URGANCH DAVLAT UNIVERSITETI HUZURIDAGI ILMIY DARAJA BERUVCHI PhD.03/30.12.2019.FM.55.01 RAQAMLI ILMIY KENGASH

URGANCH DAVLAT UNIVERSITETI

Azamatov Azizbek Shavkatovich

MOSLANGAN MANBALI KAUP-BOUSINESQ VA DISKRET SINUS-GORDON TENGLAMALAR SISTEMALARINI TEZ KAMAYUVCHI FUNKSIYALAR SINFIDA INTEGRALLASH

01.01.02 – Differensial tenglamalar va matematik fizika

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFERATI

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Dissertatsiya mavzusining dolzarbligi va zarurati. Jahon miqyosida olib borilayotgan ko‘plab ilmiy-amaliy tadqiqotlar matematik fizikaning muhim amaliy tatbiqlariga ega bo‘lgan nochiziqli evolyutsion tenglamalar sistemalari va ularning diskret analoglari bilan tavsiflanadi. Ushbu turdagi nochiziqli evolyutsion tenglamalar orasida maxsus yechimlarga ega tenglamalar sinfi, soliton yechimlarga ega tenglamalar alohida ajralib turadi. Masalan, gidromexanika, qattiq jismlar fizikasi, plazma fizikasida giperbolik sirtlar, ion akustik solitonlar va shu kabi masalalarni o‘rganishda Kaup-Bousinesq va sinus-Gordon tenglamalari muhim rol o‘ynaydi. Shu boisdan Kaup-Bousinesq va sinus-Gordon tenglamalarining soliton yechimlarini topish masalasi zamonaviy matematik fizikaning muhim masalalaridan biri bo‘lib qolmoqda.

Hozirgi kunda jahonda manbali nochiziqli evolyutsion tenglamalarni integrallash keng tadqiq etilmoqda. Odatda manbasiz tenglamalar model tenglamalar bo‘lib, ular ideal sharoitlarda keltirib chiqarilgan. Tabiiy jarayonlarda qo‘shimcha ta’sirlarni inobatga olish zarurati paydo bo‘ladi. Bunday jarayonlarda nochiziqli evolyutsion tenglamalar sistemalarini o‘rganishda tenglama tarkibiga yuklangan hadlarni va moslangan manbalarni kiritishga ehtiyoj tug‘iladi. Shu sababli bu jarayonlarning matematik modeli sifatida yuklangan hadli yoki moslangan manbali nochiziqli evolyutsion tenglamalar hosil bo‘ladi va bu tenglamalarning ayrim yechimlari soliton to‘lqinlarni tavsiflaydi. Hozirgi kunda nochiziqli to‘lqinlar dinamikasining asosiy ob’ektlaridan biri solitonlardir. Shu sababli tez kamayuvchi funksiyalar sinfida moslangan manbali, yuklangan hadlarga ega Kaup-Bousinesq va diskret sinus-Gordon tenglamalar sistemalarini integrallash va ularning soliton yechimlarini topish algoritmlarini keltirib chiqarish maqsadli ilmiy tadqiqotlardan hisoblanadi.

Mamlakatimizda fundamental fanlarning ilmiy va amaliy tatbiqiga ega bo‘lgan sohalarida ilmiy tadqiqotlar olib borish va ularning natijadorligi, amaliy ahamiyatini oshirishga alohida e’tibor qaratilmoqda. Xususan, nochiziqli to‘lqinlar nazariyasi, sochilish nazariyasining to‘g‘ri va teskari spektral masalalari va xususiy hosilali differensial tenglamalarning amaliy ahamiyatga ega bo‘lgan masalalarini o‘rganishga e’tibor kuchaydi. Buning natijasida matematik fizikaning nochiziqli evolyutsion tenglamalari uchun qo‘yilgan Koshi masalasini to‘g‘ri va teskari spektral masalalar usuli yordamida integrallash bo‘yicha salmoqli natijalarga erishildi.

“Funksional analiz, algebra, differensial tenglamalar, matematik fizika, matematik modellashtirish, hisoblash matematikasi va diskret matematika, ehtimollar nazariyasi va matematik statistika” ustuvor yo‘nalishlar bo‘yicha halqaro standartlar darajasidagi ilmiy izlanishlar olib borish ilmiy tadqiqot va oliy ta’lim muassasalari faoliyatining asosiy vazifalaridan biri hisoblanadi[1]. Ushbu qarorlar ijrosini ta'minlash maqsadida moslangan manbali yuklangan hadli nochiziqli evolyutsion tenglamalar sistemalarini va ularning diskret analoglarini integrallashni rivojlantirish muhim ahamiyatga ega.

O'zbekiston Respublikasi Prezidentining 2017 yil 7 fevraldagi PQ-4947-son "O'zbekiston Respublikasini yanada rivojlantirish bo'yicha harakatlar strategiyasi to'g'risida"gi, 2017 yil 17 fevraldagi PQ-2789-son "Fanlar akademiyasi faoliyati, ilmiy-tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari to'g'risida"gi, 2017 yil 20 apreldagi PQ-2909-son "Oliy ta'lim tizimini yanada rivojlantirish chora chora-tadbirlari to'g'risida"gi, 2018 yil 27 apreldagi PQ-3682-son "Innovatsion g'oyalar, texnologiyalar va loyihalarni amaliyotga joriy qilish tizimini yanada takomillashtirish chora-tadbirlari to'g'risida"gi, 2020 yil 7 maydagi PQ-4708-son "Matematika sohasidagi ta'lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to'g'risida"gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy xujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalar rivojlanishining ustuvor yo'nalishlariga mosligi. Mazkur tadqiqot respublika fan va texnologiyalari rivojlanishining IV. "Matematika, mexanika va informatika" ustuvor yo'nalishi doirasida bajarilgan.

Muammoning o'rganilganlik darajasi. Shturm-Liuvill operatorining kvadratik dastasi uchun teskari spektral masalalar nazariyasining eng muhim natijalari M. Jaulent, I. Miodek, F.G. Maksudov, M.G. Gasimov, G.Sh. Guseynov, A.O. Smirnov, D.H. Sattinger, J. E. Szmigielski, A. Laptev, R. Shterenberg, V. Sukhanov, A.B. Khasanov, A. Kabada, A.B. Yaxshimuratov va B.A. Babajanovlar va boshqa olimlar tomonidan olingan.

Shturm-Liuvill operatorining kvadratik dastasi uchun qo'yilgan sochilish nazariyasining teskari spektral masalalar usuli yordamida D.J. Kaup quyidagi

$$\begin{cases} \eta_\tau = \Phi_{xx} + \beta^2 \Phi_{xxxx} - \varepsilon \cdot (\Phi_x \eta)_x \\ \eta = \Phi_\tau + \dfrac{1}{2}\varepsilon \cdot \Phi_x^2 \end{cases}$$

sayoz suvda to'lqinlarning tarqalishini ifodalovchi tenglamalar sistemasini tez kamayuvchi funksiyalar sinifida to'la integrallanuvchanligini isbotlagan. Shundan so'ng, M. Jaulent and I. Miodeklar tomonidan Kaup sistemasi va uning yuqori tartibli analoglari uchun qo'yilgan Koshi masalasi yechimini tez kamayuvchi funksiyalar sinfida topish algoritmi keltirilgan. V.B. Matveev va I.V. Yavorning ishida, Kaup

[1] O'zbekiston Respublikasi Prezidentining 2020 yil 7 maydagi "Matematika sohasidagi ta'lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to'g'risida"gi PQ-4708-son qarori.

sistemaning chekli zonali, ko‘p fazali, Riman teta-funksiyalari orqali ifodalanuvchi kompleks yechimlarini tuzish usuli olingan, bundan tashqari bu sistemaning soliton yechimlari topilgan hamda ularning asimptotikalari o‘rganilgan. A.O. Smirnov, Y.A. Mitropolskiy, N.N. Bogolyubov, A.K. Prikarpatskiy va V.G. Somoylenkolar ishlarida Kaup sistemasining chekli zonali haqiqiy yechimlari o‘rganilgan. Davriy funksiyalar sinfida moslangan manbali Kaup tenglamalar sistemasi A. Kabada va A. Yaxshimuratovlar tomonidan yechilgan va yechimning davriyligi hamda x o‘zgaruvchi bo‘yicha analitikligi haqidagi muhim natija olingan.

Diskret sinus-Gordon tenglamasining integrallanuvchiligi Hirota tomonidan ko‘rsatilgan bo‘lib u bu masala uchun Laks juftligini, Beklund almashtirishini va N-soliton yechimlarni topgan. Hirota tomonidan keltirilgan diskterlashtirish sxemasining umumlashmasi Orfanidis tomonidan o‘rganilgan. D.Levi, O.Ragnisco va M. Bruschilarning ishida sochilish nazariyasining teskari masalasi usuli yordamida diskret sinus-Gordon tenglamasining soliton yechimlari keltirib chiqarilgan va bu masala bilan bog‘liq bo‘lgan chiziqli spektral masala L. Pilloni va D. Levining ishida o‘rganilgan.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya Urganch davlat universiteti “Amaliy matematika va matematik fizika” kafedrasining ilmiy-tadqiqot ishlari rejasiga muvofiq OT-F4-04 (05) “Spektral usulni matritsaviy nochiziqli evolyusion tenglamalarni yechishga tadbiqlari, Yurak-qon tomir tizimining biomexanikasi” (2017-2020 y.) mavzusidagi fundamental ilmiy tadqiqot loyihasi doirasida bajarilgan.

Tadqiqotning maqsadi:

Moslangan manbali Kaup-Bousinesq va diskret sinus-Gordon tenglamalar sistemalarini sochilish nazariyasining teskari masalalari usulidan foydalangan holda tez kamayuvchi funksiyalar sinfida integrallash. Sochilish nazariyasi berilganlarining vaqt bo’yicha evolyutsiyasini topish va qaralayotgan masalalarni yechish algoritmini keltirib chiqarishdan iborat.

Tadqiqotning vazifalari:

vaqtga bog‘liq koeffitsiyentli hadlarga ega bo‘lgan Kaup-Boussinesq tenglamalar sistemasini yechish algoritmini keltirib chiqarish;

yuklangan manbali Kaup-Boussinesq turidagi tenglamalar sistemasini integrallash;

moslangan manbali Kaup-Boussinesq tenglamalar sistemasi yechimlariga bog‘liq bo‘lgan Shturm-Liuvill operatorlari kvadratik dastasining sochilish nazariyasi berilganlarining vaqt bo‘yicha o‘zgarish dinamikasini ifodalovchi chiziqli differensial tenglamalarni keltirib chiqarish;

moslangan manbali Kaup-Boussinesq turidagi tenglamalar sistemasiga qo‘yilgan Koshi masalasining yechimini topish;

diskret sinus-Gordon tenglamasi uchun manba qurish algoritmini keltirib chiqarish;

moslangan manbali diskret sinus-Gordon tenglamasiga qo‘yilgan Koshi masalasining yechimini topish.

Tadqiqotning obyekti: manbali Kaup-Boussinesq tenglamalar sistemasi va uning yuqori tartibli analoglari, moslangan manbali diskret sinus-Gordon tenglamasi, diskret Dirak turidagi operator, Shturm-Liuvill operatorining kvadratik dastasi.

Tadqiqotning predmeti. Shturm-Liuvill operatorining kvadratik dastasi va diskret Dirak turidagi operator uchun qo‘yilgan to‘g‘ri va teskari spektral masalalarni moslangan manbali diskret sinus-Gordon, Kaup-Boussinesq va Kaup-Boussinesq turidagi tenglamalar sistemalariga qo‘yilgan Koshi masalasining yechimini topishga tatbiq qilishdan iborat.

Tadqiqotning usullari. Nochiziqli soliton tenglamalarni integrallash nazariyasi, sochilish nazariyasining teskari masalasi, integral tenglamalar nazariyasi, oddiy va xususiy hosilali differensial tenglamalar nazariyasi, funksional analiz, kompleks o‘zgaruvchili funksiyalar nazariyasidagi usullar qo‘llanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

Shturm-Liuvill operatorlari kvadratik dastasi uchun qo‘yilgan teskari masala usulidan foydalanib, vaqtga bog‘liq qo‘shimcha hadlarga ega bo‘lgan Kaup-Boussinesq tenglamalar sistemasini yechish algoritmi keltirib chiqarilgan;

Shturm-Liuvill operatorlari kvadratik dastasi uchun qo‘yilgan teskari masala usuli yordamida yuklangan hadli Kaup-Boussinesq turidagi tenglamalar sistemasi keltirib chiqarilgan va “tez kamayuvchi” funksiyalar sinfida integrallangan;

moslangan manbali Kaup-Boussinesq tenglamalar sistemasi yechimiga bog‘liq bo‘lgan Shturm-Liuvill operatorlari kvadratik dastasining sochilish nazariyasi berilganlarining vaqt bo‘yicha o‘zarish dinamikasini ifodalovchi chiziqli differensial tenglamalar keltirib chiqarilgan;

Shturm-Liuvill operatorlari kvadratik dastasi uchun qo‘yilgan teskari masala usuli foydalangan holda moslangan manbali Kaup-Boussinesq turidagi tenglamalar sistemasining to‘la integrallanuvchanligi isbotlangan;

diskret Dirak turidagi sistema uchun teskari spektral masala usulini qo‘llab diskret sinus-Gordon tenglamasi uchun manba qurish algoritmi keltirib chiqarilgan;

diskret Dirak turidagi sistema uchun qo‘yilgan teskari spektral masala usuli moslangan manbali diskret sinus-Gordon tenglamasiga qo‘yilgan Koshi masalasini yechimini topishga tatbiq etilgan.

Tadqiqotning amaliy natijalari moslangan manbali Kaup-Boussinesq tenglamalar sistemasi, yuklangan hadli Kaup-Boussinesq tenglamalar sistemasi, moslangan manbali diskret sinus-Gordon tenglamasi uchun qo‘yilgan Koshi masalasini yechish algoritmlari keltirib chiqarilgan.

Tadqiqot natijalarining ishonchliligi moslangan manbali Kaup-Boussinesq tenglamalar sistemasi va moslangan manbali diskret sinus-Gordon tenglamalarini integrallashda qo‘llanilgan usullar sochilish nazariyasining teskari masalasi, nochiziqli soliton tenglmalarni integrallash nazariyasi, integral tenglamalar nazariyasi, oddiy differensial va xususiy hosilali differensial tenglamalar nazariyasi usullariga tayanganligi hamda matematik mulohazalar va isbotlarning qat’iyligi bilan asoslanadi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati shundan iboratki, olingan ilmiy natijalar plazma fizikasi, qattiq jismlar fizikasi, kvant optikasi, elementar zarrachalar fizikasining masalalarini o‘rganishda qo‘llanilishi mumkin.

Dissertatsiyaning amaliy ahamiyati shundan iboratki, olingan ilmiy natijalardan matematik fizikaning turli nochiziqli evolyutsion tenglamalarini tez kamayuvchi funksiyalar sinfida yechishda foydalanish mumkin.

Tadqiqot natijalarining joriy qilinishi. Dissertatsiyada olingan natijalar quyidagi loyihalarda qo‘llanilgan:

Dissertatsiyada ishlab chiqilgan moslangan manbali Kaup-Bousinesq va moslangan manbali diskret sinus-Gordon tenglamalar sistemalarini yechish algoritmidan Rossiya ilmiy fondi tomonidan qo‘llab quvvatlanadigan 22-11-00196 “Matrix of monodromy and hierarchy of integrable nonlinear equations” mavzusidagi loyihasida 2×2 o‘lchamli matritsaviy koeffitsiyentli Lax juftliklari ierarxiyasini keltirib chiqarishda foydalanilgan (Sankt-Peterburg davlat aerokosmik qurilmalar qurish universitetining 2023 yil 6-apreldagi ma’lumotnomasi). Ilmiy natijalarining qo‘llanilishi nochiziqli evolyutsion tenglamalar iearxiyalarini integrallash imkonini bergan;

Diskret Dirak turidagi sistema uchun teskari spektral masala usulini qo‘llab diskret sinus-Gordon tenglamasi uchun manba qurish algoritmidan OT-F4-04 (05) “Spektral usulni matritsaviy nochiziqli evolyutsion tenglamalarni yechishga tatbiqlari, yurak-qon tomir tizimining biomexanikasi” mavzusidagi grant doirasida umumiy Toda tenglamasi uchun tez kamayuvchi funksiyalar sinfida moslangan manba qurish algoritmini keltirib chiqarishda hamda yuklangan hadli moslangan manbali Korteveg-de Friz tenglamasi uchun qo‘yilgan Koshi masalasini yechimini topish algoritmini keltirib chiqarishda foydalanilgan (Urganch davlat universitetining 2023 yil 12-maydagi ma’lumotnomasi). Ilmiy natijalarining qo‘llanilishi moslangan manbali matritsaviy nochiziqli evolyutsion tenglamalarni integrallash imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi.

Ushbu tadqiqot natijalari 8 ta xalqaro va 1 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinganligi.

Dissertatsiya mavzusi bo‘yicha jami 15 ta ilmiy ishlar chop etilgan bo‘lib, shulardan, O‘zbekiston Respublikasi Oliy ta’lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasining doktorlik dissertatsiyalari asosiy natijalarini chop etish bo‘yicha tavsiya etilgan ilmiy nashrlarda 6 ta maqola, jumladan 4 tasi respublika va 2 tasi xorijiy (SCOPUS ma’lumotlar bazasidagi) jurnallarda nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya tarkibi kirish, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan iborat. Dissertatsiyaning hajmi 88 bet.

DISSERTATSIYANING ASOSIY MAZMUNI

Dissertatsiyaning birinchi bobi **“Qo‘shimcha hadlarga ega bo‘lgan Kaup-Boussinesq tenglamalar sistemasini integrallash”** mavzusiga bag‘ishlangan bo‘lib, uning birinchi paragrafida Shturm-Liuvill operatorining kvadratik dastasi uchun teskari masala yechishning Maqsudov-Guseynov usuli haqida zaruriy ma’lumotlar keltirilgan.

Ushbu Shturm-Liuvill operatorlari kvadratik dastasini qaraymiz

$$L(k)y \equiv -y'' + v(x)y + 2ku(x)y - k^2 y = 0,\ x \in R, \tag{1}$$

bunda $v(x)$ va $u(x)$ haqiqiy funksiyalar, $u(x)$ absolyut uzluksiz va ular ushbu shartlarni qanoatlantiradi:

$$\int_{-\infty}^{\infty} |u(x)|\,dx < \infty,\quad \int_{-\infty}^{\infty} (1+|x|)[|v(x)|+|u'(x)|]dx < \infty\,. \tag{2}$$

(2) shart bajarilganda (1) tenglama $Imk \geq 0$ yuqori yarim tekislikka tegishli barcha k lar uchun quyidagi

$$f_+(x,k) = e^{ikx}[1+o(1)],\ x \to +\infty, \tag{3}$$

$$f_-(x,k) = e^{-ikx}[1+o(1)],\ x \to -\infty \tag{4}$$

asimptotikalarni qanoatlantiruvchi $f_+(x,k),\ f_-(x,k)$ yechimlarga ega. Noldan farqli haqiqiy $k \neq 0$ larda, (1) tenglama $f_+(x,k),\ \overline{f}_+(x,k)$ va $f_-(x,k),\ \overline{f}_-(x,k)$ kabi yechimlar fundamental sistemasiga ega bo‘ladi, va bu fundamental sistemalar o‘zaro quyidagi ko‘rinishda bog‘langan:

$$f_+(x,k) = b(k)\, f_-(x,k) + a(k)\, \overline{f}_-(x,k), \tag{5}$$

$$f_-(x,k) = -\overline{b}(k)\, f_+(x,k) + a(k)\, \overline{f}_+(x,k). \tag{6}$$

Bu yerda

$$a(k) = -\frac{1}{2ik} W\left\{ f_+(x,k),\, f_-(x,k) \right\}, \tag{7}$$

$$b(k) = \frac{1}{2ik} W\left\{ f_+(x,k),\, \overline{f}_-(x,k) \right\}. \tag{8}$$

Bunda $a(k)$ funksiya $Imk > 0$ yuqori yarim tekislikka analitik davom qiladi va cheklita $k_1, k_2, ..., k_N$ nollarga ega bo‘ladi hamda quyidagi tengliklar o‘rinli bo‘ladi

$$f_\mp(x,k_n) = B_n^\pm f_\pm(x,k_n), \tag{9}$$

bu yerdagi $B_n^\pm$ kattaliklar x o‘zgaruvchiga bog‘liq bo‘lmaydi.

Ta’rif 1. Ushbu

$$\left\{ r_-(k) = \frac{b(k)}{a(k)},\ k \in R \setminus \{0\},\ k_1, k_2, ..., k_N,\ \gamma_1^-, \gamma_2^-, ..., \gamma_N^- \right\} \tag{10}$$

va

$$\left\{ r_+(k) = -\frac{\overline{b}(k)}{a(k)},\ k \in R \setminus \{0\}, k_1, k_2, ..., k_N, \gamma_1^+, \gamma_2^+, ..., \gamma_N^+ \right\} \tag{11}$$

jamlanmalarga (1) tenglamaning mos ravishda chap va o‘ng sochilish nazariyasining berilganlari deyiladi, bunda $\gamma_n^{\pm}$ quyidagicha aniqlanadi:

$$\gamma_n^{\pm} = B_n^{\pm} \left(\left. \frac{da(k)}{dk} \right|_{k=k_n} \right)^{-1},\ n = 1, 2, ..., N.$$

Chap yoki o‘ng sochilish nazariyasining berilganlari orqali $u(x)$ va $v(x)$ koeffitsiyentlarni tiklash masalasiga (1) tenglama uchun qo‘yilgan teskari masala deyiladi.

O‘ng sochilish nazariyasi berilganlari (11) yordamida $u(x)$ va $v(x)$ koeffitsiyentlarni tiklash masalasini ko‘rib chiqamiz.

O‘ng sochilish nazariyasining berilganlari (11) yordamida $F_+(x)$ funksiyani qurib olamiz

$$F_+(x) = -i\sum_{n=1}^{N} \gamma_n^+ e^{ik_n x} + \frac{1}{2\pi}\int_{-\infty}^{\infty} r_+(k) e^{ikx} dk. \tag{12}$$

Topilgan $F_+(x)$ funksiyani quyidagi integral tenglamalarga qo‘yamiz

$$F_+(x+y) + \overline{K_+^{(0)}(x,y)} + \int_x^{\infty} K_+^{(0)}(x,\tau) F_+(\tau + y) d\tau = 0, x \le y < \infty, \tag{13}$$

$$iF_+(x+y) + \overline{K_+^{(1)}(x,y)} + \int_x^{\infty} K_+^{(1)}(x,\tau) F_+(\tau + y) d\tau = 0, x \le y < \infty. \tag{14}$$

Bu integral tenglamalarni yechib $K_+^{(0)}(x,y)$ va $K_+^{(1)}(x,y)$ larni topamiz. Bular yordamida quyidagi

$$K_+(x,y) = K_+^{(0)}(x,y)\cos\alpha_+(x) + K_+^{(1)}(x,y)\sin\alpha_+(x) \tag{15}$$

funksiyani qurib olamiz. Bu tenglikdagi $\alpha_+(x)$ funksiya quyidagi

$$\alpha_+(x) = \int_x^{\infty} \Phi(s, \alpha_+(s)) ds,\ -\infty < x < \infty \tag{16}$$

Volterra integral tenglamasining yechimidan iborat bo‘ladi. Bu yerda

$$\Phi(s,z) = \left[\mathrm{Re}\, K_+^{(0)}(s,s) - \mathrm{Im}\, K_+^{(1)}(s,s) \right] \sin 2z +$$
$$+2[\mathrm{Re}\, K_+^{(1)}(s,s)]\sin^2 z - 2[\mathrm{Im}\, K_+^{(0)}(s,s)]\cos^2 z. \tag{17}$$

Natijada, $u(x)$ va $v(x)$ koeffitsiyentlar ushbu tengliklar orqali topiladi

$$u(x) = -\alpha_+'(x), \tag{18}$$

$$v(x) = -u^2(x) - 2\frac{d}{dx}\left\{ \left[\mathrm{Re}\, K_+(x,x) \right] \cos\alpha_+(x) + \left[\mathrm{Im}\, K_+(x,x) \right] \sin\alpha_+(x) \right\}. \tag{19}$$

Birinchi bobning ikkinchi paragrafida quyidagi vaqtga bog‘liq koeffitsiyentli hadlarga ega bo‘lgan Kaup-Boussinesq tenglamalar sistemasini

$$\begin{cases} v_t - u_{xxx} + 4vu_x + 2uv_x = \mu(t)v_x, \\ u_t + 6uu_x + v_x = \mu(t)u_x \end{cases} \tag{20}$$

ushbu

$$v(x,t)\big|_{t=0} = v_0(x),\ u(x,t)\big|_{t=0} = u_0(x),\ x \in R, \tag{21}$$

boshlang'ich shartlar bilan qaraymiz, bunda $\mu(t)$ berilgan ixtiyoriy uzluksiz funksiya bo‘lib, $v_0(x)$, $u_0(x)$ funksiyalar haqiqiy va quyidagi shartlarni qanoatlantiradi:

i) $u_0(x)$ absolyut uzluksiz va ushbu tengsizliklar o‘rinli:

$$\int_{-\infty}^{\infty} |u_0(x)|\, dx < \infty,\ \int_{-\infty}^{\infty} (1+|x|)[|v_0(x)| + |u_0'(x)|]dx < \infty; \tag{22}$$

ii) Shturm-Liuvill operatorlarining kvadratik dastasi

$$L(0,k)y \equiv -y'' + v_0(x)y + 2ku_0(x)y - k^2 y = 0,\ x \in R$$

$2N$ ta $k_1(0), k_2(0), ..., k_{2N}(0)$ oddiy xos qiymatlarga ega.

Bu paragrafning asosiy natijasi quyidagi teoremadan iborat.

Teorema 1. *Agar* $v = v(x,t),\ u = u(x,t)$ *funksiyalar juftligi* (20)-(22) *masalaning yechimi bo‘lsa, u holda*

$$L(t,k)y \equiv -y'' + v(x,t)y + 2ku(x,t)y - k^2 y = 0,\ x \in R$$

Shturm-Liuvill operatorlari kvadratik dastasining sochilish nazariyasi berilganlari t *bo‘yicha quyidagicha o‘zgaradi*

$$\dot{r}_+(t,k) = (4ik^2 - 2ik\mu(t))r_+(t,k), \tag{23}$$

$$\dot{k}_n(t) = 0,\ n = 1,2,...,N, \tag{24}$$

$$\dot{\gamma}_n^+(t) = (2ik_n\mu(t) - 4ik_n^2)\gamma_n^+(t). \tag{25}$$

Olingan natijalar sochilish nazariyasi berilganlarining vaqt bo‘yicha evolyutsiyasini to‘la aniqlaydi va (20)-(22) masalani teskari masala usulida yechish imkonini beradi.

Misol. Teorema 1 ning qo‘llanilishini quyidagi misol yordamida ko‘rib chiqamiz. (20)-(22) masalani ushbu

$$v(x,t)\big|_{t=0} = -\frac{2}{ch^2 x},\ u(x,t)\big|_{t=0} = 0,\ x \in R$$

boshlang‘ich shartlarda qaraymiz. $\mu(t) = 2t$ bo‘lsin.

$L(0,k)$ operatorning sochilish nazariyasi berilganlarini topamiz:

$$r_+(k,0) = 0,\ k_1(0) = i,\ \gamma_1^+(0) = 2i.$$

Teorema 1 natijalaridan foydalanib, quyidagiga ega bo‘lamiz

$$r_+(k,t) = 0,\ k_1(t) = i,\ \gamma_1^+(t) = 2ie^{4it-2t^2}.$$

Topilgan sochilish nazariyasi berilganlari yordamida $F_+(x,t)$ funksiyani (12) tenglikdan aniqlaymiz:

$$F_+(x,t) = 2e^{4it-2t^2-x}.$$

$F_+(x,t)$ funksiyani (13), (14) integral tenglamalarga qo‘yamiz va uni yechib quyidagiga ega bo‘lamiz:

$$K_+^{(0)}(x,x;t) = \frac{-2e^{-2x-2t^2}\cos 4t + 2e^{-4x-4t^2}}{1-e^{-4x-4t^2}} + i\frac{2e^{-2x-2t^2}\sin 4t}{1-e^{-4x-4t^2}},$$

$$K_+^{(1)}(x,x;t) = \frac{2e^{-2x-2t^2}\sin 4t}{1-e^{-4x-4t^2}} + i\frac{2e^{-4x-4t^2} + 2e^{-2x-2t^2}\cos 4t}{1-e^{-4x-4t^2}}.$$

Bular yordamida (17) tenglikdan $\Phi(x,z;t)$ funksiyani aniqlaymiz:

$$\Phi(x,z;t) = -\frac{2\sin(2z+4t)}{sh(2x+2t^2)}.$$

$\Phi(x,z;t)$ funksiyani (16) Volterra integral tenglamasiga qo‘yamiz va uni yechib $\alpha_+(x,t)$ ni topamiz:

$$\alpha_+(x,t) = arctg\left(\frac{th^2(x+t^2)\cdot tg\,2t}{th^2(t^2)}\right) - 2t.$$

Topilgan $\alpha_+(x,t)$ dan foydalanib (15) tenglik orqali $K_+(x,x,t)$ funksiyani qurib olamiz:

$$K_+(x,x,t) = \frac{-2e^{-2x}\cos(4t+\alpha) + 2e^{-4x}\cos\alpha}{1-e^{-4x}} + i\frac{2e^{-4x}\sin\alpha + 2e^{-2x}\sin(4t+\alpha)}{1-e^{-4x}},\ x \neq 0.$$

Natijada (18) va (19) tengliklardan qaralayotgan masalaning yechimi $u(x,t)$ va $v(x,t)$ lar topiladi. Ular quyidagicha bo‘ladi:

$$u(x,t) = -\frac{sh2(x+t^2)\sin 4t}{ch^4(x+t^2)\cos^2 2t + sh^4(x+t^2)\sin^2 2t},$$

$$v(x,t) = -3u^2 - \frac{4ch2(x+t^2)\cos 2(2t+\alpha_+(x,t)) - 4}{sh^2 2(x+t^2)}.$$

Birinchi bobning uchinchi paragrafida yuklangan hadli Kaup-Boussinesq turidagi sistema tez kamayuvchi funksiyalar sinfida integrallangan.

Ushbu yuklangan hadli Kaup-Boussinesq turidagi sistemani

$$\begin{cases} v_t - v_{xxx} - 6uu_{xxx} - 18u_x u_{xx} + 6vv_x + 24vuu_x + 6v_x u^2 = H(t)v_x, \\ u_t - u_{xxx} + 6vu_x + 6v_x u + 30u_x u^2 = H(t)u_x,\ x \in R,\ t > 0 \end{cases} \quad (26)$$

quyidagi boshlang‘ich shartlarda qaraymiz

$$v(x,t)\big|_{t=0} = v_0(x),\ u(x,t)\big|_{t=0} = u_0(x),\ x \in R, \quad (27)$$

Bunda $H(t) = \mu(t)v(0,t)u(0,t)$ bo‘lib, $v_0(x),\ u_0(x)$ funksiyalar (22) shartni qanoatlantiradi.

Ushbu paragrafning asosiy natijasi quyidagi teoremada bayon qilinadi.

Teorema 2. *Agar* $v = v(x,t),\ u = u(x,t)$ *funksiyalar* (26)-(27) *masalaning yechimlari bo'lsa, u holda*

$$L(t,k)y \equiv -y'' + v(x,t)y + 2ku(x,t)y - k^2 y = 0,\ x \in R$$

Shturm-Liuvill operatorlari kvadratik dastasining sochilish nazariyasi berilganlari t *bo'yicha quyidagicha o'zgaradi*

$$\dot{r}_+(t,k) = (8ik^3 - 4kH(t))r_+(t,k), \tag{28}$$

$$\dot{k}_n(t) = 0, \tag{29}$$

$$\dot{\gamma}_n^+(t) = (2ik_n H(t) - 8ik_n^3)\gamma_n^+(t). \tag{30}$$

Ikkinchi bob **"Moslangan manbali Kaup-Boussinesq tenglamalar sistemasini integrallash"** deb nomlangan bo'lib, uning birinchi paragrafida Shturm-Liuvill operatorlari kvadratik dastasi uchun qo'yilgan to'g'ri va teskari masalalarni yechishning Jaulent-Jean usuli haqida zarur ma'lumotlar keltirilgan.

Ushbu Shturm-Liuvill operatorlari kvadratik dastasini qaraymiz

$$L(k)y = -y'' + (V - k^2)y = 0,\ x \in R, \tag{31}$$

bu yerda $V(x,k) = v(x) + 2ku(x)$ hamda $v(x)$ va $u(x)$ funksiyalar kompleks qiymatli funksiyalar bo'lib quyidagi shartlarni qanoatlantiradi:

$$\int_{-\infty}^{+\infty} x^2 \left[|v(x)| + |u'(x)|\right] dx < \infty,\ \int_{-\infty}^{+\infty} |x| \left[|v'(x)| + |u''(x)|\right] dx < \infty. \tag{32}$$

(32) shart bajarilganda, (31) tenglama ixtiyoriy $k \in R$ larda ushbu asimptotikalarni

$$\left[f_1(x,k), g_1(x,k)\right] \sim \left[e^{-ikx}, e^{ikx}\right],\ x \to \infty, \tag{33}$$

$$\left[f_2(x,k), g_2(x,k)\right] \sim \left[e^{ikx}, e^{-ikx}\right],\ x \to -\infty \tag{34}$$

qanoatlantiruvchi $\{f_1(x,k),\ g_1(x,k)\}$ va $\{f_2(x,k),\ g_2(x,k)\}$ Yost yechimlariga ega. Noldan farqli haqiqiy $k \neq 0$ da $\{f_1(x,k), g_1(x,k)\}$ va $\{f_2(x,k), g_2(x,k)\}$ funksiyalar (31) tenglamaning yechimlar fundamental sistemasini tashkil qiladi. Bu yechimlar uchun quyidagi tengliklar o'rinli

$$f_2 = c_{11} f_1 + c_{12} g_1,\ g_2 = d_{12} f_1 + d_{11} g_1, \tag{35}$$

$$f_1 = c_{22} f_2 + c_{21} g_2,\ g_1 = d_{21} f_2 + d_{22} g_2, \tag{36}$$

$$c_{12} = c_{21} = (2ik)^{-1} W[f_1, f_2],\quad c_{11} = -d_{22} = (2ik)^{-1} W[f_2, g_1], \tag{37}$$

$$d_{12} = d_{21} = (2ik)^{-1} W[g_2, g_1],\quad d_{11} = -c_{22} = (2ik)^{-1} W[f_1, g_2], \tag{38}$$

bunda $c_{11}, c_{12}, c_{21}, c_{22}, d_{11}, d_{12}, d_{21}, d_{22}$ funksiyalar x o'zgaruvchiga bog'liq emas hamda $c_{21}(k),\ (Imk < 0)$ funksiya quyi yarim tekislikka analitik davom qiladi. (31) tenglamada $V(x,k)$ o'rniga $V(x, \pm k)$ ni qarasak, u holda (31) ni ikkita tenglama deb qarash mumkin. Shu sababli yuqorida keltirilgan barcha tengliklarda "$\pm$" indeks bor deb tushunamiz. $c_{21}^{\pm}(k)(Imk<0)$ funksiya cheklita $N^{\pm}$ nollarga ega va bu nollarni $k_n^{\pm},\ n=1,2,...,N^{\pm}$ orqali belgilaymiz.

Ta'rif 2. Ushbu

$$\left\{R^{\pm}(k)=\frac{c_{11}^{\pm}(-k)}{c_{21}^{\mp}(-k)},\ k\in R\setminus\{0\},\ k_n^{\pm},\ C_n^{\pm},\ n=1,2,...,N^{\pm}\right\}$$

jamlanmaga (31) tenglamaning sochilish nazariyasining berilganlari deyiladi, bunda

$$C_n^{\pm}=\left[c_{11}^{\pm}(k_n^{\pm})\right]^{-1}\left[i\frac{d}{dk}c_{21}^{\pm}(k)\right]_{k=k_n^{\pm}}.$$

$u(x)$ va $v(x)$ koeffitsiyentlar sochilish nazariyasi berilganlari orqali yagona aniqlanadi.

Ikkinchi bobning ikkinchi paragrafida moslangan manbali Kaup-Boussinesq tenglamalar sistemasi uchun sochilish nazaryasi berilganlarining vaqt bo'yicha o'zgarish tenglamasi keltirib chiqarilgan va moslangan manbali Kaup-Boussinesq tenglamalar sistemasi uchun qo'yilgan Koshi masalasini teskari masala usuli yordamida yechish algoritmi keltirilgan.

Quyidagi moslangan manbali Kaup-Boussinesq tenglamalar sistemasini

$$\begin{cases} v_t=u_{xxx}-4vu_x-2uv_x+2\sum\limits_{m=1}^{N}\left[-u_x\varphi_m^2+(k_m-2u)\dfrac{\partial}{\partial x}\varphi_m^2\right], \\ u_t=-6uu_x-v_x+\sum\limits_{m=1}^{N}\dfrac{\partial}{\partial x}\varphi_m^2,\ x\in R,\ t>0, \\ (\varphi_m)_{xx}+\left[k_m^2-v-2k_mu\right]\varphi_m=0,\ m=1,2,...,N \end{cases} \tag{39}$$

ushbu

$$v(x,t)\big|_{t=0}=v_0(x),\ u(x,t)\big|_{t=0}=u_0(x),\ x\in R \tag{40}$$

boshlang'ich va quyidagi normallovchi

$$\int_{-\infty}^{+\infty}(2k_m-2u)\varphi_m^2dx=A_m(t),\ m=1,2,...,N, \tag{41}$$

shart bilan qaraymiz. Bunda $\varphi_1=\varphi_1(x,t),\ \varphi_2=\varphi_2(x,t),...,\varphi_N=\varphi_N(x,t)$ funksiyalar (31) tenglamaning $k_1=k_1(t),\ k_2=k_2(t),...,k_N=k_N(t)$, $\operatorname{Im}k_m<0,\ m=1,2,...,N$ xos qiymatlariga mos keluvchi xos funksiyalari. Hamda $A_1(t),\ A_2(t),...,\ A_N(t)$ lar oldindan berilgan ixtiyoriy uzluksiz funksiyalar. $v_0(x), u_0(x)$ funksiyalar quyidagi shartlarni qanoatlantiradi:

i) $v_0(x)$, $u_0(x)$ funksiyalar kompleks qiymatli funksiyalar bo'lib quyidagi shartlarni qanoatlantiradi:

$$\int_{-\infty}^{\infty} x^2 \left[|v_0(x)| + |u_0'(x)| \right] dx < \infty, \ \int_{-\infty}^{\infty} |x| \left[|v_0'(x)| + |u_0''(x)| \right] dx < \infty, \tag{42}$$

ii) Shturm-Liuvill operatorlari kvadratik dastasi

$$L(0,k)y \equiv -y'' + v_0(x)y + 2ku_0(x)y - k^2 y = 0, \ x \in R$$

chekli N ta oddiy xos qiymatlarga ega.

Qulaylik uchun (39) tenglamani vektor ko‘rinishda yozamiz. Buning uchun quyidagi belgilashlarni kiritamiz:

$$U = \begin{pmatrix} v \\ u \end{pmatrix}, \ G = \begin{pmatrix} G_1 \\ G_2 \end{pmatrix}, \tag{43}$$

$$G_1 = 2\sum_{m=1}^{N} \left[-u_x \varphi_m^2 + (k_m - 2u) \frac{\partial}{\partial x} \varphi_m^2 \right], \ G_2 = \sum_{m=1}^{N} \frac{\partial}{\partial x} \varphi_m^2, \tag{44}$$

$$T^* = \begin{pmatrix} 0 & -\dfrac{\partial^2}{\partial x^2} + 4v - 2v_x \displaystyle\int_x^{\infty} d\tau \\ 1 & 4u - 2u_x \displaystyle\int_x^{\infty} d\tau \end{pmatrix}. \tag{45}$$

U holda (39) sistema quyidagi ko‘rinishda yoziladi:

$$U_t + T^* U_x = G. \tag{46}$$

Ushbu $V(x) = \begin{pmatrix} V_1(x) \\ V_2(x) \end{pmatrix}$ va $W(x) = \begin{pmatrix} W_1(x) \\ W_2(x) \end{pmatrix}$ funksiyalar uchun quyidagi skalyar ko‘paytmani kiritamiz:

$$\langle V(x), W(x) \rangle = \int_{-\infty}^{\infty} [V_1(x)W_1(x) + V_2(x)W_2(x)] dx.$$

Lemma 1. *Agar*

$$\Phi_1^{\pm}(x,k) = \begin{pmatrix} f_1^{\pm}(x,k) \, f_2^{\pm}(x,k) \\ \pm 2k f_1^{\pm}(x,k) \, f_2^{\pm}(x,k) \end{pmatrix}, \tag{47}$$

$$\Phi_2^{\pm}(x,k) = \begin{pmatrix} g_1^{\pm}(x,k) \, f_2^{\pm}(x,k) \\ \pm 2k g_1^{\pm}(x,k) \, f_2^{\pm}(x,k) \end{pmatrix}, \tag{48}$$

bo‘lsa, u holda barcha t lar uchun quyidagi tengliklar o‘rinli

$$2ik \frac{d}{dt} c_{21}^{\pm}(t,k) = \left\langle U_t(x,t), \Phi_1^{\pm}(x,t,k) \right\rangle, \left(\mathrm{Im} k \le 0, k \ne 0 \right), \tag{47}$$

$$2ik \frac{d}{dt} c_{11}^{\pm}(t,k) = \left\langle U_t(x,t), \Phi_2^{\pm}(x,t,k) \right\rangle, \ k \in R \setminus \{0\}. \tag{48}$$

Lemma 2. *Barcha t lar uchun ushbu tengliklar o‘rinli*

$$0 = \left\langle U_x(x,t), \Phi_1^{\pm}(x,t,k) \right\rangle, \ (\mathrm{Im} k \le 0, k \ne 0), \tag{49}$$

$$-4k^2 c_{11}^{\pm}(t,k) = \left\langle U_x(x,t), \Phi_2^{\pm}(x,t,k) \right\rangle, \quad k \in R \setminus \{0\}. \tag{50}$$

Lemma 3. *Ushbu*

$$T=\begin{pmatrix} 0 & 1 \\ -\dfrac{\partial^2}{\partial x^2}+4v-2\int\limits_{-\infty}^{x} v_\tau d\tau & 4u-2\int\limits_{-\infty}^{x} u_\tau d\tau \end{pmatrix}$$

operator

$$T^*=\begin{pmatrix} 0 & -\dfrac{\partial^2}{\partial x^2}+4v-2v_x\int_x^\infty d\tau \\ 1 & 4u-2u_x\int_x^\infty d\tau \end{pmatrix}$$

operatorning qo'shmasi bo'lsa, u holda har bir fiksirlangan t *larda ushbu tengliklar o'rinli*

$$\left\langle U_x(x,t), T\Phi_1^\pm(x,t,k)\right\rangle = \pm 2k\left\langle U_x(x,t), \Phi_1^\pm(x,t,k)\right\rangle,\ Imk\le 0,\ k\ne 0, \qquad (51)$$

$$\left\langle U_x(x,t), T\Phi_2^\pm(x,t,k)\right\rangle = \pm 2k\left\langle U_x(x,t), \Phi_2^\pm(x,t,k)\right\rangle,\ Imk\le 0,\ k\ne 0. \qquad (52)$$

Bu paragrafning asosiy natijasi quyidagi teoremadan iborat.

Teorema 3. *Agar* $v=v(x,t), u=u(x,t)$ va $\phi_1(x,t),\phi_2(x,t),...,\phi_{N^+}(x,t)$ *funksiyalar* (39)-(42) *masalaning yechimi bo'lsa, u holda*

$$L(t,\pm k)y \equiv -y_{xx}+v(x,t)y\pm 2ku(x,t)y=k^2y,\ x\in R,$$

Shturm-Liuvill operatorlari kvadratik dastasining sochilish nazariyasi berilganlari quyidagi tenglamalarni qanoatlantiradi

$$\frac{dR^\pm(t,k)}{dt}=\mp 4ik^2R^\pm(t,k), k\in R, \qquad (53)$$

$$\frac{dk_n^\pm(t)}{dt}=0, n=1,2,...,N^\pm, \qquad (54)$$

$$\frac{dC_n^+(t)}{dt}=-[4i(k_n^+)^2+2ik_n^+A_n(t)]C_n^+(t), n=1,2,...,N^+, \qquad (55)$$

$$\frac{dC_n^-(t)}{dt}=4i(k_n^-)^2C_n^-(t),\ n=1,2,...,N^-. \qquad (56)$$

Olingan natijalar sochilish nazariyasi berilganlarining vaqt bo'yicha evolyutsiasini to'liq aniqlaydi, bu esa (39)-(42) masalani teskari spektral masala usulida yechish imkonini beradi.

Endi (31) tenglamaning sochilish nazariyasi berilganlari yordamida $u(x,t)$ va $v(x,t)$ larni tiklash usulini ko'rsatamiz.

Bizga $u_0(x)$, $v_0(x)$ va $A_m(t), m=1,2,...,N^+$ lar berilgan bo'lsin.

1. Berilgan $u_0(x)$ va $v_0(x)$ funksiyalar yordamida $L(0,\pm k)$ operatorlar kvadratik dastasining sochilish nazariyasi berilganlarini topamiz

$$\{R^\pm(0,k),\ k\in R\setminus\{0\},\ k_n^\pm(0),\ C_n^\pm(0),\ n=1,2,...,N^\pm\};$$

2. Teorema 3 ning natijalaridan foydalanib, $L(t,\pm k)$ operatorlar kvadratik dastasining sochilish nazriyasi berilganlariga ega bo‘lamiz

$$\{R^{\pm}(t,k),\ k\in R\setminus\{0\},\ k_n^{\pm}(t),\ C_n^{\pm}(t),\ n=1,2,...,N^{\pm}\};$$

3. Sochilish nazriyasi berilganlari yordamida $r^{\pm}(x,t)$ funksiyalarni tuzib olamiz

$$r^{\pm}(x,t)=\sum_n (C_n^{\mp}(t))^{-1}e^{-ik_n^{\mp}(t)x}-\frac{1}{2\pi}\int_{-\infty}^{\infty}R^{\pm}(k,t)e^{ikx}dk.$$

4. Topilgan $r^{\pm}(x,t)$ funksiyalarni quyidagi integral tenglamalar sistemasiga qo‘yamiz

$$h^{+}(x,t)F^{-}(x,t)=h^{-}(x,t)F^{+}(x,t),\ \ F^{+}(x,t)F^{-}(x,t)=1, \qquad (59)$$

$$A^{+}(x,y;t)=F^{-}(x,t)r^{+}(x+y;t)+\int_x^{\infty}r^{+}(y+s;t)A^{-}(x,s;t)ds, \qquad (60)$$

$$A^{-}(x,y;t)=F^{+}(x,t)r^{-}(x+y;t)+\int_x^{\infty}r^{-}(y+s;t)A^{+}(x,s;t)ds \qquad (61)$$

bunda

$$h^{\pm}(x,t)=(F^{\pm}(x,t))_{xx}-2(A^{\pm}(x,x;t))_x+2A^{\pm}(x,x;t)(F^{\pm}(x,t))'(F^{\pm}(x,t))^{-1}. \qquad (62)$$

5. Har bir fiksirlangan $x\in R$ da (59),(60) masala yagona yechimga ega bo‘lishini ta’minlash maqsadida quyidagi shartni kiritamiz:

$$a^{\pm}(y,t)=\int_x^{\infty}r^{\pm}(y+s,t)a^{\mp}(s,t)ds\Rightarrow(a^{+}(y,t),\ a^{-}(y,t))=(0,0),\ \ (y\geq x).$$

6. (60),(61) da quyidagi almashtirishni bajaramiz:

$$A^{\pm}(x,y,t)=F^{\mp}(x,t)\alpha^{\pm}(x,y,t)+F^{\pm}(x,t)\beta^{\mp}(x,y,t),\ y\geq x,\ x\in R.$$

7. Bu almashtirish natijasida biz $F^{\pm}(x,t)$ lar qatnashmaydigan $\alpha^{\pm}$ va $\beta^{\mp}$ larga nisbatan yangi integral tenglamalar sistemasiga ega bo‘lamiz. Bu integral tenglamalar sistemasini yechib $\alpha^{\pm}$ va $\beta^{\mp}$ larni topamiz.

8. (59) tenglikga (62) va $F^{\pm}(x,t)=\exp(\mp iz(x,t))$ ni qo‘yib $z=z(x,t)$ ga nisbatan quyidagi tenglamaga ega bo‘lamiz:

$$z_x=2i\alpha^{+}(x,x,t)e^{iz}-2i\alpha^{-}(x,x,t)e^{-iz}-2i\beta^{+}(x,x,t)+2i\beta^{-}(x,x,t),\ z(\infty)=0,$$

va uni yechib $z(x,t)$ ni topamiz.

9. Natijada (39)-(42) masalaning yechimi bo‘lgan $u(x,t)$ va $v(x,t)$ funksiyalar quyidagi tengliklar orqali topiladi:

$$u(x,t)=\mp 2i(F^{\pm}(x,t))'(F^{\pm}(x,t))^{-1}, \qquad (63)$$

$$v(x,t)=h^{\pm}(x,t)(F^{\pm}(x,t))^{-1}. \qquad (64)$$

Ikkinchi bobning uchinchi paragrafida quyidagi moslangan manbali Kaup-Boussinesq turidagi sistema

$$\begin{cases} v_t = v_{xxx} + 6uu_{xxx} + 18u_x u_{xx} - 6vv_x - 24vuu_x - 6v_x u^2 + \\ \quad + 2\sum_{m=1}^{N}\left[-u_x\varphi_m^2 + (k_m - 2u)\frac{\partial}{\partial x}\varphi_m^2\right], \\ u_t = u_{xxx} - 6vu_x - 6v_x u - 30u_x u^2 + \sum_{m=1}^{N}\frac{\partial}{\partial x}\varphi_m^2, \ x\in R, \ t>0, \\ \varphi_m'' + \left[k_m^2 - v - 2k_m u\right]\varphi_m = 0, \ m = 1,2,...,N \end{cases} \tag{65}$$

tez kamayuvchi funksiyalar sinfida ushbu

$$v(x,t)\big|_{t=0} = v_0(x), \ u(x,t)\big|_{t=0} = u_0(x), \ x\in R \tag{66}$$

boshlang‘ich va (41) normallovchi shartlarda integrallangan. Bunda $v_0(x)$ va $u_0(x)$ funksiyalar (42) shartni qanoatlantiradi.

Bu paragrafning asosiy natijasi quyidagi teoremadan iborat.

Teorema 4. Agar $v = v(x,t), \ u = u(x,t)$ va $\phi_1(x,t),\phi_2(x,t),...,\phi_{N^+}(x,t)$ *funksiyalar* (65),(66) *masalaning yechimi bo‘lsa, u holda*

$$L(t,\pm k)y \equiv -y_{xx} + v(x,t)y \pm 2ku(x,t)y = k^2 y, x\in R,$$

Shturm-Liuvill operatorlari kvadratik dastasining sochilish nazariyasi berilganlari quyidagi tenglamalarni qanoatlantiradi

$$\frac{dR^{\pm}(t,k)}{dt} = -8ik^3R^{\pm}(t,k), \ k\in R, \tag{67}$$

$$\frac{dk_n^{\pm}(t)}{dt} = 0, \ n = 1,2,...,N^{\pm}, \tag{68}$$

$$\frac{dC_n^{+}(t)}{dt} = [8i(k_n^{+})^3 + 2ik_n^{+}A_n(t)]C_n^{+}(t), \ \ n = 1,2,...,N^{+}, \tag{69}$$

$$\frac{dC_n^{-}(t)}{dt} = 8i(k_n^{-})^3C_n^{-}(t), \ \ n = 1,2,...,N^{-}. \tag{70}$$

Uchunchi bob **“Moslangan manbali diskret sinus-Gordon tenglamasini integrallash”** ga bag‘ishlangan.

Uchinchi bobning birinchi paragrafida diskret Dirak sistemasi turidagi sistema uchun qo‘yilgan sochilish nazariyasining to‘g‘ri va teskari masalalari haqida zaruriy ma‘lumotlar keltirilgan. Ushbu diskret Dirak sistemasi turidagi sistemani qaraymiz

$$\chi_{n+1} = L_n(z)\chi_n, n\in\mathbb{Z}, \tag{71}$$

$$L_n(z) = zP_n + \frac{1}{z}Q_n, z\neq 0, \tag{72}$$

bunda

$$P_n = \frac{1}{2}\begin{pmatrix} 1+\cos\theta_n & \sin\theta_n \\ \sin\theta_n & 1-\cos\theta_n \end{pmatrix}, \tag{73}$$

$$Q_n = \frac{1}{2}\begin{pmatrix} 1-\cos\theta_n & -\sin\theta_n \\ -\sin\theta_n & 1+\cos\theta_n \end{pmatrix}, \tag{74}$$

$$\theta_n = 0(mod\, 2\pi),\ |n| \to \infty. \tag{75}$$

Ushbu

$$\phi_n^+(z) \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} z^{-n},\ n \to -\infty, \tag{76}$$

$$\phi_n^-(z) \sim \begin{pmatrix} 0 \\ -1 \end{pmatrix} z^{n},\ n \to -\infty, \tag{77}$$

$$\psi_n^+(z) \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix} z^{-n},\ n \to \infty, \tag{78}$$

$$\psi_n^-(z) \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} z^{n},\ n \to \infty \tag{79}$$

asimptotikalarni qanoatlantiruvchi, birlik aylana $\Gamma_1 = \{z : |z| = 1\}$ da aniqlangan Yost yechimlarini $\phi_n^+(z)$, $\phi_n^-(z)$ va $\psi_n^+(z)$, $\psi_n^-(z)$ lar orqali belgilaymiz, ular uchun quyidagi yoyilmalar o‘rinli

$$\phi_n^+(z) = a^+(z)\psi_n^- + b^+(z)\psi_n^+(z), \tag{80}$$

$$\phi_n^-(z) = -a^-(z)\psi_n^+ + b^-(z)\psi_n^-(z). \tag{81}$$

Ushbu $a^-(z)$ ($a^+(z)$) funksiya Γ_1 birlik aylananing ichkarisiga (tashqarisiga) analitik davom qiladi va cheklita oddiy nollarga ega $\pm z_k^-, (\pm z_k^+), k = 1,...,N$ va quyidagi munosabatlar o‘rinli

$$R^+\left(\frac{1}{z}\right) = R^-(z) = R(z) = \frac{b^-(z)}{a^-(z)},\ z_k^+ = \frac{1}{z_k^-} = \frac{1}{z_k},$$

$$C_k^- = -\frac{C_k^+ z_k^-}{z_k^+} = C_k,\quad C_k^\pm = \frac{b^\pm(z_k^\pm)}{\left.\frac{da^\pm(z)}{dz}\right|_{z_k^\pm}} + \frac{b^\pm(-z_k^\pm)}{\left.\frac{da^\pm(z)}{dz}\right|_{-z_k^\pm}}.$$

Ta’rif 3. Ushbu $\{R(z),\ z_k,\ C_k,\ k = \overline{1,N}\}$ jamlanmaga (71) masalaning sochilish nazariyasi berilganlari deyiladi.

θ_n potensial sochilish nazariyasi berilganlari orqali yagona aniqlanadi.

Uchinchi bobning ikkinchi paragrafida, diskret sinus-Gordon tenglamalar sistemasi uchun manba qurish algoritmi keltirib chiqarilgan.

Uchinchi bobning uchinchi paragrafida quyidagi moslangan manbali diskret sinus-Gordon tenglamalar sistemasini

$$\dot{\theta}_{n+1} - \dot{\theta}_n = 2\left(\sin\theta_{n+1} + \sin\theta_n\right) + \sum_{k=1}^{N}\left(f_{1,n+1}^k f_{1,n}^k + f_{2,n+1}^k f_{2,n}^k \right), \; n \in \mathbb{Z}, \qquad (82)$$

$$L_n(z_k, t) f_n^k = f_{n+1}^k, \; n \in \mathbb{Z} \qquad (84)$$

ushbu

$$\theta_n(t)\big|_{t=0} = \theta_n^0, \; n \in \mathbb{Z} \qquad (83)$$

boshlang‘ich va quyidagi

$$\left(\sum_{i=-\infty}^{\infty} \left(f_i^k \right)^T \left(Q_i - P_i \right) \sigma_2 f_i^k \right) = \beta_k(t), \qquad (85)$$

$$\left(\sum_{i=-\infty}^{\infty} \left(\hat{f}_i^k \right)^T \left(z_k^2 Q_i - P_i \right) \sigma_2 \hat{f}_i^k \right) = \hat{\beta}_k(t) \qquad (86)$$

normallashtirish shartlari bilan qaraymiz. Bunda $\beta_k(t)$, $\hat{\beta}_k(t)$ lar oldindan berilgan uzluksiz skalyar funksiyalar, $\sigma_i\,(i = 1,2,3)$ lar Pauli matritsalari va $\hat{f}_n^k = \sigma_2 f_n^k$. θ_n^0 funksiya (75) shartni qanoatlantiradi.

Ushbu paragrafning asosiy natijasi quyidagi teoremadan iborat.

Teorema 5. Agar $\{\theta_n(t), f_n^k(t)\}$, $n \in \mathbb{Z}$, $k = 1,\dots,N$ funksiyalar to‘plami (82)-(86) masalaning yechimi bo‘lsa, u holda $L_n(z,t) = zP_n(t) + \frac{1}{z} Q_n(t)$ operatorning sochilish nazariyasi berilganlari quyidagi tenglamalarni qanoatlantiradi

$$\dot{R}(z,t) = 2\frac{z^2+1}{z^2-1} R(z,t), \; |z| = 1, \; z \neq \pm 1,$$

$$\dot{z}_k(t) = 0, \; k = 1,\dots,N,$$

$$\dot{C}_k(t) = \left(2\frac{z_k^2+1}{z_k^2-1} - \frac{z_k^2-1}{4z_k}\beta_k(t) - \frac{z_k^2-1}{2z_k(z_k^2+1)}\hat{\beta}_k(t) \right) C_k(t), \; k = 1,\dots,N.$$

Olingan natijalar (82)-(86) masalani teskari masalalar usulida yechish imkonini beradi.

Endi Teorema 5 ning natijasidan foydalanib (82)-(86) masalani yechish algoritmini keltiramiz.

Bizga $\theta_n^0, n \in \mathbb{Z}$ va $\beta_k(t)$, $\hat{\beta}_k(t)$, $k = 1,\dots,N$ funksiyalar berilgan bo‘lsin.

1. Berilgan $\theta_n^0, n \in \mathbb{Z}$ orqali to‘g‘ri masalani yechib $L_n(0)$ operator sochilish nazariyasining berilganlarini topamiz

$$\left\{ R(z,0), \; z_k(0), \; C_k(0), \; k = \overline{1,N} \right\}.$$

2. Teoremaning natijalaridan foydalanib $L_n(z,t)$ operatorning sochilish nazariyasining berilganlarini topamiz

$$\left\{ R(z,t), \; z_k(t), \; C_k(t), \; k = \overline{1,N} \right\}.$$

3. Topilgan sochilish nazariyasining berilganlari orqali ushbu $F^{\pm}(m,t)$ funksiyani tuzib olamiz

$$F^{\pm}(m,t)=\frac{1}{2\pi i}\oint_{\Gamma_1}R^{\pm}(z,t)z^{\mp 2m-1}dz\pm\sum_{k=1}^{N}C_k^{\pm}(t)(z_k^{\pm}(t))^{\mp 2m-1},$$

$$F^{-}(m,t)=F^{+}(m,t).$$

4. $F^{\pm}(m,t)$ funksiyalarni ushbu

$$K_2^{+}(n,m,t)-F^{-}(m,t)-\sum_{n'=n+1}^{\infty}K_1^{+}(n,n',t)F^{-}(m+n'-n,t)=0,(m>n),$$

$$K_1^{+}(n,m,t)+\sum_{n'=n+1}^{\infty}K_2^{+}(n,n',t)F^{-}(m+n'-n,t)=0,\ (m>n),$$

tenglamalar sistemasiga qo‘yamiz va uni yechib $K_1^{+}(n,m,t)$ va $K_2^{+}(n,m,t)$ larni topamiz.

5. $\theta_n(t)$ yechimni quyidagi tengliklardan aniqlaymiz

$$K_2^{+}(n,n+1,\,t)=-K_1^{-}(n,n+1,\,t)=tg\left\{\frac{1}{2}\left[\theta_n(t)-\theta_{n+1}(t)\right]\right\},$$

$$K_1^{+}(n,n+1,\,t)=K_2^{-}(n,n+1,\,t)=-\sum_{k=n}^{+\infty}\gamma(k+1,k,t)\gamma(k+2,k+1,t),$$

$$\gamma(j,n,t)=tg\left(\frac{1}{2}\left[\theta_j(t)-\theta_n(t)\right]\right).$$

6. Yost yechimi uchun ushbu

$$\psi_n^{\pm}(z)=z^{\mp n}\prod_{j=n}^{\infty}\alpha(j+1,\,j)\Omega(n)\times\sum_{j=0}^{\infty}K^{\pm}(n,n+j)z^{\mp 2j},$$

tasvirdan foydalanib $f_n^k(t)$ vektor-funksiyalarni tuzamiz.

Muallif o‘zining ilmiy rahbari, f.-m.f.d. Babajanov Bazar Atajanovichga doimiy e’tibori hamda mazkur dissertatsiya natijalarini muhokamasidagi qimmatli maslahatlari uchun samimiy minnatdorchiligini bildiradi.

XULOSA

Dissertatsiya ishining birinchi bobida vaqtga bog‘liq koeffitsiyentli hadlarga ega bo‘lgan Kaup-Boussinesq sistemasi va yuklangan hadli Kaup-Boussinesq turidagi sistemaga qo‘yilgan Koshi masalasi tez kamayuvchi funksiyalar sinfida Shturm-Liuvill operatorlari kvadratik dastasi uchun qo‘yilgan teskari masala usuli yordamida integrallangan.

Ikkinchi bobda moslangan manbali Kaup-Boussinesq tenglamalar sistemasiga qo‘yilgan Koshi masalasini sochilish nazariyasining teskari masalalar usulidan foydalanib yechish hamda moslangan manbali Kaup-Boussinesq turidagi tenglamalar sistemasining to‘la integrallanuvchanligini isbotlangan.

Dissertatsiya ishining uchinchi bobida diskret sinus-Gordon tenglamasi uchun moslangan manba qurilgan va moslangan manbali diskret sinus-Gordon tenglamasi “tez kamayuvchi” funksiyalar sinfida to‘la integrallanuvchanligini isbotlangan.

Tadqiqotning asosiy natijalari quyidagilardan iborat:

vaqtga bog‘liq qo‘shimcha hadlarga ega bo‘lgan Kaup-Boussinesq tenglamalar sistemasini yechish algoritmi keltirib chiqarilgan;

yuklangan manbali Kaup-Boussinesq tipidagi tenglamalar sistemasi uchun sochilish nazariyasi berilganlarining vaqt bo‘yicha evolyutsiyasi aniqlangan;

moslangan manbali Kaup-Boussinesq tenglamalar sistemasiga qo‘yilgan Koshi masalasi sochilish nazariyasining teskari masalasi usulidan foydalanib yechilgan;

moslangan manbali Kaup-Boussinesq tipidagi tenglamalar sistemasining to‘la integrallanuvchanligi isbotlangan;

diskret Dirak turidagi sistema uchun teskari spektral masala usulini qo‘llab diskret sinus-Gordon tenglamasi uchun manba qurish algoritmi keltirib chiqarilgan;

moslangan manbali diskret sinus-Gordon tenglamasiga qo‘yilgan Koshi masalasi diskret Dirak turidagi sistema uchun qo‘yilgan teskari spektral masala usulidan foydalangan holda yechilgan.

SCIENTIFIC COUNCIL AWARDING SCIENTIFIC DEGREE PhD.03/30.12.2019.FM.55.01 URGENCH STATE UNIVERSITY

URGENCH STATE UNIVERSITY

Azamatov Azizbek Shavkatovich

INTEGRATION OF THE SYSTEMS OF KAUP-BOUSSINESQ AND DISCRETE SINE-GORDON EQUATIONS WITH A SELF-CONSISTENT SOURCE IN THE CLASS OF RAPIDLY DECREASING FUNCTIONS

01.01.02 – Differential Equations and Mathematical Physics

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOFHY (PhD) ON PHYSICAL AND MATEMATICAL SCIENCES

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INTRODUCTION (abstract of the PhD thesis)

The actuality (relevance) and demand of the theme of the dissertation. Many scientific and practical researches, those carried out on a global scale, involve the integration of systems of nonlinear evolutionary equations and their discrete analogs, which have important practical applications in mathematical physics. In particular, the integration of analogues of Kaup-Bousinesq (KB) and discrete sine-Gordon (sG) systems of equations in the class of rapidly decreasing functions by the method of inverse problems and finding their explicit soliton-like solutions are one of the current issues.

Now days, the integration of nonlinear evolution equations with sources is widely researched in the world. Usually sourceless equations are model equations, derived under ideal conditions. There is a need to take into account additional effects in natural processes. In such processes, in the study of systems of nonlinear evolution equations, there is a need to include loaded terms and self-consistent sources into the equation. Therefore, as a mathematical model of these processes, nonlinear evolution equations with loaded term or self-consistent source are derived. Nowadays, solitons are one of the main objects of nonlinear wave dynamics. That’s why, the integration of KB and discrete sG systems of equations with self-consistent source, loaded terms in the class of rapidly decreasing functions and deriving algorithms for finding their soliton solutions are targeted scientific researches.

In our country, special attention is being paid to fundamental sciences with scientific and practical application, in particular, to the study of practical issues of nonlinear wave theory, direct and inverse spectral problems of scattering theory, and partial differential equations. As a result, significant results were achieved on the integration of the Cauchy problem for nonlinear evolution equations of mathematical physics using the method of direct and inverse spectral problems. Conducting scientific research at the level of international standarts in the field of mathematical physics and modern methods of this field was defined as the main task and direction. In order to ensure the execution of these decisions, it is important to research the integration of systems of nonlinear evolution equations with self-consistent source, loaded terms and their discrete analogues.

Conducting scientific research at the level of international standards in the priority directions of "Functional analysis, Algebra, Differential equations, Mathematical physics, Mathematical modeling, Computational mathematics and discrete mathematics, Probability theory and Mathematical statistics" is one of the main tasks at the activity of scientific and higher education institutions[2]. In order to

[2] Decree of the President of the Republic of Uzbekistan dated May 7, 2020 No. PP-4708 "On measures to improve the quality of education and the development of research in the field of mathematics"

ensure the execution of these resolutions, it is important to develop the integration of systems of nonlinear evolution equations and their discrete analogues with self-consistent source and loaded term.

The decrees of the President of the Republic of Uzbekistan No. PF-4947 dated February 7, 2017 "On the strategy of actions for the further development of the Republic of Uzbekistan", No. PF-60 dated January 28, 2022 "On the development strategy of New Uzbekistan for years 2022-2026", and the resolutions of the President of the Republic of Uzbekistan No. PQ-2789 dated February 17, 2017, "On measures to further improve the activities, organization, management and financing of scientific research works of the Academy of Sciences", No. PQ-2909 of April 20, 2017 "On measures to further develop the higher education system", No. PQ-3682 of April 27, 2018 "On measures to further improve the system of implementation of innovative ideas, technologies and projects", No. PQ-4708 dated May 7, 2020 "On measures to improve the quality of education and develop scientific research in the field of mathematics", and the implementation of the tasks outlined in other normative legal documents pertinent to this activity is helped to some extent by this dissertation research.

Connection of the research to the priority areas of the development of science and technology of the Republic. This study was carried out in accordance with the priority area of the development of science and technology in the Republic of Uzbekistan IV. "Mathematics, mechanics and computer science."

The degree of scrutiny of the problem. The most important results of the theory of inverse spectral problems for the quadratic pencil of the Sturm-Liouville operator are taken by M. Jaulent, I. Miodek, F.G. Maksudov, M.G. Gasymov, G.Sh. Gusienov, A.O. Smirnov, D. H. Sattinger, J.E Szmigielski, A. Laptev, R. Shterenberg, V. Sukhanov, A.B. Hasanov, A. Cabada, A.B. Yakhshimuratov and B.A. Babajanov and other scientists.

Using the method of inverse spectral problems of the scattering theory for the quadratic pencil of the Sturm-Liouville operator, D. J. Kaup showed the complete integrability of the system of equations

$$\begin{cases} \eta_\tau = \Phi_{xx} + \beta^2 \Phi_{xxxx} - \varepsilon \cdot (\Phi_x \eta)_x \\ \eta = \Phi_\tau + \dfrac{1}{2}\varepsilon \cdot \Phi_x^2, \end{cases}$$

in the class of rapidly decreasing functions, which describe the wave propagation in shallow water. Later, the algorithm for solving the Cauchy problem for the Kaup systems and its higher order analogs in the rapidly decreasing functions are derived by M. Jaulent and I. Miodek.

V.B.Matveev and M.I. Yavor obtained complex finite-gap multiphase solutions of this system, expressed in terms of the Rieman theta functions, found multisoliton solutions and investigated the asymptotic behavior of these solutions.

In the works of A.O. Smirnov, Y.A. Mitropolsky, N.N. Bogolyubov, A.K. Prikarpatsky and V.G. Somoylenko finite-zone real solutions of the Kaup system were studied. A system of Kaup equations with the self-consistent source in the class of periodic functions was integrated by A. Cabada and A.Yakhshimuratov the important results about the periodicity of the solution and the analyticity in terms of the variable x were obtained.

The integrability of the discrete sine-Gordon equation was shown by Hirota, who found the Lax pair, Backlund transformation, and N-soliton solutions for this equation. The discretization scheme, which is presented by Hirota, was generalized by Orfanidis. In the work of D. Levi, O. Ragnisco and M. Bruschi, soliton solutions of the discrete sine-Gordon equation were derived using the inverse problem method of the scattering theory, and the linear spectral problem related to this problem was studied in the work of L. Pilloni and D. Levi.

Relevance of the dissertation with the research works of higher education, where the dissertation is carried out. The dissertation has been executed according to the planned theme of the scientific research work of the “Applied mathematics and mathematical physics” department of the Urgench State University and in the frame of the scientific research project OT-F4-04 (05) “Applications of the spectral method in solving matrix nonlinear evolutionary equations. Biomechanics of the cardiovascular system” (2017-2020 yy).

The aims of research work are:

Integration of the Kaup-Boussinesq systems and discrete Sine-Gordon equations with a self-consistent source in the class of rapidly decreasing functions by the method of inverse problems of scattering theory. Finding the time evolution of the scattering data and developing an algorithm to solve the considering problems.

Research problems are:

deriving an algorithm for solving the system of KB equations with time-dependent coefficients;

integrating of the loaded KB type system;

deriving of a differential equation for the time scattering data of a quadratic pencil of Sturm-Liouville operators associated with the solution of the KB system with a source;

finding a solution to the Cauchy problem for a KB type system with a source;

deriving the source construction algorithm for the discrete sG equation;

find a solution to the Cauchy problem for a discrete sG equation with a source.

The research objects are the KB system with a source and its high-order analogues, discrete sG equation with a source, discrete Dirac type operator, quadratic pencil of Sturm-Liouville operators.

The subject of research consists of forward and inverse scattering problems for the quadratic pencil of the Sturm-Liouville operators and the discrete Dirac type operator and their application to the solving the Cauchy problem for a discrete sG equation and KB type system with a source.

Research methods. Methods of theory of integration of the nonlinear soliton equations, inverse problems of scattering theory, theory of integral equations, theory of ordinary and partial differential equations, functional analysis and functions of a complex variable are used.

Scientific novelty of research work consists of the followings:

derived the algorithm for solving the KB system with time dependence coefficients using the inverse problems of scattering theory for the quadratic pencil of the Sturm-Liouville operators;

constructed and integrated the loaded KB type system in the class of "rapidly decreasing" functions via inverse scattering theory of the quadratic pencil of the Sturm-Liouville operators;

derived linear ordinary differential equation for the time scattering data of a quadratic pencil of Sturm-Liouville operators associated with the solution of the KB system with a self-consistent source;

proved the complete integrability of the KB type system with a self-consistent source via inverse problems of scattering theory for the quadratic pencil of Sturm-Liouville operators;

by employing the inverse problems of scattering theory for a discrete system of the Dirac type, derived the algorithm for constructing a source for the discrete sG equation;

solved the Cauchy problem for a discrete sG equation with a source using the method of the inverse scattering theory of a discrete Dirac-type system.

The practical results of the research consist in the application of algorithms for solving the Cauchy problem for the system of KB equations with a self-consistent source, the system of KB equations with a loaded source, and the discrete sG equation with a self-consistent source.

Reliability of research results. Used methods at the integrating KB and discrete sG equations with source is based on inverse problems of scattering theory, theory of integration of the nonlinear soliton equations, theory of integral equations, theory of ordinary and partial differential equations as well as the rigor of mathematical considerations and proofs.

Scientific and practical significance of research results. The scientific significance of the research results is that the obtained scientific results can be used in studying the problems of plasma physics, physics of solid bodies, quantum optics, physics of elementary particles, theory of Josephson junctions, nonlinear excitations in condensed matter physics.

The practical significance of the dissertation is that the obtained scientific results can be used to solve various nonlinear evolutionary equations of mathematical physics in the class of rapidly decreasing functions.

Implementation of research results. The results obtained in the dissertation were used in the following areas:

the solving algorithm of the Kaup-Bousinesq and the discrete sG equation with a self-consistent source were used in the project 22-11-00196 "Matrix of monodromy and hierarchy of integrable nonlinear equations" of Saint Petersburg State University of Aerospace Instrumentation, supported by the Russian Science Fond (April 06, 2023) for constructing a hierarchy of Lax pairs with 2×2 matrix coefficients and allowed to integrate the hierarchy of the nonlinear evolution equations;

the algorithm construction of source for the discrete sG equation via inverse spectral problem for discrete Dirac type sistem was used for constructing the self-consistent source for the general Toda equation in the class of rapidly decreasing functions and for solving initial-value problem for the Korteweg-de Vries equation with loaded term and self-consistent source in the project OT-F4-04 (05) "Applications of the spectral method in solving matrix nonlinear evolutionary equations. Biomechanics of the cardiovascular system" (May 12, 2023) and allowed to integrate the matrix nonlinear evolution equations.

Approbation of the research results. The research results were discussed at 9 scientific and practical conferences, including 8 international conferences.

Publication of the research results. A total of 6 scientific works were published on the subject of the dissertation, 2 of them were published in international scientific journals included in the Scopus list, and 4 were published in national journals recommended by the Supreme Attestation Comission at the_Ministry of Higher education, science and innovations of the Republic of Uzbekistan for the defense of doctoral dissertations.

The structure and volume of the dissertation. The dissertation consists of an introduction, three chapters, a conclusion and a list of the used literatures. The volume of the dissertation is 88 pages.

BASIC CONTENT OF THE DISSERTATION

The first chapter concerns "Integration of the Kaup-Boussinesq system of equations with additional terms". In the first section of the first chapter, the well-known information about the Maksudov-Guseynov method of solving the inverse scattering problem for the quadratic pencil of Sturm-Liouville equations is presented.

We consider

$$L(k)y \equiv -y'' + v(x)y + 2ku(x)y - k^2 y = 0,\ x \in R, \tag{1}$$

where the functions $v(x)$ and $u(x)$ are real, moreover, $u(x)$ is absolutely continuous and the inequalities hold:

$$\int_{-\infty}^{\infty} |u(x)|\, dx < \infty,\ \int_{-\infty}^{\infty} (1+|x|)[|v(x)| + |u'(x)|]dx < \infty. \tag{2}$$

Under condition (2), Eq. (1) for all k from the half-plane $Imk \geq 0$ has solutions $f_+(x,k),\ f_-(x,k)$ satisfying asymptotics

$$f_+(x,k) = e^{ikx}[1+o(1)], x \to +\infty, \tag{3}$$

$$f_-(x,k) = e^{-ikx}[1+o(1)], x \to -\infty. \tag{4}$$

For real $k \neq 0$, the pairs $f_+(x,k),\ \overline{f}_+(x,k)$ and $f_-(x,k),\ \overline{f}_-(x,k)$ form two fundamental systems of solutions to equation (1). The following relations hold

$$f_+(x,k) = b(k)\, f_-(x,k) + a(k)\, \overline{f}_-(x,k), \tag{5}$$

$$f_-(x,k) = -\overline{b}(k)\, f_+(x,k) + a(k)\, \overline{f}_+(x,k), \tag{6}$$

$$a(k) = -\frac{1}{2ik} W\left\{ f_+(x,k),\, f_-(x,k)\right\}, \tag{7}$$

$$b(k) = \frac{1}{2ik} W\left\{ f_+(x,k),\, \overline{f}_-(x,k)\right\}. \tag{8}$$

The function $a(k)$ admits an analytic continuation to the half-plane $Imk > 0$ and can have at most a finite number of zeros $k_1, k_2, ..., k_N$, besides, at $k = k_n,\ n = 1,2,...,N$ the following equality holds

$$f_{\mp}(x,k_n) = B_n^{\pm} f_{\pm}(x,k_n), \tag{9}$$

where the quantities $B_n^{\pm}$ are independent of x.

Definition 1. The set of the quantities

$$\left\{ r_-(k) = \frac{b(k)}{a(k)},\ k \in R \setminus \{0\},\ k_1, k_2, ..., k_N,\ \gamma_1^-, \gamma_2^-, ..., \gamma_N^- \right\} \tag{10}$$

and

$$\left\{ r_+(k) = -\frac{\overline{b}(k)}{a(k)},\ k \in R \setminus \{0\}, k_1, k_2, ..., k_N,\ \gamma_1^+, \gamma_2^+, ..., \gamma_N^+ \right\} \tag{11}$$

are called the left and right scattering data of Eq. (1), respectively, here

$$\gamma_n^{\pm} = B_n^{\pm}\left(\frac{da(k)}{dk}\Big|_{k=k_n}\right)^{-1}, \; n = 1,2,...,N .$$

The problem of finding the coefficients $u(x)$ and $v(x)$ through the left or right scattering data is called the inverse problem for equation (1).

We now turn to the question of constructing $u(x)$ and $v(x)$ from scattering data (11).

We constructing the function $F_+(x)$ using the given right scattering data as follows

$$F_+(x) = -i\sum_{n=1}^{N}\gamma_n^+ e^{ik_n x} + \frac{1}{2\pi}\int_{-\infty}^{\infty} r_+(k)e^{ikx}dk. \tag{12}$$

Puting $F_+(x)$ into the following integral equations

$$F_+(x+y) + \overline{K_+^{(0)}(x,y)} + \int_x^{\infty} K_+^{(0)}(x,\tau)F_+(\tau+y)d\tau = 0, x \le y < \infty, \tag{13}$$

$$iF_+(x+y) + \overline{K_+^{(1)}(x,y)} + \int_x^{\infty} K_+^{(1)}(x,\tau)F_+(\tau+y)d\tau = 0, x \le y < \infty \tag{14}$$

and solving them we find $K_+^{(0)}(x,y)$ and $K_+^{(1)}(x,y)$. Using these, we construct the following function

$$K_+(x,y) = K_+^{(0)}(x,y)\cos\alpha_+(x) + K_+^{(1)}(x,y)\sin\alpha_+(x). \tag{15}$$

Here the function $\alpha_+(x)$ is the solution of the following Volterra integral equation

$$\alpha_+(x) = \int_x^{\infty}\Phi(s,\alpha_+(s))ds, \; -\infty < x < \infty, \tag{16}$$

where

$$\Phi(s,z) = \left[\mathrm{Re}\, K_+^{(0)}(s,s) - \mathrm{Im}\, K_+^{(1)}(s,s)\right]\sin 2z +$$

$$+2\left[\mathrm{Re}\, K_+^{(1)}(s,s)\right]\sin^2 z - 2\left[\mathrm{Im}\, K_+^{(0)}(s,s)\right]\cos^2 z. \tag{17}$$

Finally, the coefficients $u(x)$ and $v(x)$ are determined as follows

$$u(x) = -\alpha_+'(x), \tag{18}$$

$$v(x) = -u^2(x) - 2\frac{d}{dx}\left\{\left[\mathrm{Re}\, K_+(x,x)\right]\cos\alpha_+(x) + \left[\mathrm{Im}\, K_+(x,x)\right]\sin\alpha_+(x)\right\}. \tag{19}$$

In the second section of the first chapter**,** we consider the following Kaup-Boussinesq system with a time-dependent coefficients

$$\begin{cases} v_t - u_{xxx} + 4vu_x + 2uv_x = \mu(t)v_x, \\ u_t + 6uu_x + v_x = \mu(t)u_x \end{cases} \tag{20}$$

under initial condition

$$v(x,t)\big|_{t=0} = v_0(x), \; u(x,t)\big|_{t=0} = u_0(x), \; x \in R, \tag{21}$$

where $\mu(t)$ are given arbitrary continuous function and the functions $v_0(x)$, $u_0(x)$ are real and satisfy the following conditions:

i) $u_0(x)$ is absolutely continuous and the following inequalities hold:

$$\int_{-\infty}^{\infty} |u_0(x)|\,dx < \infty, \int_{-\infty}^{\infty} (1+|x|)[|v_0(x)|+|u_0'(x)|]dx < \infty; \qquad (22)$$

ii) the quadratic pencil of Sturm-Liouville operators

$$L(0,k)y \equiv -y'' + v_0(x)y + 2ku_0(x)y - k^2 y = 0, \ x \in R$$

has exactly $2N$ simple eigenvalues $k_1(0), k_2(0), ..., k_{2N}(0)$.

The main result of this section is included in the theorem below.

Theorem 1. *If the functions $v = v(x,t), u = u(x,t)$ are solutions to the problem (20)-(22), then the scattering data of the quadratic pencil of Sturm-Liouville operators*

$$L(t)y \equiv -y'' + v(x,t)y + 2ku(x,t)y - k^2 y = 0, \ x \in R$$

change on t as follows

$$\dot{r}_+(t,k) = (4ik^2 - 2ik\mu(t))r_+(t,k), \qquad (23)$$

$$\dot{k}_n(t) = 0, \ n = 1,2,...,N, \qquad (24)$$

$$\dot{\gamma}_n^+(t) = (2ik_n\mu(t) - 4ik_n^2)\gamma_n^+(t). \qquad (25)$$

The obtained results completely define the time evolution of the scattering data, which allows us to solve the problem (20)-(22).

Example. Let us illustrate the application of Theorem 1 for solving problem (20)-(22) for a given initial condition

$$v(x,t)\big|_{t=0} = -\frac{2}{ch^2 x}, \ u(x,t)\big|_{t=0} = 0, \ x \in R,$$

Let $\mu(t) = 2t$.

In this case, it is easy to find the scattering data for the operator $L(0)$:

$$r_+(k,0) = 0, \ k_1(0) = i, \ \gamma_1^+(0) = 2i.$$

By Theorem 1, we have

$$r_+(k,t) = 0, \ k_1(t) = i, \ \gamma_1^+(t) = 2ie^{4it-2t^2}.$$

Using the obtained scattering data, we uniquely determine the function $F_+(x,t)$ from the equality (12):

$$F_+(x,t) = 2e^{4it-2t^2-x}.$$

Substituting $F_+(x,t)$, into the integral equations (13), (14) and solving this system, we obtain

$$K_+^{(0)}(x,x;t) = \frac{-2e^{-2t^2}\cos 4te^{-2x} + 2e^{-4t^2}e^{-4x}}{1 - e^{-4t^2}e^{-4x}} + i\frac{2e^{-2t^2}\sin 4te^{-2x}}{1 - e^{-4t^2}e^{-4x}},$$

$$K_{+}^{(1)}(x,x;t)=\frac{2e^{-2t^2}\sin 4te^{-2x}}{1-e^{-4t^2}e^{-4x}}+i\frac{2e^{-4t^2}e^{-4x}+2e^{-2t^2}\cos 4te^{-2x}}{1-e^{-4t^2}e^{-4x}}.$$

Next, from (17) we find

$$\Phi(x,z;t)=-\frac{2\sin(2z+4t)}{sh(2x+2t^2)}.$$

Now, substituting $\Phi(x,z;t)$ into the Volterra integral equation (16) and solving it, we obtain

$$\alpha_{+}(x,t)=arctg\left(\frac{th^2(x+t^2)\cdot tg2t}{th^2(t^2)}\right)-2t.$$

Using the $\alpha_{+}(x,t)$, we construct the function $K_{+}(x,x,t)$ by the equality (15):

$$K_{+}(x,x,t)=\frac{-2e^{-2x}\cos(4t+\alpha_{+})+2e^{-4x}\cos\alpha_{+}}{1-e^{-4x}}+i\frac{2e^{-4x}\sin\alpha_{+}+2e^{-2x}\sin(4t+\alpha_{+})}{1-e^{-4x}},\ x\neq 0.$$

From (18) and (19) we find that:

$$u(x,t)=-\frac{sh2(x+t^2)\sin 4t}{ch^4(x+t^2)\cos^2 2t+sh^4(x+t^2)\sin^2 2t},$$

$$v(x,t)=-3u^2-\frac{4ch2(x+t^2)\cos 2(2t+\alpha_{+}(x,t))-4}{sh^2 2(x+t^2)}.$$

In the third section of the first chapter was integrated the loaded Kaup-Boussinesq type system in the class of rapidly decreasing functions.

We consider the following problem

$$\begin{cases} v_t-v_{xxx}-6uu_{xxx}-18u_xu_{xx}+6vv_x+24vuu_x+6v_xu^2=H(t)v_x, \\ u_t-u_{xxx}+6vu_x+6v_xu+30u_xu^2=H(t)u_x,\ x\in R,\ t>0 \end{cases} \quad (26)$$

under initial condition

$$v(x,t)\big|_{t=0}=v_0(x),\ \ u(x,t)\big|_{t=0}=u_0(x),\ \ x\in R, \quad (27)$$

where

$$H(t)=\mu(t)v(0,t)u(0,t).$$

Here $v_0(x),u_0(x)$ satisify the condition (22).

The main result of this section given by this theorem:

Theorem 2. *If the functions* $v=v(x,t)$ *and* $u=u(x,t)$ *is solution of* the problem (26)-(27), *then the scattering data of the quadratic pencil of Sturm-Liouville operators*

$$L(t,k)y\equiv -y''+v(x,t)y+2ku(x,t)y-k^2y=0,\ \ x\in R$$

change on t *as follows*

$$\dot{r}_+(t,k) = (8ik^3 - 4kH(t))r_+(t,k), \tag{28}$$

$$\dot{k}_n(t) = 0, \tag{29}$$

$$\dot{\gamma}_n^+(t) = (2ik_n H(t) - 8ik_n^3)\gamma_n^+(t). \tag{30}$$

The second chapter is called **"Integration of the Kaup-Boussinesq system with a self consistent source"**. In the first section of the second chapter, is given necessary information about the Jaulent-Jean method of the solving the inverse problem for the quadratic pencil of the Sturm-Liouville operator.

We consider equation

$$L(k)y = -y'' + (V - k^2)y = 0,\ x \in R, \tag{31}$$

where $V(x,k) = v(x) + 2ku(x)$, and $v(x)$, $u(x)$ are continuously differentiable complex valued functions and the following inequalities hold:

$$\int_{-\infty}^{+\infty} x^2 \left[|v(x)| + |u'(x)| \right] dx < \infty,\ \int_{-\infty}^{+\infty} |x| \left[|v'(x)| + |u''(x)| \right] dx < \infty. \tag{32}$$

Under condition (32), Eq. (31) for all $k \in R$ has Jost solutions $\{f_1(x,k), g_1(x,k)\}$ and $\{f_2(x,k), g_2(x,k)\}$ which satisfy the conditions

$$\left[f_1(x,k), g_1(x,k) \right] \sim \left[e^{-ikx}, e^{ikx} \right],\ x \to \infty, \tag{33}$$

$$\left[f_2(x,k), g_2(x,k) \right] \sim \left[e^{ikx}, e^{-ikx} \right],\ x \to -\infty. \tag{34}$$

For real $k \neq 0$, the pairs $\{f_1(x,k), g_1(x,k)\}$ and $\{f_2(x,k), g_2(x,k)\}$ form two fundamental systems of solutions to equation (31).

The following relations hold

$$f_2 = c_{11} f_1 + c_{12} g_1,\ \ g_2 = d_{12} f_1 + d_{11} g_1, \tag{35}$$

$$f_1 = c_{22} f_2 + c_{21} g_2,\ \ g_1 = d_{21} f_2 + d_{22} g_2, \tag{36}$$

$$c_{12} = c_{21} = (2ik)^{-1} W[f_1, f_2],\quad c_{11} = -d_{22} = (2ik)^{-1} W[f_2, g_1], \tag{37}$$

$$d_{12} = d_{21} = (2ik)^{-1} W[g_2, g_1],\quad d_{11} = -c_{22} = (2ik)^{-1} W[f_1, g_2], \tag{38}$$

where $c_{11}, c_{12}, c_{21}, c_{22}, d_{11}, d_{12}, d_{21}, d_{22}$ are independent on x. Moreover, the function $c_{21}(k)$ admits an analytic continuation to the half-plane $Imk < 0$. We can take the point of view that (31) is a pair of equations $(31)^{\pm}$ having potentials $V^{\pm}(x,k) = V(x, \pm k)$. Now all the above equations can be understood to have superscripts "$\pm$". $c_{21}^{\pm}(k)\ (Imk < 0)$ each have a finite number of zeros $N^{\pm}$, located at the points $k = k_n^{\pm},\ n = 1, 2, ..., N^{\pm}$.

Definition 2. The set of the quantities

$$\left\{ R^{\pm}(k) = \frac{c_{11}^{\pm}(-k)}{c_{21}^{\mp}(-k)},\ k \in R \setminus \{0\},\ k_n^{\pm},\ C_n^{\pm},\ n = 1, 2, ..., N^{\pm} \right\}$$

is called the scattering data of Eq. (31), where

$$C_n^{\pm} = [c_{11}^{\pm}(k_n^{\pm})]^{-1}\left[i\frac{d}{dk}c_{21}^{\pm}(k)\right]_{k=k_n^{\pm}}.$$

The coefficients $u(x)$ and $v(x)$ are uniquely recovered by the scattering data.

In the second section of the second chapter**,** we consider the following Kaup-Boussinesq system with a self-consistent source

$$\begin{cases} v_t = u_{xxx} - 4vu_x - 2uv_x + 2\sum_{m=1}^{N}\left[-u_x\varphi_m^2 + (k_m - 2u)\frac{\partial}{\partial x}\varphi_m^2\right], \\ u_t = -6uu_x - v_x + \sum_{m=1}^{N}\frac{\partial}{\partial x}\varphi_m^2,\ x\in R,\ t>0, \\ (\varphi_m)_{xx} + \left[k_m^2 - v - 2k_m u\right]\varphi_m = 0,\ m = 1,2,...,N \end{cases} \tag{39}$$

under the initial condition

$$v(x,t)\big|_{t=0} = v_0(x),\ u(x,t)\big|_{t=0} = u_0(x),\ x\in R \tag{40}$$

and the normalizing conditions

$$\int_{-\infty}^{+\infty}(2k_m - 2u)\varphi_m^2 dx = A_m(t),\ m = 1,2,...,N, \tag{41}$$

where $\varphi_1 = \varphi_1(x,t),\ \varphi_2 = \varphi_2(x,t),...,\varphi_N = \varphi_N(x,t)$ are eigenfunctions corresponding to the eigenvalues $k_1 = k_1(t),\ k_2 = k_2(t),...,k_N = k_N(t)$, $\operatorname{Im} k_m < 0,\ m = 1,2,...,N$ of the equation (31). Moreover, $A_1(t),\ A_2(t),...,\ A_N(t)$ are given arbitrary continuous functions and $v_0(x),\ u_0(x)$ are complex valued functions satisfying conditions:

1. $$\int_{-\infty}^{+\infty} x^2\left[|v_0(x)| + |u_0'(x)|\right] dx < \infty,\ \int_{-\infty}^{+\infty}|x|\left[|v_0'(x)| + |u_0''(x)|\right] dx < \infty, \tag{42}$$

2. The quadratic pencil of Sturm-Liouville operators

$$L(0,k)y \equiv -y'' + v_0(x)y + 2ku_0(x)y - k^2 y = 0,\ x\in R$$

has exactly N simple eigenvalues.

Now we rewrite system (39) in the vector form. For this we set

$$U = \begin{pmatrix} v \\ u \end{pmatrix},\ G = \begin{pmatrix} G_1 \\ G_2 \end{pmatrix}, \tag{43}$$

$$G_1 = 2\sum_{m=1}^{N}\left[-u_x\varphi_m^2 + (k_m - 2u)\frac{\partial}{\partial x}\varphi_m^2\right],\ G_2 = \sum_{m=1}^{N}\frac{\partial}{\partial x}\varphi_m^2, \tag{44}$$

$$T^* = \begin{pmatrix} 0 & -\frac{\partial^2}{\partial x^2} + 4v - 2v_x\int_x^{\infty} d\tau \\ 1 & 4u - 2u_x\int_x^{\infty} d\tau \end{pmatrix} \tag{45}$$

then after some calculations, (39) can be rewritten as follows

$$U_t + T^* U_x = G. \tag{46}$$

Now, we introduce the "scalar product" for the functions $V(x) = \begin{pmatrix} V_1(x) \\ V_2(x) \end{pmatrix}$ and $W(x) = \begin{pmatrix} W_1(x) \\ W_2(x) \end{pmatrix}$:

$$\langle V(x), W(x) \rangle = \int_{-\infty}^{+\infty} \left[V_1(x) W_1(x) + V_2(x) W_2(x) \right] dx.$$

Lemma 1. *If*

$$\Phi_1^{\pm}(x,k) = \begin{pmatrix} f_1^{\pm}(x,k)\, f_2^{\pm}(x,k) \\ \pm 2k f_1^{\pm}(x,k)\, f_2^{\pm}(x,k) \end{pmatrix}, \tag{47}$$

$$\Phi_2^{\pm}(x,k) = \begin{pmatrix} g_1^{\pm}(x,k)\, f_2^{\pm}(x,k) \\ \pm 2k g_1^{\pm}(x,k)\, f_2^{\pm}(x,k) \end{pmatrix}, \tag{48}$$

then, for all t the following equalities hold

$$2ik \frac{d}{dt} c_{21}^{\pm}(t,k) = \left\langle U_t(x,t), \Phi_1^{\pm}(x,t,k) \right\rangle, \ (\mathrm{Im}k \le 0, k \ne 0), \tag{49}$$

$$2ik \frac{d}{dt} c_{11}^{\pm}(t,k) = \left\langle U_t(x,t), \Phi_2^{\pm}(x,t,k) \right\rangle, \ k \in R \setminus \{0\}. \tag{50}$$

Lemma 2. *For all t the following equalities hold*

$$0 = \left\langle U_x(x,t), \Phi_1^{\pm}(x,t,k) \right\rangle, \ (\mathrm{Im}k \le 0, k \ne 0), \tag{51}$$

$$-4k^2 c_{11}^{\pm}(t,k) = \left\langle U_x(x,t), \Phi_2^{\pm}(x,t,k) \right\rangle, \qquad k \in R \setminus \{0\}, \ k = k_n^{\pm}. \tag{52}$$

Lemma 3. *Let the operator*

$$T = \begin{pmatrix} 0 & 1 \\ -\dfrac{\partial^2}{\partial x^2} + 4v - 2\displaystyle\int_{-\infty}^{x} v_\tau d\tau & 4u - 2\displaystyle\int_{-\infty}^{x} u_\tau d\tau \end{pmatrix}$$

be the "adjoint" operator of the operator (45) then, for a fixed t, the following equalities true

$$\left\langle U_x(x,t), T\Phi_1^{\pm}(x,t,k) \right\rangle = \pm 2k \left\langle U_x(x,t), \Phi_1^{\pm}(x,t,k) \right\rangle, \ Imk \le 0, \ k \ne 0, \tag{53}$$

$$\left\langle U_x(x,t), T\Phi_2^{\pm}(x,t,k) \right\rangle = \pm 2k \left\langle U_x(x,t), \Phi_2^{\pm}(x,t,k) \right\rangle, \ Imk \le 0, \ k \ne 0. \tag{54}$$

The main result of this section is included in the theorem below.

Theorem 3. *Let $v = v(x,t)$, $u = u(x,t)$ and $\varphi_1 = \varphi_1(x,t), \varphi_2 = \varphi_2(x,t), \ldots, \varphi_N = \varphi_N(x,t)$ be solution of (39)-(41), then the scattering data of*

$$L(t, \pm k) y \equiv -y_{xx} + v(x,t) y \pm 2k u(x,t) y = k^2 y, \ x \in R,$$

satisfy the following equations

$$\frac{dR^{\pm}(t,k)}{dt} = \mp 4ik^2 R^{\pm}(t,k),\ \ k \in R, \tag{55}$$

$$\frac{dk_n^{\pm}(t)}{dt} = 0,\ \ n = 1,2,...,N^{\pm}, \tag{56}$$

$$\frac{dC_n^{+}(t)}{dt} = -[4i(k_n^{+})^2 + 2ik_n^{+}A_n(t)]C_n^{+}(t),\ \ \ n = 1,2,...,N^{+}, \tag{57}$$

$$\frac{dC_n^{-}(t)}{dt} = 4i(k_n^{-})^2 C_n^{-}(t),\ \ n = 1,2,...,N^{-}. \tag{58}$$

The obtained results completely define the time evolution of the scattering data, which allows us to solve the problem (39)-(41).

We now turn to the question of constructing $u(x,t)$ and $v(x,t)$ from scattering data of Eq. (31).

Let us given $u_0(x)$, $v_0(x)$ and $A_m(t), m = 1,2,...,N^{+}$.

1. With the given $u_0(x)$ and $v_0(x)$, we find scattering data

$$\{R^{\pm}(0,k), k \in R\setminus\{0\},\ k_n^{\pm}(0),\ C_n^{\pm}(0),\ n = 1,2,...,N^{\pm}\}$$

for $L(0,\pm k)$;

2. According to the results of Theorem 3, we obtain the time evolution of the scattering data

$$\{R^{\pm}(t,k),\ k \in R\setminus\{0\},\ k_n^{\pm}(t),\ C_n^{\pm}(t),\ n = 1,2,...,N^{\pm}\}$$

for $L(t,\pm k)$;

3. With the obtained scattering data, we uniquely define the function $r^{\pm}(x,t)$ from the equality

$$r^{\pm}(x,t) = \sum_n \left(C_n^{\mp}(t)\right)^{-1} e^{-ik_n^{\mp}(t)x} - \frac{1}{2\pi}\int_{-\infty}^{\infty} R^{\pm}(k,t)e^{ikx}dk.$$

4. We put $r^{\pm}(x,t)$ into the following system of integral equations

$$h^{+}(x,t)F^{-}(x,t) = h^{-}(x,t)F^{+}(x,t),\ \ F^{+}(x,t)F^{-}(x,t) = 1, \tag{59}$$

$$A^{+}(x,y;t) = F^{-}(x,t)r^{+}(x+y;t) + \int_x^{\infty} r^{+}(y+s;t)A^{-}(x,s;t)ds, \tag{60}$$

$$A^{-}(x,y;t) = F^{+}(x,t)r^{-}(x+y;t) + \int_x^{\infty} r^{-}(y+s;t)A^{+}(x,s;t)ds, \tag{61}$$

where

$$h^{\pm}(x,t) = (F^{\pm}(x,t))_{xx} - 2(A^{\pm}(x,x;t))_x + 2A^{\pm}(x,x;t)(F^{\pm}(x,t))'(F^{\pm}(x,t))^{-1}. \tag{62}$$

5. We add the following assumption

$$a^{\pm}(y,t) = \int_x^{\infty} r^{\pm}(y+s,t)a^{\mp}(s,t)ds \Rightarrow (a^{+}(y,t),\ a^{-}(y,t)) = (0,0),\ \ (y \ge x),$$

for any $x \in R$. This assumption ensures the existence of a unique solution of the system (59)-(61).

6. In equations (60)-(61) we do the following replacement

$$A^{\pm}(x,y,t) = F^{\mp}(x,t)\alpha^{\pm}(x,y,t) + F^{\pm}(x,t)\beta^{\mp}(x,y,t),\ y \geq x,\ x \in R.$$

7. After this replacement we get new system of integral equations for $\alpha^{\pm}$ and $\beta^{\mp}$ which does not contain $F^{\pm}(x,t)$. Solving it we find $\alpha^{\pm}$ and $\beta^{\mp}$.

8. Substituting (62) and $F^{\pm}(x,t) = exp\left(\mp iz(x,t)\right)$ into equation (59), we get the following equation for $z = z(x,t)$:

$$z_x = 2i\alpha^{+}(x,x,t)e^{iz} - 2i\alpha^{-}(x,x,t)e^{-iz} - 2i\beta^{+}(x,x,t) + 2i\beta^{-}(x,x,t),\ z(\infty) = 0,$$

solving them we find $z(x,t)$.

9. After that, the solution of the equation (39)-(42) defined as follows

$$u(x,t) = \mp 2i(F^{\pm}(x,t))'(F^{\pm}(x,t))^{-1}, \tag{63}$$

and

$$v(x,t) = h^{\pm}(x,t)(F^{\pm}(x,t))^{-1}. \tag{64}$$

In the third section of the second chapter, is integrated the Kaup-Boussinesq type system with self consistent source in the class of rapidly decreasing functions.

We consider the following problem

$$\begin{cases} v_t = v_{xxx} + 6uu_{xxx} + 18u_x u_{xx} - 6vv_x - 24vuu_x - 6v_x u^2 + \\ \qquad + 2\sum_{m=1}^{N}\left[-u_x\varphi_m^2 + (k_m - 2u)\frac{\partial}{\partial x}\varphi_m^2\right], \\ u_t = u_{xxx} - 6vu_x - 6v_x u - 30u_x u^2 + \sum_{m=1}^{N}\frac{\partial}{\partial x}\varphi_m^2,\ x \in R,\ t > 0, \\ \varphi_m'' + \left[k_m^2 - v - 2k_m u\right]\varphi_m = 0,\ m = 1,2,...,N \end{cases} \tag{65}$$

$$v(x,t)\big|_{t=0} = v_0(x),\ u(x,t)\big|_{t=0} = u_0(x),\ x \in R \tag{66}$$

with normalizing conditions (41). Here the functions $v_0(x)$ and $u_0(x)$ satisfy the conditions (42).

The main result of this section is included in the theorem below.

Theorem 4. *Let* $v = v(x,t)$, $u = u(x,t)$ *and* $\varphi_1 = \varphi_1(x,t), \varphi_2 = \varphi_2(x,t),...,\varphi_N = \varphi_N(x,t)$ *be solution of (65)-(66), then the scattering data of*

$$L(t,\pm k)y \equiv -y_{xx} + v(x,t)y \pm 2ku(x,t)y = k^2 y,\ x \in R,$$

satisfy the following equations

$$\frac{dR^{\pm}(t,k)}{dt} = -8ik^3 R^{\pm}(t,k), \; k \in R, \tag{67}$$

$$\frac{dk_n^{\pm}(t)}{dt} = 0, \; n = 1,2,..., N^{\pm}, \tag{68}$$

$$\frac{dC_n^{+}(t)}{dt} = [8i(k_n^{+})^3 + 2ik_n^{+} A_n(t)] C_n^{+}(t), \;\; n = 1,2,..., N^{+}, \tag{69}$$

$$\frac{dC_n^{-}(t)}{dt} = 8i(k_n^{-})^3 C_n^{-}(t), \;\; n = 1,2,..., N^{-}. \tag{70}$$

The third chapter of the dissertation is devoted **"Integration of the discrete sine-Gordon equation with a self-consistent source"**. The first section of this chapter, we present the necessary information about the direct and inverse problems of scattering theory for the discrete Dirac type system

$$\chi_{n+1} = L_n(z)\chi_n, \;\; n \in \mathbb{Z}, \tag{71}$$

$$L_n(z) = zP_n + \frac{1}{z} Q_n, \; z \neq 0, \tag{72}$$

where

$$P_n = \frac{1}{2}\begin{pmatrix} 1+\cos\theta_n & \sin\theta_n \\ \sin\theta_n & 1-\cos\theta_n \end{pmatrix}, \tag{73}$$

$$Q_n = \frac{1}{2}\begin{pmatrix} 1-\cos\theta_n & -\sin\theta_n \\ -\sin\theta_n & 1+\cos\theta_n \end{pmatrix}, \tag{74}$$

$$\theta_n = 0(mod\, 2\pi), |n| \to \infty. \tag{75}$$

Denote by $\phi_n^{+}(z)$, $\phi_n^{-}(z)$ and $\psi_n^{+}(z)$, $\psi_n^{-}(z)$ the solutions of (71) defined for z on the unit circle $\Gamma_1 = \{z : |z| = 1\}$ satisfying the following asymptotics

$$\phi_n^{+}(z) \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} z^{-n}, \; n \to -\infty, \tag{76}$$

$$\phi_n^{-}(z) \sim \begin{pmatrix} 0 \\ -1 \end{pmatrix} z^{n}, \; n \to -\infty, \tag{77}$$

$$\psi_n^{+}(z) \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix} z^{-n}, \; n \to \infty, \tag{78}$$

$$\psi_n^{-}(z) \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} z^{n}, \; n \to \infty. \tag{79}$$

The following relations hold

$$\phi_n^{+}(z) = a^{+}(z)\psi_n^{-}(z) + b^{+}(z)\psi_n^{+}(z), \tag{80}$$

$$\phi_n^{-}(z) = -a^{-}(z)\psi_n^{+}(z) + b^{-}(z)\psi_n^{-}(z), \tag{81}$$

the function $a^{-}(z)(a^{+}(z))$ analytic extends to the inside (outside) of the unit circle Γ_1 and has a finite number of simple zeros $\pm z_k^{-},(\pm z_k^{+})(k=1,...,N)$ and following relations hold

$$R^{+}\left(\frac{1}{z}\right)=R^{-}(z)=R(z)=\frac{b^{-}(z)}{a^{-}(z)},\ z_k^{+}=\frac{1}{z_k^{-}}=\frac{1}{z_k},$$

$$C_k^{-}=-\frac{C_k^{+}z_k^{-}}{z_k^{+}}=C_k,\ \ C_k^{\pm}=\frac{b^{\pm}(z_k^{\pm})}{\left.\frac{da^{\pm}(z)}{dz}\right|_{z_k^{\pm}}}+\frac{b^{\pm}(-z_k^{\pm})}{\left.\frac{da^{\pm}(z)}{dz}\right|_{-z_k^{\pm}}}.$$

Definition 3. The set of $\{R(z),\ z_k,\ C_k,\ k=\overline{1,N}\}$ is called the scattering data of the problem (71).

The potential θ_n can be recovered by the spectral data in a unique way.

The second section of the third chapter is presented the source construction algorithm for the discrete sine-Gordon equation.

The third section of the third chapter**,** we consider the following discrete sine-Gordon equation with a self consistent source

$$\dot{\theta}_{n+1}-\dot{\theta}_n=2\left(\sin\theta_{n+1}+\sin\theta_n\right)+\sum_{k=1}^{N}\left(f_{1,n+1}^{k}f_{1,n}^{k}+f_{2,n+1}^{k}f_{2,n}^{k}\right),\ n\in\mathbb{Z}, \tag{82}$$

$$\left.\theta_n(t)\right|_{t=0}=\theta_n^0,\ n\in\mathbb{Z}, \tag{83}$$

$$L_n(z_k,t)f_n^k=f_{n+1}^k,\ n\in\mathbb{Z}. \tag{84}$$

$$\beta_k(t)=\left(\sum_{i=-\infty}^{\infty}\left(f_i^k\right)^T\left(Q_i-P_i\right)\sigma_2 f_i^k\right), \tag{85}$$

$$\hat{\beta}_k(t)=\left(\sum_{i=-\infty}^{\infty}\left(\hat{f}_i^k\right)^T\left(z_k^2Q_i-P_i\right)\sigma_2\hat{f}_i^k\right). \tag{86}$$

Here $\beta_k(t)$, $\hat{\beta}_k(t)$ are given scalar continuous functions, $\sigma_i(i=1,2,3)$ are the Pauli matrices of rank 2 and $\hat{f}_n^k=\sigma_2 f_n^k$. The function θ_n^0 satisfy the condition (75) .

The main result of this section is included in the theorem below:

Theorem 5. Let $\{\theta_n(t),f_n^k(t)\},\ n\in\mathbb{Z},\ k=1,...,N$ be solution to the problem (82)-(86). Then the scattering data of $L_n(z,t)=zP_n(t)+\frac{1}{z}Q_n(t)$ satisfy the following differential equations

$$\dot{R}(z,t)=2\frac{z^2+1}{z^2-1}R(z,t),\ |z|=1,\ z\neq\pm1$$

$$\dot{z}_k(t)=0,k=1,...,N,$$

$$\dot{C}_k(t)=\left(2\frac{z_k^2+1}{z_k^2-1}-\frac{z_k^2-1}{4z_k}\beta_k(t)-\frac{z_k^2-1}{2z_k(z_k^2+1)}\hat{\beta}_k(t)\right)C_k(t),\ \ k=1,...,N.$$

The obtained results completely define the time evolution of the spectral data, which allows us to solve the problem (82)–(86) by using the method of the inverse scattering problem.

Now we give an algorithm for solving the problem (82)–(86).

Let us given $\theta_n^0,\, n\in\mathbb{Z}$ and $\beta_k(t)$, $\hat{\beta}_k(t),\, k=\overline{1,N}$.

1. By solving the direct spectral problem with the given $\theta_n^0, n\in\mathbb{Z}$, we find scattering data

$$\left\{R(z,0), z_k(0), C_k(0),\ k=\overline{1,N}\right\} \text{ for } L_n(0);$$

2. According to the results of Theorem 5, we obtain the time evolution of the scattering data

$$\left\{R(z,t),\ z_k(t),\ C_k(t),\, k=\overline{1,N}\right\}$$

for $L_n(t)$;

3. With the obtained scattering data, we uniquely define the function $F^{\pm}(m,t)$ from the equality

$$F^{\pm}(m,t)=\frac{1}{2\pi i}\oint_{\Gamma_1}R^{\pm}(z,t)z^{\mp 2m-1}dz\pm\sum_{k=1}^{N}C_k^{\pm}(t)(z_k^{\pm}(t))^{\mp 2m-1},$$

$$F^{-}(m,t)=F^{+}(m,t);$$

4. Substituting $F^{\pm}(m,t)$ into the analogue of the Gelfand-Levitan-Marchenko "integral" equation

$$K_2^{+}(n,m,t)-F^{-}(m,t)-\sum_{n'=n+1}^{\infty}K_1^{+}(n,n',t)F^{-}(m+n'-n,t)=0,\ \ (m>n),$$

$$K_1^{+}(n,m,t)+\sum_{n'=n+1}^{\infty}K_2^{+}(n,n',t)F^{-}(m+n'-n,t)=0,(m>n),$$

and solving the system we define $K_1^{+}(n,m,t)$ and $K_2^{+}(n,m,t)$;

5. The solution $\theta_n(t)$ defined by the formulas

$$K_2^{+}(n,n+1,\,t)=-K_1^{-}(n,n+1,\,t)=tg\left\{\frac{1}{2}\left[\theta_n(t)-\theta_{n+1}(t)\right]\right\},$$

$$K_1^{+}(n,n+1,\,t)=K_2^{-}(n,n+1,\,t)=-\sum_{k=n}^{+\infty}\gamma(k+1,k,t)\gamma(k+2,k+1,t),$$

$$\gamma(j,n,t)=tg\left(\frac{1}{2}\left[\theta_j(t)-\theta_n(t)\right]\right).$$

6. Using the representations

$$\psi_n^{\pm}(z) = z^{\mp n}\prod_{j=n}^{\infty}\alpha(j+1, j)\Omega(n)\times\sum_{j=0}^{\infty}K^{\pm}(n, n+j)z^{\mp 2j},$$

for the Jost solutions, we construct the vector functions $f_n^k(t)$.

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CONCLUSIONS

The first chapter of the dissertation is devoted to the application of the inverse scattering theory for the quadratic pencil of Shturm-Liuville operator to the integration of the Kaup-Boussinesq system with time dependence coefficients and the loaded Kaup-Boussinesq type system in the class of the rapidly decreasing functions.

The second chapter concerned on the integration of the Kaup-Boussinesq system with a self-consistent source and the Kaup-Boussinesq type system with a self-consistent source in the class of the rapidly decreasing functions by the application of the inverse spectral problem for the quadratic pencil of Shturm-Liuville operator.

The third chapter of the dissertation we give algorithm for the source construction for the discrete sine-Gordon equation and proved the integrability of the discrete sine-Gordon equation with a self-consistent source in the class of "rapidly decreasing" functions.

The main results of the research are as follows:

the algorithm for solving the Kaup-Boussinesq system with time-dependent coefficients has been derived;

the loaded Kaup-Boussinesq type system has been integrated in the class of "rapidly decreasing" functions;

the time evolution equations of the scattering data for the Kaup-Boussinesq system with a self-consistent source have been determined in the class of "rapidly decreasing" functions;

the complete integrability of the Kaup-Boussinesq type system with a self-consistent source has been proven;

by employing the inverse problems of scattering theory for a discrete system of the Dirac type, derived the algorithm for constructing a source for the discrete sG equation;

solved the Cauchy problem for a discrete sG equation with a source using the method of the inverse scattering theory of a discrete Dirac-type system.

**НАУЧНЫЙ СОВЕТ PhD.03/30.12.2019.FM.55.01
ПО ПРИСУЖДЕНИЮ УЧЕНОЙ СТЕПЕНИ ПРИ УРГЕНЧСКОМ ГОСУДАРСТВЕННОМ УНИВЕРСИТЕТЕ**

УРГЕНЧСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

Азаматов Азизбек Шавкатович

ИНТЕГРИРОВАНИЕ СИСТЕМ УРАВНЕНИЙ КАУПА-БУССИНЕСКА И ДИСКРЕТНОГО УРАВНЕНИЯ СИНУС-ГОРДОНА С САМОСОГЛАСОВАННЫМ ИСТОЧНИКОМ В КЛАССЕ БЫСТРОУБЫВАЮЩИХ ФУНКЦИЙ

01.01.02 – Дифференциальные уравнения и математическая физика

**АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD)
ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ**

УРГЕНЧ – 2023

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при министерстве Высшего образования, науки и инноваций Республики Узбекистан за № B2022.3.PhD/FM746.

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Цель исследования:

Интегрирование системы Каупа-Буссинеска и дискретного уравнения синус-Гордон с самосогласованным источником в классе быстроубывающих функций методом обратных задач теории рассеяния. Нахождение эволюции данных рассеяния во времени и разработка алгоритма решения рассматриваемых задач.

Задачи исследования:

вывести алгоритм решения системы Каупа-Буссинеска с коэффициентами, зависящими от времени;

интегрировать нагруженную систему типа Каупа-Буссинеска;

вывести дифференциальные уравнения для данных рассеяния для квадратичного пучка операторов Штурма-Лиувилля, связанного с решением системы Каупа-Буссинеска с источником;

найти решение задачи Коши для системы типа Каупа-Буссинеска с источником;

вывести алгоритм построения источника для дискретного уравнения синус-Гордона;

найти решение задачи Коши для дискретного уравнения синус-Гордона с источником.

Научная новизна исследовательской работы состоит в следующем:

выведен алгоритм решения системы Каупа-Буссинеска с коэффициентами зависящими от времени с использованием обратных задач теории рассеяния для квадратичного пучка операторов Штурма-Лиувилля;

построена и интегрирована нагруженная система типа Каупа-Буссинеска в классе "быстроубывающих" функций методом обратных задач теории рассеяния для квадратичного пучка операторов Штурма-Лиувилля;

выведено линейное обыкновенное дифференциальное уравнение для данных рассеяния квадратичного пучка операторов Штурма-Лиувилля, связанных с решением системы Каупа-Буссинеска с самосогласованным источником;

доказана полная интегрируемость системы типа Каупа-Буссинеска с самосогласованным источником с помощью обратных задач теории рассеяния для квадратичного пучка операторов Штурма-Лиувилля;

исследована обратная задача теории рассеяния для дискретной системы типа Дирака и выведен алгоритм построения источника для дискретного уравнения синус-Гордона;

решена задача Коши для дискретного уравнения синус-Гордона с источником методом обратной задачи рассеяния для дискретной системы типа Дирака.

Внедрение результатов исследований. Полученные в диссертационной работе результаты были применены в следующих проектах:

алгоритм решения системы Каупа-Буссинеска и дискретного уравнения синус-Гордона с самосогласованным источником использованы в проекте 22-11-00196 «Матрица монодромии и иерархия интегрируемых нелинейных уравнений» Санкт-Петербургского государственного университета аэрокосмического приборостроения, поддержанного РНФ (справка от 06.04.2023) для построения иерархии пары Лакса с матричными коэффициентами 2×2. Применение научных результатов позволило интегрировать иеархии нелинейных эволюционных уравнении.

алгоритм построения источника для дискретного уравнения синус-Гордона был использован в научном проекте ОТ-F4-04 (05) «Применения спектрального метода в решении матричных нелинейных эволюционных уравнений. Биомеханика сердечно-сосудистой системы», позволившая построить самосогласованный источник для общего уравнения Тоды в классе быстроубывающих функций (справка от 12.05.2023). Применение научных результатов позволило интегрировать матричные нелинейные эволюционные уравнения.

Структура и объем диссертации. Диссертация состоит из введения, трех глав, заключения и списка использованной литературы. Объем диссертации 88 страниц.

E'LON QILINGAN ISHLAR RO'YXATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ
I bo'lim (Part I; Часть I)

1. Babajanov B.A., Babadjanova A.K., Azamatov A.Sh. Integration of the differential-difference sine-Gordon equation with a self-consistent source// Theoretical and Mathematical Physics, 2022. Vol. 210., Issue 3, pp. 327-336. (№3, Scopus, CiteScore 1.4)
2. Babajanov B.A., Azamatov A.Sh. Integration of the Kaup-Boussinesq system with a self-consistent source via inverse scattering method// Vestnik Udmurtskogo Universiteta. Matematika. Mekhanika. Komp'uternye nauki, 2022. Vol. 32. Issue 2, pp. 153-170. (№3, Scopus, CiteScore 1.0)
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5. Azamatov A.Sh. Интегрирование системы Каупа-Буссинеска методом обратной задачи рассеяния// Ilm Sarchashmalari, 2022, 11-son, Urganch, 23-28 betlar. (01.00.00, №12)
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II bo'lim (Part II; Часть II)

1. Babajanov B.A., Babadjanova A.K., Azamatov A.Sh. Integration of the discrete Sine-Gordon equation with a self-consistent source// Материалы международной научной конференции «Современные проблемы математики и физики», 12 – 15 сентября 2021 г., г. Стерлитамак, с. 267-271.
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4. Babajanov B.A., Babadjanova A.K., Azamatov A.Sh. Integration of the discrete sine-Gordon equation with a self-consistent source// Материалы международной научной конференции «Уфимская осенняя математическая школа – 2021», 6 - 9 октября 2021 г., г. Уфа, с. 28-30.

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