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GEOMETRI'YA



9

*O'zbeki'stan Respubli'kasi' Xali'q bi'li'mlendi'ri'w
mi'ni'strli'gi' uli'uwma bi'li'm beretug'i'n
mekteplerinin' 9-klasi' ushi'n sabaqli'q
si'pati'nda usi'ni'lg'an*

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9-klasta geometriyani’n’ planimetriya bo’limin — tegis geometriyalig’i’ figurallarni’n’ qasbiyatlarni u’yreniw dawam yettiriledi. Bunda siz geometriyalig’i’ figurallarni’n’ uqsasli’g’i’, u’shmu’yeshliklerdin’ ta’replari ha’m mu’yeshlari arasi’ndag’i’ qatnaslar, shen’ber uzi’nli’g’i’ ha’m do’n’gelektin’ maydani’, u’shmu’yeshlik ha’m shen’berdegi metrikaliq qatnaslar menen tani’sasi’z.

Bul “Geometriya” sabaqli’g’i’ni’n’ mazmuni’ qatan’ aksiomatikalig’i’ sistema tiykari’nda quri’lg’an. Bunda teoriyalig’i’ materiallar mu’mkinchiligi bolg’ansha a’piwayi’ ha’m ani’q tilde bayan yetilgen. Barli’q tema ha’m tu’siniklerdi ha’r tu’rli turmi’sta ushi’rasatug’i’n mi’sallar arqali’ ashi’p beriwge ha’reket yetilgen. Ha’r bir temadan son’ berilgen sorawlar, da’liyllewler, yesaplawlarga ha’m jasawlarga tiyisli ma’sele ha’m mi’sallar woqi’wshi’ni’ do’retiwshilik pikirlewge jeteleydi, wog’an wozlestirilgen bilimlerde teren’lestiriwge ha’m bekkemlep bari’wg’a ja’rdem beredi. Sabaqli’q wo’zini’n’ wo’zgeshe dizayn ha’m sabaq materialini’n’ ko’rgizbeli yetip usi’ni’li’wi’ menen de aji’rali’p turadi’. Sabaqli’qta keltirilgen su’wret ha’m si’zi’lma sabaqli’q materialin’ jaqsi’lap wo’zlestiriwge xi’zmet yetedi.

**Respublika maqsetli kitap qori’ qarji’lari’ yesabi’nan ijara ushi’n
basi’p shi’g’ari’ldi’.**

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SO'Z BASI

A'ziz oqiwshilar!

Bizler informatsion texnologiyalar a'sirinde jasap atirmiz. Zamanago'y rawajlaniw da'wirinde boli'p atirg'an du'nya ju'zilik o'zgerisler tiykarinda a'llette pa'n ha'm texnika rawajlaniwi jatir. Bunday sharayatta siz jaslardin' tiykarg'i' waziypasi ulli babalarimizg'a mu'na'sip a'wlad boli'p, zaman menen ten'dey qa'dem taslaw, ilim-pa'n shin'larin qunt penen u'yreniwden ibarat. Bul orinda matematikanin' orni ayriqsha boladi.

Matematika siz jaslardin' kamalg'a jetisiwin'izde oqiw pa'ni sipatinda ken' imkaniyatlarg'a iye ekeni ma'lim. Ol pikirlewin'izdi rawajlandirip, aqilin'izdi toliqtiradi, logikaliq pikirlew, tapqirliq qa'siyetlerin qa'liplestiredi ha'm tu'rli jag'daylarda aqilliliq penen qarar qabil etiw, analizlew ha'm juwmaq shig'ariw ko'nlikpelerin qa'liplestiredi.

Qolin'izdag'i 9-klass "Geometriya" sabaqlig'inin' tiykarg'i' waziypasi — tegis geometriyaliq figuralardi u'yreniw menen bir qatarda sizde izbe-iz logikaliq pikirlewdi o'sirip bariw na'tijesinde aqilin'izdi rawajlandiriwdan ibarat. Ol o'zlestirilgen bilim, ta'jiriybe ha'm ko'nlikpelerdi ku'ndelik turmista qollaniwin'izg'a ko'meklesedi.

Sabaqliqti jaratiwda du'nya ju'zinde toplang'an ag'la ta'jiriybe u'lgilerinen paydalandi. Sonin' menen bir qatarda elimizge ta'n bolg'an shig'is ha'm o'mirbaqiy qa'diriyatlarimizg'a, ulli babalarimiz miyrasinan da paydalaniwg'a ha'reket ettik.

Bul sabaqliqtan bilim alar ekinsiz, sizge bul juwapkershilikli, sonin' menen birge zawiqli jolda qunt ha'm qatan'liq tilep qalamiz. Geometriya tiykarlari boyi'nsha alg'an sabaqlarin'iz sizdi keleshekke jetelep, Watanimizdin' rawajlaniwi jolinda xizmet etiwge sizge ko'mekshi boladi dep, isenim bildiremiz!

Sabaqliqta qollanilg'an belgiler ha'm olardin' tu'sindirmesi:



— taza kirgizilgen geometriyalıq tu'siniktin' anıqlaması



— teoremanin' sıpatlaması



— u'lgı retinde sheship ko'rsetilgen ma'sele.



— soraw, ma'sele ha'm tapsırmalar



— oqıwshılardin' belsendiligin asırıwshı o'z betinshe yamasa toparlarda talqılanatug'in tapsırmalar



— jeke ta'rtipte yamasa toparlarda orınlanatug'in a'meliy jumis



— tariyxıy mag'liwmatlar ha'm ma'seleler



— qızıqlı ma'seleler ha'm bas-qatırmalar



— internetten usinis etiletug'in mag'liwmatlar ma'nzili.

8.

— u'yg'e sheshiwge usinis etiletug'in ma'seleler basqa ren'de berilgen

Teorema yamasa ma'selelerdin' sxematikaliq talqilaniwi

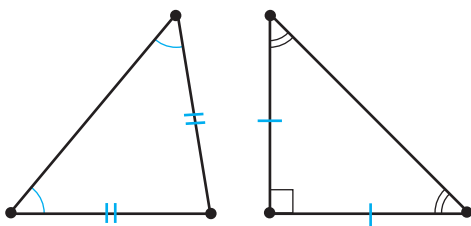


Teorema yamasa ma'selenin' sha'rtinde berilgen mag'liwmatlar



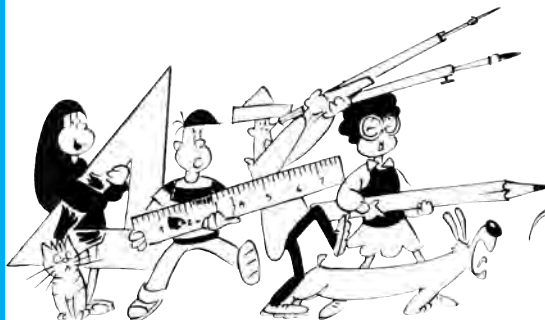
Da'lillew gerek bolg'an qa'siyet yamasa tabiliw talap etiletug'in elementler

Si'zi'lmalarda qabil etilgen o'z aldına belgiler



Sizilmalarda ten' mu'yeshler bir qiyli sandag'i dog'ashalar menen ajratiladi.

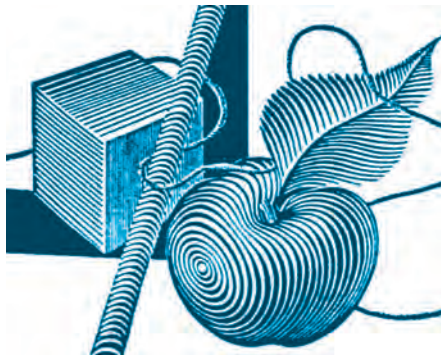
Sizilmalarda uzunlig'i ten' bolg'an kesindiler bir qiyli sandag'i sızıqshalar menen ajratiladi.



Geometriyani iyelep aliw ushi'n alg'a!



TA’KIRARLAW



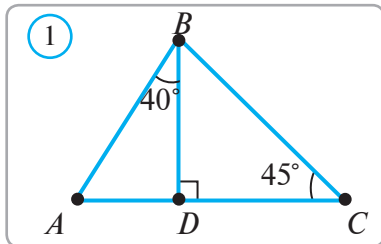
7 — 8-KLASLARDA WO’TILGENLERDI TA’KIRARLAW

- √ 7—8-klaslarda geometriyadan wo’tilgen temalardi’ ta’kirlap, alg’an bilimlerin ’izdi yeske alasi’z ha’m yerisken ko’nlikpelerin ’izdi bekkemleysiz.
- √ Bul sizge 9-klasta geometriyani’ u’yreniwdi tabi’sli’ dawam yettiriwin ’izge qolayli’li’q jaratadi’.

1

U'SHMU'YESHLIKLER

Bul bo'limdagi ma'seleler 7-8 klaslarda u'yrenilgen geometrik figuralar ha'm wolardi'n' qa'siyetlerin yadqa ali'w ushi'n berilmekte. Maselelerdi sheshi'w ushi'n sabaqli'qti'n' aqi'ri'nda keltirilgin tiykarg'i' geometriyalig' figuralarg'a tiyisli mag'liwmatlar ha'mde wolardi'n' qa'siyetlerin ani'qlawshi' figuralardan paydalani'wi'mi'z mu'mkin



1-ma'sele. ABC u'shmu'yeshliktin' BD biyikligi ju'rgizilgen (*1-su'wret*). Yeger $\angle ABD = 40^\circ$, $\angle BCD = 45^\circ$ bolsa u'shmu'yeshliktin' A ha'm B to'besindigi mu'yeshlerin tabi'n'.

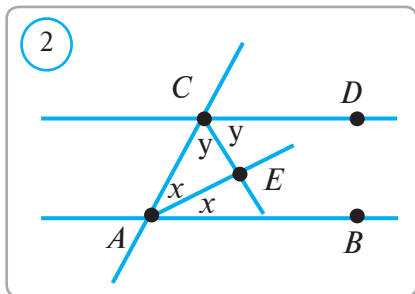
Sheshiliwi. 1) Tuvri' mu'yeshli ABD u'shmu'yeshlikte $\angle ABD = 40^\circ$ ha'm u'shmu'yeshlik ishki mu'yeshlerinin' qosi'ndi'si 180° ga ten' bolg'ani' ushi'n $\angle A = 180^\circ - (90^\circ + 40^\circ) = 50^\circ$.

2) Tuvri' mu'yeshli BCD u'shmu'yeshlikte $\angle BCD = 45^\circ$ bolg'ani' ushi'n

$$\angle DBC = 180^\circ - (90^\circ + 45^\circ) = 45^\circ.$$

$$\angle ABC = \angle ABD + \angle DBC \text{ bolg'ani' ushi'n } \angle B = 40^\circ + 45^\circ = 85^\circ.$$

Juwabi': $50^\circ, 85^\circ$.



2-ma'sele. Yeki parallel tuwri' si'zi'qti' kesiwshi menen keskende payda bolg'an ishki bir ta'repleme mu'yeshlerdin' bissektisalarini' arasi'ndagi mu'yeshi tabi'n'.

Sheshiliwi. AC tuwri' si'zi'q AB ha'm CD -parallel tuwri' si'zi'qlardi' 2-su'wrette su'wretlengendey kesip wotken bolsi'n. Ishki bir ta'replemen BAC ha'm ACD mu'yeshlerdin' bissektisalarini' E noqatta kesilisken boli'p $\angle EAC$

$= x$, $\angle ECA = y$ bolsi'n wonda mu'yesh bissektisa ani'qlamasini' boyi'nsha,

$$\angle BAC = x + x = 2x, \quad \angle ACD = y + y = 2y.$$

$AB \parallel CD$ bolg'ani' ushi'n ishki ta'replemeli mu'yeshlerdin' qa'siyeti boyi'nsha,

$$2x + 2y = 180^\circ, \quad x + y = 90^\circ.$$

Yendi, ACE u'shmu'yeshlik ishki mu'yeshlerinin' qosi'ndi'si 180° ga ten' bolg'ani' ushi'n $\angle AEC = 180^\circ - (x + y) = 180^\circ - 90^\circ = 90^\circ$. **Juwabi':** 90° .

3-ma'sele. ABC u'shmu'yeshliktin' AB ta'repi 6 sm A ha'm B mu'yeshlerine saykes tu'rde 30° ha'm 60° bolsa, ABC u'shmu'yeshliktin' maydani'n tabi'n'.

Sheshiliwi. U'shmu'yeshliktin' C mu'yeshin tabi'n'

$$\angle C = 180^\circ - (\angle A + \angle B) = 180^\circ - (60^\circ + 30^\circ) = 90^\circ.$$

Demek, tuwri' mu'yeshli ABC u'shmu'yeshliktin' AB gepotenuzasi' 6 sm ha'm A mu'yeshi 30° yeken. Tuwri' mu'yeshli u'shmu'yeshlikte 30° li mu'yesh qarsi'si'ndag'i katet gepotenuzani'n' yari'mi'na ten' bolg'ani' ushi'n $BC = 3\text{ sm}$ (3-su'wret).

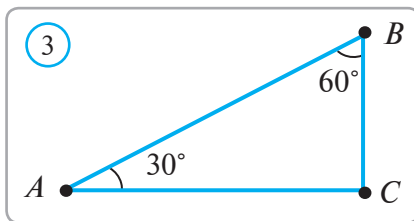
Pifagor teoremasi'nan paydalani'p AC katetin tabami'z:

$$AC^2 = AB^2 - BC^2 = 6^2 - 3^2 = 27, AC = 3\sqrt{3}\text{ sm}.$$

Yendi u'shmu'yeshliktin' maydani'n tabami'z:

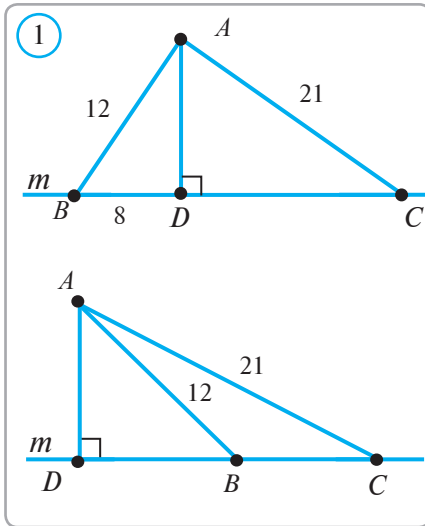
$$S_{ABC} = \frac{1}{2} AC \cdot BC = \frac{1}{2} \cdot 3\sqrt{3} \cdot 3 = \frac{9\sqrt{3}}{2} (\text{sm}^2).$$

Juwabi': $\frac{9\sqrt{3}}{2} \text{ sm}^2.$



? Soraw, ma'sele ha'm tapsi'rmalar

- ABC u'shmu'yeshlikte $\angle A = 47^\circ$, $\angle C = 83^\circ$ bolsa, u'shmu'yeshliktin' u'shinshi ishki mu'yeshin ha'm si'rtqi' mu'yeshlerin tabi'n'.
- Katetleri 15 sm ha'm 20 sm bolg'an tuwri' mu'yeshli u'shmu'yeshlik gepotenuzasi'na tu'sirilgen biyikligin tabi'n'.
- ABC u'shmu'yeshliktin' AC ta'repine parallel tuwri' si'zi'q AB ha'm BC ta'replerin saykes tu'rde E ha'm F noqatlarinda kesip wotedi. Yeger $\angle BEF = 65^\circ$ ha'm $\angle EFC = 135^\circ$ bolsa, ABC u'shmu'yeshliginin' mu'yeshlerin tabi'n'.
- ABC u'shmu'yeshlik bissektrisalari' I noqatta kesilisedi. Yeger $\angle A = 80^\circ$ ha'm $\angle B = 70^\circ$ bolsa, AIB , BIC ha'm CIA mu'yeshlerin tabi'n'.
- Ten' ta'repli u'shmu'yeshliktin' bir si'rtqi' mu'yeshi 70° qa ten'. U'shmu'yeshliktin' mu'yeshlerin tabi'n'.
- ABC u'shmu'yeshliktin' AK bissektrisasi' ju'rgizilgen. Yeger $\angle BAK = 47^\circ$ ha'm $\angle AKC = 103^\circ$ bolsa, u'shmu'yeshlik mu'yeshlerin tabi'n'.
- ABC u'shmu'yeshlik biyiklikleri H noqatta kesilisedi. Yeger $\angle A = 50^\circ$, $\angle B = 60^\circ$ bolsa, AHB , BHC ha'm CHA mu'yeshlerin tabi'n'.
- U'shmu'yeshliktin' worta si'zi'qlari woni ten'dey to'rtmu'yeshliklerge aji'ratatugi'ni'n da'liyllen'.
- ABC u'shmu'yeshlikte CD mediana dawam yettirilip bul medianag'a ten' DE kesindisi qoyi'ladi'. AF mediana dawam yettirilip AF medianag'a ten' FH kesindisi qoyi'lg'an. B, H, E noqatlari bir tuwri'da jatatug'i'nli'g'i'n da'liyllen'.
- ABC ten' qaptalli' u'shmu'yeshlikte ($AB = BC$) AN ha'm CK bissektrisalar ju'rgizilgen. a) KN kesindisi AC ta'repke parallel yekenligin ko'rsetin'. b) $AK = KN = NC$ ten'lik wori'nli' boli'wi'n da'liyllen'.



1-ma'sele. A noqattan m tuwri' si'zi'qqa uzi'nli'qlari' 12 sm ha'm 21 sm bolg'an yeki qi'ya tu'sirilgan. Yeher birinshi qi'yani'n' m tuwri'si'na proekciyasi' 8 sm bolsa, yekinshi qi'yani'n' proekciyasi'n tabi'n'.

Sheshiliwi. m tuwri' si'zi'qti'n si'rti'ndag'i A noqattan usi' tuwri' si'zi'qqa AB ha'm AC qi'yalar ha'mde AD perpendikulyar tu'sirilgan boli'p, $AB=12$ sm ha'm $AC=21$ sm bolsi'n (1-su'wret). Wonda ma'sele sha'rti boyi'nsha $BD=8$ sm boladi' ha'm CD kesindi uzi'nli'g'i'n tabi'w kerek.

1) Pifagor teoremasi'nan paydalani'p tuwri' mu'yewli ABD u'shmu'yeshlikti'n AD katetin tabami'z.

$$AD^2 = AB^2 - BD^2 = 12^2 - 8^2 = 80, AD = \sqrt{80} \text{ sm.}$$

2) Tuwri' mu'yewli ACD u'shmu'yeshlikten Pifagor teoremasi'nan paydalani'p CD kesindi uzi'nli'g'i'n tabami'z.

$$CD^2 = AC^2 - AD^2 = 21^2 - (\sqrt{80})^2 = 441 - 80 = 361, CD = 19 \text{ sm.}$$

Juwabi': 19 sm.

2-ma'sele. Tarepleri 13, 14 ha'm 15 ke ten' bolg'an u'shmu'yeshlikti'n maydani'n ha'm biyikliklerin tabi'n'.

Sheshiliwi. Yeron formulasinan paydalani'p, ta'repleri $a = 13$, $b = 14$, $c = 15$ bolg'an u'shmu'yeshlikti'n maydan'n tabami'z:

$$p = \frac{a+b+c}{2} = \frac{13+14+15}{2} = \frac{42}{2} = 21,$$

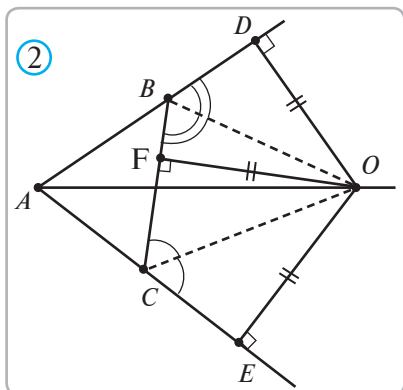
$$S = \sqrt{p(p-a)(p-b)(p-c)} = \sqrt{21 \cdot (21-13) \cdot (21-14) \cdot (21-15)} = \\ = \sqrt{21 \cdot 8 \cdot 7 \cdot 6} = \sqrt{7 \cdot 3 \cdot 8 \cdot 7 \cdot 2 \cdot 3} = 3 \cdot 4 \cdot 7 = 84.$$

Yendi, u'sgmu'yeshlikti'n maydani'n yesaplaw formulasi' $S = \frac{1}{2} a \cdot h_a$ dan paydalni'p, u'shmu'yeshlikti'n h_a biyikligin tabami'z:

$$h_a = \frac{2S}{a} = \frac{2 \cdot 84}{13} = \frac{168}{13} = 12 \frac{12}{13}.$$

Tap usi' jol menen h_b ha'm h_c biyikliklerin.

Juwabi': 84; $12 \frac{12}{13}$; 12; $11 \frac{1}{5}$.



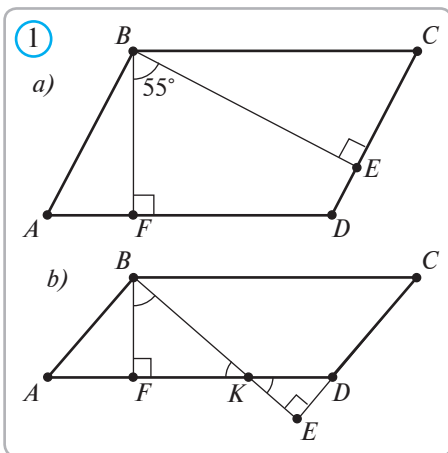
3-ma'sele. ABC u'shmu'yeshliginin' B ha'm C ushlarindag'i si'rtqi' mu'yeshlerinin' bissektisarlari O noqatta kesilisedi. O noqattin' BAC mu'yesh bissektisarisinda jati'wi'n da'liylen'.

Da'lillew. O noqattin' AB , AC ha'm BC tuwri' si'zi'qlardag'i' proekciyalari' sa'ykes tu'rde D , E ha'm F noqatlar bolsi'n (2-su'wret). Onda, birinshiden O noqat DBC mu'yeshstin' bissektisarisinda jatqani' ushi'n $OD=OF$ boladi'. Yekinshiden, O noqat BCE mu'yeshinin' bissektisarisinda jatqani' ushi'n $OF=OE$ boladi. Son-

li'qtan, $OD=OF=OE$. Demek, O noqat BAC mu'yesh ta'replerinen ten' uzaqli'qta jaylasqan yeken. Soni'n' ushi'n O noqat BAC mu'yesh bissektisarisinda jatadi'.

? Soraw, ma'sele ha'm tapsi'rmalar

1. Ta'repleri 5, 6 ha'm 7 bolg'an u'shmu'yeshliktin' maydani'n tabi'n'.
2. Berilgen noqattan a tuwri'ga uzi'nli'qlari'ni'n' ayi'rmasi' 6 g'a ten' bolg'an yeki qi'ya tu'sirilgen. Qi'yalardi'n'a a tuwri'dag'i' proekciyalari' 27 ha'm 15 ge ten'. Berilgen noqattan a tuwri'g'a shekemgi arali'qti' tabi'n'.
- 3*. ABC u'shmu'yeshliktin' A ha'm B ushlarindag'i' si'rtqi' mu'yeshlerinin' bissektisarlari' D noqatta kesilisedi. Yeger $ADB=75^\circ$ bolsa, u'shmu'yeshliktin' ACB mu'yeshin tabi'n'.
4. Ultani' AC bolg'an ABC ten' ta'repli u'shmu'yeshlikte CD bissektisa ju'r-gizilgen. ADC mu'yesh: a) 60° ; b) 75° ge ten' bolsa, u'shmu'yeshliktin' mu'yeshlerin tabi'n'.
5. Bir kateti 7 sm ge, gipotenuzasi' 25 sm ge ten' bolg'an tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasi'na tu'sirilgen biyiklikti tabi'n'.
6. ABC u'shmu'yeshliktin' BD biyikligi wo'tkerilgen ($D \in AC$). Yeger $BD=12$, $AD=5$ ha'm $DC=16$ bolsa, u'shmu'yeshliktin' perimetrin ha'm maydani'n' tabi'n'.
7. Ten' qaptalli' u'shmu'yeshliktin' qaptal ta'repi 10 sm, ultani' bolsa $10\sqrt{3}$ sm. U'shmu'yeshliktin' ultanina tu'sirilgen biyiklikti, maydani'n ha'm mu'yeshlerin tabi'n'.
8. Su'yir mu'yeshli ABC u'shmuyeshlikke si'rtlay si'zi'lg'an shen'ber orayi' O noqatta boli'p $\angle AOB=120^\circ$, $\angle BOC=110^\circ$ bolsa, ABC u'shmu'yeshlik mu'yeshlerin tabi'n'.
9. Yeger ABC u'shmu'yeshliginin' CD medianasi AB ta'repten yeki yese kishi bolsa, ACB mu'yeshin tabi'n'.
10. ABC u'shmu'yeshliktin' biyiklikleri O noqatta kesilisedi. Yeger $\angle A=60^\circ$, $\angle B=80^\circ$ bolsa, AOB mu'yeshin tabi'n'.



1-ma'sele. Yeger parallelogrammni'n' bir to'besinen woni'n' yekinshi ta'repine tu'sirilgen biyikliklarini'n' arasi'ndag'i' mu'yesh 55° qa ten' bolsa, parallelogrammni'n' mu'yeshlerin tabi'n'.

Sheshiliwi. Parallelogrammni'n' BF ha'm BE biyikliklarini'n' arasi'ndag'i' mu'yesh 55° bolsi'n (1-su'wret). Su'wrette su'wretlengen yeki jag'day: a) BE biyikligi CD ta'repine; b) BE biyikligi CD ta'repinin' dawami'na tu'sken boli'wi' mu'mkin.

a) jag'dayda $BEDF$ to'rtmu'yeshliktin' mu'yeshliklarini'n' qosi'ndi'si' 360° bolg'ani' ushi'n, $55^\circ + 90^\circ + \angle D + 90^\circ = 360^\circ$.

Bunnan $\angle D = 125^\circ$.

b) jag'dayda BE biyikligi AD ta'repi menen kesilisken noqat K bolsi'n. Wonda,

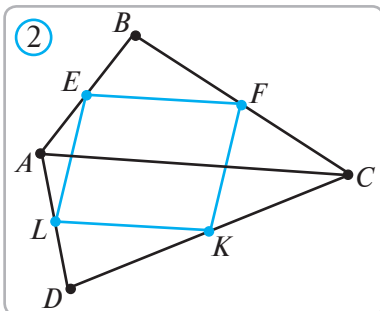
$$\angle DKE = \angle BKF = 90^\circ - 55^\circ = 35^\circ.$$

U'shmu'yeshliktin' si'rtqi' mu'yeshlarini'n' qa'siyetleri boyi'nsha,

$$\angle ADC = \angle DKE + \angle KED = 35^\circ + 90^\circ = 125^\circ.$$

Demek, ha'r yeki jag'dayda da $D=125^\circ$. Wonda, $\angle A = \angle C = 180^\circ - \angle D = 55^\circ$, $\angle B = \angle D = 125^\circ$.

Juwabi': $55^\circ, 125^\circ, 55^\circ, 125^\circ$.



2-ma'sele. To'rtmu'yeshliktin' ta'replarinin' wortalari' parallelogrammni'n' to'beleri bolatug'i'ni'n da'liyllen'.

Sheshiliwi. $ABCD$ to'rtmu'yeshliginin' AB , BC , CD ha'm DA ta'replarinin' wortalari' sa'ykes tu'rde E , F , K ha'm L noqatlari' bolsi'n (2-su'wret). $EFKL$ — parallelogramm yekenligin ko'rsetemiz.

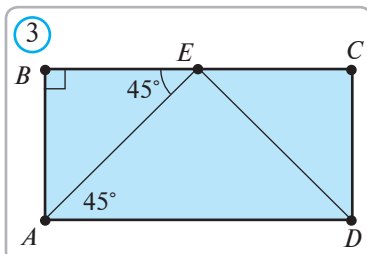
EF kesindisi ABC u'shmu'yeshliginin', KL kesindisi ACD u'shmu'yeshliginin' worta si'zi'g'i' boladi'. U'shmu'yeshliktin' worta si'zi'g'i'ni'n' qa'siyetlarinen,

$$EF \parallel AC, KL \parallel AC, \quad EF = \frac{1}{2} AC, KL = \frac{1}{2} AC.$$

Bunnan $EF \parallel KL$ ha'm $EF = LK$. Soni'n' ushi'n, parallelogrammni'n' belgilari boyi'nsha $EFKL$ — parallelogramm boladi'.

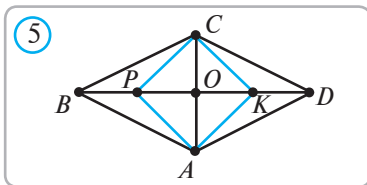
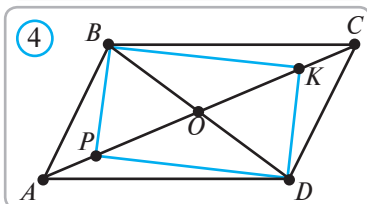
3-ma'sele. $ABCD$ tuwri'mu'yeshliginin' A ha'm D to'besindagi mu'yeshlerinin' bissektoralari' BC ta'repinde kesilisedi. Yeger $AB = 4$ sm bolsa, tuwri'mu'yeshliktin' maydani'n tabi'n'.

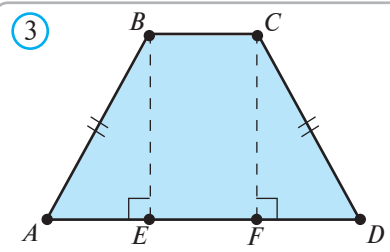
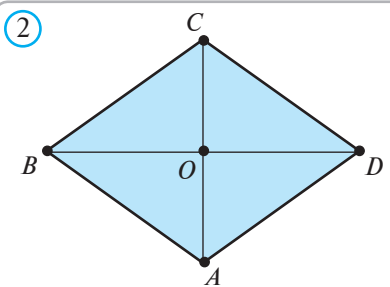
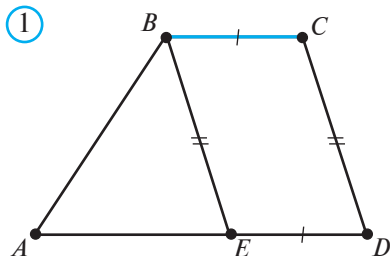
Sheshiliwi. Tuwri'mu'yeshliktin' A ha'm D mu'yeshlerinin' bissektoralari' kesilisen noqat E bolsi'n (3-su'wret). Wonda, $\angle B = 90^\circ$, $\angle BAE = 45^\circ$ bolg'ani' ushi'n, $\angle AEB = 180^\circ - 90^\circ - 45^\circ = 45^\circ$. Yag'ni'y, ABE — ten' qaptalli' u'shmu'yeshlik Bunda, $AB = BE = 4$ (sm). Da'l usi'g'an uqsas $EC = CD = 4$ (sm) yekenligin ko'rsetiw mu'mkin. Bunnan $BC = BE + EC = 8$ (sm) ha'm $S_{ABCD} = AB \cdot BC = 4 \cdot 8 = 32$ (sm²). **Juwabi':** 32 sm².



? **Soraw, ma'sele ha'm tapsi'rmarlar**

1. To'rtmu'yeshliktin' u'sh mu'yeshi: 47° , 83° ha'm 120° qa ten'ligi belgili. Woni'n' to'rtinshi mu'yeshin tabi'n'.
2. Parallelogrammni'n' yeki mu'yeshinin' qosi'ndi'si' 156° qa ten'. Woni'n' mu'yeshlerin tabi'n'.
3. Tuwri'mu'yeshliktin' diagonallari'ni'n' arasi'ndag'i mu'yesh 74° . Woni'n' bir diagonali menen ta'repleri arasi'ndag'i mu'yeshlerin tabi'n'.
4. Ten' qaptalli' trapeciyani'n' yeki mu'yeshinin' ayi'rmasi' 40° qa ten'. Woni'n' mu'yeshlerin tabi'n'.
5. Rombi'ni'n' mu'yeshlerinen biri yekinshisinen u'sh ma'rte u'lken. Rombi'ni'n' mu'yeshlerin tabi'n'.
6. $ABCD$ tuwri'mu'yeshliginin' A mu'yeshinin' bissektoralasi' BC ta'repin 2 sm ha'm 6 sm ge ten' kesindilerge aji'ratadi'. Tuwri'mu'yeshliktin' perimetrin tabi'n'.
7. Ta'repleri 3 sm ha'm 6 sm, u'lken ta'repleri arasi'ndag'i arali'q 2 sm bolg'an parallelogramm jasan'.
8. $ABCD$ parallelogrammni'n' AC diagonali'nda P ha'm K noqatlari' belgilep aling'an (4-su'wret). Yeger $OP = OB = OK$ bolsa, $BKDP$ tuwri'mu'yeshlik yekenligin da'liyllen'.
- 9*. $ABCD$ rombi'ni'n' BD u'lken diagonali'nda P ha'm K noqatlari' belgilep aling'an (5-su'wret). Yeger $OA = OP = OK$ bolsa, $APCK$ to'rtmu'yeshligi kvadrat yekenligin da'liyllen'.
- 10*. $ABCD$ parallelogrammni'n' BD diagonali'nda P ha'm K noqatlari' belgilep aling'an (6-su'wret). Yeger $BP = KD$ bolsa, $APCK$ to'rtmu'yeshligi parallelogramm yekenligin da'liyllen'.





1-ma'sele. $ABCD$ trapeciyasi'ndag'i' BC kishi ultanni'n' B to'besinen CD ta'repke parallel tuwri' ju'rgizilgen. Na'tiyjede payda bolg'an u'shmu'yeshliktin' perimetri 24 sm ge ten'. Yeger trapeciyani'n' perimetri 36 sm bolsa, BC ta'repinin' uzi'nli'g'i'n' tabi'n'.

Sheshiliwi. Ma'selenin' sha'rti boyi'nsha ju'rgizilgen tuwri' kesindisi BE bolsi'n, E noqati' AD ta'repinde jatadi' (*1-su'wret*). BE kesindisi trapeciyani ABE u'shmu'yeshlikke ha'm $BCDE$ parallelogrammg'a aji'ratadi'. Sonday aq, $BC = ED$ ha'm $CD = BE$. Ma'selenin' sha'rti boyi'nsha,

$$P_{ABCD} = AB + BC + CD + DA = AB + BC + CD + DE + EA = AB + BE + EA + 2BC = P_{ABE} + 2BC = 24 + 2BC = 36\text{ (sm)}.$$

Bunnan, $2BC = 12$, yamasa $BC = 6\text{ sm}$ yekengligin tabami'z. **Juwabi':** 6 sm .

2-ma'sele. Rombi'ni'n' diagonallari'nan biri 14 sm , ta'repi bolsa 25 sm . Rombi'ni'n' maydani'n' tabi'n'.

$ABCD$ — romb,
 $AC = 14\text{ sm}, AB = 25\text{ sm}.$ $\Rightarrow S_{ABCD} = ?$

Sheshiliwi. Romb diagonallari'ni'n' kesilisiw

noqati' O bolsi'n (*2-su'wret*). Wonda rombi'ni'n' qa'siyeti boyi'nsha,

$$AO = \frac{1}{2} AC = \frac{1}{2} \cdot 14 = 7\text{ (sm)}, \quad \angle AOB = 90^\circ.$$

Pifagor teoremasi'nan paydalani'p OB kesindisin tabami'z:

$$OB^2 = AB^2 - AO^2 = 25^2 - 7^2 = 576 \text{ yoki } OB = 24\text{ sm}.$$

Wonda $BD = 2 \cdot OB = 2 \cdot 24 = 48\text{ (sm)}$. Rombi'ni'n' maydani'n' yesaplaw formulasi'

boyi'nsha, $S_{ABCD} = \frac{1}{2} AC \cdot BD = \frac{1}{2} \cdot 14 \cdot 48 = 7 \cdot 48 = 336\text{ (sm}^2\text{)}$. **Juwabi':** 336 sm^2 .

3-ma'sele. Ten' qaptalli trapeciyani'n' qaptal ta'repi 20 sm , ultanlari' bolsa 12 sm ha'm 36 sm . Trapeciyani'n' maydani'n' tabi'n'.

Sheshiliwi. $ABCD$ trapeciyada $AB = CD = 20 \text{ sm}$, $BC = 12 \text{ sm}$, $AD = 36 \text{ sm}$ bolsi'n. Trapeciyanin' BE ha'm CF biyikliklerin ju'rgizemiz (3-su'wret).

Wonda,

$$EF = BC = 12 \text{ (sm)}, AE = FD = \frac{AD - EF}{2} = \frac{36 - 12}{2} = 12 \text{ (sm)}.$$

Tuwri' mu'yeshli ABE u'shmu'yeshlikke Pifagor teoremasin qollanip, BE - biyikligin tabami'z: $BE^2 = AB^2 - AE^2 = 20^2 - 12^2 = 256$ yaki $BE = 16 \text{ sm}$.

Trapeciyanin' maydani'n tabami'z:

$$S_{ABCD} = \frac{BC + AD}{2} \cdot BE = \frac{12 + 36}{2} \cdot 16 = 24 \cdot 16 = 384 \text{ (sm}^2\text{)}. \quad \textbf{Juwabi': } 384 \text{ sm}^2.$$



Soraw, ma'sele ha'm tapsi'rmalar

- $ABCD$ trapeciyasi'ni'n' kishi BC ultani' 7 sm ge ten'. Wo'ni'n' B to'besinen CD ta'repine parallel tuwri' ju'rgizilgen. Payda bolg'an u'shmu'yeshliktin' perimetri 16 sm ge ten'. Trapeciyanin' perimetrin tabi'n'.
- Tuwri'ni' kesip wo'tpeytug'i'n kesindinin' ushlari' bul tuwri'dan 8 sm ha'm 18 sm qashikli'qta jaylasqan. Kesindi wortasi'nan tuwri'g'a shekem bolg'an arali'qti' tabi'n'.
- Ta'repleri 4 sm ha'm 5 sm , maydani' bolsa 10 sm^2 bolg'an parallelogramm jasan'.
- Rombi'ni'n' diagonallari'ni'n' biri 80 sm , ta'repi bolsa 81 sm . Rombi'ni'n' maydani'n tabi'n'.
- Parallelogrammni'n' 135° qa ten' bolg'an dog'al mu'yeshinin' to'besinen tu'sirilgen biyikligi 4 sm ge ten' boli'p, wol wo'zi tu'sken ta'repin ten' yekige bo'ledi.
 - Usi ta'repti tabi'n'.
 - Parallelogrammni'n' dog'al mu'yeshlerinin' to'belerin tutasti'ri'wshi diagonali' menen ta'repleri arasi'ndag'i mu'yeshlerin tabi'n'.
 - Parallelogrammni'n' perimetrin ha'm maydani'n tabi'n'.
- Rombi'ni'n' dog'al mu'yeshi to'besinen tu'sirilgen biyiklik rombi'ni'n' ta'repin ten' yekige bo'ledi. Yeger rombi'ni'n' ta'repi 6 sm bolsa, wonda rombi'ni'n' maydani'n tabi'n'.
- Tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasi' 13 sm , katetlerinin' qosi'ndi'si' bolsa 17 sm . U'shmu'yeshliktin' maydani'n tabi'n'.
- Tuwri' mu'yeshli trapeciyanin' bir mu'yeshi 135° qa, worta si'zi'g'i bolsa 18 sm ge ten'. Yeger trapeciyanin' ultanlari'ni'n' qatnasi' $1:8$ ge ten' bolsa, trapeciyanin' qaptal ta'replerin tabi'n'.
- $ABCD$ ($AB \parallel CD$) trapeciyasi O worayi'na iye bolg'an shen'berge si'rtlay si'zi'lg'an. $\angle AOD = 90^\circ$ yekenligin da'liyllen'.

MATEMATIKALIQ MA'SELELER G'A'ZIYNESI

Keyingi waqitlarda informacion kommunikaciya texnologiyalari ju'da' tez pa't penen rawajlanip barmaqta. Internette barg'an sayin alis awillardi da qamtip almaqta. Usi ku'nge kelip, internettin' World-Wide-Web — Ja'ha'n informatciya tarmag'inda sonshelli ko'p informaciya derekleri jaylastirilg'an boli'p, bul g'a'ziyneden paydalaniw zamanimizdin' ha'r bir puqarasi ushi'n ha'm qariz, ham pariz yesaplanadi. Sonday-aq, bir-birinen qiziq sonday web-betler bar boli'p, wolardan qa'legen pa'ndi, sonin' ishinde geometriyani u'yreniw barisinda boli'p paydalaniw mu'mkin. To'mende bul informaciya dereklerinin' jaylasiw worinlarin beriwdi lazim taptiq. Bul web-betlerden siz wo'zbek, rus, ingliz ha'm basqa tillerde matematika a'lemindegi yen' son'g'i jan'aliqlar, yelektron kitapxanalar bazasinda saqlanip atirg'an ko'plep sabaqliqlar, arali'qtan turip matematikadan bilim aliw kurslari ha'm wolardi'n' materiallari, bul sabaqliq betlerine kirgen ha'm de kirmegen tu'rli teoriyalik materiallar, matematikadan sabaq berip atirg'an ta'jiriyebeli woqitiwshilardin' sabaq usillari ha'm metodikalik usinislari, yesap-sansiz ma'seleler, misallar ha'm wolardi'n' sheshimleri, tu'rli ma'mleketlerde wo'tkerilip atirg'an matematikalik ko'rik tan'lawlar ha'm olimpiadalar haqqindag'i mag'liwmatlar ha'm wolarda keltirilgen ma'seleler ha'm de wolardi'n' sheshimleri, qi'zi'qli' matematikalik ma'seleler ha'm wolardi'n' sheshimleri menen tanisiwin'iz mu'mkin.

<http://www.eduportal.uz> — Xali'q bilimlendiriw ministrliginin' bilimlendiriw portali'

<http://www.multimedia.uz> — Xali'q bilimlendiriw ministirligi qasindag'i Multimedia worayi' sayti'

<http://www.uzedu.uz> — Xaliq bilimlendiriw sayti'

<http://www.edu.uz> — Joqari' ha'm worta arnawli bilimlendiriw ministrliginin' bilimleniriw portali'

<http://www.pedagog.uz> — Pedagogika bilimlendiriw sho'lkemleri portali'

<http://ziyo.edu.uz> — Bilimlendiriw sho'lkemlerinin' portali'

<http://www.matematika.uz> — Matematikadan qosi'msha materiallar sayti'

<http://ziyonet.uz> — Informacion-bilimlendiriw resurslari' tarmag'i'

<http://cde.sakha.ru> — Arali'qtan turi'p woqi'ti'w sayti' (rus tilinde)

<http://www.iro.sakha.ru> — Bilimin jetilistiriw instituti' sayti' (rus tilinde)

<http://www.school.edu.ru> — Uli'wma bilimlendiriw portali' (rus tilinde)

<http://www.alledu.ru> — "Internetten bilim" portali' (rus tilinde)

<http://www.rsl.ru> — Rossiyama'mleketlik kitapxanasi' portali' (rus tilinde)

<http://www.rostest.runnet.ru> — Test ali'w worayi' servari (rus tilinde)

<http://www.allbest.ru> — Internet resurslari elektron kitapxanasi' (rus tilinde)

http://int-edu.ru/soft/base_geom.html — «Живая геометрия» bag'darlamasin qollap-quwatlaw sayti'

<http://matematika.mgdt.ru/> — Matematika ha'm informatikadan si'rtqi' tan'law (rus tilinde)

<http://www.mathtype.narod.ru/> — Online-sabaqli'qlar (rus tilinde)

<http://www.e-pi.narod.ru/> — Ha'mmesi e ha'm π sanlari' haqqi'nda (rus tilinde)

<http://mschool.kubsu.ru/> — Elektron qollanbalar kitapxanasi. Si'rtqi' matematikalik olimpiadalar

<http://matematika.agava.ru/> — Matematikadan 2000 nan arti'q ma'seleler (rus tilinde)

<http://mat-game.narod.ru/> — Matematikali'q gimnastika. Matematikali'q ma'seleler ha'm basqatirmalar

I BAP



UQSAS GEOMETRIYALIQ FIGURALAR

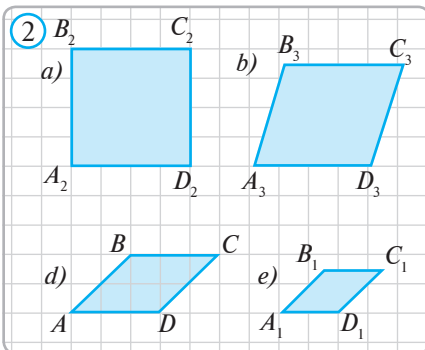
Bul bapti' u'yreniw na'tiyjesinde siz to'mendegi bilim ha'm a'meliy ko'nlikpelerge iye bolasi'z:

Bilimler:

- √ *uqsas figuralardi'n' ani'qlamasi'n ha'm belgileniwin biliw;*
- √ *u'shmu'yeshliklerdin' uqsasli'q belgilerin biliw;*
- √ *gomotetiya tu'sinigin biliw.*

A'meliy ko'nlikpeler:

- √ *yeki uqsas u'shmu'yeshliklerden sa'ykes yelementlerin taba ali'w;*
- √ *u'shmu'yeshliklerdin' uqsasli'q belgilerin da'liyllewge ha'm yesaplawg'a tiyisli ma'selelerdi sheshiwde qollana ali'w;*
- √ *gomotetiyadan paydalani'p uqsas ko'pmu'yeshliklerdi jasay ali'w.*



Ku'ndelikli turmi'sta ten' figuralardan basqa formasi' (ko'rinisi) bir qi'yli', lekin wo'lshemleri tu'rlishe bolg'an figuralarg'a da dus kelemiz. Tariyx ha'm geografiya pa'nlerinde tu'rli masshtabta islangen kartalardan paydalang'anbi'z. Klass taxtasi'na ildirilgen ha'm sabaqli'qta su'wretlengen respublikami'zdin' kartalari' tu'rli wo'lsheimde, lekin wolar bir qi'yli' formada (ko'riniste). Sonday-aq, bir fotoplyonkadan tu'rli wo'lsheimdegi fotosu'wretler tayarlanadi'. Bul su'wretlerdin' wo'lshemleri tu'rlishe bolsa da, bir qi'yli' ko'riniste, yag'niy wolar bir-birine uqsaydi (*1-su'wret*).

Shinig'iw. 3-su'wrette to'rt romb su'wretlengen. Wo'lardan tek d) ha'm e) romblar bir qi'yli' ko'riniske iye. Bul romblar nesi menen basqa romblardan ajralip turadi'? Kelin', birgelikte ani'qlayi'q.

1. Su'wretten ko'rinip turg'ani'nday, $AD=3$, $A_1D_1=2$. Rombi'ni'n' ta'replari ten' bolg'ani' ushi'n,

$$\frac{AD}{A_1D_1} = \frac{BC}{B_1C_1} = \frac{CD}{C_1D_1} = \frac{AB}{A_1B_1} = \frac{3}{2} = 1,5$$

ten'ligin payda yetemiz. Bul jag'dayda romblardi'n' sa'ykes ta'replari *proporcional* dep ayti'ladi'.

2. $ABCD$ ha'm $A_1B_1C_1D_1$ romblardi'n' sa'ykes mu'yeshleri wo'z ara ten'. Haqi'yqattan da $\angle A = \angle A_1 = 45^\circ$, $\angle B = \angle B_1 = 135^\circ$, $\angle C = \angle C_1 = 45^\circ$, $\angle D = \angle D_1 = 135^\circ$.

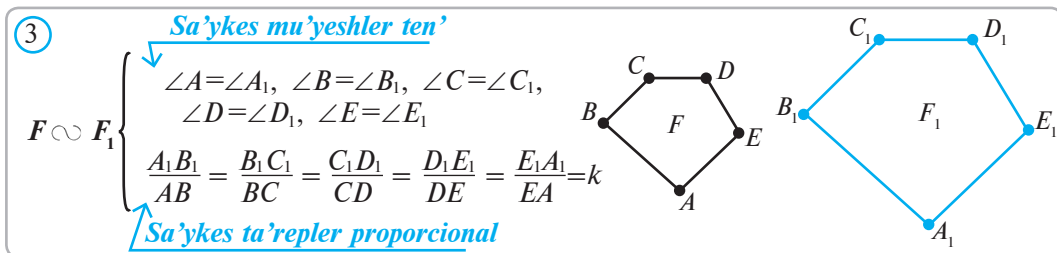
Solay yetip, bul romblardi'n' bir-birine uqsasli'g'i'ni'n' sebebi — sa'ykes ta'replerinin' proporcionalli'g'i' ha'm sa'ykes mu'yeshlerinin' ten'ligi dep ayta alami'z. Qa'legen ko'pmu'yeshliklerdin' uqsasli'g'i' tu'sinigi de usi' tiykari'nda kiritiledi. Mu'yeshlerinin' sani bir qi'yli' (demek, ta'replerinin' sani' da bir qi'yli') bolg'an ko'pmu'yeshlikler **bir qi'yli' atamadag'i ko'pmu'yeshlikler** dep ayti'ladi'.

Yeki bir qi'yli' atamali $ABCDE$ ha'm $A_1B_1C_1D_1E_1$ ko'pmu'yeshliklerinin' mu'yeshleri sa'ykes tu'rde ten' bolsi'n: $\angle A = \angle A_1$, $\angle B = \angle B_1$, $\angle C = \angle C_1$, $\angle D = \angle D_1$,

$\angle E = \angle E_1$. Bunday mu'yeshlar **sa'ykes mu'yeshlar dep ayti'ladi'**. Wonda, AB ha'm A_1B_1 , BC ha'm B_1C_1 , CD ha'm C_1D_1 , DE ha'm D_1E_1 , EA ha'm E_1A_1 ta'replari **sa'ykes ta'repler** delinedi.

✓ **Ani'qlama.** Bir qi'yli' atamali' ko'pmu'yeshliklerdin' birinin' mu'yeshleri yekinishinin' mu'yeshlerine sa'ykes tu'rde ten', sa'ykes ta'replari bolsa proporcional bolsa, bunday ko'pmu'yeshlikler **uqsas ko'pmu'yeshlikler** dep ataladi' (3-su'wret).

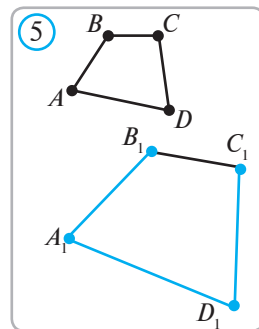
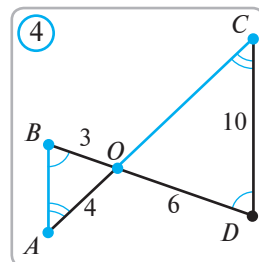
Ko'pmu'yeshliklerdin' uqsasli'g'i' $\sim$ belgisi menen belgilenedi.



Uqsas ko'pmu'yeshliklerdin' sa'ykes ta'replerinin' qatnasi'na ten' bolg'an san **uqsasli'q koefficienti** delinedi.

? **Soraw, ma'sele ha'm tapsi'rmalar**

1. Uqsas ko'pmu'yeshliklerdin' ani'qlamasi'n ayti'n'.
2. Uqsasli'q koefficienti degen ne ha'm wol qanday ani'qlanadi?
3. Yeger ABC ha'm DEF u'shmu'yeshliklerinde $\angle A = 105^\circ$, $\angle B = 35^\circ$, $\angle E = 105^\circ$, $\angle F = 40^\circ$, $AC = 4,4$ sm, $AB = 5,2$ sm, $BC = 7,6$ sm, $DE = 15,6$ sm, $DF = 22,8$ sm, $EF = 13,2$ sm bolsa, wolar uqsas bolama?
4. 2-su'wrettegi su'wretlengen a) ha'm b) romblar ne sebepten uqsas yemes? b) ha'm d) romblar-she?
5. 4-su'wrettegi ABO ha'm CDO u'shmu'yeshlikleri uqsas bolsa, AB , OC ta'replari uzi'nli'g'i'n ha'm uqsasli'q koefficientin tabi'n'.
6. 5-su'wrette $ABCD \sim A_1B_1C_1D_1$. $AB = 24$, $BC = 18$, $CD = 30$, $AD = 54$, $B_1C_1 = 54$. A_1B_1 , D_1A_1 ha'm C_1D_1 lardi' tabi'n'.
- 7*. ABC u'shmu'yeshliginin' AB ha'm AC ta'replerinin' wortalari' sa'ykes tu'rde P ha'm Q bolsi'n. $\triangle ABC \sim \triangle APQ$ yekenligin da'liyllen'.



Yen' a'piwayi' ko'pmu'yeshliklerden bolg'an u'shmu'yeshliklerdin' uqsasli'g'i'n u'yrenemiz.

Teorema. Yeki uqsas ushmu'yeshliktin' perimetrlerinin' qatnasi' uqsasli'q ko'efficientine ten'.

Bul teoremani wo'z betin'izshe da'liyllen'.

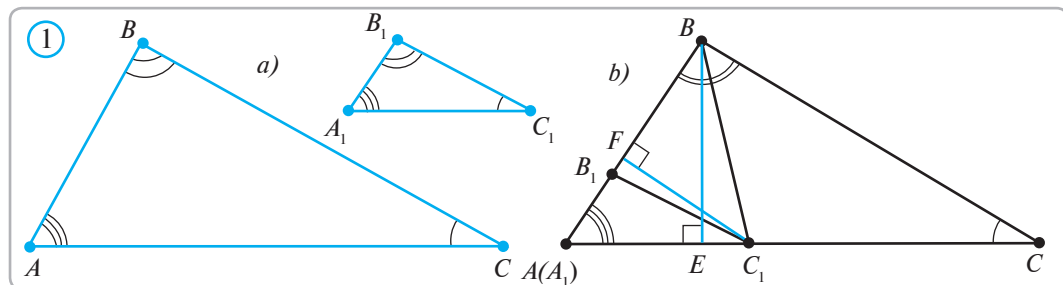
Teorema. Yeki uqsas u'shmu'yeshlik maydanlari'ni'n' qatnasi uqsasli'q ko'efficientinin' kvadratiga ten'.

$\triangle ABC \sim \triangle A_1 B_1 C_1$ (1-su'wret),
 k — uqsasli'q ko'efficienti



$$S_{ABC} : S_{A_1 B_1 C_1} = k^2$$

Da'liyllew: Teoremani'n' sha'rti boyi'nsha, $\triangle ABC \sim \triangle A_1 B_1 C_1$. Demek, ko'p-mu'yeshliklerdin' uqsasli'g'i'ni'n' ani'qlamasi' boyi'nsha, $\angle A = \angle A_1$, $\angle B = \angle B_1$, $\angle C = \angle C_1$ ha'm $\frac{AB}{A_1 B_1} = \frac{BC}{B_1 C_1} = \frac{CA}{C_1 A_1} = k$.



$\angle A = \angle A_1$ yekenliginen paydalani'p, wolardi' 1-b, su'wrettegidey u'stpe-u'st qoyami'z ha'm tiyisli jasaw ha'mde belgilewlerdi a'melge asi'rami'z.

To'mendegi u'shmu'yeshliklerdin' maydanlari'n tabami'z ha'm wolardi'n' qatnaslari'n qaraymiz:

$$\left[\begin{array}{l} S_{ABC_1} = \frac{A_1 C_1 \cdot BE}{2}; \\ S_{ABC} = \frac{AC \cdot BE}{2}; \end{array} \right] \Rightarrow \frac{S_{ABC}}{S_{ABC_1}} = \frac{AC}{A_1 C_1} \quad (1),$$

$$\left[\begin{array}{l} S_{A_1 B_1 C_1} = \frac{A_1 B_1 \cdot C_1 F}{2}; \\ S_{ABC_1} = \frac{AB \cdot C_1 F}{2}; \end{array} \right] \Rightarrow \frac{S_{A_1 B_1 C_1}}{S_{ABC_1}} = \frac{A_1 B_1}{AB} \quad (2).$$

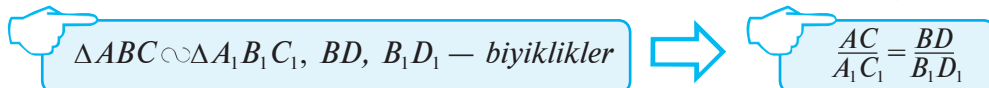
$$\frac{S_{ABC}}{S_{A_1 B_1 C_1}} = \frac{AB \cdot AC}{A_1 B_1 \cdot A_1 C_1} \quad (3)$$

(1) ten'likti (2) ten'likke bo'lsak, ten' mu'yeshke iye bolg'an u'shmu'yeshliklarning maydani'nin' qatnasi' ushi'n (3) ten'likni payda yetemiz.

Bul jerde sha'rt boyi'nsha, $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{CA}{C_1A_1} = k$ yekenligin yesapqa alsaq,

$$\frac{S_{ABC}}{S_{A_1B_1C_1}} = \frac{AB \cdot AC}{A_1B_1 \cdot A_1C_1} = \frac{AB}{A_1B_1} \cdot \frac{AC}{A_1C_1} = k \cdot k = k^2 \text{ kelip shig'adi. } \textbf{Teorema da'liylendi.}$$

1-ma'sele. Uqsas u'shmu'yeshliklarning sa'ykes ta'replerinin' qatnasi' usi' ta'replerga tu'sirilgan biyikliklarning qatnasi'na ten' yekenligin da'liyllen' (2-su'wret).

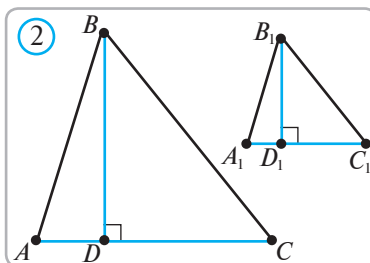


Sheshiliwi. Berilgen u'shmu'yeshliklarning uqsasli'q koefficienti k bolsi'n. Wonda, $AC : A_1C_1 = k$; $S_{ABC} : S_{A_1B_1C_1} = k^2$ (1) boladi'. Yekinishi ta'repten,

$$\frac{S_{ABC}}{S_{A_1B_1C_1}} = \frac{\frac{1}{2} AC \cdot BD}{\frac{1}{2} A_1C_1 \cdot B_1D_1} = \frac{AC}{A_1C_1} \cdot \frac{BD}{B_1D_1} = k \cdot \frac{BD}{B_1D_1} \quad (2)$$

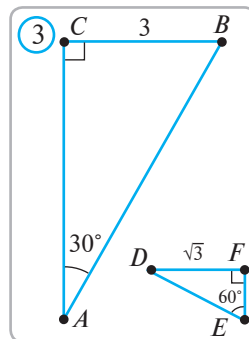
(1) ha'm (2) ten'liklardan $k \cdot \frac{BD}{B_1D_1} = k^2$ yaki $\frac{BD}{B_1D_1} = k$.

Solay yetip, $\frac{BD}{B_1D_1}$ qatnasi' da, $\frac{AC}{A_1C_1}$ qatnasi' da k g'a ten', yag'niy $\frac{AC}{A_1C_1} = \frac{BD}{B_1D_1}$.



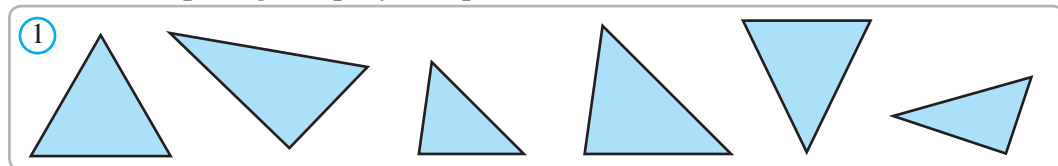
3 Soraw, ma'sele ha'm tapsi'rmalar

1. Uqsas u'shmu'yeshliklarning maydanlarinin' qatnasi haqqindagi teoremani ayting ha'm da'liyllen'.
2. Yeki ABC ha'm $A_1B_1C_1$ uqsas u'shmu'yeshliklari berilgen. Yeger $S_{ABC} = 25 \text{ sm}^2$ ha'm $S_{A_1B_1C_1} = 81 \text{ sm}^2$ bolsa, uqsasli'q koefficientin tabi'n'.
3. Yeki uqsas u'shmu'yeshlikning maydanlari 65 m^2 ha'm 260 m^2 . Birinshi u'shmu'yeshlikning bir ta'repi 6 m bolsa, yekinishi u'shmu'yeshlikning wog'an sa'ykes ta'repin tabi'n'.
4. Berilgen u'shmu'yeshlikning ta'replari 15 sm , 25 sm ha'm 30 sm . Yeger perimetri 35 sm bolg'an u'shmu'yeshlik berilgen u'shmu'yeshlikke uqsas bolsa, woni'n ta'replerin tabi'n'.
5. $\Delta ABC \sim \Delta A_1B_1C_1$ ha'm bul u'shmu'yeshliklarning sa'ykes ta'replerinin' qatnasi $7:5$ ke ten'. Yeger ABC u'shmu'yeshliginin' maydani $A_1B_1C_1$ u'shmu'yeshliginin' maydani'nan 36 sm^2 ge artiq bolsa, bul u'shmu'yeshliklarning maydanlarin tabi'n'.
6. 3-su'wrette berilgenlerden paydalani'p, u'shmu'yeshliklarning uqsas yaki uqsas yemesligin aniqlang'.



**Jedellestiriwshi shinig'iw**

1-su'wrette su'wretlengen u'shmu'yeshliklerdin' ishinen uqsaslari'n ani'qlan'. Wolardi'n' uqsasli'g'i'n qalay ani'qladi'n'i'z?



Ani'qlama boyi'nsha yeki u'shmu'yeshliktin' uqsasli'g'i'n ani'qlaw ushi'n wolardi'n' mu'yeshlerinin' ten'ligin ha'm sa'ykes ta'replerinin' proporsional yek-enligin tekseriw kerek boladi.' U'shmu'yeshlikler ushi'n bul is an'satlasadi yeken. To'mende keltirilgen teoremlar usi' tuwrali' boli'p, olar "u'shmu'yeshliklerdin' uqsasli'g'i'ni'n' belgileri" dep ataladi.'



Teorema. (U'shmu'yeshliklerdin' uqsasli'g'i'ni'n' MM belgisi). Yeger bir u'shmu'yeshliktin' yeki mu'yeshi yekinshi u'shmu'yeshliktin' yeki mu'yeshine sa'ykes tu'rde ten' bolsa, wonda bunday u'shmu'yeshlikler uqsas boladi' (2-su'wret).



$$\triangle ABC, \triangle A_1B_1C_1, \angle A = \angle A_1, \angle C = \angle C_1$$



$$\triangle ABC \sim \triangle A_1B_1C_1$$

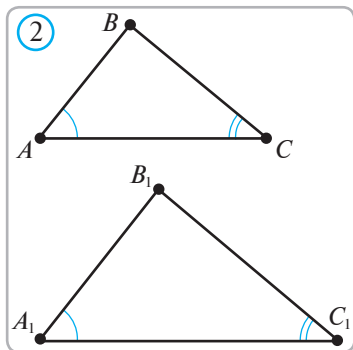
Da'liyllew. 1) U'shmu'yeshliktin' ishki mu'yeshlerinin' qosi'ndi'si' haqqindag'i teorema boyi'nsha,

$$\angle B = 180^\circ - (\angle A + \angle C), \angle B_1 = 180^\circ - (\angle A_1 + \angle C_1) \Rightarrow \angle B = \angle B_1$$

Demek, $\triangle ABC$ ha'm $\triangle A_1B_1C_1$ u'shmu'yeshliklerinin' mu'yeshleri sa'ykes tu'rde ten'.

2) Sha'rt boyi'nsha, $\angle A = \angle A_1, \angle C = \angle C_1$. Ten'dey mu'yeshke iye bol'gan u'shmu'yeshliklerdin' maydanlari'ni'n' qatnasi haqqindag'i teorema boyi'nsha,

$$\frac{S_{ABC}}{S_{A_1B_1C_1}} = \frac{AB \cdot AC}{A_1B_1 \cdot A_1C_1} \quad \text{ha'm} \quad \frac{S_{A_1B_1C_1}}{S_{A_1B_1C_1}} = \frac{CA \cdot CB}{C_1A_1 \cdot C_1B_1}.$$



Bul ten'liklerdin' won' bo'limlerin ten'lestirip, bir qi'yli' ag'zalar qisqartilsa, $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1}$ ten'ligi payda boladi.

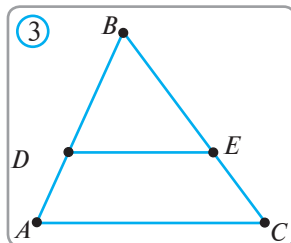
Sunday-aq, $\angle A = \angle A_1$ ha'm $\angle B = \angle B_1$ ten'liklerinen paydalani'p, $\frac{BC}{B_1C_1} = \frac{CA}{C_1A_1}$ ten'ligine iye bolami'z. Solay yetip, $\triangle ABC$ ha'm $\triangle A_1B_1C_1$ u'shmu'yeshliklerinin' mu'yeshleri ten' ha'm sa'ykes ta'repleri propor-

cional, yag'niy bul u'shmu'yeshlikler uqsas boladi.' **Teorema da'liyllendi.**

Ma'sele. ABC u'shmu'yeshliginin' yeki ta'repin kesip wo'tiwshi ha'm u'shinshi ta'repine parallel bolg'an DE tuwri'si u'shmu'yeshlikten wog'an uqsas u'shmu'yeshlik aji'ratatug'i'ni'n da'liyllen' (3-su'wret).

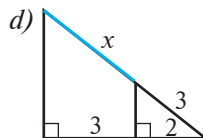
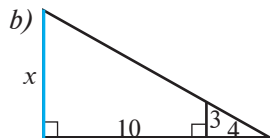
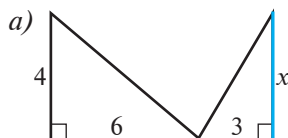
Da'liyllew. ABC ha'm DBE u'shmu'yeshliklerde $\angle B$ — uli'wma, $\angle CAB = \angle EDB$ (AC ha'm DE parallel tuwri'lari'n AB kesiwshi menen keskende payda bolg'an sa'ykes mu'yeshler bolg'ani' ushi'n) (3-su'wret).

Demek, u'shmu'yeshliklerdin' uqsaslig'i'ni'n' MM belgisi boyi'nsha, $ABC \sim DBE$.

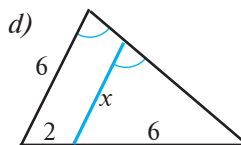
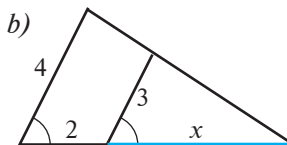
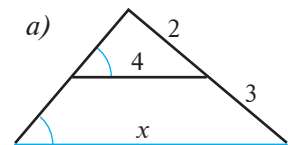


? Soraw, ma'sele ha'm tapsi'rmalar

1. U'shmu'yeshliklerdin' uqsasli'q ani'qlamasi' ha'm MM belgisin wo'z ara salistirin'.
2. U'shmu'yeshliklerdin' uqsaslig'inin' MM belgisin da'liyllen'.
3. Su'wrettegi mag'liwmatlarga tiykarlanip x ti tabin.



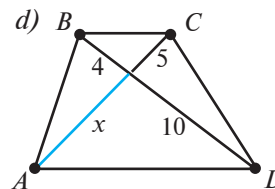
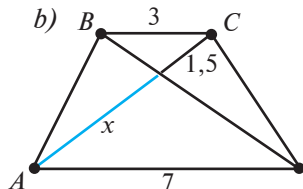
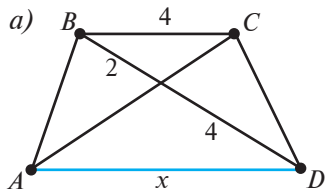
4. Su'wrettegi mag'liwmatlarga tiykarlanip x ti tabi'n'.



5. $ABCD$ parallelogrammnin' CD ta'repinen E noqati aling'an. AE ha'm BC nurlari F noqatinda kesilisedi.

- a) Yeger $DE = 8$ sm, $EC = 4$ sm, $BC = 7$ sm, $AE = 10$ sm bolsa, EF ha'm FC ni;
- b) Yeger $AB = 8$ sm, $AD = 5$ sm, $CF = 2$ sm bolsa, DE ha'm EC ni tabin.

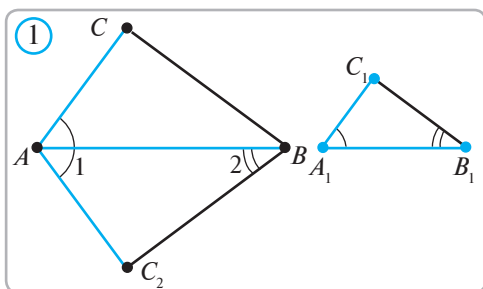
6. Su'wrette $ABCD$ — trapeciyasi' berilgen. Su'wrettegi mag'liwmatlarga tiykarlanip, x ti tabin.



- 7*. Bir su'yir mu'yeshleri ten' bolg'an yeki tuwri' mu'yeshli u'shmu'yeshlikler uqsas yekenligin da'liyllen'.

Teorema. (U'shmu'yeshliklerdin' uqsaslig'i'ni'n' TMT belgisi). Yeger bir u'shmu'yeshliktin' yeki ta'repi yekinshi u'shmu'yeshliktin' yeki ta'repine proporcional ha'm bul ta'repler payda yetken mu'yeshler ten' bolsa, onda bunday u'shmu'yeshlikler uqsas boladi' (1-su'wret).

$$\Delta ABC, \Delta A_1B_1C_1, \angle A = \angle A_1, \frac{AB}{A_1B_1} = \frac{AC}{A_1C_1} \Rightarrow \Delta ABC \sim \Delta A_1B_1C_1$$



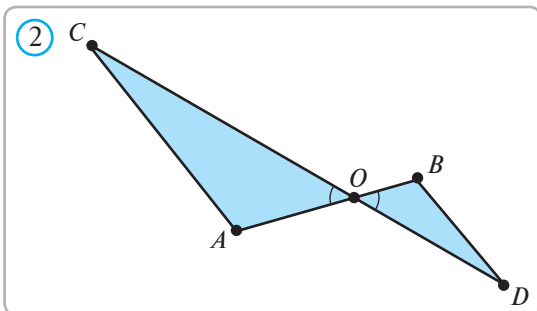
Da'liyllew. $\angle 1 = \angle A_1$, $\angle 2 = \angle B_1$ bolatu-g'i'nday yetip ΔABC_2 u'shmu'yeshligin jasay-mi'z (1-su'wret). Ol MM belgisi boyi'nsha $\Delta A_1B_1C_1$ u'shmu'yeshligine uqsas boladi'.

$$\frac{AB}{A_1B_1} = \frac{AC_2}{A_1C_1} \Leftrightarrow (\Delta A_1B_1C_1 \sim \Delta ABC_2)$$

$$\frac{AB}{A_1B_1} = \frac{AC}{A_1C_1} \Leftrightarrow (\text{sha'rtke ko're}).$$

Bul yeki ten'likten, $AC_2 = AC$ yekenligin ani'qlaymi'z. Wonda u'shmu'yeshliklerdin' ten'liginin' TMT belgisi boyi'nsha $\Delta ABC = \Delta ABC_2$. Sonliqtan, $\angle 2 = \angle B$. Lekin, jasaw boyi'nsha $\angle 2 = \angle B_1$ yedi. Demek, $\angle B = \angle B_1$. Onda, $\angle A = \angle A_1$ ha'm $\angle B = \angle B_1$ bolg'ani ushi'n, u'shmu'yeshliklerdin' uqsaslig'inin' MM belgisi boyi'nsha, $\Delta ABC \sim \Delta A_1B_1C_1$. **Teorema da'liyillendi.**

Ma'sele. AB ha'm CD kesindileri O noqatinda kesilisedi, $AO = 12sm$, $BO = 4sm$, $CO = 30sm$, $DO = 10sm$ bolsa, ΔAOC , ΔBOD u'shmu'yeshliklerinin' maydanlari'ni'n' qatnasi'n tabi'n'.



Sheshiliwi. Sha'rt boyi'nsha

$$\left. \begin{aligned} \frac{OA}{OB} &= \frac{12}{4} = 3 \\ \frac{OC}{OD} &= \frac{30}{10} = 3 \end{aligned} \right\} \Rightarrow \frac{OA}{OB} = \frac{OC}{OD} = 3.$$

Demek, ΔAOC u'shmu'yeshliginin' yeki ta'repi ΔBOD u'shmu'yeshliginin' yeki ta'repine proporcional ha'm bul ta'replerdin' arasi'ndag'i' sa'ykes

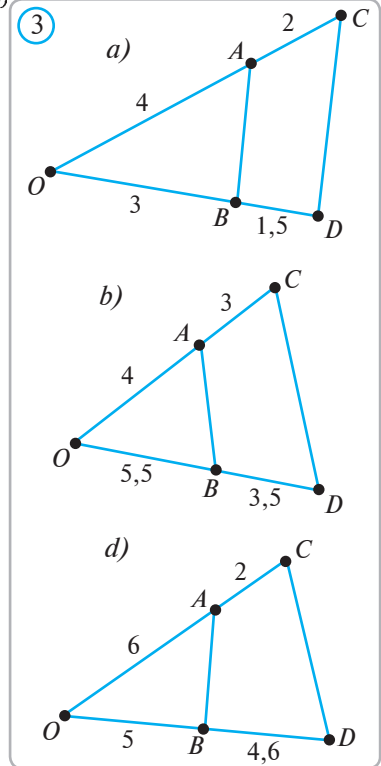
mu'yeshler vertikal mu'yeshler bolg'ani' ushi'n: $\angle AOC = \angle BOD$. Soni'n' ushi'n, u'shmu'yeshliklerdin' uqsaslig'i'ni'n' TMT belgisi boyi'nsha, $\Delta AOC \sim \Delta BOD$ ha'm

uqsasli'q koefficienti $k = \frac{OA}{OB} = 3$. Yendi uqsas u'shmu'yeshliklerdin' maydanlari'ni'n'

qatnasi haqqindag'i teoremani qollanemiz. $\frac{S_{AOC}}{S_{BOD}} = k^2 = 9$. **Juwabi':** 9.

? Soraw, ma'sele ha'm tapsi'rmalar

1. U'shmu'yeshliklerdin' uqsasli'g'i'nin' ani'qlaması' ha'm TMT belgilerin wo'z ara salisti'ri'n'.
2. U'shmu'yeshliklerdin' uqsasli'g'inin' TMT belgisin da'liyllen'.
3. To'besindegi mu'yeshleri ten' bolg'an ten' qaptalli' u'shmu'yeshliklerdin' uqsaslig'in a) MM, b) TMT belgisinen paydalani'p da'liyllen'.
4. 3-su'wrette su'wretlengen OAB ha'm OCD u'shmu'yeshlikleri uqsas pa?
5. AC ha'm BD nurlari' O noqati'nda kesilisedi. Yeger $AO : CO = BO : DO = 3$, $AB = 7$ sm bolsa, CD kesindisin ha'm de AOB ha'm COD u'shmu'yeshliklerinin' maydanlati'ni'n' qatnasi'n tabi'n'.
6. ABC ha'm $A_1B_1C_1$ u'shmuyeshliklerde $\angle A = \angle A_1$, $AB : A_1B_1 = AC : A_1C_1 = 4 : 3$.
 - a) Yeger AB kesindisi A_1B_1 den 5 sm arti'q bolsa, AB ha'm A_1B_1 ta'replerin tabi'n';
 - b) Yeger A_1B_1 kesindi AB dan 6 sm qi'sqa bolsa, AB ha'm A_1B_1 ta'replerin tabi'n';



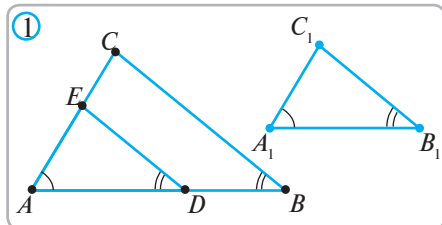
- d) Yeger berilgen u'shmu'yeshliklerdin' maydanlari'ni'n' qosi'ndi'si' 400 sm² bolsa, onda ha'r bir u'shmu'yeshlikdin' maydani'n tabi'n'.
7. Yeger bir tuwri' mu'yeshli u'shmu'yeshlikdin' katetleri yekinshi tuwri' mu'yeshli u'shmu'yeshlikdin' sa'ykes katetlerine proporcional bolsa, onda u'shmu'yeshliklerdin' uqsas ekenligin da'liyllen'.
 8. ABC u'shmu'yeshliginde $AB = 15$ m, $AC = 20$ m, $BC = 32$ m. U'shmu'yeshlikdin' AB ta'repine $AD = 9$ m kesindi, AC ta'repine $AE = 12$ m kesindi qoyi'ldi. DE kesindisin tabi'n'.
 9. Katetleri 3 dm ha'm 4 dm bolg'an tuwri'mu'yeshli u'shmu'yeshlik penen bir kateti 8 dm ha'm gipotenuzasi 10 dm bolg'an tuwri'mu'yeshli u'shmu'yeshlikdin' uqsas yekenligin da'liyllen'.
 - 10*. AB kesindisi ha'm l tuwri'si O noqatinda kesilisedi. l tuwri'sina AA_1 ha'm BB_1 perpendikulyarlari' tu'sirilgen. Yeger $AA_1 = 2$ sm, $OA_1 = 4$ sm ha'm $OB_1 = 3$ sm bolsa, BB_1 , OA ha'm AB kesindilerin tabin.

U'SHMU'YESHLIKLERDIN' UQSASLIG'ININ' U'SHINSHI BELGISI

Teorema. (U'shmu'yeshliklerdin' uqsasli'g'i'ni'n' TTT belgisi). Yeger bir u'shmu'yeshliktin' u'sh ta'repi yekinshi u'shmu'yeshliktin' u'sh ta'repine proporcional bolsa, wonda bunday u'shmu'yeshlikler uqsas boladi.

$$\triangle ABC, \triangle A_1B_1C_1, \frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{CA}{C_1A_1} \text{ (1-su'wret).}$$

$$\triangle ABC \sim \triangle A_1B_1C_1$$



Da'liyllew. ABC u'shmu'yeshliginin' AB ta'repinde $AD=A_1B_1$ bolatug'i'nday yetip D noqatin belgileymiz. D noqati'nan BC ta'repine parallel yetip ju'rgizilgen tuwri' AC ta'repin E noqati'nda kesip wo'tsin. Wonda u'shmu'yeshliklerdin' MM belgisi boyi'nsha $\triangle ADE$ ha'm $\triangle ABC$ uqsas boladi'. Bul jag'dayda

ani'qlama boyi'nsha: $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1}$ ha'm $\frac{AB}{AD} = \frac{BC}{DE}$.

Biraq jasaw boyi'nsha $A_1B_1=AD$. Wonda joqari'dag'i' ten'liklerden, $B_1C_1=DE$ ten'ligi payda boladi'. Solay yetip, u'shmu'yeshliklerdin' ten'liginin' TMT belgisi boyi'nsha $\triangle ADE$ ha'm $\triangle A_1B_1C_1$ ten' ha'm $\triangle ADE \sim \triangle ABC$. Demek, $\triangle ABC \sim \triangle A_1B_1C_1$. **Teorema da'lyllendi.**

Ma'sele. Yeger yeki ten' qaptalli' u'shmu'yeshliktin' birewinin' ultani' ha'm qaptal ta'repi yekinshisinin' ultani ha'm qaptal ta'repine proporcional bolsa, wonda bul u'shmu'yeshliklerdin' uqsas yekenligin da'liyllen'.

$$\triangle ABC, AB = BC, \frac{AB}{A_1B_1} = \frac{AC}{A_1C_1}$$

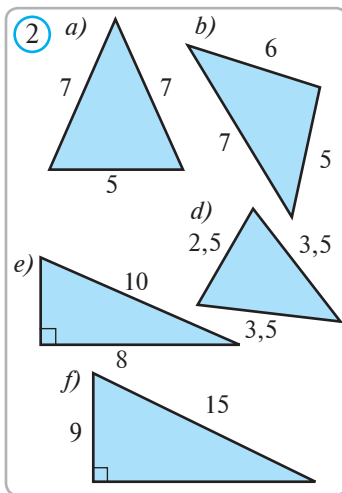
$$\triangle ABC \sim \triangle A_1B_1C_1$$

Da'liyllew. Berilgen $AB=BC$, $A_1B_1=B_1C_1$ ten'likler ha'm $\frac{AB}{A_1B_1} = \frac{AC}{A_1C_1}$ qatnastan $\frac{AB}{A_1B_1} = \frac{AC}{A_1C_1} = \frac{BC}{B_1C_1}$ ten'liklerin payda yetemiz. Demek, u'shmu'yeshliklerdin' uqsasli'g'i'ni'n' TTT belgisi boyi'nsha, $\triangle ABC \sim \triangle A_1B_1C_1$.

Soraw, ma'sele ha'm tapsi'rmalar

1. U'shmu'yeshliklerdin' uqsasli'g'i'ni'n' TTT belgisin ayti'n' ha'm da'liyllewin tu'sindirip berin'.
2. $AC=14$ sm, $AB=11$ sm, $BC=13$ sm, $A_1C_1=28$ sm, $A_1B_1=22$ sm, $B_1C_1=26$ sm yekenligi belgili. ABC ha'm $A_1B_1C_1$ u'shmu'yeshlikleri uqsas bolama?

3. 2-su'wrettegi uqsas u'shmu'yeshlikler jupli'qlari'n ko'rsetin'.
4. BCD trapeciyani'n' AB ha'm CD qaptal ta'replari dawam ettirilse, E noqati'nda kesilisedi. Yeger $AB=5$ sm, $BC=10$ sm, $CD=6$ sm, $AD=15$ sm bolsa, AED u'shmu'yeshliginin' maydani'n tabi'n.
5. Trapeciyani'n' ultanlari' 6 sm ha'm 9 sm, biyikligi 10 sm. Trapeciyani'n' diagonallari' kesisken noqati'nan ultanlari'na shekemgi arali'qlari'n tabi'n.
6. Qa'legen yeki ten' ta'repli u'shmu'yeshliktin' uqsas yekenligin da'liyllen'.
7. Ultani 12 sm, biyikligi 8 sm bolg'an ten' qaptalli' u'shmu'yeshliktin' ishine kvadrat sonday yetip ishley si'zi'lg'an, kvadrattin' yeki to'besi u'shmu'yeshliktin' ultani'nda, al qalgan yeki to'besi bolsa qaptal ta'replerde jatadi'. Kvadratti'n' ta'repin tabi'n.
- 8*. Su'yir mu'yeshli ABC u'shmu'yeshliginin' AA_1 ha'm BB_1 biyikliklari ju'rgizilgen. $\triangle ABC \sim \triangle A_1B_1C$ yekenligin da'liyllen.
9. Yeki uqsas u'shmu'yeshliktin' maydanlari 6 ha'm 24 ge ten'. Wolardi'n birewinin' perimetri yekinshisinen 6 g'a artiq. U'lken u'shmu'yeshliktin' perimetritin tabi'n.



Tariyxi'y waqi'yalar. Bul waqi'ya erami'zdan aldi'n'g'i VI a'sirde bolg'an yedi. Bul waqi'tta grekler geometriya menen derlik shug'i'llanbaytug'in yedi. Grek filosofi' Fales mi'si'r pa'ni menen tani'si'w ushi'n wol jerge barg'an. Mi'si'rli'lar og'an qi'yi'n ma'sele bergin yeken: u'lken piramidalardan birinin' biyikligin qalay tabi'w mu'mkin? Fales bul ma'selenin' a'piwayi' ha'm jen'il sheshimin tapti'. Wol tayaqshani' jerge qaqt'i ha'm sonday dedi: "Qashan bul tayaqsha sayasi'ni'n' uzi'nli'g'i' tayaqshani'n' uzi'nli'g'i' menen ten' bolsa, piramida sayasi'ni'n' uzi'nlig'i' piramida biyikligi menen ten' boladi'". Falestin' pikirin tiykarlawg'a ha'reket yetin'!



Tuwri' mu'yeshli u'shmu'yeshliklerdin' bir-birden mu'yeshleri tuwri' mu'yeshden ibarat boladi. Soni'n' ushi'n bunday u'shmu'yeshlikler ushi'n uqsasli'q belgileri a'piwayilastiriladi.

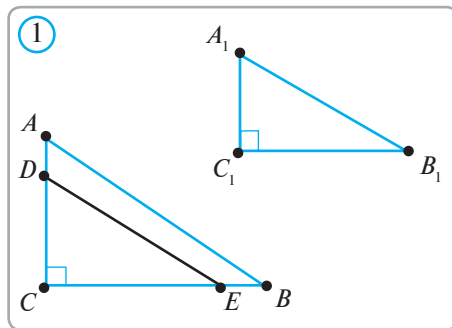
1-Teorema. Tuwri' mu'yeshli u'shmu'yeshliklerdin' bir-birden su'yir mu'yeshi sa'ykes tu'rde ten' bolsa, wonda wolar uqsas boladi'.

2-Teorema. Tuwri' mu'yeshli u'shmu'yeshliklerdin' katetleri sa'ykes tu'rde proporcional bolsa, wonda wolar uqsas boladi'.

3-Teorema. Tuwri' mu'yeshli u'shmu'yeshliklerdin' birinin' gipotenuzasi' ha'm kateti, yekinshisinin' gipotenuzasi ha'm katetine sa'ykes tu'rde proporcional bolsa, wonda wolar uqsas boladi'.

Bul belgilerdin' birinshi yekewinin' duri's yekenligi wo'z-wo'zinen ayqi'n. U'shinshi belgisin bolsa da'liyllewimiz kerek.

$$\triangle ABC, \triangle A_1B_1C_1, \angle C = 90^\circ, \angle C_1 = 90^\circ, \frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} \Rightarrow \triangle ABC \sim \triangle A_1B_1C_1$$



Da'liyllew. ABC u'shmu'yeshliginin' BC ta'repinde $CE = C_1B_1$ bolatug'inday yetip C_1B_1 kesindisin qoyamiz ha'm $DE \parallel AB$ ni ju'rgizemiz (*I-su'wret*). Wonda u'shmu'yeshliklerdin' uqsasli'g'i'ni'n' MM belgisi boyi'nsha, $\triangle DEC$ ha'm $\triangle ABC$ uqsas boladi'. Uqsas u'shmu'yeshliklerdin' sa'ykes ta'replerinin' proporcionalig'inan:

$$\frac{AB}{DE} = \frac{CB}{CE}.$$

Jasaliwi boyi'nsha, $CE = C_1B_1$. Demek, $\frac{AB}{DE} = \frac{CB}{C_1B_1}$ (1) ten'ligi ori'nli boladi.

Basqa ta'repten, teorema sha'rti boyi'nsha, $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1}$ (2)

(1) ha'm (2) ten'liklerden $DE = A_1B_1$ yekenligin ani'qlaymiz.

$A_1B_1C_1$ ha'm DEC u'shmu'yeshliklerdi qaraymiz. Wolarda:

1. $CE = C_1B_1$ (jasaw boyi'nsha); 2. $DE = A_1B_1$ (da'lillengen ten'lik boyi'nsha).

Demek, tuwri' mu'yeshli u'shmu'yeshliklerdin' bir-birden kateti ha'm de gipotenuzasinin' ten'lik belgisi boyi'nsha, $\triangle A_1B_1C_1 = \triangle DEC$. Yekinshi ta'repten bolsa $\triangle ABC \sim \triangle DEC$. Wonda $\triangle ABC \sim \triangle A_1B_1C_1$ boladi. **Teorema da'liyllendi.**

Ma'sele. Yeger yeki ten' qaptalli u'shmu'yeshlikten birewinin' qaptal ta'repi ha'm biyikligi yekinshisinin' qaptal ta'repi ha'm biyikligine proporcional bolsa, wonda bul u'shmu'yeshliklerdin' uqsas yekenligin da'liyllen' (2-su'wret).

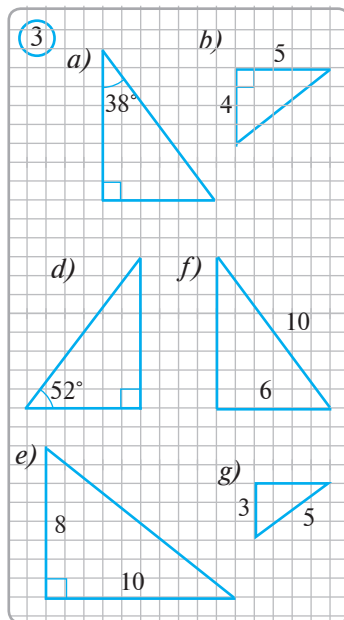
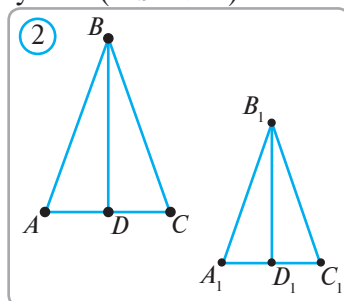
Da'liyllew. Tuwri' mu'yeshli ABD ha'm $A_1B_1D_1$ u'shmu'yeshliklerin qaraymi'z. Sha'rt boyi'nsha woldardi'n' birewden kateti ha'm gipotenuzasi wo'z ara proporcional. Demek, 3-teoremag'a tiykarlanip $\triangle ABD \sim \triangle A_1B_1D_1$. Wonda $\angle DBA \sim \angle D_1B_1A_1$.

Ten' qaptalli u'shmu'yeshliktin' ultaninatu'sirilgen biyikliktin' bissektrisasi da boli'wi'n yesapqa alsaq, $\angle B = 2\angle DBA = 2\angle D_1B_1A_1 = \angle B_1$ boladi.

Na'tiyjede, ABC ha'm $A_1B_1C_1$ u'shmu'yeshliklerde $\angle B = \angle B_1$ ha'm $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1}$ ten'liklerine iye bolami'z.

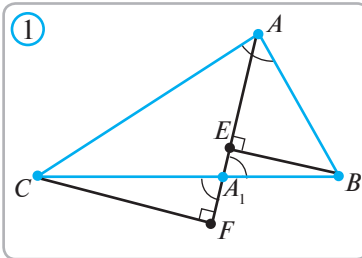
Demek, u'shmu'yeshliklerdin' uqsasli'g'i'ni'n' TMT belgisi boyi'nsha, $\triangle ABC \sim \triangle A_1B_1C_1$.

Teorema da'liyllendi.



Soraw, ma'sele ha'm tapsi'rmalar

1. Tuwri' mu'yeshli u'shmu'yeshliklerdin' uqsasli'q belgilerin ayti'n' ha'm da'liyllen'.
2. 3-su'wrettegi uqsas u'shmu'yeshliklerdi tabi'n'.
3. Katetleri 3 m ha'm 4 m bolg'an tuwri' mu'yeshli u'shmu'yeshlikke uqsas u'shmu'yeshliktin' bir kateti 27 m bolsa, yekinshi kateti neshe m boladi?
4. Maydanlari' 21 m² ha'm 84 m² bolg'an yeki tuwri' mu'yeshli u'shmu'yeshlikler wo'z ara uqsas. Yeger birinshi u'shmu'yeshliktin' bir kateti 6 m bolsa, yekinshi u'shmu'yeshliktin' katetlerin tabi'n'.
5. Bir shen'berge yeki uqsas tuwri' mu'yeshli u'shmu'yeshlik ishley si'zi'lg'an. Bul u'shmu'yeshliklerdin' ten'ligin da'liyllen'.
- 6*. Katetleri 10 sm ha'm 12 sm bolg'an tuwri' mu'yeshli u'shmu'yeshlikke bir mu'yeshi uliwma bolg'an kvadrat ishley si'zi'lg'an. Yeger kvadrattin' bir to'besi gipotenuzada yekenligi belgili bolsa, kvadrattin' ta'repin tabi'n'.
- 7*. ABC u'shmu'yeshlik berilgen. Wog'an $ADEF$ sonday etip ishley si'zi'lg'an D , E ha'm F noqatlar sa'ykes tarizde u'shmu'yeshliktin' AB , BC ha'm CA tareplerinde jatadi? Yeger $AB=c$, $AC=b$, bolsa, rombni'n ta'repin tabi'n'.



1-ma'sele. U'shmu'yeshliktin' bissektrisasi' wo'zi tu'sken ta'repti qalgan yeki ta'repke proporcional bolgan kesindilerga aji'ratatug'i'ni'n da'liyllen'.

$$\triangle ABC, AA_1 - \text{bissektrisa (1-su'wret)} \Rightarrow \frac{AB}{A_1B} = \frac{AC}{A_1C}$$

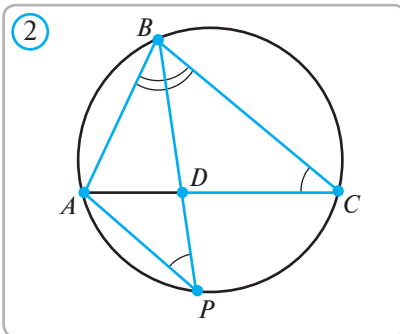
Da'liyllew. AA_1 tuwri'si'na BE ha'm CF perpendikulyari'n tu'siremiz. Wonda $\angle CAF = \angle BAE$ bolg'ani' ushi'n tuwri' mu'yeshli CAF ha'm BAE u'shmu'yeshlikler uqsas boladi'. Uqsas u'shmu'yeshliklerdin' sa'ykes ta'replerinin' proporcionalli'g'i'nan

$$\triangle CAF \sim \triangle BAE \Rightarrow \frac{AC}{AB} = \frac{CF}{BE}. \quad (1)$$

Bug'an uqsas,

$$\triangle CA_1F \sim \triangle BA_1E \Rightarrow \frac{CA_1}{BA_1} = \frac{CF}{BE}. \quad (2)$$

(1) ha'm (2) ten'liklerdi sali'sti'rsa, $\frac{AC}{AB} = \frac{CA_1}{BA_1}$ yaki $\frac{AB}{A_1B} = \frac{AC}{A_1C}$ boladi'. Bul A_1B ha'm A_1C kesindilari AB ha'm AC kesindilerga proporcional yekenligin an'latadi'.



2-ma'sele. ABC u'shmu'yeshliginin' BD bissektrisasi u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdi B ha'm P noqati'nda kesip wo'tedi. $\triangle ABP \sim \triangle BDC$ li'gi'n da'liyllen' (2-su'wret).

Sheshiliwi. $\triangle ABP$ ha'm $\angle BDC$ da:

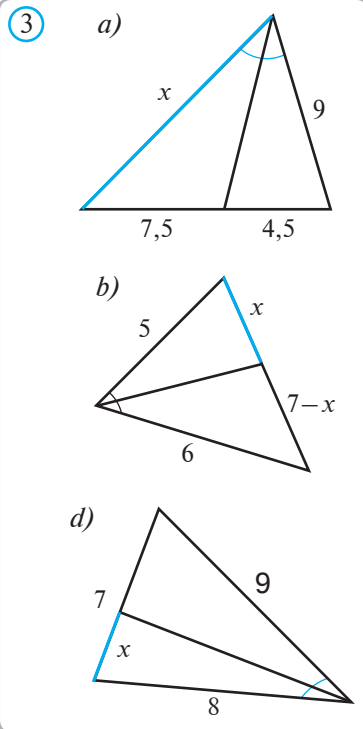
1. $\angle DBC = \angle ABP \Leftarrow$ sha'rt boyi'nsha;
2. $\angle DCB = \angle APB \Leftarrow$ sebebi wolar bir dog'ag'a tirelgen. Demek, u'shmu'yeshliklerdin' uqsaslig'i'ni'n MM belgisi boyi'nsha, $\triangle ABP \sim \triangle BDC$.

? Soraw, ma'sele ha'm tapsi'rmalar

1. U'shmu'yeshliktin' bissektrisasi' wo'zi tu'sken ta'repte aji'ratqan kesindilari ha'm u'shmu'yeshliktin' qalg'an ta'repleri arasi'ndag'i' proporcionalli'qti jazi'p ko'rsetin'.
2. Tuwri' mu'yeshli ABC u'shmu'yeshliginin' C tuwri' mu'yeshinen CD biyikligi ju'rgizilgen. $\angle ACD = \angle CBD$ yekenligin da'liyllen'. Paydabolg'an figuradan eshe

wo'z ara uqsas u'shmu'yeshliklerin ko'rsete alasiz?

3. 3-su'wrettegi mag'liwmatlarga tiykarlanip x ti tabi'n'.
4. ABC u'shmu'yeshliginin' AD bissektrisasi' ju'r-gizilgen. Yeger $CD = 4,5 m$; $BD = 13,5 m$ ha'm ABC u'shmu'yeshliktin' perimetri $42 sm$ bolsa, AB ha'm AC ta'replerin tabi'n'.
5. ABC u'shmu'yeshliginin' medianalari' N noqatindakesilisedi. Yeger ABC u'shmu'yeshliginin' maydani' $87 dm^2$ bolsa, ANB u'shmu'yeshliginin' maydani'n tabi'n'.
6. ABC u'shmu'yeshliginin' medianalari' kesilisen N noqati'nan AB ha'm BC ta'replerine shekem bolg'an arali'qlar sa'ykes tu'rde $3 dm$ ha'm $4 dm$ ge ten'. Eger $AB = 8 dm$ bolsa, BC ta'repin tabi'n'.
- 7*. Trapeciyani'n' ultani'na parallel tuwri' qaptal ta'replerinin' birewin $m:n$ qatnasta bo'liwi mu'mkin. Bul tuwri' woni'n' yekinshi qaptal ta'repin qanday qatnasta bo'ledi?



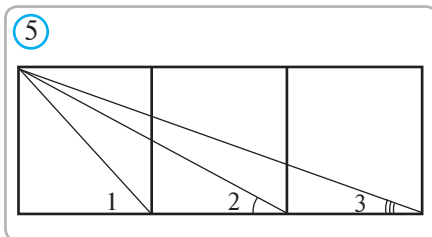
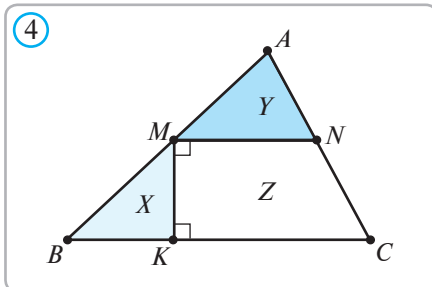
Qi'ziqli' ma'seleler

Geometriya ha'm ingliz tili. To'mendegi ingliz tilinde berilgen geometriyalik ma'seleni sheship ko'rin'shi! Buni'n' menen ha'm ingliz tili, ha'm geometriyadan bilimin'izdi bilip alasi'z.

1) Dissection Puzzle: Let M be the midpoint of the side AB of a given triangle ABC . The triangle has been dissected into parts X , Y , Z along the lines MN and MK passing through M such that MN is parallel while MK is perpendicular to the base BC (picture 4). Show how the three pieces can be fitted together to make a rectangle, respectively two different parallelograms.

2) Look at the picture 5 and proof

$$\angle 1 + \angle 2 + \angle 3 = 90^\circ.$$

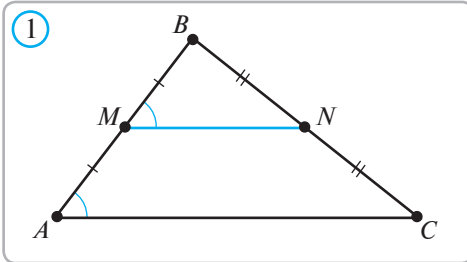


1-ma'sele. U'shmu'yeshliktin' uqsasli'g'i'nan paydalani'p, u'shmu'yeshliktin' worta sizig'i u'shmu'yeshliktin' bir ta'repine parallel ha'm usi' ta'reptin' yari'mi'na ten' ekenligin da'liyllen'.

$\triangle ABC$, MN — worta si'zi'q
(1-su'wret): $MA=MB$, $NC=NB$



$MN \parallel AC$, $MN = \frac{1}{2} AC$



Sheshiliwi. $\triangle ABC$ ha'm $\triangle MBN$ ushi'n:

$$\angle B \text{ — uliwma, } \frac{BM}{AB} = \frac{BN}{BC} = \frac{1}{2}.$$

Soni'n' ushi'n, u'shmu'yeshliktin' uqsasli'g'inin' TMT belgisi boyi'nsha, bul yeki u'shmu'yeshlik uqsas. Yendi talqi'lawdi' usilay yetip dawam yettirez:

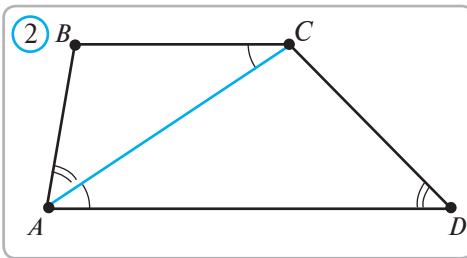
$$\triangle MBN \sim \triangle ABC \Rightarrow \begin{cases} \angle BMN = \angle A, \\ \frac{MN}{AC} = \frac{1}{2} \end{cases} \Rightarrow \begin{cases} MN \parallel AC, \\ MN = \frac{1}{2} AC. \end{cases}$$

2-ma'sele. Ultanlari BC ha'm AD bolg'an $ABCD$ trapeciyasi'ni'n' AC diagonalini' yeki uqsas u'shmu'yeshlikke ajiratadi. $AC^2 = BC \cdot AD$ yekenligin da'liyllen'.

$ABCD$ — trapetsiya, $BC \parallel AD$,
 $\triangle ABC \sim \triangle DCA$ (2-su'wret)



$AC^2 = BC \cdot AD$



Sheshiliwi. 1-qa'dem. ABC ha'm ACD

u'shmu'yeshliklerinin' mu'yeshlerin sali'sti'-ramiz. $\angle ACB = \angle CAD$, sebebi bul mu'yeshler — ishki ayqi'sh mu'yeshler. $\angle B \neq \angle D$, sebebi $ABCD$ — trapeciya (keri jag'dayda,

$$\angle D + \angle A = \angle B + \angle A = 180^\circ,$$

yag'ni'y $AB \parallel CD$ boli'p, $ABCD$ trapeciya bolmay qalar yedi). Solay etip, $\angle D = \angle BAC$

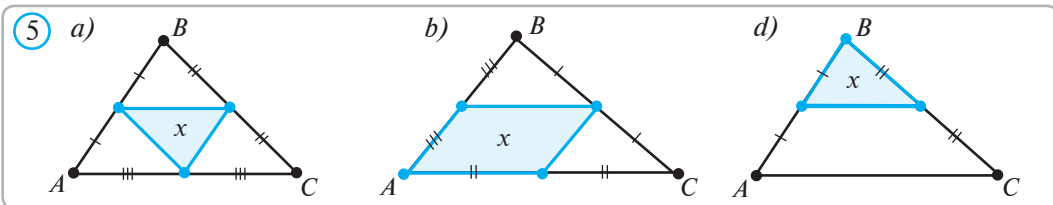
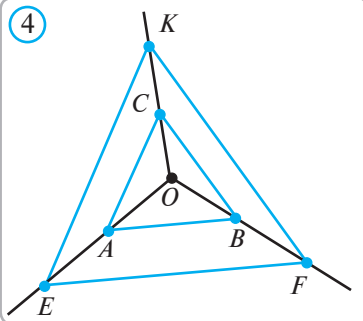
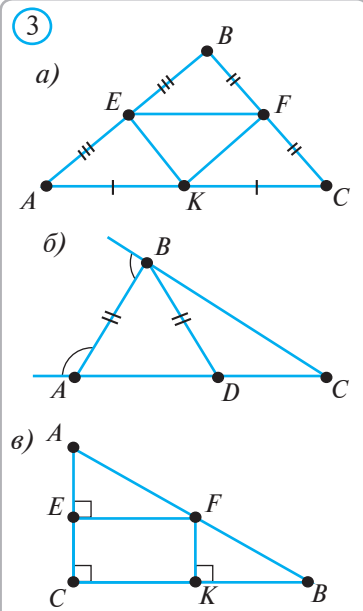
ha'm $\angle ACD = \angle B$.

2-qa'dem. Yendi ABC ha'm DCA u'shmu'yeshliktin' sa'ykes ta'replerinin' qatnasi'n jazamiz: $\frac{AC}{BC} = \frac{AD}{AC}$, bunnan $AC^2 = BC \cdot AD$.

? Soraw, ma'sele ha'm tapsi'rmalar

- a) Boyi' 170 sm bolg'an adam sayasi'ni'n' uzi'nli'g'i 1 m bolsa, biyikligi 5,4 m bolg'an terek sayasi'ni'n' uzi'nli'g'i'n' tabi'n'.

b) Yeki ten' qaptali u'shmu'yeshliktin' to'besin-degi mu'yeshleri ten'. Birinshi u'shmu'yeshliktin' qaptal ta'repi 17 sm, ultani' 10 sm ge, yekinshi u'shmu'yeshliktin' ultani' 8 sm ge ten'. Yekinshi u'shmu'yeshliktin' qaptal ta'repin tabi'n'.
- 3-su'wrettegi ha'r bir si'zi'lmadan uqsas u'shmu'yeshliklerdi ko'rsetin'.
- ABC u'shmu'yeshliginin' AP medianasi' BC ta'repine parallel ha'm to'beleri AB ha'm AC ta'replerinde jatqan qa'legen kesindini ten' yekige bo'letug'i'ni'n' da'liyllen'.
- U'shmu'yeshliktin' to'beleri woni'n' worta si'zi'g'i'n' wo'z ishine alg'an tuwri'dan ten'dey arali'qta jatatug'i'ni'n' da'liyllen'.
- Shen'berge ishley si'zi'lg'an $ABCD$ to'rtmu'yeshliktin' diagonallari O noqatta kesilisedi. $\triangle AOB \sim \triangle COD$ yekenligin da'liyllen'.
- ABC u'shmu'yeshliktin' ishinen O noqati' ha'm OA , OB , OC nurlari'ndasa'ykes tu'rde E , F , K noqatlari ali'ng'an (4-su'wret). Yeger $AB \parallel EF$ ha'm $BC \parallel FK$ bolsa, ABC ha'm EFK u'shmu'yeshliklerinin' uqsas yekenligin da'liyllen'.
- *. Trapeciyani'n' diagonallari' kesilisiw noqati'nan wo'tiwshi tuwri' trapeciya ultanlari'nan birin $m:n$ qatnastabo'ledi. Bul tuwri' yekinshi ultani'n qanday qatnasta bo'ledi?
- Yeger ABC u'shmu'yeshliklerdin' maydani' S g'a ten' bolsa, 5-su'wrettegi x penen belgilengen maydandi' tabi'n'.



I. Testler**1. 1. To'mendegi tasti'yi'qlawlardan qaysi biri duri's?**

- A) Yeki u'shmu'yeshliktin' mu'yeshleri sa'ykes tu'rde ten' bolsa, wolar uqsas delinedi;
- B) Yeki u'shmu'yeshliktin' ta'repleri sa'ykes tu'rde ten' bolsa, wolar uqsas delinedi;
- D) Yeki u'shmu'yeshliktin' sa'ykes ta'repleri proporcional ha'm sa'ykes mu'yeshleri ten' bolsa, wolar uqsas delinedi;
- E) Yeki u'shmu'yeshliktin' sa'ykes ta'repleri ha'm sa'ykes mu'yeshleri ten' bolsa, wolar uqsas delinedi.

2. Yeki uqsas u'shmu'yeshliktin' maydanlari'ni'n' qatnasi' nege ten'?

- A) Uqsasli'q koefficientine;
- B) Wolardi'n' sa'ykes ta'replerinin' qatnasi'na;
- D) Wolardin' perimetrlerinin' qatnasi'na;
- E) Uqsasli'q koefficientinin' kvadrati'na.

3. To'mendegi tasti'yi'qlawdan qaysi' biri duri's?

- A) U'shmu'yeshliklerdin' birinin' yeki mu'yeshi yekinshisinin' yeki mu'yeshine ten' bolsa, wolar uqsas boladi';
- B) U'shmu'yeshliklerdin' birinin' yeki ta'repi yekinshisinin' yeki ta'repine ten' bolsa, wolar uqsas boladi';
- D) Yeki u'shmu'yeshliktin' bir-birden mu'yeshleri ten' ha'm yeki ta'repleri proporcional bolsa, wolar uqsas boladi';
- E) Yeki u'shmu'yeshliktin' bir-birden mu'yeshleri ten' ha'm bir-birden ta'repleri proporcional bolsa, wolar uqsas boladi'.

4. Duri'si'n tabi'n. Eger yeki u'shmu'yeshlik uqsas bolsa, olardi'n':

- A) Biyiklikleri ten' boladi';
- B) Ta'repleri proporcional boladi';
- D) Ta'repleri ten' boladi';
- E) Maydanlari ten' boladi'.

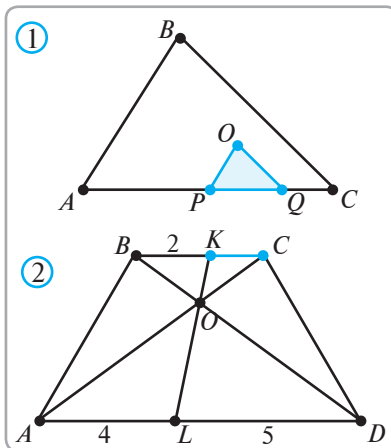
5. Uqsas u'shmu'yeshliklerdin' perimetrlerinin' qatnasi' nege ten'?

- A) Sa'ykes ta'repleri qatnasi'ni'n' kvadrati'na;
- B) Uqsasli'q koefficientine;
- D) Uqsasli'q koefficientinin' qatnasi'na;
- E) Maydanlari'ni'n' qatnasi'na.

II. Ma'seleler

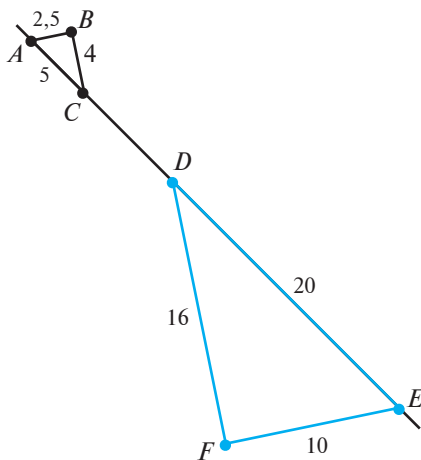
1. ABC u'shmu'yeshliginin' AB ha'm AC ta'replerinin' wortalari' sa'ykes tu'rde E ha'm F noqatlari bolsi'n. Yeger AEF u'shmu'yeshliginin' maydani' 3 sm^2 bolsa, ABC u'shmu'yeshliginin' maydani'n tabi'n'.
2. ABC u'shmu'yeshliginin' AC ta'repine parallel tuwri' AB ha'm BC ta'replerin

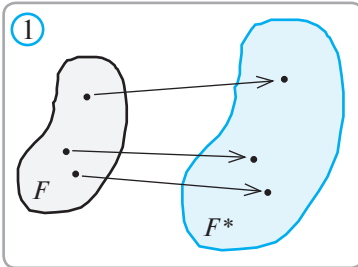
- sa'ykes tu'rde N ha'm P noqatlarda kesip wo'tedi. Yeger $AN = 4$, $NB = 3$, $BP = 3,6$ bolsa, BC ta'repin tabi'n'.
3. Su'yir mu'yeshli ABC u'shmu'yeshliginin' AB ta'repinde A noqati' ali'ng'an. Yeger $AK = 3$, $BK = 2$ ha'm u'shmu'yeshliktin' BD biyikligi 4 ke ten' bolsa, K noqati'n AC kesindisine shekem bolg'an arali'qti' tabi'n'.
 4. $ABCD$ parallelogrammi'n' BC ta'repinin' ortasi'ndag'i K noqati'n AC kesindisine shekem bolg'an arali'qti' tabi'n'.
 5. ABC u'shmu'yeshligi ishinde ali'ng'an O noqati'n u'shmu'yeshliktin' AB ha'm BC ta'replerine parallel tuwri'lar ju'rgizilgen. Bul tuwri'lar AC ta'repin sa'ykes tu'rde P ha'm Q noqatlarda kesip wo'tedi. Yeger $PQ = 2$, $AC = 7$ ha'm ABC u'shmu'yeshliginin' maydani' 98 ge ten' bolsa, POQ u'shmu'yeshliginin' maydani'n ani'qlan'.
 6. $ABCD$ trapesiyasi'ni'n' BC ha'm AD ultanlari'nda K ha'm L noqatlari' sa'ykes tu'rde ali'ng'an. KL kesindisi trapetsiyani'n' diagonallari' kesisken noqattan wo'tedi. Yeger $AL = 4$, $LD = 5$ ha'm $BK = 2$ bolsa, KC kesindisin tabi'n'.



III. Wo'zin'izdi sinap ko'rin' (u'lgi ushi'n baqlaw jumi'si').

1. $ABCD$ trapesiyani'n' AC diagonali' woni' yeki uqsas $\triangle ABC$ ha'm $\triangle ACD$ u'shmu'yeshlikke aji'ratadi'. Bunda $BC = 4$ m, $AD = 9$ m bolsa, AC diagonali'ni'n' uzi'nli'g'i'n yesaplan'.
2. Yeki uqsas u'shmu'yeshliktin' maydanlari' 50 dm^2 ha'm 32 dm^2 , wolardi'n' perimetrlarinin' qosi'ndi'si' 117 dm bolsa, ha'r bir u'shmu'yeshliktin' perimetrin tabi'n'.
3. Su'wrette su'wretlengen u'shmu'yeshliklerdin' uqsasli'g'i'n da'liyllen'. BC ha'm DF tuwri'larini'n' wo'z ara jaylasi'wi' haqqi'nda ne ayta alasi'z.
4. (Qosi'msha). Su'yir mu'yeshli ABC u'shmu'yeshliginin' BD ha'm AE biyikliklari' ju'rgizilgen. $DC \cdot AC = EC \cdot BC$ yekenligin da'liyllen'.

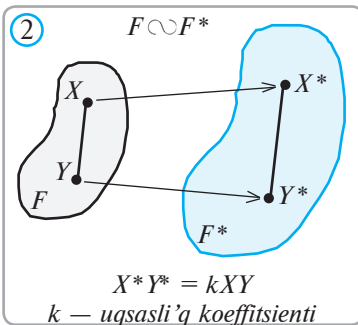




Wo'tken sabaqlarda ko'pmu'yeshliklerdin' uqsasli'g'i tu'sinigi menen tani'sti'q. Uqsasli'q tu'sinigin tek ko'pmu'yeshlikler ushi'n yemes, ba'лки qa'legen geometriyaliq figuralar ushi'n da kiritiw mu'mkin. Yeger F ha'm F^* figuralari' berilgen boli'p, F figurasi'ni'n' ha'r bir noqati'nda F^* figurasi'ni'n' qaysi' bir noqati' sa'ykes qoyi'lg'an bolsa ha'm bunda F^* figurasi'ni'n' ha'r bir noqati' F figurasi'ni'n' tek bir noqati' sa'ykes

kelse, **F figurasi F^* figurasina tu'rlendiriw delinedi.**

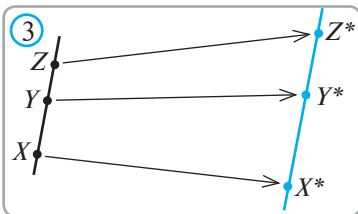
✓ **Ani'qlama.** Eger F figurasi'n F^* figurasi'na tu'rlendiriwde noqatlar arasi'ndag'i' arali'qlar 0 den wo'zgeshe ani'q bir sang'a ko'beyse, bunday tu'rlendiriw **uqsasli'q tu'rlendiriw** delinedi (2-su'wret).



Bul ani'qlamani' to'mendegishe talqi'law mu'mkin: Aytayi'q, qandayda bir tu'rlendiriw na'tiyjesinde F figurasi'ni'n' qa'legen X, Y noqatlari'na F^* figurasi'nin' X^*, Y^* noqatlari sa'ykes qoyi'lg'an bolsi'n. Yeger $X^*Y^* = k \cdot XY$, $k > 0$ bolsa, bunday tu'rlendiriwge **uqsasli'q tu'rlendiriw** delinedi. Bunda k — barli'q X ha'm Y noqatlari ushi'n bir qi'yli' san boli'p, ol **uqsasli'q ko'effitsienti** delinedi. Yeger F ha'm F^* figuralari berilgen boli'p, bul figuralardin' birin ekinshisine o'tkeretug'in uqsasli'q tu'rlendiriwi bar

bolsa, F ha'm F^* figuralari **o'z ara uqsas** delinedi. Figuralardin' uqsaslig'i $F \cap F^*$ siyaqli jaziladi. Yeger uqsasli'q ko'effitsienti k ni dako'rsetiw lazim bolsa, $F \cap F^*$ tu'rinde belgilenedi. Yeger uqsasli'q tu'rlendiriwinde X noqatiga' X^* noqati sa'ykes qoyi'lg'an bolsa, X noqati **X^* noqatiga tu'rlendi** yaki **wo'tti delinedi**. Uqsasli'q tu'rlendiriwi to'mendegi qa'siyetlerge iye:

📐 **Teorema.** Uqsasli'q tu'rlendiriwi a) tuwri' si'zi'qti' tuwri' si'zi'qqa; b) nurdi nurg'a; d) mu'yeshi (woni'n' u'ljenligin saqlag'an halda) mu'yeshke; e) kesindini (uzi'nli'g'i' bul kesindiden k ma'rte uzi'n bolg'an) kesindige wo'tkeredi.



Da'iiyllew. a) Uqsasli'q ko'effitsienti k bolg'an uqsas tu'rlendiriwde bir tuwri'da jatqan tu'rli X, Y ha'm Z noqatlari'na sa'ykes tu'rde X^*, Y^* ha'm Z^* noqatlarga' tu'rlendiriw (3-su'wret). X, Y, Z noqat-

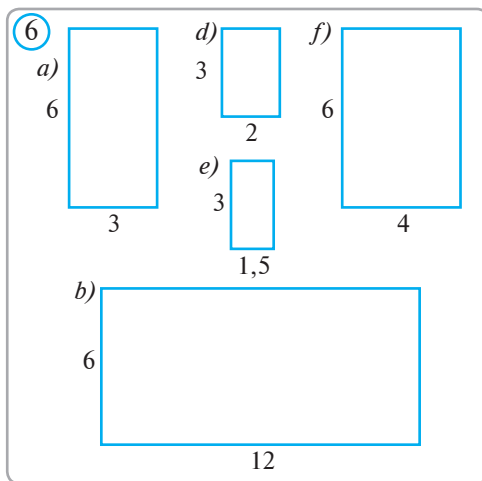
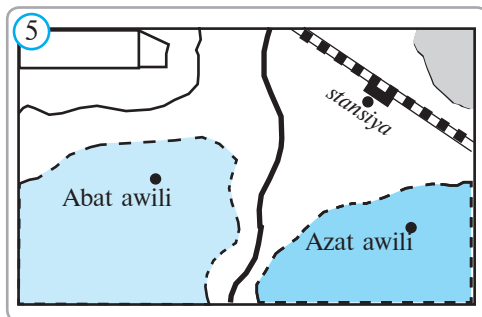
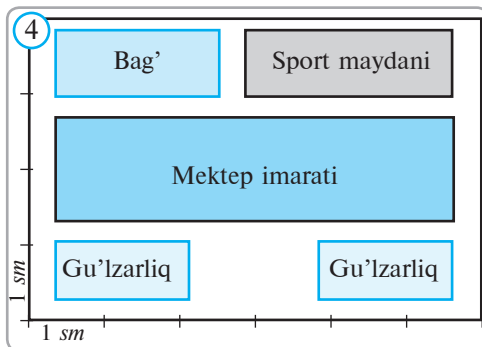
lari'nan biri, aytayi'q, Y qalg'an yekewinin' wortasi'nda jatsin. Wonda $XZ = XY + YZ$. Uqsasli'q tu'rlendiriwdin' ani'qlamasi' boyi'nsha:

$$X^*Z^* = k \cdot XZ = k \cdot (XY + YZ) = k \cdot XY + k \cdot YZ = X^*Y^* + Y^*Z^*.$$

Bul ten'likten X^* , Y^* ha'm Z^* noqatlari' bir tuwri' da jatatur'ini kelip shi'g'adi'. Bul teoremani'n' da'liyllewin tek: a) tasti'yi'qlaw ushi'n keltirdik. Qalg'an jag'daylarin wo'z betin'izshe da'liyllen'.

? *Soraw, ma'sele ha'm tapsi'rmalar*

1. Uqsasli'q tu'rlendiriw degen ne?
2. Qanday figuralar uqsas figuralar delinedi?
3. Yeni 3 sm, biyikligi 4 sm bolg'an tuwri' mu'yeshlikke uqsas, uqsasli'q koefficienti 2 ge ten' bolg'an to'rtmu'yeshlik jasan'.
4. 4-su'wrette mektep ha'wlsinin' sxemasi' 1:1000 masshtabta su'wretlengen wo'lshew islerin wori'nlan'. a) ha'wlinin'; b) mektep imarati'ni'n'; d) gu'lzarlar-di'n'; e) sport maydani'ni'n'; f) bag'-di'n' haqi'yqi'y wo'lshemlerin tabi'n'.
5. Yeger karta 1:50000 masshtabta su'wretlengen bolsa, Abat ha'm Azat awi'llari' arasi'ndag'i' arali'qti tabi'n'.
6. Uqsasli'q tu'rlendiriwde nurlar arasi'ndag'i' mu'yesh saqlanatug'i'ni'n' da'liyllen'.
- 7*. Uqsasli'q tu'rlendiriwde nurlar arasi'ndag'i' mu'yesh saqlanatug'i'ni'n' da'liyllen'.
- 8*. ABC u'shmu'yeshliginin' uqsasli'q tu'rlendiriwde $A^*B^*C^*$ u'shmu'yeshligine tu'rlendiriledi. Yeger uqsasli'q koefficienti 0,6 g'a ha'm ABC u'shmu'yeshliginin' perimetri 12 sm ge ten' bolsa, $A^*B^*C^*$ u'shmu'yeshliginin' perimetrin tabi'n'.
9. 6-su'wretten uqsas tuwri' mu'yeshliklerdin' jupli'qlari'n tabi'n' ha'm uqsasli'q koefficientlerin ani'qlan'.

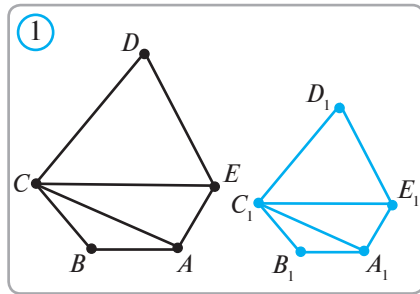


1-teorama. Uqsas ko'pmu'yeshliklerdin' perimetrlerinin' qatnasi' uqsasli'q ko'efficientine ten'.

Da'liyllew. Haqi'yqattan da, $A_1A_2...A_n$ ha'm $B_1B_2...B_n$ ko'pmu'yeshlikleri uqsas ha'm uqsasli'q ko'efficienti k bolsa, $B_1B_2=k \cdot A_1A_2$, $B_2B_3=k \cdot A_2A_3$, ... , $B_nB_1=k \cdot A_nA_1$ boladi.' Bunnan

$P=B_1B_2+B_2B_3+...+B_nB_1=k \cdot A_1A_2+k \cdot A_2A_3+...+k \cdot A_nA_1=k \cdot (A_1A_2+A_2A_3+...+A_nA_1)=k \cdot P_1$ yeken'ligin payda yetemiz. **Teorema da'liyillendi.**

2-teorema. Uqsas ko'pmu'yeshliklerdi bir qi'yli' sandag'i uqsas u'shmu'yeshliklarga ajirati'w mu'mkin.



Da'liyllew. Aytayi'q, $ABCDE$ ha'm $A_1B_1C_1D_1E_1$ ko'pmu'yeshlikleri uqsas boli'p, uqsasli'q ko'efficienti k bolsi'n.

Wo'z ara sa'ykes C ha'm C_1 to'belerinen CA , CE ha'm C_1A_1 , C_1E_1 diagonallari'n ju'rgizemiz. Na'tiyjede ko'pmu'yeshlikler bir qi'yli' sandag'i' u'shmu'yeshliklarga ajirati'ldi'. Payda bolg'an u'sh jup sa'ykes u'shmu'yeshliklerdin' uqsasli'g'i'n ko'rsetemiz.

1. $\triangle ABC \sim \triangle A_1B_1C_1$. Sebebi bul u'shmu'yeshliklerde, sha'rti boyi'nsha, $\angle B = \angle B_1$, $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = k$.

U'shmu'yeshliklerdin' uqsaslig'i'ni'n' TMT belgisi boyi'nsha $\triangle ABC \sim \triangle A_1B_1C_1$.

2. $\triangle CDE \sim \triangle C_1D_1E_1$. Bul uqsasli'q 1-ba'ntindegi siyaqli' da'liyillendi.

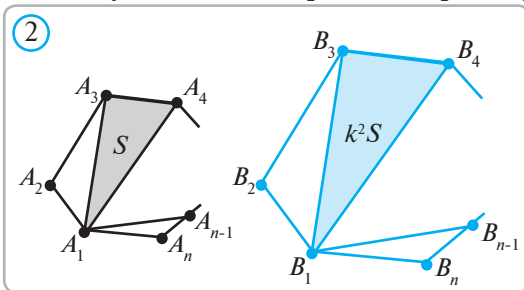
3. $\triangle ACE \sim \triangle A_1C_1E_1$. Haqi'yqattanda, $\angle CAE$ ha'm $\angle C_1A_1E_1$ mu'yeshlerin qaraymi'z. $\angle CAE = \angle BAE - \angle CAB$, $\angle C_1A_1E_1 = \angle B_1A_1E_1 - \angle C_1A_1B_1$.

Bul jerde, $\angle BAE = \angle B_1A_1E_1$ (berilgen uqsas besmu'yeshliklerdin' sa'ykes mu'yeshleri). $\angle CAB = \angle C_1A_1B_1$ (uqsas ABC ha'm $A_1B_1C_1$ u'shmu'yeshliklerinin' sa'ykes mu'yeshleri). Demek, $\angle CAE = \angle C_1A_1E_1$.

AC ha'm AE ha'm de A_1C_1 ha'm A_1E_1 ta'replerin qaraymi'z: $AC = kA_1C_1$, sebebi wolar wo'z ara uqsas ABC ha'm $A_1B_1C_1$ u'shmu'yeshliklerinin' sa'ykes ta'repleri, $AE = kA_1E_1$, sebebi wolar da berilgen uqsas besmu'yeshliklerinin' sa'ykes ta'repleri. Demek, u'shmu'yeshliklerdin' uqsaslig'i'ni'n' TMT belgisi boyi'nsha $\triangle ACE \sim \triangle A_1C_1E_1$. Qa'legen uqsas ko'pmu'yeshlikler ushi'n da usi siyaqli' talqi'lawlar paydali' boli'wi' ani'q. **Teorema da'liyillendi.**

3-teorema. Uqsas ko'pmu'yeshliklardin' maydanlari'ni'n' qatnasi' uqsasli'q coefficientinin' kvadrati'na ten'.

Da'liyllew. Aytayi'q, $A_1A_2...A_n$ ha'm $B_1B_2...B_n$ ko'pmu'yeshliklari uqsas ha'm k — uqsasli'q coefficienti bolsi'n. Wonda $A_1A_2A_3$, $A_1A_3A_4$, ..., $A_1A_{n-1}A_n$ u'shmu'yeshliklari sa'ykes tu'rde $B_1B_2B_3$, $B_1B_3B_4$, ..., $B_1B_{n-1}B_n$ u'shmu'yeshliklerine uqsas boli'p, uqsas u'shmu'yeshliklardin' maydanlari'ni'n' qatnasi' k^2 qaten' boladi' (2-su'wret).



$SA_1A_2A_3 = k^2SB_1B_2B_3$, $SA_1A_3A_4 = k^2SB_1B_3B_4$, ..., $SA_1A_{n-1}A_n = k^2SB_1B_{n-1}B_n$. Bul ten'liklardin' sa'ykes bo'limlerin qossaq, $SA_1A_2...A_n = k^2SB_1B_2...B_n$ boladi.' **Teorema da'liylandi.**

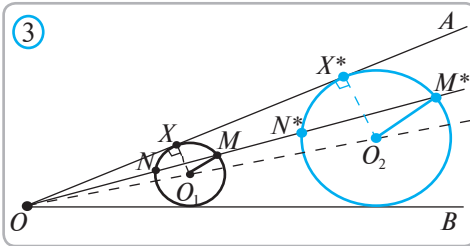
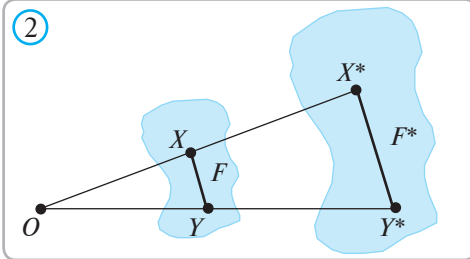
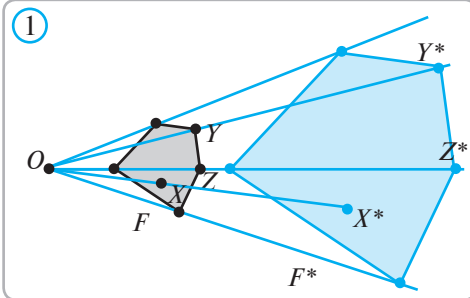
Ma'sele. Perimetrleri 18 sm ha'm 24 sm bolg'an yeki uqsas ko'pmu'yeshlik maydanlari'ni'n' qatnasi'n tabi'n'.

Sheshiliwi. 1) Uqsas ko'pmu'yeshliklari perimetrlerinin' qatnasi' uqsasli'q coefficientine ten' yekenliginen paydalani'p, $k = 24 : 18 = 4 : 3$ yekenligin tabami'z.

2) Uqsas ko'pmu'yeshliklardin' maydanlari'ni'n' qatnasi uqsasli'q coefficientinin' kvadrati'na ten' bolg'ani' ushi'n izlengen qatnas $k^2 = \frac{16}{9}$ g'aten'. **Juwabi:** $\frac{16}{9}$.

Soraw, ma'sele ha'm tapsi'rmalar

1. Uqsas ko'pmu'yeshliklardin' perimetrlerinin' qatnasi' nege ten'?
2. Uqsas ko'pmu'yeshliklardin' maydanlari'ni'n' qatnasi' haqqi'ndag'i' teoremani' tu'sindirin'.
3. U'shmu'yeshlik penen to'rtmu'yeshlik uqsas boli'wi' mu'mkin be?
4. Maydanlari' 6 m^2 ha'm 24 m^2 bolg'an yeki to'rtmu'yeshlik uqsas. Uqsasli'q coefficientin tabi'n'.
5. Yeki ko'pmu'yeshliktin' perimetrleri 18 sm ha'm 36 sm ge, maydanlari'ni'n' qosi'ndi'si' bolsa 30 sm^2 qaten'. Ko'pmu'yeshliklardin' maydanlari'n tabi'n'.
6. Perimetri 84 sm bolg'an u'shmu'yeshliktin' bir ta'repine parallel yetip ju'rgizilgen tuwri', wonnan perimetri 42 sm ge ha'm maydani' 26 sm^2 qaten' u'shmu'yeshlik ajiratadi. Berilgen u'shmu'yeshliktin' maydani'n tabi'n'.
7. O noqatina salistirg'anda simmetriyali figuralar uqsas bolama? Ko'sherge salistirg'anda simmetriyali' figuralar she? Wolardi'n' uqsasli'q coefficientin tabi'n'?
8. To'rtmu'yeshlik formasindag'i' paxta atizi kartada maydani' 12 sm^2 bolg'an u'shmu'yeshlik penen ko'rsetiledi. Yeger karta masshtabi 1:1000 bolsa, atizdin' maydani'n yesaplan'.
- 9*. Maydanlari' 8 sm^2 ha'm 32 sm^2 bolg'an yeki uqsas u'shmu'yeshlik perimetrlerinin' qosi'ndi'si' 48 sm ge ten'. U'shmu'yeshliklardin' perimetrlerin tabi'n'.



Yen' a'piwayi uqsas tu'rlandirivlerden biri gomotetiya boladi'. Aytayi'q, F — figura, O — noqat ha'm k — on' sani' berilgen bolsi'n. F figurasi'ni'n' qa'legen X noqati' arqali' OX nuri'n ju'rgizemiz ha'm bul nurda uzi'nli'g'i' $k \cdot OX$ bolg'an OX^* kesindisin qoyamiz (*1-su'wret*). Bul usi'l menen F figurasi'ni'n' ha'r bir X noqati'na X^* noqati'n sa'ykes qoyatug'in tu'rlandiriv **gomotetiya** delinedi. Bunda, O noqati' gomotetiya worayi, k sani gomotetiya koefficienti, F ha'm gomotetiya na'tijesinde F figura almasatug'in F^* figuralar bolsa **gomotetiyaliq figuralar** delinedi.

Teorema. Gomotetiya uqsasli'q tu'rlandirivi boladi.

Da'liyilew. Erkli O worayi'na iye, k koefficientli gomotetiyada F figurani'n' X ha'm Y noqatlari' X^* ha'm Y^* noqatlari'na wo'tsin (*2-su'wret*). Wonda, gomotetiya ani'qlamasi' boyi'nsha, XOY ha'm X^*OY^* u'shmu'yeshliklerinde $\angle O$ — uli'wmaha'm $\frac{OX^*}{OX} = \frac{OY^*}{OY} = k$ boladi.' Demek, XOY ha'm

X^*OY^* u'shmu'yeshlikleri yeki ta'repi ha'm wolar arasi'ndag'i' mu'yeshi boyi'nsha uqsas. Soni'n' ushi'n $\frac{X^*Y^*}{XY} = \frac{OX^*}{OX} = k$, sonli'qtan, $X^*Y^* = k \cdot XY$. **Teorema da'liyilendi.**

Ma'sele. AOB mu'yeshinin' ta'replerine uriniwshi qa'legen yeki shen'ber gomotetiya bolu'wi'n ha'm O noqati bul gomotetiyaushi'n oray yekenligin da'liyillen'.

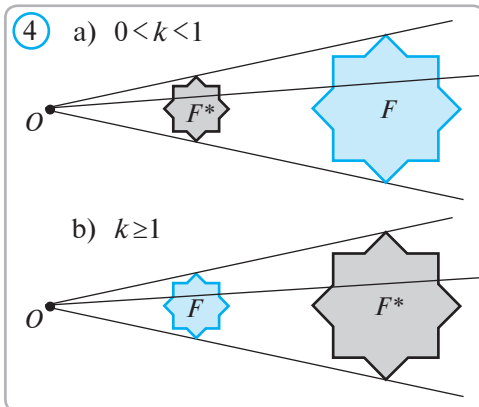
Sheshiliwi. Woraylari O_1 ha'm O_2 bolg'an shen'berler AOB mu'yeshinin' ta'replerine uri'nsi'n (*3-su'wret*). Bul shen'berlerdin' gomotetiyali'q ekenligin da'lilleymiz.

Shen'berler OA nurina sa'ykes tu'rde X ha'm X^* noqatlari'nda uri'ng'an bolsi'n (*3-su'wret*). Wonda, $\triangle OXO_1 \sim \triangle OX^*O_2$ (sebebi $\angle XO_1O = \angle X^*O_2O$ ha'm $\angle XO_1O = \angle X^*O_2O = 90^\circ$). Bunnan $\frac{OX^*}{OX} = \frac{OO_2}{OO_1}$.

Won' ta'reptegi qatnasti' k menen belgileymiz ha'm koefficienti $k = \frac{OO_2}{OO_1}$ worayi O bolg'an gomotetiyanı' qaraymı'z. Aytayi'q, bul gomotetiyada O_1 worayi'na iye shen'berdin' qa'legen M noqati' M^* noqatına tu'rlendirilgen bolsi'n Wonda

$$O_2M^* = k \cdot O_1M \text{ yaki } O_2M^* = \frac{O_2X^*}{O_1X} \cdot O_1M.$$

Bunnan, $O_1X = O_1M$ bolg'ani' ushi'n, $O_2M^* = O_2X^*$ ten'ligin payda yetemiz. Bul M^* noqati' orayi O_2 noqati'nda bolg'an radiusi' O_2X^* g'a ten' bolg'an shen'berde jatatug'i'ni'n bildiredi. Demek, qarali'p atirg'an shen'berler wo'z ara gomotetiyaliq yeken.

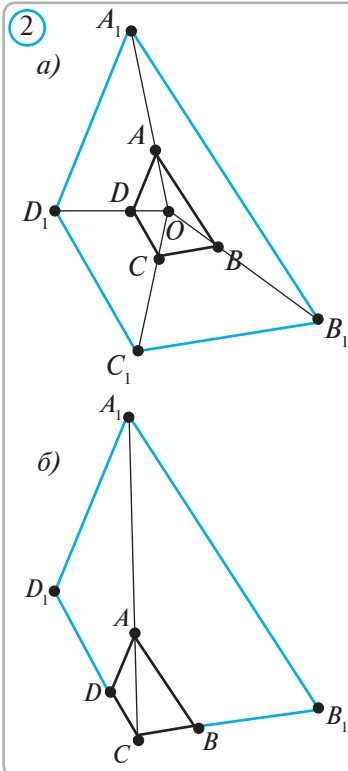
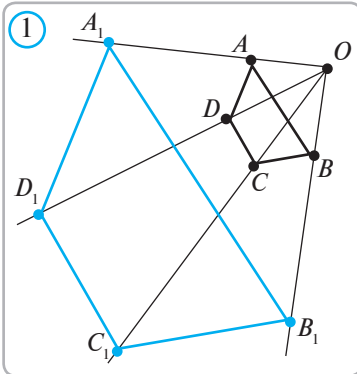


Jedellestiriwshi tapsi'rma

4-su'wrette gomotetiya koefficienti a) $0 < k < 1$; b) $k \geq 1$ bolg'an gomotetiyali'q figuralar su'wretlengen. Gomotetiya koefficientinin' shaması'na qarap gomotetiyali'q figuralardi'n "qi'si'li'wi" yaki "sozi'li'wi" haqqi'nda qanday juwmaq shi'g'ari'w mu'mkin?

Soraw, ma'sele ha'm tapsi'rmalar

1. Gomotetiya degen ne? Gomotetiya worayi', koefficienti she?
2. Gomotetiya uqsasli'q tu'rlendiriw yekenligin da'liyllen'.
3. U'shmu'yeshlik si'zi'n': a) U'shmu'yeshliktin' ishinde; b) U'shmu'yeshliktin' si'rti'nan O noqatin belgilen' ha'm koefficienti 2 ge ten' bolg'an O worayi'na iye gomotetiyanı' qarap shi'g'i'p, berilgen u'shmu'yeshlikke gomotetiyali'q u'shmu'yeshlik jasan'.
4. Perimetrleri 18 sm ha'm 27 sm bolg'an yeki romb wo'z ara gomotetiyali'q boladi'. Bul rombi'lardi'n' ta'replerinin' ha'm maydanlari'ni'n' qatnaslari'n tabi'n'.
5. Gomotetiyada X noqati' X^* noqati'na, Y noqati' Y^* noqati'na wo'tedi. Yeger X , X^* , Y , Y^* noqatlari' bir tuwri'da jatpasa, bul gomotetiyanı'n' worayin tabi'n'.
6. Koefficienti 2 ge ten' bolg'an gomotetiyada X noqati' X^* noqati'na wo'tetug'i'ni' belgili. Bul gomotetiyanı'n' worayi'n jasan'.
7. Shen'berge gomotetiyali'q figura shen'ber bolatug'i'ni'n da'liyllen'.
8. Shen'ber si'zi'n'. Worayi' shen'ber worayi'nda ha'm koefficienti a) $\frac{1}{2}$; b) 2; d) 3; e) $\frac{1}{3}$ ge ten' bolg'an gomotetiyada si'zi'lg'an shen'berge gomotetiyali'q bolg'an figuralardi' jasan'?
9. Mu'yesh ha'm woni'n' ishinde A noqati' berilgen. Mu'yesh ta'replerine uri'ni'wshi', A noqati'nan wo'tiwshi shen'ber jasan'.



Usi waqitqa shekem teoremlardi' da'liyllewde ha'm ma'selelerdi sheshiwde tu'rli uqsas u'shmu'yeshliklerdi jasad keldik. Uqsas ko'pmu'yeshlikler qanday jasaladi'? To'mende soni'n' menen tani'sami'z.

Ma'sele. Berilgen $ABCD$ to'rtmu'yeshlikke uqsas, uqsasli'q koefficienti 3 ke ten' bolg'an $A_1B_1C_1D_1$ to'rtmu'yeshligin jasan' (1-su'wret).

Jasaliwi. Tegislikte qa'legen O noqati'n alami'z. Wonnan ha'm to'rtmu'yeshliktin' to'belerinen wo'tiwshi OA , OB , OC ha'm OD nurlari'n ju'rgizemiz. Bul nurlarda O noqati'nan shi'g'atug'i'n $OA_1 = 3OA$, $OB_1 = 3OB$, $OC_1 = 3OC$ ha'm $OD_1 = 3OD$ kesindilerin qoyami'z. Payda bolg'an $A_1B_1C_1D_1$ to'rtmu'yeshligi izlenip ati'rg'an to'rtmu'yeshlik boladi'.

Tiykarlaw. $ABCD \sim A_1B_1C_1D_1$ yekenligin da'lilleymiz.

1. Sa'ykes ta'replerinin' proporcionalli'g'i'.

$$a) \triangle AOD \sim \triangle A_1OD_1 \Rightarrow \frac{A_1D_1}{AD} = \frac{O_1D_1}{OD} = \frac{OA_1}{OA} = 3; \quad (1)$$

$$b) \triangle DOC \sim \triangle D_1OC_1 \Rightarrow \frac{OD_1}{OD} = \frac{D_1C_1}{DC} = \frac{OC_1}{OC} = 3. \quad (2)$$

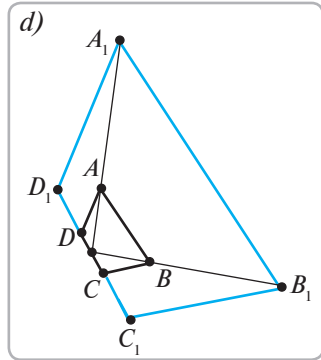
(1) ha'm (2) ten'likten $\frac{A_1D_1}{AD} = \frac{D_1C_1}{DC}$ yekenligi kelip shi'g'adi'. Da'l usi'g'an uqsas to'rtmu'yeshliklerdin' basqa sa'ykes ta'replerinin' proporcionalli'g'i'n da'liyllew mu'mkin.

2. Sa'ykes mu'yeshlerdin' ten'ligi.

Uqsas u'shmu'yeshliklerdin' sa'ykes mu'yeshleri ten' bolg'ani' ushi'n $\angle A_1D_1O = \angle ADO$, $\angle C_1D_1O = \angle CDO$. Wonda $\angle A_1D_1C_1 = \angle A_1D_1O + \angle C_1D_1O = \angle ADO + \angle CDO = \angle ADC$, yag'ni'y to'rtmu'yeshliklerdin' sa'ykes $A_1D_1C_1$ ha'm ADC mu'yeshleri wo'z ara ten'. Da'l usi'g'an uqsas to'rtmu'yeshliklerdin' basqa sa'ykes mu'yeshlerinin' ten' yekenligi da'liyllenedi. Demek, $ABCD$ ha'm $A_1B_1C_1D_1$ to'rtmu'yeshlikleri uqsas yeken.

Ta'replari qa'legen sanda bolg'an ko'pmu'yeshlikke uqsas ko'pmu'yeshlikte da'l usi' si'yaqli' jasaladi'.

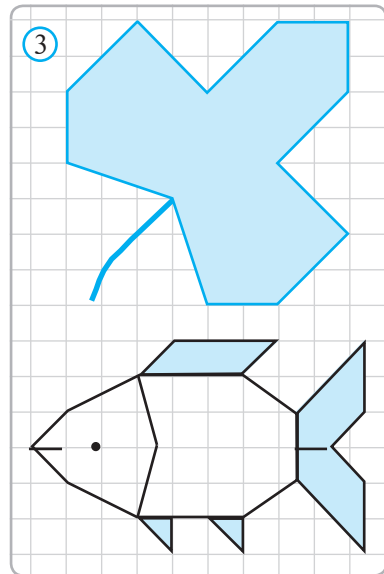
Gomotetiya worayi'n bul ma'selede to'rtmu'yeshlik-tin' si'rti'nan tan'lap aldi'q. Uli'wma alg'anda gomote-tiya worayi'n to'rtmu'yeshliktin' ishki oblasti'nda (2, *a-su'wret*), qaysi' bir to'besinde (2-*b su'wret*) yaki qaysi bir ta'repinde (2-*d su'wret*) jatatug'i'nday yetip tan'lap ali'wi'mi'zg'a da bolatug'i'n yedi. Gomote-tiya worayi'n qay jerden almayi'q, berilgen $ABCD$ to'rtmu'yeshlikke uqsas ha'm uqsasli'q koefficienti 3 ke ten' bolg'an to'rtmu'yeshlikler wo'z ara ten' boladi'.

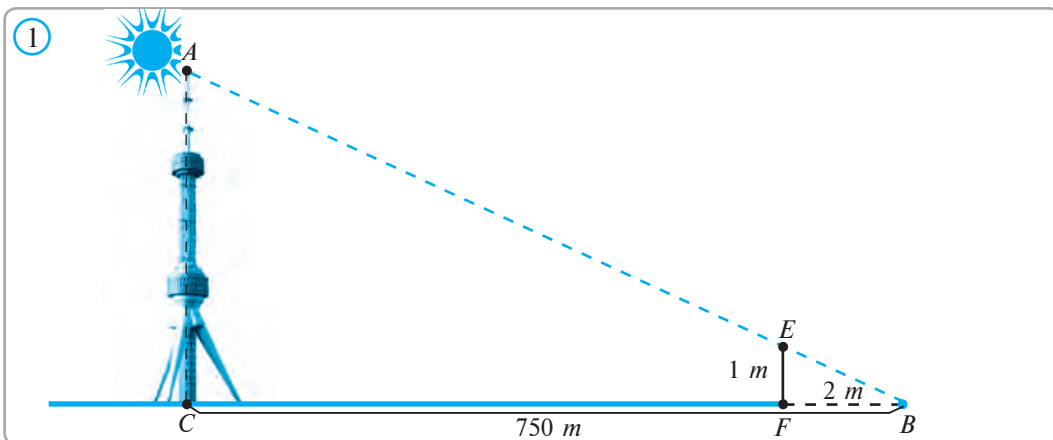


? Soraw, ma'sele ha'm tapsi'rmalar

1. Berilgen ko'pmu'yeshlikke uqsas ko'pmu'yeshlikti jasaw izbe-izligin ayti'p berin'.
2. Da'pterin'izge qanday da bir $ABCDE$ besmu'yeshligin si'zi'n'. Gomotetiya ja'rdeminde bul besmu'yeshlikke uqsas, uqsasli'q koefficienti 0,5 ke ten' bolg'an besmu'yeshlik jasan'. Gomotetiya worayi' a) C noqati'nda; b) besmu'yeshliktin' ishinde; c) AB ta'repinde bolg'an jag'daylardi' wo'z aldi'na ko'rip shi'g'i'n'.
3. Keteklerdi yesapqa alg'an halda 3-su'wrette berilgen figuralardi' da'pterin'izge si'zi'n': a) japi'raqqa uqsasli'q koefficienti 3 ke ten' bolg'an japi'raq; b) bali'qshag'a uqsasli'q koefficienti 0,8 ge ten' bolg'an baliqshani' si'zi'n'.
4. F_1 ko'pmu'yeshligi F_2 ko'pmu'yeshligine uqsas, k —uqsasli'q koefficienti. P_1, P_2, S_1, S_2 ha'ripleri menen sa'ykes tu'rde bul ko'pmu'yeshliklerdin' perimetrleri ha'm maydanlari' belgilengen. To'mendegi kesteni da'pterin'izge ko'shirin' ha'm woni' tolti'ri'n'.

	P_1	P_2	S_1	S_2	k
a)	84		100	25	
b)	14	28		48	
d)		150	200	100	
e)		30	24		3





1. Biyiklikni aniqlaw.

Jerde turi'p, Tashkent teleminarasi'ni'n' biyikligin tabayi'q. Minarani'n' ushi' — A noqati'ni'n' sayasi' B noqati'nda bolsi'n. EF tayag'i'n vertikal hali'nda sonday yetip jaylasti'rami'z (*1-su'wret*), bunda tayaqtin' E ushi'ni'n' sayasi' da B noqati'nda bolsi'n. A noqati'ni'n' jerdegi proekciyasi'n C menen belgileymiz. Wonda, tuwri' mu'yeshli ABC ha'm EBF u'shmu'yeshlikleri wo'z - ara uqsas boladi. Soni'n' ushi'n

$$\frac{AC}{EF} = \frac{BC}{BF} \quad \text{yaki} \quad AC = \frac{BC \cdot EF}{BF}.$$

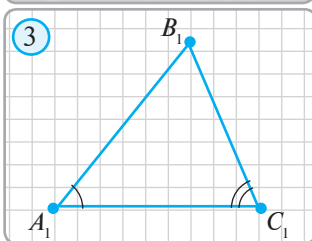
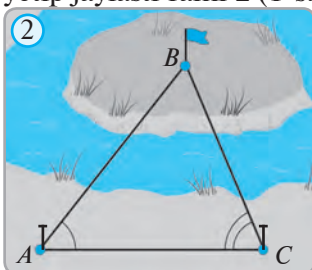
BC , BF arali'qlarin ha'm EF tayaqtin' uzi'nli'g'i'n wo'lshep, payda bolg'an formuladan teleminara biyikligi — AC kesindisinin' uzi'nli'g'i'n tabami'z. Mi'sali', yeger $EF = 1\text{ m}$, $BC = 750\text{ m}$, $FB = 2\text{ m}$ bolsa, wonda $AC = 375\text{ m}$ boladi'.

2. Bari'p bolmaytug'in jerge shekemgi bolg'an arali'qti' wo'lshew.

Aytayi'q, A noqati'nan bari'w mu'mkin bolmag'an B noqati'na shekemgi bolg'an arali'qti' aniqlaw kerek bolsi'n (*2-su'wret*). A noqati'nan bari'wg'a bolatug'i'n

gerge C noqati'n belgileymiz. Bunda C noqati'nan qarag'anda A ha'm B noqatlari' ko'rinip tursi'n ja'nede AC arali'qti' wo'lshep ali'w mu'mkin bolsi'n.

A'sbaplar ja'rdeminde BAC ha'm ACB mu'yeshlerin wo'lsheymiz. Aytayi'q, $\angle BAC = \alpha$ ha'm $\angle ACB = \beta$ bolsi'n. Qag'azg'a $\angle A_1 = \alpha$, $\angle C_1 = \beta$ bolg'an $A_1B_1C_1$



u'shmu'yeshligin jasaymi'z. Bunda ABC ha'm $A_1B_1C_1$ u'shmu'yeshliklarining' yeki mu'yeshi boyi'nsha uqsas boladi' (2-ha'm 3-su'wretler). Bunnan,

$$\frac{AB}{A_1B_1} = \frac{AC}{A_1C_1} \quad \text{yaki} \quad AB = \frac{AC \cdot A_1B_1}{A_1C_1}.$$

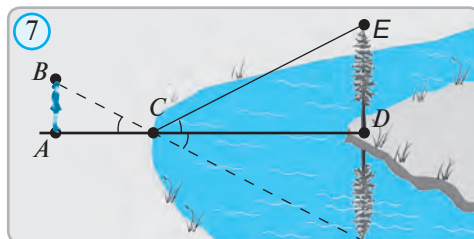
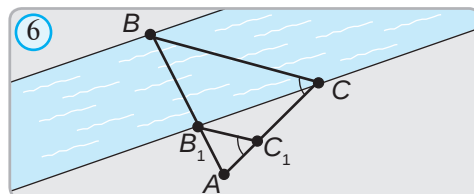
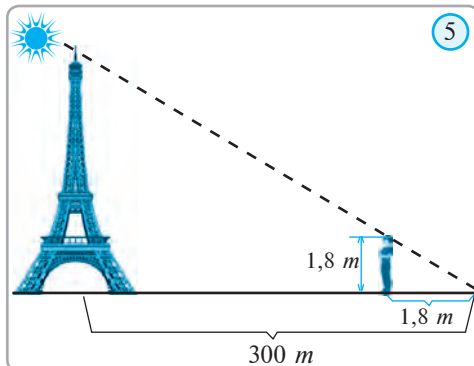
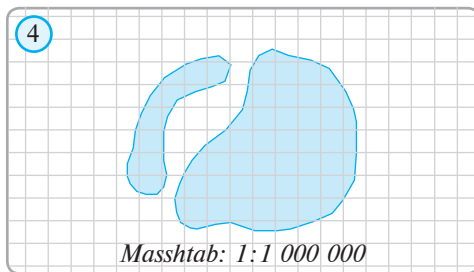
AC arali'g'i'n ha'm A_1B_1 , A_1C_1 kesindilerin wo'lshep, na'tiyjede payda bolg'an formula ja'rdeminde AB kesindisi yesaplanadi'. Yesaplaw jollari'n an'satlasti'ri'w ushi'n $AC:A_1C_1$ qatnasi'n 100:1, 1000:1 siyaqli' qatnasta ali'w mu'mkin. Mi'sali', $\angle A = 73^\circ$, $\angle C = 58^\circ$ bolsa, qag'azda $A_1B_1C_1$ u'shmu'yeshligin $\angle A_1 = 73^\circ$, $\angle C_1 = 58^\circ$, $A_1C_1 = 130 \text{ mm}$ yetip si'zami'z. A_1B_1 kesindisin wo'lshep, woni 153 mm yekenligin tabami'z. Demek, izlenip ati'rg'an arali'q 153 m boladi'.

3. Aral ten'izi haqqinda a'meliy jumis.

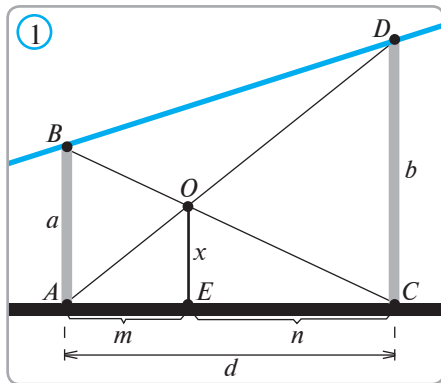
4-su'wrette Aral ten'izinin' kosmik kemesinen ali'ng'an su'wreti ko'rsetilgen. Wol tiykari'nda wo'lshep ha'm yesaplawlardi' wori'nlap, suw saqlanip qalg'an maydani'ni'n juwi'q ma'nisin tabi'n'.

2 Soraw, ma'sele ha'm tapsi'rmalar

1. Yeger boyi' 1,7 m bolg'an adam sayasi'ni'n' uzi'nli'g'i' 2,5 m bolsa, sayasinin' uzinlig'i 10,2 m bolg'an terektin' biyikligi qansha boladi'?
2. 5-su'wrette su'wretlengen minaralardin' biyikligin ani'qlan'.
3. 6-su'wrettegi yeki uqsas AB_1C_1 ha'm ABC u'shmu'yeshliklerdin' ja'rdeminde da'ryani'n' ken'ligin (yenin) ani'qlaw kerek. Yeger $AC = 100 \text{ m}$, $AC_1 = 32 \text{ m}$ ha'm $AB_1 = 34 \text{ m}$ bolsa, da'ryani'n' yenin (BB_1) tabi'n'.
4. Jap jag'asi'ndag'i' DE tereginin' suwdag'i' sa'wleleniwi A noqati'ndag'i' adamg'a ko'rinip tur. Yeger $AB = 165 \text{ m}$, $AC = 120 \text{ m}$, $CD = 4,8 \text{ m}$ bolsa, terektin' biyikligin tabi'n' (7-su'wret).
5. U'yin'izdin' jani'ndag'i' bir terekti tan'lan' ha'm woni'n' biyikligin ani'qlan'. Bul jumisti' qalay ori'nlag'ani'n'i'z haqqi'nda yesabat tayarlan'.



1-ma'sele. Uzi'nli'qlari' sa'ykes tu'rde a ha'm b bolg'an AB ha'm CD bo'reneler bir-birinen d arali'qta vertikal ta'rizde wornati'lg'an. Wolardin' bekkemligin asi'ri'w ushi'n A ha'm D , B ha'm C ushlari'n O noqati'nda kesilisiwshi polat si'mlar menen bekkemlengen (*1-su'wret*). Su'wrette berilgen mag'liwmatlar boyi'nsha a) $\frac{m}{m+n} = \frac{x}{b}$ ha'm $\frac{n}{m+n} = \frac{x}{a}$ ten'liklerin da'liyllen'; b) $\frac{x}{a} + \frac{x}{b} = 1$ ten'liktin' duri's yekenligin ko'rsetin' ha'm tu'sindirin'.



Sheshiliwi.

a) Ma'selenin' sha'rti boyi'nsha

1. $AOE \sim ADC$. Sonin' ushi'n,

$$\frac{AE}{AC} = \frac{OE}{DC}, \quad \text{yag'ni'y} \quad \frac{m}{m+n} = \frac{x}{b}. \quad (1)$$

2. $\triangle EOC \sim \triangle ABC$. Sonin' ushi'n,

$$\frac{EC}{AC} = \frac{OE}{AB}, \quad \text{yag'ni'y} \quad \frac{n}{m+n} = \frac{x}{a}. \quad (2)$$

b) (1) ha'm (2) ten'liklerdi ag'zama-ag'za qossaq, $\frac{m}{m+n} + \frac{n}{m+n} = \frac{x}{b} + \frac{x}{a}$ yaki $\frac{x}{a} + \frac{x}{b} = 1$

ten'ligin payda yetemiz. Demek, bag'analar qanday wornatilmasin, polat simlar kesiksen O noqat jerden birdey biyiklikte boladi yeken.

2-ma'sele. $ABCD$ trapeciyani'n' AB ha'm CD qaptal ta'replerinde M ha'm N noqatlari' belgilengen. Bunda MN kesindisi trapeciyanin' ultanlari'na parallel ha'm trapeciyanin' diagonallari' kesiksen O noqati' arqali' wo'tedi. Yeger $BC = a$, $AD = b$ bolsa, a) MO ; b) ON ; d) MN kesindilerin tabi'n (*2-su'wret*).

Sheshiliwi. 1) AOD ha'm BOC u'shmu'yeshliklerdin' BB belgisi boyi'nsha uqsas, sebebi $\angle BOC = \angle AOD$, $\angle OBC = \angle ADO$. Bunnan,

$$\frac{OC}{OA} = \frac{BC}{AD} \quad \text{yaki} \quad \frac{OC}{OA} = \frac{a}{b}. \quad (1)$$

2) ABC ha'm AOM u'shmu'yeshlikleri de BB belgisi boyi'nsha uqsas, sebebi $\angle AMO = \angle ABC$, $\angle ACB = \angle AOM$. Bunnan,

$$\frac{AC}{OA} = \frac{BC}{MO} \quad \text{yaki} \quad \frac{OA + OC}{OA} = \frac{a}{MO} \Rightarrow 1 + \frac{OC}{OA} = \frac{a}{MO}, \quad \frac{OC}{OA} = \frac{a}{MO} - 1. \quad (2)$$

3) (1) ha'm (2) ten'liklerinin' won' bo'limin ten'lestirip, $\frac{a}{MO} - 1 = \frac{a}{b}$

ten'ligin ha'm wonnan

$$MO = \frac{ab}{a+b}$$

yekenligin tabami'z.

4) Joqaridag'iday jol tutip

$$ON = \frac{ab}{a+b}$$

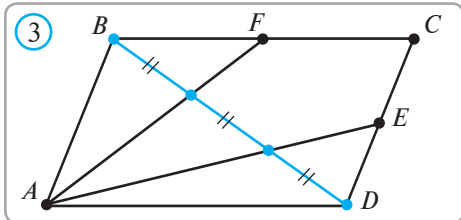
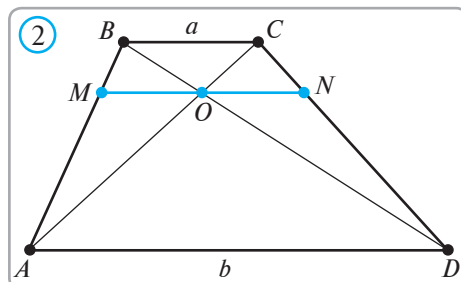
ten'ligin, keyin bolsa(3) ha'm (4) ten'likle-
rinin' sa'ykes bo'limlerin qosi'p

$$MN = \frac{2ab}{a+b}$$

ten'ligin payda yetemiz.

Juwabi: a) $\frac{ab}{a+b}$; b) $\frac{ab}{a+b}$; d) $\frac{2ab}{a+b}$.

(3)

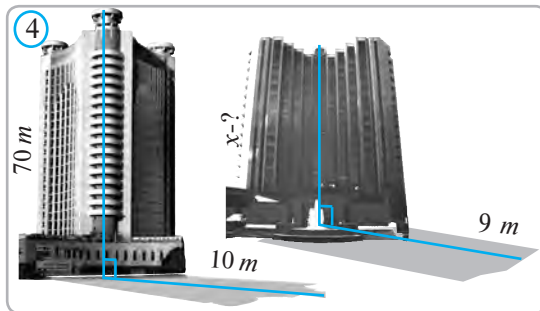
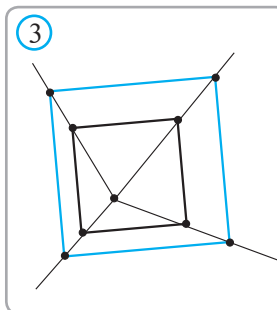
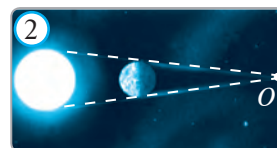
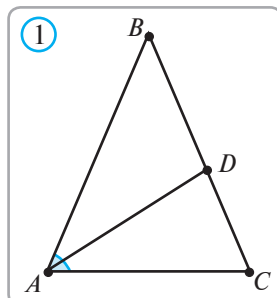


Yesletpe. Bul ma'selenin' sheshiminen $MO = ON$ yekenligi kelip shi'g'adi'.

? Soraw, ma'sele ha'm tapsi'rmalar

1. ABC u'shmu'yeshliginin' AB ha'm BC qaptal ta'replerinde D ha'm E noqatlari belgilengen. Yeger $AC \parallel DE$, $AC = 6$, $DB = 3$ ha'm $DE = 2$ bolsa, AB ta'repin tabi'n'.
2. Yeki uqsas u'shmu'yeshliktin maydanlari' 8 dm^2 ha'm 72 dm^2 qa ten', wolardan birinin' perimetri yekinishinen 26 dm ge kem. U'lken ko'pmu'yeshliginin' perimetrin tabi'n'.
3. Perimetri 1 m bolg'an $A_1B_1C_1$ u'shmu'yeshligi $A_2B_2C_2$ u'shmu'yeshliginin' ta'replerinin' wortalari'n, $A_2B_2C_2$ u'shmu'yeshlik $A_3B_3C_3$ u'shmu'yeshliginin' wortalari'n, $A_3B_3C_3$ u'shmu'yeshligi $A_4B_4C_4$ u'shmu'yeshliginin' ta'replerinin' wortalari'n tutasti'ri'wdan payda bolg'an bolsa, $A_4B_4C_4$ u'shmu'yeshliginin' perimetri qansha boladi'?
4. Yeki uqsas u'shmu'yeshliktin' perimetrleri 18 dm ha'm 36 dm ge, maydanlari'ni'n' qosi'ndi'si' 30 dm^2 qa ten'. U'lken u'shmu'yeshliktin' maydani'n tabi'n'.
5. Rombi'ni'n' ta'replerinin' wortalari' tuwri'mu'yeshliktin' to'beli bolatug'i'ni'n da'liyllen'.
6. ABC u'shmu'yeshligin jasan'. Bul u'shmu'yeshlikke uqsas ha'm maydani' ABC u'shmu'yeshliginin' maydani'nan 9 ma'rte kishi bolg'an $A_1B_1C_1$ u'shmu'yeshlikti jasan'.
- 7*. E ha'm F noqatlari sa'ykes tu'rde $ABCD$ parallelogrammni'n' CD ha'm BC ta'replerinin' wortalari', AF ha'm AE tuwri'lari' BD diagonali'n ten'dey u'sh bo'lekke bo'letug'i'ni'n da'liyllen' (3-su'wret).

1. Ten' qaptalli u'shmu'yeshliktin' ultani'ndag'i' mu'yeshstin' bissektrisasi' bul u'shmu'yeshlikten wo'zine uqsas u'shmu'yeshlik ajiratadi'. U'shmu'yeshliktin' mu'yeshlerin ani'qlan' (1-su'wret, $AB = BC$, $\triangle ABC \sim \triangle CAD$).
2. Shen'ber jasan' ha'm wonnan O noqati'n belgilen'. Orayi' O noqatinda ha'm koeffitcianti 2 ge ten' bolg'an gomotetiyada berilgen shen'berge gomotetiyaliq bolg'an shen'ber jasan'.
3. Yeki uqsas ko'pmu'yeshliktin' perimetrlerinin' qatnasi' 2:3 siyaqli' bolsi'n. U'lken ko'pmu'yeshliktin' maydani' 27, kishi ko'pmu'yeshliktin' maydani'n tabi'n'.
4. 2-su'wrette Quyashti'n' toli'q tuti'lg'an jag'dayi' su'wretlengen. Yeger Quyash radiusi' 686 784 km, Ay radiusi' 1760 km ha'm Jerden Ayg'a shekem bolg'an arali'q 384 400 km bolsa, Jerden Quyashqa shekem bolg'an arali'qti' tabi'n'.
5. a) Bir shen'berge yeki uqsas ko'pmu'yeshlik ishley si'zi'lg'an. Bul ko'pmu'yeshlikler wo'z ara ten' bolama?
b) Bir shen'berge yeki uqsas ko'pmu'yeshlik si'rtlay si'zi'lg'an. Bul ko'pmu'yeshlikler o'z ara ten' bolama?
- 6*. Bir kvadratti'n' ta'replari ekinshi kvadrattin' ta'replerine parallel. Yeger kvadratlar bir-birine ten' bolmasa, wonda wolar gomotetiyaliq bolatug'inin da'liyllen' (3-su'wret).
7. ABC u'shmu'yeshliginin' AB ha'm BC ta'replari to'rt ten'dey kesindilerga bo'lindi ha'm bo'liniw noqatlari' AC ta'repine parallel bolg'an kesindiler menen tutasti'ri'ldi' (4-su'wret). Yeger $AC=24$ sm bolsa, payda bolg'an kesindilerdin' uzi'nli'qlari'n tabi'n'.
8. Yeger su'wretler da'l bir waqi'ti'n' wo'zinde su'wretke ali'ng'an bolsa, berilgen mag'li'wmatlar boyi'nsha yekinshi imaratti'n' biyikligin tabi'n' (5-su'wret).



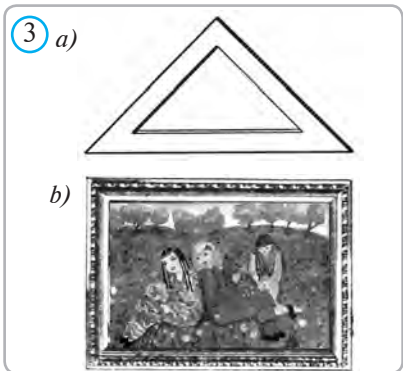
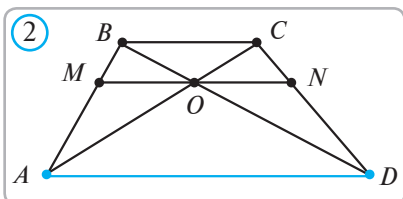
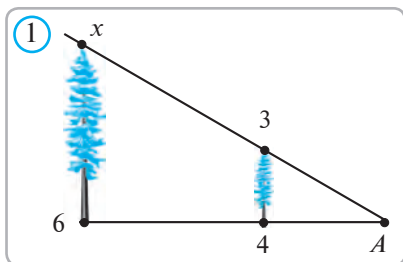
I. Testler

- Yeki uqsas u'shmu'yeshlikler ushi'n naduri's tasti'yi'qlawdi' tabi'n':**
 A. Maydanlari'ni'n' qatnasi'ni'n' uqsasli'q coefficientine ten';
 B. Sa'ykes medianalari'ni'n' qatnasi' uqsasli'q coefficientine ten';
 D. Sa'ykes bissektisalarini'n' qatnasi' uqsasli'q coefficientine ten';
 E. Sa'ykes biyiklikleri'ni'n' qatnasi' uqsasli'q coefficientine ten'.
- Yeki gomotetiyaq ko'pmu'yeshlik ushi'n duri's tasti'yi'qlawdi' tabi'n':**
 A. Wolar ten'; B. Wolar uqsas;
 D. Wolar ten' o'lsheqli; E. Duri's juwap joq.
- U'shmu'yeshliktin' medianalari' ushi'n naduri's tasti'yi'qlawdi' ko'rsetin':**
 A. Bir noqatta kesilisedi; B. Kesilisiw noqati'nda 2:1 qatnasta bo'linedi;
 D. Bir-birine ten'; E. Ha'r biri u'shmu'yeshlikti yeki ten'dey bo'lekke bo'ledi.
- U'shmu'yeshliktin' bissektisalarini' ushi'n naduri's tasti'yi'qlawdi' ko'rsetin':**
 A. Bir noqatta kesilisedi; B. Kesilisiw noqati'nda 2:1 qatnasta bo'linedi;
 D. Wo'zi tu'sken ta'repti qalg'an yeki ta'repke proporcional kesindilerga aji'ratadi';
 E. Wo'zi shi'qqan to'bedegi mu'yeshti ten' yekige bo'ledi.
- Yeki uqsas ko'pmu'yeshlik ushi'n naduri's tasti'yi'qlawdi' tabi'n':**
 A. Wolardi'n' ta'replerinin' sani' ten'; B. Wolardi'n' mu'yeshlerinin' sani' ten';
 D. Sa'ykes ta'repleri proporcional
 E. Maydanlari'ni'n' qatnasi' uqsasli'q coefficientine ten'.

II. Ma'seleler

- Ultanlari' 6 m ha'm 12 m bolg'an trapeciyani'n' diagonallari' kesilisken noqattan ultanlarga parallel tuwri' ju'rgizilgen. Tuwri'ni'n' trapeciya ishindeg'i bo'leginin' uzi'nli'g'i'n' tabi'n'.
- ABC u'shmu'yeshliginde $BC = BA = 10$, $AC = 8$. Yeger AA_1 ha'm CC_1 u'shmu'yeshliktin' bissektisalarini' bolsa, A_1C_1 kesindisin tabi'n'.
- A noqati'nan bari'p bolmaytug'i'n' B noqati'na shekemgi bolg'an arali'qti' ani'qlaw ushi'n tegis jerde C noqati' tan'lap ali'ndi'. Keyin AC arali'q, BAC ha'm ACB mu'yeshler wo'lsheendi ha'm ABC u'shmu'yeshlikke uqsas $A_1B_1C_1$ u'shmu'yeshlik jasaldi. Yeger $AC = 42$ m, $A_1C_1 = 6,3$ sm, $A_1B_1 = 7,2$ sm bolsa, AB arali'g'i'n' tabi'n'.
- Koefficienti $k=3$ bolg'an gomotetiyada F ko'pmu'yeshligi F_1 ko'pmu'yeshligine tu'rlendiriledi. Yeger F_1 ko'pmu'yeshliginin' perimetri 12 sm ha'm maydani' 4,5 sm² bolsa, F ko'pmu'yeshliginin' perimetrin ha'm maydani'n' tabi'n'.

5. Boyi' 180 sm bolg'an adam sayasi'ni'n' uzi'nli'g'i' 2,4 m bolg'an waqitta uzi'nli'g'i' 4 m bolg'an si'm ag'ashti'n' uzi'nli'g'i' neshe metr boladi'?
6. Kartada Tashkent ha'm U'rgenish qalalari'ni'n' ko'rinisleri arasi'ndag'i' arali'q 8,67 sm. Yeger karta 1:100 000 bolsa, Tashkent ha'm U'rgenish qalalari' arasi'ndag'i' arali'qti' tabi'n'.



III. Wo'zin'izdi si'nap ko'rin' (u'lg'i ushi'n baqlaw jumi'si')

1. 1-su'wrette berilgen mag'li'wmatlar tiy-kari'nda terektin' biyikligin tabi'n'.
2. ABC u'shmu'yeshliginin' ta'replari $AB=5$ sm, $AC=6$ sm, $BC=7$ sm. Bul u'shmu'yeshliktin' AC ta'repine parallel tuwri' AB ta'repin P noqati'nda, BC ta'repin bolsa K noqatinda kesip wo'tedi. Yeger $PK=2$ sm bolsa, PBK u'shmu'yeshliginin' perimetrin tabi'n'.
3. 2-su'wrette $AD \parallel BC \parallel MN$. Yeger $BC = 6$ sm, $AD = 10$ sm bolsa, MN kesindisin tabi'n'.
4. (*Qosi'msha*). Romb ta'replerinin' wortalari' tuwri' to'rtmuyeshliktin' to'leleri yekenligin daliyllen'.

⌚ Qi'ziqli' ma'seleler

1. 4 ma'rte u'lkeytirilip ko'rsetilgen lupa ayna menen qaralg'anda 2^0 li mu'yesh shamasi' qanshag'a wo'zgeredi (*3-su'wret*)?
2. a) u'shmu'yeshli sizg'ishtin' su'wretinde ko'rsetilgen ishki ha'm si'rtqi' u'shmu'yeshlikleri uqsaspa (*3-a-su'wret*)?
b) 3 b-su'wrettegi ramani'n' ishki ha'm si'rtqi' qirlarin ko'rsetiwshi to'rtmuyeshlikler uqsaspa?
3. To'mendegi rus tilinde berilgen ma'seleni sheship ko'rin'. Buni'n' menen ha'm rus tilinen, ha'm geometriyadan nege tayar yekenligin'izdi bilip alasi'z.

На 4-рисунке изображена русская игрушка “матрёшка”. Выполняющие соответствующие измерения, найти коэффициент подобия игрушек:

- a) A и B ; b) A и D ; d) C и F ; e) B и E .

II BAP



U'SHMU'YESHЛИKTIN' TA'REPLERI HA'M MU'YESHLERI ARASI'NDAG'I QATNASLAR

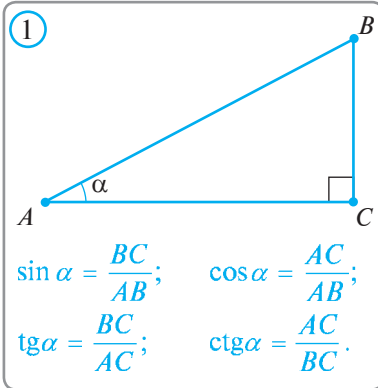
Bul bapti' u'yreniw na'tiyjesinde Siz to'mendegi bilim ha'm a'meliy ko'nlikpelerge iye bolasi'z:

Bilimler:

- √ *qa'legen mu'yeshlin' sinusi', kosinusi', tangensi ha'm kotangensinin' ani'qlamalari'n biliw;*
- √ *mu'yeshlin' radian wo'ishemin biliw;*
- √ *tiykarg'i' trigonometriyali'q birdeyliklerdi biliw;*
- √ *u'shmu'yeshliktin' maydani'n mu'yeshlin' sinusi' ja'rdeminde yesaplaw formulasi'n biliw;*
- √ *sinuslar ha'm kosinuslardi'n' teoremasi'n biliw.*

A'meliy ko'nlikpeler:

- √ *bazi bir mu'yeshlerdin' sinusi', kosinusi', tangensi ha'm kotangensin yesaplay ali'w;*
- √ *tiykarg'i' trigonometriyali'q birdeyliklerdi mi'sallar sheshiwde qollana ali'w;*
- √ *u'shmu'yeshliktin' maydani'ni'n' yeki ta'repi ha'm wolar arasi'ndag'i' mu'yeshi boyi'nsha yesaplay ali'w;*
- √ *sinuslar, kosinuslar teoremasi'nan paydalani'p yesaplawg'a ha'm da'liyllewge tiyisli ma'selelerdi sheshiw.*



Tuwri' mu'yeshli ABC u'shmu'yeshliginde $\angle C = 90^\circ$ bolsa AB ta'repi gipotenuza, BC ta'repi — A mu'yeshinin' qarsi'si'ndag'i' katet, AC ta'repi bolsa A mu'yeshine irgeles jatqan katet delinedi (1-su'wret).

Tuwri' mu'yeshli u'shmu'yeshliktin' su'yir mu'ye-shinin' **sinusi** dep usi' mu'yeshstin' qarsi'si'ndag'i' katettin' gipotenuzasina qatnasi'na, **kosinusi** dep, usi mu'yeshke irgeles jatqan katettin' gipotenuzag'a qatnasi'na ayti'ladi'.

Tuwri' mu'yeshli u'shmu'yeshliktin' su'yir mu'yeshinin' **tangensi** dep, usi mu'yeshstin' qarsi'si'ndag'i' katettin' irgeles jatqan katetke qatnasi'na, **kotangensi** dep, usi mu'yeshke irgeles jatqan katettin' qarsi'si'ndag'i' katetke qatnasi'na ayti'ladi'.

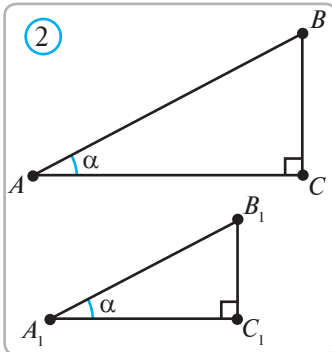
α mu'yeshinin' sinusi', kosinusi', tangensi ha'm kotangensi sa'ykes tu'rde **sin α** , **cos α** , **tg α** ha'm **ctg α** tu'rinde belgilenedi (woqi'li'wi': «**sinus alfa**», «**kosinus alfa**», «**tangens alfa**», «**kotangens alfa**»).

Joqari'dag'i' ani'qlamalardan to'mendegi formulalar kelip shig'adi':

$$1. \left[\frac{\sin A}{\cos A} = \frac{BC}{AB} \cdot \frac{AB}{AC} = \frac{BC}{AC}; \quad \operatorname{tg} A = \frac{BC}{AC} \right] \Rightarrow \boxed{\operatorname{tg} A = \frac{\sin A}{\cos A}} \quad 2. \left[\frac{\cos A}{\sin A} = \frac{AC}{AB} \cdot \frac{AB}{BC} = \frac{AC}{BC}; \quad \operatorname{ctg} A = \frac{AC}{BC} \right] \Rightarrow \boxed{\operatorname{ctg} A = \frac{\cos A}{\sin A}}$$

$$3. \operatorname{tg} A \cdot \operatorname{ctg} A = \frac{BC}{AC} \cdot \frac{AC}{BC} = 1 \Rightarrow \boxed{\operatorname{tg} A \cdot \operatorname{ctg} A = 1}.$$

Teorema. Bir tuwri' mu'yeshli u'shmu'yeshliktin' su'yir mu'yeshi yekinshi tuwri' mu'yeshli u'shmu'yeshliktin' su'yir mu'yeshine ten' bolsa, wonda bul su'yir mu'yeshlerdin' sinuslari' (kosinusi', tangensi ha'm kotangensi) da ten' boladi.



Da'liyllew. Tuwri' mu'yeshli ABC ha'm $A_1B_1C_1$ u'shmu'yeshliklerinde ($\angle C = \angle C_1 = 90^\circ$) $\angle A = \angle A_1$ bolsi'n (2-su'wret). Wonda, ABC ha'm $A_1B_1C_1$ u'shmu'yeshliktin' MM belgisi boyi'nshauqsas boladi'.

Soni'n' ushi'n, $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{AC}{A_1C_1}$. Bul ten'liklerden $\frac{BC}{AB} = \frac{B_1C_1}{A_1C_1}$ yaki $\sin A = \sin A_1$ yekenligin tabami'z.

Bul su'yir mu'yeshlerdin' kosinusi', tangensi ha'm kotangensleri de ten' boli'wi' joqari'dag'ig'a uqsas da'liyllenedi. **Teorema da'liyllendi.**

Ma'sele. ABC u'shmu'yeshlikte $\angle C = 90^\circ$, $AC = 8$ sm, $BC = 15$ sm bolsa, woni'n' B mu'yeshinin' sinusi', kosinusi', tangensi ha'm kotangensin tabi'n'.

Sheshiliwi. Pifagor teoremasi'nan paydalani'p, u'shmu'yeshliktin' gipotenuzasin tabami'z:

$$AB^2 = AC^2 + BC^2 = 8^2 + 15^2 = 289, AB = 17 \text{ (sm)}.$$

U'shmu'yeshliktin' B mu'yeshi qarsi'si'ndag'i' katet AC , B mu'yeshi qarsi'si'ndag'i' katet BC (1-su'wret). Wonda, ani'qlamalar boyi'nsha

$$\sin B = \frac{AC}{AB} = \frac{8}{17}; \quad \cos B = \frac{BC}{AB} = \frac{15}{17};$$

$$\operatorname{tg} B = \frac{AC}{BC} = \frac{8}{15}; \quad \operatorname{ctg} B = \frac{BC}{AC} = \frac{15}{8}.$$

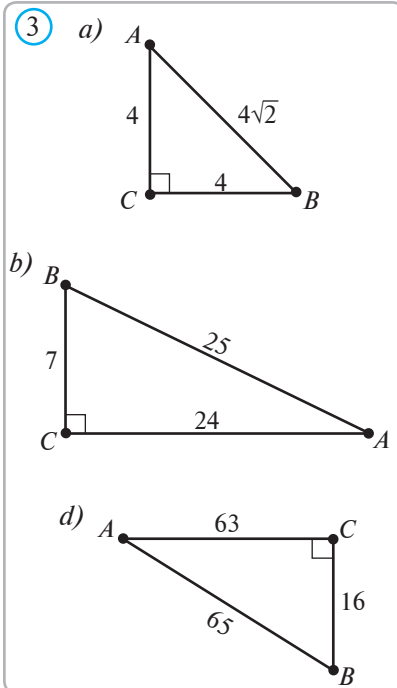
Yaki
$$\operatorname{tg} B = \frac{\sin B}{\cos B} = \frac{8}{17} \cdot \frac{17}{15} = \frac{8}{15};$$

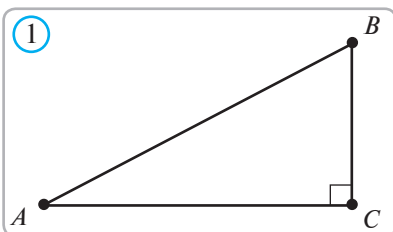
$$\operatorname{ctg} B = \frac{\cos B}{\sin B} = \frac{15}{17} \cdot \frac{17}{8} = \frac{15}{8}.$$

Juwabi': $\frac{8}{17}, \frac{15}{17}, \frac{8}{15}, \frac{15}{8}.$

? Soraw, ma'sele ha'm tapsi'rmalar

1. Tuwri' mu'yeshli u'shmu'yeshliktin' su'yir mu'yeshinin' sinusi', kosinusi', tangensi ha'm kotangensi dep nege ayti'ladi'?
2. Su'yir mu'yeshlin' sinusi', kosinusi', tangensi ha'm kotangensi nege baylani'sli', nege baylani'sli' yemes?
3. 4-su'wrettegi mag'li'wmatlar boyi'nsha $\sin A$, $\cos A$, $\sin B$, $\cos B$ ni tabi'n'.
4. Tuwri' mu'yeshli ABC u'shmu'yeshliginin' AB gipotenuzasi 13 sm ge, AC kateti bolsa 12 sm ge ten'. U'shmu'yeshliktin' A mu'yeshinin' sinusi', kosinusi', tangensi ha'm kotangensin tabi'n'.
5. Yeger tuwri' mu'yeshli ABC ($\angle C = 90^\circ$) u'shmu'yeshliginde a) $AB = 25$, $BC = 7$; b) $AC = 5$, $BC = 12$; d) $AB = 41$, $AC = 40$; e) $AC = 24$, $AB = 25$ bolsa, A ha'm B mu'yeshlerinin' sinusi', kosinusi', tangensi ha'm kotangenslerin tabi'n'.
6. Yeger ABC u'shmu'yeshliginde $\angle C = 90^\circ$, $\cos A = \frac{60}{61}$ ha'm $AC = 3$ sm bolsa, u'shmu'yeshliktin' qalg'an ta'replerin tabi'n'.
7. Yeger ABC u'shmu'yeshliginde $\angle C = 90^\circ$, $\sin A = \frac{8}{17}$ va $BC = 16$ sm bolsa, u'shmu'yeshliktin' qalg'an ta'replerin tabi'n'.





Ma'seleler sheshiwde ju'da' kerekli bolg'an ja'ne bir a'hmiyetli ten'liktin' duri'sli'g'i'n ko'rseteyik: tuwri' mu'yeshli ABC u'shmu'yeshlikte (1-su'wret) Pifagor teoremasi boyi'nsha $AB^2 = BC^2 + AC^2$. Wonda

$$\sin^2 A + \cos^2 A = \frac{BC^2}{AB^2} + \frac{AC^2}{AB^2} = \frac{BC^2 + AC^2}{AB^2} = \frac{AB^2}{AB^2} = 1.$$

$$\sin^2 A + \cos^2 A = 1$$

ten'ligi trigonometriyani'n tiyarg'i' birdeyligi dep ataladi' ("trigonometriya" so'zi grekshe "u'shmu'yeshliklerdi wo'lsheyman" degen ma'nisti an'latadi').

1-ma'sele. Yeger $\cos \alpha = \frac{1}{2}$ bo'lsa, $\sin \alpha$, $\operatorname{tg} \alpha$ ha'm $\operatorname{ctg} \alpha$ ni tabi'n'.

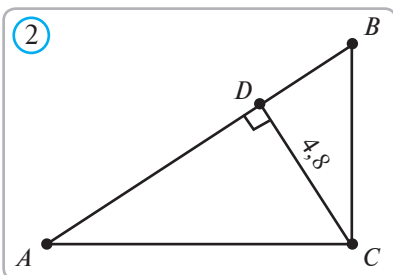
Sheshiliwi. Tiyarg'i' trigonometriyali'q birdeylik boyi'nsha:

$$\sin^2 \alpha = 1 - \cos^2 \alpha \Rightarrow \sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}.$$

$$\text{Wonda, } \operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}, \quad \operatorname{ctg} \alpha = \frac{1}{\operatorname{tg} \alpha} = \frac{1}{\sqrt{3}}.$$

2-ma'sele. ABC u'shmu'yeshlikte $\angle C = 90^\circ$ ha'm $\sin A = 0,6$. Yeger u'shmu'yeshliktin' CD biyikligi $4,8 \text{ sm}$ bolsa, woni'n' AC katetin ha'm usi' katettin' gipotenuzadag'i' proekciyasini tabi'n'.

Sheshiliwi. Tuwri' mu'yeshli ADC u'shmu'yeshligin qaraymi'z (2-su'wret).



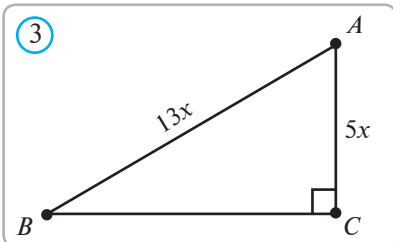
Bunda, $\sin$ usti'n' ani'qlamasini boyi'nsha

$$\sin A = \frac{DC}{AC}. \quad \text{Bunnan, } AC = \frac{DC}{\sin A} = \frac{4,8}{0,6} = 8 \text{ (sm)}.$$

Pifagor teoremasi'nan paydalani'p, AC katetinin' gipotenuzadag'i' proektsiyasi' AD ni tabami'z:

$$AD = \sqrt{AC^2 - CD^2} = \sqrt{8^2 - 4,8^2} = 6,4 \text{ (sm)}.$$

Juwabi': 8 sm ; $6,4 \text{ sm}$.



3-ma'sele. Yeger ABC u'shmu'yeshliginde $\angle C = 90^\circ$ ha'm $\cos A = \frac{5}{13}$ bolsa, u'shmu'yeshliktin' ta'replari qanday qatnasta boladi' (3-su'wret).

Sheshiliwi. Mu'yeshtin' kosinusi'ni'n' ani'qlamasini boyi'nsha

$$\cos A = \frac{AC}{AB}. \quad \text{Demek, } \frac{AC}{AB} = \frac{5}{13}.$$

Yeger $AC = 5x$ desek, wonda

$$AB = \frac{13 \cdot AC}{5} = 13x.$$

Pifagor teoremasi bo'yi'nsha,

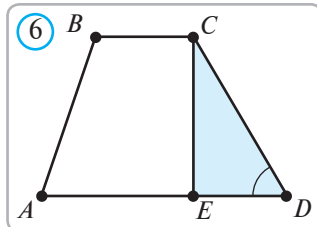
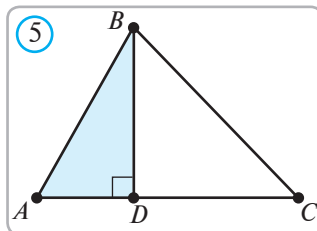
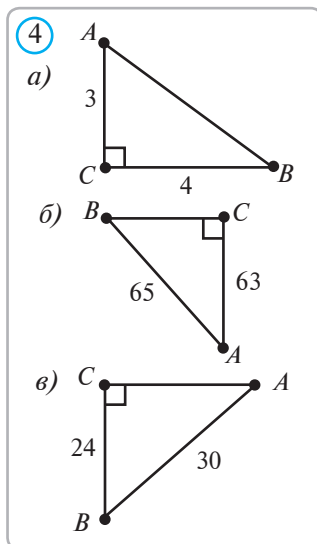
$$BC = \sqrt{AC^2 - BC^2} = \sqrt{169x^2 - 25x^2} = 12x.$$

Solay yetip, $AC:BC:AB=5:12:13$.

Juwabi': 5:12:13 siyaqli.

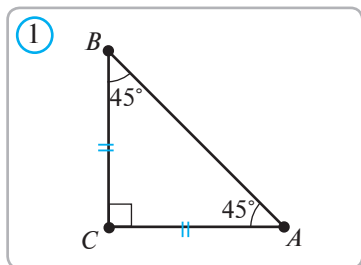
? Soraw, ma'sele ha'm tapsi'r malar

- 4-su'wrettegi mag'li'wmatlar bo'yi'nsha to'mendegilerdi a) $\sin A$, $\cos A$, $\operatorname{tg} A$, $\operatorname{ctg} A$; b) $\sin B$, $\cos B$, $\operatorname{tg} B$, $\operatorname{ctg} B$ ni tabi'n'.
- Yeger $\sin \alpha = 0,5$ bolsa, $\cos \alpha$, $\operatorname{tg} \alpha$ ha'm $\operatorname{ctg} \alpha$ ni' tabi'n'.
- Yeger $\cos \alpha = 0,6$ bolsa, $\sin \alpha$, $\operatorname{tg} \alpha$ ha'm $\operatorname{ctg} \alpha$ ni' tabi'n'.
- Tuwri' mu'yeshli ABC ($\angle C = 90^\circ$) u'shmu'yeshliginde $BC = 17 \text{ sm}$ ha'm $\sin B = \frac{15}{17}$ bolsa: a) u'shmu'yeshliktin' CD biyikligi; b) BC katetinin' gipotenuzadag'i' proekciyasi'n; c) gipotenuzasi'n; d) yekinshi katetin tabi'n'.
- Yeger ABC u'shmu'yeshliginde $\angle C = 90^\circ$, $\sin A = \frac{3}{8}$ ha'm $BC = 15 \text{ sm}$ bolsa, u'shmu'yeshliktin' gipotenuzasina tu'sirilgen biyiklikni tabi'n'.
- Yeger a) $\sin \alpha = \frac{2}{3}$; b) $\cos \alpha = \alpha$; d) $\operatorname{tg} \alpha = \frac{3}{4}$; e) $\operatorname{ctg} \alpha = \frac{4}{5}$ bolsa, α mu'yeshin jasan'.
- ABC u'shmu'yeshliginde $AC = 12 \text{ sm}$, $AB = 10 \text{ sm}$, $\sin A = 0,7$ bolsa, u'shmu'yeshliktin' maydani'n tabi'n' (5-su'wret).
- ABC u'shmu'yeshliginde BD — biyiklik, $AC = 7 \text{ sm}$, $AD = 2 \text{ sm}$ ha'm $\operatorname{tg} A = 3$ bolsa, u'shmu'yeshliktin' maydani'n tabi'n' (5-su'wret).
- $ABCD$ ($BC \parallel AD$) trapeciyada $\sin D = 0,5$; $CD = 8$, $BC = 6$, $AD = 10$ bolsa, trapeciya maydani'n tabi'n' (6-su'wret).
- $ABCD$ rombi'da $\sin A = 0,8$ ha'm $AB = 15 \text{ sm}$ bolsa, rombi'ni'n' maydani'n tabi'n'.
- Ten' qaptalli u'shmu'yeshliktin' ultanina tu'sirilgen biyikligi 5 sm , ultani' $10\sqrt{3} \text{ sm}$ bolsa, u'shmu'yeshliktin' a) mu'yeshlerin; b) qaptal ta'repin; d) maydani'n tabi'n'.
- Tuwri' mu'yeshli ABC u'shmu'yeshlikte $\sin A = \frac{3}{7}$ ha'm $\sin B = \frac{4}{7}$ boliwi mu'mkin be?



1. 45 gradusli' mu'yeshlin' sinusi', kosinusi', tangensi ha'm kotangensin yesaplan'.

Ten' qaptalli' tuwri' mu'yeshli ABC u'shmu'yeshligin qaraymi'z (1-su'wret).



$AC=BC$, $\angle A=\angle B=45^\circ$ bolsi'n. Pifagor teoremasi' boyi'nsha $AB^2=AC^2+BC^2=2AC^2$ yaki $AB=AC\sqrt{2}$.

Bunnan $AC=BC=\frac{AB}{\sqrt{2}}=\frac{AB\sqrt{2}}{2}$ ni payda yetemiz.

Solay yetip,

$$\sin 45^\circ = \sin A = \frac{BC}{AB} = \frac{\sqrt{2}}{2}; \quad \cos 45^\circ = \cos A = \frac{AC}{AB} = \frac{\sqrt{2}}{2};$$

$$\operatorname{tg} 45^\circ = \operatorname{tg} A = \frac{BC}{AC} = 1; \quad \operatorname{ctg} 45^\circ = \operatorname{ctg} A = \frac{AC}{BC} = 1.$$

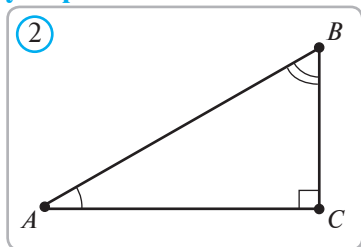
1-ma'sele. Tuwri' mu'yeshli ABC ($\angle C=90^\circ$) u'shmu'yeshliginde $\angle A=45^\circ$ ha'm $BC=6$ sm. U'shmu'yeshliktin' qalg'an ta'replerin tabi'n' (1-su'wret).

Sheshiliwi. $\frac{AC}{BC} = \operatorname{ctg} 45^\circ$ yaki $\frac{AC}{BC} = 1$, $AC=BC=6$ (sm);

$$\frac{BC}{AB} = \sin 45^\circ \text{ yaki } \frac{BC}{AB} = \frac{\sqrt{2}}{2}, \quad AB=BC\sqrt{2}=6\sqrt{2} \text{ (sm)}.$$

Juwabi': 6 sm; $6\sqrt{2}$ sm.

2. 30° ha'm 60° mu'yeshlerdin' sinusi', kosinusi', tangensi ha'm kotangensin yesaplan'.



Mu'yeshleri $\angle A=30^\circ$, $\angle B=60^\circ$ ha'm $\angle C=90^\circ$ bolg'an ABC u'shmu'yeshligin qaraymi'z (2-su'wret). 30 gradusli' mu'yeshlin' qarsi'si'nda jatqan katet gipotenuzani'n' yari'mi'na ten' bolg'ani' ushi'n AB yaki $BC = \frac{1}{2}$. Bunnan $\frac{BC}{AB} = \frac{1}{2}$

$$\sin 30^\circ = \sin A = \frac{BC}{AB} = \frac{1}{2}; \quad \cos 60^\circ = \cos B = \frac{BC}{AB} = \frac{1}{2}$$

ten'liklerin tabamiz. Tiykarg'i trigonometriyalik birdeylikten

$$\cos 30^\circ = \sqrt{1 - \sin^2 30^\circ} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}, \quad \sin 60^\circ = \sqrt{1 - \cos^2 60^\circ} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}.$$

$$\text{Tabilg'anlar boyi'nsha } \operatorname{tg} 30^\circ = \frac{\sin 30^\circ}{\cos 30^\circ} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}; \quad \operatorname{tg} 60^\circ = \frac{\sin 60^\circ}{\cos 60^\circ} = \sqrt{3};$$

$$\operatorname{ctg} 30^\circ = \frac{\cos 30^\circ}{\sin 30^\circ} = \sqrt{3}; \quad \operatorname{ctg} 60^\circ = \frac{\cos 60^\circ}{\sin 60^\circ} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}.$$

3. α nin' 30° , 45° , 60° qaten' ma'nislerde tabilg'an $\sin \alpha$, $\cos \alpha$, $\operatorname{tg} \alpha$ ha'm $\operatorname{ctg} \alpha$ ushi'n ma'nislerdi keste ko'riniside ji'ynaymi'z.

 **2-ma'sele.** Tuvri' mu'yeshli u'shmu'yeshliktin' gipotenuzasi' 10 sm ha'm mu'yeshlerinen biri 60° . Woni'n' qalg'an ta'replerin tabi'n'.

Sheshiliwi. 2-su'wretten paydalanami'z. Bunda

$$BC = AB \sin A = 10 \cdot \sin 30^\circ = 10 \cdot \frac{1}{2} = 5 \text{ (sm)},$$

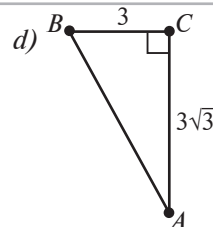
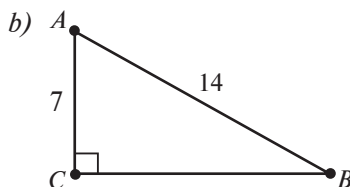
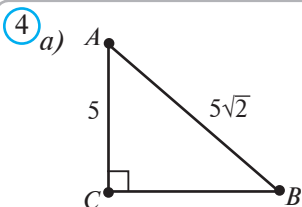
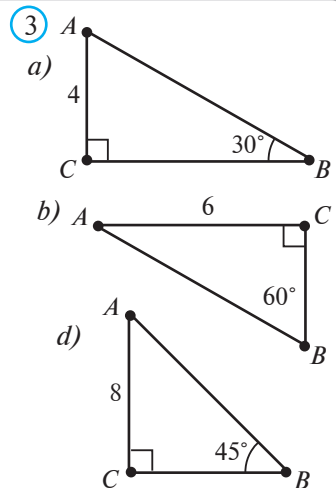
$$AC = AB \cos A = 10 \cdot \cos 30^\circ = 10 \cdot \frac{\sqrt{3}}{2} = 5\sqrt{3} \text{ (sm)}.$$

Juwabi': 5 sm ; $5\sqrt{3} \text{ sm}$.

 **Soraw, ma'sele ha'm tapsi'rmalar**

1. α mu'yesh 30° , 45° , 60° qa ten' bolsa, $\sin \alpha$, $\cos \alpha$, $\operatorname{tg} \alpha$ ha'm $\operatorname{ctg} \alpha$ ma'nisleri nege ten'? Juwaplarin'izdi da'liyllen'.
2. 3-su'wrettegi u'shmu'yeshliklerdin' perimetrlerin tabi'n'.
3. 4-su'wrettegi u'shmu'yeshliklerdin' mu'yeshlerin tabi'n'.
4. α nin' 30° , 45° , 60° qaten' ma'nisinde $\sin \alpha$, $\cos \alpha$, $\operatorname{tg} \alpha$ ha'm $\operatorname{ctg} \alpha$ ushi'n ma'nisler kestesin yadlap ali'n'.
5. Tuvri' mu'yeshli u'shmu'yeshliktin' bir su'yir mu'yeshi 30° qa ten' boli'p, wog'an irgeles jatqan katet 6 dm . Woni'n' qalg'an ta'replerin tabi'n'.
6. Ten' qaptalli' u'shmu'yeshliktin' ultani' 10 sm ge, bir mu'yeshi bolsa 120° qa ten'. Woni'n' maydani'n tabi'n'.
7. ABC u'shmu'yeshlikte $\angle C = 90^\circ$, $AB = 25 \text{ sm}$, $\sin A = \frac{7}{25}$. U'shmu'yeshliktin' qalg'an ta'replerin ha'm $\cos A$, $\operatorname{tg} A$ ha'mde $\operatorname{ctg} A$ ni tabi'n'.
8. Diagonallari' $5\sqrt{3} \text{ sm}$ ha'm 5 sm bolg'an rombi'ni'n mu'yeshlerin tabi'n'.

α	30°	45°	60°
$\sin \alpha$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos \alpha$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\operatorname{tg} \alpha$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
$\operatorname{ctg} \alpha$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$



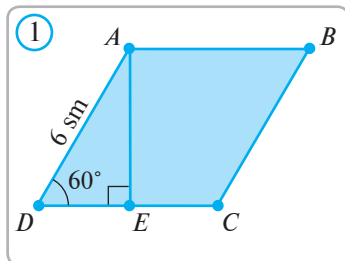
**Jedellestiriwshi shinig'iw.**

Kestenin' bos ketekshelerin tolti'ri'n'.

α	$\sin \alpha$	$\cos \alpha$	$\operatorname{tg} \alpha$	$\operatorname{ctg} \alpha$
	$\frac{\sqrt{3}}{2}$			
			$\frac{\sqrt{3}}{3}$	
		$\frac{\sqrt{2}}{2}$		



1-ma'sele. Yeger $ABCD$ rombida $\angle A = 120^\circ$ ha'm $AB = 6$ sm bolsa, rombi'ni'n' biyikligin ha'm maydani'n tabi'n' (1-su'wret).



Sheshiliwi. 1) Rombi'ni'n' bir ta'repine irgeles jatqan mu'yeshlerinin' qosi'ndi'si' 180° qa ten' bolg'ani' ushi'n $\angle D = 180^\circ - \angle A = 60^\circ$. Rombi'ni'n' AE biyikligin ju'rgizip (1-su'wret), tuwri' mu'yeshli AED u'shmu'yeshligin paydayetemiz. Wonda

$$\frac{AE}{AD} = \sin D = \sin 60^\circ = \frac{\sqrt{3}}{2} \text{ yaki } AE = \frac{\sqrt{3}}{2} \cdot AD = 3\sqrt{3} \text{ (sm).}$$

2) Yendi rombi'ni'n' maydani'n tabami'z:

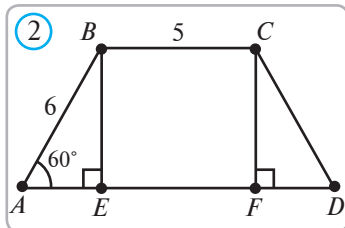
$$S_{ABCD} = DC \cdot AE = 6 \cdot 3\sqrt{3} = 18\sqrt{3} \text{ (sm}^2\text{)}.$$

Juwabi': $h = 3\sqrt{3}$ sm; $S_{ABCD} = 18\sqrt{3}$ sm².



2-ma'sele. $ABCD$ ten' qaptalli' trapeciyani'n' BC kishi ultani' 5 sm. Yeger $\angle A = 60^\circ$, $AB = 6$ sm bolsa, trapeciyani'n' maydani'n tabi'n'.

Sheshiliwi. Trapeciyanin' BE ha'm CF biyikliklerin ju'rgizemiz (2-su'wret). Bunda tuwri' mu'yeshli ABE u'shmu'yeshliginen



$$AE = AB \cos 60^\circ = 6 \cdot \frac{1}{2} = 3 \text{ (sm),}$$

$$BE = AB \sin 60^\circ = 6 \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3} \text{ (sm).}$$

Bunnan ti'sqari' $AE = FD$, $EF = BC$ bolg'ani' ushi'n,

$$AD = AE + EF + FD = 3 + 5 + 3 = 11 \text{ (sm).}$$

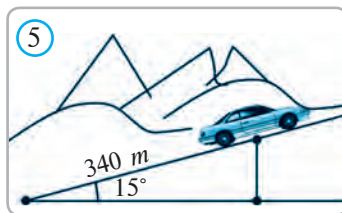
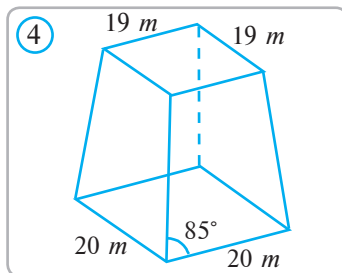
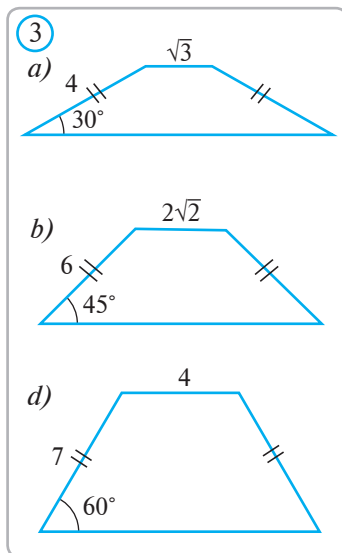
Trapeciyani'n' maydani'n tabi'w formulasi' boyi'nsha,

$$S_{ABCD} = \frac{BC + AD}{2} \cdot BE = \frac{5 + 11}{2} \cdot 3\sqrt{3} = 24\sqrt{3} \text{ (sm}^2\text{)}.$$

Juwabi': $24\sqrt{3} \text{ sm}^2$.

? Soraw, ma'sele ha'm tapsi'rmalar

1. Ten' qaptalli' tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasi 12 sm. Woni'n' maydanin' yesaplan'.
2. Biyikligi $4\sqrt{3}$ sm bolg'an ten' ta'repli u'shmu'yeshliktin' perimetrin' tabi'n'.
3. 3-su'wrette berilgenler boyi'nsha ten' qaptalli' trapeciyalardi'n' maydani'n' tabi'n'.
4. Tuwri' mu'yeshli trapeciyanin' su'yir mu'yeshi 30° qa, biyikligi 4 sm ge ha'm kishi ultani' 6 sm ge ten'. Trapeciyani'n' perimetrin' ha'm maydani'n' tabi'n'.
5. Shen'ber xordasi' 120 gradusli' dog'ani' kerip turadi'. Yeger shen'ber radiusi' 10 sm bolsa, xordani'n' uzi'nli'g'i'n' tabi'n'.
- 6*. Ten' qaptalli' u'shmu'yeshliktin' to'besindegi mu'yeshi a) 120° , b) 90° , c) 60° . U'shmu'yeshlik biyikliginin' ultani'na qatnasi'n' yesaplan'.
- 7*. 4-su'wrette su'wretlengen paxta qi'rmani'ni'n' qaptal jaqlari' ten' qaptalli' trapeciya, u'sti bolsa kvadrat formasi'nda. Su'wrette berilgenlerden paydalani'p, qi'rmandi' toli'q jabi'w ushi'n qansha material kerek yekenligin' ani'qlan'.
8. Jen'il mashina shig'arli'qti'n' joqari'g'a ko'teriliw bo'liminde 340 m jol basti'. Yeger joldi'n' gorizontqa sali'sti'rg'anda ko'teriliw mu'yeshi 15° bolsa, jen'il mashina neshe metr biyiklikke ko'terilgen (5-su'wret)?



Arnawli kalkulyatorda trigonometriyalı'q funkciyalardin' ma'nislerin' tabi'w

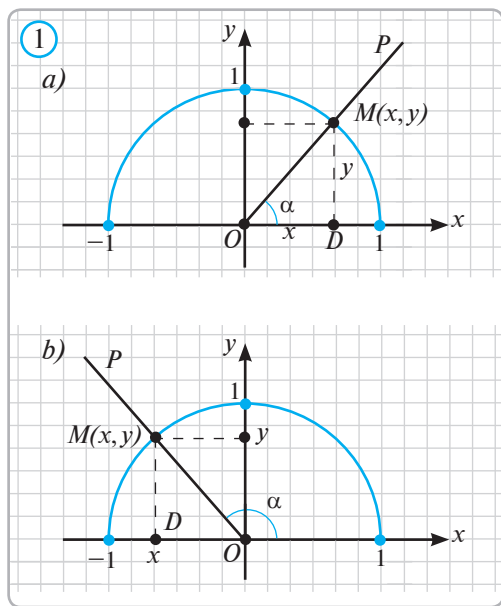
(sin) ha'm (cos) tu'ymeleri bar arnawli' kalkulyatorda trigonometriyalı'q funkciyalardi'n' ma'nisleri to'mendegishe saplanadi:

Mu'yeshlik graduslarda berilgen bolsi'n: mi'sali', $\sin 30^\circ$:

Kalkulyator jalg'anip **DEG** (gradus) tu'ymesı bası'ladi'.

2. Keyin tu'ymeler **C 3 0 Sin** ta'rtibinde bası'ladi' ha'm tiyisli juwap: 0,5 ali'nadi'. $\sin 30^\circ = 0,5$.

Yeger arnawli' kalkulyator bolmasa, sabaqli'qti'n' keynindegi qosi'mshada keltirilgen trigonometriyalı'q funkciyalardin' ma'nislerinin' kesesinen paydalani'wi'mi'z mu'mkin.



Tuwri' mu'yeshli Oxy koordinatalar sistemasi'n ha'm woni'n' I ha'm de II shereklerinde jaylasqan, radiusi birlik kesindige ten', worayi' koordinatalar basi'nda bolg'an yari'm shen'ber jasaymi'z (*I-su'wret*). Shen'berdi $M(x, y)$ noqati'nda kesip wo'tiwshi OP nuri'n ju'rgizemiz. Bul nurdi'n' Ox nuri' menen payda yetken mu'yeshin α menen belgileymiz. OP nuri'ni'n' Ox nuri' menen u'stpe-u'st tu'sken haldag'i mu'yeshin 0° li mu'yesh si'pati'nda qabi'l yetemiz.

α su'yir mu'yesh bolg'anda (*1.a-su'wret*), tuwri' mu'yeshli ODM u'sh-mu'yeshlikten

$$\sin \alpha = \frac{DM}{MO}; \quad \cos \alpha = \frac{OD}{MO};$$

$$\operatorname{tg} \alpha = \frac{DM}{OD}; \quad \operatorname{ctg} \alpha = \frac{OD}{DM}$$

ten'likleri ja'rdeminde ani'qlanadi'. Yeger $MO = 1$, $DM = y$, $OD = x$ yekenligin yesapqa alsaq,

$$\sin \alpha = y, \quad \cos \alpha = x, \quad \operatorname{tg} \alpha = \frac{y}{x}, \quad \operatorname{ctg} \alpha = \frac{x}{y} \quad (1)$$

ten'liklerine iye bolami'z.

Uli'wma jag'dayda, mu'yeshinin' 0° tan 180° qa shekemgi bolg'an barli'q ma'nislerinin' sinusi', tangensi ha'm kotangenslerin de (1) formula arqali ani'qlaymi'z.

Qa'legen α ($0^\circ \leq \alpha \leq 180^\circ$) mu'yeshinin' **sinusi'** dep, M noqatinin' ordinatasi' — y ke aytiladi'. Qa'legen α ($0^\circ \leq \alpha \leq 180^\circ$) mu'yeshinin' **kosinusi'** dep M noqati'ni'n' abscissasi — x qa ayti'ladi'. Qa'legen α ($0^\circ \leq \alpha \leq 180^\circ$, $\alpha \neq 90^\circ$) mu'yeshinin' **tangensi** dep, M noqati'ni'n' ordinatasi abscissasi'na qatnasi'na ayti'ladi'. Qa'legen α ($0^\circ < \alpha < 180^\circ$) mu'yeshinin' **kotangensi** dep, M noqati'ni'n' abscissasinin' ordinatasi'na qatnasi'na ayti'ladi'.

ODM u'shmu'yeshlik $OD^2 + DM^2 = MO^2$ yaki $x^2 + y^2 = 1$. $\sin \alpha = y$ ha'm $\cos \alpha = x$ yekenligin yesapqa alsaq, qa'legen α ($0^\circ \leq \alpha \leq 180^\circ$) mu'yesh ushi'n

$$\sin^2 \alpha + \cos^2 \alpha = 1 \quad (2)$$

ten'lik payda boladi'. Bul ten'lik, **tiykarg'i' trigonometriyalik birdeylik** dep ataladi', alding'i sabaqlarda su'yir mu'yeshler ushi'n da'liyllengen yedi.

A'meliy tapsirma

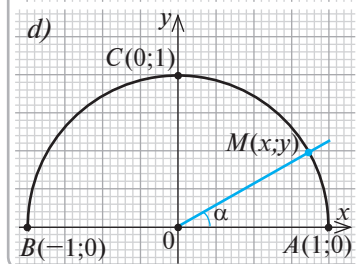
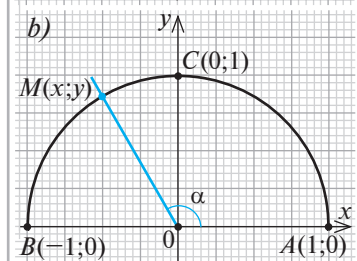
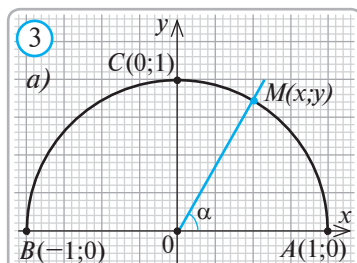
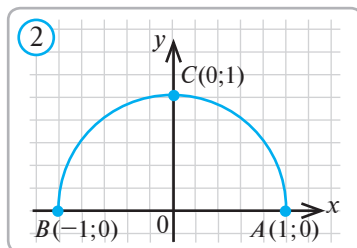
1. Birlikkesindini 5 sm ge ten' dep, tuwri' mu'yeshli koordinatalar sistemasi'n si'zi'n'.
2. Worayi' koordinatalar basindaha'm radiusi' birlik kesindige ten', I ha'm II shereklerde jaylasqan yari'm shen'ber si'zi'n'.
3. Yari'm shen'berdi M noqatta kesip wo'tetug'in ha'm Ox ko'sherinin' on' bag'i'ti' menen a) $\alpha=67^\circ$; b) $\alpha=118^\circ$; d) $\alpha=150^\circ$ qaten' mu'yesh payda yetetug'in OM nurin jasan'.
4. Wo'lshe'mler ja'rdeminde M noqati'ni'n koordinatalarin ha'm de $\sin\alpha$, $\cos\alpha$, $\operatorname{tg}\alpha$ ha'm $\operatorname{ctg}\alpha$ nin' ma'nislerin tabi'n'.

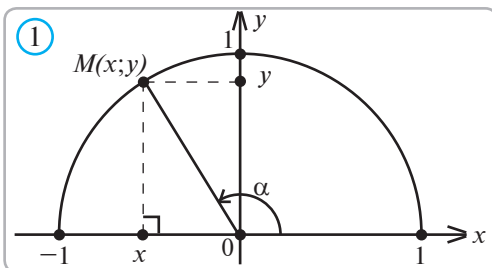
 **Ma'sele.** 0° , 90° ha'm 180° li mu'yeshlerdin' sinusin tabi'n'.

Sheshiliwi. 0° li' mu'yesh OA , 90° li' mu'yesh OC , 180° li mu'yesh OB nur ja'rdeminde ani'qlanadi' (2-su'wret). Ani'qlama boyi'nsha $\sin 0^\circ$ — $A(1;0)$ noqati'ni'n' ordinatasi' si'pati'nda 0 ge, $\sin 90^\circ$ — $C(0;1)$ noqati'ni'n' ordinatasi' si'pati'nda 1 ge, $\sin 180^\circ$ bolsa $B(-1;0)$ noqati'ni'n' ordinatasi' si'pati'nda 0 ge ten' boladi'. **Juwabi':** $\sin 0^\circ=0$, $\sin 90^\circ=1$, $\sin 180^\circ=0$.

Soraw, ma'sele ha'm tapsi'rmalar

1. 0° tan 180° qa shekem bolg'an mu'yeshstin' sinusi' ha'm kosinusi' degende ne tu'siniletug'i'ni'n ayti'p berin'.
2. α mu'yeshinin' tangensi ha'm kotangensi degen ne? α mu'yeshinin' tangensi ha'm kotangensi α nin' qanday ma'nislerinde ani'qlanbag'an?
3. Yeger $90^\circ < \alpha < 180^\circ$ bolsa, $\sin\alpha$, $\cos\alpha$, $\operatorname{tg}\alpha$ va $\operatorname{ctg}\alpha$ ma'nislerinin' belgisin ani'qlan'.
4. Yeger $0^\circ \leq \alpha \leq 180^\circ$ bolsa, $0 \leq \sin\alpha \leq 1$ va $-1 \leq \cos\alpha \leq 1$ ten'sizlikleri wori'nli' bolatug'i'ni'n tu'sindirin'.
5. 3-su'wrettegi α mu'yeshin wo'lshe'n ha'm woni'n' sinusi', kosinusi' ha'm kotangensin tiyisli wo'lshe'mler ja'rdeminde ani'qlan'.
- 6*. 1. a - su'wrette su'wretlengen yari'm shen'berdi si'zi'n'. Ox nuri' menen 45° ha'm 135° li' mu'yesh payda yetiwshi nurlardi' jasan'. Si'zi'lg'an su'wretten paydalani'p, $\sin 45^\circ$ ti' $\sin 135^\circ$ penen ha'm $\cos 45^\circ$ ti' $\cos 135^\circ$ penen wo'z arasali'sti'ri'n'.





Jedellestiriwshi shinig'iw

1-su'wretten paydalani'p noqatlardin' wornin toltirin':

$$\sin \alpha = \dots; \quad \cos \alpha = \dots;$$

$$\operatorname{tg} \alpha = \frac{\dots}{\dots}; \quad \operatorname{ctg} \alpha = \frac{\dots}{\dots}.$$

Ani'qlama boyi'nsha, ha'r bir su'yir mu'yeshke bul mu'yeshnin' sinusi'ni'n' (kosinusi', tangensi ha'm kotangensinini') bir ma'nisi sa'ykes qoyi'li'p ati'r. Bul sa'ykeslikler su'yir mu'yeshnin' trigonometriyali'q funkciyalari': sinus, kosinus, tangens ha'm kotangens funkciyalarin' ani'qlaydi'. Bul funkciyalar ko'binshe u'shmu'yeshliklerdi sheshiwde qollani'li'wi' sebepli, wolar trigonometriyaliq funkciyalar dep ataladi'.

“Trigonometriya” so'zi — grekshe “u'shmu'yeshliklerdi sheshiw” degen ma'nisti an'latadi.

Yendi α ($0^\circ \leq \alpha \leq 180^\circ$) mu'yeshinin' sinusi', kosinusi', tangensi ha'm kotangensi arasi'ndag'i qatnaslardi' ani'qlayi'q.

1. **Trigonometriyani'n' tiykarg'i' birdeyligi** dep atali'wshi', α ni'n' $0^\circ \leq \alpha \leq 180^\circ$ ma'nisleri ushi'n wori'nli' bolg'an mina

$$\sin^2 \alpha + \cos^2 \alpha = 1 \quad (1)$$

formula menen aldin'g'i' sabaqlarda tani'sqan yedik.

2. Ani'qlama boyi'nsha, $\operatorname{tg} \alpha = \frac{y}{x}$, $\operatorname{ctg} \alpha = \frac{x}{y}$, $x = \cos \alpha$, $y = \sin \alpha$ bolg'ani' ushi'n

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} \quad (\alpha \neq 90^\circ), \quad \operatorname{ctg} \alpha = \frac{\cos \alpha}{\sin \alpha} \quad (\alpha \neq 0, \alpha \neq 180^\circ), \quad (2)$$

$$\operatorname{tg} \alpha \cdot \operatorname{ctg} \alpha = 1 \quad (\alpha \neq 0, \alpha \neq 90^\circ, \alpha \neq 180^\circ)$$

birdeylikleri wori'nli' boladi'.

3. (1) ten'likтин' ha'r yeki bo'limin aldi'n $\cos^2 \alpha$ g'a, keyin bolsa $\sin^2 \alpha$ g'a bo'lip

$$1 + \operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha} \quad (\alpha \neq 90^\circ), \quad 1 + \operatorname{ctg}^2 \alpha = \frac{1}{\sin^2 \alpha}, \quad (\alpha \neq 0, \alpha \neq 180^\circ) \quad (3)$$

birdeyliklerin payda yetemiz.



Ma'sele. Yeger $\sin \alpha = 0,6$ ha'm $90^\circ \neq \alpha \neq 180^\circ$ bolsa, $\cos \alpha$, $\operatorname{tg} \alpha$ ha'm $\operatorname{ctg} \alpha$ ma'nisin tabi'n'.

Sheshiliwi. Tiykarg'i' trigonometriyalik birdeylikten paydalani'p $\cos \alpha$ ni yesaplaymiz:

$$\cos \alpha = -\sqrt{1 - \sin^2 \alpha} = -\sqrt{1 - 0,6^2} = -\sqrt{1 - 0,36} = -\sqrt{0,64} = -0,8.$$

$90^\circ \leq \alpha \leq 180^\circ$, yag'ni'y α II sherekte bolg'anda, $\cos \alpha \leq 0$. Sol sebepli koren "—" belgisi menen ali'ndi'.

(2) formulag'a tiykarlani'p

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} = -\frac{0,6}{0,8} = -\frac{3}{4}; \quad \operatorname{ctg} \alpha = \frac{1}{\operatorname{tg} \alpha} = -\frac{4}{3}.$$

Juwabi': $\cos \alpha = -0,8$; $\operatorname{tg} \alpha = -\frac{3}{4}$; $\operatorname{ctg} \alpha = -\frac{4}{3}$.

? Soraw, ma'sele ha'm tapsi'rmalar

1. $\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}$, $\operatorname{ctg} \alpha = \frac{\cos \alpha}{\sin \alpha}$, $\operatorname{tg} \alpha \cdot \operatorname{ctg} \alpha = 1$ birdeylikleri α nin' qanday ma'nislari ushi'n wori'nli'?

2. An'latpalardi' a'piwayi'lasti'ri'n':

1) $1 - \cos^2 \alpha$;

4) $1 - \sin^4 \alpha - \sin^2 \alpha \cdot \cos^2 \alpha$;

2) $(1 - \sin \alpha)(1 + \sin \alpha)$;

5) $\operatorname{ctg}^2 \alpha (2 \sin^2 \alpha + \cos^2 \alpha - 1)$;

3) $\sin^4 \alpha + 2 \sin^2 \alpha \cdot \cos^2 \alpha + \cos^4 \alpha$;

6) $\operatorname{tg}^2 \alpha - \sin^2 \alpha \cdot \operatorname{tg}^2 \alpha$.

3. Yeger a) $\sin \alpha = \frac{4}{5}$ ha'm $90^\circ < \alpha < 180^\circ$ bolsa, $\cos \alpha$ nege ten' yekenligin tabi'n';
b) $\cos \beta = -\frac{2}{3}$ ha'm $90^\circ < \beta < 180^\circ$ bolsa, $\sin \beta$ nege ten'; d) $\cos \alpha = 1$ bolsa, $\sin \alpha$ ni'n' ma'nisin yesaplan'.

4. Su'yir mu'yeshi 60° qa, biyikligi bolsa 3 sm ge ten' rombi'ni'n' maydani'n tabi'n'.

5. Ten' qaptalli' u'shmu'yeshliktin' ultani' $4,8 \text{ sm}$, ultani'ndag'i mu'yeshi bolsa 30° . U'shmu'yeshliktin' biyikligin ha'm qaptal ta'repin tabi'n'.

6. Yeger a) $\cos \alpha = \frac{1}{2}$; b) $\cos \alpha = -\frac{2}{3}$; d) $\cos \alpha = -1$ bolsa, $\sin \alpha$ nege ten'?

7. a) $\sin A = \frac{2}{3}$; b) $\cos A = \frac{3}{4}$; d) $\cos \alpha = \frac{2}{5}$ yekenligi belgili bolsa, A mu'yeshin jasan'.

8*. α ha'm β mu'yeshleri $0^\circ < \alpha < \beta < 90^\circ$ sha'rtin qanaatlandi'radi'. 2-su'wretten paydalani'p yesaplan':

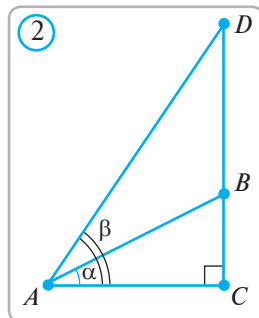
a) $\sin \alpha < \sin \beta$; b) $\cos \alpha > \cos \beta$;

d) $\operatorname{tg} \alpha < \operatorname{tg} \beta$; e) $\operatorname{ctg} \alpha > \operatorname{ctg} \beta$.

9*. OA nuri menen Ox nuri' arasi'ndag'i mu'yesh α g'aten'. Yeger

a) $OA = 3$, $\alpha = 45^\circ$; b) $OA = 1,5$, $\alpha = 90^\circ$; d) $OA = 5$, $\alpha = 150^\circ$;

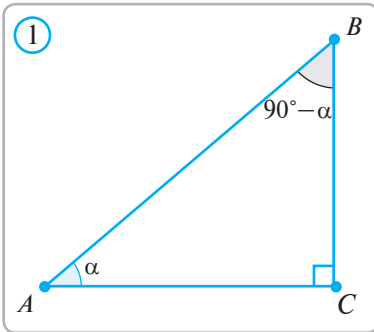
e) $OA = 2$, $\alpha = 180^\circ$; f) $OA = 4$, $\alpha = 30^\circ$ bolsa, A noqati'ni'n' koordinatalari'n tabi'n'.





1-teorema. Ha'r qanday su'yir α mu'yeshi ushi'n:

$$\sin(90^\circ - \alpha) = \cos \alpha, \quad \cos(90^\circ - \alpha) = \sin \alpha. \quad (1)$$



Da'liyllew. A to'besindagi su'yir mu'yeshi α g'a ten' bolg'an tuwri' mu'yeshli ABC u'shmu'yeshligin qaraymi'z (1-su'wret). Wondaoni'n' B to'besindagi su'yir mu'yeshi $\beta = 90^\circ - \alpha$ ge ten'. Ani'qlama boyi'nsha

$$\sin(90^\circ - \alpha) = \sin \beta = \frac{AC}{AB} = \cos \alpha,$$

$$\cos(90^\circ - \alpha) = \cos \beta = \frac{BC}{AB} = \sin \alpha.$$

Teorema da'liyllendi.



1-ma'sele. To'mendegi sanlar ishinde wo'z-ara ten'lerin tabi'n': $\sin 10^\circ$, $\cos 10^\circ$, $\sin 80^\circ$, $\cos 80^\circ$.

Sheshiliwi. $80^\circ = 90^\circ - 10^\circ$ ($\alpha = 10^\circ$) ha'm $50^\circ = 90^\circ - 40^\circ$ ($\alpha = 40^\circ$) bolg'ani' ushi'n 1-teorema boyi'nsha

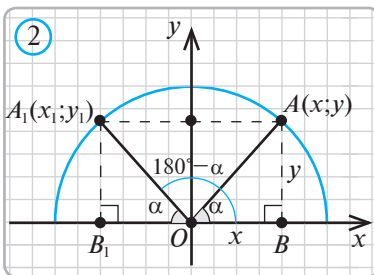
$$\sin 80^\circ = \sin(90^\circ - 10^\circ) = \cos 10^\circ, \quad \cos 80^\circ = \cos(90^\circ - 10^\circ) = \sin 10^\circ.$$

$$\text{Juwabi': } \sin 80^\circ = \cos 10^\circ, \quad \cos 80^\circ = \sin 10^\circ.$$



2-teorema. Ha'r qanday α ($0 \leq \alpha \leq 180^\circ$) mu'yeshi ushi'n:

$$\sin(180^\circ - \alpha) = \sin \alpha, \quad \cos(180^\circ - \alpha) = -\cos \alpha. \quad (2)$$



Da'liyllew. Tuwri' mu'yeshli Oxy koordinatalar sistemasi'nda worayi' O noqati'nda, radiusi' 1 ge ten' bolg'an yari'm shen'berdi jasaymi'z (2-su'wret). Shen'berdin' OA radiusi' menen Ox nuri' arasi'ndag'i' mu'yesh $180^\circ - \alpha$ ge ten' mu'yesh payda yetiwshi OA_1 radiusi'n ju'rgizemiz. OAA_1 ha'm OBB_1 tuwri' mu'yeshli u'shmu'yeshlikleri ten'. Sonday-aq $OB = OB_1$ ha'm $AB = A_1B_1$ yaki $x_1 = x$ ha'm $y_1 = y$ ten'liklerine iye bolami'z. Solay yetip,

$$\sin(180^\circ - \alpha) = y_1 = y = \sin \alpha;$$

$$\cos(180^\circ - \alpha) = x_1 = -x = -\cos \alpha.$$

Teorema da'liyllendi.

(1) ha'm (2) formulalari' *keltiriw formulalari'* delinedi.

 **2-ma'sele.** $\alpha = 120^\circ$ bolsa, $\sin \alpha$, $\cos \alpha$, $\operatorname{tg} \alpha$ ha'm $\operatorname{ctg} \alpha$ ni'n' ma'nislerin yesaplan'.

Sheshiliwi. a) (2) formula boyi'nsha

$$\sin 120^\circ = \sin(180^\circ - 60^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}; \quad \cos 120^\circ = \cos(180^\circ - 60^\circ) = -\cos 60^\circ = -\frac{1}{2}.$$

Wonda

$$\operatorname{tg} 120^\circ = \frac{\sin 120^\circ}{\cos 120^\circ} = -\sqrt{3}; \quad \operatorname{ctg} 120^\circ = \frac{\cos 120^\circ}{\sin 120^\circ} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}.$$

Juwabi': $\sin 120^\circ = \frac{\sqrt{3}}{2}; \quad \cos 120^\circ = -\frac{1}{2}; \quad \operatorname{tg} 120^\circ = -\sqrt{3}; \quad \operatorname{ctg} 120^\circ = -\frac{\sqrt{3}}{3}.$

 **Soraw, ma'sele ha'm tapsi'rmalar**

- $\operatorname{tg}(90^\circ - \alpha) = \operatorname{ctg} \alpha$ ($\alpha \neq 0^\circ$) ha'm $\operatorname{ctg}(90^\circ - \alpha) = \operatorname{tg} \alpha$ ($\alpha \neq 0^\circ$) birdeyliklerin da'liylen'.
- $\operatorname{tg}(180^\circ - \alpha) = -\operatorname{tg} \alpha$ ($\alpha \neq 90^\circ$) ha'm $\operatorname{ctg}(180^\circ - \alpha) = -\operatorname{ctg} \alpha$ ($\alpha \neq 0^\circ$ va $\alpha \neq 180^\circ$) birdeyliklerin da'liylen'.
- Kesteni tolti'ri'n'.

α	0°	30°	45°	60°	90°	120°	135°	150°	180°
$\sin \alpha$									
$\cos \alpha$									
$\operatorname{tg} \alpha$									
$\operatorname{ctg} \alpha$									

- Yeger $90^\circ < \alpha < 180^\circ$ ha'm a) $\sin \alpha = \frac{1}{2}$; b) $\cos \alpha = -\frac{\sqrt{2}}{2}$; d) $\operatorname{tg} \alpha = -1$; e) $\operatorname{ctg} \alpha = -\sqrt{3}$ bolsa, α mu'yeshinin' shamasini'n tabi'n'.
- Yesaplan':
 a) $\sin 180^\circ + 2\cos 90^\circ$; b) $4\sin 150^\circ + \sqrt{3}\operatorname{tg} 150^\circ$;
 d) $\cos 40^\circ + \cos 50^\circ - \sin 40^\circ - \sin 50^\circ$; e) $3\cos 120^\circ - 2\sqrt{3}\operatorname{ctg} 60^\circ$.
- A'piwayi'lasti'ri'n':
 a) $\cos^2(180^\circ - \alpha) + \cos^2(90^\circ - \alpha)$; b) $\sin^2(180^\circ - \alpha) + \sin^2(90^\circ - \alpha)$;
 d) $\operatorname{tg} \alpha \cdot \operatorname{tg}(90^\circ - \alpha)$; e) $\operatorname{ctg} \alpha \cdot \operatorname{ctg}(90^\circ - \alpha)$.
- ABC u'shmu'yeshliginde $\angle A = 150^\circ$ ha'm $AC = 7$ sm bolsa, u'shmu'yeshlikтин' C to'besinen tu'sirilgen biyiklikti tabi'n'.
- Tuwri'mu'yeshlikтин' 12 sm ge ten' diagonalini' bir ta'repi menen 30° qa ten' mu'yesh payda yetedi. Tuwri'mu'yeshlikтин' maydani'n tabi'n'.
- Yeger a) $\sin \alpha = \frac{\sqrt{3}}{2}$; b) $\sin \alpha = \frac{1}{4}$; d) $\sin \alpha = 1$ bo'lsa, $\cos \alpha$ ni tabi'n'.
- Yeger a) $\sin \alpha = \frac{1}{2}$; b) $\operatorname{tg} \alpha = -1$; d) $\cos \alpha = -\frac{\sqrt{3}}{2}$ bo'lsa, α ni tabi'n'.

I. Shep bag'anada berilgen atamalg'a won' bag'anada berilgen ani'qlamalardan duri'si'n sa'ykes qoyi'n'.

- | | |
|--------------------------------------|--|
| 1. α mu'yeshinin' sinusi | a) α mu'yeshinin' qarsi'si'ndag'i' katettin' gipotenuzag'a qatnasi'; |
| 2. α mu'yeshinin' kosinusi | b) α mu'yeshinin' qarsi'si'ndag'i' katettin' yekinshi katetke qatnasi'; |
| 3. α mu'yeshinin' tangensi | d) α mu'yeshinin' qarsi'si'ndag'i' katettin' yekinshi katetke qatnasi'; |
| 4. α mu'yeshinin' kotan-gensi | e) α mu'yeshke irgeles katettin' yekinshi katetke qatnasi'. |

II. Testler

1. Naduri's formulani' tabi'n':

A. $\sin(90^\circ - \alpha) = \cos \alpha$;

B. $\cos(90^\circ - \alpha) = \sin \alpha$;

D. $\sin(180^\circ - \alpha) = \sin \alpha$;

E. $\cos(180^\circ - \alpha) = \cos \alpha$.

2. Yeger $90^\circ < \alpha < 180^\circ$ bolsa, to'mendegilerden qaysi' biri won'?

A. $\sin \alpha$;

B. $\cos \alpha$;

D. $\operatorname{tg} \alpha$;

E. $\operatorname{ctg} \alpha$.

3. Duri's ten'likti tabi'n':

A. $\sin^2 \alpha = 1 + \cos^2 \alpha$;

B. $\operatorname{tg}^2 \alpha = 1 + \cos^2 \alpha$;

D. $\frac{1}{\cos^2 \alpha} = 1 + \operatorname{tg}^2 \alpha$ ($\alpha \neq 90^\circ$);

E. $\sin^2 x \cdot \cos^2 x = 1$.

4. $\sin 70^\circ$ nege ten':

A. $\sin 20^\circ$;

B. $-\sin 20^\circ$;

D. $\cos 70^\circ$;

E. $\cos 20^\circ$.

5. $\sin \alpha = \frac{1}{2}$ bolg'an α su'yir mu'yeshin ko'rsetin':

A. 30° ;

B. 45° ;

D. 90° ;

E. 60° .

6. $\cos \alpha = \frac{1}{2}$ bolsa, α su'yir mu'yeshin tabi'n':

A. 30° ;

B. 45° ;

D. 90° ;

E. 60° .

7. $\operatorname{tg} \alpha = 1$ bolsa, α su'yir mu'yeshin tabi'n':

A. 30° ;

B. 45° ;

D. 90° ;

E. 60° .

8. $\operatorname{ctg} \alpha = 1$ bolsa, α su'yir mu'yeshin tabi'n':

A. 30° ;

B. 45° ;

D. 90° ;

E. 60° .

9. Qaysi su'yir α mu'yesh ushin' $\sin \alpha = \cos \alpha$ ten'lik wori'nli'?

A. 30° ;

B. 45° ;

D. 90° ;

E. 60° .

10. Yeger $\sin B = \frac{2}{5}$ bolsa, $\cos B$ ni' tabi'n'.

A. $\frac{4}{25}$; B. $\frac{\sqrt{29}}{5}$; D. $\frac{\sqrt{21}}{5}$; E. $\frac{\sqrt{10}}{5}$.

11. Yeger $\cos A = 0,2$ bolsa, $\operatorname{tg} A$ ni' tabi'n'.

A. $\sqrt{96}$; B. $2\sqrt{6}$; D. $\sqrt{15}$; E. $\frac{\sqrt{6}}{12}$.

12. Tuwri' to'rtmu'yeshliktin' diagonali' woni'n' bir ta'repinen 2 ma'rte uzi'n. Tuwri' to'rtmu'yeshliktin' diagonallari' arasi'ndag'i mu'yeshin tabi'n'.

A. 30° ; B. 60° ; D. 90° ; E. 150° .

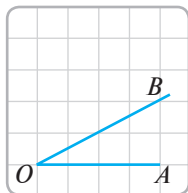
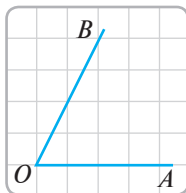
13. Ten' qaptalli' u'shmu'yeshliktin' ultani'na tu'sirilgen biyikligi 3 sm, ultani' bolsa 8 sm. U'shmu'yeshliktin' ultani'na irgeles jatqan mu'yeshin' sinusi'n tabi'n'.

A. $\frac{3}{5}$; B. $\frac{3}{4}$; D. $\frac{\sqrt{73}}{73}$; E. $\frac{4}{5}$.

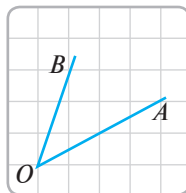
III. Ma'seleler

1. Su'wrette sa'wlelengen mu'yeshlerdin' sinusi', kosinusi', tangensi ha'm kotangensin tabi'n'.

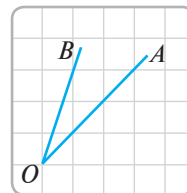
a) b)



d)



e)



2. Ashirafjon u'yinen shi'g'i's ta'repke qarap 800 m, son' arqa ta'repke qarap 600 m jola ju'rди. Wol u'yinen neshe metr uzaqliqqa keldi? Yendi wol u'yine tuwri' si'zi'q boylap jetip aliw ushi'n batisqa sali'sti'rg'anda qanday mu'yesh asti'nda ju'riwi kerek?

3. Poyezd ha'r 30 m jol ju'rgende 1 m biyiklikke ko'teriledi. Temir joldin' gorizontqa sali'stirg'andag'i' ko'teriliw mu'yeshin tabi'n'.

4. Yeger biyikligi 30 m bolg'an imarattin' sayasinan uzi'nlig'i 45 m bolsa, quyash nuri'ni'n' sol imarat jaylasqan wori'ng'a tiyisli mu'yeshin tabi'n'.

5. Tuwri' mu'yeshli u'shmu'yeshliktin' bir mu'yeshi 60° qa, u'lken kateti bolsa 6 g'a ten'. Woni'n' kishi katetin ha'm gipotenuzasi'n tabi'n'.

6. O worayina iye shen'berdin' A noqati'nan ju'rgizilgen uri'nbadan B noqati' ali'ng'an. Yeger $AB=9$ sm, $\angle ABO=30^\circ$ bolsa, shen'ber radiusi'n ha'm BC kesindisi uzi'nli'g'i'n tabi'n'.

7. m tuwri'si' ha'm woni' kesip wo'tpeytug'i'n AB kesindisi berilgen. Bunda $AB=10$, AB ha'm m tuwrlari arasi'ndag'i' mu'yesh 60° . AB kesindisi ushlari'nan m tuwri'sina AC ha'm BD perpendikulyarlari' tu'sirilgen. CD kesindisin tabi'n'.

8. Rombi'ni'n' su'yir mu'yeshi 60° qa, biyikligi bolsa 6 g'a ten'. Rombi'ni'n' diagonalin ha'm maydani'n tabi'n'.

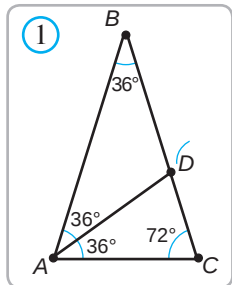
9. Radiusi' 5 sm bolg'an shen'berge ten' qaptalli' trapeciya si'rtlay si'zi'lg'an. Yeger trapeciyani'n' su'yir mu'yeshi 30° bolsa, woni'n' qaptal ta'replerin ha'm maydani'n tabi'n'.
10. Yeger $ABCD$ tuwri'mu'yeshliginde $AB=4$, $\angle CAD=30^\circ$ bolsa, wog'an si'rtlay si'zi'lg'an shen'ber radiusi'n ha'm tuwri'mu'yeshliktin' maydani'n yesaplan'.
11. Tuwri'mu'yeshliktin' ta'repleri 3 sm ha'm $\sqrt{3}$ sm. Woni'n' bir diagonali' menen ta'repleri payda yetken mu'yeshlerin tabi'n'.
12. Yeger a) $\sin A = \frac{4}{7}$; b) $\cos A = \frac{4}{7}$; d) $\cos A = -\frac{4}{7}$ bolsa, A mu'yeshin jasan'.
13. Tuwri' mu'yeshli u'shmu'yeshliktin' bir mu'yeshi 30° , gipotenuzasina tu'sirilgen biyikligi 6 sm. U'shmu'yeshliktin' ta'replerin tabi'n'.
14. Su'yir mu'yeshi 30° qa, biyikligi bolsa 4 sm ge ten' bolg'an rombi'ni'n maydani'n yesaplan'.
15. Yeger $\sin A = \frac{8}{17}$ ha'm $90^\circ < \alpha < 180^\circ$ bolsa, $\cos \alpha$, $\operatorname{tg} \alpha$ ha'm $\operatorname{ctg} \alpha$ nin' ma'nisin tabi'n'.
16. Tuwri' mu'yeshli ABC u'shmu'yeshliginin' AB gipotenuzasi'na CD biyikligi tu'sirilgen. Yeger $\angle A = 60^\circ$ ha'm $BD = 2$ bolsa, BC katetin tabi'n'.
17. ABC u'shmu'yeshliginde $\angle A = 30^\circ$, $\angle C = 45^\circ$. Yeger u'shmu'yeshliktin' BD biyikligi 12 sm bolsa, woni'n' AC ta'repin ha'm maydani'n tabi'n'.

IV. Wo'zin'izdi sinap ko'rin' (u'lgi ushi'n baqlaw jumi'si')

1. Yeger $\cos \alpha = -\frac{8}{17}$ ha'm $90^\circ < \alpha < 180^\circ$ bolsa, $\sin \alpha$, $\operatorname{tg} \alpha$, $\operatorname{ctg} \alpha$ nege ten'?
2. Tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasi' $c = 18$ sm ha'm kateti $a = 4$ sm bolsa, woni'n' yekinshi kateti ha'm su'yir mu'yeshlerin tabi'n'.
3. Ten' ta'repli u'shmu'yeshliktin' medianasi' woni'n' ta'repinen kishi bolatug'i'ni'n da'liyllen'.
4. (*Qosi'msha*). To'rtmu'yeshliktin' ha'r bir ta'repi qalg'an ta'replerinin' qosi'ndi'si'nan kishi yekenin da'liyllen'.



Tariyx betlerinen. "Alti'n u'shmu'yeshlik"



A'yyemgi grekler, mu'yeshleri 36° , 72° , 72° bolg'an ten' qaptalli' u'shmu'yeshlikti — "alti'n u'shmu'yeshlik" dep atag'an. Wo'ni'n' ultani'ndag'i' mu'yesh bissektrisasi AD woni' yeki ten' qaptalli' u'shmu'yeshlikke bo'ledi (*I-su'wret*).

Haqi'yqattan da, AD bissektrisa bolg'ani' ushi'n, BAD ha'm DAC mu'yeshleri de 36° tan. Demek, ABD u'shmu'yeshligi ten' qaptalli'. ADC u'shmu'yeshliginde ADC mu'yeshi $180^\circ - 36^\circ - 72^\circ = 72^\circ$ boli'p, ACD mu'yeshine ten'. Demek, ADC u'shmu'yeshligi de ten' qaptalli'.

Na'tiyje. ABC u'shmu'yeshligi ACD u'shmu'yeshligine uqsas ha'm

$$\frac{AC}{AB} = \frac{CD}{AC}. \quad (1)$$

Yeger ABC u'shmu'yeshliginin' qaptal ta'replari $AB = BC = 1$ dep alsaq, woni'n' ultani' to'mendegishe tabi'ladi' (2-su'wret). $AC = a$ bolsi'n. Wonda

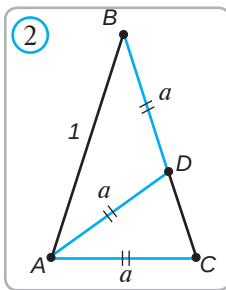
1. $AD = a$ boladi, sebebi $\triangle ACD$ ten' qaptalli'.

2. $BD = a$ boladi, sebebi $\triangle ABD$ ten' qaptalli'.

3. $CD = BC - BD = 1 - a$.

$$(1) \text{ ten'lik boyi'nsha: } \frac{a}{1} = \frac{1-a}{a}$$

$$\text{Bunnan } a^2 + a - 1 = 0 \text{ yaki } a = \frac{\sqrt{5} - 1}{2} \text{ yekenligin tabami'z.}$$



Ma'sele. $\sin 18^\circ$, $\cos 18^\circ$, $\sin 72^\circ$, $\cos 72^\circ$ ma'nislerin yesaplan'.

Sheshiliwi. Qaptal ta'repi $AB = BC = 1$ ha'm ultani $AC = a = \frac{\sqrt{5} - 1}{2}$ ten' bolg'an ABC "alti'n u'shmu'yeshlik" ti qaraymi'z (3-su'wret).

Woni'n' BE biyikligin ju'rgizemiz.

Tuwri' mu'yeshli ABE u'shmu'yeshlikten

$$\sin 18^\circ = \frac{AE}{AB} = \frac{a}{2} = \frac{\sqrt{5} - 1}{4}$$

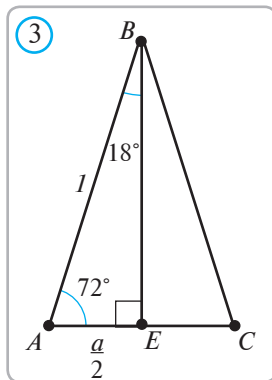
Bunnan paydalani'p, tabi'li'wi' talap yetilgen basqa ma'nislerdi yesaplaymi'z:

$$\cos 18^\circ = \sqrt{1 - \sin^2 18^\circ} = \frac{\sqrt{5} + 1}{4};$$

$$\sin 72^\circ = \sin(90^\circ - 18^\circ) = \cos 18^\circ = \frac{\sqrt{5} + 1}{4};$$

$$\cos 72^\circ = \sin 18^\circ = \frac{\sqrt{5} - 1}{4}.$$

$$\text{Juwabi': } \sin 18^\circ = \cos 72^\circ = \frac{\sqrt{5} - 1}{4}; \cos 18^\circ = \sin 72^\circ = \frac{\sqrt{5} + 1}{4};$$



Tariyx betlerinen

Ulug'bek (1394—1449) — ulli wo'zbek ali'mi' ha'm ma'mleket oyshi'li'. Negizgi ati Muxammed Tarag'ay. Wol sahibqiran Amir Temurdi'n' aqli'g'i'. Ulug'bektin' atasi' Shaxruxta ma'mleket oyshi'li' bolg'an. Ulug'bek shama menen 1425—1428-ji'llari' Samarqandti'n' a'tirapi'ndag'i' Obi Rahmat to'beshiginde wo'zinin' du'nyag'a belgili observatoriyasi'n quradi'. Observatoriya-ni'n' imarati' u'sh qabatli' boli'p, woni'n' tiykarg'i' a'sbabi' — kvadrantti'n' biyikligi 50 metr yedi. Ulug'bektin' yen' du'nyag'a belgili miyneti "Zijiy kuragoniy" dep atali'wshi' astronomiyali'q keste boladi'. Wol 1018 juldi'zdi' wo'z ishine alg'an.



Ulug'bek
(1394 — 1449)

1-teorema. U'shmuyeshliktin' maydani' woni'n' eki ta'repi menen usi' yeki ta'rep arasi'ndag'i' mu'yeshtin' sinusi'ni'n' ko'beymesi'ni'n' yari'mi'na ten'.

$\triangle ABC, BC=a, AC=b, \angle C$ (1-su'wret)

$$S_{ABC} = \frac{1}{2} ab \sin C$$

Da'liyllew. ABC u'shmuyeshliginin' BD biyikligin tu'siremiz. Wonda 1-su'wrette ko'rsetilgen u'sh jag'day boli'wi' mu'mkin.

Birinshi jag'daydi' qaraymi'z. BCD u'shmuyeshliginde $\sin C = \frac{BD}{BC}$. Bunnan $BD = BC \cdot \sin C = a \cdot \sin C$. Solay yetip,

$$S_{ABC} = \frac{1}{2} \cdot AC \cdot BD = \frac{1}{2} \cdot b \cdot a \cdot \sin C = \frac{1}{2} ab \sin C.$$

Yekinshi ha'm u'shinshi jag'daylardin' da'liylleniwin wo'z betin'izshe worinlan'. **Teorema da'liyllendi.**

1-teorema boyi'nsha, u'shmuyeshliktin' maydani' ushi'n

$$S_{ABC} = \frac{1}{2} bc \sin A \quad \text{ha'm} \quad S_{ABC} = \frac{1}{2} ac \sin B$$

formulalari' da wori'nli' boladi'.

1-ma'sele. ABC u'shmuyeshliginin' maydani' 24 sm^2 . Yeger $AC = 8 \text{ sm}$ ha'm $\angle A = 30^\circ$ bolsa, BC ta'repin tabi'n'.

Sheshiliwi. U'shmuyeshliktin' maydani'n mu'yeshtin' sinusi' arqali' tabi'w formulasi boyi'nsha,

$$S_{ABC} = \frac{1}{2} \cdot AB \cdot AC \cdot \sin A$$

Bunnan,

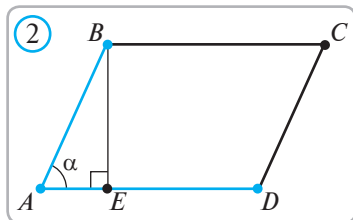
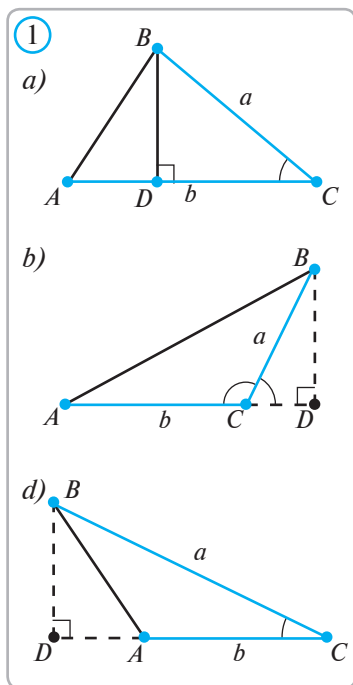
$$AB = \frac{2S_{ABC}}{AC \cdot \sin A} = \frac{2 \cdot 24}{8 \cdot \sin 30^\circ} = \frac{2 \cdot 24}{8 \cdot 0,5} = 12 \text{ (sm)}.$$

Juwabi': 12 sm.

2-ma'sele. Parallelogrammni'n' maydani' woni'n' yeki qon'si'las ta'repi ha'm bul ta'repler arasi'ndag'i' mu'yeshtin' sinusi'ni'n' ko'beymesine ten' yekenligin da'liyllen'.

$ABCD$ parallelogramm,
 $AB=a, AD=b, \angle A=\alpha$
(2-su'wret)

$$S_{ABCD} = ab \sin \alpha$$



Sheshiliwi. BE biyikligin tu'siremiz. ABE u'shmu'yeshliginde $\sin A = \frac{BE}{AB}$ yaki

$BE = AB \sin A = a \sin \alpha$. Wonda $S_{ABCD} = AD \cdot BE = ab \sin \alpha$.

2-teorema. To'rtmu'yeshliktn' maydani' woni'n' diagonal-lari' arasi'ndag'i' mu'yesh-tin' sinusi' ko'beymesinin' yari'mi'na ten'.

Da'liyilew. Diagonal-lardi'n' kesilisiwinen payda bolg'an mu'yeshlerdi qaraymi'z (3-su'wret):

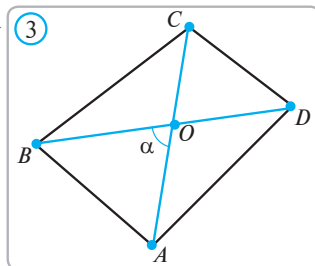
$\angle AOB = \alpha \quad \Leftarrow$ sha'rt boyi'nsha,

$\angle COD = \alpha \quad \Leftarrow \angle AOB$ g'a vertikal bolg'ani' ushi'n,

$\angle BOC = 180^\circ - \alpha \quad \Leftarrow \angle AOB$ g'a qon'silas bolg'ani' ushi'n,

$\angle DOA = 180^\circ - \alpha \quad \Leftarrow \angle BOC$ ke vertikal bolg'ani' ushi'n.

U'shmu'yeshliktn' maydani'n mu'yesh-tin' sinusi ja'rdeminde yasaplaw formulasi' boyi'nsha:



$$S_{AOB} = \frac{1}{2} AO \cdot OB \sin \alpha; \quad S_{BOC} = \frac{1}{2} BO \cdot OC \sin(180^\circ - \alpha) = \frac{1}{2} BO \cdot OC \sin \alpha;$$

$$S_{COD} = \frac{1}{2} CO \cdot OD \sin \alpha; \quad S_{DOA} = \frac{1}{2} DO \cdot OA \sin(180^\circ - \alpha) = \frac{1}{2} DO \cdot OA \sin \alpha.$$

Maydannin' qa'siyeti boyi'nsha: $S_{ABCD} = S_{AOB} + S_{BOC} + S_{COD} + S_{DOA} =$

$$\begin{aligned} &= \frac{1}{2} AO \cdot OB \sin \alpha + \frac{1}{2} BO \cdot OC \sin \alpha + \frac{1}{2} CO \cdot OD \sin \alpha + \frac{1}{2} DO \cdot OA \sin \alpha = \\ &= \frac{1}{2} (AO \cdot OB + BO \cdot OC + CO \cdot OD + DO \cdot OA) \sin \alpha = \frac{1}{2} \{ (OB \cdot (AO + OC) + \\ &\quad + OD \cdot (CO + OA)) \} \sin \alpha = \frac{1}{2} (OB \cdot AC + OD \cdot AC) \sin \alpha = \frac{1}{2} AC \cdot BD \sin \alpha. \end{aligned}$$

? **Soraw, ma'sele ha'm tapsi'r-malar**

Teorema da'liyillendi.

- 1-teoremanni' 1.b - ha'm 1.d - su'wrette su'wretlengen halda da'liyillen'.
2. Yeger a) $AB = 6$ sm, $AC = 4$ sm, $\angle A = 30^\circ$; b) $AC = 14$ sm, $BC = 7\sqrt{3}$ sm, $\angle C = 60^\circ$; d) $BC = 3$ sm, $AB = 4\sqrt{2}$ sm, $\angle B = 45^\circ$ bolsa, ABC u'shmu'yeshliginin' maydani'n tabi'n'.
3. Diagonali' 12 sm ha'm diagonal-lari' arasi'ndag'i' mu'yeshi 30° bolg'an tuwri'-mu'yeshliktn' maydani'n tabi'n'.
4. Ta'repi $7\sqrt{2}$ sm ha'm dog'al mu'yeshi 135° bolg'an romb maydani'n tabi'n'.
5. Rombi'ni'n' u'lken diagonali' 18 sm ha'm dog'al mu'yeshi 120° . Rombi'ni'n' maydani'n tabi'n'.
6. Maydani' $6\sqrt{2}$ sm² qaten' bolg'an ABC u'shmu'yeshliginde $\angle A = 45^\circ$. U'shmu'yeshliktn' AC ta'repin ha'm usi' ta'repke tu'sirilgen biyikligin tabi'n'.
- 7*. ABC u'shmu'yeshliginde $\angle A = \alpha$, B ha'm C to'belerinen tu'sirilgen biyiklikleri bolsa sa'ykes tu'rde h_b ha'm h_c bolsa, u'shmu'yeshliktn' maydani'n tabi'n'.
- 8*. ABC u'shmu'yeshliginde $AB = 8$ sm, $AC = 12$ sm ha'm $\angle A = 60^\circ$ bolsa, woni'n' AD bissektrisasi'n tabi'n' (ko'rsetpe: $S_{ABC} = S_{ABD} + S_{ADC}$).

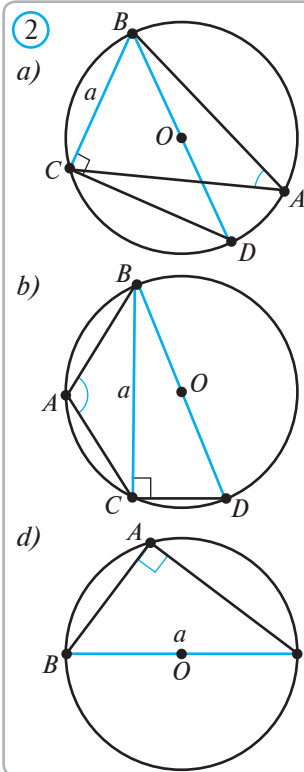
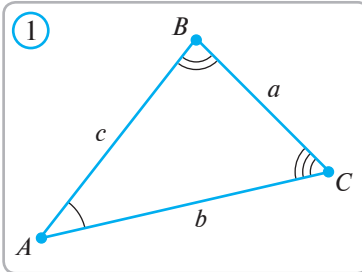


Teorema. (Sinuslar teoremasi'). U'shmu'yeshliktin' ta'repleri qarsisindag'i mu'yeshlerdin' sinuslarina proporcional.

$\triangle ABC, AB=c, BC=a, CA=b$ (1-su'wret)



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Da'liyllew. U'shmu'yeshliktin' maydani'n mu'yeshstin' sinusi' arqali' tabi'w formulasi' boyi'nsha,
 $S = \frac{1}{2}ab \sin C, S = \frac{1}{2}bc \sin A, S = \frac{1}{2}ac \sin B.$ (❖)

Bul ten'liklerdin' birinshi ekewi boyi'nsha
 $\frac{1}{2}ab \sin C = \frac{1}{2}bc \sin A,$ demek $\frac{a}{\sin A} = \frac{c}{\sin C}.$

Sunday-aq (❖) ten'liklerinin' birinshi ha'm u'sh-inshisinen $\frac{c}{\sin C} = \frac{b}{\sin B}$ ten'ligin paydayetemiz.

Solay yetip, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$

Teorema da'liyllendi.

1-ma'sele. ABC u'shmu'yeshliginde $AB = 14$ dm, $\angle A = 30^\circ, \angle C = 65^\circ$ (1-su'wret). BC ta'repin tabi'n'.

Sheshiliwi. Sinuslar teoremasi boyi'nsha

$$\frac{AB}{\sin C} = \frac{BC}{\sin A}.$$
 Wonnan

$$BC = \frac{AB \cdot \sin A}{\sin C} = \frac{14 \cdot \sin 30^\circ}{\sin 65^\circ} \approx \frac{14 \cdot 0,5}{0,9} \approx 7,78 \text{ (dm)}.$$

Yesletpe: Trigonometriyalig' funkciyalardi'n' ma'nisleri arnawli' kalkulyator yaki kesteler ja'rdeminde tabi'ladi'. Bul jerde $\sin 65^\circ \approx 0,9$ yekenligin sabaqli'qti'n' 153-betindegi kesteden ani'qladi'q.

Juwabi': 7,78 dm.

2-ma'sele. U'shmu'yeshliktin' ta'repinin' usi' ta'reptin' qarsi'si'ndag'i' mu'yeshinin' sinusi'na qat'nasi', u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'ber diametrine

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

ten' yekenligin da'liyllen'. (1-su'wret).

Sheshiliwi: Ani'q, sinuslar teoremasi' boyi'nsha $\frac{a}{\sin A} = 2R$ ten'ligin da'liyllew jetkilikli yekenligi ko'rinip tur. U'sh jag'day boli'wi' mu'mkin:

1-jag'day: $\angle A$ — su'yir mu'yesh (2.a-su'wret); 2-jag'day: $\angle A$ — dog'al mu'yesh (2.b-su'wret); 3-jag'day: $\angle A$ — tuwri' mu'yesh (2.d-su'wret).

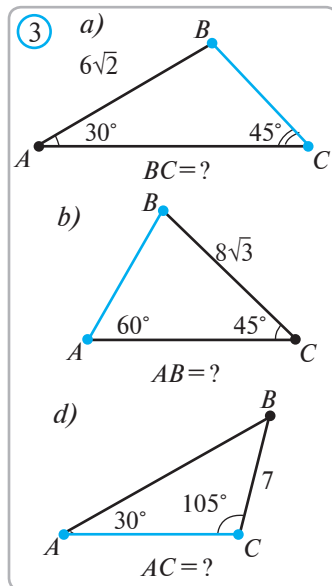
1-jag'daydi' qaraymi'z: C ha'm D noqatlari'n tutasti'rami'z. BCD — tuwri' mu'yeshli u'shmu'yeshlik, sebebi $\angle BCD$ mu'yeshi BD diametrine uri'nadi'. $\triangle BCD$ da: $BC = BD \cdot \sin D = 2R \sin D$. Biraq, $\angle D = \angle A$, sebebi wolar bir BC dog'ag'a tirelgen ishley si'zi'lg'an mu'yeshler. Wonda

$$BC = 2R \sin A \quad \text{yaki} \quad \frac{a}{\sin A} = 2R.$$

Qalg'an jag'daylardi' wo'z yerkin'izshe da'liyllen'. (Ko'rsetpe: $\angle D = 180^\circ - \angle A$ yekenliginen, 3-jag'dayda $a = 2R$ yekenliginen paydalanin')

? Soraw, ma'sele ha'm tapsi'rmalar

- U'shmu'yeshliktin' q'legen ta'repinin' usi' ta'repin' qarama-qarsi'si'ndag'i mu'yeshlin' sinusi'na qatnasi u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'ber diametrine ten' yekenligin 2-ma'selede keltirilgen 2-ha'm 3-jag'daylar ushi'n da'liyllen'.
- 3-su'wrette berilgenler boyi'nsha, soralg'an kesindilerdi tabi'n'.
- Yeger ABC u'shmu'yeshliginde:
 - $\sin A = 0,4$; $BC = 6$ sm ha'm $AB = 5$ sm bolsa, $\sin C$ ni
 - $\sin B = \frac{3}{5}$; $AC = 8$ dm ha'm $BC = 7$ dm bolsa, $\sin A$ ni
 - $\sin C = \frac{4}{7}$; $AB = 6$ m ha'm $AC = 8$ m bolsa, $\sin B$ ni tabi'n'.
- U'shmu'yeshliktin' mu'yeshi 30° qa ten'. Woni'n' qarama-qarsisindag'i ta'rep 4,8 dm. U'shmu'yeshlikke si'rtlay si'zi'lg'an shen'ber radiusin yesaplan'.
- U'shmu'yeshliktin' bir ta'repi u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'ber radiusina ten'. U'shmu'yeshliktin' usi' ta'repinin' qarama-qarsi'si'ndag'i mu'yeshin tabi'n'. Bunda yeki jag'daydi qarawg'a tuwra keliwine itibar berin'.
- ABC u'shmu'yeshligi ushi'n $AB:BC:CA = \sin C:\sin A:\sin B$ ten'ligi orinli bolatuginin tiykarlap berin'. $\sin A:\sin B:\sin C = 3:5:7$ ten'ligi duri's boliwi mu'mkin be?
- Yeger ABC u'shmu'yeshliginde $BC = 20$ m, $AC = 13$ m ha'm $\angle A = 67^\circ$ bolsa, u'shmu'yeshliktin' AB ta'repin, B ha'm C mu'yeshlerin tabi'n'.
- Yeger ABC u'shmu'yeshliginde $BC = 18$ dm, $\angle A = 42^\circ$, $\angle B = 62^\circ$ bolsa, u'shmu'yeshliktin' C mu'yeshin, AB ha'm AC ta'replerin tabi'n'.



Tuwri' mu'yeshli u'shmu'yeshlikte tuwri' mu'yeshtin' qarsi'si'ndag'i' ta'rep (gipotenuza) tin' kvadrati' qalg'an ta'repler (katetler) din' kvadratlari'ni'n' qosi'ndi'si'na ten'.

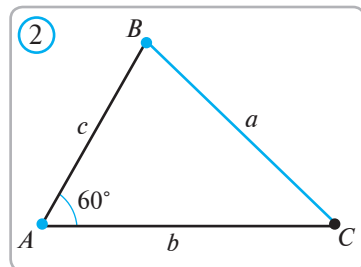
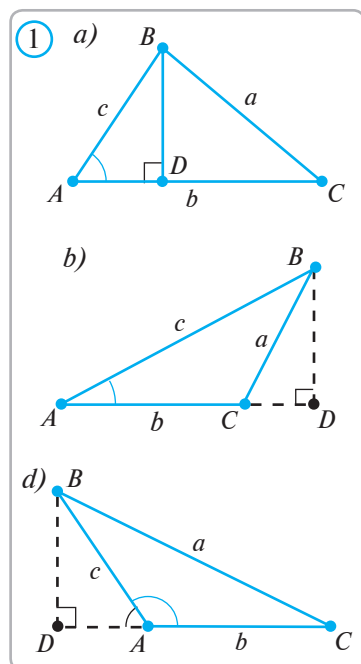
Al, tuwri' bolmag'an mu'yesh ushi'n-she? To'mendegi teorema usi' tuwrali.

Teorema. (Kosinuslar teoremasi'). U'shmu'yeshliktin' qa'legen ta'repinin' kvadrati', qalg'an yeki ta'repinin' kvadratlari'ni'n' qosi'ndi'si' usi yeki ta'rep penen wolar arasi'ndag'i' mu'yeshtin' kosinusi' ko'beymesinin' yeki yeselengen ayi'rmasi'na ten'.

$$\triangle ABC, AB=c, BC=a, CA=b \text{ (1-su'wret)}$$



$$a^2 = b^2 + c^2 - 2bc \cos A$$



Da'tiyllew. ABC u'shmu'yeshliginin' BD biyikligin ju'rgizemiz. D noqati' AC ta'repte (1.a-su'wretler) yaki woni'n' dawami'nda (1.b- ha'm 1.d-su'wretler) boli'wi' mu'mkin. Birinshi jag'daydi qaraymi'z. Tuwri' mu'yeshli BCD u'shmu'yeshlikte Pifagor teoremasi' boyi'nsha,

$$BC^2 = BD^2 + DC^2.$$

$$DC = AC - AD \text{ bolg'ani' ushi'n:}$$

$$BC^2 = BD^2 + (AC - AD)^2 = BD^2 + AC^2 - 2 \cdot AC \cdot AD + AD^2.$$

Tuwri' mu'yeshli ABD u'shmu'yeshliginde $BD^2 + AD^2 = AB^2$ ha'm $AD = AB \cos A$ yekenligin yesapqa ali'p, keyingi ten'likten

$$BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cdot \cos A,$$

yag'niy $a^2 = b^2 + c^2 - 2bc \cos A$ ten'ligine iye bolami'z.

Teorema da'liyllendi.

1. b-su'wrette su'wretlengen jag'dayda $DC = AD - AC$, 1.d-su'wrette su'wretlengen jag'dayda $DC = AD + AC$ ha'm $\cos(180^\circ - A) = -\cos A$ ten'liklerinen paydalani'p, kosinuslar teoremasi'n wo'z betin'izshe da'liyllen'.

Yeslertpe. Kosinuslar teoremasi' Pifagor teoremasi'ni'n' uliwmalasqan tu'ri boladi'. $\angle A = 90^\circ$ bolg'anda ($\cos 90^\circ = 0$ bolg'ani' ushi'n) kosinuslar teoremasi'nan Pifagor teoremasi' kelip shig'adi.

1-ma'sele. ABC u'shmu'yeshliginde $AB = 6sm$, $AC = 7sm$, $\angle A = 60^\circ$ (2-su'wret). BC ta'repin tabi'n'.

Sheshiliwi. Kosinuslar teoremasiboyi'nsha, $a^2 = b^2 + c^2 - 2bc \cos A$ yaki $BC^2 = AC^2 + AB^2 - 2AC \cdot AB \cdot \cos A$ bolg'anligi ushi'n,

$$BC^2 = 7^2 + 6^2 - 2 \cdot 7 \cdot 6 \cdot \cos 60^\circ = 49 + 36 - 84 \cdot \frac{1}{2} = 43,$$

ya'g'niy $BC = \sqrt{43}$ sm. **Juwabi':** $\sqrt{43}$ sm.

Sunday-aq, kosinuslar teoremasi'nan paydalani'p, ta'repleri belgili bolg'an u'shmu'yeshliktin' mu'yeshlerin tabiw mu'mkin:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}. \quad (1)$$

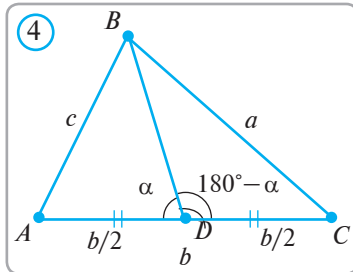
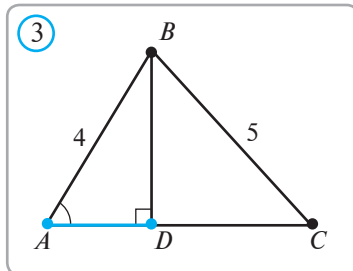
2-ma'sele. ABC u'shmu'yeshliginin' ta'repleri $a = 5$ m, $b = 6$ m ha'm $c = 4$ m. Kishi ta'reptin' u'lken ta'reptegi proekciyasini tabi'n' (3-su'wret).

Sheshiliwi. (1) formula tiykarinda $\cos A$ ni' tabami'z:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{6^2 + 4^2 - 5^2}{2 \cdot 6 \cdot 4} = \frac{9}{16}.$$

Tuwri' mu'yeshli ABD u'shmu'yeshliginde $AD = AB \cdot \cos A$ bolg'ani' ushi'n $AD = 4 \cdot \frac{9}{16} = 2,25$ (m).

Juwabi': 2,25 m.



? Soraw, ma'sele ha'm tapsi'rmalar

- Kosinuslar teoremasi'n 1.b-ha'm 1.d-su'wrette su'wretlengen jag'daylarda da'liyllen'.
- ABC u'shmu'yeshliginde: a) $AC=3$ sm, $BC=4$ sm ha'm $\angle C=60^\circ$ bolsa, AB ni'; b) $AB=4$ m, $BC=4\sqrt{2}$ m ha'm $\angle B=45^\circ$ bolsa AC ni'; d) $AB=7$ dm, $AC=6\sqrt{3}$ dm ha'm $\angle A=150^\circ$ bolsa BC ni tabi'n'.
- Ta'repleri 5 sm, 6 sm, 7 sm bolg'an u'shmu'yeshliktin' mu'yeshlerinin' kosinuslari'n tabi'n'.
- ABC u'shmu'yeshliginde $AB=10$ sm, $BC=12$ m, ha'm $\sin B=0,6$ bolsa, AC ta'repin tabi'n'.
- Parallelogrammni'n' diagonallari' 10 sm, 12 sm ha'm wolar arasi'ndag'i mu'yesh 60° qa ten'. Parallelogrammnin' ta'replerin tabi'n'.
- Ta'repleri 5 sm ha'm 7 sm bolg'an parallelogrammni'n' bir mu'yeshi 120° qa ten'. Woni'n' diagonallari'n tabi'n'.
- Ta'repleri a, b, c bolg'an ABC u'shmu'yeshliginin' BD medianasi $BD = \frac{1}{2} \sqrt{2a^2 + 2c^2 - b^2}$ formulasi menen yesaplanatug'i'ni'n da'liyllen' (4-su'wret).
- Ta'repleri 6 m, 7 m ha'm 8 m bolg'an u'shmu'yeshliktin' medianalari'n tabi'n'.
- 3-ma'seledagi u'shmu'yeshlik bissektrisalarin tabi'n'.
- 3-ma'seledagi u'shmu'yeshliktin' biyikliklerin tabi'n'.

Aldi'n'g'i' sabaqlarda da'liyllengen sinuslar ha'm kosinuslar teoremlari'nan u'shmu'yeshliklarga tiyisli ha'r qi'yli' ma'selelerdi sheshiwde na'tiyjeli paydalani'w mu'mkin. Bul sabaqta bul teoremlardi'n' ayi'ri'm qollani'wlari'na toqtap wo'temiz.

1. Kosinuslar teoremasi' u'shmu'yeshliktin' mu'yeshlerin tappastan, wonin' mu'yeshleri boyi'nsha tu'rin (su'yir, dog'al yaki tuwri' mu'yeshli yekenligin) ani'qlawg'a imkaniyat beredi. Haqiqattan da,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

formulasi'nda

- 1) yeger $b^2 + c^2 > a^2$ bolsa, $\cos A > 0$. Demek, A — su'yir mu'yesh;
- 2) yeger $b^2 + c^2 = a^2$ bolsa, $\cos A = 0$. Demek, A — tuwri' mu'yesh;
- 3) yeger $b^2 + c^2 < a^2$ bolsa, $\cos A < 0$. Demek, A — dog'al mu'yesh;

$b^2 + c^2 = a^2$ ten'ligi yaki $b^2 + c^2 < a^2$ ten'sizligi a — u'shmu'yeshliktin' yen' u'lken ta'repi bolg'an jag'dayda g'ana ori'nlanadi. Demek, u'shmu'yeshliktin' tuwri' yaki dog'al mu'yeshi woni'n' yen' u'lken ta'repinin' qarama-qarsi'si'nda jatadi.

U'shmu'yeshlin' yen' u'lken ta'repinin' shamasina qarap, bul u'shmu'yeshliktin' qanday (su'yir, dog'al, tuwri' mu'yeshli) u'shmu'yeshlik yekenligi haqqindag'i' juwmaqqa keliw mu'mkin.

 **1-ma'sele.** Ta'repleri 5 m, 6 m ha'm 7 m bolg'an u'shmu'yeshliktin' mu'yeshlerin tappastan, wonin' tu'rin ani'qlan'.

Sheshiliwi. Yen' u'lken mu'yesh qarsi'si'nda yen' u'lken ta'rep jatadi'. Soni'n' ushi'n, yeger $a=7$, $b=6$, $c=5$ bolsa, $\angle A$ yen' u'lken mu'yesh boladi'.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{36 + 25 - 49}{2 \cdot 6 \cdot 5} = \frac{12}{60} = \frac{1}{5} > 0.$$

Demek, A — su'yir mu'yesh, berilgen u'shmu'yeshlik bolsa su'yir mu'yeshli boladi'.

2. U'shmu'yeshliktin' maydani'n woni'n' yeki ta'repi ha'm wolar arasi'ndag'i' mu'yeshi arqali' yesaplaw formulasi',

$$S = \frac{1}{2}bc\sin A$$

ha'm $\sin A = \frac{a}{2R}$ formulalardan u'shmu'yeshlik maydani'n yesaplaw ushi'n

$$S = \frac{abc}{4R}$$

formulani ha'm u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusin yesaplaw ushi'n

$$R = \frac{abc}{4S}$$

formulasi'n payda yetemiz.

 **2-ma'sele.** Ta'repleri $a=5$, $b=6$, $c=10$ bolg'an u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'ber radiusi'n tabi'n'.

Sheshiliwi. Geron formulasi'nan paydalani'p, u'shmu'yeshliktin' maydani'n tabami'z:

$$p = \frac{a+b+c}{2} = \frac{5+6+10}{2} = 11,$$

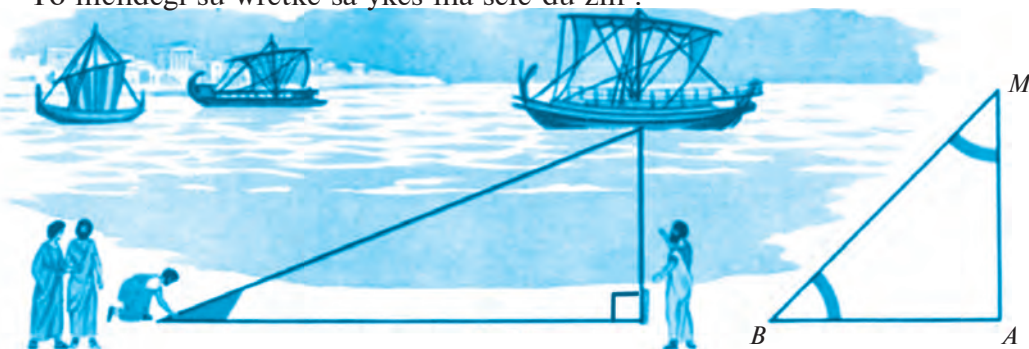
$$S = \sqrt{p(p-a)(p-b)(p-c)} = \sqrt{11(11-5)(11-6)(11-10)} = \sqrt{11 \cdot 6 \cdot 4} = \sqrt{264} \approx 16,3.$$

Wonda, $R = \frac{abc}{4S} \approx \frac{5 \cdot 6 \cdot 10}{4 \cdot 16,3} \approx 5,4.$

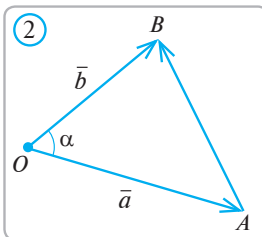
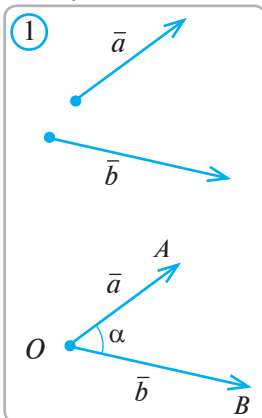
Juwabi': $\approx 5,4.$

Soraw, ma'sele ha'm tapsi'rmalar

1. Yeger $AB=7$ sm, $BC=8$ sm, $CA=9$ sm bolsa, ABC u'shmu'yeshliginin' yen' u'lken ha'm yen' kishi mu'yeshin tabi'n'.
2. Yeger ABC u'shmu'yeshliginde $\angle A=47^\circ$, $\angle B=58^\circ$ bolsa, u'shmu'yeshliktin' yen' u'lken ha'm yen' kishi ta'replerin ani'qlan'.
3. U'shmu'yeshliktin' u'sh ta'repi berilgen:
a) $a=5$, $b=4$, $c=4$; b) $a=17$, $b=8$, $c=15$; d) $a=9$, $b=5$, $c=6$.
U'shmu'yeshliktin' su'yir mu'yeshli, tuwri' mu'yeshli yaki dog'al mu'yeshli yekenligin ani'qlan'.
4. Ta'repleri a) 13, 14, 15; b) 15, 13, 4; d) 35, 29, 8; e) 4, 5, 7 bolg'an u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusi'n tabi'n'.
5. ABC u'shmu'yeshliginin' AB ta'repinde D noqati belgilengen. CD kesindisi AC ha'm BC kesindilerinin' keminde birewinen kishi yekenligin da'liyllen'.
6. U'shmu'yeshliktin' u'lken mu'yeshi qarsi'si'nda u'lken ta'repi jatatug'i'ni'n da'liyllen'.
7. U'shmu'yeshliktin' u'lken ta'repi qarsi'si'nda u'lken mu'yeshi jatatug'i'ni'n da'lillen'.
- 8*. ABC u'shmu'yeshliginin' CD medianasi' ju'rgizilgen. Yeger $AC > BC$ bolsa, ACD mu'yeshi BCD mu'yeshinen kishi bolatug'i'ni'n da'liyllen'.
9. To'mendegi su'wretke sa'ykes ma'sele du'zin'.



Vektorlardin' skalyar ko'beymesini tu'sinigi ha'm qa'siyetleri menen 8-klasta tani'sqan yedin'iz. Yeki vektordi'n' skalyar ko'beymesini wolardi'n' koordinatalari arqali' an'lati'lg'an yedi. To'mende kosinuslar teoremasi' ja'rdeminde vektorlardi'n' skalyar ko'beymesini ushi'n ja'ne bir a'hmiyetli formula keltirip shi'g'ari'ladi. Bunda skalyar ko'beyme vektorlardi'n' uzi'nli'g'i ha'm wolar arasi'ndag'i mu'yesh arqali' an'lati'ladi'.



Nol vektordan wo'zgeshe $\vec{a}$ ha'm $\vec{b}$ vektorlari' berilgen bolsi'n. Qa'legen O noqattan $\vec{OA} = \vec{a}$ ha'm $\vec{OB} = \vec{b}$ vektorlari'n qoyami'z. $\vec{a}$ ha'm $\vec{b}$ vektorlari' arasi'ndag'i mu'yesh dep AOB mu'yeshke ayti'ladi' (1-su'wret). Bir qi'yli' bag'itlang'an vektorlar arasi'ndag'i mu'yesh 0° qa'ten' dep yesaplanadi'.

Yeger yeki vektor arasi'ndag'i mu'yesh 90° qaten' bolsa, wolar **perpendikulyar** delinedi.

Yesletip wo'temiz:

1. $\vec{a}(a_1; a_2)$ vektori'ni'n' uzi'nli'g'i':

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2}.$$

2. $\vec{a}(a_1; a_2)$ ha'm $\vec{b}(b_1; b_2)$ vektorlari'ni'n' skalyar ko'beymesini,

$$\vec{a}\vec{b} = a_1b_1 + a_2b_2$$

formulalari' menen ani'qlanatug'in yedi.

Kollinear yemes $\vec{a}$ ha'm $\vec{b}$ vektorlari'n qaraymi'z. Qa'legen O noqatinan $\vec{OA} = \vec{a}$ ha'm $\vec{OB} = \vec{b}$ vektorlari'n qaraymi'z (2-su'wret). $\angle AOB = \alpha$ bolsi'n. Wonda, bir ta'repten

kosinuslar teoremasi' boyi'nsha,

$$AB^2 = OA^2 + OB^2 - 2 \cdot OA \cdot OB \cos \alpha. \quad (1)$$

Yekinshi ta'repten

$$AB^2 = |\vec{AB}|^2 = |\vec{OB} - \vec{OA}|^2 = (\vec{OA} - \vec{OB})^2 = \vec{OA}^2 + \vec{OB}^2 - 2\vec{OA} \cdot \vec{OB}. \quad (2)$$

Demek, (1) ha'm (2) boyi'nsha, $\vec{OA} \cdot \vec{OB} = OA \cdot OB \cos \alpha$ yaki $\vec{a}\vec{b} = |\vec{a}| \cdot |\vec{b}| \cos \alpha$.

Na'tiyje. Nol vektordan wo'zgeshe $\vec{a}(a_1; a_2)$ ha'm $\vec{b}(b_1; b_2)$ vektorlari' arasi'ndag'i mu'yeshi ushi'n

$$\cos \alpha = \frac{\vec{a}\vec{b}}{|\vec{a}| \cdot |\vec{b}|} \quad \text{yaki} \quad \cos \alpha = \frac{a_1b_1 + a_2b_2}{\sqrt{a_1^2 + a_2^2} \sqrt{b_1^2 + b_2^2}}$$

formulasi' orinli'.



Ma'sele. $\vec{a}(1;2)$ ha'm $\vec{b}(4;-2)$ vektorlari' arasi'ndag'i' mu'yeshti tabi'n'.

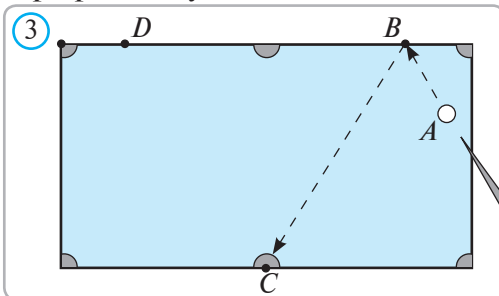
Sheshiliwi. Berilgen vektorlar arasi'ndag'i' mu'yeshti α dep belgilesek, formula boyi'nsha,

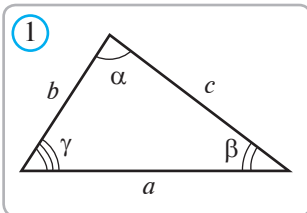
$$\cos \alpha = \frac{1 \cdot 4 + 2 \cdot (-2)}{\sqrt{1^2 + 2^2} \cdot \sqrt{4^2 + (-2)^2}} = \frac{4 - 4}{\sqrt{5} \cdot \sqrt{20}} = 0. \quad \alpha = 90^\circ. \quad \text{Juwabi': } 90^\circ.$$



Soraw, ma'sele ha'm tapsi'rmalar

- Yeger $\vec{a}$ ha'm $\vec{b}$ vektorlari ushi'n **a)** $a=4, b=5, \alpha=30^\circ$; **b)** $a=8, b=7, \alpha=45^\circ$; d) $a=2,4, b=10, \alpha=60^\circ$; e) $a=0,8, b=\frac{1}{4}, \alpha=40^\circ$ bolsa, bul vektorlardin' skalyar ko'beymesin tabi'n' (bul jerde α — $\vec{a}$ ha'm $\vec{b}$ vektorlari' arasi'ndag'i' mu'yesh).
- a) $\vec{a}(\frac{1}{4};-1)$ ha'm $\vec{b}(2;3)$; b) $\vec{a}(-5;6)$ ha'm $\vec{b}(6;5)$; d) $\vec{a}(1,5;2)$ ha'm $\vec{b}(4;-2)$ vektorlari'ni'n' skalyar ko'beymesin yesaplan' ha'm wolar arasi'ndag'i' mu'yeshti tabi'n'.
- $ABCD$ rombi'ni'n' diagonallari O noqatindakesilisedi ha'm bunda $BD=AB=4$ sm.
a) $\vec{AB}$ ha'm $\vec{AD}$; b) $\vec{AB}$ ha'm $\vec{AC}$; d) $\vec{AD}$ ha'm $\vec{DC}$; e) $\vec{AC}$ ha'm $\vec{OD}$ vektorlari'ni'n' skalyar ko'beymesin ha'm bul vektorlardin' arasi'ndag'i' mu'yeshti tabi'n'.
- Nol vektordan wo'zgeshe $\vec{a}$ ha'm $\vec{b}$ vektorlari' berilgen bolsi'n. $\vec{a} \cdot \vec{b} = 0$ bolg'anda bul vektorlar perpendikulyar bolatug'i'ni'n' ha'm kerisinshe a ha'm b vektorlari' perpendikulyar bolsa, $a \cdot b = 0$ bolatug'i'ni'n' da'liyllen'.
- x tin' qanday ma'nisinde a) $\vec{a}(4;5)$ ha'm $\vec{b}(x;6)$; **b)** $\vec{a}(x;1)$ ha'm $\vec{b}(3;2)$; **d)** $\vec{a}(0;-3)$ ha'm $\vec{b}(5;x)$ vektorlari wo'z ara perpendiculyar boladi?
- $\vec{a}(3;3), \vec{b}(2;-2), \vec{c}(-1;-4)$ ha'm $\vec{d}(-4;1)$ vektorlari arasi'nan wo'z-ara perpendiculyar jupli'qlari'n tabi'n'.
- $a^2 = |\vec{a}|^2$ ten'ligin da'liyllen'.
- Bilyard oyi'ni'nda A noqatta turg'an shar soqqi'dan keyin bilyard stoli' jaqlawi'na B noqatta uri'ldi' ha'm bag'dari'n wo'zgertip C noqattag'i sebetshage tu'sti (3-su'wret). Yeger $AB=40$ sm, $BC=150$ sm ha'm $\angle ABD=120^\circ$ bolsa $\vec{AB} \cdot \vec{BC}$ skalyar ko'beymeni tabin.
- $F(-3, 4)$ ku'sh ta'siri asti'nda noqat $A(5,-1)$ jag'daydan $B(2, 1)$ jag'dayg'a wo'tti. Bul processte qanday jumis orinlandi?





U'shmu'yeshliktin' ta'replerin a, b, c menen, al bul ta'replerdin' qarama-qarsi'si'ndag'i' mu'yeshlerdi sa'ykes tu'rde α, β, γ menen belgileymiz (*1-su'wret*). U'shmu'yeshliktin' ta'replerin ha'm mu'yeshlerin bir atama menen woni'n' **elementleri** dep ataydi'.

U'shmu'yeshlikti ani'qlawshi' beri'lgen elementi boyi'nsha, woni'n' qalg'an elementin tabii u'shmu'yeshlikti sheshiw dep ayti'ladi'.

1-ma'sele. (U'shmu'yeshlikti berilgen bir ta'repi ha'm wog'an irgeles jatqan mu'yeshleri boyi'nsha sheshiw). Yeger u'shmu'yeshlikte $a=6, \beta=60^\circ$ ha'm $\gamma=45^\circ$ bolsa, woni'n' u'shinshi mu'yeshin ha'm qalg'an yeki ta'repin tabi'n'.

Sheshiliwi. 1. U'shmu'yeshliktin' mu'yeshlerinin' qosi'ndi'si' 180° bolg'ani' ushi'n

$$\alpha = 180^\circ - \beta - \gamma = 180^\circ - 60^\circ - 45^\circ = 75^\circ.$$

Sinuslar teoremasi'nan paydalani'p, qalg'an yeki ta'repin tabami'z:

$$2. \frac{a}{\sin \alpha} = \frac{b}{\sin \beta} \quad \text{ten'likten} \quad b = a \cdot \frac{\sin \beta}{\sin \alpha} = 6 \cdot \frac{\sin 60^\circ}{\sin 75^\circ} \approx 6 \cdot \frac{0,8660}{0,9659} \approx 5,3794 \approx 5,4.$$

($\sin 60^\circ$ ha'm $\sin 75^\circ$ ma'nislari mikrokalkulyatordayaki sabaqli'qti'n' 153-betin-degi kesteden tawi'p qoyi'ldi').

$$3. \frac{a}{\sin \alpha} = \frac{c}{\sin \gamma} \quad \text{ten'liginen} \quad c = a \cdot \frac{\sin \gamma}{\sin \alpha} = 6 \cdot \frac{\sin 45^\circ}{\sin 75^\circ} \approx 6 \cdot \frac{0,7071}{0,9659} \approx 4,3924 \approx 4,4.$$

Juwabi': $\alpha=75^\circ$; $\beta \approx 5,4$; $c \approx 4,4$.

2-ma'sele. (U'shmu'yeshlikti berilgen yeki ta'repi ha'm wolar arasi'ndag'i' mu'yeshi boyi'nsha sheshiw). Yeger u'shmu'yeshlikte $a=6, b=4$ ha'm $\gamma=120^\circ$ bolsa, woni'n' u'shinshi c ta'repin ha'm qalg'an mu'yeshlerin tabi'n'.

Sheshiliwi. 1. Kosinuslar teoremasi'nan paydalani'p, u'shmu'yeshliktin' u'shinshi c ta'repin tabami'z:

$$c = \sqrt{a^2 + b^2 - 2ab \cos \gamma} = \sqrt{36 + 16 - 2 \cdot 6 \cdot 4 \cdot (-0,5)} = \sqrt{76} \approx 8,7.$$

2. Yendi, u'shmu'yeshliktin' u'sh ta'repin bilgen halda, kosinuslar teoremasi'nan paydalani'p, u'shmu'yeshliktin' qalg'an mu'yeshlerin tabami'z:

$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc} = \frac{4^2 + 76 - 6^2}{2 \cdot 4 \cdot \sqrt{76}} \approx 0,8046.$$

$\cos \alpha \approx 0,8046$ ten'ligi tiykarinda α mu'yeshinin' ma'nisin 153-bettegi kesteden ani'qlaymi'z (α — su'yir mu'yesh): $\alpha \approx 36^\circ$.

$$3. \beta = 180^\circ - \alpha - \gamma \approx 180^\circ - (36^\circ + 120^\circ) = 24^\circ.$$

Juwabi': $c \approx 8,7$; $\alpha \approx 36^\circ$, $\beta \approx 24^\circ$.



3-ma'sele. (U'shmu'yeshlikti berilgen u'sh ta'repi boyi'nsha sheshiw). Yeger u'shmu'yeshlikte $a=10$, $b=6$ ha'm $c=13$ bolsa, woni'n' mu'yeshlerin tabi'n'.

Sheshiliwi. 1. U'shmu'yeshlik su'yir mu'yeshli boliwi' yaki bolmasli'g'i'n u'lken ta'reptin' qarama-qarsi'si'ndag'i' mu'yesh tin' kosinusi'ni'n' belgisine qarap ani'qlaymi'z:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{100 + 36 - 169}{2 \cdot 10 \cdot 6} = -\frac{33}{120} \approx -0,275 < 0.$$

Demek, C — dog'al mu'yesh yeken. Buni' 153-bettegi kestden C mu'yeshinin' shamasini' ani'qlawda yesapqa alami'z. Kesteden kosinusi' 0,275 ge ten' mu'yesh $\angle C_1 = 74^\circ$ yekenligin tabami'z. Wonda $\cos(180^\circ - \alpha) = -\cos \alpha$ formulasi' boyi'nsha,

$$\angle C = 180^\circ - \angle C_1 = 180^\circ - 74^\circ = 106^\circ.$$

2. Sinuslar teoremasi' boyi'nsha, $\frac{a}{\sin A} = \frac{c}{\sin C}$. Bunnan

$$\sin A = \frac{a \cdot \sin C}{c} = \frac{10 \cdot \sin 106^\circ}{13} = \frac{10 \cdot \sin 74^\circ}{13} \approx \frac{10 \cdot 0,9615}{13} \approx 0,7396. A — su'yir mu'yesh bolg'a-ni' ushi'n 153-bettegi kestden $\angle A \approx 47^\circ$ yekenligin ani'qlaymiz.$$

3. $\angle B \approx 180^\circ - (106^\circ + 47^\circ) = 26^\circ$.

Juwabi': $\angle A \approx 47^\circ$, $\angle B \approx 26^\circ$, $\angle C \approx 106^\circ$.



Soraw, ma'sele ha'm tapsi'rmalar

1. U'shmu'yeshlik tin' bir ta'repi ha'm wog'an irgeles jatqan yeki mu'yeshi berilgen:

a) $a=5$ sm, $\beta=45^\circ$, $\gamma=45^\circ$; b) $a=20$ sm, $\alpha=75^\circ$, $\beta=60^\circ$;

d) $a=35$ sm, $\beta=40^\circ$, $\gamma=120^\circ$; e) $b=12$ sm, $\alpha=36^\circ$, $\beta=25^\circ$.

U'shmu'yeshlik tin' to'besinin' mu'yeshin ha'm qalg'an yeki ta'repin tabi'n'.

2. U'shmu'yeshlik tin' yeki ta'repi ha'm wolar arasi'ndag'i' mu'yeshi berilgen:

a) $a=6$, $b=4$, $\gamma=60^\circ$; b) $a=14$, $b=43$, $\gamma=130^\circ$;

d) $b=17$, $c=9$, $\alpha=85^\circ$; e) $b=14$, $c=10$, $\alpha=145^\circ$.

U'shmu'yeshlik tin' qalg'an mu'yeshlerin ha'm u'shinshi ta'repin tabi'n'.

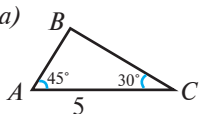
3. U'shmu'yeshlik tin' u'sh ta'repi berilgen: a) $a=2$, $b=3$, $c=4$; b) $a=7$, $b=2$, $c=8$; d) $a=4$, $b=5$, $c=7$; e) $a=15$, $b=24$, $c=18$.

U'shmu'yeshlik tin' mu'yeshlerin tabi'n'.

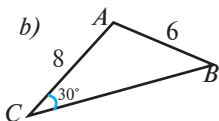
4. U'shmu'yeshlik tin' yeki ta'repi ha'm bul ta'replerden birinin' qarama-qarsisindag'i' mu'yeshi berilgen. U'shmu'yeshlik tin' qalg'an ta'repi ha'm mu'yeshlerin tabi'n': a) $a=12$, $b=5$, $\alpha=120^\circ$; b) $a=27$, $b=9$, $\alpha=138^\circ$; d) $b=2$, $c=2$, $\alpha=60^\circ$; e) $b=6$, $c=8$, $\alpha=30^\circ$.

5. 2-su'wrette berilgen mag'li'wmatlar tiykarinda u'shmu'yeshlikti sheshin'.

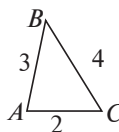
2 a)



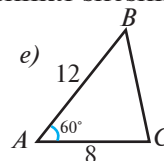
b)



d)



e)

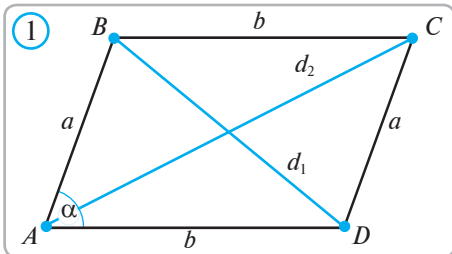


1-ma'sele. Parallelogramm diagonallari'ni'n' kvadratlari'ni'n' qosi'ndi'si' ta'replerinin' kvadratlari'ni'n' qosi'ndi'si'nin' ten' yekenligin da'liyllen'.

$ABCD$ — parallelogramm, $AB=a$,
 $AD=b$, $BD=d_1$, $AC=d_2$ (1-su'wret).



$$d_1^2 + d_2^2 = 2(a^2 + b^2)$$



Sheshiliwi. $ABCD$ parallelogrammni'n' A mu'yeshi α g'a ten' bolsi'n. Wonda $\angle B = 180^\circ - \alpha$. ABD ha'm ABC u'shmu'yeshliklerine kosinuslar teoremasi'n qollanami'z (1-su'wret):

$$d_1^2 = a^2 + b^2 - 2ab \cos \alpha, \quad (1)$$

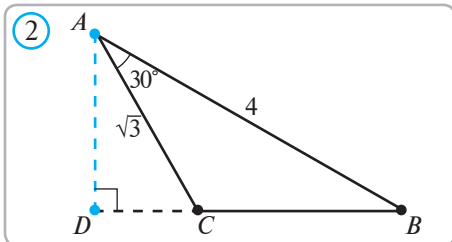
$$d_2^2 = a^2 + b^2 - 2ab \cos(180^\circ - \alpha).$$

$\cos(180^\circ - \alpha) = -\cos \alpha$ ten'ligin yesapqa alsaq,

$$d_2^2 = a^2 + b^2 + 2ab \cos \alpha. \quad (2)$$

(1) ha'm (2) ten'liklerinin' sa'ykes ma'nislerin qosip $d_1^2 + d_2^2 = 2(a^2 + b^2)$ ten'ligin payda yetemiz.

2-ma'sele. ABC u'shmu'yeshliginde $\angle A = 30^\circ$, $AB = 4$, $AC = \sqrt{3}$ bolsa, u'shmu'yeshliktin' A to'besinen tu'sirilgan AD biyikligin tabi'n' (2-su'wret).



Sheshiliwi. 1) Kosinuslar teoremasi'nan paydalani'p, u'shmu'yeshliktin' BC ta'repin tabami'z:

$$BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cdot \cos A = 4^2 + (\sqrt{3})^2 - 2 \cdot 4 \cdot \sqrt{3} \cdot \cos 30^\circ = 7, \quad BC = \sqrt{7}.$$

2) Yendi u'shmu'yeshliktin' maydani'n tabami'z:

$$S = \frac{1}{2} \cdot AB \cdot AC \cdot \sin A = \frac{1}{2} \cdot \sqrt{3} \cdot 4 \cdot \sin 30^\circ = \sqrt{3}.$$

3) Tabilg'anlardan paydalani'p, u'shmu'yeshliktin' AD biyikligin tabami'z:

$$S = \frac{1}{2} \cdot BC \cdot AD \quad \text{formuladan} \quad AD = \frac{2S}{BC} = \frac{2\sqrt{3}}{\sqrt{7}} = \frac{2\sqrt{21}}{7}. \quad \text{Juwabi': } \frac{2\sqrt{21}}{7}.$$

3-ma'sele. Aydawshi' jol ha'reketi qag'iydalari'n buzi'p, saat 12⁰⁰ de ko'shenin' A noqati'nan Almazar ko'shesine qarap burildi' ha'm 140 km/saat tezlikte ha'reketin davam yetti (3-su'wret). Saat 12⁰⁰ de MAI xizmetkeri B noqati'nan taslaq jol boylap 70 km/saat tezlikte qag'i'yda buzg'an aydawshinin' jolin kesip

shi'g'i'w ushi'n jolg'a shi'qti'. MAI xizmetkeri kesilispede yag'ni'y C noqati'nda qag'iyda buzi'wshi'ni' toqtati'p qala ala ma?

Sheshiliwi: ABC u'shmu'yeshliginde

$$\angle C = 180^\circ - (\angle A + \angle B) = 180^\circ - (20^\circ + 50^\circ) = 180^\circ - 70^\circ = 110^\circ.$$

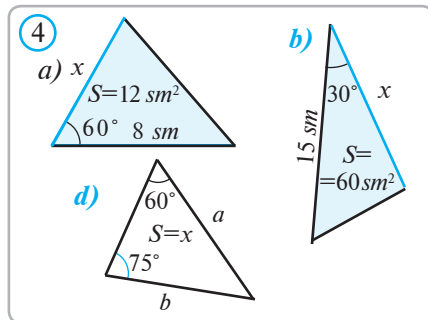
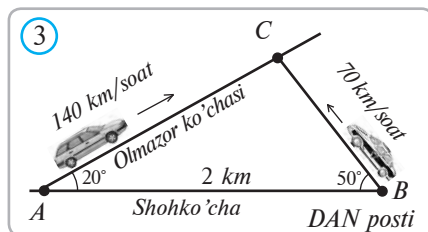
1. Almazar ko'shesindeki joldin' AC bo'liminin' uzi'nli'g'i'n tabami'z: Sinuslar teoremasi boyi'nsha, $\frac{AC}{\sin B} = \frac{AB}{\sin C}$. Bul ten'likten $AC = \frac{AB \cdot \sin B}{\sin C} = \frac{2 \cdot \sin 50^\circ}{\sin 110^\circ} = \frac{2 \cdot \sin 50^\circ}{\sin(90^\circ + 20^\circ)} = \frac{2 \cdot \sin 50^\circ}{\cos 20^\circ} \approx \frac{2 \cdot 0,766}{0,940} = \frac{1,532}{0,94} \approx 1,630$ (km). Bul joldi qag'iyda buzi'wshi aydawshi $\frac{1,630 \text{ km}}{140 \text{ km/saat}} \approx 0,0116 \text{ saat} = 0,012 \cdot 3600 \text{ sekund} \approx 42 \text{ sekundta}$ basip wo'tedi

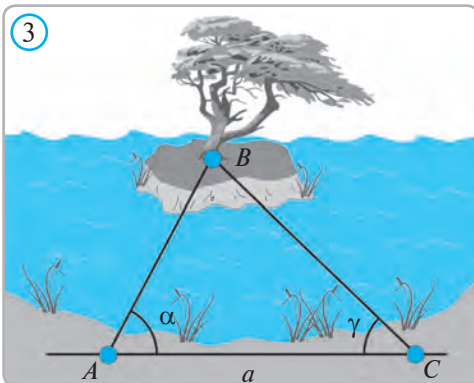
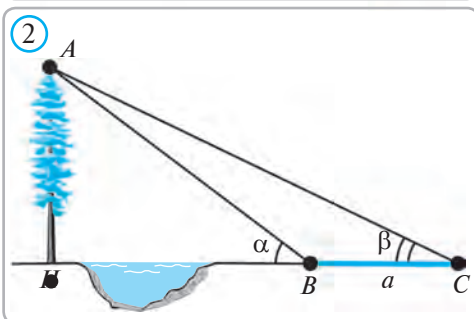
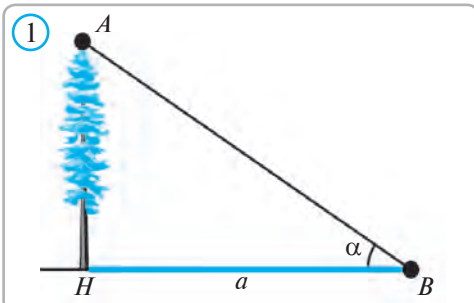
2. Yendi taslaq joldin' BC bo'legi uzi'nli'g'i'ni tabami'z: sinuslar teoremasi boyi'nsha, $\frac{BC}{\sin A} = \frac{AC}{\sin B}$. Bul ten'likten $BC = \frac{AC \cdot \sin A}{\sin B} = \frac{2 \cdot \sin 20^\circ}{\sin 50^\circ} = \frac{2 \cdot 0,342}{0,766} \approx 0,893$ (km).

Bul joldi MAI xizmetkeri $\frac{0,893 \text{ km}}{70 \text{ km/saat}} \approx 0,0128 \text{ saat} = 0,0128 \cdot 3600 \text{ sekund} \approx 46 \text{ sekundta}$ basip wo'tedi. Demek, C kesispesine MAI xizmetkeri aydawshidan keshirekjetip keledi yeken. **Ju'wabi':** Yaq.

? Soraw, ma'sele ha'm tapsi'rmlar

- 4-su'wrettegi mag'liwmatlar boyi'nshax tin' ma'nisin tabi'n'.
- ABC u'shmu'yeshliginin' CD biyikligi 4 m. Yeger $\angle A = 45^\circ$, $\angle B = 30^\circ$ bolsa, u'shmu'yeshliktin' ta'replerin tabi'n'.
- Bir noqatqa shamasi birdey bolg'an yeki ku'sh qoyi'lg'an (5-su'wret). Yeger bul ku'shlerdin' bag'itlari arasi'ndag'i mu'yesh 60° , bul ku'shlerdin' ten' ta'sir yetiwshisi 150 kg bolsa, bul ku'shlerdin' shamasin tabi'n'.
- U'shmu'yeshliktin' yeki ta'repi 7 dm ha'm 11 dm, u'shinshi ta'repine tu'sirilgen medianasi bolsa 6 dm. U'shmu'yeshliktin' u'shinshi ta'repin tabi'n'.
- Ta'repleri 6 sm ha'm 8 sm bolg'an parallelogrammnin' bir diagonal 12 sm bolsa, woni'n' yekinshi diagonalin' tabi'n'.
- U'shmu'yeshliktin' 17 sm ge ten' ta'repi qarsisindag'i mu'yeshi 60° qa ten'. U'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusin tabi'n'.
- Ten' qaptalli trapeciyanin' kishi ultani qaptal ta'repine ten', u'lken ultani bolsa 20 sm. Yeger trapeciyanin' bir mu'yeshi 120° bolsa, woni'n' perimetrin tabi'n'.





1. Biyiklikni wo'lshew. Aytayiq, nenin'dur (misali', terektin') AH biyikligin wo'lshew za'ru'r bolsi'n (*1-su'wret*).

a) Bunin' ushi'n B noqati'n belgileymiz ha'm BH aralig'i' a ni' ha'm HBA mu'yeshi α ni wo'lsheymiz. Wonda, tuwri' mu'yeshli ABH u'shmu'yeshliginde

$$AH = BH \operatorname{tg} \alpha = a \operatorname{tg} \alpha.$$

b) Yeger biyikliktin' ultani' H noqati' bari'p bolmaytug'in noqat bolsa (*2-su'wret*), joqari'dag'i' usil menen AH biyikligin ani'qlay almaymiz. Wonda, to'mendegishe jol tutami'z:

1) H noqati' menen bir tuwri'da jatqan B ha'm C noqatlari'n belgileymiz;

2) BC arali'qti' wo'lshep a ni' tabami'z;

3) ABH ha'm ACH mu'yeshlerin wo'lshep $\angle ABH = \alpha$ ha'm $\angle ACH = \beta$ lardi' tabami'z;

4) ABC u'shmu'yeshligine sinuslar teoremasi'n qollan ($\angle BAC = \alpha - \beta$)

$$\frac{AB}{\sin \beta} = \frac{a}{\sin(\alpha - \beta)} \quad \text{yag'niy} \quad AB = \frac{a \sin \beta}{\sin(\alpha - \beta)}.$$

5) Tuwri' mu'yeshli ABH u'shmu'yeshliginde AH biyikligin tabami'z:

$$AH = AB \sin \alpha = \frac{a \sin \alpha \cdot \sin \beta}{\sin(\alpha - \beta)}.$$

2. Bari'p bolmaytug'i'n noqatqa shekem bolg'an arali'qti' yesaplaw. Aytayi'q, A noqati'nan bari'p bolmaytug'in B noqati'na

shekem bolg'an arali'qti' yesaplaw kerek (*3-su'wret*). Bul ma'seleni u'shmu'yeshliklerdin' uqsasli'q belgilerinen paydalani'p sheshkenimizdi yesletip wo'temiz. Yendi bul ma'seleni sinuslar teoremasinan paydalani'p sheshemiz.

1) A ha'm B noqatlarinan ko'rinip turg'an tegis jerde C noqati'n belgileymiz.

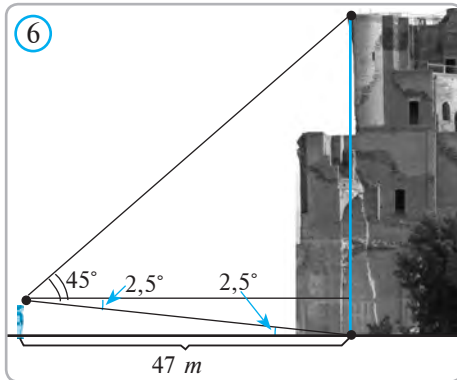
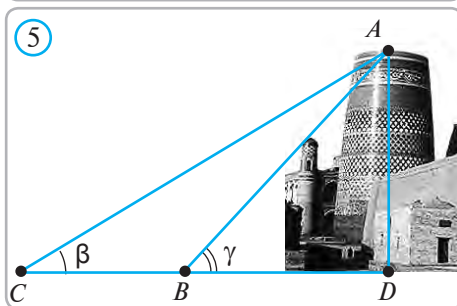
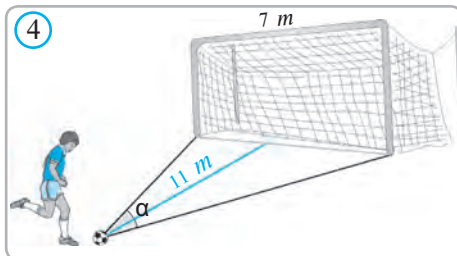
2) AC aralig'in wo'lsheymiz: $AC = a$. 3) A'sbaplar ja'rdeminde ACB ha'm BAC mu'yeshlerin wo'lsheymiz: $\angle BAC = \alpha$, $\angle BAC = \gamma$.

4) ABC u'shmu'yeshliginde $\angle B = 180^\circ - \alpha - \gamma$ bolg'ani' ushi'n, $\sin B = \sin(180^\circ - \alpha - \gamma) = \sin(\alpha + \gamma)$.

Sinuslar teoremasi boyi'nsha $\frac{AB}{\sin C} = \frac{AC}{\sin B}$ yaki $AB = \frac{a \sin \gamma}{\sin(\alpha + \gamma)}$.

? Soraw, ma'sele ha'm tapsi'rmalar

- 1-su'wrette $a = 12 \text{ m}$, $\alpha = 42^\circ$ bolsa, terektin' biyikligin yesaplan'.
- 2-su'wrette $a = 8 \text{ m}$, $\alpha = 43^\circ$, $\beta = 32^\circ$ bolsa, terektin' biyikligin yesaplan'.
- 3-su'wrette $a = 60 \text{ m}$, $\alpha = 62^\circ$, $\gamma = 44^\circ$ bolsa, AB aralig'in tabi'n'.
4. Futbol oyi'ni'nda 11 metrlik ja'riyma tobi'n da'rwazag'a bag'darlaw mu'yeshi α ni' tabi'n' (4-su'wret). Da'rwazani'n ken'ligi 7 m .
- 5-su'wrette Xiywa qalasindag'i' Kelteminara su'wretlengen. Yeger $\beta = 45^\circ$, $\gamma = 24^\circ$, $BC = 50 \text{ m}$ bolsa, Kelteminara biyikligin tabi'n'.
6. Sayaxatshi Shaxrisabz qalasi'ndag'i' Aqsaraydi wonnan 47 m arali'qtan tamashalap tur (6-su'wret). Yeger wog'an Aqsaray ultani' gorizontqa sali'sti'rg'anda $2,5^\circ$ qa ten' mu'yesh astinda, joqari' bo'limi bolsa 45° qa ten' mu'yesh asti'nda bolsa, Aqsaraydi'n' biyikligin tabi'n'.
7. U'sh jol ABC u'shmu'yeshligin quraydi'. Bul u'shmu'yeshlikte $\angle A = 20^\circ$, $\angle B = 150^\circ$. A noqati'nan jolg'a shi'qqan aydawshi' C noqati'na imkaniyati' bari'nsha tezirek jetip barmaqshi'. AC ha'm CB jollari' taslaq, AB asfalt jol boli'p, asfalt jolda taslaq jolg'a qarag'anda 2 barabar tezi'rek ha'reketleniw mu'mkin. Aydawshig'a qaysi' joldan ju'riwdi ma'sla'ha't beresiz?



⌚ Qi'ziqli' ma'sele. Pifagor teoremasi'ni'n' ja'ne bir "da'liyllemesi".

Tuwri' mu'yeshli ABC u'shmu'yeshliginde $a = c \sin \alpha$, $b = c \cos \alpha$. Bul yeki ten'likti kvadratqa ko'terip ag'zama-ag'za qossaq ha'm $\sin^2 \alpha + \cos^2 \alpha = 1$ yekenligin yesapqa alsaq, $a^2 + b^2 = c^2 \sin^2 \alpha + c^2 \cos^2 \alpha = c^2 (\sin^2 \alpha + \cos^2 \alpha) = c^2$.

Demek, $a^2 + b^2 = c^2$. Bul "da'liyllew" logikaliq jaqtan natuwri' yekenligin da'liyllen'.

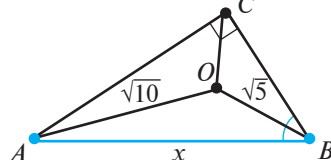
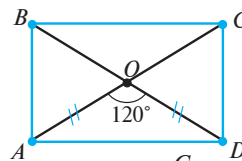
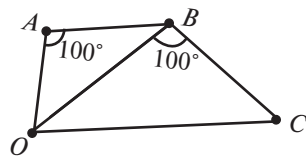
I. Testler

1. Ta'repleri a, b, c , sa'ykes mu'yeshleri α, β, γ , maydani' S bolg'an u'shmu'yeshlik ushi'n qaysi' formula naduri's?
 A. $a^2=b^2+c^2-2bccos\alpha$;
 D. $S=\frac{1}{2}absin\gamma$;
 B. $\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$;
 E. $S=\frac{1}{2}absin\alpha$.
2. Naduri's formulani tabi'n'.
 A. $\sin^2\alpha+\cos^2\alpha=1$;
 D. $\cos(180^\circ-\alpha)=\cos\alpha$;
 B. $\sin(180^\circ-\alpha)=\sin\alpha$;
 E. $\sin(90^\circ-\alpha)=\cos\alpha$.
3. U'shmu'yeshliktin' u'sh ta'repi belgili bolsa, qaysi teoremadan paydalani'p, woni'n' mu'yeshlerin tabi'w mu'mkin?
 A. Sinuslar teoremasi';
 D. Fales teoremasi';
 B. Kosinuslar teoremasi';
 E. Geron formulasi';
4. U'shmu'yeshliktin' bir mu'yeshi 137° qa, yekinshi mu'yeshi 15° qa ten'. Yeger bul u'shmu'yeshliktin' u'lken ta'repi 22 ge ten' bolsa, wonin' kishi ta'repin tabi'n'.
 A. 8,3; B. 9,3; D. 3,8; E. 6,5.
5. U'shmu'yeshliktin' 14 ha'm 19 g'a ten' bolg'an ta'repleri arasi'ndag'i' mu'yeshi 26° . Usi u'shmu'yeshliktin' u'shi'nshi ta'repin tabi'n'.
 A. 1,2; B. 5,4; D. 6,9; E. 19,7.
6. Yeger yeki vektordin' uzi'nli'qlari' $|\vec{a}|=2$, $|\vec{b}|=5$ ha'm wolar arasi'ndag'i' mu'yesh 45° bolsa, $\vec{a}$ ha'm $\vec{b}$ vektorlari'ni'n' skalyar ko'beymesin tabi'n'.
 A. 52; B. 32 D. 102; E. 2.
7. $\vec{a}(4; -1)$ ha'm $\vec{b}(2; 3)$ vektorlari'ni'n' skalyar ko'beymesin tabi'n'.
 A. 5; B. 3; D. 4; E. 9.
8. $\vec{a}(-\frac{1}{2}; \frac{\sqrt{3}}{2})$ ha'm $\vec{b}(\sqrt{3}; 1)$ vektorlar arasi'ndag'i' mu'yeshi tabi'n'.
 A. 30° ; B. 60° ; D. 90° ; E. 45° .
9. U'shmuyeshlik mu'yeshlerinin' qatnasi 3:2:1 si'yaqli' bolsa, wonin' ta'repleri qatnasi'n' tabi'n'.
 A. 3:2:1; B. 1:2:3; D. $2:\sqrt{3}:1$; E. $\sqrt{3}:\sqrt{2}:1$.
10. Ta'repi 3 sm bolg'an duri's u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'ber radiusin tabi'n'.
 A. $\sqrt{3}$; B. $\frac{\sqrt{3}}{3}$ D. $2\sqrt{3}$; E. $\frac{\sqrt{3}}{2}$

II. Ma'seleler

1. ABC u'shmu'yeshliginde $AB=6$ sm, $\angle A=60^\circ$, $\angle B=75^\circ$. BC ta'repin ha'm de ABC u'shmu'yeshligine si'rtlay si'zi'lg'an shen'berdin' radiusi'n tabi'n'.

2. Ta'repleri 5 sm, 6 sm ha'm 10 sm bolg'an u'shmu'yeshliktin' mu'yeshlerinin' kosinuslari'n tabi'n'.
3. ABC u'shmu'yeshliginde $\angle B = 60^\circ$, $AB = 6$ sm, $BC = 4$ sm. AC ta'repin ha'm de ABC u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusi'n tabi'n'.
4. Ta'repleri 51 sm, 52 sm ha'm 53 sm bolg'an u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusi'n tabi'n'.
5. U'shmu'yeshliktin' yeki ta'repi 14 sm ha'm 22 sm, u'shinshi ta'repine ju'rgizilgen medianasi' bolsa 12 sm. U'shmu'yeshliktin' u'shinshi ta'repin tabi'n'.
6. Parallelogarmmnin' diagonallari' 4 sm, $4\sqrt{2}$ sm ha'm wolar arasi'ndag'i' mu'yesh 45° . Parallelogrammnin' a) maydani'n; b) perimetrin; d) biyikliklerin tabi'n'.
7. Ta'repleri 3 ha'm 5 bolg'an parallelogarmmnin' bir diagonali 4 ke ten'. Onin' yekinshi diagonali'n tabi'n'.
8. Ta'repleri a) 2, 2 ha'm 2,5; b) 24, 7 ha'm 25; d) 9, 5 ha'm 6 bolg'an u'shmu'yeshliktin' tu'rin ani'qlan'.
9. Parallelogrammnin' ta'repleri $7\sqrt{3}$ ha'm 6 sm. Yeger oni'n' dog'al mu'yeshi 120° bolsa, woni'n' maydani'n tabi'n'.
10. ABC u'shmu'yeshliginin' AB , BC ta'replerinde N , K noqatlari' ali'ng'an. Wonda $BN = 2AN$, $3BK = 2KC$. Yeger $AB = 3$, $BC = 5$, $CA = 6$ bolsa, NK kesindisin tabi'n'.
11. ABC u'shmu'yeshliginde $\angle A = 30^\circ$, $BC = 7$ sm. U'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusi'n tabi'n'.
12. ABC u'shmu'yeshliginin' BE bissektrisasi' ju'rgizilgen. E noqati'nan BC ta'repke EF perpendikulyari tu'sirilgen. Yeger $EF = 3$, $\angle A = 30^\circ$ bolsa, AE ni tabi'n'.
13. $ABCD$ tuwri'mu'yeshliginin' AD ta'repinin' wortasi' N noqati'nda. Yeger $AB = 3$, $BC = 6$ bolsa, $\vec{NB} \cdot \vec{NC}$ skalyar ko'beymesin tabi'n'.
14. $\vec{a}(2; x)$, $\vec{b}(-4; 1)$ boli'p, $\vec{a} + \vec{b}$ ha'm $\vec{b}$ vektorlari perpendiculyar x ti' tabi'n'.
15. $\vec{m}(7; 3)$ ha'm $\vec{n}(-2; -5)$ vektorlari' arasi'ndag'i' mu'yeshi tabi'n'.
16. Su'wrette berilgenlerden paydalani'p, su'wretteg'i' yen' u'lken kesindini ani'qlan'.
17. $ABCD$ tuwri'mu'yeshliktin' diagonallari' O noqati'nda kesilisedi. Yeger $AO = 12$ sm, $\angle AOD = 120^\circ$ bolsa, to'rtmu'yeshliktin' perimetrin tabi'n'.
18. Tuwri' mu'yeshli ABC u'shmu'yeshliginin' bissektrisalari' O noqatinda kesilisedi ($\angle C = 90^\circ$). Yeger $OA = \sqrt{10}$, $OB = \sqrt{5}$ bolsa, AB gi potenuzani' tabi'n'.



III. Wo'zin'izdi si'nap ko'rin' (u'lgi ushi'n tekseriw jumi'si')

1. Ta'repleri $a=45$, $b=70$, $c=95$ bolg'an u'shmu'yeshliktin' yen' u'lken mu'yeshin tabi'n'.
2. U'shmu'yeshlikte $b=5$, $\alpha=30^\circ$, $\beta=50^\circ$ bolsa, u'shmu'yeshlikti sheshin'.
3. PKH u'shmu'yeshliginde $PK=6$, $KH=5$, $\angle PKH=100^\circ$. HF medianani'n' uzi'nli'g'i'n ha'm PFN u'shmu'yeshliginin' maydani'n tabi'n'.
4. (*Qosi'msha*). U'shmu'yeshlikte $a=\sqrt{3}$, $b=1$, $\alpha=135^\circ$ bolsa, β mu'yeshti tabi'n'.



Tariyix betlerinen. Sinus haqqinda

Sinus haqqi'ndag'i' mag'li'wmat da'slep IV–V a'sirlerdegi hind astronomlari'ni'n' shig'armalari'nda ushi'raydi.

Worta Aziyali'q ali'mlar al-Xorazmiy, Beruniy, Ibn Sino, Abdurahmon al-Haziniy (XII a'sir) sinus ushi'n «*al-jayb*» atamasi'n isletken.

Ha'zirgi sinus belgisin Simpson, Eyler, D'alamber, Lagranj (XVII a'sir) ha'm basqalar qollag'an.

«*Kosinus*» atamasi' lati'nsha «komplimenti sinus» atamasi'ni'n' qi'sqarti'ri'lg'ani', wol «qosi'msha sinus», ani'g'i'rag'i' «qosi'msha dog'ani'n' sinusi» dep ataladi'.

Kosinuslar teoremasi'n greklerde bilgen, wonin' da'liyili Yevklidtin "Negizler" shi'g'armasi'nda keltirilgen. Sinuslar teoremasi'ni'n' wo'zine tan da'liylin Abu Rayhan Beruniy ayti'p ketken.



Tariyix betlerinen. Beruniy (toli'q ati' — Abu Rayxan Muxammad ibn Axmad)



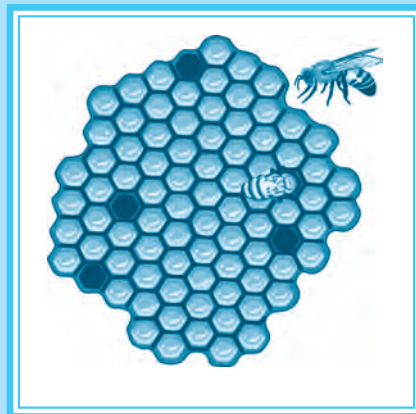
Beruniy
(973 — 1048)

(973–1048)— worta a'sirdin' ulli' enciklopedist ali'mi'. Wol Xorezm u'lkesinin' Qiyat qalasi'nda tuwi'lg'an. Qi'yat A'miwda'ryani'n' won' jag'asi' — ha'zirgi Beruniy qalasi'ni'n' worni'nda bolg'an, wol jaqi'n ku'nlerge shekem Shabbaz dep atalg'an. Beruniydin' matematika ha'm pa'nnin' basqa tarawlari'na qosqan u'lesin jazi'p qaldi'rg'an 150 den aslam miynetinen de ko'riw mu'mkin. Wolardan yen' ko'lemlilari — "Hindstan", "Estelikler", "Masud ni'zamlari", "Geodeziya", "Mineralogiya" ha'm "Astronomiya".

Beruniydin' ulli' miyneti "Masud ni'zamlari" tiykari'nan astronomiyag'a tiyisli bolsa da woni'n' matematikag'a tiyisli ko'plegen ashi'li'wlari' usi' miynetinde bayan yetilgen.

Bul miynetinde Beruniy yeki mu'yeshtin' qosi'ndi'si' ha'm ayirmasini'n' sinuslari', yeki eselengen ha'm yarim mu'yeshtin' sinuslari' haqqi'ndag'i' teoremlar menen ten' ku'shli bolg'an xordalar haqqi'nda teoremlari'n da'liyllegen, yeki gradusli' dog'ani'n' xordalari'n yesaplap tapqan, sinuslar ha'm tangensler kestelerin du'zgen, sinuslar teoremasi'n da'liyllegen.

III BAP



SHEN'BER UZI'NLI'G'I HA'M DO'N'GELEKTIN' MAYDANI'

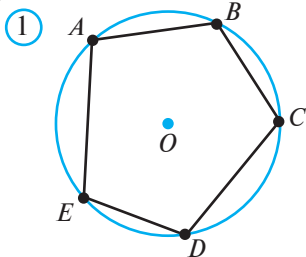
Bul bapti' u'yreniw na'tijjesinde siz to'mendegi bilim ha'm a'meliy ko'nlikpelerge iye bolasiz.

Bilimler:

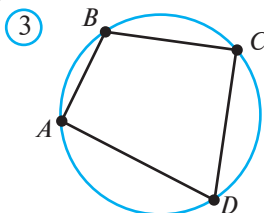
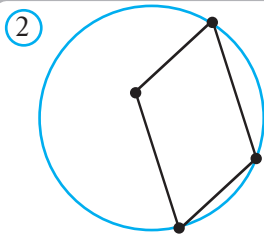
- √ *ko'pmu'yeshlikke si'rtlay ha'm ishley si'zi'lg'an shen'berlerdin' qa'siyetlerin biliw.*
- √ *duri's ko'pmu'yeshliklerdin' qa'siyetlerin biliw;*
- √ *duri's ko'pmu'yeshliktin' maydani'n yesaplaw formulalari'n biliw;*
- √ *shen'ber ha'm wonin' dog'asi'ni'n' uzi'nli'g'i'n yesaplaw formulalari'n biliw;*
- √ *do'n'gelek ha'm wonin' bo'leklerinin' maydani'n tabi'w formulalarin biliw;*
- √ *mu'yeshitin' radian wo'lshemin biliw.*

A'meliy ko'nlikpeler:

- √ *duri's ko'pmu'yeshliklerdi su'wretley aliw;*
- √ *duri's ko'pmu'yeshlikke si'rtlay ha'm ishley si'zi'lg'an shen'berlerdin' radiuslari'n taba ali'w;*
- √ *shen'ber ha'm dog'a uzi'nli'g'i'n yesaplay ali'w;*
- √ *do'n'gelek ha'm woni'n' bo'leklerinin' maydani'n yesaplay ali'w.*

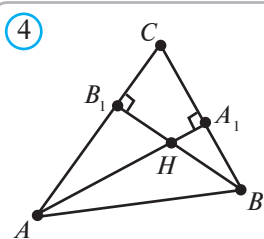


Shen'berge ishley si'zi'lg'an
besmu'yeshlik.
Besmu'yeshlikke si'rtlay
si'zi'lg'an shen'ber.



$$\angle A + \angle C = 180^\circ$$

$$\angle B + \angle D = 180^\circ$$



Ani'qlama. Yeger ko'pmu'yeshliktin' mu'yeshleri shen'berde jatsa, onda bul ko'pmu'yeshlik shen'berge **ishley si'zi'lg'an**, shen'ber bolsa ko'pmu'yeshlikke **si'rtlay si'zi'lg'an** delinedi (1-su'wret).

Qa'legen u'shmu'yeshlikke si'rtlay shen'ber si'zi'w mu'mkin yekenligi ha'm bul shen'berdin' worayi' u'shmu'yeshlik ta'replerinin' worta perpendiculylarlari' kesisken noqatta jatatug'i'ni'n 8-klasta u'yrengensiz.

Yeger ko'pmu'yeshliktin' mu'yeshlerinin' sani' u'shewden arti'q bolsa, ko'pmu'yeshlikke ha'r qashan da si'rtlay shen'ber si'zi'wg'a bola bermeydi. Mi'sali', tuwri' mu'yeshlikten wo'zgeshe parallelogramm ushi'n si'rtlay si'zi'lg'an shen'ber boli'wi' mu'mkin yemes (2-su'wret).

8-klastan ma'lim bolg'ani'nday, to'rtmu'yeshlikte qarama-qarsi' mu'yeshlerinin' qosi'ndi'si' 180° qa ten' bolg'anda ha'm tek usi'da jag'dayda g'ana si'rtlay shen'ber si'zi'w mu'mkin (3-su'wret).

1-ma'sele. Su'yir mu'yeshli ABC u'shmu'yeshliktin' AA_1 ha'm BB_1 biyikliklari H noqati'nda kesilisedi. A_1HB_1C to'rtmu'yeshligi shen'berge ishley si'zi'lg'an yekenligin da'liyl'llen'.

Sheshiliwi. $AA_1 \perp B$ ha'm $BB_1 \perp AC$ bolg'ani' ushi'n (4-su'wret). $\angle HB_1C = \angle HA_1C = 90^\circ$. Wonda $\angle HB_1C + \angle HA_1C = 180^\circ$. to'rtmu'yeshliktin' ishki mu'yeshlerinin' qosi'ndi'si' 360° bolg'ani' ushi'n: $\angle B_1CA_1 + \angle B_1HC = 180^\circ$. Demek, A_1HB_1C to'rtmu'yeshlikke si'rtlay shen'ber siziw mu'mkin. Shen'berge ishley si'zi'lg'an ko'pmu'yeshliktin' to'beleri shen'ber worayi'nan ten'dey qashiqliqta jatqani ushi'n shen'berdin' worayi ko'pmu'yeshliktin' ta'replerinin' worta perpendikulyari'nda jatadi' (5-su'wret). Demek, shen'berge ishley si'zi'lg'an ko'pmu'yeshliktin' ta'replerinin' worta perpendiculylarlari' bir noqatta kesilisiwi sha'rt.

2-ma'sele. Radiusi' 10 sm bolg'an shen'berge biyikligi 16 sm bolg'an ten' qaptalli su'yir mu'yeshli u'shmu'yeshlik ishley si'zi'lg'an. Woni'n' ta'replerin tabi'n'.

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Sunday-aq, tuwri' mu'yeshli ABD u'shmu'yeshliginde

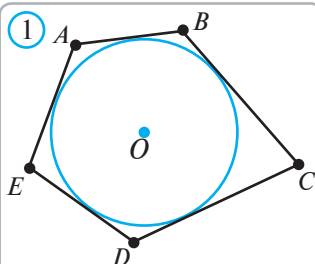
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The diagram shows an equilateral triangle ABC inscribed in a circle with center O . The altitude BD is drawn from vertex B to the base AC , meeting AC at D . The radius OA is also drawn. The diagram illustrates that the altitude BD passes through the center O and is perpendicular to AC at D . Tick marks on AD and DC indicate they are equal in length.

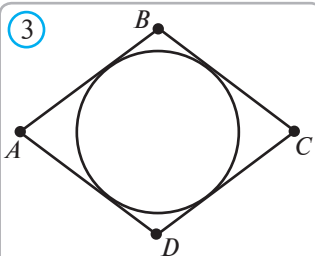
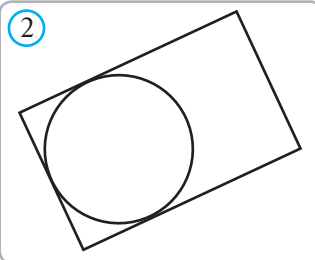
? Soraw, ma'sele ha'm tapsi'rmalar

-  **Qo'zi'qli' ma'sele.** Won alti' jasli' Galua (E. Galua—francuz matematigi, 1811—1832) kolledjde woqi'p ju'rgen waqi'tlari'nda, wog'an oqi'ti'wshi'si bir saat ishinde u'sh ma'seleni sheship beriwdi sorag'an. Wol sheshimi an'sat bolmag'an bul ma'selelerdi 15 minutta sheship, ha'mmeni hayran qaldirg'an.

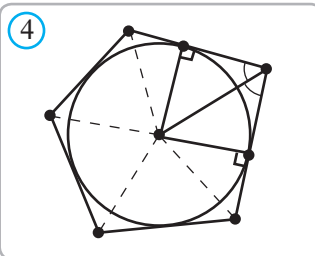
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Shen'berge si'rtlay si'zi'lg'an
ABCDE besmu'yeshlik.
ABCDE besmu'yeshlikke
ishley si'zi'lg'an shen'ber.



$$AB + CD = AD + BC$$



✓ Ani'qlama. Yeger ko'pmu'yeshliktin' barli'q ta'replari shen'berge uri'nba jasasa, wonda wol ko'p-mu'yeshlik shen'berge **si'rtlay si'zi'lg'an**, al shen'ber bolsa ko'pmu'yeshlikke **ishley si'zi'lg'an** delinedi (1-su'wret).

Qa'legen u'shmu'yeshlikke ishley shen'ber si'zi'w mu'mkin yekenligin ha'm bul shen'berdin' worayi' u'shmu'yeshliktin' bissektisalarini' kesiskan noqati'nda yekenligi menen 8-klasta tani'sqansi'z.

Yeger ko'pmu'yeshliktin' mu'yeshlerinin' sani' u'shten arti'q bolsa, bul ko'pmu'yeshlikke ha'r qashan da ishley shen'ber si'zi'w mu'mkin bola bermeydi. Mi'sali', kvadrattan wo'zgeshe tuwri' mu'yeshlikke ishley shen'ber si'zi'wg'a bolmaydi' (2-su'wret).

Ja'ne 8-klastan belgili, to'rtmu'yeshlikke tek birg'ana qarama-qarsi' ta'replerinin' qosi'ndi'si' ten' bolg'anda ishley shen'ber si'zi'w mu'mkin (3-su'wret).

Shen'berge si'rtlay si'zi'lg'an ko'pmu'yeshliktin' ta'replari shen'berge uri'ng'ani' ushi'n shen'ber worayi' usi' mu'yeshstin' bissektisiasi'nda jatadi' (4-su'wret). Demek, shen'berge si'rtlay si'zi'lg'an ko'pmu'yeshliktin' mu'yeshlerinin' bissektisalarini' bir noqatta kesilisedi.

Teorema. Yeger r radiusli shen'berge si'rtlay si'zi'lg'an ko'pmu'yeshliktin' maydani' S , yari'm perimetri p bolsa, $S = pr$ boladi'.

Da'liyllew. Teoremani'n da'liylleniwin shen'berge si'rtlay si'zi'lg'an $ABCDEF$ alti'mu'yeshlik ushi'n kel-tiremiz. Shen'ber worayi' O noqati'n ko'pmu'yeshliktin' to'beleri menen tutastiri'p, ko'pmu'yeshlikti u'shmu'yeshliklarga aji'ratamiz. Bul u'shmu'yeshliklerdin' biyikliklari r ge ten' (5-su'wret). Wonda

$$S = S_{AOB} + S_{BOC} + \dots + S_{FOA} = \frac{1}{2}AB \cdot r + \frac{1}{2}BC \cdot r + \dots + \frac{1}{2}FA \cdot r = \frac{AB + BC + \dots + FA}{2} \cdot r = pr.$$

Teorema da'liyllendi.

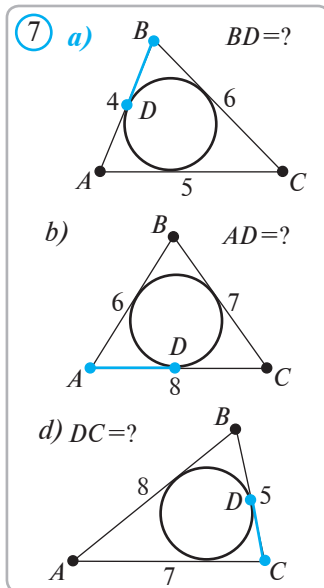
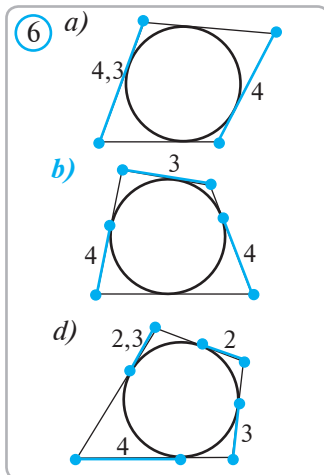
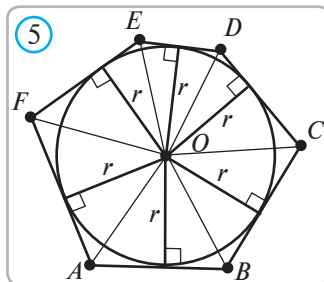
Ma'sele. Shen'berge si'rtlay si'zi'lg'an to'rtmu'yeshliktin' maydani' 21 sm^2 qa, perimetri bolsa 7 sm ge ten'. Shen'berdin' radiusi'n tabi'n'.

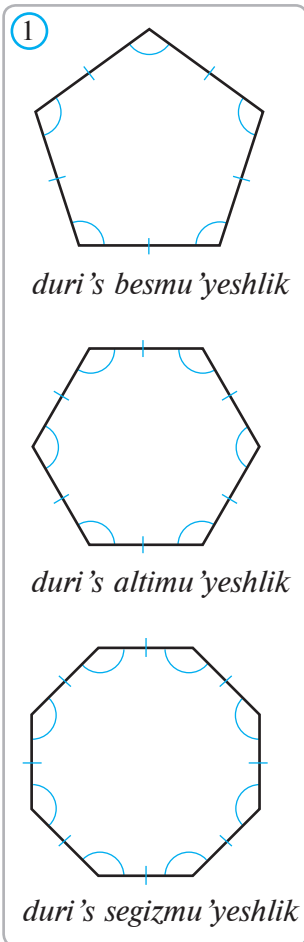
Sheshiliwi: $S = pr$ formula boyi'nsha

$$r = \frac{S}{p} = \frac{21}{3,5} = 6 \text{ (sm)}. \quad \text{Juwabi': } 6 \text{ sm.}$$

Soraw, ma'sele ha'm tapsi'rmalar

1. Ta'repi 6 sm bolg'an a) ten' ta'repli u'shmu'yeshlikke; b) kvadratqa si'rtlay si'zi'lg'an shen'ber radiusi'n tabi'n'.
2. Radiusi 5 sm bolg'an shen'berge si'rtlay si'zi'lg'an ko'pmu'yeshliktin' maydani' 18 sm^2 . Ko'pmu'yeshliktin' perimetrin tabi'n'.
3. To'rtmu'yeshliklerdin' perimetrin tabi'n' (6-su'wret).
4. 7-su'wret tegi mag'li'wmatlar tiykari'nda soralg'an kesindini tabi'n'.
5. Shen'berge si'rtlay si'zi'lg'an parallelogamm romb bolatug'i'ni'n da'liyllen'.
6. Tuwri' mu'yeshli u'shmu'yeshlikke ishley si'zi'lg'an shen'berdin' radiusi katetler qosi'ndi'si' menen gipotenuza ayi'rmasinin' yari'mi'na ten' yekenligin da'liyllen'.
7. Shen'berge si'rtlay si'zi'lg'an ten' qaptalli trapeciyanin' wortasizig'i woni'n' qaptal ta'repine ten' yekenligin da'liyllen'.
8. Ultanlari' 9 sm ha'm 16 sm bolg'an ten' qaptalli trapeciya shen'berge si'rtlay si'zi'lg'an. Shen'berdin' radiusi'n tabi'n'.
- 9*. $ABCD$ to'rtmu'yeshligi O worayi'na iye shen'berge si'rtlay si'zi'lg'an. AOB ha'm COD u'shmu'yeshliklerdin' maydanlari'ni'n' qosi'ndi'si' to'rtmu'yeshliktin' maydani'nin' yari'mi'na ten' yekenligin da'liyllen'.
- 10*. Shen'berge si'rtlay si'zi'lg'an trapeciyanin' ultanlari a ha'm b bolsa, woni'n' biyikligi $\sqrt{ab}$ g'a ten' yekenligin da'liyllen'.
- 11*. To'beleri $ABCD$ to'rtmu'yeshliktin' bissektritsalari'ni'n' kesilisiwinen payda bolg'an noqatlardan ibarat. $EF PQ$ to'rtmu'yeshlikke si'rtlay shen'ber si'zi'w mu'mkin yekenligin da'liyllen'.



**Jedellestiriwshi shi'ni'g'i'w**

1. Qanday figuralar ko'pmu'yeshlik delinedi?
2. Ko'pmu'yeshliktin' mu'yeshleri, qon'si'las ta'repleri, diagonallari' dep nege ayti'ladi'?
3. Do'n'es ko'pmu'yeshlik dep qanday ko'pmu'yeshlikke ayti'ladi'?
4. Do'n'es ko'pmu'yeshliktin' ishki mu'yeshlerinin' qosi'ndi'si' haqqi'ndag'i teoremani' ayti'n'.



Ani'qlama. Barli'q ta'repleri ten' ha'm barli'q mu'yeshleri ten' bolg'an do'n'es ko'pmu'yeshlik **duri's ko'pmu'yeshlik** delinedi.

Ten' ta'repli u'shmu'yeshlik, kvadrat duri's ko'pmu'yeshlikke mi'sal boladi'. 1-su'wrette duri's besmu'yeshlik, alti'mu'yeshlik ha'm segizmu'yeshlikler su'wretlengen.



Teorema. Duri's n mu'yeshlin' ha'r bir mu'yeshi $\frac{n-2}{n} \cdot 180^\circ$ qa ten'.

Da'liyllew. Duri's n mu'yeshlin' mu'yeshlerinin' qosi'ndi'si' $(n-2) \cdot 180^\circ$ qa ten' (8-klass). Demek, woni'n' ha'r-bir mu'yeshi $\frac{n-2}{n} \cdot 180^\circ$ qa ten'. **Teorema da'liyllendi.**



Ma'sele. Duri's $A_1A_2A_3A_4A_5$ besmu'yeshlikte A_1A_3 ha'm A_1A_4 diagonallari' ten' yekenligin ko'rsetin' (2-su'wret).



$A_1A_2A_3A_4A_5$ — duri's besmu'yeshlik



$A_1A_3 = A_1A_4$

Sheshiliwi. U'shmu'yeshliklerdin' ten'liginin' TMT belgisi boyi'nsha, $A_1A_2A_3$ ha'm $A_1A_5A_4$ u'shmu'yeshlikleri wo'z ara ten'. Haqi'yqattan da, duri's ko'pmu'yeshliktin' ta'repleri ten' ha'm mu'yeshleri ten' bolg'ani' ushi'n,

$$A_1A_2 = A_1A_5, A_2A_3 = A_5A_4 \text{ ha'm } \angle A_1A_2A_3 = \angle A_1A_5A_4.$$

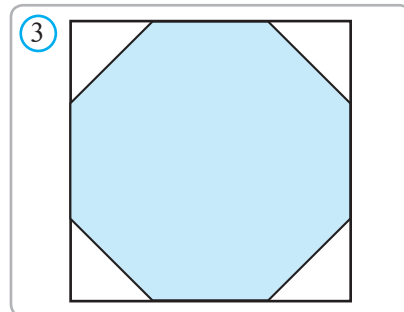
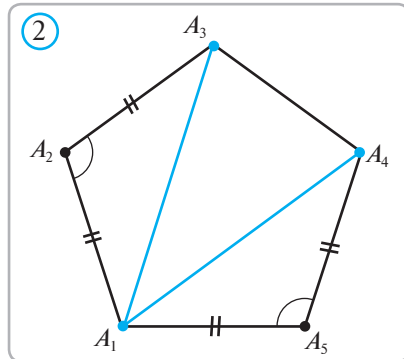
Demek, $\triangle A_1A_2A_3 = \triangle A_1A_5A_4$. Bunnan

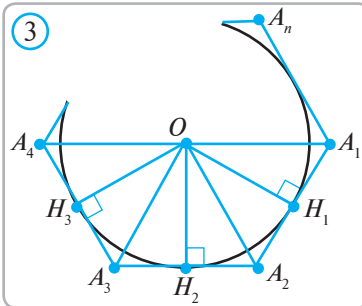
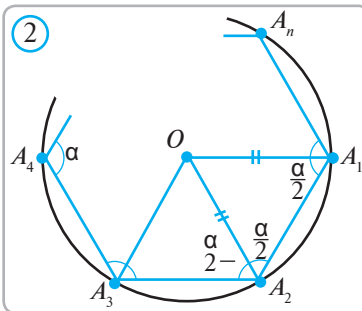
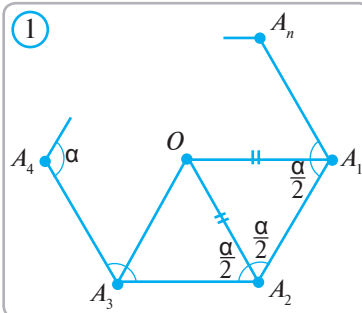
$$A_1A_3 = A_1A_4 \text{ yekenligi kelip shig'adi'}$$

Na'tiyje. Duri's besmu'yeshliktin' barli'q diagonallari' wo'z ara ten'.

? Soraw, ma'sele ha'm tapsi'rmalar

1. Duri's bolmag'an ko'pmu'yeshliklarga mi'sallar ayti'n' ha'm ne ushi'n duri's yemeligin tu'sindirin'.
2. To'mendegi tasti'yi'qlawlardan duri'slari'n tabi'n':
 - a) barli'q ta'repleri ten' bolg'an u'shmu'yeshlik duri's boladi';
 - b) barli'q ta'repleri ten' to'rtmu'yeshlik duri's boladi';
 - d) barli'q mu'yeshleri ten' to'rtmu'yeshlik duri's boladi';
 - e) barli'q mu'yeshleri ten' romb duri's boladi';
 - f) barli'q ta'repleri ten' tuwri' mu'yeshlik duri's boladi'.
3. Yeger a) $n=3$; b) $n=5$; d) $n=6$; e) $n=10$; f) $n=18$ bolsa, duri's n -mu'yeshliktin' mu'yeshlerin tabi'n'.
4. Duri's n mu'yeshliktin' si'rtqi' mu'yeshi nege ten' boladi? Yeger a) $n=3$; b) $n=5$; d) $n=6$; e) $n=10$; f) $n=12$ bolsa, duri's n mu'yeshliktin' si'rtqi' mu'yeshin tabi'n'.
5. Duri's n mu'yeshliktin' ha'r to'besinen birewden ali'ng'an si'rtqi' mu'yeshlerinin' qosi'ndi'si' 360° qa ten' yekenligin da'liyllen'.
6. Yeger duri's ko'pmu'yeshliktin' ha'r bir mu'yeshi a) 60° ; b) 90° ; d) 135° ; e) 150° bolsa, bul ko'pmu'yeshliktin' ta'replerinin' sani'n tabi'n'.
7. Duri's $ABCDEF$ altimu'yeshligi berilgen.
 - a) AC ha'm BD diagonallari'ni'n' ten'ligin da'liyllen'.
 - b) ACE — duri's u'shmu'yeshlik bolatug'i'ni'n' da'liyllen'.
 - d) AD , BE ha'm CF diagonallar wo'z ara ten'ligin da'liyllen'.
8. Ta'repi 10 sm bolg'an duri's a) besmu'yeshliktin'; b) altimu'yeshliktin'; d) segizmu'yeshliktin'; e) won eki mu'yeshliktin'; f) won segiz mu'yeshliktin' kishi diagonali'n yesaplan'.
9. Duri's to'rtmu'yeshliktin' kvadrat bolatug'i'ni'n' da'liyllen'.
- 10*. Kvadratti'n' ta'repi a g'a ten'. Woni'n' ta'replerine ha'r bir to'besinen baslap diagonali'ni'n' yari'mi'na ten' kesindiler qoy'ldi. Na'tiyjede 3-su'wrette su'wretlengen segizmu'yeshlik payda boldi'. Woni'n' tu'rin ani'qlan' ha'm maydani'n tabi'n'.





Jedellestiriwshi shinig'iw

1. Shen'berge ishley si'zi'lg'an ko'pmu'yeshlikdep, qanday ko'pmu'yeshlikke ayti'ladi'?
2. Shen'berge si'rtlay si'zi'lg'an ko'pmu'yeshlikdep, qanday ko'pmu'yeshlikke ayti'ladi'?
3. Qa'legen ko'pmu'yeshlik shen'berge ishley (si'rtlay) si'zi'lg'an boliwi mu'mkin be?

Teorema. Ha'r qanday duri's ko'pmu'yeshlikke ishley shen'ber de, si'rtlay shen'ber de si'zi'w mu'mkin.

Da'liyllew. Aytayi'q, $A_1A_2 \dots A_n$ — duri's ko'pmu'yeshlik, O — A_1 ha'm A_2 mu'yeshleri bissektrisalari'ni'n' kesilisiw noqati' bolsi'n. Bul duri's ko'pmu'yeshliktin' mu'yeshin α menen belgileyik.

1. $OA_1 = OA_2 = \dots = OA_n$ yekenligin da'liylleymiz (1-su'wret). Mu'yeshitin' bissektrisasi'ni'n' ta'riyplemesi boyi'nsha,

$$\angle OA_1A_2 = \angle OA_2A_1 = \frac{\alpha}{2}.$$

Demek, A_1OA_2 — ten' qaptalli' u'shmu'yeshlik. Bunnan, $OA_1 = OA_2$ kelip shig'adi'. $\triangle A_1A_2O$ ha'm $\triangle A_3A_2O$ u'shmu'yeshliklerdin' ten'liginin' TMT belgisi boyi'nsha ten', sebebi $A_1A_2 = A_3A_2$, A_2O — ta'repi uli'wma ha'm de

$$\angle OA_1A_2 = \angle OA_2A_1 = \frac{\alpha}{2}.$$

Soni'n' ushi'n $OA_3 = OA_1$. Da'l usinday jol tuti'p $OA_4 = OA_2$, $OA_5 = OA_3$ ha'm t. b. ten'likleri wori'nli bolatug'i'ni' ko'rsetiledi. Solay yeti'p, $OA_1 = OA_2 = \dots = OA_n$, yag'ni'y worayi O ha'm radiusi OA_1 bolg'an

shen'ber ko'pmu'yeshlikke si'rtlay si'zi'lg'an shen'berden ibarat boladi' (2-su'wret).

2. Joqari'da ayti'lg'anlar boyi'nsha ten' qaptalli A_1OA_2 , A_2OA_3 , ... A_nOA_1 u'shmu'yeshlikleri ten'. Sonin' ushi'n, bul u'shmu'yeshliklerdin' O to'besinen tu'sirilgen biyiklikleri de ten' boladi (3-su'wret).

$$OH_1 = OH_2 = \dots = OH_n.$$

Demek, O worayina iye ha'm radiusi' OH_1 , kesindige ten' bolg'an shen'ber ko'pmu'yeshliktin' barli'q ta'replerine uri'nadi. Yag'ni'y, bul shen'ber ko'pmu'yeshlikke ishley si'zi'lg'an shen'ber boladi'. **Teorema da'liyilendi.**

Na'tiyje. Duri's ko'pmu'yeshlikke ishley si'zi'lg'an ha'm si'rtlay si'zi'lg'an shen'berlerdin' oraylari bir noqatta boladi.

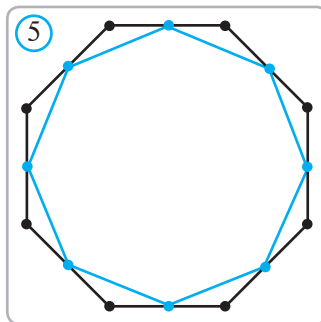
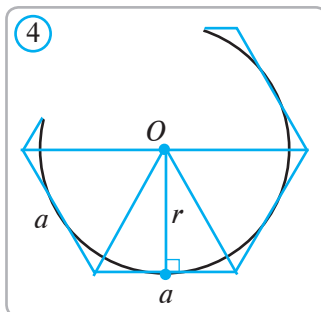
Bul noqat duri's ko'pmu'yeshliktin' **worayi** delinedi. Ko'p-mu'yeshliktin' worayin woni'n' yeki qon'silas to'beleri menen tutastiriwshi nurlardan ibarat mu'yesh (1-su'wrettegi A_1OA_2 , A_2OA_3 ... mu'yeshler) woni'n' **worayliq mu'yeshi** delinedi. Duri's ko'pmu'yeshliktin' worayinan ta'repine tu'sirilgen perpendikulyar (3-su'wrettegi OH_1 , OH_2 , ... kesindiler) woni'n' **apo-femasi** delinedi.

Ma'sele. Yeger duri's n mu'yeshliktin' ta'repi a , wog'an ishley si'zi'lg'an shen'berdin' radiusi r bolsa, woni'n' S maydani $S = \frac{1}{2}nar$ formulasi menen yesaplaw mu'mkin yekenligin da'liyllen'. (4-su'wret)

Sheshiliwi. Ko'pmu'yeshliktin' yarim perimetri $p = \frac{1}{2}na$ bolg'ani' ushi'n (shen'berge si'rtlay si'zi'lg'an, ko'pmu'yeshliktin' maydani'n tabiw) $S = pr$ formulasi boyi'nsha $S = \frac{1}{2}nar$ boladi.

? Soraw, ma'sele ha'm tapsi'rmalar

1. Maydani' 36 sm^2 bolg'an kvadratqa ishley ha'm si'rtlay si'zi'lg'an shen'berlerdin' radiuslarin tabi'n'.
2. Perimetri 18 sm bolg'an duri's u'shmu'yeshlikke ishley ha'm si'rtlay si'zi'lg'an shen'berlerdin' radiuslarin yesaplan'.
3. Duri's altimu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusi woni'n' ta'repine ten' bolatug'inin da'liyllen'.
4. Duri's ko'pmu'yeshliktin' ta'replerinin' wortalari ja'ne duri's ko'pmu'yeshlik payda yetetug'inin da'liyllen' (5-su'wret).
5. Duri's ko'pmu'yeshliktin' qa'legen yeki ta'repinin' worta perpendikulyarlari yaki bir noqatta kesilisiwi yaki bir tuwri'da jatatug'inin da'liyllen'.
- 6*. Duri's ko'pmu'yeshliktin' qa'legen yeki ta'repinin' worta perpendikulyarlari yaki bir noqatta kesilisiwi yaki bir tuwri'g'a jatatug'inin da'liyllen'.
7. Shen'berge ishley si'zi'lg'an duri's ko'pmu'yeshliktin' bir ta'repi shen'berden a) 60° ; b) 30° ; d) 36° ; e) 18° ; f) 72° qa ten' dog'a ajiratadi. Ko'pmu'yeshliktin' neshe ta'repi bar?
8. Qag'azdan alti ten'dey duri's u'shmu'yeshlik qirqi'p alin'. Wolardan paydalani'p, duri's altimu'yeshlik jasan'. Ta'repleri ten' bolg'an duri's altimu'yeshliktin' ha'm u'shmu'yeshliktin' maydanlarinin' qatnasin tabi'n'.



DURI'S KO'PMU'YESHLIKTIN' TA'REPMENEN SI'RTLAY HA'M ISHLEY SI'ZI'LG'AN SHEN'BERLER RADIUSLARI ARASI'NDAG'I BAYLANIS



c) Tuwri' mu'yeshli u'shmu'yeshliktin' su'yir mu'yeshinin' a) sinusi; b) kosinusi; c) tangensi dep nege ayti'ladi'?

Ta'repi a_n ge ten' bolg'an duri's n mu'yeshke si'rtlay si'zi'lg'an shen'berdin' R radiusi ha'm ishley si'zi'lg'an shen'berdin' r radiusin yesaplaw ushi'n tuwri' mu'yeshli ACO u'shmu'yeshliginen paydalanamiz. Bul jerde O — ko'pmu'yeshlikтин' AB ta'repinin' wortasi (*I-su'wret*). Wonda,



$$R=OA=\frac{AC}{\sin \beta}=\frac{a_n}{2 \sin \frac{180^\circ}{n}}; \quad r=OC=\frac{AC}{\operatorname{tg} \beta}=\frac{a_n}{2 \operatorname{tg} \frac{180^\circ}{n}};$$

$$r = OC = OA \cdot \cos \beta = R \cos \frac{180^\circ}{n}.$$

Bul formulalardan paydalani'pⁿ, yari'm duri's ko'pmu'yeshliklerdin' ta'repi, ishley ha'm si'rtlay si'zi'lg'an shen'berlerdin' radiuslari arasi'ndag'i baylani'slardi tabami'z.

1. Duri's u'shmu'yeshlik ushi'n ($n=3$):

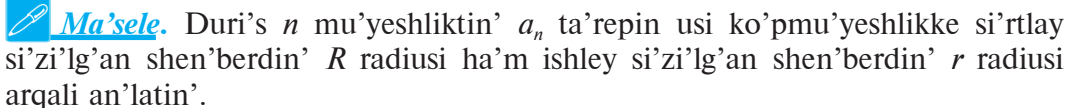
$$\beta = \frac{180^\circ}{3} = 60^\circ; \quad R = \frac{a_3}{2 \sin 60^\circ} = \frac{a_3}{\sqrt{3}}; \quad r = \frac{a_3}{2 \operatorname{tg} 60^\circ} = \frac{a_3}{2\sqrt{3}}; \quad R = 2r.$$

2. Kvadrat ushi'n ($n=4$):

$$\beta = \frac{180^\circ}{4} = 45^\circ; \quad R = \frac{a_4}{2 \sin 45^\circ} = \frac{a_4}{\sqrt{2}}; \quad r = \frac{a_4}{2 \tan 45^\circ} = \frac{a_4}{2}; \quad R = r\sqrt{2}.$$

3. Duri's altimu'yeshlik ushi'n ($n=6$):

$$\beta = \frac{180^\circ}{6} = 60^\circ; \quad R = \frac{a_6}{2 \sin 30^\circ} = a_6; \quad r = \frac{a_6}{2 \operatorname{tg} 30^\circ} = \frac{a_6 \sqrt{3}}{2}; \quad R = \frac{2r}{\sqrt{3}}.$$



Sheshiliwi. $R = \frac{a_n}{2 \sin \frac{180^\circ}{n}}$ ha'm $r = \frac{a_n}{2 \operatorname{tg} \frac{180^\circ}{n}}$ formulalardan $a_n = 2R \sin \frac{180^\circ}{n}$ ha'm $a_n = 2r \operatorname{tg} \frac{180^\circ}{n}$

formulaların payda yetemiz. Sonlu'qtan, $n=3$ bolsa, $a_3=R\sqrt{3}=2r\sqrt{3}$.

? Soraw, ma'sele ha'm tapsi'rmalar

1. Ta'repi 15 sm bolg'an a) duri's u'shmu'yeshlikke; b) duri's to'rtmu'yeshlikke; d) duri's altimu'yeshlikke ishley ha'm si'rtlay si'zi'lg'an shen'berlerdin' radiuslarin yesaplan'.
2. 2-su'wrettin' won' ta'repinde R radiusli' shen'berge ishley si'zi'lg'an kvadrat, duri's u'shmu'yeshlik ha'm duri's altimu'yeshlik su'wretlengen. Da'pterin' izge berilgen kestelerdi ko'shirip, woni'n' bos keteklerin toltiri'n' (a_n — ko'pmu'yeshliktin' ta'repi, P — ko'pmu'yeshliktin' perimetri, S — woni'n' maydani, r — wog'an ishley si'zi'lg'an shen'ber radiusi).

2) a)

b)

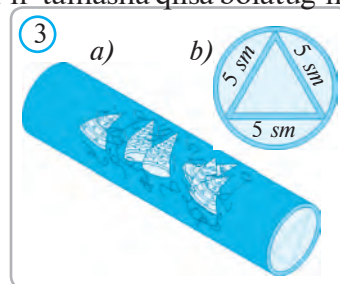
d)

	R	r	a_4	P	S
1.			6		
2.		2			
3.	4				
4.				28	
5.					16

	R	r	a_3	P	S
1.	3				
2.					10
3.		2			
4.			5		
5.				6	

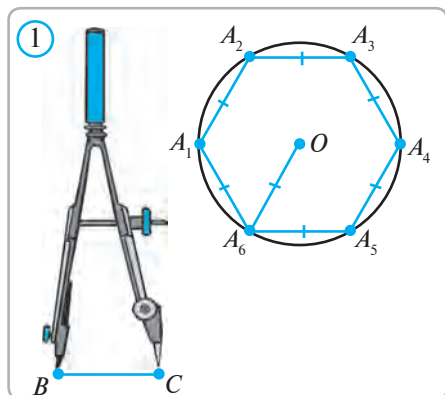
	R	r	a_6	P	S
1.	4				
2.		5			
3.			6		
4.				42	
5.					48

3. Radiusi 8 sm bolg'an shen'berge ishley si'zi'lg'an duri's won yeki mu'yeshliktin' bir to'besinen shi'qqan diagonallari'n tabi'n'.
4. Shen'berge ishley si'zi'lg'an duri's u'shmu'yeshliktin' perimetri 24 sm. Bul shen'berge ishley si'zi'lg'an kvadrattin' ta'repin tabin.
5. Csilindr formasindag'i' ag'ashtan ultani'ni'n' ta'repi 20 sm bolg'an: a) kvadrat; b) duri's altimu'yeshlik bolg'an prizma tu'rindagi bag'ana tayarlaw kerek. Ag'ashtin' kese-kesiminin' diametri keminde qansha boliwi za'ru'r?
6. 3. a-su'wrette su'wretlengen, tu'rlish nag'i'slari'n tamasha qilsa bolatug'in "Kaleydoskop" dep atalg'an woyinshiq sizge tanis bolsa kerek. Woyinshiq truba ha'm 3 ayna bo'leklerinen ibarat. 3.b-su'wrette woni'n' kese-kesimi su'wretlengen ha'm wo'lishemleri berilgen. Kaleydoskoptin' kesekesiminin' radiusin tabi'n'.



I. Testler

- To'mendegi ko'pmu'yeshliklardin' qaysi birinde ishley si'zi'lgan shen'ber joq?
A) U'shmu'yeshlikte; D) Kvadratdan wo'zgeshe rombida;
B) Kvadratta; E) Rombdan wo'zgeshe tuwri' mu'yeshli to'rtmu'yeshlikte;
- To'mendegi ko'pmu'yeshliklardin' qaysi birinde si'rtlay si'zi'lgan shen'ber joq?
A) U'shmu'yeshlikte; D) Kvadratdan wo'zgeshe rombida;
B) Kvadratta; E) Rombdan wo'zgeshe tuwri' mu'yeshli to'rtmu'yeshlikte;
- Shen'berge ishley si'zi'lg'an barliq $ABCD$ to'rt mu'yeshlikler ushi'n naduri's ten'likti tabi'n'?
A) $\angle A + \angle B + \angle C + \angle D = 360^\circ$; D) $AB + CD = BC + AD$;
B) $\angle A + \angle C = 180^\circ$; E) $\angle B + \angle D = 180^\circ$.
- Shen'berge si'rtlay si'zi'lg'an barliq $ABCD$ to'rt mu'yeshlikler ushi'n naduri's ten'likti tabi'n'?
A) $\angle A + \angle B + \angle C + \angle D = 360^\circ$; D) $AB + CD = BC + AD$;
B) $\angle A + \angle C = 180^\circ$; E) $AB - BC = AD - CD$.
- Ta'repleri 5 sm ha'm 12sm bolg'an tuwri' mu'yeshli to'rtmu'yeshlikke si'rtlay si'zi'lg'an shen'ber radiusin tabi'n'?
A) 6 sm; B) 6,5 sm; D) 7 sm; E) 7,5 sm.
- Duri's 24 mu'yeshliktin' ishki mu'yeshlerin tabi'n'?
A) 120° ; B) 135° ; D) 150° ; E) 165° .
- Ha'r bir si'rtqi' muyeshi 60° bolg'an duri's ko'pmu'yeshliktin' ishki mu'yeshlerinin' qosi'ndi'si'n tabi'n'?
A) 540° ; B) 360° ; D) 90° ; E) 720° .



II. Siziwg'a tiyisli ma'seleler.

1. Ta'repi berilgen kesindige ten' duris altimu'yesh si'zi'n. Bunda turaqli altimu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusi altimu'yeshliktin ta'repine ten' yekenliginen ha'm 1-su'wretten paydalani'n'.

2. 2-4-su'wretlerdegi mag'liwmatlardan paydalani'p, berilgen shen'berge ishley si'zi'lg'an a) duris u'shmu'yeshlik; b) kvadrat; d) duris segizmu'yeshlik si'zi'n'.

3. 5-su'wretten paydalani'p, berilgen shen'berge si'rtlay si'zi'lg'an duris altimu'yeshlik

si'zi'n' (5-su'wrette su'wretlengen shen'berge si'rtlay si'zi'lg'an altimu'yeshlik

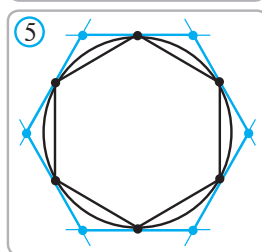
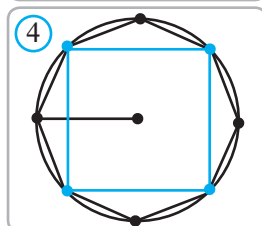
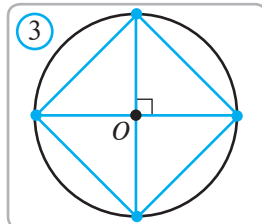
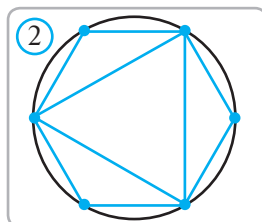
ta'replari sol shen'berge ishley si'zi'lg'an duri's altimu'yeshliktin' tobelerinen shen'berge ju'rgizilgen urinbalarda jatadi).

III. Yesaplawg'a tiyisli ma'seleler

1. Duri's u'shmu'yeshlik, kvadrat ha'm duri's altimu'yeshliklerdin' ta'replari bir-birine ten'. Wolardi'n' maydanlari'ni'n' qatnasi'n tabi'n'.
2. Bir shen'berge ishley si'zi'lg'an duri's altimu'yeshlik ha'm si'rtlay si'zi'lg'an altimu'yeshliktin' maydanlari'ni'n' qatnasi'n tabi'n'.
3. Duri's a) altimu'yeshlik; b) segizmu'yeshlik; c) won yeki mu'yeshliktin' parallel ta'replari arasi'ndag'i arali'q 10 sm ge ten'. Ko'pmu'yeshliktin' ta'repin tabi'n'.
4. Radiusi' R bolg'an shen'berge $A_1A_2 \dots A_8$ duri's segizmu'yeshlik ishley si'zi'lg'an. $A_3A_4A_7A_8$ to'rtmu'yeshliginin' tuwri'mu'yeshlik yekenligin da'liyllen ha'm woni'n' maydani'n tabi'n'.
5. Shen'berge si'rtlay si'zi'lg'an tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasi' sol shen'berge uriniw noqatinda 4 sm ha'm 6 sm uzi'nli'qtag'i kesindilerge ajiraladi. U'shmu'yeshliktin' maydani'n tabi'n'.
6. Duri's ushmu'yeshliktin' bir to'besinen shiqqan yen' u'lken ha'm yen' kishi diagonallari arasi'ndag'i mu'yeshti tabi'n'.

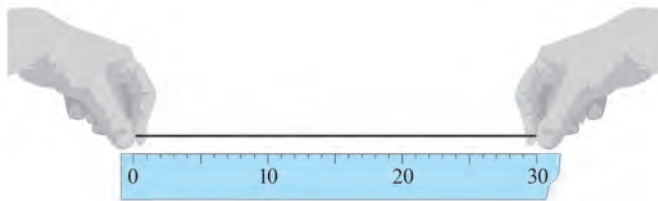
IV. Wo'zin'di sinap ko'rin' (baqlaw jumi'si' u'lgisi).

1. Katetleri 10 sm ha'm 24 sm bolg'an tuwri' mu'yeshli u'shmu'yeshlikke ishley si'zi'lg'an ha'm si'rtlay si'zi'lg'an shen'berdin' radiusin tabi'n'.
3. Ta'replari 4 sm bolg'an duri's altimu'yeshliktin' bir ushi'nan shiqqan diagonallarini tabi'n'.
2. Radiusi 5 sm bolg'an shen'berge si'rtlay si'zi'lg'an rombnin' bir mu'yeshi 150° qa ten'. Rombi'nin': a) perimetrim b) diagonallarini; d) maydani'n tabi'n'.
4. (Qosi'msha). Radiusi 3 sm bolg'an shen'berge ishley si'zi'lg'an duri's altimu'yeshlik ha'm duri's u'shmu'yeshlikler maydanlarinin' ayirmasin tabi'n'.



Tariyx betlerinen.

Qalegen duri's ko'pmu'yeshlikti de cirkul ha'm sizg'ish ja'rdeminde jasawg'a bola bermeydi yeken. Buni 1801-jili nemis matematigi Karl Gauss (1777—1855) algebralıq usılda da'liyllegen. Wol yeger n saninin' $2^m p_1 p_2 \dots p_n$ jayılmasında $p_1, p_2, \dots, p_n$ tu'rli tu'bir sanlar $2^{2^k} + 1$ ko'rinishinde bolsa g'ana duri's n -mu'yeshti tcirkul ha'm sizg'ish ja'rdeminde jasaw mu'mkin yekenligin da'liylledi. Bul jerde m ha'm k teris bolmag'an pu'tin sanlar.



Jedellestirwshi shinig'iw

1. A'dette truba bo'leginin' kese-kesimi shen'berden ibarat boladi. Jin'ishke jipti bir ushi'nan baslap, trubag'abir ma'rte woran'. Bir ma'rte worawg'a ketken jip bo'legi trubaning' kese-kesimi, yag'ni'y shen'berdin' uzi'nlig'i boladi. Woni' su'wrette ko'rsetilgende yetip si'zg'ish ja'rdeminde wo'lshe'n'.
2. Joqari'dag'i usil menen trubaning' kese-kesiminin' diametrin ani'qlan'.
3. Ani'qlang'an shen'ber uzi'nli'g'i'n woni'n' diametrine qatnasin yesaplan'.
4. Joqaridakeltirilgen wo'lshe'w ha'm yesaplaw ja'ne bir neshe tu'rli wo'lshe'mdegi trubabo'lekleri ushi'n daworinlap, shen'ber uzi'nli'g'i'nin' woni'n' diametrine qatnaslarin tabi'n'.
5. Ta'jiriybe na'tiyjesinde shen'ber uzi'nli'g'i'nin' woni'n' diametrine qatnasi haqqinda qanday juwmaq shi'g'ari'w mu'mkin?



Teorema. Shen'ber uzi'nli'g'i'nin' shen'ber diametrine qatnasi shen'berdin' radiusina baylanisli yemes, yag'niy ha'r qanday shen'ber ushi'n bul qatnas bir qi'yli' san boladi.

Da'liyllew. Qa'legen yeki shen'ber alamiz. Wolardi'n' radiuslari' R_1 ha'm R_2 , uziniqlari bolsa sa'ykes tu'rde C_1 ha'm C_2 bolsi'n. $\frac{C_1}{2R_1} = \frac{C_2}{2R_2}$ ten'ligin da'liyllewimiz kerek. Ha'r yeki shen'berge ishley duri's n -mu'yeshti sizamiz. Wolardi'n' perimetrlarin sa'ykes tu'rde P_1 ha'm P_2 dep belgileyik. Wonda,

$$P_1 = n \cdot 2R_1 \sin \frac{180^\circ}{n}, P_2 = n \cdot 2R_2 \sin \frac{180^\circ}{n} \text{ bolg'ani' ushi'n } \frac{P_1}{P_2} = \frac{2R_1}{2R_2} (*) \text{ boladi.}$$

Bul ten'lik qa'legen n ushi'n duri's boladi. n sani u'lkeyip barsa, berilgen shen'berge ishley si'zi'lg'an n -mu'yeshliktin' perimetri P_1 usi shen'ber uzunlig'i C_1 ge jaqinlasip baradi. Sol shiyaqli P_2 ha'm C_2 ge jaqinlasip baradi.

Sonin' ushi'n $\frac{P_1}{P_2}$ qatnasi $\frac{C_1}{C_2}$ qatnasina ten' boladi (bunin' toliq da'liyli matematicanin' joqari basqishlarinda u'yreniledi). Solay yetip, $(*)$ ten'liginen $\frac{C_1}{C_2} = \frac{2R_1}{2R_2}$, bunnan bolsa $\frac{C_1}{2R_1} = \frac{C_2}{2R_2}$ ten'ligi kelip shig'adi.

Teorema da'liylendi.

Shen'ber uzi'nli'g'i'n woni'n' diametrine qatnasi grek a'lipbesinin' π ha'ribi menen belgilew qabil yetilgen ("pi" dep oqiladi). Shen'ber uzi'nli'g'i'n woni'n' diametrine qatnasin " π " ha'ribi menen belgilewdi ulli matematik Leonard Eyler (1707—1783) ilimge kiritken. Grekshede "shen'ber" so'zi usi ha'rip penen baslanadi. π irrattional san boli'p, a'meliyatta woni'n' 3,1416 g'a ten' bolg'an juwq ma'nisinen paydalaniladi.

Solay yetip, $\frac{C}{2R} = \pi$. Bul ten'likten radiusi' R ge ten' shen'berdin' uzunlig'i ushi'n $C = 2\pi R$ formulasin paydayetemiz.

 **Ma'sele.** Ta'repi 6 sm bolg'an duri's u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' uzi'nli'g'i'n tabi'n'.

Sheshiliwi. Duri's u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusin tabiw formulasi $R = \frac{a_3}{\sqrt{3}}$ boyi'nsha $R = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ (sm). Yendi, shen'ber uzi'nli'g'i'n tabiw formulasinan $C = 2\pi R = 2\pi \cdot 2\sqrt{3} = 4\pi\sqrt{3}$ (sm). **Juwabi':** $4\pi\sqrt{3}$ sm.

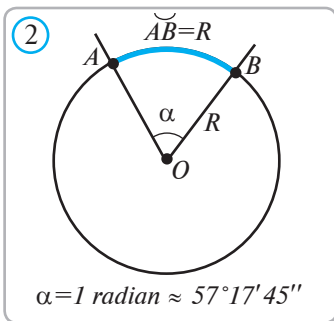
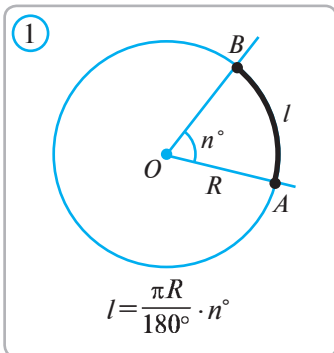
 **Soraw, ma'sele ha'm tapsi'rmarlar**

1. Qanday san π menen belgilenedi? Radiusi' R ge ten' shen'berdin' uzi'nli'g'i'n tabiw formulasinan paydalani'p kesteni toltirin' ($\pi \approx 3,14$ dep yesaplan').

C			82	18π		6,28	
R	4	3			0,7		101,5

2. Yeger shen'ber radiusi' a) 3 ma'rte artsa; b) 3 sm ge artsa; d) 3 ma'rte kemise; e) 3 sm ge kemise, shen'ber uzunlig'i' qanshag'a wo'zgeredi?
3. Yeger jer shari ekvatorinin' 40 millionnan bir bo'limi 1 m ge ten' bolsa, Jer sharinin' radiusin tabi'n'.
4. a) Ta'repi a g'a ten' bolg'an duri's u'shmu'yeshlikke; b) katetleri a ha'm b bolg'an tuwri' mu'yeshli u'shmu'yeshlikke; c) ultani' a ha'm qaptal ta'repi b bolg'an ten' qaptalli u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' uzi'nli'g'i'n tabi'n';
5. a) Ta'repi a g'a ten' kvadratqa; b) gipotenuzasi c g'a ten' bolg'an ten' qaptalli tuwri' mu'yeshli u'shmu'yeshlikke; c) gipotenuzasi c , su'yir mu'yeshi α bolg'an tuwri' mu'yeshli u'shmu'yeshlikke ishley si'zi'lg'an shen'berdin' uzi'nli'g'i'n tabi'n'.
6. Teplovoz 1413 m jol ju'rди. Bunda wonin do'n'gelegi 300 ma'rte aylandi. Teplovoz do'n'geleginin diametrin tabi'n'.
7. "Nexiya" avtomobili do'n'gelegi shen'berinin' radiusi 24 sm ge ten'. Avtomobil 100 km jol ju'rse, woni'n' do'n'gelegi neshe ma'rte aylanadi (1-su'wret)?





1. n° li' worayliq mu'yeshke tirelgen dog'anin' uzunlig'i.

Aytayiq, radiusi R ge ten' bolg'an shen'berde n° li AOB worayliq mu'yesh berilgen bolsi'n (I -su'wret).

Bunda shen'berdin' AOB worayliq mu'yeshine tirelgen AB dog'asiinin' gradus wolsheemin n° yaki n° li dog'a dep ju'rtiliwiw yesletip wotemiz.

Radiusi R ge ten' bolg'an shen'ber, yag'ini'y 360° li dog'a uzunlig'i $2\pi R$ ge ten' bolg'ani' ushi'n,

1° li dog'auzinlig'i $\frac{2\pi R}{360^\circ} = \frac{\pi R}{180^\circ}$ boladi.

Wonda, n° li dog'auzinlig'i $l = \frac{\pi R}{180^\circ} \cdot n^\circ$ formula menen ani'qlanadi (I -su'wret).

2. Mu'yeshtin' radian wo'lsheми.

Mu'yeshtin' gradus wo'lsheми menen bir qatarda wonin' radian wo'lsheми de qollaniladi.

Shen'ber dog'asi uzi'nli'g'i'nin' radiusqa qatnasin joqaridag'i formulag'a tiykarlanip $\frac{l}{R} = \frac{\pi}{180^\circ} \cdot n^\circ$ g'a ten'.

Demek, shen'ber dog'asi uzi'nli'g'i'nin' radiusina qatnasi tekusi dog'ag'a tirelgen worayliq mu'yeshtin' shamasina baylani'sli yeken. Bul qa'siyetten paydalani'p, mu'yeshtin' radian wo'lsheми sipatinda da'l usi qatnasti alamiz:

$$\alpha = \frac{l}{R} = \frac{\pi}{180^\circ} \cdot n^\circ.$$

A'dette, radian so'zi jazilmaydi. Ma'selen: 5 rad wornina 5 dep jaziladi.

Bir radian $\frac{180^\circ}{\pi}$ gradusqa ten': $1 \text{ radian} = \frac{180^\circ}{\pi} \approx 57^\circ 17' 45''$. Mu'yeshtin' gradus wo'lsheминен radian wo'lsheминe wo'tiw ushi'n

$$\alpha = \frac{\pi}{180^\circ} \cdot n^\circ$$

formuladan paydalaniladi.

Solay yetip, n° li mu'yeshtin' radian wo'lsheмин tabiw ushi'n woni'n' gradus wo'lsheмин $\frac{180^\circ}{\pi}$ ge ko'beytiw jetkilikli yeken. Jeke halda, 180° mu'yeshtin' radian wo'lsheми π ge ten', 90° li, yag'iny tuwri' mu'yeshtin' radian wo'lsheми $\frac{\pi}{2}$ ge ten' boladi.

α radiang'a ten' worayliq mu'yeshke sa'ykes dog'asinin' uzunlig'i $l = \alpha R$ formulasimenen yesaplanadi.

 **Ma'sele.** Mu'yeshleri 30° ha'm 45° bolg'an u'shmu'yeshliktin' mu'yeshlerinin' radian wo'lshemlerin tabi'n'.

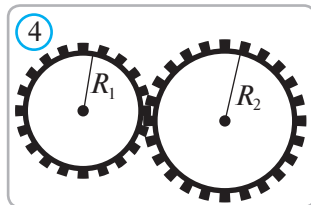
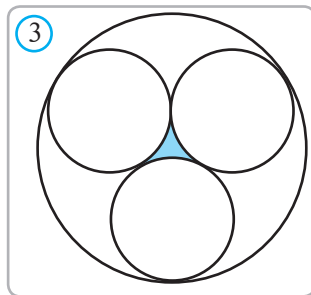
Sheshiliwi. U'shmu'yeshliktin' 30° li mu'yeshinin' radian wo'lsheми $30^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{6}$, 45° li mu'yeshinin' radian wo'lsheми $45^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{4}$. U'shmu'yeshliktin' ishki mu'yeshlerinin' qosi'ndi'si' 180° qa, yag'niy π ge ten' yekenligi haqqindag'i teoremag'a tiykarlanip, u'shmu'yeshliktin' u'shinshi mu'yeshinin' radian wo'lsheмин tabami'z.

$$\pi - \frac{\pi}{6} - \frac{\pi}{4} = \frac{7\pi}{12}.$$

Juwabi': $\frac{\pi}{6}$, $\frac{\pi}{4}$ ha'm $\frac{7\pi}{12}$

 **Soraw, ma'sele ha'm tapsi'rmalar**

1. Radiusi 6 sm bolg'an shen'berdin' gradus wo'lsheми a) 30° ; b) 45° ; d) 90° ; e) 120° bolg'an dog'a uzi'nli'g'i'n tabi'n'.
2. a) 40° ; b) 60° ; d) 75° qa ten' mu'yeshtin' radian wo'lsheмин tabi'n'.
3. a) 1,2; b) $\frac{2\pi}{3}$; d) $\frac{5\pi}{6}$ radiang'a ten' mu'yeshtin' gradus wo'lsheмин tabi'n'.
4. Yeger shen'berdin' radiusi 5 sm bolsa, woni'n' a) $\frac{\pi}{8}$; b) $\frac{2\pi}{5}$; d) $\frac{3\pi}{4}$ radiang'a ten' bolg'an dog'auzi'nli'g'i'n tabi'n'.
5. Radiusi 12 sm bolg'an shen'berge ABC u'shmu'yeshligi ishley si'zi'lg'an. Yeger a) $\angle A = 30^\circ$; b) $\angle A = 120^\circ$ bolsa, A noqatin wo'z ishine almag'an BC dog'a uzi'nli'g'i'n tabi'n'.
6. Shen'berdin' ten' xordalari shen'berden ten' dog'alar ajiratug'inin da'liyllen'.
- 7*. Yeki shen'ber bir-birinin' worayinan wo'tedi. Bul shen'berlerdin' uliwma xordasi ha'r yeki shen'berden ajiratqan dog'alar uzunliqlarinin' qatnasin tabi'n'.
- 8*. Radiuslari ten' bolg'an u'sh shen'berler bir-birine si'rttan ha'm radiusi R ge ten' bolg'an shen'berge ishley urinadi (3-su'wret): a) shen'berlerdin' radiusin tabi'n'; b) boyalg'an figurani shegaralawshi dog'alardin' uzunliqlarinin' qosi'ndi'si'n tabi'n'.

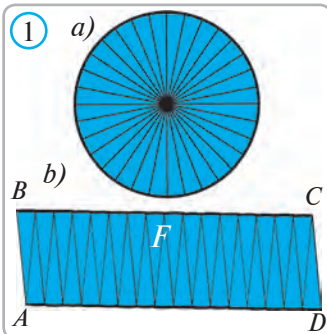


 **Qi'ziqli' ma'sele**

4-su'wrette su'wretlengen yeki tisli do'n'gelekler bir-birine "tisletilgen". Do'n'gelekler radiusi R_1 ha'm R_2 . Birinshi do'n'gelek n ma'rte aylang'anda yekinshi do'n'gelek neshe ma'rte aylanadi?

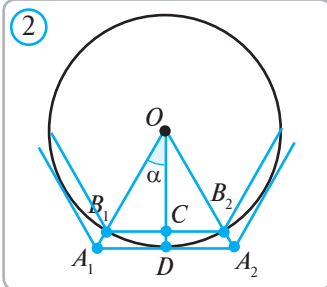
✓ **Ani'qlama.** Shen'ber ha'm tegisliktin' usi shen'ber menen shegarang'an (ishki) bo'limine **do'n'gelek** dep ataladi.

Worayi O noqat'inda ha'm radiusi R ge ten' bolg'an do'n'gelek tegisliktin' O noqatinan R den aspaytug'in arali'qta jatqan barliq noqatlardan quralg'an boladi.



Jedellestirivshi shinig'iw

Bir bet qag'azg'a qali'n' si'zi'q penen shen'ber si'zi'n' ha'm 1.a-su'wrette ko'rsetilgendey, woni'n' bir neshe diametrlerin ju'rgizip, do'n'geleklerdi ten'dey bo'leklerge bo'lin'. Son' bul bo'leklerdi qiyip alin' ha'm 1.b-su'wrette ko'rsetilgendey yetip terip, F figurasi'n payda yetin'. Yeger do'n'gelek qa'legenshe ko'p ten'dey bo'leklerge bo'linip, bul bo'lekler su'wrette ko'rsetilgen ta'rtipte terilgende, na'tiyjede tuwri' mu'yeshlikke ju'da' jaqin F figura payda boladi.



a) F figurani tuwri' mu'yeshlikformasina ju'da' jaqin yekenligin yesapqaalip, woni'n' AB ta'repi shamamenen nege ten' bolatug'inin tabi'n'. (Ko'rsetpe: AB ta'repin do'n'gelek radiusi menen salistirin').

b) F figuranin' BC "ta'repi" shama menen nege ten' boladi? (Ko'rsetpe: BC ha'm AD ta'repleri qalin' si'zi'q penen si'zi'lg'anina, yag'niy shen'ber dog'alarinan ibarat yekenligine itibar berin').

d) F figuranin' $ABCD$ tuwri' mu'yeshlik formasina ju'da' jaqin yekenligin yesapqa alip, woni'n' maydani'n juwiq yesaplan'. F figurasi'nin' maydani' do'n'gelek maydani'na ju'da' jaqin yekenligin na'zerde tutip, do'n'gelek maydani' haqqinda juwmaq shig'arin'.

📐 **Teorema.** Radiusi' R ge ten' bolg'an do'n'gelektin' maydani' R^2 qa ten'.

Da'liyllew. Radiusi R ha'm worayi O noqatta bolg'an shen'berdi qaraymiz.

Shen'berge si'rtlay si'zi'lg'an $A_1A_2 \dots A_n$ ha'm ishley si'zi'lg'an $B_1B_2 \dots B_n$ duri's n mu'yeshlerdin' maydanlari sa'ykes tu'rde S_n^I ha'm S_n^{II} bolsi'n (2-su'wrette).

A_1OA_2 ha'm B_1OB_2 u'shmu'yeshliklerdin' maydanlarin tabami'z:

$$S_{A_1OA_2} = \frac{1}{2}A_1A_2 \cdot OD = \frac{1}{2}A_1A_2 \cdot R; \quad S_{B_1OB_2} = \frac{1}{2}B_1B_2 \cdot OC = \frac{1}{2}B_1B_2 \cdot OB_1 \cos \alpha = \frac{1}{2}B_1B_2 \cdot R \cos \alpha.$$

$$\text{Wonda } S_n^I = n \cdot \frac{1}{2}A_1A_2 \cdot R = \frac{1}{2}P_n^I R, \quad S_n^{II} = n \cdot \frac{1}{2}B_1B_2 \cdot R \cos \alpha = \frac{1}{2}P_n^{II} R \cos \alpha \quad (1)$$

Bul jerde P_n ha'm P_n sa'ykes tu'rde $A_1A_2...A_n$ ha'm $B_1B_2...B_n$ ko'pmu'yeshliktin' perimetrleri $\alpha = \frac{180^\circ}{n}$ bolg'ani' ushi'n n nin' jeterlishe u'lken ma'nislerinde $\cos \alpha$ nin' ma'nisi birden, $\frac{P_n}{P_n}$ ha'm P_n lerdin' ma'nisleri shen'ber uzunlig'i yag'niy $2\pi R$ den qa'legenshe kem parq qiladi. Wonda (1) ten'likler boyi'nsha, n nin' jeterlishe u'lken ma'nislerinde ko'pmu'yeshliklerdin' maydani' πR^2 qa jaqinlasip baradi. Bunnan, do'n'gelektin' maydani' ushi'n $S = \pi R^2$ formulakelip shig'adi. **Teorema da'liylandi**

 **Ma'sele.** Cirk arenasi shen'berinin' uzunlig'i 41 m. Arena radiusin ha'm maydani'n tabi'n'.

Sheshiliwi 1) Shen'berdin' uzi'nli'g'i'n tabiw formulasin radiusin tabamiz (3-su'wret):

$$R = \frac{C}{2\pi} \approx \frac{41}{2 \cdot 3,14} \approx 6,53 \text{ (m)}.$$

2) Do'n'gelektin' maydani'n yesaplaw formulasin arenanin' maydani'n tabami'z:

$$S = \pi R^2 \approx 3,14 \cdot 6,53^2 \approx 133,84 \text{ (m}^2\text{)}.$$

Juwabi': $R \approx 6,53 \text{ m}$; $S = 133,84 \text{ m}^2$.

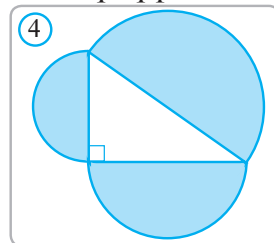


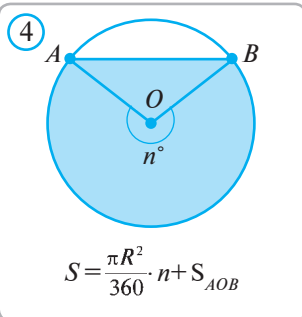
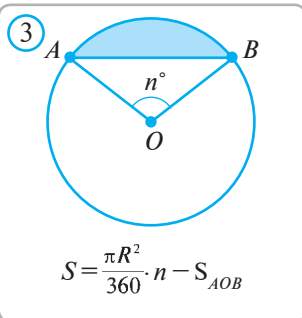
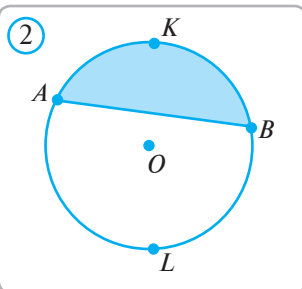
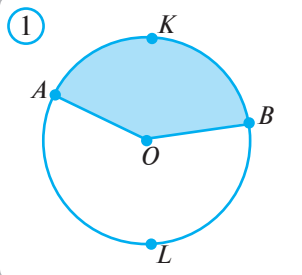
Soraw, ma'sele ha'm tapsi'rmalar

1. Do'n'gelek maydani'n yesaplaw formulasin tiykarlan'.
2. Radiusi R ge ten' bolg'an do'n'gelektin' S maydani'n tabiw formulasin paydalani'p, kesteni toltirin' ($\pi = 3,14$ dep alin').

R	2	5		$\frac{2}{7}$		54,3		6,25
S			9		49π		$\sqrt{3}$	

3. Yeger do'n'gelek radiusi a) k ma'rte wo'sse; b) k ma'rte kemise, do'n'gelektin' maydani' qalay wo'zgeredi?
4. Ta'repi 5 sm bolg'an kvadratqa ishley si'zi'lg'an ha'm si'rtlay si'zi'lg'an do'n'geleklerdin' maydani'n tabi'n'.
5. Ta'repi $3\sqrt{3}$ sm bolg'an duri's u'shmu'yeshlikke ishley ha'm si'rtlay si'zi'lg'an do'n'geleklerinin' maydani'n tabi'n'.
6. Radiusi' R bolg'an do'n'gelekten yen' u'lken kvadrat qirqip alindi. Do'n'gelektin' qalg'an bo'liminin' maydani'n tabi'n'.
7. Ta'repleri 6 sm ha'm 7 sm bolg'an tuwri' mu'yeshlikke si'rtlay si'zi'lg'an do'n'gelektin' maydani'n tabi'n'.
8. Ta'repi 10 sm ha'm su'yir mu'yeshi 60° bolg'an rombig'a ishley si'zi'lg'an do'n'gelektin' maydani'n tabi'n'.
- 9*. Tuwri' mu'yeshli u'shmu'yeshliktin' ta'replerin diametr yetip yarim do'n'gelekler si'zi'lg'an. Gipotenuzag'a si'zi'lg'an yarim do'n'gelektin' maydani' katetlerge si'zi'lg'an yarim do'n'geleklerdin' maydanlarinin' qosi'ndi'si'na ten' bolatug'inin ko'rsetin' (4-su'wret).





✓ **Ani'qlama.** Do'n'gelektin' dog'asi ha'm bul dog'a aqirlarin do'n'gelek worayi menen tutastirivshi yeki radiusi menen shegaralang'an bo'limi **sektor** delinedi. Sektordin' shegarasi bolg'an dog'**asektor dog'asi** delinedi.

1-su'wrette AKB ha'm BLA dog'ali yeki sektor su'wretlengen (*wolardan birinshisi boyalg'an*).

Radiusi R ge ha'm dog'anin' gradus wo'lishemi n° qa ten' bolg'an sektordin' S maydani'n tabiw ushi'n formula keltirip shig'aramiz. Dog'asi 1° qa ten' sektordin' maydani' do'n'gelek (yag'niy dog'asi 360° qa ten' sektor) maydani'nin' $\frac{1}{360}$ bo'limine ten' bolg'ani' ushi'n, dog'asi n° bolg'an sektordin' maydani'

$$S = \frac{\pi R^2}{360} \cdot n \text{ yoki } S = \frac{1}{2} Rl$$

formula arqali tabiladi. Bul jerda l — n° sektor jayinin' uzunligi.

✓ **Ani'qlama.** Do'n'gelektin' dog'asi ha'm bul dog'a aqirlarin tutastirivshi xordasi menen shegaralang'an bo'limi **segment** delinedi.

2-su'wrette AKB ha'm BLA dog'ali yeki segment su'wretlengen (*wolardan birinshisi boyalg'an*). Yarim do'n'gelektan parqli segmenttin' S maydani'

$$S = S_{\text{sektor}} \pm S_{\Delta} = \frac{\pi R^2}{360} \cdot n \pm S_{AOB}$$

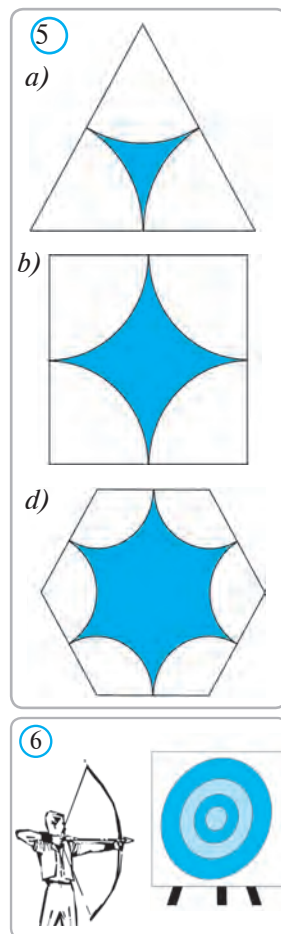
formula boyi'nsha yesaplanadi (*3-ha'm 4-su'wretlerge qaran*).

✎ **Ma'sele.** Dog'anin' gradus wo'lishemi 72° bolg'an sektordin' maydani' 45π ge ten'. Sektor radiusin tabi'n'.

Sheshiliwi. Sektor maydani'n tabiw formulasi boyi'nsha, $\frac{\pi R^2}{360} \cdot 72 = 45\pi$. Bunnan, $R^2 = \frac{45\pi \cdot 360}{72\pi} = 225$, demek $R = 15$. **Juwabi':** 15.

? Soraw, ma'sele ha'm tapsi'rmalar

1. Sektor maydani'n tabiw formulasin keltirip shig'arin'.
2. Segment maydani'n tabiw formulasin keltirip shig'arin'.
3. Radiusi 7 sm bolg'an sektor ha'm segment maydanlarin tabi'n'. Bunda, woni'n' dog'asinin' gradius wo'lshegi a) 30°; b) 45°; d) 120°; e) 90°.
4. 5-su'wrette ta'repi a g'a ten' bolg'an duri's u'shmu'yeshlik, kvadrat ha'm duri's altimu'yeshlik su'wretlengen. Boyalg'an figuralardin' maydani'n tabi'n'. Bunda sektorlardin' radiuslari ko'p mu'yeshlik ta'repinin' yarimina ten'.
5. Nishanda radiuslari 1, 2, 3, 4 ke ten' bolg'an to'rt shen'ber bar. Yen' kishi do'n'gelektin' maydani'n ha'm ha'r bir jag'day da maydani'n tabi'n' (6-su'wret).
6. Radiusi 10 sm ge ten' bolg'an do'n'gelekke radiusqa ten' xorda ju'rgizilgen. Payda bolg'an segmentlardin' maydani'n yesaplan'.
7. Radiuslari 15 sm bolg'an yeki do'n'gelektin' woraylari arasi'ndag'i arali'q 15 sm. Do'n'geleklerdin' uliwma bo'liminin' maydani'n tabi'n'.
8. Radiusi 10 sm bolg'an do'n'gelekke ishley ha'm si'rtlay si'zi'lg'an duri's won yeki mu'yeshliklerdin' maydani'n yesaplan'. Na'tiyjelerdi do'n'gelektin' maydani' menen salisti'ri'n'.

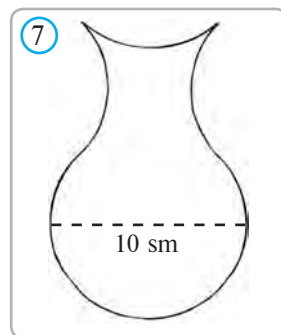


⌚ Qi'ziqli' ma'sele

7-su'wrette su'wretlengen gu'ltu'bektin' su'wretin u'sh tuwri' si'zi'q penen:

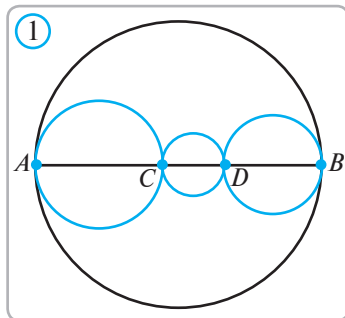
a) sonday to'rt bo'lekke bo'lin', na'tiyjede wolardan tuwri' mu'yeshlik jiynaw mu'mkin bolsi'n;

b) yeki tuwri' si'zi'q penen sonday u'sh bo'limge bo'lin', na'tiyjede wolardan kvadrat jasaw mu'mkin bolsi'n.



📖 Tariyx betlerinen. Uzaq waqitlar dawaminda du'nyanin' ko'plep matematiklari "do'n'gelek kvadraturasi" dep at alg'an to'mendegi ma'seleni sheshiwge ha'reket yetken: tcirkul ha'm sizg'ish ja'rdeminde maydani' berilgen do'n'gelek maydani'na ten' bolg'an kvadrat jasaw. Tek XIX a'sirdin' aqirinda bul ma'sele sheshimga iye yemesligi da'liyellengen.

1-ma'sele. C ha'm D noqatlari shen'berdin' AB diametrin u'sh AC , CD ha'm DB kesindilerga ajiratadi. AC , CD ha'm DB diametrli shen'berlerdin' uzunliqlarinin' qosi'ndi'si' AB diametrli shen'ber uzi'nli'g'i'na ten' yekenligin da'liylen' (*1-su'wret*).

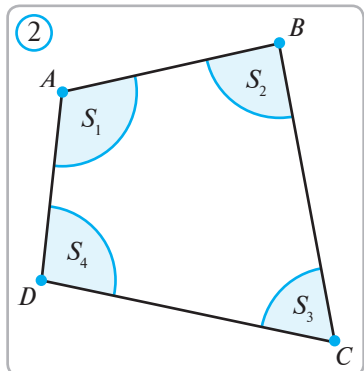


Sheshiliwi. Shen'ber uzi'nli'g'i'n tabiw formulasin paydalani'p, AC , CD ha'm DB diametrli shen'berlerdin' C_1 , C_2 , C_3 uzunliqlarinin' qosi'ndi'si'n tabami'z:

$$C_1 + C_2 + C_3 = AC \cdot \pi + CD \cdot \pi + DB \cdot \pi = \pi(AC + CD + DB).$$

$AC + CD + DB = AB$ ha'm AB diametrli shen'berdin' C uzunlig'i $AB \cdot \pi$ ge ten' bolg'ani' ushi'n $C_1 + C_2 + C_3 = C$. Ushi ten'likni da'liylen talap yetilgen yedi.

2-ma'sele. $ABCD$ to'rtmu'yeshliktin' to'belerin woray yetip birdey radiusli sektorlar jasalg'an (*2-su'wret*). Bul sektorlardan qa'legen yekewi uliwmanoqatqaiye yemes ha'm barlig'inin' radiusi 1 sm . Sektorlardin' maydanlarinin' qosi'ndi'si'n tabi'n'.



Sheshiliwi. 1) To'rtmu'yeshliktin' A , B , C , D mu'yeshleri sa'ykes tu'rde α_1 , α_2 , α_3 , α_4 bolsi'n. Wonda, ko'p-mu'yeshliktin' ishki mu'yeshlerinin' qosi'ndi'si' haqqindag'i teorema boyi'nsha,

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 360^\circ.$$

2) Sektor maydani'n tabiw formulasi ($R = 1 \text{ sm}$),

$$S_1 = \frac{\pi}{360^\circ} \cdot \alpha_1, \quad S_2 = \frac{\pi}{360^\circ} \cdot \alpha_2, \quad S_3 = \frac{\pi}{360^\circ} \cdot \alpha_3, \quad S_4 = \frac{\pi}{360^\circ} \cdot \alpha_4. \quad (1)$$

3) (1) ten'liklerdin' sa'ykes bo'leklerin qosamiz. Wonda,

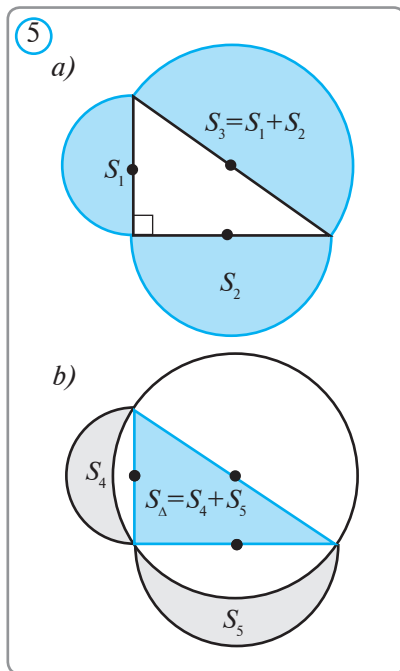
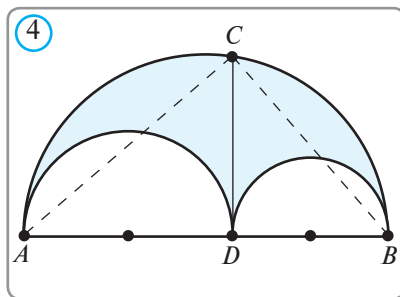
$$S_1 + S_2 + S_3 + S_4 = \frac{\pi}{360^\circ} (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4) = \frac{\pi}{360^\circ} \cdot 360^\circ = \pi \text{ (sm}^2\text{)}.$$

Juwbai': $\pi \text{ sm}^2$.

? Soraw, ma'sele ha'm tasirmalar

1. Perimetri 1 m bolg'an kvadrat ha'm uzunlig'i 1 m bolg'an shen'ber berilgen. Bul shen'ber menen shegaralang'an do'n'gelektin' may-

- dani' menen kvadrattin' maydani'n salistirin'.
2. Radiusi 8 sm bolg'an do'n'gelektan 60° li sektor qirqip aling'an. Do'n'gelektin' qalg'an bo'leginin' maydani'n tabi'n'.
 3. Diagonallari 6 sm ha'm 8 sm bolg'an rombig'a ishley si'zi'lg'an do'n'gelektin' maydani'n yesaplan'.
 4. 3-su'wrette boyalip ko'rsetilgen figura maydani'n tabi'n'. Wonda $ABCD$ — kvadrat, $AB=4$ sm.
 - 5*. 4-su'wrette "Arximed pishag'i" dep ataliwshi figura boyalip ko'rsetilgen. Woni'n' maydani'n $\frac{\pi \cdot CD^2}{4}$ formula menen yesplawdi da'liylen' (bunda $\angle ACB=90^\circ$ ha'm $CD^2=AD \cdot DB$ yek-enliginen paydalanin').
 6. Yeger $AD=6$ sm, $BD=4$ sm bolsa, 4-su'wrette boyalip ko'rsetilgen figuranin' maydani'n ha'm perimetrin tabi'n'.



Tariyx betlerinen. Gippokrat ayshalari.

Gippokrat ayshalari — yeki shen'ber dog'alari menen shegaralang'an ha'm to'mendegi qa'siyetke iye bolg'an figuralar boladi. Bul shen'berler (ayshalar) radiuslari ha'm dog'alarinin' xordalari boyi'nsha ayshalarg'a ten'dey kvadratlar jasaw mu'mkin.

Pifagor teoremasi' qollani'lsa, 5-a su'wretlengen gipotenuzag'a jasalg'an yarim do'n'geleklerdin' maydanlarinin' qosi'ndi'si'na ten' boladi' (107-bet, 9*-ma'selege qaran'). Soni'n' ushi'n 5.b-su'wrettegi ayshalardin' maydanlarinin' qosi'ndi'si' u'shmu'yeshliktin' maydani'na ten' (baqlap ko'rin'!). Yeger su'wrettegi *u'shmu'yeshliktin'* wornina ten' qaptalli tuwri' mu'yeshli u'shmu'yeshlikti alsaq, payda bolg'an yeki ayshadan ha'r birinin' maydani' u'shmu'yeshliktin' maydani'nin' yarimina ten' boladi. Do'n'gelek kvadraturasi' haqqindag'i ma'seleni sheshiwge uri'ni'p, grek matematigi Gippokrat (b.e.sh. V a'sir) ko'pmu'yeshlik penen ten'dey bir neshe tu'rli ayshalardi' jasag'an.

Gippokrat ayshalarinin' toliq kestesi tek XIX—XX a'sirlerde du'zilgen.

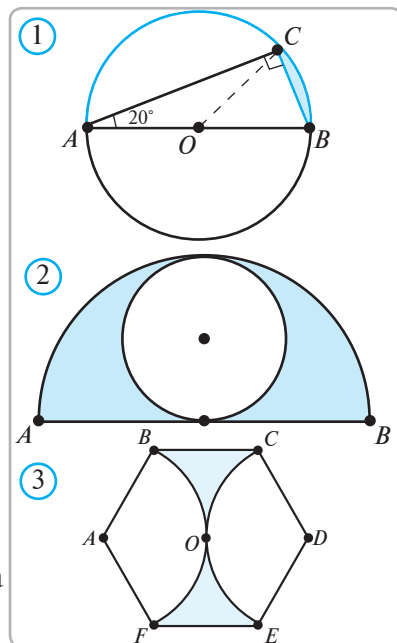
I. Testler

1. 45° gradusli mu'yeshstin' radian wo'lishemi nege ten'?
A. 1 ge ten' B. $\frac{\pi}{2}$ ge ten' D. $\frac{\pi}{4}$ ge ten' E. $\sqrt{2}$ ge ten'.
2. Radiusi 3 sm bolg'an shen'berdin' gradus wo'lishemi 150° gradus bolg'an worayliq mu'yeshke tirelgen dog'a uzi'nli'g'i'n tabi'n'.
A. $\frac{5\pi}{2}$ sm; B. $\frac{5\pi}{3}$ sm; D. $\frac{10\pi}{3}$ sm; E. $\frac{5\pi}{4}$ sm.
3. Radiusi 6 sm bolg'an shen'berde $\frac{5\pi}{4}$ radiang'a ten' worayliq mu'yesh tirelgen dog'anin' uzi'nli'g'i'n tabi'n'.
A. $\frac{15\pi}{2}$ sm; B. $\frac{5\pi}{6}$ sm; D. $\frac{4\pi}{3}$ sm; E. $\frac{5\pi}{2}$ sm.
4. Ta'repi 5 sm ge ten' bolg'an kvadratqa si'rtlay si'zi'lg'an shen'berdin' maydani'n tabi'n'.
A. $5\sqrt{2}\pi$; B. $\sqrt{2}\pi$; D. $3\sqrt{2}\pi$; E. 5π .
5. Diametri 6 g'a ten' do'n'gelektin' maydani'n tabi'n'.
A. 9π ; B. 6π ; D. $3\sqrt{2}\pi$; E. 12π .
6. Dog'anin' gradus wo'lishemi 150° , radiusi 6 sm bolg'an do'gelek sektordin maydani'n tabi'n'.
A. 15π sm²; B. 6π sm²; D. $30\sqrt{2}\pi$ sm²; E. 24π sm².
7. Dog'anin' uzunlig'i 12 sm ha'm radiusi 6 sm bolg'an do'n'gelek sektordin maydani'n tabi'n'.
A. 15π sm²; B. 6π sm²; D. $30\sqrt{2}\pi$ sm²; E. 24π sm².
8. Dog'anin' gradus o'lishemi 120° , radiusi 3 ke ten' bolg'an do'n'gelek segmenttin' maydani'n tabi'n'.
A. $6\pi - 4\sqrt{3}$; B. $6\pi + 4\sqrt{3}$; D. $3\pi - 4\sqrt{3}$; E. $3\pi + 4\sqrt{3}$.

II. Ma'seleler.

1. ABCDEFKL duri's segizmu'yeshliktin' ta'repi 6 sm. Woni'n' AC diagonalin' tabi'n'.
2. Kvadrat radiusi 4 dm bolg'an shen'berge ishley si'zi'lg'an. Kvadrattin' qon'si'las ta'replerinin' wortalarinan wo'tiwshi xordanin' shen'berden aji'ratqan dog'alarinin' uzi'nli'g'i'n tabi'n'.
3. Shen'berdin' 90° li dog'asinin' uzunlig'i 15 sm. Shen'berdin' radiusin tabi'n'.
4. Radiusi 20 g'a ten' shen'berden uzunlig'i 10π ge ten' dog'a ajiratiladi. Bul dog'ag'a sa'ykes worayliq mu'yeshi tabi'n'.
5. Yeki dog'anin' uliwma xordasi bul do'n'geleklerdi shegaralawshi shen'berlerden 60° li ha'm 120° li dog'alar ajiratadi. Do'n'geleklerdin' maydanlarinin' qatnasin tabi'n'.
6. Ta'repleri 3, 4, 5 bolg'an u'shmu'yeshlikke ishley ha'm si'rtlay si'zi'lg'an do'n'geleklerdin' maydanlarin tabi'n'.

7. Do'n'gelek xordasi 60° li dog'ani kerip turadi. Bul xorda aji'ratqan segmentlardin' maydanlarinin' qatnasin tabi'n'.
8. Duri's altimu'yeshliktin' maydani'nin' wog'an ishley si'zi'lg'an do'n'gelektin' maydani'na qatnasin tabi'n'.
9. Ta'repi a g'a ten' bolg'an $ABCDEF$ duri's altimu'yeshlik berilgen. Worayi A noqati ha'm radiusi a bolg'an shen'ber usi altimu'yeshlikni yeki bo'lekke ajiratadi. Ha'r bir bo'lektin' maydani'n tabi'n'.
10. Tuwri' mu'yeshli ABC u'shmu'yeshlikte $\angle A = 72^\circ$, $\angle C = 90^\circ$, $BC = 15$ sm. BC diametrli shen'berdin' ABC u'shmu'yeshlik ishinde jatqan dog'asinin' uzi'nli'g'i'n tabi'n'.
11. Do'n'gelekke ishley si'zi'lg'an duri's segizmu'yeshlik berilgen. Woni'n' yeki qon'silas to'belerine ju'rgizilgen radiuslar do'n'gelekti yeki sektorg'a ajiratadi. Bul sektorlardin' maydanlarinin' qatnasin tabi'n'.
12. Tuwri' mu'yeshli ABC u'shmu'yeshlikte $\angle A = 20^\circ$, $\angle C = 90^\circ$, $AB = 18$ sm. BC kesindisi u'shmu'yeshlikke si'rtlay si'zi'lg'an do'n'gelekti yeki segmentke ajiratadi. Boyap ko'rsetilgen segment maydani'n tabi'n'. (1-su'wret).
13. Kishi shen'ber u'lken shen'berge ha'm de woni'n' AB diametrine urinadi'. Yeger diametrge uriniw noqati' shen'ber worayi' ha'm $AB = 4$ bolsa, su'wrette boyalg'an figuranin' maydani'n tabi'n'. (2-su'wret).
14. Duri's $ABCDEF$ altimu'yeshliktin' ta'repi 6 g'a ten' ha'm worayi O noqati'nda. Woraylari A ha'm D noqatinda ha'm radiuslari ten' bolg'an shen'berler O noqatinda urinadi'. Boyalg'an oblast maydani'n tabi'n' (3-su'wret).
15. Tuwri' mu'yeshli ABC u'shmu'yeshlikte $\angle C = 90^\circ$, $AC = 4$, $CB = 2$. Worayi gipotenuzada bolg'an shen'ber u'shmu'yeshlik katetlerine urinadi'. Usi shen'berdin' uzi'nli'g'i'n tabi'n'.



III. Wo'zin'izdi sinap ko'rin' (u'lgi ushi'n baqlaw jumi'si')

1. Ta'repleri 6 sm bolg'an kvadratqa si'rtlay si'zi'lg'an shen'ber uzunlig'i' ha'm ishley si'zi'lg'an do'n'gelektin' maydani'n tabi'n'.
2. Ta'repi 24 sm bolg'an duri's ko'pmu'yeshlikke ishley si'zi'lg'an shen'ber radiusi $4\sqrt{3}$ sm ge ten' bolsa, wog'an si'rtlay si'zi'lg'an shen'ber radiusin tabi'n'.

3. 240° li shen'ber dog'asinin' uzunlig'i 24 sm bolsa,
 a) shen'ber radiusin; b) dog'asi 240° bolg'an sektordin' maydani'n;
 d) dog'asi 240° bolg'an segmenttin' maydani'n tabi'n'.



Qi'zi'qli' ma'sele

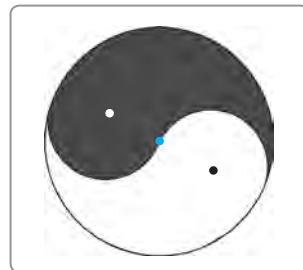
In ha'm Yan

Su'wrette ta'biyattag'i qarama-qarsiliqlardi sa'wlelendiriwshi "In ha'm Yan" dep atalg'an xitay belgisi (tamg'asi) su'wretlengen.

a) In ha'm Yan belgileri maydanlarinin ten'ligin ko'rsetin';

b) bir tuwri' si'zi'q penen bul belgilerdin' ha'r birin maydanlari ten' bo'lgan yeki bo'lekke ajiratin'.

d) In ha'm Yan nishanlari perimetrlerin (wolardi qorshap turg'an dog'alar uzunliqlarinin qosi'ndi'si'n) tabi'n'.



Tariyx betlerinen. Shen'ber uzi'nli'g'i'n yesaplaw ju'da' a'yyemnen ken' ko'lemdegi mashqala bolg'an. Shen'ber uzi'nli'g'i'n wog'an ishley si'zi'lg'an ko'pmu'yeshlik perimetrine almastiriw usi'li' ken' tarqalg'an.

Worta Aziyalı matematikler de do'n'gelekke ishley si'zi'lg'an duri's ko'pmu'yeshliklerdi jasaw, wolardi'n' ta'replerin do'n'gelektin' radiusi arqali an'latiw ma'seleleri menen shug'illang'an. Abu Rayxan Beruniy "Qonuniy Masudiy" miynetinde do'n'gelekke ishley si'zi'lg'an ko'pmu'yeshliklerdin' ta'repin ani'qlaw menen shug'illanip, ishley si'zi'lg'an besmu'yeshlik, altimu'yeshlik, jetimu'yeshlik, ..., wonmu'yeshliktin' ta'replerin ani'qlaw usi'li'n ko'rsetedi. Bul yesaplaw na'tiyjesinde wol $\pi \approx 3,14$ ma'nisine iye boladi.

A'yyemgi Grek ha'm Misir qoljazzbalarında π di u'shke ten' dep alg'an. Bul sol da'wirdin' ani'qliq talabi ushi'n jeterli bolg'an. Keyinshelik rimliler π ushi'n $3,12$ ni paydalang'an, π sani ushi'n Arximed bergen ma'nis $3,14$ boli'p, bul a'meliy ma'selelerdi sheshiwde ju'da' qolayli.

Mirza Ulug'bektin' "Astronomiya mektebi"—talabalarinin' biri Jamshid Giyasiddin al-Koshiy 1424-jili jazg'an "Shen'ber uzunlig'i haqqinda kitap" atamasindag'i traktatinda shen'berge ishley ha'm si'rtlay si'zi'lg'an duri's ko'pmu'yeshlik ta'repleri sanin yeki yeseletiw joli menen $3 \cdot 2^{28} = 800\,335\,168$ ta'repli duri's ko'pmu'yeshliklerdin' perimetrlerin yesaplap, π ushi'n $\pi = 3,1\,415\,826\,535\,897\,932$ ma'nisin payda yetken. Bul 16 wonliq sang'a shekem ani'q boladi.

Biraq al-Koshiydin' miyneti uzaq waqitqa shekem Yevropada belgisiz bolg'an. Yevropalilardan Belgiyalı Van Romen 1597-jili 2^{30} ta'repli duri's ko'pmu'yeshlikke Arximed usi'li'n qollanip, π ushi'n 17 wonliq sanlari ani'q bolg'an ma'nisti tapqan. Gollandiyalı Rudolf Van Seylon (1540—1610) bul ani'qliqti 35 wonliq sanlarga shekem alip barg'an. Ha'zirgi da'wirde elektron yesaplaw mashinalari ja'rdeminde π ushi'n millionnan artiq wonliq sanlari ani'q bolg'an ma'nisleri tabilg'an. Ku'ndelik yesaplawlar ushi'n $3,14$ ma'nisi, matematikalıq yesaplawlar ushi'n $3,1416$ ma'nisi, ha'tte astronomiya ha'm kosmonavtika ushi'n $3,1415826$ ma'nisi jetkilikli boladi'.

IV BAP



U'SHMU'YESHLIK HA'M SHEN'BERDEGI METRIKALIQ QATNASLAR

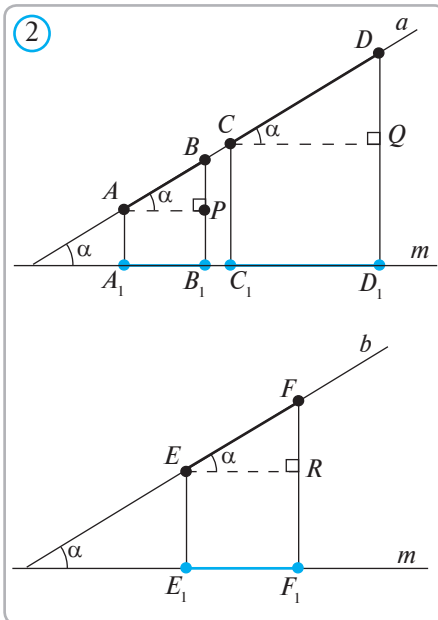
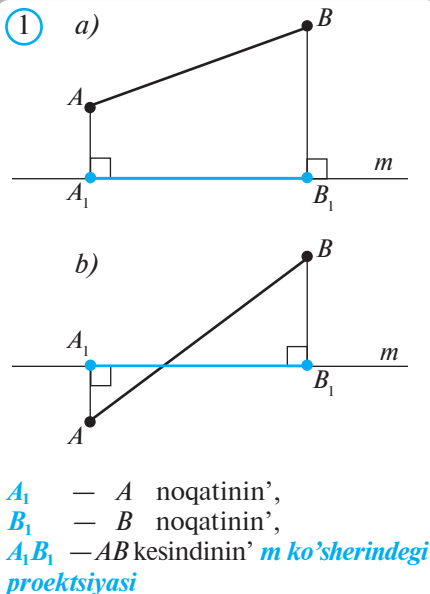
Usi bapni u'yreniw na'tiyjesinde siz to'mendegi bilim ha'm ko'nlik-pelerga iye bolasiz:

Bilimler:

- ✓ *proporcional kesindilerdin' qa'siyetlerin biliw;*
- ✓ *tuwri' mu'yeshli u'shmu'yeshlikte gipotenuzag'a tu'sirilgen biyikliktin' qa'siyetlerin biliw;*
- ✓ *wo'z ara kesilisiwshi xordalar kesindiler haqqindag'i ha'm de shen'berdi kesip wo'tiwshi tuwri'nin' kesindileri haqqindag'i qa'siyetlerdi biliw.*

Ko'nlikpeler:

- ✓ *kesindilerdin' qatnasin ha'm proporcional kesindilerga tiyisli ma'selelerdi sheshe aliw;*
- ✓ *tuwri' mu'yeshli u'shmu'yeshlikte gipotenuzag'a tu'sirilgen biyikliktin' qa'siyetlerinen paydalani'p ma'seleler sheshe aliw;*
- ✓ *kesilisiwshi xordalar kesindilerinin' ha'm kesiwshi tuwri'lar kesindilerinin' qa'siyetlerinen paydalani'p ma'seleler sheshiw.*



Jedellestirish shing'iw

1. Kesindilerdin' qatnasi neni an'latadi?
2. Qanday kesindiler proporcional delinedi?
3. Fales teoremasin aytin'.

Tegislikte m tuwri'si ha'm AB kesindisi berilgen bolsi'n. A ha'm B noqatlardan m tuwri'sina AA_1 ha'm BB_1 perpendikulyar tu'siremiz (1-su'wret). A_1B_1 kesindisi AB kesindisinin' m ko'sherindeki **proektsiyasi (sayasi)** delinedi.

AB kesindisinin' m tuwri'dag'i A_1A_1 proektsiyasin jasaw a'meli AB kesindisin m tuwri'sina **proektsiyalaw** delinedi.

Teorema. Bir tuwri'da yaki parallel tuwri'larda jatatug'in kesindiler berilgen bolsi'n. Wolardi'n' tap bir tuwri'g'a proektsiyalari berilgen kesindilerge proporcional boladi.

$a \parallel b$,
 A_1B_1 — AB nin',
 C_1D_1 — CD nin',
 E_1F_1 — EF nin'
 m tuwri'si'zi'qqa
 proyektsiyalari
 (2-su'wret)

$$\frac{A_1B_1}{AB} = \frac{C_1D_1}{CD} = \frac{E_1F_1}{EF} \quad (1)$$

Da'liyllew. a) Yeger a ha'm b tuwri'lari m tuwri'sina parallel bolsa, $AB = A_1B_1$, $CD = C_1D_1$, $EF = E_1F_1$ boladi ha'm de (1) ten'lik wori'nli' yekenligi ani'q.

b) Yeger de a ha'm b tuwri'lari m tuwri'sina perpendikulyar bolsa, A_1 ha'm B_1 , C_1 ha'm D_1 , E_1 ha'm F_1 noqatlari u'stpe-u'st tu'sedi. Sonin' ushi'n A_1B_1 , C_1D_1 , E_1F_1 kesindilerinin' uzunlig'i nolge ten' boladi ha'm (1) ten'lik worinlanadi.

d) Yendi basqa jag'daylardi qaraymiz. 2-su'wrette su'wretlengenindey tuwri' mu'yeshli ABP , CDQ , EFR u'shmu'yeshliklerin jasaymiz.

Bunda $a \parallel b$ bolg'ani ushi'n $\angle BAP = \angle DCQ = \angle FER$. Demek, ABP , CDQ ha'm EFR tuwri' mu'yeshli u'shmu'yeshlikleri uqsas. Bunnan

$\frac{A_1B_1}{AB} = \frac{C_1D_1}{CD} = \frac{E_1F_1}{EF}$ ten'ligin payda yetemiz. **Teorema da'liyllendi.**

Ma'sele. AB ha'm CD kesindileri parallel tuwri'larda jatadi. Yeger $AB = 12 \text{ sm}$, $CD = 15 \text{ sm}$ ha'm AB kesindinin' qanday da bir m tuwri'sindag'i proekciyasi 8 sm bolsa, CD kesindinin' usi m tuwri'sindag'i proektciyasin tabi'n'.

Sheshiliwi. CD kesindisinin' m tuwri'sindag'i proekciyasi x bolsi'n. Wonda, da'liyllengen teorema ha'm ma'sele sha'rtinen paydalani'p, proekciya du'zemiz:

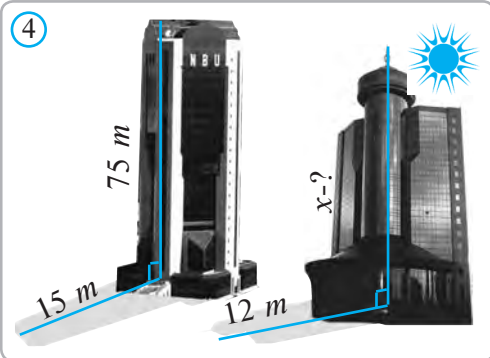
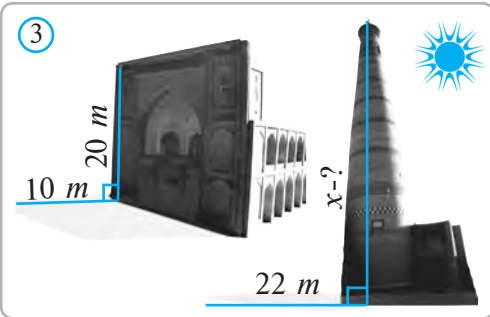
$$\frac{x}{15} = \frac{8}{12}.$$

Bul ten'likten $x = 10$ bolatug'inin tabami'z.

Juwabi': 10 sm .

? Soraw, ma'sele ha'm tapsi'rmalar

1. Kesindinin' berilgen tuwri'dag'i proekciyasi degen ne?
2. Bir tuwri'da yaki parallel tuwri'larda jatqan kesindilerdin' da'l basqa bir tuwri'g'a proekciyalari berilgen kesindilerge proporcional yekenligin da'liyllen'.
3. a ha'm b tuwri'lari arasi'ndag'i mu'yesh 45° qa ten', a tuwri'sinda uzunlig'i 10 sm bolg'an AB kesindisi aling'an. AB kesindisinin' b tuwri'si'ndag'i proektciyasin tabi'n'.
4. AB kesindisinin' ushlari' l tuwri'sinan 9 sm ha'm 14 sm qashqliqta jatadi. Yeger AB kesindisi l tuwri'sin kesip wo'tpese ha'm $AB = 13 \text{ sm}$ bolsa, AB kesindisinin' l tuwri'sindag'i proekciyasin tabi'n'.
5. 3-ha'm 4-su'wretlerdegi mag'liwmatlar tiykarinda imaratlardin' biyikliklerin tabi'n'.
6. Tuwri' ha'm wog'an parallel bolmag'an kesindi si'zi'n'. Kesindinin' tuwri'dag'i proekciyasi'n jasan'.
7. Koordinatar tegisliginde $A(2;3)$ ha'm $B(3;-4)$ noqatlari belgilengen. AB kesindisinin' koordinata ko'sherlerindeki proekciyalari uzunliqlarin tabi'n'.
8. a ha'm b tuwri'lari' arasi'ndag'i mu'yesh α yekenligi belgili, a tuwri'sinda AB kesindisi aling'an. AB kesindisinin' b tuwri'sindag'i proekciyasin tabi'n'.



Fales teoremasinin' uluwmalastiril'g'aninan ibarat a'hmiyetli qa'siyetin da'liylleymiz.

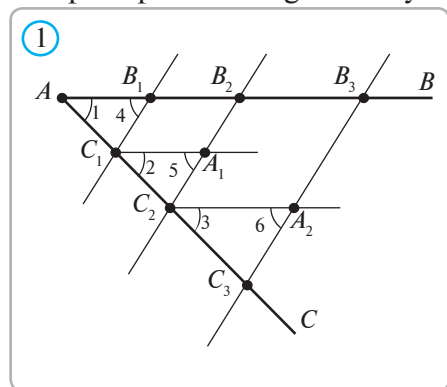
Teorema. Mu'yeshstin' ha'r yeki ta'repin kesip wo'tken parallel tuwri'lar woni'n' ta'replerinen proporcional kesindiler ajiratadi.

$\angle BAC, B_1C_1 \parallel B_2C_2 \parallel B_3C_3$ (1-su'wret)



$$\frac{AB_1}{AC_1} = \frac{B_1B_2}{C_1C_2} = \frac{B_2B_3}{C_2C_3}$$

Da'liyllew. C_1 ha'm C_2 noqatlaridan AB g'a parallel C_1A_1 ha'm C_2A_2 tuwri'larin ju'rgizemiz. Bul jag'dayda, birinshiden, $\angle 1 = \angle 2 = \angle 3$ boladi, sebebi wolar wo'z arapara'lel bolg'an AB , C_1A_1 ha'm C_2A_2 tuwri'larin AC kesip wo'tkende payda bolg'an sa'ykes mu'yeshler boladi'. Yekinishiden, $\angle 4 = \angle 5 = \angle 6$, sebebi wolar ta'repleri para'lel bolg'an mu'yeshler boladi.



Demek, u'shmu'yeshliklerdin' uqsaslig'inin' MM belgisi boyi'nsha $\triangle AB_1C_1 \sim \triangle C_1A_1C_2 \sim \triangle C_2A_2C_3$ boladi.

Bul jag'dayda $\frac{AB_1}{AC_1} = \frac{C_1A_1}{C_1C_2} = \frac{C_2A_2}{C_2C_3}$ (1) ten'liklerin payda yetemiz.

Bunnan tisqari, $B_1C_1A_1B_2$ ha'm $B_2C_2A_2B_3$ to'rtmu'yeshlikleri parallelogramm, sebebi

$B_1C_1 \parallel B_2C_2 \parallel B_3C_3$ — sha'rt boyi'nsha

$AB \parallel C_1A_1 \parallel C_2A_2$ — jasaw boyi'nsha.

Sonin' ushi'n, bul parallelogrammnin' qarama-qarsi ta'repleri wo'zara ten' boladi:

$$C_1A_1 = B_1B_2 \text{ ha'm } C_2A_2 = B_2B_3. \quad (2)$$

(1) ha'm (2) ten'liklerden $\frac{AB_1}{AC_1} = \frac{B_1B_2}{C_1C_2} = \frac{B_2B_3}{C_2C_3}$ bolatug'inlig'i kelip shig'adi.

Teorema da'liyllendi.

A'meliy shi'ni'g'iw. Kesindini berilgen qatnasta bo'liw.

Berilgen a kesindisin to'rt bo'lekke sonday yetip bo'lin', bo'leklerdin' wo'zara qatnasi $m:n:k:l$ siyaqli bolsi'n.

Bunin' ushi'n to'mendegilerdi qa'dembe-qa'dem worinlaymiz:

1-qa'dem. Qa'legen su'yir mu'yesh sizip, woni'n' bir ta'repine uzunliqlari $OA=m$, $AB=n$, $BC=l$ ha'm $CD=k$ g'a ten' bolg'an kesindilerdi 2-su'wrettegidey yetip, izbe-iz qoyip shig'amiz.

2-qa'dem. Mu'yeshstin' yekinshi ta'repindegi a kesindisine ten' OD_1 kesindisin qoyamiz.

3-qa'dem. D ha'm D_1 noqatlarin tutastiramiz.

4-qa'dem. A, B, C noqatlari arqali DD_1 ge parallel AA_1, BB_1 ha'm CC_1 kesindilerin ju'rgizemiz.

Joqaridag'i teorema boyi'nsha, berilgen $a = OD_1$ kesindisi A_1, B_1, C_1 ha'm D_1 noqatlari menen $m:n:l:k$ qatnasta bo'lingan boladi.

Tapsi'rma. Bul tasiyiqlawdi' wo'z betin'izshe tiykarlan'.

A'meliy tapsi'rma. To'rtinshi proporcional kesindini jasaw.

a, b ha'm c kesindileri berilgen. a ha'm b kesindilar c ha'm d kesindilerine proporcional, yag'niy $a:b=c:d$ yekenligin ma'lim. d kesindisin jasan' (3-rasm).

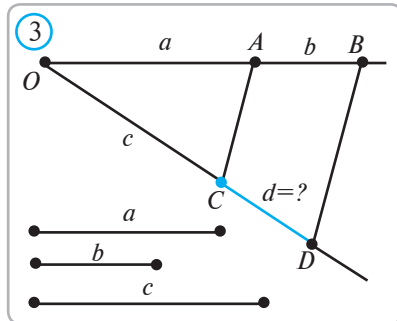
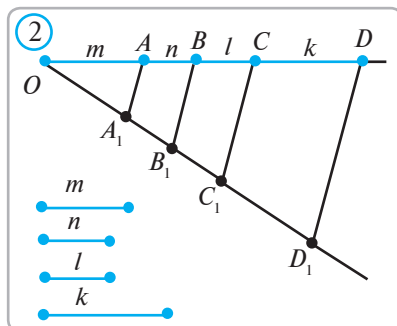
1-qa'dem. Qa'legen su'yir mu'yesh sizip, woni'n' bir ta'repine $OA=a$ ha'm $AB=b$ kesindilerin 3-su'wrette ko'rsetilgende yitip qoyamiz.

2-qa'dem. Yekinshi ta'repine bolsa $OC=c$ kesindisin qoyami'z.

3-qa'dem. A ha'm C noqatlarin tutastiramiz.

4-qa'dem. B noqatidan AC g'aparallel BD tuwri'sin ju'rgizemiz.

Tapsi'rma: CD izlenip atirg'an d kesindisi bolatug'inin tiykarlan'.



? Soraw, ma'sele ha'm tapsi'rmalar

1. Uzunlig'i 42 sm bolg'an kesindi berilgen. Woni a) 5:2; b) 3:4:7; d) 1:5:1:7 qatnastag'i bo'lekshelerga bo'lin'.
2. Su'wrette ha'r bir bo'lek birlik kesindiden ibarat bolsa, AB ha'm CD, EF ha'm MN, AC ha'm DF, AN ha'm CE, EN ha'm BM kesindilerin' qatnaslarin tabi'n'.



3. m, n kesindileri l ha'm k kesindilerine proporcional. Yeger a) $m=4$ sm, $n=3$ sm ha'm $l=8$ sm; b) $m=2$ sm, $n=3$ sm ha'm $l=7$ sm bolsa, k — to'rtinshi kesindini jasan' ha'm uzi'nli'g'i'n tabi'n'.
4. To'rtmu'yeshliktin' perimetri 54 sm ha'm ta'replari 3:4:5:6 siyaqli qatnasta bolsa, woni'n' ha'r bir ta'repin ani'qlan'.
5. To'rtmu'yeshliktin' mu'yeshleri wo'zara 3:4:5:6 siyaqli qatnasta bolsa, woni'n' kishi mu'yeshi nege ten' yekenligin tabi'n'.
6. Uzunligi 4, 5 ha'm 6 bolg'an kesindiler berilgen. Uzunligi 4,8 ge ten' kesindi jasan'.

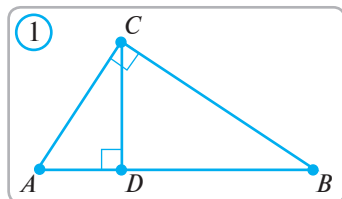
 **Qa'siyeti.** Tuwri' mu'yeshli u'shmu'yeshliktin' tuwri' mu'yeshi to'besinen tu'sirilgen biyikligi woni wo'zine uqsas yeki u'shmu'yeshlikke ajiratadi.



$\triangle ABC, \angle C = 90^\circ,$
 CD — biyiklik (1-su'wret)



$\triangle ABC \sim \triangle ACD, \triangle ABC \sim \triangle CBD$



Da'lyillew. ABC ha'm ACD u'shmu'yeshlikleri tuwri' mu'yeshli boli'p, A mu'yeshi bolsa, wolar ushi'n ulawma. Demek, $\triangle ABC \sim \triangle ACD$. Ushi siyaqli, $\triangle ABC$ ha'm $\triangle CBD$ datuwri' mu'yeshli boli'p, wolar ushi'n $\angle B$ uliwma. Demek, $\triangle ABC \sim \triangle CBD$.

1-su'wrettegi AD ha'm DC kesindilerge sa'ykes ra'wishte AC ha'm BC katetlerdin' gipotenuzadag'i proekciyasi dep ju'ritiledi.

 **Ani'qlama.** Yeger a, b ha'm c kesindilari ushi'n $a:b=b:c$ bolsa, b kesindisi a ha'm c kesindilari arasi'ndag'i **worta proporcional kesindi** dep ataladi.

Worta proporcionalliq sha'rtin $b^2=ac$ yaki $b=\sqrt{ac}$ ko'riniside de jaziw mu'mkin.

Joqarida da'liyllengen qa'siyet ha'm 1-su'wretke tiykarlanatug'in bolsaq, worta proporcional kesindiler haqqindag'i to'mendegi teoremlar an'sat da'liyllenedi.

 **1-teorema.** Tuwri' mu'yeshli u'shmu'yeshliktin' tuwri' mu'yeshi ushi'nan tu'sirilgen biyiklik katetlerdin' gipotenuzadag'i proekciyalari arasi'nda worta proporcional boladi.

Haqiyqattan da'liyllengen qa'siyet boyi'nsha: $\triangle ACD \sim \triangle CBD$. Bunnan,

$$\frac{AD}{CD} = \frac{CD}{BD} \Rightarrow CD^2 = AD \cdot BD \Rightarrow CD = \sqrt{AD \cdot BD}.$$

 **2-teorema.** Tuwri' mu'yeshli u'shmu'yeshliktin' kateti gipotenuza menen usi katettin' gipotenuzasindag'i proekciyasi arasi'nda worta proporcionali boladi (1-su'wret).

Haqiyqattan da, da'liyllengen qa'siyet boyi'nsha: $\triangle ABC \sim \triangle ACD$. Bunnan,

$$\frac{AB}{AC} = \frac{AC}{AD} \Rightarrow AC^2 = AB \cdot AD \Rightarrow AC = \sqrt{AB \cdot AD}.$$

Usig'an uqsas $BC = \sqrt{BD \cdot AB}$ yekenligin da'lyillew mu'mkin.

 **Ma'sele.** Katetleri 15 sm ha'm 20 sm bolg'an tuwri' mu'yeshli u'shmu'yeshliktin' kishi katetinin' gipotenuzasindag'i proekciyasini tabi'n.



$\triangle ABC, \angle C = 90^\circ, CD$ — biyiklik, $AC = 15$ sm,
 $BC = 20$ sm (1-su'wret)



$AD = ?$

Sheshiliwi. 1) Pifagor teoremasi'nan paydalani'p, u'shmu'yeshliktin' gipotenuzasin tabami'z: $AC^2 = AC^2 + BC^2 = 15^2 + 20^2 = 625$, yag'niy $AB = 25$ sm.

2) Yekinshi teoremadan paydalani'p AD ni tabami'z:

$$AC^2 = AB \cdot AD \Rightarrow AD = \frac{AC^2}{AB} = \frac{15^2}{25} = 9 \text{ (sm)}.$$

Juwabi': 9 sm.

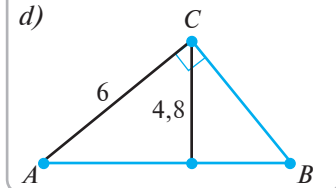
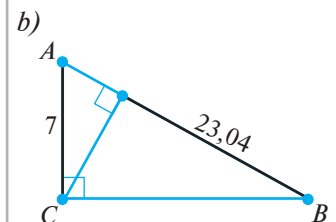
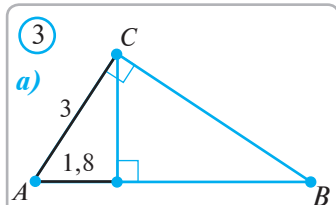
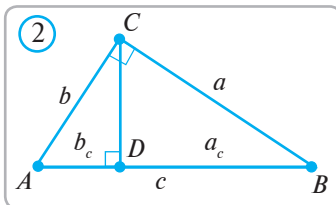
Bul yeki teoremadan na'tiyje sipatinda Pifagor teoremasinin' **Pifagordin' wo'zi jazip qaldirg'an da'liyli** kelip shig'adi (*1-su'wret*): 2-teorema boyi'nsha,

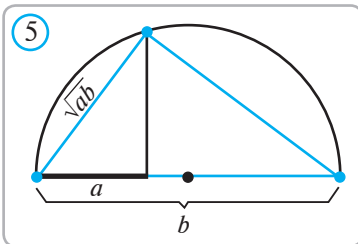
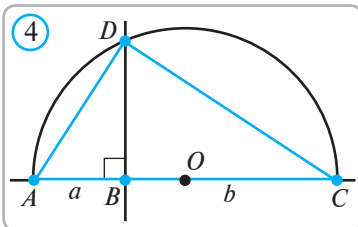
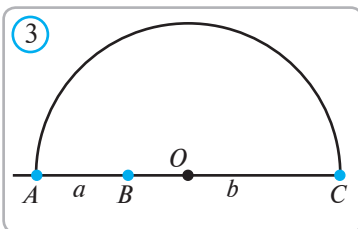
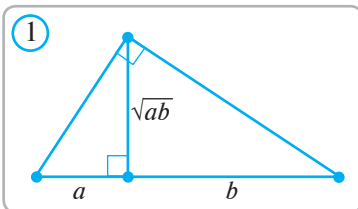
$$\left. \begin{array}{l} AC^2 = AD \cdot AB \\ BC^2 = BD \cdot AB \end{array} \right\} \Rightarrow AC^2 + BC^2 = AD \cdot AB + BD \cdot AB = AB \cdot \underbrace{(AD + BD)}_{AB} = AB \cdot AB = AB^2.$$

Solay yetip, $AC^2 + BC^2 = AB^2$.

? Soraw, ma'sele ha'm tapsi'rmalar

- Da'liyllen' (*2-su'wret*): a) $\triangle ACD \sim \triangle CBD \sim \triangle ABC$; b) $b^2 = b_c \cdot c$, $a^2 = a_c \cdot c$; d) $h_c^2 = a_c \cdot b_c$.
- Tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasina tu'sirilgen biyikligi gipotenuzani 9 sm ha'm 16 sm ge ten' kesindilerge bo'ledi. U'shmu'yeshliktin' ta'replerin tabi'n'.
- Tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasi 15 sm ge, bir kateti bolsa 9 sm ge ten'. Yekinshi katetinin' gipotenuzasindag'i proekciyasin tabi'n'.
- 3-su'wrettegi mag'liwmatlar tiykarinda ABC u'shmu'yeshliktin' ta'replerin tabi'n'.
- 5***. Katetlerinin' qatnasi 4:5 siyaqli bolg'an tuwri' mu'yeshli u'shmu'yeshliktin' katetlerinin' gipotenuzasindag'i proekciyalarinin' qatnasin tabi'n'.
- 6***. Katetlerinin' qatnasi 3:2 siyaqli bolg'an tuwri' mu'yeshli u'shmu'yeshlik berilgen. Katetlerinin' gipotenuzasindag'i proekciyalarinin' biri yekinshisinen 6 sm ge uzun. U'shmu'yeshliktin' maydani'n tabi'n'.
- Katetlerinin' gipotenuzasindag'i proekciyalari 2 sm ha'm 18 sm bolg'an tuwri' mu'yeshli u'shmu'yeshliktin' maydani'n tabi'n'.
- 8***. ABC u'shmu'yeshliginde $\angle C = 90^\circ$, CD — biyiklik, CE — bissektissa ha'm $AE:EB = 2:3$. a) $AC:BC$; b) $S_{ACE}:S_{BCE}$; d) $AD:BD$ qatnaslari'n tabi'n'.





Tuwri' mu'yeshli u'shmu'yeshliktin' tuwri' mu'yeshinen tu'sirilgen biyikligi gipotenuzani a ha'm b kesindilerge bo'lse, biyiklik $\sqrt{ab}$ g'aten' bolatug'inin ko'rgen yedik (*1-su'wret*).

Demek, berilgen yeki kesindige worta proporcional kesindini jasaw ushi'n:

- 1) gipotenuzasinin' uzunlig'i $a + b$ g'aten' (*2-su'wret*);
- 2) tuwri' mu'yeshinen tu'sirilgen biyikligi usi gipotenuzani a ha'm b bo'leklerge bo'letug'in tuwri' mu'yeshli u'shmu'yeshlik jasaw jetkilikli.

Bunin' ushi'n tuwri' mu'yeshli u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'ber worayi gipotenuzanin' wortasinda jaylasqaninan paydalanamiz (*3-su'wret*).

Jasaliwi:

- 1) tuwri' si'zi'q sizip, wog'an $AB = a$ ha'm $BC = b$ bolatug'inday yetip A , B ha'm C noqatlarin belgileymiz (*3-su'wret*).

- 2) AC kesindisinin' wortasi — O noqatin tabami'z. worayi O noqatinda bolg'an AC diametrli yarim shen'ber jasaymiz (*3-su'wret*).

- 3) B noqatinan AC tuwri'sina perpendikulyar tuwri' ju'rgizemiz (*4-su'wret*). Bul tuwri' yarim shen'berdi D noqatinda kesip wo'tken bolsi'n. Wonda $\triangle ADC$ — tuwri' mu'yeshli u'shmu'yeshlik, $BD = \sqrt{ab}$ — biz jasawimiz kerek bolg'an kesindi boladi.

Jasaw wori'nlandi'.

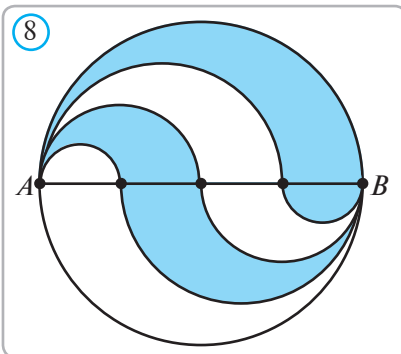
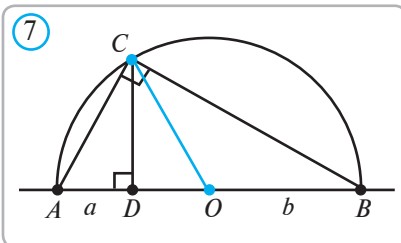
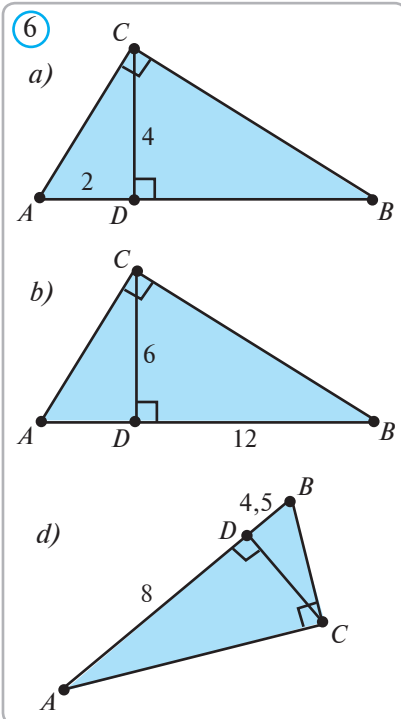
Worta proporcional kesindini jasawda tuwri' mu'yeshli u'shmu'yeshliktin' kateti gipotenuza menen usi katettin' gipotenuzadag'i proekciyasi arasi'nda worta proporcional yekenligin paydalaniw da mu'mkin (*5-su'wret*).

? Soraw, ma'sele ha'm tapsi'rmalar

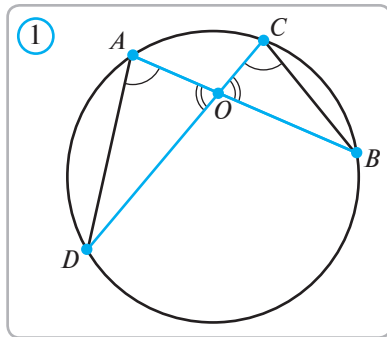
1. Uzunliqlari a ha'm b bolg'an kesindiler berilgen. Uzunlig'i $\sqrt{ab}$ bolg'an kesindini jasan'.
2. Uzunlig'i a ha'm b g'aten' kesindiler berilgen. Pifagor teoremasinan paydalani'p, uzunlig'i
a) $\sqrt{a^2 + b^2}$; b) $\sqrt{a^2 - b^2}$
bolg'an kesindilerdi jasan'.
3. Uzunlig'i 1 ge ten' bolg'an kesindi berilgen. Uzunlig'i a) $\sqrt{2}$; b) $\sqrt{3}$; d) $\sqrt{5}$; e) $\sqrt{6}$; f) $\sqrt{18}$; g) $\sqrt{30}$ bolg'an kesindilerdi jasan'.
4. 6-su'wrettegi mag'li'wmatlar tiykarinda ABC u'shmu'yeshliktin' maydani'n tabi'n'.
5. Shen'berdegi C noqatidan AB diametrge CD perpendikulyari' tu'sirilgen. Yeger $CD=12$ sm, $AD=24$ sm bolsa, do'n'gelektin' maydani'n tabi'n'.
6. Aldin'g'i ma'seledegi ABC u'shmu'yeshliktin' maydani'n tabi'n'.
7. Tuwri' mu'yeshli u'shmu'yeshliktin' tuwri' mu'yeshinin' bissektrisasi' gipotenuzani 5:3 siyaqli qatnasta bo'ledi. Tuwri' mu'yeshstin' to'besinen tu'sirilgen biyikliktin' gipotenuzadan ajiratqan kesindilerinin' qatnasin tabi'n'.
8. Radiusi 8 sm ge ten' do'n'gelekke bir mu'yeshi 30° bolg'an tuwri' mu'yeshli u'shmu'yeshliki shley si'zi'lg'an. Do'n'gelektin' u'shmu'yeshlikten si'rttag'i' bo'limi 3 segmentten ibarat. Usi segmentlerdin' maydanlarin tabi'n'.
- 9*. 7-su'wrette $AD=a$, $DB=b$, demek, $OC = \frac{a+b}{2}$ (O — shen'ber worayi). Su'wretten paydalani'p, $\frac{a+b}{2} \geq \sqrt{ab}$ ten'sizligin da'liyllen'.

⌚ Qi'ziqli' ma'sele

Shen'berdin' AB diametri to'rt ten' bo'lekke bo'lindi ha'm 8-su'wrette ko'rsetilgendey yarim shen'berler jasaldi. Yeger $AB=d$ bolsa, su'wrette boyap ko'rsetilgen ha'r bir figuranin' maydani'n yasplan'.



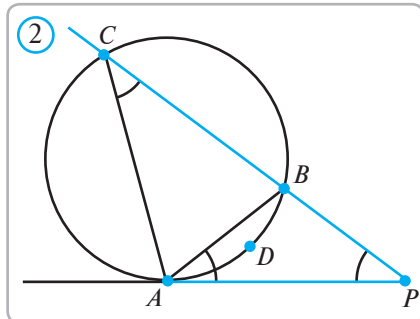
1-teorema. Shen'berdin' AB ha'm CD xordalari' O noqatinda kesilisse, $AO \cdot OB = CO \cdot OD$ ten'ligi wori'nli' boladi.



Da'liyllew. AB ha'm CD xordalari' (1-su'wret) ko'rsetilgan ta'rtipte joylasqan bolsi'n. To'belarin AD ha'm BC xordalari menen tutastiramide. Sonda BAD ha'm BCD mu'yeshleri bir dog'ag'atireledi, demek, $\angle BAD = \angle BCD$. Ja'ne, $\angle AOD = \angle BOC$ yekenligi belgili. Bul yeki ten'likten MM belgisi boyi'nsha AOD ha'm COB u'shmu'yeshliklardin' uqsaslig'i kelip shig'adi. Uqsas u'shmu'yeshliklardin' sa'ykes ta'replari bolsa proporcional: $\frac{OD}{OB} = \frac{AO}{CO}$ yaki

$AO \cdot OB = CO \cdot OD$. **Teorema da'liyillendi.**

2-teorema. Shen'berdin' si'rtindag'i P noqatidan shen'berge PA urinba (A – uriniw noqati) ha'm shen'berdi B ha'm C noqatlarinda kesip wo'tiwshi tuwri' ju'rgizilgen bolsa, $PA^2 = PB \cdot PC$ boladi.

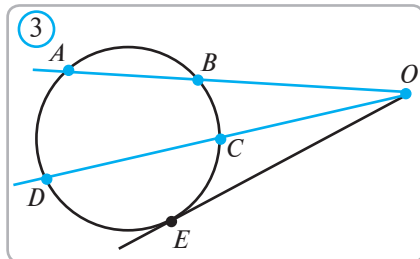


Da'liyllew. ABP ha'm CPA u'shmu'yeshliklerin qaraymiz (2-su'wret). Wonda,

$\angle C = \frac{\widehat{ADB}}{2} = \angle BAP$ ha'mde $\angle P$ — bul u'shmu'yeshlikler ushi'n uliwma mu'yesh. Demek, ABP ha'm CPA u'shmu'yeshlikleri yeki mu'yeshi boyi'nshauqsas. Bunnan, $\frac{PA}{PC} = \frac{PB}{PA}$ yaki

$PA^2 = PB \cdot PC$. **Teorema da'liyillendi.**

Ma'sele. A, B, C ha'm D noqatlari shen'berdi AB, BC, CD ha'm AD dog'alarg'a ajratadi. Yeger AB ha'm DC nurlari O noqatinda kesilisse, wonda $OA \cdot OB = OC \cdot OD$ ten'ligi wori'nli' bolatug'inin da'liyllen'.

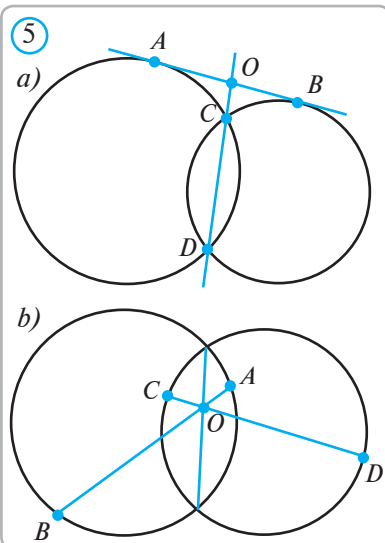
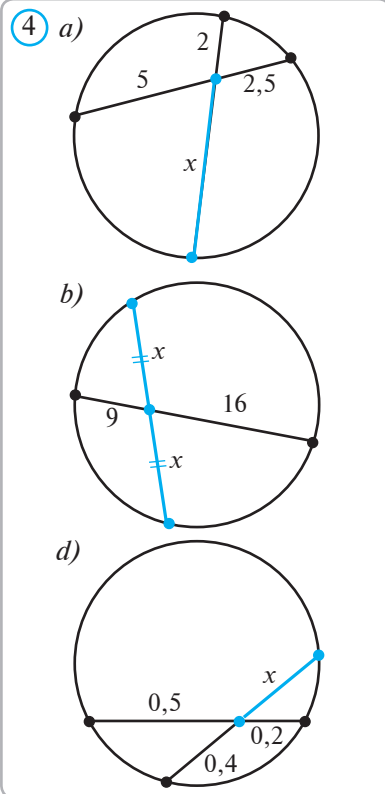


Sheshiliwi. Ma'sele sha'rtine sa'ykes si'zi'lma sizamiz (3-su'wret) ha'm O noqatidan OE urinba ju'rgizemiz. Wonda, 2-teoremag'a tiykarlanip,

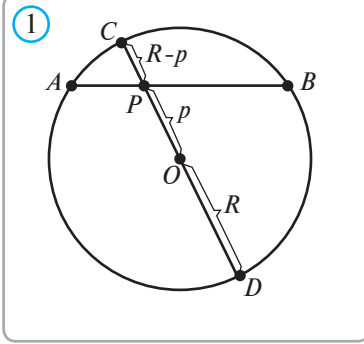
$$\left. \begin{array}{l} OB \cdot OA = OE^2 \\ OC \cdot OD = OE^2 \end{array} \right\} \Rightarrow OA \cdot OB = OC \cdot OD.$$

? Soraw, ma'sele ha'm tapsi'rmalar

1. 4-su'wrette x penen belgilengen belgisiz kesindini tabi'n'.
2. A noqatidan shen'berge AB urinba (B — uriniw noqati) ha'm shen'berdi C ha'm D noqatlarinda kesip wo'tetug'in kesiwshi ju'rgizilgen. Yeger
 - a) $AB = 4$ sm, $AC = 2$ sm bolsa, AD kesindisin;
 - b) $AB = 5$ sm, $AD = 10$ sm bolsa, AC kesindisin;
 - d) $AC = 3$ sm, $AD = 2,7$ sm bolsa, AB kesindisin tabi'n'.
3. Shen'berge $ABCD$ to'rtmu'yeshligi ishley si'zi'lg'an. AB ha'm DC nurlari' O noqatinda kesilisedi. Yeger
 - a) $AO = 10$ dm, $BO = 6$ dm, $DO = 15$ dm bolsa, OC kesindisin;
 - b) $CD = 10$ dm, $OD = 8$ dm, $AB = 4$ dm bolsa, OB kesindisin tabi'n'.
4. Shen'berdin' AB diametri ha'm bul diametrge perpendikulyar CD xordasi E noqatinda kesilisedi. Yeger $AE = 2$ sm, $EB = 8$ sm bolsa, CD xordani tabi'n'.
5. AB ha'm CD kesindileri O noqatindakesilisedi. Yeger $AO \cdot OB = BO \cdot OD$ bolsa, A, B, C ha'm D noqatlarinin' bir shen'berde jatatug'inin da'liyllen'.
6. Radiusi 13 dm bolg'an shen'ber worayinan 5 dm qashqliqta P noqati aling'an. P noqatidan uzinlig'i 25 dm bolg'an AB xorda ju'rgizilgen. AP ha'm PB kesindilerin tabi'n'.
7. 3-su'wrette AOD ha'm BOC u'shmu'yeshliklerdin' uqsas yekenliginen paydalani'p, $AO \cdot OB = CO \cdot OD$ ten'ligin da'liyllen'.
- 8*. 5-su'wretlerdegi mag'liwmatlar tiykarinda $AO \cdot OB = CO \cdot OD$ ten'ligin da'liyllen'.
- 9*. Yeki shen'ber C noqatindaurinadi'. AB tuwri' si'zi'q birinshi shen'berge B noqatindaurinadi. $\angle ACB = 90^\circ$ yekenligin da'liyllen'.



Aldi'n'g'i' sabaqta shen'ber kesilisiwshileri ha'm xordalarinin' qa'siyetlerin da'liyllegen yedik. Yendi bul qa'siyetlerdin' ayi'rim jag'daylari' menen tanisamiz.



1-ma'sele. P noqati R radiusli shen'berdin' ishki oblastindag'i wonin' worayinan p arali'qta jaylasqan bolsi'n. Wonda P noqatidan wo'tiwshi qa'legen AB xorda ushi'n

$$AP \cdot PB = R^2 - p^2 \quad (1)$$

ten'ligi worinli' boladi.

Da'liyllew. P noqati arqali shen'berdin' CD diametrin ju'rgizemiz. Wonda, $PC = R - p$, $PD = R + p$ ($1-su'wret$). Kesip o'tiwshi xordalar haqqindag'i teorema boyi'nsha,

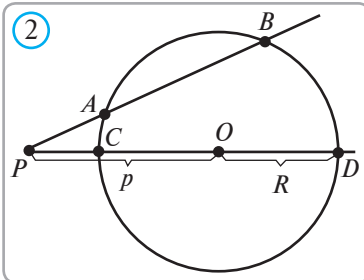
$$AP \cdot PB = CP \cdot PD = (R - p)(R + p) = R^2 - p^2.$$

(1) ten'lik da'ylillendi.

2-ma'sele. Radiusi 6 sm bolg'an shen'berdin' O worayinan 4 sm qashi'qli'qta P noqati alindi. P noqati' arqali' AB xorda ju'rgiziledi. Yeger $AP = 2\text{ sm}$ bolsa, PB kesindisin tabi'n'.

Sheshiliwi. Ma'sele sha'rti boyi'nsha $R = 6\text{ sm}$, $d = 4\text{ sm}$, $AP = 2\text{ sm}$. Wonda (1) ten'lik boyi'nsha, $2 \cdot PB = 6^2 - 4^2 = 36 - 16 = 20$. Bunnan, $PB = 10\text{ sm}$.

Juwabi': $PB = 10\text{ sm}$.



3-ma'sele. P noqati R radiusli shen'berdin' si'rtqi' oblastinda woni'n' worayinan p aralig'inda jaylasqan bolsi'n. Wonda P noqat arqali' wo'tiwshi ha'm shen'berdi A ha'm B noqatlardakesip wo'tiwshi qa'legen tuwri' si'zi'q ushi'n ,

$$PA \cdot PB = p^2 - R^2 \quad (2)$$

ten'ligi wori'nli' boli'wi'n da'liyllen.

Da'liyllew. Shen'berdin' O worayi arqali wo'tiwshi PO tuwri'si shen'ber menen C ha'm D noqatlarda kesilissin ($2-su'wret$). Onda, sha'rt boyi'nsha, $PC = p - R$, $PD = p + R$. Shen'berdin' si'rtqi' oblasti'ndag'i' noqattan ju'rgizilgen kesiwshiler haqqindag'i Teorema boyi'nsha,

$$PA \cdot PB = PC \cdot PD = (p - R)(p + R) = p^2 - R^2.$$

Solay yetip (2) ten'lik da'liyllendi.

4-ma'sele. Radiusi' 7 sm bolg'an shen'berdin' worayinan 13 sm qashqliqtag'i P noqatidan wo'tiwshi tuwri' shen'berdi A ha'm B noqatlarda kesip wo'tedi. Yeger $PA=10$ sm bolsa, AB xordani tabi'n'.

Sheshiliwi. Sha'rt boyi'nsha, $R=7$ sm, $p=13$ sm. Wonda, (2) formulag'a ko're,

$$PA \cdot PB = p^2 - R^2 = 13^2 - 7^2 = 169 - 49 = 120.$$

$$\text{Bunnan, } PB = \frac{120}{PA} = \frac{120}{10} = 12 \text{ (sm). Demek,}$$

$$AB = PB - PA = 12 - 10 = 2 \text{ (sm). Juwabi': } 2 \text{ sm.}$$

? Soraw, ma'sele ha'm tapsi'rmalar

1. Radiusi 5 sm bolg'an shen'ber worayinan 3 sm qashqliqta P noqati aling'an. AB xorda P noqati arqali wo'tedi. Yeger $PA=2$ sm bolsa, AB xorda uzi'nli'g'i'n tabi'n'.

2. Radiusi 5 m bolg'an shen'ber worayinan 7 m qashqliqta P noqati aling'an. P noqati arqali wo'tiwshi tuwri' shen'berdi A ha'm B noqatinda kesip wo'tedi. Yeger $PA=4$ m bolsa, AB xorda uzi'nli'g'i'n tabi'n'.

3. 3-su'wrettegi mag'liwmatlar tiykarinda x penen belgilengen kesindini tabi'n' (O —shen'ber worayi).

4. 4-su'wretten paydalani'p, ma'seleni sheshin'. Wonda

a) $PC=5$ dm, $OD=7$ dm, $AB=2$ dm, $PA=?$

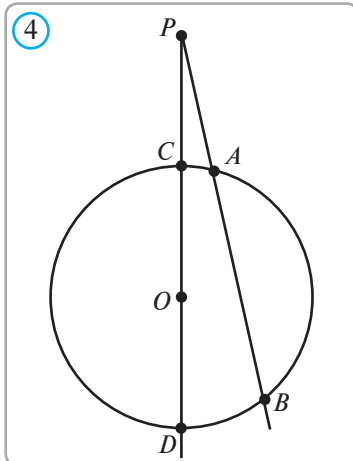
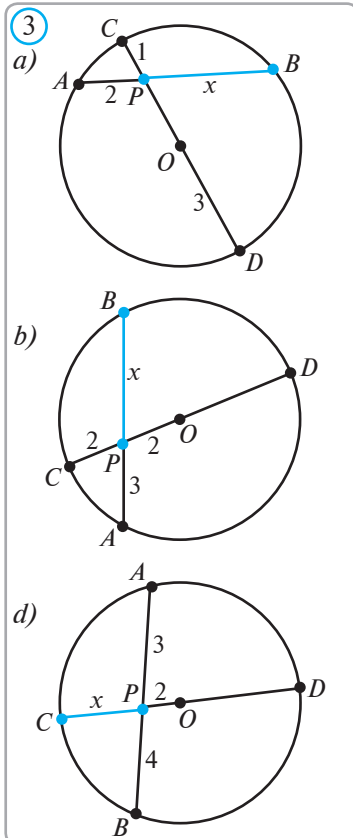
b) $PA=5$ dm, $AB=4$ dm, $PC=3$ dm, $OD=?$

5. Shen'berdin' $AB=7$ sm ha'm $CD=5$ sm xordalari P noqatinda kesilisedi. Yeger $CP:PD=2:3$ bolsa, P noqati AB xordasin qanday qatnasta bo'ledi?

6. Shen'berdin' C noqatidan AB diametrge CD perpendikulyari tu'sirilgen. Yeger $AD=2$ sm, $DB=18$ sm bolsa, CD kesindisin tabi'n'.

7*. Shen'berge ishley si'zi'lg'an $ABCD$ to'rt-mu'yeshliktin' diagonallari K noqatinda kesilisedi. Yeger $AB=2$, $BC=1$, $CD=3$ ha'm $CK:KA=1:2$ bolsa, AD kesindisin tabi'n'.

8*. Shen'berge ishley si'zi'lg'an $ABCD$ to'rt-mu'yeshlikte $AB:DC=1:2$ ha'm $BD:AC=2:3$ bolsa, $DA:BC$ qatnasi'n tabi'n'.



I. Testler

1. Tuvri' mu'yeshli u'shmu'yeshliktin' gipotenuzasina tu'sirilgen biyikligi haqqinda naduri's tastiyoqlawdi ko'rsetin':
 A. Katetlerinen kishi;
 B. U'shmu'yeshlikti yeki uqsas u'shmu'yeshliklerge ajiratadi;
 D. Katetlerinin' gipotenuzadag'i proekciyalari arasi'nda worta proporcional;
 E. Gipotenuzanin' yarimina ten'.
2. AB ha'm CD xordalar O noqatinda kesilisedi. Naduri's tastiyoqlawdi tabi'n':
 A. $\angle DAB = \angle DCB$; B. AO ha'm CO u'shmu'yeshlikler uqsas;
 D. $AO \cdot OB = CO \cdot OD$; E. $AO = CO$.
3. Duri's tastiyoqlawdi tabi'n':
 A. Ten' kesindilerdin' proektciyalari da ten' boladi;
 B. U'lken kesindinin' proekciyasi u'lken boladi;
 D. Bir tuvri' si'zi'qtag'i ten' kesindilerdin' proektciyalari ten' boladi;
 E. Proekciya uzunlig'i proektciyalaniwshi kesindi uzi'nli'g'i'na ten' boladi;
4. Tuvri' mu'yeshli u'shmu'yeshliktin' gipotenuzasina tu'sirilgen biyiklik woni yeki u'shmu'yeshlikke bo'ledi. Bul u'shmu'yeshlikler:
 A. Ten'; B. Ten'les; D. Uqsas; E. Ten' qaptalli.
5. Uzunlig'i a ha'm b bolg'an kesindilerdin' worta proporcionali nege ten'?
 A. $a + b$; B. $\sqrt{ab}$; D. $\frac{a+b}{2}$; E. $a : b$.
6. $ABCD$ to'rtmu'yeshligi O worayli shen'berge ishley si'zi'lg'an. Naduri's tastiyoqlawdi ko'rsetin':
 A. $\triangle AOB \sim \triangle COD$; B. $\angle A + \angle C = \angle B + \angle D$;
 D. $AO \cdot OB = CO \cdot OD$; E. $AB \cdot CD = BC \cdot AD$.

II. Ma'seleler

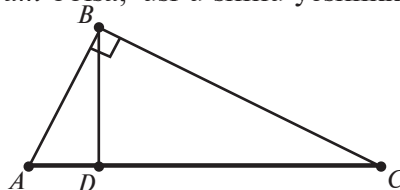
1. Tuvri' mu'yeshli u'shmu'yeshlik katetlerinin' qatnasi 3:4 ke ten'. Bul u'shmu'yeshliktin' gipotenuzasi 50 sm. U'shmu'yeshliktin' tuvri' mu'yeshi to'besinen tu'sirilgen biyikligi gipotenuzadan qanday uzunliqtag'i kesindiler ajiratadi?
2. Shen'berdin' AB ha'm CD xordalarita E noqatinda kesilisedi. Yeger $AE = 5$ sm, $BE = 2$ sm ha'm $EC = 2,5$ sm bolsa, ED ni tabi'n'.
3. Radiusi 6 m bolg'an shen'berdin' worayinan 10 m qashikliqta K noqati alindi ha'm K noqatidan shen'berge urinba ju'rgizildi. Urinbanin' uriniw noqati P menen K noqati arasi'ndag'i arali'qti tabi'n'.
4. ABC u'shmu'yeshliginde $\angle C = 90^\circ$ ha'm CD biyikligi 4,8 dm. Yeger $AD = 3,6$

dm bolsa, AB ta'repin tabi'n'.

5. Shen'berdin' AB ha'm CD xordalari O noqatta kesilisedi. Yeger $AO=6$, $OB=4$ ha'm $CO=3$ bolsa, OD kesindisin tabi'n'.
6. Shen'berde A, B, C, D noqatlari belgilengen, BA ha'm CD nurlari O noqatinda kesilisedi. Yeger $OA=5$, $AB=4$, $OD=6$ bolsa, DC xordasin tabi'n'.
7. Shen'berge B noqatinda uriniwshi tuwri'sinin' u'stinen A noqati belgilendi. Yeger $AB=12$ ha'm A noqatinan shen'berge shekem yen' qisqaarali'q 8 bolsa, shen'ber radiusin tabi'n'.
8. Yarim shen'berdegi C noqatinan AB diametrge tu'sirilgen CD perpendikulyar AB kesindide 4 ha'm 9 g'aten' kesindilerdi ajiratadi. CD kesindisin tabi'n'.
9. Tuwri' mu'yeshli u'shmu'yeshliktin' biyikligi gipotenuzani $3 dm$ ha'm $12 dm$ ge ten' kesindilerge bo'ledi. U'shmu'yeshliktin' maydani'n tabi'n'.
10. Radiusi $5 sm$ bolg'an O worayina iye shen'berdin' AB xordasinda D noqati aling'an. Yeger $AD=2 sm$, $DB=4,5 sm$ bolsa, OD kesindisin tabi'n'.
11. Radiusi $5 sm$ bolg'an O worayina iye shen'berdi A ha'm B noqatlarda kesip wo'tiwshi tuwri'da P noqati alindi. Yeger $PA=5 m$, $AB=2,8 m$ bolsa, OP aralig'in tabi'n'.
12. To'rt parallel tuwri' berilgen. Wolar mu'yeshstin' ta'replerin A ha'm A_1 , B ha'm B_1 , C ha'm C_1 ha'm de D ha'm D_1 noqatlarindakesip wo'tedi. Yeger $AB=8$, $CD=12$ ha'm $C_1D_1=9$ bolsa, A_1B_1 kesindisin tabi'n'.
13. Shen'ber mu'yeshke ishley si'zi'lg'an. Yeger mu'yesh ushi'nan shen'berge shekem bolg'an arali'q radiusqa ten' bolsa, mu'yeshstin' shamasin tabi'n'.
14. Shen'berge AB diametrinin' B ushi'nan BC urinba ha'm AC kesiwshi ju'rgizilgen. AC kesiwshisi shen'ber menen D noqatindakesilisedi. Yeger $AD=DC$ bolsa, CBD mu'yeshin tabin.
15. Tuwri' mu'yeshli u'shmu'yeshliktin' katetlerinin' qatnasi $2:3$ tu'rinde. U'shmu'yeshliktin' gipotenuzag'a tu'sirilgen biyiklik woni yeki ushmu'yeshlikke bo'ledi. Wolardi'n' maydanlarinin' qatnasin tabi'n'.

III. Wo'zin'izdi sinap ko'rin' (u'lgi ushi'n baqlaw jumi'si')

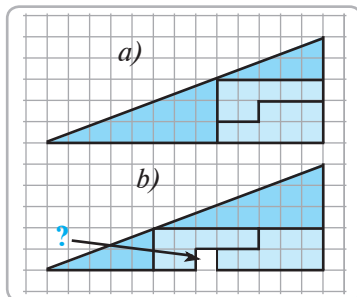
1. Shen'berdin sirti'ndag'i' noqattan wog'an shekem bolg'an yen' qisqa arali'q $2 sm$ ge, uriniw noqatinashekem bolg'an arali'q bolsa $6 sm$ ge ten'. Shen'berdin' radiusin tabi'n'.
2. $\triangle ABC$ tuwri' mu'yeshli, $AD=9 dm$, $BD=16 dm$ bolsa, usi u'shmu'yeshlikke ishley si'zi'lg'an shen'ber radiusi'n yesaplan'.
3. Noqattan tuwri'g'a yeki qiya ju'rgizilgen. Yeger qiyalar $1:2$ qatnasta boli'p, wolardin' proektsiyalari $1 m$ ha'm $7 m$ bolsa, qiyalardi'n' uzi'nliqlari'n tabin'.



- 4.* (*Qosi'msha ma'sele*). PQ ha'm wonnan uzin ET kesindilari berilgen. Sonday $ABCD$ to'rtmu'yeshligin jasan', na'tiyjede $AB=BC=PQ$; $BD=ET$ boli'p, diagonallari kesilisetug'in O noqati ushi'n $AO \cdot OC = BO \cdot OD$ ten'ligi wori'nli' bolsi'n.

Qi'ziqli' ma'seleler

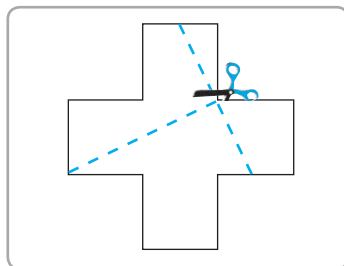
U'shmu'yeshlik 4-a su'wrette ko'rsetilgenindey yetip, to'rt bo'lekke bo'lingan ha'm 4-b su'wrette ko'rsetilgende yetip qayta jiynalg'an. Artiq kvadrat qalay payda bolg'ani'n aytib berin'.



(Yunon) Grek

Eramizdan a'ldi'n'gi 500-jillarda payda bolg'an bul forma wo'mirdin' belgisi sipatinda nan u'stinde si'zi'lg'an. (4 su'wret)

Bul formani qalin' qag'azg'a sizip alip woni su'wrette ko'rsetilgen si'zi'qlar boylap qirqin'. Payda bolg'an bo'leklerden kvadrat jasaw mu'mkinshiligine isenim payda yetin'.



V BAP

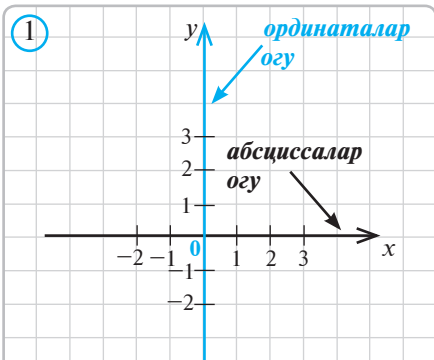


PLANIMETRIYA KURSI BOYI'NSHA TA'KIRARLAW

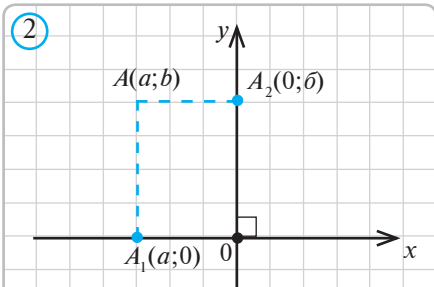
**Usi bapti' u'yreniw na'tiyjesinde siz to'mendegi bilim ha'm ko'nlikpe-
lerge iye bolasiz:**

- √ *geometriyani'n' planimetriya bo'limi boyi'nsha wo'tilgen temalardi yeske
aliw;*
- √ *planimetriya kursi' boyi'nsha wo'zlestirilgen bilim, ta'jiriybe ha'm
ko'nlikpelerdi bekkemlew;*
- √ *juwmaqlawshi baqlaw jumi'si'na tayarliq ko'riw.*

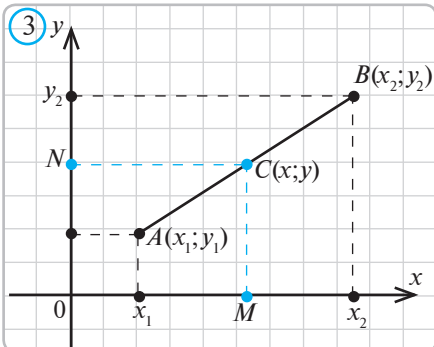
Tegisliktegi tuwri' mu'yeshli koordinatalar sistemasi menen 7-klass algebra kursinda tanisqansiz (1—2-su'wret). To'mende usi temag'a tiyisli geometriyalıq ma'selelerdi qaraymız.



0 — координаталардын башталышы



(a, b) — A чекитинин координаталары: a — анын абсциссасы; b — анын ординатасы.



1-ma'sele. To'beleri koordinatalar tegisliginin' birinshi shereginde bolg'an AB kesindisi berilgen bolsi'n: $A(x_1; y_1)$ ha'm $B(x_2; y_2)$, $x_1 > 0$, $y_1 > 0$, $x_2 > 0$, $y_2 > 0$ (3-su'wret). AB kesindisinin' wortasi bolg'an $C(x; y)$ noqatinin' koordinatalarin tabi'n'.

Sheshiliwi. Bul jag'dayda CN kesindi ultanlarinin' uzunliqlari x_1 ha'm y_1 bolg'an trapeciyanin' worta sizig'i, CM kesindisi bolsa, ultanlarinin' uzunliqlari x_2 ha'm y_2 bolg'an trapeciyanin' worta sizig'i boladi.

Trapeciyanin' worta sizig'inin' qa'siyeti boyi'nsha,

$$x = \frac{x_1 + x_2}{2}; \quad y = \frac{y_1 + y_2}{2} \quad (1)$$

boladi.

Bul formulalardin' duri's yekenligin AB kesindisinin' basqa jag'daylari ushi'n da usig'an uqsas ko'rsetiw mu'mkin.

2-ma'sele. To'beleri $A(-1; -2)$, $B(2; -5)$, $C(1; -2)$, $D(-2; 1)$ noqatlarında bolg'an $ABCD$ to'rtmu'yeshliginin' parallelogramm yekenligin da'liyllen'.

Sheshiliwi. (1) formuladan paydalani'p, to'rtmu'yeshliktin' AC ha'm BD diagonallari wortasinin' koordinatalarin tabami'z:

$$AC: \quad x = \frac{-1+1}{2} = 0, \quad y = \frac{-2+(-2)}{2} = -2;$$

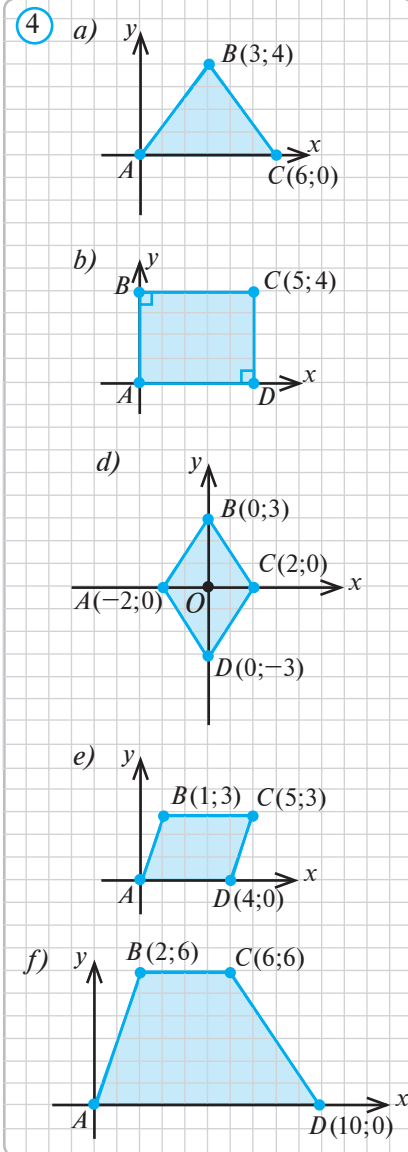
$$BD: \quad x = \frac{2+(-2)}{2} = 0, \quad y = \frac{-5+1}{2} = -2$$

Demek, $ABCD$ to'rtmu'yeshliginin' ha'r yeki diagonali'nin' wortasi da bir $(0; -2)$ noqati boladi yeken. Basqasha aytqanda $ABCD$

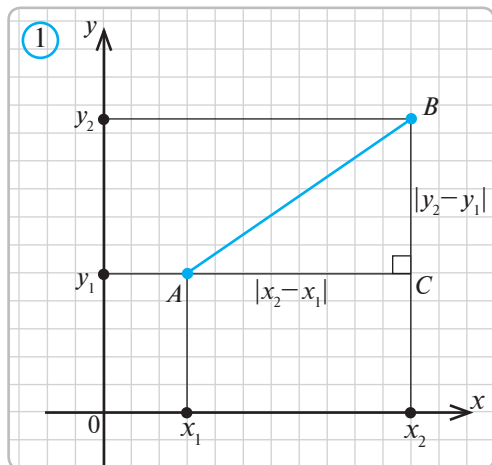
to'rtmu'yeshliginin' diagonallari $(0; -2)$ noqatida kesilisedi ha'm usi noqatta ten' yekige bo'linedi. Bul $ABCD$ to'rtmu'yeshliginin' parallelogramm bolivi belgilarinin' biri boli'p yesaplanadi.

? Soraw, ma'sele ha'm tapsi'rmalar

- Ko'pmu'yeshliklerdin' maydanlarin yesaplan' (4-su'wret).
- Shen'berdin' 8 sm ge ten' xordasi shen'berden 90° qa ten' dog'ani ajiratadi. Shen'ber worayinan xordag'a shekemgi bolg'an qashikliqi tabi'n'.
- Ta'repleri a) 5,5 ha'm 6; b) 17, 65, 80 bolg'an u'shmu'yeshliktin' maydani'n tabi'n'.
- Ta'repleri a) 13, 13, 12; b) 35, 29, 8 bolg'an u'shmu'yeshlikke ishley si'zi'lg'an shen'berdin' radiusin tabi'n'.
- Ushlari to'mendegishe bolg'an kesindilerdin' wortasinin' koordinatalarin tabi'n':
a) $A(1; -2)$, $B(5; 6)$;
b) $A(4; -3)$, $B(1; 2)$;
d) $A(-4; 5)$, $B(2; 3)$;
e) $A(-0,7; 2)$, $B(-0,3; 4,2)$.
- Yeger $A(1; 0)$, $B(2; 3)$, $C(3; 2)$ bolsa, $ABCD$ parallelogrammnin' D to'besinin' koordinatalarin tabi'n'.
- Parallelogramm mu'yeshlerinin' bissektisarlari' kesiliken noqatlar tuwri'mu'yeshliktin' to'beleri bolatug'inin da'liyllen'.
- Katetleri 40 sm ha'm 30 sm bolg'an tuwri'mu'yeshli u'shmu'yeshlikke ishley ha'm si'rtlay si'zi'lg'an shen'berlerdin' radiuslarin tabi'n'.
- Shen'berge ishley si'zi'lg'an to'rtmu'yeshliktin' u'sh mu'yeshi 2:3:4 siyaqli qatnas payda yetedi. Woni'n' mu'yeshlerin tabi'n'.
- Radiusi 6 sm bolg'an shen'berdin' 60° qaten' dog'asin kerip turg'an xordasin tabi'n'.
- Radiuslari 6 sm bolg'an shen'berlerdin' woraylari arasi'ndag'i arali'q $6\sqrt{2}$ sm ge ten'. Shen'berlerdin' uliwma xordasinin' uzi'nli'g'i'n tabi'n'.



1-ma'sele. Koordinatalar tegisliginde berilgen $A(x_1; y_1)$ ha'm $B(x_2; y_2)$ noqatlari ara-sindag'i arali'q $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ formula menen yesaplanatug'inin ko'rsetin'.



Sheshiliwi. Aytayiq, A ha'm B noqatlari 1-su'wrettegidey joylasqan bolsi'n ($x_1 \neq x_2, y_1 \neq y_2$). A ha'm B noqatlardan koordinatako'sherlerine parallel tuwri'lar ju'rgizemiz ha'm wolardi'n' kesilisiw noqatin C menen belgileymiz.

Wonda, $AC = |x_2 - x_1|$ ha'm de $BC = |y_2 - y_1|$. ABC tuwri' mu'yeshli u'shmu'yeshlikke Pifagor teoremasin qollansaq,

$AB^2 = AC^2 + BC^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ boladi. Bunnan,

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

formulani payday ytemiz. Bul formulani

$x_1 = x_2$ yaki $y_1 = y_2$ bolg'andada duri'r bolatug'inin tekserip ko'rin'.

2-ma'sele. Yeger $A(-3; -1)$, $B(1; -1)$, $C(1; -3)$, $D(-3; -3)$ bolsa, $ABCD$ tuwri'mu'yeshlik yekenligin da'liylen'.

Sheshiliwi. 1) AC diagonal'i wortasinin' x, y koordinatalarin tabami'z:

$$x = \frac{-3 + 1}{2} = -1; \quad y = \frac{-1 - 3}{2} = -2.$$

BD diagonal'i wortasinin' x, y koordinatalarin tabami'z:

$$x = \frac{1 - 3}{2} = -1; \quad y = \frac{-1 - 3}{2} = -2.$$

Demek, $ABCD$ to'rtmu'yeshligin diagonal'lari bir $(1; 2)$ noqatinda kesilisedi ha'm usi noqatta ten' yekige bo'linedi yeken. Bul, $ABCD$ parallelogramm yekenligin ko'rsetedi.

2) $ABCD$ parallelogramm diagonal'lari'ni'n' uzi'nli'g'i'n tabami'z:

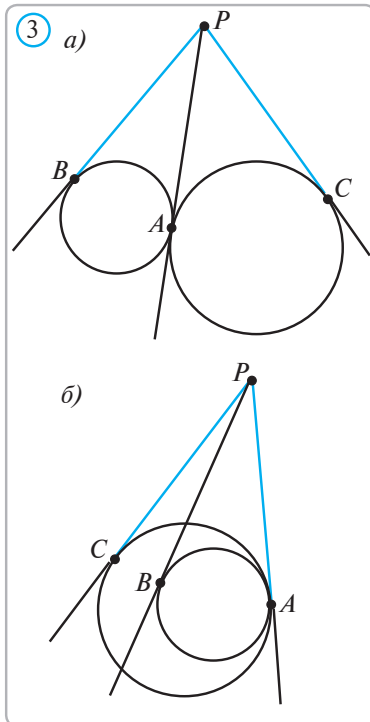
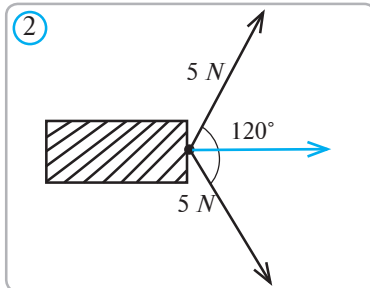
$$AC = \sqrt{(1 - (-3))^2 + (-3 - (-1))^2} = \sqrt{4^2 + (-2)^2} = \sqrt{20};$$

$$BD = \sqrt{(1 - (-3))^2 + (-1 - (-3))^2} = \sqrt{4^2 + 2^2} = \sqrt{20}.$$

Demek, $ABCD$ parallelogrammnin' diagonal'lari wo'z ara ten'. Bul (tuwri'-mu'yeshliktin' belgisi boyi'nsha), $ABCD$ — tuwri'mu'yeshlik yekenligin bildiredi.

? Soraw, ma'sele ha'm tapsi'rmalar

- Yeger a) $A(2;7)$, $B(-2;7)$; b) $A(-5;-1)$, $B(5;-7)$; d) $A(-3;0)$, $B(0;4)$; e) $A(0;3)$, $B(-4;0)$ bolsa, AB kesindisinin' uzi'nli'g'i'n yesaplan'.
- Yeger $M(4;0)$, $N(12;-12)$, $P(5;-9)$ bolsa, MNP u'shmu'yeshliginin' perimetrin tabi'n'.
- Kollinear $\bar{x}$ ha'm $\bar{y}$ vektorlarin si'zi'n' ha'm $2\bar{x}+3\bar{y}$ vektorin jasan'.
- Yeger A , B , C ha'm D noqatlari' bir tuwri'da jatpasa ha'm $\overline{AB}=0,7\overline{DC}$ bolsa, $ABCD$ to'rtmu'yeshliginin' tu'rin ani'qlan'.
- Kollinear yemes $\bar{a}$ ha'm $\bar{b}$ vektorlari' berilgen. Yeger $3\bar{a}-x\bar{b}=\bar{y}\bar{a}+4\bar{b}$ bolsa, x ha'm y sanlarin tabi'n'.
- Yeger AA_1 , BB_1 ha'm CC_1 kesindileri ABC u'shmu'yeshliklerinin' medianalari ha'm O —qa'legen noqat bolsa, $\overline{OA}+\overline{OB}+\overline{OC}=\overline{OA_1}+\overline{OB_1}+\overline{OC_1}$ ten'ligin da'liyllen'.
- ABC u'shmu'yeshliginin' medianalari O noqatinda kesilisedi. $\overline{AB}$, $\overline{BC}$ ha'm $\overline{CA}$ vektorlarin $\bar{a}=\overline{OA}$ ha'm $\bar{b}=\overline{OB}$ vektorlari arqali an'latin'.
- Denege ha'r biri $5N$ bolg'an yeki ku'sh ta'sir ko'rsetip atir (*2-su'wret*). Yeger bul ku'shlerdin' bag'itlari arasi'ndag'i mu'yesh 120° bolsa, wolardi'n' ten' ta'sir yetiwshisinin' shamasin tabi'n'.
- Ten' ta'repli u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusi 6 sm. U'shmu'yeshliktin' perimetrin ha'm maydani'n tabi'n'.
- Shen'berge A noqatidan ju'rgizilgen urinbada B noqati belgilendi. B noqatidan shen'berdin' yen' jaqin noqatiga shekem bolg'an arali'q 4 sm ge, yen' uzaq noqatiga shekem bolg'an arali'q 8 sm ge ten'. AB kesindisin tabi'n'.
- * Radiuslari ha'r tu'rli bolg'an yeki shen'ber A noqatinda PA tuwri'sinauriniadi. Bul shen'berlerge sa'ykes tu'rde PA dan wo'zgeshe PB ha'm PC urinbalar ju'rgizilgen. Yeger B ha'm C bul urinbalardin' shen'berge uriniw noqatlari bolsa, $PC=PB$ ten'ligin da'liyllen' (*3-su'wret*).

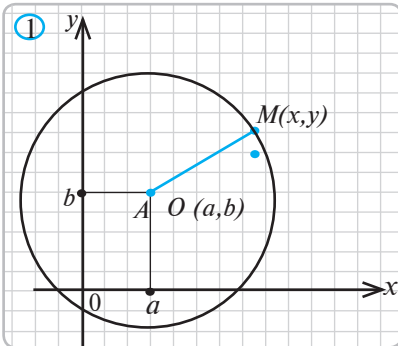




1-ma'sele. Koordinatalar tegisliginde worayi $O(a; b)$ noqatta ha'm radiusi R bolg'an shen'berdegi qa'legen $M(x; y)$ noqattin' x ha'm y koordinatalari

$$(x - a)^2 + (y - b)^2 = R^2 \quad (1)$$

ten'likti qanaatlandiriwin da'liyllen'.



Sheshiliwi. $O(a; b)$ — berilgen shen'berdin' worayi, $M(x; y)$ — ysi shen'berdin' qa'legen noqati bolsa, wonda $OM=R$ boladi. Koordinatalar tegisliginde berilgen yeki noqat arasi'ndag'i arali'qti tabiw formulasina tiykarlanip (134-bettegi 1- maselege qaran')

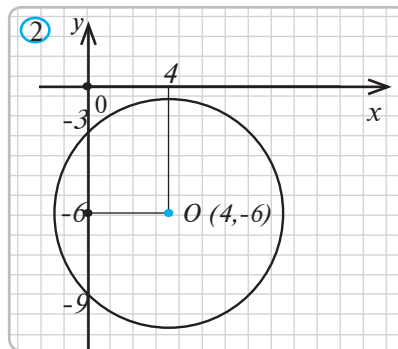
$$OM = \sqrt{(x - a)^2 + (y - b)^2}.$$

Solay yetip,

$$\sqrt{(x - a)^2 + (y - b)^2} = R.$$

Son'g'i ten'liktin' eki jag'inda kvadratqa ko'terip, (1) ten'likti payda yetemiz.

Yesletpe. (1) ten'leme worayi (a, b) noqatta bolg'an R radiusli shen'ber ten'lemesi delinedi.



2-ma'sele. Koordinatalar tegisliginde usi

$$(x - 4)^2 + (y + 6)^2 = 25$$

ten'leme menen ani'qlang'an shen'berdin' koordinatalar ko'sherinen ajiratqan kesindinin' ushlarini'n koordinatalari'n tabi'n'

Sheshiliwi. Berilgen shen'ber menen ordinatalar ko'sheri kesiliskan noqatlardin' abscisalari nolge ten' boladi. $x=0$ bolg'anda, berilgen ten'lemeden paydalani'p, bul noqatlardin' ordinatasin tabamiz: $(0 - 4)^2 + (y + 6)^2 = 25$, $(y + 6)^2 = 9$, $y = -9$

yoki $y = -3$. Demek, shen'ber ha'm ordinatalar ko'sheri $(0; -9)$ ha'm $(0; -3)$ noqatta kesilisedi. Bul noqatlar arasi'ndag'i arali'q 6 birlikke ten'. **Juwabi':** 6.



3-masele. Worayi O noqatta joylasqan eki don'gelek jag'dayinda duziledi. U'lken don'gelektin' 32 sm ge ten' AB xordasi kishi don'gelekke C noqatta urinadi. (3-suwret) Yeger sagi'ynanin' ken'ligi 8 sm bolsa, wol jag'dayda halqanin' sagi'ynani'n tabi'n'.

Sheshiliwi. U'lken don'gelektin' radiusi' R menen, kishkenesin bolsa r menen belgileymiz. Maselenin' sha'rtine ko're $OA=R=r+8(\text{sm})$ ha'm $OC=r$. Bunnan

basqa, C noqatta AB xordanin' wortasi yag'niy $AC=16sm$, OCA u'shmu'yeshlik bolsa tuwri' mu'yeshli boladi.

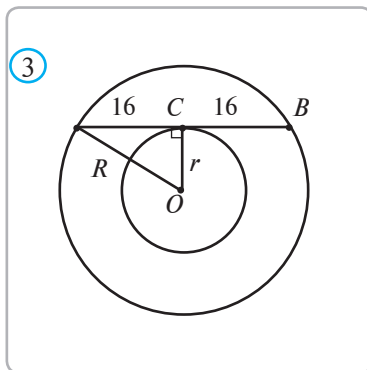
Pifagor teoremasinan paydalani'p,

$$OC^2 + CA^2 = OA^2 \text{ bolg'ani' ushi'n,}$$

$$r^2 + 16^2 = (r + 8)^2$$

ten'lemesin payda yetemiz. Bul ten'lemeni sheship, $r=12 sm$ yelkenligin tabami'z. Wonda $R=r+8=20 (sm)$ boladi. Ulken don'gelektin' maydani'nan kishkenesin ayi'ri'p, berilgen jag'dayda maydani' S ti tabami'z:

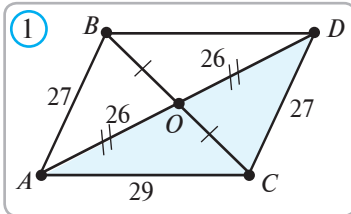
$$S = \pi R^2 - \pi r^2 = 20^2\pi - 12^2\pi = 400\pi - 144\pi = 256\pi (sm^2). \text{ Juwabi': } 256\pi sm^2.$$



? Soraw, ma'sele ha'm tapsi'r malar

- To'mendegi ten'lemeler menen berilgen shen'berler worayinin' koordinatalarin ha'm radiusin tabi'n'. Usi shen'berlerdi jasan'.
 - $(x - 1)^2 + (y + 2)^2 = 4$;
 - $(x - 4)^2 + (y - 3)^2 = 16$;
 - $x^2 + y^2 = 25$;
 - $x^2 + (y - 2)^2 = 9$.
- Shen'berge ishiley si'zi'lg'an $ABCD$ to'rtmu'yeshliktin' A, B ha'm C ushlarindag'i mu'yeshlerinin' qatnaslari 1:2:3 tu'rinde to'rtmu'yeshliktin' ishki mu'yeshlerin tabi'n'.
- Shen'berdin' 1:8 bo'limine sa'ykes orayliq mu'yeshti tabi'n'.
- Worayi A noqatda bolg'an shen'berge B noqat aling'an. Worayi B noqattan bolg'an basqa shen'ber A noqattan wo'tedi. Bul yeki shen'ber C noqatta kesilisedi. ACB mu'yeshti tabi'n'.
- Shen'berdin' AB ha'm CD xordalari O noqatta kesilisedi. Yeger $AO=4 sm$, $BO=6 sm$ ha'm $CD=11 sm$ bolsa, OC ha'm OD kesindilerdi tabi'n'.
- Shen'berge ishley si'zi'lg'an tuwri' to'rtmu'yeshliktin' diagonali bir ta'repinen yeki ma'rte u'lken. Bul to'rtmu'yeshlik ushlarinin' shen'berden ajiratqan bo'leginin' gradi'us wo'lshe'mlerin tabi'n'.
- Shen'berge si'rtlay si'zi'lg'an trapeciyanin' worta sizig'i $7 sm$. Trapeciyanin' perimetrin tabi'n'.
- *. Radiusi $15 sm$ bolg'an do'n'gelektin' worayinan $7 sm$ uzaqli'qtag'i K noqattan $27 sm$ uzaqliqtag'i AB xorda wo'tkerilgen AK ha'm BK kesindilerdi tabi'n'.
- Duri's segiz mu'yeshliktin' bir ushi'nan shiqqan yen' u'lken ha'm yen' kishi diagonal arasi'ndag'i mu'yeshti tabi'n'.
- To'beleri koordinatalar tegisligindagi $A(-3; 4)$, $B(3; 4)$, $C(3; -8)$ noqatlarda bolg'an u'shmu'yeshlik berilgen. a) $\angle ABC=90^\circ$ yekenligin ko'rsetin'; b) ABC u'shmu'yeshlikke si'rtlay si'zi'lg'an do'n'gelektin' worayin, radiusi ha'm maydani'n tabi'n'.

Ma'sele. ABC u'shmu'yeshliginde AO mediana, $AO=26$, $AB=27$ ha'm $AC=29$. U'shmu'yeshliktin' maydani'n tabi'n'.



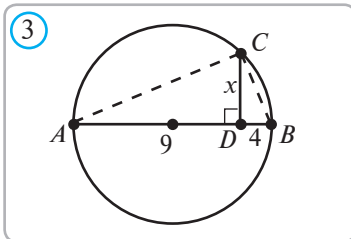
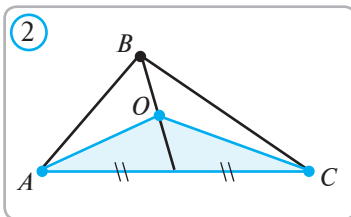
Sheshiliwi. AO nurinda A noqatinan $AD=2AO=52$ bolatug'inday yetip D noqatin tan'laymiz (*1-su'wret*). Bunda $BO=OC$, $AO=OD$ bolg'anliqtan $ABCD$ — parallelogramm boladi.

ABC ha'm ADC u'shmu'yeshliginin' maydanlari ten'. Geron formulasinan paydalani'p, ADC u'shmu'yeshliginin' maydani'n yesaplaymiz:

$$P = \frac{29+52+27}{2} = 54; \quad S = \sqrt{54 \cdot (54-29)(54-52)(54-27)} = 270. \quad \text{Juwabi': } 270.$$

Soraw, ma'sele ha'm tapsi'rmalar

- ABC ha'm EFK u'shmu'yeshliklari uqsas: AB ha'm EF , BC ha'm FK wolardi'n' sa'ykes ta'replari. Yeger $AB=4$ sm, $BC=5$ sm, $CA=7$ sm ha'm $EF:AB=2,1$ bolsa, EFK u'shmu'yeshliginin' ta'replerin tabi'n'.
- ABC ha'm $A_1B_1C_1$ u'shmu'yeshliklari uqsas ha'm wolardi'n' sa'ykes ta'replerinin' qatnasi $6:5$ ke ten'. ABC u'shmu'yeshliginin' maydani' $A_1B_1C_1$ u'shmu'yeshlik maydani'nan 77 dm^2 ge artiq. U'shmu'yeshliklerdin' maydani'n tabi'n'.
- ABC u'shmu'yeshliginin' medianalari kesilisken noqat O bolsi'n. Yeger AOC u'shmu'yeshliginin' maydani' 4 sm^2 bolsa, ABC u'shmu'yeshliginin' maydani'n tabi'n' (*2-su'wret*).
- Shen'berdin' C noqatinan AB diametrine CD perpendikulyari ju'rgizilgen. Yeger



$AD=9$, $DB=4$ bolsa, CD kesindisin tabi'n' (*3-su'wret*).

5. Ta'repi 6 m, bul ta'repine irgeles jatqan mu'yeshleri 30° ha'm 45° bolg'an u'shmu'yeshliktin' maydani'n tabi'n'.

6. Ultanlari 28 dm ha'm 16 dm, qaptal ta'replari bolsa 25 dm ha'm 17 dm bolg'an trapeciyanin' biyikligin tabi'n'.

7. Radiusi 2 sm bolg'an shen'berge maydani' 20 sm^2 bolg'an ten' qaptalli trapeciya si'rtlay si'zi'lg'an. Trapeciyata'replerinin' uzunliqlarin tabi'n'.

8. Tuvri' mu'yeshli u'shmu'yeshlikke ishley si'zi'lg'an shen'berdi gipotenuzag'auriniw noqati, gipotenuzani 2 sm ha'm 3 sm bolg'an kesindilerga ajiratadi. U'shmu'yeshliktin' katetlerin tabi'n'.

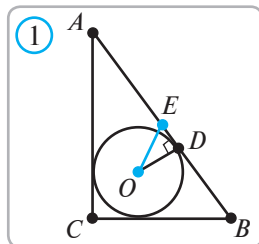
Ma'sele. Katetleri 3 ha'm 4 bolg'an tuwri' mu'yeshli u'shmu'yeshlikke ishley ha'm si'rtlay si'zi'lg'an shen'berlerdin' woraylari arasi'ndag'i arali'qti tabi'n' (1-su'wret).

Sheshiliwi. 1) ABC u'shmu'yeshliginde $\angle C = 90^\circ$, $AC = 4$ ha'm $BC = 3$ bolsi'n. Bunda, Pifagor teoremasi' boyi'nsha,

$$AB = \sqrt{3^2 + 4^2} = 5.$$

Tuwri' mu'yeshli u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' E worayi gipotenuzanin' ortasindaboladi:

$$BE = \frac{AB}{2} = \frac{5}{2}.$$



3) U'shmu'yeshlikke ishley si'zi'lg'an shen'berdin' radiusi OD ni tabami'z (D —ishley si'zi'lg'an shen'berdin' gipotenuzag'a uriniw noqati):

$$OD = \frac{AC + BC - AB}{2} = \frac{4 + 3 - 5}{2} = 1.$$

4) BD ha'm DE kesindilerin tabami'z:

$$BD = \frac{AB + BC - AC}{2} = \frac{5 + 3 - 4}{2} = 2; \quad ED = BE - DE = \frac{5}{2} - 2 = \frac{1}{2}.$$

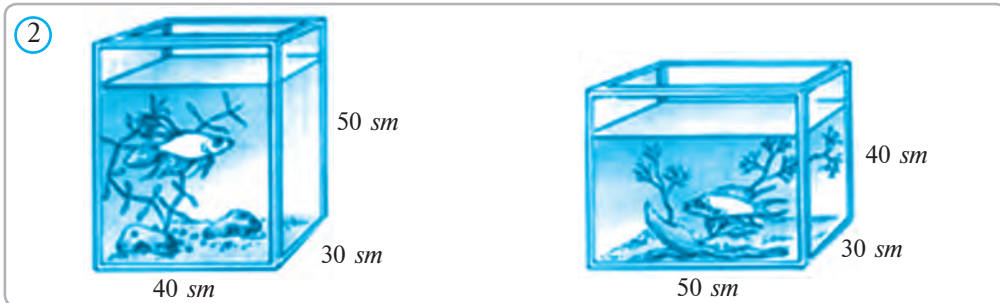
5) Tuwri' mu'yeshli ODE u'shmu'yeshliginen OE kesindilerin tabami'z:

$$OE = \sqrt{OD^2 + ED^2} = \sqrt{1 + \frac{1}{4}} = \frac{\sqrt{5}}{2}. \quad \text{Juwabi': } \frac{\sqrt{5}}{2}.$$

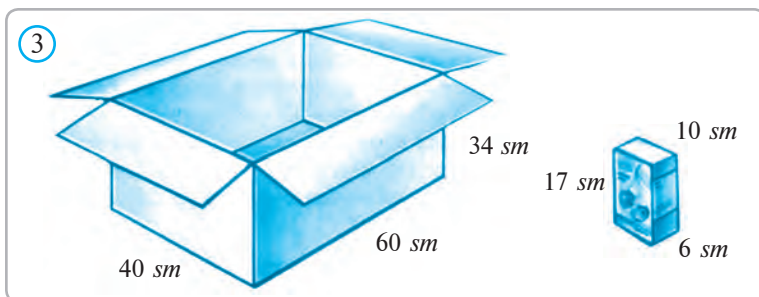
? Soraw, ma'sele ha'm tapsi'rmalar

1. Ten' qaptalli ABC u'shmu'yeshliginde $AB = AC = 4$ sm ha'm $\angle A = 30^\circ$ bolsa, onin' BE biyikligin tabi'n'.
2. Trapeciyandin' ultanlari 5 dm ha'm 8 dm, qaptal ta'repleri bolsa 3,6 dm ha'm 3,9 dm. Trapeciyandin' qaptal ta'replerinin' dawami O noqatindakesilisedi. O noqatidan trapeciyandin' ushlarina shekem bolg'an arali'qlardi tabi'n'.
3. A mu'yeshinin' bir ta'repine $AB = 5$ sm ha'm $AC = 16$ sm kesindiler, yekinshi ta'repine bolsa $AD = 8$ sm ha'm $AF = 10$ sm kesindiley qoyi'lg'an. ACD ha'm AFB u'shmu'yeshlikleri uqsaspa? Juwabi': n'izdi da'liyllen'.
4. Tuwri' mu'yeshliktin' maydani' 9 dm², diagonallari payda yetken mu'yeshlerden biri bolsa 120° qaten'. Tuwri' mu'yeshliktin' ta'replerin tabi'n'.
5. Yeger ten' qaptalli u'shmu'yeshliktin' ultani 24 sm ha'm qaptal ta'repi 13 sm bolsa, wonday jag'daydau'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusin tabi'n'.

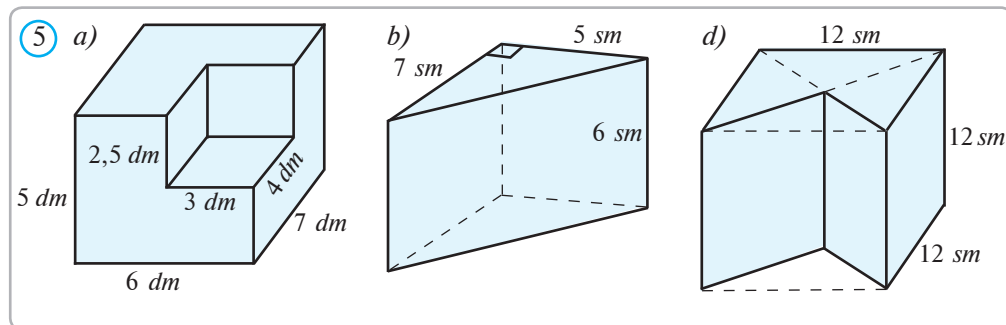
6. Rombi'ni'n' biyikligi 12 sm boli'p, diagonallaridan biri 15 sm . Rombi'ni'n' maydani'n' tabi'n'.
7. Yeger $ABCD$ parallelogrammda $A(1;-3)$, $B(-2;4)$ ha'm $C(-3;1)$ bolsa, woni'n' D to'besinin' koordinatalarin tabi'n'.
8. Yeki akvariumg'a joqarg'i shetinen 10 sm to'menlew suw quyildi (2-su'wret). Qaysi akvariumda suw ko'p?



9. Qutig'a neshe paket miywe sherbeti siyadi' (3-su'wret)?
10. 1 litrli miywe sherbeti paketi tuwri' mu'yeshli parallelepiped formasinda (4-su'wret). Bir qadaq ushi'n qansha material kerek boladi?



11*.5-su'wrette ko'rsetilgan ag'ash bo'leklerinin' ko'lemin yesaplan'.



Ma'sele. Rombi'ni'n' dog'al mu'yeshinin' to'besinen ju'rgizilgen biyiklik romb ta'replerinen birin su'yir mu'yeshinin' to'besinen baslap yesaplag'anda 5 sm ha'm 8 sm bolg'an kesindilerge ajiratadi. Rombi'nin' maydani'n yesaplan'.

Sheshiliwi. $ABCD$ romb, $\angle B > 90^\circ$, BE — biyiklik, $AE = 5$ sm, $ED = 8$ sm bolsi'n (1-su'wret).

1) Rombi'ni'n' ta'replerin tabami'z:

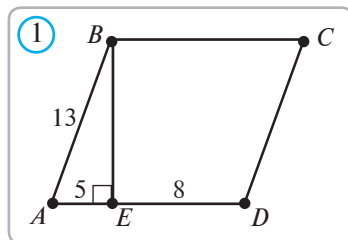
$$AD = AE + ED = 5 + 8 = 13 \text{ (sm)}.$$

2) Tuvri' mu'yeshli ABE u'shmu'yeshlikke Pifagor teoremasin qollanip, BE biyikligin tabami'z:

$$BE = \sqrt{AB^2 - AE^2} = \sqrt{13^2 - 5^2} = 12 \text{ (sm)}.$$

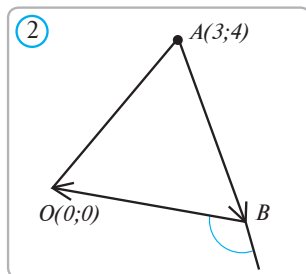
3) Rombi'ni'n' maydani'n tabami'z: $S = AD \cdot BE = 13 \cdot 12 = 156 \text{ (sm}^2\text{)}.$

Juwabi': 156 sm².



? Soraw, ma'sele ha'm tapsi'rmalar

1. Yeger ten' ta'repli AOB u'shmu'yeshliginde $O(0;0)$ ha'm $A(3;4)$ yekenligi belgili bolsa, $\overrightarrow{AB} \cdot \overrightarrow{BO}$ skalyar ko'beymeni tabi'n' (2-su'wret).
2. Ultanlari AB ha'm CD bolg'an $ABCD$ trapeciyanin' diagonallari' O noqatindakesilisedi. Yeger $OB=8$ sm, $OD=20$ sm ha'm $OC=50$ sm bolsa, AO kesindisin tabi'n'.
3. Yeger $AB=1,7$ sm, $BC=3$ sm, $CA=4,2$ sm, $A_1B_1=34$ dm, $B_1C_1=60$ dm ha'm $C_1A_1=84$ dm bolsa, ABC ha'm $A_1B_1C_1$ u'shmu'yeshlikleri uqsaspa?
4. Perimetri 36 sm bolg'an parallelogrammnin' diagonallari kesilisiwden payda bolg'an yeki u'shmu'yeshliktin' birin perimetri yekinshisikininen 8 sm artiq bolsa, parallelogrammnin' ta'replerin tabi'n'.
5. 60° qa ten' mu'yeshke bir-birine si'rttan uriniwshi yeki shen'ber ishley si'zi'lg'an. Kishi shen'berdin' radiusi 1 sm bolsa, u'lken shen'berdin' radiusin tabi'n'.
6. U'lken ultani AD bolg'an $ABCD$ trapeciyanin' AC diagonalini CD ta'repine perpendikular ha'm $\angle BAC = \angle CAD$. Yeger trapeciyanin' perimetri 20 sm ha'm $\angle D = 60^\circ$ bolsa, AD ta'repinin' uzi'nli'g'i'n tabi'n'.
7. Shen'ber diametrinin' ushlari shen'berdin' qanday da bir urinbasinan 18 sm ha'm 12 sm uzaqliqta yekenligi belgili bolsa, shen'berdin' uzi'nli'g'i'n tabi'n'.
8. Ultanlarinin' uzunliqlari ha'm maydani' sa'ykes tu'rde 8 sm, 14 sm ha'm 44 sm² bolg'an ten' qaptalli trapeciyanin' qaptal ta'repin tabi'n'.



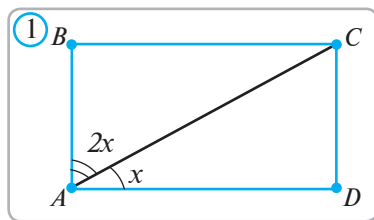
Ma'sele. Tuvri'mu'yeshliktin' diagonallari 12 sm ge ten' ha'm to'rtmu'yeshliktin' mu'yeshi $2:1$ qatnasta bo'ledi. Tuvri'mu'yeshliktin' perimetrin tabi'n'.

ABCD — tuvri'mu'yeshlik,
 $AC = 12\text{ sm}, \angle BAC : \angle CAD = 2:1$



$P = ?$

Sheshiliwi. 1) Yeger $\angle CAD = x$ desek, $\angle BAC = 2x$ ha'm $\angle CAD + \angle BAC = x + 2x = 90^\circ$ boladi. Bunnan $x = 30^\circ$.



2) Tuvri' mu'yeshli ADC u'shmu'yeshliktin' katetlerin tabami'z:

$$CD = AC \sin CAD = 12 \cdot \sin 30^\circ = 12 \cdot \frac{1}{2} = 6\text{ (sm)},$$

$$AD = AC \cdot \cos CAD = 12 \cdot \cos 30^\circ = 12 \cdot \frac{\sqrt{3}}{2} = 6\sqrt{3}\text{ (sm)}.$$

3) To'rtmu'yeshliktin' perimetrin tabami'z:

$$P = 2(AD + CD) = 2(6 + 6\sqrt{3}) = 12(1 + \sqrt{3})\text{ (sm)}.$$

Juwabi'z: $12(1 + \sqrt{3})\text{ sm}$.

Soraw, ma'sele ha'm tapsi'rmalar

- ABC ha'm $A_1B_1C_1$ u'shmu'yeshlikler uqsas, $AB=6\text{ sm}$, $BC=9\text{ sm}$, $CA=10\text{ sm}$. $A_1B_1C_1$ u'shmu'yeshliktin' u'lken ta'repi $7,5\text{ sm}$. Wonin' qalg'an ta'replerin tabi'n'.
- ABC u'shmu'yeshliginin' AB ta'repine parallel tuvri' AC ta'repin A to'besinen baslap yesaplag'anda $2:7$ siyaqli qatnastabo'ledi. Yeger $AB=10\text{ sm}$, $BC=18\text{ sm}$ ha'm $CA=21,6\text{ sm}$ bolsa, tuvri' ABC u'shmu'yeshliginen ajiratqan u'shmu'yeshliktin' ta'replerin tabi'n'.
- Ten' qaptalli trapeciyanin' qaptal ta'repi wortasizig'inaten' ha'm perimetri 48 sm . Trapeciyanin' qaptal ta'repinin' uzi'nli'g'i'n tabi'n'.
- Ultanlari 6 sm ha'm 3 sm bolg'an tuvri' mu'yeshli trapetsiyag'aishley si'zi'lg'an shen'berdin' radiusin tabi'n'.
- Yeger $A_1A_4=2,24$ bolsa, onda $A_1A_2A_3A_4A_5A_6$ duri's alt'imu'yeshliktin' perimetrin tabi'n'.
- $N(7;3)$ ha'm $M(-3;5)$ bolsa, NM diametrli shen'berdin' uzi'nli'g'i'n tabi'n'.
- To'besindagi mu'yeshi 120° boli'p, radiusi 10 sm bolg'an shen'berge ishley si'zi'lg'an ten' qaptalli u'shmu'yeshliktin' maydani'n tabi'n'.
- Yeger $ABCD$ to'rtmu'yeshliginde $AB=5\text{ sm}$, $BC=13\text{ sm}$, $CD=9\text{ sm}$, $DA=15\text{ sm}$ ha'm $AC=12\text{ sm}$ bolsa, $ABCD$ to'rtmu'yeshliginin' maydani'n tabi'n'.
- 90° li worayliq mu'yeshke sa'ykes dog'anin' uzinlig'i 15π ge ten'. Shen'berge si'rtlay si'zi'lg'an duri's u'shmu'yeshliktin' maydani'n tabi'n'.

Ma'sele. Shen'berdin' AB xordasi 10 sm. Xordanin' A ushi'nda AD urinba, B ushi'nan bolsa usi urinbag'a parallel BC xorda ju'rgizildi. Yeger $BC=12$ sm bolsa, shen'berdin' radiusin tabi'n'.

Sheshiliwi: 1) A noqati ha'm shen'ber worayi — O noqati arqali wo'tiwshi tuwri' BC xordasin K noqatindakesip o'tsin. AD urinba bolg'anliqtan $AK \perp AD$, $AD \parallel BC$ bolg'ani' ushi'n $AK \perp BC$.

2) $AK \perp BC$, yag'niy $OK \perp BC$ bolg'ani' ushi'n $CK = KB$. AK kesindisi ABC u'shmu'yeshliginin' medianasi ha'm biyikligi yeken. Demek, $AC = AB = 10$ sm.

3) Geron formulasinan paydalani'p ABC u'shmu'yeshliginin' maydani'n tabami'z:

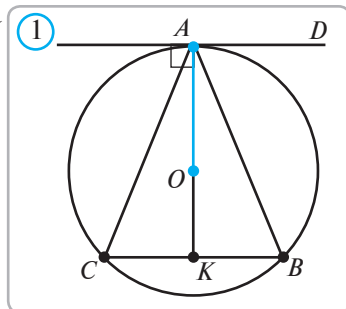
$$p = \frac{a+b+c}{2} = \frac{10+10+12}{2} = 16 \text{ (sm)},$$

$$S = \sqrt{p \cdot (p-a)(p-b)(p-c)} = \sqrt{16 \cdot (16-10)(16-10)(16-12)} = 48 \text{ (sm}^2\text{)}.$$

4) ABC u'shmu'yeshligine si'rtlay si'zi'lg'an shen'berdin' radiusin tabami'z:

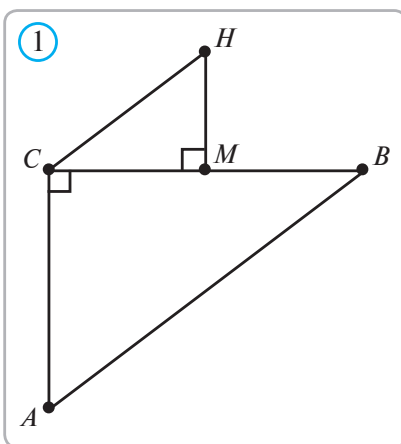
$$R = \frac{abc}{4S} = \frac{12 \cdot 10 \cdot 10}{4 \cdot 48} = 6,25 \text{ (sm)}.$$

Juwabi': 6,25 sm.



? Soraw, ma'sele ha'm tapsi'rmlar

1. U'shmu'yeshliktin' ta'repleri sa'ykes tu'rde 13 sm, 14 sm ha'm 15 sm ge ten'. U'shmu'yeshlikke ishley ha'm si'rtlay si'zi'lg'an do'n'gelektin' maydanlarinin' qatnasin tabi'n'.
2. Yeger $\angle BDC = 40^\circ$ ha'm $\angle CBD = 60^\circ$ bolsa, shen'berge ishley si'zi'lg'an $ABCD$ to'rtmu'yeshliginin' A ha'm C mu'yeshlerin tabi'n'.
3. Shen'berge ishley si'zi'lg'an ten' qaptalli trapeciyanin' ultanlari 4 sm ha'm 16 sm bolsa, shen'berdin' radiusin tabi'n'.
4. Tuwri' mu'yeshli u'shmu'yeshliktin' katetlerine tu'sirilgen medianalari $\sqrt{52}$ sm ha'm $\sqrt{73}$ sm ge ten'. U'shmu'yeshliktin' maydani'n tabi'n'.
5. Katetleri 6 m ha'm 8 m bolg'an tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasina tu'sirilgen biyikligin tabi'n'.
6. Tuwri' mu'yeshli u'shmu'yeshliktin' bir kateti 5 sm ha'm gipotenuzasi 13 sm bolsa, wonin' maydani'n tabi'n'.
7. x tin' qanday ma'nislerinde $\vec{a}(x; 7)$ ha'm $\vec{b}(5; 2-x)$ vektorlari perpendikulyar boladi?
8. U'shmu'yeshliktin' yeki ta'repi 10 sm ha'm 12 sm, wolardi'n' arasi'ndag'i su'yir mu'yeshlin' sinusi 0,8 ge ten'. U'shmu'yeshliktin' u'shinshi ta'repin tabi'n'.



I. U'lg'i ushi'n baqlaw jumi'si'

1. $ABCD$ parallelogrammda $\angle A = 45^\circ$, $AD = 4$. parallelogrammnin' AB ta'repinin' dawamina $\angle PDA = 90^\circ$ qa ten' bolatug'in BP kesindi qoyi'ldi. BC ha'm PD kesindiler T noqatinda kesilisedi, bunda $PT : TD = 3 : 1$.

a) $\triangle BPT \sim \triangle CDT$ yekenligin da'liyllen', bul u'shmu'-yeshliklerdin' maydanlarinin' qatnasin tabi'n'.

b) $ABCD$ parallelogrammnin' maydani'n tabi'n'.

d) AB ha'm TD kesindilerinin' wortalarin tutastirish kesindinin' uzi'nli'g'i'n tabi'n'.

e) $\overrightarrow{AB}$ vektorin $\overrightarrow{CA}$ ha'm $\overrightarrow{TB}$ vektorlari arqali

an'latin'.

f) CAD mu'yeshinin' sinusin tabi'n'.

2. (*Qosi'msha*) 1-su'wrette $BC \perp AC$, $MH \perp BC$, $2MC = BC$, $MH = 0,5AC$ bolsa, $AB \parallel CH$ yekenligin da'liyllen'.

II. Baqlaw jumi'si' ushi'n ko'rsetpeli testler.

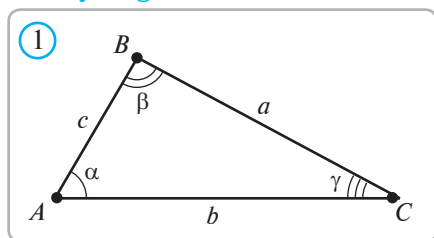
- Yeger tuwri' mu'yeshli u'shmu'yeshliktin' biyikligi gipotenuzasin 6 sm ha'm 54 sm kesindilerga ajiratsa, bul u'shmu'yeshliktin' maydani'n tabi'n':
A) 648 sm^2 ; B) 324 sm^2 ; D) 1080 sm^2 ; E) 540 sm^2 .
- C noqatidan wo'tkerilgen bir kesiwshi shen'berdi A ha'm B , yekinshisi bolsa D ha'm E noqatlarinda kesedi. Yeger $CD = 18 \text{ sm}$, $CB = 8 \text{ sm}$, $CD = 8 \text{ sm}$ bolsa, DE kesindisinin' uzi'nli'g'i'n tabi'n':
A) 17 sm ; B) 1 sm ; D) 9 sm ; E) duri's juwap ko'rsetilmegen.
- Yeger $A(-5; 2\sqrt{3})$, $B(-4; 2)$, $C(-2; \sqrt{3})$, $D(0; 2)$ bolsa, $ABCD$ to'rtmu'yeshliginin' diagonallari arasi'ndag'i mu'yeshi tabi'n':
A) 30° ; B) 60° ; D) 90° ; E) duri's juwap ko'rsetilmegen.
- Yeger parallelogrammnin' diagonallari 10 sm ha'm $8\sqrt{2} \text{ sm}$ ge ten' ha'm wolar arasi'ndag'i mu'yesh 45° bolsa, parallelogrammnin' ta'replerin tabi'n':
A) $\sqrt{17} \text{ sm}$ ha'm $\sqrt{97} \text{ sm}$; B) 5 sm ha'm 6 sm ;
D) $\sqrt{34} \text{ sm}$ ha'm $\sqrt{63} \text{ sm}$; E) duri's juwap ko'rsetilmegen.
- Radiusi 8 sm bolg'an shen'berge ishley si'zi'lg'an duri's altimu'yeshstin' maydani'n tabi'n'.
A) $48\sqrt{3} \text{ sm}^2$; B) $192\sqrt{3} \text{ sm}^2$; D) $96\sqrt{2}$; E) duri's juwap ko'rsetilmegen.

6. Worayliq mu'yeshi 140° , maydani' $31,5\pi \text{ sm}^2$ bolg'an sektordin' radiusin ani'qlan':
 A) 9 sm ; B) 18 sm ; D) $9\pi \text{ sm}$; E) duri's juwap ko'rsetilmegen.
7. Ultaninin' uzunlig'i 15 sm bolg'an u'shmu'yeshlikтин' ultanina parallel kesindi ju'rgizilgen. Yeger payda bolg'an trapeciyani'n' maydani' u'shmu'yeshlikтин' maydani'nin' $\frac{3}{4}$ bo'legin quraytug'inlig'i belgili bolsa, kesindinin' uzi'nli'g'i'n tabi'n':
 A) $6,5$; B) 7 ; D) $7,5$; E) 5 .
8. Qaptal ta'repi $2\sqrt{39} \text{ sm}$ bolg'an ten' qaptalli u'shmu'yeshlikтин' biyikliginin' ultanina qatnasi' $3:4$ ke ten' bolsa, u'shmu'yeshlikтин' maydani'n tabi'n':
 A) 260 ; B) 245 ; D) 310 ; E) 72 .
9. $a(4; 4\sqrt{3})$ ha'm $b(8\sqrt{3}; 8)$ vektorlari arasi'ndag'i mu'yeshi tabi'n':
 A) 45° ; B) 90° ; D) 30° ; E) 60° .
10. Ten' qaptalli trapeciyanin' ultanlari 10 sm ha'm 16 sm , qaptal ta'repi bolsa 5 sm . Trapeciyanin' maydani'n tabi'n':
 A) 45 ; B) 50 ; D) 48 ; E) 52 .
11. Tuwri' mu'yeshli u'shmu'yeshlikтин' gipotenuzasi 13 sm boli'p, katetlerinen biri yekinishinen 7 sm u'lken. U'shmu'yeshlikтин' maydani'n tabi'n':
 A) 30 sm^2 ; B) 25 sm^2 ; D) 45 sm^2 ; E) 40 sm^2 .
12. Ta'repi 5 sm bolg'an rombi'ni'n' bir diagonalni 6 sm ge ten'. Rombi'ni'n' maydani'n tabi'n':
 A) 24 sm^2 ; B) 30 sm^2 ; D) 29 sm^2 ; E) 40 sm^2 .
13. Diagonalni $6\sqrt{2}$ bolg'an kvadratqa ishley si'zi'lg'an shen'berdin' uzi'nli'g'i'n tabi'n':
 A) 10π ; B) 8π ; D) 9π ; E) 6π .
14. Ta'repi $6\sqrt{2} \text{ sm}$ bolg'an kvadratqa si'rtlay si'zi'lg'an do'n'gelektin' maydani'n tabi'n':
 A) 9π ; B) 12π ; D) 15π ; E) 18π .
15. Biyiklikleri 4 sm ha'm 6 sm bolg'an parallelogrammnin' maydani' 36 sm^2 ge ten'. Woni'n' perimetrin tabi'n':
 A) 26 sm ; B) 30 sm ; D) 29 sm ; E) 36 sm .
16. Perimetri 30 sm bolg'an parallelogrammnin' ta'replari $2:3$ qatnasta. Yeger parallelogrammnin' su'yir mu'yeshi 30° bolsa, woni'n' maydani'n tabi'n':
 A) 26 sm^2 ; B) 27 sm^2 ; D) 29 sm^2 ; E) 30 sm^2 .
17. Yeger ABC u'shmu'yeshliginde $AB=6\sqrt{3} \text{ sm}$, $BC=12 \text{ sm}$ ha'm $\angle C=60^\circ$ bolsa, u'shmu'yeshlikтин' A mu'yeshin tabi'n':
 A) 45° ; B) 90° ; D) 30° ; E) 60° .

PLANIMETRIYAG'A TIYISLI TIYKARG'I' TU'SINIK HA'M MAG'LIWMATLAR

U'SHMU'YESHLIKLER

1°. Tiykarg'i' tu'sinikler



Tegislikte u'sh noqat berilgen boli'p, u'shewi bir tuwri'da jatpasin. Usi noqatlardin' ha'r yekewin kesindiler menen tutastiramiz. Payda bolg'an forma *u'shmu'yeshlik* dep ataladi. Noqatlar u'shmu'yeshliktin' to'beleri *kesindiler ta'repleri* dep ataladi. Belgileniwi: A, B, C – to'beleri, a, b, c – ta'repleri (*I-su'wret*).

U'shmu'yeshliku'sh mu'yeshke iye: $\angle BAC$, $\angle CBA$, $\angle ACB$. Belgileniwi: α, β, γ .

Mediana — U'shmu'yeshliktin' to'besi menen qarsi ta'repi ortasin tutastiriwshi kesindi. U'shmu'yeshlikte 3 medianaboli'p, olar m_a, m_b, m_c tu'rinde belgilenedi.

Bissektrisa — u'shmu'yeshlik to'besin woni'n qarsi'ndag'i ta'rep penen tutashti'ri'wshi' ha'm usi' to'bedegi mu'yesh bissektrisasi'nda jati'wshi' kesindi. Ushu'yeshlikte u'sh bessektrisiw boli'p, wolar l_a, l_b, l_c ko'rinishinde belgilenedi.

Biyiklik — U'shmu'yeshliktin' to'besinen wonin' qarsisindag'i ta'repte jatqan tuwri'g'a tu'sirilgen perpendikulyar.

U'shmu'yeshlikte u'sh biyiklik boli'p, wolar h_a, h_b, h_c tu'rinde belgilenedi.

Worta si'zi'q — yeki ta'rep wortalarin tutastiriwshi kesindi.

Wortasi'zi'qlardin' sani 3.

Perimetr — u'sh ta'reptin' uzunliqlarinin' qosi'ndi'si'. Belgileniwi P .

U'shmu'yeshlikler ta'replerine qarap u'sh tu'rge bo'linadi:

a) ten' ta'repli ($a=b=c$); b) ten' qaptalli (a, b, c lardin' qanday dabir ekewi ten'); d) tu'rli ta'repli (a, b, c lardin' hesh qanday yekewi ten' emes).

U'shmu'yeshliktin' u'sh ta'repine de urinip wo'tiwshi shen'ber wog'an ishley si'zi'lg'an shen'ber delinedi (bunday shen'ber boladi ha'm jalg'iz). Ishley si'zi'lg'an shen'berdin' radiusi r menen belgilenedi.

U'shmu'yeshliktin' u'sh to'besinen, de wo'tiwshi shen'ber wog'an si'rtlay si'zi'lg'an shen'ber delinedi ha'm woni'n radiusi R menen belgilenedi (bunday shen'ber bar ha'm jalg'iz).

2°. Tiykarg'i' tu'sinikler

1) $\alpha + \beta + \gamma = 180^\circ$. U'shmu'yeshliktin' ishki mu'yeshlerinin' qosi'ndi'si' 180° qaten'. 2) U'sh medianabir noqattakesilisedi. Bul noqat medianani' 2:1 qatnasta bo'ledi. Medianau'shmu'yeshlikti yeki maydani' ten' u'shmu'yeshliklerge bo'ledi. Medianalardin' uzunliqlari

$m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$; $m_b = \frac{1}{2}\sqrt{2a^2 + 2c^2 - b^2}$; $m_c = \frac{1}{2}\sqrt{2a^2 + 2b^2 - c^2}$
formulalarinan tabiladi.

3) U'sh bissektrisa bir noqatta kesilisedi. Bul noqat ishley si'zi'lg'an shen'ber worayi boladi. Bissektrisa wo'zi tu'sirilgen ta'repti qalg'an ta'replerge proporcional bolg'an bo'leklerge bo'ledi (2-su'wret).

$$BD \text{ bissektrisa bolsa, } \frac{AB}{AD} = \frac{BC}{DC}.$$

Bissektrisa uzunliqlari:

$$l_a = \frac{2\sqrt{bc}}{b+c} \sqrt{p(p-a)}; \quad l_b = \frac{2\sqrt{ac}}{a+c} \sqrt{p(p-b)};$$

$$l_c = \frac{2\sqrt{ab}}{a+b} \sqrt{p(p-c)}, \quad p = \frac{1}{2}(a+b+c) \text{ formulalaridan tabiladi.}$$

4) U'shmu'yeshliktin' biyikliklari yaki wolardin' dawamlari bir noqatta kesilisedi. Biyikliktin' uzunliqlari

$$h_a = \frac{2S}{a}; \quad h_b = \frac{2S}{b}; \quad h_c = \frac{2S}{c}$$

formulalaridan tabiladi. Bul jerde S - u'shmu'yeshliktin' maydani'.

5) U'shmu'yeshliktin' ta'replerinin' worta perpendikulyarlari bir noqatta kesilisedi. Bul noqat u'shmu'yeshlikke *si'rtlay si'zi'lg'an shen'ber worayi* boladi.

6) U'shmu'yeshliktin' worta sizig'i u'shinshi ta'repke parallel ha'm yarimintan'.

7) Sinuslar teoremasi:

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2R.$$

8) Kosinuslar teoremasi:

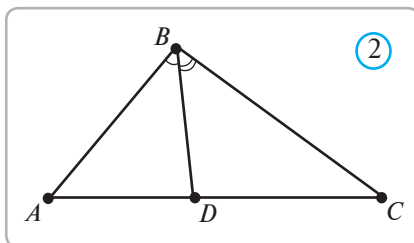
$$a^2 = b^2 + c^2 - 2bc \cos \alpha, \quad b^2 = a^2 + c^2 - 2ac \cos \beta, \quad c^2 = a^2 + b^2 - 2ab \cos \gamma$$

9) U'shmu'yeshliktin' maydani'n yesaplaw formulalari:

$$S = \frac{1}{2}ah_a = \frac{1}{2}bh_b = \frac{1}{2}ch_c; \quad S = \frac{1}{2}ab \sin \gamma = \frac{1}{2}bc \sin \alpha = \frac{1}{2}ac \sin \beta;$$

10) Geron formulasi:

$$S = \sqrt{p(p-a)(p-b)(p-c)}, \quad p = \frac{a+b+c}{2}; \quad S = \frac{abc}{4R}, \quad S = pr.$$



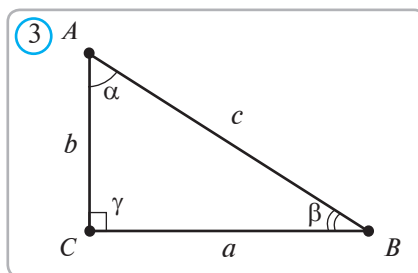
3°. Za'ru'r jeke jag'daylar

a) Tuvri' mu'yeshli u'shmu'yeshlik (3-su'wret).

$\angle \gamma = 90^\circ$, $\alpha + \beta = 90^\circ$, AC ha'm BC — katetler, AB — gipotenuza. Pifagor teoremasi: $a^2 + b^2 = c^2$.

$$S = \frac{1}{2}ab; \quad R = \frac{c}{2}; \quad r = \frac{a+b-c}{2};$$

$$\frac{a}{c} = \sin \alpha; \quad \frac{a}{c} = \cos \beta; \quad \frac{b}{c} = \sin \beta; \quad \frac{b}{c} = \cos \alpha.$$

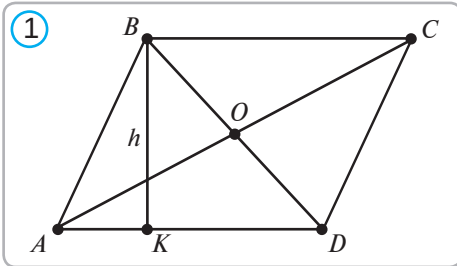


$$\frac{a}{b} = \operatorname{tg} \alpha; \quad \frac{a}{b} = \operatorname{ctg} \beta; \quad \frac{b}{a} = \operatorname{ctg} \alpha; \quad \frac{b}{a} = \operatorname{tg} \beta.$$

b) Ten' ta'repli u'shmu'yeshlik

$$\alpha = \beta = \gamma = 60^\circ, \quad S = \frac{a^2 \sqrt{3}}{4}, \quad r = \frac{a \sqrt{3}}{6}, \quad R = \frac{a \sqrt{3}}{3}.$$

TO'RTMU'YESHLIKLER



1°. Parallelogramm

Qarama-qarsi ta'replari parallel bolg'an to'rt-mu'yeshlik *parallelogramm* delinedi (1-su'wret).

Qon'silas bolmag'an to'belerin tutastirish kesindi *diagonal* delinedi.

AB ha'm CD ; AD ha'm BC parallel ta'replari; BD ha'm AC diagonal.

Tiykarg'i' qat'isietler ha'm qatnasiqlar:

1) Diagonalardin' kesilisiw noqati parallelogrammnin' simmetriyaworayi boladi.

2) Qarama-qarsi ta'replardin' uzunliqlari wo'z-ara ten':

$$AB = CD \quad \text{va} \quad AD = BC.$$

3) Parallelogrammnin' qarama-qarsi mu'yeshleri wo'z-ara ten':

$$\angle BAD = \angle BCD \quad \text{va} \quad \angle ABC = \angle ADC.$$

4) Qon'silas mu'yeshlerdin' qosi'ndi'si' 180° qaten':

5) Diagonal kesilisiw noqatinda ten' yekige bo'linedi: $BO = OD$ ha'm $AO = OC$

6) Ta'replerinin' kvadratlarinin' qosi'ndi'si', diagonalari'ni'n' kvadratlarinin' qosi'ndi'si'naten':

$$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2 \quad \text{yamasa} \quad 2(AB^2 + BC^2) = AC^2 + BD^2.$$

7) Parallelogramm maydani':

a) $S = ah_a$, bul jerde $a = AD$ ta'rep, $h_a = BK$ — biyiklik;

b) $S = absin\alpha$, bul jerde $b = AB$ — ta'rep, $\alpha = \angle BAD$ — AB ha'm AD ta'replari arasi'ndag'i mu'yesh.

2°. Romb

Barliq ta'replari wo'z-ara ten' bolg'an parallelogramm *romb* dep ataladi.

Parallelogramm ushi'n orinli bolg'an barliq qat'isietler romb ushi'n da worinli.

Rombinin' qosi'msha qat'isietleri.

1) Rombi'ni'n' diagonalari wo'z-ara perpendikulyar.

2) Rombi'ni'n' diagonalari ishki mu'yeshlerdin' bissektrisalari boladi.

3) Rombi'ni'n' maydani' $S = \frac{1}{2} d_1 d_2$, bul jerde d_1, d_2 — rombi'ni'n' diagonalari.

3°. Tuwri' mu'yeshlik

Barliq mu'yeshleri 90° qaten' bolg'an parallelogramm *tuwri' mu'yeshlik* dep ataladi.

- 1) Tuvri'mu'yeshliktin' diagonallari wo'z-ara ten'.
- 2) Tuvri'mu'yeshliktin' maydani' $S = ab$, bul jerde a ha'm b — tuwri'mu'yeshliktin' qon'silas ta'replari.

4°. Kvadrat

Barliq ta'replari wo'z-ara ten' bolg'an tuwri'mu'yeshlik *kvadrat* delinedi.

Romb ha'm tuwri'mu'yeshlikler ushi'n wori'nli' bolg'an barliq qa'siyetler kvadrat ushi'n da wori'nli'.

Yeger a — kvadrattin' ta'repi, d diagonali bolsa: $S = a^2$; $S = \frac{d^2}{2}$; $d = a\sqrt{2}$.

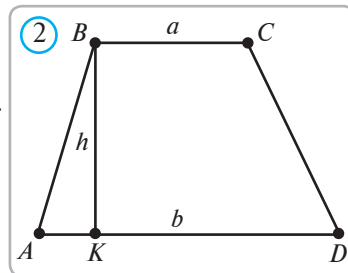
5°. Trapeciya

Ultanlar dep atalatug'in yeki ta'repi wo'z-ara parallel ha'm qaptal ta'repler dep atalatug'i'n, qalg'an yeki ta'repi bolsa parallel bolmag'an to'rtmu'yeshlik *trapetsiya* dep ataladi.

Qaptal ta'replerinin' wortalarin tutastiriwshi kesindi trapeciyanin' *worta sizig'i* dep ataladi.

Tiykarg'i' qa'siyetler

- 1) Trapeciyanin' wortasizig'i ultanlarga parallel boladi ha'm ultanlardin' qosi'ndi'si'nin' yariminen'.
- 2) Trapeciyanin' maydani' $S = \frac{a+b}{2}h$, bul jerde a ha'm b — ultanlar, h bolsabiyiklik (*2-su'wret*).



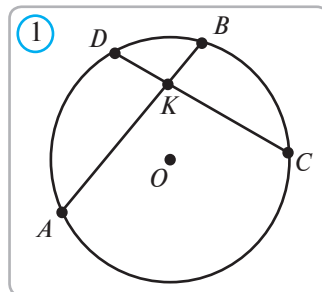
SHEN'BER, DO'N'GELEK

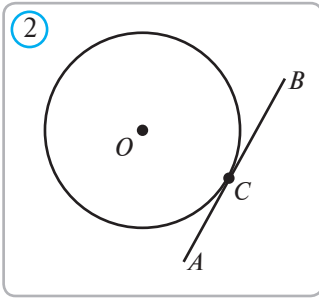
1°. Tegislikte R on' sani ha'm O noqati berilgen bolsi'n. O noqatidan R aralig'inda jaylasqan noqatlardan quralg'an figura *shen'ber* dep ataladi. O noqati *shen'berdin' worayi*, woray menen shen'berdegi noqatti tutastiriwshi kesindi *radius*, R sani bolsar *radius uzunlig'i* dep ataladi. Shen'berdegi yeki noqatti tutastiriwshi kesindi *xorda*, woraydan wo'tiwshi xorda bolsa *diametr* dep ataladi.

Tegisliktin' shen'ber menen shegaralang'an shekli bo'limi — *do'n'gelek* dep ataladi.

Tiykarg'i' qatnaslar

- 1) $D = 2R$, bul jerde: D — diametrdin' uzunlig'i.
- 2) $L = 2\pi R$ — shen'berdin' uzunlig'i.
- 3) $S = \pi R^2$ — do'n'gelektin' maydani'.
- 4) AB ha'm CD xordalar K noqatinda kesilisse (*1-su'wret*), $AK \cdot KB = CK \cdot KD$ qatnaslar worinlanadi.
- 5) Xordani ten' yekige bo'liwshi diametr usi xordag'aperpendikulyar boladi.





6) Ten' xordalar woraydan ten' arali'qta joylasqan ha'm kerisinshe woraydan ten' arali'qta joylasqan xordalar wo'z-ara ten'.

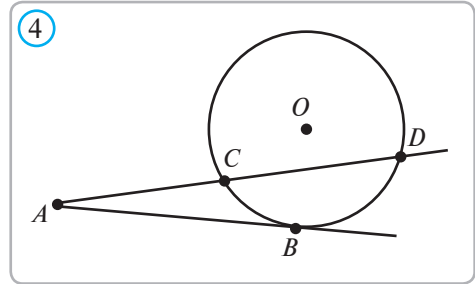
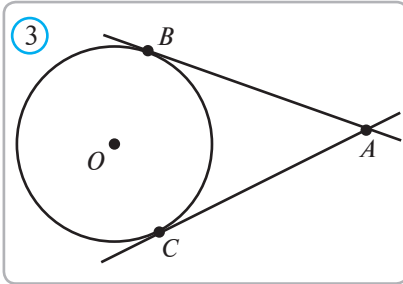
2°. Urinba

Shen'ber (yamasa do'n'gelek) menen birden-bir uliwma noqatqa iye bolg'an tuwri' *urinba* dep ataladi. Noqat bolsa *uriniv noqati* dep ataladi (2-su'wret).

Shen'ber menen 2 uliwma noqatqa iye bolg'an tuwri' *kesiwshi* dep ataladi.

Urinbanin' qa'siyetleri

- 1) Uriniw noqatiga wo'tkerilgen radius urinbag'aper pendikulyar boladi.
- 2) Do'n'gelek si'rtindag'i noqattan usi do'n'gelekke yeki urinba wo'tkeriw mu'mkin. Bul urinbalardin' kesindileri wo'z-ara ten' (3-su'wret): $AB=AC$.

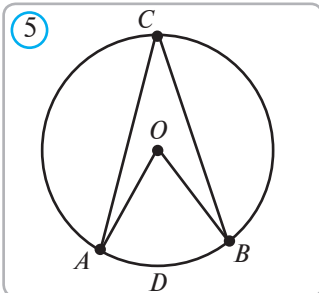


- 3) Yeger AC kesiwshi boli'p, shen'berdi C ha'm D noqatlarda kesip wo'tse, AB urinba bolsa, $AB^2=AD \cdot AC$ ten'ligi worinli boladi (4-su'wret).

3°. Orayliq ha'm ishley si'zi'lg'an mu'yeshler.

Shen'berdegi yeki noqat ja'rdeminde shen'ber yeki bo'lekke bo'linedi. Bul bo'lekke *dog'alar* dep ataladi. Belgileniwi: $\angle ADB$; $\angle ACB$.

$\angle AOB$ mu'yeshi $\angle ADB$ dog'ag'a tirelgen *worayliq mu'yesh*, (5-su'wret), $\angle ACB$ mu'yeshi bolsa $\angle ADB$ dog'ag'a tirelgen ha'm shen'berge *ishley si'zi'lg'an mu'yesh* delinedi. Bul mu'yeshlar arasi'nda,



$$\angle ACB = \frac{1}{2} \angle AOB$$

qatnasi' wori'nli'.

Demek, yari'm shen'berge (diametrge) tirelgen ishki mu'yesh tuwri' mu'yesh boladi (6-su'wret). Bir dog'ag'a tirelgen shen'berge ishley si'zi'lg'an mu'yeshlar ten' boladi.

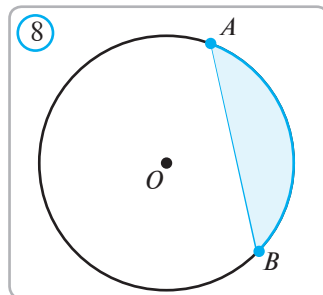
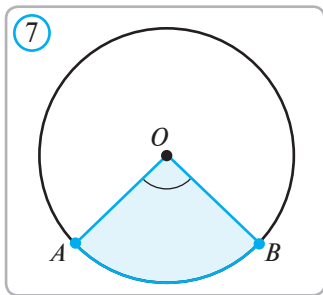
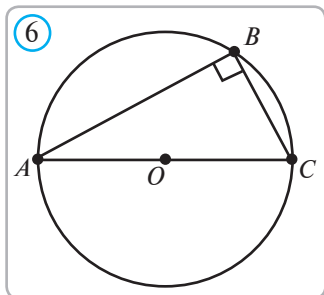
4°. Sektor ha'm segment

Do'n'gelektin' yeki radius penen shegaralang'an

bo'legi *sektor* delinedi (7-su 'wret). Sektor dog'asinin' uzunlig'i: $l = \frac{\pi R \alpha}{180}$, bul jerde, α — worayliq mu'yeshitin' gradius wo'lsheimi.

Sektor maydani': $S = \frac{\pi R^2 \alpha}{360}$; $S = \frac{1}{2} Rl$.

Segment — do'n'gelektin' xordasi ha'm usi xorda tirelgen dog'a menen shegaralang'an bo'legi (8-su 'wret).



Segment maydani': $S = S_{\text{sektor}} \pm S_{\Delta} = \frac{\pi R^2}{360} \cdot \alpha \pm \frac{1}{2} R^2 \sin \alpha$

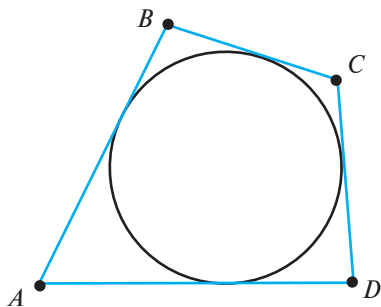
DURI'S KO'PMU'YESHLIKLER

Duri's n mu'yeshitin' ta'repi a_n , perimetri P_n , maydani' S_n , ishley si'zi'lg'an shen'ber radiusi r_n , si'rtlay si'zi'lg'an shen'ber radiusi R_n , ishki mu'yeshi n bolsa,

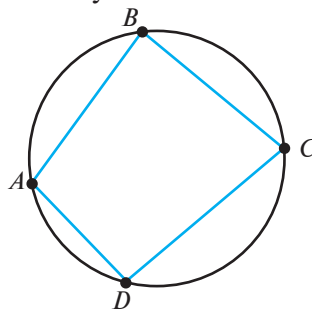
$$P_n = na_n, \quad S_n = \frac{1}{2} P_n r_n = \frac{1}{2} na_n r_n, \quad \alpha_n = \frac{(n-2) \cdot 180^\circ}{n}$$

$$R_n = \frac{a_n}{2 \sin \frac{180^\circ}{n}}, \quad r_n = \frac{a_n}{2 \operatorname{tg} \frac{180^\circ}{n}}$$

Shen'berge ishley ha'm si'rtlay si'zi'lg'an to'rtmu'yeshlikler.



$$BC + AD = AB + CD$$



$$\angle A + \angle C = 180^\circ$$

$$\angle B + \angle D = 180^\circ$$

**10 nan 99 g'a shekem bolg'an natural sanlardin'
kvadratlarinin' kestesı**

<i>onliq</i> <i>birlik</i>	1	2	3	4	5	6	7	8	9
0	100	400	900	1600	2500	3600	4900	6400	8100
1	121	441	961	1681	2601	3721	5041	6561	8281
2	144	484	1024	1764	2704	3844	5184	6724	8464
3	169	529	1089	1849	2809	3969	5329	6889	8649
4	196	576	1156	1936	2916	4036	5476	7056	8836
5	225	625	1225	2025	3025	4225	5625	7225	9025
6	256	676	1296	2116	3136	4356	5776	7396	9216
7	289	729	1369	2209	3249	4489	5929	7569	9409
8	324	784	1444	2304	3364	4624	6084	7744	9604
9	361	841	1521	2401	3481	4761	6241	7921	9801

Ayirim shamalardin' kestesı

$\pi \cong 3,1416$	$\sqrt{8} \cong 2,8284$
$\sqrt{2} \cong 1,4142$	$\sqrt{10} \cong 3,1623$
$\sqrt{3} \cong 1,7320$	$\frac{1}{\sqrt{2}} \cong 0,7071$
$\sqrt{5} \cong 2,2360$	$\frac{1}{\sqrt{3}} \cong 0,5774$
$\sqrt{6} \cong 2,4495$	$\frac{1}{\sqrt{\pi}} \cong 0,3183$
$\sqrt{7} \cong 2,6457$	

Trigonometriyalik funktsiyalardi'n' ma'nislerinin' kestesesi									
α°	$\sin\alpha$	$\cos\alpha$	$\operatorname{tg}\alpha$	$\operatorname{ctg}\alpha$	α°	$\sin\alpha$	$\cos\alpha$	$\operatorname{tg}\alpha$	$\operatorname{ctg}\alpha$
1	0,0175	1,000	0,0175	57,3	46	0,719	0,695	1,036	0,966
2	0,0349	0,999	0,0349	28,6	47	0,731	0,682	1,072	0,933
3	0,0523	0,999	0,0524	19,1	48	0,743	0,669	1,111	0,900
4	0,0698	0,998	0,0699	14,3	49	0,755	0,656	1,150	0,869
5	0,0872	0,996	0,0875	11,4	50	0,766	0,643	1,192	0,839
6	0,1045	0,995	0,1051	9,51	51	0,777	0,629	1,235	0,810
7	0,1219	0,993	0,1228	8,14	52	0,788	0,616	1,280	0,781
8	0,139	0,990	0,141	7,11	53	0,799	0,602	1,327	0,754
9	0,156	0,988	0,158	6,31	54	0,809	0,588	1,376	0,727
10	0,174	0,985	0,176	5,67	55	0,819	0,574	1,428	0,700
11	0,191	0,982	0,194	5,145	56	0,829	0,559	1,483	0,675
12	0,208	0,978	0,213	4,507	57	0,839	0,545	1,540	0,649
13	0,225	0,974	0,231	4,331	58	0,848	0,530	1,600	0,625
14	0,242	0,970	0,249	4,011	59	0,857	0,515	1,664	0,601
15	0,259	0,966	0,268	3,732	60	0,866	0,500	1,732	0,577
16	0,276	0,961	0,287	3,487	61	0,875	0,485	1,804	0,554
17	0,292	0,956	0,306	3,271	62	0,883	0,469	1,881	0,532
18	0,309	0,951	0,325	3,078	63	0,891	0,454	1,963	0,510
19	0,326	0,946	0,344	2,904	64	0,899	0,438	2,050	0,488
20	0,342	0,940	0,364	2,747	65	0,906	0,423	2,145	0,466
21	0,358	0,934	0,384	2,605	66	0,914	0,405	2,246	0,445
22	0,375	0,927	0,404	2,475	67	0,921	0,391	2,356	0,424
23	0,391	0,921	0,424	2,356	68	0,927	0,375	2,475	0,404
24	0,405	0,914	0,445	2,246	69	0,934	0,358	2,605	0,384
25	0,423	0,906	0,466	2,145	70	0,940	0,342	2,747	0,364
26	0,438	0,899	0,488	2,050	71	0,946	0,326	2,904	0,344
27	0,454	0,891	0,510	1,963	72	0,951	0,309	3,078	0,325
28	0,469	0,883	0,532	1,881	73	0,956	0,292	3,271	0,306
29	0,485	0,875	0,554	1,804	74	0,961	0,276	3,487	0,287
30	0,500	0,866	0,577	1,732	75	0,966	0,259	3,732	0,268
31	0,515	0,857	0,601	1,664	76	0,970	0,242	4,011	0,249
32	0,530	0,848	0,625	1,600	77	0,974	0,225	4,331	0,231
33	0,545	0,839	0,649	1,540	78	0,978	0,208	4,507	0,213
34	0,559	0,829	0,675	1,483	79	0,982	0,191	5,145	0,194
35	0,574	0,819	0,700	1,428	80	0,985	0,174	5,67	0,176
36	0,588	0,809	0,727	1,376	81	0,988	0,156	6,31	0,158
37	0,602	0,799	0,754	1,327	82	0,990	0,139	7,11	0,141
38	0,616	0,788	0,781	1,280	83	0,993	0,1219	8,14	0,1228
39	0,629	0,777	0,810	1,235	84	0,995	0,1045	9,51	0,1051
40	0,643	0,766	0,839	1,192	85	0,996	0,0872	11,4	0,0875
41	0,656	0,755	0,869	1,150	86	0,998	0,0698	14,3	0,0699
42	0,669	0,743	0,900	1,111	87	0,999	0,0523	19,1	0,0524
43	0,682	0,731	0,933	1,072	88	0,999	0,0349	28,6	0,0349
44	0,695	0,719	0,966	1,036	89	1,000	0,0175	57,3	0,0175
45	0,707	0,707	1,000	1,000	90	1,000	0,0000	-	0,0000

JUWAPLAR HA'M KO'RSETPELER

- 1-sabaq.** 1. 50° ; 130° ; 133° ; 97° . 2. 12 sm. 3. 65° ; 70° ; 45° . 4. 105° ; 130° ; 125° . 5. 35° ; 35° ; 110° . 6. 94° ; 56° ; 30° . 7. 110° ; 130° ; 120° . 8. *Ko'rsetpe:* to'rtu'shmu'yeshliktin' ha'r birinin' ta'replari da'slepki u'shmu'yeshliktin' sa'ykes ta'replerinin' yarmina ten'. 9. *Ko'rsetpe:* DF kesindi ABH u'shmu'yeshliktin'de, CEB u'shmu'yeshliktin' de worta sizig'i boladi. 10. *Ko'rsetpe:* ANC ha'm CKA u'shmu'yeshliklerdin' ha'mde ishki almasiwshi mu'yeshlerdin' ten'liginen paydalanin'.
- 2-sabaq.** 1. $6\sqrt{6}$. 2. 36. 3. 30° . 4. a) 80° ; 80° ; 20° ; b) 70° ; 70° ; 40° . 5. 6,72 sm. 6. 54. 7. 5 sm; $25\sqrt{3}$ sm²; 120° ; 30° ; 30° . 8. 55° ; 60° ; 65° . 9. 90° . 10. 140° .
- 3-sabaq.** 2. 78° ; 102° ; 78° ; 102° . 3. 53° ; 37° . 4. 110° ; 70° ; 110° ; 70° . 5. 45° ; 135° ; 45° ; 135° . 6. 20 sm yoki 28 sm. 7. *Ko'rsetpe:* Da'slep ta'replari $AB=2$ sm, $BC=6$ sm bolg'an $ABCD$ tuwri'mu'yeshlik jasan'. Keyin woraylari B ha'm C noqatlarda radiusi 3 sm bolg'an shen'berler jasan'.
- 4-sabaq.** 1. 30 sm. 2. 13 sm. 3. *Ko'rsetpe:* 3-sabaqtan 7-ma'selege qaran'. 4. $880\sqrt{41}$ sm². 5. a) 4 sm, 8 sm; b) 45° , 90° ; d) $16+8\sqrt{2}$ sm, 32 sm². 6. $18\sqrt{3}$ sm². 7. 30 sm². 8. 28 sm; $28\sqrt{2}$ sm.
- 5-sabaq.** 3. U'shmu'yeshlikler uqsas. 5. 5; 8; $\frac{1}{2}$. 6. 72; 162; 90.
- 6-sabaq.** 3. 12 m. 4. 7,5 sm; 12,5 sm; 15 sm. 5. $73,5$ m²; $37,5$ m². 6. U'shmu'yeshlikler uqsas.
- 7-sabaq.** 3. a) 4,5; b) 10,5; d) 4,5. 4. a) 10; b) 6; d) 4,5. 5. a) 5 sm, 3,5 sm; b) $5\frac{5}{7}$ sm, $2\frac{2}{7}$ sm. 6. a) 8; b) 3,5; d) 12,5.
- 8-sabaq.** 4. a) awa; b) awa; d) yaq. 5. $2\frac{1}{3}$ sm, 9. 6. a) 15 sm; 20 sm; b) 24 sm; 18 sm; d) 144 sm²; 256 sm². 8. 19,2 m.
- 9-sabaq.** 2. awa; 3. a) ha'm d); e) ha'm f). 4. 108 sm². 5. 4 sm; 6 sm. 7. 4,8 sm. 9. 12.
- 10-sabaq.** 2. a) ha'm d); b) ha'm e); g) ha'm f). 3. 36 m yoki 20,25 m. 4. 12 sm; 14 sm. 6. $5\frac{5}{11}$ sm.
- 11-sabaq.** 3. a) 15; b) $3\frac{2}{11}$; d) $3\frac{5}{17}$. 4. 18 sm; 6 sm. 5. 29 dm². 6. 6 dm. 7. m:n.
- 12-sabaq.** 1. $3\frac{3}{17}$ m; 13,6 sm. 7. n:m. 8. a) $S:4$; b) $S:2$; d) $S:4$.
- 13-sabaq.** II. 1. 12 sm². 2. 8,4. 3. 2,4. 4. 24. 5. 8. 6. 1,6. III. 1. 6 sm. 2. 65 dm; 52 dm. 3. $AB \parallel EF$.
- 14-sabaq.** 5. 1 km 750 m. 8. 7,2 sm. 9. $k=\frac{1}{2}$ yoki $k=2$.
- 15-sabaq.** 4. $k=2$. 5. 6 sm²; 24 sm². 6. 104 sm². 7. Ha'r yekijag'dayda $k=1$. 8. 1,2 m². 9. 16 sm, 32 sm.
- 16-sabaq.** 4. $\frac{2}{3}$; $\frac{4}{9}$. 5. $X \cdot X$ ha'm $Y \cdot Y$ nurlarinin' kesilisiw noqati gomotetiyaorayi boladi. 6. $OX_1=2 \cdot OX$. 7. *Ko'rsetpeler:* Temadag'i ma'selelerdin' sheshiminen paydalanin'.

8. a) $OA_1 = \frac{2}{3}OA$; b) $OA_1 = 20A$; d) $OA_1 = 30A$; e) $OA_1 = OA$. 9. *Ko'rsatpe*: Temadag'i 3-su'wretten paydalanin'.

17-sabaq. 4. a) $P_2 = 42$; $k = \frac{1}{2}$; b) $S_1 = 12$, $k = 2$; d) $P_1 = 150\sqrt{2}$, $k = \sqrt{2}$; e) $P_1 = 10$, $S_2 = 216$.

18-sabaq. 1. $\approx 6,97$ m. 2. 300 m. 3. ≈ 72 m. 4. 6,6 m.

19-sabaq. 1. 9. 2. 12 dm. 3. 8 m. 4. 24 dm². 6. *Ko'rsatpe*: ABC u'shmu'yeshlik si'zi'n', ko'pmu'yeshliklerdi jasaw temasindag'i 1-ma'seleden paydalani'p si'zi'lg'an u'shmu'yeshlik ta'replerinen u'sh ma'rte kishi u'sh mu'yeshlik jasan'.

20-sabaq. 1. 72°; 72°; 36°. 3. 12 sm². 4. 15000000 km. 5. a) Awa; b) Awa; 7. 6 sm, 12 sm, 18 sm. 8. 63 m.

21-sabaq. II. 1. 8 sm. 2. $4\frac{4}{9}$ sm. 3. 48 m. 4. 4 sm; 0,5 sm². 5. $5\frac{1}{3}$ m. 6. 867 km. III. 1. 7,5 m.

2. 6 sm. 3. a) 7,5 sm; b) 6 sm; d) 16,2 sm. Qi'ziqli' ma'seleler: 1. Wo'zgermeydi. 2. a) Awa; b) Yaq. 3. *Ko'rsatpe*: Sizg'ish penen ha'r bir quwirshaqtin' boyin wo'lshen' ha'm olardin' qatnasin tabi'n'.

22-sabaq. 4. $\sin A = \frac{5}{13}$; $\cos A = \frac{12}{13}$; $\operatorname{tg} A = \frac{5}{12}$; $\operatorname{ctg} A = \frac{12}{5}$. 5. a) $\sin A = \frac{7}{25}$; $\cos A = \frac{24}{25}$; $\operatorname{tg} A = \frac{7}{25}$; $\operatorname{ctg} A = \frac{25}{7}$. $\sin B = \frac{24}{25}$; $\cos B = \frac{7}{25}$; $\operatorname{tg} B = \frac{25}{7}$; $\operatorname{ctg} B = \frac{7}{25}$. 6. $BC = \frac{11}{20}$; $AB = \frac{61}{20}$. 7. $AB = 34$; $AC = 30$.

23-sabaq. 4. a) 15 sm; b) 8 sm; d) 36,125 sm; e) 31,875 sm. 5. $\frac{15\sqrt{55}}{8}$ sm. 7. 42 sm². 8. 21 sm². 9. 32 sm². 10. 180 sm².

24-sabaq. 3. $2\sqrt{3}$ dm, $4\sqrt{3}$ dm. 4. a) $12 + 4\sqrt{3}$; b) $6 + 6\sqrt{3}$; d) $16 + 8\sqrt{2}$. 5. a) $\angle A = 45^\circ$, $\angle B = 45^\circ$; b) $\angle A = 60^\circ$; $\angle B = 30^\circ$; d) $\angle A = 30^\circ$, $\angle B = 60^\circ$. 6. $\frac{25\sqrt{3}}{3}$ sm². 7. 7 sm; 24 sm, $\cos A = \frac{24}{25}$, $\operatorname{tg} A = \frac{7}{24}$; $\operatorname{ctg} A = \frac{24}{7}$. 8. 120°; 120°; 60°; 60°.

25-sabaq. 1. 36 sm². 2. 24 sm. 3. a) $6\sqrt{3}$; b) 30; d) $\frac{105\sqrt{3}}{4}$. 4. $(24 + 4\sqrt{3})$ sm; $(24 + 8\sqrt{3})$ sm². 5. $10\sqrt{3}$ sm. 6. a) $\frac{\sqrt{3}}{6}$; b) $\frac{1}{2}$; d) $\frac{\sqrt{3}}{2}$. 7. ≈ 807 m². 8. ≈ 88 m.

26-sabaq. 2. tangens 90° ta, kotangens 0° ha'm 180° ta. 3. $\sin \alpha > 0$, $\cos \alpha < 0$, $\operatorname{tg} \alpha < 0$, $\operatorname{ctg} \alpha < 0$. 6. $\sin 45^\circ = \sin 135^\circ$; $\cos 45^\circ > \cos 135^\circ$.

27-sabaq. 2. 1) $\sin^2 \alpha$; 2) $\cos^2 \alpha$; 3) 1; 4) $\cos^2 \alpha$; 5) $\cos^2 \alpha$; 6) $\sin^2 \alpha$. 3. a) $-\frac{3}{5}$; b) $\frac{\sqrt{5}}{3}$; d) 0. 4. $6\sqrt{3}$ sm². 5. $0,8\sqrt{3}$ sm, $1,6\sqrt{3}$ sm. 6. a) $\frac{\sqrt{3}}{2}$; b) $\frac{\sqrt{5}}{3}$; e) 0. 9. a) $A\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$; e) $A(-2; 0)$; f) $A\left(\frac{4\sqrt{3}}{3}, 2\right)$.

28-sabaq. 4. a) 150°; b) 135°; d) 135°; e) 150°. 5. a) 0; b) 1; d) 0; e) -3,5; 6. a) 1; b) 1; d) 1. 7. 3,5 sm. 8. $36\sqrt{3}$ sm². 9. a) $\frac{1}{2}$, $-\frac{1}{2}$; b) $\pm \frac{\sqrt{15}}{4}$; d) 0. 10*. a) 30°; b) 135°; d) 150°.

29-sabaq. III. 2. 1000, 37°. 3. 2°. 4. 34°. 5. $2\sqrt{3}$; $4\sqrt{3}$. 6. $3\sqrt{3}$ sm. 7. 5 sm. 8. 12, $24\sqrt{3}$. 9. 20 sm, 100 sm². 10. 4, $16\sqrt{3}$. 11. 30°; 60°. 13. 12 sm; $4\sqrt{3}$ sm; $8\sqrt{3}$ sm. 14. 32 sm². 15. $-\frac{15}{17}$, $-\frac{8}{15}$, $-\frac{15}{8}$. 16. $\frac{4\sqrt{3}}{3}$. 17. $12(\sqrt{3}+1)$, $72(\sqrt{3}+1)$. IV. 1. $\frac{15}{17}$, $-\frac{8}{15}$. 2. $2\sqrt{77}$; 13°; 77°. 4. *Ko'rsatpe*: U'shmu'yeshliktin' ten'ligi haqqindag'i teo-remadan paydalanin'.

- 30-sabaq.** 2. a) 6 sm^2 ; b) $73,5 \text{ sm}^2$; d) 6 sm^2 . 3. 36 sm^2 . 4. $49\sqrt{2} \text{ sm}^2$. 5. $54\sqrt{3} \text{ sm}^2$. 6. $\frac{2}{3} \text{ sm}$; $4,5\sqrt{2} \text{ sm}$. 7. $\frac{h_a h_b}{2 \sin \alpha}$. 8. $4,8\sqrt{3} \text{ sm}$.
- 31-sabaq.** 2. a) $BC=6$; b) $AB=8\sqrt{2}$; d) $AC=7\sqrt{2}$. 3. a) $\sin C=\frac{1}{3}$; b) $\sin A=\frac{21}{40}$; d) $\sin B=\frac{16}{21}$. 4. $4,8 \text{ dm}$. 5. 30° yamasa 150° . 6. Mu'mkin. 7. $AB \approx 21,1 \text{ m}$; $\angle B \approx 37^\circ$, $\angle C \approx 76^\circ$. 8. 76° ; $26,1 \text{ sm}$; $23,8 \text{ sm}$.
- 32-sabaq.** 2. a) $\sqrt{13} \text{ sm}$; b) 4 m ; d) $\sqrt{283} \text{ dm}$. 3. $\frac{1}{5}$; $\frac{19}{35}$; $\frac{5}{7}$. 4. $2\sqrt{13} \text{ sm}$ yoki $2\sqrt{109} \text{ sm}$. 5. $\sqrt{31} \text{ sm}$, $\sqrt{91} \text{ sm}$. 6. $\sqrt{109} \text{ sm}$, $\sqrt{39} \text{ sm}$.
7. *Ko'rsatpe:* ADC ha'm BDC u'shmu'yeshliklarga kosinuslar teoremesidan paydalanib, a^2 ha'm c^2 ni tabin, keyin bul ten'liklerdi ag'zama-ag'za qosin'.
8. $\frac{\sqrt{106}}{2} \text{ sm}$; $\frac{\sqrt{151}}{2} \text{ sm}$; $\frac{\sqrt{190}}{2} \text{ sm}$.
- 33-sabaq.** 1. $\angle B$ ha'm $\angle C$. 2. AB ha'm BC . 3. a) su'yir muyeshli; b) tuwri' mu'yeshli; d) dog'al mu'yeshli. 4. a) $8\frac{1}{8}$; b) $8\frac{1}{8}$; d) $24\frac{1}{6}$; e) $\frac{35\sqrt{6}}{24}$. 6. *Ko'rsatpe:* Sinuslar teoremesidan paydalanin'. 7. *Ko'rsatpe:* 6-ma'selege uqsas sheshiledi.
8. *Ko'rsatpe:* Sinuslar teoremesidan paydalanin'.
- 34-sabaq.** 1. a) $10\sqrt{3}$; b) $28\sqrt{2}$; d) 12 ; e) $\approx 0,1532$. 2. a) $-2,5$; b) 0 ; d) 2 . 3. a) 8 ; b) 24 ; d) 8 ; e) 0 . 5. a) $-7,5$; d) 0 . 6. $a \perp b$, $c \perp d$.
- 35-sabaq.** 1. a) $\alpha=90^\circ$, $a=b=5$, $c=5\sqrt{2}$. b) $\gamma \approx 45^\circ$; $b \approx 17,9$, $c \approx 14,6$; d) $\alpha=20^\circ$; $b \approx 65,8$; $c \approx 88,6$; e) $\gamma=119^\circ$; $a \approx 16,7$; $b \approx 11,2$. 2. a) $c \approx 5,29$; $\alpha \approx 79^\circ 6'$; $\beta \approx 138^\circ 21'$; b) $c \approx 53,09$; $\alpha \approx 11^\circ 39'$; $\beta \approx 38^\circ 21'$; d) $a \approx 19,9$; $\beta \approx 58^\circ 19'$; $\gamma \approx 936^\circ 41'$; e) $a \approx 22,9$; $\beta \approx 21^\circ$; $\gamma \approx 15^\circ$. 3. a) $\alpha \approx 29^\circ$; $\beta \approx 47^\circ$; $\gamma \approx 104^\circ$; b) $\alpha \approx 54^\circ$; $\beta \approx 13^\circ$; $\gamma \approx 113^\circ$; d) $\alpha \approx 34^\circ$; $\beta \approx 44^\circ$; $\gamma \approx 102^\circ$; e) $\alpha \approx 39^\circ$; $\beta \approx 93^\circ$; $\gamma \approx 48^\circ$.
- 36-sabaq.** 1. a) $2\sqrt{3} \text{ sm}$; b) 16 sm ; d) $\frac{ab\sqrt{2}}{4}$. 2. $4\sqrt{2} \text{ m}$; 8 m va $4+4\sqrt{3} \text{ m}$. 3. $50\sqrt{3} \text{ kg}$. 4. 14 sm . 5. $2\sqrt{14} \text{ sm}$. 6. $6\sqrt{3} \text{ sm}$. 7. 50 sm .
- 37-sabaq.** 1. $\approx 10,8 \text{ m}$. 2. $\approx 15 \text{ m}$. 3. $\approx 43,4 \text{ m}$. 4. $\approx 35^\circ$. 5. $\approx 73,2 \text{ m}$. 6. $\approx 49 \text{ m}$. 7. Asfalt jayilg'an.
- 38-39-sabaq.** II. 1. $3\sqrt{6}$, $3\sqrt{2}$. 2. $\frac{111}{120}$; $0,89$; $-0,65$. 3. $2\sqrt{7} \text{ sm}$; $\frac{2\sqrt{21}}{3} \text{ sm}$. 4. $30\frac{1}{30} \text{ sm}$. 5. 28 sm . 6. 8 sm^2 ; $(4+4\sqrt{5}) \text{ sm}$; $h_a=4 \text{ sm}$, $h_b=0,8\sqrt{5} \text{ sm}$. 7. $2\sqrt{13}$. 8. a) su'yir mu'yeshli; b) tuwri' mu'yeshli, d) dog'al mu'yeshli. 9. 63 sm^2 . 10. $\approx 3,7 \text{ sm}$. 11. 7 sm . 12. 6 . 13. 0 . 14. -9 . 15. 135° . 16. $OC \approx 9,6$. 17. $(24+24\sqrt{3}) \text{ sm}$. 18. 5. III. 1. $\approx 109^\circ$. 2. $\gamma=100^\circ$, $a \approx 3,25$; $c \approx 6,43$. 3. $6,25$; $14,76$.
- 40-sabaq.** 2. a) Ha'r qanday ush'mu'yeshlik shen'berge ishley si'zi'liwi mu'mkin. b) Qarama-qarsi mu'yeshlirinin' qosi'ndi'si' 180° bolg'an to'rtmu'yeshlikler. 3. Bir dog'ag'auring'an mu'yeshleri ten'. 4. 10 sm . 5. 672 sm^2 . 6. a) $10\sqrt{3} \text{ sm}$; b) $10\sqrt{2} \text{ sm}$; d) $10\sqrt{2} \text{ sm}$; $10\sqrt{2} \text{ sm}$; 20 sm . 7. $8\frac{1}{3} \text{ sm}$. 8. $\triangle ABF$ da, $\angle BAF + \angle AFB = 90^\circ$, $\angle ABF = 90^\circ$. Demek, AF — diametr. 9. Qarama-qarsi

mu'yeshlerinin' qosi'ndi'si' 180° , yag'niy shen'berge ishley siziw mu'mkin.
10. *Ko'rsetpe:* bir u'ltan ha'm bir qaptal ta'reptin' worta perpendekulyari' kesisken noqat shen'ber worayi' boladi'.

41-sabaq. **2.** 7,2 sm. **3.** a) 16,6; b) 22; d) 22,6. **4.** a) 2,5; b) 3,5; d) 2. **8.** 6 sm.

42-sabaq. **3.** a) 60° ; b) 108° ; d) 120° ; e) 144° ; f) 160° . **4.** a) 120° ; b) 72° ; d) 120° ; e) 36° ; f) 30° . **5.** a) 3; b) 4; d) 8; e) 12.

43-sabaq. **1.** 3 sm ha'm $3\sqrt{2}$ sm. **2.** $\sqrt{3}$ ha'm $2\sqrt{3}$. **7.** a) 6; b) 12; d) 10; e) 20; f) 5.

44-sabaq. **3.** 8 sm; $8\sqrt{2}$ sm; $8\sqrt{3}$ sm; $8\sqrt{2}+3$ sm; 16 sm.

4. $\frac{8\sqrt{6}}{3}$ sm; **5.** a) $20\sqrt{2}$ sm; b) 40 sm. **6.** $\frac{5\sqrt{3}}{3}$ sm.

45-sabaq. **1.** E; **2.** D; **3.** D; **4.** B; **5.** B; **6.** E; **7.** E. **III.** **1.** $\sqrt{3}:4$; $6\sqrt{3}$. **2.** $3:4$. **3.** a) $\approx 5,780$ sm; b) $\approx 4,142$ sm; d) $\approx 2,679$ sm. **4.** $S=\sqrt{2}R^2$. **5.** 24 sm^2 . **IV.** **1.** 4 sm; 13 sm. **2.** $4\sqrt{3}$ sm; 8 sm. **3.** a) 80 sm; b) $20\sqrt{2-\sqrt{3}}$ sm; $40\sqrt{2-\sqrt{3}}$ sm; d) 200 sm^2 . **4.** $\frac{27\sqrt{3}}{4}\text{ sm}^2$.

46-sabaq. **2.** a) 3 ma'rte artadi; b) 6π sm ge artadi; d) 3 ma'rte kemeyedi; e) 6π sm ge kemeyedi. **3.** 6369 km. **4.** a) $\frac{2\pi\sqrt{3}a}{3}$; b) $\pi\sqrt{a^2+b^2}$; d) $\frac{2\pi b^2}{\sqrt{4b^2-a^2}}$. **5.** a) $\pi\alpha$; b) $\pi c(\sqrt{2}-1)$; d) $\pi c(\sin\alpha + \cos\alpha - 1)$. **6.** 1,5 m. **7.** 66348 ma'rte.

47-sabaq. **1.** a) π sm; b) $1,5\pi$ sm; d) 3π sm; e) 4π sm. **2.** a) $\frac{2\pi}{9}$; b) $\frac{\pi}{3}$; d) $\frac{5\pi}{12}$. **3.** a) $\approx 69^\circ$; b) 120° ; d) 150° . **4.** a) $\frac{5\pi}{8}$ sm; b) 2π sm; d) $\frac{15\pi}{4}$ sm; **5.** a) 4π ; b) 16π . **7.** 2.

48-sabaq. **3.** k^2 ma'rte artti; b) k^2 ma'rte kemeyedi. **4.** $6,25\pi\text{ sm}^2$; $12,5\pi\text{ sm}^2$. **5.** $2,25\pi\text{ sm}^2$; $9\pi\text{ sm}^2$. **6.** $(\pi-2)R^2$. **7.** $21,25\pi\text{ sm}^2$. **8.** $7,5\text{ sm}^2$.

49-sabaq. **3.** a) $\frac{49}{12}\pi\text{ sm}^2$; $\frac{49(\pi-3)}{12}\text{ sm}^2$; b) $6,125\pi\text{ sm}^2$; $\frac{49(\pi-2\sqrt{2})}{8}\text{ sm}^2$; d) $\frac{49\pi}{3}\text{ sm}^2$; $\frac{49(4\pi-3\sqrt{3})}{12}\text{ sm}^2$; e) $\frac{49\pi}{4}\text{ sm}^2$; $\frac{49(\pi-2)}{4}\text{ sm}^2$. **4.** a) $a^2\left(\frac{\sqrt{3}}{4}-\frac{\pi}{8}\right)$; b) $a^2\left(1-\frac{\pi}{4}\right)$; d) $\frac{3\sqrt{3}-\pi}{2}a^2$; **5.** $\pi\text{ sm}^2$; $3\pi\text{ sm}^2$; $5\pi\text{ sm}^2$; $7\pi\text{ sm}^2$. **6.** $\frac{25(2\pi-3\sqrt{3})}{3}\text{ sm}^2$; $\frac{25(10\pi+3\sqrt{3})}{3}\text{ sm}^2$; **7.** $\frac{75\cdot(4\pi-3\sqrt{3})}{2}\text{ sm}^2$. **8.** $S_1 < S < S_2$; $300\text{ sm}^2 < 314\text{ sm}^2 < 321,48\text{ sm}^2$.

50-sabaq. **1.** Do'n'gelektin' maydani' u'lken. **2.** $\frac{160}{3}\pi\text{ sm}^2$. **3.** $5,76\pi\text{ sm}^2$. **4.** $8\cdot(\pi-2)\text{ sm}^2$. **6.** $6\pi\text{ sm}^2$; $10\pi\text{ sm}$.

51-sabaq. **II.** **1.** $6\sqrt{2+\sqrt{2}}$. **2.** $\frac{8\pi}{3}\text{ dm}$. **3.** 30 sm. **4.** 90° . **5.** 3. **6.** π va $6,25\pi$. **7.** $\frac{10\pi+3\sqrt{3}}{2\pi-3\sqrt{3}}$. **8.** $\frac{2\sqrt{3}}{6}$. **9.** $\frac{9\sqrt{3}-2\pi}{6}a^2$. **10.** $1,5\pi$. **11.** 7. **12.** $\approx 9\pi-26,04$. **13.** π . **14.** $54\sqrt{3}-24\pi$. **15.** $\frac{3\pi}{8}$. **III.** **2.** $8\sqrt{3}\text{ sm}$. **3.** a) $\frac{18}{\pi}\text{ sm}$; b) $\frac{216}{\pi}\text{ sm}^2$; d) $\frac{216\pi+81\sqrt{3}}{\pi^2}\text{ sm}^2$.

52-sabaq. **3.** $5\sqrt{2}\text{ sm}$. **4.** 12 sm. **5.** 44 m, 60 m. **7.** 1:7. **8.** $AB\cos\alpha$.

53-sabaq. **1.** a) 30 sm, 12 sm; b) 9 sm, 12 sm, 21 sm; d) 3 sm, 15 sm, 3 sm, 21 sm.

3. 6 sm; 10,5 sm. 4. 9 sm, 12 sm, 15 sm, 18 sm. 5. 60° . 6. 21 sm.
- 54-sabaq. 1. Ko'rsatpe: $\triangle ACD \sim \triangle CBD \sim \triangle ABC$. 2. 25 sm, 15 sm, 20 sm. 3. $9\frac{3}{5}$ sm. 4. a) 5, 4; b) 24, 25; d) 8, 10. 5. 16:25. 6. 56,16 sm². 7. 60 sm². 8. $\frac{2}{3}; \frac{4}{9}; \frac{2}{3}$.
- 55-sabaq. 2. Ko'rsatpe: a) katetleri a ha'm b bolg'an tuwri' mu'yeshli u'shmu'yeshlik jasan'; b) gipotenuzasi a , bir kateti b bolg'an tuwri' mu'yeshli u'shmu'yeshlik jasan'. 3. Ko'rsatpe: Katetleri $AB=BC=1$ bolg'an $\triangle ABC$ jasan'. Keyin kateti $CC_1=1$ ha'm $\angle C_1=90^\circ$ bolg'an $\triangle BCC_1$ jasan' ha'm t.b. 4. a) 20; b) 45; d) 37,5. 5. 225 sm². 6. 180 sm². 7. 25:9. 9. $OC \geq OD$ bolg'ani' ushi'n ten'sizlik ha'r qashan duri's.
- 56-sabaq. 1. a) 6,25; b) 12; d) 0,25. 2. a) 8 sm; b) 2,5 sm; d) 0,9 sm. 3. a) 4 dm; b) 4 dm. 4. 4 sm. 6. 9 dm; 16 dm.
- 57-sabaq. 1. 10 sm. 2. 2 sm. 3. a) 2,5; b) 4; d) 2. 4. a) $4\sqrt{6}-1$ sm; b) 6 sm. 5. 1:6. 6. 6 sm. 7. 3. 8. 1:4.
- 58-sabaq. II. 1. 18 sm; 32 sm. 2. 4 sm; 3. 8 sm; 4. 6,4 dm. 5. 8 sm. 6. 1,5. 7. 5. 8. 6. 9. 45 dm². 10. 4 sm. 11. 8 sm. 12. 6. 13. 60° . 14. 45° . 15. 4:9. III. 1. 8 sm. 2. 5 dm. 3. 4 sm; 8 sm.
- 59-sabaq. 1. a) 12 (kv.b.); b) 20 (kv.b.); d) 12 (kv.b.); e) 12 (kv.b.); f) 42 (kv.b.). 2. 4 sm. 3. a) 12; b) 288. 4. a) $\frac{6\sqrt{133}}{19}$; b) $2\frac{1}{3}$. 5. a) (3;2); b) (2,5;-0,5); d) (-1;4); e) (-0,5;3,1). 6. D(2;-1). 8. 10 sm; 25 sm. 9. 60° ; 90° ; 120° ; 90° . 10. 6 sm. 11. $6\sqrt{2}$ sm.
- 60-sabaq. 1. a) 4; b) 6; d) 5; e) 5. 2. $4\sqrt{13} + \sqrt{82} + \sqrt{58}$. 4. Trapeciya. 5. $x=4$, $y=3$. 7. $b-a$; $-a-2b$; $2a+b$. 8. 5N. 9. $18\sqrt{3}$; $27\sqrt{3}$. 10. $4\sqrt{2}$ sm. 11. $PA=PB$ ha'm $PA=PC$ bolg'ani' ushi'n $PB=PC$.
- 61-sabaq. 2. 45° ; 90° ; 135° ; 90° . 3. 45° . 4. 60° . 5. 3 sm; 8 sm. 7. 28 sm. 9. 45° .
- 62-sabaq. 1. 8,4 sm, 10,5 sm, 14,7 sm. 2. 175 dm²; 252 dm². 3. 12 sm². 4. 6. 5. $9(3-\sqrt{3})$ sm². 6. 8 sm. 7. 5 sm; 2 sm; 5 sm; 8 sm. 8. 3 sm, 4 sm.
- 63-sabaq. 1. 2 sm. 2. 6 dm; 9,6 dm; 6,5 dm; 10,4 dm. 3. Awa. 4. $\sqrt[4]{27}$; $3\sqrt[4]{3}$. 5. 16,9 sm. 6. 150 sm². 7. (0;-6). 8. Birinshisinde. 9. 80 ta. 10. 7 dm². 11. a) 180 dm³; b) 105 sm³; d) 1296 sm³.
- 64-sabaq. 1. -12,5. 2. 20 sm. 3. Awa. 4. 5 sm, 13 sm. 5. 3 sm. 6. 8 sm. 7. 30π sm. 8. 5 sm. 9. 25 sm yaki $20\sqrt{2}$ sm.
- 65-sabaq. 1. 4,5 sm; 6,75 sm. 2. $\frac{20}{9}$ sm, 4 sm; 4,8 sm. 3. 12 sm. 4. 2 sm. 5. 6,72. 6. $2\sqrt{26}\pi$ 7. $25\sqrt{3}$ sm². 8. 84 sm². 9. $675\sqrt{3}$ sm².
- 66-sabaq. 1. $4\frac{129}{1024}$. 2. 100° ; 80° . 3. 4 sm. 4. 24 sm². 5. 4,8 m. 6. 30 sm². 7. 7. 8. 10 sm yaki $2\sqrt{97}$ sm.
- 67-68-sabaq. 1. a) 9; b) 4 sm²; d) 3,5 sm; e) $\frac{4}{3}$ TB-CA; f) 0,2. 2. $\triangle CMH \sim \triangle BCA$.

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Ijarag'a berilgen sabaqliq jag'dayin ko'rsetiwshi keste

T/r	Woiwshinin' ati ha'm familiyasi	Woqiw jili	Sabaqliqtin' aling'andag'i jag'dayi	Klass basshi-sinin' qoli	Sabaqliqtin' tapsiril-g'andag'i jag'dayi	Klass basshi-sinin' qoli
1						
2						
3						
4						
5						
6						

Sabaqliq ijarag'a berilip, woqi'w ji'li' aqi'ri'nda qaytari'p ali'ng'anda joqari'dag'i' keste klass basshi'si' ta'repinen to'mendegi bahalaw wo'lsheplerine tiykarlanip toltiriladi.

Taza	Sabaqliqtı birinshi ret paydalaniwg'aberilgendegi jag'dayi.
Jaqsı	Muqabapu'tin, sabaqliqtin' tiykarǵı' bo'liminen ajiralmag'an. Barlıq betleri bar. Jirtilmag'an, betleri almastirilmag'an, betlerin-de jazıw ha'm si'zi'qlar joq.
Qanaatlandirarli	Muqabajelingen, biraz si'zi'lip, shetleri qayrilg'an, sabaqliqtin' tiykarǵı' bo'liminen alinip qaliw jag'dayi bar, paydalaniwshi ta'repinen qanaatlanarli qa'lpine keltirilgen. Aling'an betler qayta islengen, ayrim betleri si'zi'lg'an.
Qanaatlandirarsız	Muqabag'asi'zi'lg'an, jirtilg'an, tiykarǵı' bo'liminen ajiralg'an yamasa pu'tinley joq, qanaatlandirarsız islengen. Betleri jirtilg'an, betleri toliq emes, sizip, boyap taslang'an. Sabaqliqtı qayta tiklewge bolmaydı.