of static BEC. We assume that the interaction between condensate and polaron is relatively weak ($\tilde{v} \ll 1$), that can't provide deformation of condensate and it is always at rest. As a result, the dynamics of the particle will be governed only by the solution of the Eq. (21). By using the interaction energy approximation Eq. (19) with static solution GPE (18) we rewrite Eq. (21) in the form

$$M \ddot{\mathbf{R}}(t) = -2\pi\tilde{V} \frac{\partial |\psi(\mathbf{R}(t))|^2}{\partial \mathbf{R}(t)} \equiv 4\tilde{V}\mathbf{R}(t)\exp(-R^2(t)), \quad (22)$$

Since the potential Eq. (19) describes attraction between the polaron and condensate, Eq. (20) can provide the centripetal force for the particle. From this, it follows that for any initial velocity $\mathbf{v}(0)$ perpendicular to the vector $\mathbf{R}(0)$ the equation of the circular trajectory of the particle is defined by

$$M \frac{v_0^2}{R_0} = F(R_0), \quad (23)$$

where $v_0 \equiv |\mathbf{v}(0)|$ and $R_0 \equiv |\mathbf{R}(0)|$. Actually, due to the circular trajectory we have $v(t) \equiv v_0 = \text{const}$ consequently $R(t) \equiv R_0 = \text{const}$. Using Eq. (22) and (23) we derive an equation

$$Mv_0^2 = 4|\tilde{V}|R_0^2 \exp(-R_0^2). \quad (24)$$

As a result, Eq. (24) determines the circular trajectory of the embedded particle in a static BEC. Now we assume that the given initial velocity, $\mathbf{v}(0)$, and position, $\mathbf{R}(0)$, don't satisfy Eq. (24). In this case, if $Mv_0^2 < 4|\tilde{V}|R_0^2 \exp(-R_0^2)$, then the trajectory of the particle demonstrates quasi-apsidal-precession inside the circular orbit or if $Mv_0^2 > 4|\tilde{V}|R_0^2 \exp(-R_0^2)$, then the trajectory of the particle demonstrates quasi-apsidal-precession outside the circular orbit. Fig. 9 is presenting three particle trajectories: circular, elliptic precession outside the circle, and elliptic precession inside the circle, which corresponds to a different initial velocity.

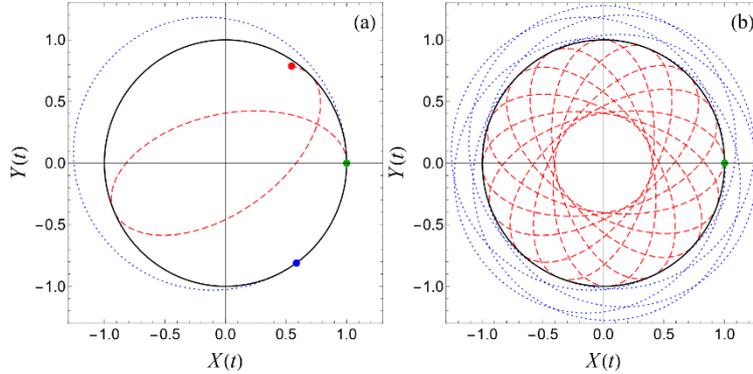


Fig. 9. Plot of embedded particle's trajectory in a plane (x, y) for a static BEC. Initial parameters of particle given by $M = 1$, $\mathbf{R}(0) = (1, 0)$, $\tilde{V} = -10^{-2}$. The lines correspond to different initial velocities given as $\mathbf{v}(0) = (0, 0.5v_0)$ -red dashed, $\mathbf{v}(0) = (0, v_0)$ -black solid and $\mathbf{v}(0) = (0, 1.1v_0)$ -blue dotted.

In Fig. 9 velocity $v_0 = 2(|\tilde{V}|R_0^2 \exp(-R_0^2))^{1/2} \approx 0.121$ is defined for circular trajectory of particle and a rotation period with a circular orbit is $t_0 = 2\pi R_0 / v_0 \approx 51.8$. In plot (a) the trajectories correspond to the time t_0 and in plot (b) the trajectories correspond to the time t_f when the particle returns to the initial position (blue point) $\mathbf{R}(t_f) \approx \mathbf{R}(0)$. In panel (a) one can see that the red dashed trajectory makes more than full rotation at one circular rotation time t_0 of the particle even though the initial

velocity decreased to $0.5v_0$. Correspondingly, blue dotted trajectory cannot make a full rotation even though its velocity is increased to $1.1v_0$.

For polaron-formed particles the Eqs. (22)-(24) are not satisfied, because due to the deformation of the initial state of BEC, the potential for polarons becomes time-dependent. Therefore, the trajectory of polarons becomes strongly irregular due to condensate dynamics that are performed based on the initial parameters of polaron potential.

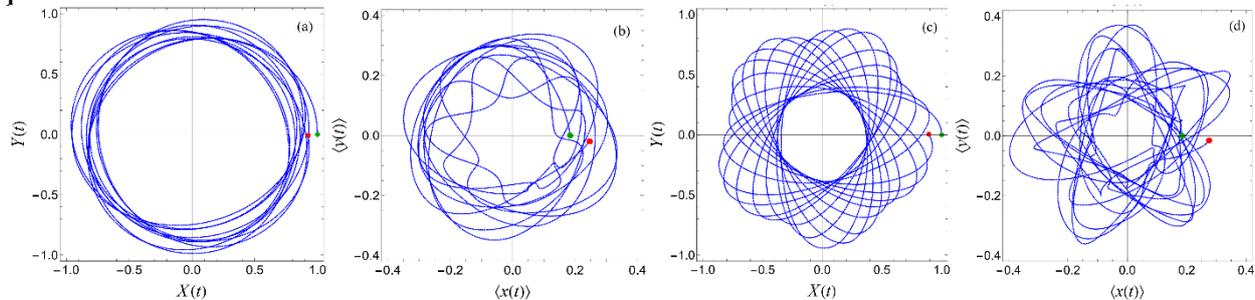


Fig. 10. Plots of the polaron's trajectories (a), (c) and center of mass of the condensate (b), (d) in a plane (x, y) . Initial parameters are given by $M = 1$, $\mathbf{R}(0) = (1, 0)$, $\tilde{V} = -10^{-1}$. The plots (a), (b) correspond to $\mathbf{v}(0) = (0, v_0)$ and (c), (d) correspond to $\mathbf{v}(0) = (0, 0.5v_0)$ initial velocities.

The trajectory of polaron and center of mass of quasi-two-dimensional BEC are presented in Fig. 10. Corresponding circular orbit velocity for polaron is $v_0 = (4|\tilde{V}|R_0^2 \exp(-R_0^2))^{1/2} \approx 0.384$. Plot times for (a), (b) is $t_f = 133$ and for (c), (d) is $t_f = 187.7$ which is taken thus the $\mathbf{R}(t_f)$ - red point becomes close to $\mathbf{R}(0)$ - initial blue point. The plots demonstrate the nonuniform trajectory of the particle and irregular center of mass of the condensate due to condensate dynamics caused by the polaron effect.

The fourth chapter of the thesis entitled “**Gravitating polarons in Bose-Einstein condensates**” is devoted to study the effect of the one-dimensional BEC on the dynamics induced by a model Newton's "gravitational" interaction between the particles embedded in the condensate located in a harmonic oscillator potential. We consider in a simple model polaron in the BEC as a possible manifestation of the dark matter-related interactions, concentrating mostly on the analytical and numerical calculations of the “fall-onto-the-center” dynamics. In addition, to understand the effects of the evolution of the condensate, we study the particles' drag by the oscillating condensate in the dynamics caused by the simulated gravitational forces.

The system is described by the GPE in the form:

$$i \frac{\partial}{\partial t} \psi(x, t) = \left(-\frac{1}{2} \frac{\partial^2}{\partial x^2} + \frac{\omega^2(t)}{2} x^2 \right) \psi(x, t) + g |\psi(x, t)|^2 \psi(x, t) + \mathcal{V}_{\text{ext}}(x) \psi(x, t), \quad (25)$$

with

$$\mathcal{V}_{\text{ext}}(x) = \mathcal{V}_1(x - X_1(t)) + \mathcal{V}_2(x - X_2(t)), \quad (26)$$

where $\omega(t)$ is frequency of the nonstationary harmonic trap. The $\mathcal{V}_1(x - X_1(t))\psi(x, t)$ and $\mathcal{V}_2(x - X_2(t))\psi(x, t)$ terms describe the local interaction of the BEC with

embedded movable particles located at the time-dependent positions $X_1(t)$ and $X_2(t)$, with the masses M_1 and M_2 , respectively. Eqs. (19) and (20) can be represented for both particles. The time-dependent force between the condensate and the external particles depends on the positions of both particles and has the form

$$\mathcal{F}_c^{[i]}(X_i(t), X_j(t)) = -\partial_{X_i(t)} \int_{-\infty}^{\infty} \mathcal{V}_{\text{ext}}(x) |\psi(x, t)|^2 dx, \quad (27)$$

where indices $(i, j) = (1, 2)$ correspond to the two embedded particles.

We assume that the principal interaction between the embedded particles is described by the model **Newtonian "gravitation"-like force**

$$\mathcal{F}_G^{[i]}(X_i(t), X_j(t)) = GM_1 M_2 \frac{X_j(t) - X_i(t)}{|X_i(t) - X_j(t)|^3}, \quad (28)$$

where G is Newton's "gravitational" interaction constant. Then particle positions are described by the classical Newton equation defined by

$$M_i \frac{d^2 X_i(t)}{dt^2} = \mathcal{F}_c^{[i]}(X_i(t), X_j(t)) + \mathcal{F}_G^{[i]}(X_i(t), X_j(t)). \quad (29)$$

Since the parametric space of the problem of interest is very large, we consider a star-like realization with a relatively heavy particle ($M_2 \gg M_1$) located in the origin and a relatively light particle M_1 falling onto the origin. Thus, we will neglect the motion of the particle located at the origin assuming $X_2(t) \equiv 0$ and concentrate on the $X_1(t)$ - dependence.

We begin with a stationary harmonic trap with $\omega(t) \equiv \text{const}$. To get insight into the dynamics of interest, we begin with the classical mechanic's problem of a particle moving in the sum of the one-dimensional gravitational and harmonic potentials in the form

$$U(x) = -G \frac{M_1 M_2}{x} + M_1 \frac{\Omega^2}{2} x^2, \quad (30)$$

where the term $M_1 \Omega^2 x^2 / 2$ corresponds to the interaction with the BEC and we assume that $M_2 \gg M_1$, meaning that the M_2 particle is always at rest. At $\Omega = 0$ the time and position are related by:

$$t(x) = \sqrt{\frac{x(0)}{2\gamma M_2}} \left(\sqrt{x(x(0) - x)} + x(0) \arccos \frac{x}{x(0)} \right), \quad (31)$$

with $x(0)$ being the initial position and the initial velocity $dx/dt|_{t \rightarrow 0} = 0$. Thus, the system is characterized by two main timescales such as the gravitational fall time T_G , corresponding to the third Kepler's law for motion in the gravitational field:

$$T_G = \frac{\pi}{2\sqrt{2}} \sqrt{\frac{1}{GM_2}} x^{3/2}(0) \quad (32)$$

and the oscillation period $T_\Omega = 2\pi / \Omega$. We assume here that $T_G \ll T_\Omega$ for the M_1 particle is initially sufficiently close to the origin. In this realization, the fall time T_f , where $x(T_f) = 0$ is close to T_G and the corresponding correction is given by:

$$\frac{T_f - T_G}{T_G} = -\frac{11}{2\pi^2} (T_G \Omega)^2. \quad (33)$$

The correction to the fall time due to the polaronic effect producing the potential $M_1 \Omega^2 x^2 / 2$ is one of the quantities of interest for the role of the BEC in gravitational dynamics.

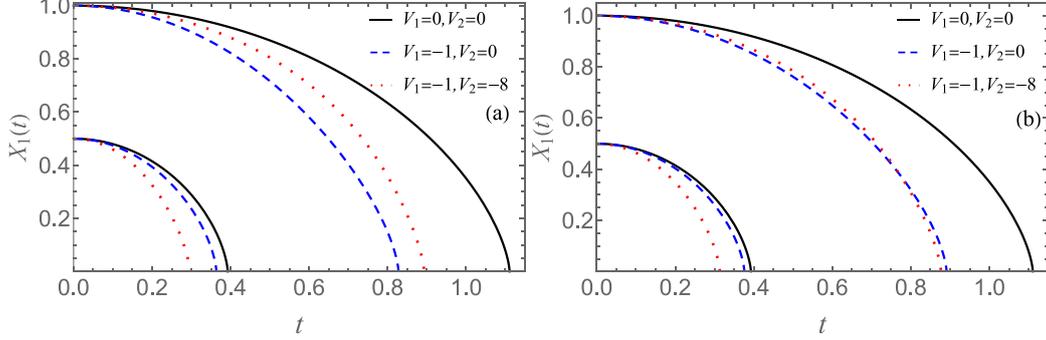


Fig. 11. Time-dependent position of the particle $X_1(t)$ at different initial conditions and coupling strength (a) $g = 0$, (b) $gN = 2$. The black line corresponds to purely gravitational interaction and is described by Eq. (31). The blue line corresponds to the effect of the condensate, and the red line shows the additional effect of interaction via the condensate. Here and in the numerical results presented below we consider fixed parameters $M_2 = 10^9$, $M_1 = 10^3$, $G = 10^{-9}$ and $\delta = 0.1$.

Figure 11 is presenting the time dependence of the moving particle position, $X_1(t)$, for various types of interactions and initial conditions. These effects correspond to our qualitative analysis and demonstrate a strong dependence on the initial position. For $X_1(0) = 0.5$ the additional potential is approximately parabolic and corresponds to the increased attraction force. Therefore, the fall time T_f decreases with $T_f < T_G$. For $X_1(0) = 1$, the initial position corresponds to the depleted density of the condensate and, therefore, the fall time increases.

Now we consider the realizations where at $t = 0$ the potential frequency ω suddenly changes from $\omega_- \equiv \omega(t < 0)$ to a different $\omega_+ \equiv \omega(t > 0)$. This change in the trap frequency causes compression ($\omega_+ > \omega_-$) or expansion ($\omega_+ < \omega_-$) of the condensate. Then, oscillations in the condensate shape begin and one can expect that the resulting time and position-dependent density can drag the embedded particle influencing its motion $X_1(t)$ and modifying the fall time.

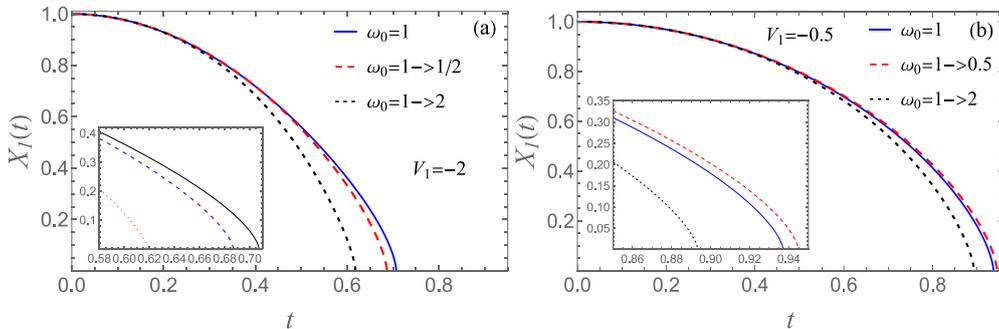


Fig.12. Effect of the change in the trap frequency ω on the time dependence $X_1(t)$ and the fall time. Panel (a) characterizes the effect of anti-drag where the fall time decreases both for the expanding and compressing condensate due to a strong polaronic effect of the M_1 - particle. Panel

(b) characterizes the effect of drag where the fall time decreases for compressing and increases for the expanding condensate. Here the polaronic effect of the M_1 - particle is relatively weak and the main effect is due to the BEC dynamics. Note that the difference $|T_f - T_G|$ in panel (a) is considerably larger than that in panel (b). The insets show zoom on the time interval near the fall time T_f . Here $V_2 = g = 0$.

The corresponding numerical results are presented in Fig. 12. As one can see, the oscillating condensate shape modifies the fall of the particles onto the center. Figure 12(a) shows the effects of the drag and the back flow (anti-drag) due to polaronic effects. The effect of the drag is clearly seen in Fig. 12(b), where expansion of the condensate drags the polaron out (with the fall time $T_f > T_G$) while compression of the BEC drags the polaron in (with $T_f < T_G$).

CONCLUSION

According to the results of the research carried out on the theme of the doctoral dissertation “Nonlinear dynamics, polarons and mechanisms of self-organization in Bose-Einstein Condensates”, the following conclusions are presented:

1. It has been shown, the presence of the Dresselhaus SOC prevents the collapsing condensate from compressing, causing an "anomalous" rotation-dependent velocity. Weak SOC allowed for a distant temporal collapse, while strong SOC caused the flow to leave the center of the BEC, resulting in an annular density. This prevented condensate from squeezing out and reduced particle attraction. These results show that SOC can prevent BEC collapse resulting in stable condensates.

2. It has been shown that the Rabi magnetic field can affect the collapse of condensate with intra-spin interaction by modifying the number of atoms in each spin component. This leads to oscillations in the self-interaction energy, stabilizing the condensate. With cross-spin interaction, the spin rotation enhances self-interaction, causing the condensate to collapse. We determined the collapse and stability diagram for both interactions. Intra-spin coupling modifies the critical number of atoms causing the collapse, while for cross-spin coupling, only double spin-component collapse occurs.

3. We have shown that for given SOC, the soliton motion strongly depends on the Zeeman splitting and the self-interaction of the condensate. In particular, the Zeeman interaction can lead to localization or delocalization of the soliton due to the spin-dependent anomalous velocity proportional to the SOC. A sufficiently strong Zeeman field can cause localization of the soliton near the random potential minima. If the Zeeman frequency is close to the typical frequency of the soliton oscillations in the random potential, this resonance can cause its delocalization.

4. We have shown that the motion of the center-of-mass of a self-attractive BEC can be driven by the joint effect of spin precession in a Zeeman field and the presence of an external potential. The broadening of the condensate due to the spin rotation creates a net force that induces its motion, even without SOC.

5. We have demonstrated that the BEC softness plays a critical role in the dynamics of polaron-based embedded external particles. In the weak interaction regime, the condensate provides a static potential for the particle, resulting in circular and quasi-periodic trajectories. However, in the relatively strong interaction regime, the condensate becomes a time-dependent potential for the polaron-forming particles, leading to a shift in the center of mass of the condensate from the origin of the parabolic potential. The BEC behaves as a soft quantum matter and the interaction with embedded particles can lead to highly nontrivial coupled dynamics of the particle and condensate.

6. We have modelled two gravitating particles in a BEC with polaron effects. The polaronic effect modifies the dynamics caused by Newton's "gravitational" forces between particles and leads to a modification of the gravitational fall time.

**НАУЧНЫЙ СОВЕТ DSc.03/31.03.2022.T/FM.10.04 ПО ПРИСУЖДЕНИЮ
УЧЕНОЙ СТЕПЕНИ ПРИ ИНСТИТУТЕ ФУНДАМЕНТАЛЬНЫХ И
ПРИКЛАДНЫХ ИССЛЕДОВАНИЙ НАЦИОНАЛЬНОГО
ИССЛЕДОВАТЕЛЬСКОГО УНИВЕРСИТЕТА «ТИИИМСХ»**

**ИНСТИТУТ ФУНДАМЕНТАЛЬНЫХ И ПРИКЛАДНЫХ
ИССЛЕДОВАНИЙ**

МАРДОНОВ ШУХРАТ НУМОНЖОНОВИЧ

**НЕЛИНЕЙНАЯ ДИНАМИКА, ПОЛЯРОНЫ И МЕХАНИЗМЫ
САМООРГАНИЗАЦИИ В БОЗЕ-ЭЙНШТЕЙНОВЫХ КОНДЕНСАТАХ**

01.04.02 – Теоретическая физика

**АВТОРЕФЕРАТ ДИССЕРТАЦИИ
доктора физико-математических наук (DSc)**

Ташкент – 2023

Тема диссертации доктора физико-математических наук (DSc) зарегистрирована Высшей аттестационной комиссией при Министерстве высшего образования, науки и инноваций Республики Узбекистан под номером B2022.2.DSc/FM195.

Докторская (DSc) диссертация выполнена в Институте фундаментальных и прикладных исследований Национального исследовательского университета «ТИИИМСХ».

Автореферат диссертации на трех языках (узбекском, английском и русском (резюме)) размещен на сайтах Ученого совета (www.ifag.uz), Национального информационного агентства (www.uza.uz) и Информационно-образовательном портале «Ziyonet» (www.ziyonet.uz).

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ВВЕДЕНИЕ (Аннотация к диссертации доктора наук (DSc))

Целью исследования является разработка новых нелинейных динамических свойств конденсата Бозе-Эйнштейна на основе анализа решений уравнения Гросса-Питаевского при различных внутренних и внешних нелинейных системах, исследование динамических свойств конденсата Бозе-Эйнштейна как макроскопической материи с использованием законов классической физики (механики Ньютона).

Задачи исследования:

определить влияние спин-орбитальной связи (СОС) на коллапс конденсата Бозе-Эйнштейна (КБЭ) и найти ее критическое значение, препятствующее коллапсу конденсата;

исследовать влияние внешнего магнитного поля, приложенного к спину конденсата с меж- и внутриспиновыми нелинейностями в уравнении Гросс-Питаевского (УГП), на коллапс конденсата;

проанализировать движение яркого солитона в КБЭ с притягивающими взаимодействиями, на динамику которых могут сильно влиять беспорядок и СОС;

исследовать связанную динамику внедренной частицы с поляронным эффектом в одно- и двумерных КБЭ, характеризующихся УГП и законами классической физики;

проанализировать модель неоднородной плотности темной материи, моделируемую динамикой «гравитационного» взаимодействия двух частиц в одномерном КБЭ.

Объектом исследования являются конденсат Бозе-Эйнштейна, спин, солитон, случайный потенциал, внедренные частицы и поляроны.

Предметом исследования являются коллапс спин-орбитально-связанного конденсата Бозе-Эйнштейна, спин-орбитально-связанный солитон в случайном потенциале, поляроны и гравитирующие поляроны в бозе-эйнштейновском конденсате.

Методами исследования являются математический аппарат теории КБЭ с применением законов классической физики, аналитические и численные методы решения систем дифференциальных уравнений в частных производных.

Научная новизна исследования заключается в следующем:

впервые изучено влияние спина конденсата под действием СОС и внешнего магнитного поля на коллапс квазидвумерного КБЭ и показано, что аномальные спин-индуцированные скорости полностью контролируют коллапс конденсата;

впервые показано, что из-за аномальной спин-зависимой скорости синтетическая зеемановская связь может играть критическую роль в динамике солитона, вызывая его локализацию или делокализацию в случайном потенциале;

показано, что смещение самопритягивающегося КБЭ может быть вызвано совместным действием прецессии спина в зеемановском поле и наличием случайного потенциала;

впервые продемонстрирована связанная динамика поляронов в одномерном и двумерном КБЭ, характеризующаяся УГП и законами классической ньютоновской механики;

впервые представлена упрощенная модель неоднородной плотности темной материи, моделируемая динамикой «гравитационного» взаимодействия двух поляронов в одномерном КБЭ;

было обнаружено, что поляронный эффект, то есть модификация плотности КБЭ в окрестности внедренной частицы, существенно модифицирует динамику, вызванную «гравитационными» силами между частицами, и приводит к модификации времени гравитационного спада.

Практические результаты исследования заключаются в следующем:

численно получено критическое значение аномальной спин-зависимой скорости, вызванной симметризованной связью Дрессельхауса, которая может предотвратить скорость коллапса и стабилизировать квазидвумерный КБЭ.

установлена зависимость поля коллапса конденсата от напряженности внешнего магнитного поля и параметра взаимодействия конденсата для соответствующих внутри- и межспиновых взаимодействий компонентов;

получен аналитический и численный критерий локализации или делокализации солитона за счет спин-зависимой аномальной скорости, которая пропорциональна СОС и зеемановскому взаимодействию. Показано, что достаточно сильное зеемановское поле приводит к локализации яркого солитона вблизи случайных минимумов потенциала;

показана совместная динамика полярона и конденсата, описываемая системой уравнений Гросса-Питаевского и классических уравнений Ньютона. Было обнаружено, что КБЭ ведет себя как мягкая квантовая материя для поляронов;

было установлено, что эффект полярона существенно изменяет динамику, вызванную ньютоновскими «гравитационными» силами между частицами, и приводит к модификации времени гравитационного падения;

проанализировано влияние динамики конденсата в нестационарном потенциале на движение «гравитирующих» внедренных частиц.

Достоверность результатов исследования обеспечивается тем, что в диссертационной работе использовались стандартные методы математической и теоретической физики, в том числе высокоэффективные численные методы и программы; проведена тщательная проверка непротиворечивости полученных теоретических результатов с результатами исследований других авторов; выводы хорошо согласуются с основными положениями теории конденсатов Бозе-Эйнштейна.

Научная и практическая значимость результатов исследования. Научная значимость результатов исследования определяется способностью разработанного в диссертации формализма анализировать нелинейные

динамические свойства конденсата Бозе-Эйнштейна как макроскопической материи, характеризуемой УГП, для широкого применения КБЭ в таких областях, как квантовые вычисления и квантовая информация, а также пролить свет на более глубокое понимание макроскопических свойств квантовой материи. Кроме того, исследования различных эффектов в атомарных конденсатах могут пролить свет на свойства и динамику темной материи с множеством пока неизвестных основных характеристик.

Практическая значимость результатов исследования заключается в том, что они могут быть использованы для получения оценок нелинейной динамики КБЭ, таких как коллапсирующая материя, спин-зависимая динамика, параметры деформации, смещения, а также темной материи, которая появляется из-за поправок в гравитационной модели Ньютона. Результаты также могут быть полезны для анализа природы и динамики конденсированного состояния, при разработке предлагаемых экспериментов.

Внедрение результатов исследования. Диссертационная работа выполнена в рамках гранта Швейцарского национального фонда («Schweiz. Nationalfonds») в рамках программы SCOPES по поддержке институционального партнерства с партнерами из Восточной Европы IZ74Z0_160527/1 (2016-2017 гг.); 2019 год - стипендия Фонда «Эл-Юрт умиди» для обучения специалистов за рубежом и связи с соотечественниками при Кабинете Министров Республики Узбекистан; 2022 год - Стипендия (Договор № 2 от 19 января) по программе «Организация и финансирование краткосрочных научных стажировок молодых ученых в зарубежных научных организациях» финансируемой Фондом финансирования науки и поддержки инноваций при Министерстве инновационного развития Республики Узбекистана.

Полученные результаты коллапса КБЭ со спин-орбитальной связью были использованы более чем в 40 научных статьях, опубликованных в журналах с наиболее высокими импакт-факторами (H. Sakaguchi, B. Li, and B.A. Malomed, Phys. Rev. E 89, 032920; Y. Zhang, M.E. Mossman, T. Busch, et al. Front. Phys. 11, 118103; H. Sakaguchi, E.Ya. Sherman, and B.A. Malomed, Phys. Rev. E 94, 032202; B.A. Malomed 2018 EPL 122, 36001, и др.). Результаты спин-вызванной динамики солитона в случайном потенциале использованы более чем в 10 статьях (J. Sun, Y. Chen, X. Chen, and Y. Zhang, Phys. Rev. A 101, 053621; J. Yang and Y. Zhang, Phys. Rev. A 107, 023316; J. Fan, G. Chen, and S. Jia, Phys. Rev. A 102, 063311, и др.) для разработки динамических свойств аномальных скоростей, зависящих от вращения, в различных системах.

Апробация результатов исследования. Результаты исследований докладывались и обсуждались на 8 международных и республиканских научных конференциях.

Основные результаты исследования апробированы на научных семинарах Института фундаментальных и прикладных исследований (2023 г.), Астрономического института (2020-2022 гг.), кафедры теоретической физики Самаркандского государственного университета Узбекистана (2019 г.),

кафедры Химия-физика в Университете Страны Басков (Испания, 2017-2018 гг.), кафедры теоретической физики в Университете Цюриха (Швейцария, 2016 г.) и др.

Опубликованность результатов исследования. По теме диссертации опубликовано 19 научных работ, в том числе 11 научных статей в международных журналах с высокими импакт-фактором и рекомендованных Высшей аттестационной комиссией Республики Узбекистан для публикации основных научных результатов докторских диссертаций.

Объем и структура диссертации. Диссертация состоит из введения, четырех глав, заключения, приложения и списка литературы. Объем диссертации составляет 165 страниц.

ЗАКЛЮЧЕНИЕ

По результатам исследований, проведенных по теме докторской диссертации «Нелинейная динамика, поляроны и механизмы самоорганизации в конденсатах Бозе-Эйнштейна», представлены следующие выводы:

1. Показано, что наличие СОС Дрессельхауса предотвращает сжатие коллапсирующего конденсата, вызывая «аномальную» скорость, зависящую от вращения. Слабый СОС допускает отдаленный временной коллапс, в то время как сильный СОС заставляет поток покидать центр КБЭ, что приводит к кольцевой плотности. Это предотвращает выдавливание конденсата и уменьшает притяжение частиц. Эти результаты показывают, что СОС может предотвратить коллапс КБЭ, что приводит к образованию стабильных конденсатов.

2. Показано, что магнитное поле Раби может влиять на коллапс конденсата с внутриспиновым взаимодействием, изменяя число атомов в каждой компоненте спина, что приводит к осцилляциям энергии самовоздействия, стабилизирующим конденсат. При кросс-спиновом взаимодействии вращение спина усиливает самовзаимодействие, вызывая коллапс конденсата. Определены диаграммы коллапса и устойчивости для обоих взаимодействий. Внутриспиновая связь изменяет критическое число атомов, вызывающих коллапс, в то время как при межспиновой связи происходит только двойной коллапс спиновой компоненты.

3. Показано, что для данного СОС движение солитона сильно зависит от зеemanовского расщепления и самовоздействия конденсата. В частности, зеemanовское взаимодействие может приводить к локализации или делокализации солитона за счет зависящей от спина аномальной скорости, пропорциональной СОС. Достаточно сильное зеemanовское поле может вызвать локализацию солитона вблизи случайных минимумов потенциала. Если зеemanовская частота близка к типичной частоте колебаний солитона в случайном потенциале, то этот резонанс может вызвать его делокализацию.

4. Показано, что движение центра масс самопритягивающегося КБЭ может быть вызвано совместным действием прецессии спина в зеemanовском поле и

наличием внешнего потенциала. Расширение конденсата из-за вращения спина создает результирующую силу, вызывающую его движение даже без СОС, что предполагает потенциальное расширение исследования на многомерные и многосолитонные настройки.

5. Продемонстрировано, что мягкость КБЭ играет решающую роль в динамике внедренных внешних частиц на основе поляронов. В режиме слабого взаимодействия конденсат обеспечивает статический потенциал для частицы, что приводит к круговым и квазипериодическим траекториям. Однако, в режиме относительно сильного взаимодействия конденсат становится зависящим от времени потенциалом для частиц, образующих поляроны, что приводит к смещению центра масс конденсата от источника параболического потенциала.

6. Смоделированы две гравитирующие частицы в КБЭ с поляронными эффектами. Поляронный эффект изменяет динамику, вызванную ньютоновскими «гравитационными» силами между частицами, и приводит к изменению времени гравитационного падения. Степень этого эффекта сильно зависит от начальных условий и параметров системы.

E'LON QILINGAN ISHLAR RO 'UXATI
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