V.I.ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

O‘ZBEKISTON MILLIY UNIVERSITETI

HOMIDOV MUHRIDDIN KARIMJON O‘G‘LI

MAXSUSLIKKA EGA BO‘LGAN DINAMIK SISTEMALAR KUTISH VAQTLARI UCHUN LIMIT TEOREMALAR

01.01.05 – Ehtimollar nazariyasi va matematik statistika

FIZIKA-MATEMATIKA FANLARI bo‘yicha falsafa doktori (PhD) dissertatsiyasi AVTOREFERATI

TOSHKENT – 2023

UDK: 519.214

Fizika-matematika fanlari bo'yicha falsafa doktori (PhD)
Dissertatsiyasi avtoreferati mundarijasi

Оглавление автореферата диссертации
доктора философии (PhD) по физико-математическим наукам

Contents of dissertation abstract of doctor of philosophy (PhD) on
physical-mathematical sciences

V.I.ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

O‘ZBEKISTON MILLIY UNIVERSITETI

HOMIDOV MUHRIDDIN KARIMJON O‘G‘LI

MAXSUSLIKKA EGA BO‘LGAN DINAMIK SISTEMALAR KUTISH VAQTLARI UCHUN LIMIT TEOREMALAR

01.01.05 – Ehtimollar nazariyasi va matematik statistika

FIZIKA-MATEMATIKA FANLARI bo‘yicha falsafa doktori (PhD) dissertatsiyasi AVTOREFERATI

TOSHKENT – 2023

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O‘zbekiston Respublikasi Oliy ta’lim, Fan va Innovatsiyalar Vazirligi huzuridagi Oliy attestatsiya komissiyasida № B2022.3.PhD/FM752 raqam bilan ro‘yxatga olingan.

Dissertatsiya Mirzo Ulug‘bek nomidagi O‘zbekiston Milliy universitetida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o‘zbek, rus, ingliz (rezyume)) Ilmiy kengash vebsahifasi: (http://kengash.mathinst.uz) va “ZiyoNet” ta’lim axborot tarmog‘ida (http:www.ziyonet.uz) joylashtirilgan.

Ilmiy rahbar:	**Djalilov Ahtam Abduraxmanovich** Fizika-matematika fanlari doktori, professor
Rasmiy opponentlar:	**Miraxmedov Sherzod Adilovich** fizika-matematika fanlari doktori, professor
	Azimov Jahongir Bahromovich fizika-matematika fanlari nomzodi, dotsent
Yetakchi tashkilot:	**Namangan muhandislik-qurilish instituti**

Dissertatsiya himoyasi V.I.Romanovskiy nomidagi Matematika instituti huzuridagi DSc.02/30.12.2019.FM.86.01 raqamli Ilmiy kengashning 2023 yil “ ___ ”__________soat ____ dagi majlisida bo‘lib o‘tadi. (Manzil: 100174, Toshkent shahar., Olmazor tumani, Universitet ko‘chasi, 9-uy. Tel.: (+99871)-207-91-40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz).

Dissertatsiya bilan V.I.Romanovskiy nomidagi Matematika institutining Axborot-resurs markazida tanishish mumkin (_____-raqami bilan ro‘yxatga olingan). (Manzil: 100174, Toshkent shahar., Olmazor tumani, Universitet ko‘chasi, 9-uy. Tel.: (+99871)-207-91-40.

Dissertatsiya avtoreferati 2023-yil “ ___ ” __________ kuni tarqatildi.

(2023 yil “ ___ ” ______________dagi -raqamli reestr bayonnomasi).

A.Azamov
Ilmiy darajalar beruvchi
Ilmiy kengash rais o‘rinbosari,
f.-m.f.d., akademik

J.K.Adashev
Ilmiy darajalar beruvchi
Ilmiy kengash ilmiy kotibi,
f-m.f.d., katta ilmiy xodim

SH.K.Formanov
Ilmiy darajalar beruvchi
Ilmiy kengash huzuridagi
Ilmiy seminar raisi,
f.-m.f.d., akademik

KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahon miqyosida fundamental fanlar sohasida olib borilayotgan ko‘plab ilmiy-amaliy tadqiqotlar zamonaviy dinamik sistemalar xususan bir o‘lchamli dinamik sistemalarni tadqiq qilishga keltiriladi. Zamonaviy bir o‘lchamli diskret vaqtli dinamik sistemalar nazariyasida, maxsuslikka ega bo‘lgan aylana akslantirishlarini, ularning orbitalari yordamida hosil qilingan dinamik parchalanishlarning asimptotik holatini tadqiq qilish dolzarb masala hisoblanadi. Dastlab, XIX asr oxirida osmon mexanikasining muammolarini tadqiq qilish, aylana akslantirishlari va qaytish vaqti masalalari bilan bog‘liq ravishda tadqiq qilingan. Nochiziqli jarayonlarning ko‘plab masalalari va amaliyotning, jumladan, yurak kasalliklarining, elektr tarmoqlarining va tabiiy fanlarning matematik modellarini tadqiq qilish aylana akslantirishlari bilan uzviy bog‘liq.

Hozir kunda, dinamik sistemalarning muhim yo‘nalishlaridan biri bo‘lgan aylananing silliq akslantirishlarini, bo‘lakli silliq va kritik akslantirishlarini va nochiziqli jarayonlarni matematik modellashtirish ustida muhim izlanishlar olib borilmoqda. Shu kabi jarayonlar murakkab tuzilishga va xaotik xarakterga ega bo‘lgani uchun bu masalalarni hal etishda ehtimollar nazariyasining metodlarini qo‘llash salmoqli natijalar bermoqda. Bunda, olib borilayotgan ilmiy tadqiqotlar dinamik sistemalar orqali hosil qilingan tasodifiy miqdorlar ketma-ketliklari uchun limit teoremalar isbotlashning alohida ahamiyat kasb etishini ko‘rsatib bermoqda. Ushbu yo‘nalishda olib borilayotgan izlanishlar dinamik sistemalarning qaytish va kutish vaqtlari orqali hosil qilingan tasodifiy miqdorlar ketma-ketligining asimptotik holatlarini aniqlash kabi masalalar maqsadli ilmiy tadqiqotlardan hisoblanadi.

Mamlakatimizda matematikaning fundamental va ilmiy va amaliy tadbiqiga ega bo‘lgan amaliy matematika, informatika, raqamli iqtisodiyot, ehtimollar nazariyasi va dinamik sistemalar nazariyasiga keng e’tibor berilmoqda. Jumladan so‘nggi yillarda dinamik sistemalar va ehtimollar nazariyasi zamonaviy matematikaning muhim tashkil etuvchisi sifatida, amaliy va nazariy masalalarni yechishda ko‘plab natijalarga erishildi. Shu bilan birga, ehtimollar nazariyasi va matematik statistika fanining ustuvor yo‘nalishlari bo‘yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish asosiy vazifalar va faoliyat etib belgilandi[1]. Bu borada maxsuslikka ega aylana akslantirishlarini tadqiq qilish nazariy va amaliy jihatdan alohida ahamiyatga egadir.

O‘zbekiston Respublikasi Prezidentining 2017 yil 7 fevraldagi “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha Harakatlar strategiyasi to‘g‘risida” gi PF-4947 Farmoni, 2019 yil 9 iyuldagi “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora tadbirlari to‘g‘risida”gi PQ-4387-sonli va 2020 yil 7 maydagi “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy

[1] O‘zbekiston Respublikasi Vazirlar mahkamasining 2017 yil 18 maydagi “O‘zbekiston Respublikasi Fanlar akademiyasining yangidan tashkil etilgan ilmiy tadqiqot muassasalari faoliyatini tashkil etish to‘g‘risida”gi 292-sonli qarori.

tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi PQ-4708 sonli Qarorlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishi ustuvor yo‘nalishlariga bog‘liqligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. “Matematika, mexanika va informatika” ustivor yo‘nalishi aylanasida bajarilgan.

Muammoning o‘rganilganlik darajasi. Ehtimollik o‘lchovlarini o‘zaro taqqoslash masalasi matematikaning ko‘plab sohalari xususan o‘lchovlar nazariyasi, ehtimollar nazariyasi va dinamik sistemalar uchun muhim hisoblanadi. Dinamik sistemalar nazariyasining kutish va qaytish vaqtlarining xatti-harakatlarini o‘rganish ergodiklik nazariyasining muhim muammosidir. Bu muammolar dinamik Borel-Kantelli masalasi bilan bevosita bog‘liq. Olimlar dinamik sistemalar uchun qaytish vaqtlari muammosining tezligini o‘rganishga katta qiziqish bildirmoqda.

O‘lchovni saqlovchi dinamik sistemalar uchun fundamental natijalar Anri Puankare tomonidan olingan. Keyinchalik nuqtaning o‘zi yotgan to‘plamga qaytish vaqtining tezligini baholash haqida juda ko‘plab maqolalar chiqarildi. Fundamental natijalar A. Puankare, M. Katz, A.D. Viyner, J. Ziv, D. Ornsteyn B. Veyss va boshqalar tomonidan olingan. Musbat entropiyali dinamik sistemalar uchun uchun qaytish va kutish vaqtilarining yaqinlashish masalasi yaxshi o‘rganilgan. Oxirgi muhim natijalar musbat entropiyali dinamik sistemalar uchun L. Barreyra va B. Saussol tomonidan olingan. Ma’lumki chiziqli irratsional burish $T_\rho x = x + \rho \mod 1$ akslantirishlarining entropiyasi nolga teng. D. Kim va B. Seo tomonidan chiziqli irratsional burish uchun $f_2(x) = 2x \mod 1$ xaotik akslantirish orqali hosil qilingan $\{Q_n\}$ parchalanishlar ketma-ketliklariga tushish vaqti tasodifiy miqdorlarining asimptotik holatini o‘rganilgan. Koelo va de Faria tomonidan chiziqli irratsional burish akslantirishi uchun x_0 nuqtaning renorm $V_n = [x_{q_{n-1}}, x_{q_n}]$ atroflariga tushish vaqti tasodifiy miqdorlarini yaqinlashish masalasini o‘rganilgan. Nochiziqli aylana gomeomorfizmlari va chiziqli irratsional burish orasidagi qo‘shma gomeomorfizmning silliqlik masalasi aylana akslantirishlari nazariyasining muhim masalalaridan biri hisoblanadi. Aylananning analitik diffeomorfizmlar sinfi dastlab V.I. Arnold tomonidan o‘rganilgan. Aylana diffeomorfizmlari va chiziqli irratsional burish akslantirishlari orasidagi qo‘shma gomeomorfizmning absolyut uzluksiz bo‘lishi M. Erman, Yu. Mozer, J. Yokkoz, X. Katsnelson, D. Ornsteyn, Ya. Sinai, K.Hanin va boshqalarning ishlarida isbotlangan. So‘nggi eng muhim natija X. Katsnelson va D. Ornsteyn tomonidan isbotlangan. Qo‘shma gomeomorfizm absolyut uzluksiz bo‘lganligi uchun aylana diffeomorfizmining invariant ehtimollik o‘lchovi Lebeg o‘lchoviga nisbatan absolyut uzluksiz bo‘ladi. Shu sababli tushish vaqti masalalari uchun chiziqli irratsional burish akslantirishlarida olingan natijalar aylana diffeomorfizmlarida ham o‘rinli bo‘ladi. Sinish tipidagi yoki kritik tipdagi maxsuslikka ega aylana akslantirishlari aylana diffeomorfizmlarining tabiiy umumlashmasidir. Aylananing sinish tipidagi maxsuslikka ega akslatirishlari M. Erman, I. Lious I. Koelo va A. Lopes ishlarida o‘rganilgan. Aylananing sinish tipidagi maxsuslikka ega akslatirishlarining invariant ehtimollik o‘lchovi A. Djalilov, K. Hanin, D. Mayer, I. Lious U. Safarov A. Teplinskiy va boshqalarning ishlarida o‘rganilgan. Kritik tipdagi maxsuslikka ega aylana akslantirishlari dastlab D. Yokkoz tomonidan o‘rganilgan va bu akslantirishlar chiziqli irratsional burishga topologik ekvivalent bo‘lishi isbotlangan. Keyinchalik Grachek va G. Sventek tomonidan kritik aylana akslantirishlarining invariant o‘lchovi Lebeg o‘lchoviga nisbatan singular bo‘lishi isbotlangan.

Dastlab, A. Djalilov tomonidan bitta nuqtada kritik maxsuslikka ega va burish soni (“oltin kesim”) ga teng bo‘lgan aylana akslantirishlari uchun kritik nuqtaning renorm $V_n = [x_{q_{n-1}}, x_{q_n}]$ atroflariga tushish vaqti tasodifiy miqdorlar ketma-ketligi normallashtirib Lebeg o‘lchoviga nisbatan taqsimot funksiyasi yaqinlashishi hamda hosil bo‘lgan limit funksiya ham taqsimot funksiya bo‘lishi, uzluksizligi qat’iy o‘suvchiligi va singular funksiya bo‘lishi isbotlangan. Huddi shu akslantirishlar uchun Sh. Ayupov va A. Jalilov tomonidan x_{cr} kritik nuqtaning $0<\theta<1$ parametrga bog‘liq bo‘lgan $[x_{cr}, c_n(\theta)]$ atroflariga tushish vaqti tasodifiy miqdorlari normallashtirib taqsimot bo‘yicha yaqinlashishi va limit funksiya ham taqsimot funksiya bo‘lishi uzluksizligi, qat’iy monotonligi hamda singular bo‘lishi isbotlangan. Yuqoridagi keltirilgan ko‘plab natijalarga qaramasdan tushish va qaytish vaqti tasodifiy miqdorlarining asimptotikasini o‘rganish masalasi maxsuslikka ega aylana akslantirishlari uchun to‘la o‘rganilmagan. Shu sababli bu sohadagi tadqiqotlar dolzarb hisoblanadi

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya mavzusi Mirzo Ulug‘bek nomidagi O‘zbekiston milliy universiteti ilmiy kengashida tasdiqlangan va Matematika fakulteti “Ehtimollar nazariyasi va matematik statistika” kafedrasining rejalashtirilgan mavzusiga muvofiq bajarilgan.

Tadqiqotning maqsadi chiziqli irratsional burish akslantirishlari uchun tushish va qaytish vaqtlari tasodifiy miqdorlarini tadqiq qilish, kritik nuqtaga ega aylana akslantirishlari uchun tushish vaqti tasodifiy miqdorlarining taqsimot funksiyalarini yaqinlashishi va limit funksiyani silliqlikka tekshirishdan iborat.

Tadqiqotning vazifalari:

dinamik va aralash parchalanishlar uchun tushish vaqti funksiyalarining asimptotik holatini aniqlash;

chiziqli irratsional burish akslantirishlari uchun qaytish vaqti funksiyalarining yaqinlashish masalasini o‘rganish;

umumlashgan dinamik parchalanishlar ketma-ketligi uchun torni ustida aniqlangan tushish vaqti tasodifiy miqdorlarining taqsimot funksiyalarining yaqinlashish masalasini o‘rganish;

bitta kritik nuqtaga ega aylana akslantirishlari uchun parametrga bog‘liq tushish vaqti tasodifiy miqdorlarining taqsimot bo‘yicha yaqinlashish masalasini o‘rganish va parametr bo‘yicha taqqoslash.

Tadqiqotning obyekti. Chiziqli irratsional burish, kritik tipdagi maxsuslikka ega aylana gomeomorfizmlari, tasodifiy miqdorlar ketma-ketligi, invariant ehtimollik o‘lchovlari, dinamik sistemaning entropiyasi.

Tadqiqotning predmeti. Ehtimollar nazariyasi, dinamik sistemalar nazariyasi, aylana gomeomorfizmlar nazariyasi.

Tadqiqotning usullari. Dissertatsiya ishida ehtimollar nazariyasi, ergodik nazariya, sonlar nazariyasi, dinamik sistemalar va matematik analiz usullari qoʻllanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

chiziqli irratsional burish akslantirishlari uchun qaytish vaqtlari orqali hosil qilingan tasodifiy miqdorlar ketma-ketligining bir ehtimollik bilan yaqinlashishi isbotlangan;

aylananing dinamik parchalanishlari uchun kutish vaqtlari orqali hosil qilingan tasodifiy miqdorlar ketma-ketligining bir ehtimollik bilan oʻzgarmas songa yaqinlashishi isbotlangan;

bitta kritik nuqtaga ega aylana akslantirishlari uchun tushish vaqtlari orqali hosil qilingan tasodifiy miqdorlar ketma-ketligining taqsimot boʻyicha yaqinlashishi va limit taqsimot funksiyaning singular boʻlishi isbotlangan;

bitta kritik nuqtaga ega aylana akslantirishlari orqali hosil qilingan limit taqsimot funksiyalarini parametrning turli qiymatlari boʻyicha taqqoslangan.

Tadqiqotning amaliy natijalari. Olingan natijalar nuqtaning boshlangʻich kichik atrofiga qaytib kelish tezligini aniqlash va Shennon entropiyasini hisoblash imkonini beradi. Hamda aylana akslantirishlarini modellashtirishda, kritik tipdagi maxsuslikka ega aylana gomeomorfizmlari uchun gipotezalarni tekshirishda va dasturlash yordamida invariant oʻlchovining sonli xarakteristikalarini hisoblashda foydalaniladi.

Tadqiqotning natijalarining ishonchliligi matematik fikrlash va isbotlarning qat’iyligi, ehtimollar nazariyasi, sonlar nazariyasi dinamik sistemalar nazariyasi, ergodiklik nazariyasi va matematik analiz usullaridan foydalanish orqali asoslanadi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqotning ilmiy ahamiyati olingan natijalar bir oʻlchamli dinamik sistemalarda qaytish nuqtaning dastlabki turgan atrofiga qaytish tezligini aniqlashda, shuningdek kritik aylana gomeomorfizmlari tushish vaqtlari orqali hosil qilingan singular ehtimollik oʻlchovlarining oʻzaro ekvivalent boʻlishi oʻlchovlarni oʻzaro tadqiq qilishda foydalaniladi.

Tadqiqotning amaliy ahamiyati olingan natijalaridan aylana akslantirishlarini modellashtirishda, axborot havfsizligidagi ma’lumotlarni tahlil qilishda foydalanish mumkinligi bilan izohlanadi.

Tadqiqot natijalarining joriy qilinishi. Maxsuslika ega boʻlgan dinamik sistemalar kutish vaqtlari uchun limit teoremalar boʻyicha olingan natijalar asosida:

kritik tipdagi aylana akslantirishlari uchun parametrga bogʻliq kutish vaqtlari taqsimot funksiyalarining limit taqsimotlari singular boʻlishi va ularning ekvivalentligi haqidagi teoremadan OT-F-4-42 raqamli “Yarim additiv τ -silliq va Radon funksionallar fazolarining kardinal va topologik xossalari” mavzusidagi fundamental loyihada ehtimollik oʻlchovlari fazolarining topologik xossalarini isbotlashda foydalanilgan. (Oʻzbekiston Milliy universitetining 2023 yil 4 fevraldagi № 04/11-550-sonli ma’lumotnomasi). Ilmiy natijaning qoʻllanilishi ehtimollik o'lchovlari fazolarida aniqlangan singulyar ehtimollik o'lchovlarini taqqoslash imkonini bergan;

bitta kritik nuqtaga ega aylana akslantirishlariga mos chiziqli transfer-operatorlarni spektri xossalaridan OT–F-4-03 raqamli “Uzluksiz hamda diskret vaqtli aniq dinamik sistemalar, qismiy integral operatorlar spektrlari” mavzusidagi fundamental loyihada diskret vaqtli dinamik sistemalarning integral operatorlari spektrining xossalarini isbotlashda foydalanilgan (Qarshi davlat universitetining 2023 yil 10 iyundagi 04/2165-son ma’lumotnomasi). Ilmiy natijaning qo'llanilishi diskret vaqtli aylana dinamik sistemalarining integral operator spektrlari mavjudligini isbotlash va bu spektrlarning absolyut qiymatini baholash imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqot natijalari 3 ta xalqaro va 4 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o'tkazilgan.

Tadqiqot natijalarining e’lon qilinganligi Dissertatsiya tadqiqot mavzusi bo'yicha jami 13 ta ilmiy ish chop etilgan, shulardan, O'zbekiston Respublikasi Oliy Attestatsiya komissiyasining falsafa doktorlik dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 6 ta, jumladan 2 tasi xorijiy va 4 tasi respublika jurnallarida, shuningdek, 7 ta ma’ruza tezislari ilmiy konferensiya materiallarida nashr etilgan.

Dissertatsiya tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, 10 paragraf, xulosa va foydalanilgan adabiyotlar ro'yxatidan tashkil topgan. Dissertatsiyaning umumiy hajmi 101 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Dissertatsiya kirish qism, uchta bob, xulosa va foydalanilgan adabiyotlar ro'yxatidan iborat.

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo'nalishlariga mosligi ko'rsatilgan, mavzu bo'yicha xorijiy ilmiy tadqiqotlar sharhi, muammoning o'rganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, obyekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo'yicha ma’lumotlar keltirilgan.

Dissertatsiyaning “**Aylana akslantirishlari va limit teoremalar**” nomli birinchi bobida aylana akslantirishlari nazariyasining boshlang'ich tushunchalari, undagi dinamik parchalanishlar, qaytish va tushish vaqtlari, dinamik sistemalarning entropiyasi va tasodifiy miqdorlarning yaqinlashish turlari haqida boshlang'ich ma’lumotlar keltirilgan.

Birinchi bobning birinchi paragrafida qoldiq algebrasiga tegishli hodisalar va Borel-Kantelli lemmasi keltirilgan.

1-ta’rif. Aytaylik, $(\Omega, \mathcal{F}, P)$ ehtimolliklar fazosida $\{X_n : n \in \mathbb{N}\}$ tasodifiy miqdorlar ketma-ketligi va X tasodifiy miqdor berilgan bo'lsin. Agar $\forall \varepsilon > 0$ uchun

$$\lim_{n\to\infty} P\left(\left|X_n - X\right| > \varepsilon\right) = 0$$

o‘rinli bo‘lsa, $\{X_n :\ n \in \mathbb{N}\}$ tasodifiy miqdorlar ketma-ketligi X tasodifiy miqdorga ehtimol bo‘yicha yaqinlashadi deyiladi.

Ehtimol bo‘yicha yaqinlashish $X_n \xrightarrow{p} X$ kabi belgilanadi. Funksional analiz kursida bu yaqinlashishni o‘lchov bo‘yicha yaqinlashish deb aytiladi.

2-ta’rif. Aytaylik, $(\Omega, \mathcal{F}, P)$ ehtimolliklar fazosida $\{X_n :\ n \in \mathbb{N}\}$ tasodifiy miqdorlar ketma-ketligi va X tasodifiy miqdor berilgan bo‘lsin. Agar

$$P\left(\omega :\ \lim_{n\to\infty} X_n = X\right) = 1$$

o‘rinli bo‘lsa, $\{X_n :\ n \in \mathbb{N}\}$ tasodifiy miqdorlar ketma-ketligi X tasodifiy miqdorga bir ehtimol bilan (deyarli muqarrar) yaqinlashadi deyiladi.

Ya’ni $X_n(\omega)$ ketma-ketlik $X(\omega)$ ga yaqinlashmaydigan ω elementar hodisalar to‘plamining ehtimoli nolga teng. Bir ehtimol bilan yaqinlashish $X_n \xrightarrow{1\ eht} X$ kabi belgilanadi. Bir ehtimol bilan yaqinlashish

$$\lim_{n\to\infty} P\left(\omega :\ \sup_{k\geq n}\left|X_k - X\right| \geq \varepsilon\right) = 0$$

munosabatga teng kuchli.

3-ta’rif. Aytaylik, $(\Omega, \mathcal{F}, P)$ ehtimolliklar fazosida $\{X_n :\ n \in \mathbb{N}\}$ tasodifiy miqdorlar ketma-ketligi va X tasodifiy miqdor berilgan bo‘lsin va ularning chekli $EX, EX_n,\ n \geq 1$ matematik kutilmalari mavjud bo‘lsin. Agar ixtiyoriy uzluksiz $f(x)$ funksiya uchun quyidagi

$$\lim_{n\to\infty} Ef\left(X_n\right) = Ef\left(X\right)$$

tenglik o‘rinli bo‘lsa, $\{X_n :\ n \in \mathbb{N}\}$ tasodifiy miqdorlar ketma-ketligi va X tasodifiy miqdorga taqsimot bo‘yicha yaqinlashadi deyiladi. Bunday yaqinlashish $X_n \xrightarrow{d} X$ kabi belgilanadi.

Yuqorida keltirilgan yaqinlashish turlari orasida quyidagi munosabatlar o’rinli:

$$\xi_n \xrightarrow{1} \xi \ \Rightarrow\ \xi_n \xrightarrow{P} \xi \Rightarrow \xi_n \xrightarrow{d} \xi .$$

Aytaylik, $(\Omega, \mathcal{F}, P)$ ehtimolliklar fazosida $\{X_n :\ n \in \mathbb{N}\}$ tasodifiy miqdorlar ketma-ketligi berilgan bo‘lsin. Quyidagi belgilashlarni kiritamiz:

$$\mathcal{F}_n^{\infty} := \sigma(X_n, X_{n+1}, ...) \ \text{ va } \ \mathcal{X} = \bigcap_{n=1}^{\infty} \mathcal{F}_n^{\infty}.$$

$\sigma-$algebralarning kesishmasi yana $\sigma-$algebra bo‘lganligi uchun $\mathcal{X}$ ham $\sigma-$algebra bo‘ladi va “qoldiq $\sigma-$algebra” deb ataladi. Ixtiyoriy $A \in \mathcal{X}$ hodisa har bir chekli n uchun $\{X_n :\ n \in \mathbb{N}\}$ tasodifiy miqdorlarga bog‘liq bo‘lmaydi.

Klassik Borel-Kantelli lemmalarini keltiramiz.

1-lemma. Aytaylik (Ω, F, P) ehtimolliklar fazosi va $\{A_n,\, n \geq 1\}$ unda aniqlangan hodisalar bo‘lsin. Agar $\{A_n,\, n \geq 1\}$ hodisalarning ehtimolliklaridan tuzilgan qator yaqinlashuvchi bo‘lsa, yani $\sum\limits_{n=1}^{\infty} P\left(A_n\right) < \infty,$ u holda

$$P\left(\limsup A_n\right) = 0$$

bo‘ladi.

2-lemma. Faraz qilaylik (Ω, F, P) ehtimolliklar fazosi va $\{A_n,\, n \geq 1\}$ bog‘liqsiz hodisalar ketma-ketligi bo‘lsin. Agar $\sum\limits_{n=1}^{\infty} P\left(A_n\right) = \infty,$ bo‘lsa, u holda

$$P\left(\limsup A_n\right) = 1$$

bo‘ladi.

Aytaylik, $S^1 = \mathbb{R}^1 / \mathbb{Z}^1 \cong [0,1)$ birlik aylana va $T: S^1 \to S^1$ unda aniqlangan yo‘nalishni saqlovchi aylana gomeomorfizmi berilgan bo‘lsan. Quyidagi shartlarni qanoatlantiruvchi $f: \mathbb{R}^1 \to \mathbb{R}^1$ funksiyani qaraymiz

- f funksiya $\mathbb{R}^1$ da qat’iy o‘suvchi va uzluksiz;
- Ixtiyoriy $x \in \mathbb{R}^1$ uchun $f(x+1) = f(x) + 1$.

Ushbu f funksiya orqali yo‘nalishni saqlovchi T_f aylana gomeomorfizmini quyidagicha aniqlash mumkin

$$T_f x = f(x) \bmod 1.$$

f funksiyaga T_f aylana gomeomorfizmning aniqlovchi funksiyasi deyiladi.

Eng sodda misol sifatida $f_\rho(x) = x + \rho$ funksiya orqali hosil qilingan $T_\rho x = x + \rho \bmod 1$ aylana gomeomorfizmini keltirishimiz mumkin. T_ρ aylana gomeomorfizmiga **chiziqli burish** deyiladi.

4-ta’rif. Aytaylik, $T: S^1 \to S^1$ aylana gomeomorfizmi berilgan bo‘lsin. Agar $T^{(i)}(x_0) \neq T^{(j)}(x_0), \quad 0 \leq i < j \leq k-1,\ k > 1$ va $T^{(k)} x_0 = x_0$ bo‘lsa, $x_0 \in S^1$ nuqta $k-$davriy nuqta deyiladi.

Ushbu $T x_0 = x_0$ tenglik bajarilsa, u holda x_0 nuqtaga T akslantirishning qo‘zg‘almas nuqtasi deyiladi.

A.Puankarening klassik teoremasini keltiramiz.

1-teorema (Puankare). Aytaylik, $T: S^1 \to S^1$ yo‘nalishni saqlovchi aylana gomeomorfizmi bo‘lib, f- uning aniqlovchi funsiyasi berilgan bo‘lsin. U holda ixtiyoriy $x \in \mathbb{R}^1$ uchun quyidagi

$$\lim_{n \to \infty} \frac{f^{(n)}(x)}{n} = \rho_T(f)$$

limit mavjud va uning qiymati $x \in \mathbb{R}^1$ nuqtaning tanlab olinishiga bog‘liq emas. $\rho_T(f)$ ratsional bo‘lishi uchun T davriy nuqtaga ega bo‘lishi zarur va yetarli.

Agar $f_1(x) = f_2(x) + k,\ k \in Z$ bo‘lsa, u holda

$$\rho_T(f_1) = \rho_T(f_2) + k$$

bo‘ladi.

5-ta’rif. Aytaylik, $T: S^1 \to S^1$ yo‘nalishni saqlovchi aylana gomeomorfizmi bo‘lib, f - uning aniqlovchi funsiyasi bo‘lsin.

$$\rho_T = \lim_{n\to\infty} \frac{f^{(n)}(x)}{n} \mod 1$$

soni T gomeomorfizmning burish soni deyiladi.

Har bir haqiqiy $x \in (0, 1)$ sonini yagona ravishda zanjir kasrga yoyish mumkin

$$x = \cfrac{1}{k_1 + \cfrac{1}{k_2 + \ldots}}, \qquad (1)$$

bu yerda $k_1, k_2, \ldots$ natural sonlar. (1) munosabatga x sonining **uzluksiz kasr**ga yoyilmasi deyiladi va $x = [k_1, k_2, \ldots]$ kabi belgilanadi. Uzluksiz kasr chekli bo‘lsa, u ratsional

$$\frac{p}{q} = [k_1, k_2, \ldots, k_n]$$

sonni ifodalaydi. Bu yerda p, q o‘zaro tub sonlar. Faraz qilaylik $x = [k_1, k_2, \ldots]$ va ushbu

$$\frac{p_n}{q_n} = [k_1, k_2, \ldots, k_n],\ n \geq 1$$

kasrlarga x **yaqinlashuvchi kasrlar**i deyiladi. Ushbu kasrlarning surati va maxraji uchun quyidagi rekkurent munosabatlar o‘rinli

$$p_0 = 0,\ p_1 = 1 \text{ va } p_{n+1} = k_{n+1} p_n + p_{n-1},\ n \geq 1,$$

$$q_0 = 1,\ q_1 = k_1 \text{ va } q_{n+1} = k_{n+1} q_n + q_{n-1},\ n \geq 1.$$

Bu yerda $q_n, n \geq 1$ sonlariga **birinchi qaytish vaqtilari** deyiladi.

Har bir haqiqiy $t \in \mathbb{R}^1$ soni uchun $\|t\|$ norma orqali eng yaqin butun songacha bo‘lgan masofani kiritamiz

$$\|t\| = \min_{n \in \mathbb{Z}^1} |t - n|.$$

6-ta’rif. Quyidagi

$$h = \sup\left\{b : \liminf_{n\to\infty} n^b \| n\theta \| = 0\right\}$$

shartni qanoatlantiruvchi θ irratsional soniga h- tipli irratsional soni deyiladi.

M_h orqali $[0,1]$ segmentga tegishli, h- tipli barcha irratsional sonlardan iborat to‘plamni belgilaymiz. $[0,1]$ segmentdagi 1- tipga tegishli barcha irratsional sonlar to‘plami M_1 ning Lebeg o‘lchovi birga teng. Shuni aytib o‘tish joizki, tipi ∞ bo‘lgan Liuvill sonlari ham mavjud.

Aytaylik, $T_\rho x = x + \rho \bmod 1$ irratsional burish berilgan bo‘lsin. Ixtiyoriy $x_0 \in S^1$ nuqta uchun $\Delta_0^{(n)}(x_0)$ orqali chetki nuqtalari x_0 va $x_{q_n} = T_f^{(q_n)} x_0$ bo‘lgan yarim ochiq intervalni belgilaymiz. Agar n toq son bo‘lsa, x_{q_n} nuqta x_0 nuqtaning chap tomonida joylashgan bo‘ladi va n juft bo‘lsa u holda o‘ng tomonida joylashgan bo‘ladi. $\Delta_i^{(n)}(x_0) = T_f^{(i)}\left(\Delta_0^{(n)}(x_0)\right)$, $i \ge 1$ deb belgilash kiritamiz. x_0 nuqtaning orbitalaridan tuzilgan $\left\{x_i = T_f^{(i)} x_0,\ 0 \le i < q_n + q_{n+1}\right\}$ to‘plam aylanani kesishmaydigan yarim intervallarga ajratadi: $\Delta_i^{(n)}(x_0),\ 0 \le i < q_{n+1}$, $\Delta_j^{(n+1)}(x_0),\ 0 \le j < q_n$. Hosil bo‘lgan parchalanishni $\Delta_n(x_0)$ orqali belgilaymiz. $\Delta_n(x_0)$ parchalanishga n**-chi dinamik parchalanish** deyiladi.

Aytaylik, $(\Omega, \mathcal{F}, \mu)$ ehtimolliklar fazosi va $T: \Omega \to \Omega$ o‘lchovni saqlovchi akslantirish bo‘lsin, ya’ni

$$\mu(T^{-1}A) = \mu(A), \forall A \in \mathcal{F}.$$

Faraz qiliylik $(\Omega, \mathcal{F}, \mu)$ ehtimolliklar fazosining T endomorfizmi va $A \in \mathcal{F}$ o‘lchovli to‘plam bo‘lsin. Agar $x \in A$ nuqta uchun kamida bitta $n > 0$ topilib $T^n x \in A$ bo‘lsa, u holda $x \in A$ nuqtaga **qaytuvchi nuqta** deyiladi.

$A \in \mathcal{F}$, $\mu(A) > 0$ uchun quyidagi $R_A : A \to \mathbb{N}$ funksiyani aniqlaymiz:

$$R_A(x) = \min\{j \ge 1 : T^j x \in A\}. \tag{2}$$

$R_A(\cdot)$ funksiyaga A to‘plamning **birinchi qaytish vaqti** funksiyasi deyiladi.

Aytaylik, $T_\rho x = x + \rho \bmod 1$ irratsional burish bo‘lsin. $[0,1)$ yarim ochiq intervalning $\alpha_n = \bigvee_{j=0}^{n-1} S^{-j}\alpha$ bilan aniqlanuvchi α_n parchalanishlar ketma-ketligini qaraymiz, bu yerda α – o‘lchovli parchalanish va $S:[0,1) \to [0,1)$ – endomorfizm.

Quyidagi $H_n(\alpha_n; x, y) : [0,1) \times [0,1) \to \mathbb{N}$ funksiyani aniqlaylik:

$$H_n(\alpha_n; x, y) = \min\{j : T_\rho^j y \in I_n(x)\},$$

bu yerda $I_n(x)$ – x nuqtani o‘z ichiga olgan α_n parchalanishning elementi. $H_n(\alpha_n; x, y)$ funksiya α_n parchalanishlarga nisbatan **kutish vaqti funksiyasi** deyiladi. $K_n(\cdot)$ orqali logarifmik normallangan kutish vaqti funksiyasini quyidagicha kiritamiz:

$$K_n(\alpha_n; x, y) = \frac{\log H_n(\alpha_n; x, y)}{n}.$$

Dissertatsiyaning “**Irratsional burishlar uchun logarifmik kutish vaqtlarining deyarli muqarrar yaqinlashishi**” nomli ikkinchi bobida irratsional burish uchun qaytish va kutish vaqtlari tadqiq qilinadi. Shuningdek, logarifmik normallangan qaytish va kutish vaqtlariga bog‘langan tasodifiy miqdorlar ketma-ketligi uchun limit teoremalar isbotlanadi.

$T_\rho x = x + \rho \bmod 1$ irratsional burish va $b_0 \in (0,1)$ bo‘lsin. $[0,1)$ yarim ochiq intervalning $\mathrm{B}_0 = \{[0,b_0)\,;\,[b_0,1)\}$ parchalanishni qaraylik. Endi quyidagi parchalanishlar ketma-ketligini aniqlaymiz:

$$\mathrm{B}_n := \mathrm{B}_n(b_0) = \bigvee_{k=0}^{n-1} T_\rho^k \mathrm{B}_0 \; n \geq 1.$$

2-teorema. Aytaylik, $\rho \in (0,1)$ son $\eta-$tipga tegishli irratsional son va $T_\rho x = x + \rho \bmod 1$ irratsional burish bo‘lsin. U holda $\mu-$Lebeg o‘lchoviga nisbatan deyarli barcha $x \in [0,1)$ uchun quyidagi munosabatlar o‘rinli:

$$\liminf_{n\to\infty} \frac{\log R_{B_n(x)}(x)}{\log n} = \frac{1}{\eta}, \quad \limsup_{n\to\infty} \frac{\log R_{B_n(x)}(x)}{\log n} = 1.$$

Bu yerda $B_n(x)$ interval x ni o‘z ichiga oluvchi B_n parchalanishning elementidir. Aytaylik D_n parchalanish $n-$dinamik parchalanish bo‘lsin. Bu parchalanishga nisbatan kutish vaqti funksiyasi va logarifmik normallangan kutish vaqti funksiyalarini aniqlaymiz:

$$H_n(\mathrm{D}_n; x, y) := \inf\{ j \geq 1 : T_\rho^j(y) \in \Delta^{(n)}(x) \},$$

$$K_n(\mathrm{D}_n; x, y) := \frac{\log H_n(\mathrm{D}_n; x, y)}{n}.$$

$n \to +\infty$ da $K_n(\mathrm{D}_n; x, y)$ tasodifiy miqdorning asimptotik holatini o‘rganish muhim masalalardan biridir.

3-teorema. Aytaylik, $T_\rho(x) = x + \rho \bmod 1$ aylanadagi irratsional burish bo‘lsin. U holda Lebeg o‘lchovi $\mu_1(M) = 1$ bo‘lgan shunday $M \subset [0,1)$ to‘plam mavjudki, agar $\rho \in M$ bo‘lsa, Deyarli barcha $(x, y) \in S^1 \times S^1$ quyidagi

$$\lim_{n\to\infty} K_n(\mathrm{D}_n; x, y) = \frac{\pi^2}{12 \log 2}$$

limit o‘rinli bo‘ladi.

Aytaylik $\rho \in [0,1)$ irratsional sonining uzluksiz kasrga yoyilmasi davriy bo‘lsin, ya’ni

$$\rho = [a_1, a_2, \ldots, a_m, k_1, k_2, \ldots, k_s, k_1, k_2, \ldots, k_s, \ldots,), s \geq 1.$$

Bunday yoyilmaga ega irratsional songa **kvadratik irratsional son** deyiladi.

Quyidagi belgilashlarni kiritamiz:

$$\rho_j^{-1} := [k_j, k_{j-1}, \ldots, k_1, k_s, k_{s-1}, \ldots, k_{j+1}, k_j, k_{j-1}, \ldots, k_1, k_s, k_{s-1}, \ldots, k_{j+1}, \ldots), \; 1 \leq j \leq s,$$

$$\bar{\rho} := \sqrt[s]{\rho_1 \cdot \rho_2 \cdot \ldots \cdot \rho_s}.$$

4-teorema. Aytaylik, T_ρ aylananing ρ burchakka irratsional burish bo‘lsin. Faraz qilaylik ρ – kvadratik irratsional son bo‘lib, uning uzluksiz kasrga yoyilmasi quyidagicha bo‘lsin:

$$\rho = [a_1, a_2, \ldots, a_m, k_1, k_2, \ldots, k_s, k_1, k_2, \ldots, k_s, \ldots), s \geq 1.$$

U holda deyarli barcha $(x, y) \in S^1 \times S^1$ quyidagi

$$\lim_{n\to\infty} \log K_n(\mathrm{D}_n; x, y) = \log \bar{\rho}.$$

limit o‘rinli bo‘ladi.

Endi $[0,1)$ yarim ochiq intervalning $\left[0, \frac{1}{2}\right)$ va $\left[\frac{1}{2}, 1\right)$ qismlarida mos ravishda Q_n^l va D_n^r parchalanishlarni qaraymiz va bu parchalanishni τ_n orqali belgilaymiz.

5-teorema. Aytaylik, T_ρ – birlik aylanada irratsional burish va ρ burish soni M_0 to‘plamga tegishli bo‘lsin. U holda shunday $A_1(\rho),\ A_2(\rho) \subset S^1 \times S^1$: $\mu_2(A_1) = \mu_2(A_2) = \frac{1}{2}$ to‘plamlar mavjudki quyidagi limit o‘rinli:

$$\lim_{n\to\infty} K_n(\tau_n; x, y) = \begin{cases} \log 2, & (x, y) \in A_1 \\ \dfrac{\pi^2}{12 \log 2}, & (x, y) \in A_2 \end{cases}$$

bu yerda $A_1(\rho) \subset \left[0, \frac{1}{2}\right) \times S^1$ va $A_2(\rho) \subset \left[\frac{1}{2}, 1\right) \times S^1$.

6-teorema. Aytaylik, T_ρ – birlik aylanada irratsional burish bo‘lsin. Faraz qilaylik ρ son kvadratik irratsional son bo‘lib uning uzluksiz kasrga yoyilmasi quyidagicha bo‘lsin:

$$\rho = [a_1, a_2, \ldots, a_m, k_1, k_2, ..., k_s, k_1, k_2, ..., k_s, ...].$$

U holda, shunday $A_1(\rho),\ A_2(\rho) \subset S^1 \times S^1$: $\mu_2(A_1) = \mu_2(A_2) = \frac{1}{2}$ to‘plamlar mavjud bo‘lib quyidagi limit o‘rinli bo‘ladi:

$$\lim_{n\to\infty} K_n(\tau_n; x, y) = \begin{cases} \log 2, & (x, y) \in A_1 \\ \log \bar{\rho}, & (x, y) \in A_2 \end{cases}$$

bu yerda $A_1(\rho) \subset \left[0, \frac{1}{2}\right) \times S^1$ va $A_2(\rho) \subset \left[\frac{1}{2}, 1\right) \times S^1$.

Dissertatsiyaning **“Aylana akslantirishlari uchun normallangan kutish vaqtilarining kuchsiz yaqinlashishi”** deb nomlangan uchinchi bobida chiziqli irratsional burishlar va aylananing bitta kritik nuqtaga ega akslantirishlari uchun tushish vaqtlari tadqiq qilinadi. Xususan, normallangan kutish vaqtlari tasodifiy miqdorlari uchun limit teoremalar isbotlanadi.

Quyidagi belgilashlarni kiritamiz:

n – juft bo‘lganda,

$$L_0^{(n)} := [0, c_n + (K+1)\Delta_{n+1} - \Delta_n);\ M_0^{(n)} := [c_n + (K+1)\Delta_{n+1} - \Delta_n, \Delta_{n+1});$$

$$R_0^{(n)} := [\Delta_{n+1}, c_n).$$

n – toq bo‘lganda,

$$L_0^{(n)} := [\Delta_n - (K+1)\Delta_{n+1}, c_n);\ M_0^{(n)} := [c_n - \Delta_{n+1}, \Delta_n - (K+1)\Delta_{n+1});$$

$$R_0^{(n)} := [0, c_n - \Delta_{n+1}),$$

bu yerda $K = \left[\dfrac{(1-\theta)\Delta_n}{\Delta_{n+1}}\right]$ va $\theta \in (0,1)$.

Har bir $n \geq 1$ uchun quyidagi yarim-ochiq intervallar birlashmasi aylanada quyidagi

$$G_n = \left\{L_0^{(n)}, L_1^{(n)}, ..., L_{q_n+(K+1)q_{n+1}-1}^{(n)}\right\} \bigcup \left\{M_0^{(n)}, M_1^{(n)}, ..., L_{q_n+(K+2)q_{n+1}-1}^{(n)}\right\} \bigcup \left\{R_0^{(n)}, R_1^{(n)}, ..., R_{q_{n+1}-1}^{(n)}\right\}$$

parchalanishni ifodalaydi. G_n parchalanish aylananing $n-$ **umumlashgan dinamik parchalanishi** deyiladi. G_n umulashgan dinamik parchalanishga nisbatan kutish vaqti funksiyasini qaraymiz va normallangan kutish vaqti funksiyasini quyidagicha aniqlaymiz:

$$\tilde{H}_n(G_n; x, y) = \frac{H_n(G_n; x, y)}{q_{n+m^*} + (K^* + 2) q_{n+m^*+1}},$$

bu yerda $m^* = \max\{m_1, m_2, m_3\}$.

7-teorema. Aytaylik, T_ρ – aylanadagi irratsional burish bo‘lsin. Faraz qilaylik $\rho = [k_1, k_2, ..., k_s, k_1, k_2, ..., k_s, ...]$ burish soni kvadratik irratsional son bo‘lsin. $\theta \in (\max\{\alpha_1, \alpha_2, ..., \alpha_s\}, 1)$ soni uchun $\{c_n = \theta\Delta_n\}_{n\geq 1}$ ketma-ketlikni qaraymiz. U holda $\tilde{H}_{ns+i}(\tau_{ns+i}; x, y)$ tasodifiy miqdorlar ketma-ketligining $\Phi_{ns+i,\theta}(t), 1 \leq i \leq s$ taqsimot funksiyalaridan iborat ketma-ketligi $[0,1]$ kesmada bo‘lakli chiziqli funksiyaga tekis yaqinlashadi.

Aytaylik, $(\xi(x), \eta(x))$ jufliklar quyidagi shartlarni qanoatlantirsin:

a) $0 < \xi(0) < 1$;

b) $\xi(0) = \eta(0) + 1$;

c) $\xi(\eta(0)) = \eta(\xi(0))$;

d) $\xi(\eta(0)) < 0, \xi^2(\eta(0)) < 0, ..., \xi^{k-1}(\eta(0)) < 0$;

e) $\xi^k(\eta(0)) > 0$;

f) $\xi'(0) = \eta'(0) = \xi''(0) = \eta''(0) = 0; \xi'''(0) \neq 0, \eta'''(0) \neq 0$;

g) $(\xi \circ \eta)'''(0) = (\eta \circ \xi)'''(0)$.

Bu jufliklar orqali hosil qilingan kritik aylana gomomorfizmlar fazosini X_{cr}^k bilan belgilaymiz.

Y_k orqali burish soni $\rho_k = \rho_k(T_{\xi,\eta}(x)) = [k, k, ..., k, ...] = \dfrac{-k + \sqrt{k^2 + 4}}{2}$ ga teng bo'lgan X_{cr}^k fazoning (ξ, η) juftliklardan iborat qism fazosini belgilaymiz.

8-teorema. Aytaylik, $T_k \in Y_k$ Har bir $\theta \in (0,1)$ uchun $[x_0, c_n)$ intervallarga mos kutish vaqti funksiyalari ketma-ketligi $\{\Phi_{\theta,n}(t), n \in \mathrm{N}\}$ qaraylik. U holda

1. har bir haqiqiy t son uchun $\lim\limits_{n\to+\infty} \Phi_{\theta,n}(t) = \Phi_\theta(t)$ limit mavjud va $\Phi_\theta(t) = 0, t \leq 0, \Phi_\theta(t) = 1, t \geq 1$;
2. $\Phi_\theta(t)$ taqsimot funksiya R^1 to'g'ri chiziqda uzluksiz bo'ladi.

9-teorema. Aytaylik, $T_k \in Y_k$. Har bir $\theta \in (0,1)$ uchun $[x_0, c_n)$ intervallarga mos kutish vaqti funksiyalari ketma-ketligi $\{\Phi_{\theta,n}(t), n \in \mathrm{N}\}$ qaraylik. $\Phi_\theta(t)$ taqsimot funksiya bu ketma-ketlikning limit funksiyasi bo'lsin. U holda

1. $\Phi_\theta(t)$ funksiya $[0,1]$ da qat'iy monoton funksiya;
2. $\Phi_\theta(t)$ funksiya $[0,1]$ da singulyar bo'ladi.

$[0,1)$ yarim ochiq intervalni quyidagi parchalanishini qaraymiz:

$$\Theta = \{I_{s,j} := \left[\rho_k^{2s} - j\rho_k^{2(s+1)}, \rho_k^{2s} - (j-1)\rho_k^{2(s+1)}\right), j = \overline{1..k}, s = \overline{0..\infty}\}.$$

Har bir $j \in \{1, 2, ..., k\}$ uchun $I_j = \bigcup\limits_{s=0}^{\infty} I_{s,j}$ birlashma bilan aniqlanuvchi $I_j \subset [0,1)$ qism intervallarni qaraylik. Bu qism to'plamlar o'zaro kesishmaydi va $\bigcup\limits_{j=1}^{k} I_j = [0,1)$. $\theta \in (0,1)$ son uchun $\Phi_\theta(t)$ taqsimot funksiya $[0,1]$ da ehtimollik o'lchovi hosil qiladi. $a, b \in [0,1]$ sonlar uchun quyidagicha o'lchov aniqlanadi

$$\mu_\theta([a,b]) = \Phi_\theta(b) - \Phi_\theta(a),$$

10-teorema. Aytaylik, $T_k \in Y_k$. Har bir $\theta \in (0,1)$ uchun $[x_0, c_n)$ intervallarga mos kutish vaqti funksiyalari ketma-ketligi $\{\Phi_{\theta,n}(t), n \in \mathrm{N}\}$ qaraylik. $\Phi_\theta(t)$ taqsimot funksiya bu ketma-ketlikning limit funksiyasi bo'lsin. U holda

- $\Phi_{\theta_1}(t) = \Phi_{\theta_2}(t), \forall t \in R^1$, agar $\dfrac{\theta_1}{\theta_2} = \rho_k^{2(s_1 - s_2)}, \theta_1 \in I_{s_1,j}, \theta_2 \in I_{s_2,j}$;

• $\Phi_{\theta_2}(t) \le \Phi_{\theta_1}(t), \forall t \in \mathrm{R}^1$, agar $\dfrac{\theta_1}{\theta_2} < \rho_k^{2(s_1-s_2)}, \theta_1 \in I_{s_1,j}, \theta_2 \in I_{s_2,j}$. Shuningdek, $\Phi_{\theta_1}(t_0) \ne \Phi_{\theta_2}(t_0)$ bo‘ladigan shunday $t_0 \in (0,1)$ mavjud.

11-teorema. Aytaylik, $T_k \in Y_k$. Har bir $\theta \in (0,1)$ uchun $[x_0, c_n)$ intervallarga mos kutish vaqti funksiyalari ketma-ketligi $\{\Phi_{\theta,n}(t), n \in \mathrm{N}\}$ qaraylik. $\Phi_\theta(t)$ taqsimot funksiya bu ketma-ketlikning limit funksiyasi bo‘lsin. Har bir $j \in \{1,2,..,k\}$ uchun agar $\theta_1, \theta_2 \in I_j$ bo‘lsa, u holda μ_{θ_1} va μ_{θ_2} ehtimollik o‘lchovlari ekvivalent bo‘ladi.

XULOSA

Dissertatsiya ishining asosiy maqsadi aylana akslantirishlari uchun kutish va qaytish vaqti tasodifiy miqdorlarining asimptotik holatini tadqiq qilishga bag‘ishlangan. Xususan aylana akslantirishlari tushish va qaytish vaqti tasodifiy miqdorlari uchun limit teoremlar isbotlashdan iborat.

Tadqiqotning asosiy natijalari quyidagilardan iborat:

1. chiziqli irratsional burish uchun logarifmik normallangan qaytish vaqtlari orqali hosil qilingan tasodifiy miqdorlar ketma-ketligi bir ehtimol bilan yaqinlashishi isbotlangan;
2. dinamik parchalanishlar uchun logarifmik normallangan tushish vaqtlari orqali hosil qilingan tasodifiy miqdorlar ketma-ketligi bir ehtimol bilan yaqinlashishi isbotlangan;
3. aylanadagi aralash parchalanishlari uchun logarifmik normallangan tushish vaqtlari orqali hosil qilingan tasodifiy miqdorlar ketma-ketligi bir ehtimol bilan yaqinlashishi isbotlangan;
4. bitta kritik nuqtaga ega bo‘lgan aylana akslantirishlari uchun invariant o‘lchov yordamida normallangan kutish vaqtlari orqali hosil qilingan tasodifiy miqdorlar taqsimot bo‘yicha yaqinlashishi isbotlangan;
5. kritik gomomorfizmning limit taqsimot funksiyasi singulyarligi isbotlangan;
6. bitta kritik nuqtaga ega bo‘lgan aylana gomomorfizmlarining limit taqsimot funksiyalari hosil qilgan ikkita ehtimollik o‘lchovlarining ekvivalent bo‘lishi isbotlangan.

SCIENTIFIC COUNCIL AWARDING OF THE SCIENTIFIC DEGREES DSc.02/30.12.2019.FM.86.01 INSTITUTE OF MATHEMATICS NAMED AFTER V.I.ROMANOVSKIY

NATIONAL UNIVERSITY OF UZBEKISTAN

KHOMIDOV MUKHRIDDIN KARIMJON UGLI

LIMIT THEOREMS FOR HITTING TIMES OF DYNAMICAL SYSTEMS WITH SINGULARITIES

01.01.05-Probability theory and mathematical statistics

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT-2023

The theme of dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Commission at the of Ministers of Higher education, Science and Innovations of the Republic of Uzbekistan under number B2022.3.PhD/FM752.

Dissertation has been prepared at National university of Uzbekistan named after Mirzo Ulugbek.

The abstract of the thesis is posted in three languages (Uzbek, English, Russian (summary)) on the website http://kengash.mathinst.uz and in the website of "ZiyoNet" Information and educational portal http://www.ziyonet.uz/.

Scientific supervisor:	**Dzhalilov Akhtam Abdurakhmanovich** Doctor of physical and mathematical sciences, Professor
Official opponents:	**Mirakhmedov Sherzod Adilovich** Doctor of physical and mathematical sciences, Professor **Azimov Jakhongir Bahromovich** Candidate of physical and mathematical sciences, Docent
Leading organization:	**Namangan engineering-construction institute**

Defense will take place " __ " _________ 2023 at _____ at the meeting of Scientific Council number DSc.02/30.12.2019.FM.86.01 at Institute of Mathematics named after V.I. Romanovskiy. (Address: University str. 9, Almazar area, Tashkent city, 100174, Uzbekistan, Ph.: (99871) 207-91-40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz)

Thesis is possible to review in Information-resource center at Institute of Mathematics named after V.I.Romanovskiy (is registered №_____). (Address: University str. 9, Almazar area, Tashkent city, 100174, Uzbekistan, Ph.: (99871)-207-91-40).

Abstract of the thesis sent out on "____" ____________ 2023 year

(Mailing report № ___ on "____" ____________ 2023 year)

A.Azamov
Deputy Chairman of Scientific
Council on award of scientific
degrees, D.F.-M.S., Academician

J.K.Adashev
Scientific secretary of Scientific
Council on award of scientific
degrees, D.F.-M.S., Senior researcher

Sh.K.Formanov
Chairman of Scientific seminar under
Scientific Council on award of scientific
degrees, D.F.-M.S., Academician

INTRODUCTION (abstract of PhD thesis)

Actuality and demand of the theme of the thesis. Many scientific and applied researches of fundamental science in the world have been devoted to the problems of modern dynamical systems, in particular one-dimensional dynamical systems. Actual researches of modern one-dimensional discrete-time dynamical systems are related to the study of circle maps with singularities and the asymptotic behaviour of dynamical partitions associated with their orbits. Circle homeomorphisms and return times problems were first studied at the end of the 19th century in connection with the problem of celestial mechanics. Many problems of nonlinear processes, abnormal rhythms in heart disease, random noise in information theory, and problems of natural sciences are closely related to circle homeomorphisms.

Scientific research is currently being conducted worldwide on the mathematical modeling of smooth circle maps, piecewise-smooth and critical circle maps, and nonlinear processes, all of which present actual challenges in the field of dynamic systems theory. Due to their complex and chaotic nature, these processes yield significant results when tackled with probability theory methods. Recent studies have emphasized the particular importance of investigating the asymptotic behaviour of sequences of random variables associated with dynamical systems featuring singularities. In this way, problems such as the study of the asymptotic behaviour of the sequence of random variables associated with the return and hitting times of dynamical systems require targeted scientific research.

In our country, special attention is given to applied mathematics, computer science, digital economy, probability theory, and dynamic systems. These fields have both fundamental and practical applications, extending beyond mathematics. Investigations on the international level in such important areas as the functional analysis, mathematical physics, theory of probability and theory of dynamical systems considered as the main task of fundamental research[1]. Circle homeomorphisms and the theory of dynamical systems are not only important for natural sciences but also find applications in economics, information theory, biology, the study of various heart diseases, blood tests, and more.

The subject and object of research of this dissertation are in line with tasks identified in the Decrees and Resolutions of the President of the Republic of Uzbekistan of February 7, 2017, PF-4947, “On the strategy of action for the further development of the Republic of Uzbekistan”, PQ-4387 dated July 9, 2019 “On state support for the further development of mathematics education and science, as well as measures to radically improve the activities of the Institute of Mathematics named after V.I. Romanovsky of the Academy of Sciences of the Republic of Uzbekistan”, PQ-4708 of May 7, 2020 “On measures to improve the quality of education and research in the field of mathematics” as well as in other regulations related to basic sciences.

---

[1] Decree of the Cabinet of Ministers of the Republic of Uzbekistan dated May 18, 2017 No. 292 “On measures to organize the activities of newly created scientific research institutions of the Academy of Sciences of the Republic of Uzbekistan”

Connection of research to priority directions of development of science and technologies of the Republic. This study was performed in accordance with the priority areas of science and technology of the Republic of Uzbekistan IV, “Mathematics, Mechanics and Computer Science”.

The degree of scrutiny of the problem. Problems related to equivalence problems of probability measures are important in many branches of mathematics, such as measure theory, probability theory, and dynamical systems. Studying the behaviour of the waiting and hitting times of dynamical systems is a significant problem in ergodic theory. These issues are closely related to dynamical Borel-Cantelli and shrinking target problems. Scientists are currently displaying significant interest in studying the speed of the return times problem for dynamical systems. The foundational results for measure-preserving systems were obtained in the seminal works of A. Poincaré. Further a great number of articles have been published about the speed with which a point returns to a set it started in. Fundamental results are obtained by A. Poincaré, M. Kac, A.D. Wyner, J. Ziv, D. Ornstein, B. Weiss et. al. Convergence problems for return and hitting times have been well studied for dynamical systems with positive entropy. The most recent significant results were proven by L. Barreira and B. Saussol for dynamical systems with positive entropy, and by D. Kim and B. Seo, Coelho and de Faria for linear irrational rotations $T_{\rho}x = x + \rho \mod 1$. D. Kim and B. Seo proved that for typical irrational numbers on $(0,1)$, the sequence of random variables associated with normalized hitting times, with respect to the partitions $\{Q_n, n \geq 1\}$ which obtained by the chaotic map $f_2(x) = 2x \mod 1$, converges to a constant number with probability one. Coelho and de Faria investigated the convergence problem of random variables associated with rescaled hitting times for a sequence of renormalized intervals. It has been proven that for Lebesgue almost every rotation number, the rescaled hitting times do not converge in law as the renormalized intervals tend to zero. Additionally, all possible limit laws under a subsequence of $V_n = [x_{q_{n-1}}, x_{q_n}]$ are obtained. The first results for the class of analytic circle diffeomorphisms were obtained in the works of V.I. Arnold. Subsequently, fundamental contributions on the smoothness of diffeomorphisms and irrational rotations were made by M. Herman, Y. Moser, J. Yoccoz, K. Katznelson, D. Ornstein, Y. G. Sinai, K.M. Khanin, and others. The most recent significant results were proven in the works of K. Katznelson and D. Ornstein. Since the conjugate homeomorphism is absolutely continuous, the invariant probability measure of the circle diffeomorphism is also absolutely continuous with respect to the Lebesgue measure. Therefore, the results obtained for the hitting time problem with irrational rotations also hold true for circle diffeomorphisms, as they are natural generalizations of such rotations. The study of piecewise-smooth homeomorphisms of the circle was initiated in the works of M. Erman, I. Lious, I. Coelho, and A. Lopez. Subsequently, the investigation of invariant measures for piecewise-smooth homeomorphisms of the circle was undertaken by A. Dzhalilov, K. Khanin, D. Mayer, I. Lious, U. Safarov, A. Teplinsky, and others.

Critical circle maps were first studied in the works of D. Yoccoz, and it was proven that these maps are topologically conjugate to linear irrational rotations. J.

Graczyk and G. Świątek later demonstrated that the invariant measure of critical circle maps is singular with respect to the Lebesgue measure. In his work, A. A. Dzhalilov investigated the problem of the convergence of rescaled random variables, denoted by $\overline{E}_n(x)$, of critical circle homeomorphisms with respect to the Lebesgue measure λ on the renormalization interval $V_n = [x_{q_{n-1}}, x_{q_n}]$. It is shown that for critical circle homeomorphisms with a single critical point and a golden mean rotation number, the distribution function of the random values $\overline{E}_n(x)$ on the renormalization interval $V_n = [x_{q_{n-1}}, x_{q_n}]$ converges to a singular distribution function as n tends to infinity. Sh. Ayupov and A. Zhalilov proved that for a critical circle homeomorphism with a single critical point and a golden mean rotation number, the distribution function of random values $\overline{E}_{n,\theta}(x)$ on the interval $[x_{cr}, c_n(\theta)]$, $0<\theta<1$ with respect to the Lebesgue measure λ converges to a singular distribution function as n tends to infinity.

It should be noted that, despite the many important results mentioned above, there are still some open problems, such as the asymptotic behaviour of random variables for the return and hitting times of circle maps with singularities. Therefore, research in this area remains relevant.

The connection of the theme of the thesis with the research plans of the higher education institute, where the research on the thesis is carried out.

The theme of the dissertation was approved by the Scientific Council of the National University of Uzbekistan named after Mirzo Ulugbek and was carried out in accordance with the planned theme of the Department of "Probability Theory and Mathematical Statistics" of the Faculty of Mathematics.

The aim of the research work is to prove the limit theorems for random variables associated by return and hittiing times of irrational rotations and to investigate the smoothness of the limit function.

Research problems:

to study the asymptotic behaviour of return time for linear irrational rotations;

to prove limit theorems for hitting times associated with dynamical partitions of the circle;

to investigate the distribution functions of hitting times for critical circle maps and examine their smoothness;

to compare parameter-dependent limit distribution functions of critical circle maps.

The research object: Linear irrational rotations, circle homeomorphisms with a single critical point, invariant probability measure, singular measure, absolute continuous measure, random variable, distribution function, limit theorem.

The research subject: probability theory, one-dimensional dynamical systems, Entropy, Theory of circle maps.

Research methods. In the work used the methods of probability theory, ergodic theory, dynamical systems and mathematical analysis.

Scientific novelty of the research work is as follows:

the convergence with probability one of the sequence of random variables associated with log normalized hitting times for both dynamical and mixed partitions has been proven;

the convergence with distribution of the sequence of random variables associated with hitting times for circle maps with a single critical point has been proven;

the singularity of distribution function of critical homeomorphism is proved;

two probability measures of circle homeomorphism with a single critical point corresponding to different values of parameter depending limit distribution functions are compared.

Practical results of the research:

The obtained results allow determining the speed with which a point returns to the initial small vicinity and enable the calculation of Shannon entropy. Additionally, they are utilized for modeling circle homeomorphisms with a critical singularity, testing hypotheses for such homeomorphisms, and calculating finite characteristics of the invariant measure.

The reliability of the results of the study. The results have been obtained using a combination of methods from probability theory, mathematical analysis, dynamical systems, and ergodic theory. The obtained results are mathematically well-founded and strongly proved.

Scientific and practical significance of the research results.

The scientific significance of the research results is that the obtained results are used to estimate the speed of return time to initial position and to calculate Shannon entropy.

The practical significance of the research is emphasized by the possibility of using the obtained results in modeling circle homeomorphisms and analyzing data security in information systems.

Implementation of the research results. The scientific results obtained during the research of dissertation are implemented in the following research projects:

the limit theorem of rescaled hitting time for critical circle homeomorphisms was used to compare probability measures in probability measures spaces in the fundamental project OT-F-4-42 “Cardinal and topological properties of spaces of semi-additive τ – smooth and Radon functionals” (Reference No. 04/11-550 of the National University of Uzbekistan dated February 4, 2023). This scientific result enabled the comparison of singular probability measures within probability measure spaces;

the results on the spectrum of linear transfer operators corresponding to circle maps were used to study asymptotic properties and integral spectra of discrete time dynamical systems in the fundamental project OT–F-4-03 on the topic “Continuous and discrete time exact dynamic systems, spectra of partial integral operators” (Reference No. 04/2165 of Karshi State University dated June 10, 2023). The application of the scientific result enabled the proof of the existence of spectra of integral operators in some dynamical systems and the estimation of the absolute value of these spectra.

Approbation of the research results. The main results of the research have been discussed at 3 international and 4 national scientific conferences.

Publications of the research results: On the topic of the dissertation 13 research papers have been published in the scientific journals, 6 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the PhD thesis; in addition 2 of them were published in international journals and 4 papers published in national mathematical journals.

The structure and volume of the thesis. The dissertation consists of an introduction, three chapters, conclusion and bibliography. The general volume of the thesis is 101 pages.

THE MAIN CONTENT OF THE THESIS

The **introduction** part of the thesis includes the actuality and the demand of the research, the relevance of the research to the priority areas of science and technology. We showed the degree of study of the problem, formulates goals and objectives, identifies the object and subject of research, sets out scientific novelty and practive the results of the research, the theoretical and practical significance of the results obtained is disclosed, information is given on the implementation of the research results, on the published works and on the structure of the dissertation.

The first chapter of the dissertation, titled "**Limit theorems and circle maps**" contains preliminary and necessary information for theory of circle maps, dynamical partitions associated with them, the return and hitting times, entropy of dynamical systems and convergence types of random variables.

In the first paragraph of the first chapter, tail algebra of events and Borel-Cantelli lemma are introduced.

Definition 1. The sequence of random variables $\{X_n : n \in \mathbb{N}\}$ converges in probability to the random variable X which we indicate by writing $(X_n \xrightarrow{p} X)$, if for every $\varepsilon > 0$

$$\lim_{n\to\infty} P(|X_n - X| > \varepsilon) = 0.$$

In analysis this is known as convergence in measure.

Definition 2. The sequence of random variables $\{X_n : n \in \mathbb{N}\}$ is called convergent with probability one (almost surely, almost everywhere) to the random variable X, if

$$P(\omega : X_n \to X) = 1, \ n \to \infty$$

i.e. if the set of sample points ω for which $X_n(\omega)$ does not converge to X has probability zero. This convergence is denoted by $X_n \xrightarrow{a.e.} X$.

Definition 3. The sequence of random variables $\{X_n : n \in \mathbb{N}\}$ is called convergent in mean of order $p, 0 < p < \infty$, to the random variable X if

$$\lim_{n\to\infty} E|X_n - X|^p = 0,$$

where EX is denoted by the expectation of random variable X.

In analysis this is known as convergence in L^p, and it is denoted by $X_n \xrightarrow{L^p} X$. The special case $p = 2$ it is called mean square convergence.

Relations between different types of convergence:

$$X_n \xrightarrow{a.e.} X \Rightarrow X_n \xrightarrow{p} X \Rightarrow X_n \xrightarrow{d} X,$$
$$X_n \xrightarrow{L^p} X \Rightarrow X_n \xrightarrow{p} X \Rightarrow X_n \xrightarrow{d} X.$$

Let $(\Omega, \mathcal{F}, P)$ be a probability space, and let $\{X_n : n \in \mathbb{N}\}$ be a sequence of random variables. Define

$$\mathcal{F}_n^{\infty} := \sigma(X_n, X_{n+1}, \ldots) \quad \mathcal{X} = \bigcap_{n=1}^{\infty} \mathcal{F}_n^{\infty}.$$

Then $\mathcal{X}$ is a $\sigma-$algebra, called the **tail $\sigma-$algebra** of $\{X_n : n \in \mathbb{N}\}$. It contains the events which depend only on the limiting behaviour of the sequence.

The classical Borel-Cantelli lemmas are important and useful for proving the laws of large numbers in strong form. Suppose that $\{A_n, n \geq 1\}$ is a sequence of events in a probability space $(\Omega, \mathcal{F}, P)$.

Now we formulate the classical Borel-Cantelli lemmas.

Lemma 1. Let $\{A_n, n \geq 1\}$ is a sequence of events in a probability space. If $\sum_{n=1}^{\infty} P(A_n) < \infty$, then $P(\limsup A_n) = 0$.

Lemma 2. Let $(\Omega, \mathcal{F}, P)$ be a probability space and let $\{A_n, n \geq 1\}$ be independent events in $\mathcal{F}$. If $\sum_{n=1}^{\infty} P(A_n) = \infty$, then $P(\limsup A_n) = 1$.

Consider the unit circle $S^1 = \mathbb{R}^1 / Z^1 ;\ [0,1)$ and an orientation preserving homeomorphism $T : S^1 \to S^1$. Recall that the positive direction on S^1 is direction from 0 to 1. By fixing the positive direction determine an order on the circle. We denote by $\prec$ the order's symbol .Consider the function $f : \mathbb{R}^1 \to \mathbb{R}^1$ with properties:

- $f(x)$ strictly increasing and continuous function on $\mathbb{R}^1$;
- $f(x+1) = f(x) + 1$ for all $x \in \mathbb{R}^1$.

The function $f(x)$ uniquely defines the orientation preserving circle homeomorphism T:

$$Tx = f(x) \bmod 1, \forall x \in S^1.$$

The function f is called the **lift function** of the homeomorphism T.

The most elementary example is given by a translation $T_\rho x = x + \rho \bmod 1$ (or, a rotation by $e^{2\pi i\rho}$ on the unit circle), which is represented by $f_\rho(x) = x + \rho$. Homeomorphism T_ρ is called **linear rotation**.

Definition 4. Let $T: S^1 \to S^1$ be a circle homeomorphism. The point $x_0 \in S^1$ is called the periodic point with period $k, k > 1$, if

$$T^i x_0 \neq T^j x_0, 0 \leq i < j < k, \text{and } T^k x_0 = x_0.$$

In the case, $Tx_0 = x_0$ the point x_0 is called **a fixed point** of the map T.

Next, we formulate the classical theorem of A. Poincaré.

Theorem 1. Let $T: S^1 \to S^1$ be an orientation preserving circle homeomorphism and f be any lift of the homeomorphism T. Then for any $x \in \mathrm{R}^1$ there exists a finite limit

$$\lim_{n\to\infty} \frac{f^n(x)}{n} = \rho_T(f).$$

The value of the limit does not depend on the choosing $x \in \mathrm{R}^1$. The number $\rho_T(f)$ is rational if and only if, when T has at least one periodic orbit. The number $\rho_T(f)$ depends on the lift function f of the homeomorphism T. However, if $f_1(x) = f_2(x) + k, k \in \mathrm{Z}^1$, then it is easy to see that $\rho_T(f_1) = \rho_T(f_2) + k$.

Defenition 5. Let T be an orientation preserving circle homeomorphism and f be its lift function. The number $\rho_T := \rho_T(f) \bmod 1$ is called the **rotation number** of the homeomorphism T.

Any real number $x \in (0,1)$ can be uniquely presented in the form:

$$x = \cfrac{1}{k_1 + \cfrac{1}{k_2 + \dots}}, \tag{1}$$

where $k_1, k_2, \dots$ are natural numbers. The expression (1) is said to be a **continued fraction** expansion for the number x and it is denoted as $x := [k_1, k_2, \dots]$. A finite continued fraction can be uniquely written as a rational number:

$$\frac{p}{q} = [k_1, k_2, \dots, k_n],$$

where p, q are natural numbers relatively prime to each other. The fractions

$$\frac{p_n}{q_n} = [k_1, k_2, \dots, k_n], n \geq 1,$$

are called **convergent** for the continued fraction $[k_1, k_2, \dots, k_n] =: x$. The numerators and denominators of the convergence we can find by following recurrent relations

$$\begin{cases} p_{n+1} = k_{n+1} p_n + p_{n-1}, & p_0 = 0, p_1 = 1, \\ q_{n+1} = k_{n+1} q_n + q_{n-1}, & q_0 = 1, q_1 = k_1. \end{cases}$$

The numbers q_n are called the **first return times** of x.

A norm $\|t\|$ of $t \in \mathrm{R}$ is defined as follows

$$\|t\| = \min_{n \in \mathrm{Z}} |t - n|,$$

i.e. the distance to the nearest integer.

Definition 6. An irrational number $\theta, 0 < \theta < 1,$ is called to be type $\eta,$ if

$$\eta = \sup\{\beta : \lim_{n\to\infty} \inf n^{\beta} \|n\theta\| = 0\}.$$

Let us denote by M_{η} the set of all irrational numbers of type $\eta,$ belonging to segment $[0,1]$. The set M_1 of all irrational numbers of type 1 has Lebesgue measure 1 and includes the set of irrational numbers of "bounded type" (i.e. the sequence of elements the continued fraction expansion is bounded), which is of measure 0. Notice, that there are the numbers of type ∞, called the Liouville numbers.

Let $T_{\rho}x = x + \rho \bmod 1$ be an irrational rotation of the circle. Then ρ can be uniquely represented as a continued fraction as

$$\rho := [k_1, k_2, ..., k_n, ...] := \cfrac{1}{k_1 + \cfrac{1}{k_2 + \cdots}}.$$

The forward infinite orbit of the point x_0 defined by

$$O^{+}(x_0) = \{x_n = T_{\rho}^{n} x_0 : n = 0, 1, 2, ...\}.$$

We define the sequence of dynamical partitions of T associated by orbit $O^{+}(x_0)$. Indeed, denote by $\Delta_0^{(n)} := \Delta_0^{(n)}(x_0)$ semi-closed interval of S^1 with the endpoints x_0 and $x_{q_n} = T_{\rho}^{q_n} x_0$. In the clockwise orientation of the circle, the point x_{q_n} lies on the left of x_0 for odd n, and on the right for even n. We set $\Delta_j^{(n)} = T_{\rho}^{j} \Delta_0^{(n)}, j > 0$. It is well known that the partition $\mathrm{D}_n := \mathrm{D}_n(x_0)$ of the circle S^1 appeared with mutually disjoint intervals, defined as,

$$\mathrm{D}_n = \{\Delta_i^{(n)}, 0 \le i < q_{n+1}\} \bigcup \{\Delta_j^{(n+1)}, 0 \le j < q_n\}.$$

The partition D_n is called the $n-$**th dynamical partition of the circle** associated by $x_{0.}$

Let $(\Omega, \mathrm{F}, \mu)$ be a probability space and $T : \Omega \to \Omega$ be a measure preserving transformation ($\mu(T^{-1}A) = \mu(A), \forall A \in \mathrm{F}$).Suppose T is an endomorphism of the probability space $(\Omega, \mathrm{F}, \mu)$ and $A \in \mathrm{F}$. Then the point $x \in A$ is said to be a **recurrence point** (in the set A) if $T^{n}x \in A$ for at least one $n > 0$.

Suppose T is an endomorphism of the probability space $(\Omega, \mathrm{F}, \mu)$. For $A \in \mathrm{F}, \mu(A) > 0,$ we define the function $R_A : A \to \mathrm{N}$:

$$R_A(x) = \min\{j \ge 1 : T^{j}x \in A\}. \qquad (2)$$

The function $R_A(\cdot)$ is called the **first return time** function to the subset of A.

Let $T_\rho x = x + \rho \bmod 1$ be an irrational rotation. Consider the sequence of partitions α_n of the interval $[0,1)$ which defined by $\alpha_n = \bigvee_{j=0}^{n-1} S^{-j}\alpha$ Where α is measurable partition of $[0,1)$ and $S:[0,1)\to[0,1)$ is an endomorphism.

We define the function $H_n(\alpha_n;x,y):[0,1)\times[0,1)\to \mathrm{N}$

$$H_n(\alpha_n;x,y) = \min\{j : T_\rho^j y \in I_n(x)\},$$

where $I_n(x)$ is an element of the partition α_n containing the point *x*. The function $H_n(\alpha_n;x,y)$ is called **hitting times function** with respect to partition α_n. Now we define the **log hitting times** $K_n(\cdot)$ as

$$K_n(\alpha_n;x,y) = \frac{\log H_n(\alpha_n;x,y)}{n}.$$

The second chapter of the dissertation, titled **"The almost everywhere convergence of log hitting times for irrational rotations"** is devoted to investigate return and hitting times for irrational rotations. It is proved the limit theorems for the sequence of random variables associated by log normilized return and hitting times.

Consider irrational rotation $T_\rho x = x + \rho \bmod 1$. Let $b_0 \in (0,1)$. We define the partition $\mathrm{B}_0 = \{[0,b_0);[b_0,1)\}$ of the unit interval $[0,1)$. Next define the sequence of partitions:

$$\mathrm{B}_n := \mathrm{B}_n(b_0) = \bigvee_{k=0}^{n-1} T_\rho^k \mathrm{B}_0 \; n \ge 1.$$

Theorem 2. Let $\rho \in (0,1)$ be an irrational number of type η and $T_\rho = x + \rho \bmod 1$ be an irrational rotation. For almost every $x \in [0,1)$ with respect to Lebesgue measure μ

$$\liminf_{n\to\infty} \frac{\log R_{B_n(x)}(x)}{\log n} = \frac{1}{\eta}, \quad \limsup_{n\to\infty} \frac{\log R_{B_n(x)}(x)}{\log n} = 1,$$

where $B_n(x)$ is the interval of B_n containing *x*.

Let D_n be n^{th} dynamical partition. Define the sequence of hitting times $H: S^1 \times S^1 \to \mathrm{N}$ and log hitting times $K_n(\cdot)$ functions with respect to dynamical partition as:

$$H_n(\mathrm{D}_n;x,y) := \inf\{j \ge 1 : T_\rho^j(y) \in \Delta^{(n)}(x)\},$$

$$K_n(\mathrm{D}_n;x,y) := \frac{\log H_n(\mathrm{D}_n;x,y)}{n}.$$

An important problem is to determine the asymptotic behaviour of the random variable $K_n(\mathrm{D}_n;x,y)$, as $n \to +\infty$.

Theorem 3. Let $T_\rho(x) = x + \rho \bmod 1, x \in S^1$ be an irrational rotation of circle S^1. There exists a subset of irrational numbers $M \subset [0,1)$ with a full Lebesgue measure i.e. $\mu_1(M) = 1$, such that if $\rho \in M$, then

$$\lim_{n\to\infty} K_n(\mathrm{D}_n; x, y) = \frac{\pi^2}{12\log 2},$$

almost everywhere in the Lebesgue measure μ_2 on $S^1 \times S^1$.

Let an irrational number $\rho \in [0,1)$ have an eventually periodic decomposition into a continued fraction i.e.

$$\rho = [a_1, a_2, \ldots, a_m, k_1, k_2, \ldots, k_s, k_1, k_2, \ldots, k_s, \ldots,), s \geq 1,$$

which is called **a quadratic irrational number**.
Define the numbers $\rho_1, \rho_2, \ldots, \rho_s$ and $\bar{\rho}$ as

$$\rho_j^{-1} := [k_j, k_{j-1}, \ldots, k_1, k_s, k_{s-1}, \ldots, k_{j+1}, k_j, k_{j-1}, \ldots, k_1, k_s, k_{s-1}, \ldots, k_{j+1}, \ldots),$$

$$1 \leq j \leq s,$$

$$\bar{\rho} := \sqrt[s]{\rho_1 \cdot \rho_2 \cdot \ldots \cdot \rho_s}.$$

Theorem 4. Let T_ρ be an irrational rotation of the circle through the angle ρ. Assume that ρ is a quadratic irrational number and

$$\rho = [a_1, a_2, \ldots, a_m, k_1, k_2, \ldots, k_s, k_1, k_2, \ldots, k_s, \ldots), s \geq 1.$$

Then

$$\lim_{n\to\infty} \log K_n(\mathrm{D}_n; x, y) = \log \bar{\rho},$$

almost everywhere w.r.t. Lebesgue measure μ_2 on $S^1 \times S^1$.

Combining the partitions Q_n^l and D_n^r of the segments $[0, \frac{1}{2})$ and $[\frac{1}{2}, 1)$, respectively, we obtain a partition τ_n of the circle S^1.

Theorem 5. Let T_ρ be an irrational rotation of the circle and the number ρ belongs to the subset of M_1. Then, there exist subsets $A_1(\rho), A_2(\rho) \subset S^1 \times S^1, \mu_2(A_1) = \mu_2(A_2) = \frac{1}{2}$, such that

$$\lim_{n\to\infty} K_n(\tau_n; x, y) = \begin{cases} \log 2, & (x,y) \in A_1, \\ \dfrac{\pi^2}{12\log 2}, & (x,y) \in A_2. \end{cases}$$

where $A_1(\rho) \subset \left[0, \frac{1}{2}\right) \times S^1$ and $A_2(\rho) \subset \left[\frac{1}{2}, 1\right) \times S^1$.

Theorem 6. Let T_ρ be an irrational rotation of the circle. Suppose ρ is an irrational number of quadratic type and its continued fraction expansion has the form $\rho=[a_1,a_2,\dots,a_m,k_1,k_2,\dots,k_s,k_1,k_2,\dots,k_s,\dots],s\geq 1$. Then, there are subsets $A_1(\rho),A_2(\rho)\subset S^1\times S^1$, $\mu_2(A_1)=\mu_2(A_2)=\frac{1}{2}$, such that

$$\lim_{n\to\infty}K_n(\tau_n;x,y)=\begin{cases}\log 2, & (x,y)\in A_1,\\ \log\bar{\rho}, & (x,y)\in A_2.\end{cases}$$

where $A_1(\rho)\subset\left[0,\frac{1}{2}\right)\times S^1$ and $A_2(\rho)\subset\left[\frac{1}{2},1\right)\times S^1$.

In the third chapter of the thesis, titled "**The weak convergence of rescaled hitting times for circle maps**" we investigate return and hitting times for irrational rotations and circle maps with a single singular point. Moreover, we prove the limit theorems for sequence of random variables associated by rescailed return and hitting times.

Let us denote the following notations:
In the case n is even,

$$L_0^{(n)}:=[0,c_n+(K+1)\Delta_{n+1}-\Delta_n);$$
$$M_0^{(n)}:=[c_n+(K+1)\Delta_{n+1}-\Delta_n,\Delta_{n+1});$$
$$R_0^{(n)}:=[\Delta_{n+1},c_n).$$

If n is odd,

$$L_0^{(n)}:=[\Delta_n-(K+1)\Delta_{n+1},c_n);$$
$$M_0^{(n)}:=[c_n-\Delta_{n+1},\Delta_n-(K+1)\Delta_{n+1});$$
$$R_0^{(n)}:=[0,c_n-\Delta_{n+1}),$$

where $K=\left[\frac{(1-\theta)\Delta_n}{\Delta_{n+1}}\right]$ and $\theta\in(0,1)$.

For every $n\geq 1$, the collection of semi-intervals

$$G_n=\left\{L_0^{(n)},L_1^{(n)},\dots,L_{q_n+(K+1)q_{n+1}-1}^{(n)}\right\}\bigcup\left\{M_0^{(n)},M_1^{(n)},\dots,L_{q_n+(K+2)q_{n+1}-1}^{(n)}\right\}\bigcup\left\{R_0^{(n)},R_1^{(n)},\dots,R_{q_{n+1}-1}^{(n)}\right\}$$

constitutes the partition of the circle. The partition G_n is called $n-$**generalized dynamical partition** of circle. We consider the hitting function $H_n(G_n;x,y):[0,1)\times[0,1)\to\mathbb{N}$ with respect to generalized dynamical partition G_n and define the rescaled hitting time as

$$\tilde{H}_n(G_n;x,y)=\frac{H_n(G_n;x,y)}{q_{n+m^*}+(K^*+2)q_{n+m^*+1}},$$

where $m^*=\max\{m_1,m_2,m_3\}$.

Theorem 7. Let T_ρ be irrational rotation of the circle. Suppose the rotation number $\rho=[k_1,k_2,\dots,k_s,k_1,k_2,\dots,k_s,\dots]$ is algebraic irrational number. Let $c_n=\theta\Delta_n$

with $\theta \in (\max\{\alpha_1, \alpha_2, ..., \alpha_s\}, 1)$. The distribution functions $\Phi_{ns+i,\theta}(t), 1 \le i \le s$ of $\tilde{H}_{ns+i}(\tau_{ns+i}; x, y)$ converge uniformly to the continuous piecewise linear function on interval $[0,1]$.

Let pairs $(\xi(x), \eta(x))$ satisfy the following conditions:

a) $0 < \xi(0) < 1$;

b) $\xi(0) = \eta(0) + 1$;

c) $\xi(\eta(0)) = \eta(\xi(0))$;

d) $\xi(\eta(0)) < 0, \xi^2(\eta(0)) < 0, ..., \xi^{k-1}(\eta(0)) < 0$;

e) $\xi^k(\eta(0)) > 0$;

f) $\xi'0) = \eta'(0) = \xi''(0) = \eta''(0) = 0;\ \ \xi'''(0) \ne 0, \eta'''(0) \ne 0$;

g) $(\xi \circ \eta)'''(0) = (\eta \circ \xi)'''(0)$.

Let us consider the space of X^k_{cr} consists of critical circle homeomorphisms associated by pairs $(\xi(x), \eta(x))$.

We define by Y_k the subset of X^k_{cr}, consisting the pairs (ξ, η), that rotation number

$$\rho_k = \rho_k(T_{\xi,\eta}(x)) = [k, k, ..., k, ...] = \frac{-k + \sqrt{k^2 + 4}}{2}.$$

Theorem 8. Let $T_k \in Y_k$. For every $\theta \in (0,1)$, we consider the sequence of distribution functions $\{\Phi_{\theta,n}(t), n \in \mathrm{N}\}$ corresponding to hitting times in the intervals $J_{\theta,n}$. Then:

1. for every real number t, there exists the finite limit

$$\lim_{n \to +\infty} \Phi_{\theta,n}(t) = \Phi_\theta(t),$$

and $\Phi_\theta(t) = 0, t \le 0, \Phi_\theta(t) = 1, t \ge 1$;

2. $\Phi_\theta(t)$ is a continuous distribution function on real line R^1.

Theorem 9. Let $T \in X^{cr}_k(T_k)$ and $\{\Phi_{\theta,n}(t)\}, n \in \mathrm{N}$ be the sequence of distribution functions, corresponding hitting times in the intervals $J_{\theta,n}$ and $\Phi_\theta(t)$ be its limit function. Then:

1. $\Phi_\theta(t)$ is strictly monotone function on [0,1];
2. $\Phi_\theta(t)$ is singular function on [0,1].

Let us we introduce the following partition of the interval $[0,1)$ as $\Theta = \{I_{s,j} := \left[\rho_k^{2s} - j\rho_k^{2(s+1)}, \rho_k^{2s} - (j-1)\rho_k^{2(s+1)}\right), j = \overline{1..k}, s = \overline{0..\infty}\}$. For every $j \in \{1, 2, ..., k\}$ we define the union of intervals $I_j \subset [0,1)$ as $I_j = \bigcup_{s=0}^{\infty} I_{s,j}$. These subsets are nonintersecting and $\bigcup_{j=1}^{k} I_j = [0,1)$ i.e. these system of subsets determine the partition of [0,1). For each $\theta \in (0,1)$ the distribution function Φ_θ produce a probability measure on $[0,1]$. Setting

$$\mu_\theta([a,b]) = \Phi_\theta(b) - \Phi_\theta(a),$$

for any $a,b \in [0,1]$, we have a measure on algebra of intervals.

Theorem 10. Let $T \in Y_k$ and $\{\Phi_{\theta,n}(t)\}, n \in \mathrm{N}$ be the sequence of distribution functions, corresponding to hitting times in the intervals $J_{\theta,n}$ and $\Phi_\theta(t)$ be its limit function. Then:

- $\Phi_{\theta_1}(t) = \Phi_{\theta_2}(t), \forall t \in R^1$, if $\dfrac{\theta_1}{\theta_2} = \rho_k^{2(s_1 - s_2)}, \theta_1 \in I_{s_1,j}, \theta_2 \in I_{s_2,j}$;
- $\Phi_{\theta_2}(t) \le \Phi_{\theta_1}(t), \forall t \in \mathrm{R}^1$, if $\dfrac{\theta_1}{\theta_2} < \rho_k^{2(s_1 - s_2)}, \theta_1 \in I_{s_1,j}, \theta_2 \in I_{s_2,j}$.

Moreover there exists $t_0 \in (0,1)$ such that $\Phi_{\theta_1}(t_0) \ne \Phi_{\theta_2}(t_0)$.

Theorem 11. Let $T \in Y_k$, $\theta \in (0,1)$ and $\{\Phi_{\theta,n}(t), n \in \mathrm{N}\}$ be the sequence of probability distribution functions corresponding to hitting times in the intervals $J_{\theta,n}$ and $\Phi_\theta(t)$ be its limit function. Then for every $j \in \{1,2,..,k\}$, the probability measures μ_{θ_1} and μ_{θ_2} are equivalent, if $\theta_1, \theta_2 \in I_j$.

CONCLUSION

This thesis is devoted to study dynamics of circle homeomorphisms with singularity from the point of view of probability theory. In particular, limit theorems for random variables of the hitting and return times of circle maps are proved.

Main results of the thesis are followings:

1. convergence with probability one the sequence of random variables associated by return time for linear irrational rotations is proved;
2. convergence with probability one the sequence of random variables associated by log normalized hitting times for dynamical partitions is proved;
3. convergence with probability one the sequence of random variables associated by log normalized hitting times for mixed partitions is proved;
4. convergence with distribution the sequence of random variables associated by rescaled return times for circle maps with a single critical point is proved;
5. the singularity of limit distribution function of critical homeomorphism is proved;
6. equivalence of two probability measures of parameter depending limit distribution functions of circle homeomorphisms with a single critical point is proved.

**НАУЧНЫЙ СОВЕТ DSc.02/30.12.2019.FM.86.01
ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ
ИНСТИТУТЕ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО**

НАЦИОНАЛЬНЫЙ УНИВЕРСИТЕТ УЗБЕКИСТАНА

ХОМИДОВ МУХРИДДИН КАРИМЖОН УГЛИ

ПРЕДЕЛЬНЫЕ ТЕОРЕМЫ ДЛЯ ВРЕМЕНИ ПОПАДАНИЯ ДИНАМИЧЕСКИХ СИСТЕМ С ОСОБЕННОСТЯМИ

01.01.05 –Теория вероятностей и математическая статистика

**АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD)
ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ**

ТАШКЕНТ-2023

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Министерстве Высшего образавания, Науки и Инноваций Республики Узбекистан за № B2022.3.PhD/FM752.

Диссертация выполнена в Национальном университете Узбекистана имени Мирзо Улугбека.

Автореферат диссертации на трех языках (узбекский, английский, русский, (резюме)) размещен на веб-странице по адресу http://kengash.mathinst.uz и на Информационно-образовательном портале «ZiyoNet» по адресу http://www.ziyonet.uz.

Научный руководитель:	**Джалилов Ахтам Абдурахманович** доктор физико-математических наук, профессор
Официальные оппоненты:	**Мирахмедов Шерзод Адилович** доктор физико-математических наук, профессор
	Азимов Джахонгир Бахромович кандидат физико-математических наук, доцент
Ведущая организация:	**Наманганский инженерно-строительный институт**

Защита диссертации состоится «____»___________ 2023 года в ______ на заседании Научного совета DSc.02/30.12.2019.FM.86.01 при Институте Математики имени В.И.Романовского. (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 9.Тел.: (+99871) 207-91-40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz)

С диссертацией можно ознакомиться в Информационно-ресурсном центре Института Математики имени В.И.Романовского (зарегистрирована за № _____). (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 9.Тел.: (+99871) 207-91-40).

Автореферат диссертации разослан «____» ____________ 2023 года.
(протокол рассылки № ____ от «____» _____________ 2023 года).

А.Азамов
Заместитель председателя Научного
совета по присуждению ученых
степеней, д.ф.-м.н., академик

Ж.К.Адашев
Ученый секретарь Научного
совета по присуждению ученых
степеней, д.ф.-м.н., старший
научный сотрудник

Ш.К.Форманов
Председатель Научного семинара
при Научном совете по присуждению
ученых степеней, д.ф.-м.н., академик

ВВЕДЕНИЕ (аннотация диссертации доктора философии(PhD))

Целью исследования является доказательство предельных теорем для случайных величин, связанных со временем возврата и попадания иррациональных вращений, а также исследование гладкости предельной функции.

Объект исследования: Линейные иррациональные вращения, гомеоморфизмы окружности с одной критической точкой, инвариантная вероятностная мера, сингулярная мера, абсолютная непрерывная мера, случайная величина, функция распределения, предельная теорема.

Научная новизна исследования состоит в следующем:

Доказана сходимость с вероятностью единица последовательности случайных величин, связанных с моментами попадания для динамических и смешанных разбиений;

Доказана сходимость с распределением последовательности случайных величин, связанных временами возврата для круговых карт с одной критической точкой;

Доказана особенность функции распределения критического гомеоморфизма;

Сравниваются две вероятностные меры зависящих от параметра предельных функций распределения гомеоморфизма окружности с одной критической точкой.

Внедрение результатов исследования. Научные результаты, полученные в ходе исследования диссертации, использованы в следующих научно-исследовательских проектах;

предельная теорема ремасштабированного времени попадания для критических гомеоморфизмов окружности использовалась для сравнения вероятностных мер пространств вероятностных мер в фундаментальном проекте ОТ-Ф-4-42 Кардинальные и топологические свойства пространств полуаддитивных τ – гладких и радоновских функционалы (справка № 04/11-550 Национального университета Узбекистана от 4 февраля 2023 года). Применение научного результата позволило сравнивать сингулярные вероятностные меры в пространствах вероятностных мер;

результаты о спектре линейных трансфер-операторов, соответствующих отображениям окружности, были использованы для исследования асимптотических свойств и интегральных спектров динамических систем с дискретным временем в фундаментальном проекте ОТ–Ф-4-03 по теме «Точные динамические системы с непрерывным и дискретным временем, спектры частных интегральных операторов» (справка № 04/2165 Каршинского государственного университета от 10 июня 2023 г.). Применение научного результата позволило доказать существование спектров интегральных операторов в некоторых динамических системах и оценить абсолютное значение этих спектров.

Структура и объем диссертации. Диссертация состоит из введения, трёх глав, заключения и списка использованной литературы. Объем диссертации составляет 101 страниц.

ЭЪЛОН ҚИЛИНГАН ИШЛАР РЎЙХАТИ
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ
LIST OF PUBLISHED WORKS

1. Dzhalilov A.A., Khomidov M.K. Hitting functions for mixed partitions// The Bulletin of Udmurt University. Mathematics. Mechanics. Computer Science 2023, Volume 33, Issue 2. – P.197-211 (3. Scopus IF=0.342)
2. Dzhalilov A.A., Khomidov M.K. Weak convergency of hitting functions for circle maps // AIP Conference Proceedings 2023, Volume 2781, Issue 1, 020033. (3. Scopus IF=0,189).
3. Dzhalilov A.A., Khomidov M.K. The waiting time and dynamic partitions // Bulletin of National university of Uzbekistan: Mathematics and Natural sciences 2019, Volume 2, Issue 1, – P. 35-51. (01.00.00; №8)
4. Khomidov M.K. A note on behaviour of first return times for irrational rotations // Uzbek Mathematical Journal 2021, Volume 65, Issue 4, – P. 79-88. (01.00.00; №6)
5. Khomidov M.K. The limit theorems for hitting time functions of circle maps with a single critical point // Doklady Akad Nauk RUz, 2022, No 4. – P. 14-20. (01.00.00; №7)
6. Khomidov M.K. Absolute continuity of the limit distributions associated by critical circle maps // Uzbek Mathematical Journal 2022, Volume 66, Issue 3 – P. 75-84. (01.00.00; №6)

II бўлим (Часть 2 ; Part 2)

7. Dzhalilov A.A., Khomidov M.K. Dynamical partitions and waiting times for irrational rotations // Abstract of the International Conference “Mathematical analysis and its applications to mathematical physics”. – Samarkand, 2018. – P. 75-76.
8. M.Khomidov, The behavior of waiting times for irrational rotations // Abstract of the Republican scientific conference “New theorems of young mathematicians-2018”. – Namangan, October, 2018. – P. 45-46.
9. Dzhalilov A.A., Aliev A.F., Khomidov M.K. Limit theorems for hitting times of critical maps // Abstract of the Republican scientific conference “Modern Problems of Probability Theory and Mathematical Statistics”. – Tashkent, 30 April – 1 May, 2019. – P. 112-114.
10. Dzhalilov A.A. Khomidov M.K. A note on hitting times for irrational rotations // Abstract of the International Conference “Modern problems of applied mathematics and information technologies Al-Khwarizmi-2021”. – Fergana, 15-17 November, 2021. – P. 77-78.
11. Dzhalilov A.A., Khomidov M.K., Aliyev A.F. Regularity properties of hitting time processes for circle maps // Abstracts of communications, International conference “Limit theorems of probability theory and mathematical statistics”. – Tashkent, 26-28 September, 2022. – P. 34-36.

12. Dzhalilov A.A., Khomidov M.K. Return time asymptotics for an irrational rotation of a circle // Abstracts of the Republican scientific conference with the participation of foreign scientists “Sarimsakov’s readings”. – Tashkent, 16-18 September, 2021. – P. 53-55.

13. Khomidov M.K. The limit behaviour of hitting times for a quadratic irrational rotations // Abstracts of the Republican Scientific and Practical Conference II of Young Scientists “Mathematics, mechanics and intellectual technologies”. – Tashkent, 2023, – P. 41-43.

Avtoreferat “O‘zbekiston matematika jurnali” tahririyatida 2023 yil 31 iyulda tahrirdan o‘tkazilib, o‘zbek, rus va ingliz tillaridagi matnlar o‘zaro muvofiqlashtirildi.

Bosmaxona litsenziyasi:

9338

Bichimi: 84x60 $^{1}/_{16}$. «Times New Roman» garniturasi.
Raqamli bosma usulda bosildi.
Shartli bosma tabog‘i: 3,5. Adadi 100 dona. Buyurtma № 47/23.

Guvohnoma № 851684.
«Tipograff» MCHJ bosmaxonasida chop etilgan.
Bosmaxona manzili: 100011, Toshkent sh., Beruniy ko‘chasi, 83-uy.