

**“TIQXMMI” MILLIY TADQIQOT UNIVERSITETI HUZURIDAGI
FUNDAMENTAL VA AMALIY TADQIQOTLAR INSTITUTI
HUZURIDAGI ILMIY DARAJALAR BERUVCHI
DSc.03/31.03.2022.T/FM.10.04 RAQAMLI ILMIY KENGASH**

FUNDAMENTAL VA AMALIY TADQIQOTLAR INSTITUTI

TO‘RAYEV YUNUSBEK SHAVKATOVICH

**QORA O‘RALAR ATROFIDAN ENERGIYA AJRALISHI VA
ASTROFIZIK FENOMENLAR**

01.04.02 - Nazariy fizika

01.03.01 - Astronomiya

**FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD)
DISSERTATSIYASI AVTOREFERATI**

Toshkent – 2023

**Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertasiyasi
avtoreferati mundarijasi**

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Mathematics Sciences**

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Fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O'zbekiston Respublikasi Oliy ta'lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasida DSc.03/31.03.2022.T/FM.10.04 raqam bilan ro'yxatga olingan.

Dissertatsiya "TIQXMMI" Milliy tadqiqotlar universiteti huzuridagi Fundamental va Amaliy tadqiqotlar institutida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o'zbek, rus, ingliz, (xulosa)) Ilmiy Kengashning inteReissner-Nordstromet sahifasida (www.ifar.uz) va "Ziyonet" axborot-ta'lim portalida (www.ziyonet.uz) joylashtirilgan.

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zaruriyati. Hozirgi vaqtda dunyoda turli fizik va astrofizik hodisalaReissner-Nordstroming matematik modellarini ishlab chiqish va takomillashtirishga katta e'tibor berilmoqda. Xususan, Event Horizon teleskopi tomonidan olingan so'nggi kuzatuv ma'lumotlari galaktikalar markazida qora o'ralar soyasi mavjudligidan dalolat beradi. Boshqa tomondan, fundamental fizikada yagona maydon nazariyasini izlash, mavjud umumiy nisbiylik nazariyasiga yangi alteReissner-Nordstromativ nazariyalar ishlab chiqish jihatidan ham dolzarb mavzulardan hisoblanadi. Ushbu nazariyalaReissner-Nordstromi kuzatuv va eksperimental ma'lumotlar bilan taqqoslash va yangi nazariyalaReissner-Nordstroming parametrlari uchun kiritik qiymatlaReissner-Nordstromi olish, astrofizik qora o'ralar yaqinidagi optik-energetik jarayonlaReissner-Nordstromi matematik modellashtirish yordamida amalga oshiriladi.

Astrofizik kuzatuvlaReissner-Nordstroming eksperimental texnikalari yordamida erishilayotgan zamonaviy yutuqlar ko'plab kundalik astronomik kuzatishlar natijalariga umumiy relyativistik va postrelyativistik ta'sirlaReissner-Nordstromi hisobga olish talab qildi. Boshqa tomondan, kompakt obyektlar yaqinidagi geometriyaning Rimann tabiatining astronomik kuzatishlar natijalariga ta'sirini o'rganish va modellashtirish ham dolzarb masalalardandir. So'ngi paytlarda relyativistik astrofizikada koinotda materiyaning yangi shakllari kashf etilishi munosabati bilan tabiatdagi qorong'u energiya yoki qorong'u materiyani hisobga oladigan yechimlarga qiziqish ortdi. Qorong'u materiya va qorong'u energiyani hisobga olgan holda gravitatsiyaviy o'zaro ta'siReissner-Nordstroming mavjud modellarini o'zgartirish masalasi ham dolzarbdir.

Mamlakatimizda relyativistik astrofizika sohasida eksperimental va nazariy ishlar olib borishga katta e'tibor qaratilmoqda, shuningdek, jahon miqyosidagi fundamental tadqiqotlaReissner-Nordstromi olib borishga ham. Respublikamiz Prezidenti tomonidan ilgari surilgan 2017–2021-yillarda O'zbekiston Respublikasini yanada rivojlantirish bo'yicha harakatlar strategiyasida¹ mamlakatimizda ilm-fanning yuksalishi, fundamental tadqiqotlaReissner-Nordstroming muhim yo'nalishlari va ulaReissner-Nordstroming natijalarini hayotga tatbiq qilish yo'llari ko'rsatib berilgan. Buni isboti o'laroq, shuni aytib o'tish mumkinki, oxirgi yillarda relyativistik astrofizikada kompakt obyektlar atrofida magnitlangan va zaryadlangan zarralaReissner-Nordstroming harakati,

¹ O'zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi № PF-4947 sonli Farmoni "2017—2021-yillarda O'zbekiston Respublikasini rivojlantirishning beshta ustuvor yo'nalishi bo'yicha harakatlar strategiyasi".

ulaReissner-Nordstroming qora o'ra tomonidan tutib qolinishi, zarralaReissner-Nordstroming fizik hususiyatlari, kompakt ob'ektlar atrofidagi tashqi magnit maydon va ulaReissner-Nordstroming tabiati chuqur o'rganildi. Qora oralaReissner-Nordstromi kuzatishda qanchalik ulkan ishlar bajarilganligiga qaramasdan, bu boradagi ayrim masalalarga hali javob topilgancha yo'q.

Mazkur dissertatsiya ishi davlat me'yoriy hujjatlarida, O'zbekiston Respublikasi Prezidentining "O'zbekiston Respublikasini yanada rivojlantirish bo'yicha Harakatlar strategiyasi to'g'risida"gi PF-4947-son farmonlarida, 2017-yil 7-fevraldagi, shuningdek, O'zbekiston Respublikasi Hukumatining 2018-yil 29-noyabrdagi qarori bilan e'lon qilingan "2019-2021-yillarda O'zbekistonda tarkibiy o'zgarishlaReissner-Nordstroming asosiy yo'nalishlarining "Yo'l xaritasi"da tasdiqlangan vazifalaReissner-Nordstromi ma'lum darajada amalga oshirishga xizmat qilmoqda.

Tadqiqotning respublika fan va texnologiya rivojlanishining ustuvor yo'nalishlariga mosligi. Mazkur dissertatsiya ishi O'zbekiston Respublikasi fan va texnologiyalarini rivojlantirishning IV ustuvor yo'nalishi – "Axborotlashtirish va axborot-kommunikatsiya texnologiyalarini rivojlantirish"ga muvofiq amalga oshirildi.

Muammoning o'rganilganlik darajasi. Qora o'ralar soyasining matematik modeli va tortishish to'lqinlarining xossalari o'rganish bo'yicha ko'plab olimlar, jumladan yapon olimlari K. Xioki, K. Maeda, rus olimlari A.Zaxarov, V.Frolov, I.Novikovla, Germaniyalik olimlar C. Laemmerzahl, J. Kunz, A. Grezenbach, Argentinalik olimlardan L. Amarilla, E. Eiroa, Chexiyalik olimlar Z. Stuchlik, J. Schee va boshqalar, Hindistonlik olimlar N. Dadhich, S. Ghosh, P. Joshi, M. Patil va boshqa xorijiy olimlar R. Vald, F. de Felice, F. Sorge va boshqalar tomonidan o'rganilgan. Umumiy nisbiylik nazariyasi doirasida elektromagnit maydon va undagi magnitlangan zarracha harakatining matematik modeli respublikamiz olimlaridan B. Ahmedov, A. Abdujabborov, B. Toshmatov va boshqalar, tomonidan o'rganilgan.

Elektromagnit maydonlar va magnitlangan zarrachalar harakatining matematik modelini ishlab chiqish va o'rganish asosida ko'plab ilmiy muammolar, jumladan, qora o'ra atrofidagi kvazidavriy tebranishlar va energiya ajralish effektivligiga qora o'ra turli parametrlarini ta'siri o'rganish, qora o'raning massasini fotometrik kuzatuvlar asosida baholash, qora o'ra soyasining o'lchamining turli parametrlarga bog'liqligi va ko'plab ilmiy muammolar mavjud.

Dissertatsiya mavzusining dissertatsiya bajarilgan ilmiy-tadqiqot muassasasining ilmiy-tadqiqot ishlari bilan bog'liqligi. Dissertatsiya "Fundamental va amaliy tadqiqotlar" institutining ilmiy-tadqiqot ishlari rejasiga muvofiq, ilmiy-tadqiqot mavzusiga binoan olib borilgan.

Tadqiqotining maqsadi – kompakt obyektlar atrofidagi kvazidavriy tebranishlar va gravitatsion linzalanish xodisalari haqidagi bilimlaReissner-Nordstromi takomillashtirish, aktiv yadroli gallaktikalaReissner-Nordstroming markaziy qismidagi supermassiv qora o‘ra massasini fotometrik kuzatuvlar orqali baholash usulini ishlab chiqishdan iborat.

Tadqiqot vazifalari: Yuqoridagi maqsadlardan kelib chiqib quyidagi vazifalar belgilandi:

- qora o‘ralar atrofidagi zaryadli va neytral sinov zarralar harakatini tahlil qilish,
- qora o‘ra atrofidagi kvazidavriy tebranishlaReissner-Nordstromi va energiya chiqish effektivligini nazariy tadqiq etish,
- qora o‘ralar va “yumronqoziq uyasi (wormhole)” atrofidagi gravitatsion linzalanishni o‘rganish,
- aktiv yadroli gallaktikalaReissner-Nordstroming tayyor kuzatuv ma’lumotlari asosida ulaReissner-Nordstroming markaziy qismidagi supermassiv qora o‘ra massasini baholash usulini ishlab chiqish.

Tadqiqot ishining obyekti qora o‘ralar atrofidagi zaryadli va neytral sinov zarralar, qora o‘ralar atrofidagi kvazidavriy tebranishlar, qora o‘ra atrofidan energiya ajralish effektivligi.

Tadqiqot ishining predmeti qora o‘ralar atrofidagi jarayonlarini tavsiflovchi matematik modellar, differensial tenglamalaReissner-Nordstromi yechishning sonli va analitik usullari.

Tadqiqot usullari hisoblash matematikasi usullari, nazariy astrofizika usullari, matematik-fizikaning zamonaviy usullari, maydon va zarralar harakati uchun differentsial tenglamalaReissner-Nordstromi yechishning analitik va sonli usullari.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

- birinchi marta Bardeen qora o‘rasi atrofidagi kvazidavriy tebranishlaReissner-Nordstroming magnit zaryadiga va Gauss-Bonnet paramateriga bog‘ligi o‘rganildi,
- birinchi marta “yumronqoziq uyasi” atrofidagi gravitatsion linzalanish o‘rganildi va eksponensial metrikada kuzatuv uchun zarur bo‘lgan kattalashtirish qiymati uchun olindi,
- birinchi marta optik diapazondagi kuzatuvlar orqali aktiv yadroli gallaktikalaReissner-Nordstroming markazidagi qora o‘ra massasini “baholash” uchun optimallashtirish usul ishlab chiqildi.

Tadqiqotning amaliy natijalari quyidagilardan iborat: Ushbu tadqiqot ishining amaliy natijalari optik diapazondagi kuzatuvlar orqali aktiv yadroli

gallaktikalaReissner-Nordstroming markazidagi qora o'ra massasini "baholash" optimallashtirish usuli ishlab chiqildi.

Tadqiqot natijalarining ishonchliligi matematik-fizika, hisoblash matematikasi va relyativistik astrofizikaning zamonaviy isbotlangan usullarini qo'llandi. Biz taklif qilgan qora o'ra massasini baholash usuli orqali hisoblangan natijalar spektral kuzatuvdan olingan natijalar bilan solishtirildi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati quyidagilar: birinchidan, qora o'ralar atrofidagi zarralar harakatida yuzaga keladigan quzidavriy tebranishlaReissner-Nordstroming qora o'ra parametrlariga bog'liqligi ko'rsatildi, ikkinchidan, aktiv yadroli gallaktikalaReissner-Nordstroming fotometrik kuzatuvdan ulaReissner-Nordstroming markaziy qismidagi supermassiv qora o'ra massasini baholash usuli ishlab chiqildi.

Tadqiqot natijalarining joriy qilinishi. Ushbu dissertatsiya ishi doirasida chop etilgan ilmiy ishlardan turli xalqaro ishlarda foydalanilgan, jumladan,

«*Quasi-periodic oscillation around regular Bardeen black holes in 4D Einstein–Gauss–Bonnet gravity*» ilmiy ishimizdan quyidagi xalqaro darajadagi ilmiy ishlarda foydalanilgan:

1. Z. Stuchlík, J. Vrba "Geodesic Model of HF QPOs Tested for Black Holes in Spacetimes Reflecting the Effect of Surrounding Dark Matter" *The Astrophysical Journal* Reissner-Nordstromal, Volume 935, Number 2.
2. Javlon Rayimbaev, Dilshodbek Bardiev, Temurbek Mirzaev, Ahmadjon Abdujabbarov, Akram Khalmirzaev, Shadow and massless particles around regular Bardeen black holes in 4D Einstein Gauss–Bonnet gravity, *International Journal of Reissner-Nordstromal of ModeReissner-Nordstrom Physics D* Vol. 31, No. 07, 2250055 (2022).
3. Javlon Rayimbaev, Ahmadjon Abdujabbarov, Farukh Abdulkhamidov, Vokhid Khamidov, Sherzod Djumanov, Javohir Toshov, Shukurillo Inoyatov, Quasiperiodic oscillation around charged black holes in Einstein–Maxwell-scalar theory, *The European Physical Journal Reissner-Nordstromal C* volume 82, Article number: 1110 (2022) .

Bundan tashqari bizning natijalarimizdan «Czech Science Foundation grants SGS/26/2022 and GACR, grant number 23-07043S» ilmiy loyihalarida ham qo'llanilgan (xat ilova qilinadi).

Tadqiqot natijalarining aprobatyasi. Tadqiqot natijalari 4 ta respublika va 3 ta xalqaro miqyosdagi ilmiy anjumanlarida, O'zR FA Astronomiya instituti Nazariy astrofizika laboratoriyasi seminarlarida hamda Qozoqistonlik hamkorlar bilan tashkil etilgan doimiy "UZB-KAZ SEMINAR" seminarlarida muhokama qilingan.

Tadqiqot natijalarining e'lon qilinganligi. Dissertatsiya mavzusi bo'yicha jami 12 ta ilmiy ishlar chop etilgan, jumladan, 5 ta ilmiy maqola va 7 tezis-ma'ruzalar. Shulardan, O'zbekiston Respublikasi Oliy attestatsiya komissiyasi tomonidan doktorlik dissertatsiyalarining asosiy ilmiy natijalarini chop etish uchun tavsiya etilgan ilmiy jurnal Reissner-Nordstromlarda 3 ta maqola, shu jumladan, 3 ta xorijiy ilmiy jurnal Reissner-Nordstromlarda maqolalar chop etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish, to'rt bob, xulosa va foydalanilgan adabiyotlar ro'yxati hamda ilovalardan iborat. Dissertatsiya hajmi ilovalar bilan birgalikda 105 bet.

DISSERTATSIYANING ASOSIY MAZMUNI

Dissertatsiyaning «Kirish» qismida mavzuning dolzarbligi, tadqiqot ishining obyekti, predmeti, maqsadi va vazifalari bayon qilingan. Bundan tashqari qora o'ra atrofida energiya ajralishi bo'yicha mavjud modellar haqida qisqacha ma'lumot berilgan.

Dissertatsiyaning «Zaryadlangan qora tuynuk atrofida sinov zarralar dinamikasi» deb nomlangan 1-bobida sinov zarralar Reissner-Nordstroming singulyar va regulyar qora o'ra atrofidagi harakatlari qarab chiqilgan.

Reissner-Nordstrom va Ayon-Beato-Garcia qora o'ralari atrofida sinov zarralar harakati. Umumiy xolda Reissner-Nordstrom metrika quyidagicha ifodalanadi:

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (1)$$

bu yerda

$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}. \quad (2)$$

Reissner-Nordstrom elektr maydon uchun to'rt potensial quyidagicha:

$$A_\alpha = \frac{Q}{r} \{1, 0, 0, 0\}. \quad (3)$$

Zarra uchun Lagranjian va saqlanuvchi kattaliklari Reissner-Nordstrom umumiy xolda quyidagicha yoziladi:

$$\mathcal{L} = \frac{1}{2} m g_{\mu\nu} u^\mu u^\nu + e u^\mu A_\mu, \quad (4)$$

$$g_{tt} \dot{t} + q A_t = E \text{ va } g_{\phi\phi} \dot{\phi} = L. \quad (5)$$

Endi yuqoridagi (1) - (5) ifodalardan foydalanib zarra uchun effektiv potensialni quyidagicha yozamiz:

$$V_{eff}(r) = \frac{qQ}{r} \pm \sqrt{f \left(1 + \frac{L^2}{r^2} \right)}. \quad (6)$$

Xuddi shunga o'xshash tarzda Ayon-Beato-Garcia qora o'radi atrofidagi zarraning effektiv potensial energiyasi uchun ifodani yozishimiz mumkin. Umumiy xolda Ayon-Beato-Garcia qora o'radi uchun metrika quyidagicha yoziladi:

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (7)$$

$$f = 1 - \frac{2Mr^2}{(r^2+Q^2)^{1.5}} + \frac{Q^2 r^2}{(r^2+Q^2)^2}, \quad (8)$$

va ushbu metrika bilan birlashgan elektr maydon quyidagicha beriladi:

$$E_r(r) = Q r^4 \left[\frac{r^2 - 5Q^2}{(r^2 + Q^2)^4} + \frac{15}{2} \cdot \frac{M}{(r^2 + Q^2)^{3.5}} \right]. \quad (9)$$

Elektr maydon uchun to'rt potensialni (9) ifoda orqali quyidagicha hisoblaymiz:

$$A_t = \int E_r dr = \frac{Qr^5}{2} \left[\frac{3M}{Q^2(r^2+Q^2)^{2.5}} - \frac{2}{(r^2+Q^2)^3} \right] + C, \quad (10)$$

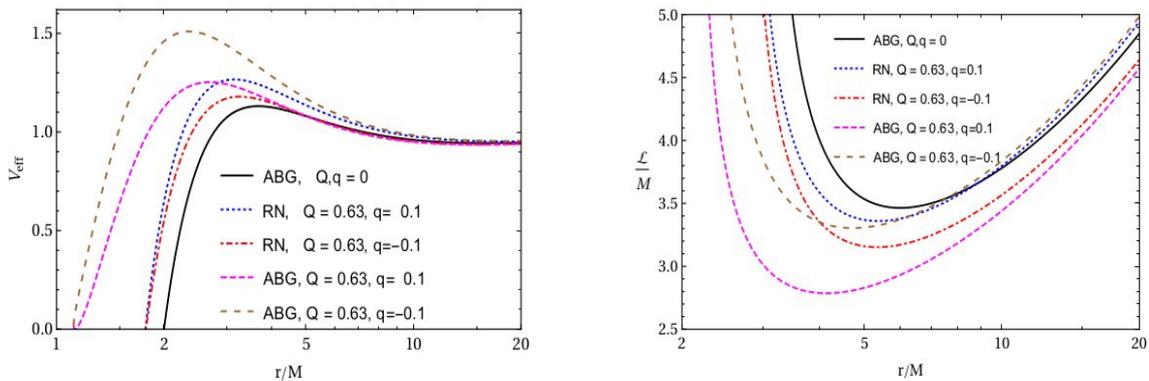
bu yerda (10) ifodadagi C doimiyni ni quyidagicha aniqlaymiz:

$$\lim_{Q \rightarrow 0} \frac{Qr^5}{2} \left[\frac{3M}{Q^2(r^2+Q^2)^{2.5}} - \frac{2}{(r^2+Q^2)^3} \right] + C = 0 \rightarrow C = -\frac{3M}{2Q}. \quad (11)$$

Endi xuddi yuqoridagiga o'xshash Ayon-Beato-Garcia qora o'rasi atrofidagi harakat qilayotgan zarra uchun effektiv potensialni yozishimiz mumkin:

$$V_{eff}(r) = qA_t + \sqrt{f(r) \left(1 + \frac{L^2}{r^2} \right)}. \quad (12)$$

Quyidagi 1-rasmda sinov zarrasining turli xolatlar uchun effektiv potentsiali va



impuls momentining radial o'zgarishlari keltirilgan.

1-rasm. Sinov zarrasining Reissner-Nordstrom va Ayon-Beato-Garcia qora o'rasi atrofidagi harakati uchun effektiv potentsiali.

Endi effektiv potensial ifodasidan zarra harakati uchun muhim bo'lgan ISCO radiusini quyidagicha topiladi:

$$V_{eff} = E \quad V'_{eff} = 0 \quad V''_{eff} = 0. \quad (13)$$

(13) ifodalardan hisoblashimiz va uning qora o‘raning qaysi parametrlariga bog‘liqligini ko‘rishimiz mumkin.

Dissertatsiyaning «Kvazidavriy tebranishlar. Aylanuvchi qora tuynuklar atrofida energiya ajralish mexanizmlari» deb nomlangan 2-bobida Bardeen qora o‘rasi atrofida kvazidavriy tebranishlar va energiya ajralish effektivligi o‘rganilgan.

Umumiy xolda Bardeen qora o‘rasi uchun metrika quyidagicha yoziladi:

$$ds^2 = -f(r) dt^2 + \frac{1}{f(r)} dr^2 + d\Omega^2, \quad (14)$$

$$f(r) = 1 + \frac{r^2}{2\alpha} \left(1 \pm \sqrt{1 + \frac{8M\alpha}{(r^2+g^2)^{1.5}}} \right) \quad (15)$$

Energiya ajralish effektivligi ISCO (eng kichik stabil doiraviy orbita) dagi energiya orqali aniqlanadi:

$$\eta = 1 - E_{isco} \quad (16)$$

(16) ifodadan Bardeen qora o‘rasi uchun energiya ajralish effektivligi turli α va g lar uchun bimalol hisoblashimiz mumkin.

Endi kvazidavriy tebranishlar chastotasini hisoblashga o‘tamiz. Umumiy xolda qora o‘ra atrofida yuzaga keladigan fundamental tebranishlar chastotasi (garmonik tebranishlarga yaqin deb tasavvur qilinsa) quyidagicha aniqlanadi:

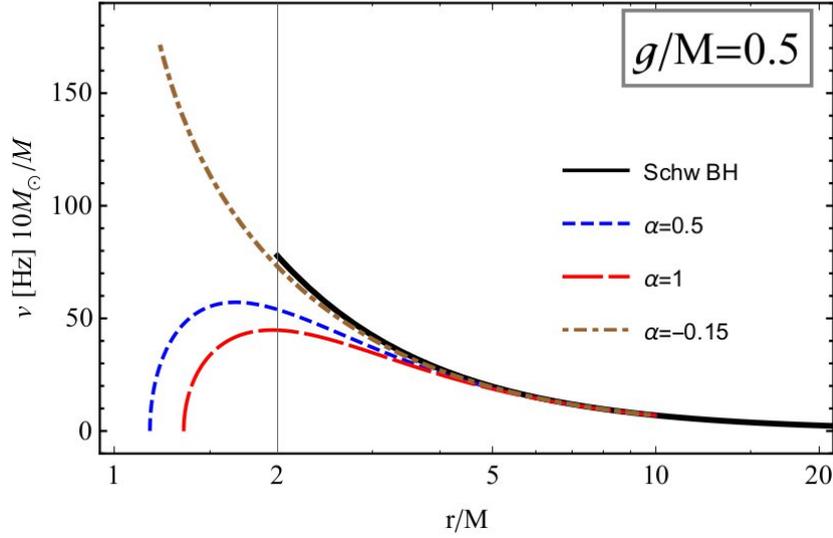
$$\Omega_K = \frac{d\phi}{dt} = \sqrt{\frac{f'(r)}{2r}}, \quad (17)$$

Yoki (17) ifodani biroz soddalashtirib quyidagicha yozib olsak bo‘ladi:

$$\Omega_K^2 = \frac{P(r)}{2} + \frac{1}{r^2} f(r), \quad (18)$$

(18) ifodadan oddiy Herz ga o‘tib yozsak :

$$\nu = \frac{1}{2\pi} \frac{c^3}{GM} \Omega \quad (\text{Hz}). \quad (19)$$



2-rasm. Kvazi-davriy tebranishlar chastotasi

Yuqoridagi 2-rasmdan ko‘rishimiz mumkinki α ning qiymati ortishi bilan chastota maksimumi kamayib boradi.

Dissertatsiyaning «Qora o‘ra va "yumronqoziq uyasi" (wormhole) atrofida gravitatsion linzalanish» deb nomlangan 3-bobida aylanuvchi «yumronqoziq uyasi» (wormhole) atrofidagi eksponensial metrikada gravitatsion linzalanish qarab chiqilgan va tipik hisoblashlar qilingan.

«Yumronqoziq uyasi» dagi gravitatsion linzalanish. Umumiy xolda metrika berilgan bo‘lsa, gravitatsion linzalanishdagi og‘ish burchagi quyidagicha hisoblanadi:

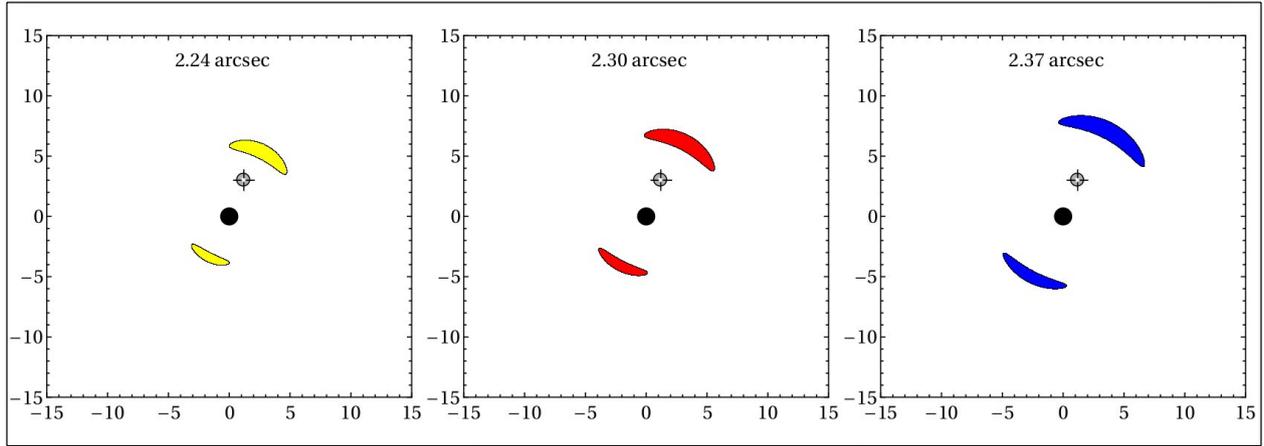
$$ds^2 = ds_0^2 + \frac{2M}{r}(dt^2 + dx^2 + dy^2 + dz^2), \quad (20)$$

$$\alpha = \frac{1}{2} \int_{-\infty}^{\infty} \left(\frac{\partial h_{tt}}{\partial x^\alpha} + \frac{\partial h_{zz}}{\partial x^\alpha} \right) dz = \int_{-\infty}^{\infty} \frac{d}{db} \left\{ \frac{2M}{\sqrt{z^2 + b^2}} \right\} dz = -\frac{4M}{b}, \quad (21)$$

Bu yerda $h_{tt} = h_{zz} = 2M/r$ va $r = \sqrt{z^2 + b^2}$. Enshteyn og‘ish burchagini ham hisobga olsak ($\Theta = \sqrt{\frac{4MD_s}{D_d D_s}}$), linzalanish natijasida hosil bo‘ladigan tasvir uchun kattalashtirishni quyidagicha hisoblashimiz mumkin bo‘ladi:

$$\mu_{\pm} = \frac{1}{4} \left(\frac{y}{\sqrt{y^2+4}} + \frac{\sqrt{y^2+4}}{y} \pm 2 \right) \quad (22)$$

Bu yerda $y = \frac{\beta}{\theta}$ va $\beta = \theta - \frac{\theta^2}{\theta}$, θ - obyektning kuzatuvchi-linza o‘qidan tasvirining ko‘rinish burchagi. Quyidagi rasmda turli massali «yumronqoziq uyasi» uchun tipik hisobashlar natijasi tasvirlangan.



3-rasm. Chapdan o'ngga qarab $10^6 M_0$, $5 \cdot 10^6 M_0$, va $7 \cdot 10^6 M_0$ (M_0 – quyosh massasi) massali «yumronqoziq uyasi» uchun gravitatsion linzalanish xolatlari.

Dissertatsiyaning «Qora o'ra massasini baholash» deb nomlangan 4-bobida aktiv yadroli gallaktikaReissner-Nordstroming yorqinligi hamda «bulge»dagi yulduzlaReissner-Nordstroming tezlik dispersiyasi asosida ulaReissner-Nordstroming markaziy qismidagi supermassiv qora o'ra massasini baholash usuli o'rganilgan.

Aktiv yadroli gallaktikaReissner-Nordstroming yorqinligiga ko'ra ulaReissner-Nordstroming markaziy qismidagi supermassiv qora o'ra massasini baholash. Buning uchun biz dastlab yaxshi o'rganilgan har xil tipdagi 300 ga yaqin aktiv yadroli gallaktikaReissner-Nordstroming ma'lumotlarini to'pladik. UlaReissner-Nordstroming yorqinligi turli olimlar tomonidan o'rganilgan va yorqinlikni hisoblashda asosan spektral kuzatuvlar natijalariga asoslangan. Bizning maqsadimiz esa yorqinlik va supermassiv qora o'ra massasi orasidagi empirik bog'lanish bor yo'qligini aniqlash va bor bo'lsa uni topishdan iborat. Korrelyatsion bog'lanish mavjud yoki yo'qligi korrelyatsiya koeffitsienti orqali aniqlaymiz:

$$k = \frac{\sum_{n=1}^N (x_i - \langle x \rangle)(y_i - \langle y \rangle)}{\sqrt{\sum_{n=1}^N (x_i - \langle x \rangle)^2} \cdot \sqrt{\sum_{n=1}^N (y_i - \langle y \rangle)^2}}, \quad (23)$$

bu yerda x -aktiv yadroli gallaktika yorqinligi, y – supermassiv qora o'ra massasi. Korellatsiya koeffitsientidan foydalanib regressiya koeffitsientini hisoblaymiz:

$$R_{yx} = k \frac{\sigma_y}{\sigma_x} = k \cdot \frac{\sqrt{\sum_{n=1}^N (y_i - \langle y \rangle)^2 / N}}{\sqrt{\sum_{n=1}^N (x_i - \langle x \rangle)^2 / N}}. \quad (24)$$

(24) ifodadagi regressiya koeffitsientidan foydalanib aktiv yadroli gallaktika va uning markazidagi supermassiv qora o'ra massasi orasidagi empirik bog'lanishni chiziqli regressiya tenglamasini tuzishimiz mumkin:

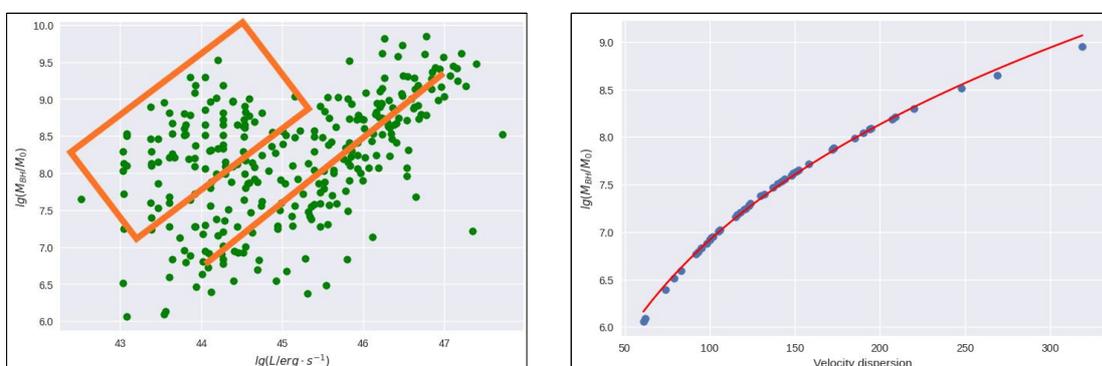
$$y - \langle y \rangle = R_{yx} (x - \langle x \rangle) \rightarrow y = ax + b, \quad (25)$$

bu yerda $y = \lg\left(\frac{M_{bh}}{M_0}\right)$ va $x = \lg\left(\frac{L}{\text{erg/s}}\right)$ M_0 – quyosh massasi, L – aktiv yadroli gallaktika yorqinligi.

Bizning xolatimizda $a = 0.9 \pm 0.173$ va $b = -32.8 \pm 2.332$ ga teng bo‘ldi. Biz bundan tashqari tezlik dispersiya orqali ham markaziy supermassiv qora o‘ra massasini baholashni ko‘rib chiqdik va quyidagi emperik munosabatni aniqladik (yorqinlik va tezlik dispersiyasi bo‘yicha tanlab olingan ayrim aktiv yadroli gallaktikalar haqida ma‘lumot 1-jadvalda berilgan.) :

$$\lg(M_{bh}/M_0) \approx c \cdot \sigma^d, \quad (26)$$

bu yerda $c = 2.359 \pm 0.04$ va $d = 0.23 \pm 0.00034$. Quyidagi 4-rasmda yorqinlik va supermassiv qora o‘ra massasi orasida bog‘lanish hamda tezlik dispersiyasi orasidagi bog‘lanish keltirilgan.



4-rasm. Supermassiv qora o‘ra massasi va aktiv yadroli gallaktika yorqinligi orasidagi emperik munosabat (chapda) va supermassiv qora o‘ra massasi va tezlik dispersiyasi orasidagi emperik bog‘lanish (o‘ngda)

1-Jadval

Name of the AGN	Type of the AGN	Redshift	Luminosity (lg(L/L _s))	Mass of the black hole (lg(M _{bh} / M _s))	Velocity dispersion (km/s)
NGC1566	SY1	0.005	44.45	6.92	100
NGC2841	SY1	0.002	43.67	8.21	209
NGC3982	SY1	0.004	43.54	6.09	62
NGC3998	SY1	0.003	43.54	8.95	319
Mrk10	SY1	0.029	44.61	7.47	137
NGC3223	SY1	0.016	44.27	7.02	106
NGC513	SY2	0.002	42.52	7.65	152
NGC788	SY2	0.014	44.33	7.51	140
NGC1052	SY2	0.005	43.84	8.19	207
NGC1275	SY2	0.018	45.04	8.51	248
NGC1320	SY2	0.009	44.02	7.18	116
NGC1358	SY2	0.013	44.37	7.88	173
NGC1386	SY2	0.003	43.38	7.24	120
NGC1667	SY2	0.015	44.69	7.88	173
NGC2110	SY2	0.008	44.1	8.3	220
NGC2273	SY2	0.006	44.05	7.3	124
NGC2992	SY2	0.008	43.92	7.72	158
NGC3185	SY2	0.004	43.08	6.06	61
NGC3362	SY2	0.028	44.27	6.77	92
NGC3786	SY2	0.009	43.47	7.53	142
NGC4117	SY2	0.003	43.64	6.83	95
NGC4339	SY2	0.004	43.38	7.4	132
NGC5194	SY2	0.002	43.79	6.95	102
NGC5252	SY2	0.023	45.39	8.04	190
NGC5273	SY2	0.004	43.03	6.51	79
NGC5347	SY2	0.008	43.81	6.79	93
NGC5427	SY2	0.009	44.12	6.39	74
NGC5929	SY2	0.008	43.04	7.25	121
NGC5953	SY2	0.007	44.05	6.94	101
NGC6104	SY2	0.028	43.6	7.6	148
NGC7213	SY2	0.006	44.3	7.99	185
NGC7319	SY2	0.023	44.19	7.38	130
NGC7603	SY2	0.03	44.66	8.08	194
NGC7672	SY2	0.013	43.86	6.88	98
NGC7682	SY2	0.017	43.93	7.28	123
NGC7743	SY2	0.006	43.6	6.59	83
Mrk1	SY2	0.016	44.2	7.16	115
Mrk3	SY2	0.014	44.54	8.65	269
Mrk78	SY2	0.037	44.59	7.87	172
Mrk270	SY2	0.01	43.37	7.6	148
Mrk348	SY2	0.015	44.27	7.21	118
Mrk533	SY2	0.029	45.15	7.56	144

ASOSIY NATIJALAR VA XULOSALAR

Ushbu "Qora o'ra atrofidan energiya ajralishi va astrofizik fenomenlar" dissertatsiya ishi bo'yicha quyidagi ilmiy va amaliy natijalar olindi:

- Ilk bor magnit zaryad va Gauss-Bonnet parametrlari qora tuynukning hodisa gorizonti radiusining pasayishiga olib keladi. Energiya ajralish effektivligini tahlil qilish shuni ko'rsatdiki, Bardeen qora o'rasining magnit zaryadi ortishi bilan effektivlik qiymati oshadi. Bundan tashqari, maksimal samaradorlik qora o'ra magnit zaryadining ekstremal qiymatlarining kamayishi natijasida yuzaga keladigan Gauss-Bonnet parametrining ortishi bilan kamayishini ko'rsatdi.
- Yuqori va quyi chastotalar fazosini qurish asosida Relyativistik Presessiya modeli doirasida "qo'shaloq cho'qqili" kvazidavriy tebranishlaReissner-Nordstroming kuzatuv ma'lumotlaridan foydalangan holda tortishish modellarini sinash uchun yangi model ishlab chiqildi. Yuqori va pastki chastotalar oralig'ida kuzatilgan egizak cho'qqilar yordamida markaziy obyekt turini ajratishga yordam beradigan maydon mavjudligi aniqlandi.
- «Yumronqoziq uyasi» soyasining kattaligi bir xil massadagi Schwarzschild qora tuynugining soyasidan kattaroq ekanligi aniqlandi. Bundan tashqari, qurt teshigining energiya samaradorligi bir xil massadagi Schwarzschild qora tuynugiga qaraganda nisbatan katta.
- Birinchi marta Papapetru fazosidagi Eynshteyn og'ish burchagining aniq ifodasi topildi.
- Birinchi marta aktiv yadroli gallaktikalaReissner-Nordstroming markaziy qismida joylashgan supermassiv qora o'ralar massasini baholashning optimallashtirilgan metodi taklif etildi.

Avtoreferat "Fan va innovatsiyalar" xalqaro ilmiy jurnali (International scientific journal "Science and Innovation") tahririyatida tahrirlan o'kazilib, o'zbek, ingliz va rus tillaridagi matnlari o'zaro muvofiqlashtirildi (05.07.2023).



**SCIENTIFIC COUNCIL DSc.03/31.03.2022.T/FM.10.04 ON AWARD OF
SCIENTIFIC DEGREE AT INSTITUTE OF FUNDAMENTAL AND
APPLIED RESEARCH «TIAMI» NATIANOL RESEARCH UNIVERSITY**

INSTITUTE OF FUNDAMENTAL AND APPLIED RESEARCH

TURAEV YUNUSBEK SHAVKATOVICH

**ENERGY EXTRACTION AND ASTROPHYSICAL PHENOMENA
AROUND BLACK HOLES**

01.03.01 - Astronomy

01.04.02 - Theoretical physics

**ABSTRACT OF THESIS OF THE DOCTOR OF PHILOSOPHY (PhD)
ON PHYSICAL AND MATHEMATICAL SCIENCES**

Tashkent – 2023

The topic of the Doctor of Philosophy (PhD) dissertation in physical and mathematical sciences is registered in the Supreme Attestation Commission at the Ministers of Higher education, science and innovations of the Republic of Uzbekistan under the number B2023.2.PhD/FM761.

The dissertation was completed at the Institute of Fundamental and Applied Research under the National Research University "TIAMI".

The abstract of the dissertation is posted in three languages (Uzbek, Russian, English (summary)) on the website of the Scientific Council (www.ifar.uz) and on the information and education portal "Ziyonet" (www.ziyonet.uz).

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The dissertation will be held at the meeting of the Scientific Council numbered DSc.03/31.03.2022.T/FM.10.04 at the Institute of Fundamental and Applied Research under the National Research University "TIAMI" at 22, 2023 at 16⁰⁰. (Address: 100000, Tashkent city, 39 Qori Niyoziy str., Institute of Fundamental and Applied Research, Hall 108; tel.: +998 71 237-09-61.; e-mail: info@ifar.uz

The dissertation can be viewed at the Information Resource Center of the Institute of Fundamental and Applied Research under the National Research University "TIAMI" (registered number _____). (Address: 100000, Tashkent city, 39, Qori Niyoziy str., Institute of Fundamental and Applied Research, Hall 108; tel.: 71 237-09-61

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INTRODUCTION

Topicality and demand of the theme of the dissertation. The development and improvement of mathematical models of various astrophysical processes in the vicinity of black holes is one of the most demanded tasks of modern Reissner-Nordstrom relativistic astrophysics. Revolutionary discoveries during the last ten years in observable astronomy have accelerated the rate of development of theoretical and mathematical models of astrophysical phenomena in the vicinity of compact gravitational objects. On the other hand, the search for a unified field theory in fundamental physics requires the development of new alternative Reissner-Nordstromative energy or modified gravity theories to the existing general relativity. Comparison of these theories with observational and experimental data and obtaining critical values for parameters of new theories is carried out using mathematical modelling of optical - energetic processes near astrophysical black holes.

Attempts to develop alternative Reissner-Nordstromative and modified theories of gravity are justified, in particular, by being cosmologically comprehensive, related, for example, to the presence of dark matter and dark energy in the Universe. At the same time, this theory is constructed in such a way that the general theory of relativity as a standard theory becomes the initial one as the initial theory, but when calculating the deviation from it in the early Universe and in the strong field regime. Thus, the study and analysis of the study of curved space in astrophysical research approaches compact objects, and, therefore, the results of astronomical observations are relevant.

In recent years, in our country, more and more attention has been paid to the development of current directions of fundamental and applied research. In particular, the development of astrophysical research, which is one of the promising areas, is an important issue today. The main directions of fundamental research and development and their practical application for the successful development of science in our country are reflected in the Strategy for the further development of the Republic of Uzbekistan from 2022-2026. Therefore, the research of scalarized solutions in higher curvature gravity theories remains one of the urgent issues in the field of fundamental research.

This dissertation work corresponds to the tasks by the following state regulatory documents: Decree of the President of the Republic of Uzbekistan No. PD-4947 "On the Strategy of Actions for the Further Development of the Republic of Uzbekistan" dated February 07, 2017, Resolution of the President of the Republic of Uzbekistan No. PR-2789 "On measures for further improvement of the

activities of the Academy of Sciences, organization, management, and financing of research activities" dated February 18, 2017, and others.

Compliance of the study with the priority areas of development of science and technology of the Republic.

This dissertation work was carried out in accordance with the IV priority direction of the development of science and technology of the Republic of Uzbekistan - "Informatization and development of information and communication technologies".

Degree of study of the problem. The mathematical model of the shadow of black holes and the study of the properties of gravitational waves the studied are many scientists, including Japanese scientists K. Hioki, K. Maeda, Russian scientists A. Zakharov, V. Frolov, I. Novikov, German scientists C. Laemmerzahl, J. Kunz, A. Grezenbach, Argentine L. Amarilla, E. Eiroa, Czech scientists Z. Stuchlik, J. Schee, Indian scientists N. Dadhich, S. Ghosh, P. Joshi, M. Patil another foreign authors such as R. Wald, F. de Felice, F. Sorge, and many others. The mathematical model for the electromagnetic field and the motion of a magnetized particle in it within the framework of the general theory of relativity was studied by such domestic scientists as B. Akhmedov, A. Abdujabbarov, B. Toshmatov, and others. Based on the development and study of the mathematical model of electromagnetic fields and the movement of magnetized particles, many scientific problems, including the problem of checking the mass and individual parameters of the black hole, checking the size of the black hole shadow, and many other problems are remains open. In addition, several other scientific problems related to compact objects remain open, for example, energetic processes around the wormhole, various issues of the electromagnetic field, and etc.

Connection of the topic of the dissertation topic to the scientific works of higher education and research institutions, where the dissertation is carried out. The dissertation has been done on the research topics of the Institute of Fundamental and Apply Research.

The aim of the research improvement of knowledge about quasi-periodic fluctuations and gravitational lensing phenomena around compact objects, development of a method for estimating the mass of the supermassive black hole in the central part of active galactic nuclei through photometric observations.

The tasks of the research are

- analysis of the motion of charged and neutral test particles around charged black holes,
- theoretical study of quasi-periodic oscillations and energy extraction efficiency around black holes,

- the study of gravitational lensing around black holes and wormholes,
- development of a method for estimating the mass of the supermassive black hole in their central part based on available observational data of AGNs.

Objects of research are charged and neutral test particles around black holes, quasi-periodic oscillations around black holes, energy extraction efficiency from black holes.

Subjects of research are mathematical models for studying processes around black holes, numerical and analytical methods for solving differential equations.

The methods of the research are methods of computational mathematics, methods of theoretical astrophysics, modeReissner-Nordstrom methods of mathematical physics, analytical and numerical methods of solving differential equations for field and particle motion.

Scientific novelties in the research are as follows:

- for the first time, the dependence of the quasi-periodic oscillations around the Bardeen black hole on the magnetic charge and the Gauss-Bonnet parameter was studied,
- for the first time, gravitational lensing around the wormholes was studied and obtained for the magnification value necessary for observation in the exponential metric,
- for the first time, a method was developed to estimate the mass of the supermassive black hole at the center of an AGN using only photometric observations in the optical range.

Practical results of the research are as follows: The practical results of this research work have developed an optimized method of "estimating" the black hole mass in the center of active galactic nuclei through observations in the optical band.

Reliability of the research results. The results were published in several scientific journals, participated in various conferences with abstracts, and were discussed at several seminars in the Department of Theoretical Astrophysics at the Institute of Astronomy.

The scientific and practical significance of the research results.

- the obtained constraints for the parameters of BHs may allow getting information about the gravitational feature of the spacetime around the BHs, giving the possibility of determining which gravitational effect plays a dominant role;
- the obtained results may help to determine the parameters of stellar mass BHs together with the space-time features around them based on observational data from QPOs and shadows of supermassive BHs, and the

observation of QPOs and black hole accretion made it possible to explain the physical mechanisms in the accretion disc;

- the obtained results can be useful in determining the identifications of singular and regular BHs in modified and quantum theories of gravity.

Application of the research results. Scientific works published within the framework of this dissertation work were used in various inteReissner-Nordstromational works, including our scientific work "Quasi-periodic oscillation around regular Bardeen black holes in 4D Einstein–Gauss–Bonnet gravity" was used in the following inteReissner-Nordstromational scientific works:

1. Z Stuchlík, J Vrba“Geodesic Model of HF QPOs Tested for Black Holes in Spacetimes Reflecting the Effect of Surrounding Dark Matter” The Astrophysical JouReissner-Nordstromal, Volume 935, Number 2
2. Javlon Rayimbaev, Dilshodbek Bardiev, Temurbek Mirzaev, Ahmadjon Abdujabbarov, Akram Khalmirzaev, Shadow and massless particles around regular Bardeen black holes in 4D Einstein Gauss–Bonnet gravity, InteReissner-Nordstromational JouReissner-Nordstromal of ModeReissner-Nordstrom Physics DVol. 31, No. 07, 2250055 (2022)
3. Javlon Rayimbaev, Ahmadjon Abdujabbarov, Farukh Abdulkhamidov, Vokhid Khamidov, Sherzod Djumanov, Javohir Toshov, Shukurillo Inoyatov, Quasiperiodic oscillation around charged black holes in Einstein–Maxwell-scalar theory, The European Physical JouReissner-Nordstromal C volume 82, Article number: 1110 (2022)

In addition, our results were used in the "Czech Science Foundation grants SGS/26/2022 and GACR, grant number 23-07043S" scientific projects (letter is attached).

Approbation of the research results. The results of this thesis were analyzed at 4 national and 3 inteReissner-Nordstromational conferences, and also discussed in seminars in the "Theoretical Astrophysics" laboratory of Astronomical Institute.

Publication of research results. A total of 12 scientific works have been published in this dissertation work, of which 5 are scientific articles and 7 are thesis reports in the materials of national and inteReissner-Nordstromational scientific conferences.

Volume and structure of the dissertation. This dissertation consists of an introduction, four chapters, the main result's bibliography, and appendixes. The volume of this thesis is 105 pages with appendixes.

THE MAIN CONTENT OF THE DISSERTATION

The "Introduction" part of the dissertation describes the relevance of the topic, the object, subject, purpose and tasks of the research work. The **chapter I** of the dissertation entitled "Dynamics of test particles around a charged black hole environment" examines the movements of test particles around a singular and regular black hole.

Motion of test particles around the Reissner-Nordstrom and Ayon-Beato-Garcia black holes. In general, the Reissner-Nordstrom metric is expressed as:

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (1)$$

where

$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}. \quad (2)$$

The four potential for electric field in the Reissner-Nordstrom metric as follows:

$$A_\alpha = \frac{Q}{r} \{1, 0, 0, 0\}. \quad (3)$$

The Lagrangian and conserved quantities for a particle are generally written as:

$$\mathcal{L} = \frac{1}{2} m g_{\mu\nu} u^\mu u^\nu + e u^\mu A_\mu, \quad (4)$$

$$g_{tt} \dot{t} + q A_t = E \quad \text{va} \quad g_{\phi\phi} \dot{\phi} = L. \quad (5)$$

Now, using expressions (1) - (5) above, we write the effective potential for the particle as follows:

$$V_{eff}(r) = \frac{qQ}{r} \pm \sqrt{f \left(1 + \frac{L^2}{r^2} \right)}. \quad (6)$$

In the same way, we can write for the Ayon-Beato-Garcia black hole. In general, the metric for the Ayon-Beato-Garcia black hole is written as:

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (7)$$

$$f = 1 - \frac{2Mr^2}{(r^2+Q^2)^{1.5}} + \frac{Q^2 r^2}{(r^2+Q^2)^2}, \quad (8)$$

and the electric field combined with this metric is given by:

$$E_r(r) = Q r^4 \left[\frac{r^2 - 5Q^2}{(r^2 + Q^2)^4} + \frac{15}{2} \cdot \frac{M}{(r^2 + Q^2)^{3.5}} \right]. \quad (9)$$

We calculate the four potentials for the electric field using expression (9) as follows:

$$A_t = \int E_r dr = \frac{Qr^5}{2} \left[\frac{3M}{Q^2(r^2+Q^2)^{2.5}} - \frac{2}{(r^2+Q^2)^3} \right] + C, \quad (10)$$

here we define the constant C in expression (10) as follows:

$$\lim_{Q \rightarrow 0} \frac{Qr^5}{2} \left[\frac{3M}{Q^2(r^2+Q^2)^{2.5}} - \frac{2}{(r^2+Q^2)^3} \right] + C = 0 \rightarrow C = -\frac{3M}{2Q}. \quad (11)$$

Now we can write the effective potential for a particle moving around the black hole Ayon-Beato-Garcia, similar to the above:

$$V_{eff}(r) = qA_t + \sqrt{f(r) \left(1 + \frac{L^2}{r^2} \right)}. \quad (12)$$

In the follow Fig.1 given radial variations of the effective potential and angular momentum of the test particle for different conditions.

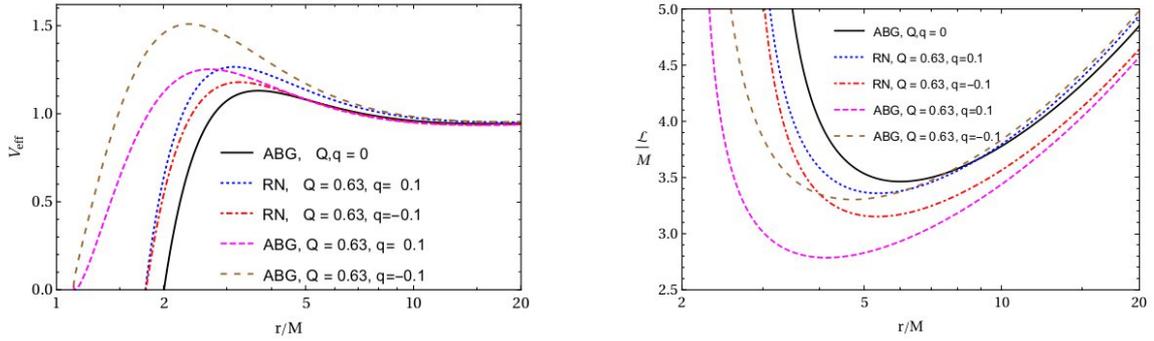


Figure 1. Effective potential of test particle around Reissner-Nordstrom and Ayon-Beato-Garcia black holes.

Now from the expression of the effective potential, the ISCO radius, which is important for the particle motion

$$V_{eff} = E \quad V'_{eff} = 0 \quad V''_{eff} = 0 \quad (13)$$

From the expression (13), we can see what parameters of the black hole the ISCO radius depends on.

In chapter II, which entitled «Quasiperiodic oscillations and energy extraction around black holes» quasi-periodic oscillations around the Bardeen black hole and the efficiency of energy extraction are studied.

In general, the metric for the Bardeen black hole is written as follows:

$$ds^2 = -f(r) dt^2 + \frac{1}{f(r)} dr^2 + d\Omega^2, \quad (14)$$

$$f(r) = 1 + \frac{r^2}{2\alpha} \left(1 \pm \sqrt{1 + \frac{8M\alpha}{(r^2+g^2)^{1.5}}} \right) \quad (15)$$

The energy extraction efficiency is determined by the energy in the ISCO :

$$\eta = 1 - E_{isco} \quad (16)$$

From expression (16) we can easily calculate the energy extraction efficiency for Bardeen black hole for different values of α and g .

Now let's calculate the frequency of quasi-periodic oscillations. In general, the frequency of the fundamental oscillations around the black hole (assumed to be close to harmonic oscillations) is defined as:

$$\Omega_K = \frac{d\phi}{dt} = \sqrt{\frac{f'(r)}{2r}} \quad (17)$$

or we can simplify expression (17) and write it as follows:

$$\Omega_K^2 = \frac{P(r)}{2} + \frac{1}{r^2} f(r) \quad (18)$$

from (18) expression to Herz, we can rewrite:

$$\nu \approx \frac{1}{2\pi} \frac{c^3}{GM} \Omega \quad (\text{Hz}) \quad (19)$$

In the follow given radial dependence of frequency

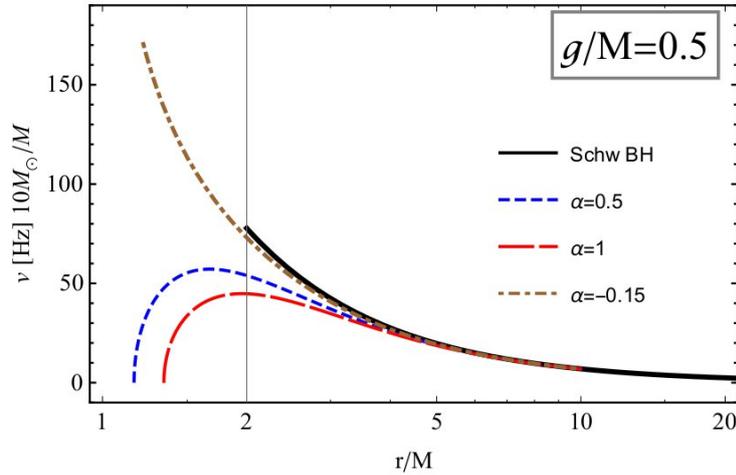


Figure 2. Frequency of quasi-periodic oscillations

From the figure 2 above, we can see that as the value of α increases, the maximum frequency decreases.

In the third chapter of the dissertation entitled "Gravitational lensing around wormhole", gravitational lensing in the exponential metric around the "wormhole" was observed and typical calculations were performed. *Gravitational lensing around a wormhole*. In general, if the metric is given, the deflection angle in gravitational lensing is calculated as follows:

$$ds^2 = ds_0^2 + \frac{2M}{r}(dt^2 + dx^2 + dy^2 + dz^2), \quad (20)$$

$$\alpha = \frac{1}{2} \int_{-\infty}^{\infty} \left(\frac{\partial h_{tt}}{\partial x^\alpha} + \frac{\partial h_{zz}}{\partial x^\alpha} \right) dz = \int_{-\infty}^{\infty} \frac{d}{db} \left\{ \frac{2M}{\sqrt{z^2 + b^2}} \right\} dz = -\frac{4M}{b}, \quad (21)$$

where $h_{tt} = h_{zz} = 2M/r$ and $r = \sqrt{z^2 + b^2}$, β is the real object from the observer-lens axis and angle θ is the apparent image of the object from the observer-lens axis, D_d is distance between observer and lens, D_s is distance between observer and source, D_{ds} is distance between lens and source.

If we also take into account the Einstein deflection angle ($\Theta = \sqrt{\frac{4MD_{ds}}{D_d D_s}}$), we can calculate the magnification for the image produced by the lensing:

$$\mu_{\pm} = \frac{1}{4} \left(\frac{y}{\sqrt{y^2 + 4}} + \frac{\sqrt{y^2 + 4}}{y} \pm 2 \right), \quad (22)$$

Where $y = \frac{\beta}{\Theta}$ and $\beta = \theta - \frac{\Theta^2}{\theta}$. The figure below shows the result of typical calculations for wormholes of different masses.

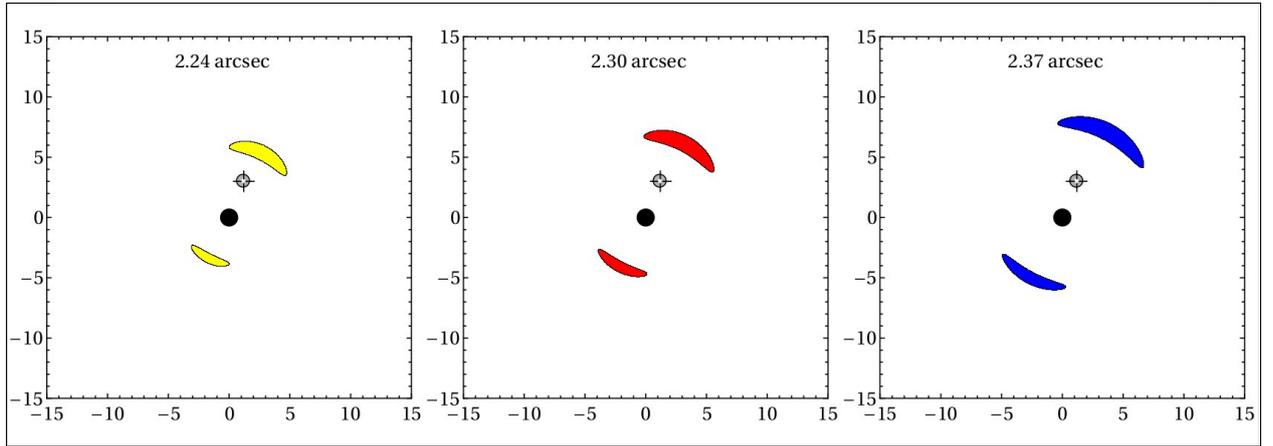


Figure.3 Gravitational lensing around different mass wormholes, from left to right $10^6 M_0$, $5 \cdot 10^6 M_0$, and $7 \cdot 10^6 M_0$ (M_0 is mass of the Sun).

Chapter 4 of the dissertation entitled "Estimating the mass of the black hole" deals with the method of estimating the mass of the supermassive black hole in their central part based on the brightness of active galactic nuclei and the velocity dispersion of the stars in the "bulge".

Estimation of the supermassive black hole mass in the central part of active galactic nuclei based on their luminosities. To do this, we initially collected data from about 300 well-studied active galactic nuclei of various types. Their luminosities has been studied by various scientists and the calculation of luminosity is mainly based on the results of spectral observations. Our goal is to determine whether there is an empirical connection between the luminosity and the

mass of the supermassive black hole, and if so, to find it. We determine whether there is a correlation connection or not using the correlation coefficient:

$$k = \frac{\sum_{n=1}^N (x_i - \langle x \rangle)(y_i - \langle y \rangle)}{\sqrt{\sum_{n=1}^N (x_i - \langle x \rangle)^2} \cdot \sqrt{\sum_{n=1}^N (y_i - \langle y \rangle)^2}} \quad (23)$$

where x is luminosity of the active galactic nuclei and y is mass of the supermassive black hole which located in central part of the active galactic nuclei.

We calculate the regression coefficient using the correlation coefficient:

$$R_{yx} = k \frac{\sigma_y}{\sigma_x} = k \cdot \frac{\sqrt{\sum_{n=1}^N (y_i - \langle y \rangle)^2 / N}}{\sqrt{\sum_{n=1}^N (x_i - \langle x \rangle)^2 / N}} \quad (24)$$

Using the regression coefficient in expression (24), we can construct a linear regression equation for the empirical relationship between the an active galactic nuclei and mass of the supermassive black hole which located central part of active galactic nuclei:

$$y - \langle y \rangle = R_{yx} (x - \langle x \rangle) \rightarrow y = ax + b \quad (25)$$

Where $y = \lg(\frac{M_{bh}}{M_0})$ and $x = \lg(\frac{L}{\text{erg/s}})$; M_0 is mass of the Sun, L is luminosity of active galactic nuclei. In our case $a = 0.9 \pm 0.173$ and $b = -32.8 \pm 2.332$. We also considered the estimation of the mass of the central supermassive black hole by means of the velocity dispersion and determined the following empirical relation (information about selected active galactic nuclei according to luminosity and velocity dispersion is given in Table 1) :

$$\lg(M_{bh}/M_0) \approx c \cdot \sigma^d \quad (26)$$

Where $c = 2.359 \pm 0.04$ and $d = 0.23 \pm 0.00034$. Figure 4 below shows the relationship between luminosity and supermassive black hole mass, as well as the relationship between velocity dispersion.

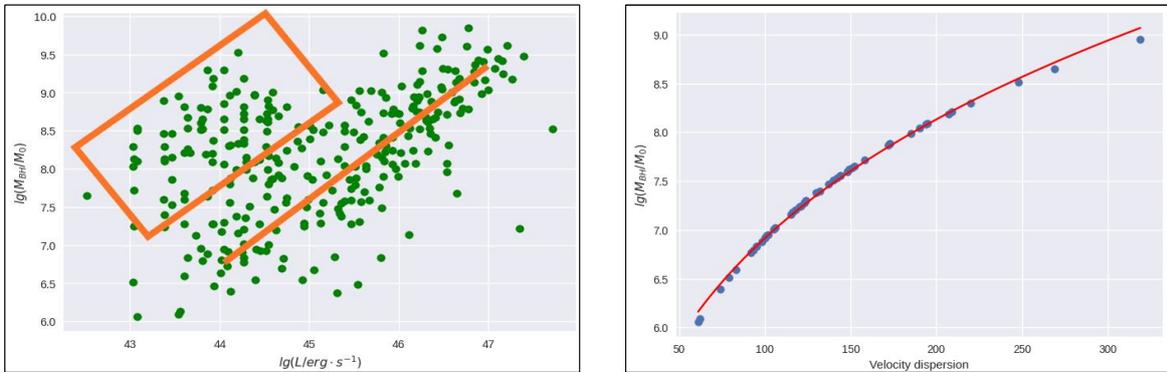


Figure. 4 Empirical relation between supermassive black hole mass and luminosity of active galactic nuclei (on the left) and empirical relation between supermassive black hole mass and velocity dispersion (on the right).

Table-1.

Name of the AGN	Type of the AGN	Redshift	Luminosity (lg(L/L_s))	Mass of the black hole (lg(M_{bh} / M_s))	Velocity dispersion (km/s)
NGC1566	SY1	0.005	44.45	6.92	100
NGC2841	SY1	0.002	43.67	8.21	209
NGC3982	SY1	0.004	43.54	6.09	62
NGC3998	SY1	0.003	43.54	8.95	319
Mrk10	SY1	0.029	44.61	7.47	137
NGC3223	SY1	0.016	44.27	7.02	106
NGC513	SY2	0.002	42.52	7.65	152
NGC788	SY2	0.014	44.33	7.51	140
NGC1052	SY2	0.005	43.84	8.19	207
NGC1275	SY2	0.018	45.04	8.51	248
NGC1320	SY2	0.009	44.02	7.18	116
NGC1358	SY2	0.013	44.37	7.88	173
NGC1386	SY2	0.003	43.38	7.24	120
NGC1667	SY2	0.015	44.69	7.88	173
NGC2110	SY2	0.008	44.1	8.3	220
NGC2273	SY2	0.006	44.05	7.3	124
NGC2992	SY2	0.008	43.92	7.72	158
NGC3185	SY2	0.004	43.08	6.06	61
NGC3362	SY2	0.028	44.27	6.77	92
NGC3786	SY2	0.009	43.47	7.53	142
NGC4117	SY2	0.003	43.64	6.83	95
NGC4339	SY2	0.004	43.38	7.4	132
NGC5194	SY2	0.002	43.79	6.95	102
NGC5252	SY2	0.023	45.39	8.04	190
NGC5273	SY2	0.004	43.03	6.51	79
NGC5347	SY2	0.008	43.81	6.79	93
NGC5427	SY2	0.009	44.12	6.39	74
NGC5929	SY2	0.008	43.04	7.25	121
NGC5953	SY2	0.007	44.05	6.94	101
NGC6104	SY2	0.028	43.6	7.6	148
NGC7213	SY2	0.006	44.3	7.99	185
NGC7319	SY2	0.023	44.19	7.38	130
NGC7603	SY2	0.03	44.66	8.08	194
NGC7672	SY2	0.013	43.86	6.88	98
NGC7682	SY2	0.017	43.93	7.28	123
NGC7743	SY2	0.006	43.6	6.59	83
Mrk1	SY2	0.016	44.2	7.16	115
Mrk3	SY2	0.014	44.54	8.65	269
Mrk78	SY2	0.037	44.59	7.87	172
Mrk270	SY2	0.01	43.37	7.6	148
Mrk348	SY2	0.015	44.27	7.21	118
Mrk533	SY2	0.029	45.15	7.56	144

MAIN RESULTS AND CONCLUSIONS

The following scientific and practical results were obtained in this thesis:

- For the first time showed that the magnetic charge and the Gauss-Bonnet parameters lead to a decrease in the radius of the event horizon of the black hole. The analysis of the energy extraction efficiency showed that the efficiency value increases with the increase of the magnetic charge of the Bardeen black hole. In addition, the maximum efficiency was shown to decrease with the increase of the Gauss-Bonnet parameter, which is caused by the decrease in the extreme values of the BH magnetic charge.
- A new model has been developed to test gravity models using observational data of twin-peak QPOs in the framework of the Relativistic Precession model based on the construction of the space of upper and lower frequencies. It has been found that there is an area in the space of upper and lower frequencies that may help to distinguish the type of the central object using observed twin-peaked.
- For the first time has been obtained that the size of shadow of wormhole is greater than shadow of the Schwarzschild black hole of the same mass. Moreover, the energy efficiency of the wormhole is relatively large than the Schwarzschild black hole of the same mass.
- For the first time the explicit expression for the Einstein deflection angle in the Papapetrou spacetime has been found.
- For the first time, an optimized method for estimating the mass of supermassive black holes located in the central part of galaxies with active nuclei was proposed.



**УЧЕНЫЙ СОВЕТ DSc.03/31.03.2022.T/FM.10.04 ПО ПРИСУЖДЕНИЮ
УЧЕНОЙ СТЕПЕНИ ПРИ ИНСТИТУТЕ ФУНДАМЕНТАЛЬНЫХ И
ПРИКЛАДНЫХ ИССЛЕДОВАНИЙ НАЦИОНАЛЬНОГО
ИССЛЕДОВАТЕЛЬСКОГО УНИВЕРСИТЕТА «ТИИИМСХ»**

**ИНСТИТУТ ФУНДАМЕНТАЛЬНЫХ И ПРИКЛАДНЫХ
ИССЛЕДОВАНИЙ**

ТУРАЕВ ЮНУСБЕК ШАВКАТОВИЧ

**ИЗВЛЕЧЕНИЕ ЭНЕРГИИ И АСТРОФИЗИЧЕСКИЕ ЯВЛЕНИЯ
ВОКРУГ ЧЕРНЫХ ДЫР**

01.04.02 – Теоретическая физика

01.03.01 - Астрономия

Диссертация доктора философии (Ph.D.) по физико-математическим наукам.

Ташкент – 2023

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшая аттестационная комиссия при Министерстве Высшего образования, науки и инноваций Республики Узбекистан за номером № DSc.03/31.03.2022.T/FM.10.04

Диссертация выполнена в Института фундаментальных и прикладных исследований национального исследовательского .

Автореферат диссертации на трех языках (узбекский, английский, русский (резюме)) размещен на веб-странице Научного совета (www.ifar.uz) и Информационно-образовательном портале «Ziyonet» (www.ziyonet.uz).

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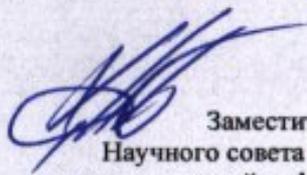
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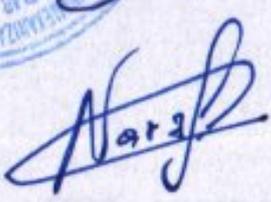
Диссертация зарегистрирована в Информационно-ресурсном центре Института фундаментальных и прикладных исследований Национального исследовательского университета "ТИИИМСХ". (регистрационный номер 10). С диссертацией можно ознакомиться в Библиотеке Института фундаментальных и прикладных исследований Национального исследовательского университета "ТИИИМСХ". Адрес: 100000, г.Ташкент, улица Кори Ниязова 39, Тел.: +998 71 237-09-61

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ВВЕДЕНИЕ (аннотация докторской диссертации)

Цель исследования – совершенствование знаний о квазипериодических флуктуациях и явлениях гравитационного линзирования вокруг компактных объектов, разработка метода оценки массы сверхмассивной черной дыры в центральной части галактик с активными ядрами посредством фотометрических наблюдений.

Задачи исследования: Исходя из поставленных целей, были определены следующие задачи:

- анализ движения заряженных и нейтральных пробных частиц вокруг черной дыры,
- теоретическое исследование квазипериодических колебаний и эффективности вывода энергии вокруг черной дыры,
- изучение гравитационного линзирования вокруг черных дыр и «wormhole»,
- разработка метода оценки массы сверхмассивных черных дыр в их центральной части на основе имеющихся данных наблюдений галактик с активными ядрами.

Объект исследования заряженных и нейтральных пробных частиц вокруг черных дыр, квазипериодические колебания вокруг черных дыр, эффективность энерговыделения черных дыр.

Предмет исследования математические модели для изучения процессов вокруг черных дыр, численные и аналитические методы решения дифференциальных уравнений.

Научная новизна исследования заключается в следующем:

- впервые изучена зависимость квазипериодических колебаний вокруг черной дыры Бардина от магнитного заряда и параметра Гаусса-Боннет,
- впервые изучено и получено гравитационное линзирование вокруг "веретенообразного гнезда" для необходимой для наблюдения величины увеличения в экспоненциальной метрике,
- впервые разработан оптимизированный метод "оценки" массы черной дыры в центре галактик с активными ядрами по наблюдениям в оптическом диапазоне.

Внедрение результатов исследования. Научные работы, опубликованные в рамках данной диссертации, использовались в различных международных работах, в том числе наша научная работа «Quasi-periodic oscillation around regular Bardeen black holes in 4D Einstein–Gauss–Bonnet gravity» использовалась в следующих международных научных работах:

1. Z Stuchlík, J Vrba “Geodesic Model of HF QPOs Tested for Black Holes in Spacetimes Reflecting the Effect of Surrounding Dark Matter” The Astrophysical Journal: Reissner-Nordstrom, Volume 935, Number 2
2. Javlon Rayimbaev, Dilshodbek Bardiev, Temurbek Mirzaev, Ahmadjon Abdujabbarov, Akram Khalmirzaev, Shadow and massless particles around regular Bardeen black holes in 4D Einstein Gauss–Bonnet gravity, International Journal of Reissner-Nordstrom of Mode Reissner-Nordstrom Physics D Vol. 31, No. 07, 2250055 (2022)
3. Javlon Rayimbaev, Ahmadjon Abdujabbarov, Farukh Abdulhamidov, Vokhid Khamidov, Sherzod Djumanov, Javohir Toshov, Shukurillo Inoyatov, Quasiperiodic oscillation around charged black holes in Einstein–Maxwell-scalar theory, The European Physical Journal: Reissner-Nordstrom C volume 82, Article number: 1110 (2022) ;

Кроме того, наши результаты были использованы в научных проектах «Czech Science Foundation grants SGS/26/2022 и GACR, grant number 23-07043S» (письмо прилагается).

Структура и объем диссертации. Диссертация состоит из введения, четырех глав, заключения, списка литературы и приложений. Объем диссертации вместе с приложениями составляет 105 страниц.

ОСНОВНЫЕ РЕЗУЛЬТАТЫ И ВЫВОДЫ

В диссертации были получены следующие научно-практические результаты:

- Впервые показал, что магнитный заряд и параметры Гаусса-Бонне приводят к уменьшению радиуса горизонта событий черной дыры. Анализ эффективности отбора энергии показал, что значение эффективности увеличивается с увеличением магнитного заряда черной дыры Бардина. Кроме того, было показано, что максимальная эффективность уменьшается с увеличением параметра Гаусса-Бонне, что связано с уменьшением экстремальных значений магнитного заряда ЧД.
- Разработана новая модель для проверки гравитационных моделей по данным наблюдений двухпиковых КПО в рамках модели релятивистской прецессии, основанной на построении пространства верхних и нижних частот. Было обнаружено, что в пространстве верхних и нижних частот есть область, которая может помочь различить тип центрального объекта с помощью наблюдаемого двойного пика.
- Впервые получено, что размер тени червоточины больше тени черной дыры Шварцшильда той же массы. Более того, энергетическая эффективность червоточины относительно велика, чем у черной дыры Шварцшильда той же массы.
- Впервые найдено явное выражение для угла отклонения Эйнштейна в пространстве-времени Папапетру.
- Впервые предложен оптимизированный метод оценки массы сверхмассивных черных дыр, находящихся в центральной части галактик с активными ядрами.



E'lon qilingan ishlar ro'yxati
List of published works
Список опубликованных работ

I bo'lim (Part I, Часть I)

1. Javlon Rayimbaev, Dilshodbek Bardiev, Ahmadjon Abdujabbarov, **Yunus Turaev**, and Zdeněk Stuchlík "Quasi-periodic oscillation around regular Bardeen black holes in 4D Einstein-Gauss-Bonnet gravity" *JouReissner-Nordstromal: InteReissner-Nordstromational JouReissner-Nordstromal of ModeReissner-Nordstrom Physics D*, Volume 31, Issue 2, id. 2250004-450, 2022y, (IF = 2.547).
2. Bobur Turimov, **Yunus Turaev**, Bobomurat Ahmedov, Zdeněk Stuchlík "Circular motion of test particles around wormhole represented by exponential metric" *Physics of the Dark Universe*, Volume 35, article id. 100946, 2022y, (IF = 5.09).
3. A. Demyanova, O. Rakhimov, Y. Turaev, N. Kurbonov, J. Rayimbaev "Characteristic orbits of charged particles around charged black holes, Proceedings of RAGtime 20-22: Workshops on Black Holes and Neutron Stars" Том 2020-December. 2020
4. **Yunus Turaev**, Nodira Hojiakbarova, Aksungul Arzikulova, and Gulzaman Koshetrova , "Test particle motion around regular black holes" *JouReissner-Nordstromal of Fundamental and Applied Research* Vol. 2, Issue 1 (2022) 20220006
5. **Yunus Turaev**, Bobur Turimov, Mahzuna Qorjobova, and Maksud Umaraliyev «Black hole mass estimating from optical band observation of AGN» *JouReissner-Nordstromal of Fundamental and Applied Research* Vol. 3, Issue 1 (2023) 20230006

II bo'lim (Part II, Часть II)

6. **Y.SH.Turaev**, M.A.Xayrullayeva - "Test particle motion around regular black hole" , "Kondensirlangan muhit fizikasining rivojlanish istiqbollari" , *Xalqaro ilmiy va ilmiy-texnik anjuman materiallari* 134-136-bet, QDU 2022-yil, 14-15-oktyabr
7. V.S.Hamidov, **Y.SH.Turaev** , K.B.Haydarov - "Observational properties of black hole candidate objects" *Technology of New Materials: Prospects for the Development of Polymer Composite Materials Used in Mechanical Engineering*", Andijan Machine-Building Institute, 329-330 bet, 2022-yil, 19-20-oktyabr
8. **Y.SH.Turaev**, V.S.Hamidov, K.B.Haydarov - "Estimation of black hole mass from optical band observation" *Technology of New Materials: Prospects for the Development of Polymer Composite Materials Used in Mechanical Engineering*", Andijan Machine-Building Institute, 331-333 bet, 2022-yil, 19-20-oktyabr

9. **Y.SH.Turaev**, M.F.Qorjobova, V.S.Hamidov, M.Sobitbekova - "Shadow of the rotating Ayon-Beato-Garcia black hole" "Fizika fanini axborot va innovatsion texnologiyalar muhitida o'qitishning zamonaviy tendensiyalari: muammo va yechimlar" Respublika ilmiy-amaliy anjumani materiallari 488-490-bet, Navoiy Pedagogika Instituti, 2022-yil, 20-noyabr
10. **Y.SH.Turaev**, M.F.Qorjobova, D.O.Ochilov - "Penrose process around regular black holes" "Fizika fanini axborot va innovatsion texnologiyalar muhitida o'qitishning zamonaviy tendensiyalari: muammo va yechimlar" Respublika ilmiy-amaliy anjumani materiallari 544-545-bet, Navoiy Pedagogika Instituti, 2022-yil, 20-noyabr.
11. **Y.SH.Turaev**, J.R.Raimbayev, K.B.Haydarov - "Black hole mass estimation from luminosity of active galactic nuclei" "Innovatsion texnologiyalar: Ilmiy g'oyalar va ishlanmalaReissner-Nordstromi amaliyotga joriy etish masalalar va yechimlari" mavzusidagi xalqaro qo'shma ilmiy-amaliy konferensiya materiallari 110-113-bet, Toshkent amaliy fanlar universiteti, 28-29-may.
12. **Y.SH.To'rayev**, A.A.Abdujabbarov, K.B.Haydarov - "Penrose mexanizmi va qora o'ra massasining nazariy maksimal qiymati" "Innovatsion texnologiyalar: Ilmiy g'oyalar va ishlanmalaReissner-Nordstromi amaliyotga joriy etish masalalar va yechimlari" mavzusidagi xalqaro qo'shma ilmiy-amaliy konferensiya materiallari 164-167-bet, Toshkent amaliy fanlar universiteti, 28-29-may.

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