**O‘ZBEKISTON MILLIY UNIVERSITETI
HUZURIDAGI ILMIY DARAJALAR BERUVCHI
DSc.03/30.12.2019.FM.01.02 RAQAMLI ILMIY KENGASH**

V.I. ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI

XAYRIYEV UMEDJON NARMON O‘G‘LI

DIFFERENSIALLANUVCHI DAVRIY FUNKSIYALARNING GILBERT FAZOSIDA INTEGRALLARNI TAQRIBIY HISOBLASHNING OPTIMAL METODLARI

**01.01.03 – Hisoblash matematikasi va diskret matematika
(fizika-matematika fanlari)**

**FIZIKA-MATEMATIKA FANLARI
bo‘yicha falsafa doktori (PhD) dissertatsiyasi
AVTOREFERATI**

Toshkent-2023

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Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

Content of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

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Fizika-matematika fanlari bo‘yicha falsafa doktori (Doctor of Philosophy) dissertatsiyasi mavzusi O‘zbekiston Respublikasi Oliy ta’lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasida B2022.4.PhD/FM794 raqam bilan ro‘yxatga olingan.

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Dissertatsiya bilan O‘zbekiston Milliy universitetining Axborot-resurs markazida tanishish mumkin (___ raqami bilan ro‘yxatga olingan). (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko‘chasi, 4-uy. Tel.: (+99871) 246-02-24).

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahon miqyosida olib borilayotgan koʻplab ilmiy-amaliy tadqiqotlar natijasida tasvirlani tahlil qilish, kommunikatsiya tizimlari uchun signallarni modulyatsiya va demodulyatsiya qilish, sanoatda va tibbiyotda kompyuter tomografiyasi masalalarining yechimlari kuchli tebranuvchi funksiyalarning maʼlum integrallarini hisoblashga olib kelinadi. Kuchli tebranuvchi integrallarni taqribiy hisoblash uchun sonli integrallashning standart metodlari katta miqdordagi hisoblash ishlarini talab qiladi va ularni amaliyotda toʻgʻridan-toʻgʻri qoʻllash samarali natija bermaydi. Shuning uchun, bunday integrallarni taqribiy hisoblashning samarali usullarini ishlab chiqish, funksiyalarning turli sinflarida eksponensial vaznli integrallarni taqribiy hisoblashning yangi metodlarini yaratish, hamda ularning xatoliklarini baholash hisoblash matematikasining muhim vazifalaridan biri boʻlib hisoblanadi.

Hozirgi kunda dunyoda kuchli tebranuvchi integrallarni taqribiy hisoblashda optimal kvadratur formulalar qurish muhim ahamiyat kasb etmoqda. Xususan, Furye koeffisiyentlari va integrallarini sonli integrallashda davriy funksiyalar fazolarida optimal kvadratur formulalar qurish, ularning xatoliklarini baholash keng tatbiq etilmoqda. Hisoblash matematikasining eng muhim muammolaridan biri bu – kuchli tebranuvchi integrallarni taqribiy hisoblash uchun differensiallanuvchi funksiyalarning turli fazolarida samarali kvadratur formulalar qurishdan iborat. Shu munosabat bilan kuchli tebranuvchi integrallarni taqribiy hisoblash, shuningdek, differensiallanuvchi davriy funksiyalar fazosida ularning xatoliklarini baholash uchun asimptotik optimal va optimal kvadratur formulalar qurish maqsadli ilmiy tadqiqotlardan biri hisoblanadi.

Mamlakatimizda fundamental fanlarning ilmiy va amaliy tatbiqiga ega boʻlgan tasvirlar tahlili, elektrodinamika, suyuqliklar mexanikasi va matematik fizika, geolegiya, geofizika masalalarini sonli-analitik yechish va kompyuter tomografiyasi tasvirlarini samarali qayta tiklash kabi muhim yoʻnalishlarga katta eʼtibor qaratilmoqda. Xususan, hisoblash matematikasining sonli integrallash nazariyasiga katta eʼtibor qaratilgan boʻlib, ayniqsa, bir va koʻp oʻzgaruvchili, davriy va davriy boʻlmagan funksiyalarning Banax va Gilbert fazolarida regulyar va singulyar vaznli integrallarni taqribiy hisoblash uchun panjarali optimal kvadratur va kubatur formulalar qurish boʻyicha muhim natijalarga erishildi. “Funksional analiz, matematik fizika, dinamik sistemalar nazariyasi, differensial tenglamalar, amaliy va hisoblash matematikasi” kabi fanlarining ustuvor yoʻnalishlari boʻyicha xalqaro standartlar darajasida ilmiy izlanishlar olib borish OʻzR FA V.I.Romanovskiy nomidagi Matematika instituti faoliyatining asosiy vazifalaridan biri hisoblanadi [1]. Qaror ijrosini taʼminlash maqsadida kuchli tebranuvchi integrallarni taqribiy hisoblash uchun optimal kvadratur formulalar qurish va differensiallanuvchi davriy funksiyalarning turli Gilbert fazolarida ularning xatoliklarini baholash muhim ahamiyatga ega.

[1] Oʻzbekiston Respublikasi Vazirlar Mahkamasining 2017-yil 18-maydagi “Oʻzbekiston Respublikasi Fanlar akademiyasining yangi tashkil etilayotgan ilmiy-tadqiqot muassasalari faoliyatini tashkil etish chora-tadbirlari toʻgʻrisida”gi 292-son qarori va tuzilmaviy masalalar

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevral PF-4947-sonli «O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha harakatlar strategiyasi to‘g‘risida»gi, 2022-yil 28-yanvar PF-60 sonli «2022-2026 yillarga mo‘ljallangan Yangi O‘zbekistonning taraqqiyot strategiyasi to‘g‘risida»gi farmonlari, 2017-yil 17-fevral PQ-2789-sonli «Fanlar akademiyasi faoliyati, ilmiy-tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari to‘g‘risida»gi, 2017-yil 20-aprel PQ-2909-sonli «Oliy ta’lim tizimini yanada rivojlantirish chora-tadbirlari to‘g‘risida»gi, 2018-yil 27-aprel PQ-3682-sonli «Innovatsion g‘oyalar, texnologiyalar va loyihalarni amaliyotga joriy qilish tizimini yanada takomillashtirish chora-tadbirlari to‘g‘risida»gi, 2020-yil 7-may PQ-4708-sonli «Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida»gi qarorlari, hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga bog‘liqligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV «Matematika, mexanika va informatika» ustuvor yo‘nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. Ma’lumki, fan va texnikaning ko‘plab muammolari kuchli tebranuvchi funksiyalarning ma’lum integrallarini, xususan, Furye koeffisiyentlari va integrallarini hisoblashga olib kelinadi. Kuchli tebranuvchi integrallarni taqribiy hisoblash uchun bir qancha maxsus usullar ishlab chiqilgan. Jumladan, Faylon usuli, asimptotik yoyish usuli, Levinning kollokatsiya usuli, eng tez tushish va optimal kvadratur va kubatur formulalar usullarini misol qilib aytish mumkin. Dastlab, integrallash oralig‘ida mos funksiyani parabola yoylari bilan bo‘lakli yaqinlashtirishga asoslangan Faylon usuli taqdim etilgan. Keyinchalik turli tipdagi kuchli tebranuvchi funksiyalarga ega integrallarni taqribiy hisoblash uchun Faylon tipidagi, Klenshou-Kurtis-Faylon tipidagi, Levin tipidagi, takomillashgan Klenshou-Kurtis, umumlashgan kvadratur formulalar va Gauss-Lagerr kvadratur formulalar usullari ishlab chiqildi. Bu usullar bilan, oxirgi yillarda, L. Zhang, A. Asheim, V. Dominguez, E.A. Flinn, J. Gao, J.M. Melenk, K.N. Melnik va R.V.N. Melnik, H. Mo va Sh. Xiang, L.F. Shampine, G. He va Y.J. Cho, H. Kang, M.I. Isroilov, B. Eshdavlatov va S.A. Baxromov kabi olimlar ko‘plab tadqiqotlarni amalga oshirishgan. D. Huybrechs, Sh. Olver, A. Iserles, S.P. Norsett, S.-I.S. Zaman, S.I.U. Nasib, S. Olver, H. Wang kabi olimlar asimptotik yoyish usuli bo‘yicha ilmiy tadqiqotlar olib borishgan.

Ma’lum Banax fazolarida berilgan funksiyalar aniq integrallarini taqribiy hisoblash uchun optimal kvadratur va kubatur formulalar qurishda Sobolev, splayn funksiyalar va ϕ - funksiya usullari mavjud. Dastlab, $L_2(\Omega)$ fazoda chiziqli differensial operatorning diskret analogi tushunchasidan foydalanib, optimal kvadratur va kubatur formulalar qurish nazariyasi bilan S.L. Sobolev shug‘ullangan. Akademik S.L. Sobolev tomonidan taklif qilingan optimal kvadratur va kubatur formulalar qurish algoritmni amalga oshirish bilan $L_2(\Omega)$, $K_2(P_m)$, $K_2^{(\overline{m,m-2})}$ va

$W_2^{(m,0)}$ fazolarida Z.J. Jamalov, F.Ya. Zagirova, X.M. Shadimetov, A.R. Hayotov, G.V. Milovanović, F. Lanzara, F.A. Nuraliev, D.M. Axmedov, S.S. Azamov va A.K. Bolatevlar shug'ullanishgan. 1999-yilda X.M. Shadimetovning ishida, davriy funksiyalarning Sobolev fazosida vaznli integrallar uchun panjarali kubatur formulalar qurilgan. Ushbu ishdan, xususan, vazn funksiyasi $\exp(i\sigma x)$ bo'lganda Furye koeffisiyentlarini taqribiy hisoblash uchun qurilgan I. Babuškaning optimal kvadratur formulasi kelib chiqadi. 2015-yilda E. Novak, M. Ulrih va H.Vajniakovskiylar Sobolevning standart H^s davriy va davriy bo'lmagan funksiyalar fazosida bir o'zgaruvchili tebranuvchi integrallarning taqribiy hisoblashlarini o'rganishgan.

Ta'kidlash joizki, so'ngi yillarda, $L_2^{(m)}$ va $W_2^{(m,m-1)}$ Gilbert fazolarida X.M. Shadimetov, G.V. Milovanović, A.R. Hayotov, N.D. Boltaev, C.-O. Lee, S. Jeon, S.S. Babaev va B.I. Bozarovlar kuchli tebranuvchi integrallarni taqribiy hisoblash uchun optimal kvadratur formulalar qurish va ularni amaliyotga tatbiq qilish bo'yicha ilmiy izlanishlar olib borishgan. Buning natijasida, sanoat va tibbiyot sohalarida kompyuter tomografiyasi tasvirlarini, laboratoriya sharoitida yuqori aniqlikda qayta tiklashga erishishgan.

Dissertatsiya mavzusining dissertatsiya bajarilayotgan oliy ta'lim muassasasining ilmiy-tadqiqot ishlari bilan bog'liqligi. Dissertatsiya tadqiqoti O'zbekiston Respublikasi Fanlar Akademiyasi V.I. Romanovskiy nomidagi Matematika instituti Hisoblash matematikasi laboratoriyasining "Gilbert fazolarida optimal kvadratur, interpolyatsion, ayirmali formulalar qurish va ularni integral tenglamalarni yechishga tatbiqlari" mavzusidagi kalendar reja doirasida bajarilgan.

Tadqiqotning maqsadi differensiallanuvchi davriy funksiyalarning $\tilde{W}_2^{(m,m-1)}(0,1]$ Gilbert fazosida Furye koeffisiyentlarini taqribiy hisoblash uchun optimal kvadratur fomulalar qurish va ularning xatoliklarini aniq yuqori chegarasini topish, hamda differensiallanuvchi funksiyalarning $W_2^{(m,m-1)}(a,b)$ kompleks qiymatli Gilbert fazosida Furye integrallarini taqribiy hisoblash uchun effektiv kvadratur formulalar olishdan iborat.

Tadqiqotning vazifalari:

differensiallanuvchi davriy funksiyalarning $\tilde{W}_2^{(m,m-1)}(0,1]$ kompleks qiymatli Gilbert fazosida Furye koeffisiyentlarini sonli hisoblash uchun kvadratur fomulalarning ekstremal funksiyasini topish;

differensiallanuvchi davriy funksiyalarning $\tilde{W}_2^{(m,m-1)}(0,1]$ kompleks qiymatli Gilbert fazosida Furye koeffisiyentlarini sonli hisoblash uchun kvadratur fomula xatolik funksionali normasining analitik ko'rinishini topish;

differensiallanuvchi davriy funksiyalarning $\tilde{W}_2^{(m,m-1)}(0,1]$ kompleks qiymatli Gilbert fazosida Furye koeffisiyentlarini sonli hisoblash uchun kvadratur fomulalarning xatolik funksionali normasiga eng kichik qiymat beruvchi optimal koeffisiyentlarini topish;

differensiallanuvchi davriy funksiyalarning $\tilde{W}_2^{(m,m-1)}\left(0,1\right]$ kompleks qiymatli Gilbert fazosida Furye koeffisiyentlarini sonli hisoblash uchun optimal kvadratur fomulalarning xatolik funksionali normasini hisoblash;

$W_2^{(m,m-1)}\left(a,b\right)$ kompleks qiymatli funksiyalarning Gilbert fazosida Furye integrallarini taqribiy hisoblash uchun effektiv kvadratur formulalarning koeffisiyentlarini topish.

Tadqiqotning obyekti differensiallanuvchi davriy funksiyalarning Gilbert fazosi, optimal va effektiv kvadratur formulalar, kuchli tebranuvchi integrallar.

Tadqiqotning predmeti ekstremal funksiyalar, differensiallanuvchi davriy funksiyalarning $\tilde{W}_2^{(m,m-1)}$ Gilbert fazosi, Furye koeffitsientlarini va integrallarini taqribiy hisoblash uchun eksponensial vaznli optimal va effektiv kvadratur formulalardan iborat.

Tadqiqotning usullari. Ilmiy tadqiqot ishida hisoblash matematikasi, funksional analiz, umumlashgan funksiyalar nazariyasi, differensiallanuvchi tenglamalar nazariyasi, diskret argumentli funksiyalar nazariyasi usullaridan foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

differensiallanuvchi davriy funksiyalarning $\tilde{W}_2^{(m,m-1)}\left(0,1\right]$ kompleks qiymatli Gilbert fazosida Furye koeffisiyentlarini sonli hisoblash uchun kvadratur fomulalarning ekstremal funksiyalari topilgan;

differensiallanuvchi davriy funksiyalarning $\tilde{W}_2^{(m,m-1)}\left(0,1\right]$ kompleks qiymatli Gilbert fazosida Furye koeffisiyentlarini sonli hisoblash uchun kvadratur fomula xatolik funksionali normasining analitik ko‘rinishi topilgan;

differensiallanuvchi davriy funksiyalarning $\tilde{W}_2^{(m,m-1)}\left(0,1\right]$ kompleks qiymatli Gilbert fazosida Furye koeffisiyentlarini sonli hisoblash uchun optimal kvadratur fomulalarning koeffisiyentlari topilgan;

differensiallanuvchi davriy funksiyalarning $\tilde{W}_2^{(m,m-1)}\left(0,1\right]$ kompleks qiymatli Gilbert fazosida Furye koeffisiyentlarini sonli hisoblash uchun optimal kvadratur fomulalarning xatolik funksionalining normasi hisoblangan va $m=1$ va $m=2$ uchun sonli natijalari berilgan;

$W_2^{(m,m-1)}\left(a,b\right)$ kompleks qiymatli Gilbert fazosida Furye integrallarini taqribiy hisoblash uchun effektiv kvadratur formulalarning koeffisiyentlari topilgan.

Tadqiqotning amaliy natijasi quyidagilardan iborat:

qurilgan optimal kvadratur formulalar yordamida Furye koeffisiyentlari taqribiy hisoblangan;

olingan effektiv kvadratur formulalar yordamida Furye integrallari sonli hisoblangan. Olingan natijalar signal modulyatsiya va demodulyatsi qilishda, tasvirlarni qayta ishlashda va boshqa amaliy fanlarida qo‘llanilishi mumkin.

Tadqiqot natijalarining ishonchliligi kvadratur formulalar nazariyasi, hisoblash matematikasi, funksional analiz va diskret argumentli funksiyalar nazariyasi usullari qo‘llanilganligi, matematik mulohazalarning qat’iyligi, shuningdek, olingan sonli natijalar bilan asoslangan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati Gilbert fazolarida kuchli tebranuvchi integrallarni taqribiy hisoblash uchun optimal va effektiv kvadratur formulalarni qurish algoritmi, hamda qurilgan formulalarning yaqinlashish tezligi yuqoriligi bilan izohlanadi.

Tadqiqot natijalarining amaliy ahamiyati qurilgan optimal va effektiv kvadratur formulalar kompyuter tomografiyasi va tasvirlar tahlili masalalarini kuchli tebranuvchi integrallar yordamida taqribiy yechishda qo‘llaniladi.

Tadqiqot natijalarining joriy qilinishi. Davriy funksiyalarning Gilbert fazosida kuchli tebranuvchi integrallarni taqribiy hisoblash uchun optimal kvadratur formulalarni qurish bo‘yicha olingan ilmiy natijalar asosida:

Davriy funksiyalarning $\tilde{W}_2^{(1,0)}(0,1]$ va $\tilde{W}_2^{(2,1)}(0,1]$ Gilbert fazolarida kuchli tebranuvchi integrallarni taqribiy hisoblash uchun qurilgan optimal kvadratur formulalardan PZ-20170930257 raqamli “Mahalliy bug‘doy donlaridan navli un tortish texnologiyasida gidrometrik ishlov berish jarayonini takomilllashtirish” (Toshkent kimyo-texnologiya institutining 2023-yil 14-apreldagi 1/04-1177- sonli ma’lumotnomasi) nomli amaliy loyihada gidrotermik ishlov berish jarayonida bug‘doy donlarining o‘lchamlarini (bo‘yi, eni va uzunligi) o‘lchashning aniqligini oshirishda foydalanilgan. Buning natijasida, gidrotermik qurilmalarida sodir bo‘luvchi namlik tarqalishini ifodalovchi matematik modellar yechimlarini ushbu formulalar yordamida sonli hisoblash, ularning geometrik o‘lchamlari optimal qiymatlarini yanada aniqlik bilan topish imkonini bergan.

$\tilde{W}_2^{(3,2)}(0,1]$ kompleks qiymatli Gilbert fazosida qurilgan optimal kvadratur formuladan OT-F4–02 – “Matematik fizikaning holatlar to‘plami cheksiz bo‘lgan modellari termodinamikasi” (Buxoro davlat universitetining 2023-yil 25-apreldagi 04/752-sonli ma‘lumotnomasi) nomli fundamental loyihada matematik fizika tenglamalari uchun qo‘yilgan aralash masalalardagi integrallarni taqribiy hisoblashda qo‘llanilgan. Buning natijasida, qo‘yilgan aralash masalalarning sonli yechimini yuqori aniqlikda topish imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqotning asosiy natijalari 11 ta ilmiy-amaliy anjumanlarda, jumladan 7 ta xalqaro va 4 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinishi.

Dissertatsiya mavzusi bo‘yicha jami 19 ta ilmiy ish chop etilgan, shulardan, O‘zbekiston Respublikasi Oliy Attestatsiya komissiyasining doktorlik dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 6 ta maqola, jumladan, 2 tasi xorijiy va 4 tasi respublika jurnallarida nashr etilgan, hamda elektron hisoblash mashinalari uchun dasturni rasmiy ro‘yxatdan o‘tkazish to‘g‘risidagi bitta guvohnoma olingan.

Dissertatsiyaning hajmi va tuzilishi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan. Dissertatsiyaning umumiy hajmi 92 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, muammoning o‘rganilganlik darajasi, mavzu bo‘yicha dunyo miqyosidagi ilmiy-tadqiqotlar sharhi keltirilgan, tadqiqot maqsadi, vazifalari, obyekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo‘yicha ma’lumotlar keltirilgan.

Dissertatsiyaning “**Gilbert fazolarida sonli integrallash formulalari**” deb nomlangan birinchi bobi asosan kirish xarakteriga ega bo‘lib, unda dissertatsiyada qo‘llaniladigan asosiy tushuncha va ta’riflar keltirilgan. Shuningdek, ushbu ilmiy tadqiqot mavzusi doirasida tadqiq etilgan ilmiy izlanishlar va olingan natijalar bayon qilingan.

Bu bobning birinchi paragrafida Gilbert fazolari haqida bayon qilingan va bu fazolarga misollar keltirilgan. Jumaladan, biz ish olib boradigan $W_2^{(m,m-1)}(0,1)$ Gilbert fazosi va undagi skalyar ko‘paytma va norma tushunchalari keltirilgan.

Quyidagi fazoni qaraymiz

$$W_2^{(m,m-1)}(0,1) := \{\varphi : [0,1] \to \mathrm{C} \mid \varphi^{(m-1)} \text{ abs. uzl. va } \varphi^{(m)} \in L_2(0,1)\}$$

bu kompleks qiymatli funksiyalarning Gilbert fazosi bo‘lib, elementlari $m-1$ chi tartibli hosilasi va m chi tartibli umumlashgan hosilalari yig‘indisining absolyut qiymati kvadrati bilan integrallanuvchi funksiyalardir. Ushbu fazoda ikkita φ va ψ funksiyalarning skalyar ko‘paytmasi quyidagicha aniqlangan

$$\langle \varphi, \psi \rangle_{W_2^{(m,m-1)}} = \int_0^1 (\varphi^{(m)}(x) + \varphi^{(m-1)}(x))(\overline{\psi}^{(m)}(x) + \overline{\psi}^{(m-1)}(x))dx, \quad (1)$$

bu yerda $\overline{\psi}$ bu ψ funksiyaning kompleks qo‘shma funksiyasi. (1)-skalyar ko‘paytma bilan birgalikda $W_2^{(m,m-1)}(0,1)$ fazo Gilbert fazosini tashkil etadi. (1)-skalyar ko‘paytmaga mos biror $\varphi \in W_2^{(m,m-1)}(0,1)$ funksiyaning normasi quyidagi kiritiladi

$$\|\varphi\|_{W_2^{(m,m-1)}} = \langle \varphi, \varphi \rangle_{W_2^{(m,m-1)}}^{1/2}, \quad (2)$$

bunda $\int_0^1 \left|\varphi^{(m)}(x) + \varphi^{(m-1)}(x)\right|^2 dx < \infty$.

Ushbu $W_2^{(m,m-1)}(0,1)$ fazoning har bir elementi bu – biri ikkinchisidan e^{-x} va $(m-2)$ darajaligacha bo‘lgan ko‘phadlarning chiziqli kombinatsiyasi bilan farq qiluvchi funksiyalar sinfidir, ya’ni $W_2^{(m,m-1)}(0,1)$ fazo faktor fazodir.

Birinchi bobning ikkinchi paragrafida ushbu ilmiy ishda muhim bo‘lgan Bernulli sonlari va ko‘phadlari va ularning xossalari, hamda davriy Bernulli ko‘phadlari keltirilgan.

Quyidagi tenglikda

$$\frac{z}{e^z-1}=\sum_{n=0}^{\infty}\frac{B_n}{n!}z^n, \quad |z|<2\pi,$$

B_n - Bernulli soni bo‘lib, uning B_1 elementidan boshqa qolgan barcha toq nomerdagi sonlari nolga teng, ya’ni

$$B_{2k+1}=0, \ k=1,2,\ldots$$

va dastlabki bir nechta hadlari quyidagicha

$$B_0=1, \ B_1=-\frac{1}{2}, \ B_2=\frac{1}{6}, \ B_4=-\frac{1}{30}, \ B_6=\frac{1}{42}, \ B_8=-\frac{1}{30}, \ B_{10}=\frac{5}{66}.$$

Bernulli ko‘phadlari quyidagi formula orqali aniqlanadi

$$\frac{ze^{xz}}{e^z-1}=\sum_{n=0}^{\infty}\frac{B_n(x)}{n!}z^n, \quad |z|<2\pi.$$

Davriy Bernulli ko‘phadlari

$$\tilde{B}_n(x)=B_n(x), \quad 0\le x<1,$$

va

$$\tilde{B}_n(x+1)=B_n(x), \quad x\in\mathbb{R}$$

ga teng.

Birinchi bobning uchinchi paragrafida davriy funksiyalar va ularning xossalari keltirib o‘tilgan.

1-ta’rif. Agar x o‘zgaruvchining barcha haqiqiy qiymatlarida aniqlangan $f(x)$ funksiya uchun noldan farqli shunday haqiqiy T son mavjud bo‘lsaki

$$f(x+T)=f(x), \ \forall x\in\mathbb{R},$$

tenglik o‘rinli bo‘lsa, u holda $f(x)$ *davriy funksiya*, T son *uning davri* deyiladi.

Birinchi bobning to‘rtinchi paragrafida B - Banax fazosida optimal kvadratur formulalar qurish masalasi keltirilgan.

S.L. Sobolev tomonidan quyidagi ko‘rinishdagi kvadratur formulalar

$$\int_a^b p(x)\varphi(x)dx\cong\sum_{\beta=0}^{N}C_\beta\varphi(x_\beta), \tag{3}$$

ushbu xatolik funksionali bilan birga qaralgan

$$\ell_N(x)=p(x)\varepsilon_{[a,b]}(x)-\sum_{\beta=0}^{N}C_\beta\delta(x-x_\beta),$$

bunda $p(x)$ – vazn funksiyasi bo‘lib, $\int\limits_a^b p(x)dx<\infty$ o‘rinli, x_β – tugun nuqtalar, C_β – koeffisiyentlar, $\varphi(x)\in B$ va B Banax fazosi uzluksiz funksiyalar sinfida kompakt joylashgan, ya’ni $B\to C(a,b)$.

(3)-kvadratur formulaning xatoligi quyidagicha bo‘lib

$$(\ell_N,\varphi)=\int_a^b p(x)\varphi(x)dx-\sum_{\beta=0}^{N}C_\beta\varphi(x_\beta), \tag{4}$$

ℓ_N xatolik funksionalining φ dagi qiymatiga teng.

Koshi-Shvarts tengsizligiga ko‘ra (3)-kvadratur formulaning (4)-xatoligi ℓ_N xatolik funksionalining normasi yordamida yuqoridan quyidagicha baholanadi

$$\left|(\ell_N,\varphi)\right| \le \|\ell_N\|_{B^*} \cdot \|\varphi\|_B .$$

2-ta'rif. Barcha $\varphi \in B$ lar uchun, (4)-xatolikning absolyut qiymatining yuqori chegarasiga, ya'ni $\|\ell_N\|_{B^*}$ normaga koeffisiyentlar va tugun nuqtalar bo'yicha eng kichik qiymat beradigan kvadratur formulaga, qaralayotgan fazoda *Nikolskiy ma'nosida optimal kvadratur formula* yoki *eng yaxshi optimal kvadratur formula* deyiladi.

3-ta'rif. Tugun nuqtalar fiksirlangan holda, $\|\ell_N\|_{B^*}$ normaga minimum qiymat beruvchi C_β koeffisiyentlar optimal koeffisiyentlar deyiladi va $\overset{\circ}{C}_\beta$ kabi belgilanadi. Bu optimal koeffisiyentlar bilan birgalikda (3)-ko'rinishdagi kvadratur formulaga *Sard ma'nosida optimal kvadratur formula* deyiladi.

Biz bundan keyin Sard ma'nosida optimal kvadratur formulalar qurish masalasini qaraymiz. Bu masalani yechish uchun quyidagi masalalarni ketma-ket hal qilish kerak bo'ladi.

Masala A. (3)-kvadratur formula ℓ_N xatolik funksionalining $\|\ell_N\|_{B^*}$ normasini hisoblash.

Masala B. ℓ_N xatolik funksionalining $\|\ell_N\|_{B^*}$ normasiga minimum qiymat beruvchi $C_\beta = \overset{\circ}{C}_\beta$ optimal koeffisiyentlarni topish

$$\left\|\overset{\circ}{\ell}_N\right\|_{B^*} := \inf_{C_\beta} \|\ell_N\|_{B^*} .$$

Birinchi bobning beshinchi paragrafida

$$I_\varphi(\omega) = \int_\Omega e^{2\pi i\omega g(x)}\varphi(x)dx \tag{5}$$

ko'rinishdagi kuchli tebranuvchi integrallarni taqribiy hisoblash usullari muhokama qilinadi. (5)-ko'rinishdagi integralda φ va g lar tebranmaydigan funksiyalar, ω – tebranish chastotasi va Ω – bo'lakli uzluksiz chegaraga ega.

(5)-ko'rinishdagi integrallarni taqribiy hisoblashda sonli integrallashning standart metodlar ko'pincha ko'proq hisoblash ishlarini talab qiladi va ularni amaliyotda qo'llash samarali natija bermaydi. Shu bois, maxsus metodlar ishlab chiqish zarurati paydo bo'ldi. φ va g funksiyalarning xossalariga qarab, $I_\varphi(\omega)$ kuchli tebranuvchi integrallarni sonli hisoblashning turli metodlari ishlab chiqilgan.

Bu paragrafda (5)-ko'rinishdagi kuchli tebranuvchi integrallarni taqribiy hisoblashning maxsus usullari: asimptotik yoyish, Filon tipidagi usullari, Levinning kolakatsiya usuli, eng tez tushish usuli, hamda optimal kvadratur va kubatur formulalar usullari tahlil qilingan.

Dissertatsiyaning **"Davriy funksiyalarning $\tilde{W}_2^{(m,m-1)}(0,1]$ Gilbert fazolarida optimal kvadratur formulalar"** deb nomlangan ikkinchi bobi differensiallanuvchi davriy funksiyalarning Gilbert fazosida optimal kvadratur formulalar qurishga bag'ishlanadi.

Ushbu bobning birinchi paragrafida $\tilde{W}_2^{(m,m-1)}(0,1]$ fazoda Furye koeffisiyentlarini sonli hisoblash uchun optimal kvadratur formulalar qurish masalasi qo‘yilgan.

Biz ushbu

$$W_2^{(m,m-1)}(0,1) := \{\varphi : [0,1] \to \mathbb{C} \mid \varphi^{(m-1)} \text{ abs.uzl.va } \varphi^{(m)} \in L_2(0,1)\}$$

fazoning quyidagi tenglikni qanoatlantiruvchi

$$\varphi(x+\beta) = \varphi(x),\ x \in \mathbb{R} \text{ va } \beta \in \mathbb{Z},$$

ya’ni, eng kichik musbat davri 1 bo‘lgan elementlaridan tashkil topgan qism fazosini $\tilde{W}_2^{(m,m-1)}(0,1]$ deb belgilaymiz.

Bu fazoda, quyidagi ko‘rinishdagi kvadratur formulani qaraymiz

$$\int_0^1 e^{2\pi i\omega x}\varphi(x)dx \cong \sum_{k=1}^{N} C_k\varphi(hk), \tag{6}$$

bu yerda $\omega \in \mathbb{Z}\setminus\{0\}$, C_k – koeffisiyentlar, $h = 1/N$ va N – tugun nuqtalar soni.

(6)-kvadratur formulaga mos xatolik

$$(\ell,\varphi) = \int_0^1 e^{2\pi i\omega x}\varphi(x)dx - \sum_{k=1}^{N} C_k\varphi(hk) = \int_0^1 \ell(x)\varphi(x)dx \tag{7}$$

va unga mos davriy xatolik funksionali quyidagicha

$$\ell(x) = e^{2\pi i\omega x} - \sum_{k=1}^{N} C_k \sum_{\beta=-\infty}^{\infty} \delta(x - hk - \beta),$$

bu yerda δ – Dirakning delta-funksiyasi.

Ushbu $\tilde{W}_2^{(m,m-1)*}$ qo‘shma fazo $m \geq 2$ uchun birga ortogonal bo‘lgan davriy funksionallardan tashkil topgan, ya’ni

$$(\ell,1) = 0. \tag{8}$$

(8)-shart (6)-ko‘rinishdagi kvadratur formulani ixtiyoriy o‘zgarmas songa aniq ekanligini ko‘rsatadi va uni quyidagicha yozish mumkin

$$\int_0^1 e^{2\pi i\omega x}dx = \sum_{k=1}^{N} C_k.$$

Koshi-Shvarts tengsizligidan ko‘rinadiki, (7)-xatolikning absolyut qiymati ℓ xatolik funksionalining normasi yordamida yuqoridan quyidagicha baholanadi

$$|(\ell,\varphi)| \leq \|\ell\|_{\tilde{W}_2^{(m,m-1)*}} \cdot \|\varphi\|_{\tilde{W}_2^{(m,m-1)}},$$

bu yerda

$$\|\ell\|_{\tilde{W}_2^{(m,m-1)*}} = \sup_{\varphi, \|\varphi\| \neq 0} \frac{|(\ell,\varphi)|}{\|\varphi\|}.$$

Shuni ta’kidlaymizki, xatolik funksionalining normasiga minimum qiymat beruvchi C_k koeffisiyentlarga *optimal koeffiiyentlar* deyiladi va $\mathring{C}_k$ kabi belgilanadi. Bu koeffisiyentlar bilan birgalikda (6)-ko‘rinishdagi kvadratur formulaga esa *optimal kvadratur formula* deyiladi.

Optimal kvadratur formulaning xatolik funksionali normasining minimumi quyidagicha belgilanadi

$$\left\|\overset{\circ}{\ell}\right\|_{\widetilde{W}_2^{(m,m-1)*}} := \inf_{C_k} \|\ell\|_{\widetilde{W}_2^{(m,m-1)*}}.$$

Ushbu $\widetilde{W}_2^{(m,m-1)}(0,1]$ fazosida (6)-ko'rinishdagi optimal kvadratur formulalar qurish uchun quyidagi masalani yechishimiz talab etiladi.

1-masala. (6)-ko'rinishidagi kvadratur formulaning ℓ xatolik funksionali $\|\ell\|_{\widetilde{W}_2^{(m,m-1)*}}$ normasining analitik ko'rinishini topish.

2-masala. (6)-ko'rinishidagi kvadratur formulaning ℓ xatolik funksionali normasiga minimum qiymat beruvchi $\overset{\circ}{C}_k$ optimal koeffisiyentlarni topish.

3-masala. (6)-optimal kvadratur formulaning $\overset{\circ}{\ell}$ xatolik funksionali normasini hisoblash.

Yuqoridagi masalani ixtiyoriy $m \in \mathrm{N}$ va $\omega \in \mathrm{Z}$ da to'liq yechishimiz uchun, quyidagi hollarda bu masalani alohida qarab chiqishimizga to'g'ri keladi:

1. $m = 1$ va $\omega \in \mathrm{Z}$;
2. $m \geq 2$ va $\omega \in \mathrm{Z} \setminus \{0\}$, $\omega h \notin \mathrm{Z}$;
3. $m \geq 2$ va $\omega h \in \mathrm{Z} \setminus \{0\}$;
4. $m \geq 2$ va $\omega = 0$.

Ikkinchi bobning ikkinchi paragrafida 1-masalani yeshish bilan shug'ullanamiz. Bunig uchun, ℓ xatolik funksionalining ekstremal funksiyasidan foydalanamiz.

Koshi-Shvarts tengsizligini tenglikka aylantiruvchi, ya'ni quyidagi tenglikni qanoatlantiruvchi ℓ xatolik funksionaliga mos ψ_ℓ funksiyaga *ekstremal funksiya* deyiladi

$$(\ell, \psi_\ell) = \|\ell\|_{\widetilde{W}_2^{(m,m-1)*}} \cdot \|\psi_\ell\|_{\widetilde{W}_2^{(m,m-1)}}.$$

Ekstremal funksiya uchun quyidagi tasdiq o'rinli.

Chiziqli uzluksiz funksionalning umumiy ko'rinishi haqidagi Riss teoremasidan foydalanib, quyidagi differensial tenglamani olamiz

$$\overline{\psi_\ell}^{(2m)}(x) - \overline{\psi_\ell}^{(2m-2)}(x) = (-1)^m \ell(x). \tag{9}$$

Teorema 1. (9)-tenglamaning umumlashgan yechimi ℓ xatolik funksionaliga mos ψ_ℓ ekstremal funksiya bo'lib, u quyidagicha ifodalanadi

a) $m = 1$ va $\omega \in \mathrm{Z}$ uchun

$$\psi_\ell(x) = e^{-2\pi i\omega x} \cdot \kappa_1(\omega) - \sum_{k=1}^{N} \overline{C}_k \sum_{\beta=-\infty}^{\infty} e^{2\pi i\beta(x-hk)} \cdot \kappa_1(\beta),$$

b) $m \geq 2$ va $\omega \in \mathrm{Z} \setminus \{0\}$ uchun

$$\psi_\ell(x) = e^{-2\pi i\omega x} \cdot \kappa_m(\omega) - \sum_{k=1}^{N} \overline{C}_k \sum_{\beta\neq 0} e^{2\pi i\beta(x-hk)} \cdot \kappa_m(\beta) + d_0,$$

bunda d_0 – biror o'zgarmas va

$$\kappa_m(\omega) = \frac{1}{(2\pi\omega)^{2m} + (2\pi\omega)^{2m-2}}. \tag{10}$$

(8)-ortogonallik shartidan ko‘rinadiki, ℓ xatolik funksionalining normasini $m=1$ va $m\geq 2$ hollarida alohida hisoblash talab etiladi.

$m=1$ da ℓ xatolik funksionali normasinining analitik ko‘rinishi quyidagicha ifodalanadi

$$\|\ell\|^2_{\tilde{W}_2^{(1,0)*}}=\kappa_1(\omega)-\kappa_1(\omega)\sum_{k'=1}^{N}\overline{C}_{k'}e^{2\pi i\omega hk'}-\kappa_1(\omega)\sum_{k=1}^{N}C_k e^{-2\pi i\omega hk}$$
$$+\sum_{k=1}^{N}\sum_{k'=1}^{N}C_k\overline{C}_{k'}\sum_{\beta=-\infty}^{\infty}\kappa_1(\beta)\cdot e^{2\pi i\beta h(k-k')}, \qquad (11)$$

bunda $\omega\in\mathbb{Z}$ va $\kappa_1(\cdot)$ miqdor (10)-formula bilan aniqlangan.

Ushbu $\tilde{W}_2^{(m,m-1)*}$ qo‘shma fazoda $m\geq 2$ uchun ℓ xatolik funksionali normasi kvadratining analitik ko‘rinishi quyidagicha aniqlanadi

$$\|\ell\|^2_{\tilde{W}_2^{(m,m-1)*}}=\kappa_m(\omega)-\kappa_m(\omega)\sum_{k'=1}^{N}\overline{C}_{k'}e^{2\pi i\omega hk'}-\kappa_m(\omega)\sum_{k=1}^{N}C_k e^{-2\pi i\omega hk}$$
$$+\sum_{k=1}^{N}\sum_{k'=1}^{N}C_k\overline{C}_{k'}\sum_{\beta\neq 0}\kappa_m(\beta)\cdot e^{2\pi i\beta h(k-k')}, \qquad (12)$$

bunda $\omega\in\mathbb{Z}\setminus\{0\}$.

Shunday qilib, 1-masala yechildi.

Ikkinchi bobning uchinchi paragrafida ushbu dissertatsiya ishining asosiy natijalari olinadi, ya’ni ℓ xatolik funksionalining normasiga minimum qiymat beruvchi, (6)-kvadratur formulaning $\overset{\circ}{C}_k$ optimal koeffisiyentlarini topamiz. Buning uchun, bu masalani $m=1$ va $\omega\in\mathbb{Z}$, hamda $m\geq 2$ va $\omega\in\mathbb{Z}\setminus\{0\}$ hollarida alohida qaraymiz.

Dastlab, $m=1$ uchun quyidagi chiziqli tenglamalar sistemasini olamiz

$$\frac{\partial L_1}{\partial\overline{C}_{k'}}=-e^{2\pi i\omega hk'}\kappa_1(\omega)+\sum_{k=1}^{N}C_k\sum_{\beta=-\infty}^{\infty}e^{2\pi i\beta h(k-k')}\kappa_1(\beta)=0,\ k'=1,2,...,N. \qquad (13)$$

Ta’kidlash kerakki, (13)-tenglamalar sistemasi diskret Viner-Xopf tipidagi tenglamalar sistemasi bo‘lib, bu turdagi sistemalarning yechimi mavjud va yagonaligi va bu yechim $\|\ell\|$ normaga mininimum berishi S.L. Sobolev va X.M. Shadimetovlar tomonidan isbotlangan.

Teorema 2. Kompleks-qiymatli, eng kichik musbat davri 1 bo‘lgan funksiyalarning $\tilde{W}_2^{(1,0)}(0,1]$ fazosida (6)-ko‘rinishidagi kvadratur formulalar orasida, koeffisiyentlari quyidagicha ko‘rinishda bo‘lgan yagona optimal kvadratur formula mavjud

$$\overset{\circ}{C}_k=\frac{2K_{\omega,1}}{4\pi^2\omega^2+1}\cdot e^{2\pi i\omega hk}\quad k=1,2,...,N, \qquad (14)$$

bunda

$$K_{\omega,1}=\frac{e^{2h}+1-2e^h\cos(2\pi\omega h)}{e^{2h}-1}.$$

Teorema 2 dan quyidagi natijalar kelib chiqadi:

Natija 1. Teorema 2 dagi, (14)-ko‘rinishdagi optimal koeffisiyentlar, $\omega h\in\mathbb{Z}$ bo‘lganda quiyidacha ifodalanadi

$$\overset{\circ}{C}_k = \frac{2}{4\pi^2\omega^2+1} \cdot \frac{e^h-1}{e^h+1}, \quad k=1,2,...,N.$$

Natija 2. (14)-ko‘rinishdagi optimal koeffisiyentlar, $\omega = 0$ da quyidagi ko‘rinishga ega

$$\overset{\circ}{C}_k = \frac{2\left(e^h-1\right)}{e^h+1}, \quad k=1,2,...,N.$$

$\tilde{W}_2^{(m,m-1)}(0,1]$ fazosida $m \ge 2$ da (6)-kvadratur formulaning koeffisiyentlari uchun quyidagi chiziqli tenglamalar sistemasini olamiz

$$\frac{\partial L}{\partial \overline{C}_{k'}} = -e^{2\pi i \omega h k'} \cdot \kappa_m(\omega) + \sum_{k=1}^{N} C_k \sum_{\beta \ne 0} e^{2\pi i \beta h(k-k')} \kappa_m(\beta) = 0, \; k'=1,2,\ldots,N, \tag{15}$$

$$\frac{\partial L}{\partial \mu} = \int_0^1 e^{2\pi i \omega x} dx - \sum_{k'=1}^{N} C_{k'} = 0, \tag{16}$$

bu yerda $\kappa_m(\cdot)$ kattalik (10)-formula bilan aniqlangan.

(15),(16)-tenglamalar sistemasining yechimi uchun quyidagi teoremalar o‘rinli.

Teorema 3. Kompleks-qiymatli davriy funksiyalarning $\tilde{W}_2^{(m,m-1)}(0,1]$ fazosida $m \ge 2$, $\omega \in Z \setminus \{0\}$ va $\omega h \notin Z$ holida (6)-optimal kvadratur formulaning koeffisiyentlari quyidagicha ko‘rinishda bo‘ladi

$$\overset{\circ}{C}_k = \frac{2K_{\omega,m}}{(2\pi\omega)^{2m} + (2\pi\omega)^{2m-2}} \cdot e^{2\pi i \omega h k}, \quad k=1,2,...,N. \tag{17}$$

bu yerda

$$K_{\omega,m} = (-1)^{m-1}\left[\frac{e^{2h}-1}{e^{2h}+1-2e^h\cos(2\pi\omega h)} + \sum_{n=1}^{m-1} \frac{2h^{2n-1} \cdot \lambda E_{2n-2}(\lambda)}{(2n-1)!(1-\lambda)^{2n}}\right]^{-1},$$

$\lambda = e^{2\pi i \omega h}$ va $E_{2n-2}(\lambda)$ bu – $(2n-2)$ darajali Eyler-Frobenius ko‘phadi.

Teorema 3 dan quyidagi natijani olamiz:

Natija 3. (6)-optimal kvadratur formulaning (17)-koeffisiyentlari $\omega \in \square \setminus \{0\}$ va $\omega h \notin Z$ holi uchun $\tilde{W}_2^{(2,1)}\left(0,1\right]$ fazoda quyidagicha ko‘rinishda bo‘ladi

$$\overset{\circ}{C}_k = \frac{K_{\omega,2}}{8\pi^4\omega^4 + 2\pi^2\omega^2} \cdot e^{2\pi i \omega h k}, \quad k=1,2,...,N,$$

bunda

$$K_{\omega,2} = -\left[\frac{e^{2h}-1}{e^{2h}+1-2e^h\cos(2\pi\omega h)} + \frac{h}{\cos(2\pi\omega h)-1}\right]^{-1}.$$

Teorema 4. Ushbu $\tilde{W}_2^{(m,m-1)}(0,1]$ fazoda $m \ge 2$, $\omega \ne 0$ va $\omega h \in Z$ uchun (6)-optimal kvadratur formulaning koeffisiyentlari nolga teng

$$\overset{\circ}{C}_k = 0, \quad k=1,2,...,N. \tag{18}$$

Ushbu paragrafda shuningdek, $m \ge 2$ va $\omega = 0$ holi alohida qaralgan.

Tasdiq 1. Davriy funksiyalarning $\overset{\circ}{W}{}_2^{(m,m-1)}(0,1]$ haqiqiy qiymatli fazosida $m \geq 2$ uchun, quyidagi ko‘rinishdagi kvadratur formulaning

$$\int_0^1 \varphi(x)dx \cong \sum_{k=1}^{N} C_k \varphi(x_k), \qquad (19)$$

optimal koeffisiyentlari uchun quyidagi tenglik o‘rinli

$$\overset{\circ}{C}_k = h, \qquad k = 1, 2, \ldots N.$$

Shunday qilib, 2-masala yechildi.

Ikkinchi bobning to‘rtinchi paragrafida $m \in \mathrm{N}$ va $\omega \in \mathrm{Z}$ ning barcha holatlarini qamrab oladigan qilib qurilgan optimal kvadratur formulalarning xatoliklarining aniq yuqori chegaralarini, ya’ni xatolik funksionallari normalarini hisoblaymiz.

Dastlab $m = 1$ holatini qaraymiz. Bu holat uchun quyidagi teorema bajariladi.

Teorema 5. Ushbu $\overset{\circ}{W}{}_2^{(1,0)*}(0,1]$ fazoda $\omega \in \mathrm{Z}$ uchun (6)-optimal kvadratur formulaning xatolik funksionali normasi quyidagi ko‘rinishga ega

$$\left\| \overset{\circ}{\ell} \right\|^2_{\overset{\circ}{W}{}_2^{(1,0)*}} = \frac{1}{4\pi^2\omega^2 + 1}\left[1 - \frac{2}{4\pi^2\omega^2 + 1} \cdot \frac{e^{2h} + 1 - 2e^h \cos(2\pi\omega h)}{h\left(e^{2h} - 1\right)}\right]. \qquad (20)$$

Endi, $m \geq 2$ holida, $\|\ell\|_{\overset{\circ}{W}{}_2^{(m,m-1)*}}$ norma uchun quyidagi natijani olamiz.

Teorema 6. Ushbu $\overset{\circ}{W}{}_2^{(m,m-1)*}$ fazoda $m \geq 2$, $\omega \in \mathrm{Z} \setminus \{0\}$ va $\omega h \notin \mathrm{Z}$ uchun (6)-optimal kvadratur formulaning xatolik funksionali normasi quyidagicha aniqlanadi

$$\left\| \overset{\circ}{\ell} \right\|^2_{\overset{\circ}{W}{}_2^{(m,m-1)*}} = \frac{1}{(2\pi\omega)^{2m} + (2\pi\omega)^{2m-2}} \cdot \left[1 - \frac{1}{h} \cdot \frac{2K_{\omega,m}}{(2\pi\omega)^{2m} + (2\pi\omega)^{2m-2}}\right], \qquad (21)$$

bunda

$$K_{\omega,m} = (-1)^{m-1}\left[\frac{e^{2h} - 1}{e^{2h} + 1 - 2e^h \cos(2\pi\omega h)} + \sum_{n=1}^{m-1} \frac{2h^{2n-1} \cdot \lambda E_{2n-2}(\lambda)}{(2n-1)!(1-\lambda)^{2n}}\right]^{-1},$$

$\lambda = e^{2\pi i \omega h}$ va $E_{2n-2}(\lambda)$ – $(2n-2)$ darajali Eyler-Frobenius ko‘phadi.

Natija 4. Ushbu limitni inobatga olib

$$\lim_{h \to 0} \frac{K_{\omega,m}}{h} = \left[\frac{2}{(2\pi\omega)^{2m} + (2\pi\omega)^{2m-2}}\right]^{-1},$$

quyidagilarga ega bo‘lamiz

1. $h \to 0$ *da*

$$\left\| \overset{\circ}{\ell} \right\|^2_{\overset{\circ}{W}{}_2^{(m,m-1)*}} \to 0,$$

2. $\omega \to \infty$ va fiksirlangan h larda

$$\left\| \overset{\circ}{\ell} \right\|^2_{\overset{\circ}{W}{}_2^{(m,m-1)*}} \to 0.$$

Teorema 7. Ushbu $\tilde{W}_2^{(m,m-1)}$ fazoda $m \ge 2$ va $\omega h \in Z \setminus \{0\}$ holda (6)-optimal kvadratur formulaning xatolik funksionali normasi quyidagi ko‘rinishda bo‘ladi

$$\left\| \overset{\circ}{\ell} \right\|^2_{\tilde{W}_2^{(m,m-1)*}} = \frac{1}{(2\pi\omega)^{2m} + (2\pi\omega)^{2m-2}}.$$

Teorema 6 va 7, hamda Natija 4 dan $\tilde{W}_2^{(m,m-1)}$ fazoda qurilgan (6)-optimal kvadratur formulalarning yaqinlashish tartibi $|\omega| < N$ uchun $O(h^m)$ va $|\omega| \ge N$ uchun $O(|\omega|^{-m})$ ekanini xulosa qilishimiz mumkin.

Teorema 8. Ushbu $\tilde{W}_2^{(m,m-1)}$ fazoda $m \ge 2$ va $\omega = 0$ holda (19)-optimal kvadratur formulaning xatolik funksionali normasi quyidagi ko‘rinishda bo‘ladi

$$\left\| \overset{\circ}{\ell} \right\|^2_{\tilde{W}_2^{(m,m-1)*}} = (-1)^{m-1} \sum_{n=2m}^{\infty} \frac{B_n}{n!} \cdot h^n = \frac{|B_{2m}|}{(2m)!} h^{2m} - \frac{|B_{2m+2}|}{(2m+2)!} h^{2m+2} + O(h^{2m+4}),$$

bu yerda B_n bu – Bernulli soni.

Shunday qilib, 3-masala to‘liq yechildi.

Ikkinchi bobning beshinchi paragrafida $\tilde{W}_2^{(m,m-1)}$ fazoda qurilgan optimal kvadratur formulalarning xatolik funksionali normasining qiymati, $m=1$ va $m=2$ hollari uchun sonli natijalarda keltirilgan. Natijalar Maple dasturidan foydalanib olingan.

Dissertatsiyaning “ $W_2^{(m,m-1)}(a,b)$ **fazoda Furye integrallarini sonli hisoblash uchun effektiv kvadratur formulalar”** deb nomlanuvchi uchinchi bobida differensiallanuvchi funksiyalarning $W_2^{(m,m-1)}(a,b)$ Gilbert fazosida Furye integrallarini taqribiy hisoblash uchun effektiv kvadratur formulalar qurish bilan shug‘ullanamiz.

Uchinchi bobning birinchi paragrafida $W_2^{(m,m-1)}(a,b)$ fazoda $\omega \in R$ da

$$I_\varphi(\omega) = \int_a^b e^{2\pi i \omega x} \varphi(x) dx \tag{22}$$

integrallarni taqribiy hisoblash uchun quyidagi ko‘rinishdagi effektiv kvadratur formulalar beramiz

$$\int_a^b e^{2\pi i \omega x} \varphi(x) dx \cong \sum_{k=0}^{N} C_{k,\omega}[a,b] \varphi(hk + a), \tag{23}$$

bunda $\omega \in R$, $C_{k,\omega}[a,b]$ - koeffisiyentlar va $h = (b-a)/N$.

(23)-effektiv kvadratur formulaning koeffisiyentlarini topish uchun ikkinchi bobda qurilgan (6)-optimal kvadratur formulalarning koeffisiyentlarini ω ning funksiyasi sifatida $\omega \in$ gacha davom ettirib, quyidagi natijani olamiz.

Teorema 9. Ushbu $W_2^{(m,m-1)}(a,b)$ fazoda $m \ge 2$ uchun (23)-effektiv kvadratur formulalarning $C_{k,\omega}[a,b]$ koeffisiyentlari $\omega \in R \setminus \{0\}$ va $\omega h \notin Z$ da

$$C_{0,\omega}[a,b] = \frac{(b-a)K_{\omega,m}}{(2\pi\omega_1)^{2m} + (2\pi\omega_1)^{2m-2}} \cdot e^{2\pi i \omega a},$$

$$C_{k,\omega}\left[a,b\right]=\frac{2(b-a)K_{\omega,m}}{(2\pi\omega_1)^{2m}+(2\pi\omega_1)^{2m-2}}\cdot e^{2\pi i\omega(hk+a)},\ k=1,2,\ldots,N-1, \tag{24}$$
$$C_{N,\omega}\left[a,b\right]=\frac{(b-a)K_{\omega,m}}{(2\pi\omega_1)^{2m}+(2\pi\omega_1)^{2m-2}}\cdot e^{2\pi i\omega b},$$

va $\omega=0$ uchun quyidagi ko‘rinishga ega

$$C_{0,\omega}\left[a,b\right]=\frac{h}{2},$$
$$C_{k,\omega}\left[a,b\right]=h,\quad k=1,2,\ldots,N-1, \tag{25}$$
$$C_{N,\omega}\left[a,b\right]=\frac{h}{2},$$

bunda

$$K_{\omega,m}=(-1)^{m-1}\cdot\left[\frac{e^{\frac{2h}{b-a}}-1}{e^{\frac{2h}{b-a}}+1-2e^{\frac{h}{b-a}}\cos(2\pi\omega h)}+\sum_{n=1}^{m-1}\frac{2h^{2n-1}\cdot\lambda E_{2n-2}(\lambda)}{(2n-1)!\cdot(b-a)^{2n-1}\cdot(1-\lambda)^{2n}}\right]^{-1},$$

va $\omega_1=(b-a)\omega,\ h=\dfrac{b-a}{N},$ $E_{2n-2}(\lambda)$ bu – $(2n-2)$ darajali Eyler-Frobinius ko‘phadi va $\lambda=e^{2\pi i\omega h}$.

Teorema 10. Ushbu $W_2^{(m,m-1)}\left(a,b\right)$ fazoda $m\geq 2$ uchun (23)-effektiv kvadratur formulalarning $C_{k,\omega}[a,b]$ koeffisiyentlari $\omega\in\mathbb{R}\setminus\{0\}$ va $\omega h\in Z$ da quyidagicha ifodalanadi

$$C_{k,\omega}=0,\quad k=0,1,\ldots,N.$$

Uchinchi bobning ikkinchi paragrafida diskret Furye almashtirishi formulasi va xossalari keltirilgan.

Uchinchi bobning uchinchi paragrafida (23)-ko‘rinishidagi effektiv kvadratur formulalarning $m=2$ holi ayrim funksiyalarning Furye almashtirishlarini taqribiy hisoblashda qo‘llanilgan. Natijalar MATLAB dasturining *fft* standart funksiyasi yordamida hisoblangan qiymatlari bilan solishtirilib, effektiv kvadratur formula yordamida olingan natijalarning xatoliklari *fft* funksiyasi ning xatoligiga nisbatan kichik ekanligi ko‘rsatilgan.

XULOSA

Dissertatsiya ishi ikki qismdan iborat bo‘lib, birinchi qism kompleks qiymatli, differensiallanuvchi davriy funksiyalarning $\tilde{W}_2^{(m,m-1)}(0,1]$ fazosida Furye koeffisiyentlarini taqribiy hisoblash uchun optimal kvadratur formulalar qurish va ularning aniq yuqori chegaralarini baholashga, hamda ikkinchi qism kompleks qiymatli funksiyalarning $W_2^{(m,m-1)}(a,b)$ fazosida Furye integrallarini taqribiy hisoblash uchun effektiv kvadratur formulalar qurishga bag‘ishlangan.

Tadqiqot ishining asosiy natijalari quyidagilardan iborat:

1. Kompleks qiymatli, differensiallanuvchi davriy funksiyalarning $\tilde{W}_2^{(m,m-1)}(0,1]$ Gilbert fazosida optimal kvadratur formulalarning extremal funksiyalari topilgan.
2. Davriy funksiyalarning $\tilde{W}_2^{(m,m-1)}(0,1]$ Gilbert fazosida kvadratur formulalarning ekstremal funksiyalaridan foydalanib xatolik funksionallari normalarining analitik ko‘rinishlari $m=1$ va $m\geq 2$ holatlari uchun alohida topilgan.
3. Davriy funksiyalarning $\tilde{W}_2^{(1,0)}(0,1]$ Gilbert fazosida $\omega \in \mathbb{R}$ holatida Furye koeffisiyentlarini taqribiy hisoblash uchun optimal kvadratur formulaning koeffisiyentlari topilgan.
4. Davriy funksiyalarning $\tilde{W}_2^{(m,m-1)}(0,1]$ Gilbert fazosida $m\geq 2$ da quyidagi uchta $\omega \in \mathbb{R}\setminus\{0\}$ va $\omega h \notin \mathbb{Z}$; $\omega \in \mathbb{R}\setminus\{0\}$ va $\omega h \in \mathbb{Z}$, hamda $\omega = 0$ alohida hollarida Furye koeffisiyentlarini taqribiy hisoblash uchun optimal kvadratur formulalarning koeffisiyentlari topilgan.
5. $\tilde{W}_2^{(m,m-1)*}$ qo‘shma fazosida optimal kvadratur fomulaning $\mathring{\ell}$ xatolik funksionali normasining kvadrati hisoblangan va $m=1$ va $m=2$ hollarida sonli natijalar olingan.
6. O‘tkazilgan sonli eksprimentlar natijalarida optimal kvadratur fomulalarning yaqinlashish tartibi $O\left(\left(\frac{1}{N+|\omega|}\right)^m\right)$ ekanligi ko‘rsatilgan.
7. $W_2^{(m,m-1)}(a,b)$ fazosida Furye integrallarini taqribiy hisoblash uchun effektiv kvadratur formulalar qurilgan.
8. $W_2^{(2,1)}(-1,1)$ fazosida qurilgan effektiv kvadratur formuladan foydalanib, ayrim funksiyalarning Furye almashtirishlari hisoblangan.

SCIENTIFIC COUNCIL AWARDING SCIENTIFIC DEGREES DSc.03/30.12.2019.FM.01.02 NATIONAL UNIVERSITY OF UZBEKISTAN

V.I.ROMANOVSKIY INSTITUTE OF MATHEMATICS

KHAYRIEV UMEDJON NARMON UGLI

OPTIMAL METHODS FOR APPROXIMATE CALCULATION OF INTEGRALS OF DIFFERENTIABLE PERIODIC FUNCTIONS IN A HILBERT SPACE

01.01.03 – Computational and discrete mathematics

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT – 2023

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Introduction (abstract of doctoral dissertation)

Actuality and demand of the theme of the dissertation. As a result of many scientific and practical studies carried out on a global scale, solving problems of image analysis, modulation and demodulation of signals for communication systems, computed tomography in industry and medicine is reduced to the calculation of some integrals of strongly oscillating functions. Standard methods of numerical integration for the approximate calculation of strongly oscillatory integrals require a large amount of computational work, and their direct application in practice does not give effective results. Therefore, the development of special methods for the approximate calculation of such integrals, the creation of new methods for the approximate calculation of these integrals in various classes of functions, and the estimation of their errors are considered one of the important problems of Computational Mathematics.

Nowadays, the construction of optimal quadrature formulas in the approximate calculation of strongly oscillating integrals is of great importance in the world. In particular, in the numerical integration of Fourier coefficients and integrals, the construction of optimal quadrature formulas in the space of periodic functions and the estimation of their errors are widely used. Actual problems of Computational Mathematics is the construction of effective quadrature formulas in various spaces of differentiable functions for approximating strongly oscillating integrals. In this regard, the construction of asymptotically optimal and optimal quadrature formulas for the approximate calculation of strongly oscillating integrals, as well as estimating their errors in the space of differentiable periodic functions, is one of the targeted scientific studies.

In our country, great attention is paid to such important directions as image analysis, electrodynamics, fluid mechanics and mathematical physics, geology, geophysics, and effective reconstruction of computed tomography images, which have scientific and practical applications of fundamental sciences. Especially, a lot of attention is paid to the theory of numerical integration in computational mathematics, especially the construction of optimal quadrature and cubature formulas, special attention is paid to the evaluation of their errors in the Banach and Hilbert spaces of periodic and non-periodic functions. Important results were achieved on the construction of lattice optimal cubature formulas for the approximate calculation of regular and singular weighted integrals in the Sobolev spaces of univariate and multivariable, periodic and non-periodic functions. in the activities of V.I.Romanovskiy Institute of Mathematics named after of the Academy of Sciences of the Republic of Uzbekistan, is one of the main problems[2]. In order to ensure decision execution, it is important to construct optimal quadrature formulas for the approximate calculation of strongly oscillating integrals and to estimate their errors in different Hilbert spaces of differentiable, periodic functions.

[2] Resolution of the Cabinet of Ministers of the Republic of Uzbekistan No. 292 "On measures to organize the activities of the newly created research institutions of the Academy of Sciences of the Republic of Uzbekistan" dated May 18, 2017.

This dissertation work is intended to solve the problems outlined in the Decree of the President of the Republic of Uzbekistan No.DP - 4947 dated February 7, 2017 “About the Action Strategy for the further development of the Republic of Uzbekistan”, in resolutions No.RP - 2789 dated February 17, 2017 “On measures to further improve the activities of the Academy of Sciences, organization, management and financing of research activities”, No.RP - 2909 dated April 20, 2017 “On measures for the further development of the higher education system”, No.RP - 3682 dated April 27, 2018 “On measures to further improve the system for the practical implementation of innovative ideas, technologies and projects”, №RP - 4708 of May 07, 2020 “On measures to improve the quality of education and the development of scientific research in the field of mathematics”, as well as in other regulatory legal acts related to this area activities.

Connection of research to priority directions of development of science and technologies of the Republic. This study was performed in accordance with the priority areas of science and technology of Republic of Uzbekistan IV “Mathematics, Mechanics and Computer Science”.

The degree of scrutiny of the problem. It is known that many problems of science and technology are brought to the calculation of certain integrals of strongly oscillating functions, especially, Fourier coefficients and integrals. Since it is not always possible to calculate these integrals using analytical methods, their numerical integration is required. Several special methods have been developed for the approximate calculation of strongly oscillatory integrals. Examples include the Filon method, asymptotic expansion method, Levin's collocation method, methods of steepest descent, and optimal quadrature and cubature formulas methods. Initially, Filon's method, based on piecewise approximation of the appropriate function with parabolic arcs in the interval of integration. Later, Filon-type, Clanshaw-Curtis-Filon-type, modified Clanshaw-Curtis, Levin-type, Gauss-Laguerre quadrature formulas and generalized quadrature formulas were developed for numerical calculation of integrals with different types of strongly oscillating functions. In recent years, scientists such as L. Zhang, A. Asheim, V. Dominguez, E.A. Flinn, J. Gao, J.M. Melenk, K.N. Melnik and R.V.N. Melnik, H. Mo and Sh. Xiang, L.F. Shampine, G. He, and Y.J. Cho, H. Kang, M.I. Israilov, B. Eshdavlatov and S.A. Bakhromov carried out research in these methods. Scientists such as D.Huybrechs, Sh. Olver, A. Iserles, S.P. Nørsett, S.-I.S. Zaman, S.I.U. Nasib, S. Olver, and H. Wang conducted scientific research on the asymptotic expansion method.

There are the Sobolev method, spline functions, and ϕ -functions methods for constructing optimal quadrature and cubature formulas for approximate calculation of definite integrals of given functions in certain Banach spaces. Initially, S.L. Sobolev was engaged in the theory of constructing optimal quadrature and cubature formulas using the concept of a discrete analogue of a linear differential operator in the space $L_2^{(m)}(\Omega)$. The algorithm for construction of optimal quadrature and cubature formulas proposed by Academician S.L. Sobolev was developed by Z.J. Jamalov, F.Y. Zagirova, Kh.M. Shadimetov, A.R. Hayotov, G.V. Milovanović,

F. Lanzara, F.A. Nuraliev, D.M. Akhmedov, S.S. Azamov and A.K. Bolatev in spaces $L_2^{(m)}(\Omega)$, $K_2(P_m)$, $K_2^{\overline{(m,m-2)}}$ and $W_2^{(m,0)}$. In 1999, in Kh.M. Shadimetov's work, lattice cubature formulas for weighted integrals in the Sobolev space of periodic functions were constructed. In particular, I. Babuška's optimal quadrature formula is derived from this work, which is constructed for the approximate calculation of Fourier coefficients when the weight function is $\exp(i\sigma x)$. In 2015, E. Novak, M. Ullrich and H. Woźniakowski studied the approximate calculations of one-variable oscillating integrals in the space H^s of standard Sobolev of periodic and non-periodic functions.

It should be noted that in recent years, in the Hilbert spaces $L_2^{(m)}$ and $W_2^{(m,m-1)}$ Kh.M. Shadimetov, G.V. Milovanović, A.R. Hayotov, N.D. Boltaev, C.-O. Lee, S. Jeon, S.S. Babaev, and B.I. Bozarov carried out scientific research on the construction of optimal quadrature formulas for approximate calculation for strongly oscillatory integrals and their practical applications. As a result, they have achieved high-resolution reconstruction of computed tomography images in the industrial and medical fields under laboratory conditions.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out.

The dissertation research was carried out within the framework of the calendar plan of the Laboratory of Computational Mathematics of V.I. Romanovskiy Institute of Mathematics of the Academy of Sciences of the Republic of Uzbekistan on the topic "Construction of optimal quadrature, interpolation, difference formulas and their application to solving of integral equations in Hilbert spaces".

The aim of research work consists of constructing optimal quadrature formulas for the numerical calculation of Fourier coefficients in the complex-valued Hilbert space $\tilde{W}_2^{(m,m-1)}(0,1]$ of differentiable periodic functions and estimating their errors, and obtaining effective quadrature formulas for the numerical calculation of Fourier integrals in the complex-valued Hilbert space $W_2^{(m,m-1)}(a,b)$ of differentiable functions.

Research problems:

finding the extremal function of quadrature formulas for numerical calculation of Fourier coefficients in complex-valued Hilbert space $\tilde{W}_2^{(m,m-1)}(0,1]$ of differentiable, periodic functions;

finding the analytic form of the norm for the error functional of quadrature formulas for numerical calculation of Fourier coefficients in complex-valued Hilbert space $\tilde{W}_2^{(m,m-1)}(0,1]$ of differentiable, periodic functions;

finding optimal coefficients of quadrature formulas that give the smallest value to the norm of the error functional for numerical calculation of Fourier coefficients in the complex-valued Hilbert space $\tilde{W}_2^{(m,m-1)}(0,1]$ of differentiable, periodic functions;

calculating the norm of the error functional of optimal quadrature formulas for numerical calculation of Fourier coefficients in the complex-valued Hilbert space $\widetilde{W}_2^{(m,m-1)}(0,1]$ of differentiable periodic functions;

finding the coefficients of the effective quadrature formulas for the approximate calculation of Fourier integrals in the Hilbert space $W_2^{(m,m-1)}(a,b)$ of complex-valued functions.

The research object is the Hilbert space of differentiable, periodic functions, optimal and effective quadrature formulas, strongly oscillatory integrals.

The research subject consists of extremal functions, Hilbert space $\widetilde{W}_2^{(m,m-1)}(0,1]$ of differentiable periodic functions, and exponentially weighted optimal and effective quadrature formulas for approximate calculation of Fourier coefficients and integrals.

Research methods. The methods of computational mathematics, functional analysis, theory of generalized functions, theory of differential equations, and theory of functions with discrete arguments are used in the scientific research work.

Scientific novelty of the research work:

the extremal functions of quadrature formulas for numerical calculation of Fourier coefficients in complex-valued Hilbert space $\widetilde{W}_2^{(m,m-1)}(0,1]$ of differentiable periodic functions are found;

in order to numerical calculation of the Fourier coefficients in the Hilbert space $\widetilde{W}_2^{(m,m-1)}(0,1]$ of complex-valued, differentiable periodic functions, the analytic form of the norm for the error functional of the quadrature formulas are found;

the coefficients of the optimal quadrature formulas for numerical calculation of Fourier coefficients in the Hilbert space $\widetilde{W}_2^{(m,m-1)}(0,1]$ of complex-valued, differentiable periodic functions is found;

the norm of the error functional of optimal quadrature formulas for numerical calculation of Fourier coefficients in the Hilbert space $\widetilde{W}_2^{(m,m-1)}(0,1]$ of complex-valued, differentiable periodic functions is calculated, and numerical results are given for $m=1$ and $m=2$;

the coefficients of the effective quadrature formulas for the approximate calculation of Fourier integrals in the Hilbert space $W_2^{(m,m-1)}(a,b)$ of complex-valued functions are found.

Practical results of the research are as follows:

Fourier coefficients are numerically calculated using the constructed optimal quadrature formulas;

using the obtained effective quadrature formulas, Fourier integrals are numerically calculated. The obtained results can be used in modulation and demodulation of signals, image processing and other practical sciences.

The reliability of the results of the study is based on the application of the theory of quadrature formulas, computational mathematics, functional analysis and the theory of functions with discrete arguments, the rigidity of mathematical considerations, as well as numerical experiments.

Scientific and practical significance of the research results. The scientific significance of the research results is explained by the algorithm for constructing optimal and effective quadrature formulas for the approximate calculation of strongly oscillating integrals in Hilbert spaces, and the high convergence speed of the constructed formulas.

The practical significance of the research results is that optimal and effective quadrature formulas can be used in the approximate solution of computed tomography and image analysis problems with the help of strongly oscillatory integrals.

Implementation of the research results. Based on the obtained scientific results on the construction of optimal quadrature formulas for the approximate calculation of strongly oscillating integrals in the Hilbert space of periodic functions:

constructed optimal quadrature formulas in Hilbert spaces $\tilde{W}_2^{(1,0)}(0,1]$ and $\tilde{W}_2^{(2,1)}(0,1]$ are used in the practical project no. PZ-20170930257 “Improvement of the hydrometric treatment process in the technology of extraction of graded flour from local wheat grains” (reference of Tashkent Institute of Chemical Technology dated April 14, 2023, No. 1/04-1177) to increase the accuracy of measuring the size of wheat grains (height, width and length). As a result, the numerical calculation of solutions of mathematical models representing the distribution of moisture occurring in hydrothermal devices, using these formulas, made it possible to find the optimal values of their geometric dimensions with high accuracy;

in the Hilbert space $\tilde{W}_2^{(3,2)}(0,1]$ of complex-valued functions, constructed optimal quadrature formula is used in the fundamental project no. OT-F4–02 – “Thermodynamics of models of mathematical physics with an infinite set of states” (reference of Bukhara State University dated April 25, 2023, No. 04/752) for the approximate calculation of integrals in mixed problems posed for mathematical physics equations. As a result, it made it possible to find a numerical solution to the set of mixed problems with high accuracy.

Approbation of the research results. The main results of this research were discussed at 11 scientific and practical conferences, including 7 international and 4 national scientific and practical conferences.

Publications of the research results. On the topic of the dissertation, 19 scientific papers were published, 6 of which are included in the list of scientific publications proposed by the Higher Attestation Commission of the Republic of Uzbekistan for the defence of theses of the Doctor of Philosophy, including 2 of them published in foreign journals and 4 in national scientific journals, as well as the certificate of authorship for a computer program.

The structure and volume of the dissertation. The dissertation work consists of the introduction, three chapters, conclusion, bibliography. The general volume of the thesis is 92 pages.

THE MAIN CONTENT OF THE DISSERTATION

In the introduction the motivation of the research theme and correspondence to the priority research areas of science and technology of the Republic is given, we present a review of international research on the theme of the dissertation and degree of scrutiny of the problem, formulate our goals and objectives, identify the object and subject of study, and state the scientific novelty and practical results of the research. Furthermore, we give the theoretical and practical importance of the obtained results, and also give information on the implementation of the research results, the published works and the structure of the dissertation.

Chapter I of the PhD thesis entitled **"Numerical integration formulas in Hilbert spaces"** is mainly of an introductory nature, it presents the main concepts and definitions used in the dissertation. Furthermore, the scientific research and the obtained results are described within the scope of this scientific research theme.

The first section of this chapter introduces Hilbert spaces and gives examples of these spaces. First, the Hilbert space $W_2^{(m,m-1)}(0,1)$ we work with and the concepts of an inner product and a norm in it are presented.

We consider the following space

$W_2^{(m,m-1)}(0,1) := \{\varphi : [0,1] \to \mathrm{C} \mid \varphi^{(m-1)}$ abs. cont. and $\varphi^{(m)} \in L_2(0,1)\}$,

is the Hilbert space of complex-valued functions which are $(m-1)^{\text{st}}$ order derivative is absolute continuous and m^{the} order derivative (in the generalized sense) are square integrable, with the inner product

$$\langle \varphi, \psi \rangle_{W_2^{(m,m-1)}} = \int_0^1 (\varphi^{(m)}(x) + \varphi^{(m-1)}(x))(\overline{\psi}^{(m)}(x) + \overline{\psi}^{(m-1)}(x))dx, \tag{1}$$

where $\overline{\psi}$ is the complex conjugate of the function ψ. The space $W_2^{(m,m-1)}(0,1)$ together with the inner product (1) forms the Hilbert space. The norm of a function $\varphi \in W_2^{(m,m-1)}(0,1)$ corresponding to the inner product (1) is introduced as follows

$$\|\varphi\|_{W_2^{(m,m-1)}} = \langle \varphi, \varphi \rangle_{W_2^{(m,m-1)}}^{1/2}, \tag{2}$$

where $\int\limits_0^1 \left|\varphi^{(m)}(x) + \varphi^{(m-1)}(x)\right|^2 dx < \infty$.

Equality (2) is a semi-norm and $\|\varphi\|_{W_2^{(m,m-1)}} = 0$ if and only if $\varphi(x) = P_{m-2}(x) + de^{-x}$, where $P_{m-2}(x)$ is a polynomial of degree $(m-2)$ and d is a constant. Every element of the space $W_2^{(m,m-1)}$ is a class of functions that are differ from each other by a linear combination of a polynomial of degree $(m-2)$ and e^{-x}.

The second section of the first chapter presents Bernoulli numbers and polynomials and their properties, as well as periodic Bernoulli polynomials, which are important in this scientific work.

We consider the well-known formula

$$\frac{z}{e^z - 1} = \sum_{n=0}^{\infty} \frac{B_n}{n!} z^n, \quad |z| < 2\pi,$$

where B_n is the Bernoulli number. All Bernoulli numbers with odd index except B_1 vanish

$$B_{2k+1} = 0 \quad \text{for} \quad k = 1, 2, \dots, .$$

and the first nonvanishing Bernoulli numbers are

$$B_0 = 1,\ B_1 = -\frac{1}{2},\ B_2 = \frac{1}{6},\ B_4 = -\frac{1}{30},\ B_6 = \frac{1}{42},\ B_8 = -\frac{1}{30},\ B_{10} = \frac{5}{66}.$$

The Bernoulli polynomials are expressed as follows

$$\frac{ze^{xz}}{e^z - 1} = \sum_{n=0}^{\infty} \frac{B_n(x)}{n!} z^n, \quad |z| < 2\pi.$$

The periodic Bernoulli polynomials are defined by the following formula

$$\tilde{B}_n(x) = B_n(x), \quad 0 \le x < 1,$$

and

$$\tilde{B}_n(x+1) = B_n(x), \quad x \in \mathbb{R}.$$

Periodic functions and their properties are mentioned in the third section of the first chapter.

Definition 1. If there exists a non-zero real number T for the function $f(x)$ defined in all real values of the variable x which satisfies the equality

$$f(x+T) = f(x) \ \text{ for all } x \in \mathbb{R},$$

then $f(x)$ is a periodic function, and the number T is called its period.

The fourth section of Chapter I presents the problem of constructing optimal quadrature formulas in Banach space B.

Quadrature formulas of the following form

$$\int_a^b p(x)\varphi(x)dx \cong \sum_{\beta=0}^{N} C_\beta \varphi(x_\beta), \tag{3}$$

were considered by S.L. Sobolev together with the error functional

$$\ell_N(x) = p(x)\varepsilon_{[a,b]}(x) - \sum_{\beta=0}^{N} C_\beta \delta(x - x_\beta),$$

where $p(x)$ is weigted function, and it satisfies $\int\limits_a^b p(x)dx < \infty$, x_β are nodes, C_β are coefficients, $\varphi(x) \in B$ and B is a Banach space is compactly embedded in the class of continuous functions, i.e., $B \to C(a,b)$.

The error of the quadrature formula (3) is as follows

$$\left(\ell_N, \varphi\right) = \int_a^b p(x)\varphi(x)dx - \sum_{\beta=0}^{N} C_\beta \varphi(x_\beta), \tag{4}$$

where $\left(\ell_N, \varphi\right)$ is the value of the error functional ℓ_N on the function φ.

The absolute value of the error (4) for the quadrature formula (3) is estimated by the Cauchy-Schwarz inequality as follows

$$\left|\left(\ell_N, \varphi\right)\right| \le \|\ell_N\|_{B^*} \cdot \|\varphi\|_B .$$

Definition 2. For all $\varphi \in B$ the upper bound of the absolute value of error (4), i.e., the quadrature formula that gives the smallest value in terms of the coefficients and nodes of the norm $\|\ell_N\|_{B^*}$, in the considered space is called *the optimal quadrature formula in the sense of Nikolskii* or *the best optimal quadrature formula*.

Definition 3. With the fixed nodes, coefficients C_β that give the minimum value to the norm $\|\ell_N\|_{B^*}$ are called optimal coefficients and are denoted as $\overset{\circ}{C}_\beta$. Together with these optimal coefficients, the quadrature formula of the form (3) is called *the optimal quadrature formula in the sense of Sard*.

Next, we consider the problem of constructing optimal quadrature formulas in the sense of Sard. To solve this problem, it is necessary to solve the following problems in a row.

Problem A. Calculate the norm $\|\ell_N\|_{B^*}$ for error functional ℓ_N of the quadrature formula (3).

Problem B. Find the coefficients C_β that give the minimum value to the quantity $\|\ell\|_{B^*}$, and calculate the following quantity

$$\|\overset{\circ}{\ell}\|_{B^*} = \inf_{C_\beta}\|\ell\|_{B^*}.$$

The fifth section of the first chapter discusses the methods of approximate calculation of strong oscillatory integrals of the following form

$$I_{\varphi,g}(\omega) = \int_\Omega e^{2\pi i\omega g(x)}\varphi(x)dx, \tag{5}$$

where functions φ and g are non-oscillating functions, ω is oscillatory frequency and Ω a domain with piecewise continuous bound.

Since the standard quadrature formulas are inefficient, there is a direct alternative in the form of an asymptotic expansion. Based on the properties of the functions φ and g, various methods have been developed for the numerical calculation of strongly oscillating integrals (5). In the present section, we discuss the following methods: *the asymptotic expansion, the Filon method, Levin's collocation method*, *the steepest descent method,* and *optimal quadrature and cubature formulas*.

Chapter II of the PhD thesis entitled "**Optimal quadrature formulas in the Hilbert space $\tilde{W}_2^{(m,m-1)}(0,1]$ of periodic functions**" is devoted to the construction of optimal quadrature formulas in the Hilbert space of differentiable periodic functions.

In the first section of the chapter, the problem of constructing optimal quadrature formulas for the numerical calculation of Fourier coefficients in the space $\tilde{W}_2^{(m,m-1)}(0,1]$ is posed.

We denote by $\tilde{W}_2^{(m,m-1)}(0,1]$ the subspace of the space $W_2^{(m,m-1)}(0,1)$ consisting of complex-valued, 1-periodic functions $\varphi(x)$ on $(0,1]$, that is, every element of the space $\tilde{W}_2^{(m,m-1)}(0,1]$ satisfies the following condition of 1-periodicity

$$\varphi(x+\beta)=\varphi(x) \text{ for } x\in \mathbb{R} \text{ and } \beta\in \mathbb{Z}.$$

We consider a quadrature formula of the form

$$\int_0^1 e^{2\pi i\omega x}\varphi(x)dx \cong \sum_{k=1}^{N} C_k\varphi(hk), \tag{6}$$

where $\omega\in \mathbb{R}\setminus\{0\}$ is a parameter, φ belongs to the space $\tilde{W}_2^{(m,m-1)}(0,1]$, and C_k $(k=1,2,...,N)$ are coefficients of the quadrature formula, they are complex numbers, $i^2=-1$, and $N\in \mathbb{N}$, $h=1/N$.

The difference between the integral and the quadrature sum is called *the error* of the quadrature formula (6)

$$(\ell,\varphi)=\int_0^1 e^{2\pi i\omega x}\varphi(x)dx-\sum_{k=1}^{N} C_k\varphi(hk)=\int_0^1 \ell(x)\varphi(x)dx. \tag{7}$$

This difference defines a linear functional

$$\ell(x)=e^{2\pi i\omega x}-\sum_{k=1}^{N} C_k\sum_{\beta=-\infty}^{\infty}\delta(x-hk-\beta),$$

where $\delta(x)$ is the Dirac delta-function.

Since the error functional ℓ is defined on the space $\tilde{W}_2^{(m,m-1)}(0,1]$, the following equality is valid

$$(\ell,1)=0 \quad \text{for } m\geq 2. \tag{8}$$

This condition means the exactness of the quadrature formula (6) for any constant, and it can be written as follows $\int_0^1 e^{2\pi i\omega x}dx=\sum_{k=1}^{N} C_k$.

Using the Cauchy-Schwarz inequality, we obtain the sharp upper bound for the absolute value of the error (7)

$$|(\ell,\varphi)|\leq \|\ell\|_{\tilde{W}_2^{(m,m-1)*}}\cdot\|\varphi\|_{\tilde{W}_2^{(m,m-1)}},$$

where

$$\|\ell\|_{\tilde{W}_2^{(m,m-1)*}}=\sup_{\varphi,\|\varphi\|\neq 0}\frac{|(\ell,\varphi)|}{\|\varphi\|}.$$

We note that the coefficients C_k that give the minimum value to the norm of the error function are called optimal coefficients and are denoted as $\overset{\circ}{C}_k$. Together with these optimal coefficients, the quadrature formula of the form (6) is said to be *an optimal quadrature formula*.

The minimum of the norm of the error functional corresponding to the optimal quadrature formula is denoted as follows

$$\left\|\overset{\circ}{\ell}\right\|_{\tilde{W}_2^{(m,m-1)*}}:=\inf_{C_k}\|\ell\|_{\tilde{W}_2^{(m,m-1)*}}.$$

Thus, to construct the optimal quadrature formula in form (6) in the space $\tilde{W}_2^{(m,m-1)}$ we should solve the following problems.

Problem 1. Find the analytical representation of the norm $\|\ell\|_{\tilde{W}_2^{(m,m-1)*}}$ for the error functional ℓ of the quadrature formula of the form (6).

Problem 2. Find the optimal coefficients $\overset{\circ}{C}_k$ that give the minimum value to the quantity $\|\ell\|_{\tilde{W}_2^{(m,m-1)*}}$.

Problem 3. Calculate the norm for the error functional $\overset{\circ}{\ell}$ of the optimal quadrature formula (6), that is, calculate the quantity $\inf\limits_{C_k}\|\ell\|_{\tilde{W}_2^{(m,m-1)*}}$.

In order to solve the above posted problem for all values of arbitrary $m \in \mathrm{N}$ and $\omega \in \mathrm{Z}$, we have to consider these problems separately in the following cases:

1. $m=1$ and $\omega \in \mathrm{Z}$;
2. $m \geq 2$ and $\omega \in \mathrm{Z}\setminus\{0\}$, $\omega h \notin \mathrm{Z}$;
3. $m \geq 2$ and $\omega h \in \mathrm{Z}\setminus\{0\}$;
4. $m \geq 2$ and $\omega = 0$.

In the second section of the second chapter, we deal with solving of Problem 1. For this, we use the extremal function of the error functional ℓ.

For finding the analytic form of the norm for the error functional ℓ, we use *the extremal function* ψ_ℓ satisfying the following equality

$$(\ell,\psi_\ell) = \|\ell\|_{\tilde{W}_2^{(m,m-1)*}} \cdot \|\psi_\ell\|_{\tilde{W}_2^{(m,m-1)}}.$$

The following assertion holds for the extremal function.

Using the Riesz representation theorem, we get the following differential equation

$$\overline{\psi_\ell}^{(2m)}(x) - \overline{\psi_\ell}^{(2m-2)}(x) = (-1)^m \ell(x). \tag{9}$$

Theorem 1. *The generalized solution of equation (9) is the extremal function* ψ_ℓ *of the error functional* ℓ *and it is expressed as*

a) *for* $m=1$ *with* $\omega \in \mathbb{Z}$

$$\psi_\ell(x) = e^{-2\pi i\omega x}\cdot\kappa_1(\omega) - \sum_{k=1}^{N}\overline{C}_k\sum_{\beta=-\infty}^{\infty} e^{2\pi i\beta(x-hk)}\cdot\kappa_1(\beta),$$

b) *for* $m \geq 2$ *with* $\omega \in \mathbb{Z}\setminus\{0\}$

$$\psi_\ell(x) = e^{-2\pi i\omega x}\cdot\kappa_m(\omega) - \sum_{k=1}^{N}\overline{C}_k\sum_{\beta\neq 0} e^{2\pi i\beta(x-hk)}\cdot\kappa_m(\beta) + d_0,$$

where d_0 *is a constant term and*

$$\kappa_m(\omega) = \frac{1}{(2\pi\omega)^{2m} + (2\pi\omega)^{2m-2}}. \tag{10}$$

Orthogonality condition (8) shows that it is necessary to calculate the norm of the error functional separately in cases $m=1$ and $m \geq 2$.

The analytic form of the norm for the error functional for $m=1$ is expressed as follows

$$\|\ell\|^2_{\tilde{W}_2^{(1,0)*}} = \kappa_1(\omega) - \kappa_1(\omega)\sum_{k'=1}^{N}\overline{C}_{k'}e^{2\pi i\omega hk'} - \kappa_1(\omega)\sum_{k=1}^{N}C_k e^{-2\pi i\omega hk} +$$

$$+\sum_{k=1}^{N}\sum_{k'=1}^{N}C_k\overline{C}_{k'}\sum_{\beta=-\infty}^{\infty}\kappa_1(\beta)\cdot e^{2\pi i\beta h(k-k')}, \tag{11}$$

where $\omega\in\mathbb{Z}$ and $\kappa_1(\cdot)$ is defined by (10).

In the space $\overset{\circ}{W}_2^{(m,m-1)*}$ of functionals with $m\geq 2$, the analytical representation of the norm $\|\ell\|_{\overset{\circ}{W}_2^{(m,m-1)*}}$ is expressed as follows

$$\|\ell\|^2_{\overset{\circ}{W}_2^{(m,m-1)*}}=\kappa_m(\omega)-\kappa_m(\omega)\sum_{k'=1}^{N}\overline{C}_{k'}e^{2\pi i\omega hk'}-\kappa_m(\omega)\sum_{k=1}^{N}C_k e^{-2\pi i\omega hk}+$$

$$+\sum_{k=1}^{N}\sum_{k'=1}^{N}C_k\overline{C}_{k'}\sum_{\beta\neq 0}\kappa_m(\beta)\cdot e^{2\pi i\beta h(k-k')}, \tag{12}$$

where $\omega\in\mathbb{Z}\setminus\{0\}$.

Thus, Problem 1 has been solved.

In the third section of Chapter II, the main results of this dissertation work are presented, i.e., we find the optimal coefficients $\overset{\circ}{C}_k$ of the quadrature formula (6) which give the minimum value to the norm of the error functional ℓ. To do this, we consider. this problem in cases $m=1$ with $\omega\in\mathbb{Z}$ and $m\geq 2$ with $\omega\in\mathbb{Z}\setminus\{0\}$, separately.

Initially, we obtain the following linear system of equations for $m=1$

$$\frac{\partial L_1}{\partial\overline{C}_{k'}}=-e^{2\pi i\omega hk'}\kappa_1(\omega)+\sum_{k=1}^{N}C_k\sum_{\beta=-\infty}^{\infty}e^{2\pi i\beta h(k-k')}\kappa_1(\beta)=0,\ k'=1,2,...,N. \tag{13}$$

It should be noted that the system of equations (13) is a system of discrete Wiener-Hopf type, S.L. Sobolev and Kh.M. Shadimetov proved that the solution of this type of system exists and is unique and that this solution gives a minimum to the norm $\|\ell\|$.

For the solution of the system (12) the following holds.

Theorem 2. Let $\varphi\in\overset{\circ}{W}_2^{(1,0)}(0,1]$, the following formula is valid for the optimal coefficients of the quadrature formula (6) with $\omega\in\mathbb{R}$

$$\overset{\circ}{C}_k=\frac{2K_{\omega,1}}{4\pi^2\omega^2+1}\cdot e^{2\pi i\omega hk}\qquad\text{for each}\quad k=1,2,...,N, \tag{14}$$

where

$$K_{\omega,1}=\frac{e^{2h}+1-2e^h\cos(2\pi\omega h)}{e^{2h}-1}.$$

The following results follow from Theorem 2.

Corollary 1. For $\omega h\in\mathbb{Z}$, optimal coefficients in the form (14) have the following form

$$\overset{\circ}{C}_k=\frac{2}{4\pi^2\omega^2+1}\cdot\frac{e^h-1}{e^h+1}\qquad\text{for each}\quad k=1,2,...,N.$$

Corollary 2. In the case $\omega=0$, optimal coefficients in the form (14) have the following form

We obtain the following system of linear equations for the coefficients of the quadrature formula (6) in the space $\tilde{W}_2^{(m,m-1)}(0,1]$ with $m \geq 2$

$$\frac{\partial L}{\partial \overline{C}_{k'}} = -e^{2\pi i\omega hk'} \cdot \kappa_m(\omega) + \sum_{k=1}^{N} C_k \sum_{\beta \neq 0} e^{2\pi i\beta h(k-k')} \kappa_m(\beta) = 0,\ k' = 1,2,\ldots,N, \tag{15}$$

$$\frac{\partial L}{\partial \mu} = \int_0^1 e^{2\pi i\omega x} dx - \sum_{k'=1}^{N} C_{k'} = 0, \tag{16}$$

where $\kappa_m(\cdot)$ is defined by formula (10).

The following theorem is true for the solution of the system (15),(16).

Theorem 3. In the space $\tilde{W}_2^{(m,m-1)}(0,1]$ of periodic functions with $m \geq 2$, among the quadrature formulas of the form (6) with $\omega \in \mathbb{R} \setminus \{0\}$ and $\omega h \notin Z$ there is a unique quadrature formula with coefficients having the representation

$$\mathring{C}_k = \frac{2K_{\omega,m}}{(2\pi\omega)^{2m} + (2\pi\omega)^{2m-2}} \cdot e^{2\pi i\omega hk} \text{ for each } k = 1,2,\ldots,N, \tag{17}$$

where

$$K_{\omega,m} = (-1)^{m-1} \cdot \left[\frac{e^{2h} - 1}{e^{2h} + 1 - 2e^h \cos(2\pi\omega h)} + \sum_{n=1}^{m-1} \frac{2h^{2n-1} \cdot \lambda E_{2n-2}(\lambda)}{(2n-1)! \cdot (1-\lambda)^{2n}} \right]^{-1},$$

$\lambda = e^{2\pi i\omega h}$ and $E_{2n-2}(\lambda)$ is the Euler-Frobenius polynomial of degree $(2n-2)$.

The following corollary follows from Theorem 3.

Corollary 3. In the space $\tilde{W}_2^{(2,1)}(0,1]$ of periodic functions, the analytical representation of coefficients of the optimal quadrature formula (6) for $\omega \in \mathbb{R} \setminus \{0\}\backslash$ and $\omega h \notin Z$ as follows

$$\mathring{C}_k = \frac{K_{\omega,2}}{8\pi^4\omega^4 + 2\pi^2\omega^2} \cdot e^{2\pi i\omega hk} \quad \text{for each } k = 1,2,...,N,$$

where

$$K_{\omega,2} = -\left[\frac{e^{2h} - 1}{e^{2h} + 1 - 2e^h \cos(2\pi\omega h)} + \frac{h}{\cos(2\pi\omega h) - 1} \right]^{-1}.$$

Theorem 4. In the space $\tilde{W}_2^{(m,m-1)}(0,1]$ of periodic functions with $m \geq 2$, for coefficients of the optimal quadrature formula of the form (6), the following equality holds when $\omega h \in Z$ and $\omega \neq 0$

$$\mathring{C}_k = 0 \text{ for each } k = 1,2,\ldots,N. \tag{18}$$

The case $m \geq 2$ with $\omega = 0$ is also considered separately in this section.

Remark 1. In the space $\tilde{W}_2^{(m,m-1)}(0,1]$ of real-valued, periodic functions for $m \geq 2$, the optimal coefficients of the following quadrature formula

$$\int_0^1 \varphi(x) dx \cong \sum_{k=1}^{N} C_k \varphi(x_k), \tag{19}$$

have the following form

$$\overset{\circ}{C}_k = h \quad \text{for each } k = 1,2,\ldots N.$$

Thus, Problem 2 has been solved.

In the fourth section of the second chapter, we find the sharp upper bound of the error of the constructed optimal quadrature formulas to cover all cases of $m \in \mathrm{N}$ and $\omega \in \mathrm{Z}$, that is, we calculate the norm of the error functional.

Firstly, we consider the case $m = 1$. The following theorem holds for this case.

Theorem 5. On the space $\widetilde{W}_2^{(1,0)*}(0,1]$ of functionals, for $\omega \in \mathbb{Z}$ the norm for the error functional ℓ of the optimal quadrature formula (6) has the following form

$$\left\|\overset{\circ}{\ell}\right\|^2_{\widetilde{W}_2^{(1,0)*}} = \frac{1}{4\pi^2\omega^2+1}\left[1-\frac{2}{4\pi^2\omega^2+1}\cdot\frac{e^{2h}+1-2e^h\cos(2\pi\omega h)}{h\left(e^{2h}-1\right)}\right]. \quad (20)$$

Now, the following holds for the norm of the error functional at $m \geq 2$ with $\omega \in \mathbb{R}\setminus\{0\}$ and $\omega h \notin \mathbb{Z}$.

Theorem 6. On the space $\widetilde{W}_2^{(m,m-1)*}(0,1]$ of functionals with $m \geq 2$ the norm for the optimal error functional $\overset{\circ}{\ell}$ for $\omega \in \mathbb{R}\setminus\{0\}$ and $\omega h \notin \mathbb{Z}$ has the following form

$$\left\|\overset{\circ}{\ell}\right\|^2_{\widetilde{W}_2^{(m,m-1)*}} = \frac{1}{(2\pi\omega)^{2m}+(2\pi\omega)^{2m-2}}\cdot\left[1-\frac{1}{h}\cdot\frac{2K_{\omega,m}}{(2\pi\omega)^{2m}+(2\pi\omega)^{2m-2}}\right], \quad (21)$$

where

$$K_{\omega,m} = (-1)^{m-1}\left[\frac{e^{2h}-1}{e^{2h}+1-2e^h\cos(2\pi\omega h)}+\sum_{n=1}^{m-1}\frac{2h^{2n-1}\cdot\lambda E_{2n-2}(\lambda)}{(2n-1)!(1-\lambda)^{2n}}\right]^{-1},$$

$\lambda = e^{2\pi i\omega h}$ and $E_{2n-2}(\lambda)$ is the Euler-Frobenius polynomial of degree $(2n-2)$.

Corollary 4. Taking into account that the following equality

$$\lim_{h\to 0}\frac{K_{\omega,m}}{h} = \left[\frac{2}{(2\pi\omega)^{2m}+(2\pi\omega)^{2m-2}}\right]^{-1},$$

we have the following

a) $\left\|\overset{\circ}{\ell}\right\|^2_{\widetilde{W}_2^{(m,m-1)*}} \to 0$ as $h \to 0$;

b) $\left\|\overset{\circ}{\ell}\right\|^2_{\widetilde{W}_2^{(m,m-1)*}} \to 0$ as $\omega \to \infty$ and h is fixed.

Theorem 7. On the space $\widetilde{W}_2^{(m,m-1)*}(0,1]$ with $m \geq 2$, the norm of the optimal error functional $\overset{\circ}{\ell}$ for $\omega h \in \mathbb{Z}\setminus\{0\}$ has the following form

$$\left\|\overset{\circ}{\ell}\right\|^2_{\widetilde{W}_2^{(m,m-1)*}} = \frac{1}{(2\pi\omega)^{2m}+(2\pi\omega)^{2m-2}}.$$

From Theorems 6 and 7, and Corollary 4, in the space $\overset{\circ}{W}_2^{(m,m-1)}$ we conclude that the order of convergence of optimal quadrature formulas (6) is $O(h^m)$ when $|\omega|< N$ and is $O(|\omega|^{-m})$ when $|\omega|\geq N$.

Theorem 8. On the space $\overset{\circ}{W}_2^{(m,m-1)*}(0,1]$ of functionals with $m\geq 2$, the norm for the error functional of the optimal quadrature formula (19) has the following form

$$\left\|\overset{\circ}{\ell}\right\|^2_{\overset{\circ}{W}_2^{(m,m-1)*}} = (-1)^{m-1}\sum_{n=2m}^{\infty}\frac{B_n}{n!}\cdot h^n = \frac{|B_{2m}|}{(2m)!}h^{2m} - \frac{|B_{2m+2}|}{(2m+2)!}h^{2m+2} + O(h^{2m+4}),$$

where B_n is the Bernoulli number.

Thus, Problem 3 is completely solved.

In the fifth section of the second chapter, the value of the norm of the error functional of the constructed optimal quadrature formulas in the space $\overset{\circ}{W}_2^{(m,m-1)}$ is presented in the numerical results for the cases $m=1$ and $m=2$. These results are obtained using Maple software.

In Chapter III of the PhD thesis, known as "**Effective quadrature formulas for the numerical calculation of the Fourier integrals in the space $W_2^{(m,m-1)}(a,b)$**", we deal with construction of effective quadrature formulas for the numerical integration of the Fourier integrals in the Hilbert space $W_2^{(m,m-1)}(a,b)$ of complex-valued functions.

In the first section of the third chapter, for the approximate calculation of integrals

$$I(\omega)=\int_a^b e^{2\pi i\omega x}\varphi(x)dx, \tag{22}$$

we give the following effective quadrature formula

$$\int_a^b e^{2\pi i\omega x}\varphi(x)dx \cong \sum_{k=0}^{N} C_{k,\omega}[a,b]\varphi(hk+a), \tag{23}$$

in the space $W_2^{(m,m-1)}(a,b)$, here $\omega\in\mathrm{R}$, $C_{k,\omega}[a,b]$ are coefficients and $h=(b-a)/N$.

One of the expansions of the optimal quadrature formulas (6) for the case real ω is the approximation formula obtained by assuming the coefficients in Theorem 3 as continuous functions with respect to $\omega\in\mathrm{R}$.

Theorem 9. The coefficients $C_{k,\omega}[a,b]$ of the effective quadrature formula (23) with $\omega\in\mathbb{R}\setminus\{0\}$ and $\omega h\notin \mathrm{Z}$ in the space $W_2^{(m,m-1)}(a,b)$ with $m\geq 2$ has the following forms

$$C_{0,\omega}[a,b]=\frac{(b-a)K_{\omega,m}}{(2\pi\omega_1)^{2m}+(2\pi\omega_1)^{2m-2}}\cdot e^{2\pi i\omega a},$$

$$C_{k,\omega}\left[a,b\right]=\frac{2(b-a)K_{\omega,m}}{(2\pi\omega_1)^{2m}+(2\pi\omega_1)^{2m-2}}\cdot e^{2\pi i\omega(hk+a)},\ k=1,2,\ldots,N-1, \quad (24)$$

$$C_{N,\omega}\left[a,b\right]=\frac{(b-a)K_{\omega,m}}{(2\pi\omega_1)^{2m}+(2\pi\omega_1)^{2m-2}}\cdot e^{2\pi i\omega b},$$

and for $\omega=0$ they take the form

$$C_{0,\omega}\left[a,b\right]=\frac{h}{2},$$

$$C_{k,\omega}\left[a,b\right]=h,\quad k=1,2,\ldots,N-1, \quad (25)$$

$$C_{N,\omega}\left[a,b\right]=\frac{h}{2},$$

where

$$K_{\omega,m}=(-1)^{m-1}\cdot\left[\frac{e^{\frac{2h}{b-a}}-1}{e^{\frac{2h}{b-a}}+1-2e^{\frac{h}{b-a}}\cos(2\pi\omega h)}+\sum_{n=1}^{m-1}\frac{2h^{2n-1}\cdot\lambda E_{2n-2}(\lambda)}{(2n-1)!\cdot(b-a)^{2n-1}\cdot(1-\lambda)^{2n}}\right]^{-1},$$

and $\omega_1=(b-a)\omega,\ h=\frac{b-a}{N},$ $E_{2n-2}(\lambda)$ is the Euler-Frobenius polynomial of degree $(2n-2)$ and $\lambda=e^{2\pi i\omega h}$.

The following is true.

Theorem 10. In the space $W_2^{(m,m-1)}\left(a,b\right)$ with $m\geq 2,$ for coefficients $C_{k,\omega}[a,b]$ of the effective quadrature formula of the form (23), the following equality is valid when $\omega h\in Z$ and $\omega\neq 0$

$$C_{k,\omega}[a,b]=0 \text{ for each } k=0,1,\ldots,N.$$

The second section of the third chapter presents the formulation and properties of the discrete Fourier transform.

In the third section of the third chapter, the case $m=2$ of the effective quadrature formula of form (23) is used in the approximate calculation of Fourier transforms of some functions. The results are compared with the values calculated using the built-function *fft* in MATLAB, and it is shown that the error of the results obtained using the effective quadrature formula is smaller than the error of the function *fft*.

CONCLUSIONS

The dissertation consists of two parts, the first part is devoted to the construction of optimal quadrature formulas for the numerical calculation of Fourier coefficients in the space $\widetilde{W}_2^{(m,m-1)}(0,1]$ of complex-valued, differentiable periodic functions and the estimation of their sharp upper bounds, and the second part is devoted to the effective quadrature formulas for approximate calculation of the Fourier integrals in the space $W_2^{(m,m-1)}(a,b)$ of complex-valued functions.

The main results of the research are as follows:

1. The extremal functions of quadrature formulas in Hilbert space $\widetilde{W}_2^{(m,m-1)}(0,1]$ of complex-valued, differentiable, periodic functions are found;
2. Using the extremal functions of the quadrature formulas in the Hilbert space $\widetilde{W}_2^{(m,m-1)}(0,1]$ of periodic functions, the analytical expressions of the norms of the error functionals have been found separately for the cases $m=1$ and $m\geq 2$.
3. The coefficients of the optimal quadrature formula for the approximate calculation of the Fourier coefficients in the Hilbert space $\widetilde{W}_2^{(m,m-1)}(0,1]$ of periodic functions have been found for $\omega\in\mathbb{R}$.
4. The coefficients of the optimal quadrature formulas for the approximate calculation of the Fourier coefficients in the Hilbert space $\widetilde{W}_2^{(m,m-1)}(0,1]$ of periodic functions have been found separately for $\omega\in\mathbb{R}\setminus\{0\}$ with $\omega h\notin\mathbb{Z}$, $\omega\in\mathbb{R}\setminus\{0\}$ with $\omega h\in\mathbb{Z}$, and $\omega=0$.
5. The square of the norm of the error functional $\mathring{\ell}$ of the optimal quadrature formula in the space of functionals has been calculated and the numerical results have been obtained in the case $m=1$ and $m=2$.
6. The results of the conducted numerical experiments show that the order of convergence of the optimal quadrature formulas is $O\left(\left(\frac{1}{N+|\omega|}\right)^m\right)$.
7. Effective quadrature formulas have been constructed for the approximate calculation of Fourier integrals in the space $W_2^{(m,m-1)}(a,b)$.
8. Using constructed effective quadrature formula in the space $W_2^{(2,1)}(-1,1)$, Fourier transforms of some functions are calculated.

**НАУЧНЫЙ СОВЕТ DSc.03/30.12.2019.FM.01.02
ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ
НАЦИОНАЛЬНОМ УНИВЕРСИТЕТЕ УЗБЕКИСТАНА**

ИНСТИТУТ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО

ХАЙРИЕВ УМЕДЖОН НАРМОН УГЛИ

ОПТИМАЛЬНЫЕ МЕТОДЫ ПРИБЛИЖЕННОГО ВЫЧИСЛЕНИЯ ИНТЕГРАЛОВ ДИФФЕРЕНЦИРУЕМЫХ ПЕРИОДИЧЕСКИХ ФУНКЦИЙ В ГИЛЬБЕРТОВОМ ПРОСТРАНСТВЕ

01.01.03 – Вычислительная и дискретная математика

**АВТОРЕФЕРАТ
диссертации доктора философии (PhD) по
ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ**

ТАШКЕНТ-2023

Тема диссертации доктора философии (Doctor of Philosophy) по физико-математическим наукам зарегистрированна в Высшей аттестационной комиссии при Министерстве высшего образования, науки и инноваций Республики Узбекистан за № B2022.4.PhD/FM794.

Диссертация выполнена в Институте Математики им. В.И.Романовского АН РУз.

Автореферат диссертации на трех языках (узбекский, английский, русский (резюме)) размещен на веб-странице Научного совета (http://ik-fizmat.nuu.uz/) и на Информационно-образовательном портале «Ziyonet» (www.ziyonet.uz)

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Защита диссертации состоится «____» _____________2023 года в ____ часов на заседании Научного совета DSc.03/30.12.2019.FM.01.02 при Национальном университете Узбекистана. (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 4. Тел.: (+99871)227-12-24, факс: (+99871) 246-53-21, e-mail: nauka@nuu.uz).

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования состоит в построении оптимальных квадратурных формул для численного вычисления коэффициентов Фурье в комплекснозначном гильбертовом пространстве $\widetilde{W}_2^{(m,m-1)}(0,1]$ дифференцируемых периодических функций и оценка их погрешностей, и получение эффективных квадратурных формул для приближенного вычисления интегралов Фурье в комплекснозначном гильбертовом пространстве $W_2^{(m,m-1)}(a,b)$ дифференцируемых функций.

Объект исследования – гильбертово пространство дифференцируемых периодических функций, квадратурные и эффективные формулы, коэффициенты и интегралы Фурье.

Научная новизна исследования состоит из следующих:

найдены экстремальные функции квадратурных формул для численного вычисления коэффициентов Фурье в комплекснозначном гильбертовом пространстве $\widetilde{W}_2^{(m,m-1)}(0,1]$ дифференцируемых периодических функций;

найдено аналитическое выражение нормы функционала погрешности квадратурных формулы для численного вычисления коэффициентов Фурье в гильбертовом пространстве $\widetilde{W}_2^{(m,m-1)}(0,1]$ комплекснозначных дифференцируемых периодических функций;

найдены коэффициенты оптимальных квадратурных формул для численного вычисления коэффициентов Фурье в гильбертовом пространстве $\widetilde{W}_2^{(m,m-1)}(0,1]$ комплекснозначных дифференцируемых периодических функций;

вычислена норма функционала погрешности оптимальных квадратурных формул для численного вычисления коэффициентов Фурье в гильбертовом пространстве $\widetilde{W}_2^{(m,m-1)}(0,1]$ комплекснозначных дифференцируемых периодических функций и приведены численные результаты при $m=1$ и $m=2$;

найдены коэффициенты эффективных квадратурных формул для приближенного вычисления интегралов Фурье в гильбертовом пространстве $W_2^{(m,m-1)}(a,b)$ комплекснозначных дифференцируемых функций.

Внедрение результатов исследования. На основе полученных научных результатов по построению оптимальных квадратурных формул для приближенного вычисления сильно осциллирующих интегралов в гильбертовом пространстве периодических функций:

построенные оптимальные квадратурные формулы в гильбертовых пространствах $\widetilde{W}_2^{(1,0)}(0,1]$ и $\widetilde{W}_2^{(2,1)}(0,1]$ использованы в практическом проекте №. ПЗ-20170930257 «Совершенствование процесса гидрометрической обработки в технологии выделения сортовой муки из местных зерен пшеницы» (справка Ташкентского химико-технологического института от 14 апреля 2023 года №1/04-1177) для повышения точности измерения размеров зерен пшеницы (высоты, ширины и длины). Результат численного расчета решений математических моделей, представляющих распределение влаги, происходящее в гидротермальных устройствах, по этим формулам позволил с большей точностью найти оптимальные значения их геометрических размеров;

Оптимальная квадратурная формула построенная в гильбертовом пространстве комплекснозначных функций $\tilde{W}_2^{(3,2)}(0,1]$, построенная использована в фундаментальном проекте № ОТ-Ф4–02 – «Термодинамика моделей математической физики с бесконечным множеством состояний» (справка Бухарского государственного университета от 25 апреля 2023 года № 04/752) для приближенного вычисления интегралов в комплексе смешанных задач уравнения математической физики. В результате это позволило найти численное решение комплекса смешанных задач с высокой точностью.

Структура и объем диссертации. Диссертационная работа состоит из введения, трех глав, заключения, списка использованной литературы. Общий объем диссертации составляет 92 страниц.

E'LON QILINGAN ISHLAR RO'YXATI
LIST OF PUBLISHED WORKS
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