FARG‘ONA DAVLAT UNIVERSITETI
HUZURIDAGI ILMIY DARAJALAR BERUVCHI
PhD.03/30.12.2019.FM.05.04 RAQAMLI ILMIY KENGASH

NAMANGAN DAVLAT UNIVERSITETI

XORILOV MAXMUD ABDUMALIKOVICH

BOSHQARUVLAR NOSTATSIONAR CHEGARALANISHLI DIFFERENSIAL O‘YINLAR

01.01.02 – Differensial tenglamalar va matematik fizika

FIZIKA-MATEMATIKA FANLARI
bo‘yicha falsafa doktori (PhD) dissertatsiyasi
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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahon miqyosida olib borilayotgan koʻp sonli ilmiy-amaliy tadqiqotlar natijasida yuzaga keladigan koʻplab amaliy va nazariy muammolarni hal qilishda vaqt, resurs, xarajat, sifat va imkoniyatlarni optimallashtirish masalalari yetakchi oʻrinlardan birini egallamoqda. Dunyo miqyosidagi bu kabi masalalar boshqaruv jarayonlarining matematik modellarini qurish va masalalarning optimal yechimlarini aniqlanishni taqozo etadi. Shu jihatdan differensial oʻyinlar nazariyasining asosiy obyekti hisoblangan qarama-qarshi boshqaruvli jarayonlarni tadqiq qilish, bu boʻyicha boshqaruvlari nostatsionar chegaralanishlarga ega boʻlgan quvish-qochish masalalari uchun yangi yetarli shartlar mavjudligini aniqlash hamda qaralayotgan obyektlarning yetishish toʻplamini tavsiflash muhim ahamiyatga ega hisoblanadi.

Jahonda boshqaruvlari nostatsionar chegaralanishli differensial oʻyin masalalarini tadqiq qilishga yoʻnaltirilgan ilmiy-tadqiqot ishlari olib borilmoqda. Bu borada differensial oʻyinlarning klassik masalalari natijasining samaradorligini oshirish uchun boshqaruvlari turli koʻrinishdagi chegaralanishlarga ega koʻp oʻyinchili differensial oʻyin masalalarini oʻrganish, differensial oʻyinlarning boshqaruvlari nostatsionar yoki turli koʻrinishdagi chegaralanishlarga ega boʻlgan quvish-qochish masalalari uchun optimal strategiyalarni qurish, yangi yetarli shartlarni topish, oʻyinchilarning yetishish toʻplamini tavsiflash shu bilan birga Ayzeksning “Qutulish chizigʻi” oʻyinini hal etishga alohida eʼtibor berilmoqda.

Respublikamizda fundamental fanlarning ilmiy va amaliy tatbiqlariga ega boʻlgan dolzarb yoʻnalishlariga eʼtibor berish yuzasidan keng qamrovli chora-tadbirlar amalga oshirilib, muayyan natijalarga erishilmoqda. Jumladan, “algebra va uning tatbiqlari, differensial tenglamalar va uning tatbiqlari, chiziqsiz tizimlar, dinamik sistemalar va ularning tatbiqlarini matematik modellashtirish, stoxastik tahlil, tibbiy-biologik informatika, hisoblash matematikasi” kabi ustuvor yoʻnalishlar boʻyicha muhim vazifalar belgilab berilgan[1]. Ushbu vazifalarni amalga oshirishda, jumladan, differensial tenglamalar, dinamik sistemalar va differensial oʻyinlar nazariyalarini rivojlantirish maqsadida differensial oʻyinlarning boshqaruvlari nostatsionar yoki turli koʻrinishdagi chegaralanishlarga ega boʻlgan quvish-qochish masalalari uchun optimal strategiyalarni qurish muhim ahamiyat kasb etmoqda.

Oʻzbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PF-4947-son “Oʻzbekiston Respublikasini yanada rivojlantirish boʻyicha Harakatlar strategiyasi toʻgʻrisida”gi va 2022-yil 28-yanvardagi PF-60-son “2022-2026-yillarga moʻljallangan Yangi Oʻzbekistonning Taraqqiyot strategiyasi toʻgʻrisida”gi Farmonlari, 2019-yil 9-iyuldagi PQ-4387-son “Matematika taʼlimi va fanlarini yanada rivojlantirishni davlat tomonidan qoʻllab-quvvatlash, shuningdek Oʻzbekiston Respublikasi Fanlar akademiyasining V.I.Romanovskiy nomidagi

[1] Oʻzbekiston Respublikasi Prezidentining 2019-yil 9-iyuldagi №PQ-4387 “Matematika taʼlimi va fanlarini yanada rivojlantirishni davlat tomonidan qoʻllab-quvvatlash, shuningdek, Oʻzbekiston Respublikasi Fanlar akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari toʻgʻrisida”gi qarori.

Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi va 2020-yil 7-maydagi PQ-4708-son “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalarni rivojlantirishning ustivor yo‘nalishlariga bog‘liqligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. “Matematika, mexanika va informatika” ustivor yo‘nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. “Differensial o‘yin” tushunchasi XX asrning 50-yillar boshlarida Amerikalik matematik R.Ayzeks tomonidan qo‘llanilgan. 1965 yilda R.Ayzeksning izlanishlari monografiya shaklida nashr etilgan. Bu monografiya Gamilton-Yakobi nazariyasidan kelib chiqadigan evristik mulohazalar va ko‘plab misollardan iborat bo‘lib, umumiy nazariyani o‘z ichiga olmagan. O‘sha yillarda L.S.Pontryagin va uning shogirdlari havo hujumida samolyotni boshqarish vazifalarini soddalashtirishdan kelib chiqqan holda boshqariladigan jarayonlarning matematik nazariyasini yaratdilar. Shuni ta’kidlash kerakki, O‘zbekistonda ham differensial o‘yinlar nazariyasi bo‘yicha ilmiy maktab shakllantirilgan bo‘lib, unga N.Yu.Satimov asos solgan va hozirda A.A.Azamov rahbarlik qilib kelmoqda.

L.S.Pontryaginning birinchi to‘g‘ri usuli M.S.Nikolskiy tomonidan integral cheklovlar holatiga o‘tkazilgan. Ushbu yondashuv A.Ya.Azimov, F.V.Guseynov va boshqalarning ishlarida keng rivojlantirilgan. N.N.Krasovskiyning integral cheklovlarga ega differensial o‘yinlarni yechishda ekstremal maqsadga asoslangan usuli asosida mustaxkam asosga ega usul ishlab chiqilgan. Regulyar holat uchun pozitsion yondashish usuli B.N.Pshenichniy, A.A.Chikriy, I.J.Rappoport, Yu.N.Onopchuk va boshqalarning ishlarida turli ko‘rinishdagi cheklovlar integral cheklovli o‘yinlarga o‘tkazilgan. Keyinchalik, N.Yu.Satimov, G.I.Ibragimov, B.T.Samatov va boshqalar ishlarida integral cheklovlar bilan differensial va diskret o‘yinlarida jamoaviy quvlovchilar o‘rtasida boshqarish resurslarini taqsimlash usuliga asoslanib yechishni taklif etdilar. So‘nggi yillarda A.A.Azamov va A.Sh.Kuchkarovlar tomonidan quvlovchining boshqaruvi funksiyasiga geometrik, qochuvchining boshqaruviga esa integral cheklovlar qo‘yilganda quvish, boshqaruvchanlik va chiziqli sistemalarda turg‘unlik masalalarining o‘zaro bog‘liqligi bo‘yicha tadqiqotlarga asos solingan. Differensial o‘yinlarning umumiy nazariyasida quvish-qochish masalalari bir qator o‘ziga xosliklari bilan alohida ahamiyatga ega. Bular masala qo‘yilishidagi xilma-xillik, qo‘llaniladigan usullar va natijalarning xususiyatlaridan iborat. Yetishish sohasining mahsus xossasiga asoslanib, R.Ayzeksning “Qutulish chizig‘i” masalasi oddiy harakatlar bo‘lgan holida L.A.Petrosyan tomonidan yechilgan. Bunda parallel quvish strategiyasi optimal strategiya ekanligini isbotlangan. Yuqoridagi kabi ko‘plab ilmiy ishlar bajarilganiga qaramasdan, hozirgi kunda boshqaruv funksiyalari nostatsionar chegaralanishli differensial o‘yinlarni yetarlicha o‘rganilmaganligini ta’kidlash joiz.

Shu bois boshqaruv funksiyalari nostatsionar chegaralanishli differensial oʻyinlarni oʻrganish katta ahamiyatga ega.

Mazkur dissertatsiya boshqaruv funksiyalari nostatsionar chegaralanishli differensial oʻyinlarni va R.Ayzeksning “Qutulish chizigʻi” muammosini hal qilishga bagʻishlangan.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bogʻliqligi. Dissertatsiya tadqiqoti Namangan davlat universitetining ilmiy-tadqiqot ishlari rejasining “Fundamental tadqiqotlar” tarmogʻi doirasida bajarilgan.

Tadqiqotning maqsadi boshqaruvlarga nostatsionar chegaralanishlar qoʻyilgan hollarda quvish-qochish masalalarini va R.Ayzeksning “Qutulish chizigʻi” muammosini yechishdan iborat.

Tadqiqotning vazifalari:

oʻyinchilarning boshqaruvlariga chiziqli chegaralanishlar qoʻyilgan differensial oʻyinda quvish masalasi yechimga ega boʻlishining yangi yetarli shartlarini topish;

boshqaruvlarga eksponensial integral chegaralanishlar qoʻyilgan holda quvish-qochish masalalari yechimga ega boʻlishining yangi yetarli shartlarini topish;

boshqaruvlarga nostatsionar geometrik chegaralanishlar qoʻyilgan holda quvish-qochish masalalari yechimga ega boʻlishining yangi yetarli shartlarini aniqlash;

boshqaruvlari nostatsionar geometrik chegaralanishli quvish-qochish masalalari uchun R.Ayzeksning “Qutulish chizigʻi” masalasini toʻliq hal etish.

Tadqiqotning obyekti. Optimal boshqarish nazariyasi, differensial tenglamalar nazariyasi, differensial oʻyinlar nazariyasi.

Tadqiqotning predmeti. Boshqaruv funksiyalari geometrik, nostatsionar geometrik, chiziqli va nostatsionar integral chegaralanishlarga ega boʻlgan quvish-qochish masalalari.

Tadqiqotning usullari. Dissertatsiyada quvish masalalarini yechish uchun oʻyinchilarning parallel yaqinlashish usullari, qochish masalalarini yechish uchun esa yoʻnalish boʻyicha qochish usullaridan foydalanilgan. Shu bilan birgalikda, funksional tahlil, optimal boshqaruv nazariyasi va koʻp qiymatli akslantirishlar nazariyasi keng qoʻllanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

boshqaruv funksiyalari chiziqli chegaralanishli differensial oʻyinda quvishni tugallash uchun yangi yetarli shartlar quvlovchilar uchun parallel yaqinlashish metodi yordamida olingan hamda oʻyinchilar yetishish sohasining monotonligi toʻplamning tirgak funksiyasi yordamida isbotlangan;

boshqaruv funksiyalari eksponensial integral chegaralanishga ega holda quvish-qochish differensial oʻyin masalalari quvlovchi uchun parallel yaqinlashish metodi va qochuvchi uchun esa yoʻnalish boʻyicha qochish metodi yordamida yechilgan;

boshqaruv funksiyalari nostatsionar geometrik chegaralanishli differensial oʻyinda quvish-qochish masalalarini yechish uchun yangi yetarli shartlar quvlovchi uchun parallel yaqinlashish metodi va qochuvchi uchun esa yo‘nalish bo‘yicha qochish metodi yordamida aniqlangan;

boshqaruv funksiyalari nostatsionar geometrik chegaralanishli differensial o‘yinda o‘yinchilar yetishish sohasining yangi monotonlik shartlari topilgan va o‘yinchilar yetishish sohasining monotonlik xossasi yordamida Ayzeksning “Qutulish chizig‘i” masalasi to‘liq yechilgan.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

differensial o‘yinlar nazariyasi asoslarining hayotiy jarayonlar bilan o‘zaro mutanosibligi nuqtai-nazaridan, qarama-qarshi boshqaruvli jarayonlarga mos masalalarning matematik modellari bo‘lgan quvish-qochish masalalari yechilgan.

Tadqiqot natijalarining ishonchliligi. Ishda differensial tenglamalar nazariyasi, funksional tahlil, optimal boshqaruv nazariyasi hamda differensial o‘yinlar nazariyasining quvish-qochish masalalari haqidagi teoremalaridan foydalanilgan. Olingan natijalar qat’iy matematik mulohazalarga asoslanib isbotlangan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati boshqaruv va differensial o‘yinlar nazariyasini rivojlantirish hamda ularning tatbiqlariga doir masalalarning samarali yechish usullarini aniqlashda foydalanish bilan izohlanadi.

Tadqiqot natijalarining amaliy ahamiyati dinamik o‘yinlarning matematik modellaridagi resurs boshqaruvlari uchun optimal taqsimotning berilishi, hamda ularning sonli algoritmlarini qurish orqali texnika, tibbiyot va iqtisodiyotga tatbiq etish mumkinligi bilan izohlanadi.

Tadqiqot natijalarining joriy qilinishi. Boshqaruvlari nostatsionar chegaralanishlarga ega differensial o‘yinlar bo‘yicha olingan natijalar asosida:

boshqaruvlari nostatsionar chegaralanishli quvish-qochish masalalar va Ayzeksning “Qutulish chizig‘i” masalasi yechimining mavjudligi bo‘yicha olingan natijalar OT-F4-33 raqamli “Differensial tenglamalar bilan tavsiflanuvchi ziddiyatli holatlarni boshqarishning yangi usullarini yaratish va ularni sonli amalga oshirish” mavzusidagi fundamental loyihada boshqaruvlari nostatsionar chegaralanishli qarama-qarshi boshqaruvli dinamik sistemalarda foydalanilgan (Mirzo Ulug‘bek nomidagi O‘zbekiston Milliy universitetining 2023-yil 9-iyundagi 04/11-3587-sonli ma’lumotnomasi). Natijada, nostatsionar chegaralanishli dinamik sistemalarda ziddiyatli boshqaruv (dinamik o‘yin) masalalarining umumlashgan yechimlarini aniqlash imkonini bergan;

boshqaruvlari chiziqli chegaralanishli differensial o‘yin masalasi yechimining mavjudligi va optimalligi bo‘yicha olingan natijalar 374874-2015 raqamli “Fazaviy o‘tishlar va tahliliy hodisalar masalalari. Ularning tez o‘tish tenglamalari va asimptotikalarining matematik xususiyatlari” mavzusidagi xorijiy grant loyihasida oddiy differensial tenglamalar uchun analogik masalalarni yechishda foydalanilgan (O‘sh davlat universitetining 2023-yil 27-iyundagi 838-sonli ma’lumotnomasi, Qirg‘iziston Respublikasi). Natijada, yangi nolokal umumlashgan spektral masalalarning yechimini qurish imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqot natijalari 18 ta ilmiy-amaliy anjumanlarda, jumladan 9 ta xalqaro va 9 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya tadqiqoti mavzusi bo‘yicha jami 24 ta ilmiy ish chop etilgan, shulardan O‘zbekiston Respublikasi Oliy Attestasiya komissiyasining falsafa doktorlik dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 6 ta, jumladan 2 tasi xorijiy va 4 tasi respublika jurnallarida nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan. Dissertatsiyaning umumiy hajmi 93 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, dissertatsiya mavzusi bo‘yicha xorijiy ilmiy tadqiqotlarning tahlili berilgan, muammoning o‘rganilganlik darajasi yoritilgan, tadqiqotning maqsad va vazifalari, obyekti va predmeti ko‘rsatilgan, tadqiqot natijalarining ilmiy yangiligi ochib berilgan, olingan natijalarning nazariy va amaliy ahamiyati ko‘rsatilgan, tadqiqot natijalarining tatbiqi, shuningdek nashr etilgan ilmiy ishlar va dissertatsiyaning tuzilishi haqida ma’lumotlar keltirilgan.

Birinchi bob “**Boshqaruvlari** $LG-$**chegaralanishlarga ega differensial o‘yinlar**” deb nomlangan bo‘lib, ushbu bob boshqaruvlari chiziqli va geometrik chegaralanishlarga ega bir nechta quvlovchi va bitta qochuvchili quvish differensial o‘yinini tadqiq qilishga bag‘ishlangan.

$\mathbb{R}^n$ fazoda chekli sondagi P_i, $i=1,2,...,m$ quvlovchilar va E qochuvchi harakatlanayotgan bo‘lib, x_i va y mos ravishda quvlovchilar va qochuvchining radius vektorlari bo‘lsin. Agar u_i va v ularning tezlik vektorlari bo‘lsa, u holda quvlovchilar va qochuvchining harakatlari mos ravishda quyidagi tenglamalar bilan berilgan differensial o‘yinni ko‘rib chiqamiz:

$$P_i: \ \dot{x}_i = u_i, \quad x_i(0) = x_{i0}, \tag{1}$$

$$E: \ \dot{y} = v, \quad y(0) = y_0, \tag{2}$$

bu yerda $x_i, y, u_i, v \in \mathbb{R}^n$, $n \geq 2$, va x_{i0}, $i=1,2,...,m$, vektorlar P_i obyektlarning boshlang‘ich holatlari va y_0 esa E obyektning boshlang‘ich holati. Har bir P_i quvlovchining u_i boshqaruv parametrlari vaqt bo‘yicha $u_i(\cdot):[0,\infty) \to \mathbb{R}^n$ o‘lchovli funksiyalar bo‘lishi kerak va har bir $u_i(\cdot)$ funksiya quyidagi integral ko‘rinishdagi chegaralanishni qanoatlantirishini talab qilamiz:

$$\int_0^t |u_i(s)|^2 \, ds \leq L_i(t) \ \text{deyarli barcha} \ t \geq 0, \tag{3}$$

bu yerda $L_i(t) = k_i t + \rho_{i0}$, $i=1,2,...,m$, funksiya resursning har bir t vaqtdagi chiziqli o‘zgarishi, k_i, ρ_{i0} lar manfiy bo‘lmagan parametrik sonlar. (3) ko‘rinishdagi chegaralanishni chiziqli chegaralanish yoki qisqacha, L_i – chegaralanish deb ataymiz va $\mathbb{U}_L^i$ orqali P_i quvlovchilarning joiz boshqaruvlari, ya’ni (3) tengsizlikni qanoatlantiruvchi barcha o‘lchovli funksiyalar sinfini belgilaymiz.

Fizik nuqtai nazaridan, (3) ning o‘ng tomoni $t \geq 0$ vaqtga bog‘liq holda berilgan resursning chiziqli o‘zgarishiga mos keladi. Shu sababli, $L_i(t)$ chiziqli funksiyani berilgan P_i quvlovchilar resurslarining ayni vaqtdagi o‘zgarishi deb nomlash mumkin. Ravshanki, quvlovchilarning boshlang‘ich resurslari $k_i > 0$ bo‘lsa ortadi, $k_i < 0$ bo‘lsa kamayadi va $k_i = 0$ bo‘lsa o‘zgarmaydi. Oxirgi holatda (3) ni integral chegaralanish deyiladi. Agar $\rho_{i0} = 0$ va $k_i > 0$ bo‘lsa, u holda (3) ni geometrik chegaralanish deyiladi.

Yuqoridagi kabi, E qochuvchining v boshqaruvi ham vaqt bo‘yicha $v(\cdot):[0, +\infty) \to \mathbb{R}^n$ o‘lchovli funksiya sifatida tanlanadi va

$$|v(t)| \leq \beta \text{ deyarli barcha } t \geq 0, \tag{4}$$

geometrik chegaralanishni (qisqacha, G-chegaralanish) qanoatlantirishini talab qilinadi, bu yerda β manfiy bo‘lmagan parametrik son bo‘lib, qochuvchining maksimal tezligining qiymatini ifodalaydi. Biz $\mathbb{V}_G$ orqali (4) tengsizlikni qanoatlantiruvchi qochuvchining barcha $v(\cdot)$ joiz boshqaruvlari sinfini belgilaymiz.

(1)–(4) differensial o‘yinni L_iG – o‘yin deb ataylik.

L_iG – o‘yinda P_i, $i = 1, 2, ..., m$ quvlovchilarning asosiy maqsadi E qochuvchini tutishdir, ya’ni chekli vaqtda biror $i = 1, 2, ..., m$ uchun $x_i(t) = y(t)$ tenglikka erishish, bu yerda $x_i(t)$, $i = 1, 2, ..., m$ va $y(t)$ vektor funksiyalar L_iG – o‘yin davomida hosil bo‘ladigan trayektoriyalar. E qochuvchi esa quvlovchi bilan to‘qnashishning oldini olishga harakat qiladi va agar buning iloji bo‘lmasa, u holda to‘qnashish vaqtini imkon boricha kechiktirishga harakat qiladi.

Ushbu bobda, biz asosan qochuvchi uchun $\mathbb{R}^n$ fazoning “Qutilish chizig‘i” deb nomlangan A qism to‘plami bilan berilgan holat chegaralanishlariga ega o‘yinni ko‘rib chiqamiz.

1-ta’rif. (1) va (2) ga ko‘ra, $(x_{i0}, u_i(\cdot))$, $u_i(\cdot) \in \mathbb{U}_L^i$, $i = 1, 2, ..., m$ va $(y_0, v(\cdot))$, $v(\cdot) \in \mathbb{V}_G$ juftliklar mos ravishda $x_i(t) = x_{i0} + \int_0^t u_i(s)ds$ va $y(t) = y_0 + \int_0^t v(s)ds$ yechimlarni hosil qiladi. Bu holda, $x_i(t)$, $i = 1, 2, ..., m$ ni P_i quvlovchining harakat trayektoriyasi, $y(t)$ ni esa E qochuvchining harakat trayektoriyasi deb ataymiz.

2-ta'rif. Har bir $(k_i, \rho_{i0}, u_i(\cdot))$, $u_i(\cdot) \in \mathbb{U}_L^i$, $i = 1,2,...,m$ uchlik uchun ushbu $\rho_i(t) = L_i(t) - \int_0^t |u_i(s)|^2 ds$, $\rho_i(0) = \rho_{i0}$, $t \geq 0$ skalyar kattalik P_i quvlovchining har bir t momentdagi qoldiq resursi deyiladi.

3-ta'rif. Ushbu $\mathbf{u}_{L_iG}^i : \mathbb{V}_G \to \mathbb{U}_L^i$, $i = 1,2,...,m$ akslantirish P_i quvlovchining strategiyasi deyiladi, agar: 1^o. (Joizlik) Ixtiyoriy $v(\cdot) \in \mathbb{V}_G$ boshqaruv funksiya uchun $u_i(\cdot) = \mathbf{u}_{L_iG}^i[v(\cdot)] \in \mathbb{U}_L^i$ tegishlilik bajarilsa; 2^o. (Volterra) Agar ixtiyoriy $v_1(\cdot), v_2(\cdot) \in \mathbb{V}_G$ boshqaruv funksiyalar va har bir $t \in [0, \infty)$ uchun deyarli barcha $[0,t]$ kesmada $v_1(s) = v_2(s)$ tenglik deyarli barcha $[0,t]$ kesmada $u_{i1}(s) = u_{i2}(s)$ tenglikning bajarilishini ifodalasa, bu yerda $u_{ij}(\cdot) = \mathbf{u}_{L_iG}^{ij}[v_j(\cdot)] \in \mathbb{U}_L^i$, $j = 1, 2$.

4-ta'rif. Agar har bir $v(\cdot) \in \mathbb{V}_G$ uchun biror $t_i^* \in \left[0, T_{L_iG}^i\right]$ payt mavjud bo'lib, $x(t_i^*) = y(t_i^*)$ tenglik hosil bo'lsa, u holda $L_iG -$o'yinda $\mathbf{u}_{L_iG}^i(v)$, $i = 1,2,...,m$ strategiya $\left[0, T_{L_iG}^i\right]$ vaqt oralig'ida P_i quvlovchilar uchun yutuqli deyiladi, bu yerda $T_{L_iG}^i$ kafolatlangan quvish vaqti deyiladi.

5-ta'rif. Faraz qilaylik, $y_0 \notin \bar{A} \subset \mathbb{R}^n$ bo'lsin. Agar quyidagi shartlar bajarilsa: a) biror $t_i^* \in \left[0, T_{L_iG}^i\right]$ vaqt mavjud bo'lib, $x(t_i^*) = y(t_i^*)$ o'rinli bo'lsa; b) $y_t(\cdot) = \left\{y(s): 0 \leq s \leq t_i^*\right\} \notin \bar{A}$ bo'lsa, u holda "Qutilish chizig'i" ga ega $L_iG -$o'yinda $\left[0, T_{L_iG}^i\right]$ vaqt oralig'ida $\mathbf{u}_{L_iG}^i(v)$, $i = 1,2,...,m$ strategiya P_i quvlovchilar uchun yutuqli deyiladi.

$x_{i0} \neq y_0$ bo'lsin va $v(t)$, $t \geq 0$, $v(t) \in \mathbb{V}_G$ boshqaruvning qiymati berilgan bo'lsin. Faraz qilaylik, (μ_{i0}, k_i, β) uchlik $L_iG -$o'yining parametrik holati bo'lsin va uni p_i bilan belgilaylik. Bunday p_i holatlarning bo'sh bo'lmagan va o'zaro sodda bog'langan $\mathbf{P}_{L_iG}^i = \mathbf{P}_1^i \cup \mathbf{P}_2^i \cup \mathbf{P}_3^i$ to'plamini topamiz, bu yerda

$$\mathbf{P}_1^i = \left\{p_i : \mu_{i0} \geq 0, k_i > \beta^2, \beta \geq 0\right\}, \ \mathbf{P}_2^i = \left\{p_i : \mu_{i0} > 2\beta, \ k_i = \beta^2, \ \beta \geq 0\right\},$$

$$\mathbf{P}_3^i = \left\{p_i : \mu_{i0} \geq 2\left(\beta + \sqrt{\beta^2 - k_i}\right), k_i < \beta^2, \beta \geq 0\right\}$$

va $\mathbf{P}_1^i$, $\mathbf{P}_2^i$, $\mathbf{P}_3^i$ o'zaro kesishmaydigan to'plamlar.

6-ta'rif. $L_iG -$o'yinda $\mathbf{u}_{L_iG}^i(v) = v - \lambda_{L_iG}^i(v)\xi_{i0}$ funksiyani P_i quvlovchining parallel quvish strategiyasi (qichqacha, $\Pi_{L_iG}^i$ – strategiya) deb ataymiz, bu yerda

$$\lambda_{L_iG}^i(v) = \mu_{i0}/2 + \langle v, \xi_{i0}\rangle + \sqrt{\left(\mu_{i0}/2 + \langle v, \xi_{i0}\rangle\right)^2 + k_i - |v|^2},$$

$$\xi_{i0} = z_{i0}/|z_{i0}|, \quad z_{i0} = x_{i0} - y_0, \quad \mu_{i0} = \rho_{i0}/|z_{i0}|.$$

Faraz qilaylik, E qochuvchi t, $t \ge 0$ vaqt momentida y holatdan v, $|v| \le \beta$ o‘zgarmas boshqaruv yordamida harakatlansin. P_i quvlovchilar esa ρ_i, $\rho_i > 0$ resursga asosan x_i holatdan boshlab $\Pi^i_{L_iG}$ – strategiyani qo‘llasin. U holda agar w nuqta P_i quvlovchilar va E qochuvchi biror $T > 0$ vaqtda to‘qnashishi kerak bo‘lgan nuqta bo‘lsa, barcha shunday w nuqtalar to‘plamini quyidagi munosabatlar yordamida beriladi:

$$|w - x_i| = T|u_i(v)|, \quad |w - y| = T|v|, \quad T|u_i(v)|^2 = k_i T + \rho_i,$$

va bu tengliklardan quyidagini topamiz:

$$W^i_{L_iG}(x_i, y, \rho_i) = \left\{ w : |w - x_i|^2 \ge (k_i / \beta^2)|w - y|^2 + (\rho_i / \beta)|w - y| \right\}.$$

1-teorema. L_iG–o‘yinda $p_i \in \mathbf{P}^i_{L_iG}$ o‘rinli bo‘lsin. U holda ixtiyoriy $t_1, t_2 \in [0, t_i^*]$, $0 \le t_1 \le t_2$ lar uchun $W^i_{L_iG}(t_2) \subset W^i_{L_iG}(t_1)$ munosabat o‘rinli bo‘ladi.

2-teorema. Agar biror $i = 1, 2, ..., m$ uchun $p_i \in \mathbf{P}^i_{L_iG}$ bo‘lsa, u holda $[0, T_{L_iG}]$ da $y(t) \in W$ bajariladi, bu yerda

$$W = \bigcap_{i=1}^{m} W^i_{L_iG}(0), \ T_{L_iG} = d / \beta, \ d = \max\{|w_1 - w_2| : w_1, w_2 \in W\}.$$

3-teorema. Agar $W \cap A = \varnothing$ bo‘lsa, u holda, “Qutilish chizig‘i”ga ega L_iG – o‘yinda $\Pi^i_{L_iG}$ – strategiya $[0, T_{L_iG}]$ da P_i, $i = \overline{1, m}$ quvlovchilar uchun yutuqli bo‘ladi.

Ikkinchi bob “**Boshqaruvlari nostatsionar integral chegaralanishli differensial o‘yinlar**” deb nomlangan bo‘lib, ushbu bob boshqaruvlari nostatsionar integral chegaralanishlarga ega sodda harakatli differensial o‘yinlarni yechishga bag‘ishlangan.

$\mathbb{R}^n$ fazoda quvlovchi va qochuvchining harakatlari mos ravishda

$$P: \quad \dot{x} = u, \quad x(0) = x_0, \tag{5}$$

$$E: \quad \dot{y} = v, \quad y(0) = y_0 \tag{6}$$

tenglamalar bilan berilgan bo‘lsin, bu yerda x_0 va y_0 nuqtalar $x_0 \ne y_0$ shart asosida quvlovchi va qochuvchining boshlang‘ich vaziyatlarini bildiradi.

$\mathbb{U}^1_{I_k}$ va $\mathbb{V}^1_{I_k}$ orqali mos ravishda quyidagi shartlarni qanoatlantiruvchi barcha $u(\cdot)$ va $v(\cdot)$ o‘lchovli funksiyalar sinflarini belgilaymiz:

$$\int_0^t |u(s)|^2 \, ds \le \frac{\rho_1^2}{2k}\left(1 - e^{-2kt}\right), \ \rho_1 > 0, \ k > 0, \ t \ge 0, \tag{7}$$

$$\int_0^t |v(s)|^2 \, ds \le \frac{\sigma_1^2}{2k}\left(1 - e^{-2kt}\right), \ \sigma_1 > 0, \ k > 0, \ t \ge 0. \tag{8}$$

(7) yoki (8) ko‘rinishdagi chegaralanishni noldan eksponensial chegaralanish yoki qisqacha, I_k^1-chegaralanish deb ataymiz.

(5)–(8) differensial o‘yinni I_k^1–o‘yin deb ataymiz. Bundan tashqari, ikkinchi bobda (7) va (8) chegaralanishlarning umumlashmasi sifatida quvlovchi va qochuvchining boshqaruvlari $u(\cdot)$ va $v(\cdot)$ o‘lchovli funksiyalar sifatida tanlanadigan va mos ravishda quyidagi integral chegaralanishlar (qisqacha, I_k^2- chegaralanishlar) ni qanoatlantiradigan holdagi (5) va (6) sodda harakatlarga ega differensial o‘yinni qaraymiz:

$$\int_0^t |u(s)|^2\, ds \le \frac{\rho_1^2}{2k}(1-e^{-2kt})+\rho_0^2,\ \ \rho_1>0,\ \rho_0>0,\ k>0,\ t\ge 0, \tag{9}$$

$$\int_0^t |v(s)|^2\, ds \le \frac{\sigma_1^2}{2k}(1-e^{-2kt})+\sigma_0^2,\ \ \sigma_1>0,\ \sigma_0>0,\ k>0,\ t\ge 0 \tag{10}$$

va (9) yoki (10) ko‘rinishidagi integral chegaralanishni boshlang‘ich resursga ega eksponensial chegaralanish deb ataymiz. Bunda ρ_0^2 va σ_0^2 mos ravishda P va E obyektlarning boshlang‘ich resurslarini ifodalaydi. $\mathbb{U}_{I_k}^2$ va $\mathbb{V}_{I_k}^2$ orqali (9) va (10) chegaralanishlarni qanoatlantiruvchi barcha $u(\cdot)$ va $v(\cdot)$ o‘lchovli funksiyalar sinflarini belgilaymiz.

(5), (6), (9), (10) differensial o‘yinni I_k^2-o‘yin deb ataymiz.

7-ta’rif. I_k^1-quvish o‘yinida (I_k^2-quvish o‘yinida) Borel ma’nosida o‘lchovli $\mathbf{u}_{I_k^1}:\mathbb{R}_+\times\mathbb{V}_{I_k}^1\to\mathbb{U}_{I_k}^1$ $\left(\mathbf{u}_{I_k^2}:\mathbb{R}_+\times\mathbb{V}_{I_k}^2\to\mathbb{U}_{I_k}^2\right)$ funksiyaga quvlovchining strategiyasi deyiladi.

8-ta’rif. Agar ixtiyoriy $v(\cdot)\in\mathbb{V}_{I_k}^1$ $\left(v(\cdot)\in\mathbb{V}_{I_k}^2\right)$ boshqaruv uchun biror $t^*\in\left[0,\,T\left(\mathbf{u}_{I_k^1}\right)\right]$ $\left(t^*\in\left[0,\,T\left(\mathbf{u}_{I_k^2}\right)\right]\right)$ da $x(t^*)=y(t^*)$ bajarilsa, u holda $\mathbf{u}_{I_k^1}=\mathbf{u}_{I_k^1}(t,v)$ $\left(\mathbf{u}_{I_k^2}=\mathbf{u}_{I_k^2}(t,v)\right)$ strategiya $T\left(\mathbf{u}_{I_k^1}\right)$ $\left(T\left(\mathbf{u}_{I_k^2}\right)\right)$ chekli vaqtda quvishni yakunlashni kafolatlaydi deb aytamiz, bu yerda $x(t)$ va $y(t)$, $t\ge 0$ lar mos ravishda (5) va (6) masalalarning yechimlari.

9-ta’rif. $\rho\ge\sigma$ uchun $\mathbf{u}_{I_k^1}(t,v)=v-\lambda_{I_k^1}(t,v)\xi_0$ funksiyani I_k^1-quvish o‘yinida quvlovchining $\Pi_{I_k^1}$-strategiyasi deb ataymiz, bu yerda

$$\lambda_{I_k^1}(t,v)=\langle v,\xi_0\rangle+\sqrt{\langle v,\xi_0\rangle^2+\delta_1 e^{-2kt}},\ \ \delta_1=\rho_1^2-\sigma_1^2,\ \ \xi_0=z_0/|z_0|$$

va $\langle v,\xi_0\rangle-$ esa $\mathbb{R}^n$ fazodagi v va ξ_0 vektorlarning skalyar ko‘paytmasini ifodalaydi.

Ta’kidlash kerakki, $t\ge 0$ da $\left|\mathbf{u}_{I_k^1}(t,v)\right|^2=|v|^2+\delta_1 e^{-2kt}$ o‘rinli bo‘ladi.

10-ta’rif. Agar ushbu

$$\sqrt{\Phi_P(t)+\Psi_P(t)}-\sqrt{\Phi_P(t)}=|z_0| \tag{11}$$

tenglamaning t bo‘yicha kamida bitta musbat ildizi mavjud bo‘lsa, u holda (11) tenglamaning eng kichik musbat ildizini kafolatlangan quvish vaqti deb ataymiz va uni $T_{I_k^1}$ orqali belgilaymiz, bu yerda

$$\Phi_P(t) = t\sigma_1^2(1-e^{-2kt})/2k, \quad \Psi_P(t) = (\rho_1^2 - \sigma_1^2)(1-e^{-kt})^2/k^2.$$

4-teorema. Agar I_k^1–quvish o‘yinida $\rho_1 > \sigma_1$ va (11) tenglamaning biror musbat ildizi mavjud bo‘lsa, u holda $\Pi_{I_k^1}$-strategiya $\left[0, T_{I_k^1}\right]$ vaqt oralig‘ida quvishni yakunlashni kafolatlaydi.

11-ta’rif. I_k^1–qochish o‘yinida $\sigma_1 \geq \rho_1$ bo‘lsin. U holda qochuvchining $E_{I_k^1}$-strategiyasi deb,

$$v_{I_k^1}(t, u_\varepsilon(t)) = \begin{cases} 0, & \text{if} \quad 0 \leq t < \varepsilon, \\ -\sqrt{|u(t-\varepsilon)|^2 + \theta e^{-2k(t-\varepsilon)}}\,\xi_0, & \text{if} \quad t \geq \varepsilon \end{cases}$$

funksiyaga aytamiz, bu yerda

$$u_\varepsilon(t) = \begin{cases} 0, & \text{if} \quad 0 \leq t < \varepsilon, \\ u(t-\varepsilon), & \text{if} \quad t \geq \varepsilon, \end{cases} \quad u_\varepsilon(t) \in \mathbb{R}^n, \quad \theta = \sigma_1^2 - \rho_1^2, \quad \xi_0 = z_0/|z_0|.$$

12-ta’rif. Ixtiyoriy $u(\cdot) \in \mathbb{U}_{I_k}^1$ boshqaruv uchun $E_{I_k^1}$- strategiya qochuvchi uchun yutuqli deyiladi agar:

a) $\dot{z} = u(t) - v_{I_k^1}(t, u_\varepsilon(t)),\ z(0) = z_0$ Koshi masalasining $z(t)$ yechimi noldan farqli bo‘lsa, ya’ni ixtiyoriy t, $t \geq 0$ lar uchun $z(t) \neq 0$ bo‘lsa;

b) ixtiyoriy t, $t \geq 0$ lar uchun $v_{I_k^1}(t, u_\varepsilon(\cdot)) \in \mathbb{V}_{I_k}^1$ tegishlilik o‘rinli bo‘lsa.

5-teorema. Agar I_k^1–o‘yinda $\rho_1 \leq \sigma_1,\ k > 0$ va $0 < \varepsilon \leq \dfrac{2k|z_0|^2}{\rho_1^2}$ bo‘lsa, u holda $E_{I_k^1}$-strategiya qochuvchi uchun yutuqli bo‘ladi hamda P va E obyektlar orasidagi masofa uchun quyidagi baholash o‘rinli bo‘ladi:

$$|z(t)| > \begin{cases} 0, & \text{if} \quad 0 \leq t < \varepsilon, \\ \sqrt{\Phi_E(t-\varepsilon) + \Psi_E(t-\varepsilon)} - \sqrt{\Phi_E(t-\varepsilon)}, & \text{if} \quad t \geq \varepsilon, \end{cases}$$

bu yerda $\Phi_E(t) = t\rho_1^2(1-e^{-2kt})/(2k), \quad \Psi_E(t) = (\sigma_1^2 - \rho_1^2)q(1-e^{-kt})^2/k^2.$

13-ta’rif. I_k^2–o‘yinda quvlovchining $\Pi_{I_k^2}$-strategiyasi deb, $\mathbf{u}_{I_k^2}(t, v) = v - \lambda_{I_k^2}(t, v)\xi_0$ funksiyaga aytamiz, bu yerda

$$\lambda_{I_k^2}(t, v) = \mu_0 + \langle v, \xi_0 \rangle + \sqrt{(\mu_0 + \langle v, \xi_0 \rangle)^2 + \delta_1 e^{-2kt}}, \ \mu_0 = \delta_0/2|z_0|, \ \delta_0 = \rho_0^2 - \sigma_0^2,$$

$\delta_1 = \rho_1^2 - \sigma_1^2$, $\xi_0 = z_0/|z_0|$, $z_0 = x_0 - y_0$ va $\langle v, \xi_0 \rangle$–ifoda $\mathbb{R}^n$ fazodagi v va ξ_0 vektorlarning skalyar ko‘paytmasini bildiradi.

14-ta’rif. Agar, ushbu

$$\sqrt{\Phi(t) + \Psi(t)} - \sqrt{\Phi(t)} = |z_0| \tag{12}$$

tenglamaning t bo‘yicha eng kamida bitta musbat ildizi mavjud bo‘lsa, u holda (12) tenglamaning birinchi musbat ildizini kafolatlangan tutish vaqti deb ataymiz va uni $T_{I_k^2}$ orqali belgilaymiz, bu yerda

$$\Phi(t) = t\left(\frac{\sigma_1^2}{2k}(1-e^{-2kt}) + \sigma_0^2\right), \quad \Psi(t) = \delta_0 t + \delta_1\left(\frac{1-e^{-kt}}{k}\right)^2.$$

6-teorema. a) $\rho_1 > \sigma_1$ bo‘lsin; b) (12) tenglama kamida bitta musbat ildizga ega bo‘lsin. U holda I_k^2 –o‘yinda $\Pi_{I_k^2}$-strategiya $\left[0, T_{I_k^2}\right]$ vaqt oralig‘ida quvishni yakunlashini kafolatlaydi.

Uchinchi bob “**Boshqaruvlari nostatsionar geometrik chegaralanishli differensial o‘yinlar**” deb nomlangan bo‘lib, mazkur bob boshqaruvlari nostatsionar geometrik chegaralanishlarga ega sodda va inersion harakatli differensial o‘yinlarda quvish-qochish va “Qutulish chizig‘i” masalalarini tadqiq qilishga bag‘ishlangan.

$\mathbb{R}^n$ fazoda quvlovchi va qochuvchi mos ravishda (5) va (6) tenglamalar bilan harakatlanadigan holda quvish-qochish masalalarini ko‘rib chiqamiz. Obyektlarning boshqaruvlari mos ravishda

$$|u(t)| \le \alpha(t) \text{ deyarli barcha } t \ge 0, \tag{13}$$

$$|v(t)| \le \beta(t) \text{ deyarli barcha } t \ge 0, \tag{14}$$

nostatsionar geometrik chegaralanishlarga (yoki qisqacha, G_t –chegaralanishlar) bo‘ysunadi, bu yerda $\alpha(t)$ va $\beta(t)$ mos ravishda quvlovchi va qochuvchining har bir ayni $t \ge 0$ paytdagi maksimal tezlik qiymatlarini ifodalovchi manfiy bo‘lmagan integrallanuvchi funksiyalar. $\mathbb{U}_G^t$ va $\mathbb{V}_G^t$ orqali mos ravishda (13) va (14) G_t – chegaralanishlarni qanoatlantiruvchi barcha $u(\cdot)$ va $v(\cdot)$ o‘lchovli funksiyalar to‘plamlarini belgilaymiz.

Ushbu bobda asosan, qochuvchi uchun “Qutilish chizig‘i” deb nomlanuvchi $M \subset \mathbb{R}^n$ yopiq qism to‘plam bilan berilgan holat chegaralanishlariga ega o‘yinni o‘rganamiz. “Qutilish chizig‘i”ga ega differensial o‘yinda P quvlovchining maqsadi E qochuvchini $\mathbb{R}^n \setminus M$ sohada tutish, ya’ni biror $t, t > 0$ vaqtda $x(t) = y(t)$ tenglikni amalga oshirish, bu yerda $x(t)$ va $y(t)$ mos ravishda (5) va (6) tenglamalarning yechimlari. E qochuvchining maqsadi esa quvlovchi tutishidan oldin M to‘plamga yetib borish yoki barcha $t \ge 0$ uchun $x(t) \ne y(t)$ tengsizlikni amalga oshirish. M to‘plam P quvlovchining harakatini cheklamasligini aytib o‘tishimiz kerak. Bundan tashqari x_0 va y_0 boshlang‘ich holatlar $x_0 \ne y_0$ va $y_0 \notin M$ shartlar bilan berilgan deb faraz qilamiz.

$B(c;r)$ markazi c nuqtada bo‘lgan r radiusli $\mathbb{R}^n$ dagi sharni ifodalasin.

$z = x - y,\ z_0 = x_0 - y_0$ belgilaymiz, u holda (5) va (6) tenglamalardan ushbu $\dot{z} = u - v,\ z(0) = z_0$ ko‘rinishdagi tenglamaga ega bo‘lamiz.

15-ta’rif. Agar ixtiyoriy $v(\cdot) \in \mathbb{V}_G^t$ boshqaruv uchun shunday $t^* \in [0, T]$ vaqt mavjud bo‘lib, bunda: 1) $x(t^*) = y(t^*)$ bo‘lsa; 2) barcha $t \in [0, t^*]$ uchun $y(t) \notin M$ bo‘lsa, u holda “Qutilish chizig‘i” o‘yinida $\mathbf{u}_{G_t}(v,t)$ strategiya chekli $[0, T]$ vaqt oralig‘ida P quvlovchi uchun yutuqli deyiladi.

16-ta'rif. $\alpha(t)\ge\beta(t)$, $t\ge 0$ lar uchun ushbu $\mathbf{u}_{G_t}(v,t)=v-\lambda_{G_t}(v,t)\xi_0$ funksiyani quvlovchining parallel quvish strategiyasi yoki Π_{G_t}-strategiya deb ataymiz, bu yerda $\lambda_{G_t}(v,t)=\langle v,\xi_0\rangle+\sqrt{\langle v,\xi_0\rangle^2+\alpha^2(t)-|v|^2}$, $\xi_0=z_0/|z_0|$ va $\langle v,\xi_0\rangle-$ifoda $\mathbb{R}^n$ fazodagi v va ξ_0 vektorlarning skalyar ko'paytmasi.

7-teorema. Barcha $t\ge 0$ uchun $\alpha(t)\ge\beta(t)$ o'rinli, T_{G_t} esa $\int_0^t(\alpha(s)-\beta(s))ds=|z_0|$ tenglamaning birinchi musbat ildizi bo'lsin. U holda G_t-o'yinda: a) Π_{G_t}-strategiya P quvlovchi uchun $\left[0,T_{G_t}\right]$ vaqt oralig'ida yutuqli bo'ladi; b) P quvlovchining ixtiyoriy boshqaruvi uchun ushbu $v^*(t)=-\beta(t)\xi_0$ strategiya E qochuvchi uchun yutuqli bo'ladi, ya'ni $\left[0,T_{G_t}\right)$ vaqt oralig'ida $|z(t)|\ge|z_0|-\int_0^t(\alpha(s)-\beta(s))ds>0$ tengsizlik o'rinli bo'ladi.

$(x(t),y(t))$ juftlik uchun quyidagi to'plamni qaraylik:

$$\mathbf{P}(t)=\mathbf{P}(x(t),y(t))=\{w:\beta(t)|w-x(t)|\ge\alpha(t)|w-y(t)|\}.$$

Bunda har bir $t\in[0,t^*]$ uchun $y(t)\in\mathbf{P}(t)$ munosabat bajariladi.

1-lemma. Agar, barcha $t\in\left[0,T_{G_t}\right]$ uchun $\alpha(0)>\beta(0)$, $\alpha'(t)\beta(t)\ge\alpha(t)\beta'(t)$ tengsizliklar bajarilsa, u holda $\mathbf{P}(t)$ to'plam $\left[0,T_{G_t}\right]$ oraliqda kamayuvchi bo'ladi, ya'ni har bir $t_1,t_2\in\left[0,T_{G_t}\right]$ lar uchun $t_1<t_2$ bo'lganda $\mathbf{P}(t_1)\supset\mathbf{P}(t_2)$ o'rinli bo'ladi.

8-teorema. 7-teorema va 1-lemmaning shartlari bajarilsin. Agar $M\cap\mathbf{P}(x_0,y_0)=\varnothing$ bo'lsa, u holda G_t-chegaralanishlarga ega "Qutilish chizig'i" o'yinida Π_{G_t}-strategiya quvlovchi uchun yutuqli bo'ladi.

9-teorema. Agar 7-teoremaning shartlari bajarilsa, u holda $v^*(t)=-\beta(t)\xi_0$ strategiya G_t-chegaralanishlarga ega "Qutilish chizig'i" o'yinida qochuvchi uchun yutuqli bo'ladi.

Endi o'yinchilarning harakat dinamikasi

$$P:\ \ddot{x}=u,\ x(0)=x_0,\ \dot{x}(0)=x_1, \tag{15}$$

$$E:\ \ddot{y}=v,\ y(0)=y_0,\ \dot{y}(0)=y_1, \tag{16}$$

ikkinchi tartibli differensial tenglamalar bilan tavsiflangan differensial o'yinni qaraymiz, bu yerda $x,y,x_0,y_0,x_1,y_1,u,v\in\mathbb{R}^n$, $n\ge 1$; x_0 va y_0- P va E o'yinchilarning boshlang'ich vaziyatlari, x_1 va y_1 lar mos ravishda ularning boshlang'ich tezlik vektorlari. O'yinning boshida $x_0\ne y_0$ va $x_1=y_1$ shartlarni talab qilamiz. O'yinchilarning boshqaruvlari $u(\cdot):[0,\infty)\to\mathbb{R}^n$ va $v(\cdot):[0,\infty)\to\mathbb{R}^n$ o'lchovli funksiyalar sifatida tanlanadi va mos ravishda

$$|u(t)|\le\rho e^{-kt}\ \text{deyarli barcha}\ t\ge 0, \tag{17}$$

$$|v(t)| \le \sigma e^{-kt} \quad \text{deyarli barcha } t \ge 0 \tag{18}$$

geometrik chegaralanishlarga (qisqacha, $G_k^- -$chegaralanish) bo‘ysunadi.

$\mathbb{U}_k^G$ $\left(\mathbb{V}_k^G\right)$ orqali (17) ((18)) ni qanoatlantiruvchi barcha $u(\cdot)$ $\left(v(\cdot)\right)$ o‘lchovli funksiyalar to‘plamini belgilaymiz.

Keyinchalik, qisqalik uchun (15)–(18) o‘yinni ikkinchi tartibli $G_k^- -$o‘yin deb ataymiz.

17-ta’rif. $\rho \ge \sigma$ uchun, $|\mathbf{u}(t,v)| = \rho e^{-kt}$ qiymatga ega

$$\mathbf{u}(t,v) = v - \phi(t,v)\zeta_0, \;\; \phi(t,v) = \langle v, \zeta_0 \rangle + \sqrt{\langle v, \zeta_0 \rangle^2 + \rho^2 e^{-2kt} - |v|^2} \tag{19}$$

funksiyaga quvlovchining parallel quvish strategiyasi yoki qisqacha, Π-strategiya deyiladi, bu yerda $\zeta_0 = z_0 / |z_0|$.

1-tasdiq. $\rho > \sigma$ va $h(t) = 1 + \dfrac{\sigma - \rho}{k^2 |z_0|}\left(e^{-kt} + kt - 1\right)$, $t \ge 0$ bo‘lsin. U holda $h(t) = 0$ tenglama yagona musbat ildizga ega va uni T_* orqali ifodalaymiz.

10-teorema. Agar $\rho > \sigma$ bo‘lsa, u holda (19) Π-strategiya $[0, T_*]$ intervalda P uchun yutishni kafolatlaydi.

Qochish o‘yinida qochuvchi uchun

$$v_*(t) = -\sigma e^{-kt}\zeta_0, \;\; \zeta_0 = z_0 / |z_0| \tag{20}$$

ko‘rinishdagi joiz boshqaruv funksiyani taklif qilamiz.

11-teorema. a) Agar $\rho > \sigma$ bo‘lsa, u holda (20) boshqaruv $[0, T_*)$ intervalda E uchun yutishni kafolatlaydi, bu yerda T_* kafolatlangan quvish vaqti (10-teoremaga qarang);

b) Agar $\rho \le \sigma$ bo‘lsa, u holda (20) boshqaruv $[0, +\infty)$ intervalda E uchun yutishni kafolatlaydi.

$W(x,y)$ − x nuqtadan chiquvchi P quvlovchining y nuqtadan chiquvchi E qochuvchiga nisbatan oldinroq yetib boradigan barcha w nuqtalar to‘plami bo‘lsin, ya’ni

$$W(x,y) = \left\{ w : |w - x| \ge (\rho / \sigma)\,|w - y| \right\}.$$

$\rho \ne \sigma$ bo‘lgan holda, $W(x,y)$ ning chegarasi

$$S(x,y) = \left\{ w : |w - x| = (\rho / \sigma)\,|w - y| \right\}$$

Apolloniy sferasi bilan beriladi.

2-lemma. $W_*(t, x(t), y(t)) = W(x(t), y(t)) - tx_1$ bo‘lsin. U holda $W_*(t, x(t), y(t))$ t, $t \in [0, t^*]$ bo‘yicha ichma-ich joylashishga nisbatan kamayuvchi bo‘ladi, ya’ni agar $t_1, t_2 \in [0, t^*]$ va $t_1 < t_2$ bo‘lsa, u holda $W_*(t_2, x(t_2), y(t_2)) \subset W_*(t_1, x(t_1), y(t_1))$ bo‘ladi.

18-ta’rif. $W^*(x_0, y_0, T_*) = \bigcup\limits_{t=0}^{T_*} \left\{ W(x_0, y_0) + tx_1 \right\}$ to‘plamni ikkinchi tartibli $G_k^- -$ quvish o‘yinida qochuvchining yetishish sohasi deb ataymiz.

$\mathbb{R}^n$ fazoda “Qutilish chizig‘i” deb nomlangan M yopiq qism to‘plam berilgan bo‘lsin.

12-teorema. Agar $\rho > \sigma$ va $W^*(x_0, y_0, T_*) \cap M = \varnothing$ bo‘lsa, u holda (19) Π-strategiya $[0, T_*]$ intervalda P uchun yutishni kafolatlaydi.

Har bir $w \in W(x_0, y_0)$ uchun $\Theta(w, y_0) = \left\{ \theta : e^{-k\theta} + k\theta - 1 = \frac{k^2 |w - y_0|}{\sigma} \right\}$

to‘plamni va har bir $w \in W(x_0, y_0)$ va $\theta \in \Theta(w, y_0)$ uchun esa

$$W_E(w, \theta) = \{w_* : w_* = w + \theta x_1\} \tag{21}$$

to‘plamni qaraylik.

19-ta’rif. (21) to‘plamni M “Qutilish chizig‘i” ga ega ikkinchi tartibli G_k^- – o‘yinda qochuvchining yetishish sohasi deb ataymiz.

13-teorema. Agar $\rho > \sigma$ va $W_E(w, \theta) \cap M \neq \varnothing$ bo‘lsa, u holda E uchun yutuqli bo‘lgan $v_*(\cdot) \in \mathbb{V}_k^G$ boshqaruv mavjud.

XULOSA

Dissertatsiya ishi o‘yinchilarning boshqaruvlariga nostatsionar chegaralanishlar qo‘yilgan holda R. Ayzeksning “Qutilish chizig‘i” o‘yini va quvish-qochish muammolarini to‘liq o‘rganishga bag‘ishlangan.

Tadqiqotning asosiy natijalari quyidagilardan tarkib topgan:

1. Quvlovchilarning boshqaruvlariga chiziqli chegaralanish, qochuvchining boshqaruviga esa geometrik chegaralanish qo‘yilgan holdagi sodda harakatli differensial o‘yinda quvlovchilar uchun parallel quvish strategiyasi qurilgan hamda har bir quvlovchining yetishish to‘plami topilib, bu to‘plamlarning monotonlik shartlari asosida quvish masalasining yechimi va R. Ayzeks “Qutulish chizig‘i” masalasi yechimi mavjudligining yangi yetarli shartlari topilgan;
2. Boshqaruvlari eksponensial integral chegaralanishlarga ega differensial o‘yinda quvlovchi uchun parallel quvish strategiyasi qurilgan va quvish-qochish masalasi yechilishining yangi yetarli shartlari aniqlangan;
3. Boshqaruvlari nostatsionar geometrik chegaralanishlarga ega differensial o‘yinlarda quvlovchi uchun parallel quvish strategiyasi qurilgan, qochuvchi uchun esa alohida boshqaruv funksiya topilgan va quvish-qochishning yangi yetarli shartlari aniqlangan;
4. Boshqaruvlari nostatsionar geometrik chegaralanishli differensial o‘yinlarda o‘yinchilarning yetishish to‘plami qurilgan va bu to‘plamning monotonlik shartlari asosida R.Ayzeksning “Qutulish chizig‘i” masalasi to‘liq hal qilingan.

SCIENTIFIC COUNCIL AWARDING SCIENTIFIC DEGREES PhD.03/30.12.2019.FM.05.04 FERGANA STATE UNIVERSITY

NAMANGAN STATE UNIVERSITY

HORILOV MAXMUD ABDUMALIKOVICH

DIFFERENTIAL GAMES WITH NON-STATIONARY CONSTRAINTS ON CONTROLS

01.01.02 – Differential equations and mathematical physics

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

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INTRODUCTION (abstract of the PhD thesis)

Actuality and demand of the theme of dissertation. The optimization problems of time, resource, cost, quality and capacity are taking one of the leading roles in solving numerous applied and theoretical issues arising due to multitudinous scientific-practical researches, which are being conducted around the world. Such problems on a global scale involve constructing mathematical models of control processes and determining optimal solutions of the problems. In this regard, researching on conflict-controlled processes being the main object of the theory of differential games, determining the existence of new sufficient conditions for pursuit-evasion problems with non-stationary constraints on controls, and description of the attainability set of the considered objects are considered to have great significance.

Worldwide, researches oriented towards exploring differential game problems with non-stationary constraint on controls are being conducted. In order to improve the efficiency of the results of classical problems of differential games, special attention is being paid to the study of multi-player differential game problems with constraints of different forms on controls, constructing optimal strategies for pursuit-evasion problems in differential games with constraints of non-stationary or various forms on controls, finding new sufficient conditions, describing the attainability set of players, including solving the “Life-line” game of Isaacs.

In our country, being performed comprehensive measures in terms of paying attention to momentous areas with scientific and practical applications of fundamental sciences, certain results are being achieved. The main tasks[1] have been defined with regard to the priority directions of, such as, “algebra and its applications, differential equations and their applications, nonlinear systems, mathematical modeling of dynamical systems and their applications, stochastic analysis, medical-biological informatics, mathematics of calculus”. In the implementation of these tasks, including for the purpose of developing the theories of differential equations, dynamical systems and differential games, constructing optimal strategies for pursuit-evasion problems in differential games with constraints of non-stationary or various forms on controls is taking a crucial role.

The subject and object of research of this dissertation are in line with tasks identified in the Decrees of the President of the Republic of Uzbekistan UP-4947 of February 7, 2017 “On the strategy of action for the further development of the Republic of Uzbekistan”, UP-2789 dated February 17, 2017 “On measures to further improvement of the activities of the Academy of Sciences, organization, management and financing of research activities, PP-3682 from April 27, 2018 “On measures to further improve the system of practical implementation of innovative ideas, technologies and projects” and PP-4387 from July 9, 2019

[1] Decree of the President of the Republic of Uzbekistan dated July 9, 2019 №PQ-4387 “On state support for the further development of mathematics education and subjects, as well as measures to fundamentally improve the activities of the Institute of Mathematics named after V.I. Romanovsky of the Academy of Sciences of the Republic of Uzbekistan”.

“On measures to further development of mathematical education and science, and also root improvement of the activity of the Uzbekistan Academy of Sciences V.I.Romanovsky Institute of Mathematics”, as well as in other regulations related to basic science.

Connection of research to priority directions of development of science and technologies of the Republic. This research was conducted in connection with the priority areas of science and technology of the Republic of Uzbekistan IV, “Mathematics, Mechanics and Computer Science”.

The degree of scrutiny of the problem. The concept of “differential game” was introduced by an American mathematician R.Isaacs in the early 1950s. In 1965, the researches of R.Isaacs were published in the form of a monograph. This monograph consists of heuristics and many examples derived from the Hamilton-Jacobi theory, and does not include the general theory. In those years, L.S.Pontryagin and his students created a mathematical theory of controlled processes based on the simplification of the tasks of controlling an aircraft in an air attack. It should be noted that a scientific school on the theory of differential games was formed in Uzbekistan, which was founded by N. Yu. Satimov, and currently, A.A.Azamov is leading this school.

The first correct method of L.S.Pontryagin was transferred to the case of integral constraints by M.S.Nikolsky. This approach was widely developed in the works of A.Ya.Azimov, F.V.Guseinov, and others. Based on N.N.Krasovskii’s method relying on extremal goal for solving differential games with integral constraints, a well-founded method was developed. A positional approach for the regular case was transferred to games of integral restriction from restrictions of different forms in the works of B.N.Pshenichnyi, B.N.Pshenichnyi, A.A.Chikrii, and I.J.Rappoport, B.N.Pshenichnyi and Yu.N.Onopchuk, and others. Later on, N.Yu.Satimov, G.I.Ibragimov, B.T.Samatov, and others proposed to solve differential and discrete games with integral constraints based on the method of distribution of control resources between team pursuers. In recent years, A.A.Azamov and A.Sh.Kuchkarov have created the foundation for the research on interdependence of problems of pursuit, controllability, and stability in linear systems when geometric constraints are imposed on the control vector of a pursuer, and integral constraints are imposed on the control function of an evader. In the general theory of differential games, pursuit-evasion problems are of special importance with a number of identities. These include the diversity in formulation of a problem, the applicable methods, and the characteristics of the results. Based on the special property of the attainability domain, the “Life-line” problem of R.Isaacs was solved by L.A.Petrosyan in the case of simple motions. In this case, the parallel pursuit strategy was proven to be the optimal strategy. Despite the fact that many scientific works as above have been carried out, it should be noted that differential games with non-stationary constraints on control functions have not been investigated enough. Therefore, the research on differential games under non-stationary constraints on control functions is of great importance. This dissertation is dedicated to solve differential games, which are of control functions under non-stationary constraints, and the “Life-line” problem of R. Isaacs.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. This research was performed within the “Fundamental research” network of the plan of scientific research works of Namangan State University.

The aim of the research is to solve the pursuit-evasion problems and the “Life-line” game of R.Isaacs when non-stationary constraints are imposed on controls of players.

Problems of the research:

find new sufficient conditions for the pursuit problem to have a solution in a differential game with linear constraints on players' controls

find new sufficient conditions for the solvability of the pursuit-evasion problems with exponential integral constraints on controls;

determine new sufficient conditions for the solvability of the pursuit-evasion problems with non-stationary geometric constraints on controls;

Solve the “Life-line” problem of R.Isaacs in differential games with non-stationary geometric constraints on controls.

The object of the research is the theory of optimal control, the theory of differential equations, the theory of differential games.

The subject of the research is pursuit-evasion problems with geometric, non-stationary geometric, linear and non-stationary integral constraints on controls.

Methods of the research. In this dissertation, to solve the pursuit problems the method of parallel approach of players is implemented, and to solve the evasion problems the method of evasion in terms of the direction is used. Moreover, elements of convex analysis, optimal control theory and multi-valued mappings are applied in proofs of theorems and lemmas.

Scientific novelty of the research consists of the following:

In order to end the pursuit in a differential game of linear constraints on control functions, new sufficient conditions are obtained for pursuers, and the monotonicity of the players' attainability domain is substantiated by means of the support function of a set;

Pursuit-evasion differential game problems in the case of control functions of exponential integral restrictions are solved for pursuer with the help of the method of parallel convergence and for evader by virtue of the method of directional escape;

In order to solve pursuit-evasion problems in a differential game of non-stationary geometric restrictions on control functions, new sufficient conditions are defined for pursuer with the help of the method of parallel convergence and for evader by virtue of the method of directional escape;

In a differential game of non-stationary geometric restrictions on control functions, new monotonicity conditions of the players' attainability domain are found, and the "Life-line" problem of Isaacs is fully solved by means of the monotonicity property of the players' attainability domain.

Practical results of the research are to evolve the scope of the theory of pursuit-evasion differential games, contributing to solving many problems related to mathematical modeling and prognosis of real-life processes depending on the technical, economical, biological problems.

The reliability of the results of the study. The results in this dissertation have been attained by means of the methods of the theory of differential equations, functional and convex analysis, optimal control theory, the theory of differential games.

The scientific and practical significance of the research results. The scientific importance of the results of the present research work is to establish admissible strategies satisfying non-stationary constraints on the control and expenditure. It is worth noting that the obtained results will serve as the motivation for the interest to the theory of optimal control and differential games, and they can be effective for solving problems associated with dynamic games and their applications.

The practical importance of this research work appears clearly in the construction of control frameworks in mathematical models of dynamic games and their proportion for the expansion of numerical algorithms and computer science.

Implementation of the research results. On the basis of the results obtained in differential games with non-stationary constraints on controls:

the obtained results in terms of existence of a solution to pursuit-evasion problems with non-stationary constraints on controls and to the Isaacs "Life-line" problem were employed in conflict-controlled dynamical systems with non-stationary constraints on controls in the fundamental project named "Creation of new methods of control of conflict situations characterized by differential equations and their numerical implementation", with number OT-F4-33 (the reference with number 04/11-3587 of the National University of Uzbekistan named after Mirzo Ulugbek dated June 9, 2023). As a result, it was possible to determine generalized solutions of the conflicted control problems (dynamic games) in dynamical systems with non-stationary constraints;

the obtained results in terms of existence and optimality of a solution to differential game problem with linear constraints on controls were employed in solving analogical problems for ordinary differential equations in the foreign grant project named "Problems of phase transitions and analytical phenomena. Mathematical properties of their fast transition equations and asymptotics", with number 374874-2015 (the reference with number 838 of Osh State University dated June 27, 2023, Kyrgyz Republic). As a consequence, it was possible to construct a solution of new non-local generalized spectral problems.

Approbation of the research results. The results of the research were discussed at 18 scientific and practical conferences, including 9 international and 9 Republican scientific and practical conferences.

Publications of the research results. On the topic of the dissertation, 24 scientific papers were published, 6 of which are included in the list of scientific publications proposed by the Higher Attestation Commission of the Republic of

Uzbekistan for the defense of theses of the Doctor of Philosophy. 2 of them published in foreign journals, 4 in national scientific journals and 18 abstracts.

The structure and volume of the dissertation. The dissertation consists of the introduction, three chapters, conclusion and bibliography. The total volume of the dissertation consists of 93 pages.

THE MAIN CONTENT OF THE DISSERTATION

In introduction the motivation of research topic and correspondence to the priority research areas of science and technology of the Republic are given, we present a review of international research on the topic of the dissertation and degree of scrutiny of the problem, formulate our goals and objectives, identify the object and subject of study, and state scientific novelty and practical results of the research. Moreover, we give the theoretical and practical importance of the obtained results, and also give information on the implementation of the research results, the published works and the structure of the dissertation.

The first chapter is called "**Differential games with the $LG-$constraints on controls**", and this chapter is devoted to investigate simple motion differential games with the non-stationary integral constraints on controls.

Let us examine the differential game when pursuers P_i, $i=1,2,...,m$ and evader E having radius vectors x_i and y correspondingly move in space $\mathbb{R}^n$. If their velocity vectors are u_i and v, which function as control parameters of the Pursuers and the Evader, respectively, then we consider the game described as

$$P_i: \ \dot{x}_i = u_i, \quad x_i(0) = x_{i0}, \tag{1}$$

$$E: \ \dot{y} = v, \quad y(0) = y_0, \tag{2}$$

where $x_i, y, u_i, v \in \mathbb{R}^n$, $n \geq 2$, and x_{i0}, $i=1,2,...,m$, are the initial positions of the objects P_i and y_0 is that of the object E. Here the temporal variation of the control u_i of each pursuer P_i should be a measurable function $u_i(\cdot):[0,\infty) \to \mathbb{R}^n$, and on each control function $u_i(\cdot)$ we will impose the integral constraint of the form

$$\int_0^t |u_i(s)|^2 \, ds \leq L_i(t) \ \text{ for almost every } t \geq 0, \tag{3}$$

admitting a linear change $L_i(t) = k_i t + \rho_{i0}$, $i=1,2,...,m$, at each time t, where k_i, ρ_{i0} are non-negative parametric numbers. We call the constraint (3) the linear constraint or briefly, the L_i-constraint, and denote by $\mathbb{U}_L^i$ the class of admissible controls of Pursuers P_i, i.e. of all measurable functions satisfying (3).

From the physical point of view, the right-hand side of (3) corresponds to the linear change of the given resource depending on time $t \geq 0$. Therefore, the linear function $L_i(t)$ can be called the current change of the given resources of pursuers P_i. It is clear that the pursuers' resources increase if $k_i > 0$, decrease if $k_i < 0$ and remain unchanged if $k_i = 0$. In the last case, (3) is called the integral constraint. If $\rho_{i0} = 0$ and $k_i > 0$, then (3) is called the geometric constraint.

Analogously, it is regarded that the temporal variation of the control v of Evader E becomes also a measurable function $v(\cdot):[0,\infty) \to \mathbb{R}^n$ and it is required that this control function satisfies the G-constraint

$$|v(t)| \le \beta \text{ for almost every } t \ge 0, \tag{4}$$

where β is a non-negative parametric number, which expresses the maximal value of velocity of the evader. We refer by $\mathbb{V}_G$ the class of all the evader's admissible controls satisfying (4).

We call the differential game (1)–(4) the L_iG-Game.

In the L_iG-Game, the main objective of Pursuers P_i, $i = 1,2,...,m$ is to capture Evader E, i.e., to reach the equality $x_i(t) = y(t)$ for some $i = 1,2,...,m,$ where $x_i(t), i = 1,2,...,m$ and $y(t)$ are the trajectories generated during the L_iG-Game. Whereas Evader E strives to avoid the meeting, and if it is impossible, postpone the moment of the meeting as far as possible.

In this chapter, we are going to study mainly the game with phase constraints for the Evader being given by a subset A of $\mathbb{R}^n$ which is called the "Life-Line".

Definition 1. By (1) and (2), the pairs $(x_{i0}, u_i(\cdot))$, $u_i(\cdot) \in \mathbb{U}_L^i$, $i = 1,2,...,m$ and $(y_0, v(\cdot))$, $v(\cdot) \in \mathbb{V}_G$ bring about the solutions $x_i(t) = x_{i0} + \int_0^t u_i(s)ds$ and respectively. In this case, we call $x_i(t)$ the motion trajectory of Pursuers P_i, and $y(t)$ the motion trajectory of Evader E.

Definition 2. For every triple $(k_i, \rho_{i0}, u_i(\cdot))$, $u_i(\cdot) \in \mathbb{U}_L^i$, $i = 1,2,...,m$, the scalar quantity $\rho_i(t) = L_i(t) - \int_0^t |u_i(s)|^2 ds, \ \rho_i(0) = \rho_{i0}, \ t \ge 0$ is called the residual resource of pursuer P_i at each moment t.

Definition 3. A mapping $\mathbf{u}_{L_iG}^i : \mathbb{V}_G \to \mathbb{U}_L^i$, $i = 1,2,...,m$ is called the strategy for pursuers P_i if: 1^o. (Admissibility) For an arbitrary control function $v(\cdot) \in \mathbb{V}_G$, the inclusion $u_i(\cdot) = \mathbf{u}_{L_iG}^i[v(\cdot)] \in \mathbb{U}_L^i$ holds; 2^o. (Volterranianity) For any controls $v_1(\cdot), v_2(\cdot) \in \mathbb{V}_G$ and for each $t \in [0, \infty)$, the equality $v_1(s) = v_2(s)$ a.e. on $[0, t]$ implies $u_{i1}(s) = u_{i2}(s)$ a.e. on $[0, t]$, where $u_{ij}(\cdot) = \mathbf{u}_{L_iG}^{ij}[v_j(\cdot)] \in \mathbb{U}_L^i$, $j = 1, 2$.

Definition 4. A strategy $\mathbf{u}_{L_iG}^i(v)$, $i = 1,2,...,m$ is called winning for Pursuers P_i on the time interval $\left[0, T_{L_iG}^i\right]$ in the L_iG-Game if, for every $v(\cdot) \in \mathbb{V}_G$, there exists a moment $t_i^* \in \left[0, T_{L_iG}^i\right]$ such that the equality $x(t_i^*) = y(t_i^*)$ takes place.

Definition 5. Assume that $y_0 \notin \bar{A} \subset \mathbb{R}^n$. Then a strategy $\mathbf{u}^i_{L_iG}(v)$, $i=1,2,...,m$ is called winning for Pursuers P_i on the time interval $\left[0, T^i_{L_iG}\right]$ in the L_iG-Game with a "Life-Line" if the following conditions are satisfied: a) there is some time $t_i^* \in \left[0, T^i_{L_iG}\right]$ at which $x(t_i^*) = y(t_i^*)$ is valid; b) $y_t(\cdot) = \left\{y(s): 0 \le s \le t_i^*\right\} \not\subset \bar{A}$.

Let $x_{i0} \neq y_0$ and the current value of control $v(t)$, $t \ge 0$, $v(t) \in \mathbb{V}_G$ be given. Suppose that the triple (μ_{i0}, k_i, β) is a parametric state of the L_iG-Game and let us denote it by p_i. We find the following nonempty and simply connected set of such states p_i: $\mathbf{P}^i_{L_iG} = \mathbf{P}^i_1 \cup \mathbf{P}^i_2 \cup \mathbf{P}^i_3$, where $\mathbf{P}^i_1 = \left\{p_i : \mu_{i0} \ge 0, k_i > \beta^2, \beta \ge 0\right\}$,

$$\mathbf{P}^i_2 = \left\{p_i : \mu_{i0} > 2\beta,\ k_i = \beta^2,\ \beta \ge 0\right\},$$

$$\mathbf{P}^i_3 = \left\{p_i : \mu_{i0} \ge 2\left(\beta + \sqrt{\beta^2 - k_i}\right), k_i < \beta^2, \beta \ge 0\right\}$$

and $\mathbf{P}^i_1$, $\mathbf{P}^i_2$, $\mathbf{P}^i_3$ are mutually disjoint sets.

Definition 6. We call the function $\mathbf{u}^i_{L_iG}(v) = v - \lambda^i_{L_iG}(v)\xi_{i0}$ the strategy of parallel pursuit (briefly, $\Pi^i_{L_iG}-$strategy) for Pursuer P_i in the L_iG-game, where

$$\lambda^i_{L_iG}(v) = \mu_{i0}/2 + \langle v, \xi_{i0}\rangle + \sqrt{\left(\mu_{i0}/2 + \langle v, \xi_{i0}\rangle\right)^2 + k_i - |v|^2},$$

$$\xi_{i0} = z_{i0}/|z_{i0}|, \quad z_{i0} = x_{i0} - y_0, \quad \mu_{i0} = \rho_{i0}/|z_{i0}|.$$

Let us suppose that at the moment t, $t \ge 0$, Evader E moves from a position y holding a constant vector v, $|v| \le \beta$. Pursuers P_i use the $\Pi^i_{L_iG}-$strategy from a position x_i basing on the resource ρ_i, $\rho_i > 0$. Then if w is a point where the Pursuers P_i should meet with Evader E at some time $T > 0$, then the set of all such points w will be given by the relations:

$$|w - x_i| = T\left|u_i(v)\right|, \quad |w - y| = T|v|, \quad T|u_i(v)|^2 = k_iT + \rho_i,$$

and from these relations we find

$$W^i_{L_iG}(x_i, y, \rho_i) = \left\{w : |w - x_i|^2 \ge (k_i/\beta^2)|w - y|^2 + (\rho_i/\beta)|w - y|\right\}.$$

Theorem 1. Let $p_i \in \mathbf{P}^i_{L_iG}$ be true in the L_iG-Game. Then the relation $W^i_{L_iG}(t_2) \subset W^i_{L_iG}(t_1)$ is true for any $t_1, t_2 \in [0, t_i^*]$, $0 \le t_1 \le t_2$.

Theorem 2. If $p_i \in \mathbf{P}^i_{L_iG}$ holds for some $i = 1,2,...,m$, then $y(t) \in W$ holds on $[0, T_{L_iG}]$, where

$$W = \bigcap_{i=1}^{m} W^i_{L_iG}(0), \ T_{L_iG} = d/\beta, \ d = \max\{|w_1 - w_2| : w_1, w_2 \in W\}.$$

Theorem 3. If $W \cap A = \varnothing$, then the $\Pi^i_{L_iG}-$strategy is winning for Pursuers P_i, $i = 1,2,...,m$ on $[0, T_{L_iG}]$ in the L_iG-Game with a "Life-line".

The second chapter is called "**Differential games with non-stationary**

integral constraints on controls", and this chapter is devoted to solve simple motion differential games with the non-stationary integral constraints on controls.

In space $\mathbb{R}^n$, let motions of the Pursuer and the Evader be given by the equations

$$P: \ \dot{x} = u, \quad x(0) = x_0, \tag{5}$$

$$E: \ \dot{y} = v, \quad y(0) = y_0, \tag{6}$$

respectively, where x_0 and y_0 describe Pursuer and Evader's initial states under the condition $x_0 \neq y_0$.

We denote the classes of all measurable functions $u(\cdot)$ and $v(\cdot)$ satisfying the conditions

$$\int_0^t |u(s)|^2 \, ds \leq \frac{\rho_1^2}{2k}\left(1 - e^{-2kt}\right), \ \ \rho_1 > 0, \ \ k > 0, \ \ t \geq 0, \tag{7}$$

$$\int_0^t |v(s)|^2 \, ds \leq \frac{\sigma_1^2}{2k}\left(1 - e^{-2kt}\right), \ \ \sigma_1 > 0, \ \ k > 0, \ \ t \geq 0 \tag{8}$$

by $\mathbb{U}_{I_k}^1$ and $\mathbb{V}_{I_k}^1$ respectively. We call the restriction of the form (7) or (8) an exponential constraint from scratch, or briefly, I_k^1-constraint.

We call the differential game (5)–(8) the I_k^1–Game. Furthermore, in the second chapter, as a generalization of the constraints (7) and (8) we consider the differential game with simple motions (5) and (6) when the Pursuer and the Evader have the controls $u(\cdot)$ and $v(\cdot)$, which are taken as measurable functions, being subjected to the integral constraints (briefly, the I_k^2 –constraints) of the forms

$$\int_0^t |u(s)|^2 \, ds \leq \frac{\rho_1^2}{2k}(1 - e^{-2kt}) + \rho_0^2, \ \ \rho_1 > 0, \ \ \rho_0 > 0, \ \ k > 0, \ \ t \geq 0, \tag{9}$$

$$\int_0^t |v(s)|^2 \, ds \leq \frac{\sigma_1^2}{2k}(1 - e^{-2kt}) + \sigma_0^2, \ \ \sigma_1 > 0, \ \ \sigma_0 > 0, \ \ k > 0, \ \ t \geq 0, \tag{10}$$

respectively, and we call integral constraint of the form (9) or (10) the exponential constraint with initial resource. Here ρ_0^2 and σ_0^2 represent the initial resources of the objects P and E. Denote the sets of all measurable functions satisfying the constraints (9) and (10) by $\mathbb{U}_{I_k}^2$ and $\mathbb{V}_{I_k}^2$, respectively.

We call the differential game (5), (6), (9), (10) the I_k^2 –Game.

Definition 7. In the I_k^1 –Game of Pursuit (the I_k^2 –Game of Pursuit) a Borel measurable function $\mathbf{u}_{I_k^1} : \mathbb{R}_+ \times \mathbb{V}_{I_k}^1 \to \mathbb{U}_{I_k}^1$ $\left(\mathbf{u}_{I_k^2} : \mathbb{R}_+ \times \mathbb{V}_{I_k}^2 \to \mathbb{U}_{I_k}^2\right)$ is called a strategy of the Pursuer.

Definition 8. We say that a strategy $\mathbf{u}_{I_k^1} = \mathbf{u}_{I_k^1}(t, v)$ $\left(\mathbf{u}_{I_k^2} = \mathbf{u}_{I_k^2}(t, v)\right)$ guarantees completion of pursuit at a finite time $T\left(\mathbf{u}_{I_k^1}\right)$ $\left(T\left(\mathbf{u}_{I_k^2}\right)\right)$ if $x(t^*) = y(t^*)$ holds for any

$v(\cdot)\in\mathbb{V}^1_{I_k}$ $\left(v(\cdot)\in\mathbb{V}^2_{I_k}\right)$ at some $t^*\in\left[0, T\left(\mathbf{u}_{I_k^1}\right)\right]$ $\left(t^*\in\left[0, T\left(\mathbf{u}_{I_k^2}\right)\right]\right)$, where $x(t)$ and $y(t)$, $t\geq 0$ are the solutions of the problems (5) and (6), respectively.

Definition 9. For $\rho\geq\sigma$, we call the function $\mathbf{u}_{I_k^1}(t,v)=v-\lambda_{I_k^1}(t,v)\xi_0$ the $\Pi_{I_k^1}$-strategy of the Pursuer in the I_k^1-Game of Pursuit, where

$$\lambda_{I_k^1}(t,v)=\langle v,\xi_0\rangle+\sqrt{\langle v,\xi_0\rangle^2+\delta_1 e^{-2kt}},\quad \delta_1=\rho_1^2-\sigma_1^2,\quad \xi_0=z_0/|z_0|,$$

and $\langle v,\xi_0\rangle$ denotes the inner product of the vectors v and ξ_0 in $\mathbb{R}^n$.

Note that $\left|\mathbf{u}_{I_k^1}(t,v)\right|^2=|v|^2+\delta_1 e^{-2kt}$ is satisfied for $t\geq 0$.

Definition 10. If there exists at least one positive root of the equation

$$\sqrt{\Phi_P(t)+\Psi_P(t)}-\sqrt{\Phi_P(t)}=|z_0| \tag{11}$$

with t, where $\Phi_P(t)=t\sigma_1^2(1-e^{-2kt})/2k,\ \ \Psi_P(t)=(\rho_1^2-\sigma_1^2)(1-e^{-kt})^2/k^2$, then we call the smallest positive root of equation (11) the guaranteed pursuit time and denote it by $T_{I_k^1}$.

Theorem 4. If $\rho_1>\sigma_1$ and there is some positive root of equation (11), then the $\Pi_{I_k^1}$-strategy guarantees completion of pursuit in the I_k^1-Game of Pursuit on the time interval $\left[0, T_{I_k^1}\right]$.

Definition 11. Let $\sigma_1\geq\rho_1$ in the I_k^1-Game of Evasion. Then as the $E_{I_k^1}$-strategy of the Evader we mean the function

$$v_{I_k^1}(t,u_\varepsilon(t))=\begin{cases}0, & \text{if}\quad 0\leq t<\varepsilon,\\ -\sqrt{|u(t-\varepsilon)|^2+\theta e^{-2k(t-\varepsilon)}}\,\xi_0, & \text{if}\quad t\geq\varepsilon,\end{cases}$$

where

$$u_\varepsilon(t)=\begin{cases}0, & \text{if}\quad 0\leq t<\varepsilon,\\ u(t-\varepsilon), & \text{if}\quad t\geq\varepsilon,\end{cases}\quad u_\varepsilon(t)\in\mathbb{R}^n,\ \ \theta=\sigma_1^2-\rho_1^2,\ \ \xi_0=z_0/|z_0|.$$

Definition 12. The $E_{I_k^1}$-strategy is called winning for the Evader if, for any $u(\cdot)\in\mathbb{U}_I^k$: a) the solution $z(t)$ of $\dot z=u(t)-v_{I_k^1}(t,u_\varepsilon(t)),\ z(0)=z_0$ is nonzero, i.e. $z(t)\neq 0$ for all t, $t\geq 0$; b) $v_{I_k^1}(t,u_\varepsilon(\cdot))\in\mathbb{V}^1_{I_k}$ for all t, $t\geq 0$.

Theorem 5. If $\rho_1\leq\sigma_1,\ k>0$ and $0<\varepsilon\leq\dfrac{2k|z_0|^2}{\rho_1^2}$ in the I_k^1-Game, then the $E_{I_k^1}$-strategy is winning for the Evader, and the following estimate is true for the distance between the objects P and E:

$$|z(t)|>\begin{cases}0, & \text{if}\quad 0\leq t<\varepsilon,\\ \sqrt{\Phi_E(t-\varepsilon)+\Psi_E(t-\varepsilon)}-\sqrt{\Phi_E(t-\varepsilon)}, & \text{if}\quad t\geq\varepsilon,\end{cases}$$

for all $t \geq 0$, where $\Phi_E(t) = t\rho_1^2(1-e^{-2kt})/(2k),\ \Psi_E(t) = (\sigma_1^2 - \rho_1^2)q(1-e^{-kt})^2/k^2$.

Definition 13. We say the function $\mathbf{u}_{I_k^2}(t,v) = v - \lambda_{I_k^2}(t,v)\xi_0$ the $\Pi_{I_k^2}$-strategy of the Pursuer in the I_k^2 −Game, where

$$\lambda_{I_k^2}(t,v) = \mu_0 + \langle v, \xi_0 \rangle + \sqrt{(\mu_0 + \langle v, \xi_0 \rangle)^2 + \delta_1 e^{-2kt}},\ \mu_0 = \delta_0 / 2|z_0|,\ \delta_0 = \rho_0^2 - \sigma_0^2,$$

$\delta_1 = \rho_1^2 - \sigma_1^2$, $\xi_0 = z_0/|z_0|$, $z_0 = x_0 - y_0$, and $\langle v, \xi_0 \rangle$ means the inner product of the vectors v and ξ_0 in $\mathbb{R}^n$.

Definition 14. If there exists at least one positive root of the equation

$$\sqrt{\Phi(t) + \Psi(t)} - \sqrt{\Phi(t)} = |z_0| \tag{12}$$

with t, where $\Phi(t) = t\left(\frac{\sigma_1^2}{2k}(1-e^{-2kt}) + \sigma_0^2\right)$, $\Psi(t) = \delta_0 t + \delta_1\left(\frac{1-e^{-kt}}{k}\right)^2$, then we call the first positive root of (12) the guaranteed capture time and denote it by $T_{I_k^2}$.

Theorem 6. Let a) $\rho_1 > \sigma_1$; b) Equation (12) has at least one positive root. Then the $\Pi_{I_k^2}$-strategy guarantees completion of pursuit on the time interval $\left[0, T_{I_k^2}\right]$ in the I_k^2 −Game.

The third chapter is called "**Differential games with non-stationary geometric constraints on controls**", and the present chapter is devoted to investigate the pursuit-evasion and "Life-Line" problems in differential games with the simple and the inertial motions under non-stationary geometric constraints on the controls of players.

In the space $\mathbb{R}^n$, consider the Pursuit-Evasion problems when the objects move in accordance with equations (6) and (7), respectively. The controls of the objects are subjected to the non-stationary geometric constraints (in brief, the G_t – constraints)

$$|u(t)| \leq \alpha(t) \text{ for almost every } t \geq 0, \tag{13}$$

$$|v(t)| \leq \beta(t) \text{ for almost every } t \geq 0 \tag{14}$$

correspondingly, where $\alpha(t)$ and $\beta(t)$ are non-negative integrable functions, which mean the maximal values of velocities of the Pursuer and the Evader at each current moment $t \geq 0$. We denote the sets of all measurable functions satisfying the G_t – constraints (13) and (14) by $\mathbb{U}_G^t$ and $\mathbb{V}_G^t$, respectively.

In the current chapter, we are mainly going to study the game with phase constraints for the Evader being given by a closed subset $M \subset \mathbb{R}^n$, which is called a "Life-Line". In the differential game with a "Life-Line", Pursuer P aims to catch Evader E, i.e. to realize the equality $x(t) = y(t)$ for some $t > 0$, where $x(t)$ and $y(t)$ are the solutions of equations (5) and (6), respectively, while Evader E stays in the zone $\mathbb{R}^n \setminus M$. The aim of Evader E is to reach the zone M before being caught by the Pursuer, or to keep the relation $x(t) \neq y(t)$ for all $t,\ t \geq 0$. Notice that M doesn't restrict motion of Pursuer P. Besides, we will assume that the initial positions x_0 and y_0 are given such that $x_0 \neq y_0$ and $y_0 \notin M$.

Let $B(c;r)$ denote the ball of radius r and centered at the point c in $\mathbb{R}^n$.

Write $z = x - y,\ z_0 = x_0 - y_0$, then from (5) and (6) we have the equation of the form $\dot{z} = u - v,\ z(0) = z_0$.

Definition 15. A strategy $\mathbf{u}_{G_t}(v,t)$ is called winning for the Pursuer on the time interval $[0, T]$ in the game with a "Life-line" if, for every $v(\cdot) \in \mathbb{V}_G^t$, there exists some moment $t^* \in [0, T]$ such that: 1) $x(t^*) = y(t^*)$; 2) $y(t) \notin M$ while $t \in [0, t^*]$.

Definition 16. For $\alpha(t) \geq \beta(t)$, $t \geq 0$, we call the function $\mathbf{u}_{G_t}(v,t) = v - \lambda_{G_t}(v,t)\xi_0$ the Π_{G_t}-strategy of the Pursuer in the G_t-game, where

$$\lambda_{G_t}(v,t) = \langle v, \xi_0 \rangle + \sqrt{\langle v, \xi_0 \rangle^2 + \alpha^2(t) - |v|^2},\ \xi_0 = z_0 / |z_0|,$$

and $\langle v, \xi_0 \rangle$ is a scalar product of v and ξ_0 in the space $\mathbb{R}^n$

Theorem 7. Let $\alpha(t) \geq \beta(t)$ for all $t \geq 0$ and T_{G_t} be first positive root of the equation $\int_0^t (\alpha(s) - \beta(s))ds = |z_0|$. Then in the G_t−Game:

a) the Π_{G_t}-strategy is winning for Pursuer P on the time interval $\left[0, T_{G_t}\right]$;

b) for an arbitrary control of the Pursuer, the strategy $v^*(t) = -\beta(t)\xi_0$ is winning for Evader E, i.e. $|z(t)| \geq |z_0| - \int_0^t (\alpha(s) - \beta(s))ds > 0$ is true in the time interval $\left[0, T_{G_t}\right)$.

Now let us consider the sets

$$\mathbf{P}(t) = \mathbf{P}(x(t), y(t)) = \left\{ w : \beta(t)|w - x(t)| \geq \alpha(t)|w - y(t)| \right\}$$

for the pair of $(x(t), y(t)))$. Here the relation $y(t) \in \mathbf{P}(t)$ holds for each $t \in [0, t^*]$.

Lemma 1. If $\alpha(0) > \beta(0)$ and $\alpha'(t)\beta(t) \geq \alpha(t)\beta'(t)$ for all $t \in \left[0, T_{G_t}\right]$, then the set $\mathbf{P}(t)$ is decreasing on the interval $\left[0, T_{G_t}\right]$, i.e. $\mathbf{P}(t_1) \supset \mathbf{P}(t_2)$ is valid when $t_1 < t_2$ for any $t_1, t_2 \in \left[0, T_{G_t}\right]$.

Theorem 8. Let the conditions of Theorem 7 and Lemma 1 take place. If $M \cap \mathbf{P}(x_0, y_0) = \varnothing$, then the Π_{G_t}-strategy is winning for the Pursuer in the "Life-Line"−Game with G_t−constraints.

Theorem 9. If the conditions of Theorem 7 take place, then the strategy $v^*(t) = -\beta(t)\xi_0$ is winning for the Evader in the "Life-Line"−Game with G_t −constraints.

Now consider the differential game where players' motion dynamics is described by the second order differential equations

$$P:\ \ddot{x}=u,\ x(0)=x_0,\ \dot{x}(0)=x_1, \tag{15}$$

$$E:\ \ddot{y}=v,\ y(0)=y_0,\ \dot{y}(0)=y_1, \tag{16}$$

where $x, y, x_0, y_0, x_1, y_1, u, v \in \mathbb{R}^n$, $n \geq 1$; x_0, y_0 are the initial locations of P and E, and x_1, y_1 are their initial velocity vectors, respectively. In this game, we require the conditions $x_0 \neq y_0$ and $x_1 = y_1$. The controls of the players are picked as measurable functions $u(\cdot):[0,\infty)\to\mathbb{R}^n$ and $v(\cdot):[0,\infty)\to\mathbb{R}^n$, which are subjected to the geometric constraints (briefly, the G_k^- –constraints)

$$|u(t)| \leq \rho e^{-kt} \text{ for almost every } t \geq 0, \tag{17}$$

$$|v(t)| \leq \sigma e^{-kt} \text{ for almost every } t \geq 0, \tag{18}$$

correspondingly. We denote the set of all measurable functions $u(\cdot)$ $\left(v(\cdot)\right)$ satisfying (17) ((18)) by $\mathbb{U}_k^G$ $\left(\mathbb{V}_k^G\right)$.

In the sequal, for brevity we will call the game (15)–(18) the second order G_k^- –Game.

Definition 17. For $\rho \geq \sigma$, the function

$$\mathbf{u}(t,v) = v - \phi(t,v)\zeta_0,\ \phi(t,v) = \langle v,\zeta_0\rangle + \sqrt{\langle v,\zeta_0\rangle^2 + \rho^2 e^{-2kt} - |v|^2} \tag{19}$$

with $|\mathbf{u}(t,v)| = \rho e^{-kt}$ is called the Π-strategy, where $\zeta_0 = z_0/|z_0|$.

Proposition 1. Let $\rho > \sigma$ and $h(t) = 1 + \dfrac{\sigma-\rho}{k^2|z_0|}\left(e^{-kt}+kt-1\right)$, $t \geq 0$. Then the equation $h(t) = 0$ has a unique positive root, which will be represented by T_*.

Theorem 10. If $\rho > \sigma$, then Π-strategy (19) guarantees winning for P on the interval $[0, T_*]$.

In the evasion game, for the Evader we offer an admissible control function in the form

$$v_*(t) = -\sigma e^{-kt}\zeta_0,\ \zeta_0 = z_0/|z_0|. \tag{20}$$

Theorem 11. a) If $\rho > \sigma$, then the control (20) guarantees winning for E on the interval $[0, T_*)$, where T_* is the guaranteed pursuit time (see Theorem 10);

b) If $\rho \leq \sigma$, then the control (20) guarantees winning for E in the interval $[0, +\infty)$.

Let $W(x, y)$ be the set of all points w such that Pursuer P can get to w from the position x earlier than Evader E from the position y, i.e.

$$W(x,y) = \{w : |w-x| \geq (\rho/\sigma)|w-y|\}.$$

In the case $\rho \neq \sigma$, the boundary of $W(x, y)$ is given by

$$S(x,y) = \{w : |w-x| = (\rho/\sigma)|w-y|\},$$

which is usually called Apollonius' sphere.

Lemma 2. Let

$$W_*(t, x(t), y(t)) = W(x(t), y(t)) - tx_1.$$

Then $W_*(t, x(t), y(t))$ is decreasing in respect to inclusion with t, $t \in [0, t^*]$, i.e. if t_1, $t_2 \in [0, t^*]$ and $t_1 < t_2$, then $W_*(t_2, x(t_2), y(t_2)) \subset W_*(t_1, x(t_1), y(t_1))$.

Definition 18. We call the set

$$W^*(x_0, y_0, T_*) = \bigcup_{t=0}^{T_*} \{W(x_0, y_0) + tx_1\}$$

the attainability domain of the Evader in the second order G_k^-−Game of Pursuit.

Let a closed subset M called the "Life-line" be given in $\mathbb{R}^n$.

Theorem 12. If $\rho > \sigma$ and $W^*(x_0, y_0, T_*) \cap M = \varnothing$, then Π-strategy (19) guarantees winning for P on the interval $[0, T_*]$.

Let, for every $w \in W(x_0, y_0)$, $\Theta(w, y_0) = \left\{ \theta : e^{-k\theta} + k\theta - 1 = \dfrac{k^2 |w - y_0|}{\sigma} \right\}$ and let, for every $w \in W(x_0, y_0)$ and for each $\theta \in \Theta(w, y_0)$

$$W_E(w, \theta) = \{w_* : w_* = w + \theta x_1\}. \qquad (21)$$

Definition 19. We call the set (21) the attainability domain of the Evader in the second order G_k^-−Game with the "Life-line" M.

Theorem 13. If $\rho > \sigma$ and $W_E(w, \theta) \cap M \neq \varnothing$, then there exists a control $v_*(\cdot) \in \mathbb{V}_k^G$ such that guarantees winning for E.

CONCLUSION

The dissertation work is devoted to the complete study of the pursuit-evasion problems and the "Life-line" game of R. Isaacs when non-stationary constraints are imposed on the controls of the players.

The main results of the research consist of the following:

1. When the controls of the Pursuers are subject to linear constraints and the control of the Evader are subject to geometric constraint, the parallel pursuit strategy has been constructed for the Pursuers and the attainability set of each pursuer has been determined, and on the basis of monotonicity conditions of this set, new sufficient conditions have been found for solving the pursuit problem and the "Life-line" problem of R.Isaacs;
2. In the differential game with exponential integral constraints on the controls, the parallel pursuit strategy has been constructed for the Pursuer and new sufficient conditions for the solvability of the pursuit-evasion problem have been determined;
3. In the differential games with non-statioary geometric constraints on the controls, the parallel pursuit strategy has been constructed for the Pursuer,

the specific control function has been found for the Evader and new sufficient conditions of the pursuit-evasion have been determined;

4. In the differential games with non-statioary geometric constraints on the controls, the attainability set of the players has been constructed and on the basis of the monotonicity conditions of this set, the “Life-line” problem of R.Isaacs has been completely solved.

НАУЧНЫЙ СОВЕТ PhD.03/30.12.2019.FM.05.04 ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ФЕРГАНСКОМ ГОСУДАРСТВЕННОМ УНИВЕРСИТЕТЕ

НАМАНГАНСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

ХОРИЛОВ МАХМУД АБДУМАЛИКОВИЧ

ДИФФЕРЕНЦИАЛЬНЫЕ ИГРЫ С НЕСТАЦИОНАРНЫМИ ОГРАНИЧЕНИЯМИ НА УПРАВЛЕНИЯ

01.01.02 – Дифференциальные уравнения и математическая физика

АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD) ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

Фергана – 2023

Тема диссертации доктора философии (Doctor of Philosophy) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Министерстве высшего образования, науки и инноваций Республики Узбекистан за №B2022.4.PhD/FM194.

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Защита диссертации состоится «11» 01 2024 года в 11:00 часов на заседании Научного совета PhD.03/30.12.2019.FM.05.04 при Ферганском государственном университете. (Адрес: 150100, г. Фергана, ул. Мураббийлар, 19. Тел.: (+99873) 244-44-02, факс: (+99873) 244-44-93, e-mail: fardu_info@umail.uz).

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования является решения задач преследования-убегания и задачи Р. Айзекса об игре с «линией жизни», когда на управления игроков накладываются нестационарные ограничения.

Объектом исследования является теория оптимального управления, теория дифференциальных уравнений и теория дифференциальных игр.

Научная новизна исследования заключается в следующем:

получены новые достаточные условия для решения задачи преследования в дифференциальной игре с линейными ограничениями на функции управления игроков методом параллельного сближения, при этом с помощью свойств опорных функций доказана монотонность области достижимости убегающего;

решены задачи преследования-убегания с экспоненциальными интегральными ограничениями на функции управления игроков, при этом для решения задачи преследования применяется метод параллельного сближения игроков, а для решения задачи убегания метод по заданному направлению;

определены новые достаточные условия для решений задач преследования-убегания с нестационарными геометрическими ограничениями на функции управления игроков, при этом для решения задачи преследования применяется метод параллельного сближения игроков, а для решения убегания метод уклонения по заданному направлению;

найдены новые условия монотонности области достижимости убегающего в дифференциальной игре с нестационарными геометрическими ограничениями на функции управления игроков, при этом с помощью свойства монотонности области достижимости полностью решена задача Р.Айзекса игра с «линией жизни».

Внедрение результатов исследования. На основе результатов, полученных в дифференциальных играх с нестационарными ограничениями на управления:

полученные результаты в части существования решения задач преследования-убегания на управлении с нестационарными ограничениями и задачи «Линии жизни» Айзекса были использованы в конфликтно-управляемых динамических системах на управлении с нестационарными ограничениями в фундаментальном проекте под названием «Создание новых методов управления конфликтными ситуациями, характеризующимися дифференциальными уравнениями и их численная реализация», под номером ОТ-Ф4-33 (справка № 04/11-3587 от 9 июня 2023 года Национального университета Узбекистана имени Мирзо Улугбека). В результате удалось определить обобщенные решения конфликтующих задач управления (динамических игр) в динамических системах с нестационарными ограничениями;

полученные результаты с точки зрения существования и оптимальности решения дифференциальной игровой задачи на управлении с линейными ограничениями были использованы при решении аналогичных задач для обыкновенных дифференциальных уравнений в рамках зарубежного грантового проекта «Задачи фазовых переводов и критической явления. Математической аспекты их уравнений быстрые переходы и асимптотика» под номером 374874-2015 (справка № 838 от 27 июня 2023 года Ошского государственного университета Кыргызской Республики). Как следствие, стало возможным построить решение новых нелокальных обобщенных спектральных задач.

E’LON QILINGAN ISHLAR RO‘YXATI
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